Impact of Ambiguity on Stock Markets

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Abstract

Quantitative studies have provided evidence showing that ambiguity can help to explain the equity premium puzzle and the excess volatility puzzle of the equity market. In addition, it also plays an important role in the 2008 financial crisis. However, empirical studies remain few. Anderson et al. (2009) develop an empirical measure based on the Survey of Professional Forecasters (SPF). The survey data are collected from part of the professionals in the US finance industry, which might result in biased findings. Viale et al. (2014) develop another empirical measure of ambiguity based on the reference model calculated using the smooth transition autoregressive (STAR) model and assumptions about the confidence level of investors. It may be improper to use the STAR model as the reference model because it is difficult to find out a forecasting model that is used by all investors. As such, the first empirical study in Chapter 3 focuses on high-frequency forecasting using linear AR models, exponential smoothing models and nonlinear AR models. The findings suggest that the best-performing forecasting model changes from one period to another and the STAR model cannot beat the AR model, suggesting that the calculation of the ambiguity measure of Viale et al. (2014) is improper. Therefore, the other two empirical studies in Chapters 4 and 5 develop two new empirical ambiguity measures with inspiration from theoretical works. The results support the theoretical proposition that ambiguity can explain the equity premium puzzle and the excess volatility puzzle. In addition, the degree of ambiguity of the equity market can be affected by investors' expectations on macroeconomic conditions and default risks. On the other hand, Chapter 5 shows that ambiguity plays an important role in the 2008 financial crisis. Last but not least, the thesis also provides an ambiguity indicator for regulators and financial market practitioners.

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Declaration

I declare that this thesis is a presentation of original work and I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as References.

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Chapter 1. Introduction

1.1. Introduction

This thesis mainly focuses on the impact of ambiguity on stock markets from empirical perspectives. It first investigates and points out the disadvantages of existing empirical measures of ambiguity in Chapters 2 and 3. Then in Chapters 4 and 5, two empirical ambiguity measures are developed to find empirical evidence that uncovers the impact of ambiguity on stock markets as well as its role in the 2008 financial crisis.

1.2. Background of Study

The three empirical studies are conducted with inspirations from quantitative studies. According to Epstein and Schneider (2010), ambiguity plays an important role in asset pricing theoretically. Although there is a large number of quantitative studies in the field of study, the number of empirical studies remain few. Anderson et al. (2009) develop an ambiguity measure using the Survey of Professional Forecasts (SPF), which is also used by other researchers in ambiguity studies. However, this measure is based on forecasts of professionals and thus it neglects the perceptions of individual investors. Viale et al. (2014) develop another ambiguity measure using a smooth transition autoregressive (STAR) model. The STAR model is employed as the reference model in their study and the ambiguity measure is calculated by the distance between the distorted value due to ambiguity and the reference model. The calculation of the distorted value is based on assumptions on investors' confidence level. As such, their measure does not purely rely on real-life data. In addition, whether the STAR model is representative of the reference model is doubtful. Therefore, the thesis begins with a study of forecasting, followed by two empirical chapters where two new empirical measures of ambiguity are developed.

1.3. Research Gap

The thesis attempts to fill in a few research gaps. The first empirical chapter, namely Chapter 3, attempts to uncover whether sample size can affect forecasting accuracy. It also investigates the adaptive market hypothesis (AMH) using a forecasting method that has not been used in existing literature. In addition, it also helps to figure out whether it is proper to use the STAR model as the reference model in ambiguity literature.

The second and third empirical chapters, Chapters 4 and 5, attempt to develop two new empirical measures of ambiguity using bid and ask prices and intraday highest and lowest prices respectively. In addition, these two chapters provide empirical evidence on how ambiguity affects stock markets, and more importantly, it investigates how macroeconomic conditions and default risks can interact with ambiguity in stock markets, which helps to shed light on the role of ambiguity in the 2008 financial crisis.

1.4. Methodology

The forecasting chapter applies high-frequency data, including minute data, hourly data and daily data, to compare forecasting performances of linear autoregressive models, exponential smoothing models and nonlinear autoregressive models. The linear autoregressive models include the simple autoregressive (AR) model and the autoregressive integrated moving average (ARIMA) model. Nonlinear models include the additive autoregressive model, the threshold autoregressive (TAR) model and the STAR model. Furthermore, the full sample is split into subsamples to investigate the impact of sample size on forecasting accuracy.

The two empirical chapters on ambiguity mainly develop the ambiguity measures with inspirations from theoretical works. The econometric model applied is the vector autoregressive (VAR) model, which can uncover interactions among different variables even if no prior knowledge is known about the relationships among the variables. Since there are few empirical studies on ambiguity so far, the empirical relationship between ambiguity and variables of interest remain unclear. As such, the VAR model serves the purpose of the study best, which is the reason why it is selected to conduct the empirical analyses.

1.5. Summary of Empirical Chapters

As is mentioned before, Chapter 3 focuses on high-frequency forecasting of stock prices. The full-sample results indicate that the linear models involved in the study generally provide better forecasting performances than the nonlinear models. This suggests that it is improper to assume the STAR model to be the reference model in Viale et al. (2014). On the other hand, the modified Diebold-Mariano (MDM) test results show that the UK stock market is not in a weak form of efficiency with significant evidence from minute data and weakly significant evidence from hourly data and daily data. The subsample forecasting results suggest that increasing sample size does not necessarily result in more accurate forecasts. In addition, the subsample analyses also indicate that the AMH characterises the UK stock market better than the EMH.

Chapter 4 uses the ambiguity measure calculated from bid and ask prices of ETF FTSE100 to investigate the impact of ambiguity on the UK stock market. The findings from the analysis between the ambiguity measure and market return suggest that the interaction between the ambiguity measure and market return and the interaction between the ambiguity measure and excess market return are statistically weak while ambiguity tends to have a significant impact on the volatility index, which measures investors' expectation on future volatility. The findings also confirm that existence of ambiguity can explain the excess volatility puzzle of the equity market, which is consistent with the proof of Epstein and Schneider (2010). In addition, apart from return, volatility is also shown to be source of ambiguity. Interactions between the ambiguity measure and the two term structure measures suggest that macroeconomic conditions can affect the degree of ambiguity of the equity market. When investors are more optimistic about the future economic state, the degree of ambiguity of the equity market tends to decrease. On the other hand, if investors are more worried about future default risks, the degree of ambiguity of the equity market would also increase.

Chapter 5 develops another ambiguity measure, which is calculated from the gap between the intraday highest and lowest prices. This chapter investigates both the UK stock market and the US stock market. In addition, the full sample is split into the pre-crisis period and the post-crisis period, which helps to uncover the role of ambiguity in the 2008 financial crisis. The full-sample results and the post-crisis results are consistent with the results in Chapter 4. However, the pre-crisis results show some differences. Firstly, for the UK stock market, the ambiguity measure does not seem to affect the volatility index during the pre-crisis period from 2004 to 2008, which suggests that investors did not pay attention to the degree of ambiguity before crisis. The US result suggests that investors started to realise the existence of ambiguity from 2007. On the other hand, before the crisis, investors viewed ambiguous information and signals as signs of better economic conditions, which contributes to the bubble. In addition, investors did not realise the default risks were high until 2007 when the crisis was about to happen. The two situations together led to the collapse of the financial markets and investors started to become aware of the importance of ambiguity after the crisis. As such, the results suggest that ambiguity contributed to the 2008 financial crisis.

1.6. Conclusion

The empirical chapters of the thesis serve the purposes of study and fill the research gaps defined from existing literature. As such, the thesis makes original contributions to the field of study of ambiguity as well as that of high-frequency forecasting. The rest of the thesis is constructed as follows. Chapter 2 provides a review of ambiguity literature. Chapters 3 – 5 are the empirical studies on high-frequency forecasting, ambiguity study using bid and ask prices and ambiguity study using intraday highest and lowest prices. The last chapter, which is Chapter 6 provides an in-depth discussion of the empirical results obtained from the three empirical chapters. More importantly, it links the results of the empirical chapters. Last but not least, conclusions and further studies are illustrated at the end of Chapter 6.

Chapter 2. Review of Ambiguity Literature

2.1. Introduction

Since the 2008 subprime mortgage crisis, ambiguity becomes an important research topic because some researchers find that the financial crisis is associated with ambiguity (Guidolin and Rinaldi, 2013; Guidolin and Rinaldi, 2014). Ambiguity refers to the uncertainty in probability distribution of asset prices due to misinterpretation or lack of information. The uncertainty of ambiguity arises from the unknown mean and hence the unknown probability distribution of the mean while for risk the uncertainty originates from the unknown variance.

Studies of ambiguity originate from the Ellsberg's Paradox. According to the decision experiment of Ellsberg (1961), different combinations of indifferent acts are meaningful because by combining the indifferent acts ambiguity can be removed. Hence, a decision maker prefers to take risks in order to hedge the uncertainty that arises from ambiguity. Thus, decision makers are ambiguity-averse, and the impact of ambiguity is found to be larger than that of risks in quantitative studies (Epstein and Schneider, 2010). Preference theories and utility models are therefore set up to understand ambiguity and its impact on portfolio choice and asset pricing. The three main utility models are the multiple-prior model, the smooth model and the multiplier model. In addition to the static preference models, researchers have also been working on dynamic utility models, which reflect the learning and updating processes of decision makers in a dynamic setting.

So far, there are several literature reviews on preference theories and ambiguity models, comparing the models in relation to their applications in portfolio formation and asset. Most of them attempt to explain the theories and models from a theoretical perspective by illustrating the formulas of the models. Epstein and Schneider (2010) first prove the ambiguity-aversion models with examples and mainly focus on the illustration of the multiple-prior model. They provide a comparison among the multiple-prior model, the smooth models and the multiplier model and their implications for financial markets. Etner et al. (2012) provide a review of ambiguity literature, which mainly explains all the major ambiguity models and the rationale behind them. Another more recent literature review, done by Guidolin and Rinaldi (2013), focuses on the effect of ambiguity and ambiguity aversion on portfolio choice and asset pricing. However, due to lack of empirical works in the field of study, the reviews mainly focus on theoretical implications from quantitative studies to illustrate the impact of ambiguity on financial markets. This literature review attempts to explain the intuitions behind the different utility models based on ambiguity rather than providing a review of the theories. More importantly, it also tries to identify research gaps from the empirical perspective, with a review of recent empirical works on ambiguity measures.

The structure of the literature review is as follows. Section 2.2 explains ambiguity into details using the Ellsberg's experiment (1961). Section 2.3 describes the main ambiguity models and their implications. Section 2.4 briefly discusses the learning and updating process of decision-makers. In Section 2.5 and 2.6, the effect of ambiguity on portfolio choice and asset pricing are explained respectively, with evidence from both quantitative studies and empirical works. In the end, a short conclusion is provided in Section 2.6.

2.2. Ellsberg's Experiment (1961)

In order to illustrate what is ambiguity, it is necessary to introduce the concept of risk first. Suppose we are told there are 50 black balls and 50 red balls in an urn, then we know that there are two possible outcomes if we were to prick a ball from the urn, namely either picking a black ball or picking a red ball. Since we are told the number of balls of each colour, it is easy to calculate the probability of picking a black ball, which is 50%, and the probability of picking a red ball, which is also 50%. In this case, we are faced with risks when betting on the colour of the ball that is picked since we know probability distribution of the possible outcomes. Now suppose the only information we know about the urn is that there is a total number of 100 black balls and red balls and we are told to bet on the colour of the ball that is picked from the urn, then we cannot calculate the probabilities of the possible outcomes although there are still two possible outcomes, namely picking a black ball and picking a red ball. In this case, we are faced with ambiguity because we don't know the probability distribution. The first urn introduced in this section, which is denoted by Urn 2 in the Ellsberg's experiment (1961), is known as the risky urn, and the second urn, which is denoted by Urn 1 in the experiment, is known as the ambiguous urn.

During the experiment, the participants were asked two questions. First, which is more likely to happen, picking a black ball from Urn 1 or picking a red ball from Urn 1? The second question is, which is more likely to happen, picking a black ball from Urn 2 or picking a red ball from Urn 2? As is mentioned before, the probability of picking a black ball from Urn 2, the risky urn, and the probability of picking a red ball from Urn 2 are both 50%, so they should be equally likely to happen. On the other hand, the chance of picking a black ball from Urn 1, the ambiguous urn, and the chance of picking a red ball from Urn 1 should also seem equal because the probabilities can take any value from 0 to 1. Following the previous two questions, the participants were asked two further questions. The first question is, which is more likely to happen, picking a black ball from Urn 1 or picking a black ball from Urn 2? The second one is, which is more likely to happen, picking a red ball from Urn 1 or picking a red ball from Urn2? The experiment suggests that the participants thought picking a black ball from Urn 2 was more likely to happen than picking a black ball from Urn 1 and picking a red ball from Urn 2 was also more likely to happen than picking a red from Urn 1. This implies that the participants thought the probability of picking a black ball from Urn 1 was less than 50% and the probability of picking a red ball from Urn 1 was also less than 50%. The two results contradict because according to probability theory, probabilities of all possible outcome should add up to 1. Picking a black ball and picking a red ball from the same urn were the only two possible outcomes and they did not add up to 1 according to the responses. Since this situation cannot be justified by the subjective expected utility (SEU) theory, there must be something missing in the theory.

In the second set of experiment, the participants were informed that there were 30 red balls and a total of 60 black balls and yellow balls in the urn and they were asked to choose between two options. Option 1 pays $1 if a red ball is picked from the urn and nothing otherwise. Option 2 pays $1 if a black ball is picked and nothing otherwise. The experiment result was that most participants showed preference towards option 1. Then they were offered another two options. Option 1 pays $1 if either a red ball or a yellow ball is picked from the urn and nothing otherwise. Option 2 pays $1 if either a black ball or a yellow ball is picked from the urn and nothing otherwise. The result turned out to be that most participants showed preference towards option 2 this time. According to the SEU theory, if a decision-maker chooses option 1 in the first decision-making problem, he should also choose option 1 in the second problem because choosing option 1 in the first problem indicates that he strictly prefers betting on a red ball to betting on a black ball and the two options in the second problem are just a combination of the outcomes in the first problem with another outcome, which is picking a yellow ball from the urn. However, since the participants chose option 2 in the second problem, this suggests that there must be something wrong with the SEU theory, and this is the beginning of development of ambiguity literature.

To put it in a more formal way, we need some preliminaries. Let Ω represent the state space, C denote a set of possible outcomes and $\Delta(C)$ be the probability distribution of the set of outcomes C, where Ω and C can be assumed to be finite without losing generality (Epstein and Schneider, 2010). An Anscombe-Aumann (AA) act is a function that maps the state space, which means a set of all the contingencies or events, into the probability measure of the set of outcomes $f: \Omega \longrightarrow \Delta(C)$. On the other hand, a Savage act is a function that maps the state space into the possible outcomes $f: \Omega \longrightarrow C$. In decision theories, $\succcurlyeq$ refers to strictly preferred to or indifferent from; $\succ$ represents strict preference on one act to another; and $\backsim$ means the two acts are indifferent. In addition, strictly preference means that if an act A is strictly preferred to an act B, A is preferred to B and B cannot replace A. On the other hand, if A and B are indifferent, they can replace each other.

The SEU theory is based on the expected utility theory first proposed by Bernoulli (1738) and formalised by von Neumann and Morgenstern (1944). The expected utility theory states that a decision-maker behaves in a way such that he maximises the expected utility. Thus, preference of a decision-maker is modelled by the expected value of a set of utility functions on all possible outcomes, which are called the von Neumann-Morgenstern (vMN) utility function. Based on the expected utility theory, Savage (1954) introduced the SEU theory by adding a subjective probability measure to the expected utility function in order to incorporate risk aversion. They characterise rational behaviour of a decision-maker by four axioms, which are completeness, transitivity, continuity and independence. The completeness axiom states that for two acts A and B, either one of the two is strictly preferred or they are indifferent. This means that a decision-maker is always able to tell his preference.

The transitivity axiom states that if an act A is preferred to or indifferent to an act B and act B is preferred to or indifferent to an act C, act A is preferred to or indifferent to act C, and it can be written as:

$$If\ A \succcurlyeq B\ and\ B \succcurlyeq C, then\ A \succcurlyeq C.$$

$$If\ A \backsim B\ and\ B \backsim C, then\ A \backsim C. \qquad 2.1$$

The continuity axiom states that if an act A is preferred to or indifferent to an act B and act B is preferred to or indifferent to an act C, there exists a non-negative probability p, which is smaller than or equal to 1, such that a combination of A and C, denoted by $pA + (1 - p)C$, is indifferent from B. Formally, it can be written as:

$$If\ A \succcurlyeq B \succcurlyeq C, there\ exists\ p \in [0,1]\ such\ that\ pA + (1 - P)C \backsim B. \qquad 2.2$$

The independence axiom states that if an act A is strictly preferred to or indifferent from an act B, there exists an act C such that a combination of A and C, with weightages p and $1 - p$ respectively, is strictly preferred to or indifferent from a combination of B and C with the same weightages p and $1-p$, where p is a non-negative number smaller than or equal to 1. It can be formally written as:

$$If\ A \geqslant B, pA + (1-P)C \geqslant pB + (1-P)C\ where\ p \in [0,1]. \qquad 2.3$$

According to SEU, preference of a decision-maker depends on the subjective expected utility, which can be written as:

$$U(f) = \sum_i u(f_i)p(f_i) \qquad 2.4$$

where u is the utility of possible outcomes; and p is the probability of the possible outcomes.

With the formal settings and the preliminaries, the Ellsberg's experiment, which is also known as the Ellsberg's paradox, can be explained in a more mathematical way where the utility models of ambiguity aversion come from. The first Ellsberg's experiment can be summarised by the contingency table in Table 2.1. The implication from the responses of the participants are that the risky urn is preferred to the ambiguity because they chose Urn 2 in each contingency.

Table 2.1 Contingency Table of Ellsberg's Experiment 1

	Red	Black
Urn 1 (Ambiguous)	Red from Urn 1	Black from Urn 1
Urn 2 (Risky)	Red from Urn 2	Black from Urn 2

Table 2.2 shows the payoff table of the second Ellsberg's experiment. In the first problem, the participants chose option 1, and this means that they strictly preferred option 1 to option 2, which in turn suggests that they strictly preferred red balls to black balls. This can be written as:

$$f_R \succ f_B \qquad 2.5$$

According the independence axiom of the SEU theory, the responses of the participants in the second problem should indicate the following preference based on their responses in option 1:

$$f_R + f_Y \succ f_B + f_Y \tag{2.6}$$

However, in the second problem, they chose option 2, indicating that they strictly preferred option 2 to option 1, which in turn suggests that they strictly preferred a combination of black balls and yellow balls to a combination of red balls to yellow balls. This can be written as:

$$f_B + f_Y \succ f_R + f_Y \tag{2.7}$$

Since Equation 2.7 contradicts Equation 2.6, the SEU cannot justify the result of Ellsberg's experiment, and according to the first experiment, a decision maker differentiates ambiguity from risk, which implies that ambiguity is missing from the SEU theory. As such, theoretical papers develop utility models of ambiguity aversion to incorporate ambiguity aversion into the process of decision-making.

Table 2.2 Payoff Table of Ellsberg's Experiment 2. This table shows the payoff of each option that corresponds to the outcomes indicated in the first column.

Panel A: Payoffs of Problem 1			
	Red ($\boldsymbol{f_R}$)	Black ($\boldsymbol{f_B}$)	Yellow ($\boldsymbol{f_Y}$)
Option 1	\$1	0	-
Option 2	0	\$1	-
Panel B: Payoffs of Problem 2			
Option 1	\$1	0	\$1
Option 2	0	\$1	\$1

2.3. Ambiguity Models

This section discusses ambiguity aversion utility models and the intuition behind the models. The development is based on the illustrations of the multiple-prior model, the smooth model and the multiplier model.

2.3.1. Multiple-Prior Model

With inspiration from the Ellsberg's Paradox, Gilboa and Schmeidler (1989) develop the multiple-prior model to incorporate ambiguity. The basic idea of ambiguity aversions models is that a decision maker considers a set of prior probability distributions of the possible outcomes to support his decision-making instead of using a single prior. The utility function of the multiple-prior model can be written as follows:

$$U(f) = (1-\alpha)\int_{\Omega} u(f)dp^{*} + \alpha \min_{p\in\Delta(C)} \int_{\Omega} u(f)dp \qquad 2.8$$

where Ω is the space state; $\Delta(C)$ is a set of priors; f stands for an act; p^{*} represents the reference probability measure; p represents the alternative probability measure; α is the weight assigned to the alternative probability measure; and u represents a von-Neumann-Morgenstern (vMN) utility function.

The intuition behind the multiple-prior model is that a decision-maker forms a set of priors about the possible outcomes and takes the worst-case scenario more seriously. The minimum function in Equation 2.8 represents the worst case among the alternative probability measures to the reference measure. The decision-maker assigns a higher weight to the worst case when he becomes less confident about his reference model. As such, a larger α indicates that the decision-maker is more ambiguity-averse. Once he solves this minimisation problem, he calculates the expected utility using Equation 2.8 and ranks the options according to the expected utility. The final decision is based on the one that generates the highest expected utility and hence, he makes decision by maximising the expected utility based on the worst case. Thus, the decision process is made up of a minimisation problem and a maximisation problem, and this is the reason why this model is also called the maxmin expected utility model.

The potential problem of this model arises from the worst case. Although solving the minimisation can embody ambiguity-aversion, it may be too extreme for a decision maker to make decision based on the worst-case scenario (Epstein and Schneider, 2010). Nevertheless, the multiple-prior model has been widely applied in financial markets. For instance, Dow and Werlang (1992) and Garlappi et al. (2007) investigate the effect of ambiguity on portfolio choice based on the multiple-prior model. In particular, Garlappi et al. (2007) show that the multiple-prior model performs better than the classic mean-variance analysis and the Bayesian approach empirically. Routledge and Zin (2009) and Ozsoylev and Werner (2011) investigate the impact of ambiguity on liquidity and find that investors behave under multiple-prior preferences. Another reason why the multiple-prior utility model is popular is that it makes it possible to carry out empirical studies compared to other ambiguity aversion models. Thus, so far, the empirical measures are all based on the multiple-prior model (Anderson et al. 2009; Viale et al., 2014; Antoniou et al., 2015)

2.3.2. Smooth Model

Klibanoff et al. (2003) developed another ambiguity aversion model to make the kinked indifferent curves of the multiple-prior model smooth. As such, this model is called the smooth utility model, and it accommodates the multiple-prior model, which is a special case of the smooth model. The utility function of the smooth model can be written as follows:

$$U(f) = \int_{\Delta} \phi\left(\int_{\Omega} u(f)dp\right)d\mu \qquad 2.9$$

where f represents an act; Ω is the state space; Δ is the set of probability measures of the state given subjective information; u represents a von Neumann-Morgenstern (vNM) utility function; μ is the probability measure subject to the possible probabilities Δ; and ϕ measures the degree of ambiguity-aversion.

In this model, a concave ϕ means the decision-maker is ambiguity-averse and the larger the concavity, the more ambiguity-averse the decision-maker is. If ϕ is linear, the decision-maker is said to be ambiguity-neutral. On the other hand, one advantage of this model is that it separates the level of ambiguity, which is measured by μ, from the extent of ambiguity-aversion, which is captured by ϕ. Thus, the model can be interpreted in the following way. The risk tolerance of a decision-maker is captured by the subjective expected utility function. Then with respect to the subjective information that the decision-maker has, he penalises the expected utility according to his extent of ambiguity aversion. On the other hand, if the decision-maker is ambiguity-neutral, he becomes the subjective expected utility agent, who only cares about risks. In addition, the model also implies that a decision-maker prefers risks to ambiguity under the smooth model, which is consistent with the Ellsberg's Paradox. The difference between the smooth models and the multiple-prior model is that change in ambiguity resembles change in risk in the smooth models while in the multiple-prior model change ambiguity suggests change in the mean (Epstein and Schneider, 2010).

