What Prompts the Transmission of Exchange Rate Movements into International Prices?

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Abstract

What is the causal effect of exchange rates on international prices over time when the state of the economy is continuously shifting? This thesis demonstrates that the existing reduced-form estimates of exchange rate pass-through are biased, at the very least over the longer term, and raises concerns over the long-standing disconnect between the average causal effect and the dynamic causal effect. A unique methodology is then developed to quantify an unbiased measure of exchange rate transmission at the firm-level from their observed co-movements at the aggregate level. In this framework, exchange rate impact on the transition path of prices from vintages to the inter-temporal optimum is determined by a compromise between economic structure and stochastic innovations. The blueprint builds on a micro-founded multi-country business cycle model that disciplines the structural parameters by the data on macroeconomic fundamentals using the method of moments. The substance of the quantitative predictions are exposited by addressing two broad research questions at the forefront of the policy debate in international economics. Specifically, (i) "*What Drives the Terms of Trade Neutrality to Exchange Rates?*"; and (ii) "*Why Are Import Prices More Elastic To Local Currency Depreciations Than Appreciations?*"

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Declaration

I declare that this thesis is a presentation of original work and I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as references.

Chapter 1

Introduction

The collapse of the Bretton-Woods system in March 1973 marked the end of the gold standard era. Since then, exchange rates began to float against one another, although some with greater flexibility than others. While dominant currencies such as the U.S. Dollar (USD) or the Deutsche Mark were allowed to float freely, other currencies were either closely tied to these anchor currencies or they were gradually abandoned altogether.

The design of the international monetary system guiding the policy makers during that transition period was the ubiquitous Mundell-Fleming paradigm. It captured the essence of the international economic policy domain, which later became known as the 'impossible trinity' or simply 'trilemma'. It states that the simultaneous adoption of the fixed exchange rate regime, liberalisation of the international capital account, and an active role of the local monetary policy are incompatible with the goal of stabilising local real output over time. According to the model, the paramount tool when liberalising the international flows of capital is the effectiveness of monetary policy in terms of smoothing out the local business cycle, but only if exchange rates are allowed to float freely by absorbing external shocks.

The political appeal for such an unprecedented theoretical prediction could not have been greater at the time. Mostly because it disowned the notion of adopting the price of gold or other commodities as nominal anchors. It also sent a powerful message to the monetary authorities around the world that the stability of local real output over time in an open economy can be maintained by simply controlling the growth of monetary base in nominal terms. Yet despite the transparency of the qualitative policy implications that are difficult to refute to this day, the quantitative significance of the theory has become increasingly at odds with the data due to a number of structural developments that challenge the implicit assumptions imposed at the core of the mechanism.

One of the crucial channels through which expansionary monetary policy is deemed highly effective in the Mundell-Fleming model is the anticipation of ensuing capital outflows that ultimately depreciate the real value of the local currency in the short-run. In turn, the local terms of trade are expected to deteriorate, thereby boosting the local net exports and, in the medium-run, inducing an upward pressure on price inflation at home. However, there are at least three long-standing empirical irregularities characterising this conventional line of reasoning. First, Mendoza (1995) shows that the correlation between the terms of trade and the net exports in many advanced and developing economies is mostly positive, as it is implied by the Mundell-Fleming model, but it is generally weak. The caveat of the so-called

Harberger-Laursen-Metzler effect is the fact that in practice most terms of trade shocks are viewed as permanent rather than transitory. Consequently, households optimising over long periods of time are less inclined to shift their financial capital overseas in order to leverage a contemporaneous increase in their standard of living. The positive effect on the local net exports is therefore dampened through the balance of payments condition.

Second, the model assumes that all goods and services are internationally traded in a perfectly integrated market structure, such that no parallel trade takes place. Indeed, Engel (1999) shows that the CPI-based real exchange rate of the USD vis-à-vis other major currencies is mostly explained by the volatility in the relative price of tradable goods, while the relative price of non-tradable goods and services are entirely disconnected. This implies that the PPI- and the CPI-based real exchange rates are roughly equally as volatile as is later confirmed by Obstfeld & Rogoff (2001) for other major currencies. The problem is that Atkeson & Burstein (2008) also document a stark discrepancy between the high volatility of the real exchange rate and the low volatility of the terms of trade measured by the relative price of manufactured goods. The pinnacle of these two findings is that they cannot be rationalised by the home-bias of consumption towards non-tradable goods and services, but instead necessitate the consideration of more sophisticated global structures of segmented product markets famously coined ‘pricing-to-market’ by Krugman (1986).

Third, all prices are sticky in the currency units of the source country in the short-run - a custom commonly referred to as Producer Currency Pricing (PCP). In this environment, import prices in the currency units of the destination country would move one-to-one with the nominal exchange rate. Combining this notion with the results of Meese & Rogoff (1983), who show that a freely floating nominal exchange rate time series closely resembles a unit root, implies that nominal import prices of all goods and services ought to adjust over time almost as immediately and erratically as asset prices associated with private equity. Yet Goldberg & Knetter (1997) among many others demonstrate that import prices of goods and services other than energy-related commodities are sluggish to adjust in spite of large movements in exchange rates. The rationale is simply that both export and import prices behave in the Keynesian fashion, otherwise known as the custom of Local Currency Pricing (LCP), dampening the expansionary impetus of the exchange rate channel.

Globalisation is a major structural shift that could not have been anticipated more than half a century ago when the gold standard was abolished. The multilateral curtailment of the trade barriers under the GATT and the WTO paved the way for specialisation and international trade in not just the final goods, but also intermediate goods. Miroudot et al. (2009) show that 56% of imported goods and 74% of imported services in OECD economies during 1995-2005 were in fact intermediate. It became cheaper for some firms to outsource labour-intensive tasks from their domicile in advanced economies to the manufacturing plants in developing economies. In this process of international division of labour, production technologies of consumer goods became more intertwined across the globe than ever before. In response to these international developments, otherwise known as Global Value Chains (GVC), a large number of other empirical motives have been put forward in the more recent years in order to explain why the widely-observable exchange rate movements are only partially absorbed by international prices. The emphasis has notably shifted towards studies of firm-level data that reveal detailed accounts of micro-founded tendencies of the exporting firms. Amiti et al. (2014) use Belgian micro-level data and show that firms with greater market share are more likely to export, are more open to intermediate imports, and are more likely to adopt LCP strategies, while smaller firms are less likely to export, are more dependent on local factors of production, and are more accustomed to PCP. Berman et al. (2012) use a rich dataset of the French firm-level data and find that exporters with a high market share react to a local currency depreciation by increasing significantly more their markup and by increasing less their export volume than the firms with a small market share. The central message from the global integration of the supply-side is that the higher is the share of imported intermediate goods priced in foreign currency, the lower (greater) is the extent of exchange rate movements absorbed by PCP (LCP) import prices and the greater (lower) is the effect on net exports.

And yet some monetary aspects have changed very little. According to Gopinath (2015), some 5% of all global trade transactions were conducted directly with the U.S. in the period of 1999-2014, but more than 20% of global imports and exports were invoiced in USD. The fact that a vast amount of goods around the world are still traded in anchor currencies has important policy implications, namely – which exchange rate, if any, matters? Because even if minor currency areas decided to float *de jure*, the *de facto* international prices would still be highly influenced by innovations in major currency areas, which could also bring the infamous Harberger-Laursen-Metzler effect to a grinding halt. Monetary policy spillovers from major to minor currency areas are therefore particularly pronounced. On the other hand, expansionary monetary policy in minor currency areas is only effective when there exists a currency mismatch between intermediate imports and final exports, which leaves them more susceptible to external innovations emanating from the major currency areas. Ultimately, this discrepancy reinforces the notion first articulated by Calvo & Reinhart (2002) that emerging market economies are reluctant to adopt floating exchange rate regimes and their policies are indeed guided by rational motives. In fact, Céspedes et al. (2004) argued that an expansion of the monetary aggregates via open market operations in emerging markets may even be contractionary due to a pronounced currency mismatch in assets and liabilities, inducing non-trivial balance sheet effects when servicing debt denominated in foreign currency. However, IMF (2016) shows that the share of foreign currency denominated liabilities, otherwise known as the 'original sin' due to Eichengreen & Hausmann (2005), has subsided dramatically in many emerging market economies around the world since the East Asian crisis in the 1990s.

There are several takeaway points from this discussion: (i) if we want to understand whether monetary policy is in fact any more effective in an open rather than a closed economy setting, we need to not only recognise a compatible configuration of external policy instruments, but also be able to quantify the exchange rate channel; (ii) the supply-side indicators associated with the import-export structure, such as the invoicing currency, openness to intermediate imports, market concentration, average price duration, as well as other moments of the price adjustment distribution across industries, are crucial when modelling the dynamics of international prices; and (iii) the interest rate transmission channel depicted in the Mundell-Fleming model and the one implied by the empirical stylised facts are broadly aligned in a qualitative sense, but quantitatively they are dissimilar in advanced and developing economies.

The premise of this thesis is to bridge the gap between the theoretical and empirical strands of the literature that have made valuable progress in addressing the three aspects outlined above. The first strand refers to a number of Dynamic Stochastic General Equilibrium (DSGE) models that have been developed in order to conduct micro-founded business cycle analysis in an open economy setting. The second strand is based on (Structural) Vector Autoregressions (SVAR) that are used for the same purpose, but with a milder form of structure and more elegance, which lacks the transparency in terms of the deep parameter identifiability characterising the solutions of DSGEs. Yet unlike SVARs, very few DSGEs are developed with the express purpose of measuring the elasticity of international prices to exchange rate movements, more commonly known as exchange rate pass-through. Those that do, such as Corsetti et al. (2008) or Choudhri & Hakura (2015), are subject to a number of restrictions. Either the estimates are severely prone to attenuation bias, in which case they are hardly more transparent than the reduced-form estimates obtained from single-equation regressions, or they impose theoretically-motivated identification restrictions, but then insist on measuring exchange rate pass-through 'agnostically' as it is done in the SVAR literature, such as Shambaugh (2008) and Forbes et al. (2017). The latter is based on computing the ratio between the cumulative impulse response functions of price inflation and exchange rate changes, which effectively measures their co-movement rather than transmission. While the former is susceptible to the issue of simultaneity, the latter reflects the causal effect. The agnostic approach is also limited to estimates of pass-through that are conditional on observing a certain counterfactual state of the economy, which makes it difficult to assess the average causal effect that *de facto* formulates the core of the informed policy decision making process. And though it is important to analyse the shock-dependence properties of exchange rate pass-through, it does not supersede the underlying economic structure *per se*, since economies are rarely exposed to a single distinct innovation at any given point in time.

This thesis makes several important contributions to the literature on estimating exchange rate pass-through: (i) it presents a novel way to extract an unbiased causal effect of exchange rates on international prices at the firm level from their observed co-movements at the aggregate level; (ii) it resolves the dichotomy between the transmission and the co-movement channels by treating exchange rate pass-through itself as an endogenous variable within the system of difference equations characterising the dynamics of the economy; (iii) it simultaneously captures the salient features of the global import-export market structure as well as shock-dependence, thereby distinguishing between estimates of the dynamic and the average causal effect; and (iv) the mechanism is disciplined by the data on macroeconomic fundamentals using the method of moments. The methodology is then used to answer two novel research questions that are at the forefront of the policy debate. Specifically, (i) "*What Drives the Terms of Trade Neutrality to Exchange Rates?*"; and (ii) "*Why Are Import Prices More Elastic To Local Currency Depreciations Than Appreciations?*"

Chapter 2

Why Are Import Prices More Elastic To Local Currency Depreciations Than Appreciations?

2.1 Background

Import prices are often claimed to be sticky relative to the large movements in exchange rates, but they tend to be more responsive to local currency depreciations than appreciations. Several hypotheses for the origins of this asymmetry already exist, such as short-run capacity constraints or downward price rigidities in the export industry, but they are generally difficult to test. The nature of the data-generating process therefore remains unclear – non-linearities documented at the border could be either traced back to the first principles of the export price setting decisions or they could be driven by a time-varying volatility of aggregate innovations. But whichever view we take about the source of exchange rate pass-through asymmetry, there is no unifying modelling framework that could be used to reconcile its implications on the bilateral terms of trade in a flexible exchange rate environment.

Even if we were to accept the view that the asymmetric causal effect of exchange rates on international prices is at least in part driven by structural factors, it is still unclear how it should be measured in a pragmatic business cycle model. Pioneered by Shambaugh (2008), a growing strand of the literature, such as Choudhri & Hakura (2015) and Forbes et al. (2017), computes the magnitude of the dynamic causal effect using (cumulative) impulse response functions of price inflation and differenced exchange rate triggered by an arbitrary innovation. This chapter demonstrates an astounding bias in this 'agnostic' measurement of exchange rate pass-through, which arises primarily because it stands for the aggregate price and exchange rate co-movements rather than an aggregated firm-level transmission of exchange rate innovations. This seemingly innocuous approach is admittedly appealing for its simplicity and general applicability in both the business cycle models (DSGE) as well as structural vector autoregressions (SVAR). And yet, in equilibrium, it accounts for numerous interactions among a large number of control variables that obscure the channel of exchange rate transmission by convoluting the implicit policy function of exchange rate pass-through with irrelevant state variables from the perspective of the price setting decisions of the individual exporters. It also implies that all estimates of exchange rate pass-through are conditional on observing a certain state of the economy, which makes it difficult to obtain a sensible estimate of the average causal effect. Indeed, Forbes et al. (2017) demonstrates that arbitrary innovations can influence not only the magnitude of exchange rate pass-through, but also its sign. Aggregating the state-dependent dynamic causal effects using, say, a time-varying shock decomposition can therefore lead to counterintuitive policy implications. But the practical crux of all problems is the fact that accommodating non-linearities in the impulse response functions generated by SVAR models presents hindering computational challenges.

This chapter demonstrates a unique method to derive an *ex ante* unbiased causal effect of exchange rates on international prices directly from the first order conditions of the exporters depicted in a relatively standard New-Keynesian model that can be augmented with any non-linearity, such as occasionally binding constraints or other one-sided frictions. It can be derived in the context of any business cycle model as long as: (i) unit costs of producing exported varieties are correlated with the exchange rate; and (ii) the price adjustment costs are convex *à la* Rotemberg (1982). The method addresses all three challenges outlined above by treating exchange rate pass-through itself as an endogenous variable within the system of difference equations characterising the dynamics of all macroeconomic fundamentals. Therefore, contrary to SVAR models, the assessment of both dynamic and average causal effects becomes a relatively simple matter even in highly non-linear settings.

But what are the supply-side factors responsible for the non-linear exchange rate transmission channel? In order to abstract from the recurring structural shifts that could only be captured by a time-varying volatility of innovations, this chapter explores the stylised facts of the globalised export market structure during the Great Moderation period (i.e. 1982-2008), which is well-known for its unique environment of relatively low aggregate uncertainty. The first distinguishing feature of the data in this period is the high positive skewness of export and consumer price inflation relative to the unit labour costs in many OECD economies.[1] Secondly, large exporters in OECD economies are highly dependent on intermediate imports, which are generally priced in the U.S. Dollar (USD), creating a strong correlation between the exchange rate and the unit cost of producing final exported varieties.[2] Exchange rate depreciations are thus observationally equivalent to an aggregate decline in exporter productivity. Similar to the conjectures of Taylor (2000), if the depreciation (appreciation) is sufficiently large, then import-dependent exporters could shift the aggregate inflation into a state with a higher (lower) frequency of price adjustment, since it intensifies (subsides) the 'selection effect' even for firms that have successfully hedged their exposure to exchange rate risk.

Yet unlike the menu cost models, such as Golosov & Lucas (2007), Nakamura & Steinsson (2008), or Midrigan (2011), where aggregate innovations trigger a self-selection of the firms to adjust their prices or to keep them constant depending on idiosyncratic risk, business cycle models are based on representative agents and they are subject to a degenerate price adjustment distribution in equilibrium. The selection effect in business cycle models is

[1] The positive skewness of inflation is even greater in the period of 1960s-70s, but declines considerably in the aftermath of the Great Financial Crisis.

[2] Miroudot et al. (2009) estimate that in the period of 1995-2005 around 56% (74%) of total OECD imports consisted of intermediate goods (services). Moreover, Amiti et al. (2014) establish a strong correlation between firms choosing to export and their dependence on intermediate imports.

therefore independent of the state of the economy, unless there was a way to approximate state-dependence of price adjustment in reduced form. Indeed, this chapter integrates the aforementioned downward price rigidities by introducing a non-linear price adjustment cost function due to Varian (1975) in the context of a representative firm. The properties of these so-called LINEX price adjustment costs extend the quadratic framework originally proposed by Rotemberg (1982). Specifically, the name LINEX derives from their shape, resembling a linear (exponential) functional form for price increases (decreases), allowing for asymmetric convexity controlled by the sign and magnitude of a single structural parameter.

By no means should this framework be viewed as an exhaustive depiction of the global export market structure surrounding the exchange rate channel. The premise of the modelling strategy is to start from an already familiar New-Keynesian framework in the fashion of Corsetti et al. (2008). A small number of extensions are then introduced in order to exposit a previously unexplored channel. To that end, this chapter deliberately simplifies the modelling of 'pricing-to-market' behaviour to the case of Choudhri & Hakura (2015), where low average pass-through emanates from a constant density of exporters adopting Local Currency Pricing (LCP), while the remaining firms subscribe to Producer Currency Pricing (PCP). The focus throughout this chapter is on mapping the extent of downward price rigidities observed in the data to the price setting behaviour of the firms, thereby isolating the novel higher order effects from already well-known first order effects such as variable price mark-ups as in Atkeson & Burstein (2008), the endogenous choice of invoicing currency as in Gopinath et al. (2010), or the dominant currency paradigm advocated by Gopinath (2015) among others.

This chapter contributes to the broad theoretical and empirical literature analysing the degree of exchange rate pass-through into import prices at the border as well as high-end consumer prices. First, the theoretical model is motivated by the empirical estimates of asymmetric exchange rate pass-through by Webber (2000), Delatte & López-Villavicencio (2012), Ben Cheikh (2012), Bussière (2013), Kilic (2016), Caselli & Roitman (2016), Razafindrabe (2017) and Brun-Aguerre et al. (2017). These studies generally find a greater import (and consumer) price responsiveness to local currency depreciations than appreciations in many advanced and developing economies depending on the methodology and the sample period. Second, the hypotheses for the prevalence of asymmetric import price elasticity to exchange rate movements were first proposed by Ware & Winter (1988), Froot & Klemperer (1989), Dixit (1989) and Knetter (1994). Brun-Aguerre et al. (2017) provide a thorough literature survey of these earlier conjectures. Third, there is a vast literature aimed at estimating the degree of exchange rate pass-through based on an affine reduced form regression approach, such as Goldberg & Knetter (1997), Campa & Goldberg (2005) and Burstein & Gopinath (2014) among many others. Finally, the mechanism depicted in this chapter is influenced by other endogenous exchange rate pass-through models such as Devereux & Yetman (2010), Gopinath et al. (2010), and the menu cost model of Flodén & Wilander (2006).

2.2 Non-Linear Price Adjustment Costs

Why are exporters domiciled in OECD economies more reluctant to adjust prices downwards than in the upward direction? This observation can be extrapolated from table 2.1, which demonstrates that the export and consumer price inflation is overwhelmingly positively skewed in almost all of the OECD countries, while the skewness of wage inflation is generally less pronounced with the exception of the United States.[3] While the higher-order moments are sensitive to the selection of the time period, this chapter abstracts from the recurring structural shifts by exploring the stylised facts of the globalised export market structure during the Great Moderation period (i.e. 1982-2008), which is well-known for its unique environment of relatively low aggregate uncertainty. Although some of the skewness in these countries can be attributed to the shifting monetary policy regimes towards inflation targeting, there are two reasons to believe that nominal rigidity is the more important factor in generating downward rigidities of export prices observed in the data.

First, wages are not subject to the same extent of skewness. In fact, they are virtually normally distributed. Despite its decline in recent years, it is well-established that the labour share of income, or the wage bill, still contributes the largest amount to the production costs in advanced economies (see ILO, IMF, OECD, IBRD (2015) for G20 estimates). Downward rigidity of export prices is therefore not an artefact of downward wage rigidities, at least not outside of the United States. Secondly, large exporters in OECD economies are highly dependent on intermediate imports, which are generally priced in the U.S. Dollar (USD), creating a strong correlation between the exchange rate and the unit cost of producing final exported varieties. Miroudot et al. (2009) estimate that in the period of 1995-2005 around 56% (74%) of total OECD imports consisted of intermediate goods (services). Moreover, Amiti et al. (2014) establish a strong correlation between firms choosing to export and their dependence on intermediate imports. Exchange rate depreciations are thus observationally equivalent to an aggregate decline in exporter productivity in the short-run. The cost-push effects across a large number of import-dependent exporters translate to a greater aggregate export price inflation, thereby generating an even further ‘selection effect’ for firms that are limited in their exposure to exchange rate risk.[4]

This chapter provides a reduced form link between the state-dependent pricing of exports at home and asymmetric import price responsiveness to exchange rate changes in the destination economy. The link is established by setting out a dynamic stochastic general equilibrium model in the open economy setting, which is augmented with real and nominal rigidity in the form of wage and price adjustment costs. The adjustment costs enter the constraints of rational agents, whose goal is to maximise the present discounted value of their objective functions. The distinctive feature of the model is the choice of a non-linear functional form

[3]Workers and trade unions may be more reluctant to accept a wage cut than a pay rise, as opposed to the employers, therefore the implementation of downward adjustment of wages requires more effort and resources than an upward adjustment, which could generate positively skewed wage adjustments at the aggregate level.

[4]Using firm-level data on US export and import prices, Gopinath & Rigobon (2008) show that export prices increase almost as frequently as they decrease. However, the frequency of price increases may be partially blurred due to averaging over a number of heterogeneous sectors of goods as well as services. In fact, Nakamura & Steinsson (2008) find a significantly greater fraction of price increases compared to decreases in many different categories of goods in the US, which is indicative of the aforementioned selection effect.

Table 2.1: Wage & Price Inflation in OECD Economies (1982:Q1-2008:Q1)

	Consumer Inflation Rate (CPI)		Export Inflation Rate (EPI)		Wage Growth Rate (ULC)	
	`std.dev.`	`skewness`	`std.dev.`	`skewness`	`std.dev.`	`skewness`
Australia	0.88%	$0.72^{\dagger\dagger}$	3.14%	0.44	1.25%	$1.04^{\dagger}$
Austria	0.65%	$0.53^{\dagger}$	2.25%	$0.08^{\dagger\dagger\dagger}$	0.79%	$-0.02^{\dagger\dagger}$
Belgium	0.58%	$0.92^{\dagger}$	-	-	0.91%	$0.40^{\dagger}$
Canada	0.64%	$0.80^{\dagger}$	1.71%	$0.99^{\dagger}$	0.79%	$0.43^{\dagger\dagger}$
Denmark	0.65%	$0.99^{\dagger}$	-	-	1.83%	0.27
Finland	0.75%	$0.94^{\dagger}$	0.65%	$4.50^{\dagger}$	1.25%	0.16
France	0.64%	$1.61^{\dagger}$	1.44%	-0.07	3.07%	-0.24
Germany	0.43%	$1.09^{\dagger}$	0.50%	0.32	0.71%	$0.99^{\dagger}$
Greece	2.37%	0.40	3.58%	$0.08^{\dagger}$	2.85%	$0.43^{\dagger}$
Iceland	4.11%	$2.49^{\dagger}$	-	-	-	-
Ireland	0.83%	$2.25^{\dagger}$	2.97%	$-0.35^{\dagger}$	2.24%	$0.28^{\dagger}$
Italy	0.89%	$1.68^{\dagger}$	-	-	1.32%	$0.44^{\dagger\dagger}$
Japan	0.60%	$0.96^{\dagger}$	-	-	1.19%	0.24
Luxembourg	0.67%	$1.06^{\dagger}$	-	-	2.33%	$0.22^{\dagger}$
Mexico	7.68%	$1.69^{\dagger}$	4.89%	$-0.66^{\dagger}$	-	-
Netherlands	0.47%	$-0.74^{\dagger}$	-	-	0.96%	$-0.07^{\dagger\dagger\dagger}$
New Zealand	1.34%	$2.68^{\dagger}$	3.37%	$0.88^{\dagger}$	1.52%	$0.75^{\dagger}$
Norway	0.88%	$0.71^{\dagger}$	-	-	2.62%	$0.54^{\dagger\dagger\dagger}$
Portugal	1.96%	$1.57^{\dagger}$	2.85%	$1.89^{\dagger}$	1.10%	$0.70^{\dagger}$
Spain	1.00%	$0.76^{\dagger\dagger}$	-	-	1.83%	$-5.27^{\dagger}$
Sweden	0.99%	$1.03^{\dagger}$	1.58%	$0.90^{\dagger}$	1.37%	$-0.18^{\dagger\dagger}$
Switzerland	0.63%	$0.43^{\dagger\dagger\dagger}$	-	-	2.38%	0.03
United Kingdom	0.83%	$1.62^{\dagger}$	1.90%	$0.94^{\dagger}$	0.92%	0.09
United States	0.45%	$-0.11^{\dagger\dagger}$	0.97%	$0.93^{\dagger}$	0.69%	$1.09^{\dagger}$
OECD Average	0.67%	$1.06^{\dagger}$	-	-	0.43%	0.33

The number of †'s next to the third moment indicates a 1%, 5% and 10% p-value respectively when computing a Jacque-Bera test statistic under the null hypothesis of a normal distribution. The sample size for all CPI series starts in 1982:Q1 and ends in 2008:Q1. However, the data for ULC in New Zealand starts in 1989:Q1, while in Greece, Luxembourg, Portugal, and Ireland - 1995:Q2; EPI data for Canada starts in 1997:Q1, France - 1999:Q1, and Ireland - 2000Q1. The magnitudes of positive skewness of prices are somewhat lower if the sample is extended to include the Great Financial Crisis (i.e. 1982Q1: 2017Q1), but considerably greater when the time horizon is extended to include the 1960s-70s. The EPI data comes from the IMF International Financial Statistics database, while CPI and ULC are collected from OECD Economic Outlook and OECD Main Economic Indicators.

of these adjustment costs, rendering downward adjustments of wages and prices generally more costly than in the opposite direction. Specifically, the most convenient functional form of asymmetric adjustment costs is called LINEX due to Varian (1975), which extends the standard quadratic adjustment cost framework:

$$\varDelta(\dot{v}_t) = \frac{\kappa_v[\exp(\zeta_v(\dot{v}_t - \dot{v})) - \zeta_{i,v}(\dot{v}_t - \dot{v}) - 1]}{\zeta_v^2} \geq 0, \tag{2.1}$$

$$\varDelta'(\dot{v}_t) = \frac{\kappa_v[\exp(\zeta_v(\dot{v}_t - \dot{v})) - 1]}{\zeta_v} \lesseqgtr 0, \tag{2.2}$$

$$\varDelta''(\dot{v}_t) = \kappa_v[\exp(\zeta_v(\dot{v}_t - \dot{v}))] \lesseqgtr 0, \tag{2.3}$$

where $\dot{v}_t = v_t/v_{t-1}$ measures the gross growth rate of an arbitrary variable v_t at date $t = 0, 1, 2, ...$, and the associated long-run trend is $\dot{v} \geq 1$. The name LINEX derives from the properties of the function, namely $\varDelta(\dot{v}_t)$ rises exponentially in the case of $\zeta_v > 0$ ($\zeta_v < 0$) when $\dot{v}_t > 1$ ($\dot{v}_t < 1$), but approximately linearly when $\dot{v}_t < 1$ ($\dot{v}_t > 1$). In the special case when $\zeta_v \to 0$, the adjustment costs are quadratic *à la* Rotemberg (1982):

$$\lim_{\zeta_v \to 0} \Delta(\dot{v}_t) = \frac{\kappa_v(\dot{v}_t - \dot{v})^2}{2}. \tag{2.4}$$

The key distinguishing feature of LINEX adjustment costs, as opposed to time-dependent nominal rigidity or quadratic adjustment costs, is that they can induce significant skewness and kurtosis in the temporal distributions of macroeconomic fundamentals as it is observed in the data. However, unlike models with fixed menu costs and heterogeneous firms, such as Flodén & Wilander (2006), state-dependence in this framework refers to the asymmetric magnitudes of upward and downward adjustments of prices and wages, rather than the frequency with which they adjust depending on the size and the nature of innovations. Specifically, with LINEX adjustment costs, constant returns to scale technology, and homogeneous productivity, all prices and wages respond to all shocks in every period, but only partially if price adjustment costs are strictly convex (i.e. $\kappa_v \geq 0$) and more so for downward (upward) pressures when $\zeta_v < 0$ ($\zeta_v > 0$). Under this reduced form mechanism, a structural innovation of any given magnitude can induce asymmetrical pass-through into prices depending on whether the shock is positive or negative. Although there are a number of existing applications of LINEX adjustment costs to business cycle models in the closed economy literature, they tend to focus on the downward rigidity of wages in the United States, such as Kim & Ruge-Murcia (2009, 2011), Abbritti & Fahr (2013), Aruoba et al. (2017), yet this seems less appropriate in the context of other OECD economies. The primary contribution of this chapter is therefore to demonstrate how state-dependent exchange rate pass-through can be derived and quantified using an open economy model with LINEX adjustment costs.

2.3 Model

There are a finite number of economies in this model. Each economy comprises of three types of interacting agents: households, firms, and central banks. There are three sectors in the supply-side of each economy: wholesale, retail, and distribution. The latter is responsible for transporting the goods from the upstream to the downstream markets within an economy. International shipment of goods is subject to Samuelson's 'iceberg costs'. Households can only purchase goods from the local retail sector, but they supply differentiated labour services to the distribution sector. In turn, workers and employers determine the equilibrium real wages through a stylised process of collective bargaining. In addition, households sell a proportion of their perfectly divisible consumption basket to the firms abroad, which is then used as an intermediate input in the local wholesale industry. Households smooth their lifetime consumption in response to country-specific shocks by trading bonds. As is usual, the adjustment costs of wages and prices induce persistence of output and inflation in response to internal and external shocks, but they are gradually stabilised by the local central banks that are independently committed to an autonomous Taylor rule.

2.3.1 Import-Export Wholesalers

Consider a world evolving over discrete time $t = 0, 1, 2...$, that consists of $n = \{1, 2, ..., N\}$ number of economies populated by a continuum of monopolistically-competitive wholesalers indexed by $\omega \in [0, 1]$. All economies are open to trade with the rest of the world and all wholesale varieties are internationally traded, but they are split into two categories indexed by $\phi \in \{-\pi, \pi\}$. A fraction $\chi_{in}(\pi) \in (0, 1)$ of the firms domiciled in economy $i \in n$ sells their variety in all n destinations and sets the price for their manufactured output at the factory door (PCP), while the remaining proportion $\chi_{in}(-\pi) \in (0, 1)$ set prices at the docks of each destination (LCP), such that $\sum_{\phi} \chi_{in}(\phi) = 1$. All wholesalers import commodities from the destination economy that are used as intermediate inputs in producing the final manufactured goods. This relationship is captured by the linear import-export production technology: $y_{in,t}(\omega, \phi) = z_{i,t} m_{in,t}(\omega, \phi)/\xi_i$. In this simplified setup, the term $m_{in,t}(\omega, \phi)$ denotes the stock of intermediate imports used in the production of manufactured output $y_{in,t}(\omega, \phi)$.[5] Homogeneous exporter productivity follows a stationary autoregressive process $z_{i,t} = z_{i,t-1}^{\rho_{i,z}} \exp(\sigma_{i,z}\epsilon_{i,z,t})$, where $\rho_{i,z} \in (0, 1)$, $\sigma_{i,z} > 0$ and $\epsilon_{i,z,t} \sim iid(0, 1)$.

The real unit costs of producing final wholesale varieties are homogeneous across all firms within the ϕ market segment. Namely, $mc_{in,t}(\phi) = \xi_i Q_{in,t}(\phi)/z_{i,t}$, where $\xi_i \in [0, 1]$ measures the intermediate import intensity, $Q_{in,t}(\pi) = \min\{q_{ni,t}; n = 1, 2, ..., N\}$, $Q_{in,t}(-\pi) = \min\{q_{jn,t}; j = 1, 2, ..., N\}$, and $q_{ni,t}$ is the bilateral real exchange. Specifically, a rise in $q_{ni,t} = 1/q_{in,t}$ implies an i'th currency depreciation against the n'th currency in real terms, and a rise in the price of imported commodities in the i'th economy. The rationale behind the unit cost invoicing function $Q_{in,t}(\phi)$ is based on two empirical stylised facts. First, Chen et al. (2010) demonstrate that exchange rates of major commodity exporters have strong predictive power in forecasting movements in global commodity prices, which provides a motive for incomplete pass-through via cost-push shocks. Imposing a causal link between unit costs and exchange rates is therefore rather innocuous and common in the literature, such as Monacelli (2013) for example. Second, Amiti et al. (2014) and Chung (2016) show that intermediate import intensity is correlated with exporters choice to invoice profits in the currency units of the destination economy. Hence, $Q_{in,t}(\pi)$ depends on the value of the i'th currency (i.e. source) relative to the rest of the world, while $Q_{in,t}(-\pi)$ depends on the value of the n'th currency (i.e. destination) relative to the rest of the world. Specifically, LCP firms take into the account the most favourable bilateral real exchange rate, because wholesalers choose to import commodities from the most competitive foreign location in order to maximise the price-cost margin. It implicitly assumes that when the LCP firms sell goods to the n'th economy, their intermediate imports are shipped directly to the destination economy and assembled into final goods thereafter as though they never cross the domestic borders.

[5] Intermediate imports are treated as equity owned by foreign households, who trade a fraction of their consumption bundle as bonds in Arrow-Debreu markets. This mechanism is analogous to the intra-national roundabout structure depicted in Nakamura & Steinsson (2010), except the production function in this context is linear in intermediate goods. If capital and labour were introduced using a Cobb-Douglas production function, then a local currency depreciation would have the tendency to lower the demand for imported inputs while at the same time increasing the demand for labour and capital, resulting in a counterfactually strong negative correlation between unit costs and exchange rate. Alternatively, a more elaborate framework could establish complementarity in intermediate imports using a CES technology with IES$<$1.

In each period, the wholesale goods are produced and then either costlessly stored in a warehouse, or they are shipped to all other $N-1$ destinations. If they are shipped abroad, then the firms incur iceberg costs equivalent to a fixed proportion $\tau_{in}-1 \in (0,1)$ of their total revenue generated abroad as in Samuelson (1954).[6] Suppose that international arbitrage forces are near-perfectly efficient in terms of mitigating international price discrimination at the border. Each variety will therefore be priced according to an 'equilibrium' condition expressed in real terms that allows for a negligible amount of parallel trade in the short-run:

$$p_{in,t}(\omega,\phi) = \tau_{in} q_{in,t} \delta_{in,t}(\omega,\phi) p_{ii,t}(\omega,\phi), \tag{2.5}$$

where $p_{in,t}(\omega,\phi) = P_{in,t}(\omega,\phi)/P_{n,t}$ stands for the relative price of variety ω originating from the i'th economy and sold in the n'th economy, while $P_{n,t}$ is the consumer price index in the n'th economy. Similar to Monacelli (2005), the term $\delta_{in,t}(\omega,\phi)$ represents the endogenous dispersion of price mark-ups across borders whenever product markets are imperfectly integrated. By construction, $\delta_{in,t}(\omega,\pi) = 1$ at all times, but $\delta_{in,t}(\omega,-\pi) \neq 1$ in the short-run, since prices set in local currency terms respond not only to domestic innovations, but also to the ones characterising the destination economy, which gives rise to incomplete pass-through.[7]

When the goods arrive at the border of the n'th economy during that same period, they are aggregated into bundles of goods according to their origin using the Constant Elasticity of Substitution (CES) technology due to Dixit & Stiglitz (1977):

$$y_{in,t} = \left[\sum_{\phi} \int_{0}^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi)^{1-1/\varepsilon} d\omega \right]^{1/(1-1/\varepsilon)}, \tag{2.6}$$

where parameter $\varepsilon > 1$ stands for the intra-temporal elasticity of substitution (IES) between different wholesale varieties. The aggregate price of imports is therefore equal to a weighted average of all the wholesale import prices:

$$p_{in,t} = \left[\sum_{\phi} \int_{0}^{\chi_{in}(\phi)} p_{in,t}(\omega,\phi)^{1-\varepsilon} d\omega \right]^{1/(1-\varepsilon)}, \tag{2.7}$$

such that the density of firm types $\chi_{in}(\phi)$ determines the average import price responsiveness

[6] By definition $\tau_{ii} = \tau_{nn} = 1$, but $\tau_{ni} = \tau_{in} = \tau \geq 1$, such that only international transportation of goods are subject to iceberg costs. It is also implicitly assumed that direct shipment of goods is always the least expensive route, namely $\tau_{ni} \leq \tau_{nj}\tau_{ji} \equiv \tau^2$ for all $i, j \in n$, such that the 'triangular equation' holds.

[7] Quantitative trade models have long neglected the peculiarities of international market segmentation that generate substantial deviations from the law of one price even after accounting for the geographical dimension. For instance, Engel & Rogers (1996) examine the price dispersion of a large number of consumer goods in large cities of the U.S. and Canada. They find that price dispersion measured in USD is higher for two cities located across the border than two equidistant cities in the same country. The symptoms of imperfect product market integration are particularly noticeable in the pharmaceutical or the automobile industries, where Vogler et al. (2016) and Saridakis & Baltas (2016) among many others document stark discrepancies between international prices of cars and cancer drugs. In theory, this dictates 'free lunch' opportunities, or parallel trade, whereby arbitrageurs could make profits by shipping goods across borders. Yet in practice, differences in patent rights and insurance policies across borders – something that is rarely considered in macroeconomic models – mitigate these prospects, thus resulting in a distorted general equilibrium.

to exchange rate innovations. When $\chi_{in}(\pi) \to 0$ ($\chi_{in}(\pi) \to 1$), product markets become less (more) integrated and import prices are more (less) stable relative to the exchange rate.

2.3.2 Sticky Prices

Suppose all import-export wholesalers are characterised by rational expectations. Suppose further that every time they adjust prices, they incur price adjustment costs denoted by $\mathit{\Delta}_{in,t}(\omega,\phi) \in [0,1)$, which adopt LINEX functional form as described in equations (2.1) - (2.3). Parameters $\kappa_{i,p} \geq 0$ and $-\infty < \zeta_{i,p} < \infty$ control their average convexity as well as their pivot. On average, the greater is the difference between the optimal price and the local long-run trend of inflation $\dot{p}_n \geq 1$, the greater are the price adjustment costs. Firms therefore have the incentive to adjust their prices gradually and continuously, except when $\zeta_{i,p} < 0$ ($\zeta_{i,p} > 0$), then upward (downward) adjustments are less expensive and will generally be more sizeable. Firms choose the nominal price $P_{in,t}(\omega,\phi)$ to maximise the present discounted value of profit dividends subject to their demand schedule:

$$\max_{\{P_{in,t}(\omega,\phi)\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{i,t,t+1} \left\{ \sum_{n=1}^{N} \left[\left(1 - \mathit{\Delta}_{in,t}(\omega,\phi)\right) p_{in,t}(\omega,\phi) - mc_{in,t}(\phi) \right] y_{in,t}(\omega,\phi) \right\}$$
$$\text{s.t.} \quad y_{in,t}(\omega,\phi) = y_{in,t} \left[\frac{p_{in,t}(\omega,\phi)}{p_{in,t}} \right]^{-\varepsilon},$$

where $\mathbb{E}[\lambda_{i,t,t+1}]$ stands for the stochastic discount factor in the source country.[8]

In the symmetric equilibrium, all ϕ-type firms set identical prices due to their homogeneous productivity. The first-order condition with respect to the nominal export price $P_{ii,t}(\omega,\phi)$ evaluated at the symmetric equilibrium thus gives rise to a non-linear relationship between the export prices and the unit costs of production:

$$p_{ii,t}(\phi) = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{mc_{ii,t}(\phi)}{\mathit{\Phi}_{ii,t}(\phi)}, \tag{2.8}$$

$$\mathit{\Phi}_{ii,t}(\phi) = 1 - \mathit{\Delta}_{ii,t}(\phi) + \frac{\mathit{\Delta}'_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{\varepsilon-1} - \lambda_{i,t,t+1}\mathbb{E}_t \left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)} \frac{\mathit{\Delta}'_{ii,t+1}(\phi)}{\varepsilon-1} \frac{\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}} \right], \tag{2.9}$$

where $\mathit{\Phi}_{ii,t}(\phi)$ captures the asymmetric price stickiness, $\dot{p}_{i,t} = P_{i,t}/P_{i,t-1}$ is the gross rate of consumer price inflation, while $\dot{p}_{ii,t}(\phi) = P_{ii,t}(\phi)/P_{ii,t-1}(\phi) = \dot{p}_{i,t}(p_{ii,t}(\phi)/p_{ii,t-1}(\phi))$ denotes export price inflation. In the long-run, $\mathit{\Phi}_{ii}(\phi) = 1$, because $\dot{p}_{ii} = \dot{p}_i$. However, as long as $\zeta_{i,p} < 0$, the magnitude of $\mathit{\Phi}_{ii,t}(\phi)$ will be larger whenever $\dot{p}_{ii,t}(\phi) < \dot{p}_i$ compared to when $\dot{p}_{ii,t}(\phi) > \dot{p}_i$, which generates positively skewed import, export, and consumer price inflation.

Now that we have characterised the optimal time path of export prices in the symmetric equilibrium, computing their elasticity to exchange rate innovations should be relatively straightforward. Yet a growing strand of the literature pioneered by Shambaugh (2008) close their systems of difference equations and then compute the ratio of cumulative impulse response functions of price inflation and differenced exchange rate triggered by an arbitrary

[8] Following Kim & Ruge-Murcia (2009, 2011), price adjustment is viewed as an unproductive activity, since it does not generate any value-added. As such, price adjustment costs are deducted from the total revenue generated in each destination individually.

innovation. The problem with such an 'agnostic' approach is that it is highly susceptible to the way in which the system is closed, because it measures aggregate co-movements between prices and exchange rates. In particular, different assumptions behind the determination of exchange rate dynamics will produce different dynamic causal effect, because the implicit policy function of exchange rate pass-through depends on numerous state variables that are irrelevant from the perspective of individual exporters. Because we know for a fact that all firms in the monopolistically-competitive market structure are infinitesimally 'small', they always take the exchange rate innovations as given, but the aggregate co-movement approach treats international prices and exchange rates as though the relationship between the two is endogenous in both directions. It therefore makes an astounding difference whether exchange rate pass-through is computed as an aggregate co-movement between prices and exchange rates or aggregated firm-level transmission of exchange rate innovations. The discussion now turns to the derivation of a micro-founded mechanism of exchange rate transmission directly from the first order conditions of the exporters portrayed in equations (2.7), (2.8), and (2.9).[9]

Lemma 1. *If $1 < \varepsilon < \infty$, $\xi_i(\phi) > 0$, $\kappa_{i,p} > 0$, $\zeta_{i,p} < 0$, and import-export wholesalers are monopolistically-competitive, then the causal effect of a unilateral i'th currency depreciation on the i'th economy export price index is more sizeable than for an exact opposite appreciation:*

$$erpt_{ii,t} = \frac{\partial \ln p_{ii,t}}{\partial \ln q_{ni,t}} = \sum_{\phi} \frac{s_{ii,t}(\phi)}{1 + \Gamma_{ii,t}(\phi)}, \tag{2.10}$$

where

$$\Gamma_{ii,t}(\phi) = \Xi_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t \left[\Xi_{ii,t+1}(\phi) \frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)} \frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)} \frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}} \right], \tag{2.11}$$

$$\Xi_{ii,t}(\phi) = \frac{\dot{p}_{ii,t}(\phi) \left[\dot{p}_{ii,t}(\phi) \Delta''_{ii,t}(\phi) + (2 - \varepsilon) \Delta'_{ii,t}(\phi) \right]}{\Phi_{ii,t}(\phi)(\varepsilon - 1)}, \tag{2.12}$$

$$s_{ii,t}(\phi) = \frac{p_{ii,t}(\phi) y_{ii,t}(\phi)}{p_{ii,t} y_{ii,t}} = \chi_{ii}(\phi) \left[\frac{p_{ii,t}(\phi)}{p_{ii,t}} \right]^{1-\varepsilon}. \tag{2.13}$$

Proof. Suppose $\dot{q}_{ni,t} = q_{ni,t}/q_{ni,t-1} > 1$, then because $Q_{ii,t}(\phi) = \min\{q_{ni,t}; n = 1, ..., N\}$ and $mc_{ii,t} = \xi_i(\phi) Q_{ii,t}(\phi)/z_{ii,t}$, it follows that $\partial \ln mc_{ii,t}(\phi)/\partial \ln q_{ni,t} = 1$. In turn, observe that $\partial \ln \Phi_{ii,t}(\phi)/\partial \ln q_{ni,t} = erpt_{ii,t}(\phi)\Gamma_{ii,t}(\phi)$ and $erpt_{ii,t}(\phi) = \partial \ln mc_{ii,t}(\phi)/\partial \ln q_{ni,t} - \partial \ln \Phi_{ii,t}(\phi)/\partial \ln q_{ni,t} = 1/(1 + \Gamma_{ii,t}(\phi))$. Finally, the aggregate export price pass-through is given by $erpt_{ii,t} = \sum_\phi s_{ii,t}(\phi) erpt_{ii,t}(\phi) = \sum_\phi s_{ii,t}(\phi)/(1 + \Gamma_{ii,t}(\phi))$. Consequently, as long as $\zeta_{i,p} < 0$, $erpt_{ii,t}$ will be greater (lower) when $\dot{q}_{ni,t} > 1$ ($\dot{q}_{ni,t} < 1$), since the magnitude of $\Xi_{ii,t}(\phi)$ and $\Gamma_{ii,t}(\phi)$ will be larger (smaller). See appendix A.1 for algebraic details. □

[9] When import-export wholesalers are monopolistically-competitive, they do not exert enough market power to shift the dynamics of the consumer price index. For this reason, exchange rate pass-through derived in real or nominal terms is observationally equivalent. However, it is more convenient to showcase the results in real terms, since the model is solved for a general case in which price levels are indeterminate due to the long-run trend of inflation pursued by the central banks. Moreover, all innovations in this model are generated in a perfectly unanticipated manner. Exchange rate pass-through presented henceforth therefore treats all forward-looking variables as orthogonal to contemporaneous exchange rate movements.

The conceptual rationale for the non-linear exchange rate transmission channel goes back to the idea that exchange rate depreciations are observationally equivalent to an aggregate decline in import-dependent exporter productivity. If the depreciation (appreciation) is sufficiently large, then import-dependent exporters could shift the aggregate inflation into a state with a higher (lower) frequency of price adjustment, since it intensifies (subsides) the selection effect even for firms that have successfully hedged their exposure to exchange rate risk. The shift in the selection effect is approximated by the pivot in the LINEX price adjustment cost function, which allows export prices to be more flexible upwards than downwards.[10]

Lemma 2. *Import prices in the n'th economy are more elastic to unilateral n'th currency depreciations than appreciations as long as* $1 < \varepsilon < \infty$*,* $\xi_i(\phi) > 0$*,* $\kappa_{i,p} > 0$*,* $\zeta_{i,p} < 0$*, and import-export wholesalers are monopolistically-competitive:*

$$erpt_{in,t} = \frac{\partial \ln p_{in,t}}{\partial \ln q_{in,t}} = \frac{s_{in,t}(-\pi)}{1 + \mathit{\Gamma}_{in,t}(-\pi)} + \frac{s_{in,t}(\pi)\mathit{\Gamma}_{ii,t}(\pi)}{1 + \mathit{\Gamma}_{ii,t}(\pi)}. \tag{2.14}$$

Proof. Observe that $erpt_{in,t}(\pi) = 1 - erpt_{ii,t}(\pi) = \mathit{\Gamma}_{ii,t}(\pi)/(1 + \mathit{\Gamma}_{ii,t}(\pi))$ and $erpt_{in,t}(-\pi) = 1/(1 + \mathit{\Gamma}_{in,t}(-\pi))$. When $\zeta_{i,p} < 0$ and $\dot{q}_{in,t} > 1$, $\mathit{\Gamma}_{ii,t}(\pi)$ decreases by less ($\mathit{\Gamma}_{in,t}(-\pi)$ increases by more) than it increases (decreases) when $\dot{q}_{in,t} < 1$, such that $erpt_{in,t}(\pi)$ and $erpt_{in,t}(-\pi)$ are more responsive to $\dot{q}_{in,t} > 1$ than $\dot{q}_{in,t} < 1$. Moreover, $s_{in,t}(\pi) = 1 - s_{in,t}(-\pi)$ decreases (increases) when $\dot{q}_{in,t} > 1$ ($\dot{q}_{in,t} < 1$), therefore $erpt_{in,t} = \sum_{\phi} s_{in,t}(\phi) erpt_{in,t}(\phi) = s_{in,t}(-\pi)/(1 + \mathit{\Gamma}_{in,t}(-\pi)) + s_{in,t}(\pi)\mathit{\Gamma}_{ii,t}(\pi)/(1 + \mathit{\Gamma}_{ii,t}(\pi))$ is higher (lower) for $\dot{q}_{in,t} > 1$ ($\dot{q}_{in,t} < 1$). □

There are two opposing forces at play in the transmission mechanism: a direct currency conversion effect and an indirect cost-push effect. The latter applies to both PCP and LCP firms, while the former applies only to the PCP firms.[11] When the destination currency depreciates, the direct effect is the upward pressure on the PCP import prices in the destination economy due to a change in the relative price of the currencies. The indirect effect is the decrease (increase) in the costs of intermediate imports in the source (destination) economy, which leads to a modest decrease (sharp increase) in the PCP (LCP) import prices in the destination economy. The inflationary effects of the destination currency depreciation are thus exacerbated by the downward rigidity of PCP export prices, since the direct effect generally dominates the indirect effect. Conversely, when the destination currency appreciates, the PCP import prices in the destination economy decrease due to a change in the relative price of the currencies. However, the costs of intermediate imports in the source (destination) economy increase (decrease). Because the PCP export prices are more flexible upwards than downwards, the fall in the destination economy import prices will be smaller compared to the magnitude by which they tend to rise, since destination currency appreciations lead to a pronounced increase (moderate decrease) in the PCP (LCP) import prices.

[10] An alternative approach is to impose a short-run capacity constraint as originally conjectured by Knetter (1994). For example, $\dot{y}_{ii,t} = \min\{y_{ii,t}(\phi)/y_{ii,t-1}(\phi), \bar{y}_{ii}\}$, where $\bar{y}_{ii} > 0$ is a constant. However, only in high-frequency 'fire sale' settings' would it be plausible to assume that an unexpected favourable demand shock transformed the pricing schedule exponentially into auction, which links the prices to the remaining stock endogenously. But it is much less plausible in business cycle frequency when firms are subject to menu costs.