However, the disadvantage of the model is that it cannot rationalise the situation where a decision-maker can guess the true distribution instead of applying the vNM function to figure out the reference model, which is illustrated by Epstein and Schneider (2010) using a thought experiment. However, Klibanoff et al. (2012) showed that the thought experiment of Epstein and Schneider (2010) is misleading in the sense that they did not consider the whole state space.

Regardless of the debates, a few researchers use smooth models to investigate the impact of ambiguity on financial markets (Epstein and Schneider, 2010). For instance, Chen et al. (2014) employed the smooth model to discover the effect of ambiguity on portfolio choice. Many studies, which adopt the smooth approach, use the recursive smooth models that is explained later in this chapter. However, the smooth model is difficult to be applied in empirical studies because parameters such as concavity of the utility function, ϕ, are difficult to measure empirically.

2.3.3. Multiplier Model and Variational Utility Model

The idea of the multiplier model comes from the robust control theory in engineering, which is introduced by Anderson et al. (2003). It incorporates ambiguity aversion using an approximating model to measure the reference model, which a decision-maker believes as a true model. The utility function of the multiplier model is formally developed by Strzalecki (2011) as follows:

$$U(f) = \min_{p \in \Delta}\left(\int_{\Omega} u(f)dp + \theta R(p\|p^*)\right) \qquad 2.10$$

where $R(p\|p^*) = \begin{cases} \int_{\Omega}\left(log\frac{dp}{dp^*}\right)dp, when\ p \in \Delta \\ \infty \qquad , elsewhere \end{cases}$

In Equation 2.10, f stands for an act; Ω is the state space; Δ is the set of probability measures of the state given subjective information; u is a von Neumann-Morgenstern (vNM) utility function; θ is a positive parameter; and $R(p\|p^*)$ is a non-negative relative entropy, which measures the distance between the reference model

p^* and other possible models p, and the formula of the entropy is given above. The extent of ambiguity-aversion of a decision-maker decreases when θ increases. Thus, when θ goes into infinity, the decision-maker is completely confident about his approximating model. On the other hand, the entropy, $R(p\|p^*)$, measures the level of ambiguity. The intuition behind the model is that apart from the reference model, a decision-maker also considers other possible models because he is ambiguous-averse. As such, he also considers the worst case, but he is still confident about his reference model to some extent. Hence, he assigns a higher weight to the possible models that are close to his reference model, and his final decision is based on the highest ranking of the resulted expected utility.

The multiplier model is criticised because of its inability to rationalise situations where there is more than one ambiguous urn in the Ellsberg's experiment (Epstein and Schneider, 2010). As such, a generalised version of the multiplier model, which is also called the variational utility model, is proposed by Maccheroni et al. (2006) to solve this problem. The utility function of the variational utility preference is shown as follows:

$$U(f) = \min_{p \in \Delta}\left(\int_{\Omega} u(f)dp + c(p)\right) \quad 2.11$$

In Equation 2.11, $c(p)$ is a cost function, which can take any value from 0 to infinity. When $c(p)$ equals to $\theta R(p\|q)$, the variational utility model becomes the multiplier model.

So far, the multiplier model has not been as widely used as the multiple-prior model and the smooth model. However, some researchers use it as a different method to verify the existing literature. Barillas et al. (2009) use this model to re-examine the results from existing literature that explains composites of equity premium.

2.3.4. Dynamic Preference Models

Based on each static preference model introduced in the above subsections, there are three corresponding dynamic models. The time-consistent, intertemporal version of the multiple-prior model is proposed in Epstein and Wang (1994) and is formally set up in Epstein and Schneider (2003) as a recursive multiple-prior model. The utility function is presented as follows:

$$V_{t,\varphi}(F) = u\big(F_t(\varphi)\big) + \beta \min_{p \in P_{t,\varphi}} \int_{\Omega} V_{t+1,\omega}\, dp(\omega) \qquad 2.12$$

where $P_{t,\varphi}$ is a set of priors about time $t+1$ at time t.

A dynamic, continuous-time version of the multiple-prior model is proposed in Duffie and Epstein (1992) and is formalised in Chen and Epstein (2002), which is an important paper for asset pricing theories. The model is presented as follows:

$$V_t = \min_{p \in P^{\theta}} E_p\left(\int_t^T h\big(C_s, V_s^p\big) ds | \mathcal{F}_t\right) \qquad 2.13$$

where P^{θ} is a set of priors; h is defined as an aggregator; and $\mathcal{F}_t$ is a filtration. The interpretation of the dynamic models is similar to that of the static ones expect that the time period is now continuous.

In terms of application, the dynamic version of the multiple-prior model is widely used in quantitative papers to study the effect of ambiguity on financial markets, often involving learning and updating processes. For instance, Jeong et al. (2015) adopt the continuous-time recursive multiple-prior model to investigate the effect of ambiguity on asset pricing, which is found to be significant.

The dynamic intertemporal version of the smooth model is proposed in Klibanoff et al. (2009), and it is rearranged by Epstein and Schneider (2010) as follows:

$$V_{t,\varphi}(F) = u\big(F_t(\varphi)\big) + \beta B_{t+1}\phi^{-1} \min_{p\in P_{t,\varphi}} \int_\Delta \phi\big(B_{t+1}^{-1} \int_\Omega V_{t+1,\omega}\, dp(\omega) d\mu_{t,\varphi}\big) \quad 2.14$$

Further to this mode, Ju and Miao (2012) developed a new general recursive smooth model, which helps to solve the consumption-based equilibrium of asset prices.

The recursive multiplier utility function can be written as follows:

$$V_{t,\varphi}^{\theta}(F) = \min_{p\in\Delta} \left(\int_\Omega \left(\sum_t^T \beta^t\, u\big(F(\omega)\big)\right) dp(\omega) + \theta R(p\|q)\right) \qquad 2.15$$

2.4. Learning and Updating

Learning and updating processes in the financial markets refer to the situations where investors make observations from historical prices, learn the hidden future states from the past and update the prior probability measures accordingly. Since the learning and updating process involves different time periods, multi-stage static models and dynamic models are used to accommodating them. Epstein and Schneider (2008) developed a dynamic preference model based on the multiple-prior model. They claimed that ambiguity is not only caused by lack of information and quality of information but is also related to how the information is processed. Based on their research, investors tend to believe that bad news contains more ambiguous information or signals than good news. Moreover, Illeditsch (2009) find that putting risk-aversion and ambiguity-aversion together amplifies the effect of ambiguity due to bad news under heterogeneous agent models. Ju and Miao (2012) also find the amplified effect of ambiguity due to bad news using the generalised recursive smooth learning model.

On the other hand, learning under the multiplier model is intuitively not useful in the sense that it is quite similar to the subjective expected utility model, which does not include a learning process (Epstein and Schneider, 2010). Bayesian learning is proved to be not as good as models of learning under ambiguity by Chen et al. (2014), who use a smooth ambiguity-aversion model. This is also consistently found by Viale et al. (2014), who empirically show that their learning under ambiguity model is better than the Bayesian learning model and other classic asset pricing models.

2.5. Effect of Ambiguity on Portfolio Choice

Ambiguity affects financial markets in two ways, one of which is in terms of portfolio choice. In theory, ambiguity aversion can cause selective participation and non-participation, which is also known as portfolio inertia (Epstein and Schneider, 2010). Dow and Werlang (1992) prove that ambiguity aversion can cause portfolio inertia by using one ambiguous asset in a two-period setting. They assume that there are only one ambiguous asset and one risk-free asset, which is a bond, in the market, and their analysis is based on the multiple-prior model. The result indicates that an investor forms an estimated price interval such that if the price lies in the interval, he will not participate in the market. Thus, the investor will buy the asset only if the price goes below the lower bound of the interval and will short the asset only if the price goes beyond the upper bound of the interval. They find that the range of the interval becomes larger when there is uncertainty, or ambiguity.

Epstein and Schneider (2010) also use the multiple-prior model to illustrate portfolio inertia caused by ambiguity aversion. As is discussed in the previous section, under the multiple-prior model, a change in ambiguity resembles a change in the mean. Based on this, Epstein and Schneider (2010) show that an investor has a reference rate of return as a benchmark to evaluate the return of the asset. However, due to ambiguity-aversion, the estimate will be allowed to go up and down by an amount such that it reflects the degree of lack of confidence caused by ambiguity. Thus, ambiguous return falls into the interval $[r^e - r_L, r^e + r_L]$ where r^e represents the estimated benchmark on the rate of return and r_L stands for the dispersion due to ambiguity. They showed that an investor would take a long position of the asset if the lower bound of the interval is positive while he would take a short position if the upper bound of the interval is negative. In the case where the interval contains the zero value, the investor would neither go long nor go short. As such, ambiguity leads to nonparticipation, or portfolio inertia. In contrast, risk does not seem to affect the decision on participation. It affects the amount of the investment. Therefore, ambiguity has a first-order effect while risk has a second-order effect. This is consistently shown by Jeong et al. (2015) who find that risk-aversion is reduced when an ambiguity-aversion term is considered using a continuous-time recursive multiple-prior model. Thus, the effect of ambiguity on portfolio choice is prevailing to the effect of risk. Moreover, as is illustrated by Epstein and Schneider (2010), the first-order effect remains even if the bond becomes risky and there is no longer a risk-free asset available in the market.

In the case where there are a number of independent ambiguous assets, an investor will participate partly in the market when their estimated interval of rate of return does not contain the zero value. On the other hand, in the case where the ambiguous assets are dependent, the first-order effect of ambiguity will be reduced because a pool of mutually dependent ambiguous assets can help to hedge ambiguity and hence benefits from diversification will offset part of the effect caused by ambiguity. In addition, ambiguity has an intertemporal effect on portfolio choice. It can have an impact on the entry and exit criteria by affecting both the benchmark and the lack of confidence parameter through updating, which means that the realised return of the previous period will affect the estimated interval of the rate of return of the next period due to learning and updating from the previous period. The implication behind this is that an investor focuses on future profitability and hence he will react according to the information available now to hedge against ambiguity in the future (Epstein and Schneider, 2010). The intertemporal effect of ambiguity can also be rationalised by dynamic models (Epstein and Schneider, 2007; Miao, 2009).

Selective participation due to ambiguity is also found when the smooth ambiguity aversion model is applied. Chen et al. (2014) use the smooth model to compare the investment patterns of an ambiguity-averse investor and a Bayesian investor. They find that an ambiguity-averse investor is less involved in the stock markets than a Bayesian investor under the same circumstances. This suggests that ambiguity aversion leads to selective participation. In addition, they also show that an ambiguity-averse investor hedges against ambiguity. Liu (2011), who uses a continuous-time smooth model to investigate the effect of ambiguity on portfolio choice in an intertemporal context, finds a significance impact of ambiguity on portfolio choice and an intertemporal hedging demand against ambiguity. However, under the multiplier model, the first-order effect and hedging demand do not seem obvious because the multiplier model is similar to the subjective expected utility model in terms of reasoning on Savage acts (Epstein and Schneider, 2010).

On the empirical side, Antoniou et al. (2015) adopt the ambiguity measure developed by Anderson et al. (2009) to investigate the impact of ambiguity aversion on market participation of the US stock market. The ambiguity measure is based on the extent of inconsistency in professional forecasts using data from the Survey of Professional Forecasts (SPF). Their results are consistent with the theoretical evidence that ambiguity aversion refrains investors from participating in the stock market.

2.6. Effect of Ambiguity under Asset Pricing

Epstein and Schneider (2010) derive a theoretical formula of equity premium based on the multiple-prior model. The formula can be written as:

$$\mu^* - log\left(\frac{P_i}{D_i}\right) - r^f + \frac{1}{2}\sigma^2 = \delta\sigma^2 + \mu^* - (\bar{\mu} - \bar{x}) \qquad 2.16$$

where μ^* represents the true mean of the stock return; P_i represents the price of stock i; D_i is the dividend of stock i; r^f represents the risk-free rate; δ is the proportion of wealth invested in stock i; $\bar{x}$ measures distortion from the reference model; $\bar{\mu}$ is the reference model of the mean return; and σ^2 is the volatility of the return.

Equation 2.16 shows that the equity premium is made up of a risk premium term $\delta\sigma^2$ and an ambiguity premium term $\mu^* - (\bar{\mu} - \bar{x})$, but it does not change with the number of stocks in the cross section. On the contrary, compensation on risk changes with number of stocks. The implication behind is that diversification can remove idiosyncratic risk, but it cannot remove ambiguity. On the other hand, an increase in the level of ambiguity measured by $\bar{\mu} - \bar{x}$ can increase the equity by an equal amount. As such, the existence of ambiguity explains the equity premium puzzle.

In addition, a one-unit increase in ambiguity needs to be offset by an increase of $\frac{1}{\delta}$ in volatility, which is larger than 1, and this explains the excess volatility puzzle.

Quantitative studies on asset pricing models under ambiguity are either based on the representative agent setting or the heterogenous agent setting. Based on the representative agent setting, Epstein and Wang (1994) use Euler inequalities to solve pricing equilibrium with the multiple-prior preference model. Chen and Epstein (2002) build a continuous-time multiple-prior model under the representative agent setting. Epstein and Schneider (2008) also use the representative agent setting to study the impact of ambiguity on asset pricing under learning. Results from the representative agent models generally agree that a sudden increase in ambiguity can make return decrease and overreaction to bad news is a result of level of ambiguous information. However, these models are criticised by Guidolin and Rinaldi (2013) because market participants are assumed to be homogenous, which do not conform to the reality. As such, more recent quantitative studies have been focusing on heterogeneous agent models of ambiguity aversion. Thus, some investors are allowed to be more ambiguous than the others in the models. Under the heterogeneous agent setting, ambiguity is found to have different impact on stock returns compared to the result obtained under the representative agent setting. Ambiguity may not necessarily make asset returns increase because more ambiguity-averse investors can simply quit the market while less ambiguity-averse investors remain in the market (Cao et al., 2005; Chapman and Polkovnichenko, 2009). As such, investors may not necessarily be compensated for bearing ambiguity and hence ambiguity may not necessarily affect the equity premium in the short run. However, in the long run, ambiguity may have an impact on equity returns (Condie, 2008). On the other hand, quantitative studies have also shown that ambiguity has an impact on financial crises and bank runs (Guidolin and Rinaldi, 2014; Uhlig, 2010). Recent studies also attempt to find the relationship between ambiguity and macroeconomic conditions, and there is evidence showing that ambiguity is associated with macroeconomic conditions (Jurado et al., 2015; Collard et al., 2018).

On the empirical side, Anderson et al. (2009) used Survey of Professional Forecasters (SPF) data to measure ambiguity. Their results suggest that ambiguity tends to affect equity premium instead of risks. The issue with their ambiguity measure is that SPF data are collected from professionals and the survey records the data only if the participant provides forecast during the investigated period of time. As such, their results can only represent part of the professionals in the US finance industry. Therefore, it is reasonable to presume that their results are based on the representative agent models. Viale et al. (2014) use a nonlinear forecasting model, which is the STAR model, to derive the reference model and make assumptions about investors' confidence level to calculate distortion from the reference model, which is used as the ambiguity measure in their study. They find that ambiguity has an impact on excess returns of stocks. However, one issue with the ambiguity measure based on a forecasting model is that it is difficult to find a forecasting model that is consistently used as the reference model by every investor. Since they use a single forecasting model as the reference model and the assumed confidence level applies to every investor, their results also seem to be based on the representative agent model.

2.7. Conclusion

So far, there are a large number of quantitative studies on ambiguity. However, as is evident from the review, the number of empirical studies remain few. In addition, the existing empirical studies mainly use either forecasting methods or the SPF data to measure ambiguity. Viale et al. (2014) use a smooth transition autoregressive (STAR) model to obtain reference probability on the future economic state. However, the immediate issue with this method would be that stock returns may not be well-forecasted by STAR models. In addition, their ambiguity measure is based on assumptions of investors' confidence levels, which is not purely based on real-life data. On the other hand, one issue with the ambiguity measure based on the SPF data is that the results might be biased because the data are collected from professionals. As such, this thesis first conducts a research on forecasting, which can uncover whether it is reasonable to use an STAR model as the reference model, followed by developing ambiguity measures that are purely based on financial data.

Chapter 3. Forecasting UK Stock Prices with High-Frequency Data

3.1. Introduction

Predictability of stock prices has been of interest to many financial practitioners and scholars. Prior to the wide application of computing techniques in forecasting, the literature mainly focuses on model development and forecasting monthly data, quarterly data and yearly data. Recently, researchers have been investigating predictability of stock volatilities using high-frequency data (Poon and Granger, 2003; Blair et al., 2001). However, there are few studies on mean forecasting using high-frequency data. As such, this chapter attempts to use high-frequency data, including minute data, hourly data and daily data, to investigate the mean return forecasting performances of linear autoregressive models, nonlinear autoregressive models and exponential smoothing models. The testing assets include two UK market indices, FTSE100 index and FTSE Small Cap index, two large capitalisation stocks listed on the London stock exchange, HSBA LN Equity and GLEN LN Equity, and two small capitalisation stocks, MCLS LN Equity and DIA LN Equity. The sample period is from 13 October 2015 to 26 April 2016. Two-thirds of the data are used as in-sample data to initialise the forecasting models, and the rest are used as out-or-sample data to evaluate the forecasting performances of the models. In-sample goodness-of-fit and out-of-sample forecasting performances are evaluated based on accuracy measures, including the root-mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). Diebold-Mariano (DM) test is also applied to test whether the differences of forecasting performances among the forecasting models are statistically significant.

The results suggest that nonlinear autoregressive models do not necessarily outperform the linear models, which are statistically significant as is shown by the MDM test results. In addition, weak significance is found, showing that the autoregressive (AR) model outperforms the random walk model for FTSE100 in daily frequency as well as for SMX in hourly frequency. Moreover, the AR model significantly outperforms the random walk model for FTSE100 using minute data. As such, the UK stock market seems not in a weak form of efficiency, at least with evidence from minute data.

Fama and Malkiel (1970) defined three forms of market efficiency, namely weak, semi-strong and strong form of efficiency. Under the strong form of market efficiency, all available information is fully reflected in stock prices and hence neither public nor private information can be used to earn anomalous returns. The strong form of market efficiency incorporates the semi-strong form and the weak form. Under the semi-strong form of efficiency, stock prices reflect all publicly available information, and the semi-strong form incorporates the weak form. Under the weak form of market efficiency, past information such as historical prices cannot be used to predict stock prices and hence technical analysis cannot generate anomalous returns for investors. The weak form of market efficiency suggests that stock prices move upwards and downwards randomly and hence they follow a random walk. As a result, future prices are independent of past prices, making it impossible for investors to use past prices to predict future prices or the trend of future price movements. Further to the EMH, Lo (2004) introduces a new view of market efficiency, which is known as the adaptive market hypothesis (AMH). Under the AMH, market efficiency can change over time. Thus, when arbitrage opportunities exist, investors exploit such opportunities so that prices return to the fundamental. In addition, investors also look for new opportunities to get extra profits, driving prices away from the fundamental. Therefore, efficiency and inefficiency interchangeably appear in the market by turns. Existing literature has shown evidence that stock markets are better characterised by the AMH using linear and nonlinear tests (Urquhart and Hudson, 2013; Smith, 2012). High-frequency data make it possible to use forecasting methods to test the AMH because the dataset is large. To test the AMH, the full sample is divided into 7 subsamples by month to uncover why increasing data frequency can improve forecasting accuracy.

The subsample results indicate that increasing sample size does not necessarily result in more accurate forecasts while increasing the continuity of data increases forecasting accuracy. This result applies to the full sample as well. In addition, the results also suggest that the exponential smoothing models and the AR based models do not tend to suffer from over-fitting problems caused by inactive data. The subsample results, together with the full-sample results imply that the adaptive market hypothesis (AMH) characterise the UK stock market better than the efficient market hypothesis (EMH).

This chapter contributes to the field of study in the following aspects: 1) it compares and contrasts linear autoregressive models, exponential smoothing models and nonlinear autoregressive models using high-frequency data; 2) it splits the full sample into subsamples to investigate why increasing data frequency helps to improve forecasting accuracy; and 3) it tests the AMH in the UK context with evidence from high-frequency forecasting, which is different from the methods applied in existing AMH literature.

The rest of the chapter is structured as follows. Section 3.2 provides a review of related literature. Section 3.3 describes the forecasting model used in this study; Section 3.4 shows the data and methodology. Section 3.5 explains the forecasting results including the full-sample results and sub-sample results. In the end, a short conclusion is provided in Section 3.6.

3.2. Related Literature

Existing literature on financial high-frequency forecasting mostly focuses on performances of volatility forecasting. Blair et al. (2001) use the generalised autoregressive conditional heteroskedasticity (GARCH) model, more precisely a $GARCH(1,1)$ model, to forecast five-minute stock returns. They find that using high-frequency data can produce more accurate forecasts than daily data. Taylor (2004) finds that the smooth transition exponential smoothing model is more accurate in forecasting stock volatilities. Findings of Bluhm and Yu (2001) suggest that the stochastic volatility forecasting model perform better than GARCH models. Although the results are mixed when it comes to which model provide the best forecasting performance, it seems agreed that using high-frequency data can improve forecasting accuracy.

Matias and Reboredo (2012) compare the forecasting performances of nonlinear models with the performances of the AR model and the random walk model using high-frequency data. Their data are in 5-minute frequency, 10-minute frequency, 20-minute frequency, 30-minute frequency and hourly frequency, and the nonlinear models investigated in their study include the Markov switching (MS) model, the smooth transition autoregressive (STAR) model, the STAR-GARCH model, the nonparametric kernel (KR) model, the artificial neural network (ANN) model and the support vector machine (SVM) method. The results indicate that the KR, ANN and SVM models provide better forecasts than the AR model and the random walk model. However, the STAR model and the STAR-GARCH do not seem to outperform the AR model. Their results provide a reference for the empirical results in this chapter. In terms of exponential smoothing models, Makridakis et al. (1984) and Mills (2009) confirm the predictability of the models. In addition, according to Mills (2009), exponential smoothing models could be applied to small samples and volatile data. Leung et al. (2000) suggest that probability-based forecasting models are better than level-based models such as the exponential smoothing model.

Other studies use explanatory variables to forecast excess returns of stocks. For instance, Ang and Bekaert (2007) use dividend yields to forecast excess returns. However, whether explanatory variables can improve the predictability of stock returns remain unclear. Welch and Goyal (2007) show that univariate forecasting models provide better forecasts than forecasting models using explanatory variables while findings of Campbell and Thompson (2008) suggest that explanatory models can outperform univariate models.

Overall, forecasting performances of different models are mixed and are still under debate. In addition, although there are a large number of research papers investigating high-frequency forecasting of stock volatilities, the number of similar studies to Matias and Reboredo (2012), which focus on forecasting mean returns, remains few, and yet such studies have important implications for ambiguity literature. This motivates the research in this chapter. On the other hand, how selection of sample can affect forecasting result remain unclear from existing literature. As such, the full sample is split into subsamples to shed light on this question.

3.3. Forecasting Methods

This section provides a detailed description of the forecasting models involved in this chapter.

3.3.1. Random Walk Model

Under the random walk model, the stock prices move up and down without a pattern and hence the model can be expressed by the following equation:

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t \qquad 3.1$$

where Y_t represents the realisation at time t; α is called the drift term; and ε_t is the error term with a zero mean. If α is non-zero, the model is said to be a random walk model with a drift.

Under the weak form of market efficiency, historical information is incorporated in the current stock price and hence historical prices cannot be used to predict future prices. Thus, the best prediction of today's price should be yesterday's price. As such, the random walk model should outperform other forecasting models if the market is in the weak form of efficiency. Thus, in this study, the random walk model is used as a benchmark model to testify the weak form of market efficiency.

3.3.2. Linear Autoregressive Models

Linear autoregressive models that are investigated in this chapter include the autoregressive (AR) mode and the autoregressive integrated moving average (ARIMA) model. The AR model is selected using Akaike (1969, 1970) information criterion (AIC), which indicates a better model if it has a smaller AIC. On the other hand, selection of the ARIMA model is based on the Box-Jenkins approach.

3.3.2.1. Autoregressive (AR) Model

The equation of an $AR(p)$ model is:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + \varepsilon_t \qquad 3.2$$

where Y_{t-p} represents the realisation at time $t - p$; β_0 is a constant; and ε_t is the error term.

The idea behind the AR model is that the future value of a time series is based on its historical values. As such, if a time series follows an AR process, a pattern can be observed over time.

3.3.2.2. Autoregressive Integrated Moving Average (ARIMA) Model

An $ARIMA(p, 0, q)$ can be expressed by the following equation:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} - \gamma_1 e_1 - \cdots - \gamma_q e_q + e_t \qquad 3.3$$

where Y_{t-p} represents the realisation at time $t - p$; e_q represents the regression error at time q; and α is a constant. If the non-stationary time-series become stationary when difference of order d is taken, the model is called an $ARIMA(p, d, q)$ model.

Makridakis et al. (1984) suggest that the Box-Jenkins forecasting method can be used to improve the forecasting performance of the ARIMA model. The approach involves several steps, first of which is to determine the number of differences to be taken in the data so that the time-series are stationary without seasonality. If the series are non-stationary, differences are taken to remove non-stationary patterns. On the other hand, if seasonal patterns present, the series are de-seasonalised. When the series are ready for analysis, ACF and PACF plots are used to specify the AR and MA terms of the ARIMA model. When model specification is done, parameter coefficients of the specified model is estimated. Then the model is checked for goodness-of-fit and the AR and MA terms of the model are adjusted until the model fits the data best.

3.3.3. Exponential Smoothing Forecasting Models

Exponential smoothing models put more weights on the more recent data and the weights decrease exponentially when the data get far from the most recent point of time. Thus, they average data exponentially. One type of the exponential smoothing model is the simple exponential smoothing model, also named as single exponential smoothing model, which can be expressed as:

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha\left(Y_t - \hat{Y}_t\right) \qquad 3.4$$

where $\hat{Y}_{t+1}$ represents the forecast at time $t + 1$; $\hat{Y}_t$ represents the forecast at time t; Y_t represents the observation at time t; and α is a constant between 0 and 1.

In Equation 3.4, $Y_t - \hat{Y}_t$ is the forecast error at time t and the forecast of the next period time $t + 1$ simply equals to the current forecast plus a proportion of the forecast error of the current period time t. Whether the forecast error is adjusted to a large extent purely depends on the estimate of α. As such, the forecasts will always follow the trend of the observations during the estimation period.

The second type of the exponential smoothing model is the Holt's linear exponential smoothing model, which can be characterised by the following equations:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + S_{t-1}) \quad 3.5$$

$$S_t = \beta(L_t - L_{t-1}) + (1-\beta)S_{t-1} \quad 3.6$$

$$\hat{Y}_{t+n} = L_t + S_t n \quad 3.7$$

where L_t is the estimated level of the data at time t; S_t is the estimated slope of the data at time t; Y_t represents the observation at time t; $\hat{Y}_{t+n}$ represents the forecast at time $t+n$; and α and β are constant between 0 and 1.

Equation 3.5 is an adjustment for the level or the mean of the data in the previous period. Then in Equation 3.6, the slope or the trend is updated according to the previous slope and the adjusted level. The n-period ahead forecast is calculated using Equation 3.7. The Brown's linear exponential smoothing model is similar to the Holt's model except that the Holt's model separates the level term and the trend term while the Brown's model uses one parameter for both. As such, the Holts model should generate more accurate forecasts than the Brown's model.

The damped-trend linear exponential smoothing model, which generates forecasts with a more conservative trend than the Holt's and Brown's linear exponential smoothing models, can be characterised by the following equations:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + \gamma S_{t-1}) \quad 3.8$$

$$S_t = \beta(L_t - L_{t-1}) + (1-\beta)\gamma S_{t-1} \quad 3.9$$

$$\hat{Y}_{t+1} = L_t + (\gamma + \gamma^2 + \cdots + \gamma^n)S_t n \quad 3.10$$

where L_t is the estimated level of the data at time t; S_t is the estimated slope of the data at time t; Y_t represents the observation at time t; $\hat{Y}_{t+1}$ represents the forecast of time $t+n$; and α, β and γ are constant between 0 and 1.

Similar to the Holt's model, Equation 3.8 is an adjustment for the level of the data in the previous period, and the trend is updated in Equation 3.9. Then the n-period ahead forecast is calculated using Equation 3.10. It is noticeable that the trend term becomes proportional to the trend term of the previous period with the introduction of γ and hence γ makes the forecasts of the damped-trend model less sensitive to the trend term than the Holt's and Brown's models. When γ equals to 1, the damped-trend linear exponential smoothing model becomes the Holt's linear exponential smoothing model. The best-fit exponential smoothing model is selected from the models listed above by the software, which is then used to generate forecasts.

3.3.4. Nonlinear Autoregressive Model

Nonlinear autoregressive models used in this study include the additive autoregressive model, the threshold autoregressive model (TAR) and the smooth transition autoregressive (STAR) model. Model selection of the nonlinear models are more complicated and hence it is directly conducted by software.

3.3.4.1. Additive Autoregressive Model

An additive autoregressive model can be characterised by the following equation:

$$\hat{Y}_t = \alpha + \sum_{i=1}^{p} f_i(Y_{t-i}) + \sum_{j=1}^{m} s_j(\boldsymbol{X_{tj}}) \qquad 3.11$$

where $\hat{Y}_t$ represents the forecast at time t; α is a constant; $\sum_{i=1}^{p} f_i(Y_{t-i})$ represents the AR terns of order p; and $\sum_{j=1}^{m} s_j(\boldsymbol{X_{tj}})$ represents the smoothers of covariates $\boldsymbol{X_{tj}} = (Y_{t-i_1}, Y_{t-i_2}, \dots, Y_{t-i_j})'$.

3.3.4.2. Threshold Autoregressive (TAR) Model

The TAR model is a special case of the regime-switching model where the value of dependent variable is based on different regimes. Since this study implements univariate forecasting, the self-exciting threshold autoregressive (SETAR) model is used. As the name suggests, the SETAR model is purely based on the historical values of a time series. An $SETAR(k, p)$ model is specified by k regimes and AR terns of order p. For instance, an $SETAR(2, p)$ model can be characterised by the following equation:

$$Y_t = \begin{cases} \alpha_0 + \alpha_1 Y_{t-1} + \cdots + \alpha_p Y_{t-p} + \varepsilon_t, \ if \ z_t < \text{c} \\ \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + \varepsilon_t{}', \ if \ z_t \geq \text{c} \end{cases} \quad 3.12$$

where Y_t represents the realisation at time t; z_t is the threshold variable; c is the threshold value; α_0 and β_0 are constants; and ε_t is the error term.

As Equation 3.12 suggests, a time series is assumed to be linear in each regime under the TAR model.

3.3.4.3. Smooth Transition Autoregressive (STAR) Model

Similar to the SETAR model, the STAR model can also be used to forecast univariate time series. As the name suggests, the transition from one regime to another is continuous and smooth. An $STAR(p)$ model can be characterised by the following equation:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \cdots + \alpha_p Y_{t-p} + \left(\beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p}\right) G(z_t, \gamma, c) + \varepsilon_t \quad 3.13$$

where Y_t represents the realisation at time t; $G(z_t, \gamma, c)$ is the transition function valued between 0 and 1; z_t is the threshold variable; γ is a speed and smoothness parameter of the transition function; c is the threshold value; α_0 is a constant; and ε_t is the error term.

If the transition function $G(z_t, \gamma, c)$ is specified as a logistic function, the model is called a logistic smooth transition autoregressive (LSTAR) model. On the other hand, if it is specified as an exponential function, the model is called an exponential smooth transition autoregressive (ESTAR) model.

3.4. Data and Methodology

3.4.1. Data

The sample period started from 13 October 2015 to 26 April 2016. The test assets include the FTSE100 index (UKX), the FTSE Small Cap index (SMX), two large-cap stocks and two small-cap stocks that are traded on the London stock exchange. Stocks of HSBC Holdings Plc. (HSBA) and Glencore Plc. (GLEN) are used as large-cap stocks and those of Dialight Plc. (DIA) and McColl's Retail Group Plc. (MCLS) are used as small-cap stocks. Daily, hourly and minute prices of the test assets are downloaded from Bloomberg. Table 3.1 shows the summary statistics of the data. The ADF test statistics indicate that all the series are non-stationary and hence first-order difference are taken, after which the series become stationary. As is suggested by Atchison et al. (1987), one issue with stock predictability arises from nonsynchronous trading, which causes measurement errors because tradings may happen between two measurement time points but may not be recorded at the point of measurement. This could lead to observed autocorrelations of portfolios and stock indices in a daily frequency. The autocorrelation statistics of FTSE100 daily returns in Table 3.1 seem to conform to such a phenomenon. However, nonsynchronous trading is found to explain little proportion of autocorrelations (Atchison et al., 1987) and hence it is not considered when doing forecasting for the daily index returns. Another issue of stock predictability is related to non-trading, which results in no price change, especially for small stocks. As is evident from Table 3.1, proportion of no price change increases with data frequency, and the increases of the two small stocks are much more significant than those of the indices and large stocks. This issue is more important in volatility forecasting literature and thus modification of the Student's t distribution is frequently used to handle inactive data in volatility forecasting (Meade, 2002). However, existing literature of return forecasting does not do distribution modification, suggesting that inactive data do not seem to affect return forecasting.

Table 3.1 Summary Statistics. This table shows the summary statistics of the test asset prices.

	Daily	Hourly	Minute
FTSE100			
Mean	0.000	0.000	0.000
Standard Deviation	0.012	0.004	0.000
Skewness	0.016	-0.659	1.739
Kurtosis	0.208	9.903	296.979
ADF Test Statistic	-1.963	-1.935	-1.865
Autocorrelation	2	0	1
% No Price Change	0	0	0.388%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		
FTSE SmallCap (SMX)			
Mean	0.000	0.000	0.000
Standard Deviation	0.006	0.002	0.000
Skewness	-0.818	0.719	7.701
Kurtosis	2.281	28.767	1,950.713
ADF Test Statistic	-1.549	-1.257	-0.533
Autocorrelation	0	1	5
% No Price Change	0	0.082%	2.906%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		
HSBC Holdings Plc. (HSBA)			
Mean	-0.001	0.000	0.000
Standard Deviation	0.018	0.006	0.001
Skewness	0.110	-0.412	-0.976
Kurtosis	1.404	9.992	203.160
ADF Test Statistic	-2.208	-2.160	-2.017
Autocorrelation	0	0	0
% No Price Change	0.740%	1.399%	14.266%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		
Glencore Plc. (GLEN)			
Mean	0.002	0.000	0.000
Standard Deviation	0.053	0.017	0.002
Skewness	0.112	0.220	1.710
Kurtosis	1.189	3.071	126.618
ADF Test Statistic	-1.757	-1.925	-1.872
Autocorrelation	0	0	4
% No Price Change	0.740%	0.741%	9.021%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		
Dialight Plc. (DIA)			
Mean	-0.001	0.000	0.000
Standard Deviation	0.034	0.014	0.002
Skewness	-0.643	-1.896	-19.611
Kurtosis	4.146	29.501	2,316.740
ADF Test Statistic	-2.288	-1.681	-1.928
Autocorrelation	1	1	0
% No Price Change	2.220%	37.531%	97.128%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		
McColl's Retail Group Plc. (MCLS)			
Mean	0.001	0.000	0.000
Standard Deviation	0.022	0.011	0.002
Skewness	-0.048	0.350	0.927
Kurtosis	0.623	9.499	509.652
ADF Test Statistic	-1.116	-1.433	-2.658
Autocorrelation	1	1	0
% No Price Change	2.960%	70.041%	99.274%
Observations	135	1,215	68,743
Sample Period	13/NOV/2015 – 26/APR/2016		

3.4.2. Methodology

In forecasting literature, samples are split into in-sample data and out-of-sample data. In-sample data are also known as the training set, which are used to find a best-fit model. Out-of-sample data, on the other hand, are known as the test set, which are used to evaluate the forecasting performance of selected models. Thus, the samples are split into two subsets, one of which is used to estimate the parameters of the forecasting models and the other is used to compare the forecasting performances of different models. In this study, the first two-thirds of the data are used as the training set and the rest are used as the test set, which is a widely-adopted practice in forecasting literature. As such, the in-sample daily data have 91 observations; the in-sample hourly data have 811 observations; and the in-sample minute data have 45,839 observations. Then the in-sample data are used to initialise the forecasting model. Once model selection is done, coefficients of the parameters are estimated to generate one-step ahead forecasts for the out-of-sample period. The forecasts are calculated without re-estimating the parameters and coefficients. Hence, a newly available observation is used to forecast the value of the next time point while the forecasting model remains the same as the one that is used to forecast the value of the current time point.

The forecasts are then compared to the observation of the out-of-sample period, and forecasting performances are evaluated by the root mean-squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE), which are calculated as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2} \quad 3.14$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i| \quad 3.15$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right| \qquad 3.16$$

where X_i represents the forecasted value of the series at time i; Y_i represents the observed value at time i; and hence $Y_i - X_i$ is the forecasting error at time i.

Equations 3.14, 3.15 and 3.16 show that the three accuracy measures are different ways of calculating forecasting errors and hence a smaller value indicates a better forecasting performance. Thus, the in-sample RMSE, MAE and MAPE reflect the goodness-of-fit of the forecasting models while the out-of-sample values indicate the real forecasting performance.

3.4.2.1. Diebold-Mariano Test

Although forecasting accuracy measures can be used to evaluate forecasting performances, the differences of the measures among different forecasting models might be small, which sometimes makes it difficult to determine whether one model indeed generates superior forecasts to other models. As such, a statistical test is favoured to provide some confidence level that the results are statistically robust even if different samples are used. Diebold and Mariano (1995) developed a statistical test that allows researchers to compare forecasting performances among different forecasting models. The Diebold-Mariano (DM) test statistic can be calculated as:

$$DM = \frac{\bar{d}}{\sqrt{\widehat{VAR(\bar{d})}}} \qquad 3.17$$

where $\bar{d}$ is the sample mean of the differences between the loss functions $g(e)$ of the forecasting errors that are calculated from two forecasting models; $\widehat{VAR(\bar{d})}$ is an asymptotic approximation of the variance of $\bar{d}$; and $g(e)$ needs to be specified, which can be a loss function of either power 1 or power 2 function. The choice of loss function depends on how the forecaster evaluates losses due to forecasting errors and hence test results of both power 1 and power 2 loss functions are presented in forecasting literature. The DM statistic is asymptotically standard normal and hence can be directly compared to standard-normal critical values to make statistical inferences, which is easy to implement.

However, Harvey et al. (1997) found that the original DM test might generate inaccurate results if the forecasts are biased. This issue becomes more severe if the sample size is small and it remains a problem even if the sample becomes large. As such, they recommended a modified DM (MDM) test. The MDM test statistic can be calculated as:

$$MDM = DM\sqrt{\frac{n+1-2h+h(h-1)/n}{n}} \qquad 3.18$$

where n is the number of forecasting errors used to calculate the MDM statistic; and h is the forecasting horizon, namely $h = 1$ if forecasting errors are generated from 1-step ahead forecasts.

Harvey et al. (1997) also recommended that the MDM test statistic should be compared with Student's t-statistic with $n - 1$ degrees of freedom to make statistical inferences. The result would be especially accurate for 1-step ahead forecasts. As such, in this paper, 1-step ahead forecasts will be used to investigate the predictability of the UK stock prices and forecasting performances of different models will be compared using the MDM test. The test statistics are compared to the Student's t-statistics.

On the other hand, the large sample size of high-frequency data makes it more likely to incorrectly reject the null hypothesis of Student's t-tests, which is known as the Lindley's paradox. Connolly (1989) recommended the following modified critical t-value to make statistical inferences for large samples:

$$t^* = \sqrt{(T-K)(T^{1/T}-1)} \quad 3.19$$

where T is the sample size; and $T-K$ is the degree of freedom.

This method is based on Bayesian statistical inference, which uses prior and posterior probabilities of the null hypothesis and the alternative hypothesis. In order to make the null and alternative hypotheses have equal prior probabilities, namely not favouring either of the hypotheses, the null hypothesis needs to be rejected only if the Student's t-statistic calculated from the sample is larger than the modified critical t-value t^* in Equation 3.19. As such, in this study, the null hypotheses of Student's t-tests will be rejected when the t-statistic is larger than the corresponding t^* in Equation 3.19.

3.4.2.2. Forecasting Horizon

As is mentioned above, 1-step ahead forecasts are used in this chapter. In addition, the full sample is split into 7 subsamples, which are organised by month. Thus, the full sample period is divided into 7 months. As such, forecasting models are re-estimated every month and the forecasting performances of each month are reported. This is different from rolling forecasting because forecasting performances are not evaluated for each rolling window. Comparing and contrasting forecasting performances among different subsamples provides a clearer picture of high-frequency forecasting. In particular, the results can uncover whether the forecasting performances using high-frequency data are sensitive to sample size and whether there is a forecasting model that can outperform other models in each period. These questions remain unclear from existing literature, which provides a motivation for using subsamples. On the other hand, it is also inspired by the AMH studies, which show that the efficiency of UK stock market is adaptive over time. The implication for forecasting is that the best model for forecasting market indices might change over time and hence the whole sample is divided into subsamples to investigate whether the findings and results obtained from the full sample are robust.

3.5. Empirical Results

Results of the daily data are presented in Table 3.2. It is noticeable that nonlinear AR models provide a better in-sample goodness-of-fit while the linear models provide a better out-of-sample performance and hence better forecasting performance. For FTSE100 and HSBA, the three accuracy measures indicate that the TAR model and the STAR model provide better in-sample performances than the other models. The AR model provides a better out-of-sample performance for FTSE100 while for HSBA, the three accuracy measures show different results. The RMSE measure indicates that the AR model has better forecasting performance, MAE the exponential smoothing model, and MAPE the Box-Jenkins approach. For SMX and DIA, the STAR model provides a better in-sample performance. The AR model generates better forecasts for SMX and the exponential smoothing model generates better forecasts for DIA. The RMSE measure indicates that the STAR model has better in-sample performance for GLEN and MCLS while the MAE and MAPE measures indicate that the TAR model has better in-sample performance. However, the Box-Jenkins approach provides better forecasting performance for GLEN and the AR model provides better forecasting performance for MCLS. Overall, the three linear models provide better forecasting performance than the nonlinear models.

Table 3.2 1-Step Ahead Forecasting Accuracy of Daily Data. This table shows the accuracy measures of the daily forecasts. RMSE represents the root mean-squared error accuracy measure, which is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; MAE represents the mean absolute error measure, which is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and MAPE represents the mean absolute percentage error measure, which is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample			**Out-of-Sample**				**In-Sample**			**Out-of-Sample**		
	RMSE	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**		**RMSE**	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**
FTSE100							**FTSE SmallCap (SMX)**						
AR	76.074	60.074	0.997%	53.660	40.552	0.659%	AR	29.892	21.430	0.483%	23.883	20.000	0.446%
ARIMA	76.814	60.985	1.011%	58.471	45.221	0.736%	ARIMA	28.758	20.969	0.475%	25.403	21.374	0.476%
ES	76.811	60.917	1.010%	58.470	45.221	0.736%	ES	29.620	21.417	0.485%	24.585	20.447	0.456%
AAR	74.960	58.924	0.976%	59.617	46.743	0.760%	AAR	28.087	21.141	0.478%	28.630	23.266	0.518%
TAR	73.010	55.721	0.922%	60.283	47.650	0.774%	TAR	26.824	20.095	0.453%	31.113	25.826	0.576%
STAR	73.010	55.721	0.922%	60.283	47.650	0.774%	STAR	26.823	20.071	0.452%	31.106	25.820	0.575%
HSBC Holdings Plc. (HSBA)							**Glencore Plc. (GLEN)**						
AR	8.524	6.405	1.305%	8.126	5.792	1.291%	AR	4.704	3.699	3.970%	8.363	6.414	4.408%
ARIMA	8.449	6.457	1.315%	8.198	5.706	1.261%	ARIMA	4.728	3.756	4.006%	8.351	6.379	4.400%
ES	8.433	6.466	1.316%	8.193	5.678	1.263%	ES	4.726	3.755	4.004%	8.362	6.383	4.398%
AAR	8.237	6.273	1.273%	8.279	6.403	1.439%	AAR	4.688	3.725	3.990%	8.842	6.847	4.674%
TAR	8.085	6.183	1.253%	9.942	7.780	1.751%	TAR	4.548	3.571	3.844%	13.958	12.466	8.252%
STAR	8.085	6.183	1.253%	9.942	7.780	1.751%	STAR	4.519	3.627	3.859%	8.841	6.849	4.675%
Dialight Plc. (DIA)							**McColl's Retail Group Plc. (MCLS)**						
AR	17.415	11.811	2.446%	16.098	12.439	2.362%	AR	2.556	1.910	1.377%	3.306	2.550	1.599%
ARIMA	17.709	11.944	2.450%	16.245	11.972	2.277%	ARIMA	2.535	1.936	1.395%	3.347	2.628	1.661%
ES	17.687	12.016	2.461%	16.037	11.789	2.242%	ES	2.599	1.953	1.407%	3.482	2.781	1.769%
AAR	17.243	11.653	2.395%	17.111	13.225	2.489%	AAR	2.467	1.895	1.365%	3.498	2.801	1.776%
TAR	16.991	11.524	2.371%	17.273	13.151	2.478%	TAR	2.400	1.867	1.347%	16.305	13.849	8.454%
STAR	16.961	11.417	2.358%	17.240	13.339	2.508%	STAR	2.397	1.870	1.349%	16.974	14.467	8.834%

Table 3.3 shows the empirical results of the hourly data. The RMSE measure indicates that the TAR and STAR models fit the in-sample data better for FTSE100 while the AR model provides a better forecasting performance. On the other hand, MAE and MAPE suggest that the additive AR model has better goodness-of-fit of the in-sample data while the AR model provides a better forecasting performance. For SMX, the STAR model seems to provide a better in-sample performance as is indicated by the RMSE measure while the MAE and MAPE measures indicate that the exponential smoothing model seem to fit the in-sample data better. However, the three measures consistently indicate that the AR model has a better forecasting performance. For HSBA, RMSE indicates that the TAR and STAR models provide better in-sample performances while the AR model provides a better forecasting performance. On the other hand, MAE and MAPE suggest that the Box-Jenkins approach has a better in-sample performance while the exponential smoothing model provides a better forecasting performance. In terms of GLEN, the three measure consistently indicate that the TAR model has a better in-sample performance while the Box-Jenkins approach provides a better forecasting performance. For DIA, RMSE indicates that the Box-Jenkins approach provides a better in-sample performance while the AR model has a better forecasting performance. However, MAE and MAPE suggest that the additive AR model has a better in-sample performance while the STAR model has a better forecasting performance. For MCLS, RMSE shows that the additive AR model has a better in-sample performance while the AR model has a better forecasting performance. On the other hand, the MAE and MAPE measures indicate that the exponential smoothing model provides both better in-sample goodness-of-fit and better forecasting performance. In general, both linear and nonlinear models can fit the in-sample data well while the linear models seem to have better forecasting performance except for DIA.