[11] While there exists only one first order condition for PCP exporters in the source country, the LCP exporters set prices for all n destinations – each characterised by an analogous first order condition to the one depicted in equations (2.8) and (2.9). For this reason, the direct effect is subsumed in the wedge prevailing between international prices in the short-run. See equation (2.5).

The fundamental difference between the present mechanism and those presented in Amiti et al. (2014) or Burstein & Gopinath (2014) is that non-linearities induced by LINEX price adjustment costs generally produce a stochastic steady state of pass-through into import prices that is shifted upwards (downwards) for LCP (PCP) exports relative to the deterministic steady state, creating an additional motive for 'incompleteness'. As a result, if $s_{in,t}(\pi)$ was close to unity, the classical Mundell-Fleming predictions would hold *ex ante*, but the *ex post* relationship would be influenced by the fat tails of the distribution associated with the export price inflation. The most interesting prediction of the non-linear model is therefore that it has the capacity to statistically reject the 'complete' pass-through hypothesis even if it were explicitly embedded into the model via a counterfactually high share of PCP imports.

The quantitative predictions of exchange rate pass-through in this environment are obtained by establishing a general equilibrium in a closed model. But before moving onto the remaining parts of the model, the last segment of this section draws your attention to the simplifying assumptions that nest already familiar corner solutions.

Proposition 1. *When monopolistically-competitive exporters are autonomous to intermediate imports, such that $\xi_i(\phi) \to 0$, or equivalently prices in the source economy are perfectly rigid in the short-run, such that $\kappa_{i,p} \to \infty$, export prices are neutral to exchange rates and import prices move one-to-one with the π-type market share in the destination economy:*

$$\lim_{\xi_i(\phi)\to 0\,\forall\,\phi} erpt_{ii,t} = \lim_{\kappa_{i,p}\to\infty} erpt_{ii,t} = 0, \tag{2.15}$$

$$\lim_{\xi_i(\phi)\to 0\,\forall\,\phi} erpt_{in,t} = \lim_{\kappa_{i,p}\to\infty} erpt_{in,t} = s_{in,t}(\pi). \tag{2.16}$$

Proof. When $\xi_i(\phi) \to 0$, the model approaches the analytical construct of the endowment economy, where $\partial \ln mc_{in,t}(\phi)/\partial \ln q_{ni,t} = 0$ for $\phi = \{\pi, -\pi\}$, while $\kappa_{i,p} \to \infty$ gives rise to $\mathit{\Gamma}_{ii,t}(\phi) \to \infty$ for $\phi = \{\pi, -\pi\}$ and $\mathit{\Gamma}_{in,t}(-\pi) \to \infty$. Both imply $erpt_{ii,t}(\phi) = 0$ for $\phi = \{\pi, -\pi\}$ and $erpt_{ii,t} = \sum_\phi s_{ii,t}(\phi) erpt_{ii,t}(\phi) = 0$. Consequently, $erpt_{in,t}(\pi) = 1 - erpt_{ii,t}(\pi) \equiv 1$ and $erpt_{in,t}(-\pi) = 0$, such that $erpt_{in,t} = \sum_\phi s_{in,t}(\phi) erpt_{in,t}(\phi) = s_{in,t}(\pi)$. □

Proposition 2. *When export prices are perfectly flexible, but wholesale production technology is import-dependent, such that $\kappa_{i,p} \to 0$ and $\xi_i(\phi) > 0$, export prices fully absorb the exchange rate movements, while import prices move one-to-one with the $-\pi$-type market share.*

$$\lim_{\kappa_{i,p}\to 0} erpt_{ii,t} = 1, \tag{2.17}$$

$$\lim_{\kappa_{i,p}\to 0} erpt_{in,t} = s_{in,t}(-\pi). \tag{2.18}$$

Proof. Perfectly flexible prices imply $\mathit{\Gamma}_{ii,t}(\phi) \to 0$ for $\phi = \{\pi, -\pi\}$ and $\mathit{\Gamma}_{in,t}(-\pi) \to 0$. Consequently $erpt_{ii,t}(\phi) = \partial \ln mc_{ii,t}(\phi)/\partial \ln q_{ni,t} = 1$ for $\phi = \{\pi, -\pi\}$ and $erpt_{in,t}(-\pi) = \partial \ln mc_{in,t}(-\pi)/\partial \ln q_{in,t} = 1$. Therefore, $erpt_{ii,t} = \sum_\phi s_{ii,t}(\phi) erpt_{ii,t}(\phi) = 1$ and $erpt_{in,t} = \sum_\phi s_{in,t}(\phi) erpt_{in,t}(\phi) = s_{in,t}(-\pi)$. □

To summarise, perfect price flexibility (rigidity) leads to exchange rate pass-through into import prices driven primarily by LCP (PCP) market share.[12] The remaining parts of the modelling section set out the demand side of the economy and close the general equilibrium model by characterising the notions of aggregate income and the trade balance.

2.3.3 Consumer Prices

Once the imported goods are sorted into country-specific bundles, they are merged into an aggregate bundle of tradable goods using the standard CES technology:

$$x_{i,t} = \left[\sum_{n=1}^{N} \alpha_{ni}^{1/\eta} \, y_{ni,t}^{1-1/\eta} \right]^{1/(1-1/\eta)}, \tag{2.19}$$

such that $\eta > 1$ is the elasticity of substitution between the locally-produced and imported goods, and parameter $\alpha_{ii} = 1 - \sum_{n=1}^{N-i} \alpha_{ni} \in (0, 1)$ measures the i'th economy home-bias. The price of the tradable goods bundle is set as a trade-weighted average of all the import prices:

$$p_{i,x,t} = \left[\sum_{n=1}^{N} \alpha_{ni} \, p_{ni,t}^{1-\eta} \right]^{1/(1-\eta)}. \tag{2.20}$$

The household consumption bundle consists of tradable and non-tradable goods that are assembled in the downstream market by a competitive retailer using Cobb-Douglas technology:

$$c_{i,t} = (a_{i,t} h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i}. \tag{2.21}$$

Labour services are non-tradable, because labour is perfectly mobile within, but not across borders. The term $h_{i,t} \in (0, 1)$ stands for the hours spent in the labour force by i'th economy households relative to the total endowment of time. Similar to García-Cicco et al. (2010), labour productivity follows a random walk with a drift, such that $a_{i,t} = \gamma_i a_{i,t-1} \exp(\sigma_{i,a} \epsilon_{i,a,t})$, where $\sigma_{i,a} > 0$ and $\epsilon_{i,a,t} \sim iid(0, 1)$. The drift $\gamma_i > 1$ determines the long-run trend of labour productivity driving the perpetual growth rate of the real hourly wage denoted by $w_{i,t}$.

The retail price index is then set as a weighted average of prices of tradable and non-tradable goods, where the weights correspond to their share in the final consumption basket:

$$p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i} \right)^{\alpha_i} \left(\frac{p_{i,x,t}}{1-\alpha_i} \right)^{1-\alpha_i}. \tag{2.22}$$

The prices of non-traded goods in this model are highly stylised for two reasons. First, Engel (1999) shows that the real exchange rate volatility in most advanced economies can be accounted for by the movements in relative prices of tradable goods, while the relative price of

[12] This result is supported by the empirical evidence presented in Berman et al. (2012). But more generally, exchange rate pass-through into PCP (LCP) import prices is increasing (decreasing) in the slope of the Phillips curve. Up to the first-order, Ascari & Rossi (2012) show that the slope of the Phillips curve in a Rotemberg (1982) style pricing model is given by $(\varepsilon - 1)/\kappa_{i,p}$, which is approximately equal to 0.05-0.2 in the OECD economies, depending on the sample period and the empirical methodology. Hence, in order to obtain a sufficiently flat Phillips curve, the average convexity of the price adjustment costs $\kappa_{i,p}$ is typically calibrated to a relatively large magnitude, while $\varepsilon > 1$ remains relatively low.

non-tradable goods are largely disconnected. Second, Atkeson & Burstein (2008) argue that non-traded goods and services, such as those associated with transportation and distribution of manufactured imports, are mostly labour-intensive, such that their output is proportional to the labour input. The production costs of non-traded goods thus consist primarily of the wage bill $w_{i,t}h_{i,t}$, which mostly depends on the aggregate productivity growth and is largely orthogonal to exchange rate movements in the short-run.[13]

Lemma 3. *When the aggregate consumption bundle consists of tradable and non-tradable goods and only a fraction of tradable goods are imported, such that such that* $\alpha_i \in (0,1)$ *and* $\alpha_{ii} \in (0,1)$*, the exchange rate elasticity of consumer prices is bounded between zero and unity:*

$$erpt_{i,t} = \frac{\partial \ln p_{i,t}}{\partial \ln q_{ni,t}} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt_{ni,t} \in (0,1). \tag{2.23}$$

Proof. Let $s_{ni} = (p_{ni,t}y_{ni,t})/(p_{i,x,t}x_{i,t}) = \alpha_{ni}[p_{ni,t}/p_{i,x,t}]^{1-\eta} \in [0,1]$ measure the trade weight of the source country n in destination i. The prices of non-tradable goods are not directly related to exchange rates by construction, such that $\partial \ln w_{i,t}/\partial \ln q_{ni,t} = 0$. By contrast, equation (2.20) expresses the prices of tradable goods as a trade-weighted average of domestic export and import prices, such that $\partial \ln p_{i,x,t}/\partial \ln q_{ni,t} = \sum_{n=1}^{N} s_{ni,t} erpt_{ni,t}$. The exchange rate elasticity of consumer prices is thus proportional to the elasticity of tradable goods prices, namely $\partial \ln p_{i,t}/\partial \ln q_{ni,t} = (1-\alpha_i)\partial \ln p_{i,x,t}/\partial \ln q_{ni,t} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt_{ni,t}$. $\square$

2.3.4 Wage Bargaining

The skewness of the retail price inflation at the end of the pricing chain can also be linked to the skewness of wage inflation, since a large proportion of production costs in the supply-side comprise of the wage bill. In order to determine how much of the skewness associated with the consumer price index is attributable to the LINEX price adjustment costs, it is important to incorporate and control for the imperfections in the labour market. Not least because all of the non-traded goods in this model are produced by competitive firms characterised by labour-intensive technology, thereby attributing all of the market power over price setting decisions to the exporting firms once the terms of the labour contracts are set.

Consider a unit mass of rational households indexed by ω populating the i'th economy. Their preferences are homothetic and additively separable:

$$u_{i,t} = \log\left(\frac{c_{i,t} - \vartheta_i c_{i,t-1}}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right) h_{i,t}^{1+\varphi_i}, \tag{2.24}$$

such that households derive utility from the habit-adjusted stock of an infinitely-divisible retail good $c_{i,t}(\omega)$ and disutility from the hours of labour $h_{i,t}(\omega) \in (0,1)$. Parameter $\psi_i > 0$ controls the disutility of labour, $\varphi_i \geq 0$ is the inverse Frisch elasticity of labour supply, and $\vartheta_i \in (0,1)$ governs the strength of additive consumption habits. Each household supplies

[13]See Burstein et al. (2003) and Campa & Goldberg (2010) for a thorough literature survey and empirical estimates of the relative size of the distribution sector in OECD economies.

an imperfectly substitutable service to a competitive labour packer, who aggregates them according to the following CES technology:

$$h_{i,t} = \left[\int_0^1 h_{i,t}(\omega)^{1-1/\varepsilon} d\omega \right]^{1/(1-1/\varepsilon)}. \tag{2.25}$$

Because labour services are imperfectly substitutable, workers can exploit the labour demand schedule for their service when bargaining over wages with the employers.

The dynamic stochastic utility maximisation problem solved by a rational household can then be stated as follows:

$$\begin{aligned}
&\max_{\{c_{i,t}(\omega), w_{i,t}(\omega), b_{i,t+1}(\omega)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log \left(\frac{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)}{a_{i,t}} \right) - \left(\frac{\psi_i}{1+\varphi_i} \right) h_{i,t}(\omega)^{1+\varphi_i} \right\}, \\
&\text{s.t.} \quad h_{i,t}(\omega) = h_{i,t} \left[\frac{w_{i,t}(\omega)}{w_{i,t}} \right]^{-\varepsilon}, \\
&\text{s.t.} \quad c_{i,t}(\omega) + \lambda_{i,t,t+1} \mathbb{E}_t[b_{i,t+1}(\omega)] = b_{i,t}(\omega) + (1 - \mathit{\Delta}_{i,w,t}(\cdot)) w_{i,t}(\omega) h_{i,t}(\omega) + d_{i,t}(\omega),
\end{aligned}$$

where $\beta \in (0,1)$ is the parameter of time preference. The first constraint is the demand schedule for each variety of labour services, which households take as given. The second constraint summarises an indefinite sequence of budget restrictions facing the households, where $b_{i,t}(\omega)$ is the portfolio of bonds, and $d_{i,t}(\omega) = \sum_\phi \mathit{\Pi}_{i,t}(\omega, \phi) + \sum_\phi M_{i,t}(\omega, \phi)$ is the exogenously given stock of wealth acquired from the ownership of the firms, such that $\mathit{\Pi}_{i,t}(\omega, \phi) = \sum_{n=1}^{N} \mathit{\Pi}_{in,t}(\omega, \phi)$ are the profit dividends, while the term $M_{i,t}(\omega, \phi) = \sum_{n=1}^{N} q_{in,t} m_{ni,t}(\omega, \phi)$ measures the total sales of commodities to foreign producers.

There exists a contingent claim for any state of nature among households in any given economy, allowing them to diversify the risk arising from idiosyncratic income shocks. But the risk arising from country-specific shocks is offset only partially by trading commodities and bonds with households and firms abroad in incomplete financial markets. In this highly stylised asset pricing framework, the price of the one-period bond in real terms is equivalent to the stochastic discount factor $\lambda_{i,t,t+1}$, which is inversely related to the risk-free rate of return, while the relative price of commodities is simply equal to the real exchange rate. The portfolio choice is not modelled explicitly, since there are no constraints imposed on either liquidity or the collateral, such that supply of equity is perfectly price-elastic.

The wage bargaining outcome is state-dependent, since the negotiated change in the real wage prompts wage bargaining costs incurred by the households. They appear as $\mathit{\Delta}_{i,w,t}(\omega)$ in the budget constraint and they are deducted from the total labour income, since collective bargaining requires effort and resources. This includes hiring lobbyists and trade unions that negotiate the terms of the labour contracts with the employers on behalf of the workers, yet the process itself does not produce any value-added.

The first-order conditions in a symmetric equilibrium are given by:

$$w_{i,t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{mrs_{i,t}}{\Theta_{i,t}}, \tag{2.26}$$

$$\Theta_{i,t} = 1 - \Delta_{i,w,t} + \frac{\Delta'_{i,w,t}\dot{w}_{i,t}}{\varepsilon-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{h_{i,t+1}}{h_{i,t}}\frac{\Delta'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\varepsilon-1}\right], \tag{2.27}$$

$$\dot{w}_{i,t} = \frac{w_{i,t}}{w_{i,t-1}}, \tag{2.28}$$

$$mrs_{i,t} = -\frac{u_{i,h,t}}{u_{i,c,t}}, \tag{2.29}$$

$$u_{i,h,t} = -\psi_i h_{i,t}^{\varphi_i}, \tag{2.30}$$

$$u_{i,c,t} = \beta\mathbb{E}_t\left[\frac{u_{i,c,t+1}}{\lambda_{i,t,t+1}}\right], \tag{2.31}$$

$$u_{i,c,t} = \Psi_{i,t} - \vartheta_i\beta\,\mathbb{E}_t\left[\Psi_{i,t+1}\right], \tag{2.32}$$

$$\Psi_{i,t} = \frac{1}{c_{i,t} - \vartheta_i c_{i,t-1}}. \tag{2.33}$$

As long as $1 < \varepsilon < \infty$, the aggregate real hourly wage rate is set above the marginal rate of substitution between consumption and labour denoted by $mrs_{i,t}$, where the term $u_{i,h,t} = \partial u_{i,t}/\partial h_{i,t}$ is the marginal disutility of labour and $u_{i,c,t} = \partial u_{i,t}/\partial c_{i,t}$ is the marginal utility of consumption. However, in the short-run, $mrs_{i,t}$ and $\Theta_{i,t}$ are positively correlated, such that an increase in the former leads to a less than one-to-one increase in the later, causing a lagged response of real wages to innovations. Moreover, if wages are downwardly rigid, such that $\zeta_{i,w} < 0$, then $\Theta_{i,t}$ tends to rise disproportionally less than it falls.

2.3.5 General Equilibrium

The presence of a unit root in the stochastic labour productivity process constrains the rational expectations solution of the model to the balanced growth path narrative, which involves de-trending all of the stock variables by labour productivity (e.g. $\tilde{v}_{i,t} = v_{i,t}/a_{i,t}$ for an arbitrary variable $v_{i,t}$). The above transformation induces stationarity in the market clearing condition for goods and services, where the total output $y_{i,t}$ equals the sum of total consumption expenditure, the local trade balance, and short-run externalities:

$$\tilde{y}_{i,t} = \tilde{c}_{i,t} + \tilde{nx}_{i,t} + \Delta_{i,w,t}\tilde{w}_{i,t}h_{i,t} + \sum_{\phi}\sum_{n=1}^{N}\Delta_{in,t}(\phi)p_{in,t}(\phi)\tilde{y}_{in,t}(\phi), \tag{2.34}$$

$$\tilde{nx}_{i,t} = \sum_{n=1}^{N-i} p_{in,t}q_{ni,t}\tilde{y}_{in,t} - p_{ni,t}\tilde{y}_{ni,t}. \tag{2.35}$$

Price adjustment and wage bargaining costs impose additional restrictions on the total income in the short-run, since a small proportion of resources is used Pareto-inefficiently in order to implement the adjustment of macroeconomic fundamentals that are consistent with the rational expectations equilibrium. Furthermore, the persistence of output and inflation induced by real and nominal rigidity justifies local central banks to intervene in the bonds markets in order to stabilise economic activity using a Taylor rule:

$$r_{i,t} = (r_{i,t-1})^{\rho_{i,r}} (r^*_{i,t})^{1-\rho_{i,r}}, \tag{2.36}$$

$$r^*_{i,t} = \frac{\gamma_i \dot{p}_i}{\beta} \left(\frac{\dot{p}_{i,t}}{\dot{p}_i} \right)^{\nu_{i,p}} \left(\frac{\tilde{y}_{i,t}}{\tilde{y}_i} \right)^{\nu_{i,y}} \exp(\sigma_{i,r} \epsilon_{i,r,t}), \tag{2.37}$$

where $\sigma_{i,r} > 0$, $\rho_{i,r} \in (0,1)$, $\epsilon_{i,r,t} \sim iid(0,1)$, $\nu_{i,p} > 0$, $\nu_{i,y} > 0$, and $r_{i,t} = \mathbb{E}_t [\dot{p}_{i,t+1}/\lambda_{i,t,t+1}]$. Conventional monetary policy conduct combined with liberalised flows of financial capital across borders implies that the bilateral exchange rate is freely floating and determined by the perfect consumption risk sharing relationship:

$$q_{ni,t} = \mu_{ni} \left(\frac{\tilde{u}_{n,c,t}}{\tilde{u}_{i,c,t}} \right) e_{ni,t}, \tag{2.38}$$

such that $\mu_{ni} = 1/\mu_{in} > 0$, $e_{ni,t} = e^{\rho_e}_{ni,t-1} \exp(\sigma_e \epsilon_{e,t})$, $\rho_e \in (0,1)$, $\sigma_e > 0$ and $\epsilon_{e,t} \sim iid(0,1)$. Central banks commit to a monetary policy rule that is implicitly based on symmetrical preferences, therefore any skewness of the policy rate is directly attributable to the non-linearities in the labour and product markets. Moreover, the term $e_{ni,t}$ is a stationary and stochastic process of exchange rate noise, capturing all of the exchange rate dynamics that are not associated with innovations in the macroeconomic fundamentals. Somewhat in the fashion of Bacchetta & van Wincoop (2017), the relative price of currency in real terms is determined by both macroeconomic fundamentals as well as stochastic noise, such that exchange rates are disconnected from the fundamentals and mostly driven by forces determined outside of the model. Incorporating exchange rate noise explicitly is motivated by the lack of persistence and volatility of exchange rates typically encountered in dynamic stochastic general equilibrium models for advanced economies even if financial markets are complete. This result has been originally pointed out by Backus & Smith (1993). It should be emphasised that the novelty of the framework lies in the way it analyses the consequences, rather than the determinants, of exchange rate volatility. A more rigorous treatment of the exchange rate dynamics goes beyond the scope of this chapter.

2.4 Non-Linear Model Summary

In the end, there are three factors contributing to the incompleteness of exchange rate pass-through into consumer prices. First, the presence of non-traded distribution services (i.e. $\alpha_i \in (0,1)$), the prices of which are disconnected from exchange rates, thereby insulating consumer prices from innovations in the currency markets. Second, the presence of home-bias (i.e. $\alpha_{ii} \in (0,1)$), which reflects the fact that only a fraction of final goods $\sum_{n=1}^{N-i} \alpha_{ni} \in (0,1)$ are imported from abroad, while the majority are produced locally and priced in producer currency units. Third, a fraction $s_{ni}(-\pi)$ of the import prices are set in local currency units, which further stabilises the prices of tradable goods at the border.

The setup of the model closely follows the approach of Corsetti et al. (2008), except the supply-side in this framework incorporates intermediate imports, somewhat in the fashion of

Monacelli (2013) and Amiti et al. (2014), and the optimal price setting condition is characterised by non-linear price adjustment costs. However, unlike Corsetti et al. (2008), the magnitude of exchange rate pass-through in this multi-country model need not be determined through means of indirect inference. Instead, the theoretical measurement of exchange rate pass-through sketched out above can be simulated along with the system of difference equations characterising the dynamics of the macroeconomic fundamentals. Because of its top-down structure, the dynamic measure of exchange rate pass-through itself does not influence the evolution of the economy around the balanced growth path, but random shifts in the state variables facing the exporters can influence the optimal exchange rate pass-through endogenously. In order to infer meaningful policy implications from the simulations of the model that follow, the structural parameters of the model ought to be disciplined by the data. This allows the model-implied quantitative predictions to be compared to those reported in the empirical literature. The following chapter is dedicated to mapping the standard and non-standard parameters of the business cycle model to the key moments characterising the persistence, volatility, and higher-order moments of inflation and exchange rates observed in the data of United States and United Kingdom.

Chapter 3

Estimating Non-Linear US-UK Exchange Rate Pass-Through

3.1 Motivation

The continuous nature of LINEX price adjustment costs is appealing for its compatibility with the higher-order perturbation methods used to solve rational expectations models. Indeed, the numerical solution to the non-linear model presented in the previous chapter is obtained using a second order perturbation of the policy function as in Schmitt-Grohé & Uribe (2004), but the state-space is pruned using the Andreasen et al. (2018) method. Even in highly non-linear settings, perturbation and pruning ensures locally-stable dynamics of all the control variables around their accurately specified balanced growth paths when drawing transitory innovations from Gaussian distributions. However, due to their large and restrictive dimensions, multi-country models are generally more challenging to estimate than models in the closed economy setting. There are only a handful of studies, most notably Lubik & Schorfheide (2005), Adolfson et al. (2007), and García-Cicco et al. (2010), using full information methods that are based on the Bayesian evaluation of the likelihood function. In particular, they are applied to medium-large scale two-country models that are linearised around the steady state, which enhances the speed of the numerical computations, since it renders the policy function compatible with the Kalman Filter. On the other hand, estimation of non-linear multi-country models is even more challenging, since the evaluation of the pruned state-space commands the use of the Particle Filter due to Fernández-Villaverde & Rubio-Ramírez (2007) or alternative techniques, which are generally highly computationally-demanding and mostly applied to small scale models in the closed economy setting.

This chapter implements a less computationally-demanding alternative – Simulated Method of Moments (SMM) due to Duffie & Singleton (1993) and Ruge-Murcia (2012). SMM is a partial information methodology, which is not based on the principle of maximising the likelihood function. Instead, it makes an educated guess as to what the magnitude of the model parameters may be by targeting a finite number of selected moments associated with the control variables that have an observable real-world counterpart. As a result, it circumvents the issue of stochastic singularity and abstracts from any ad hoc shocks that would otherwise need to be added into a likelihood-based model in order to establish effective parameter identification. It also does not impose non-standard prior distributions on the initialised values of the parameters, while the 'posterior' densities are characterised by the conventional properties of a normal distribution from which standard inference can be drawn. The latter point can be viewed as both an advantage and a disadvantage. On the one hand, the estimation algorithm is more flexible and immune to any preconceptions. On the other hand, the results are more susceptible to model misspecification. In order to preserve the accuracy of the solution and to ensure saddle-path stability, the model parameters are thus disciplined by imposing lower- and upper-bounds on the estimates that discard theoretically incredible values from the objective function, which ultimately amounts to 'bounded SMM'.

3.2 Methodology

Suppose that $\boldsymbol{\theta}$ is a $g_1 \times 1$ vector of unknown parameters. Suppose further that a sample of $T > 0$ observations of economic data, $\{\varpi_t\}$, is available to estimate the model, where ϖ_t is stationary and ergodic. Similarly, there exists a synthetic counterpart of the observed data, namely $\{\varpi_\iota(\boldsymbol{\theta})\}$, obtained by simulating the model for a given draw of random shocks. Assuming that the length of simulated series is ςT, where $\varsigma > 0$ is an integer, the distance between the observed and simulated moments computed based on time averages can be denoted by a $g_2 \times 1$ vector:

$$\mathbf{M}(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{m}(\varpi_t) - \frac{1}{\varsigma T}\sum_{\iota=1}^{\varsigma T}\mathbf{m}(\varpi_\iota(\boldsymbol{\theta})). \tag{3.1}$$

The SMM estimator can then be defined as:

$$\hat{\boldsymbol{\theta}} = \underset{\underline{\boldsymbol{\theta}}<\boldsymbol{\theta}<\bar{\boldsymbol{\theta}}}{\operatorname{argmin}}\ \mathbf{M}(\boldsymbol{\theta})'\mathbf{S}^{-1}\mathbf{M}(\boldsymbol{\theta}), \tag{3.2}$$

where

$$\mathbf{S} = \lim_{T\to\infty} \operatorname{Var}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\mathbf{m}(\varpi_t)\right] \tag{3.3}$$

is a $g_1 \times g_2$ positive-definite optimal weighting matrix obtained using the Newey-West estimator with a Bartlett kernel, while $\underline{\boldsymbol{\theta}}$ and $\bar{\boldsymbol{\theta}}$ are the lower- and upper-bounds that discard the economically implausible outcomes. The weighting matrix $\mathbf{S}$ ensures the efficiency of $\hat{\boldsymbol{\theta}}$ by putting the most weight on the most accurately measured moments and simultaneously discounting the least accurately measured moments. Moreover, if $\mathbf{J} = \mathbb{E}[\partial\mathbf{m}(\varpi_\iota(\boldsymbol{\theta}))/\partial\boldsymbol{\theta}]$ is a $g_1 \times g_2$ Jacobian matrix of full column rank, then the SMM estimator is generally identified, so long as the necessary condition for the degrees of freedom $g_2 \geq g_1$ holds, and it asymptotically follows a truncated-normal distribution:

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \to N(0, (1 + 1/\varsigma)(\mathbf{J}'\mathbf{S}^{-1}\mathbf{J})^{-1}), \tag{3.4}$$

In theory, when $\varsigma \to \infty$, the variance-covariance matrix of SMM becomes fully efficient and converges to the one obtained using Generalised Method of Moments (GMM) as in Aguiar & Gopinath (2007). However, unlike GMM, SMM does not require closed form solutions to the theoretical moments associated with each control variable (i.e. $\mathbb{E}[\mathbf{m}(\omega_\iota(\boldsymbol{\theta}))]$) when minimising the distance between the observed and simulated moments. The law of large numbers ensures that SMM is able to approximate the theoretical moments accurately at a fraction of the duration it takes to implement the alternative. Furthermore, both methods deliver unbiased estimators of $\boldsymbol{\theta}$, but GMM tends to converge more rapidly and deliver more precise estimates in the context of small models or linear endowment economies, but in larger models solved using a higher-order perturbation, the computation of theoretical moments can be particularly arduous. Ruge-Murcia (2012) originally proposed using the values for ς in the range from 5 to 20 due to the associated increase in computational burden. In practice, the choice of ς depends on the frequency and the length of the time series at hand, but for quarterly data and $T > 100$, I find that the marginal gains in terms of precision beyond $\varsigma = 20$ are generally negligible compared to the marginal cost in terms of the computational speed, especially when the chosen weighting matrix $\mathbf{S}$ is optimal.

3.2.1 Data

The model is estimated using quarterly data for United Kingdom (UK) and United States (US). The UK is chosen as a robust representation of the higher-order stylised facts in OECD economies displayed in table 2.1 during 1982:Q1 and ends in 2008:Q1. The Great Moderation period is used by the most relevant literature on exchange rate pass-through asymmetry. Moreover, Great Moderation period is known to be characterised by a unique environment of low aggregate uncertainty, which helps distinguishing structural non-linearities from the artefacts of potentially time-varying distributions of exogenous innovations.[1] The time series used for both countries include the level of real exchange rate, the quarterly nominal interest rate, as well as the following list of first-differenced series: per capita consumption, unit labour costs, average hours of labour, and the consumer price index.[2] The vector of moments around which parameters are estimated includes the variance-covariance matrix and the first-order autocorrelation for all eleven time series. It also includes the skewness of the first-differenced unit labour costs and the consumer price index, such that the total number of moments used to evaluate the SMM objective function (i.e. g_2) is equal to 80.

3.2.2 Identification

Similar to GMM or a Maximum-Likelihood technique, SMM is sensitive to model misspecification, which is why it is useful to impose some natural limits on the individual elements of

[1] The UK officially operated under the Exchange Rate Mechanism (ERM) from 1990:Q3 to 1992:Q3, which allowed the Pound-Sterling to fluctuate within ±6% bands against other ERM currencies. However, unofficially, the UK shadowed the Deutsch Mark from as early as 1987. While it contradicts the flexible exchange rate regime explicitly imposed by the model, the estimations were also carried out using a shorter sample size from 1992 onwards. Although parameter estimates do in fact differ, the key qualitative predictions of exchange rate pass-through remain unchanged, therefore the results discussed below will ignore this particular caveat.

[2] From the perspective of the model, the level of real exchange rate is stationary, while it is difficult to reject the null hypothesis of a unit root in the actual time series of the GBP/USD exchange rate. As a result, the deterministic and stochastic trends of the real exchange rate in the actual data are removed using a one-sided Hodrick-Prescott filter with a standard factor of 1600. In the empirical applications, consumption and wages grow at a rate γ_i in the long-run, price inflation rises along with the target $\dot{p}_i$, whereas hours of labour, nominal interest rates and the real exchange rate remain constant.

$\boldsymbol{\theta}$ that then follow truncated-normal distributions. The rule of thumb for statistical significance is henceforth focused on the one-sided p-values relative to the bounds. Furthermore, a fraction of the model parameters are pre-calibrated, because some of the model parameters are only weakly identified and data outside of the model can provide more information about their magnitudes (see table 3.1, panel (i)). This reduces the length of the vector $\boldsymbol{\theta}$ to $g_1 = 30$.

All of the varieties of goods in this model are imperfectly substitutable, but those traded within borders are more substitutable than those traded across borders: $1 < \eta < \varepsilon < \infty$. Based on UNCTAD (2015) estimates of the average global CIF (cost, insurance and freight) relative to FOB (free on board) price levels, the parameter driving the magnitude of iceberg costs is chosen to be 10% of the total revenue (i.e. $\tau_{ij} = \tau_{ji} = \tau = 1.1$). The calibration of the remaining business cycle parameters in this framework is mainly based on the previous literature such as Smets & Wouters (2007) and Schmitt-Grohé & Uribe (2012). In this model, the share of distribution services $\alpha_i = \alpha_j = \alpha$ corresponds to the labour share of income, which is roughly 2/3 of aggregate consumption bundle, but it is close to the estimates provided in Burstein et al. (2003). The values of import penetration ratios $\alpha_{ji}, \alpha_{ij}, \xi_i, \xi_j$ are taken from Campa & Goldberg (2010). Most notably, the US is a relatively larger, yet less open economy compared to the UK, which is reflected in a lower import intensity of final goods in the US (25%) than in the UK (34%). Similarly, the share of imported intermediate goods in the UK is equal to $\alpha_{ii}(1-\alpha)\xi_i = 0.2$, which is greater than in the US (i.e. $\alpha_{jj}(1-\alpha)\xi_j = 0.08$). Due to the homogeneity of unit costs of production and the convexity of the price adjustment costs, the PCP and LCP price setting in the source country is *ex post* identical, which is why $\chi_{ii}(\pi) = \chi_{jj}(\pi) = 1$. Moreover, in order to save space, the notation henceforth expends π, such that $\chi_{ij}(\pi) = \chi_{ij}$ and $\chi_{ij}(-\pi) = 1 - \chi_{ij}$.

3.2.3 Estimation Results

Panel (ii) in table 3.1 summarises the initialised parameter values, the lower- and upper-bounds as well as the point estimates. First, the parameters controlling the superficial consumption habits, the density of PCP exporters, and the persistence of the innovations are all bounded between zero and unity. They are initialised to the magnitude of one half, which is analogous to imposing a 'beta prior' in the Bayesian maximum-likelihood methods. Second, analogous to the 'inverse gamma prior', the magnitudes of shocks other than the exchange rate noise are initialised to relatively low values, but they are capped at relatively large values. Third, in order to ensure that the Blanchard & Kahn (1980) conditions hold, $\nu_{i,y}$ is bounded between zero and unity, while the Taylor principle requires $\nu_{i,p}$ to be larger than unity. Fourth, the average convexities of the adjustment costs are bounded between 0 and 100, which allows the slope of the Phillips curve to fluctuate anywhere from 0.05 to infinity. Finally, the values of $\zeta_{i,p}$ and $\zeta_{i,w}$ are initialised at zero and they are bounded up to ± 1000, which closely resembles a flat prior. Following the reasoning of Aruoba et al. (2017), imposing the bounds on $\zeta_{i,p}$ and $\zeta_{i,w}$ are important in terms of preserving the accuracy of the approximation associated with the second-order perturbation of the policy function. Mostly because unusually large values of these parameters are not necessary in order to generate sufficient skewness of inflation, while at the same time the approximation errors are generally proportional to their magnitudes.

Most of the parameter estimates turn out to be identified in a sense that they are asymmetrical across countries, as one would reasonably expect, and they diverge from their initialised values considerably. The average convexity associated with the price adjustment costs $\kappa_{i,p}$ (wage adjustment costs $\kappa_{i,w}$) are low (high) in the UK, while the opposite is true for the the US. In spite of the differences in the slope of the Phillips curve (i.e. 0.251 in the UK and 0.065 in the US), the degree of price stickiness remains quantitatively comparable with the time-dependent nominal rigidity framework such as Lubik & Schorfheide (2005). The Phillips curve is significantly steeper in the UK compared to the US, partly due the implicit trade-off in the SMM objective function, which requires sufficient volatility of inflation, synonymous with a low magnitude of $\kappa_{i,p}$, in order to generate sufficiently large skewness. Contrary to previous studies, the degree of price adjustment cost asymmetry $\zeta_{i,p}$ is found to be highly pervasive in the UK, indicating that higher-order moments of inflation are non-trivial, especially the downward rigidity of price inflation. Despite a negative and insignificant estimated value of $\zeta_{i,w}$ in the UK, wage inflation turns out to be hardly skewed at all, which provides concrete evidence that the skewness of British price inflation is not an artefact of the labour market imperfections, but rather attributable to the asymmetric nominal rigidity.

The exact opposite turns out to be the case in the US, where the real wage is downwardly rigid, but prices adjust more symmetrically to either directions. This finding is in-line with the results in the related literature associated with LINEX adjustment costs, such as Kim & Ruge-Murcia (2009, 2011), Abbritti & Fahr (2013) and Aruoba et al. (2017). Hence $\zeta_{i,p} > 0$ in the US, since high positive skewness of wage inflation in the US due to $\zeta_{i,w} < 0$ would otherwise induce counterfactually high positive skewness of price inflation. In other words, consumer prices are a function of wages through the distribution sector, while wages are related to prices only indirectly through output and consumption. The spillover of wage skewness into prices is thus much more pronounced than the other way around, which is why the estimate of $\zeta_{i,w} < 0$ in the is UK insignificant, albeit sizeable, and overshadowed by the high wage stickiness. The results are not displayed for the special case when wages are perfectly flexible (i.e. $\kappa_{i,w} = 0$), but $\zeta_{i,p}$ in that case turn out to be negative and non-trivial for both UK and US. It is therefore important to control for asymmetric wage rigidity, otherwise the model may over-predict the extent of downward price rigidity, thus exaggerating the incompleteness and asymmetry of exchange rate pass-through into import prices.

The crucial parameter in terms of capturing the average magnitude of exchange rate pass-through into import prices is the density of PCP exporters χ_{ji}. Only 2% of all US imports from the UK turn out to be invoiced in GBP, compared to 37% in the UK from the US. This finding is generally in-line with the stylised fact that most US imports are priced in USD as advocated by Gopinath & Rigobon (2008) and Gopinath (2015) among others. That said, unlike in the linear model of Choudhri & Hakura (2015), the density of PCP exporters and their market share is not one and the same in this non-linear model due to the influence of relative prices (see equation (2.13)). Yet the market share of PCP imports in the US (UK) turns out to be only 18% (5%) with a standard deviation of 1.6% (0.6%), therefore exchange rate pass-through remains relatively low in the long-run.

Table 3.1: Calibrated & Estimated Parameters

(i) Calibrated Parameters

Structural		Country-Specific	UK	US
β	0.995	γ_i	1.005	1.00524
η	6	$\dot{p}_i$	1.00625	1.005
ε	6	q_{ji}	0.61	1/0.61
h	0.3	α_{ji}	0.34	0.25
τ	1.1	ξ_i	0.88	0.48
α	2/3			

(ii) Estimated Parameters

Parameter $\boldsymbol{\theta}$	Initialised Value $\boldsymbol{\theta}_0$		Lower Bound $\underline{\boldsymbol{\theta}}$		Upper Bound $\bar{\boldsymbol{\theta}}$		Point Estimate $\hat{\boldsymbol{\theta}}$	
	UK	US	UK	US	UK	US	UK	US
ϑ_i	0.5	0.5	0	0	1	1	0.44	0.52
φ_i	0.1	0.1	1	1	10	10	0.64	2.02
χ_{ji}	0.5	0.5	0	0	1	1	0.37	0.02
$\kappa_{i,p}$	40	40	0	0	100	100	19.90	76.80
$\zeta_{i,p}$	0	0	-1000	-300	1000	300	-172.00	105.00
$\kappa_{i,w}$	10	10	0	0	100	100	58.80	13.20
$\zeta_{i,w}$	0	0	-300	-1000	300	1000	-200.00_*	-387.00
$\nu_{i,p}$	1.5	1.5	1	1	5	5	1.34_*	1.32
$\nu_{i,y}$	0.5	0.5	0	0	1	1	0.75^*_*	0.39
$\rho_{i,r}$	0.5	0.5	0	0	1	1	0.77	0.85
$\rho_{i,z}$	0.5	0.5	0	0	1	1	0.40^*_*	0.49^*_*
ρ_e	0.5	-	0	-	1	-	0.60	-
$\sigma_{i,a}$	0.005	0.005	0	0	0.0125	0.0125	0.00203	0.00496
$\sigma_{i,r}$	0.005	0.005	0	0	0.0125	0.0125	0.00167	0.00282
$\sigma_{i,z}$	0.005	0.005	0	0	0.0125	0.0125	0.00556^*_*	0.00672^*_*
σ_e	0.050	-	0	-	0.0750	-	0.0130	-

The asterisk in the subscript $_*$ (superscript *) next to the point estimate indicates that parameter estimate is not different from the lower (upper) bound with a 95% level of confidence. All other parameters are significantly different from zero as well as the bounds at > 1% level. The SMM algorithm is implemented using Dynare 4.5.4 and Matlab 2017a. The objective function is minimised conditional on the bounds imposed on the vector of parameters using the Sequential Quadratic Programming algorithm.

The Frisch elasticity of labour supply $1/\varphi_i$ is relatively high in the UK, but not the US, which implies that the hours of labour in the UK are more responsive to changes in the real wage, yet wages are on average stickier in the UK (i.e. higher $\kappa_{i,w}$) than the US. Furthermore, significant values of parameter ϑ_i reflect the importance of superficial consumption habits in terms of generating sufficiently persistent dynamics of consumption and inflation. Interestingly, the estimates of $\rho_e = 0.6$ and $\sigma_e = 0.013$ are relatively modest compared to the persistence and volatility of residuals of the first-order autoregressive process for the one-sided Hodrick-Prescott filtered GBP/USD real exchange rate (i.e. ACF(1)=0.87 and

RMSE=0.0376). This implies that the exogenous exchange rate shocks are only partly attributable to the overall volatility and persistence of the real exchange rate in the model, while the choice of invoicing currency, imported intermediate imports, and non-linear price adjustment costs drive the remaining part of exchange rate dynamics. This is the main reason why exchange rate pass-through based on aggregate co-movement of prices and exchange rates is astoundingly biased compared to the aggregated firm-level transmission of exchange rate innovations. Analogous to the GMM estimates of Aguiar & Gopinath (2007) and Bayesian estimates of García-Cicco et al. (2010) in emerging markets, the SMM results present evidence that permanent labour productivity shocks are more sizeable than the transitory shocks during the Great Moderation period (i.e. unlike $\sigma_{i,a}$, $\sigma_{i,z}$ is not statistically significant in both the UK and US). They also contribute a greater proportion to the volatility of macroeconomic fundamentals as a whole compared to the monetary policy shocks.

TABLE 3.2: Observed and Simulated Moments

	Std. Dev.		Skewness		Kurtosis		ACF(1)	
(i) United States								
	`data`	`model`	`data`	`model`	`data`	`model`	`data`	`model`
$\dot{c}_{j,t}$	0.51%	0.52%	0.03	-0.74	3.21	3.59	0.19	0.25
$\dot{w}_{j,t}$	0.65%	0.52%	**0.78**	**0.93**	6.17	4.35	-0.13	0.22
$\dot{h}_{j,t}$	0.26%	0.27%	0.44	-0.99	2.76	8.12	-0.10	-0.24
$\dot{p}_{j,t}$	**0.45%**	**0.88%**	**-0.11**	**0.03**	4.24	3.72	0.22	-0.04
$r_{j,t}$	0.59%	0.29%	0.15	0.16	3.01	3.52	0.98	0.65
(ii) United Kingdom								
	Std. Dev.		Skewness		Kurtosis		ACF(1)	
	`data`	`model`	`data`	`model`	`data`	`model`	`data`	`model`
$\dot{c}_{i,t}$	0.77%	0.43%	0.35	-0.78	2.73	5.91	0.43	0.28
$\dot{w}_{i,t}$	0.92%	0.17%	**0.09**	**-0.33**	2.41	3.91	0.32	0.54
$\dot{h}_{i,t}$	0.41%	0.30%	-0.27	-0.65	4.11	6.47	0.12	0.20
$\dot{p}_{i,t}$	**0.82%**	**1.23%**	**1.63**	**1.88**	7.29	9.23	-0.01	0.30
$r_{i,t}$	0.78%	0.56%	0.65	0.85	2.32	5.43	0.98	0.76
(iii) GBP/USD Real Exchange Rate								
	Std. Dev.		Skewness		Kurtosis		ACF(1)	
	`data`	`model`	`data`	`model`	`data`	`model`	`data`	`model`
$\dot{q}_{ji,t}$	**3.87%**	**2.47%**	-0.103	0.00	8.01	3.34	**0.86**	**0.71**

The observed moments are based on quarterly time series during 1982:Q1-2008:Q1. Alongside are the model-implied counterparts generated by the Monte Carlo simulations of the using 4000 observations. The exchange rate persistence is measured for the level of the series, not the first-difference.

The fit of the second- and higher-order moments associated with consumer price inflation, wage inflation, and exchange rates in the UK and the US is quite remarkable (see table 3.2). The close resemblance of the model to the data established by SMM mirrors the stylised facts established in table 2.1. The extent of exchange rate pass-through asymmetry can therefore be deduced from the model without the caution about how the model is closed. And yet the moments of other variables do not fit so well, owing to the fact that the model does not incorporate public expenditure or financial frictions, which have the tendency to amplify productivity shocks and enhance the volatility of consumption and real wages. One cause for concern from the non-linear general equilibrium perspective is the fat tails of the labour hours. They are less concerning in Abbritti & Fahr (2013) due to their application to the US data including the 1970s. By contrast, in the Great Moderation period explored in this chapter, they seem to evolve in a relatively stable and normally distributed fashion. The mismatch of hours suggests that the stylised model presented above could benefit from the introduction of search and matching frictions or Nash bargaining as in Abbritti & Fahr (2013), since they could enrich the fit of the model with the positive skewness of unemployment at the extensive margin, positive skewness of wages and less skewed level of employment at the intensive margin.[3] However, for the sake of clarity and parsimony, the model is kept as simple as possible, which inevitably leaves some room for generalisations and extensions in the future applications. In summary, the main purpose of estimating the model is to fit the moments of wage and price inflation in the UK and the US. Having established a reasonably close fit of these moments, the next section uses the values of estimated parameters in order to assess how they influence the incompleteness and asymmetry of exchange rate pass-through.

3.3 Exchange Rate Pass-Through

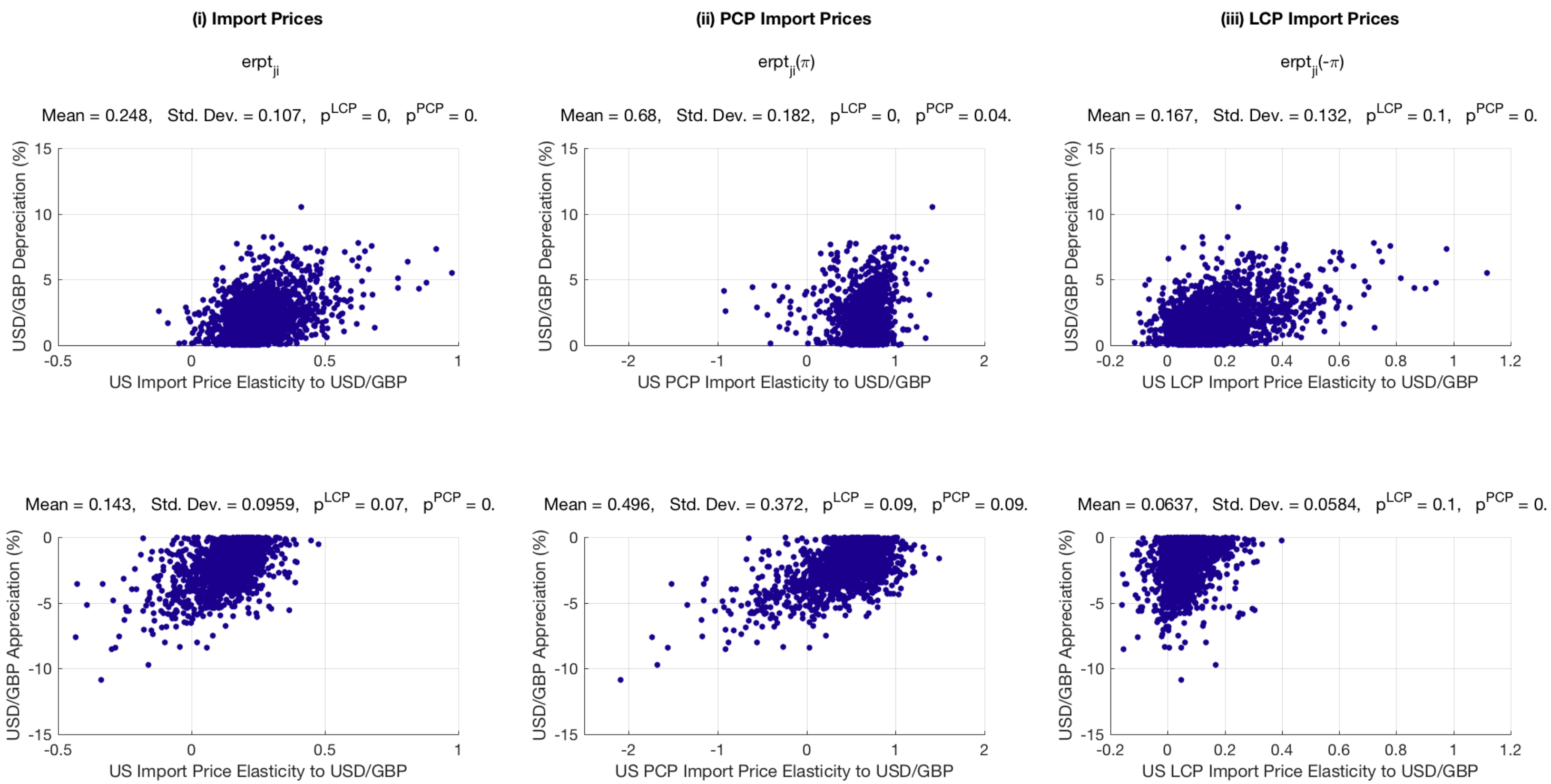
3.3.1 Average Causal Effect

What is the quantitative significance of the downward price rigidities on exchange rate pass-through into import prices? Figures 3.1 and 3.2 plot the period-by-period realisations of pass-through into US and UK import prices against the depreciations and the appreciations of the USD/GBP and GBP/USD exchange rate respectively. Panel (i) of figure 3.1 demonstrates that, on average, USD depreciations against the GBP lead to greater exchange rate pass-through (24.8%) than appreciations (14.3%). Moreover, the null hypotheses of zero and full pass-through are strongly rejected for USD depreciations (see the p-values denoted by $\mathrm{p}^{\mathrm{LCP}}$ and $\mathrm{p}^{\mathrm{PCP}}$ above each subplot in panel (i)), while zero pass-through hypothesis is more difficult to reject for appreciations - broadly consistent with the empirical findings in Brun-Aguerre et al. (2017). As expected, the average exchange rate pass-through is higher for PCP imports (58.9%) than LCP imports (11.6%), therefore the low pass-through into aggregate import prices in this model stems from a low market share of PCP imports in the US (18%). However, due to the downward price rigidities, the PCP (LCP) import prices in the US are around 18% (10%) more responsive to USD depreciations compared to appreciations.

There are two opposing forces at play in the transition mechanism: a direct currency conversion effect and an indirect cost-push effect. The indirect effect applies to both PCP and LCP firms, while the direct effect applies only to the PCP firms. When the USD depreciates, the direct effect is the upward pressure on the PCP import prices in the US due to a change in the relative price of the currencies. The indirect effect is the decrease (increase) in the costs of intermediate imports in the UK, which leads to a modest decrease (sharp increase) in the PCP (LCP) import prices in the US. The inflationary pressure of the USD depreciation are thus

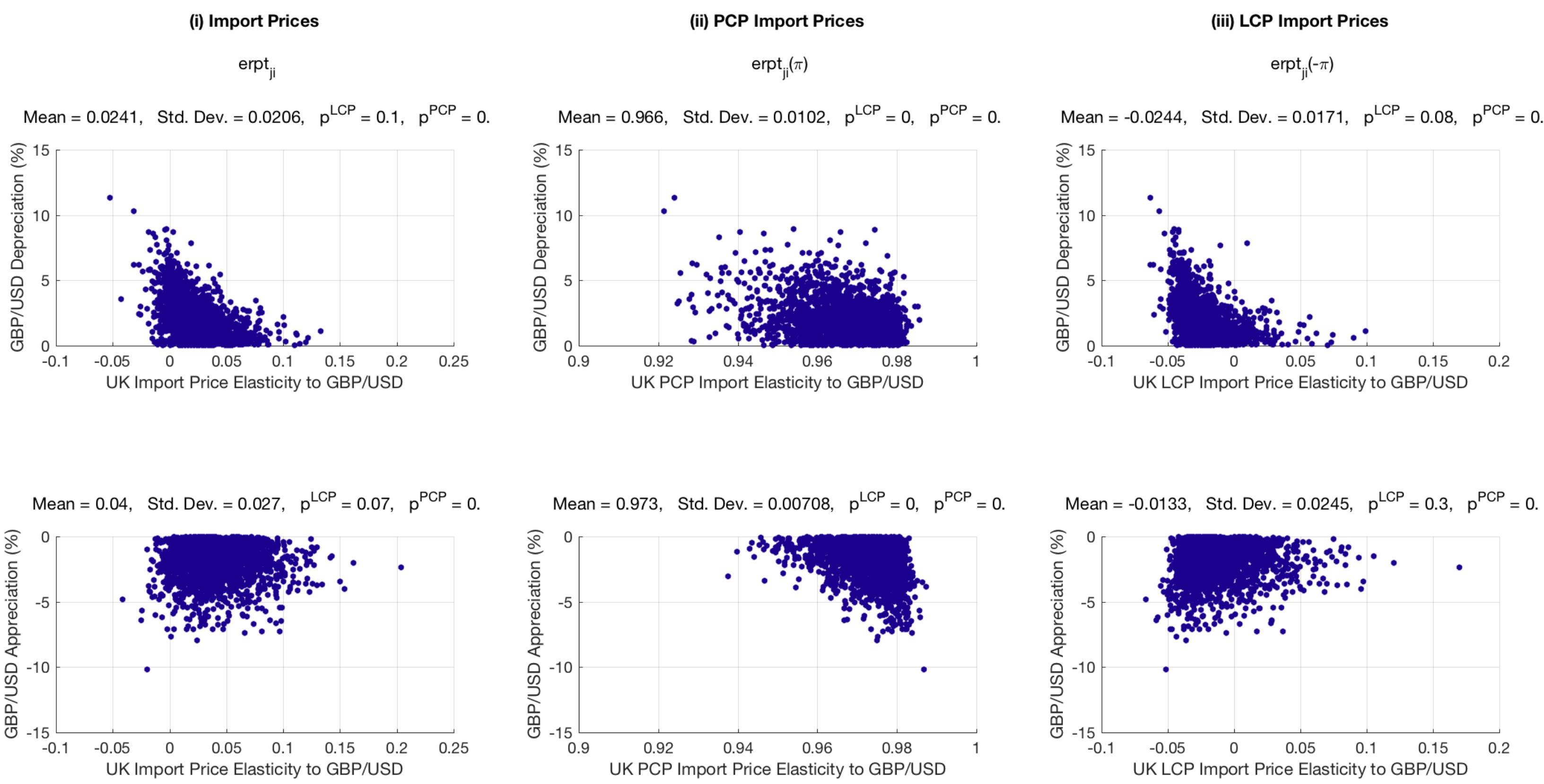
[3] See Olmedo (2014) for more stylised facts about non-linearities in the labour market – it contains a thorough literature survey on the skewness of unemployment rate in the advanced economies.

FIGURE 3.1: Simulated Exchange Rate Pass-Through into US Import Prices



The scatter plots are produced using 4000 observations of simulated data for the bilateral USD/GBP exchange rate and the associated exchange rate pass-through into US import and consumer prices. Each subplot segments the exchange rate pass-through series into depreciations and appreciations of the exchange rate. Above each subplot are the estimated mean and standard deviation of exchange rate pass-through as well as one-sided p-values associated with a t-test for the zero pass-through (LCP) and full pass-through (PCP) hypotheses respectively.

FIGURE 3.2: Simulated Exchange Rate Pass-Through into UK Import Prices



The scatter plots are produced using 4000 observations of simulated data for the bilateral GBP/USD exchange rate and the associated exchange rate pass-through into UK import prices. Each subplot segments the exchange rate pass-through series into depreciations and appreciations of the exchange rate. Above each subplot are the estimated mean and standard deviation of exchange rate pass-through as well as one-sided p-values associated with a t-test for the zero pass-through (LCP) and full pass-through (PCP) hypotheses respectively.

exacerbated by the downward rigidity of UK export prices, since the direct effect generally dominates the indirect effect. Conversely, when the USD appreciates, the PCP import prices in the US decrease due to a change in the relative price of the currencies. However, the costs of intermediate imports in the UK increase (decrease). Because the UK export prices are more flexible upwards than downwards, the fall in the US import prices will be smaller compared to the magnitude by which they tend to rise.

Contrary to Amiti et al. (2014) or Burstein & Gopinath (2014), where intermediate imports lead to an incomplete and symmetric exchange rate pass-through, the model with LINEX price adjustment costs can account for over 10% of asymmetry and around 80% of exchange rate pass-through incompleteness. Most interestingly, the partial responsiveness of import prices prevails even if product markets were fully integrated at the border, or equivalently, if arbitrage forces at the docks were perfectly efficient. Specifically, the average exchange rate pass-through into PCP is only 59% compared to 89% in the steady state. This result is chiefly attributable to the relatively low average convexity of price adjustment costs and pervasive downward price rigidity. To elaborate, the average US import price response to an exchange rate change is imperfect due to the fat-tailed distribution of inflation in the UK export prices. Whenever exchange rate movements are large, the indirect cost-push effect has the tendency to overshoot the direct currency conversion effect, which can lead to a negative magnitude of pass-through into PCP import prices in the short-run. According to Krugman (1986), variable price mark-ups and pricing-to-market is the main underlying reason why import price levels did not fall as much as anticipated in the 1980s in spite of a pro-longed episode of USD appreciation. In the light of the present framework, the other plausible contributing factor may be the rising commodity prices in the local currency terms of other OECD economies combined with a greater selection effect associated with firms choosing to adjust prices more frequently in response to higher aggregate inflation at the time.

By contrast, the outcome for the import prices in the United Kingdom is to a large extent consistent with the conventional wisdom of Engel (2000). On average, exchange rate pass-through into PCP import prices is complete, while LCP import prices are largely orthogonal to transitory exchange rate movements. Due to an overwhelmingly large market share of LCP imports (94.5%), the average pass-through into aggregate import prices at the border is virtually zero. There are three reasons why the the qualitative outcomes for the US and UK are non-identical. First, the US is less open to intermediate imports than the UK, which dampens the cost-push effects associated with the exchange rate and commodity price co-movements. Second, export price inflation is less volatile in the US compared to the UK, resulting in a greater slope of the Phillips curve in the UK compared to the US (i.e. greater average convexity of price adjustment costs in the US). Third, the high positive skewness of wage growth and symmetric distribution of inflation observed in the US during the great moderation period can only be replicated by establishing a mild degree of upward price rigidity. As a result, UK import prices are counterfactually unrelated to the relative value of the GBP. However, both the US and the UK are engaged in many trade partnerships with other OECD economies, most of which exhibit high positive skewness of price inflation (see table 2.1). If other economies were to be incorporated into the model, the effective exchange rate pass-through into UK import prices would be qualitatively similar to that of the US.

3.3.2 Dynamic Causal Effect

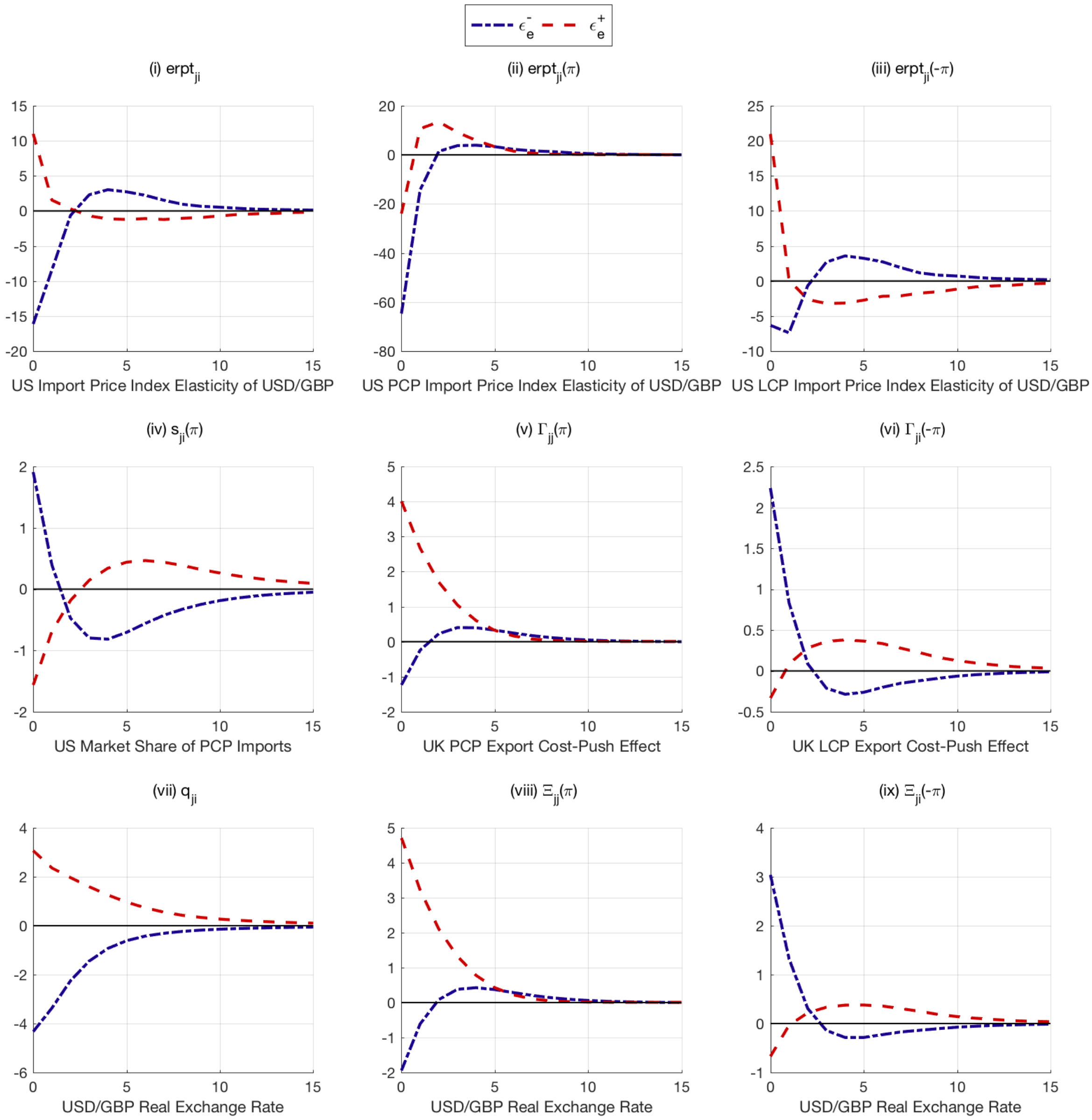
How persistent is the exchange rate pass-through asymmetry? Figure 3.3 plots the impulse response functions of the endogenously determined exchange rate pass-through to a positive and negative shock associated with an exogenous process of exchange rate noise. The size of the shock is two standard deviations. A positive (negative) shock $\epsilon^{+}_{e,t}$ ($\epsilon^{-}_{e,t}$) implies a USD depreciation (appreciation) relative to GBP in real terms, which moves the real USD/GBP exchange rate relative to the long-run steady state by around 3-4%, the influence of which takes around 16 quarters to fully dissipate (see subplot (vii)). Because of the non-linear price adjustment costs characterising the UK export prices, subplots (i)-(iii) demonstrate that the USD depreciation (GBP appreciation) leads to a greater pass-through of cost-push effects into LCP import prices than an equivalent USD appreciation (GBP depreciation), in which case they are more pronounced for PCP import prices (see subplots (v),(vi),(viii) and (ix)).

In this simple model, the price mark-ups are held fixed at all times, which therefore directly translates the cost-push effects into movements of prices and exchange rate pass-through (see equation 2.14).[4] A USD depreciation (GBP appreciation) leads to a fall in PCP pass-through in the short-run due to the price stickiness (see subplots (vi) and (viii)) and it gradually reaches its peak (equal to unity) at quarter 3. By contrast, LCP pass-through reaches its maximum immediately after the shock occurs, since prices are relatively flexible in the upward direction. Conversely, when USD appreciates (GBP depreciates) PCP pass-through falls by even more in the short-run, on account that export prices in GBP rise disproportionately. The latter effect comes from the rise in the costs of intermediate imports, thereby insulating US import prices in USD until quarter 4 when pass-through reaches its maximum of around 90%. A similar pattern is followed by LCP import price pass-through, but the maximum at quarter 4 is around 15%.

By construction, exchange rate pass-through into aggregate import prices is a weighted average of exchange rate pass-through into the PCP and LCP import prices, where the weight corresponds to the market share of each type of imports (see equation 2.14). Upon USD depreciation (appreciation) the market share of PCP imports falls (rises), thereby attaching more weight to the high (low) short-run pass-through from the LCP (PCP) imports. That said, the exchange rate pass-through asymmetry is generally short-lived and is partially inverted in the medium-run. Specifically, USD depreciations lead to high and immediate pass-through, while USD appreciations lead to low short-run pass-through and it takes longer to reach its maximum, but in the medium-run, pass-through is indeed marginally higher for appreciations than depreciations. In the long-run, when all of the short-run effects from transitory innovations dissipate, exchange rate pass-through from either depreciations or appreciations is identical and equal to the value of the steady state - asymmetry dissipates completely.

[4]More specifically, the intra-temporal elasticity of substitution is time-invariant, which implies that price mark-ups can drift over time due to price stickiness only. Indeed, an interesting extension not pursued in this chapter for the sake of simplicity and transparency would be to allow for elasticity of substitution to fluctuate in order to see how much of the non-linearities transpire into prices in that context.

Figure 3.3: Exogenous Exchange Rate Shock



Each subplot displays generalised impulse response functions following an exogenous shock to the autoregressive process of exchange rate noise equal to two standard deviations. Subplots (i) through (iv) are expressed as an absolute change over time relative to the steady state (scaled by 100), since the variables themselves are measured in percentage points. Subplot (v) - (ix) are expressed as percentage deviations from the steady state. The impulse responses are computed as conditional forecasts over 116 periods, where the initial 100 burn-in periods are dropped. The conditional forecast results are affected by structural shocks other than the one impulse in period 101. In order to average over the effect of these random draws, the exercise is replicated 500 times.

3.3.3 Terms of Trade

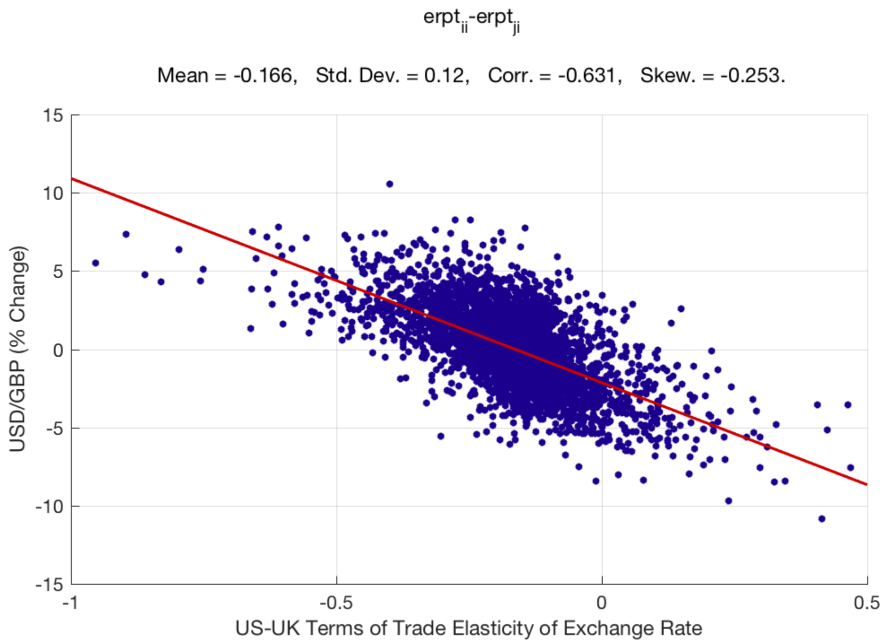
Based on the quantitative predictions of the import and the export price responsiveness to the exchange rate changes, it is possible to draw some novel quantitative implications associated with the US terms of trade. Let $T_{ji,t} = p_{ii,t}/p_{ji,t}$ denote the bilateral terms of trade between economies i and j, which measures the ratio of i'th economy export prices to the i'th economy import prices of j'th economy goods. The conventional wisdom suggests that the local terms of trade improve following a local currency appreciation, allowing the domestic households to consume more of the imported goods for every unit of exported goods. It can be shown that this will be true if and only if the exchange rate pass-through into import prices is strictly greater than into export prices. Specifically, if and only if $\partial \ln T_{ji,t}/\partial \ln q_{ji,t} = erpt_{ii,t} - erpt_{ji,t} < 0$, the terms of trade in the i'th economy improve following its currency appreciation, but they deteriorate following a depreciation.

In the Mundell-Fleming environment, where prices are sticky in producer currency units and the import penetration ratio of intermediate goods is negligible, this condition always holds, because the unit costs of producing an exported good are independent of the exchange rate (i.e. $erpt_{ji,t} = 1, erpt_{ii,t} = 0$ – see proposition 1), thus $\partial \ln T_{ji,t}/\partial \ln q_{ji,t} = -1$. This implies that the relative price of tradable goods move one-to-one with the relative prices of consumer goods, where the latter encompass both tradable and non-tradable goods. However, Atkeson & Burstein (2008) document that the U.S. terms of trade are considerably more stable than the real exchange rate. In turn, the most common way to solve this puzzle in dynamic stochastic general equilibrium models is to impose a 100% share of LCP imports, in which case the the above condition is always violated and the terms of trade are constant over time (i.e. $erpt_{ii,t} = erpt_{ji,t} = 0$ and $\partial \ln T_{ji,t}/\partial \ln q_{ji,t} = 0$).

Similar to Choudhri & Hakura (2015), the average terms of trade elasticity in this model is chiefly controlled by a non-degenerate density of LCP and PCP imports measured on the unit interval (i.e. $\chi_{ji} \in [0, 1]$). The higher is the magnitude of χ_{ji}, the closer is the terms of trade elasticity to unity and vice versa. However, unlike in Choudhri & Hakura (2015), in this framework, the market share of each import type is endogenous (see equation 2.13) and it responds to the structural innovations due to income and substitution effects associated with the exchange rate changes. As a result, the terms of trade elasticity to exchange rate is time-varying and state-dependent (see figure 3.4). In particular, the US export prices are mostly inelastic to exchange rates (i.e. $erpt_{ii,t}$ is close to zero), while the import price elasticity in the steady state closely follows the magnitude of the market share associated with the PCP imports (equal to around 18% in the US).

Contrary to the Mundell-Fleming paradigm, on average only around 16.6% of the movements in the USD/GBP exchange rate are reflected in the US terms of trade. However, the unique finding in this chapter is that the terms of trade elasticity of the exchange rate is strongly negatively correlated with the exchange rate itself (-0.63), such that regardless of the size of the USD depreciation, it leads to an overall deterioration in the US terms of trade. However, small USD appreciations imply little to no movement in the US terms of trade, while large and persistent appreciations have the capacity to somewhat reverse the US comparative disadvantage into a comparative advantage. The rationale for this result is exactly the same as before, where downward export price rigidity in the UK creates a tendency for the British exporters to over-react to persistent GBP depreciations against the USD – not only impeding, but negating the pro-competitive effects of the exchange rate channel. The notorious US current account deficit can therefore only be justified by the developments in the private and public sector savings and not the foreign exchange market. If anything, large movements in the USD – regardless of up or down – boost the competitiveness of the US firms, because other OECD economies are subject to more severe structural non-linearities.

FIGURE 3.4: Exchange Rate Pass-Through into U.S. Terms of Trade



3.3.4 Sensitivity Analysis

How sensitive are the quantitative results presented above to the changes in the structural parameters, even those that are estimated using the SMM? Table 3.3 presents the results from a number of simulations with the express purpose of checking the robustness of the results presented in the previous sections. The focus in this section is on the magnitude of exchange rate pass-through into US import and consumer prices. The sensitivity analysis is presented for the six most relevant parameters summarising the degree of downward price rigidity, average convexity of price adjustment costs, intermediate and final import intensity, iceberg costs, as well as the density of PCP and LCP imports.

First, in the special case when adjustment costs associated with UK export prices are quadratic (i.e. $\zeta_{j,p} \to 0$), the null hypothesis of symmetric exchange rate pass-through into US import and consumer prices cannot be rejected (see the first column in section (i) of table 3.3). However, as the downward rigidity of UK export prices becomes more pronounced, the pass-through into US import and consumer prices following USD appreciations (depreciations) approaches zero (remains unchanged). Conversely, increase in the average convexity of the UK price adjustment costs shrinks the extent of pass-through asymmetry for both import and consumer prices (see section (ii) of table 3.3).

If the UK was autonomous to intermediate imports (i.e. $\xi_j \to 0$), then proposition 1 proves that the only source of incompleteness and time-variability in exchange rate pass-through into import prices stems from the market share of PCP imports. This is because pass-through into PCP (LCP) import prices would be equal to unity (zero) at all times (see the first column in section (iii) of table 3.3 and equation 2.14). However, as soon as $\xi_j > 0$, $\Gamma_{jj,t}(\pi)$ and $\Gamma_{ji,t}(-\pi)$ are no longer equal to zero, such that cost-push effects associated with exchange rate and commodity price co-movement lead to incomplete exchange rate pass-through. Interestingly, as UK intermediate import intensity increases, US import prices become somewhat more

TABLE 3.3: Exchange Rate Pass-Through Robustness in the United States

(i) Downward Price Rigidity of UK Exports

	$\zeta_{j,p} = 0.00$		$\zeta_{j,p} = -100.00$		$\zeta_{j,p} = -200.00$		$\zeta_{j,p} = -300.00$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	24.43%	25.47%	24.81%	19.33%	24.76%	12.16%	25.12%	3.16%
$erpt_{i,t}$	2.34%	2.59%	2.36%	2.18%	2.36%	1.71%	2.37%	1.10%

(ii) UK Export Price Stickiness

	$\kappa_{j,p} = 10.00$		$\kappa_{j,p} = 20.00$		$\kappa_{j,p} = 30.00$		$\kappa_{j,p} = 40.00$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	32.56%	-5.36%	24.75%	14.37%	22.45%	18.07%	21.40%	19.40%
$erpt_{i,t}$	2.82%	0.55%	2.35%	1.85%	2.22%	2.09%	2.16%	2.18%

(iii) Intermediate Import Intensity in the UK

	$\xi_j = 0.00$		$\xi_j = 0.20$		$\xi_j = 0.40$		$\xi_j = 0.60$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	16.77%	18.98%	24.77%	11.27%	24.52%	12.83%	24.63%	13.70%
$erpt_{i,t}$	2.78%	3.15%	3.36%	2.06%	2.80%	1.97%	2.53%	1.90%

(iv) Density of PCP imports in the US

	$\chi_{ji} = 0.00$		$\chi_{ji} = 0.05$		$\chi_{ji} = 0.1$		$\chi_{ji} = 0.15$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	16.61%	6.59%	37.99%	26.77%	52.29%	40.07%	61.24%	48.10%
$erpt_{i,t}$	1.80%	1.33%	3.27%	2.70%	4.31%	3.66%	5.00%	4.28%

(v) Import Intensity of Final Goods in the US

	$\alpha_{ji} = 0.00$		$\alpha_{ji} = 0.15$		$\alpha_{ji} = 0.30$		$\alpha_{ji} = 0.45$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	16.76%	6.02%	22.96%	12.39%	25.44%	15.02%	27.00%	16.74%
$erpt_{i,t}$	0.54%	0.75%	1.77%	1.48%	2.60%	2.00%	3.16%	2.39%

(vi) Iceberg Costs

	$\tau = 1.00$		$\tau = 1.10$		$\tau = 1.20$		$\tau = 1.30$	
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)
$erpt_{ji,t}$	25.30%	14.81%	24.80%	14.28%	24.32%	13.81%	23.88%	13.36%
$erpt_{i,t}$	2.42%	1.88%	2.36%	1.84%	2.30%	1.81%	2.25%	1.78%

All panels display a hypothetical magnitude of exchange rate pass-through into US import prices and consumer prices following depreciations (+) and appreciations (–) of the USD/GBP exchange rate when the value of a given parameter changes. The remaining parameters are held constant at their default values. The results are based on 4000 observations of synthetic data generated by the model, where it is implicitly assumed that i = US and j = UK.

elastic to exchange rate changes, but more so for appreciations, thereby diminishing pass-through asymmetry.

Similar to Choudhri & Hakura (2015), exchange rate pass-through into US import prices is increasing in the density of PCP imports in the US (i.e. χ_{ji}). However, the relationship in this framework is non-linear, since in addition to this density, the relative prices of both PCP and LCP imports also influence the weight on pass-through from each type of import (see equation 2.13). Moreover, increase in the density of PCP imports enhances the exchange rate pass-through asymmetry, since more weight is attached to the dynamics of PCP import prices that are more volatile and subject to greater non-linearities by construction.

As expected, exchange rate pass-through into US consumer prices is increasing in the US import intensity of final goods (see see section (v) of table 3.3 and equation 2.23). The unexpected result is that exchange rate pass-through into US import prices is also increasing in the import intensity of final goods. Hence, a rise in α_{ji} leads to an increase in the market share of the PCP firms through changes in the steady state of the relative prices. The rationale is simply that greater consumption expenditure on imported goods in any given economy is associated with greater market power of the foreign firms and their tendency to choose the invoicing currency of their domicile. Finally, an increase in the iceberg costs leads to marginally lower and less asymmetric exchange rate pass-through into US import and consumer prices, but their overall influence is quantitatively negligible.

3.3.5 Alternative Reduced Form Estimates

Now that we have established all of the micro-founded properties of the model-implied exchange rate pass-through, the next natural question is – how biased are the alternative reduced form estimates? This section explores two methodologies that are widely-applied in the literature, namely: (i) the ratio of cumulative impulse response functions of nominal import price inflation and the first-differenced nominal exchange rate triggered by exchange rate noise as in Shambaugh (2008), Choudhri & Hakura (2015), and Forbes et al. (2017); and (ii) the cumulative dynamic multipliers generated by a non-linear error correction model, pioneered by Shin et al. (2014) and Brun-Aguerre et al. (2017).

First, consider the cumulative impulse response approach *à la* Shambaugh (2008), henceforth CIRF. Recall that the numerical solution to the model is obtained for real prices and real exchange rates. Because all firms are infinitely ‘small’ in this model, it does not matter if the closed-form solution to exchange rate pass-through is derived endogenously in real or nominal terms. However, computing the ratio of cumulative impulse response functions of the real import price inflation and the real exchange rate change would introduce an inconsistency associated with extraneous indirect income effects, because the deflators of these two variables are not identical.[5] And yet even after carefully mitigating these effects from the impulse response functions, the quantitative predictions of exchange rate pass-through in their reduced form bear very little resemblance to the micro-founded estimates presented above. In spite of the fact that both approaches represent the same exact data generating process, the policy implications that immediately follow are poles apart.

For instance, figure 3.5 displays the CIRF estimates of exchange rate pass-through, all of which are virtually zero in the long-run for PCP, LCP, and aggregate import prices. On the other hand, the model predicts a 28% (8%) pass-through in the steady state into US (UK) import prices at the aggregate level. Although both methods predict a gradually dissipating exchange rate pass-through asymmetry, it is considerably more persistent and more pronounced in the case of CIRF. Both methods predict import prices to be more responsive to local currency depreciations than appreciations, but the CIRF estimate over-shoots the micro-founded estimate by more than 60% in the case of depreciations (i.e. CIRF elasticity is 0.62, while the model-implied elasticity of 0.38). These results suggest that indirect income effects prevail even after carefully dissecting the price and exchange rate series from the mechanics of the model. The presumption that the same exact veil of 'smallness' applies to the CIRF estimates as it does in the case of the micro-founded measurement is thus misguided, because the co-movement of macroeconomic fundamentals in equilibrium drives their generalised impulse response functions. Yet they are irrelevant to the optimal behaviour of individual exporters, since they have no market power over the influence of aggregate developments and it would not influence their optimal exchange rate pass-through.

Second, consider the non-linear error correction model as in Brun-Aguerre et al. (2017), henceforth NECM, who show that in many advanced and developing economies around the world import prices are more responsive to local currency depreciations than appreciations, including the US and the UK.[6] Another property of the NECM is that it typically gives rise to a greater exchange rate pass-through in the long-run than in the short-run, purporting that nominal rigidity ought to delay the onset of exchange rate impact on international prices. In order to compare the theoretical predictions of the model presented above to the empirical

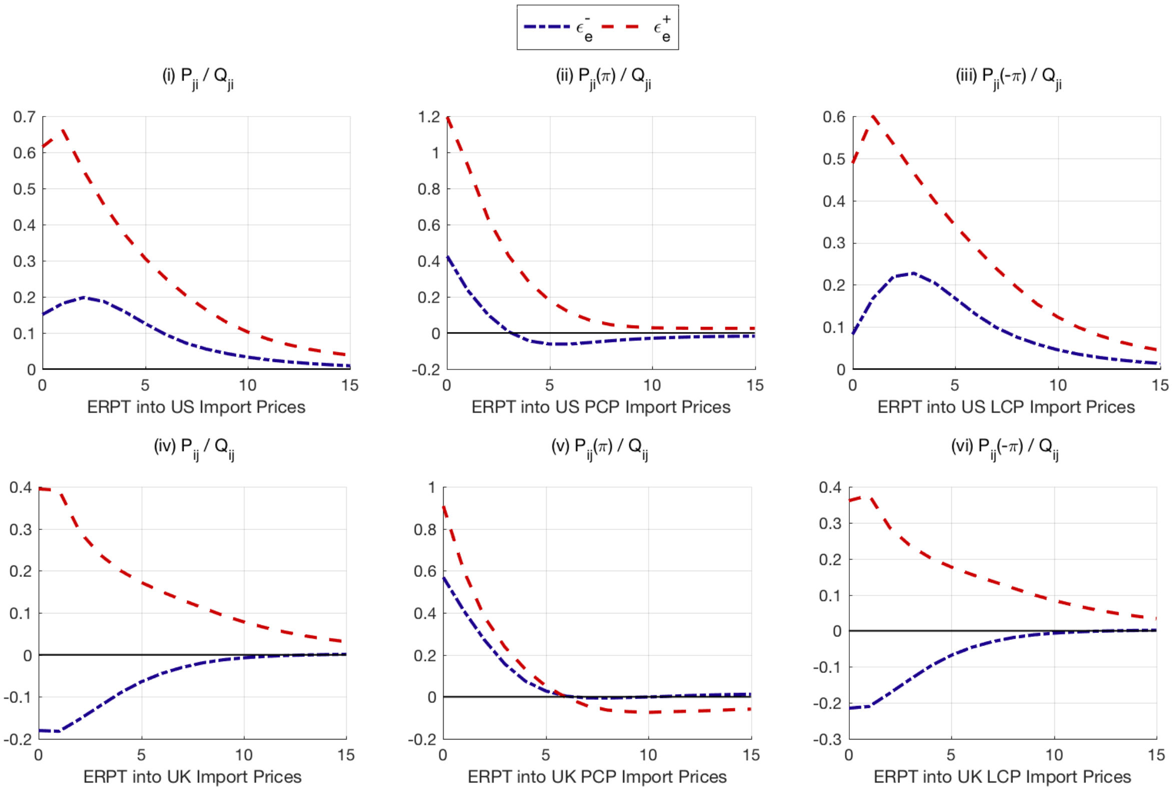
[5] In particular, real price $p_{ji,t} = P_{ji,t}/P_{i,t}$ is the real import price of source country j in destination i, while $q_{ji,t} = Q_{ji,t}P_{j,t}/P_{i,t}$ is the real exchange rate. The ratio of cumulative import price inflation and the nominal exchange rate change is therefore computed as

$$\mathbf{G}(P_{ji}, Q_{ji}) = \frac{\sum_{h=1}^{H}[\dot{p}_{ji,h} - \dot{p}_{i,h}]}{\sum_{h=1}^{H}[\dot{q}_{ji,h} - \dot{p}_{j,h} + \dot{p}_{i,h}]},$$

where $h = 1, 2, ..., H$ is the length of the impulse response function, $\mathbf{G}(\cdot)$ is an $H \times 1$ vector, while $\dot{p}_{ji,h}$, $\dot{p}_{i,h}$, $\dot{q}_{ji,h}$, and $\dot{p}_{j,h}$ are model-implied generalised impulse response functions.

[6] Error correction models in general are among the most popular reduced form methodologies in terms of estimating exchange rate pass-through. Thanks to the contributions of Campa & Goldberg (2005) and Burstein & Gopinath (2014) among many others, valuable progress has been made in terms of quantifying the speed and the extent of exchange rate shock transmission into international prices. Moreover, the seminal contribution of Shin et al. (2014) demonstrated that error correction models can indeed serve an important additional purpose without abandoning their appeal attributed to the computational simplicity. In particular, they can be used to assess the exponential adjustment of a co-integrating relationship towards an asymmetric long-run equilibrium by estimating a univariate regression function by Ordinary Least Squares (OLS). The specification of NECM used in this chapter is exactly the same as in Brun-Aguerre et al. (2017), who provide all of the remaining technical details related to the implementation. Suffice it to say that it is based on nominal prices of imports and exports as well as the bilateral nominal exchange rate.

Figure 3.5: Cumulative Impulse Response Estimates of Exchange Rate Pass-Through

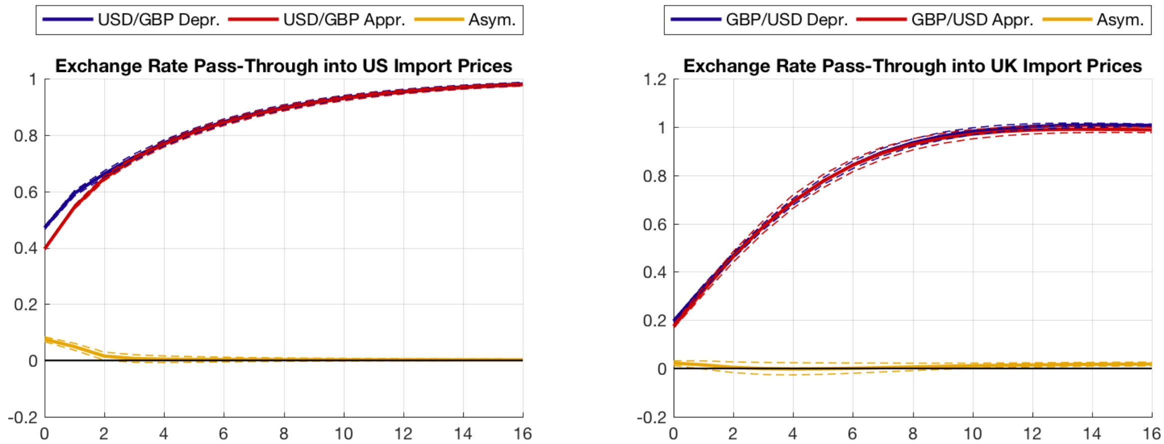


Each subplot displays the ratio of cumulative generalised impulse response functions of import price inflation and first-differenced nominal exchange rate following an exogenous shock to the autoregressive process of exchange rate noise equal to two standard deviations. The magnitude of pass-through is expressed as an elasticity bounded between zero and unity. The impulse responses are computed as conditional forecasts over 116 periods, where the initial 100 burn-in periods are dropped. The conditional forecast results are affected by structural shocks other than the one impulse in period 101. In order to average over the effect of these random draws, the exercise is replicated 500 times.

rationale *à la* Brun-Aguerre et al. (2017), this chapter adopts an indirect inference approach originally proposed by Corsetti et al. (2008). The idea is to take the model-implied data and to extract the synthetic magnitude of exchange rate pass-through using NECM – exactly as it is done in the context of actual data on macroeconomic fundamentals.

The model-implied NECM results presented in figure 3.6 are qualitatively similar to the empirical studies in a number of ways, one of which is that exchange rate pass-through is monotonically increasing over the projected horizon – it also takes time for it to fully unfold. It should be emphasised, however, that this finding has very little theoretical basis from the perspective of the true, and in this case known, data generating process, such that the transition path generated by the NECM acts as though it is a mere mechanical property of the methodology. Even if we were to take the aforementioned argument of nominal rigidity at face value, New-Keynesian models are notorious for their missing persistence of inflation to monetary innovations – one would not expect as much persistence of exchange rate pass-through as suggested by the low rate of decay in the transition path. On the other hand, NECM does detect a statistically significant albeit mild disparity between import price responsiveness to local currency depreciations and appreciations in the US, but not in the UK, much like the

Figure 3.6: Non-Linear Error Correction Model Estimates of Exchange Rate Pass-Through



The subplots display cumulative dynamic multipliers measuring the transition path of the elasticity of US (UK) import prices to USD/GBP (GBP/USD) exchange rate that is triggered by an exogenous change in the latter. The confidence intervals are bootstrapped using 2000 replications of the coefficient estimates.

micro-founded estimates and unlike CIRF. NECM also matches the stylised fact established by the business cycle model that exchange rate pass-through asymmetry is generally short-lived. However, in spite of the fact that the model embeds a number of structural factors that insulate import prices from large exchange rate movements in the steady state, the NECM predicts complete exchange rate pass-through for distant projected horizons, unlike CIRF and unlike the micro-founded estimates. A similar observation is made by Corsetti et al. (2008) in a calibrated and linearised two-country model with a number of properties distorting the purchasing power parity and the law of one price. The dichotomy between the true, and in this case known, data generating process and the NECM estimates of exchange rate pass-through implies that the policy implications derived from the latter can be remarkably misleading.

The synopsis of this section is not to advocate the applications of business cycle models as be-all and end-all in the context of measuring exchange rate pass-through *per se*. If anything, it claims that the reduced form measurements of exchange rate pass-through applied to the aggregate data on prices can be just as misleading as the the business cycle model estimates whenever the latter are postulated on the basis of uninformed presumptions about the structure of the economy. With that in mind, if we were to ever pin down the truly unbiased estimate of pass-through, it would almost certainly come from a simple structural model that is based entirely on detailed firm-level data of prices and mark-ups. The problem is that such data are still scarcely available even in OECD economies, not to mention the emerging markets, despite the growing initiative of the literature towards this direction. And so in the imperfect information environment such as the *status quo*, this chapter presents the case in favour of using business cycle models that assimilate the salient features of the globalised export market structure as a tool to transcend the interim administrative challenges and to extract more signal and less noise about the exchange rate transmission channel.

3.4 Non-Linear Estimation Summary

This chapter estimates the micro-founded measurement of exchange rate pass-through derived directly from the first order conditions of import-dependent exporters in the context of a multi-country business cycle model. When the non-linearities are driven by non-trivial higher-order moments of inflation in OECD economies, the estimated business cycle model predicts that US import prices are on average 10% more responsive to USD depreciations than appreciations. The central idea behind this result is that exchange rate depreciations are treated analogously as an aggregate decline in exporter productivity in the short-run. The cost-push effects across a large number of import-dependent exporters translate to a greater aggregate export price inflation, thereby generating an even further 'selection effect' for firms that are limited in their exposure to exchange rate risk. The shifts in the selection effect are approximated by the pivot in the LINEX price adjustment cost function in a representative firm framework.

More specifically, the estimated model predicts that US import prices at the border absorb 24.8% (14.3%) of USD depreciations (appreciations), while the UK import prices are mostly orthogonal to GBP fluctuations. This finding is based on the high share of LCP imports in the US, a significant positive skewness of UK export price inflation, and a vastly greater openness to intermediate imports in the UK compared to the US. In the absence of non-linear price adjustment costs, exchange rate pass-through into import prices becomes symmetric, which implies that intermediate imports are a necessary, but not a sufficient property of the model in terms of generating asymmetric pass-through. The reduced form measurement of exchange rate pass-through advocated by Shambaugh (2008) delivers qualitatively similar results, but the quantitative predictions are shown to be poles apart. In particular, the reduced form measure is biased upwards by more than 60% upon impact and it generates a much more persistent exchange rate pass-through asymmetry relative to the micro-founded measurement. Contrary to the Mundell-Fleming paradigm, the model-implied US terms of trade absorb an average of only 16.6% of the real exchange rate fluctuations. But when monetary authorities are equally averse to inflationary and deflationary pressures, a strong USD would not improve the US terms of trade by as much as they would deteriorate when the USD weakens - a direct consequence of the positively skewed export price inflation in the UK. The notorious US current account deficit can therefore only be justified by the developments in the private and public sector savings and not the foreign exchange market. If anything, large movements in the USD – regardless of up or down – boost the competitiveness of the US firms, because other OECD economies are subject to more severe structural non-linearities.

There are a multitude of possible extensions to the mechanism presented so far. First, the model imposes CES production technology and assumes that each firm is infinitesimally small. Consequently, absent of nominal rigidity, the price mark-ups would be counter-factually constant over time. Second, both UK and US are treated as equally important, but one would expect the US to exert dominance over small open economies, such as the UK. The next chapter therefore generalises the globalised export market structure presented so far and examines how dynamic strategic complementarities among multi-national exporters influence the channel of exchange rate transmission.

Chapter 4

What Drives the Terms of Trade Neutrality to Exchange Rates?

4.1 Background

Small objects in the close vicinity of enormous objects behave in peculiar ways. The precession of Mercury's perihelion around the Sun illustrates how a close proximity of two objects with an immensely dissimilar mass consistently violate the Newton's laws of universal gravitation. And yet the principal force of gravity driving the global patterns of commerce is most widely encapsulated by the mere geographical dimension according to the ubiquitous notion of Samuelson's 'iceberg costs'. It simply quantifies the distance between the origin of the shipment and the destination in which the merchandise is ultimately exchanged. But it disregards the unorthodox influence on the terms of trade that the hegemonic power seized by any one of those regions might entail, such as the widely-observable circulation of select-few currencies around the world as a customary medium of exchange. Indeed, a growing strand of the literature, most notably Boz et al. (2017), point out that the geographic interpretation of the gravity laws is consistently defied around the world whenever the U.S. Dollar (USD) weakens – it leads to a global intensification of trade flows (measured in gross trade volumes) as though fewer shipments are 'sunk' along the way. At the same time, the U.S. economy remains practically neutral to the pronounced turbulence in the foreign exchange markets revolving around the trajectory of the USD.

This chapter explores the structural and stochastic factors surrounding the globalised export market structure that warp the terms of trade over time. The multi-country business cycle model presented in this chapter moves away from the classical Mundell-Fleming paradigm by distinguishing between minor and major currency areas. Close attention is paid to the innovations emanating from the major currency areas that spill-over to the minor currency areas, all of which are freely floating against one another. The model emphasises that a global trade flow intensification due to a weakening of the major currency is unlikely to boost all countries identically in a flexible exchange rate environment. That channel crucially depends on country-specific structural factors, such as exporter reliance on intermediate imports, import and export price stickiness, as well as the shares of invoicing currency in which international prices are sticky. Although the choice of invoicing currency is weakly correlated with the intermediate import intensity, nominal price stickiness and strategic complementarities among exporters are formally shown to materialise in unprecedented terms of trade developments over time. Specifically, rigid (flexible) prices in minor currency areas imply that a good proxy for the exchange rate pass-through into their terms of trade is their total share of imports (exports) invoiced in USD. Strategic complementarities among exporters competing for those shares therefore transpire into a time-varying exchange rate pass-through into the terms of trade driven by a number of innovations, such as transitory distortions to labour productivity, exchange rate noise, or monetary policy surprises.

The net exchange rate effect on the prices of imports and exports in any given economy constitutes the terms of trade transmission channel. It determines whether local monetary policy measures are more effective in terms of influencing domestic output and inflation in an open economy rather than a closed economy. Yet predicting the movements in the terms of trade induced by innovations in major or minor currency areas requires a thorough understanding of what structural factors drive some exporter mark-ups to absorb more of the exchange rate volatility than others. Throughout the years, a lot of empirical analysis has been conducted in terms of quantifying the exchange rate pass-through into import prices, emphasising the role of international price discrimination, otherwise known as 'pricing-to-market'.[1] But globalisation has increasingly shifted the emphasis towards studying exchange rate pass-through into export prices, too.[2] Global Value Chains (GVC), characterising the nexus of widespread import-export industries around the world, have opened up the debate about the transmission of cost-push effects into the export prices of manufactured goods. The latter are chiefly associated with the global cost of imported commodities, which Chen et al. (2010) find to be strongly influenced by the developments in the USD market. The dramatic shift in the import content from the end of the GATT era to the present times implies that it is paramount to track the movements of the terms of trade as a whole rather than focusing on import prices alone. It also points to the fact that both pricing-to-market and intermediate imports play an equally important role in the transmission of exchange rate innovations.

Another important dimension of the terms of trade channel is the exporters choice of invoicing currency. According to Gopinath (2015), some 5% of all global trade transactions were conducted directly with the U.S. in the period of 1999-2014, but more than 20% of global imports and exports were invoiced in USD.[3] The fact that a vast amount of goods around the world are traded in very few currencies has important implications for the terms of trade transmission channel, namely – which exchange rate, if any, matters? Because even if minor currency areas decided to float *de jure*, the *de facto* international prices would still be highly influenced by innovations in major currency areas, which could bring the infamous Harberger-Laursen-Metzler effect to a grinding halt.

A lot of progress has been made in recent years in terms of assimilating these salient features into empirically-motivated theoretical models that replicate the properties of the

[1] Goldberg & Knetter (1997) and Burstein & Gopinath (2014) provide excellent literature surveys of the earlier and the more recent empirical work respectively.

[2] Miroudot et al. (2009) estimate that in the period of 1995-2005 around 56% (74%) of total OECD imports consisted of intermediate goods (services).

[3] Does it also mean that all merchandise invoiced in USD is also priced in USD? Friberg & Wilander (2008) show that in practice that usually is the case using the survey data of Swedish exporters.

terms of trade observed in the data. Much of the work builds on the quantitative trade model pioneered by Melitz (2003), where differences in firm-level productivity within any nation lead to a self-selection of the firms into the exporting industry. Atkeson & Burstein (2008) demonstrate that enhancing such models with oligopolistic competition is remarkably fruitful when modelling the stark discrepancy between the high volatility of the real exchange rate and the low volatility of the terms of trade observed in the U.S. data. Yet U.S. is a large and a relatively closed economy. By contrast, in a small open economy of Belgium, Amiti et al. (2014) show that openness to intermediate imports at the firm-level strongly correlates with the strategic flexibility and the average magnitude of price mark-ups as well as the tendency to invoice profits in the currency units of the destination market. Although quantitative trade models are perfectly suited to study the effects of sudden shifts upon opening up to international competition on the domestic 'creative destruction' and inter-sectoral factor reallocation, they are ill-suited to study the dynamic properties of the terms of trade in the context of aggregate uncertainty. With the exception of Gopinath et al. (2010) and several others, who study the endogenous choice of invoicing currency in a dynamic context, most quantitative trade models are static and they are missing the notion of short-run nominal price stickiness. But this chapter formally shows that even conditional on the choice of invoicing currency and intermediate import intensity, nominal price stickiness has important policy implications for the terms of trade channel. Specifically, if commodities are overwhelmingly priced in the USD, but international prices of manufactured goods around the world are rigid (flexible), then the minor currency areas with a higher share of imports (exports) invoiced in USD gain a greater comparative advantage against one another whenever USD depreciates.

In order to capture all three paramount dimensions of the globalised export market structure simultaneously, namely pricing-to-market, intermediate import intensity, and heterogeneous invoicing currency strategies, but also to demonstrate the qualitative and quantitative significance of the nominal price stickiness in such a context, this chapter develops a novel multi-country business cycle framework. Much like the two-country model of Choudhri & Hakura (2015), there is a continuum of firms segmented into constant densities of exporters who choose to price their goods at the factory door (PCP) or at the docks of each destination (LCP). Alas, the multi-country modelling framework presented in this chapter also allows for goods originating from minor currency areas to be priced in major currency units (DCP). The market shares of each market segment are time-varying due to strategic complementarities embedded into the Bertrand-competitive market structure borrowed from Atkeson & Burstein (2008). All firms depend on imported commodities that are exclusively priced in major currency units, but each market segment is subject to a varying intermediate import intensity. Every time prices of manufactured goods adjust, exporters are subject to industry-specific price adjustment costs *à la* Rotemberg (1982), incentivising gradual transition of international prices from vintages to the inter-temporal optimum. Cost-push shocks emanating from innovations in the major currency area thus have heterogeneous impact across different types of exporters due to peculiarities such as import-dependence, currency denomination, and strategic complementarities.

The closest to the present approach is the dominant currency paradigm developed by Casas et al. (2016), henceforth CDGG. Indeed, there are many superficial similarities between both approaches, but the seemingly subtle technical differences introduced in this chapter unravel an ample of new insights about the terms of trade channel. First, CDGG establish three regions: home, dominant, and the rest of the world, where home can influence neither the dominant region nor other small open economies. And yet if we want to study the international strategic complementarities prevalent among multi-national exporters, there needs to be an endogenous feedback mechanism between the price setters at home and abroad irrespective of domicile. The present approach therefore adopts a more general setting, where the world consists of a finite number of major and minor currency areas, such that innovations emanating from the major currency areas contaminate other minor currency areas, while transitory shocks in the minor currency areas spill-over from one to the other without feeding into the major currency areas.