Table 3.3 1-Step Ahead Forecasting Accuracy of Hourly Data. This table shows the accuracy measures of the hourly forecasts. RMSE represents the root mean-squared error accuracy measure, which is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; MAE represents the mean absolute error measure, which is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and MAPE represents the mean absolute percentage error measure, which is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model. Some numbers are shown as the same but actually have different values due to rounding.

	In-Sample			**Out-of-Sample**				**In-Sample**			**Out-of-Sample**		
	RMSE	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**		**RMSE**	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**
FTSE100							**FTSE SmallCap (SMX)**						
AR	24.411	15.886	0.263%	19.728	13.633	0.221%	AR	7.875	4.501	0.102%	6.177	3.960	0.088%
ARIMA	24.432	15.876	0.263%	20.134	13.796	0.224%	ARIMA	7.783	4.424	0.100%	6.217	4.018	0.089%
ES	24.431	15.868	0.263%	20.134	13.796	0.224%	ES	7.781	4.415	0.100%	6.216	4.018	0.089%
AAR	24.373	15.859	0.262%	20.185	13.916	0.226%	AAR	7.646	4.470	0.101%	6.707	4.446	0.099%
TAR	24.288	15.863	0.263%	20.220	13.968	0.227%	TAR	7.789	4.468	0.101%	6.552	4.353	0.097%
STAR	24.288	15.863	0.263%	20.220	13.968	0.227%	STAR	7.711	4.451	0.101%	6.337	4.078	0.091%
HSBC Holdings Plc. (HSBA)							**Glencore Plc. (GLEN)**						
AR	2.768	1.890	0.383%	2.851	1.754	0.391%	AR	1.712	1.216	1.289%	2.313	1.612	1.110%
ARIMA	2.765	1.877	0.380%	2.854	1.745	0.390%	ARIMA	1.713	1.219	1.292%	2.311	1.603	1.105%
ES	2.764	1.877	0.380%	2.855	1.743	0.389%	ES	1.711	1.217	1.290%	2.319	1.609	1.109%
AAR	2.759	1.883	0.381%	2.852	1.779	0.398%	AAR	1.702	1.206	1.281%	3.806	3.222	2.148%
TAR	2.752	1.887	0.382%	2.849	1.786	0.400%	TAR	1.699	1.201	1.276%	3.494	2.919	1.953%
STAR	2.752	1.887	0.382%	2.849	1.786	0.400%	STAR	1.699	1.201	1.276%	3.494	2.919	1.953%
Dialight Plc. (DIA)							**McColl's Retail Group Plc. (MCLS)**						
AR	6.872	3.739	0.772%	7.227	4.574	0.856%	AR	1.501	0.834	0.596%	1.547	0.863	0.546%
ARIMA	6.841	3.805	0.785%	7.281	4.730	0.886%	ARIMA	1.505	0.822	0.587%	1.589	0.920	0.583%
ES	6.850	3.758	0.775%	7.253	4.614	0.864%	ES	1.542	0.701	0.501%	1.559	0.787	0.499%
AAR	6.960	3.699	0.761%	7.388	4.569	0.852%	AAR	1.490	0.798	0.570%	5.590	4.641	2.830%
TAR	6.939	3.776	0.780%	7.382	4.563	0.853%	TAR	1.515	0.786	0.561%	1.582	0.908	0.576%
STAR	6.942	3.759	0.776%	7.375	4.542	0.849%	STAR	1.515	0.786	0.561%	1.582	0.909	0.577%

Empirical results of the minute data are shown in Table 3.4. For FTSE100, SMX and DIA, the RMSE measure indicates that the AR model provides better in-sample goodness-of-fit than other models while for HSBA, GLEN and MCLS, it favours nonlinear models with HSBA the TAR model and GLEN and MCLS, the additive AR model. On the other hand, the MAE and MAPE measures suggest that the AR model has better in-sample performance than other models for FTSE100 while the exponential smoothing model provides better in-sample performance for the rest testing assets. For out-of-sample performance, the RMSE measure indicates that the AR model outperforms other models for all testing assets. The MAE and MAPE measures suggest that the AR model provides better forecasting performance for FTSE100 and SMX, Box-Jenkins approach for HSBA and exponential smoothing model for GLEN, DIA and MCLS. Overall the findings seem to suggest that the linear models outperform the nonlinear models in terms of both in-sample performance and out-of-sample performance although the RMSE measure still favours nonlinear models.

Table 3.4 1-Step Ahead Forecasting Accuracy of Minute Data. This table shows the accuracy measures of the minute forecasts. RMSE represents the root mean-squared error accuracy measure, which is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; MAE represents the mean absolute error measure, which is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and MAPE represents the mean absolute percentage error measure, which is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model. Some numbers are shown as the same but actually have different values due to rounding.

	In-Sample			**Out-of-Sample**				**In-Sample**			**Out-of-Sample**		
	RMSE	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**		**RMSE**	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**
FTSE100							**FTSE SmallCap (SMX)**						
AR	2.9219	1.6357	0.02701%	2.5515	1.4161	0.02298%	AR	0.6738	0.2686	0.00601%	0.6564	0.2884	0.00640%
ARIMA	2.9224	1.6365	0.02703%	2.5548	1.4172	0.02300%	ARIMA	0.6757	0.2676	0.00599%	0.6570	0.2888	0.00641%
ES	2.9254	1.6381	0.02705%	2.5567	1.4185	0.02302%	ES	0.6754	0.2674	0.00599%	0.6569	0.2886	0.00640%
AAR	2.9240	1.6362	0.02702%	2.5549	1.4174	0.02300%	AAR	0.6756	0.2686	0.00601%	0.6570	0.2900	0.00643%
TAR	2.9238	1.6363	0.02702%	2.5555	1.4178	0.02301%	TAR	0.6762	0.2686	0.00601%	0.6573	0.2895	0.00642%
STAR	2.9236	1.6367	0.02703%	2.5555	1.4177	0.02300%	STAR	0.6762	0.2686	0.00601%	0.6573	0.2895	0.00642%
HSBC Holdings Plc. (HSBA)							**Glencore Plc. (GLEN)**						
AR	0.35987	0.2265	0.04571%	0.3641	0.2150	0.04816%	AR	0.2422	0.5162	0.16449%	0.3200	0.1894	0.12983%
ARIMA	0.35987	0.2261	0.04563%	0.3642	0.2146	0.04808%	ARIMA	0.2424	0.5162	0.16446%	0.3203	0.1894	0.12989%
ES	0.35983	0.2261	0.04563%	0.3642	0.2146	0.04808%	ES	0.2426	0.5160	0.16427%	0.3203	0.1891	0.12968%
AAR	0.35984	0.2265	0.04570%	0.3642	0.2150	0.04817%	AAR	0.2418	0.5162	0.16445%	0.3289	0.2040	0.13928%
TAR	0.35983	0.2266	0.04573%	0.3642	0.2151	0.04818%	TAR	0.2420	0.5162	0.16450%	0.3280	0.2028	0.13853%
STAR	0.35983	0.2266	0.04573%	0.3642	0.2151	0.04818%	STAR	0.2420	0.5162	0.16450%	0.3279	0.2027	0.13847%
Dialight Plc. (DIA)							**McColl's Retail Group Plc. (MCLS)**						
AR	1.1119	0.1318	0.02720%	1.1508	0.1557	0.02923%	AR	0.21924	0.01783	0.01274%	0.2452	0.02413	0.01521%
ARIMA	1.1141	0.1300	0.02687%	1.1523	0.1529	0.02867%	ARIMA	0.21929	0.01383	0.00988%	0.2457	0.01841	0.01162%
ES	1.1155	0.1127	0.02327%	1.1509	0.1320	0.02475%	ES	0.21929	0.01382	0.00988%	0.2457	0.01840	0.01162%
AAR	1.1128	0.1295	0.02674%	1.1539	0.1484	0.02782%	AAR	0.21917	0.01882	0.01345%	0.3116	0.17222	0.10482%
TAR	1.1140	0.1260	0.02587%	1.1523	0.1422	0.02667%	TAR	0.21921	0.01845	0.01321%	0.2486	0.05038	0.03114%
STAR	1.1140	0.1260	0.02586%	1.1523	0.1421	0.02665%	STAR	0.21920	0.01873	0.01340%	0.2570	0.08082	0.04957%

The MAPE measure makes it possible to compare the results of difference frequencies, which decreases both in sample and out of sample with an increase in data frequency. On the other hand, the linear models tend to provide both better in-sample performance and better out-of-sample performance than the nonlinear models when data frequency increases. This can be attributed to the continuity of data points. Changes in data or price movements are depicted in a more detailed and continuous way, which makes it easier for the AR based models and exponential smoothing models to capture the changes or movements. However, this does not necessarily mean that increasing sample size can improve forecasting performance and make linear models outperform nonlinear models. Nevertheless, whether the differences in forecasting performance are statistically significant or not needs to be verified by the modified Diebold-Mariano (MDM) test, which is explained in the following subsection.

3.5.1. Diebold-Mariano Test Results

Table 3.5 shows the out-of-sample MDM test statistics of daily data. Panel A presents the test statistics calculated from the power 1 loss function. The results suggest that the AR model provides superior forecasting performance than other linear models for FTSE100 at the 10% significance level. It outperforms the nonlinear models at the 5% significance level. However, for FTSE100, the differences in forecasting performance between other linear models and the nonlinear models are not statistically significant at the 10% significance level. It is also noticeable that the AR model outperforms the random walk model at the 10% significance level, which indicates that the UK stock market is not in a weak form of efficiency at the 10% significance level. For SMX, the differences in forecasting performances among the linear models are not significant at the 10% significance level. However, the forecasting performances of the nonlinear models are worse than the linear models at the 5% significance level. For HSBA, the differences in forecasting performances are insignificant among the linear models and the additive AR model at the 10% significance level. However, the TAR and STAR models have poorer forecasting performance than the other models at the 5% significance level. For GLEN, the linear models and nonlinear models provide similar out-of-sample accuracies expect for the TAR model, which is outperformed by other models at the 1% significance level. For DIA, the linear models outperform the nonlinear models at the 10% significance level. The exponential smoothing model seems to provide a better forecasting performance than the other linear models. However, the difference is only significant in comparison with the AR model at 10% significance level. For MCLS, the linear models are similar in terms of out-of-sample performance at the 10% significance level. In addition, the linear models still outperform the nonlinear models except for the additive AR model. Results in Panel B, where the test statistics calculated from a power 2 loss function are presented, are similar to those in Panel A. As such, it can be concluded from the MDM tests of daily data that the linear models outperform the nonlinear models.

Table 3.5 Out-of-Sample Modified Diebold-Mariano (MDM) Test Statistics of Daily Data. This table shows the MDM test statistics of daily data based on power 1 loss function (upper triangle) and power 2 loss function (lower triangle). Models include random walk model (RW), autoregressive model (AR), autoregressive integrated moving average model (ARIMA), exponential smoothing model (ES), nonlinear autoregressive model (NAR), additive autoregressive model (AAR), threshold autoregressive model (TAR), and smooth transition autoregressive model (STAR). A positive (negative) test statistic means the column model provides better (worse) forecasting performance than the row model. - means the models in comparison are the same. Significance level: *** 1%, ** 5%, 10%.

FTSE100	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	1.859*	-	0.434	-0.894	-0.975	-0.975
AR	-1.738*	-	-1.859*	-1.859*	-2.452**	-2.279**	-2.279**
ARIMA	-	1.738*	-	0.434	-0.894	-0.975	-0.975
ES	-0.791	1.738*	-0.791	-	-0.894	-0.976	-0.976
AAR	0.741	2.172**	0.741	0.741	-	-0.655	-0.655
TAR	0.550	1.948*	0.550	0.550	0.297	-	1.60
STAR	0.550	1.948*	0.550	0.550	0.297	-1.295	-
SMX	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	0.683	-0.927	-0.339	-2.591**	-3.061***	-3.053***
AR	-0.936	-	-1.878*	-0.683	-2.316**	-3.097***	-3.093***
ARIMA	0.815	1.918*	-	0.926	-1.209	-2.154**	-2.149**
ES	0.451	0.936	-0.814	-	-2.591**	-3.061***	-3.053***
AAR	3.113***	2.660**	1.852*	3.113***	-	-2.013*	-1.999*
TAR	3.009***	3.120***	2.399**	3.009***	1.683*	-	0.728
STAR	3.001***	3.116***	2.393**	3.001***	1.669	-0.714	-
HSBA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-0.501	-	0.345	-1.646	-2.670**	-2.670**
AR	-0.375	-	0.501	0.623	-1.513	-2.524**	-2.524**
ARIMA	-	0.375	-	0.345	-1.646	-2.670**	-2.670**
ES	-0.064	0.298	-0.064	-	-1.714*	-2.713***	-2.713***
AAR	0.128	0.308	0.128	0.137	-	-3.223***	-3.223***
TAR	1.727*	1.975*	1.727*	1.732*	3.128***	-	-0.831
STAR	1.727*	1.975*	1.727*	1.732*	3.128***	1.553	-
GLEN	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-0.212	-	-0.121	-1.119	-5.621***	-1.120
AR	0.058	-	0.212	0.187	-1.665	-6.164***	-1.662
ARIMA	-	-0.058	-	-0.121	-1.119	-5.621***	-1.120
ES	0.291	-0.002	0.291	-	-1.120	-5.677***	-1.121
AAR	1.048	6.164***	1.048	1.088	-	-6.759***	-0.581
TAR	3.992***	4.402***	3.992***	4.048***	4.834***	-	6.780***
STAR	1.036	1.677	1.036	1.075	-0.292	-4.851***	-
DIA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-1.197	-	1.563	-1.844*	-1.899*	-1.868*
AR	-0.405	-	1.197	1.696*	-1.782*	-1.547	-1.839*
ARIMA	-	0.405	-	1.563	-1.844*	-1.899*	-1.868*
ES	-2.077**	-0.171	-2.077**	-	-2.166**	-2.293**	-2.169**
AAR	1.449	2.355**	1.449	1.805*	-	0.274	-2.137**
TAR	1.367	1.558	1.367	1.723*	0.320	-	-0.638
STAR	1.553	2.436**	1.553	1.885*	2.884***	-0.066	-
MCLS	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	0.962	0.401	-0.229	-0.258	-8.767***	-8.983***
AR	-0.421	-	-0.599	-1.624	-1.289	-9.112***	-9.339***
ARIMA	-0.142	0.336	-	-1.374	-1.858*	-8.765***	-8.989***
ES	0.407	1.222	1.278	-	-0.150	-8.391***	-8.602***
AAR	0.420	1.090	1.879*	0.144	-	-8.391***	-8.613***
TAR	7.607***	7.658***	7.578***	7.496***	7.494***	-	-4.421***
STAR	7.688***	7.733***	7.659***	7.580***	7.581***	7.003***	-

The out-of-sample MDM test results of hourly data are presented in Table 3.6. Results in Panel A show that nonlinear models expect the additive AR model have worse forecasting performances than the linear models for FTSE100 at the 10% significance level. However, the differences become insignificant when the critical value is adjusted using the Bayesian method of Connolly (1989). As such, all the models seem to provide similar forecasting accuracies for FTSE100. For SMX, the AR model seems to outperform the other models at the 5% significance level, including the random walk model. However, the results become insignificant for the linear models when the Bayesian critical value is applied. Nevertheless, the nonlinear models are still outperformed by the AR model. The additive AR model and the TAR model provide poorer forecasting performances than the other models at the 1% significance level, and the result is robust to the adjustment of Bayesian statistical inference. For HSBA, the additive AR, TAR and STAR models have poorer forecasting performances than the other models at the 5% significance level. However, with adjustment to Bayesian critical value, the differences in forecasting performances are only significant when the three models are compared to the exponential smoothing model. For GLEN, the linear models have similar forecasting performance while the nonlinear models are outperformed by the linear models at the 1% significance level, which is still significant when the Bayesian critical value is applied. For DIA, the random walk model outperforms the other models at the 1% significance level, which is robust to the Bayesian adjustment. However, it seems that there is no difference in forecasting performances between other linear models and nonlinear models. For MCLS, the random walk model has better forecasting performance than the other models at the 1% significance level and the differences are still statistically significant when the Bayesian critical value is applied. Among the other linear models, the exponential smoothing model performs best, followed by the AR model and the Box-

Jenkins approach, both at the 1% significance level and with evidence from Bayesian inference. The TAR and STAR models have similar forecasting performances to the Box-Jenkins approach while the other two nonlinear models are outperformed by the linear models at the 1% significance level, which is robust to the Bayesian adjustment. In Panel B, where a power 2 loss function is applied, the results are slightly different from those in Panel A. The AR model outperforms the random walk model for SMX even if the Bayesian critical value is applied. The differences in forecasting performances among the linear and nonlinear models are no longer statistically significant for HSBA. For DIA and MCLS, the random walk model no longer outperforms the other models. Instead, it seems outperformed by the AR model at the 5% significance level and 1% level respectively. However, the differences are not significance when the Bayesian critical value is applied. Overall, the findings seem to suggest that the linear models provide better forecasting performances than the nonlinear models if the forecaster cares less about losses and hence uses a power 1 loss function. However, if he cares more about losses due to forecasting errors and penalises the losses using a power 2 function, the nonlinear models do not necessarily provide worse performances than the linear models. In addition, forecasters can use an AR model to estimate the mean price movement since it seems to outperform the other models when a power 2 loss function is applied.

Table 3.6 Out-of-Sample Modified Diebold-Mariano (MDM) Test Statistics of Hourly Data. This table shows the MDM test statistics of houtly data based on power 1 loss function (upper triangle) and power 2 loss function (lower triangle). Models include random walk model (RW), autoregressive model (AR), autoregressive integrated moving average model (ARIMA), exponential smoothing model (ES), nonlinear autoregressive model (NAR), additive autoregressive model (AAR), threshold autoregressive model (TAR), and smooth transition autoregressive model (STAR). A positive (negative) test statistic means the column model provides better (worse) forecasting performance than the row model. - means the models in comparison are the same. Sgnificaince level: *** 1%, ** 5%, 10%. § means significant when compared to the modified critical value of Connolly (1989).

FTSE100	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	0.777	-	-0.228	-1.958*	-2.031**	-2.031**
AR	-1.443	-	-0.777	-0.777	-1.327	-1.520	-1.520
ARIMA	-	1.443	-	-0.228	-1.958*	-2.031**	-2.031**
ES	0.091	1.443	0.091	-	-1.958*	-2.031**	-2.031**
AAR	0.856	1.625	0.856	0.856	-	-1.198	-1.198
TAR	0.811	1.790*	0.811	0.811	0.513	-	2.116**
STAR	0.811	1.790*	0.811	0.811	0.513	-0.675	-
SMX	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	2.301**	0.358	0.375	-4.834***§	-5.498***§	-1.351
AR	-2.706***§	-	-2.435**	-2.404**	-5.362***§	-5.444***§	-2.509**§
ARIMA	-0.783	1.784*	-	1.839*	-4.445***§	-4.294***§	-1.020
ES	-0.796	1.769*	-0.997	-	-4.451***§	-4.298***§	-1.032
AAR	4.279***§	4.784***§	4.235***§	4.236***§	-	1.116	4.615***§
TAR	4.471***§	4.599***§	4.076***§	4.077***§	-1.522	-	4.833***§
STAR	2.176**	3.129***§	2.070**	2.082**	-3.719***§	-2.968***§	-
HSBA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-1.280	-	0.815	-2.296**	-2.306**	2.306**
AR	-0.474	-	1.280	1.541	-2.165**	-2.342**	-2.342**
ARIMA	-	0.474	-	0.815	-2.296**	-2.306**	-2.306**
ES	0.410	0.545	0.410	-	-2.473**§	-2.464**§	-2.464**§
AAR	-0.133	0.093	-0.133	-0.225	-	-1.219	-1.219
TAR	-0.304	-0.162	-0.304	-0.370	-0.603	-	-2.820***§
STAR	-0.304	-0.162	-0.304	-0.370	-0.603	0.988	-
GLEN	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-1.583	-	-1.195	-15.57***§	-13.96***§	-13.96***§
AR	0.326	-	1.583	0.381	-16.04***§	-14.45***§	-14.45***§
ARIMA	-	-0.326	-	-1.195	-15.57***§	-13.96***§	-13.96**§
ES	1.140	0.689	1.140	-	-15.51***§	-13.90***§	-13.90***§
AAR	12.40***§	12.79***§	12.40***§	12.40***§	-	21.09***§	21.09***§
TAR	11.04***§	11.48***§	11.04***§	11.04***§	-17.39***§	-	22.75***§
STAR	11.04***§	11.48***§	11.04***§	11.04***§	-17.39***§	-18.77***§	-
DIA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-4.347***§	-5.243***§	-3.918***§	-7.659***§	-8.063***§	-7.764***§
AR	-2.284**	-	-5.012***§	-1.373	0.105	0.225	0.663
ARIMA	-0.999	1.634	-	6.947***§	2.335**	2.423**	2.730***§
ES	-1.424	0.873	-1.540	-	0.667	0.756	1.070
AAR	0.613	2.555**§	1.232	1.560	-	0.361	1.877*
TAR	0.473	2.430**	1.157	1.501	-0.404	-	6.801***§
STAR	0.262	2.354**	1.084	1.433	-0.911	-2.263**	-
MCLS	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-6.702***§	-8.069***§	-3.597***§	-24.43***§	-6.248***§	-6.283***§
AR	-1.955*	-	-3.968***§	6.029***§	-24.43***§	-2.422**	-2.471**§
ARIMA	-0.535	1.832*	-	7.265***§	-23.95***§	0.535	0.492
ES	-2.024**	0.798	-1.098	-	-24.38***§	-7.705***§	-7.757***§
AAR	20.03***§	20.50***§	20.36***§	20.27***§	-	23.82***§	23.80***§
TAR	-0.548	1.350	-0.196	0.868	-20.33***§	-	-1.334
STAR	-0.543	1.359	-0.190	0.877	-20.33***§	1.288	-

The out-of-sample MDM test statistics of minute data are presented in Table 3.7. Similarly, in Panel A, the results are based on a power 1 loss function. For FTSE100, the random walk model seems outperformed by the other models except the exponential smoothing model at the 5% significance level. However, the difference is only significant between the AR model and the random walk model when the Bayesian critical value is applied. The nonlinear models perform worse than the AR model and the Box-Jenkins approach and are better than or equal to the exponential smoothing model. However, the differences are not significant with Bayesian statistical inference. For SMX, the linear models outperform the nonlinear models both at the 1% significance level and with Bayesian inference. The linear models seem to have similar forecasting performances. For HSBA and MCLS, the random walk model outperforms the other models both at the 1% significance level and with Bayesian inference. The AR model generates worse forecasts than the other linear models both at the 1% significance level and with Bayesian inference. On the other hand, the nonlinear models provide worse forecasts than the linear models both at the 1% significance level and with Bayesian inference. However, the additive AR, TAR and STAR models seem to have similar performances to the AR model. For GLEN, the random walk model has better forecasting performance than the other models both at the 1% significance level and with Bayesian inference. In addition, the linear models significantly outperform the nonlinear models. For DIA, the random walk has better forecasting performances than the other models both at the 1% significance level and with Bayesian inference. The exponential smoothing model performs better than the AR model and the Box-Jenkins approach as well as the nonlinear models both at the 1% significance level and with Bayesian inference. However, the AR model and the Box-Jenkins approach are outperformed by the nonlinear models both at the 1% significance level and with Bayesian inference. In

Panel B, where a power 2 loss function is used, the statistics have slightly different indications from the results of a power 1 function. For FTSE100, the random walk model is outperformed by all the models except the exponential smoothing model both at the 1% significance level and with Bayesian inference. For SMX, the AR model seems to provide better performance than the other models including the random walk model at the 5% significance level, but the differences are not significant when the Bayesian critical value is applied. For HSBA, the differences in forecasting performances are not statistically significant among the forecasting models. For GLEN and MCLS, the AR model seems better than or equal to the other models. However, the differences are not significant when the Bayesian critical value is applied. Overall, the findings are similar to those of hourly data. Choices of the forecasting models partly depend on interpretation of losses due to forecasting errors. When a power 1 loss function is used, the nonlinear models are outperformed by at least one of the linear models while this is not the case when a power 2 function is applied. On the other hand, the AR model seems to consistently provide better performance than the other models including the random walk model when a power 2 loss function is used.