Second, and more importantly, CDGG derive a time-invariant exchange rate pass-through into international prices irrespective of the nature of innovations distorting the global economy. This result comes from a combination of three modelling choices, namely Kimball (1995) preferences, Calvo (1983) nominal price rigidity, and denomination of all intermediate input costs in home currency units. In the end, the endogenous strategic complementarity is missing, since the price mark-ups evaluated at the symmetric equilibrium solely depend on a constant elasticity of substitution and an exogenous density of exporters choosing to invoice profits in foreign currency units. But constant price mark-ups contradict the findings of Amiti et al. (2018), who document strong evidence in favour of non-trivial strategic complementarities among exporters domiciled in the small open economy of Belgium. Indeed, these admittedly restrictive modelling choices are guided by analytical tractability and elegance, but they come at a cost of creating a dichotomy between the theoretical framework, predicting a constant average causal effect on international prices upon exchange rate impact, and the empirical evidence, documenting a state-dependent and a dynamic causal effect. In particular, Shambaugh (2008) and Forbes et al. (2017) use structural vector autoregressive (SVAR) models and find that not only the magnitude, but even the sign of exchange rate pass-through depends on the nature of innovations to the macroeconomic fundamentals.

Although this chapter places itself in parallel with the policy implications procured by SVAR models, it presents a meaningful refinement of the tools that are used to reach these conclusions. Specifically, SVAR models measure exchange rate pass-through in reduced form by computing the ratio between cumulative impulse response functions to price inflation and exchange rate changes conditional on triggering some exogenous innovation to the system of difference equations. However, impulse response functions embody the properties of price and exchange rate co-movement, rather than exchange rate transmission into prices, which obscures the magnitude of exchange rate pass-through by accounting for a number of indirect (general equilibrium) effects. The latter are not necessarily relevant to the price setting decision of the firm and they are highly susceptible to abstract methodological specificities, such as the shock sign restrictions in SVAR models or how the model is closed in a general equilibrium setting. Instead, this chapter demonstrates a simple way how to derive a closed form solution to exchange rate pass-through into the terms of trade directly from the first order conditions of the exporters, which responds to a multitude of state variables and stochastic innovations in a top-down fashion without influencing the evolution of macroeconomic fundamentals. It provides a useful analytical tool that resolves the seemingly perplex disconnect between the average causal effect and the dynamic causal effect in a controlled structural environment without the use of external instruments advocated by Stock & Watson (2018). Most distinctly, it demonstrates the importance of strategic complementarities in a globalised export market structure for the conduct of monetary policy in a flexible exchange rate environment.

4.2 Model

There are three types of interacting agents in this multi-country model: firms, households, and central banks. The world economy is divided into major and minor currency areas.[4] Households supply labour to the firms and consume their final output, while firms hire workers and produce goods so as to clear the factor and product markets in all destinations. Central banks in each location are autonomous and commit to independent interest rate rules aimed at stabilising the local rate of consumer price inflation and the output gap by intervening in local bond markets. All currencies are therefore *de facto* freely floating against one another.[5] The model is presented in a general form with the introduction of several unconventional features, though a number of familiar corner solutions are exposited along the way, nesting well-established results in international macroeconomics.

4.2.1 Globalised Export Market Structure

Consider a world evolving over discrete time $t = 0, 1, 2, ...$, which consists of a finite number of interacting economies denoted by $n = \{1, 2, ..., N\}$. Each economy is populated by a continuum of manufacturers indexed by $\omega \in [0, 1]$. They import intermediate commodities from abroad and produce manufactured goods that are internationally-traded in imperfectly-competitive and segmented markets. The import-export manufacturers are categorised into different types indexed by $\phi = \{\pi, \ell, \$\}$. Each ϕ-type from the source economy $i \in n$ captures $\chi_{in}(\phi)$ density of the product market in every destination n, such that $\sum_{\phi} \chi_{in}(\phi) = 1$. The π-type firms price their final goods at the factory door in producer currency units (PCP), the ℓ-type goods are priced at the docks of each destination in local currency units (LCP), while the $\$$-type firms set prices in major currency units pertaining to locations $k = \{1, 2, ..., K\} \in n$, which exert dominance over the remaining $N - K$ minor currencies (DCP).[6]

[4] Major currency dominates the global economy via its widespread use as the invoicing currency by multinational exporters. This adaptation of the world is an intermediate case between the conventional small open economy approach, which takes the world variables as given without any feedback mechanism, and a two-country country approach, where both countries are equally important. This extension is particularly useful when modelling an endogenous feedback mechanism among trade partners in minor currency areas, while at the same time capturing top-down monetary policy spillovers from the major currency areas.

[5] An interesting extension that is not considered in this chapter is one where minor currency areas recognise the hegemonic power of the major currency areas and manage their currencies in response to external shocks.

[6] The density of each type of firms is a time-invariant parameter in the short-run. This assumption follows the empirical findings of Gopinath (2015), who shows that the annual share of imports and exports invoiced in foreign currency to be relatively stable over time in many advanced developing economies.

The production technology of each manufacturer is linear: $y_{in,t}(\omega,\phi) = z_{i,t}m_{in,t}(\omega,\phi)/\xi_i(\phi)$, where $m_{in,t}(\omega,\phi)$ denotes the stock of imported intermediate commodities used in the production of manufactured output $y_{in,t}(\omega,\phi)$, such that $\xi_i(\phi)$ controls the ϕ-type import intensity.[7] All manufacturers domiciled in the source country are subject to homogeneous productivity, which follows a stationary autoregressive process $z_{i,t} = z_{i,t-1}^{\rho_{i,z}} \exp(\sigma_{i,z}\epsilon_{i,z,t})$, where $\rho_{i,z} \in (0,1)$, $\sigma_{i,z} > 0$, and $\epsilon_{i,z,t} \sim iid(0,1)$. All intermediate imports in all countries are priced in the k'th major currency units, but if they are shipped abroad, then a fixed proportion $d_i - 1 \in (0,1)$ of the commodity is sunk in the process, thereby imposing iceberg costs *à la* Samuelson (1954).[8] Therefore, the unit costs of each exported final good in real terms are equal to $mc_{i,t}(-\$) = \xi_i(-\$)d_i q_{ki,t}/z_{i,t}$ and $mc_{i,t}(\$) = \xi_i(\$)d_i/z_{i,t}$, where $-\$ \in \{\pi,\ell\}$, and $q_{ki,t}$ is the major real exchange, such that a rise in $q_{ki,t} = 1/q_{ik,t}$ implies an i'th currency depreciation against the k'th currency in real terms, and a rise in commodity prices for all $i \neq k$.[9]

When the final goods are shipped to the remaining $N-1$ economies, the equilibrium import price in destination n is determined by the i'th factory export price, the bilateral exchange rate, the shipping costs, as well as international product market integration:

$$p_{in,t}(\omega,\phi) = d_n q_{in,t}\delta_{in,t}(\omega,\phi)p_{ii,t}(\omega,\phi), \tag{4.1}$$

where $p_{in,t}(\omega,\phi) = P_{in,t}(\omega,\phi)/P_{n,t}$ stands for the real price of variety ω originating from the i'th economy and sold in the n'th economy, while $P_{n,t}$ is the consumer price index in the n'th economy. The term $\delta_{in,t}(\omega,\phi)$ stands for the endogenously determined deviations from the law of one price, which give rise to parallel trade in the short-run due to the peculiarities over the choice of invoicing currency. By definition, the combined market density of π-type firms is perfectly integrated, such that $\int_0^{\chi_{in}(\pi)} \delta_{in,t}(\omega,\pi)d\omega = \chi_{in}(\pi)$. Conversely, the $\$$-type segment of the market is perfectly disintegrated, such that $\int_0^{\chi_{in}(\$)} \delta_{in,t}(\omega,\$)d\omega = \chi_{in}(\$)q_{ki,t}$. However, $\int_0^{\chi_{in}(\ell)} \delta_{in,t}(\omega,\ell)d\omega$ is distinct for its persistent movements due to the ℓ-type price mark-up flexibility that absorbs exchange rate fluctuations and stabilises their market share.

4.2.2 Terms of Trade

When the manufactured goods arrive at the border of the destination market during that same period, they are sold to the competitive collectors operating in the local distribution sector. The collectors in economy n aggregate all varieties of manufactured goods from each location i according to the CES technology due to Dixit & Stiglitz (1977):

$$y_{in,t} = \left[\sum_{\phi}\int_0^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi)^{1-1/\zeta}d\omega\right]^{1/(1-1/\zeta)}, \tag{4.2}$$

[7]Linear technology is a simple way of introducing a positive correlation between the unit costs of production and the relative value of the local currency against the major currency. A more elaborate framework should include capital and labour in a CES framework with elasticity of substitution bounded between zero and unity.

[8]By construction, iceberg costs apply only to shipments across borders, but it is equally costly to transport commodities and manufactured goods to their destination from any source country.

[9]It is well-known that most of the commodities are priced in very few currency units, especially the U.S. Dollar (USD). See Chen et al. (2010) for a more rigorous empirical motive behind this assumption.

where parameter $\zeta > 1$ stands for the intra-temporal elasticity of substitution between different varieties ω within sector ϕ, and equivalently, between different ϕ-types. The producer price index is equal to the unit costs of the collector given by a weighted average of prices associated with all manufactured goods sold in the destination market:

$$p_{in,t} = \left[\sum_{\phi} \int_{0}^{\chi_{in}(\phi)} p_{in,t}(\omega, \phi)^{1-\zeta} d\omega \right]^{1/(1-\zeta)}. \tag{4.3}$$

The homogeneous productivity in the manufacturing sector ensures that there exists a symmetric equilibrium in which all ω firms within ϕ market segment set identical prices. Both the export price index $p_{nn,t}$ and the import price index $p_{in,t}$ can then be used to assess the relationship between exchange rates and the net flows of goods and services. The latter primarily depend on the terms of trade:

$$T_{in,t} = \frac{p_{nn,t}}{p_{in,t}}, \tag{4.4}$$

which measure the amount of imports the n'th economy can purchase for every unit of exports it produces. A rise in the value of $T_{ij,t}$ therefore constitutes an improvement in the terms of trade of the j'th destination relative to the source country $i \in n \neq j$. Observe that the terms of trade in this simple framework are independent of shipping costs, since both import and export prices are linearly related to d_n by construction. Similarly, the terms of trade will improve (deteriorate) following a local currency appreciation (depreciation), but only if import prices are more elastic to exchange rate changes than export prices. The extent to which they move depends on the import-dependence of the export production technology (i.e. $\xi_i(\phi)$), the market density of firm types (i.e. $\chi_{in}(\phi)$, $\chi_{nn}(\phi)$), and the import-export price elasticity of demand (i.e. ζ). Before we turn to the general case in which international prices are sticky in the short-run, consider the simplest possible narrative in which the demand for exports is perfectly price-elastic, such that $\zeta \to \infty$. It nullifies the price mark-ups, insulates the terms of trade from strategic complementarities, and draws attention to the influence of the intermediate import intensity as well as the choice of invoicing currency.

Lemma 4. *In a symmetric equilibrium, where $\xi_i(\phi) > 0$ and $\zeta \to \infty$, the bilateral terms of trade between minor currency areas $j, i \neq k$ are neutral to bilateral exchange rate innovations if they originate from the source country, but not if they stem from the destination country:*

$$\lim_{\zeta\to\infty} \tau_{ij,t} = \lim_{\zeta\to\infty} \frac{\partial \ln T_{ij,t}}{\partial \ln q_{ij,t}} = \begin{cases} -s_{jj,t}(-\$) & \because \ln q_{ij,t} - \ln q_{ij,t-1} \neq 0|_{j\neq i,k} \\ 0 & \because \ln q_{ij,t} - \ln q_{ij,t-1} \neq 0|_{i\neq j,k} \end{cases}. \tag{4.5}$$

Proof. Let $s_{ij,t}(\phi) = (p_{ij,t}(\phi) y_{ij,t}(\phi))/(p_{ij,t} y_{ij,t}) = \chi_{ij}(\phi) \left[p_{ij,t}(\phi)/p_{ij,t}\right]^{1-\zeta} \in [0,1]$, $ptm_{ij,t}(\phi) = \partial \ln \delta_{ij,t}(\phi)/\partial \ln q_{ij,t}$, $erpt_{ii,t}(\phi) = \partial \ln p_{ii,t}(\phi)/\partial \ln q_{ji,t}$, and $erpt_{ij,t}(\phi) = \partial \ln p_{ij,t}(\phi)/\partial \ln q_{ij,t}$. First, suppose exchange rate innovations stem from the destination economy $j \neq i, k$. Then $erpt_{ii,t}(\phi) = 0 \,\forall\, \phi$, $erpt_{jj,t}(\$) = 0$, and $erpt_{jj,t}(-\$) = 1$, since $\partial \ln mc_{i,t}(-\$)/\partial \ln q_{ki,t} = 1$, $\partial \ln mc_{j,t}(-\$)/\partial \ln q_{kj,t} = 1$, but $\partial \ln mc_{i,t}(\phi)/\partial \ln q_{ji,t} = 0 \,\forall\, \phi$, $\partial \ln mc_{j,t}(\phi)/\partial \ln q_{ij,t} = 0 \,\forall\, \phi$.

Second, price mark-ups of all ϕ-types are constant and equal to zero, such that $ptm_{ij,t}(-\$) = 0$ and $ptm_{ij,t}(\$) = 1$. Because $erpt_{ij,t}(\phi) = 1 - ptm_{ij,t}(\phi) - erpt_{ii,t}(\phi) = 1 \, \forall \, \phi$, it implies $erpt_{ij,t} = \sum_\phi s_{ij,t}(\phi) erpt_{ij,t}(\phi) = 1$, while $erpt_{jj,t} = \sum_\phi s_{jj,t}(\phi) erpt_{jj,t}(\phi) = s_{jj,t}(\$)$. Consequently, $\tau_{ij,t} = \partial \ln T_{ij,t} / \partial \ln q_{ij,t} = erpt_{jj,t} - erpt_{ij,t} = s_{jj,t}(\$) - 1 = -s_{jj,t}(-\$)$. Analogously, if minor exchange rate innovations stem from the source $i \neq j, k$, then $erpt_{jj,t}(\phi) = 0 \, \forall \, \phi$, $erpt_{ii,t}(\$) = 0$, and $erpt_{ii,t}(-\$) = 1$. Because $ptm_{ij,t}(\$) = 1$ and $erpt_{ij,t}(\phi) = 1 - ptm_{ij,t}(\phi) - erpt_{ii,t}(\phi) = 0 \, \forall \, \phi$, it implies $erpt_{ij,t} = \sum_\phi s_{ij,t}(\phi) erpt_{ij,t}(\phi) = 0$ and $erpt_{jj,t} = \sum_\phi s_{jj,t}(\phi) erpt_{jj,t}(\phi) = 0$. Therefore, $\tau_{ij,t} = erpt_{jj,t} - erpt_{ij,t} = 0$. $\square$

In generic terms, minor exchange rate innovations emanating from the destination economy are fully absorbed by the import prices. Alas, the shift in the destination export prices stems from only the PCP and LCP firms, while DCP prices remain fixed. The terms of trade are thus non-neutral to exchange rates as long as the market share of DCP exporters is non-negligible. Conversely, when the minor exchange rate innovations stem from the source economy, then neither the export nor import prices of DCP firms fluctuate, while the PCP and LCP export prices in the source economy absorb the exchange rate innovations fully. Yet the exchange rate innovation from the source country is not transmitted into the import prices of the destination economy, because a weakening of the source currency relative to the major currency is simultaneously a weakening against all other minor currencies. Therefore, the cost-push effects in the source economy exactly outweigh the currency conversion effects in the destination economy. Lemma 4 brings us to a powerful policy implication – unless the bilateral exchange rate innovation is triggered unilaterally, all bilateral terms of trade movements between minor currency areas are ultimately attributable to sticky prices and strategic complementarities, rather than the choice of invoicing currency or the iceberg costs, but only if exporter technology is linear in intermediate imports and if symmetrical trade barriers are reciprocated multilaterally.[10] It also shows that the empirical regularity known as the 'terms of trade puzzle' can be resolved without pricing-to-market.[11]

When the world economy is subject to innovations in the k'th major currency area, the movements in the major exchange rates exert a direct influence on the global fluctuations of commodity prices. Therefore the unit costs of producing exported manufactured goods in all minor currency areas are particularly sensitive to innovations in $q_{kj,t} (= q_{ki,t} q_{ij,t})$, compared to $q_{ij,t}$. Unless exporters choose to invoice their profits in major currency units, a perfectly price-elastic demand for exports implies that they are unable to maintain prices constant when facing major currency innovations. DCP is in some sense synonymous to price stickiness, but more generally it is accompanied by subtleties clarified in section 4.2.4.

[10] The caveat of these predictions is simply that a more sophisticated production technology would give rise to more general outcomes nuanced by deep structural parameters. But the starting point of this debate is in-line with the empirical results of Boz et al. (2017), who demonstrate that minor currency fluctuations exert negligible influence on bilateral trade flows.

[11] The conventional wisdom embedded in the Mundell-Fleming paradigm is the Harberger-Laursen-Metzler effect, which implicitly assumes the terms of trade to move one-to-one with the real exchange rate. But Atkeson & Burstein (2008) among many others exposed a robust quantitative disconnect between the U.S terms of trade and the fluctuations of the USD. While they demonstrate that the primary attribute behind resolving this puzzle is pricing-to-market in a major currency area, such as the U.S., this chapter shows a viable alternative for other minor currency areas. In practice, they are almost surely not mutually exclusive.

Lemma 5. *When the demand for import-dependent exports is perfectly price-elastic, such that $\xi_i(\phi) \in (0,1)$ and $\zeta \to \infty$, the terms of trade elasticity to the major exchange rate innovations is equal to the additive inverse of the \$-type export market share in the destination economy:*

$$\lim_{\zeta\to\infty} \tau^k_{ij,t} = \lim_{\zeta\to\infty} \frac{\partial \ln T_{ij,t}}{\partial \ln q_{kj,t}} = -s_{jj,t}(\$) \leq 0. \tag{4.6}$$

Proof. $\zeta \to \infty$ implies $ptm^k_{ij,t}(-\$) = 0$ and $ptm^k_{ij,t}(\$) = 1$. Because $erpt^k_{ii,t}(\$) = erpt^k_{jj,t}(\$) = 0$ and $erpt^k_{ii,t}(-\$) = erpt^k_{jj,t}(-\$) = 1$, it follows that $erpt^k_{ij,t}(\phi) = ptm^k_{ij,t}(\phi) + erpt^k_{ii,t}(\phi) = 1\,\forall\,\phi$. Consequently, $erpt^k_{jj,t} = \sum_\phi s_{jj,t}(\phi) erpt^k_{jj,t}(\phi) = s_{jj,t}(-\$) = 1 - s_{jj,t}(\$)$, but $erpt^k_{ij,t} = \sum_\phi s_{ij,t}(\phi) erpt^k_{ij,t}(\phi) = \sum_\phi s_{ij,t}(\phi) = 1$. Therefore, $\tau^k_{ij,t} = erpt^k_{jj,t} - erpt^k_{ij,t} = -s_{jj,t}(\$)$. □

When the major currency area depreciates unilaterally (i.e. orthogonally to the bilateral real exchange rates of all possible pairs of minor economies), both $q_{kj,t}$ and $q_{ki,t}$ decrease, while $q_{ij,t}$ remains in-tact. In this case, lemma 5 proves that the terms of trade between minor currency areas i and j improve even though their bilateral exchange rate has not changed. The terms of trade improve because import prices of all goods and export prices of PCP and LCP goods absorb the movements in the major exchange rate through the cost-push effects of global commodity prices. At the same time, firms domiciled in the destination economy that choose to invoice their profits in major currency units are immune to these innovations, thereby keeping their prices fixed. Because import prices absorb more of the major currency innovations than the export prices, the difference being exactly the export market share of DCP firms, the terms of trade must improve. Another way of putting it is that a major currency depreciation is observationally equivalent to a boost in the import-dependent exporter productivity in all minor currency areas, which leads to a surge in global trade flows akin to the stylised facts established by Boz et al. (2017).

Lemma 6. *When import-dependent exports are perfect substitutes, such that $\xi_i(\phi) \in (0,1)$ and $\zeta \to \infty$, the terms of trade of major currency areas are neutral to their exchange rates:*

$$\lim_{\zeta\to\infty} \tau_{ik,t} = \lim_{\zeta\to\infty} \frac{\partial \ln T_{ik,t}}{\partial \ln q_{ik,t}} = 0. \tag{4.7}$$

Proof. Constant mark-ups imply $ptm_{ki,t}(\phi) = 0\,\forall\,\phi$, $ptm_{ik,t}(-\$) = 0$, and $ptm_{ik,t}(\$) = 1$. Irrespective of where the innovation emanates from, $erpt_{kk,t}(\phi) = 0\,\forall\,\phi$, $erpt^k_{ii,t}(\$) = 0$, and $erpt^k_{ii,t}(-\$) = 1$. Consequently, $erpt_{ik,t}(\phi) = 1 - ptm_{ik,t}(\phi) - erpt^k_{ii,t}(\phi) = 0\,\forall\,\phi$, $erpt_{ki,t}(\phi) = 1 - ptm_{ki,t}(\phi) - erpt_{kk,t}(\phi) = 1\,\forall\,\phi$, $erpt_{ik,t} = \sum_\phi s_{ki,t}(\phi) erpt_{ki,t}(\phi) = 0$, and $erpt^k_{kk,t} = \sum_\phi s_{ii,t}(\phi) erpt^k_{ii,t}(\phi) = 0$. Therefore, $\tau^k_{ik,t} = \partial \ln T_{ik,t} / \partial \ln q_{ik,t} = erpt^k_{kk,t} - erpt_{ik,t} = 0$. But $erpt_{ki,t} = \sum_\phi s_{ki,t}(\phi) erpt_{ki,t}(\phi) = 1$ and $erpt^k_{ii,t} = \sum_\phi s_{ii,t}(\phi) erpt^k_{ii,t}(\phi) = s_{ii,t}(-\$) = 1 - s_{ii,t}(\$)$. Thus, $\tau^k_{ki,t} = erpt^k_{ii,t} - erpt_{ki,t} = -s_{ii,t}(\$) \leq 0$. □

Ultimately, a major currency weakening leads to a collective improvement in the terms of trade among all minor currency areas as well as against the major currency area. On the other hand, the terms of trade in the major currency area remain neutral to all exchange rate innovations. The higher is the share of exports invoiced in major currency units, namely the closer are $s_{jj,t}(\$)$ and $s_{ii,t}(\$)$ to unity, the greater is the improvement in the terms of trade of minor currency areas. But there is no reason why the forces of gravity should be equally strong in both directions across the minor-minor borders. Using lemma 5, it can be shown that the difference in the terms of trade elasticities between two minor locations is given by $\lim_{\zeta\to\infty}(\tau^k_{ij,t}-\tau^k_{ji,t}) = s_{ii,t}(\$)-s_{jj,t}(\$)$. If this difference is positive (i.e. $s_{ii,t}(\$) > s_{jj,t}(\$)$), then a major currency weakening leads to a greater improvement in the terms of trade of location i relative to location j, thus boosting the bilateral competitiveness of the j'th exports relative to the ones originating from the i'th location. Hence, when tradable varieties are perfectly substitutable (i.e. $\zeta \to \infty$), a country with a lower (higher) share of $\$$-type exports experiences a trade surplus (deficit) following a major currency depreciation. Intuitively, the lower is the market share of $\$$-type firms, the more sensitive are the export prices to major exchange rate fluctuations (i.e. cost-push effects are more pronounced). Conversely, the location with a higher share of $\$$-type firms exercises more stable export prices due to a less pronounced currency mismatch, thus smoothing the international flows of goods and services.

The policy implications presented in lemmas 4 – 6 open up new channels in the already established literature on two-country models with perfectly segmented international product markets, such as Lubik & Schorfheide (2005), Adolfson et al. (2007), Corsetti et al. (2008), or De Walque et al. (2017). In particular, most of the New Keynesian open economy models are a special case of the more general framework presented in this chapter, since they attribute all of the export market power to LCP firms, which leads to insensitive terms of trade to both minor and major exchange rates. Somewhat in the fashion of Choudhri & Hakura (2015), who analyse a two-country model with both PCP and LCP firms, the present framework successfully replicates the muted responsiveness of the terms of trade to exchange rate movements in the major currency areas. However, unlike most open economy models, it predicts excess sensitivity of minor currency areas to the movements in the major exchange rate due to the adoption of DCP strategies. Moreover, the degree to which the terms of trade respond to minor exchange rate fluctuations also depends on where the innovation originates from. It is worth reemphasising that the conventional wisdom prevails only in the special case when exporters in each location produce manufactured goods independently from intermediate imports. In that counterfactual scenario, exchange rate pass-through into import prices would be symmetric for all currency types: $\lim_{\xi_i(\phi)\to 0} erpt^k_{ij,t} = erpt_{ij,t}$. And yet it would still respond to innovations endogenously if tradable varieties were imperfectly substitutable, since the market share and thus the price mark-up of ℓ-type firms would be time-varying.

To summarise, the heterogeneity in the choice of invoicing currency and intermediate import intensity can generate novel and unprecedented predictions in the global export market structure. Specifically, major currency depreciations (appreciations) intensify (abate) rest-of-the-world trade flows and lead to global trade imbalances. However, most of the results presented so far examine a set of corner solutions - perfect competition or autonomous production of exports - neither one of which is a fully accurate depiction of the export market structure in the light of Yeaple (2013), who documents widespread Global Value Chains (GVC) dominated by large-scale multi-national firms. Without explicitly modelling the intermediate case of imperfect competition (i.e. $1 < \zeta < \infty$) and import-dependent technology of exports (i.e. $\xi_i(\phi) > 0$), not much can be said about the dynamics of the export price mark-ups, which has come under scrutiny in recent years due to Atkeson & Burstein (2008), Berman et al. (2012), and Amiti et al. (2018) among others. The goal of the remaining parts of the chapter is therefore to generalise the stylised properties proposed so far by introducing short-run price stickiness and a time-varying elasticity of substitution between exported goods. The combination of these properties will generate persistence of export prices and strategic complementarities among different ϕ-types, which play an important role in terms of assessing the qualitative implications of the net flows of goods and services.

4.2.3 Imported Inflation

Once all of the imported manufactured goods are sorted into CES bundles by origin, they are stored in the warehouse of retail goods, where they are merged into a CES bundle of locally-produced and imported manufactured goods:

$$x_{i,t} = \left[\sum_{n=1}^{N} \alpha_{ni}^{1/\eta} \, y_{ni,t}^{1-1/\eta} \right]^{1/(1-1/\eta)}, \tag{4.8}$$

such that $\eta > 1$ is the elasticity of substitution between the domestic and foreign manufactured goods, and parameter $\alpha_{ii} = 1 - \sum_{n=1}^{N-i} \alpha_{ni} \in (0, 1)$ measures the i'th economy home-bias. The weighted average of all international price indices pertaining to the i'th economy measures the aggregate producer price index:

$$p_{i,x,t} = \left[\sum_{n=1}^{N} \alpha_{ni} \, p_{ni,t}^{1-\eta} \right]^{1/(1-\eta)} \tag{4.9}$$

The relationship between the tradable goods and non-tradable goods in the final basket of consumer goods is established by a Cobb-Douglas production technology of a competitive retailer, who distributes the goods to the households:

$$c_{i,t} = (a_{i,t} h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i}, \tag{4.10}$$

where $h_{i,t} \in (0, 1)$ are the aggregate hours that are spent in the labour force relative to the total endowment of time of the i'th economy households, while $\alpha_i \in (0, 1)$ controls the share of non-tradable goods in the basket of consumer goods.[12] Labour productivity is a unit root process with a drift $a_{i,t} = \gamma_i a_{i,t-1} \exp(\sigma_{i,a} \epsilon_{i,a,t})$, where $\gamma_i > 1$ measures the gross deterministic trend of the real hourly wage rate, while $\sigma_{i,a} > 0$ and $\epsilon_{i,a,t} \sim iid(0, 1)$. It follows that the consumer price index is a function of the variable costs facing the retailer, namely the aggregate producer price index and the effective real wage:

$$p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i} \right)^{\alpha_i} \left(\frac{p_{i,x,t}}{1-\alpha_i} \right)^{1-\alpha_i}. \tag{4.11}$$

The salient features of the global export market structure developed in the previous section are thus reflected in $p_{i,x,t}$. By contrast, the prices of non-traded goods are henceforth depicted

[12] See Burstein et al. (2003) and Campa & Goldberg (2010) for a thorough literature survey and empirical estimates of the relative size of the distribution sector in G7 and OECD economies respectively.

most parsimoniously for two reasons. First, Engel (1999) shows that real exchange rate volatility in most advanced economies can be accounted for by the movements in relative prices of tradable goods, while the relative price of non-tradable goods are largely disconnected. Second, Atkeson & Burstein (2008) argue that non-traded goods and services, such as those associated with transportation and distribution of manufactured imports, are mostly labour-intensive, such that their output is proportional to the labour input. It follows that the production costs of non-traded goods consist primarily of the wage bill $w_{i,t}h_{i,t}$, which in turn mostly depends on the long-run labour productivity growth and is largely orthogonal to exchange rate movements in the short-run. Although the role of non-traded goods and services in determining exchange rate volatility is limited, they enter the framework as one of the primary insulators of consumer prices from large movements in exchange rates.

Lemma 7. *Major exchange rate pass-through into consumer prices is proportional to the sum of trade-weighted import price elasticities from all locations:*

$$erpt^k_{i,t} = \frac{\partial \ln p_{i,t}}{\partial \ln q_{ki,t}} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt^k_{ni,t}. \tag{4.12}$$

Proof. Let $s_{ni} = (p_{ni,t}y_{ni,t})/(p_{i,x,t}x_{i,t}) = \alpha_{ni}[p_{ni,t}/p_{i,x,t}]^{1-\eta} \in [0,1]$ measure the trade weight of the source country n in destination i. The prices of non-tradable goods are not directly related to exchange rates by construction, such that $\partial \ln w_{i,t}/\partial \ln q_{ki,t} = 0$. By contrast, equation (4.9) expresses the prices of tradable goods as a trade-weighted average of domestic export and import prices, such that $\partial \ln p_{i,x,t}/\partial \ln q_{ki,t} = \sum_{n=1}^{N} s_{ni,t} erpt^k_{ni,t}$, which is non-trivially positive as long as innovations stem from the k'th major currency area. The exchange rate elasticity of consumer prices is thus proportional to the elasticity of tradable goods prices, namely $\partial \ln p_{i,t}/\partial \ln q_{ki,t} = (1-\alpha_i)\partial \ln p_{i,x,t}/\partial \ln q_{ki,t} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt^k_{ni,t}$. □

In essence, when the k'th economy exerts dominance over the remaining $N-k$ currencies, the magnitude of imported inflation in minor economies will be particularly sensitive to the movements in the major exchange rate. However, exchange rate pass-through into consumer prices is dampened by the trade weights (i.e. home bias), which measure the intensity of international commerce between location i and n, and the presence of non-traded goods, whose prices are *ex ante* orthogonal to the exchange rates. Moreover, if the majority of imports are priced in the local currency terms, then the minor exchange rate acts as a shock absorbing mechanism, in which case the magnitude of imported inflation from other minor locations is effectively zero. Analogous conclusions are drawn for the case of exchange rate pass-through following i'th currency innovation: $erpt_{i,t} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt_{ni,t}$. Although export prices in all $j \neq i, k$ locations remain fixed despite movements in the i'th ($\neq k$'th) currency, the import prices from all minor economies respond, since $\dot{q}_{ji,t} = q_{ji,t}/q_{ji,t-1}$ deviates from unity in the short-run. On the other hand, export prices in economy i respond as though it is a shock to the value of the major currency, since it implies movements in $q_{ki,t}$ and the unit costs of producing an exported good fluctuate accordingly.

The theoretical concept of exchange rate pass-through into consumer prices goes hand-in-hand with both the structural and the stochastic properties of the macroeconomic developments. Lemma 7 shows that the model encompasses both of these elements by resorting to the salient features of the global export market structure. Specifically, each individual producer of manufactured goods ω is assumed to be infinitesimally 'small', such that it can only influence the price of its manufactured good within the sector ϕ. However, the non-degenerate market density of $\chi_{in}(\phi)$ ensures that the ϕ-segment of the market as a whole is not infinitesimally 'small', in principal allowing for strategic complementarities between different ϕ-types. If exporters populating the same segment of the market indeed recognised their collective market power, then the framework would implicitly adopt an oligopolistic market structure, thereby generating additional layers of transmission.

And yet if we were to close the model and then plot the impulse response functions of import, export, or consumer price inflation against the exchange rate triggered by an exogenous innovation in the foreign exchange market – it would almost certainly give us a different magnitude of exchange rate pass-through.[13] At the very least, it would be biased in the long-run, because all impulse response functions characterising a saddle-path stable solution gradually decay to zero, while the market shares, along which the terms of trade elasticity is centred, are characterised by a well-defined and non-degenerate steady state. The idea is that the two approaches – the formal and the reduced-form – ultimately measure two vaguely related concepts: transmission and co-movement, the difference between which has nothing to do with strategic complementarity. Instead, impulse response functions of prices and exchange rates are tainted by model-specific properties controlling their aggregate co-movements. But these co-movements are irrelevant to the optimal exchange rate pass-through of the individual exporters, because they do not exert sufficient market power to take into the account their influence on aggregate prices. The next section formally characterises the exchange rate transmission channel that abstracts from the notion of aggregate co-movements.

4.2.4 Sticky Prices & Strategic Complementarity

Suppose all import-export manufacturers in each economy are characterised by rational expectations. Suppose further that every time they adjust prices relative to the local long-run trend of inflation $\dot{p}_n \geq 1$, the firms incur quadratic price adjustment costs *à la* Rotemberg (1982). The size of the price adjustment costs depend on ϕ, since it is an indicator assigning the invoicing currency in which prices are sticky. Following Kim & Ruge-Murcia (2009), price adjustment is viewed as an unproductive activity that does not produce any value-added, thus price adjustment costs are deducted from the total revenue. Yet unlike in closed economy models, price adjustment costs are deducted only from the revenue of the invoiced currency,

[13] The approach of measuring exchange rate pass-through agnostically by merely inspecting the impulse response functions of price inflation and exchange rates was pioneered by Shambaugh (2008) and later adopted by Choudhri & Hakura (2015), Casas et al. (2016), and Forbes et al. (2017) among countless others. It's appeal stems from the fact that the agnostic exchange rate pass-through is simple to measure and its magnitude is conditional on observing a certain state of the economy, thereby establishing a dichotomy between the long-standing emphasis on the importance of economic structure versus the seemingly shock-dependent nature of the data generating process.

while the imperfect arbitrage forces characterising the wedge in equation (4.1) reflect the internalised adjustment costs of the remaining international prices that are ultimately shared between the producers and the consumers in equilibrium.

The oligopolistically-competitive ϕ-type import-export manufacturer domiciled in the i'th economy chooses the nominal price $P_{in,t}(\omega,\phi)$ to maximise the expected present discounted value of the real future profit dividends generated in the n'th destination:

$$\max_{\{P_{in,t}(\omega,\phi)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{i,t,t+1} \left\{ \sum_{n=1}^{N} \left(1 - \mathit{\Delta}_{in,t}(\omega,\phi)\right) p_{in,t}(\omega,\phi)\, y_{in,t}(\omega,\phi) - mc_{i,t}(\phi)\, y_{in,t}(\omega,\phi) \right\}$$

$$\text{s.t.} \quad \mathit{\Delta}_{in,t}(\omega,\phi) = \frac{\kappa_i(\phi)(\dot{p}_{in,t}(\omega,\phi) - \dot{p}_n)^2}{2},$$

$$\text{s.t.} \quad y_{in,t}(\omega,\phi) = y_{in,t} \left[\frac{p_{in,t}(\omega,\phi)}{p_{in,t}}\right]^{-\zeta},$$

$$\text{s.t.} \quad y_{in,t} = \alpha_{in,t} x_{n,t} \left[\frac{p_{in,t}}{p_{n,x,t}}\right]^{-\eta},$$

where $\lambda_{i,t,t+1}$ stands for the stochastic discount factor, $\kappa_i(\phi) \geq 0$ measures the convexity of the quadratic price adjustment costs, while $\dot{p}_{in,t}(\omega,\phi) = P_{in,t}(\omega,\phi)/P_{in,t-1}(\omega,\phi) = \dot{p}_{n,t}(p_{in,t}(\omega,\phi)/p_{in,t-1}(\omega,\phi))$ denotes the gross rate of import price inflation. Manufacturers are subject to two types of demand constraints: firm-specific (i.e. $y_{in,t}(\omega,\phi)$) and country-specific (i.e. $y_{in,t}$). This implies that each ϕ-type is not infinitesimally 'small', unlike each individual firm ω, controlling just their individual prices. Instead, each $\chi_{in}(\phi)$ density as a whole is sufficiently 'large' to influence the country-wide import price indices. However, they are still sufficiently 'small' at the aggregate level, since they take the aggregate producer price index and the consumer price index as given. Incorporating the second demand constraint is the distinguishing feature of the Bertrand-competitive market structure as in Atkeson & Burstein (2008), which gives rise to non-trivial and endogenous market shares for all ϕ-types.

The first order condition of the ϕ-type firms evaluated at the symmetric equilibrium is summarised by the following set of equations:

$$p_{in,t}(\phi) = \frac{\mathit{\Theta}_{in,t}(\phi) mc_{i,t}(\phi)}{\mathit{\Phi}_{in,t}(\phi)}, \tag{4.13}$$

$$\mathit{\Theta}_{in,t}(\phi) = \frac{\varepsilon_{in,t}(\phi)}{\varepsilon_{in,t}(\phi) - 1}, \tag{4.14}$$

$$\varepsilon_{in,t}(\phi) = \zeta(1 - s_{in,t}(\phi)) + \eta s_{in,t}(\phi), \tag{4.15}$$

$$\mathit{\Phi}_{in,t}(\phi) = 1 - \mathit{\Delta}_{in,t}(\phi) + \frac{\mathit{\Delta}'_{in,t}(\phi)\dot{p}_{in,t}(\phi)}{\varepsilon_{in,t}(\phi) - 1} - \lambda_{i,t,t+1}\mathbb{E}_t \left[\frac{y_{in,t+1}(\phi)}{y_{in,t}(\phi)} \frac{\mathit{\Delta}'_{in,t+1}(\phi)}{\varepsilon_{in,t}(\phi) - 1} \frac{\dot{p}^2_{in,t+1}(\phi)}{\dot{p}_{n,t+1}}\right]. \tag{4.16}$$

The optimal real price $p_{in,t}(\phi)$ depends on the unit costs of producing an exported good $mc_{i,t}(\phi)$, the variable price mark-up $\mathit{\Theta}_{in,t}(\phi)$, and $\mathit{\Phi}_{in,t}(\phi)$, which denotes the persistence induced by the presence of price adjustment costs. Under the plausible assumption of $\eta < \zeta$, which implies that the varieties of goods are more substitutable within each sector than across nations, a major currency appreciation vis-à-vis the i'th currency (i.e. a rise in $q_{ki,t}$) pushes up the unit costs of producing exported goods in location i, thereby increasing the import prices of all ϕ-types in all n foreign destinations. Although a rise in the import prices shrinks the i'th economy import penetration in the foreign markets (i.e. s_{in} for all $n \neq i$), the extent to which it shrinks the ϕ-type market share abroad $s_{in,t}(\phi)$ depends on the increase in the ϕ-type elasticity of substitution $\varepsilon_{in,t}(\phi)$, since the latter effect leads to a decrease in the price mark-up $\Theta_{in,t}(\phi)$ and an incomplete pass-through into import prices. The novel part of the exchange rate channel depicted in this model is therefore the presence of strategic complementarities among all ϕ-types, which arise because adjustments of the sector-wide market shares are a zero-sum game, since $\sum_{\phi} s_{in,t}(\phi) = 1$ holds at all times by construction.

Lemma 8. *If $\kappa_i(\phi) > 0$ and $1 < \eta < \zeta < \infty$, then the major exchange rate pass-through into export prices is endogenous to the price stickiness and strategic complementarities:*

$$erpt^{k}_{ii,t}(\phi) = \frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}} = \frac{\Gamma_{ii,mc,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi) erpt^{k}_{ii,t}(-\phi)}{1 + \Gamma_{ii,\Delta,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)}, \tag{4.17}$$

$$\Gamma_{ii,\Theta,t}(\phi) = \frac{(\zeta-1)(\zeta-\eta)}{\varepsilon_{ii,t}(\phi)-1}\frac{s_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)}, \tag{4.18}$$

$$\Gamma_{ii,\Delta,t}(\phi) = \Xi_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t\left[\Xi_{ii,t+1}(\phi)\left(\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right)\right], \tag{4.19}$$

$$\Xi_{ii,t}(\phi) = \frac{\dot{p}_{ii,t}(\phi)\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\phi) + (2-\varepsilon_{ii,t}(\phi))\Delta'_{ii,t}(\phi)\right]}{\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}, \tag{4.20}$$

where $\Gamma_{ii,\Delta,t}(\phi)$ and $\Gamma_{ii,\Theta,t}(\phi)$ stand for the ϕ-type price stickiness and strategic complementarity components respectively, while $\Gamma_{ii,mc,t}(-\$) = 1$ and $\Gamma_{ii,mc,t}(\$) = 0$.

Proof. See appendix B.1. □

The presence of sticky prices and strategic complementarities complicates the determination of the period-by-period exchange rate pass-through into export, import, and consumer prices. In the static price setting depicted in lemmas 4 – 6, the only source of pass-through time variability stems from a random draw of innovations to productivity and exchange rates. By contrast, when prices are set dynamically, the persistence of pass-through is driven by the convexity of price adjustment costs and the time variability stems from the top-down influence of numerous aggregate variables. Lemma 8 proves that a closed-form solution to the transition and the balanced growth paths of optimal exchange rate pass-through for each ϕ-type can be obtained directly from the first order condition displayed in equations (4.13) – (4.16). The pretence that individual firms are too 'small' to influence the aggregate variables, while ϕ-type density as a whole can only influence the sectoral import prices, separates the transmission channel from the indirect income effects.

In order to further disentangle the exchange rate transmission channel into export and import prices by country, by sector, and over-time, the general equilibrium model must firstly be closed and subsequently equations (4.17) – (4.20) must be simulated along with all other difference equations characterising the dynamics of the macroeconomic fundamentals. While the aggregate fundamentals influence the optimal exchange rate pass-through in equilibrium, there is no feedback mechanism the other way around. This implies that all time variation in pass-through is endogenous to the state of the economy in the short-run, but the shock-dependence patterns do not supersede economic structure in the long-run when the distortionary remnants of transitory innovations fade away. Because it is particularly difficult to dissect the *ex ante* influence of strategic complementarities analytically when $\eta < \zeta$, it is paramount to abstract from the indirect general equilibrium effects when generating impulse response functions of macroeconomic fundamentals and exchange rate pass-through. Specifically, the agnostic measures of pass-through, such as the ratio of cumulative impulse response functions to prices and exchange rates triggered by an exogenous innovation as in Shambaugh (2008), are convoluted by extraneous model-specific co-movements. On the other hand, impulse response functions to $erpt^k_{ii,t}$ uncover the unbiased channel of exchange rate transmission in a controlled general equilibrium environment.

Before we close and simulate the model, consider the corner solution in which all international prices are perfectly rigid in the short-run. This simplified narrative sits on the opposite side of the spectrum to the scenario of perfectly price-elastic demand for exports that is scrutinised in lemmas 4 – 6. These extreme settings are indeed important to contemplate in that they highlight the limits to the domain of the terms of trade elasticity with respect to the major currency innovations – it is always bounded by their interval conditional on the deep structural parameters.

Lemma 9. *If international prices of minor currency areas are perfectly rigid in their invoiced currencies, such that* $\kappa_i(\phi), \kappa_j(\phi) \to \infty \,\forall\, \phi \in \{\pi, \ell, \$\}$*, then the share of imports invoiced in major currency units determines the terms of trade elasticity to major currency innovations:*

$$\lim_{\kappa_i(\phi),\, \kappa_j(\phi) \to \infty \,\forall\, \phi} \tau^k_{ij,t} = \lim_{\kappa_i(\phi),\, \kappa_j(\phi) \to \infty \,\forall\, \phi} \frac{\partial \ln T_{ij}}{\partial \ln q_{kj}} = -s_{ij,t}(\$) \leq 0. \tag{4.21}$$

Proof. A fall (rise) in $q_{kj,t}$ implies a fall (rise) in $q_{ki,t}$, while $q_{ij,t}$ remains in-tact. Thus $erpt^k_{ii,t}(-\$)$, $erpt^k_{jj,t}(-\$) \in (0,1)$ as long as $\kappa_i(-\$)$, $\kappa_n(-\$) \geq 0$. On the other hand, $erpt^k_{ii,t}(\$) = erpt^k_{jj,t}(\$) = 0$ irrespective of $\kappa_i(\$)$, $\kappa_n(\$)$. The ℓ-type firms price-to-market, since ℓ-type import prices in local currency terms respond analogously to export prices, such that $erpt^k_{ij,t}(\ell)$, $erpt^k_{ii,t}(\ell) \in (0,1)$, while $ptm^k_{ij,t}(\ell) = erpt^k_{ii,t}(\ell) \in (0,1) - erpt^k_{ij,t}(\ell)$ moves persistently in order to clear the market. The π-type firms keep their price mark-ups fixed by construction, such that $erpt^k_{ii,t}(\pi) = erpt^k_{ij,t}(\pi) \in (0,1)$, resulting in $ptm^k_{ij,t}(\pi) = 0$. The $\$$-type import prices absorb the major currency innovations fully, since $erpt^k_{ij,t}(\$) = ptm^k_{ij,t}(\$) + erpt^k_{ii,t}(\$) = 1$. Because $erpt^k_{ij,t} = \sum_\phi s_{ij,t}(\phi) erpt^k_{ij,t}(\phi)$, it follows that $erpt^k_{ij,t} = s_{ij,t}(\$) + s_{ij,t}(\ell) ptm_{ij,t}(\ell) + \sum_{-\$} s_{ij,t}(-\$) erpt_{ii,t}(-\$)$. Observe that $\lim_{\kappa_i(\phi)\to\infty} \mathit{\Gamma}_{ii,\Delta,t}(\phi) = \infty \,\forall\, \phi$, thus $\lim_{\kappa_i(\ell)\to\infty} ptm_{ij,t}(\ell) = 0$, and more generally $\lim_{\kappa_n(\phi)\to\infty} erpt_{nn,t}(\phi) = 0 \,\forall\, \phi$. Therefore $\lim_{\kappa_i(\phi),\, \kappa_j(\phi)\to\infty \,\forall\, \phi} \tau^k_{ij,t} = \lim_{\kappa_i(\phi),\, \kappa_j(\phi)\to\infty \,\forall\, \phi} [erpt^k_{jj,t} - erpt^k_{ij,t}] = -s_{ij,t}(\$)$. □

To elaborate, when export prices are perfectly rigid, the major exchange rate pass-through into export prices in minor currency areas is equal to zero as though exporters are autonomous to intermediate imports. The only difference is that the cost-push effects are internalised by the profit margins, thus generating foregone profits relative to the inter-temporal optimum. Unless the firm chooses to invoice profits in major currency units, those losses will be proportional to the size of the major currency innovations and the difference between inter-sectoral and inter-national elasticities of substitution. Hence, when the latter difference is infinitesimally small, the elasticity of substitution is constant over time, since $\lim_{\zeta\to\eta}\varepsilon_{in,t}(\phi)=\varepsilon$, thereby switching off the strategic complementarities across sectors, such that $\lim_{\zeta\to\eta}\Gamma_{ii,\Theta,t}(\phi)=0$. If prices are perfectly rigid and the elasticity of substitution is constant over time, then the price mark-ups are constant over time and there is no room for pricing-to-market to smooth out transitory fluctuations in profits. Irrespective of whether price mark-ups adjust, the perfect disintegration of the product market for goods invoiced in major currency units implies that the DCP import prices in minor currencies absorb all movements in the major exchange rate, while PCP and LCP import prices remain constant.

Lemma 10. *If all international prices are perfectly rigid in their invoiced currencies, such that $\kappa_i(\phi),\kappa_k(\phi)\to\infty\,\forall\,\phi\in\{\pi,\ell,\$\}$, then the share of imports invoiced in producer currency units determines the major economy's terms of trade elasticity to its own currency innovations:*

$$\lim_{\kappa_i(\phi),\,\kappa_k(\phi)\to\infty\,\forall\,\phi}\tau_{ik,t}=\lim_{\kappa_i(\phi),\,\kappa_k(\phi)\to\infty\,\forall\,\phi}\frac{\partial\ln T_{ik}}{\partial\ln q_{ik}}=-s_{ik,t}(\pi)\leq 0. \tag{4.22}$$

Proof. Perfect price rigidity implies $\Gamma_{ii,\Delta,t}(\phi)\to\infty\,\forall\,\phi$, therefore $erpt^{k}_{ii,t}(\phi)=0\,\forall\,\phi$ and $erpt_{ik,t}(\ell)=0$, such that $ptm_{ik,t}(\pi)=0$ and $ptm_{ik,t}(-\pi)=1$. It follows that $erpt_{ik,t}=\sum_\phi s_{ik,t}(\phi)erpt_{ik,t}(\phi)=\sum_\phi s_{ik,t}(\phi)[1-ptm_{ik,t}(\phi)-erpt^{k}_{ii,t}(\phi)]=s_{ik,t}(\pi)$. Then because $erpt_{kk,t}(\phi)=0\,\forall\,\phi$, we have $\tau_{ik,t}=\partial\ln T_{ik}/\partial\ln q_{ik}=erpt_{kk,t}-erpt_{ik,t}=-s_{ik,t}(\pi)$. By the same token, $ptm_{ki,t}(\pi)=0$ and $ptm_{ki,t}(\ell)=1$, thus $erpt_{ki,t}=s_{ki,t}(\pi)$ and $erpt^{k}_{ii,t}=0$. Consequently, $\tau_{ki,t}=\partial\ln T_{ik}/\partial\ln q_{ik}=erpt^{k}_{ii,t}-erpt_{ki,t}=-s_{ki,t}(\pi)\leq 0$. □

The major-minor terms of trade respond only by as much as there are firms pricing their exports at the factory door relative to the size of the destination market. Unlike in lemma 4, where prices are perfectly flexible, the origin of innovations is irrelevant for their comparative advantage. But heterogeneities prevail between minor-minor and major-minor terms of trade elasticities. Specifically, in the classical Mundell-Fleming paradigm, where all firms price their goods at the factory door, such that $s_{in,t}(\pi)=1\,\forall\,i\in n$, the distinction between major and minor is redundant, since the terms of trade always move one-to-one with the bilateral exchange rate due to the absence of hegemonic power seized by any one region. It delivers the full intensity of the Harberger-Laursen-Metzler effect, making expansionary monetary policy in open economies highly effective by boosting net exports. And yet in the globalised export market structure in which the share of PCP imports in major currency areas is negligible, lemma 10 proves that minor (major) currency areas are unable to boost their comparative advantage against the major (minor) currency areas. The only way for them to gain any comparative advantage against other minor currency areas when international prices are perfectly rigid in the short-run is to import more final goods invoiced in major currency units (see lemma 9).

4.2.5 Sticky Wages

Suppose each economy $i \in n$ is populated by a continuum of infinitely-lived households indexed by $\omega \in [0, 1]$. All households are characterised by rational expectations, but they develop habits over the consumption of goods and services. In each time period, households derive utility from the consumption of habit-adjusted stock of final goods and disutility from the hours spent in the labour force:

$$u_{i,t} = \ln\left(\frac{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right) h_{i,t}(\omega)^{1+\varphi_i}, \tag{4.23}$$

where $\vartheta_i \in [0, 1)$ measures the intensity of consumption habits, $\varphi_i \geq 0$ is the inverse Frisch elasticity of labour, and ψ_i stands for the relative disutility of labour. Each household provides a unique service to a competitive trade union, which aggregates these services and outsources workers to the retailer according to the following CES technology:

$$h_{i,t} = \left[\int_0^1 h_{i,t}(\omega)^{1-1/\zeta} d\omega\right]^{1/(1-1/\zeta)}. \tag{4.24}$$

Due to the imperfect substitutability of each service captured by the elasticity of substitution $\zeta > 1$, households negotiate over their hourly wages with the trade unions and the employers. However, the adjustments of wages in response to innovations occur gradually, because it takes time and resources to reach an agreement through the process of collective bargaining. Specifically, every time wages adjust, households incur one-off quadratic costs proportional to the total wage bill denoted by $\mathit{\Delta}_{i,w,t}(\omega)$.

Formally, households in each economy $i \in n$ maximise their lifetime utility subject to an indefinite sequence of budget constraints and the demand for their labour service:

$$\begin{aligned}
\max_{\{c_{i,t}(\omega), w_{i,t}(\omega), b_{i,t+1}(\omega)\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \ln\left(\frac{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right) h_{i,t}(\omega)^{1+\varphi_i} \\
\text{s.t. } & c_{i,t}(\omega) + \lambda_{i,t,t+1}\mathbb{E}_t[b_{i,t+1}(\omega)] = b_{i,t}(\omega) + (1 - \mathit{\Delta}_{i,w,t}) w_{i,t}(\omega) h_{i,t}(\omega) + v_{i,t}(\omega), \\
\text{s.t. } & h_{i,t}(\omega) = h_{i,t}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\zeta}, \\
\text{s.t. } & \mathit{\Delta}_{i,w,t}(\omega) = \frac{\kappa_{i,w}(\dot{w}_{i,t}(\omega) - \gamma_i)}{2}.
\end{aligned}$$

Parameter $\beta_i \in (0, 1)$ denotes time preference, $\dot{w}_{i,t} = (w_{i,t}/w_{i,t-1})$ stands for the gross growth rate of the real hourly wage rate, $\kappa_{i,w} \geq 0$ measures the convexity of the wage adjustment costs, $b_{i,t}$ is the stock of country-specific bonds, and $v_{i,t} = \mathit{\Pi}_{i,t} + M_{i,t}$ is the exogenously given stock of wealth acquired from the ownership of the firms and the sales of commodities to foreign producers, such that $\mathit{\Pi}_{i,t} = \sum_{n=1}^{N} \sum_{\phi} \mathit{\Pi}_{in,t}(\phi)$, while the term $M_{i,t} = \sum_{n=1}^{N} q_{kn,t} \sum_{\phi} m_{ni,t}(\phi)$ measures the total sales of commodities abroad.

The competitive market structure for commodities ensures that even if the country-specific bonds are in a zero net supply in the long-run, the trade surplus in the final goods can be sustained by a persistent trade deficit in the intermediate goods. It also imposes strong balance sheet effects that are often observed in emerging markets along the lines of the 'original sin' described in Eichengreen & Hausmann (2005). In particular, a major currency appreciation causes the value of equity to contract and the value of liabilities to expand in all minor economies. However, more generally, long-run trade imbalances are determined by the differences in the home-bias across different economies and the heterogeneities in the import penetration ratios. They can be financed by either borrowing from abroad in local currency terms or selling commodities in major currency units, thereby introducing a highly pro-cyclical stock of private debt at the national level. This proposition places itself in parallel with Benigno & Thoenissen (2008), who presented a compelling resolution to the notorious 'consumption correlation puzzle' portrayed by Backus & Smith (1993). It concerns the weak, if not negative, correlation in OECD economies between aggregate real consumption and the value of the local currency in real terms, while the opposite turns out to be true in many real business cycle models that impose perfect capital mobility and complete global financial market structures. Yet an imperfect international financial market structure in minor currency areas has become a less innocuous framework in recent times in the light of IMF (2016), who document a substantially diminished currency mismatch of liabilities.

The first order conditions of the households evaluated at the symmetric equilibrium are summarised as follows:

$$\begin{aligned}
w_{i,t} =& \left(\frac{\zeta}{\zeta-1}\right)\frac{mrs_{i,t}}{\mathit{\Psi}_{i,t}}, && (4.25)\\
\mathit{\Psi}_{i,t} =& 1-\mathit{\Delta}_{i,w,t}+\frac{\mathit{\Delta}'_{i,w,t}\dot{w}_{i,t}}{\zeta-1}-\lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{h_{i,t+1}}{h_{i,t}}\frac{\mathit{\Delta}'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\zeta-1}\right], && (4.26)\\
1 =& \beta_i\mathbb{E}_t\left[\frac{u_{i,c,t+1}}{\lambda_{i,t,t+1}u_{i,c,t}}\right], && (4.27)\\
mrs_{i,t} =& -\frac{u_{i,h,t}}{u_{i,c,t}}, && (4.28)\\
u_{i,h,t} =& -\psi_i h_{i,t}^{\varphi_i}, && (4.29)\\
u_{i,c,t} =& \frac{1}{c_{i,t}-\vartheta_i c_{i,t-1}}-\beta_i\vartheta_i\mathbb{E}_t\left[\frac{1}{c_{i,t+1}-\vartheta_i c_{i,t}}\right]. && (4.30)
\end{aligned}$$

On average, wages are set above the marginal rate of substitution between consumption and labour $mrs_{i,t}$, where the term $u_{i,h,t} = \partial u_{i,t}/\partial h_{i,t}$ is the marginal disutility of labour and $u_{i,c,t} = \partial u_{i,t}/\partial c_{i,t}$ is the marginal utility of consumption. However, $mrs_{i,t}$ fluctuates by more and also more rapidly in response to innovations in the macroeconomic fundamentals, since wage inflation is characterised by persistent movements due to $\mathit{\Psi}_{i,t}$, which differs from unity in the short-run. Specifically, $mrs_{i,t}$ and $\mathit{\Psi}_{i,t}$ are positively correlated, such that an increase in the former leads to a less than one-to-one increase in the later, causing a lagged response of real wages to innovations. The Euler equation for the stock of bonds defines the stochastic discount factor $\lambda_{i,t,t+1}$, which is a reciprocal of the gross risk-free rate of return in real terms. The introduction of additive habits is motivated by the seminal contribution of Constantinides (1990), who demonstrates that they lower the inter-temporal substitutability of consumption and lower the volatility of the stochastic discount factor, ergo resolving the well-known 'equity premium and risk-free rate puzzles'.

4.2.6 Monetary Policy

The perpetual growth of labour productivity represented by the deterministic trend in the retail production technology deems the entire system of difference equations characterising the world economy inherently unstable. In order to represent such an environment in a stationary system of difference equations, the equilibrium conditions of the households and the firms are transformed akin to the seminal contribution of Merton (1975), who normalises the stock variables by the contemporaneous level of productivity.[14] The solution to this augmented system of difference equations characterises the evolution of all variables over time around a balanced growth path rather than a deterministic steady state.

Once the mechanical non-stationarity of the system is mitigated, the only other source of aggregate instability stems from the combination of sticky prices and sticky wages. The latter source of non-stationarity is counteracted endogenously by the interventions of the local central banks in the international bonds markets, on account that it provides guidance to the product and labour market equilibria towards the balanced growth path. In this model, these interventions are depicted by a stochastic Taylor rule to which local central banks credibly commit for an indefinite period of time:

$$r_{i,t} = (r_{i,t-1})^{\rho_{i,r}} (r^*_{i,t})^{1-\rho_{i,r}}, \tag{4.31}$$

$$r^*_{i,t} = \frac{\dot{p}_i \gamma_i}{\beta} \left(\frac{\dot{p}_{i,t}}{\dot{p}_i}\right)^{\nu_{i,p}} \left(\frac{\tilde{y}_{i,t}}{\tilde{y}_i}\right)^{\nu_{i,y}} \exp(\sigma_{i,r}\epsilon_{i,r,t}), \tag{4.32}$$

where $\sigma_{i,r} > 0$, $\rho_{i,r} \in (0,1)$, $\epsilon_{i,r,t} \sim iid(0,1)$, $\nu_{i,p} > 0$, $\nu_{i,y} > 0$, and $\tilde{y}_{i,t} = y_{i,t}/a_{i,t}$.

In the absence of public or private investment, the aggregate demand in this economy consists of aggregate consumption, the trade balance, and short-run externalities such as price adjustment costs internalised by the import-export manufacturers and wage adjustment costs internalised by the households:

$$\tilde{y}_{i,t} = \tilde{c}_{i,t} + \tilde{nx}_{i,t} + \mathit{\Delta}_{i,w,t}\tilde{w}_{i,t}h_{i,t} + \sum_{n=1}^{N}\sum_{\phi} \mathit{\Delta}_{in,t}(\phi)p_{in,t}(\phi)\tilde{y}_{in,t}(\phi), \tag{4.33}$$

$$\tilde{nx}_{i,t} = \sum_{n=1}^{N-i} p_{in,t}q_{ni,t}\tilde{y}_{in,t} - p_{ni,t}y_{ni,t}. \tag{4.34}$$

When the international flows of country-specific bonds are unrestricted, the possibility of fixed exchange rates is implicitly ruled out. Consequently, the combination of incomplete financial markets, floating exchange rates and liberalised financial capital flows implies that the real exchange rate is determined by the perfect consumption risk sharing relationship:

$$q_{ni,t}\tilde{u}_{i,c,t} = \mu_i \tilde{u}_{j,c,t} e_{i,t}, \tag{4.35}$$

[14]The full list of stationarised equilibrium conditions are displayed in Appendix B.5

such that $\mu_{ni} = 1/\mu_{in} > 0$, $e_{i,t} = e_{i,t-1}^{\rho_{i,e}} \exp(\sigma_{i,e}\epsilon_{i,e,t})$, $\rho_{i,e} \in (0,1)$, $\sigma_{i,e} > 0$ and $\epsilon_{i,e,t} \sim iid(0,1)$. The term $e_{i,t}$ stands for exchange rate noise, capturing all of the exchange rate movements unexplained by innovations in the macroeconomic fundamentals. It is beyond the scope of this chapter to tackle peculiarities of the foreign exchange market, therefore exchange rate noise appears as a simplest tool to account for large movements in the real exchange rate that cannot be explained by the relatively small changes in macroeconomic fundamentals. Although it is admittedly unusual to impose persistent exogenous shocks to the perfect consumption risk sharing identity, Bacchetta & van Wincoop (2017) introduce such innovations in the context of the uncovered interest rate parity (UIP). They are also present in structural vector autoregression models, such as Forbes et al. (2017), where this so-called 'exogenous exchange rate shock' also emanates from the UIP.

4.3 Summary of the Multi-Country Model

There are a number of subtle differences between the models presented in chapters 2 and 4. First, exporters are no longer infinitesimally small within their market segment and they exert some market power over the dynamics of sectoral prices, but not aggregate consumer prices. As a result, firms set their prices strategically by taking the optimal behaviour of other sectors into the account. Second, major currency areas now contaminate the entire global economy with adverse innovations without facing exact opposite repercussions when those innovations emanate from the minor currency areas. Third, the model with dynamic strategic complementarities is based on quadratic price adjustment costs in order to distinguish their influence from the possible downward price rigidities illustrated in the previous chapters.

When the global trade in commodities and manufactured goods is invoiced in a small number of major currencies, this chapter shows that not all countries are subject to symmetrical intensity of external innovations. That channel crucially depends on country-specific structural factors, such as exporter reliance on intermediate imports, import and export price stickiness, as well as the shares of invoicing currency in which international prices are sticky. Although the choice of invoicing currency may be weakly correlated with the intermediate import intensity, this chapter shows that nominal rigidity and strategic complementarities among exporters materialise in unprecedented developments in the minor-minor as well as major-minor currency area terms of trade. The central policy implication for monetary authorities in minor currency areas is that expansionary monetary policy may be more effective in countries experiencing a currency mismatch between intermediate imports and final exports, but it leaves them more susceptible to instability attributed to innovations emanating from the major currency areas. In order to infer the above policy implications in quantitative terms, the next chapter is dedicated to estimating the model depicted above in the context of United States, Japan, and South Korea.

Chapter 5

Estimating a US-JP-KR DSGE Model With Sticky Prices & Strategic Complementarities

5.1 Synopsis

Dynamic stochastic general equilibrium environment is a particularly appealing tool for policy makers when conducting counterfactual scenario analysis, primarily because the deep structural parameters of these models can be easily disciplined by the data. And yet most of these models developed in the open economy setting are confined to a two-country scenario using Bayesian methods, such as Lubik & Schorfheide (2005), Adolfson et al. (2007), García-Cicco et al. (2010), and De Walque et al. (2017). In spite of the fact that Bayesian methods incorporate prior knowledge about economically-plausible boundaries of structural parameters, multi-country models, such as the one depicted in this chapter, are subject to a severe case of the 'curse of dimensionality'. That is, the number of parameters to be estimated grows exponentially with the number of countries N, while the number of shocks increases at a linear rate. In practice, whenever $N > 2$, stochastic singularity and issues related to parameter identification become even more prominent in the context of Bayesian analysis when compared to the case of $N \leq 2$.

Instead of incorporating a number of additional *ad hoc* stochastic variables in order to improve parameter identifiability, this chapter adopts Generalised Method of Moments (GMM) *à la* Aguiar & Gopinath (2007). GMM is a tractable alternative when facing the overwhelmingly large dimensions of multi-country business cycle models and especially useful in situations when the available set of time series on macroeconomic fundamentals is limited. It circumvents the curse of dimensionality by harnessing moments on the global connectedness from a variance-covariance matrix that increases exponentially with the number of countries. In order to discard economically intractable local maxima and in order to incorporate reliable information outside of the model, GMM algorithm depicted in this chapter is further augmented in two distinct dimensions. First, two-sided lower- and upper-bounds are imposed for each deep structural parameter. It simultaneously eliminates the possibility of indeterminacy or multiple equilibria in a rational expectations setting and leaves a large parameter space unconfined by narrow prior beliefs. Second, the 'bounded GMM' incorporates a number of non-linear constraints applied to multiple combinations of inter-sectoral parameters. The latter feature is helpful when information about the currency of invoicing across sectors is only observed for say, the USD, but unobserved for the remaining currencies. Non-linear constraints that condition available information about invoicing patters on the trade shares can thus provide a reasonable educated guess about the magnitude of the unobserved invoicing shares based entirely on the key moments of macroeconomic fundamentals, such as the average volatility and the persistence of international prices and exchange rates.

5.2 Estimation

Once the values of all structural parameters are postulated, the multi-country model can be solved numerically by perturbing the policy function up to a first order around a balanced growth path *à la* Schmitt-Grohé & Uribe (2004).[1] Some parameters may be successfully calibrated using information outside of the model. For example, Gopinath (2015) provides many useful stylised facts about nationwide invoicing of imports and exports. Yet the intra-industry data on the global export market structure is virtually scant, such that the parameters controlling the intra-sectoral dependence on imported intermediate inputs, their choice of invoicing currency, and their market shares are mostly unknown. At the same time, studies using the limited intra-sectoral data that are available, such as Berman et al. (2012) and Amiti et al. (2014), demonstrate that there are predictable systematic differences in exchange rate pass-through among individual exporters in France and Belgium respectively.

In the light of the limitations in terms of the micro-level data, this chapter maps the inter-industry parameters to the moments of the most disaggregated data on macroeconomic fundamentals that are available. In principal, the mapping could be implemented using either maximum-likelihood or moment-based techniques. Although there are many examples of open economy models estimated using Bayesian maximum-likelihood techniques, the multi-country model depicted in this framework is subject to a severe case of the 'curse of dimensionality'. Instead of incorporating a number of additional *ad hoc* stochastic variables in order to improve parameter identifiability, application of GMM is arguably more suitable, where the emphasis is placed on targeting a select-few first and second moments of key the variables such as import and export price inflation and the exchange rate. It also does not impose non-standard prior distributions on the initialised values of the parameters, while the 'posterior' densities are characterised by the conventional properties of a normal distribution from which standard inference can be drawn. The latter point can be viewed as both an advantage and a disadvantage. On the one hand, the estimation algorithm is more flexible and immune to any preconceptions. On the other hand, the results are more susceptible to model misspecification. In order to preserve the accuracy of the solution and to ensure saddle-path stability, the model parameters are thus disciplined by imposing lower- and upper-bounds on the estimates that discard theoretically incredible values from the objective function, ultimately amounting to 'bounded GMM'.

[1]The detailed derivations of the balanced growth path are subsumed within Appendix B.6.

5.2.1 Generalised Method of Moments

Suppose $\boldsymbol{\theta}$ is a $g_1 \times 1$ vector of unknown parameters. Suppose further that $T > 0$ observations of stationary and ergodic economic data, $\{\varpi_t\}$, is available to estimate a $g_2 \times 1$ vector of moments $\mathbf{m}(\varpi_t)$. Similarly, there exists an analogous $g_2 \times 1$ vector of theoretical (i.e. unconditional) moments $\mathbb{E}[\mathbf{m}(\boldsymbol{\theta})]$ implied by the structural model with Gaussian disturbances. The distance between the data- and the model-implied moments is defined as:

$$\mathbf{M}(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{m}(\varpi_t) - \mathbb{E}[\mathbf{m}(\boldsymbol{\theta})]. \tag{5.1}$$

The restricted GMM estimator can then be defined as:

$$\hat{\boldsymbol{\theta}} = \underset{\underline{\boldsymbol{\theta}}<\boldsymbol{\theta}<\bar{\boldsymbol{\theta}}}{\operatorname{argmin}}\ \mathbf{M}(\boldsymbol{\theta})'\mathbf{S}^{-1}\mathbf{M}(\boldsymbol{\theta}), \tag{5.2}$$

where $\underline{\boldsymbol{\theta}}$ and $\bar{\boldsymbol{\theta}}$ are the lower- and upper-bound vectors of $\boldsymbol{\theta}$, while

$$\mathbf{S} = \lim_{T\to\infty} \operatorname{Var}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\mathbf{m}(\varpi_t)\right] \tag{5.3}$$

is a $g_1 \times g_2$ positive-definite optimal weighting matrix obtained using the Newey-West estimator with a Bartlett kernel. It ensures the efficiency of $\hat{\boldsymbol{\theta}}$ by putting the most weight on the most accurately measured moments and simultaneously discounting the least accurately measured moments. If and only if $\mathbf{J} = \mathbb{E}[\partial\mathbf{m}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}]$ is the $g_1 \times g_2$ finite Jacobian matrix of full column rank and $g_2 \geq g_1$, then the GMM estimator is said to be 'identified'. Moreover, it follows an asymptotic normal distribution under the Hansen (1982) regularity conditions:

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \to \mathcal{N}(0, (\mathbf{J}'\mathbf{S}^{-1}\mathbf{J})^{-1}), \tag{5.4}$$

Similar to the basic maximum-likelihood technique, GMM is sensitive to model misspecification, which is why it is useful to impose some natural limits on the individual elements of $\boldsymbol{\theta}$. Consequently, they follow truncated-normal distributions, and the rule of thumb for statistical significance is one-sided relative to a pre-determined lower- or upper-bound.

5.2.2 Data Description

The model is estimated for $N = 3$ using X-13 seasonally-adjusted monthly data covering the period of 1994:M1-2016:M12, where United States is the major currency area, while Japan and South Korea are the minor currency areas. Conditional on being some of the largest exporters in the world, all economies were chosen on the basis of adopting monetary policy regimes that encompass a freely-floating exchange rate and an inflation-targeting objective throughout the sample period, which is arguably compatible with the Taylor rule depicted in the model. The vector of time series in first-order differences contains the real aggregate household consumption (CONS), the consumer price index (CPI), and the unit value indices of imports and exports (UVIM, UVIX). The vector of time series also includes the levels of the nominal interest rate (NIR) and the CPI-based real exchange rates (KRW/USD, KRW/JPY, and JPY/USD). There are 18 time series in total and 53 parameters to be estimated. GMM uses the sample average, the standard deviation, the first- and the second-order autocorrelation, as well as the covariance of each time series, such that $g_1 = 53$ and $g_2 = 225$.

5.2.3 Parameters

Calibration

Not all parameters described in the model enter into the vector $\boldsymbol{\theta}$. The goal is to use as much information outside of the model and to calibrate as many structural parameters as possible, thereby minimising the length of $\boldsymbol{\theta}$ and maximising the degrees of freedom. All calibrated parameters are displayed in table 5.1. Following Atkeson & Burstein (2008), the inter-sectoral and inter-national elasticities of substitution adopt the following ordering: $1 < \eta < \zeta < \infty$. Moreover, the magnitude of η is close to unity in order to keep the country-specific trade weights $s_{ni,t}$ relatively stable over time. Because the GMM objective function abstracts from fitting the labour market dynamics, the aggregate hours of labour are assumed to be supplied inelastically, such that $\varphi_i = 1$. The iceberg costs d are set to 10% for all source and destination economies. It is in-line with UNCTAD (2015) global estimates of the differences between the CIF and FOB price levels of internationally exchanged commodities and manufactured goods. The relative size of the distribution sector α is equal to 2/3 of the final household consumption basket – not far from the G7 estimates presented in Burstein et al. (2003).

The discount factor β_n is computed as the inverse sample mean of the gross real interest rate, while the trend of monthly consumer price inflation $\dot{p}_n$ is simply approximated by the sample average of the CPI inflation rate. The mean of the real exchange rate q_{ni} is proxied by the average bilateral nominal exchange rate. However, without loss of generality, the relative price of currencies are expressed in terms of the price of 0.01 USD rather than one unit. The magnitude of ξ_n is computed using three indicators. First, the 1994-2016 average import share of GDP obtained from the World Bank. These are 14.5%, 12.5%, and 36.9% for United States, Japan, and South Korea (i.e. US, JP, KR) respectively. Second, the proportion of imported intermediate goods relative to the total value of imports obtained from Miroudot et al. (2009). In US, JP, and KR, 51.5%, 68.0%, and 75.1% of all imports respectively are deemed to be intermediate goods. Because ξ_i measures the share of imported intermediate goods in the tradable goods sector, it follows that $(1 - \alpha)\xi_i$ for each destination is equal to the product of the above two shares. Around 83% of all exports in South Korea are estimated to be re-exports to other destinations, which is evocative of its ubiquitous role as a gateway for APEC economies. On the other hand, Japan and United States are almost four times less dependent on imported intermediate goods than South Korea.

Similarly, the average trade weights s_{ji} are computed using the IMF Direction of Trade Statistics. First, the average home-bias s_{ii} is obtained by subtracting the product of the import share of GDP and the consumption share of GDP from unity. Specifically, in US, JP, and KR, 9.7%, 7.0%, and 19.0% of all consumer goods are imported from abroad. Second, the average bilateral trade shares between each of these economies are computed. Namely,

Table 5.1: Partial Calibration of the US-JP-KR Model

Structural Parameters										
ζ					10					
η					1.1					
d					1.1					
α					0.667					
			US			JP			KR	
$(1/\beta_n - 1) \times 100$			0.043			0.025			0.209	
$(\gamma_n - 1) \times 100$			0.197			0.068			0.110	
$(\dot{p}_n - 1) \times 100$			0.18			0.01			0.25	
ξ_n			0.224			0.255			0.831	
	$n \backslash i$		US			JP			KR	
q_{ni}	US		1			1/1.364			1/12.827	
	JP		1.364			1			1/9.404	
	KR		12.827			9.404			1	
	$n \backslash i$		US			JP			KR	
s_{ni}	US		0.8383			0.0896			0.1433	
	JP		0.1246			0.8833			0.1750	
	KR		0.0371			0.0271			0.6817	
		PCP	LCP	DCP	PCP	LCP	DCP	PCP	LCP	DCP
	$n \backslash i$		US			JP			KR	
$s_{ni}(\phi)$	US	0.97	0.03	$-$	0.73	0.27	$-$	0.98	0.02	$-$
	JP	0.08	$*$	$*$	0.39	0.11	0.50	0.70	0.03	0.27
	KR	0.02	$*$	$*$	0.38	0.08	0.54	0.01	0.14	0.85

An asterisk $*$ indicates an unknown parameter value that are subsequently estimated using GMM. A hyphen $-$ indicates that parameter does not exist. For example, DCP in the US implies PCP, thus the distinction between the two is redundant.

2.8% (9.5%) of US imports originate from KR (JP); 15.0% (4.5%) of JP imports originate from US (KR); 13.9% (17.0%) of KR imports originate from US (JP). Third, the product of the aggregate import share and the bilateral import share gives the actual trade weight for each trade partner. However, because US, JP, and KR trade with other economies outside of this model, the framework is closed by adding the trade-weighted remainder to each of the two trading partners such that $\sum_{i=1}^{N} s_{ni} = 1$.

Most of the market shares are calibrated by making use of the shares of invoicing currency at the national level reported in Gopinath & Rigobon (2008), Gopinath (2015), and Boz et al. (2017). In particular, Gopinath & Rigobon (2008) estimates that around 97% (90%) of US exports (imports) are priced in USD. Moreover, Gopinath (2015) finds that 35% (5%) imports in JP (KR) are priced in JPY (SKW); 39% (1%) of JP (KR) exports are invoiced in JPY (SKW); and 50% (85%) of JP (KR) exports are invoiced in USD. Finally, Boz et al. (2017) reports that 71% (81%) of imports in JP (KR) are invoiced in USD. Admittedly, these values represent the global invoicing shares relative to all trade partners, while the model depicted in this chapter is a three-country world. The predicament is that it is not immediately clear if imports invoiced in USD in JP (KR) comes from the US or KR (JP). The shares of imports invoiced in local currency are therefore multiplied by the trade weight (e.g. for JP imports from US, $s_{ki}(\ell) = 0.35 \times s_{ki}/(1 - s_{ii}) = 0.27$), while the share invoiced in USD from either the US or other minor economy must add up to the reported magnitude (e.g. for JP imports from KR that are invoiced in USD, $s_{ji}(\$) = 0.71 - (1 - s_{ki}(\ell)) \times s_{ji}/(1 - s_{ii}) = 0.54$). The one segment of table 5.1 that the data outside of the model cannot determine is how much of the US imports invoiced in USD come from LCP or DCP firms. However, these shares can be estimated along with the remaining parameters using GMM as discussed in the next section.

A glance at the cross-country differences of the invoicing shares is already an indication that the terms of trade elasticity to exchange rate movements is likely to be heterogeneous. The exact patterns of comparative advantage are discussed in more detail in section 5.3.1. Suffice it to say that the smaller the economy, the more likely it is to invoice profits in either the local or major currency units, while the larger the economy, the more likely it is that its currency becomes a customary medium of exchange for other smaller economies.

Bounds & Non-Linear Constraints

The configuration of the bounds imposed on the GMM algorithm is displayed in table 5.2. The inter-sectoral convexity of the price adjustment costs $\kappa_i(\phi)$ is initialised to a relatively large magnitude of 75, owing to the monthly nature of the data in which prices are expected to be stickier than in the quarterly or annual settings that are more commonly encountered in the New Keynesian literature. The lower bound of zero for $\kappa_i(\phi)$ allows for the narrative of perfect price flexibility, while a relatively high upper bound of 500 ensures that the outcome of an effectively flat Phillips curve is possible. Wage stickiness $\kappa_{i,w}$ is initialised to 100, since wages are expected to be more sluggish to adjust than prices as they are less sensitive to exchange rate volatility. The remaining configurations of the nation-wide parameters are relatively standard, with the exception of the shocks to the exchange rate noise $\sigma_{i,e}$. As discussed in the modelling section, the perfect consumption risk sharing framework adopted in the household optimisation problem does not fully capture the real exchange rate volatility encountered in practice and gives rise to counterfactually stable exchange rate dynamics. Introduction of exchange rate noise shocks with a sizeable upper bound of 8% mimics the otherwise puzzling high exchange rate volatility observed in all three foreign exchange markets.

When it comes to the remaining inter-sectoral parameters, it is not enough to simply impose bounds on the individual elements of $\boldsymbol{\theta}$. Each ϕ-type openness to intermediate imports, namely $\xi_i(\phi)$, is in principle bounded between zero and unity. But it is not analogous to a 'beta prior' in the Bayesian estimation algorithms, because it needs to satisfy an additional condition for each parameter iteration, namely $\sum_{\phi} \xi_i(\phi) \times \sum_{n=1}^{N} s_{in}(\phi) = \xi_i$. Specifically, the economy-wide average intermediate import intensity must equal the sum of the inter-sectoral intermediate import intensities weighted by sectoral market shares. Guided purely by the gradient of the GMM objective function, nothing prevents the algorithm from searching for values of $\xi_i(\phi)$ that violate this non-linear constraint if the only restrictions imposed on the parameter space were the bounds on the individual elements of $\boldsymbol{\theta}$. It is therefore paramount to incorporate the non-linear constraint into the estimation routine in order to discard theoretically incredible values of unbounded GMM estimates, where $\sum_{\phi} \xi_i(\phi) \times \sum_{n=1}^{N} s_{in}(\phi) \lesseqgtr \xi_i$.

And yet if all $2(N^2-1)+N$ restrictions were to be imposed simultaneously, namely $2(N^2-1)$ number of bounds and N number of non-linear constraints, then the bounded GMM estimates of the intermediate import intensity would be over-identified, but not in the usual sense of that terminology. In practice, non-linear constraints are difficult to satisfy with exact equality, because it generally comes at an internal trade-off of attaining the local maxima. Imposing many tight non-linear constraints on a large parameter space along with the bounds on the individual elements is somewhat reminiscent to the problem of stochastic singularity as in Altug (1989) and Sargent (1989), since there may not be a feasible combination of deep structural parameters that satisfies the convergence criterion. The pragmatic solution to this problem is to impose a margin for error $\varsigma > 0$, which is somewhat analogous to Schmitt-Grohé & Uribe (2012) in the Bayesian maximum-likelihood setting. It requires transforming each equality constraint $\sum_\phi \xi_i(\phi) \times \sum_{n=1}^N s_{in}(\phi) = \xi_i$ into an inequality constraint, namely $\xi_i - \varsigma \leq \sum_\phi \xi_i(\phi) \times \sum_{n=1}^N s_{in}(\phi) \leq \xi_i + \varsigma$. Inequality constraints with a pre-specified margin for error are not as demanding on the objective function, since they discard futile computations in the neighbourhood of the local maxima that ultimately satisfy the convergence criterion at an infeasible point. The tighter is the margin for error, the less feasible is the local maxima, but also the less likely are the constraints to be violated and vice versa.

Fortunately, the margin for error in this application is not required to be particularly broad. After experimenting with a number of values for ς, which need not be symmetrical across all non-linear constraints, the final specification applies a 5% margin for error for intermediate import intensity and a 0% margin for error (i.e. equality constraint) for the sum of the market shares, namely $\sum_{n=1}^N s_{ni}(\phi) = 1$. Non-linear constraints on the bounded parameter space can be useful not only in terms of satisfying abstract theoretical constructs, but also as a viable alternative to the prior distributions in Bayesian analysis. The crucial difference here is that (non-)standard 'prior distributions' are not controlled by pre-determined hyper-parameters, but rather a pre-determined range of estimates. If the constraint consists of just a linear combination of parameters, then the 'posterior' distribution is simply a truncated normal distribution. This chapter makes use of additional non-linear constraints in order to discipline the inter-sectoral parameters and bring the model predictions as close to the available empirical literature as possible. In the end, bounded GMM estimation with 8 non-linear constraints splices the parameter space into a lower-dimensional object, whose limits may be as tight or slack as ones confidence in the information outside of the model.

Bounded GMM Estimates

All parameter estimates displayed in table 5.2 turn out to be well-identified in the sense that they diverge from their initialised values and they are generally different from the lower- and upper-bounds at less than 1% level of significance. The estimates of the market shares reveal that more US imports from both JP and KR are priced in LCP rather than DCP strategies.[2] The high share of LCP exporters thus implies that there are non-trivial differences in the

[2] Recall that more than 90% of all US imports are invoiced in dollars, while 54% (27%) of JP (KR) imports from KR (JP) are invoiced in USD, which reinforces the role of the USD as the dominant medium of exchange even in the present approximation of the world economy.

prices of tradable goods across borders in the short-run. Moreover, the price adjustment costs of LCP (PCP) export prices exhibit the greatest convexity in the US and JP (KR). One would expect the DCP export prices to be the most rigid as they are virtually immune to the global commodity price fluctuations, but that does not turn out to be the case. Instead, the data favours the outcome of a substantial product market segmentation between the US and other currency areas. By contrast, most trade that takes place between JP and KR is either conducted by PCP or DCP firms, leaving very little room for price discrimination.

TABLE 5.2: US-JP-KR Model Estimation

GMM Configuration & Estimates

Parameter $\boldsymbol{\theta}$	Initialised Value $\boldsymbol{\theta}_0$			Lower Bound $\underline{\boldsymbol{\theta}}$			Upper Bound $\bar{\boldsymbol{\theta}}$			Point Estimate $\hat{\boldsymbol{\theta}}$		
	`US`	`JP`	`KR`	`US`	`JP`	`KR`	`US`	`JP`	`KR`	`US`	`JP`	`KR`
$s_{jk}(\ell)$	1/3	*	*	0	*	*	1	*	*	0.51	*	*
$s_{jk}(\$)$	$1-\sum_{-\$} s_{jk}(-\$)$	*	*	0	*	*	1	*	*	0.41	*	*
$s_{ik}(\ell)$	1/3	*	*	0	*	*	1	*	*	0.77	*	*
$s_{ik}(\$)$	$1-\sum_{-\$} s_{ik}(-\$)$	*	*	0	*	*	1	*	*	0.21	*	*
$\xi_i(\pi)$	ξ_k	ξ_i	ξ_j	0	0	0	1	1	1	0.10⸸	0.00‡	0.92‡
$\xi_i(\ell)$	ξ_k	ξ_i	ξ_j	0	0	0	1	1	1	0.08‡	0.04‡	0.98‡
$\xi_i(\$)$	-	ξ_i	ξ_j	-	0	0	-	1	1	-	0.25‡	0.35‡
$\kappa_i(\pi)$	75	75	75	0	0	0	500	500	500	32.90	16.91	207.50
$\kappa_i(\ell)$	75	75	75	0	0	0	500	500	500	447.51	127.41	47.07
$\kappa_i(\$)$	-	75	75	-	0	0	-	500	500	-	41.76	51.91
$\kappa_{i,w}$	100	100	100	0	0	0	500	500	500	22.04	0.15	13.41
ϑ_i	0.5	0.5	0.5	0	0	0	1	1	1	0.27	0.40	0.06
$\nu_{i,p}$	1.5	1.5	1.5	1	1	1	5	5	5	3.26	3.95	3.89
$\nu_{i,y}$	0.5	0.5	0.5	0	0	0	1	1	1	0.08	0.22	0.15
$\rho_{i,r}$	0.75	0.75	0.75	0	0	0	1	1	1	0.47	0.99	0.78
$\rho_{i,z}$	0.75	0.75	0.75	0	0	0	1	1	1	0.80	0.45	0.66
$\rho_{i,e}$	0.75	0.75	0.75	0	0	0	1	1	1	0.58	0.89	0.89
$\sigma_{i,r}$	0.5%	0.5%	0.5%	0%	0%	0%	8%	8%	8%	0.30%	0.00%	0.11%
$\sigma_{i,a}$	0.5%	0.5%	0.5%	0%	0%	0%	8%	8%	8%	0.04%	0.82%	0.02%
$\sigma_{i,z}$	0.5%	0.5%	0.5%	0%	0%	0%	8%	8%	8%	5.08%	0.99%	7.72%
$\sigma_{i,e}$	2.5%	2.5%	2.5%	0%	0%	0%	8%	8%	8%	4.58%	2.69%	1.50%

The (inverted) cross † (⸸) next to the point estimate indicates that it is not different from the upper (lower) bound at a 5% level of significance. A double cross ‡ indicates that it is not different from either bounds at 5% level of significance. The GMM algorithm is implemented using `Dynare 4.5.4` and `Matlab 2017a`. The objective function is minimised conditional on the bounds and non-linear constraints using the Sequential Quadratic Programming algorithm embedded into `fmincon`, which takes around 60 minutes to converge when using a machine with a 4 GHz processor speed and 16GB memory.

There is no coherent pattern across countries in terms of their intermediate import intensity. Theoretically, one would expect the market share of the DCP firms to depend on the share of intermediate imports are denominated in USD. But GMM portrayed in this chapter captures the same idea in reverse – conditional on observing the invoicing shares, what proportion of inputs would the data suggest to be intermediate? Indeed, it is not the most efficient method to force a confession out of the data, which is reflected in large uncertainty bands around the estimates of $\xi_i(\phi)$. Yet given the micro-level data limitations, it is the only viable method available at hand aside from an outright calibration or other forms of indirect inference. The fact that most estimates of intermediate import intensity are inefficient is not a reflection on the general performance of non-linear constraints, because market shares of US imports turn out to be highly statistically significant. Instead it is mostly due to the choice of linear production technology in import-export industries, leading to insensitive consumption, interest rates, and international prices to changes in intermediate import intensity.

There are two competing views about what determines the choice of invoicing currency. First, according to the firm-level data reported in Amiti et al. (2014), the more import-dependent are the export production technologies in Belgium, the more likely are firms to adopt LCP strategies and exercise sticky prices. In that case, large and transitory movements in commodity prices are generally not reflected in import and export prices, but rather absorbed by the producer price mark-ups, thereby creating some short-run arbitrage opportunities. The first view also reinforces the earlier evidence produced by Froot & Klemperer (1989) about the sources of low exchange rate pass-through into import prices, which was thought to be motivated by so-called market share hysteresis - temporary contraction is considered to be permanent, thus transitory adjustment of the price mark-up is considered to be less costly than the present value of foregone market share. Second, Gopinath (2015) inadvertently shows that openness to intermediate imports strongly correlates with the adoption of USD as the primary invoicing currency of overseas revenues in many advanced and emerging economies. If most intermediate commodities are priced in USD, then adopting the DCP strategy renders the export price mark-up immune to major exchange rate volatility, thereby keeping the price mark-up constant at home and in the major economy. At the same time, the DCP import price mark-up adjustments occur through adverse changes in market shares in other minor currency areas. However, the latter effect may be dampened if the major currency is sufficiently widespread (i.e. 'dominant currency paradigm').

In the US, both PCP and LCP firms are estimated to be relatively closed, which supports neither of the views. Conversely, in JP, PCP firms are virtually closed, LCP firms are only somewhat open, while DCP firms import nearly a quarter of their inputs, which conforms to the view of Gopinath (2015). On the other hand, both PCP and LCP exports originating from KR are almost entirely re-exports, which is evocative of the ubiquitous role of KR as a gateway for APEC economies and somewhat supports the empirical regularities established in Amiti et al. (2014). Consequently, both views seem to be warranted by GMM, such that they are not necessarily mutually exclusive, but rather each sector tends to complement one another. Irrespective of how mixed the results are across countries, they are poles apart from the backbone of the classical Mundell-Fleming paradigm, in which international prices of final goods are orthogonal to intermediate imports and all exporters invoice their profits in producer currency units.

Wages turn out to be relatively flexible in all three economies. In fact, wage adjustments are virtually costless in JP. Although additive habits are negligible in KR, they play an important role in JP and KR in terms of lowering the inter-temporal substitution of consumption. In general, both wage stickiness and additive habits are important when matching the high volatility and persistence of exchange rates with relatively low volatility and persistence of consumption. Notice that exchange rate noise shocks adjacent to short-run productivity shocks in the import-export industries are generally larger than other sources of macroeconomic volatility. Both monetary policy surprises and long-run productivity shocks are mostly negligible. With the exception of JP, where both short-run and long-run productivity shocks are almost equally as important, much of the price mark-up variability associated with firms domiciled in the US and KR comes from innovations to short-run productivity and the USD.

The Taylor rule feedback parameters for inflation (output gap) are estimated to be larger (lower) than it is commonly found in the New-Keynesian literature. In general, the adoption of the Taylor rule in a liquidity-trapped Japanese economy is an over-simplification of the monetary policy conduct during the sample period. An extension of this model to accommodate a zero lower bound of the nominal interest rate would provide a more accurate fit of the data, especially the low volatility and very high persistence of the nominal interest rate, but it goes beyond the scope of this chapter. Although monetary policy shocks are very small in the US due to the liquidity trap characterising a large part of the last decade, they capture a large part of the nominal interest rate volatility. This result is important since it means that exogenous monetary policy shocks are not required to explain the US business cycle. Instead, transitory productivity shocks turn out to be large in the US and KR, which are capturing the large movements in the fundamentals during the Great Financial Crisis of 2008-09.

5.2.4 Assessing the Goodness of Fit

The empirical validity of the estimated parameters is based not only on their statistical significance, but also on how well they perform in terms of minimising the GMM objective function. Table 5.3 breaks down the distance between the data-implied and the model-implied moments of the fitted variables. For the sake of transparency and space, the covariance matrix of all 18 time series are not displayed. Overall, the quantitative fit of the model is quite remarkable given the simplicity of the demand-side depicted in this framework – not a single series exhibits dynamics that stray afar from those of the data generating process. However, the volatility of all series is somewhat underestimated, which is not surprising, since the model does not incorporate any form of capital formation – both physical and financial capital market imperfections tend to exacerbate the business cycle. Several other observations are particularly noteworthy.

First, the model predicts low volatility and persistence of the consumer price inflation and a high volatility and persistence of the exchange rates. The compatibility of these moments is an especially desirable feature of many open economy models, because it renders exchange rate pass-through into consumer prices relatively low – in-line with the empirical estimates, such as Campa & Goldberg (2010). Second, the model captures a large part of the exchange rate volatility and persistence in each market, which is largely driven by the autoregressive process of exchange rate noise. Some of the missing volatility in consumption accounts for the remnants of the missing volatility in exchange rates. Third and most surprisingly, the fit of the second moment of the nominal interest rate is exceptionally good, especially for JP, where the nominal interest rate is virtually constant and highly persistent as though it were a unit root. As a result, the model overcomes the risk-free rate puzzle to some extent in that

Table 5.3: Data- and Model-Implied Moments (1994:M1-2016:M12)

	(i) United States							
	Mean		Std. Dev.		ACF(1)		ACF(2)	
	Data	Model	Data	Model	Data	Model	Data	Model
Consumption (M-o-M)	0.19%	0.20%	0.39%	0.28%	-0.23	0.15	0.05	0.01
Consumer Price Index (M-o-M)	0.18%	0.18%	0.27%	0.15%	0.41	0.27	0.00	0.12
Import Unit Value Index (M-o-M)	0.10%	0.18%	1.22%	0.80%	0.60	0.29	0.29	0.13
Export Unit Value Index (M-o-M)	0.08%	0.18%	0.59%	0.32%	0.48	0.73	0.29	0.51
Nominal Interest Rate	0.23%	0.43%	0.19%	0.17%	0.99	0.55	0.99	0.43
	(ii) Japan							
	Mean		Std. Dev.		ACF(1)		ACF(2)	
	Data	Model	Data	Model	Data	Model	Data	Model
Consumption (M-o-M)	0.08%	0.07%	1.07%	0.65%	-0.48	0.46	0.09	0.22
Consumer Price Index (M-o-M)	0.01%	0.01%	0.26%	0.22%	0.13	0.46	-0.02	0.23
Import Unit Value Index (M-o-M)	0.18%	0.01%	2.66%	1.96%	0.35	0.01	0.16	0.14
Export Unit Value Index (M-o-M)	0.13%	0.01%	1.76%	0.94%	0.18	0.54	0.08	0.26
Nominal Interest Rate	0.04%	0.10%	0.03%	0.02%	0.99	0.97	0.97	0.92
	(iii) South Korea							
	Mean		Std. Dev.		ACF(1)		ACF(2)	
	Data	Model	Data	Model	Data	Model	Data	Model
Consumption (M-o-M)	0.11%	0.11%	1.09%	0.64%	0.47	-0.09	0.30	-0.04
Consumer Price Index (M-o-M)	0.25%	0.25%	0.35%	0.25%	0.31	0.02	0.06	0.13
Import Unit Value Index (M-o-M)	0.12%	0.25%	2.07%	1.26%	0.60	0.13	0.35	0.02
Export Unit Value Index (M-o-M)	-0.16%	0.25%	1.34%	0.94%	0.68	0.52	0.41	0.21
Nominal Interest Rate	0.46%	0.57%	0.37%	0.25%	0.98	0.75	0.95	0.65
	(iv) Real Exchange Rate							
	Mean		Std. Dev.		ACF(1)		ACF(2)	
	Data	Model	Data	Model	Data	Model	Data	Model
JPY/USD	1.36	1.37	0.079	0.076	0.92	0.62	0.80	0.39
SKW/USD	12.83	12.84	1.06	0.71	0.83	0.60	0.69	0.36
SKW/JPY	9.40	9.43	0.80	0.46	0.85	0.86	0.71	0.74

All variables as expressed as monthly percentage changes with the exception of the nominal interest rates and the real exchange rates, all of which are depicted in levels. The stochastic trend of the real exchange rate series is removed by means of a one-sided Hodrick-Prescott filter with the standard smoothing factor of 14400. The stationary cyclical components are then fitted to the model-implied real exchange rates.

the nominal interest rate in all economies is excessively stable in spite of large fluctuations in productivity and exchange rates.

5.3 Average Causal Effect

5.3.1 Terms of Trade Warp

How important is the notion of nominal price stickiness when quantifying the shifts in the terms of trade in response to exchange rate innovations? Now that we have obtained the estimates of deep structural parameters and simulated the model with the estimated degree of nominal rigidity, we can answer this question by making use of lemmas 4 – 10. In particular, table 5.4 presents the 95% confidence intervals of the terms of trade elasticities to major currency innovations in the US, JP, and KR under three alternative scenarios. Namely, the actual elasticity that prevails under the estimated degree of price stickiness and two other hypothetical narratives - perfect price flexibility or perfect price rigidity. These estimates represent the *ex ante* average causal effect of exchange rates on the terms of trade. It is said to be 'average' in the sense that it conditions the elasticities on the distributions of all Gaussian innovations in this model. Moreover, it stands for the 'causal' effect, since this measurement is immune to endogeneity that plagues the co-movements of international prices and exchange rates at the aggregate level.

By definition, the terms of trade measure the amount of final imports that the destination economy can purchase for every unit of final exports it sells abroad. A negative terms of trade elasticity means that a unilateral depreciation of the major currency (i.e. orthogonal to the bilateral minor-minor exchange rate) leads to an improvement in the terms of trade in the destination economy. The average terms of trade elasticity turns out to be bounded between negative unity and zero not by mere accident, but because it is primarily driven by the market shares of multi-national exporters choosing to invoice profits in heterogeneous currencies, which are bounded between zero and unity by construction.

TABLE 5.4: Sticky Prices & Terms of Trade Neutrality

n \ k	(i) United States								
	(a) Flexible Prices $\lim_{\zeta\to\infty}\tau^k_{nk}=0$			(b) Sticky Prices $\tau^k_{nk}=erpt_{kk}-erpt_{nk}$			(c) Rigid Prices $\lim_{\kappa\to\infty}\tau^k_{nk}=-s_{nk}(\pi)$		
	min.	avg.	max.	min.	avg.	max.	min.	avg.	max.
Japan	–	**0.00**	–	-0.03	**-0.02**	-0.01	-0.14	**-0.08**	-0.01
South Korea	–	**0.00**	–	-0.02	**-0.02**	-0.01	-0.04	**-0.02**	-0.00
n \ j	(ii) Japan								
	(a) Flexible Prices $\lim_{\zeta\to\infty}\tau^k_{nj}=-s_{jj}(\$)$			(b) Sticky Prices $\tau^k_{nj}=erpt_{jj}-erpt_{nj}$			(c) Rigid Prices $\lim_{\kappa\to\infty}\tau^k_{nj}=-\{s_{ij}(\$),s_{kj}(\pi)\}$		
	min.	avg.	max.	min.	avg.	max.	min.	avg.	max.
United States	-0.55	**-0.50**	-0.45	-0.83	**-0.67**	-0.51	-0.88	**-0.72**	-0.57
South Korea	-0.55	**-0.50**	-0.45	-0.55	**-0.50**	-0.44	-0.59	**-0.54**	-0.48
n \ i	(iii) South Korea								
	(a) Flexible Prices $\lim_{\zeta\to\infty}\tau^k_{ni}=-s_{ii}(\$)$			(b) Sticky Prices $\tau^k_{ni}=erpt_{ii}-erpt_{ni}$			(c) Rigid Prices $\lim_{\kappa\to\infty}\tau^k_{ni}=-\{s_{ji}(\$),s_{ki}(\pi)\}$		
	min.	avg.	max.	min.	avg.	max.	min.	avg.	max.
United States	-0.89	**-0.85**	-0.81	-0.98	**-0.97**	-0.96	-0.99	**-0.98**	-0.97
Japan	-0.89	**-0.85**	-0.81	-0.41	**-0.36**	-0.30	-0.33	**-0.27**	-0.22

All numbers in bold represent the average exchange rate pass-through into the bilateral terms of trade following a movement in the USD. The numbers on the left and the right hand-side stand for the 95% confidence interval. All predictions are conditional on a single set of GMM parameter estimates, such that transitions between the states where prices are perfectly flexible or perfectly rigid are hypothetical as shown in lemmas 4 – 10.