Table 3.7 Out-of-Sample Modified Diebold-Mariano (MDM) Test Statistics of Minute Data. This table shows the MDM test statistics of minute data based on power 1 loss function (upper triangle) and power 2 loss function (lower triangle). Models include random walk model (RW), autoregressive model (AR), autoregressive integrated moving average model (ARIMA), exponential smoothing model (ES), nonlinear autoregressive model (NAR), additive autoregressive model (AAR), threshold autoregressive model (TAR), and smooth transition autoregressive model (STAR). A positive (negative) test statistic means the column model provides better (worse) forecasting performance than the row model. - means the models in comparison are the same. Sgnificaince level: *** 1%, ** 5%, 10%. § means significant when compared to the modified critical value of Connolly (1989).

FTSE100	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	3.229***§	2.550**	0.794	2.163**	2.064**	2.057**
AR	-4.176***§	-	-1.813*	-3.253***§	-2.195**	-2.653***	-2.549**
ARIMA	-3.340***§	2.828***	-	-2.408**	-1.924*	-2.603***	-2.442**
ES	-2.199**	3.965***§	2.118**	-	1.912*	1.489	1.567
AAR	-3.258***§	2.911***	0.660	-2.025**	-	-2.025**	-1.833*
TAR	-3.361***§	3.331***§	2.235**	-1.646	2.271**	-	0.845
STAR	-3.259***§	3.252***§	2.370**	-1.649	2.418**	-0.213	-
SMX	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	0.652	-0.923	-0.291	-5.655***§	-5.280***§	-5.374***§
AR	-2.037**	-	-1.367	-0.842	-5.310***§	-4.410***§	-4.465***§
ARIMA	-1.242	1.883**	-	2.655***	-3.667***§	-2.578***	-2.621***
ES	-1.344	1.664**	-1.431	-	-4.167***§	-3.137***	-3.181***§
AAR	-0.520	1.033	-0.036	0.184	-	2.776***	2.706***
TAR	-0.325	2.601***	0.555	0.963	0.695	-	-0.874
STAR	-0.305	2.659***	0.591	1.005	0.721	1.316	-
HSBA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-6.199***§	-	-6.867***§	-11.86***§	-11.88***§	-11.88***§
AR	-1.402	-	6.199***§	6.147***§	-0.137	-0.834	-0.823
ARIMA	-	1.402	-	-6.867***§	-11.86***§	-11.88***§	-11.88***§
ES	-1.009	1.393	-1.009	-	-11.76***§	-11.79***§	-11.79***§
AAR	-0.218	1.323	-0.218	-0.187	-	-4.326***§	-4.266***§
TAR	-0.460	1.230	-0.460	-0.434	-0.832	-	11.90***§
STAR	-0.462	1.230	-0.462	-0.436	-0.838	-0.737	-
GLEN	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-8.563***§	-6.728***§	-5.988***§	-31.86***§	-30.58***§	-30.48***§
AR	-1.460	-	-1.502	3.815***§	-32.02***§	-30.82***§	-30.72***§
ARIMA	-1.079	3.045***	-	4.520***§	-30.67***§	-29.44***§	-29.33***§
ES	2.490**	2.523**	0.147	-	-31.28***§	-29.91***§	-29.81***§
AAR	15.99***§	16.68***§	15.07***§	15.10***§	-	32.05***§	33.16***§
TAR	14.70***§	15.53***§	13.91***§	13.74***§	-15.55***§	-	42.94***§
STAR	14.62***§	15.45***§	13.82***§	13.65***§	-16.24***§	-24.80***§	-
DIA	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-69.40***§	-62.20***§	-15.26***§	-33.67***§	-28.01***§	-27.81***§
AR	-0.850	-	10.07***§	68.59***§	13.63***§	28.02***§	28.23***§
ARIMA	0.998	2.933***	-	58.35***§	9.423***§	27.01***§	27.26***§
ES	-1.503	0.112	-1.781*	-	-32.95***§	-23.57***§	-23.36***§
AAR	2.141**	2.156**	1.217	2.238**	-	18.81***§	19.08***§
TAR	0.898	1.578	-0.006	2.189**	-1.172	-	7.863***§
STAR	0.908	1.590	0.001	2.210**	-1.166	0.336	-
MCLS	**RW**	**AR**	**ARIMA**	**ES**	**AAR**	**TAR**	**STAR**
RW	-	-82.08***§	-10.77***§	-10.77***§	-201.3***§	-236.7***§	-217.2***§
AR	-2.108**	-	80.72***§	80.79***§	-195.4***§	-181.8***§	-197.6***§
ARIMA	-0.320	2.138**	-	10.87***§	-201.1***§	-234.2***§	-216.4***§
ES	-0.325	2.137**	0.062	-	-201.1***§	-234.3***§	-216.4***§
AAR	55.22***§	58.48***§	55.78***§	55.77***§	-	193.5***§	191.5***§
TAR	11.15***§	11.44***§	11.71***§	11.71***§	-64.67***§	-	-199.1***§
STAR	22.55***§	24.89***§	23.52***§	23.15***§	-73.48***§	33.61***§	-

For different frequencies, at least one of the linear models outperform the nonlinear models when a power 1 loss function is applied. Under power 2 loss function, the AR model seems to have better or equal performance compared to the other models. In addition, the nonlinear models no longer necessarily provide inferior forecasting performances than the linear models where a power 2 function is applied. For daily data and hourly data, the MDM test result of one of the two market indices provides weak evidence that index price movements can be better predicted by the AR model instead of the random walk model. This implies that there is weak evidence that the UK stock market is not in a weak form of efficiency. However, the evidence is claimed to be weak because it is significant at the 10% significance level for daily data and not significant with the Bayesian adjustment but at the 5% significance level for hourly data. For minute data, the FTSE100 index price can be better predicted by the AR model both at the 1% significance level and with Bayesian inference, which implies that minute price movement of the market portfolio has a pattern and hence the market is not in a weak form of efficiency with evidence from minute observations. This provides incentives for investors to do higher-than-minute-frequency trading but probably with more advanced methods that have an AR term.

3.5.2. Subsample Forecasting Results

Table 3.8 and Table 3.9 display the best forecasting models for hourly and minute data of each testing asset in each subsample period, which are used to compare with the forecasting performance of the random walk model. The results indicate that the best-performing model changes over time. As such, there seems no forecasting model that constantly beats the other models. The implication behind is that forecasters should not rely on a single forecasting model. Instead, they can use computing techniques to develop a computer programme that update the forecasting method from one period to another. This result also provides implications for ambiguity literature, which attempts to use forecasting methods to find out the reference model. For instance, the reference model used by Viale et al. (2014) is solely based on the regime-switching model. The advantage is that it allows to calculate the transition probability from one economic state to another, which can be used as reference probability. However, according to the results in this study, it might not be proper to use the regime-switching model as a reference model in the sense that 1) it does not necessarily provide superior forecasting performances and hence may not be used by investors as a reference model; and 2) since the best-performing model changes over time, reference model of investors might also change over time.

The tables also show comparisons of forecasting performances between the selected models and the random walk model. According to Table 3.8, which shows the results from hourly data, the AR model provides better forecasts than the random walk model for the FTSE SmallCap index in month 4, which is significant at the 5% level using a loss function of power 1 and at the 10% level using a loss function of power 2. As such, evidence from small stocks seems to suggest that efficiency of the UK stock market is adaptive over time. However, evidence from large stocks, which are represented by FTSE100, suggests that the UK stock market at least has a weak form of efficiency. In combination, the findings seem to imply that efficiency of the UK stock market is adaptive. This outcome becomes more evident in Table 3.9, where minute data are applied. For the FTSE100 index, the AR model outperforms the random walk model in month 2 at the 5% significance level, as is shown in Panel A, where a power 1 loss function is used. In Panel B, where a power 2 loss function is applied, the AR model outperforms the random walk model in month 6 at the 1% significance level, month 5, the 5% level, and month 2, the 10% level. For the FTSE

SmallCap index, the AR model provides better out-of-sample performance than the random walk model in month 4, which is significant at the 5% level under a power 1 loss function and at the 1% level under a power 2 loss function. Thus, evidence from both large stocks and small stocks seems to suggest that efficiency of the UK stock market is adaptive. This outcome is consistent with the result of Urquhart and Hudson (2013) that the AMH characterises the UK stock market better than the EMH.

Table 3.8 Out-of-Sample Modified Diebold-Mariano (MDM) Test Statistics by Month (Hourly Data). This table shows the MDM test statistics between the random walk model and the best forecasting model from hourly data. The Model rows display the best model selected; and the MDM rows display the MDM test statistics. Models include random walk model (RW), autoregressive model (AR), autoregressive integrated moving average model (ARIMA), exponential smoothing model (ES), nonlinear autoregressive model (NAR), additive autoregressive model (AAR), threshold autoregressive model (TAR), and smooth transition autoregressive model (STAR). A positive (negative) test statistic means the selected model is better (worse) than the random walk model. - means the models in comparison are the same. Sgnificaince level: *** 1%, ** 5%, 10%.

Panel A	**Power 1 Loss Function**						
Month	**1**	**2**	**3**	**4**	**5**	**6**	**7**
FTSE100							
Model	AR	ES	ES	AR	ES	ES	ES
MDM	0.897	0.466	1.117	1.171	-2.263**	0.533	1.135
SMX							
Model	AR	AR	ES	AR	ARIMA	AAR	TAR
MDM	0.362	0.681	0.763	2.427**	-	1.141	0.400
HSBA							
Model	AR	ES	AR	AR	ES	ARIMA	ES
MDM	0.282	0.373	0.012	0.921	0.988	-	-0.620
GLEN							
Model	AR	AAR	AR	ES	ARIMA	TAR	AR
MDM	0.891	0.864	1.275	0.445	-	0.185	-0.091
DIA							
Model	ES	ARIMA	ES	AR	ARIMA	ES	AR
MDM	1.163	-	0.035	-0.957	-	-0.036	-1.170
MCLS							
Model	ES	ARIMA	ES	ES	ARIMA	ARIMA	ARIMA
MDM	-1.628	-2.088**	-3.477***	-1.927*	-	-0.899	-
Panel B	**Power 2 Loss Function**						
Month	**1**	**2**	**3**	**4**	**5**	**6**	**7**
FTSE100							
Model	AR	AR	AR	AR	ES	AR	AR
MDM	0.790	0.995	0.669	0.980	-2.584**	0.502	0.572
SMX							
Model	AR	AR	ES	AR	ARIMA	AAR	AAR
MDM	0.915	1.409	0.670	1.962*	-	1.442	0.690
HSBA							
Model	ES	ARIMA	AR	AR	AR	ES	ES
MDM	0.596	-	0.119	0.903	1.154	0.201	0.338
GLEN							
Model	AR	AAR	AR	AR	ARIMA	AR	AR
MDM	0.887	1.185	0.887	0.245	-	0.768	0.175
DIA							
Model	AR	ARIMA	AR	AR	ARIMA	ES	AR
MDM	1.080	-	1.873*	0.845	-	1.957*	1.489
MCLS							
Model	AR	ARIMA	AR	AR	ARIMA	ES	AR
MDM	0.605	0.588	-0.528	1.448	-	1.238	1.232

Table 3.9 Out-of-Sample Modified Diebold-Mariano (MDM) Test Statistics by Month (Minute Data). This table shows the MDM test statistics between the random walk model and the best forecasting model from minute data. The Model rows display the best model selected; and the MDM rows display the MDM test statistics. Models include random walk model (RW), autoregressive model (AR), autoregressive integrated moving average model (ARIMA), exponential smoothing model (ES), nonlinear autoregressive model (NAR), additive autoregressive model (AAR), threshold autoregressive model (TAR), and smooth transition autoregressive model (STAR). A positive (negative) test statistic means the selected model is better (worse) than the random walk model. - means the models in comparison are the same. Sgnificaince level: *** 1%, ** 5%, 10%.

Panel A	**Power 1 Loss Function**						
Month	**1**	**2**	**3**	**4**	**5**	**6**	**7**
FTSE100							
Model	AR	AR	ARIMA	ARIMA	AR	AR	AR
MDM	0.797	2.082**	1.426	0.482	1.311	0.400	0.359
SMX							
Model	AR	ARIMA	ARIMA	AR	ES	AAR	ES
MDM	-0.494	-	-1.166	2.399**	-2.595***	-0.557	-0.425
HSBA							
Model	ES	ES	ES	ES	ES	ES	ES
MDM	-5.597***	-5.851***	-6.294***	-0.951	-2.543**	-2.597***	-5.124***
GLEN							
Model	AR	ES	ES	AR	STAR	ES	ES
MDM	-3.367***	-2.294**	0.927	-2.487**	-3.658***	-2.213**	-1.904*
DIA							
Model	ARIMA	ES	ES	ARIMA	ES	ES	ES
MDM	-	-6.442***	-5.994***	-	-5.106***	-7.941***	-5.167***
MCLS							
Model	ARIMA	ES	ARIMA	ARIMA	ARIMA	ES	ARIMA
MDM	-	-3.935***	-3.860***	-	-	-4.556***	-
Panel B	**Power 2 Loss Function**						
Month	**1**	**2**	**3**	**4**	**5**	**6**	**7**
FTSE100							
Model	AR	AR	ARIMA	AR	AR	AR	AR
MDM	1.465	1.914*	0.675	1.264	2.398**	2.820***	0.807
SMX							
Model	AR	AR	AR	AR	ES	AAR	ES
MDM	1.624	0.755	-1.551	3.033***	-1.976**	0.758	0.552
HSBA							
Model	AR	AR	ES	AR	AR	ES	ARIMA
MDM	1.150	0.533	0.279	1.456	0.591	0.918	0.875
GLEN							
Model	AR	ES	AR	AR	STAR	AR	AR
MDM	0.911	-1.180	1.529	0.248	-0.733	0.703	2.657***
DIA							
Model	AR	TAR	AR	AR	AR	ES	AR
MDM	2.064**	1.573	-0.438	0.417	1.303	0.158	1.243
MCLS							
Model	AR	AR	ARIMA	AR	ARIMA	AR	ARIMA
MDM	0.582	0.021	-0.172	0.685	-	0.495	-

The detailed results regarding the forecasting accuracies for the subsamples are provided in Appendices. The results consistently show that the nonlinear models fit the in-sample data better while the linear models generally have better out-of-sample performances. In comparison with the results from the full sample, the MAPE accuracy measure suggests that the monthly out-of-sample forecasts are not necessarily less accurate than the full sample forecasts. In some months the subsample MAPEs are smaller than the full sample MAPEs while in other months the full sample MAPEs are smaller. This implies that increasing sample size does not necessarily increase forecasting accuracy indeed. Nevertheless, the MAPEs of minute data are smaller than those of hourly data for each month, which suggests that data frequency or the continuity of data points can affect forecasting performances. Thus, the fact that data with a higher frequency has better forecasting performance can be attributed to the continuity of data instead of the size of data. This provides incentives for forecasters to use high-frequency data to forecast stock prices and returns. This is probably why high-frequency trading becomes popular in recent years. In addition, results in this subsection also suggest that improving forecasting accuracy does not necessarily require an increased amount of data, which also makes it favourable to forecast using high-frequency data.

3.6. Conclusion

In this chapter, exponential smoothing models and AR based forecasting models are used to investigate their forecasting performances for predicting mean price movements. The findings suggest that nonlinear AR models do not necessarily have superior forecasting performances than the exponential smoothing models and the linear AR based models. Instead, they are generally outperformed by the linear models. The implication behind is that it may not be proper to assume nonlinear models as the reference model in ambiguity literature, which is one of the contributions of this chapter. On the other hand, the MDM test results indicate that the AR model outperforms the random walk model for FTSE100 in daily frequency and for SMX in hourly frequency are either weakly significant or not significant with adjustment to the Bayesian critical value. However, the AR model significantly outperforms the random walk model for FTSE100 in minute data. These findings suggest that the UK stock market is probably not in a weak form of efficiency, at least with evidence from minute data. As such, the EMH does not appear to characterise the UK stock market well.

The subsample forecasting results suggest that increasing sample size does not necessarily result in more accurate forecasts. Instead, increasing data frequency, more precisely, the continuity of data, increases forecasting accuracy for each month in the subsample analyses as well as for the full sample. In addition, the exponential smoothing models and the AR based models do not seem to suffer from over-fitting problems caused by inactive data. If such a problem exists, forecasts of hourly data should be more accurate than those of minute data in full sample analyses or subsample analyses or both because minute data have more non-changing data points than hourly data. This can be further confirmed by the result that in-sample MAPEs are generally similar to out-of-sample MAPEs regardless of what data frequency is used. Last but not least, the subsample results imply that the AMH characterise the UK stock market better than the EMH, which is also one of the contributions of this chapter as it tests the EMH and AMH using forecasting methods based on high-frequency data.

3.7. Appendices

Table 3.10 1-Step Ahead Forecasting Accuracy of Month 1 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 1. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	16.288	0.18402%	**12.684**	**0.13937%**	AR	3.394	0.05183%	**5.284**	**0.06762%**
ARIMA	16.364	0.18780%	13.117	0.14485%	ARIMA	3.371	0.05407%	6.269	0.07549%
ES	16.349	0.18662%	13.121	0.14495%	ES	3.368	0.05234%	5.973	0.07467%
AAR	16.072	0.18381%	12.820	0.14084%	AAR	3.312	0.05124%	6.204	0.07506%
TAR	15.649	0.18137%	13.488	0.14643%	TAR	**3.251**	0.05099%	6.285	0.08111%
STAR	**15.647**	**0.18086%**	13.505	0.14664%	STAR	3.262	**0.05032%**	6.003	0.07278%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	1.925	0.27122%	1.697	**0.27521%**	AR	1.977	1.15810%	**1.141**	**0.79096%**
ARIMA	1.965	0.27209%	1.701	0.27835%	ARIMA	2.005	1.20203%	1.184	0.82136%
ES	1.957	0.27287%	**1.686**	0.27692%	ES	1.991	1.18681%	1.207	0.84602%
AAR	1.923	0.27004%	1.705	0.27789%	AAR	1.825	1.18019%	1.210	0.83997%
TAR	1.885	0.26761%	1.735	0.28727%	TAR	1.826	1.17889%	1.246	0.85051%
STAR	**1.884**	**0.26656%**	1.737	0.28774%	STAR	**1.802**	**1.11927%**	1.316	0.92377%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	6.304	0.63776%	**18.482**	1.30489%	AR	1.068	0.41313%	**1.016**	0.50579%
ARIMA	6.336	0.60787%	19.059	1.27915%	ARIMA	1.056	0.40345%	1.069	0.55346%
ES	6.336	0.60688%	19.058	**1.27909%**	ES	1.158	**0.26737%**	1.061	**0.37055%**
AAR	5.952	0.64025%	53.713	8.84404%	AAR	1.068	0.40913%	1.065	0.54989%
TAR	**5.885**	**0.64708%**	38.237	6.02715%	TAR	**1.051**	0.40079%	1.135	0.57408%
STAR	**5.885**	**0.64708%**	37.615	5.98896%	STAR	**1.051**	0.40079%	1.135	0.57408%

Table 3.11 1-Step Ahead Forecasting Accuracy of Month 1 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 1. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	2.2547	0.02240%	**1.7658**	**0.01930%**	AR	0.4176	0.00474%	**0.4680**	**0.00491%**
ARIMA	2.2554	0.02243%	1.7710	0.01933%	ARIMA	0.4881	0.00579%	0.5470	0.00610%
ES	2.2549	0.02240%	1.7722	0.01934%	ES	0.4164	**0.00472%**	0.4688	0.00491%
AAR	**2.2442**	**0.02234%**	1.7723	0.01935%	AAR	**0.4157**	0.00473%	0.4701	0.00495%
TAR	2.2490	0.02235%	1.7717	0.01935%	TAR	0.4165	0.00473%	0.4708	0.00494%
STAR	2.2489	0.02236%	1.7738	0.01936%	STAR	0.4165	0.00473%	0.4708	0.00494%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.2882	0.03740%	**0.2322**	0.03363%	AR	0.2772	**0.15738%**	**0.2455**	**0.15176%**
ARIMA	0.2899	0.03717%	0.2326	0.03334%	ARIMA	0.2793	0.15796%	0.2477	0.15264%
ES	0.2894	**0.03700%**	0.2327	**0.03323%**	ES	0.2794	0.15805%	0.2473	0.15227%
AAR	**0.2877**	0.03738%	0.2340	0.03381%	AAR	**0.2770**	0.15752%	0.2473	0.15245%
TAR	0.2889	0.03738%	0.2332	0.03364%	TAR	0.2781	0.15778%	0.2480	0.15283%
STAR	0.2889	0.03736%	0.2358	0.03448%	STAR	0.2781	0.15778%	0.2479	0.15282%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	0.9029	0.02051%	**3.1357**	0.06103%	AR	0.14801	0.00829%	**0.1754**	0.01336%
ARIMA	0.9031	**0.01780%**	3.2026	**0.04728%**	ARIMA	0.14817	0.00475%	0.1756	**0.00853%**
ES	0.9030	0.01792%	3.2029	0.04764%	ES	0.14815	**0.00473%**	0.1756	0.00853%
AAR	**0.9009**	0.02306%	3.4341	0.24613%	AAR	0.14801	0.00806%	0.1755	0.01345%
TAR	0.9014	0.02239%	3.2044	0.06915%	TAR	**0.14798**	0.00824%	0.1758	0.01370%
STAR	0.9014	0.02275%	3.2056	0.07407%	STAR	**0.14798**	0.00824%	74.2993	5.50101%

Table 3.12 1-Step Ahead Forecasting Accuracy of Month 2 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 2. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	17.778	0.18176%	**16.550**	0.17912%	AR	3.802	0.06017%	**4.503**	**0.05798%**
ARIMA	17.813	0.18211%	16.698	0.17906%	ARIMA	3.812	0.06048%	4.530	0.05832%
ES	17.805	0.18132%	16.697	**0.17906%**	ES	3.790	0.05969%	4.529	0.05832%
AAR	17.778	0.18177%	16.596	0.18007%	AAR	3.728	0.05889%	4.599	0.06046%
TAR	17.401	0.17442%	18.801	0.19804%	TAR	**3.697**	0.05811%	4.544	0.06008%
STAR	**17.400**	**0.17417%**	19.224	0.20511%	STAR	**3.697**	**0.05809%**	4.544	0.06008%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	2.184	0.28948%	1.862	0.26514%	AR	1.942	1.37074%	1.244	1.00986%
ARIMA	2.211	0.29425%	**1.854**	0.24798%	ARIMA	1.936	1.36624%	1.250	1.01827%
ES	2.210	0.29347%	1.854	**0.24797%**	ES	1.926	1.36554%	1.242	1.01430%
AAR	2.176	0.28942%	1.883	0.27393%	AAR	1.881	1.30874%	**1.197**	**0.98348%**
TAR	2.134	0.29272%	1.910	0.28159%	TAR	**1.859**	**1.29893%**	1.446	1.26291%
STAR	**2.124**	**0.28465%**	2.047	0.32031%	STAR	1.861	1.32889%	1.214	1.01570%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	5.145	0.60311%	2.063	0.24272%	AR	**0.672**	0.30908%	1.347	0.56098%
ARIMA	5.122	0.58144%	**1.063**	**0.20937%**	ARIMA	0.726	**0.27618%**	**1.335**	**0.50395%**
ES	5.122	**0.58067%**	2.051	0.20938%	ES	0.726	0.27648%	1.336	0.50651%
AAR	5.122	0.60633%	2.042	0.21238%	AAR	0.717	0.29256%	2.612	1.57075%
TAR	5.025	0.61241%	2.044	0.21560%	TAR	0.745	0.30582%	1.414	0.55128%
STAR	**4.981**	0.58692%	2.050	0.22043%	STAR	0.711	0.30427%	2.881	1.76343%