A unilateral depreciation (appreciation) of the USD improves (deteriorates) the terms of trade around the world in almost all possible scenarios – analogous to the stylised facts established in Boz et al. (2017). But the value-added from conducting this exercise is that it shows how important price stickiness is in terms of explaining the qualitative finding of global trade flow intensification (abatement) due to a weakening (strengthening) of the USD.

For instance, the US-JP terms of trade response in the sticky price scenario is warped in that it statistically lies in-between the flexible and the rigid price setting scenarios (see table 5.4, panel (ii), first row). On the other hand, KR-JP terms of trade elasticity is practically identical to the hypothetical case of perfect price flexibility, indicating that nominal rigidities conditional on all other properties play very little role (see table 5.4, panel (ii), second row). By contrast, the US-KR terms of trade elasticity is very close to unity – a lot like when prices are perfectly rigid – inducing an average of 12% stronger terms of trade response to USD movements than in the case of perfect price flexibility (see table 5.4, panel (iii), first row). Similarly, the JP-KR terms of trade elasticity is around 50% lower when compared to the case of perfect price flexibility, but not significantly different from the perfectly rigid price scenario (see table 5.4, panel (iii), second row). However, in the US, the terms of trade are almost entirely neutral to exchange rate movements regardless of the extent of nominal rigidity, primarily because JP and KR both choose USD as their invoicing currency when exporting goods to the US (see table 5.4, panel (i), first and second rows).

As discussed in section 4.2.2, the patterns of the rest-of-the-world comparative advantage following USD innovations can be deduced by comparing the terms of trade elasticities against one another. For instance, $\hat{\tau}^k_{ij} - \hat{\tau}^k_{ji} = -0.14 < 0$, where j stands for JP and i denotes KR. Because this difference is negative, it means that the KR-JP terms of trade improve by more than the JP-KR, thereby boosting the bilateral competitiveness of the KR exports into JP by more than the other way around. In the end, although both economies gain a comparative advantage against the major economy whenever USD weakens, such that $\hat{\tau}^k_{jk} - \hat{\tau}^k_{kj} = 0.65 > 0$ and $\hat{\tau}^k_{ik} - \hat{\tau}^k_{ki} = 0.95 > 0$, KR further develops a trade surplus vis-à-vis JP. The movements in the USD therefore generate global trade imbalances, the patterns of which can be systematically predicted using a multi-country model with sticky prices such as the one presented in this chapter. The patterns of trade imbalances could also be naïvely proxied by invoicing shares as described in lemmas 4 – 10. But some questions following that approach always remain, namely 'how sticky are international prices' and 'which shares matter more: USD exports or USD imports?' Fitting a rich multi-country business cycle model to the data on international prices and exchange rates and using the simulated measures of the terms of trade elasticities presented in this chapter is one way to answer these questions pragmatically and in relative terms without having to impose value judgements onto the data.

How random are the fluctuations in the terms of trade elasticity? Are there any distinct patterns of co-movement with the macroeconomic fundamentals? The main novelty of the model-implied terms of trade elasticities is that they can be used as a 'rule of thumb' for testing non-linearities in the terms of trade channel by checking their correlation with a number of endogenous variables. In other words, we can analyse the causal effect without averaging across all innovations and instead exploring how the causal effect co-moves with the macroeconomic fundamentals conditional on all innovations simultaneously. Although there are a myriad of possibilities, one of the most obvious candidates is the exchange rate against the USD itself, since it reveals whether there are any state-dependence patterns in the transmission of exchange rate innovations. Yet before jumping to policy implications, the reader ought to be cautioned that the model presented in this chapter is linearised around

Table 5.5: Co-Movement of the Terms of Trade Elasticity and the U.S. Dollar

n \ k	(i) United States								
	(a) Flexible Prices corr.$(\dot{q}_{nk},\, \lim_{\zeta\to\infty} \tau^k_{nk})$			(b) Sticky Prices corr.$(\dot{q}_{nk},\, \tau^k_{nk})$			(c) Rigid Prices corr.$(\dot{q}_{nk},\, \lim_{\kappa\to\infty} \tau^k_{nk})$		
	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$
Japan	–	–	–	**0.46**	0.5	0.48	**0.50**	0.54	0.59
South Korea	–	–	–	**-0.22**	-0.4	-0.42	**0.52**	0.58	0.56

n \ j	(ii) Japan								
	(a) Flexible Prices corr.$(\dot{q}_{kj},\, \lim_{\zeta\to\infty} \tau^k_{nj})$			(b) Sticky Prices corr.$(\dot{q}_{kj},\, \tau^k_{nj})$			(c) Rigid Prices corr.$(\dot{q}_{kj},\, \lim_{\kappa\to\infty} \tau^k_{nj})$		
	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$
United States	**-0.15**	-0.14	-0.15	**0.28**	0.23	0.23	**0.27**	0.21	0.20
South Korea	**-0.15**	-0.14	-0.15	**0.17**	0.22	0.25	**0.15**	0.18	0.19

n \ i	(iii) South Korea								
	(a) Flexible Prices corr.$(\dot{q}_{ki},\, \lim_{\zeta\to\infty} \tau^k_{ni})$			(b) Sticky Prices corr.$(\dot{q}_{ki},\, \tau^k_{ni})$			(c) Rigid Prices corr.$(\dot{q}_{ki},\, \lim_{\kappa\to\infty} \tau^k_{ni})$		
	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$	$\eta = 1.1$	$\eta = 5$	$\eta = 10$
United States	**0.03**	0.01	>0.01	**0.23**	0.16	0.13	**0.23**	0.17	0.15
Japan	**0.03**	0.01	>0.01	**0.05**	0.05	0.04	**0.04**	0.05	0.08

All numbers in bold are conditional on a single set of GMM parameter estimates, while the remaining estimates are obtained by changing the value of intra-temporal elasticity of substitution η. Transitions between the states of perfect price flexibility or rigidity are hypothetical as shown in lemmas 4 – 10.

the balanced growth path, but in principal the co-movement patterns could also depend on the order of the perturbation. This means that higher order perturbations can lead to a non-monotonic relationship between the terms of trade elasticity and the major exchange rate, something that is not explored in due diligence in this chapter. More importantly, if the terms of trade elasticities reveal strong state-dependent tendencies, then at the same time it means that another, and indeed unobserved, channel must completely undo these effects at the aggregate level within the confines of the model for the system of difference equations is ultimately linear. It therefore speaks volumes to the literature on time-varying exchange rate pass-through – state-dependence at the disaggregated level does not always translate into time-varying transmission at the aggregate level. Because the model is not subject to any explicit forms of non-linearities, such as the zero lower-bound of the nominal interest rate or the downward rigidity of wages for example, all state-dependence patterns are essentially attributable to strategic complementarities of the firms. This is because the terms of trade elasticities are strongly correlated to the shares of invoicing currency (see lemmas 4 – 10), which in turn depend on relative prices of the underlying varieties that are optimally set by exporters who take their sectoral prices into the account.

Table 5.5 presents the unconditional correlations between the terms of trade elasticities and the value of the local currency relative to the USD, except for the US, where the exchange rate is bilateral as per usual. As expected, when the intra-temporal elasticity η is close to unity, the bilateral JPY/SKW exchange rate is mostly orthogonal to the terms of trade elasticities with respect to major currency innovations. This is because the price mark-ups are close to their theoretical maximum as shown in equation (4.15). When $\eta \to 10$, the price mark-ups decline and become time-invariant, since $\zeta = 10$ in columns (b) and (c) of table 5.5, which seems to increase the correlation between the bilateral JPW/SKW exchange rate and the terms of trade elasticity. In general, there appears to be a positive relationship between the unconditional correlations and nominal rigidity for a given value of η, but it tends to subside as η rises, except in the case of the US. Hence, when prices are flexible, the terms of trade elasticity in minor economies is mostly unrelated to the exchange rate, because it is mostly driven by the market share of the DCP exports, whose unit costs are orthogonal to the movements in the USD by construction (see lemma 5). But when prices become increasingly rigid, the terms of trade elasticity becomes increasingly positively correlated to the exchange rate, because it closely follows the dynamics of the DCP import market share (see lemma 9). It is paramount to integrate the forces of strategic complementarities at this point, since USD depreciation (appreciation) implies a decrease (increase) in the price of DCP imports, thereby crowding out (in) PCP and LCP imports and leading to a rise (fall) in the DCP market share. Because the market share of DCP imports is endogenous to the USD movements, their positive unconditional correlation reveals an unprecedented new mechanism associated with the terms of trade channel. Specifically, a USD depreciation on average leads to a greater improvement in the rest-of-the-world terms of trade than an equally large appreciation causes them to deteriorate.

The second-order findings may initially seem to suggest redefining the notion of dynamic comparative advantage altogether. On the one hand, US insulates itself from external shocks by exercising hegemonic power over the global flows of its currency. On the other hand, it comes at the expense of a decline in the US export competitiveness over time. Hence, the US terms of trade are virtually neutral to movements in the USD, but the terms of trade of JP and KR against the US are correlated with the USD, such that the US experiences a perpetually increasing current account deficit even if exchange rate changes are distributed normally. It strongly reinforces the findings of Bernanke (2005) for the US data since the 1980s. And yet a closer look at the estimates of the terms of trade elasticity suggests that their volatility is not high enough and the unconditional correlation is not strong enough to warrant abrupt shifts in the U.S. trade policy. That said, although pricing-to-market on its own may not be responsible for the non-linear transmission of exchange rate shocks, it nevertheless reveals the potential to amplify them if they were incorporated into the model explicitly.

To summarise, (i) US terms of trade are effectively neutral to the USD innovations; (ii) USD depreciations improve the rest-of-the-world terms of trade by more than USD appreciations cause them to deteriorate; (iii) sticky prices can either significantly increase or decrease the magnitude of the terms of trade elasticity to major currency innovations, depending on the shares of imports and exports invoiced in USD; and (iv) pricing-to-market alone does not automatically lend itself to non-linear exchange rate pass-through, but it could potentially amplify the non-monotonicity of the exchange rate channel.

5.3.2 Granularity

To what extent are the estimated terms of trade elasticities to USD innovations driven by the market shares as opposed to other properties of the model? If prices were either perfectly flexible or perfectly rigid, then predicting exchange rate pass-through across different sectors is relatively straight-forward as shown in lemmas 4 – 10. However, when the actual transition of prices from vintages to the inter-temporal optimum is gradual, the magnitude of sectoral pass-through could in principal move away from the neighbourhood of zero or unity, leading to partial transmission even in the long-run, thereby diminishing the important role played by the market shares encapsulated by other corner solutions. For this reason, the discussion now turns to the granularity of the exchange rate channel, which refers to the dispersion of exchange rate pass-through across sectors and across countries.

The average causal effect of exchange rates on prices from the upstream to the downstream level are presented in table 5.6, which are computed using the micro-founded measurement described in section 4.2. The table is split into two panels – one for measuring the causal effect due to major currency innovations and the other for the minor currency innovations. Consider the first panel, where the inter-sectoral causal effects are shown in the first three lines. They all depend on the pricing strategy and an indicator specifying whether the country is the source or the destination for each country pair. As expected, the exchange rate pass-through into the export and import prices of DCP firms for any country or the US firms does not move away from from the neighbourhoods of zero or unity. The most significant movements of exchange rate pass-through away from zero are in the PCP sector of JP, transmitting 13% of major currency innovations into import and export prices, as well as the LCP sector of KR, which transmits around 7-8% of major currency innovations into export and import prices in KR and JP respectively. The magnitude of exchange rate pass-through for all other sectors is very close to either zero or unity. Consequently, import and export prices aggregated across sectors indeed mostly depend on the market shares pertaining to each sector (see tables 5.1 and 5.2 for comparison purposes). In particular, exchange rate pass-through into export prices in all countries is less than 6% in spite of the pronounced openness to intermediate imports. It is worth re-emphasising that this finding is attributable to a large estimated value of the convexity of price adjustment costs displayed in table 5.2. By contrast, import prices are more responsive to major currency innovations in minor economies – they absorb 72% (55%) of USD movements in JP from US (KR) and 98% (37%) of USD movements in KR from US (JP). In order to grasp the effective exchange rate pass-through into import prices, the table also provides a trade-weighted average of the causal effects. In both JP and KR, the effective exchange rate pass-through is around 65%. On the other hand, both import and export prices in the US are practically neutral to the USD innovations, because most final and intermediate imports are invoiced in USD.

Consistent with the stylised facts presented in Boz et al. (2017), the difference between the major and minor 'own' exchange rate pass-through is very low, but positive, indicating that major currency innovations dominate the minor currency innovations. Hence, minor 'own' depreciations imply a depreciation against all other major and minor currency areas, therefore leading to considerably greater exchange rate pass-through into effective import prices by definition. When it comes to exchange rate pass-through into consumer prices, it is computed as a weighted average of pass-through into export and import prices, where the weights are the home-bias and the import penetration ratio respectively (see lemma 7).

Table 5.6: Granularity of Exchange Rate Pass-Through

(i) Major Exchange Rate Innovations

	Source \ Destination	US PCP	US LCP	US DCP	JP PCP	JP LCP	JP DCP	KR PCP	KR LCP	KR DCP
Inter-Sectoral Prices	US	0.00⸸	0.00⸸	-	1.00†	0.00⸸	-	1.000†	0.000⸸	-
	JP	0.13	0.02	> 0.01	0.13	0.03	> 0.01	0.13	0.03	< 1.00
	KR	0.02	0.02	> 0.01	0.02	0.08	< 1.000	0.02	0.07	> 0.01

	Source	US	JP	KR
Import & Export Prices	US	0.00⸸	0.72	0.98
	JP	0.02	0.06	0.37
	KR	0.02	0.55	0.01
Effective Import Prices		0.02	0.68	0.64
Consumer Prices		0.002	0.06	0.12

(ii) Minor 'Own' Exchange Rate Innovations

	Source \ Destination	JP PCP	JP LCP	JP DCP	KR PCP	KR LCP	KR DCP
Inter-Sectoral Prices	US	1.00†	0.00⸸	-	1.00†	0.00⸸	-
	JP	0.13	0.00⸸	> 0.01	1.00†	0.00⸸	1.00†
	KR	1.00†	0.00⸸	1.00†	0.02	0.00⸸	> 0.01

	Source	JP	KR
Import & Export Prices	US	0.72	0.98
	JP	0.06	0.97
	KR	0.92	0.01
Effective Import Prices		0.77	0.98
Consumer Prices		0.07	0.19

The (inverted) cross † (⸸) next to the point estimate indicates that it is not different from the upper (lower) bound at a 5% level of significance. The inequality operators indicate numbers that are beyond the third decimal point, but nevertheless statistically different from zero or unity.

Exchange rate pass-through into consumer prices is significantly different from zero and unity for all countries – a case commonly referred to as 'incompleteness'. This is because only a relatively small proportion of final goods are imported from abroad, while the majority of the consumer goods are produced domestically, priced at the factory door, and largely bundled with non-tradable services, thereby insulating consumer price inflation from exchange rate innovations. Notice that higher major exchange rate pass-through into effective import prices in JP compared to KR does not imply higher pass-through into consumer prices, since KR is relatively more open to trade in final goods and services compared to JP. Although the most open economy KR in relative terms exhibits the highest pass-through into consumer prices (i.e. 12% in response to SKW/USD movements), it is still far less volatile than the nominal exchange rate, thereby invalidating the classical Purchasing Power Parity hypothesis.

Ultimately, a compressed US-JP-KR environment portrayed in this chapter predicts that USD depreciations are expansionary in both KR and JP, but more so in KR. To put this into a broader perspective, a USD depreciation not only implies downward pressure on consumer prices in both KR and JP, but also an increase in their trade balance against the major economy. In the familiar two-dimensional space of CPI and real output, it is analogous to a rightward shift in both the aggregate supply and demand, where the supply shift outweighs the demand shift leading to consumer price deflation. The expansionary effect is stronger (weaker) in KR (JP), since it also gains (loses) comparative advantage against JP (KR). On the other hand, the US remains mostly unaffected by USD innovations, closely approximating the analytical construct of a closed economy.

In summary, (i) major currency innovations lead to greater exchange rate pass-through than minor currency innovations; (ii) import prices are more responsive to exchange rate changes than export prices; (iii) exchange rate pass-through into import prices is determined primarily by the market shares of exporters choosing heterogeneous invoicing currencies; and (iv) consumer prices are positively related, but much less responsive to exchange rates than effective import prices.

5.4 Dynamic Causal Effect

What is the causal effect of exchange rates on the terms of trade over time? Is the transition path monotonic or rather subject to J-curve effects? There are two ways to answer these questions. In the first instance, we could generate the impulse response functions to the inflation rates of international prices and first-order differenced exchange rates, find their cumulative sums, and compute the ratio between the two as suggested by Shambaugh (2008). The aforementioned agnostic measurement of exchange rate pass-through is, however, subject to at least two concerning limitations. Firstly, it is always conditional on observing a certain state of the economy, which does not bear a meaningful long-run counterpart, since all impulse response functions of saddle-path stable systems of difference equations are by default converging to zero as time goes to infinity. Second, and more importantly, all impulse response functions of endogenous variables at the aggregate level are driven by multiple state variables that may be irrelevant for the optimal price setting decision of the exporters, since they do not exert enough market power to influence their developments. Using the reduced-form measurement of exchange rate pass-through can therefore provide misleading policy implications about the magnitude of exchange rate transmission, since the dynamics of exchange rates and international prices at the aggregate level are tainted by indirect co-movements with a multitude of other macroeconomic fundamentals.

This chapter proposes a viable alternative that abstracts from the irrelevant aggregate co-movements from the perspective of individual exporters and instead allows exchange rate pass-through to respond endogenously to innovations through a limited subset of state-space captured in lemma 8. At each point in time, there is a tight compromise between economic structure, captured by the steady state of each endogenous variable, and the pattern of shock-dependence, encompassing intrinsic uncertainty about the aggregate state. Their relative importance has already been articulated by Forbes et al. (2017), but unlike their SVAR approach, this chapter adopts the view that both elements are *ex ante* equally important and in principal do not supersede one another even in the short-run.[3] This line of reasoning can be challenged by pointing out that what ultimately matters is indeed the aggregate co-movement of exchange rates and international prices, since that is explicitly observed and targeted by monetary authorities all around the world. But the response to this criticism is simple – all structural models, regardless of their scale, are subject to model-specific nuances, such as the 'true, but unknown' identification of macroeconomic innovations in SVARs or the way in which the model is closed in a general equilibrium environment. No matter how sophisticated is the system of difference equations characterising the dynamics of the macroeconomic fundamentals, the reduced-form measurement of pass-through is almost surely susceptible to model misspecification and measurement errors. Although the approach presented in this chapter is not immune to these criticisms in the short-run, it is nevertheless consistent in the long-run as long as the system of difference equations is saddle-path stable and all innovations are Gaussian. This is because observable elements of the economic structure, such as the shares of invoicing currencies for imports or exports, pin down the magnitude of the terms of trade elasticity in the long-run steady state when the transitory innovations fade away (see lemmas 4 – 10).

Figure 5.1 displays the dynamic causal effect of exchange rate noise on international prices as well as the auxiliary endogenous variables underlying the channel of exchange rate transmission. Each impulse response represents a real depreciation of JPY or SKW against the USD equal to one standard deviation. The focus here is on JP and KR, since US prices have already been shown to be disconnected from exchange rates. First, a local currency depreciation in real terms shows up as a rise in the value of the real bilateral exchange rate (subplot (6)), which leads to an increase in the price of domestically produced and imported final goods (subplots (4) and (3) respectively) due to an increase in the price of commodities priced in USD. It also renders domestic currency less valuable abroad, thus all imports not priced in domestic currency units exhibit a currency conversion effect (i.e. greater rise in the price of imports than exports). In turn, local currency depreciation creates upward pressure on consumer price inflation and a temporary decline in the aggregate consumption growth in real terms (subplots (2) and (1) respectively).

Second, according to lemma 8, transitory exchange rate innovations ought to shift the magnitude of exchange rate pass-through into export and import prices in the short-run. Indeed, exchange rate pass-through into export prices rises, but only by around 0.1% in JP (0.03% in KR) and it rapidly converges to the magnitude of the steady state, since the objective function of the firms incentivises smoothing out the present value of discounted future profits (see subplot (9)). To elaborate, ceteris paribus, $\dot{q}_{ki,t} > 1$ leads to $\dot{p}_{ii,t}(\phi) >$

[3]Unlike SVARs, where truly exogenous exchange rate shocks are unobserved, such as the external instruments advocated by Stock & Watson (2018)), business cycle models can single-out innovations without resorting to *ad hoc* internal instruments such as a Cholesky decomposition. The premise of exogeneity is the assumption that the variance-covariance matrix of innovations is diagonal. The first order conditions summarising the inter-temporal motion of the equilibria are then transformed into policy functions as in Schmitt-Grohé & Uribe (2004), which expresses each control variable at any given point in time as a function of (pre-determined) state variables and exogenous innovations, thereby providing credible identification restrictions. Each synthetic data point represents a random and independent draw of shocks from their normal distributions simultaneously. Similarly, each impulse response function is generated by drawing an innovation of a given magnitude independently of other innovations and tracing out the ensuing transition path.

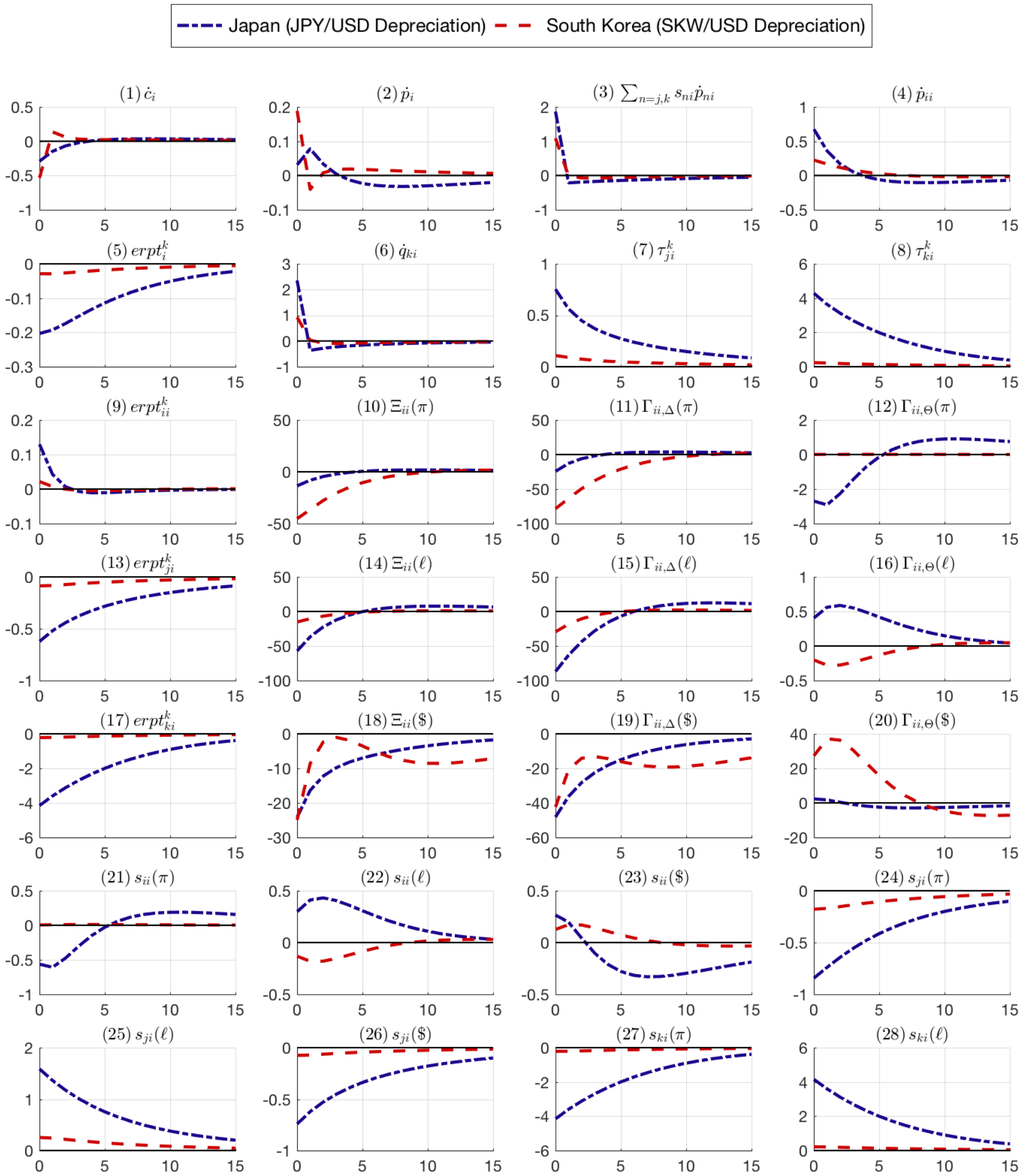
$1\,\forall\,\phi$. The extent to which it does depends on the size of the innovation and the relative importance of price stickiness compared to strategic complementarity. A rise in export prices generally implies a decrease in $\Xi_{ii,t}(\phi)\,\forall\,\phi$, because exporters must re-label the price tags of their products, thereby internalising the price adjustment costs. In turn, the magnitude of $\Gamma_{ii,\Delta,t}(\phi)\,\forall\,\phi$ decreases, which implies a rise in $erpt^{k}_{ii,t}(\phi)\,\forall\,\phi$ (see subplots (10)-(12), (14)-(16), (18)-(20)). Although the nominal rigidity component of DCP firms domiciled in KR suggests exchange rate pass-through ought to increase in the short-run as it does in other sectors, strong strategic complementarities due to a large DCP market share (i.e. 85% as shown in table 5.1) completely undo those effects leading to practically time-invariant exchange rate pass-through into export prices that is in the neighbourhood of zero as shown in table 5.6 (see subplots (19) and (20)).

Conversely, exchange rate pass-through into JP (KR) import prices from KR (JP) falls by around 0.6% (0.05%) as is seen subplot (13), which closely follows the trajectory of the USD invoicing share (subplot (26)). Hence, a local currency depreciation against the USD induces an income and a substitution effect, whereby fewer goods can be purchased in USD for a given budget without excessive borrowing, thus consumers substitute imports invoiced in USD and domestically produced goods with imports priced in local currency units (see subplots (24)-(26)). Similarly, pass-through into import prices from the US falls in both JP and KR, because USD depreciation implies a fall in the share of US imports priced in USD and the exact opposite rise in the share of US imports priced in local currency units (subplots (27) and (28)). Because the fall in exchange rate pass-through from other minor and major currency areas is considerably larger than the rise in export prices, at least in JP, the pass-through into aggregate consumer prices falls (subplot (5)). A local currency depreciation is therefore less inflationary at the consumer level than an identical appreciation is deflationary. This is because local currency depreciations lead to a fall in the amount of imported goods and an endogenous rise in the home-bias of consumption, such that pass-through into consumer prices is pro-cyclical when subject to domestic currency innovations.

The net causal effect of exchange rates on the prices of imports and exports constitutes the terms of trade transmission channel. Consistent with the volatility of the terms of trade elasticities shown in table 5.5, they are more responsive in JP than KR (see subplots (7) and (8)). In fact, the terms of trade elasticity in KR is mostly constant and equal to the steady state throughout the projected horizon. It does not imply that exchange rates exert no influence on the terms of trade. It simply reflects the large share of USD imports in KR, which enhances the average causal effect and diminishes the dynamic causal effect.

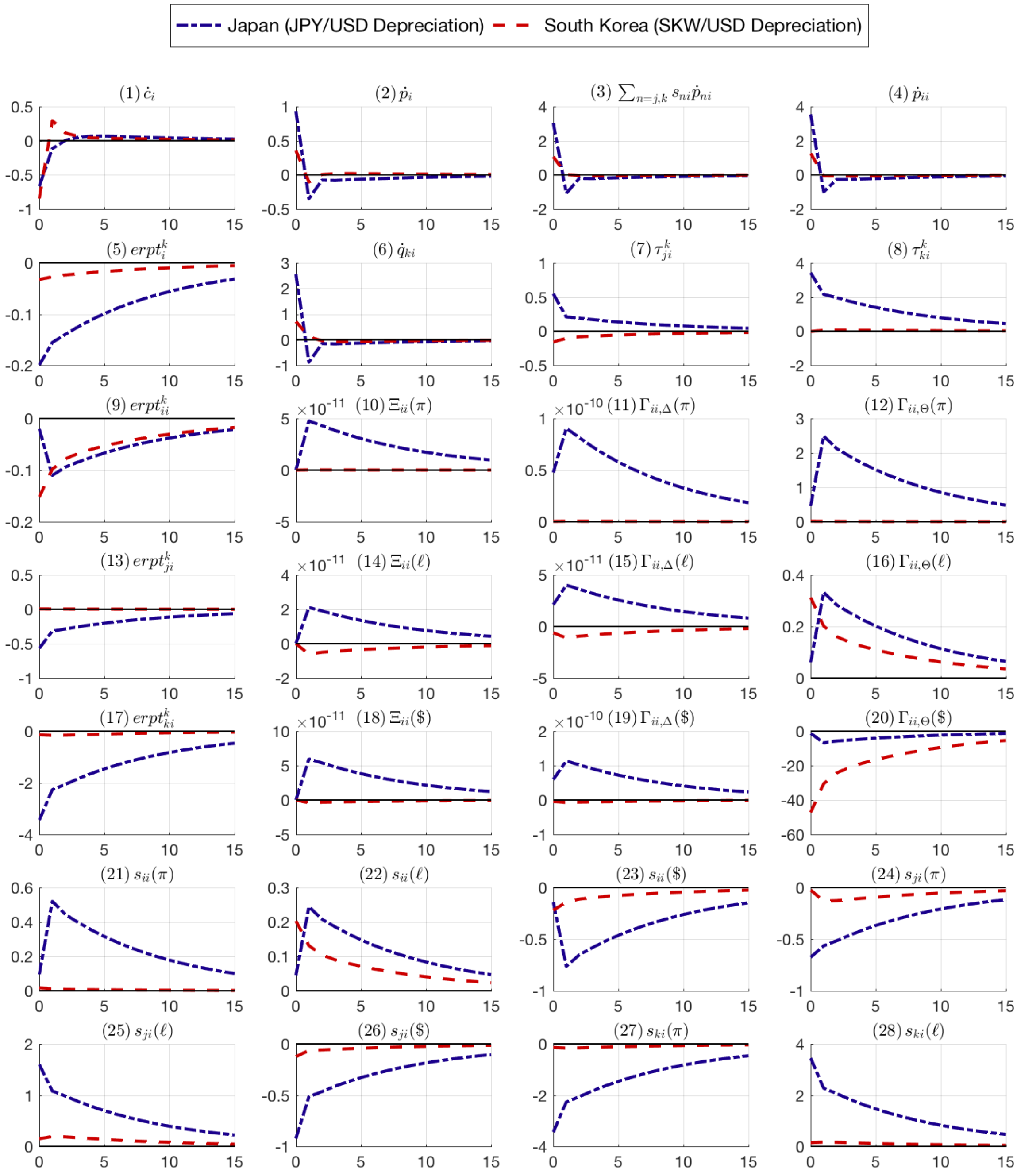
What is the relative importance of sticky prices and strategic complementarities? Figures 5.2 present the dynamic causal effects generated by setting the convexity of price adjustment costs for all firms in all countries to zero while keeping all other estimates at their default values. Although price stickiness exerts important implications for the average causal effect, it appears that the impulse response functions of exchange rate pass-through into import prices remain practically unchanged upon impact, but notably the transition path is somewhat less persistent. The main difference appears to be the change in the sign of the exchange rate pass-through into export prices. Hence, when prices are perfectly flexible, even if wage stickiness

FIGURE 5.1: Dynamic Causal Effect When Prices Are Sticky



Each subplot depicts the dynamic causal effect on a given variable due to a one-off unanticipated exogenous innovation at time $t = 0$ equal to one standard deviation in magnitude. The horizontal axis measures time in monthly intervals, while the vertical axis measures the absolute change in the variable over time.

FIGURE 5.2: Dynamic Causal Effect When Prices Are Flexible



Each subplot depicts the dynamic causal effect on a given variable due to a one-off unanticipated exogenous innovation at time $t = 0$ equal to one standard deviation in magnitude. The horizontal axis measures time in monthly intervals, while the vertical axis measures the absolute change in the variable over time.

spills over into price stickiness, strategic complementarity of PCP and LCP exporters with the DCP exporters leads to a decline in exchange rate pass-through into export prices. Because DCP export unit costs are orthogonal to the fluctuations of home currency against the USD, but PCP and LCP firms exhibit strong cost-push effects, they optimally choose to reflect some of the exchange rate innovations as indicated by a positive average causal effect in table 5.6. But the dynamic causal effect of exchange rates on export prices is pro-cyclical (counter-cyclical) when they are flexible (sticky).

To sum up, (i) dynamic causal effect of exchange rates on the terms of trade is quantitatively dominated by the average causal effect; (ii) dynamic causal effect of exchange rates on export prices is pro-cyclical (counter-cyclical) when they are flexible (sticky); and (iii) exchange rate pass-through into consumer prices is small, but positive and pro-cyclical.

5.5 Summary of Empirical Results

One may be inclined to dismiss some of the seemingly subtle differences between the US-JP-KR model presented in this thesis and a number of other open economy models as mere 'bells and whistles'. After all, the 'impossible trinity' articulated by the Mundell-Fleming paradigm still holds and the nexus of the design associated with the monetary policy conduct remains unchanged in large economies, such as the United States. But this chapter bears important implications for the literature on the international monetary system. The core of the US-JP-KR model expands the blueprint of the Dominant Currency Paradigm (DCP) in multiple dimensions and uncovers a myriad of new insights about the effectiveness of monetary policy in open economies characterised by the globalised export market structure. The most noteworthy qualitative and quantitative predictions presented in this chapter can be summarised as follows: (i) US terms of trade are effectively neutral to movements of the USD; (ii) USD depreciations improve the rest-of-the-world terms of trade by more than USD appreciations cause them to deteriorate; (iii) sticky prices can either significantly increase or decrease the magnitude of the terms of trade elasticity to major currency innovations, depending on the shares of imports and exports invoiced in USD; (iv) pricing-to-market alone does not automatically lead to non-linear exchange rate pass-through, but it could potentially amplify the non-monotonicity of the exchange rate channel; (v) dynamic causal effect of exchange rates on the terms of trade is quantitatively dominated by the average causal effect; and (vi) dynamic causal effect of exchange rates on export prices is pro-cyclical (counter-cyclical) when they are flexible (sticky).

Chapter 6

Conclusion

After all that is said and done, there remains a stark incongruity between the US-UK and the US-JP-KR models. On the one hand, when the multi-country model is linearised around the balanced growth path, the US terms of trade are virtually neutral to exchange rate movements. On the other hand, when international price inflation is subject to non-trivial higher order moments, the US terms of trade elasticity exhibits excessively strong negative correlation with the exchange rate. One thing is for certain, this discrepancy is unrelated to the properties of the exchange rate noise, because both models are remarkably successful in terms of characterising the volatility and the persistence of the real exchange rate dynamics. Indeed, the dynamic causal effect is quantitatively negligible in the linearised model, while the non-linear model exhibits substantial non-monotonicities, but only when exchange rate innovations are sufficiently large. Although the analysis of downward price rigidities combined with strategic complementarities is not carried out explicitly in this thesis, the results suggest that it has the potential to enlarge the size and the persistence of exchange rate pass-through asymmetry even further. It is also explicitly assumed that US monetary authorities do not exacerbate the non-linearities with overly hawkish or dovish responses when USD fluctuates, but that may well be the case in practice. Still, much of the skewness in international prices has largely subdued in the wake of the Great Financial Crisis, which has lead to almost a decade of persistently low rate of inflation. As a result, the selection effect that is chiefly responsible for the predicted non-linearities is mostly dampened in spite of the multiple rounds of unconventional monetary policy measures implemented in many major currency areas around the world. Only time will tell if previously observed large swings in exchange rates are able to shift inflation expectations across distinct states in the future. Until then, it seems that among many other things – 'money' cannot buy US export competitiveness.

Appendix A

A.1 Exchange Rate Pass-Through

Consider the symmetric equilibrium price setting condition for the ϕ-type firm:

$$p_{in,t}(\phi) = \tau_{in} q_{in,t} \delta_{in,t}(\phi) p_{ii,t}(\phi).$$

Differentiate both sides with respect to the real exchange rate $q_{in,t}$ and rearrange:

$$\begin{aligned}
\frac{\partial p_{in,t}(\phi)}{\partial q_{in,t}} &= \tau_{in} \left[\delta_{in,t}(\phi) p_{ii,t}(\phi) + q_{in,t} \delta_{in,t}(\phi) \frac{\partial p_{ii,t}(\phi)}{\partial q_{in,t}} + q_{in,t} p_{ii,t}(\phi) \frac{\partial \delta_{in,t}(\phi)}{\partial q_{in,t}} \right], \\
\frac{\partial p_{in,t}(\phi)}{\partial q_{in,t}} \frac{q_{in,t}}{p_{in,t}(\phi)} &= \frac{\tau_{in} \delta_{in,t}(\phi) q_{in,t} p_{ii,t}(\phi)}{p_{in,t}(\phi)} \left[1 - \frac{\partial p_{ii,t}(\phi)}{\partial q_{ni,t}} \frac{q_{ni,t}}{p_{ii,t}(\phi)} - \frac{\partial \delta_{in,t}(\phi)}{\partial q_{ni,t}} \frac{q_{ni,t}}{\delta_{in,t}(\phi)} \right], \\
\frac{\partial \ln p_{in,t}(\phi)}{\partial \ln q_{in,t}} &= 1 - \frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ni,t}} - \frac{\partial \ln \delta_{in,t}(\phi)}{\partial \ln q_{ni,t}}, \\
erpt_{in,t}(\phi) &= 1 - erpt_{ii,t}(\phi) - ptm_{in,t}(\phi),
\end{aligned} \tag{A.1}$$

since

$$q_{ni,t} = q_{in,t}^{-1}, \tag{A.2}$$

$$\frac{\partial \ln q_{in,t}}{\partial \ln q_{ni,t}} = -1. \tag{A.3}$$

By construction, the product market of PCP exports is perfectly integrated such that $\delta_{in,t}(\pi) = 1$ and $ptm_{in,t}(\pi) = 0$. As a result, pass-through into PCP import prices is inversely related to pass-through into PCP export prices:

$$erpt_{in,t}(\pi) = 1 - erpt_{ii,t}(\pi). \tag{A.4}$$

Conversely, LCP product market is imperfectly integrated such that $ptm_{in,t}(\pi) \neq 0$ and determined as the wedge between price changes and exchange rate volatility:

$$ptm_{in,t}(\phi) = 1 - erpt_{ii,t}(\phi) - erpt_{in,t}(\phi). \tag{A.5}$$

Next, recall that the aggregate import price index in the symmetric equilibrium is given by

$$p_{in,t} = \left[\sum_{\phi} \chi_{in}(\phi) p_{in,t}(\phi)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}.$$

The exchange rate pass-through into aggregate import prices is therefore obtained as a weighted average of pass-through into each ϕ-type import prices:

$$erpt_{in,t} = \frac{\partial \ln p_{in,t}}{\partial \ln q_{in,t}} = \sum_{\phi} \chi_{in}(\phi) \left[\frac{p_{in,t}(\phi)}{p_{in,t}}\right]^{1-\varepsilon} erpt_{in,t}(\phi) = \sum_{\phi} s_{in}(\phi) erpt_{in,t}(\phi). \qquad \text{(A.6)}$$

Similarly, the consumer price index is given by

$$p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i} \left(\frac{1}{1-\alpha_i}\left(\sum_{n=1}^{N} \alpha_{ni}\, p_{ni,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\alpha_i}.$$

Exchange rate pass-through into consumer prices is therefore obtained using the producer and consumer price index identities:

$$erpt_{i,t} = \frac{\partial \ln p_{i,t}}{\partial \ln q_{ni,t}} = (1-\alpha_i) \sum_{n=1}^{N} \alpha_{ni} \left(\frac{p_{ni,t}}{p_{i,x,t}}\right)^{1-\eta} erpt_{ni,t} = (1-\alpha_i) \sum_{n=1}^{N} s_{ni,t} erpt_{ni,t}. \quad \text{(A.7)}$$

It follows that if $erpt_{in,t}(\phi)$ is known, then exchange rate pass-through into all prices can be determined endogenously. Consider the optimal real export price:

$$p_{ii,t}(\phi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}.$$

Exchange rate pass-through into i'th economy export prices is derived by differentiating the above with respect to the real exchange rate using the quotient rule:

$$erpt_{ii,t}(\phi) = \frac{\partial \ln mc_{ii,t}(\phi)}{\partial \ln q_{ni,t}} - \frac{\partial \ln \Phi_{ii,t}(\phi)}{\partial \ln q_{ni,t}}. \qquad \text{(A.8)}$$

Without loss of generality it is assumed that $Q_{ii,t}(\phi) = \min\{q_{ni,t};\ n = 1, \ldots, N\} = q_{ni,t}$, such that

$$mc_{ii,t}(\phi) = \frac{\xi_i q_{ni,t}}{z_{i,t}},$$

thus

$$\frac{\partial \ln mc_{ii,t}(\phi)}{\partial \ln q_{ni,t}} = 1. \qquad \text{(A.9)}$$

The auxiliary price setting variable is given by

$$\Phi_{ii,t}(\phi) = 1 - \Delta_{ii,t}(\phi) + \frac{\Delta'_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{\varepsilon - 1} - \lambda_{i,t,t+1}\mathbb{E}_t \left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)} \frac{\Delta'_{ii,t+1}(\phi)}{\varepsilon - 1} \frac{\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}}\right].$$

Differentiating the above with respect to the real exchange rate gives:

$$
\begin{aligned}
\frac{\partial \ln \Phi_{ii,t}(\phi)}{\partial \ln q_{ni,t}} &= -\frac{\Delta'_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ni,t}} \\
&+ \frac{1}{\Phi_{ii,t}(\phi)(\varepsilon-1)}\left[\Delta''_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)^2\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ni,t}} + \Delta'_{ii,t}(\phi)q_{ni,t}\frac{\partial \dot{p}_{ii,t}(\phi)}{\partial q_{ni,t}}\right] \\
&- \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta''_{ii,t+1}(\phi)\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon-1)}\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ni,t}}\right] \\
&- \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{2\Delta'_{ii,t+1}(\phi)\dot{p}_{ii,t+1}(\phi)q_{ni,t}}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon-1)}\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ni,t}}\right] \\
&+ \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta'_{ii,t+1}(\phi)\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon-1)}\frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln q_{ni,t}}\right], \\
&= \frac{\dot{p}_{ii,t}(\phi)}{\varepsilon-1}\frac{erpt_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\phi) + (2-\varepsilon)\Delta'_{ii,t}(\phi)\right] \\
&+ \mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)^2}{r_{i,t}(\varepsilon-1)}\frac{erpt_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\left(\dot{p}_{ii,t+1}(\phi)\Delta''_{ii,t+1}(\phi) + (2-\varepsilon)\Delta'_{ii,t+1}(\phi)\right)\right], \\
&= erpt_{ii,t}(\phi)\left\{\Xi_{ii,t}(\phi) + \mathbb{E}_t\left[\Xi_{ii,t+1}(\phi)\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{r_{i,t}}\right]\right\}, \\
&= erpt_{ii,t}(\phi)\Gamma_{ii,t}(\phi), \qquad \text{(A.10)}
\end{aligned}
$$

where

$$
\Gamma_{ii,t}(\phi) = \Xi_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t\left[\Xi_{ii,t+1}(\phi)\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right], \qquad \text{(A.11)}
$$

$$
\Xi_{ii,t}(\phi) = \frac{\dot{p}_{ii,t}(\phi)\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\phi) + (2-\varepsilon)\Delta'_{ii,t}(\phi)\right]}{\Phi_{ii,t}(\phi)(\varepsilon-1)}, \qquad \text{(A.12)}
$$

$$
\frac{\partial \dot{p}_{ii,t}(\phi)}{\partial q_{ni,t}} = \frac{\dot{p}_{ii,t}(\phi)}{p_{ii,t}(\phi)}\frac{\partial p_{ii,t}(\phi)}{\partial q_{ni,t}} = \frac{erpt_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{q_{ni,t}}, \qquad \text{(A.13)}
$$

$$
\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ni,t}} = -\frac{\dot{p}_{ii,t+1}(\phi)}{p_{ii,t}(\phi)}\frac{\partial p_{ii,t}(\phi)}{\partial q_{ni,t}} = -\frac{erpt_{ii,t}(\phi)\dot{p}_{ii,t+1}(\phi)}{q_{ni,t}}, \qquad \text{(A.14)}
$$

$$
\frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln q_{ni,t}} = \frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln p_{ii,t}(\phi)}\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ni,t}} = -\varepsilon\, erpt_{ii,t}(\phi). \qquad \text{(A.15)}
$$

Substituting equation (A.10) and (A.9) into (A.8) and solving for $erpt_{ii,t}(\phi)$ gives

$$
erpt_{ii,t}(\phi) = \frac{1}{1+\Gamma_{ii,t}(\phi)}. \qquad \text{(A.16)}
$$

It follows from equation (A.4) that exchange rate pass-through into PCP import prices is given by

$$
erpt_{in,t}(\pi) = 1 - erpt_{ii,t}(\pi) = \frac{\Gamma_{ii,t}(\pi)}{1+\Gamma_{ii,t}(\pi)}. \qquad \text{(A.17)}
$$

Because the first order condition for LCP import prices is analogous to LCP export prices, the pass-through into LCP import prices is analogous to LCP export prices:

$$erpt_{in,t}(-\pi) = \frac{1}{1 + \Gamma_{in,t}(-\pi)}. \tag{A.18}$$

Notably, equation (A.18) assumes that $Q_{in,t}(\phi) = \min\{q_{in,t};\ n = 1, \ldots, N\} = q_{in,t}$, such that

$$mc_{in,t}(\phi) = \frac{\xi_i q_{in,t}}{z_{i,t}},$$

thus

$$\frac{\partial \ln mc_{in,t}(\phi)}{\partial \ln q_{in,t}} = 1. \tag{A.19}$$

Substituting the above results into equation (A.6) gives the final expression for exchange rate pass-through into aggregate import prices:

$$\begin{aligned} erpt_{in,t} &= s_{in,t}(-\pi) erpt_{in,t}(-\pi) + s_{in,t}(\pi) erpt_{in,t}(\pi), \\ &= \frac{s_{in,t}(-\pi)}{1 + \Gamma_{in,t}(-\pi)} + \frac{s_{in,t}(\pi)\Gamma_{ii,t}(\pi)}{1 + \Gamma_{ii,t}(\pi)}. \end{aligned} \tag{A.20}$$

Note that if the assumption of monopolistic competition is maintained throughout the derivations, such that each individual firm is too 'small' to influence the aggregate variables, then it makes no quantitative difference whether exchange rate pass-through is derived in nominal terms or real terms. However, it is more convenient to showcase the results in real terms, since the model is solved for a general case in which price levels are indeterminate due to the deterministic trend of inflation pursued by the central banks. Moreover, all innovations in this model are generated in a perfectly unanticipated manner. Therefore, the derivations of exchange rate pass-through presented above treat all forward-looking variables as orthogonal to contemporaneous exchange rate movements.

A.2 Production Technology

Import-export wholesalers solve the following real cost minimisation problem:

$$\min_{\{y_{in,t}(\omega,\phi)\}} tc_{ij,t}(\omega,\phi) = Q_{in,t}(\phi) m_{in,t}(\omega,\phi) = \frac{\xi_i Q_{in,t}(\phi) y_{in,t}(\omega,\phi)}{z_{i,t}}, \tag{A.21}$$

where the first-order condition is simply given by

$$\frac{\partial tc_{in,t}(\omega)}{\partial y_{in,t}(\omega)} = mc_{in,t}(\phi) = \frac{\xi_i Q_{in,t}(\phi)}{z_{i,t}}, \tag{A.22}$$

where

$$Q_{in,t}(\pi) = \min\{q_{ni,t}; n = 1, 2, ..., N\}, \quad Q_{in,t}(-\pi) = \min\{q_{jn,t}; j = 1, 2, ..., N\}. \tag{A.23}$$

The next step along the pricing chain is the relationship between the wholesale sector and the distributors. The demand facing the upstream wholesalers from the distribution sector is derived from the profit maximisation problem of the competitive collectors, who aggregate individual varieties of goods into country-specific bundles:

$$\max_{\{y_{in,t}(\phi)\}} \; p_{in,t} y_{in,t} - \sum_{\phi} \int_{0}^{\chi_{in}(\phi)} p_{in,t}(\omega,\phi) y_{in,t}(\omega,\phi) d\omega$$

$$\text{s.t.} \;\; y_{in,t} = \left[\sum_{\phi} \int_{0}^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi)^{1-1/\varepsilon} d\omega \right]^{1/(1-1/\varepsilon)} ,$$

which gives rise to the following first order condition:

$$y_{in,t}(\omega,\phi) = y_{in,t} \left[\frac{p_{in,t}(\omega,\phi)}{p_{in,t}} \right]^{-\varepsilon} . \tag{A.24}$$

The aggregate demand for the ϕ-type imports from location i to n is then given by

$$y_{in,t}(\phi) = \int_{0}^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi) d\omega = \chi_{in}(\phi) y_{in,t} \left[\frac{p_{in,t}(\phi)}{p_{in,t}} \right]^{-\varepsilon} \tag{A.25}$$

The aggregate upstream price index is derived by substituting the above first order condition into the budget constraint of the collector, which gives rise to the following expression:

$$p_{in,t} = \left[\sum_{\phi} \int_{0}^{\chi_{in}(\phi)} p_{in,t}(\omega,\phi)^{1-\varepsilon} d\omega \right]^{1/(1-\varepsilon)} , \tag{A.26}$$

or, alternatively, in the symmetric equilibrium it is equivalent to

$$p_{in,t} = \left[\sum_{\phi} \chi_{in}(\phi) p_{in,t}(\phi)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} . \tag{A.27}$$

The optimal demand for locally-produced and imported goods are derived from the second stage of aggregation:

$$\max_{\{y_{ni,t}\}} \; p_{i,x,t} x_{i,t} - \sum_{n=1}^{N} p_{ni,t} y_{ni,t}, \quad \text{s.t.} \; x_{i,t} = \left(\sum_{n=1}^{N} \alpha_{ni}^{1/\eta} y_{ni,t}^{1-1/\eta} \right)^{1/(1-1/\eta)} ,$$

The first order condition is given by

$$y_{ni,t} = \alpha_{ni} x_{i,t} \left[\frac{p_{ni,t}}{p_{i,x,t}} \right]^{-\eta} , \tag{A.28}$$

The aggregate producer price index is derived by substituting the above first order condition into the budget constraint of the distributor, which gives rise to the following expression:

$$p_{i,x,t} = \left(\sum_{n=1}^{N} \alpha_{ni}\, p_{ni,t}^{1-\eta} \right)^{1/(1-\eta)} . \tag{A.29}$$

Finally, the competitive retailer maximises profits subject to the Cobb-Douglas production technology characterising the downstream market:

$$\max_{\{h_{i,t}, x_{i,t}\}} \quad p_{i,t}c_{i,t} - w_{i,t}h_{i,t} - p_{i,x,t}x_{i,t}, \quad \text{s.t. } c_{i,t} = (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i}.$$

The cost minimisation problem is formulated as a static Lagrangian:

$$\mathbb{L} = p_{i,t}c_{i,t} - w_{i,t}h_{i,t} - p_{i,x,t}x_{i,t} - o_{i,t}\left(y_{i,t} - (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i} \right), \tag{A.30}$$

where the first order conditions are given by

$$p_{i,x,t} - (1-\alpha_i)o_{i,t}\left(\frac{c_{i,t}}{x_{i,t}} \right) = 0, \tag{A.31}$$

$$w_{i,t} - \alpha_i o_{i,t}\left(\frac{c_{i,t}}{h_{i,t}} \right) = 0, \tag{A.32}$$

$$c_{i,t} - (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i} = 0. \tag{A.33}$$

It can be shown that the shadow price of technology $o_{i,t}$ corresponds to the consumer price index. In order to find the functional form of the retail price index, consider the unconditional demand schedule for each of the factor inputs. They are obtained by firstly dividing the top two first order conditions one by the other:

$$\frac{h_{i,t}}{x_{i,t}} = \frac{\alpha_i}{1-\alpha_i}\frac{p_{i,x,t}}{w_{i,t}}, \tag{A.34}$$

and then substitute the above schedule for tradable goods bundle and labour, each in turn, into the technological constraint:

$$x_{i,t} = c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{\alpha_i} \right)^{\alpha_i} \left(\frac{p_{i,x,t}}{1-\alpha_i} \right)^{-\alpha_i}, \tag{A.35}$$

$$h_{i,t} = c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{1-\alpha_i} \right)^{1-\alpha_i} \left(\frac{p_{i,x,t}}{1-\alpha_i} \right)^{\alpha_i - 1} . \tag{A.36}$$

Then substitute the unconditional demand schedules into the expression for the total effective real costs of production to obtain

$$\begin{aligned} tc_{i,t} &= w_{i,t}h_{i,t} + p_{i,x,t}x_{i,t} = (\alpha_i + 1 - \alpha_i)o_{i,t}c_{i,t} = o_{i,t}c_{i,t} \\ &= c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}, \qquad \text{(A.37)} \\ \frac{\partial tc_{i,t}}{\partial c_{i,t}} &= p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}. \qquad \text{(A.38)} \end{aligned}$$

Therefore, the first order conditions can be written as

$$\begin{aligned} p_{i,x,t}x_{i,t} &= (1-\alpha_i)p_{i,t}c_{i,t}, \\ w_{i,t}h_{i,t} &= \alpha_i p_{i,t}c_{i,t}. \end{aligned}$$

A.3 Price Stickiness

The optimal price that the ω wholesaler decides to set is derived by maximising the present discounted value of the real profits subject to a sequence of demand schedules:

$$\begin{aligned} &\max_{\{P_{ii,t}(\omega,\phi)\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{i,t,t+1} \left\{ \sum_{n=1}^{N} \left[\left(1 - \mathit{\Delta}_{ii,t}(\omega,\phi)\right) p_{ii,t}(\omega,\phi) - mc_{ii,t}(\phi) \right] y_{ii,t}(\omega,\phi) \right\} \\ &\text{s.t.} \quad y_{ii,t}(\omega,\phi) = y_{ii,t}\left[\frac{p_{ii,t}(\omega,\phi)}{p_{ii,t}}\right]^{-\varepsilon}, \end{aligned}$$

The first-order condition is obtained using the product rule:

$$\begin{aligned} &(1-\mathit{\Delta}_{ii,t}(\phi))(1-\varepsilon)\left(\frac{y_{ii,t}(\omega,\phi)}{P_{i,t}}\right) + \varepsilon\, mc_{ii,t}(\phi)\left(\frac{y_{ii,t}(\omega,\phi)}{P_{ii,t}(\omega,\phi)}\right), \\ &- \mathit{\Delta}'_{ii,t}(\phi)\dot{p}_{ii,t}(\omega,\phi)\left(\frac{y_{ii,t}(\omega,\phi)}{P_{i,t}}\right) + \mathbb{E}_t\left[\lambda_{i,t,t+1}\mathit{\Delta}'_{ii,t+1}(\phi)\dot{p}_{ii,t+1}(\omega,\phi)^2\left(\frac{y_{ii,t+1}(\omega,\phi)}{P_{i,t+1}}\right)\right] = 0. \end{aligned}$$

Alternatively

$$\begin{aligned} &(1-\mathit{\Delta}_{ii,t}(\phi))p_{ii,t}(\omega,\phi) - \left(\frac{\varepsilon}{\varepsilon-1}\right)mc_{ii,t}(\phi), \\ &+\left(\frac{\mathit{\Delta}'_{ii,t}(\phi)\dot{p}_{ii,t}(\omega,\phi)}{\varepsilon-1}\right)p_{ii,t}(\omega,\phi) - \mathbb{E}_t\left[\frac{y_{ii,t+1}(\omega,\phi)}{y_{ii,t}(\omega,\phi)}\frac{\mathit{\Delta}'_{ii,t+1}(\phi)}{\varepsilon-1}\frac{\dot{p}_{ii,t+1}(\omega,\phi)^2}{r_{i,t}}\right]p_{ii,t}(\omega,\phi) = 0, \end{aligned}$$

where $\dot{p}_{ii,t}(\omega,\phi) = \dot{p}_{i,t}p_{ii,t}(\omega,\phi)/p_{ii,t-1}(\omega,\phi)$. Then note that linear production technology and homogenous productivity implies $p_{ii,t}(\omega,\phi) = P_{ii,t}(\omega,\phi)/P_{i,t} = P_{ii,t}(\phi)/P_{i,t} \Rightarrow$

$y_{ii,t}(\omega,\phi) = y_{ii,t}(\phi)$, therefore

$$p_{ii,t}(\phi) = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{mc_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}, \tag{A.39}$$

$$\Phi_{ii,t}(\phi) = 1 - \Delta_{ii,t}(\phi) + \frac{\Delta'_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{\varepsilon-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta'_{ii,t+1}(\phi)}{\varepsilon-1}\frac{\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}}\right]. \tag{A.40}$$

A.4 Wage Bargaining

The competitive labour packer maximises the real profits that accrue as part of the aggregation process:

$$\max_{\{h_{i,t}(\omega)\}} \quad w_{i,t}h_{i,t} - \int_0^1 w_{i,t}(\omega)h_{i,t}(\omega)d\omega,$$

$$\text{s.t.} \quad h_{i,t} = \left[\int_0^1 h_{i,t}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}d\omega\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The first-order condition gives rise to the following service-specific demand schedule:

$$h_{i,t}(\omega) = h_{i,t}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\varepsilon}. \tag{A.41}$$

The optimal inter-temporal allocation of consumption, savings and optimal wage is derived by solving the following dynamic stochastic optimisation problem:

$$\max_{\{c_{i,t}(\omega),w_{i,t}(\omega),b_{i,t+1}(\omega)\}} \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\log\left(\frac{c_{i,t}(\omega)-\vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right)h_{i,t}(\omega)^{1+\varphi_i}\right\}$$

$$\text{s.t.} \quad h_{i,t}(\omega) = \frac{h_{i,t}}{\Omega_i}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\varepsilon},$$

$$\text{s.t.} \quad c_{i,t}(\omega) + \lambda_{i,t,t+1}\mathbb{E}_t[b_{i,t+1}(\omega)] \leq b_{i,t}(\omega) + (1-\Delta_{i,w,t}(\omega))w_{i,t}(\omega)h_{i,t}(\omega) + d_{i,t}(\omega),$$

Re-writing the above in terms of the Current Value Lagrangian gives

$$\begin{aligned}\mathbb{CVL} = {} & \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\{\log\left(\frac{c_{i,t}(\omega)-\vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right)h_{i,t}(\omega)^{1+\varphi_i} \\ & + o_{i,h,t}(\omega)\left[\frac{h_{i,t}}{\Omega_i}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\varepsilon} - h_{i,t}(\omega)\right] \\ & + o_{i,c,t}(\omega)\left[b_{i,t}(\omega) + (1-\Delta_{i,w,t}(\omega))w_{i,t}(\omega)h_{i,t}(\omega) + d_{i,t}(\omega) - c_{i,t}(\omega) - \lambda_{i,t,t+1}\mathbb{E}_t[b_{i,t+1}(\omega)]\right]\}.\end{aligned}$$

Consider the first-order conditions with respect to consumption and bond holdings:

$$\frac{1}{c_{i,t}(\omega)-\vartheta_i c_{i,t-1}(\omega)}-\vartheta_i\beta\mathbb{E}_t\left[\frac{1}{c_{i,t+1}(\omega)-\vartheta_i c_{i,t}(\omega)}\right]-o_{i,c,t}(\omega)=0, \tag{A.42}$$

$$o_{i,c,t}(\omega)\mathbb{E}_t[\lambda_{i,t,t+1}]-\beta\mathbb{E}_t[o_{i,c,t+1}(\omega)]=0. \tag{A.43}$$

Alternatively, imposing preference homotheticity gives

$$u_{i,c,t}=\frac{\partial u_{i,t}}{\partial c_{i,t}}=o_{i,c,t}=\Psi_{i,t}-\vartheta_i\beta\mathbb{E}_t[\Psi_{i,t+1}], \tag{A.44}$$

$$1=\Psi_{i,t}(c_{i,t}-\vartheta_i c_{i,t-1}), \tag{A.45}$$

$$\lambda_{i,t,t+1}u_{i,c,t}=\beta\mathbb{E}_t\left[u_{i,c,t+1}\right]. \tag{A.46}$$

The first-order condition with respect to the real hourly wage rate is given by

$$(1-\Delta_{i,w,t}(\omega))(1-\varepsilon)o_{i,c,t}(\omega)h_{i,t}(\omega)-\varepsilon\left(\frac{o_{i,h,t}(\omega)h_{i,t}(\omega)}{w_{i,t}(\omega)}\right)-\Delta'_{i,w,t}(\omega)\dot{w}_{i,t}(\omega)o_{i,c,t}(\omega)h_{i,t}(\omega),$$
$$+\beta\mathbb{E}_t\left[\Delta'_{i,w,t+1}(\omega)\dot{w}_{i,t+1}(\omega)^2 o_{i,c,t+1}(\omega)h_{i,t+1}(\omega)\right]=0. \tag{A.47}$$

Preference homotheticity implies $w_{i,t}(\omega)=w_{i,t}\Rightarrow h_{i,t}(\omega)=h_{i,t}$, thus

$$(1-\Delta_{i,w,t})w_{i,t}+\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{u_{i,h,t}}{u_{i,c,t}}+\left(\frac{\Delta'_{i,w,t}\dot{w}_{i,t}}{\varepsilon-1}\right)w_{i,t}$$
$$-\beta\mathbb{E}_t\left[\frac{u_{i,c,t+1}}{u_{i,c,t}}\frac{h_{i,t+1}}{h_{i,t}}\frac{\Delta'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\varepsilon-1}w_{i,t}\right]=0, \tag{A.48}$$

or simply

$$w_{i,t}=\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{mrs_{i,t}}{\Theta_{i,t}}, \tag{A.49}$$

$$\Theta_{i,t}=1-\Delta_{i,w,t}+\frac{\Delta'_{i,w,t}\dot{w}_{i,t}}{\varepsilon-1}-\lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{h_{i,t+1}}{h_{i,t}}\frac{\Delta'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\varepsilon-1}\right], \tag{A.50}$$

where $\dot{w}_{i,t}=w_{i,t}/w_{i,t-1}$, $mrs_{i,t}=-u_{i,h,t}/u_{i,c,t}$, and $u_{i,h,t}=\partial u_{i,t}/\partial h_{i,t}=o_{i,h,t}=-\psi_i h_{i,t}^{\varphi_i}$.

A.5 De-trended Equilibrium Conditions

Consumption Smoothing:

$$\tilde{u}_{i,c,t}q_{ni,t}=\mu_i e_{ni,t}\tilde{u}_{n,c,t}, \tag{A.51}$$

$$\tilde{u}_{i,c,t}\lambda_{i,t,t+1}\mathbb{E}_t\left[\dot{a}_{i,t+1}\right]=\beta\mathbb{E}_t\left[\tilde{u}_{i,c,t+1}\right], \tag{A.52}$$

$$\tilde{u}_{i,c,t}=\tilde{\Psi}_{i,t}-\vartheta_i\beta\mathbb{E}_t\left[\frac{\tilde{\Psi}_{i,t+1}}{\dot{a}_{i,t+1}}\right], \tag{A.53}$$

$$\tilde{\Psi}_{i,t}=\frac{\dot{a}_{i,t}}{\dot{a}_{i,t}\tilde{c}_{i,t}-\vartheta_i\tilde{c}_{i,t-1}}. \tag{A.54}$$

Production Technology:

$$\tilde{c}_{i,t} = h_{i,t}^{\alpha_i} \tilde{x}_{i,t}^{1-\alpha_i}, \tag{A.55}$$

$$\frac{h_{i,t}}{\tilde{x}_{i,t}} = \frac{\alpha_i}{1-\alpha_i} \frac{p_{i,x,t}}{\tilde{w}_{i,t}}, \tag{A.56}$$

$$\xi_i \tilde{y}_{ij,t} = z_{i,t} \tilde{m}_{ij,t}, \tag{A.57}$$

$$\tilde{y}_{ii,t} = s_{ii,t} \tilde{x}_{i,t}, \tag{A.58}$$

$$\tilde{y}_{in,t} = s_{in,t} \tilde{x}_{j,t}. \tag{A.59}$$

Trade Weights:

$$s_{ii,t} = \alpha_{ii} \left(\frac{p_{ii,t}}{p_{i,x,t}} \right)^{-\eta}, \tag{A.60}$$

$$s_{in,t} = \alpha_{in} \left(\frac{p_{in,t}}{p_{n,x,t}} \right)^{-\eta}. \tag{A.61}$$

Market Clearing Conditions:

$$\tilde{y}_{i,t} = \tilde{c}_{i,t} + \tilde{nx}_{i,t} + \mathit{\Delta}_{i,w,t} \tilde{w}_{i,t} h_{i,t} + \sum_{\phi} \sum_{n=1}^{N} \mathit{\Delta}_{in,t}(\phi) p_{in,t}(\phi) \tilde{y}_{in,t}(\phi), \tag{A.62}$$

$$\tilde{nx}_{i,t} = -\tilde{nx}_{j,t} = p_{ij,t} q_{ji,t} \tilde{y}_{ij,t} - p_{ji,t} \tilde{y}_{ji,t}. \tag{A.63}$$

Wage Bargaining:

$$\tilde{w}_{i,t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{mrs_{i,t}}{\mathit{\Theta}_{i,t}}, \tag{A.64}$$

$$\mathit{\Theta}_{i,t} = 1 - \mathit{\Delta}_{i,w,t} + \frac{\mathit{\Delta}'_{i,w,t} \dot{w}_{i,t}}{\varepsilon - 1} - \lambda_{i,t,t+1} \mathbb{E}_t \left[\frac{h_{i,t+1}}{h_{i,t}} \frac{\mathit{\Delta}'_{i,w,t+1} \dot{w}_{i,t+1}^2}{\varepsilon - 1} \right], \tag{A.65}$$

$$mrs_{i,t} = -\frac{\tilde{u}_{i,h,t}}{\tilde{u}_{i,c,t}}, \tag{A.66}$$

$$\tilde{u}_{i,h,t} = -\psi_i h_{i,t}^{\varphi_i}, \tag{A.67}$$

$$\mathit{\Delta}_{i,w,t} = \frac{\kappa_{i,w} \left[\exp\left(\zeta_{i,w} \left(\dot{w}_{i,t} - \gamma_i \right) \right) - \zeta_{i,w} \left(\dot{w}_{i,t} - \gamma_i \right) - 1 \right]}{\zeta_{i,w}^2}, \tag{A.68}$$

$$\mathit{\Delta}'_{i,w,t} = \frac{\kappa_{i,w} \left[\exp\left(\zeta_{i,w} \left(\dot{w}_{i,t} - \gamma_i \right) \right) - 1 \right]}{\zeta_{i,w}}. \tag{A.69}$$

Relative Prices:

$$1 = \left(\frac{\tilde{w}_{i,t}}{\alpha_i}\right)^{\alpha_i} \left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}, \tag{A.70}$$

$$p_{i,x,t} = \left[\sum_{n=1}^{N} \alpha_{ni}\, p_{ni,t}^{1-\eta}\right]^{1/(1-\eta)}, \tag{A.71}$$

$$s_{in,t}(\phi) = \chi_{in}(\phi) \left[\frac{p_{in,t}(\phi)}{p_{in,t}}\right]^{1-\varepsilon}, \tag{A.72}$$

$$1 = \sum_{\phi} s_{in,t}(\phi), \tag{A.73}$$

$$mc_{ii,t}(\pi) = mc_{ii,t}(-\pi) = \frac{\xi_i q_{ni,t}}{z_{i,t}}, \tag{A.74}$$

$$mc_{in,t}(-\pi) = \frac{\xi_i q_{in,t}}{z_{i,t}}. \tag{A.75}$$

Producer Currency Pricing:

$$p_{in,t}(\pi) = \tau_{in} q_{in,t} p_{ii,t}(\pi), \tag{A.76}$$

$$p_{ii,t}(\pi) = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{mc_{ii,t}(\pi)}{\Phi_{ii,t}(\pi)}, \tag{A.77}$$

$$\Phi_{ii,t}(\pi) = 1 - \Delta_{ii,t}(\pi) + \frac{\Delta'_{ii,t}(\pi)\dot{p}_{ii,t}(\phi)}{\varepsilon-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{\tilde{y}_{ii,t+1}(\pi)\dot{a}_{i,t+1}}{\tilde{y}_{ii,t}(\pi)} \frac{\Delta'_{ii,t+1}(\pi)}{\varepsilon-1} \frac{\dot{p}_{ii,t+1}(\pi)^2}{\dot{p}_{i,t+1}}\right], \tag{A.78}$$

$$\Delta_{ii,t}(\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(\pi) - \dot{p}_i\right)\right) - \zeta_{i,p}\left(\dot{p}_{ii,t}(\pi) - \dot{p}_i\right) - 1\right]}{\zeta_i^2}, \tag{A.79}$$

$$\Delta'_{ii,t}(\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(\pi) - \dot{p}_i\right)\right) - 1\right]}{\zeta_{i,p}}, \tag{A.80}$$

$$\Delta''_{ii,t}(\pi) = \kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(\pi) - \dot{p}_i\right)\right)\right]. \tag{A.81}$$

Local Currency Pricing:

$$p_{in,t}(-\pi) = \tau_{in} q_{in,t} \delta_{in,t}(-\pi) p_{ii,t}(-\pi), \tag{A.82}$$

$$p_{ii,t}(-\pi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{ii,t}(-\pi)}{\Phi_{ii,t}(-\pi)}, \tag{A.83}$$

$$\begin{aligned}\Phi_{ii,t}(-\pi) = {} & 1 - \Delta_{ii,t}(-\pi) + \frac{\Delta'_{ii,t}(-\pi)\dot{p}_{ii,t}(\phi)}{\varepsilon - 1} \\ & - \lambda_{i,t,t+1}\mathbb{E}_t \left[\frac{\tilde{y}_{ii,t+1}(-\pi)\dot{a}_{i,t+1}}{\tilde{y}_{ii,t}(-\pi)} \frac{\Delta'_{ii,t+1}(-\pi)}{\varepsilon - 1} \frac{\dot{p}_{ii,t+1}(-\pi)^2}{\dot{p}_{i,t+1}}\right],\end{aligned} \tag{A.84}$$

$$p_{in,t}(-\pi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\tau_{in} mc_{in,t}(-\pi)}{\Phi_{in,t}(-\pi)}, \tag{A.85}$$

$$\begin{aligned}\Phi_{in,t}(-\pi) = {} & 1 - \Delta_{in,t}(-\pi) + \frac{\Delta'_{in,t}(-\pi)\dot{p}_{in,t}(\phi)}{\varepsilon - 1} \\ & - \lambda_{i,t,t+1}\mathbb{E}_t \left[\frac{\tilde{y}_{in,t+1}(-\pi)\dot{a}_{i,t+1}}{\tilde{y}_{in,t}(-\pi)} \frac{\Delta'_{in,t+1}(-\pi)}{\varepsilon - 1} \frac{\dot{p}_{in,t+1}(-\pi)^2}{\dot{p}_{n,t+1}}\right],\end{aligned} \tag{A.86}$$

$$\Delta_{ii,t}(-\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(-\pi) - \dot{p}_i\right)\right) - \zeta_{i,p}\left(\dot{p}_{ii,t}(-\pi) - \dot{p}_i\right) - 1\right]}{\zeta_i^2}, \tag{A.87}$$

$$\Delta'_{ii,t}(-\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(-\pi) - \dot{p}_i\right)\right) - 1\right]}{\zeta_{i,p}}, \tag{A.88}$$

$$\Delta''_{ii,t}(-\pi) = \kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{ii,t}(-\pi) - \dot{p}_i\right)\right)\right], \tag{A.89}$$

$$\Delta_{in,t}(-\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{in,t}(-\pi) - \dot{p}_n\right)\right) - \zeta_{i,p}\left(\dot{p}_{in,t}(-\pi) - \dot{p}_n\right) - 1\right]}{\zeta_i^2}, \tag{A.90}$$

$$\Delta'_{in,t}(-\pi) = \frac{\kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{in,t}(-\pi) - \dot{p}_n\right)\right) - 1\right]}{\zeta_{i,p}}, \tag{A.91}$$

$$\Delta''_{in,t}(-\pi) = \kappa_{i,p}\left[\exp\left(\zeta_{i,p}\left(\dot{p}_{in,t}(-\pi) - \dot{p}_n\right)\right)\right]. \tag{A.92}$$

Monetary Policy:

$$r_{i,t} = \mathbb{E}_t\left[\frac{\dot{p}_{i,t+1}}{\lambda_{i,t,t+1}}\right], \tag{A.93}$$

$$r_{i,t} = (r_{i,t-1})^{\rho_{i,r}} (r^*_{i,t})^{1-\rho_{i,r}}, \tag{A.94}$$

$$r^*_{i,t} = \frac{\gamma_i \dot{p}_i}{\beta} \left(\frac{\dot{p}_{i,t}}{p_i}\right)^{\nu_p} \left(\frac{\tilde{y}_{i,t}}{\tilde{y}_i}\right)^{\nu_y} \exp(\sigma_{i,r}\epsilon_{i,r,t}). \tag{A.95}$$

Exchange Rate Pass-Through:

$$erpt_{i,t} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt_{ni,t}, \tag{A.96}$$

$$erpt_{ii,t} = \sum_{\phi} \frac{s_{ii,t}(\phi)}{1+\mathit{\Gamma}_{ii,t}(\phi)}, \tag{A.97}$$

$$erpt_{in,t} = \frac{s_{in,t}(-\pi)}{1+\mathit{\Gamma}_{in,t}(-\pi)} + \frac{s_{in,t}(\pi)\mathit{\Gamma}_{ii,t}(\pi)}{1+\mathit{\Gamma}_{ii,t}(\pi)}, \tag{A.98}$$

$$\mathit{\Gamma}_{ii,t}(\phi) = \mathit{\Xi}_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t\left[\mathit{\Xi}_{ii,t+1}(\phi)\frac{\tilde{y}_{ii,t+1}(\phi)\dot{a}_{i,t+1}}{\tilde{y}_{ii,t}(\phi)}\frac{\mathit{\Phi}_{ii,t+1}(\phi)}{\mathit{\Phi}_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right], \tag{A.99}$$

$$\mathit{\Xi}_{ii,t}(\phi) = \frac{\dot{p}_{ii,t}(\phi)\left[\dot{p}_{ii,t}(\phi)\mathit{\Delta}''_{ii,t}(\phi) + (2-\varepsilon)\mathit{\Delta}'_{ii,t}(\phi)\right]}{\mathit{\Phi}_{ii,t}(\phi)(\varepsilon-1)}, \tag{A.100}$$

$$\tag{A.101}$$

A.6 Deterministic Steady State

The focus of this paper is on the possibility of long-run trend of price inflation:

$$\dot{p}_i > 1 \tag{A.102}$$

and positive labour productivity growth:

$$\dot{a}_i = \gamma_i > 1. \tag{A.103}$$

The nominal interest rate and real interest rates are related via the Fisher equation:

$$r_i = \frac{\dot{p}_i}{\lambda_i}, \tag{A.104}$$

and the stochastic discount factor is pinned down by the Euler equation:

$$\lambda_i = \frac{\beta}{\gamma_i}, \tag{A.105}$$

which implies that the real interest rate (i.e. $1/\lambda_i - 1$) is lower than the nominal interest rate in the deterministic steady state. The initial condition of the real exchange rate is set to it's long-run mean:

$$q_{ni,0} = q_{in,0}^{-1} = q_{ni} > 0, \tag{A.106}$$

which implies the following restrictions:

$$\mu_{ni} = \frac{q_{ni}\tilde{u}_{i,c}}{\tilde{u}_{n,c}}, \tag{A.107}$$

$$e_{ni} = 1, \tag{A.108}$$

and pin down the real marginal costs:

$$mc_{ii}(\pi) = mc_{ii}(-\pi) = mc_{ii} = \xi_i Q_{ii} = \xi_i \min\{q_{ni};\ n = 1, \ldots, N\} = q_{ni}, \tag{A.109}$$

$$mc_{in}(-\pi) = \xi_i Q_{in}(-\pi) = \xi_i \min\{q_{ni};\ n = 1, \ldots, N\} = q_{ni}. \tag{A.110}$$

since $z_i = 1$. Next, price adjustment and wage bargaining costs are equal to zero in the steady state such that the resource allocation is Pareto-efficient in the long-run:

$$\varDelta_{ii,p}(\pi) = \varDelta_{ii,p}(-\pi) = \varDelta_{in,p}(-\pi) = \varDelta_{i,w} = 0, \tag{A.111}$$

$$\varDelta'_{ii,p}(\pi) = \varDelta'_{ii,p}(-\pi) = \varDelta'_{in,p}(-\pi) = \varDelta'_{i,w} = 0, \tag{A.112}$$

$$\varDelta''_{ii,p}(\pi) = \varDelta''_{ii,p}(-\pi) = \varDelta''_{in,p}(-\pi) = \kappa_{i,p}. \tag{A.113}$$

As a result, the auxiliary variables associated with wage and price adjustment are given by

$$\varTheta_i = 1, \tag{A.114}$$

$$\dot{w}_i = \gamma_i, \tag{A.115}$$

$$\varPhi_{ii}(\pi) = \varPhi_{ii}(-\pi) = \varPhi_{ni}(-\pi) = 1, \tag{A.116}$$

$$\dot{p}_{ii}(\pi) = \dot{p}_{ii}(-\pi) = \dot{p}_{ni}(-\pi) = \dot{p}_i. \tag{A.117}$$

The first order conditions of the wholesalers pin down the real export prices of both PCP and LCP firms as well as LCP import prices:

$$p_{ii}(\pi) = p_{ii}(-\pi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) mc_{ii} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) q_{ni}, \tag{A.118}$$

$$p_{in}(-\pi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \tau_{in} mc_{in}(-\pi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \tau_{in} q_{in}. \tag{A.119}$$

therefore $\delta_{in}(-\pi) = q_{in}$. If the densities $\chi_{ni}(\pi)$ and $\chi_{ni}(-\pi)$ are known, then the market share of each firm type is given by

$$s_{ni}(\phi) = \chi_{ni}(\phi) \left[\frac{p_{ni}(\phi)}{p_{ni}}\right]^{1-\varepsilon} \tag{A.120}$$

where

$$p_{ni} = \left[\sum_{\phi} \chi_{ni}(\phi) p_{ni}(\phi)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{A.121}$$

Similarly, if the values of α_{ii} and α_{ni} were known, then the trade weights are determined by

$$s_{ni} = \alpha_{ni} \left[\frac{p_{in}}{p_{i,x}}\right]^{1-\eta} \tag{A.122}$$

where

$$p_{i,x} = \left[\sum_{n=1}^{N} \alpha_{ni} p_{ni}^{\eta-1}\right]^{1/(\eta-1)}. \tag{A.123}$$

Combining the above results together pins down the demand for each ϕ-type products

$$\tilde{y}_{in}(\phi) = s_{in}(\phi)\left[\frac{p_{in}\tilde{y}_{in}}{p_{in}(\phi)}\right], \tag{A.124}$$

and the demand for imported intermediate goods

$$\tilde{m}_{in}(\phi) = \xi_i \tilde{y}_{in}(\phi), \tag{A.125}$$

but only if $\tilde{y}_{in}$ is known. The demand for home production would also be known:

$$\tilde{y}_{in} = s_{in}\left[\frac{p_{n,x}\tilde{x}_n}{p_{in}}\right], \tag{A.126}$$

but only if $\tilde{x}_n$ is known. Recall that the wage bill and the total cost of tradable goods are proportional to the total consumption expenditure:

$$p_{i,x}\tilde{x}_i = (1-\alpha_i)p_i\tilde{c}_i \Rightarrow \tilde{x}_i = \frac{(1-\alpha_i)\tilde{c}_i}{p_{i,x}}, \tag{A.127}$$

$$\tilde{w}_i h_i = \alpha_i p_i \tilde{c}_i \Rightarrow \tilde{c}_i = \frac{\tilde{w}_i h_i}{\alpha_i}, \tag{A.128}$$

where $p_i = 1$. Therefore, if $\tilde{c}_i$ is known, then the steady state is well-defined. However, in order to determine consumption, one must first pin down the wage bill. The hourly wages are obtained from the consumer price index identity:

$$1 = \left(\frac{\tilde{w}_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x}}{1-\alpha_i}\right)^{1-\alpha_i} \Rightarrow \tilde{w}_i = \alpha_i\left(\frac{p_{i,x}}{1-\alpha_i}\right)^{1-1/\alpha_i}. \tag{A.129}$$

Secondly, the hours of labour are pre-determined $h_i \in (0,1)$, which puts a restriction on the parameter controlling the relative disutility of labour:

$$\begin{aligned} \tilde{w}_i &= \frac{\varepsilon}{\varepsilon-1}\tilde{mrs}_i, \\ &= \frac{\varepsilon}{\varepsilon-1}\frac{\psi_i h_i^{\varphi_i}}{\tilde{u}_{i,c}}, \\ \Rightarrow \psi_i &= \frac{\varepsilon-1}{\varepsilon}\frac{\tilde{w}_i\tilde{u}_{i,c}}{h_i^{\varphi_i}}, \end{aligned} \tag{A.130}$$

where

$$\tilde{u}_{i,c} = \frac{1-\beta_i\vartheta_i}{\tilde{c}_i}\left(\frac{\gamma_i}{\gamma_i-\vartheta_i}\right). \tag{A.131}$$

It follows that the steady state values of endogenous variables derived above are consistent with all structural equations of the model. Therefore, the aggregate output can be obtained using the income-expenditure identity:

$$\tilde{y}_i = \tilde{c}_i + \tilde{nx}_i,$$
$$\tilde{nx}_i = \sum_{n=1}^{N-i} p_{in} q_{ni} \tilde{y}_{in} - p_{ni} \tilde{y}_{ni}.$$

Now that the steady state of all the endogenous control variables is set out, the deterministic steady state of exchange rate pass-through immediately follows. Firstly, consider the auxiliary variables:

$$\Xi_{ii}(\pi) = \Xi_{ii}(-\pi) = \Xi_{ii} = \frac{\kappa_{i,p} \dot{p}_i^2}{\varepsilon - 1}, \tag{A.132}$$

$$\Xi_{in}(-\pi) = \frac{\kappa_{i,p} \dot{p}_n^2}{\varepsilon - 1}, \tag{A.133}$$

$$\Gamma_{ii}(\pi) = \Gamma_{ii}(-\pi) = \Gamma_{ii} = \Xi_{ii} \left(1 + \lambda_i \gamma_i\right) = \Xi_{ii} \left(1 + \beta\right), \tag{A.134}$$

$$\Gamma_{in}(-\pi) = \Xi_{in}(-\pi) \left(1 + \lambda_i \gamma_n\right) = \Xi_{in}(-\pi) \left(1 + \frac{\beta \gamma_n}{\gamma_i}\right). \tag{A.135}$$

Next, consider exchange rate pass-through into import, export, and consumer prices respectively:

$$erpt_{ii} = \sum_{\phi} \frac{s_{ii}(\phi)}{1 + \Gamma_{ii}(\phi)} = \frac{1}{1 + \Gamma_{ii}} = \frac{1}{1 + \Xi_{ii} \left(1 + \lambda_i \gamma_i\right)} = \frac{1}{1 + \frac{\kappa_{i,p}}{\varepsilon - 1} \left(1 + \lambda_i \gamma_i\right) \dot{p}_i^2}, \tag{A.136}$$

$$erpt_{in} = \frac{s_{in}(-\pi)}{1 + \Gamma_{in}(-\pi)} + \frac{s_{in}(\pi) \Gamma_{ii}(\pi)}{1 + \Gamma_{ii}(\pi)}, \tag{A.137}$$

$$erpt_i = (1 - \alpha_i) \sum_{n=1}^{N} s_{ni} erpt_{ni}. \tag{A.138}$$

Appendix B

B.1 Exchange Rate Pass-Through

Consider the symmetric equilibrium import price of the ϕ-type firm in economy $j \in n \neq i, k$:

$$p_{ij,t}(\phi) = d_j q_{ij,t} \delta_{ij,t}(\phi) p_{ii,t}(\phi). \tag{B.1}$$

Differentiating both sides of the above with respect to the major exchange rate in real terms $q_{kj,t}$ and rearranging gives:

$$\begin{aligned}
\frac{\partial p_{ij,t}(\phi)}{\partial q_{kj,t}} &= d_j \left[\delta_{ij,t}(\phi) p_{ii,t}(\phi) \frac{\partial q_{ij,t}}{\partial q_{kj,t}} + q_{ij,t} p_{ii,t}(\phi) \frac{\partial \delta_{ij,t}(\phi)}{\partial q_{kj,t}} + q_{ij,t} \delta_{ij,t}(\phi) \frac{\partial p_{ii,t}(\phi)}{\partial q_{kj,t}} \right], \\
\frac{\partial p_{ij,t}(\phi)}{\partial q_{kj,t}} \frac{q_{kj,t}}{p_{ij,t}(\phi)} &= d_j \left(\frac{q_{ij,t} \delta_{ij,t}(\phi) p_{ii,t}(\phi)}{p_{ij,t}(\phi)} \right) \left[\frac{\partial \delta_{ij,t}(\phi)}{\partial q_{kj,t}} \frac{q_{kj,t}}{\delta_{ij,t}(\phi)} + \frac{\partial p_{ii,t}(\phi)}{\partial q_{ki,t}} \frac{q_{ki,t}}{p_{ii,t}(\phi)} \frac{\partial q_{ki,t}}{\partial q_{kj,t}} \frac{q_{kj,t}}{q_{ki,t}} \right], \\
\frac{\partial \ln p_{ij,t}(\phi)}{\partial \ln q_{ij,t}} &= \frac{\partial \ln \delta_{ij,t}(\phi)}{\partial \ln q_{kj,t}} + \frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}}, && \text{(B.2)} \\
erpt^k_{ij,t}(\phi) &= ptm^k_{ij,t}(\phi) + erpt^k_{ii,t}(\phi), && \text{(B.3)}
\end{aligned}$$

since

$$\begin{aligned}
q_{kj,t} &= q_{ki,t} q_{ij,t}, \\
\frac{\partial q_{ki,t}}{\partial q_{kj,t}} \frac{q_{kj,t}}{q_{ki,t}} &= 1 \quad \text{if} \quad \epsilon_{k,e,t} > 0, \\
\frac{\partial q_{ij,t}}{\partial q_{kj,t}} \frac{q_{kj,t}}{q_{ij,t}} &= 0.
\end{aligned}$$

Next, consider the optimal export price setting condition:

$$p_{ii,t}(\phi) = \frac{\Theta_{ii,t}(\phi) mc_{i,t}(\phi)}{\Phi_{ii,t}(\phi)} \tag{B.4}$$

Exchange rate pass-through into i'th economy export prices is derived by differentiating the above with respect to the major exchange rate using the quotient rule:

$$erpt^k_{ii,t}(\phi) = \frac{\partial \ln \Theta_{ii,t}(\phi)}{\partial \ln q_{ki,t}} + \frac{\partial \ln mc_{i,t}(\phi)}{\partial \ln q_{ki,t}} - \frac{\partial \ln \Phi_{ii,t}(\phi)}{\partial \ln q_{ki,t}} > 0. \tag{B.5}$$

Each of the three sub-elasticities can be derived as separate endogenous variables using the structure of the general equilibrium model. First, recall that the variable price mark-up expression is given by:

$$\Theta_{ii,t}(\phi) = \frac{\varepsilon_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)-1} \tag{B.6}$$

$$\varepsilon_{ii,t}(\phi) = \zeta(1-s_{ii,t}(\phi)) + \eta s_{ii,t}(\phi), \tag{B.7}$$

$$s_{ii,t}(\phi) = \chi_{ii}(\phi)\left(\frac{p_{ii,t}(\phi)}{p_{ii,t}}\right)^{1-\zeta}, \tag{B.8}$$

$$p_{ii,t} = \left[\sum_{\phi} \chi_{ii}(\phi) p_{ii,t}(\phi)^{1-\zeta}\right]^{1/(1-\zeta)}. \tag{B.9}$$

The exchange rate elasticity of the price mark-up is therefore obtained as follows:

$$\begin{aligned}
\frac{\partial \ln \Theta_{ii,t}(\phi)}{\partial \ln q_{ki,t}} &= -\left[\frac{1}{\varepsilon_{ii,t}(\phi)-1}\right]\frac{\partial \ln \varepsilon_{ii,t}(\phi)}{\partial \ln q_{ki,t}},\\
&= \left[\frac{\zeta-\eta}{\varepsilon_{ii,t}(\phi)-1}\frac{s_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)}\right]\frac{\partial \ln s_{ii,t}(\phi)}{\partial \ln q_{ki,t}},\\
&= \left[\frac{(\zeta-1)(\eta-\zeta)}{\varepsilon_{ii,t}(\phi)-1}\frac{s_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)}\right]\left(\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}} - \frac{\partial \ln p_{ii,t}}{\partial \ln q_{ki,t}}\right),\\
&= \left[\frac{(\zeta-1)(\eta-\zeta)}{\varepsilon_{ii,t}(\phi)-1}\frac{s_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)}\right]\left(erpt^k_{ii,t}(\phi) - erpt^k_{ii,t}\right), && \text{(B.10)}\\
&= -\Gamma_{ii,\Theta,t}(\phi)\left(erpt^k_{ii,t}(\phi) - erpt^k_{ii,t}\right), && \text{(B.11)}\\
&= -\Gamma_{ii,\Theta,t}(\phi)\left((1-s_{ii,t}(\phi))erpt^k_{ii,t}(\phi) - \sum_{-\phi} s_{ii,t}(-\phi)erpt^k_{ii,t}(-\phi)\right), && \text{(B.12)}
\end{aligned}$$

where the notation $-\phi$ indicates all of the remaining types of firms (e.g. if $\phi = \pi$, then it follows that $-\phi = \{\ell, \$\}$). Second, the real marginal costs are linearly related to the major exchange rate:

$$mc_{i,t}(\phi) = \frac{\xi_i(\phi) q_{ki,t}}{z_{i,t}}, \tag{B.13}$$

$$\frac{\partial \ln mc_{i,t}(\phi)}{\partial \ln q_{ki,t}} = \Gamma_{ii,mc,t}(\phi). \tag{B.14}$$

Third, the price stickiness is subsumed with the auxiliary variable:

$$\Phi_{ii,t}(\phi) = 1 - \Delta_{ii,t}(\phi) + \frac{\Delta'_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta'_{ii,t+1}(\phi)}{\varepsilon_{ii,t}(\phi)-1}\frac{\dot{p}^2_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right]. \tag{B.15}$$

Differentiating the above with respect to the major exchange rate gives:

$$
\begin{aligned}
&\frac{\partial \ln \Phi_{ii,t}(\phi)}{\partial \ln q_{ki,t}} = -\frac{\Delta'_{ii,t}(\cdot)\dot{p}_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}} \\
&+ \left(\frac{1}{\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}\right)\left[\Delta''_{ii,t}(\cdot)\dot{p}_{ii,t}(\phi)^2\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}} + \Delta'_{ii,t}(\cdot)q_{ki,t}\frac{\partial \dot{p}_{ii,t}(\phi)}{\partial q_{ki,t}}\right] \\
&- \lambda_{i,t,t+1}\,\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta''_{ii,t+1}(\cdot)\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ki,t}}\right] \\
&- \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{2\Delta'_{ii,t+1}(\cdot)\dot{p}_{ii,t+1}(\phi)q_{ki,t}}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ki,t}}\right] \\
&+ \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Delta'_{ii,t+1}(\cdot)\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}\,\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}\frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln q_{ki,t}}\right], \\
&= \frac{\dot{p}_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)-1}\frac{erpt^k_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\cdot) + (2-\varepsilon_{ii,t}(\phi))\Delta'_{ii,t}(\cdot)\right] \\
&+ \frac{\lambda_{i,t,t+1}}{\varepsilon_{ii,t}(\phi)-1}\frac{erpt^k_{ii,t}(\phi)}{\Phi_{ii,t}(\phi)}\mathbb{E}_t\left[\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)^2}{\dot{p}_{i,t+1}}\left(\dot{p}_{ii,t+1}(\phi)\Delta''_{ii,t+1}(\cdot) + (2-\varepsilon_{ii,t}(\phi))\Delta'_{ii,t+1}(\cdot)\right)\right], \\
&= erpt^k_{ii,t}(\phi)\left\{\Xi_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t\left[\Xi_{ii,t+1}(\phi)\left(\frac{y_{ii,t+1}(\phi)}{y_{ii,t}(\phi)}\frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right)\right]\right\} \qquad \text{(B.16)} \\
&= erpt^k_{ii,t}(\phi)\Gamma_{ii,\Delta,t}(\phi), \qquad \text{(B.17)}
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi_{ii,t}(\phi) &= \frac{\dot{p}_{ii,t}(\phi)\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\phi) + (2-\varepsilon_{ii,t}(\phi))\Delta'_{ii,t}(\phi)\right]}{\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}, \\
\frac{\partial \dot{p}_{ii,t}(\phi)}{\partial q_{ki,t}} &= \left(\frac{\dot{p}_{ii,t}(\phi)}{p_{ii,t}(\phi)}\right)\frac{\partial p_{ii,t}(\phi)}{\partial q_{ki,t}} = \left(\frac{erpt^k_{ii,t}(\phi)\dot{p}_{ii,t}(\phi)}{q_{ki,t}}\right), \\
\frac{\partial \dot{p}_{ii,t+1}(\phi)}{\partial q_{ki,t}} &= -\left(\frac{\dot{p}_{ii,t+1}(\phi)}{p_{ii,t}(\phi)}\right)\frac{\partial p_{ii,t}(\phi)}{\partial q_{ki,t}} = -\left(\frac{erpt^k_{ii,t}(\phi)\dot{p}_{ii,t+1}(\phi)}{q_{ki,t}}\right), \\
\frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln q_{ki,t}} &= \frac{\partial \ln y_{ii,t}(\phi)}{\partial \ln p_{ii,t}(\phi)}\frac{\partial \ln p_{ii,t}(\phi)}{\partial \ln q_{ki,t}} = -\varepsilon_{ii,t}(\phi)\,erpt^k_{ii,t}(\phi).
\end{aligned}
$$

The structure of the sub-elasticities is now explicitly derived. Next, the solution to exchange rate pass-through into the i'th economy export prices is obtained by substituting (B.12), (B.14), and (B.17) into (B.5) and solving for $erpt^k_{ii,t}(\phi)$ as follows:

$$
erpt^k_{ii,t}(\phi) = \Gamma_{ii,mc,t}(\phi) - erpt^k_{ii,t}(\phi)\Gamma_{ii,\Delta,t}(\phi) - \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)\left(erpt^k_{ii,t}(\phi) - erpt^k_{ii,t}(-\phi)\right),
$$

$$
erpt^k_{ii,t}(\phi)\left[1 + \Gamma_{ii,\Delta,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)\right] = \Gamma_{ii,mc,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)erpt^k_{ii,t}(-\phi),
$$

$$
erpt^k_{ii,t}(\phi) = \frac{\Gamma_{ii,mc,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)erpt^k_{ii,t}(-\phi)}{1 + \Gamma_{ii,\Delta,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)} \qquad \text{(B.18)}
$$

The next step is to find the exchange rate pass-through into an aggregate import price index, which is obtained as follows:

$$
\begin{aligned}
erpt^k_{ij,t} = \frac{\partial \ln p_{ij,t}}{\partial \ln q_{kj,t}} &= \sum_{\phi} \left(\frac{p_{ij,t}(\phi)}{p_{ij,t}}\right)^{1-\zeta} \left[ptm^k_{ij,t}(\phi) + erpt^k_{ii,t}(\phi)\right] \int_0^{\chi_{ij}(\phi)} d\omega, \\
&= \sum_{\phi} \chi_{ij}(\phi) \left(\frac{p_{ij,t}(\phi)}{p_{ij,t}}\right)^{1-\zeta} \left[ptm^k_{ij,t}(\phi) + erpt^k_{ii,t}(\phi)\right], \\
&= \sum_{\phi} s_{ii,t}(\phi) \left[ptm^k_{ij,t}(\phi) + erpt^k_{ii,t}(\phi)\right]. \qquad \text{(B.19)}
\end{aligned}
$$

Finally, the the consumer price index is less elastic to exchange rate fluctuations than the price of manufactured goods due to home-bias and distribution services:

$$
p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i} \left(\frac{1}{1-\alpha_i} \left(\sum_{n=1}^{N} \alpha_{ni}\, p^{1-\eta}_{ni,t}\right)^{\frac{1}{1-\eta}}\right)^{1-\alpha_i}, \qquad \text{(B.20)}
$$

$$
erpt^k_{i,t} = \frac{\partial \ln p_{i,t}}{\partial \ln q_{ki,t}} = (1-\alpha_i) \sum_{n=1}^{N} \alpha_{ni} \left(\frac{p_{ni,t}}{p_{i,x,t}}\right)^{1-\eta} erpt^k_{ni,t} = (1-\alpha_i) \sum_{n=1}^{N} s_{ni,t} erpt^k_{ni,t} \qquad \text{(B.21)}
$$

B.2 Supply-Side

Import-export manufacturers solve the following real cost minimisation problem:

$$
\min_{\{y_{in,t}(\omega,\phi)\}} tc_{ij,t}(\omega,\phi) = q_{ki,t} m_{in,t}(\omega,\phi) = \frac{\xi_i(\phi) q_{ki,t} y_{in,t}(\omega)}{z_{i,t}}, \qquad \text{(B.22)}
$$

where the first order condition is simply given by:

$$
\frac{\partial tc_{ij,t}(\omega,\phi)}{\partial y_{ij,t}(\omega,\phi)} = mc_{i,t}(\phi) = \frac{\xi_i(\phi) d_i q_{ki,t}}{z_{i,t}}. \qquad \text{(B.23)}
$$

The next step along the pricing chain is the relationship between the manufacturing sector and the distributors. The demand facing the upstream manufacturers from the distribution sector is derived from the profit maximisation problem of the competitive collectors, who aggregate individual varieties of goods into country-specific bundles:

$$
\begin{aligned}
\max_{\{y_{in,t}(\phi)\}} \quad & p_{in,t} y_{in,t} - \sum_{\phi} \int_0^{\chi_{in}(\phi)} p_{in,t}(\omega,\phi) y_{in,t}(\omega,\phi) d\omega \\
\text{s.t.} \quad & y_{in,t} = \left[\sum_{\phi} \int_0^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi)^{1-1/\zeta} d\omega\right]^{1/(1-1/\zeta)},
\end{aligned}
$$

which gives rise to the following first order condition:

$$y_{in,t}(\omega,\phi) = y_{in,t}\left[\frac{p_{in,t}(\omega,\phi)}{p_{in,t}}\right]^{-\zeta}. \tag{B.24}$$

The aggregate demand for the ϕ-type imports from location i to n is then given by:

$$y_{in,t}(\phi) = \int_0^{\chi_{in}(\phi)} y_{in,t}(\omega,\phi)d\omega = \chi_{in}(\phi)y_{in,t}\left[\frac{p_{in,t}(\phi)}{p_{in,t}}\right]^{-\zeta} \tag{B.25}$$

The aggregate upstream price index is derived by substituting the above first order condition into the budget constraint of the collector, which gives rise to the following expression:

$$p_{in,t} = \left[\sum_{\phi}\int_0^{\chi_{in}(\phi)} p_{in,t}(\omega,\phi)^{1-\zeta}d\omega\right]^{1/(1-\zeta)}, \tag{B.26}$$

or alternatively in the symmetric equilibrium it is equivalent to:

$$p_{in,t} = \left[\sum_{\phi}\chi_{in}(\phi)p_{in,t}(\phi)^{1-\zeta}\right]^{1/(1-\zeta)}. \tag{B.27}$$

The optimal demands for locally-produced and imported goods are derived from the second stage of aggregation:

$$\max_{\{y_{ni,t}\}} \; p_{i,x,t}x_{i,t} - \sum_{n=1}^{N} p_{ni,t}y_{ni,t}$$
$$\text{s.t.} \;\; x_{i,t} = \left(\sum_{n=1}^{N}\alpha_{ni}^{1/\eta}\, y_{ni,t}^{1-1/\eta}\right)^{1/(1-1/\eta)},$$

The first order condition is given by:

$$y_{ni,t} = \alpha_{ni}x_{i,t}\left[\frac{p_{ni,t}}{p_{i,x,t}}\right]^{-\eta}, \tag{B.28}$$

The aggregate producer price index is derived by substituting the above first order condition into the budget constraint of the distributor, which gives rise to the following expression:

$$p_{i,x,t} = \left(\sum_{n=1}^{N}\alpha_{ni}\, p_{ni,t}^{1-\eta}\right)^{1/(1-\eta)}. \tag{B.29}$$