Table 3.13 1-Step Ahead Forecasting Accuracy of Month 2 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 2. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}}$; mean absolute error measure is calculated as $\boldsymbol{MAE = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|}$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|}$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	**2.2250**	0.02123%	**1.9661**	**0.01683%**	AR	**0.4147**	0.00496%	**0.4693**	0.00498%
ARIMA	2.2277	0.02125%	1.9752	0.01691%	ARIMA	0.4187	0.00497%	0.4694	**0.00497%**
ES	2.2299	0.02129%	1.9719	0.01689%	ES	0.4149	0.00496%	0.4698	0.00498%
AAR	2.2267	**0.02123%**	1.9716	0.01688%	AAR	0.4152	**0.00496%**	0.4695	0.00498%
TAR	2.2255	0.02123%	1.9708	0.01687%	TAR	0.4150	0.00496%	0.4695	0.00499%
STAR	2.2251	0.02125%	1.9711	0.01687%	STAR	0.4150	0.00496%	0.4695	0.00499%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.3150	**0.03939%**	**0.3052**	0.03391%	AR	0.2609	0.16066%	0.2054	0.14569%
ARIMA	0.3148	0.03943%	0.3061	0.03392%	ARIMA	**0.2606**	0.16064%	0.2061	0.14636%
ES	0.3153	0.03908%	0.3055	**0.03342%**	ES	0.2613	**0.16053%**	**0.2052**	**0.14540%**
AAR	**0.3132**	0.03942%	0.3082	0.03502%	AAR	0.2609	0.16066%	0.2054	0.14593%
TAR	0.3134	0.03944%	0.3077	0.03452%	TAR	0.2608	0.16078%	0.2059	0.14637%
STAR	0.3134	0.03944%	0.3077	0.03452%	STAR	0.2608	0.16081%	0.2061	0.14653%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	0.8655	0.02247%	0.43976	0.01199%	AR	**0.1051**	0.00839%	**0.1640**	0.01117%
ARIMA	0.8650	0.02250%	0.43995	0.01147%	ARIMA	0.1112	0.00544%	0.1642	0.00762%
ES	0.8664	**0.01965%**	0.43993	**0.01014%**	ES	0.1112	**0.00543%**	0.1642	**0.00762%**
AAR	**0.8627**	0.02418%	0.43973	0.01086%	AAR	0.1111	0.00822%	0.1882	0.06096%
TAR	0.8651	0.02208%	**0.43972**	0.01066%	TAR	0.1112	0.00786%	0.1641	0.01008%
STAR	0.8651	0.02208%	0.43972	0.01066%	STAR	0.1111	0.00844%	0.1930	0.06686%

Table 3.14 1-Step Ahead Forecasting Accuracy of Month 3 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 3. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	20.460	0.24004%	**19.156**	0.21150%	AR	4.784	0.07789%	5.723	0.08737%
ARIMA	20.507	0.23740%	19.316	0.21076%	ARIMA	4.794	0.07817%	5.645	0.08380%
ES	20.499	0.23657%	19.316	**0.21076%**	ES	**4.643**	**0.07396%**	**5.553**	**0.08167%**
AAR	**19.423**	**0.22409%**	20.340	0.23465%	AAR	4.748	0.07782%	5.902	0.08929%
TAR	19.649	0.22950%	19.856	0.22093%	TAR	4.688	0.07528%	6.148	0.09313%
STAR	19.649	0.22950%	19.856	0.22093%	STAR	4.688	0.07528%	6.148	0.09313%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	2.375	0.33137%	**2.042**	**0.26490%**	AR	1.486	1.18900%	**1.076**	**0.86428%**
ARIMA	2.378	0.32520%	2.046	0.26504%	ARIMA	**1.386**	1.16975%	1.197	1.06178%
ES	2.378	0.32447%	2.046	0.26503%	ES	1.499	1.18315%	1.099	0.90109%
AAR	**2.276**	**0.31913%**	2.079	0.27002%	AAR	1.463	1.15547%	1.139	0.90680%
TAR	2.300	0.32507%	2.058	0.26742%	TAR	1.429	1.13440%	1.111	0.90112%
STAR	2.298	0.32405%	2.058	0.26742%	STAR	1.429	**1.13123%**	1.107	0.89530%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	5.897	0.86365%	**5.048**	0.74422%	AR	1.961	0.89223%	**1.644**	0.78916%
ARIMA	5.674	0.84857%	5.513	0.80912%	ARIMA	1.910	0.87737%	2.202	1.26271%
ES	5.828	0.86210%	5.081	**0.72741%**	ES	2.034	0.70133%	1.676	**0.67855%**
AAR	5.350	0.83348%	5.830	0.95140%	AAR	1.844	0.88659%	2.304	1.28680%
TAR	5.440	0.82460%	6.204	1.02453%	TAR	**1.835**	**0.81786%**	1.903	1.03613%
STAR	**5.335**	**0.82355%**	6.779	1.13164%	STAR	**1.835**	**0.81786%**	1.903	1.03613%

Table 3.15 1-Step Ahead Forecasting Accuracy of Month 3 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 3. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	2.7984	0.02449%	1.9897	0.01955%	AR	**0.4870**	0.00567%	**0.6331**	0.00582%
ARIMA	2.7974	0.02449%	**1.9787**	**0.01951%**	ARIMA	0.4905	0.00569%	0.6331	**0.00581%**
ES	2.7986	0.02448%	1.9899	0.01954%	ES	0.4870	**0.00567%**	0.6337	0.00581%
AAR	**2.7856**	**0.02442%**	1.9928	0.01983%	AAR	0.4880	0.00568%	0.6335	0.00582%
TAR	2.7915	0.02447%	1.9859	0.01965%	TAR	0.4876	0.00567%	0.6336	0.00581%
STAR	2.7916	0.02445%	1.9868	0.01964%	STAR	0.4876	0.00567%	0.6335	0.00581%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.3647	0.04139%	0.2896	0.03437%	AR	0.2047	0.15828%	**0.1689**	0.12912%
ARIMA	0.3649	0.04144%	0.2901	0.03456%	ARIMA	0.2048	0.15840%	0.1694	0.12937%
ES	0.3652	**0.04125%**	**0.2896**	**0.03429%**	ES	0.2056	**0.15796%**	0.1693	**0.12892%**
AAR	**0.3631**	0.04150%	0.2902	0.03506%	AAR	**0.2034**	0.15799%	0.1703	0.12999%
TAR	0.3637	0.04145%	0.2911	0.03521%	TAR	0.2040	0.15832%	0.1708	0.13093%
STAR	0.3637	0.04145%	0.2911	0.03521%	STAR	0.2041	0.15833%	0.1708	0.13092%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	0.9745	0.03260%	**0.7480**	0.02173%	AR	0.2975	0.02297%	0.19924	0.01756%
ARIMA	0.9748	0.02881%	0.7486	0.01969%	ARIMA	0.2979	**0.01467%**	**0.19917**	**0.01069%**
ES	0.9750	**0.02795%**	0.7486	**0.01909%**	ES	0.2979	0.01468%	0.19917	0.01071%
AAR	**0.9729**	0.03695%	0.7515	0.03223%	AAR	**0.2968**	0.02539%	0.23108	0.07008%
TAR	0.9732	0.03672%	0.7602	0.03997%	TAR	0.2972	0.02273%	0.19997	0.02365%
STAR	0.9732	0.03672%	0.7602	0.03997%	STAR	0.2972	0.02273%	0.19997	0.02365%

Table 3.16 1-Step Ahead Forecasting Accuracy of Month 4 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 4. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	33.630	0.37283%	**27.916**	**0.33877%**	AR	11.649	0.17273%	**9.745**	**0.14051%**
ARIMA	33.625	0.36207%	28.641	0.35095%	ARIMA	10.044	0.13433%	11.516	0.18960%
ES	33.516	**0.36027%**	28.800	0.35356%	ES	10.037	0.13359%	10.582	0.16116%
AAR	33.294	0.36720%	28.911	0.35974%	AAR	10.003	0.13305%	12.636	0.22043%
TAR	32.750	0.36234%	29.288	0.36949%	TAR	9.860	0.13868%	12.963	0.22897%
STAR	**32.732**	0.36121%	29.324	0.37030%	STAR	**9.242**	**0.12954%**	27.936	0.54524%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	3.400	0.46795%	**3.230**	**0.47139%**	AR	1.673	1.56840%	**1.793**	1.68287%
ARIMA	3.278	**0.44431%**	3.475	0.52445%	ARIMA	1.689	1.56550%	1.802	1.66043%
ES	3.327	0.44956%	3.322	0.48691%	ES	1.682	1.55143%	1.814	**1.65138%**
AAR	**3.208**	0.44565%	3.336	0.49786%	AAR	1.667	1.55845%	1.868	1.75847%
TAR	3.243	0.45111%	3.486	0.52354%	TAR	**1.640**	**1.53404%**	1.854	1.74165%
STAR	3.242	0.45023%	3.486	0.52349%	STAR	1.644	1.53808%	1.872	1.75154%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	5.165	0.85409%	**7.285**	**1.19207%**	AR	1.432	0.60880%	**2.180**	1.07705%
ARIMA	**4.906**	0.83439%	7.688	1.27504%	ARIMA	1.437	0.59806%	2.321	1.26638%
ES	4.985	0.80591%	7.431	1.19911%	ES	1.504	**0.48766%**	2.245	**0.93061%**
AAR	5.097	0.79052%	7.746	1.23513%	AAR	1.299	0.54730%	5.768	3.12819%
TAR	5.029	0.79761%	7.776	1.26025%	TAR	1.344	0.53756%	6.246	3.38103%
STAR	5.024	**0.78646%**	7.627	1.21125%	STAR	**1.298**	0.55288%	13.911	6.52460%

Table 3.17 1-Step Ahead Forecasting Accuracy of Month 4 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 4. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	3.7618	0.03417%	**3.6561**	0.03467%	AR	0.6980	0.00711%	0.8026	**0.00744%**
ARIMA	3.7696	0.03403%	3.6608	**0.03467%**	ARIMA	0.6880	0.00690%	0.8008	0.00757%
ES	3.7555	0.03404%	3.6565	0.03467%	ES	0.6822	**0.00678%**	**0.7997**	0.00745%
AAR	**3.7424**	0.03400%	3.6749	0.03481%	AAR	**0.6780**	0.00680%	0.8166	0.00792%
TAR	3.7524	**0.03397%**	3.6612	0.03471%	TAR	0.6810	0.00688%	0.8104	0.00778%
STAR	3.7525	0.03397%	3.6612	0.03471%	STAR	0.6810	0.00688%	0.8104	0.00778%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.3838	0.05018%	**0.4090**	0.05446%	AR	0.2151	0.17360%	**0.2296**	**0.18743%**
ARIMA	0.3831	0.05015%	0.4095	0.05458%	ARIMA	0.2153	0.17343%	0.2304	0.18778%
ES	0.3834	**0.04995%**	0.4093	**0.05439%**	ES	0.2153	0.17326%	0.2299	0.18747%
AAR	**0.3820**	0.05013%	0.4147	0.05550%	AAR	**0.2142**	**0.17316%**	0.2311	0.18916%
TAR	0.3831	0.05018%	0.4101	0.05463%	TAR	0.2147	0.17321%	0.2299	0.18771%
STAR	0.3831	0.05018%	0.4101	0.05463%	STAR	0.2147	0.17322%	0.2299	0.18772%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	0.8587	0.03093%	**0.9636**	0.03917%	AR	**0.2220**	0.01841%	**0.318**	0.02886%
ARIMA	0.8587	**0.02497%**	0.9640	**0.03198%**	ARIMA	0.2240	0.00993%	0.320	**0.01759%**
ES	**0.8572**	0.02613%	0.9639	0.03250%	ES	0.2240	**0.00992%**	0.320	0.01759%
AAR	0.8576	0.03084%	0.9685	0.05065%	AAR	0.2233	0.01712%	0.336	0.07579%
TAR	0.8576	0.02906%	0.9642	0.03300%	TAR	0.2236	0.01807%	0.322	0.04351%
STAR	0.8576	0.02908%	0.9642	0.03300%	STAR	0.2234	0.01732%	0.349	0.08805%

Table 3.18 1-Step Ahead Forecasting Accuracy of Month 5 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 5. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	30.764	0.36887%	24.430	0.30154%	AR	12.289	0.16782%	6.026	0.09763%
ARIMA	30.051	0.34521%	27.874	0.37768%	ARIMA	12.390	0.17023%	**5.701**	**0.08651%**
ES	30.951	0.36490%	**24.129**	**0.29452%**	ES	12.225	0.18005%	6.073	0.09650%
AAR	**28.269**	**0.33534%**	36.766	0.47032%	AAR	10.957	0.17790%	6.378	0.10700%
TAR	29.696	0.37069%	24.851	0.31274%	TAR	11.262	0.18094%	6.403	0.10447%
STAR	29.653	0.35137%	41.970	0.52656%	STAR	**9.476**	**0.15032%**	6.438	0.10554%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	3.499	0.57827%	**4.399**	0.61565%	AR	2.018	1.48443%	2.060	1.21348%
ARIMA	3.583	0.57272%	4.491	0.60977%	ARIMA	1.983	1.41478%	**2.005**	**1.17902%**
ES	3.583	0.57195%	4.491	**0.60975%**	ES	1.982	1.41490%	2.012	1.18459%
AAR	3.484	0.57127%	4.428	0.61836%	AAR	1.882	**1.36280%**	5.142	3.23333%
TAR	3.445	0.57769%	4.482	0.62685%	TAR	1.944	1.40886%	2.236	1.38092%
STAR	**3.433**	**0.55725%**	4.428	0.62256%	STAR	**1.863**	1.37469%	9.560	6.00811%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	5.637	0.87986%	4.139	0.58904%	AR	**1.502**	0.65897%	1.188	0.44626%
ARIMA	5.718	**0.75170%**	**4.118**	**0.48628%**	ARIMA	1.577	0.46830%	**1.173**	**0.33975%**
ES	5.663	0.81621%	4.353	0.53669%	ES	1.576	**0.46766%**	1.173	0.33977%
AAR	5.488	0.84523%	4.410	0.69769%	AAR	1.545	0.62074%	1.255	0.51848%
TAR	5.445	0.88772%	4.606	0.82627%	TAR	1.523	0.62097%	1.247	0.51707%
STAR	**5.440**	0.88487%	4.628	0.83579%	STAR	1.523	0.62097%	1.247	0.51707%

Table 3.19 1-Step Ahead Forecasting Accuracy of Month 5 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 5. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	3.5850	0.03873%	**3.0953**	**0.02806%**	AR	**1.0808**	0.00778%	0.4669	0.00660%
ARIMA	4.3644	0.04806%	3.7590	0.03548%	ARIMA	1.2889	0.00933%	0.5288	0.00785%
ES	3.5816	0.03871%	3.1042	0.02812%	ES	1.1158	**0.00762%**	**0.4653**	**0.00653%**
AAR	**3.5625**	**0.03858%**	3.1129	0.02857%	AAR	1.1152	0.00762%	0.4672	0.00660%
TAR	3.5666	0.03859%	3.1085	0.02825%	TAR	1.1146	0.00767%	0.4668	0.00659%
STAR	3.5666	0.03859%	3.1085	0.02825%	STAR	1.1146	0.00767%	0.4668	0.00659%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.4456	0.06810%	**0.4633**	0.06076%	AR	0.2966	0.19129%	0.2862	0.14365%
ARIMA	0.4464	0.06824%	0.4639	0.06078%	ARIMA	0.2965	0.19143%	0.2868	0.14372%
ES	0.4468	0.06815%	0.4634	**0.06062%**	ES	0.2970	**0.19100%**	0.2866	0.14327%
AAR	0.4444	**0.06800%**	0.4642	0.06095%	AAR	**0.2950**	0.19130%	0.2915	0.15009%
TAR	**0.4443**	0.06803%	0.4645	0.06094%	TAR	0.2952	0.19144%	0.2861	0.14326%
STAR	0.4443	0.06803%	0.4645	0.06094%	STAR	0.2953	0.19145%	**0.2861**	**0.14324%**
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	0.8874	0.02777%	**0.8163**	0.02393%	AR	0.23170	0.01535%	0.1572	0.00971%
ARIMA	0.8873	0.02269%	0.8265	0.01927%	ARIMA	0.23180	**0.01057%**	**0.1571**	**0.00629%**
ES	0.8877	**0.02189%**	0.8260	**0.01850%**	ES	0.23178	0.01066%	0.1571	0.00635%
AAR	**0.8802**	0.02916%	0.8259	0.02264%	AAR	0.23165	0.01497%	0.1581	0.01539%
TAR	0.8836	0.02683%	0.8262	0.02247%	TAR	0.23162	0.01489%	0.1730	0.03682%
STAR	0.8836	0.02683%	0.8262	0.02247%	STAR	**0.23161**	0.01497%	0.1832	0.04491%

Table 3.20 1-Step Ahead Forecasting Accuracy of Month 6 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 6. Root mean-squared error accuracy measure is calculated a $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$s; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	19.940	0.23080%	**18.993**	0.23623%	AR	6.036	0.09355%	5.893	0.08888%
ARIMA	19.587	0.22966%	21.688	0.27518%	ARIMA	5.928	0.08942%	6.022	0.09183%
ES	21.252	0.23950%	19.454	**0.21687%**	ES	6.034	0.09059%	5.894	0.08787%
AAR	19.392	0.23107%	20.049	0.26522%	AAR	5.867	0.08941%	**5.826**	**0.08687%**
TAR	18.988	**0.22596%**	22.002	0.28921%	TAR	5.568	0.08747%	6.084	0.09301%
STAR	**18.973**	0.22627%	22.112	0.29039%	STAR	**5.567**	**0.08738%**	6.084	0.09301%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	2.765	0.41058%	1.808	0.31680%	AR	3.037	1.46821%	**1.795**	0.85689%
ARIMA	2.823	0.41187%	1.709	**0.28694%**	ARIMA	3.054	1.46302%	1.841	0.82604%
ES	2.823	0.41018%	**1.708**	0.28730%	ES	3.054	1.46261%	1.841	0.82607%
AAR	**2.662**	**0.39065%**	3.955	0.74493%	AAR	3.025	1.45783%	1.807	0.84808%
TAR	2.674	0.40104%	3.403	0.64789%	TAR	**2.979**	**1.41166%**	1.801	**0.81996%**
STAR	2.667	0.39965%	3.715	0.70850%	STAR	**2.979**	1.41166%	1.828	0.83893%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	7.194	0.97072%	6.187	0.75757%	AR	1.729	0.66565%	1.569	0.59733%
ARIMA	6.979	0.92004%	6.079	0.73812%	ARIMA	1.807	**0.64371%**	1.579	**0.51710%**
ES	7.086	0.90266%	**5.994**	**0.69884%**	ES	1.770	0.66372%	**1.557**	0.52481%
AAR	**6.565**	0.90931%	13.697	2.04152%	AAR	1.623	0.64749%	2.123	1.00188%
TAR	6.847	**0.89316%**	6.201	0.72165%	TAR	**1.609**	0.64448%	1.767	0.71597%
STAR	6.849	0.91585%	6.409	0.78873%	STAR	**1.609**	0.64448%	1.767	0.71597%

Table 3.21 1-Step Ahead Forecasting Accuracy of Month 6 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 6. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	2.6114	**0.02501%**	**2.5068**	**0.02187%**	AR	0.5416	0.00651%	0.8717	0.00696%
ARIMA	2.6246	0.02509%	2.5207	0.02192%	ARIMA	0.5417	0.00648%	0.8722	0.00699%
ES	2.6279	0.02508%	2.5247	0.02190%	ES	0.5394	**0.00646%**	0.8722	0.00699%
AAR	**2.6015**	0.02506%	2.5383	0.02252%	AAR	0.5402	0.00649%	**0.8714**	**0.00696%**
TAR	2.6105	0.02502%	2.5476	0.02282%	TAR	**0.5391**	0.00651%	0.8718	0.00697%
STAR	2.6105	0.02502%	2.5475	0.02282%	STAR	0.5392	0.00651%	0.8718	0.00697%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	**0.4152**	**0.05185%**	0.2717	0.04143%	AR	0.3931	0.15550%	**0.2788**	0.10754%
ARIMA	0.4181	0.05208%	0.2714	0.04136%	ARIMA	0.3935	0.15542%	0.2792	0.10721%
ES	0.4181	0.05207%	**0.2714**	**0.04136%**	ES	0.3940	0.15481%	0.2792	**0.10647%**
AAR	0.4178	0.05207%	0.2750	0.04231%	AAR	**0.3880**	**0.15463%**	0.2813	0.10844%
TAR	0.4174	0.05206%	0.2841	0.04454%	TAR	0.3913	0.15474%	0.2791	0.10719%
STAR	0.4174	0.05206%	0.2841	0.04454%	STAR	0.3909	0.15500%	0.2804	0.10800%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	1.1212	0.03255%	1.0083	0.03046%	AR	**0.2781**	0.02163%	**0.2882**	0.02213%
ARIMA	1.1182	0.03433%	1.0118	0.03355%	ARIMA	0.2811	0.01467%	0.2886	0.01596%
ES	1.1234	**0.02713%**	**1.0087**	**0.02720%**	ES	0.2815	**0.01393%**	0.2887	**0.01516%**
AAR	1.1160	0.03486%	1.0377	0.05872%	AAR	0.2813	0.01903%	0.2905	0.03467%
TAR	1.1157	0.03085%	1.0087	0.02917%	TAR	0.2812	0.01967%	0.2938	0.04562%
STAR	**1.1157**	0.03092%	1.0087	0.02918%	STAR	0.2812	0.02004%	0.2938	0.04562%

Table 3.22 1-Step Ahead Forecasting Accuracy of Month 7 (Hourly Data). This table shows the accuracy measures of the hourly forecasts of month 7. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	16.399	0.18311%	**17.056**	0.19316%	AR	5.208	0.08261%	5.307	0.07354%
ARIMA	15.621	0.16994%	17.422	0.19909%	ARIMA	5.115	0.07978%	5.408	0.08188%
ES	16.272	0.17601%	17.181	**0.19251%**	ES	5.204	0.08041%	5.291	0.07563%
AAR	**13.976**	0.16576%	36.166	0.46553%	AAR	5.086	0.07943%	**5.268**	0.07529%
TAR	14.027	0.16618%	34.590	0.48967%	TAR	4.974	0.07759%	5.301	**0.07433%**
STAR	13.991	**0.16509%**	34.590	0.48968%	STAR	**4.920**	**0.07742%**	5.271	0.07547%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	2.282	0.32208%	2.650	0.38507%	AR	1.875	0.97772%	**1.651**	**0.70734%**
ARIMA	2.214	0.31542%	2.710	0.39682%	ARIMA	1.859	0.95214%	1.680	0.74930%
ES	2.262	0.30744%	**2.611**	**0.37770%**	ES	1.864	0.94351%	1.682	0.71573%
AAR	2.201	**0.29095%**	4.872	0.90812%	AAR	1.871	0.96626%	1.700	0.74656%
TAR	2.157	0.29497%	5.571	1.05491%	TAR	**1.817**	**0.91391%**	3.901	1.84844%
STAR	**2.155**	0.29533%	5.568	1.05442%	STAR	**1.817**	**0.91391%**	3.896	1.83452%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	7.611	0.92793%	**7.918**	**1.03818%**	AR	1.250	0.46868%	**1.092**	0.41672%
ARIMA	7.615	0.93792%	8.105	1.09228%	ARIMA	1.281	**0.35735%**	1.136	**0.35799%**
ES	8.050	1.03988%	8.698	1.21142%	ES	1.277	0.37463%	1.130	0.36740%
AAR	7.638	0.92485%	8.302	1.04736%	AAR	1.242	0.46189%	1.092	0.41446%
TAR	**7.452**	**0.89044%**	8.513	1.08419%	TAR	**1.210**	0.48998%	1.269	0.55943%
STAR	**7.452**	**0.89044%**	8.513	1.08419%	STAR	**1.210**	0.48998%	1.269	0.55943%

Table 3.23 1-Step Ahead Forecasting Accuracy of Month 7 (Minute Data). This table shows the accuracy measures of the minute forecasts of month 7. Root mean-squared error accuracy measure is calculated as $\boldsymbol{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2}$; mean absolute error measure is calculated as $\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}|Y_i - X_i|$; and mean absolute percentage error measure is calculated as $\boldsymbol{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - X_i)\times 100}{Y_i}\right|$. The forecasting model with the smallest forecasting error measure is highlighted in bold. RW denotes the random walk model; AR denotes the autoregressive model; ARIMA denotes the autoregressive integrated moving average model; ES denotes the exponential smoothing model; AAR denotes the additive autoregressive model; TAR denotes the threshold autoregressive model; and STAR denotes the smooth transition autoregressive model.

	In-Sample		**Out-of-Sample**			**In-Sample**		**Out-of-Sample**	
	RMSE	**MAPE**	**RMSE**	**MAPE**		**RMSE**	**MAPE**	**RMSE**	**MAPE**
FTSE100					**FTSE SmallCap (SMX)**				
AR	2.2895	0.02064%	**1.9024**	**0.01909%**	AR	**0.5514**	**0.00595%**	0.6669	0.00615%
ARIMA	2.2835	0.02059%	1.9070	0.01915%	ARIMA	0.6388	0.00611%	0.6626	0.00598%
ES	2.2934	0.02068%	1.9043	0.01910%	ES	0.6405	0.00613%	**0.6551**	**0.00587%**
AAR	**2.2616**	**0.02055%**	2.0803	0.02235%	AAR	0.6168	0.00623%	0.6586	0.00601%
TAR	2.2722	0.02063%	2.0807	0.02263%	TAR	0.6305	0.00609%	0.6553	0.00588%
STAR	2.2722	0.02063%	2.0807	0.02263%	STAR	0.6305	0.00609%	0.6553	0.00588%
HSBC Holdings Plc. (HSBA)					**Glencore Plc. (GLEN)**				
AR	0.3041	0.04445%	0.2857	0.04172%	AR	0.2715	0.11761%	**0.2718**	0.10510%
ARIMA	0.3048	0.04451%	**0.2856**	0.04167%	ARIMA	0.2725	0.11790%	0.2737	0.10549%
ES	0.3050	**0.04435%**	0.2858	**0.04155%**	ES	0.2731	**0.11746%**	0.2738	**0.10472%**
AAR	**0.3018**	0.04443%	0.3804	0.06256%	AAR	**0.2694**	0.11747%	0.2789	0.11027%
TAR	0.3029	0.04444%	0.3124	0.04834%	TAR	0.2708	0.11748%	0.2842	0.11367%
STAR	0.3029	0.04444%	0.3125	0.04836%	STAR	0.2708	0.11748%	0.2841	0.11362%
Dialight Plc. (DIA)					**McColl's Retail Group Plc. (MCLS)**				
AR	1.2698	0.03732%	**1.3041**	0.03551%	AR	**0.2383**	0.01888%	0.14948	0.01030%
ARIMA	1.2728	0.02568%	1.3081	0.02365%	ARIMA	0.2458	**0.01166%**	**0.14860**	**0.00604%**
ES	1.2728	0.02566%	1.3080	**0.02365%**	ES	0.2456	0.01213%	0.14860	0.00605%
AAR	1.2695	0.03433%	1.3087	0.03917%	AAR	0.2449	0.01902%	0.21168	0.05306%
TAR	1.2696	0.03599%	1.3102	0.04082%	TAR	0.2443	0.01624%	0.22860	0.05307%
STAR	**1.2694**	**0.03369%**	1.3111	0.03997%	STAR	0.2447	0.01627%	0.34083	0.08841%

Chapter 4. A Spread Measure of Ambiguity: Evidence from The UK Stock Market

4.1. Introduction

Recent studies have shown that illiquidity can be attributed to ambiguity and ambiguity-aversion. Routledge and Zin (2009) investigated the impact of ambiguity on liquidity and found that investors behave under the multiple-prior utility model. Ozsoylev and Werner (2011) also studied the effect of ambiguity on liquidity and proved that ambiguity can be associated with illiquid financial markets. This provides an inspiration to develop an empirical measure of ambiguity based on the bid-ask spread, which makes it possible to investigate the impact of ambiguity on stock markets.

On the other hand, although ambiguity asset pricing theories are well-established, empirical studies remain few. Viale et al. (2014) use the multiple-prior utility theory to construct a learning model under ambiguity and then investigate the effect of ambiguity on the pricing process of the US cross-section stock returns. However, they introduce a method based on the concept of entropy, which measures the distance between the reference prior and the worst-case prior. The reference model is based on the regime switching model. As is shown in the previous chapter, regime switching model may not provide better forecasts than linear autoregressive models. As such, it is highly likely that the reference model that they use is not the real reference model. Another issue is that they have to pre-set the confidence level about the information quality to calculate the entropy because confidence of investors is hard to measure in reality. Anderson et al. (2009) and Antoniou et al. (2015) used the Survey of Professional Forecasters (SPF) data to calculate the inconsistency in forecasts, which is used as a measure of ambiguity. However, this method is based on financial professionals and hence might not be representative of the population of market participants. These empirical papers motivate researchers to develop new empirical measures of ambiguity.

In this chapter, an empirical approach to measure the degree of ambiguity is developed using the bid price and ask price, which is purely based on real-life data and does not require an estimate of the reference model. According to Epstein and Schneider (2010), investors in long position and short position have different worst-case perceptions of asset prices, and this serves as a theoretical base for the empirical measure of this chapter. The aim of this chapter is to investigate the impact of ambiguity on the UK stock market using an empirical measure. First, the ambiguity measure is regressed against a liquidity measure based on trading volume to remove the impact of market makers on the gap between bid and ask prices. Lybek and Sarr (2002) divide liquidity measures into four categories. The four kinds of measures are based on transaction cost, trading volume, equilibrium price and price impact. Bid-ask spread is mainly used to capture transaction costs, which reflect liquidity of a market. Liquidity measures based on trading volume capture information revealed from order flows and hence can potentially reflect the role of market makers in transactions. The other two kinds, namely equilibrium price and price impact liquidity measures, are used to capture price adjustment to new information, which reflects market liquidity. Goyenko et. al. (2009) find that measurement performances of liquidity measures can change over time. They suggest that although of all their tested measures, which include transaction cost measures and price impact measures, some proxies provide consistently good liquidity measures in each of the test windows, their results are yet to be tested in thin markets that have relatively fewer order flows. Their results provide an implication for this study, which is that trading volume should be controlled when analysing or applying spread-based liquidity meausres because liquidity measures based on trading volume capture whether a market is thin or broad. As such, the ambiguity measure is regressed against a trading volume liquidity measure to ensure robustness of results.

Since there is limited existing empirical literature, especially with evidence from the UK stock market, there is little knowledge on the impact of ambiguity on the UK stock market empirically. As such, a vector autoregressive (VAR) model is used to investigate the interactions between ambiguity and variables of interest. The variables included in the study are UK stock market return, implied volatility and two term structure measures, which are used to reflect investors' expectations on the future macroeconomic conditions and their perceptions on future default risks respectively.

The empirical results show that ambiguity does not affect market return and excess market return directly. Instead, it has an impact on investors' expectation on future volatility. On other hand, news regarding the future economic conditions and default risks has an impact on the degree of ambiguity. A shock to the economy due to bad news will increase the degree of ambiguity of the stock market. The main contributions of this paper to the ambiguity studies are that 1) it develops an empirical approach to measure ambiguity; and 2) it provides empirical evidence on how ambiguity affects the UK stock market.

The rest of the chapter is organised as follows. Section 4.2 explains the calculation of the ambiguity measure. Section 5.3 shows the data and summary statistics. Section 5.4 describes the methodology. Section 5.5 – 5.7 explain the empirical results, followed by a discussion of the findings and a short conclusion in Section 5.8.

4.2. Ambiguity Measure

The ambiguity measure is based on the multiple-prior utility model developed by Gilboa and Schmeidler (1989), which suggests that a subject makes decisions based on the worst-case scenario of a set of priors. Starting with a simple setting, assume that investors have a homogeneous reference model on the distribution of the price of the same asset, namely a homogeneous reference return denoted by r^*, buyers and sellers of the same asset have different "worst-case scenarios" and hence use different priors to make buy and sell decisions. Buyers are worried about a price decrease since they take a long position and will incur a loss if the price goes down. As such, their worst case is that price goes below the reference price when they enter the long position, which makes them require a compensation for the uncertainty due to lack of information. Hence, they will quote a bid price as low as possible to get compensated for ambiguity. In terms of return, they will require the reference return minus an ambiguity premium to compensate the possible loss due to ambiguity when they enter the long position. Suppose the degree of ambiguity expressed by return is κ, then the prior return of the buyers will be $r^* - \kappa$. Similarly, sellers fear price increase because they are in a short position and will incur a loss if the price goes up. Therefore, their worst case is that price goes beyond the reference price when they enter the short position. Hence, they will quote an ask price as high as possible to get compensation for ambiguity. In terms of return, they will require the reference return plus an ambiguity term, $r^* + \kappa$, to get the compensation for bearing ambiguity. Although κ may not necessarily be the same for buyers and sellers, the gap between the bid price and the ask price will be guaranteed to be widened due to ambiguity. As such, the ambiguity measure is constructed as follows:

$$ln\frac{B}{P_{t-1}} = lnB - lnP_{t-1} = r^* - \kappa \quad 4.1$$

$$ln\frac{A}{P_{t-1}} = lnA - lnP_{t-1} = r^* + \kappa \quad 4.2$$

where B and A are bid and ask price respectively; P_{t-1} is the stock market price at time t-1.

Subtracting equation 1 from equation 1, we have:

$$lnA - lnB = 2\kappa \quad 4.3$$

Equation 4.3 provides a method of measuring degree of ambiguity κ and hence the following equation is used as a proxy of degree of ambiguity or the ambiguity spread:

$$\kappa = \frac{lnA - lnB}{2} \quad 4.4$$

where κ is assumed to be the same for buyers and sellers. In the case of asymmetric ambiguity spread, κ still captures the ambiguity spread because it is proportional to $lnA - lnB$ and $lnA - lnB$ is a result of ambiguity.

As Equation 4.4 indicates, the empirical measure does not actually rely on the reference model. As such, it is also applicable to the situations where heterogenous agents exist. This empirical measure of ambiguity measure is inspired by Easley and O'Hara (2010a, 2010b), who theoretically demonstrate that ambiguity can widen the bid-ask spread. Ealier literature such as Huang and Stoll (1997) shows that bid-ask spread can be decomposed into three components, which arise from order processing, inventories costs and bad news. According to their results, a majority of the spread, over 60% is attributed to order processing while around 30% arises from inventory costs and around 10%, bad news. Thus, a small proportion of the bid-ask spread can be attributed to information, which can be further categorised as bad news, information quality and misinterpretation. Poor information quality and misinterpretation of information are associated with ambiguity and ambigutiy aversion, implying that the bid-ask spread can capture ambiguity and ambiguity aversion to some extent. In addition, intuitively, inventory costs could be larger under ambigutiy in the sense that ambigutiy and ambigutiy aversion contribute to portfolio inertia and non-participation, or a "freezing" market. Therefore, the existing framework of bid-ask spread can be further developed to accommodate ambigutiy and ambiguity aversion. Easley and O'Hara (2010a, 2010b) have shown that the bid-ask spread is indeed an ambiguity spread, which provides a theoretical support for the empirical ambiguity measure of this study that is based on bid and ask prices.

4.3. Data

In finance literature, stock indices are used to investigate the stock market as a whole. However, one issue with the spread ambiguity measure is that stock indices do not have bid and ask prices because they are not for trading. As such, equity traded fund of market index is used because it proxies the market portfolio while having bid and ask prices. Thus, bid price and ask price of FTSE100 equity traded fund (ETF FTSE100) are collected from September 23rd, 2009 to June 30th, 2016 to proxy the market portfolio of the UK stock market. Then the bid and ask price are used to calculate the ambiguity measure by Equation 4.4. Adjusted closing price of FTSE100 is also collected and hence FTSE100 is used as a proxy for the market portfolios in the UK equity market. The index price is converted into daily return using the following formula:

$$r_t = ln\frac{P_t}{P_{t-1}} \quad 4.5$$

where r_t represents the daily return at time t; and P_t is the closing price of ETF FTSE100 at time t.

Since ambiguity may also be associated with liquidity and volatility, turnover by volume of EFT FTSE100 and volatility index of FTSE100 are also collected within the same period. All data are from DataStream on a daily basis.

To calculate the excess market return, the daily one-month deposit interest rate in the UK is also collected from DataStream, which is used as the risk-free rate. To construct term structure measures in Fama and French (1993), bond returns of short-term government bond (with maturity of 1 month), long-term government bond (with maturity of over 10 years) and long-term corporate bond (with maturity of over 10 years) of the UK are collected as well. The short-term government bond return and the long-term government bond return are used to construct the term spread *Term*, which is calculated from the difference between the two returns. The long-term government bond return and the long-term corporate bond return are used to construct the default spread *Def*, which is calculated from the difference between the two returns. *Term* and *Def* are used to reflect investors' expectations on future macroeconomic conditions and future default risks respectively. Table 4.1 shows the summary statistics of the variables. The summary statistics indicate that the market return, return of FTSE100, is has a symmetric distribution centred around zero and some negative skewness.

Table 4.1 Summary Statistics. This table shows the summary statistics. *Return* represents the daily return of FTSE100; *Volume* represents the daily turnover by volume of ETF FTSE100; *Volatility* represents the daily volatility index of FTSE100; *lnA-lnB* represents the gap between the natural logarithms of ask and bid prices; *Ambiguity* represents the original ambiguity measure on a daily basis; and *Term* and *Def* represent two bond market risk factors calculated from the term structure on a daily basis.

	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
Return	0.000	0.010	-0.048	0.050	-0.166	1.840
Volume	12.742	10.813	0.000	106.700	2.480	10.842
Volatility	18.314	5.548	9.672	43.610	1.213	1.808
lnA-lnB	0.003	0.008	0.000	0.127	11.078	140.061
Ambiguity	0.002	0.004	0.000	0.063	11.078	140.061
Term	0.033	0.007	0.016	0.047	0.086	-1.057
Def	0.011	0.002	0.008	0.018	1.182	0.746

4.4. Methodology

The role of market makers is considered when the ambiguity measure is calculated because market makers make profits through the bid-ask spread. Grossman and Miller (1988) developed a market making model, taking transaction costs into consideration, which is used as the theoretical base in terms of the role of market makers in this chapter. Suppose there are two liquidity traders in the market, L1 and L2. L1 sells *m* units of stocks at time 1 and L2 buys *m* units at time 2. If the number of market makers is denoted by *n* and the stock price at time i is S_i, then the present value of the stock at time 0 will be

$$S_0 = \mu - A\sigma^2 \frac{m}{n+1} - 2c\frac{n}{n+1} \qquad 4.6$$

where the stock price is assumed to be normally distributed with mean μ and variance σ^2; *c* is the transaction costs per unit of the stock; and *A* is the degree of risk aversion of the traders, namely both the liquidity traders and the market makers.

Equation 4.6 provides an insight of the mechanism of market making. Liquidity with the presence of transaction costs can therefore be calculated as

$$L = \frac{m}{n+1} + 2\frac{cn}{A\sigma^2(n+1)} \qquad 4.7$$

Equation 4.7 shows that liquidity is associated with trading volume m, number of market makers n, degree of risk aversion A, transaction costs c and volatility of the stock price σ^2. The number of the market makers in the financial markets and the degree of risk aversion are difficult to measure empirically. Since bid-ask spread is frequently used as a liquidity measure in existing literature, liquidity should be separated from the ambiguity measure regardless of the fact that the ambiguity measure is not exact the same as the bid-ask spread that is used in existing literature. As is mentioned in the introduction section, of the four categories of liquidity measures in Lybek and Sarr (2002), a trading volume liquidity measure is applied instead of other measures that are based on transaction cost, equilibrium price and price impact. This is because the purpose of using a liquidity measure is to remove the role of market makers, and liquidity measures based on trading volume capture information revealed from order flows, which can potentially reflect the role of market makers in transactions. Therefore, turnover by volume, which is a liquidity measure based on trading volume, is used to remove the impact of market makers on the spread by the following linear regression:

$$Ambiguity_t = \beta_0 + \beta_1 Volume_t + \varepsilon_t \qquad 4.8$$

where $Ambiguity_t$ is the ambiguity measure at time t and $Volume_t$ is turnover by volume of ETF FTSE100 at time t. The regression is checked for heteroskedasticity and the Newey-West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix is used to correct the regression results for heteroskedasticity.

Table 4.2 shows the regression result with Newey-West standard errors. The result indicates that the linear relationship between the ambiguity measure that is constructed using bid and ask prices and volume by turnover is not statistically significant. As such, the ambiguity measure is used directly for subsequent analyses without removing the effect of the liquidity measure.

Table 4.2 Regression of Ambiguity Measure on Liquidity Measure (*Volume*) with Newey-West Standard Errors. This table shows the regression result of the ambiguity measure on turnover by volume of ETF FTSE100 with Newey-West standard errors. *Volume* represents the coefficient of daily turnover by volume of ETF FTSE100; t-statistics are robust to heteroskedasticity and autocorrelation; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

	Estimate	**t-Value**
Volume	0.000	1.243
Intercept	0.001	12.544***

For each part of the analysis, the interactions between the ambiguity measure and variables of interest are investigated using the following VAR model:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \epsilon_t \quad 4.9$$

where y_t is a vector of stationary time-series variables in the system, which are the ambiguity measure and variables of interest; β's are vectors of coefficients of different equations; p is the order of the lags included in the model, which is determined by information criteria; and ϵ_t is a vector of error terms. Variables of interest include the UK stock market return, implied volatility and two term structure measures, which are used to measure investors' expectations on the future economic conditions and their perceptions of future default risks.

Since the VAR model requires the underlying time-series variables to be stationary, the variables are checked for stationarity using the Augmented Dickey-Fuller (ADF) test before the VAR model is run. If the ADF test shows that the variable is non-stationary, the differenced value will be used until the ADF test indicates stationarity. Then information criteria will be used to select number of lags to be included in the VAR model. The criteria used are the Akaike information criterion (AIC), the Hannan-Quinn information criterion (HQIC), the Schwarz's Bayesian information criterion (SC or SBIC) and the final prediction error (FPE). In case that the four criteria show different numbers of lags, the maximum number of lags of the four criteria will be included in the VAR model. This is to prevent any significant results from being neglected due to model selection.

Both Granger-Causality test and orthogonalised impulse response function (OIRF) are used to uncover the interactions between the ambiguity measure and variables of interest. The 95% confidence interval of an OIRF is constructed using the wild bootstrap method, which is developed by Wu (1986) to accommodate heteroskedasticity, which frequency presents in time-series data.

4.5. Ambiguity and Market Return

Table 4.3 shows the regression result of FTSE100 return on the ambiguity measure denoted by *Ambiguity*. The result indicates that the negative coefficient of the ambiguity measure is not statistically significant. This suggests that there is no contemporary linear relationship between the market return and the ambiguity measure.

Table 4.3 Regression of FTSE100 Return on Ambiguity Measure (*Ambiguity*) with Newey-West Standard Errors. This table shows the regression results of FTSE100 return on the ambiguity measure with Newey-West standard errors. *Ambiguity* represents the coefficient of the original ambiguity measure; t-statistics are robust to heteroskedasticity and autocorrelation; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

	Estimate	t-Value
Ambiguity	-0.107	-1.188
Intercept	0.0004	1.757*

Table 4.4 shows the interactions between FTSE100 return and the ambiguity measure with Panel A showing the result of the return equation, Panel B the result of the ambiguity measure equation and Panel C the results of Granger-Causality tests. Result in Panel A indicates that market return is autoregressive in lags 4 and 8 at the 5% significance level and both of the lags have a negative impact on the future return. On the other hand, the market return does not seem to be associated with past values of the ambiguity measure, which together with the past values of the market return can only explain 0.377% of the variations in market return.

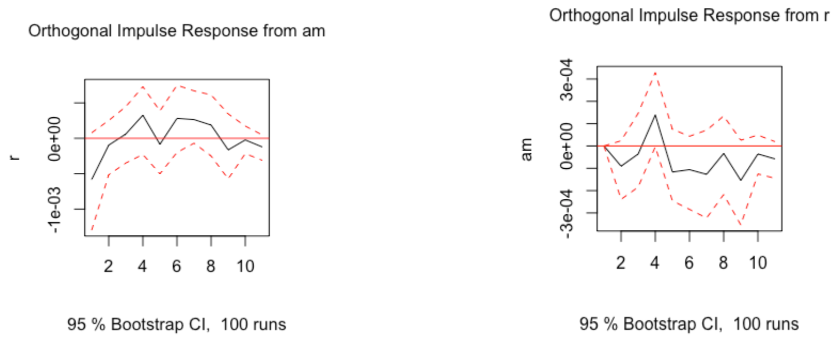
In Panel B, the ambiguity measure is autoregressive in lags 1, 2 and 8 at the 1% significance level and lag 4, at the 5% significance level. The positive coefficients of lags 1, 2 and 8 indicate that past degrees of ambiguity can have a strong accumulative impact on the future degree of ambiguity. On the other hand, lag 3 of the market return has a positive impact on the future value of the ambiguity measure at the 5% significance level, and lag 4 has a negative impact at the 10% significance level. It is also noticeable that there is a statistically significant constant in the ambiguity measure equation, which suggests that the gap between bid and ask prices still exists even if the impacts of past values of the market return and the ambiguity measure are taken into consideration. The implication behind the regression results is that the interaction between the market return and the ambiguity measure is not strong.

This is consistent with the Granger-causality test results in Panel C, which indicate that the market return can Granger-cause the ambiguity measure at the 10% significance level but not at the 5% level. The results also indicate that the ambiguity measure cannot Granger-cause the market return. The orthogonalised impulse response function (OIRF) plots in Figure 4.1 also confirms the weak interaction between market return and the ambiguity measure. It is shown in the plots that the two variables do not respond to a shock to each other at the 5% significance level. Overall, the results in Table 4.4 suggest that the interaction between the market return and the ambiguity measure is not strong. The implication for asset pricing is that ambiguity does not seem to affect market return directly and hence investors who perceive ambiguous information or information of poor quality should withdraw from the market instead of expecting an increased likelihood of a high return.