Finally, the competitive retailer maximises profits subject to the Cobb-Douglas production technology characterising the downstream market:

$$\begin{aligned} \max_{\{h_{i,t},x_{i,t}\}} \quad & p_{i,t}y_{i,t} - w_{i,t}h_{i,t} - p_{i,x,t}x_{i,t}, \\ \text{s.t.} \quad & c_{i,t} = (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i}. \end{aligned}$$

The cost minimisation problem is formulated as a static Lagrangian:

$$\mathbb{L} = p_{i,t}y_{i,t} - w_{i,t}h_{i,t} - p_{i,x,t}x_{i,t} - o_{i,t}\left(y_{i,t} - (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i}\right), \tag{B.30}$$

where the first order conditions are given by:

$$p_{i,x,t} - (1-\alpha_i)o_{i,t}\left(\frac{c_{i,t}}{x_{i,t}}\right) = 0, \tag{B.31}$$

$$w_{i,t} - \alpha_i o_{i,t}\left(\frac{c_{i,t}}{h_{i,t}}\right) = 0, \tag{B.32}$$

$$c_{i,t} - (a_{i,t}h_{i,t})^{\alpha_i} x_{i,t}^{1-\alpha_i} = 0. \tag{B.33}$$

It can be shown that the shadow price of technology $o_{i,t}$ corresponds to the consumer price index. Substituting the first two first order conditions above into the expression of the total real costs of production and taking a partial derivative with respect to retail output:

$$tc_{i,t} = (\alpha_i + 1 - \alpha_i)o_{i,t}c_{i,t} = o_{i,t}c_{i,t}, \quad \frac{\partial tc_{i,t}}{\partial c_{i,t}} = o_{i,t} = p_{i,t}. \tag{B.34}$$

In order to find the functional form of the retail price index, consider the unconditional demand schedule for each of the factor inputs. They are obtained by firstly dividing the top two first order conditions one by the other:

$$\frac{h_{i,t}}{x_{i,t}} = \frac{\alpha_i}{1-\alpha_i}\frac{p_{i,x,t}}{w_{i,t}}, \tag{B.35}$$

and then substitute the above schedule for tradable goods bundle and labour, each in turn, into the technological constraint:

$$x_{i,t} = c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{-\alpha_i}, \tag{B.36}$$

$$h_{i,t} = c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{1-\alpha_i}\right)^{1-\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{\alpha_i-1}. \tag{B.37}$$

Then substitute the unconditional demand schedules into the expression for the total effective real costs of production to obtain:

$$tc_{i,t} = c_{i,t}\left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}, \tag{B.38}$$

$$\frac{\partial tc_{i,t}}{\partial c_{i,t}} = p_{i,t} = \left(\frac{w_{i,t}/a_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}. \tag{B.39}$$

Therefore, the first order conditions can be written as:

$$p_{i,x,t}x_{i,t} = (1-\alpha_i)p_{i,t}c_{i,t}, \tag{B.40}$$

$$w_{i,t}h_{i,t} = \alpha_i p_{i,t}c_{i,t}. \tag{B.41}$$

B.3 Sticky Prices

The optimal price set by the ω'th import-export manufacturer of ϕ-type is derived by maximising the present discounted value of their real profits subject to a sequence of demand schedules:

$$\begin{aligned}
&\max_{\{P_{in,t}(\omega,\phi)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{i,t,t+1}\left\{\sum_{n=1}^{N}\left(1-\mathit{\Delta}_{in,t}(\omega,\phi)\right)p_{in,t}(\omega,\phi)\,y_{in,t}(\omega,\phi) - mc_{i,t}(\phi)\,y_{in,t}(\omega,\phi)\right\}\\
&\quad\text{s.t.}\quad \mathit{\Delta}_{in,t}(\omega,\phi) = \frac{\kappa_i(\phi)(\dot{p}_{in,t}(\omega,\phi)-\dot{p}_n)^2}{2},\\
&\quad\text{s.t.}\quad y_{in,t}(\omega,\phi) = y_{in,t}\left[\frac{p_{in,t}(\omega,\phi)}{p_{in,t}}\right]^{-\zeta},\\
&\quad\text{s.t.}\quad y_{in,t} = \alpha_{in,t}x_{n,t}\left[\frac{p_{in,t}}{p_{n,x,t}}\right]^{-\eta}.
\end{aligned}$$

The first order condition is obtained using the product rule:

$$\begin{aligned}
&(1-\mathit{\Delta}_{in,t}(\omega,\phi))\,y_{in,t}(\omega,\phi)\left[\frac{\partial p_{in,t}(\omega,\phi)}{\partial P_{in,t}(\omega,\phi)} + \frac{\partial p_{in,t}(\omega,\phi)}{\partial P_{in,t}(\omega,\phi)}\frac{\partial y_{in,t}(\omega,\phi)}{\partial p_{in,t}(\omega,\phi)}\frac{p_{in,t}(\omega,\phi)}{y_{in,t}(\omega,\phi)}\right]\\
&-\mathit{\Delta}'_{in,t}(\omega,\phi)\dot{p}_{in,t}(\omega,\phi)y_{in,t}(\omega,\phi)\left(\frac{\partial p_{in,t}(\omega,\phi)}{\partial P_{in,t}(\omega,\phi)}\right) + mc_{i,t}(\phi)\frac{\partial p_{in,t}(\omega,\phi)}{\partial P_{in,t}(\omega,\phi)}\frac{\partial y_{in,t}(\omega,\phi)}{\partial p_{in,t}(\omega,\phi)}\\
&+\mathbb{E}_t\left[\lambda_{i,t,t+1}\mathit{\Delta}'_{in,t+1}(\omega,\phi)\dot{p}_{in,t+1}(\omega,\phi)^2 y_{in,t+1}(\omega,\phi)\left(\frac{\partial p_{in,t+1}(\omega,\phi)}{\partial P_{in,t+1}(\omega,\phi)}\right)\right] = 0.
\end{aligned}$$

Alternatively:

$$\begin{aligned}
&(1-\mathit{\Delta}_{in,t}(\omega,\phi))p_{in,t}(\omega,\phi) - \left(\frac{\varepsilon_{in,t}(\omega,\phi)}{\varepsilon_{in,t}(\omega,\phi)-1}\right)mc_{i,t}(\phi) + \left(\frac{\mathit{\Delta}'_{in,t}(\omega,\phi)\dot{p}_{in,t}(\omega,\phi)}{\varepsilon_{in,t}(\omega,\phi)-1}\right)p_{in,t}(\omega,\phi)\\
&-\mathbb{E}_t\left[\frac{y_{in,t+1}(\omega,\phi)}{y_{in,t}(\omega,\phi)}\frac{\mathit{\Delta}'_{in,t+1}(\omega,\phi)}{\varepsilon_{in,t}(\omega,\phi)-1}\frac{\dot{p}_{in,t+1}(\omega,\phi)^2}{r_{i,t}}\right]p_{in,t}(\omega,\phi) = 0,
\end{aligned}$$

where

$$\frac{\partial y_{in,t}(\omega,\phi)}{\partial p_{in,t}(\omega,\phi)} = -\zeta\frac{y_{in,t}(\omega,\phi)p_{in,t}}{p_{in,t}(\omega,\phi)}\left\{\frac{1}{p_{in,t}^2}\left(p_{in,t} - p_{in,t}(\omega,\phi)\frac{\partial p_{in,t}}{\partial p_{in,t}(\omega,\phi)}\right)\right\}$$
$$- \eta\frac{y_{in,t}(\omega,\phi)p_{n,x,t}}{p_{in,t}}\left\{\frac{1}{p_{n,x,t}^2}\left(p_{n,x,t}\frac{\partial p_{in,t}}{\partial p_{in,t}(\omega,\phi)}\right)\right\},$$

$$\frac{\partial \ln y_{in,t}(\omega,\phi)}{\partial \ln p_{in,t}(\omega,\phi)} = -\zeta(1-s_{in,t}(\omega,\phi)) - \eta s_{in,t}(\omega,\phi) = -\varepsilon_{in,t}(\omega,\phi), \tag{B.42}$$

$$s_{in,t}(\omega,\phi) = \left(\frac{p_{in,t}(\omega,\phi)}{p_{in,t}}\right)^{1-\zeta} = \frac{p_{in,t}(\omega,\phi)}{p_{in,t}}\frac{y_{in,t}(\omega,\phi)}{y_{in,t}} \equiv \frac{\partial \ln p_{in,t}}{\partial \ln p_{in,t}(\omega,\phi)}. \tag{B.43}$$

Finally, solving for the optimal price in the symmetric equilibrium gives:

$$p_{in,t}(\phi) = \frac{\Theta_{in,t}(\phi)mc_{i,t}(\phi)}{\Phi_{in,t}(\phi)}, \tag{B.44}$$

$$\Theta_{in,t}(\phi) = \frac{\varepsilon_{in,t}(\phi)}{\varepsilon_{in,t}(\phi)-1}, \tag{B.45}$$

$$\varepsilon_{in,t}(\phi) = \zeta(1-s_{in,t}(\phi)) + \eta s_{in,t}(\phi), \tag{B.46}$$

$$\Phi_{in,t}(\phi) = 1 - \Delta_{in,t}(\phi) + \frac{\Delta'_{in,t}(\phi)\dot{p}_{in,t}(\phi)}{\varepsilon_{in,t}(\phi)-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{y_{in,t+1}(\phi)}{y_{in,t}(\phi)}\frac{\Delta'_{in,t+1}(\phi)}{\varepsilon_{in,t}(\phi)-1}\frac{\dot{p}_{in,t+1}(\phi)^2}{\dot{p}_{n,t+1}}\right]. \tag{B.47}$$

B.4 Sticky Wages

Competitive trade union maximises its real profits with respect to the hours of each service:

$$\max_{\{h_{i,t}(\omega)\}} w_{i,t}h_{i,t} - \int_0^1 w_{i,t}(\omega)h_{i,t}(\omega)d\omega,$$

$$\text{s.t. } h_{i,t} = \left[\int_0^1 h_{i,t}(\omega)^{\frac{\zeta-1}{\zeta}}d\omega\right]^{\frac{\zeta}{\zeta-1}}.$$

The first order condition gives rise to the following service-specific demand schedule:

$$h_{i,t}(\omega) = h_{i,t}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\zeta}. \tag{B.48}$$

The optimal inter-temporal allocation of consumption, savings and optimal wage is derived by solving the following dynamic stochastic optimisation problem:

$$\max_{\{c_{i,t}(\omega), w_{i,t}(\omega), b_{i,t+1}(\omega)\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \ln\left(\frac{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right) h_{i,t}(\omega)^{1+\varphi_i}$$

$$\text{s.t. } c_{i,t}(\omega) + \lambda_{i,t,t+1}\mathbb{E}_t[b_{i,t+1}(\omega)] = b_{i,t}(\omega) + (1 - \mathit{\Delta}_{i,w,t})w_{i,t}(\omega)h_{i,t}(\omega) + v_{i,t}(\omega),$$

$$\text{s.t. } h_{i,t}(\omega) = h_{i,t}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\zeta},$$

$$\text{s.t. } \mathit{\Delta}_{i,w,t}(\omega) = \frac{\kappa_{i,w}(\dot{w}_{i,t}(\omega) - 1)}{2}.$$

Re-writing the above in terms of the Current Value Lagrangian gives:

$$\begin{aligned}
\mathbb{CVL} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \{ &\ln\left(\frac{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)}{a_{i,t}}\right) - \left(\frac{\psi_i}{1+\varphi_i}\right) h_{i,t}(\omega)^{1+\varphi_i} \\
&+ u_{i,c,t}(\omega)\left[b_{i,t}(\omega) + (1 - \mathit{\Delta}_{i,w,t}(\omega))w_{i,t}(\omega)h_{i,t}(\omega) + v_{i,t}(\omega) - c_{i,t}(\omega) - \mathbb{E}_t[\lambda_{i,t,t+1}b_{i,t+1}(\omega)]\right]\} \\
&+ u_{i,h,t}(\omega)\left[h_{i,t}\left[\frac{w_{i,t}(\omega)}{w_{i,t}}\right]^{-\zeta} - h_{i,t}(\omega)\right].
\end{aligned} \tag{B.49}$$

Consider the first order conditions with respect to consumption and bond holdings:

$$\frac{1}{c_{i,t}(\omega) - \vartheta_i c_{i,t-1}(\omega)} - \beta_i\vartheta_i\mathbb{E}_t\left[\frac{1}{c_{i,t+1}(\omega) - \vartheta_i c_{i,t}(\omega)}\right] - u_{i,c,t}(\omega) = 0, \tag{B.50}$$

$$u_{i,c,t}(\omega)\mathbb{E}_t[\lambda_{i,t,t+1}] - \beta_i\mathbb{E}_t[u_{i,c,t+1}(\omega)] = 0. \tag{B.51}$$

Alternatively, imposing preference homotheticity gives:

$$u_{i,c,t} = \frac{\partial u_{i,t}}{\partial c_{i,t}} = \mathit{\Psi}_{i,t} - \beta_i\vartheta_i\mathbb{E}_t[\mathit{\Psi}_{i,t+1}], \tag{B.52}$$

$$\mathit{\Psi}_{i,t} = \frac{1}{c_{i,t} - \vartheta_i c_{i,t-1}}, \tag{B.53}$$

$$1 = \beta_i\mathbb{E}_t\left[\frac{u_{i,c,t+1}}{\lambda_{i,t,t+1}u_{i,c,t}}\right]. \tag{B.54}$$

The first order condition with respect to the optimal wage is given by:

$$\begin{aligned}
&(1 - \mathit{\Delta}_{i,w,t}(\omega))(1-\zeta)u_{i,c,t}(\omega)h_{i,t}(\omega) - \zeta\left(\frac{u_{i,h,t}(\omega)h_{i,t}(\omega)}{w_{i,t}(\omega)}\right) - \mathit{\Delta}'_{i,w,t}(\omega)\dot{w}_{i,t}(\omega)u_{i,c,t}(\omega)h_{i,t}(\omega), \\
&+ \beta_i\mathbb{E}_t\left[\mathit{\Delta}'_{i,w,t+1}(\omega)\dot{w}_{i,t+1}(\omega)^2 u_{i,c,t+1}(\omega)h_{i,t+1}(\omega)\right] = 0.
\end{aligned} \tag{B.55}$$

Alternatively, under preference homotheticity, $w_{i,t}(\omega) = w_{i,t} \Rightarrow h_{i,t}(\omega) = h_{i,t}$, thus:

$$\begin{aligned}
(1 - \mathit{\Delta}_{i,w,t})w_{i,t} + \left(\frac{\zeta}{\zeta-1}\right)\frac{u_{i,h,t}}{u_{i,c,t}} + \left(\frac{\mathit{\Delta}'_{i,w,t}\dot{w}_{i,t}}{\zeta-1}\right)w_{i,t} \\
-\beta_i\mathbb{E}_t\left[\frac{u_{i,c,t+1}}{u_{i,c,t}}\frac{h_{i,t+1}}{h_{i,t}}\frac{\mathit{\Delta}'_{i,w,t+1}\dot{w}_{i,t+1}^2}{\zeta-1}w_{i,t}\right] = 0,
\end{aligned} \tag{B.56}$$

or simply:

$$w_{i,t} = \left(\frac{\zeta}{\zeta-1}\right)\frac{mrs_{i,t}}{\Psi_{i,t}}, \tag{B.57}$$

$$\Psi_{i,t} = 1 - \Delta_{i,w,t} + \frac{\Delta'_{i,w,t}\dot{w}_{i,t}}{\zeta-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{h_{i,t+1}}{h_{i,t}}\frac{\Delta'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\zeta-1}\right], \tag{B.58}$$

where

$$\dot{w}_{i,t} = = \frac{w_{i,t}}{w_{i,t-1}} = \dot{a}_{i,t}\left(\frac{\tilde{w}_{i,t}}{\tilde{w}_{i,t-1}}\right), \tag{B.59}$$

$$mrs_{i,t} = -\frac{u_{i,h,t}}{u_{i,c,t}}, \tag{B.60}$$

$$u_{i,h,t} = \frac{\partial u_{i,t}}{\partial h_{i,t}} = -\psi_i h^{\varphi_i}_{i,t}. \tag{B.61}$$

B.5 Equilibrium Conditions

Consumption Smoothing:

$$q_{ji,t}\tilde{u}_{i,c,t} = \mu_i\tilde{u}_{j,c,t}e_{i,t}, \tag{B.62}$$

$$\lambda_{i,t,t+1}\tilde{u}_{i,c,t}\mathbb{E}_t[\dot{a}_{i,t+1}] = \beta_i\mathbb{E}_t\left[\tilde{u}_{i,c,t+1}\right], \tag{B.63}$$

$$\tilde{u}_{i,c,t} = \frac{\dot{a}_{i,t}}{\dot{a}_{i,t}\tilde{c}_{i,t} - \vartheta_i\tilde{c}_{i,t-1}} - \beta_i\vartheta_i\mathbb{E}_t\left[\frac{\dot{a}_{i,t+1}}{\dot{a}_{i,t+1}\tilde{c}_{i,t+1} - \vartheta_i\tilde{c}_{i,t}}\right]. \tag{B.64}$$

Monetary Policy:

$$r_{i,t}\lambda_{i,t,t+1} = \mathbb{E}_t\left[\dot{p}_{i,t+1}\right], \tag{B.65}$$

$$r_{i,t} = (r_{i,t-1})^{\rho_{i,r}}(r^*_{i,t})^{1-\rho_{i,r}}, \tag{B.66}$$

$$r^*_{i,t} = \frac{\dot{p}_i}{\beta}\left(\frac{\dot{p}_{i,t}}{\dot{p}_i}\right)^{\nu_{i,p}}\left(\frac{\tilde{y}_{i,t}}{\tilde{y}_i}\right)^{\nu_{i,y}}\exp(\sigma_{i,r}\epsilon_{i,r,t}). \tag{B.67}$$

Market Clearing Conditions:

$$\tilde{y}_{i,t} = \tilde{c}_{i,t} + \tilde{nx}_{i,t} + \Delta_{i,w,t}\tilde{w}_{i,t}h_{i,t} + \sum_{n=1}^{N}\sum_{\phi}\Delta_{in,t}(\phi)p_{in,t}(\phi)\tilde{y}_{in,t}(\phi), \tag{B.68}$$

$$\tilde{nx}_{i,t} = \sum_{n=1}^{N-i} p_{in,t}q_{ni,t}\tilde{y}_{in,t} - p_{ni,t}\tilde{y}_{ni,t}. \tag{B.69}$$

Sector-Specific Market Shares

$$p_{in,t}(\phi)\tilde{y}_{in,t}(\phi) = s_{in,t}(\phi)p_{in,t}\tilde{y}_{in,t}, \tag{B.70}$$

$$s_{in,t}(\phi) = \chi_{in}(\phi)\left[\frac{p_{in,t}(\phi)}{p_{in,t}}\right]^{1-\zeta}, \tag{B.71}$$

$$\sum_{\phi} s_{in,t}(\phi) = 1. \tag{B.72}$$

Country-Specific Trade Weights

$$p_{in,t}\tilde{y}_{in,t} = s_{in,t}p_{n,x,t}\tilde{x}_{n,t}, \tag{B.73}$$

$$s_{in,t} = \alpha_{in}\left[\frac{p_{in,t}}{p_{n,x,t}}\right]^{1-\eta}, \tag{B.74}$$

$$\sum_{n=1}^{N} s_{ni,t} = 1. \tag{B.75}$$

Production Technology:

$$p_{i,x,t}\tilde{x}_{i,t} = (1-\alpha_i)\tilde{c}_{i,t}, \tag{B.76}$$

$$\tilde{w}_{i,t}h_{i,t} = \alpha_i\tilde{c}_{i,t}, \tag{B.77}$$

$$\xi_i(\phi)\tilde{y}_{in,t}(\phi) = z_{i,t}\tilde{m}_{in,t}(\phi), \tag{B.78}$$

$$mc_{i,t}(\phi)z_{i,t} = \xi_i(\phi)d_i q_{ki,t}. \tag{B.79}$$

Price Adjustment Costs:

$$\mathit{\Delta}_{in,t}(\phi) = \frac{\kappa_i(\phi)(\dot{p}_{in,t}(\phi) - \dot{p}_n)^2}{2}, \tag{B.80}$$

$$\mathit{\Delta}'_{in,t}(\phi) = \kappa_i(\phi)(\dot{p}_{in,t}(\phi) - \dot{p}_n), \tag{B.81}$$

$$\mathit{\Delta}''_{in,t}(\phi) = \kappa_i(\phi), \tag{B.82}$$

$$\dot{p}_{in,t}(\phi) = \frac{\dot{p}_{n,t}p_{in,t}(\phi)}{p_{in,t-1}(\phi)}. \tag{B.83}$$

Producer Currency Pricing (i.e. if $\phi = \pi$):

$$p_{in,t}(\pi) = d_{n,t}q_{in,t}p_{ii,t}(\pi), \tag{B.84}$$

$$p_{ii,t}(\pi) = \frac{\mathit{\Theta}_{ii,t}(\pi)mc_{i,t}(\pi)}{\mathit{\Phi}_{ii,t}(\pi)}, \tag{B.85}$$

$$\mathit{\Theta}_{ii,t}(\pi) = \frac{\varepsilon_{ii,t}(\pi)}{\varepsilon_{ii,t}(\pi) - 1}, \tag{B.86}$$

$$\varepsilon_{ii,t}(\pi) = \zeta(1 - s_{ii,t}(\pi)) + \eta s_{ii,t}(\pi), \tag{B.87}$$

$$\mathit{\Phi}_{ii,t}(\pi) = 1 - \mathit{\Delta}_{ii,t}(\pi) + \frac{\mathit{\Delta}'_{ii,t}(\pi)\dot{p}_{ii,t}(\pi)}{\varepsilon_{ii,t}(\pi) - 1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{\dot{a}_{i,t+1}\tilde{y}_{ii,t+1}(\pi)}{\tilde{y}_{ii,t}(\pi)}\frac{\mathit{\Delta}'_{ii,t+1}(\pi)}{\varepsilon_{ii,t}(\pi) - 1}\frac{\dot{p}_{ii,t+1}(\pi)^2}{\dot{p}_{i,t+1}}\right]. \tag{B.88}$$

Local Currency Pricing (i.e. if $\phi = \ell$):

$$p_{in,t}(\ell) = d_n q_{in,t} \delta_{in,t}(\ell) p_{ii,t}(\ell), \tag{B.89}$$

$$p_{ii,t}(\ell) = \frac{\Theta_{ii,t}(\ell) mc_{i,t}(\ell)}{\Phi_{ii,t}(\ell)}, \tag{B.90}$$

$$p_{in,t}(\ell) = \frac{\Theta_{in,t}(\ell) mc_{i,t}(\ell)}{\Phi_{in,t}(\ell)}, \tag{B.91}$$

$$\Theta_{in,t}(\ell) = \frac{\varepsilon_{in,t}(\ell)}{\varepsilon_{in,t}(\ell) - 1}, \tag{B.92}$$

$$\varepsilon_{in,t}(\ell) = \zeta(1 - s_{in,t}(\ell)) + \eta s_{in,t}(\ell), \tag{B.93}$$

$$\Phi_{in,t}(\ell) = 1 - \Delta_{in,t}(\ell) + \frac{\Delta'_{in,t}(\ell)\dot{p}_{in,t}(\ell)}{\varepsilon_{in,t}(\ell) - 1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{\dot{a}_{i,t+1}\tilde{y}_{in,t+1}(\ell)}{\tilde{y}_{in,t}(\ell)} \frac{\Delta'_{in,t+1}(\ell)}{\varepsilon_{in,t}(\ell) - 1} \frac{\dot{p}_{in,t+1}(\ell)^2}{\dot{p}_{n,t+1}}\right]. \tag{B.94}$$

Major Currency Pricing (i.e. if $\phi = \$$):

$$q_{kn,t} = q_{in,t}\delta_{in,t}(\$), \tag{B.95}$$

$$p_{in,t}(\$) = d_n q_{kn,t} p_{ii,t}(\$), \tag{B.96}$$

$$p_{ii,t}(\$) = \frac{\Theta_{ii,t}(\$)\xi_i(\$)}{z_{i,t}\Phi_{ii,t}(\$)}, \tag{B.97}$$

$$\Theta_{ii,t}(\$) = \frac{\varepsilon_{ii,t}(\$)}{\varepsilon_{ii,t}(\$) - 1}, \tag{B.98}$$

$$\varepsilon_{ii,t}(\$) = \zeta(1 - s_{ii,t}(\$)) + \eta s_{ii,t}(\$), \tag{B.99}$$

$$\Phi_{ii,t}(\$) = 1 - \Delta_{ii,t}(\$) + \frac{\Delta'_{ii,t}(\$)\dot{p}_{ii,t}(\$)}{\varepsilon_{ii,t}(\$) - 1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{\dot{a}_{i,t+1}\tilde{y}_{ii,t+1}(\$)}{\tilde{y}_{ii,t}(\$)} \frac{\Delta'_{ii,t+1}(\$)}{\varepsilon_{ii,t}(\$) - 1} \frac{\dot{p}_{ii,t+1}(\$)^2}{\dot{p}_{i,t+1}}\right]. \tag{B.100}$$

Wage Adjustment Costs

$$\Delta_{i,w,t} = \frac{\kappa_{i,w}(\dot{w}_{i,t} - \gamma_i)}{2}, \tag{B.101}$$

$$\Delta'_{i,w,t} = \kappa_{i,w}(\dot{w}_{i,t} - 1), \tag{B.102}$$

$$\dot{w}_{i,t} = \dot{a}_{i,t}\left(\frac{\tilde{w}_{i,t}}{\tilde{w}_{i,t-1}}\right). \tag{B.103}$$

Collective Bargaining

$$1 = \left(\frac{\tilde{w}_{i,t}}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x,t}}{1-\alpha_i}\right)^{1-\alpha_i}, \tag{B.104}$$

$$\tilde{w}_{i,t} = \left(\frac{\zeta}{\zeta-1}\right)\frac{m\tilde{r}s_{i,t}}{\Psi_{i,t}}, \tag{B.105}$$

$$\Psi_{i,t} = 1 - \Delta_{i,w,t} + \frac{\Delta'_{i,w,t}\dot{w}_{i,t}}{\zeta-1} - \lambda_{i,t,t+1}\mathbb{E}_t\left[\frac{h_{i,t+1}}{h_{i,t}}\frac{\Delta'_{i,w,t+1}\dot{w}^2_{i,t+1}}{\zeta-1}\right], \tag{B.106}$$

$$m\tilde{r}s_{i,t} = -\frac{\tilde{u}_{i,h,t}}{\tilde{u}_{i,c,t}}, \tag{B.107}$$

$$\tilde{u}_{i,h,t} = -\psi_i h_{i,t}^{\varphi_i}, \tag{B.108}$$

$$\tag{B.109}$$

Minor Exchange Rate Pass-Through into Downstream Import Prices:

$$erpt_{i,t} = (1-\alpha_i)\left[s_{ii,t}erpt^k_{ii,t} + \sum_{n=1}^{N-i} s_{ni,t}s_{ni,t}(\pi)\right], \tag{B.110}$$

$$erpt_{ni,t} = 1 - s_{ni,t}(\ell). \tag{B.111}$$

Major Exchange Rate Pass-Through into Export Prices:

$$erpt^k_{ii,t}(\phi) = \frac{\Gamma_{ii,mc,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)\sum_{-\phi} s_{ii,t}(-\phi)erpt^k_{ii,t}(-\phi)}{1 + \Gamma_{ii,\Delta,t}(\phi) + \Gamma_{ii,\Theta,t}(\phi)(1 - s_{ii,t}(\phi))}, \tag{B.112}$$

$$\Gamma_{ii,\Theta,t}(\phi) = \frac{(\zeta-1)(\zeta-\eta)}{\varepsilon_{ii,t}(\phi)-1}\frac{s_{ii,t}(\phi)}{\varepsilon_{ii,t}(\phi)}, \tag{B.113}$$

$$\Gamma_{ii,\Delta,t}(\phi) = \Xi_{ii,t}(\phi) + \lambda_{i,t,t+1}\mathbb{E}_t\left[\Xi_{ii,t+1}(\phi)\left(\frac{\dot{a}_{i,t+1}\tilde{y}_{ii,t+1}(\phi)}{\tilde{y}_{ii,t}(\phi)}\frac{\Phi_{ii,t+1}(\phi)}{\Phi_{ii,t}(\phi)}\frac{\dot{p}_{ii,t+1}(\phi)}{\dot{p}_{i,t+1}}\right)\right], \tag{B.114}$$

$$\Xi_{ii,t}(\phi) = \frac{\dot{p}_{ii,t}(\phi)\left[\dot{p}_{ii,t}(\phi)\Delta''_{ii,t}(\phi) + (2-\varepsilon_{ii,t}(\phi))\Delta'_{ii,t}(\phi)\right]}{\Phi_{ii,t}(\phi)(\varepsilon_{ii,t}(\phi)-1)}. \tag{B.115}$$

Major Exchange Rate Pass-Through into Upstream Import Prices:

$$erpt^k_{in,t}(\pi) = erpt^k_{ii,t}(\pi), \tag{B.116}$$

$$erpt^k_{in,t}(\$) = 1 + erpt^k_{ii,t}(\$), \tag{B.117}$$

$$erpt^k_{in,t}(\ell) = \frac{\Gamma_{in,mc,t}(\ell) + \Gamma_{in,\Theta,t}(\ell)\sum_{-\phi} s_{in,t}(-\ell)erpt^k_{in,t}(-\ell)}{1 + \Gamma_{in,\Delta,t}(\ell) + \Gamma_{in,\Theta,t}(\ell)(1 - s_{in,t}(\ell))}, \tag{B.118}$$

$$ptm^k_{in,t}(\ell) = erpt^k_{in,t}(\ell) - erpt^k_{ii,t}(\ell), \tag{B.119}$$

$$\Gamma_{ii,mc,t}(\pi) = \Gamma_{ii,mc,t}(\ell) = \Gamma_{in,mc,t}(\ell) = 1, \tag{B.120}$$

$$\Gamma_{ii,mc,t}(\$) = \Gamma_{kk,mc,t}(\phi) = 0. \tag{B.121}$$

Major Exchange Rate Pass-Through into Downstream Import Prices:

$$erpt^k_{i,t} = (1-\alpha_i)\sum_{n=1}^{N} s_{ni,t} erpt^k_{ni,t}, \tag{B.122}$$

$$erpt^k_{ni,t} = \sum_{\phi} s_{ni,t}(\phi) erpt^k_{ni,t}(\phi). \tag{B.123}$$

Terms of Trade

$$T_{ni,t} = \frac{p_{ii,t}}{p_{ni,t}}, \tag{B.124}$$

$$\tau^k_{ni,t} = erpt^k_{ii,t} - erpt^k_{ni,t}, \tag{B.125}$$

B.6 Balanced Growth Path

Macroeconomic Fundamentals

The balanced growth path rate of inflation and labour productivity growth are strictly positive and pre-determined:

$$\dot{p}_i > 1, \tag{B.126}$$

$$\dot{a}_i = \gamma_i > 1. \tag{B.127}$$

The nominal interest rate is pinned down by the Fisher equation:

$$r_i = \frac{\dot{p}_i}{\lambda_i}, \tag{B.128}$$

and the stochastic discount factor is pinned down by the Euler equation:

$$\lambda_i = \frac{\beta_i}{\gamma_i}, \tag{B.129}$$

which implies that the real interest rate is lower than the nominal interest rate in the balanced growth path. Since the stochastic discount factors are not identical in all countries, the nominal exchange rate is non-stationary, albeit the initial condition of the real exchange rate, of which there are N, is set to it's long-run mean, which is assumed to be known:

$$q_{ni,0} = q^{-1}_{in,0} = q_{ni} > 0, \tag{B.130}$$

$$\mu_i = \frac{q_{ji}\tilde{u}_{i,c}}{\tilde{u}_{j,c}}, \tag{B.131}$$

$$e_i = 1. \tag{B.132}$$

Next, the menu costs and wage adjustment costs are equal to zero in the long-run, such that the resource allocation is Pareto-efficient:

$$\varDelta_{ii}(\phi) = \varDelta'_{ii}(\phi) = \varDelta_{i,w} = \varDelta'_{i,w} = 0, \tag{B.133}$$
$$\varDelta''_{ii}(\phi) = \kappa_i(\phi). \tag{B.134}$$

The auxiliary variables associated with price and wage adjustment are therefore normalised:

$$\varPhi_{in}(\phi) = \varPsi_i = 1, \tag{B.135}$$
$$\dot{p}_{in}(\phi) = \dot{p}_n. \tag{B.136}$$

For \$-type or all firms domiciled in location $i = k$, the unit costs are $mc_i(\phi) = \xi_i(\phi)d_i$, while for $\phi = \{\pi, \ell\}$ in $i \neq k$, the unit costs are proportional to the major exchange rate:

$$mc_i(\phi) = \xi_i(\phi)d_i q_{ki}, \tag{B.137}$$

since $z_i = 1$. Suppose that the balanced growth path values of the market shares at home are pre-determined, such that $s_{ii}(\pi) = 1 - s_{ii}(\ell) - s_{ii}(\$)$, where $s_{ii}(\ell)$, and $s_{ii}(\$)$ are known for all $i \in n$. This implies that the ϕ-type elasticity of substitution is also known:

$$\varepsilon_{ii}(\phi) = \zeta(1 - s_{ii}(\phi)) + \eta s_{ii}(\phi), \tag{B.138}$$

which in turn determines the balanced growth path of the price mark-up:

$$\varTheta_{ii}(\phi) = \frac{\varepsilon_{ii}(\phi)}{\varepsilon_{ii}(\phi) - 1}, \tag{B.139}$$

and relative prices of exports:

$$p_{ii}(\phi) = \varTheta_{ii}(\phi) mc_i, \tag{B.140}$$

In addition, the relative price of imports of π- and \$-type goods can also be obtained using the symmetric equilibrium identity:

$$p_{in}(\pi) = d_n q_{in} p_{ii}(\pi), \tag{B.141}$$
$$p_{in}(\$) = d_n p_{ii}(\$), \tag{B.142}$$

such that $\delta_{in}(\pi) = 1$, $\delta_{in}(\$) = q_{ni}$. Suppose further that the balanced growth path values of the market shares abroad are also pre-determined, such that $s_{in}(\pi)$, $s_{in}(\ell)$, and $s_{in}(\$)$ are known for all $n \neq i$. Then the elasticity of substitution and the price mark-up abroad (i.e. $\varepsilon_{in}(\ell)$ and $\varTheta_{in}(\ell)$) are also known. It follows that the relative price of ℓ-type goods abroad can also be determined:

$$p_{in}(\ell) = \varTheta_{in}(\ell) mc_i(\ell). \tag{B.143}$$

The deviations from the law of one price for the ℓ-type products can then be obtained as the ratio of the import and export prices:

$$\delta_{in}(\ell) = \frac{p_{in}(\ell)}{d_n q_{in} p_{ii}(\ell)} = \frac{\Theta_{in}(\ell)}{d_n q_{in} \Theta_{ii}(\ell)}. \tag{B.144}$$

If the steady-state values of ϕ-specific market shares are calibrated, then the 'change-of-variable' principle restricts the values of the associated ϕ-type market densities as follows:

$$\chi_{in}(\phi) = s_{in}(\phi) \left(\frac{p_{in}(\phi)}{p_{in}}\right)^{\zeta-1}. \tag{B.145}$$

Having determined all relative prices associated with import-export manufactured goods of each type, the economy-wide price of manufactured goods are obtained as follows:

$$\begin{aligned} 1 &= \sum_{\phi} \chi_{in}(\phi), \\ 1 &= \sum_{\phi} s_{in}(\phi) \left[\frac{p_{in}(\phi)}{p_{in}}\right]^{\zeta-1}, \\ p_{in} &= \left[\sum_{\phi} s_{in}(\phi) p_{in}(\phi)^{\zeta-1}\right]^{1/(\zeta-1)}, \end{aligned} \tag{B.146}$$

Suppose that the country-specific trade weights at home and abroad are pre-determined, such that s_{ni} is known for all $i \in n$. Then $p_{i,x}$ can be obtained as follows:

$$\begin{aligned} 1 &= \sum_{n=1}^{N} \alpha_{ni}, \\ 1 &= \sum_{n=1}^{N} s_{ni} \left[\frac{p_{ni}}{p_{i,x}}\right]^{\eta-1}, \\ p_{i,x} &= \left[\sum_{n=1}^{N} s_{ni} p_{ni}^{\eta-1}\right]^{1/(\eta-1)}. \end{aligned} \tag{B.147}$$

It follows that the downstream import penetration ratio from each location $i \in n$ is given by:

$$\alpha_{ni} = s_{ni} \left(\frac{p_{ni}}{p_{i,x}}\right)^{\eta-1}. \tag{B.148}$$

Combining the above results together pins down the demand for each ϕ-type products:

$$\tilde{y}_{in}(\phi) = s_{in}(\phi) \left[\frac{p_{in} \tilde{y}_{in}}{p_{in}(\phi)}\right], \tag{B.149}$$

and the demand for imported intermediate goods

$$\tilde{m}_{in}(\phi) = \xi_i(\phi) \tilde{y}_{in}(\phi), \tag{B.150}$$

but only if $\tilde{y}_{in}$ is known. The demand for home production would also be known:

$$\tilde{y}_{ni} = s_{ni}\left[\frac{p_{i,x}\tilde{x}_i}{p_{ni}}\right], \tag{B.151}$$

but only if $\tilde{x}_i$ is known. Recall that the wage bill and the total cost of tradable goods are proportional to the total consumption expenditure:

$$p_{i,x}\tilde{x}_i = (1-\alpha_i)p_i\tilde{c}_i \Rightarrow \tilde{x}_i = \frac{(1-\alpha_i)\tilde{c}_i}{p_{i,x}}, \tag{B.152}$$

$$\tilde{w}_i h_i = \alpha_i p_i \tilde{c}_i \Rightarrow \tilde{c}_i = \frac{\tilde{w}_i h_i}{\alpha_i}, \tag{B.153}$$

where $p_i = 1$. Therefore, if $\tilde{c}_i$ is known, then the balanced growth path is well-defined. However, in order to determine consumption, one must first pin down the wage bill. The hourly wages are obtained from the consumer price index identity:

$$1 = \left(\frac{\tilde{w}_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{p_{i,x}}{1-\alpha_i}\right)^{1-\alpha_i} \Rightarrow \tilde{w}_i = \alpha_i\left(\frac{p_{i,x}}{1-\alpha_i}\right)^{1-1/\alpha_i}. \tag{B.154}$$

Secondly, the hours of labour are pre-determined $h_i \in (0,1)$, which puts a restriction on the parameter controlling the relative disutility of labour:

$$\begin{aligned}
\tilde{w}_i &= \frac{\zeta}{\zeta-1}\tilde{mrs}_i,\\
&= \frac{\zeta}{\zeta-1}\frac{\psi_i h_i^{\varphi_i}}{\tilde{u}_{i,c}},\\
\Rightarrow \psi_i &= \frac{\zeta-1}{\zeta}\frac{\tilde{w}_i\tilde{u}_{i,c}}{h_i^{\varphi_i}},
\end{aligned} \tag{B.155}$$

where

$$\tilde{u}_{i,c} = \frac{1-\beta_i\vartheta_i}{\tilde{c}_i}\left(\frac{\gamma_i}{\gamma_i-\vartheta_i}\right) \tag{B.156}$$

It follows that the balanced growth path values of endogenous variables derived above are consistent with all structural equations of the model. Therefore, the aggregate output can be obtained using the income-expenditure identity:

$$\begin{aligned}
\tilde{y}_i &= \tilde{c}_i + \tilde{nx}_i,\\
\tilde{nx}_i &= \sum_{n=1}^{N-i} p_{in}q_{ni}\tilde{y}_{in} - p_{ni}\tilde{y}_{ni}.
\end{aligned}$$

Exchange Rate Pass-Through & Terms of Trade

Consider the auxiliary variables associated with exchange rate pass-through into the export prices. If prices are flexible in the long-run, then:

$$\varXi_{ii}(\phi) = \frac{\kappa_i(\phi)\dot{p}_i^2}{\varepsilon_{ii}(\phi) - 1}, \tag{B.157}$$

$$\varGamma_{ii,\Delta}(\phi) = \varXi_{ii}(\phi)\left(1 + \frac{\lambda_i\gamma_i\dot{p}_{ii}}{\dot{p}_i}\right), \tag{B.158}$$

$$\varGamma_{ii,\Theta}(\phi) = \frac{(\zeta-1)(\zeta-\eta)}{\varepsilon_{ii}(\phi)-1}\frac{s_{ii}(\phi)}{\varepsilon_{ii}(\phi)}. \tag{B.159}$$

Once the values of the auxiliary variables are determined, it is possible to solve for the magnitude of exchange rate pass-through into the export prices of each ϕ-type firms using the following expression:

$$\begin{aligned} erpt_{ii}^k(\phi) &= \frac{\varGamma_{ii,mc}(\phi) + \varGamma_{ii,\Theta}(\phi)\sum_{-\phi} s_{ii}(-\phi)erpt_{ii}^k(-\phi)}{1 + \varGamma_{ii,\Delta}(\phi) + \varGamma_{ii,\Theta}(\phi)(1 - s_{ii}(\phi))}, \\ &= \varLambda_{ii,1}(\phi) + \varLambda_{ii,2}(\phi)\sum_{-\phi} s_{ii}(-\phi)erpt_{ii}^k(-\phi). \end{aligned} \tag{B.160}$$

Consider evaluating the above for each ϕ-types of firms as follows:

$$\begin{aligned} erpt_{ii}^k(\pi) &= \varLambda_{ii,1}(\pi) + \varLambda_{ii,2}(\pi)\sum_{-\pi} s_{ii}(-\pi)erpt_{ii}^k(-\pi), \\ &= \varLambda_{ii,1}(\pi) + \varLambda_{ii,2}(\pi)\left\{s_{ii}(\ell)erpt_{ii}^k(\ell) + s_{ii}(\$)erpt_{ii}^k(\$)\right\}, \end{aligned} \tag{B.161}$$

$$\begin{aligned} erpt_{ii}^k(\ell) &= \varLambda_{ii,1}(\ell) + \varLambda_{ii,2}(\ell)\sum_{-\ell} s_{ii}(-\ell)erpt_{ii}^k(-\ell), \\ &= \varLambda_{ii,1}(\ell) + \varLambda_{ii,2}(\ell)\left\{s_{ii}(\pi)erpt_{ii}^k(\pi) + s_{ii}(\$)erpt_{ii}^k(\$)\right\}, \end{aligned} \tag{B.162}$$

$$\begin{aligned} erpt_{ii}^k(\$) &= \varLambda_{ii,1}(\$) + \varLambda_{ii,2}(\$)\sum_{-\$} s_{ii}(-\$)erpt_{ii}^k(-\$), \\ &= \varLambda_{ii,1}(\$) + \varLambda_{ii,2}(\$)\left\{s_{ii}(\pi)erpt_{ii}^k(\pi) + s_{ii}(\ell)erpt_{ii}^k(\ell)\right\}. \end{aligned} \tag{B.163}$$

Next, substitute the pass-through from $-type firms into that of the ℓ-type firms and solve for the latter:

$$
\begin{aligned}
erpt^k_{ii}(\ell) &= \Lambda_{ii,1}(\ell) + \Lambda_{ii,2}(\ell)s_{ii}(\pi)erpt^k_{ii}(\pi) + \Lambda_{ii,2}(\ell)s_{ii}(\$)erpt^k_{ii}(\$), \\
&= \Lambda_{ii,1}(\ell) + \Lambda_{ii,2}(\ell)s_{ii}(\pi)erpt^k_{ii}(\pi) \\
&\quad + \Lambda_{ii,2}(\ell)\left\{s_{ii}(\$)\left[\Lambda_{ii,1}(\$) + \Lambda_{ii,2}(\$)s_{ii}(\pi)erpt^k_{ii}(\pi) + \Lambda_{ii,2}(\$)s_{ii}(\ell)erpt^k_{ii}(\ell)\right]\right\}, \\
&= \Lambda_{ii,1}(\ell) + \Lambda_{ii,2}(\ell)\Lambda_{ii,1}(\$)s_{ii}(\$) \\
&\quad + \Lambda_{ii,2}(\ell)s_{ii}(\pi)erpt^k_{ii}(\pi)\left(1 + \Lambda_{ii,2}(\$)s_{ii}(\$)\right) \\
&\quad + \Lambda_{ii,2}(\ell)s_{ii}(\ell)\Lambda_{ii,2}(\$)s_{ii}(\$)erpt^k_{ii}(\ell), \\
&= \frac{\Lambda_{ii,1}(\ell) + \Lambda_{ii,2}(\ell)\Lambda_{ii,1}(\$)s_{ii}(\$) + \Lambda_{ii,2}(\ell)s_{ii}(\pi)erpt^k_{ii}(\pi)\left(1 + \Lambda_{ii,2}(\$)s_{ii}(\$)\right)}{1 - \Lambda_{ii,2}(\ell)s_{ii}(\ell)\Lambda_{ii,2}(\$)s_{ii}(\$)}, \\
&= \Omega_{ii,1}(\ell) + \Omega_{ii,2}(\ell)erpt^k_{ii}(\pi). \qquad \text{(B.164)}
\end{aligned}
$$

Now substitute the pass-through from $-type and ℓ-type firms into that of the π-type firms and solve for the latter:

$$
\begin{aligned}
erpt^k_{ii}(\pi) &= \Lambda_{ii,1}(\pi) + \Lambda_{ii,2}(\pi)s_{ii}(\ell)erpt^k_{ii}(\ell) + \Lambda_{ii,2}(\pi)s_{ii}(\$)erpt^k_{ii}(\$), \\
&= \Lambda_{ii,1}(\pi) + \Lambda_{ii,2}(\pi)s_{ii}(\ell)erpt^k_{ii}(\ell) \\
&\quad + \Lambda_{ii,2}(\pi)s_{ii}(\$)\left\{\Lambda_{ii,1}(\$) + \Lambda_{ii,2}(\$)\left\{s_{ii}(\pi)erpt^k_{ii}(\pi) + s_{ii}(\ell)erpt^k_{ii}(\ell)\right\}\right\}, \\
&= \Lambda_{ii,1}(\pi) + \Lambda_{ii,2}(\pi)s_{ii}(\$)\Lambda_{ii,1}(\$) \\
&\quad + \Lambda_{ii,2}(\pi)s_{ii}(\ell)erpt^k_{ii}(\ell)(1 + \Lambda_{ii,2}(\$)s_{ii}(\$)) \\
&\quad + \Lambda_{ii,2}(\pi)s_{ii}(\pi)\Lambda_{ii,2}(\$)s_{ii}(\$)erpt^k_{ii}(\pi), \\
&= \frac{\Lambda_{ii,1}(\pi) + \Lambda_{ii,2}(\pi)s_{ii}(\$)\Lambda_{ii,1}(\$) + \Lambda_{ii,2}(\pi)s_{ii}(\ell)erpt^k_{ii}(\ell)(1 + \Lambda_{ii,2}(\$)s_{ii}(\$))}{1 - \Lambda_{ii,2}(\pi)s_{ii}(\pi)\Lambda_{ii,2}(\$)s_{ii}(\$)}, \\
&= \Omega_{ii,1}(\pi) + \Omega_{ii,2}(\pi)erpt^k_{ii}(\ell), \qquad \text{(B.165)} \\
&= \Omega_{ii,1}(\pi) + \Omega_{ii,2}(\pi)\left\{\Omega_{ii,1}(\ell) + \Omega_{ii,2}(\ell)erpt^k_{ii}(\pi)\right\}, \\
&= \frac{\Omega_{ii,1}(\pi) + \Omega_{ii,2}(\pi)\Omega_{ii,1}(\ell)}{1 - \Omega_{ii,2}(\pi)\Omega_{ii,2}(\ell)}. \qquad \text{(B.166)}
\end{aligned}
$$

After the determination of export price pass-through, the properties of the import price pass-through can be pinned down. Because the π-type products are perfectly integrated around the global markets, the import prices in the destination economy will absorb all of the major exchange rate fluctuations in excess of the adjustments in the export prices:

$$
ptm^k_{in}(\pi) = 0, \qquad \text{(B.167)}
$$

$$
erpt^k_{in}(\pi) = erpt^k_{ii}(\pi). \qquad \text{(B.168)}
$$

Similarly, the international market for $\$$-type products is perfectly disintegrated, such that:

$$ptm_{in}(\$) = 1, \tag{B.169}$$

$$erpt_{in}(\$) = 1 + erpt_{ii}^{k}(\$). \tag{B.170}$$

However, import prices of ℓ-type products respond in a similar fashion to the export prices:

$$erpt_{in}^{k}(\ell) = \Lambda_{in,1}(\ell) + \Lambda_{in,2}(\ell) \left\{ s_{in}(\pi) erpt_{in}^{k}(\pi) + s_{in}(\$) erpt_{in}^{k}(\$) \right\}, \tag{B.171}$$

such that

$$ptm_{in}^{k}(\ell) = erpt_{in}(\ell) - erpt_{ii}(\ell) \tag{B.172}$$

As for the downstream import prices and consumer prices, they are equal to the weighted average of pass-through into import prices:

$$erpt_{i}^{k} = (1 - \alpha_i) \sum_{n=1}^{N} s_{ni} erpt_{ni}^{k}, \tag{B.173}$$

$$erpt_{ni}^{k} = \sum_{\phi} s_{ni}(\phi) erpt_{ni}^{k}(\phi). \tag{B.174}$$

The terms of trade and their elasticity with respect to the exchange rate are given by:

$$T_{ni} = \frac{p_{ii}}{p_{ni}}, \tag{B.175}$$

$$\tau_{ni}^{k} = erpt_{ii}^{k} - erpt_{ni}^{k}. \tag{B.176}$$

List of Abbreviations

CIF	Cost, Insurance, Freight
CPI	Consumer Price Index
CONS	Real Aggregate Consumption
DSGE	Dynamic Stochastic General Equilibrium
ERPT	Exchange Rate Pass-Through
G7	Group of Seven
GATT	General Agreement on Tariffs and Trade
GBP	British Pound Sterling
GMM	Generalised Method of Moments
GVC	Global Value Chains
LCP	Local Currency Pricing
FOB	Freight On Board
JP	Japan
JPY	Japanese Yen
KR	South Korea
DCP	Dominant Currency Pricing
NIR	Nominal Interest Rate
OECD	Organisation for Economic Cooperation & Development
PCP	Producer Currency Pricing
PPI	Producer Price Index
SKW	South Korean Won
SMM	Simulated Method of Moments
SVAR	Structural Vector Autoregression
USD	United States Dollar
VAR	Vector Autoregression
UK	United Kingdom
US	United States
UVIM	Unit Value Index of Imports
UVIX	Unit Value Index of Exports
WTO	World Trade Organisation

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