Table 4.4 Interaction between FTSE100 Return (*Return*) and Ambiguity Measure (*Ambiguity*). This table shows the VAR regression results between FTSE100 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *Return* represents the daily return of FTSE100; *Ambiguity* represents the original ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	0.012	0.496
	2	-0.014	-0.594
	3	-0.003	-0.112
	4	-0.075	-3.132***
	5	-0.020	-0.814
	6	0.005	0.191
	7	-0.028	-1.159
	8	-0.050	-2.091**
Ambiguity	1	-0.026	-0.365
	2	0.022	0.299
	3	0.097	1.275
	4	-0.066	-0.868
	5	0.057	0.747
	6	0.068	0.887
	7	0.020	0.277
	8	-0.097	-1.378
Constant		0.001	1.295
Trend		0.000	-1.299
Adjusted R-Squared		0.377%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.009	-1.099
	2	-0.001	-0.142
	3	0.017	2.110**
	4	-0.014	-1.707*
	5	-0.012	-1.517
	6	-0.007	-0.883
	7	0.005	0.675
	8	-0.012	-1.467
Ambiguity	1	0.255	10.788***
	2	0.308	12.613***
	3	0.020	0.769
	4	-0.056	-2.208**
	5	-0.022	-0.874
	6	0.030	1.183
	7	-0.036	-1.467
	8	0.154	6.516***
Constant		0.001	3.711***
Trend		0.000	-0.708
Adjusted R-Squared		24.370%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**Ambiguity (Predictor)**
Return		-	0.767
Ambiguity		1.689*	-

Figure 4.1 Plots of Orthogonalised Impulse Response Function (OIRF) of FTSE100 Return (*r*) and Original Ambiguity Measure (*am*)



Notes: *r* represents the daily return of FTSE100; *am* represents the original ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

The relationship between the excess market return and the ambiguity measure is shown in Table 4.5 and the interaction between the two variables is illustrated in Table 4.6 and Figure 4.2. The results are similar to those displayed in Table 4.3, Table 4.4 and Figure 4.1. Thus, the results imply that an increased level of ambiguity does not necessarily mean a higher premium in a short time horizon. This empirical evidence is consistent with conclusions of existing quantitative studies that advocate heterogenous investors (Epstein and Schneider, 2010). Thus, if some investors are more ambiguity-averse than the others, investors may not necessarily be compensated for bearing ambiguity because those who are more ambiguity-averse can simply quit the market while those who are less ambiguity-averse continue participating in the market. As such, investors may not get compensation for bearing ambiguity by receiving a higher premium in the short run.

Table 4.5 Regression of FTSE100 Excess Return on Ambiguity Measure (*Ambiguity*) with Newey-West Standard Errors. This table shows the regression results of FTSE100 excess return on the ambiguity measure with Newey-West standard errors. *Ambiguity* represents the coefficient of the original ambiguity measure; t-statistics are robust to heteroskedasticity and autocorrelation; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

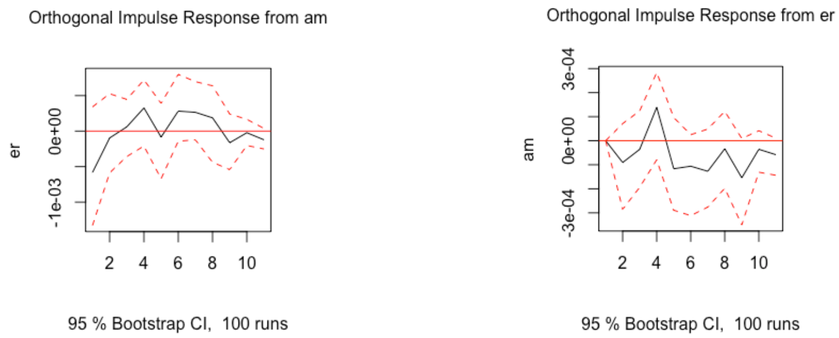
	Estimate	**t-Value**
Ambiguity	-0.107	-1.188
Intercept	0.0004	1.683*

Table 4.6 Interaction between FTSE100 Excess Return (*Er*) and Ambiguity Measure (*Ambiguity*). This table shows the VAR regression results between FTSE100 excess return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *Er* represents the daily excess return of FTSE100; *Ambiguity* represents the original ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Excess Return Equation**		
	Lag	**Estimate**	**t-Value**
Er	1	0.012	0.497
	2	-0.014	-0.594
	3	-0.003	-0.112
	4	-0.075	-3.132***
	5	-0.020	-0.813
	6	0.005	0.191
	7	-0.028	-1.159
	8	-0.050	-2.090**
Ambiguity	1	-0.026	-0.365
	2	0.022	0.300
	3	0.097	1.275
	4	-0.066	-0.868
	5	0.057	0.747
	6	0.068	0.887
	7	0.020	0.277
	8	-0.097	-1.378
Constant		0.001	1.249
Trend		0.000	-1.219
Adjusted R-Squared		0.376%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Er	1	-0.009	-1.099
	2	-0.001	-0.142
	3	0.017	2.110**
	4	-0.014	-1.707*
	5	-0.012	-1.517
	6	-0.007	-0.883
	7	0.005	0.675
	8	-0.012	-1.467
Ambiguity	1	0.255	10.788***
	2	0.308	12.613***
	3	0.020	0.769
	4	-0.056	-2.208**
	5	-0.022	-0.874
	6	0.030	1.183
	7	-0.036	-1.467
	8	0.154	6.516***
Constant		0.001	3.708***
Trend		0.000	-0.707
Adjusted R-Squared		24.370%	

Panel C	**Granger Causality Test**	
	Er (Predictor)	**Ambiguity (Predictor)**
Er	-	0.876
Ambiguity	2.155*	-

Figure 4.2 Plots of Orthogonalised Impulse Response Function (OIRF) of FTSE100 Excess Return (*er*) and Original Ambiguity Measure (*am*)



Notes: *er* represents the daily excess return of FTSE100; *am* represents the original ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

4.6. Ambiguity and Implied Volatility

The interaction between the volatility index and the ambiguity measure is presented in Table 4.7. In Panel A, the result indicates that the volatility index is autoregressive in lag 1 at the 1% significance level. The positive coefficient of lag 1 suggests that an increased level of the volatility index today is associated with an increase in the volatility tomorrow. Lag 1 of the ambiguity measure also has a positive impact on the future value of the volatility index at the 10% significance level. However, the positive relationship is not statistically significant at the 5% level. As such, past values of the ambiguity measure do not seem to have a strong impact on the future value of the volatility index from the regression result.

In Panel B, the ambiguity measure is autoregressive in lags 1, 2 and 8 at the 1% significance level and lags 4, 7 and 9, the 5% level. In addition, lags 1, 2 and 8 are all positively related to the future value of the ambiguity measure. On the other hand, lag 4 and lag 8 of the volatility index both have a positive impact on the future value of the ambiguity measure at the 5% significance level.

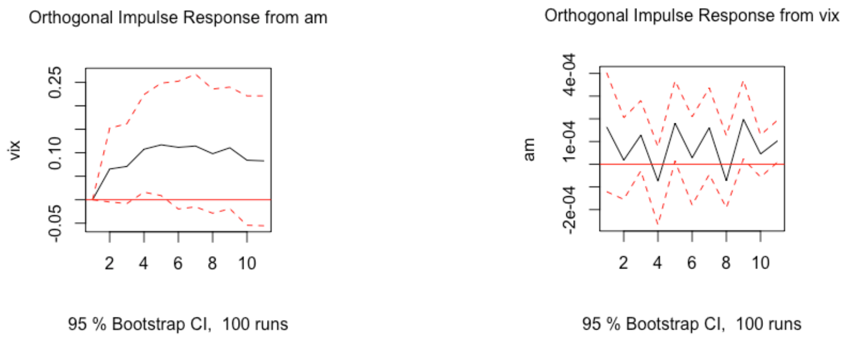
In comparison with the interaction between the ambiguity measure and market return, interaction between the ambiguity measure and the volatility index is stronger. The Granger-causality test results in Panel C indicate that the ambiguity measure can Granger-cause the volatility index at the 5% significance level while the volatility index can Granger-cause the ambiguity measure at the 10% significance level. This suggests that past signals of the ambiguity measure can help to better predict the future value of the volatility index.

The OIRF plots in Figure 4.3 suggest that the volatility index increases in periods 4 and 5 in response to a positive shock in the ambiguity measure at the 5% significance level. The ambiguity measure increases around period 5 in response to a positive shock in the volatility index at the 5% significance level and then increases again around period 9. This implies that a positive shock in the degree of ambiguity, or an unexpected increase in the ambiguity measure can lead to perceptions of a more volatile stock market, which can lead to excess volatility in the short run. This explains the excess volatility puzzle of the equity market, which is introduced by Shiller (1981). In combination with the findings of Kim et al. (2004), who report a significant positive relationship between market volatility and equity premium in the long run, the empirical evidence seems to imply that although ambiguity does directly affect market risk premium, a shock of an increased degree of ambiguity can lead to perceptions of a more volatile market, generating excess market volatility, which in turn results in a higher equity premium in the long run. As such, findings in this chapter support the theory that existence of ambiguity helps to explain the equity market premium, which is proved by Epstein and Schneider (2010) from a theoretical perspective.

Table 4.7 Interaction between Volatility Index (*Volatility*) and Ambiguity Measure (*Ambiguity*). This table shows the VAR regression results between volatility index of FTSE100 and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *Volatility* represents the daily volatility index of FTSE100; *Ambiguity* represents the original ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.964	40.172***
	2	-0.055	-1.637
	3	0.019	0.577
	4	-0.018	-0.549
	5	-0.018	-0.529
	6	0.052	1.546
	7	-0.016	-0.488
	8	0.016	0.468
	9	0.023	0.971
Ambiguity	1	18.960	1.878*
	2	-2.484	-0.241
	3	6.101	0.568
	4	0.436	0.041
	5	-2.518	-0.234
	6	2.413	0.225
	7	-3.730	-0.348
	8	7.131	0.693
	9	-9.599	-0.952
Constant		0.663	3.813***
Trend		0.000	-1.328
Adjusted R-Squared		93.060%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.0000	-0.285
	2	0.0001	0.844
	3	-0.0001	-1.636
	4	0.0002	2.472**
	5	-0.0001	-1.429
	6	0.0001	0.946
	7	-0.0002	-1.899*
	8	0.0002	2.301**
	9	-0.0001	-1.511
Ambiguity	1	0.252	10.517***
	2	0.302	12.359***
	3	0.018	0.702
	4	-0.055	-2.169**
	5	-0.019	-0.734
	6	0.028	1.121
	7	-0.056	-2.215**
	8	0.140	5.731***
	9	0.050	2.083**
Constant		0.000	0.228
Trend		0.000	0.020
Adjusted R-Squared		24.560%	
Panel C		**Granger Causality Test**	
		Volatility (Predictor)	**Ambiguity (Predictor)**
Volatility		-	4.626**
Ambiguity		1.688*	-

Figure 4.3 Plots of Orthogonalised Impulse Response Function (OIRF) of Volatility Index (*vix*) and Original Ambiguity Measure (*am*)



Notes: *vix* represents the daily volatility index of FTSE100; *am* represents the original ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

4.7. Ambiguity and Term Structure Measures

In this section, interactions between the ambiguity measure and the two term structure measures, *Term* and *Def* are explained. Since *Term* is the difference in returns between the long-term government bond and short-term government bond, it can reflect investors' expectations on future economic state. If there is an increase in the difference, which is equivalent to a positive first difference of *Term*, investors are expecting a good future economic state. Instead, if there is a decrease in the difference, which is equivalent to a negative first difference of *Term*, investors are expecting a bad future economic state. Thus, an increase in the first difference of *Term*, which is denoted by *dTerm* in the following paragraphs, indicates a more optimistic perception of the future economic state compared to the previous period while a decrease in *dTerm* indicates a more pessimistic perception compared to the previous period.

On the other hand, *Def* measures the gap between the return of long-term government bond and the return of long-term corporate bond and hence it can reflect investors' view on future default risk. If there is an increase in *Def*, which is equivalent to a positive first different of *Def*, investors are expecting a higher default risk while if there is a decrease in *Def*, which is equivalent to a negative first difference of *Def*, investors are expecting a lower default risk. As such, an increase in the first different of *Def*, which is denoted by *dDef* in the following paragraphs, suggests that investors become more worried about the future default risk while a decrease in *dDef* suggests that they become less worried about the future default risk.

4.7.1. Ambiguity and Macroeconomic Conditions

Table 4.8 shows the interaction between the ambiguity measure and *dTerm*. Result in Panel A indicates that *dTerm* is autoregressive in lags 2 and 7 at the 1% significance level, lags 8 and 9, the 5% level, and lag 3, the 10% level. Lag 10 of the ambiguity measure has a negative impact on *dTerm* at the 10% significance level, which is not significant at the 5% level. As such, past values of the ambiguity measure do not appear to have a strong impact on *dTerm*.

In Panel B, the ambiguity measure is autoregressive in lags 1, 2 and 8 at the 1% significance level, lags 4, 7 and 9, at the 5% level. Similar to the regression results of the previous sections, lags with positive coefficients, which are lags 1, 2 and 8 in this section, have a stronger impact on the future value of the ambiguity measure than those with negative coefficients, which are lags 4 and 7 in this section. This suggests that the ambiguity measure can accumulate over time. On the other hand, lags 1, 4 and 9 of *dTerm* have negative effects on the future value of the ambiguity measure with lag 4 significant at the 1% level, and lags 1 and 9, the 5% level.

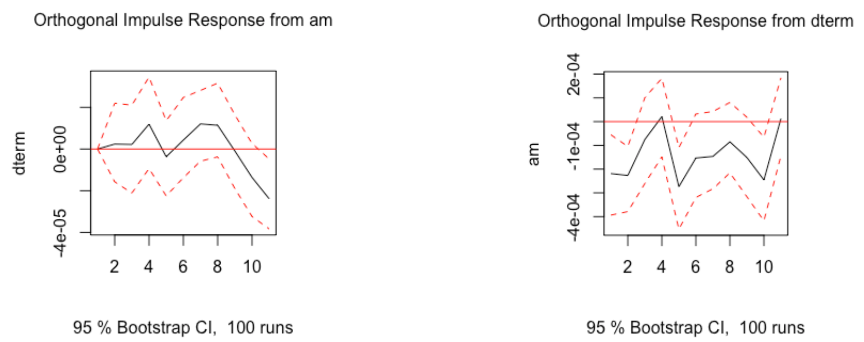
The Granger-causality test results in Panel C indicate that *dTerm* can Granger-cause the ambiguity measure at the 1% significance level while the ambiguity measure cannot Granger-cause *dTerm*. This implies that investors' expectation of the future economic state of the UK can affect the degree of ambiguity of the stock market. The OIRF plots in Figure 4.4 show that *dTerm* does not seem to respond to a positive shock to the ambiguity measure until period 10 onwards while the ambiguity measure decreases around periods 1, 2, 5 and 10 in response to a positive shock in *dTerm* at the 5% significance level. The implication behind is that a shock to the economy that makes investors more optimistic about the future economic state of the UK can lead to a decrease in the degree of ambiguity of the equity market in the future while a shock that leads to more pessimistic investors can results in an increased degree of ambiguity in the future.

Table 4.8 Interaction between First-Differenced Term Structure Measure Term (*dTerm*) and Ambiguity Measure (*Ambiguity*). This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *dTerm* represents the first-differenced value of the term structure measure *Term*; *Ambiguity* represents the original ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	0.017	0.697
	2	-0.076	-3.174***
	3	-0.045	-1.868*
	4	0.013	0.535
	5	-0.040	-1.644
	6	0.025	1.022
	7	-0.065	-2.671***
	8	-0.061	-2.505**
	9	0.050	2.051**
	10	0.030	1.214
Ambiguity	1	0.001	0.232
	2	0.000	0.150
	3	0.003	0.994
	4	-0.002	-0.671
	5	0.001	0.235
	6	0.004	1.117
	7	0.002	0.671
	8	-0.002	-0.550
	9	-0.005	-1.588
	10	-0.005	-1.799*
Constant		0.000	0.140
Trend		0.000	-0.861
Adjusted R-Squared		1.986%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-0.406	-2.098**
	2	0.120	0.623
	3	0.233	1.206
	4	-0.636	-3.287***
	5	-0.217	-1.111
	6	-0.106	-0.543
	7	-0.039	-0.197
	8	-0.213	-1.089
	9	-0.417	-2.133**
	10	0.382	1.951*
Ambiguity	1	0.245	10.198***
	2	0.313	12.709***
	3	0.014	0.556
	4	-0.060	-2.347**
	5	-0.022	-0.843
	6	0.039	1.511
	7	-0.053	-2.074**
	8	0.132	5.195***
	9	0.050	2.027**
	10	0.013	0.532
Constant		0.001	3.359***
Trend		0.000	-0.657
Adjusted R-Squared		24.980%	

Panel C	**Granger Causality Test**	
	dTerm (Predictor)	**Ambiguity (Predictor)**
dTerm	-	1.269
Ambiguity	2.644***	-

Figure 4.4 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure Term (*dterm*) and Original Ambiguity Measure (*am*)



Notes: *dterm* represents the first-differenced value of the term structure measure *Term*; *am* represents the original ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

4.7.2. Ambiguity and Default Risks

Table 4.9 shows the interaction between the ambiguity measure and *dDef*. In Panel A, the result indicates that *dDef* is autoregressive in lags 1, 4 and 6 at the 1% significance level, and lag 7, the 5% level. Past values of the ambiguity measure do not appear to have a statistically significant impact on future value of *dDef*.

Panel B shows that the ambiguity measure is autoregressive in lags 1, 2 and 8 at the 1% significance level, and lags 4, 7 and 9, the 5% level. Similar to previous results, lags 1, 2 and 8, which have positive coefficients, leave a stronger impact on the future value of the ambiguity measure than lags 4 and 7, which have negative coefficients. Lag 4 of *dDef* has a positive impact on the future value of the ambiguity measure at the 1% significance level, and lags 6 and 9, the 10% level.

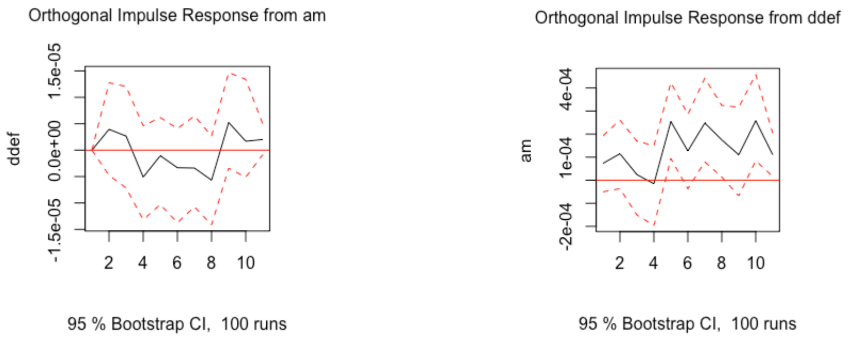
Results of the Granger-causality test in Panel C show that *dDef* can Granger-cause the ambiguity measure at the 5% significance level while the ambiguity measure does not seem to Granger-cause *dDef*. The OIRF plots in Figure 4.5 indicate that *dDef* does not respond to a positive shock to the ambiguity measure while the ambiguity measure increases around periods 5, 7 and 10 in response to a positive shock to *dDef* at the 5% significance level. This implies that a shock to the UK economy that makes investors more worried about the future default risk can lead to an increase in the degree of ambiguity of the stock market in the future while a shock that makes investors feel safer about the future default can result in a decreased degree of ambiguity.

Table 4.9 Interaction between First-Differenced Term Structure Measure Def (*dDef*) and Ambiguity Measure (*Ambiguity*). This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *dDef* represents the first-differenced value of the term structure measure *Def*; *Ambiguity* represents the original ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	-0.140	-5.846***
	2	-0.020	-0.845
	3	0.033	1.386
	4	0.072	2.981***
	5	0.020	0.830
	6	0.070	2.848***
	7	0.060	2.471**
	8	0.002	0.093
	9	0.042	1.720*
Ambiguity	1	0.001	0.799
	2	0.001	0.444
	3	-0.002	-1.266
	4	-0.001	-0.331
	5	-0.001	-0.365
	6	-0.001	-0.407
	7	-0.001	-0.809
	8	0.002	1.386
	9	0.001	0.666
Constant		0.000	-1.409
Trend		0.000	1.558
Adjusted R-Squared		2.670%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	0.470	1.177
	2	-0.061	-0.151
	3	-0.286	-0.711
	4	1.169	2.918***
	5	0.506	1.244
	6	0.767	1.890*
	7	0.402	0.989
	8	-0.061	-0.149
	9	0.771	1.915*
Ambiguity	1	0.245	10.247***
	2	0.306	12.551***
	3	0.013	0.500
	4	-0.050	-1.976**
	5	-0.023	-0.905
	6	0.029	1.138
	7	-0.054	-2.131**
	8	0.141	5.812***
	9	0.054	2.279**
Constant		0.001	3.828***
Trend		0.000	-0.961
Adjusted R-Squared		24.780%	

Panel C	**Granger Causality Test**	
	dDef (Predictor)	**Ambiguity (Predictor)**
dDef	-	0.743
Ambiguity	2.175**	-

Figure 4.5 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure Def (*ddef*) and Ambiguity Measure (*am*)



Notes: *ddef* represents the first-differenced value of the term structure measure *Def*; *am* represents the original ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

4.8. Discussion and Conclusion

The findings from the analysis between the ambiguity measure and market return suggest that the interaction between the ambiguity measure and market return and the interaction between the ambiguity measure and excess market return are statistically weak. As such, investors may not necessarily be compensated for bearing ambiguity, which provides empirical evidence to support ambiguity pricing models of heterogeneity. The implication for investors is that they should not participate the market if they perceive that the information is of poor quality or the financial markets are faced with ambiguity.

Nevertheless, weak interaction between the ambiguity measure and excess market return does not necessarily mean that ambiguity does not have an impact on equity premium. According to the interaction between the ambiguity measure and the volatility index, an unexpected increase in the degree of ambiguity can lead to perceptions of more volatile financial markets, which generates excess volatility in the short run and can in turn affect the equity premium in the long run (Kim et al., 2004). This seems consistent with the result of Condie (2008), who mathematically proves that ambiguity can have affect asset prices in the long run under heterogeneity settings. Therefore, theories of ambiguity asset pricing can help to explain the equity premium puzzle, as is illustrated by Epstein and Schneider (2010). On the other hand, the findings also suggest that similar to return, volatility can also be ambiguous.

The interactions between the ambiguity measure and the two term structure measures suggest that macroeconomic conditions can affect the degree of ambiguity of the equity market. If investors are more optimistic about the future economic state, the degree of ambiguity of the equity market tends to decrease while if they are more pessimistic, the degree of ambiguity tends to increase. On the other hand, if investors are more worried about future default risks, the degree of ambiguity of the equity market would increase while if they are less worried about future default risks, the degree of ambiguity would decrease. As such, similar to theoretical evidence, empirical results also suggest that ambiguity literature can help to explain why investors respond differently when they are faced with good news and bad news about macroeconomic conditions. Bad news on the future macroeconomic conditions can result in subsequent increases in both market volatility (Fostel and Geanakoplos, 2012) and the degree of ambiguity of the equity market and hence have an amplified impact on the market volatility in the short run, which might in turn affect equity premia in the long run, depending on the extent of the degree of ambiguity that is affected. Although good news regarding the future economic conditions can contribute to a decrease in the degree of ambiguity, it might not necessarily lead to a decrease in the market volatility and hence the effect of decreasing degree of ambiguity may cancel out with an increasing market volatility.

This chapter contributes to the ambiguity literature by providing empirical evidence that verifies theoretical studies. The findings support the theoretical results that ambiguity can help to explain the equity premium puzzle and the excess volatility puzzle, and that bad news can amplify the impact of ambiguity. On the other hand, the empirical evidence seems to favour the heterogeneous agent models of ambiguity. Last but not least, existing literature mainly focuses on ambiguity in asset returns while the empirical evidence from this chapter suggests that volatility, which is the second moment of stock price, can also be ambiguous.

Chapter 5. A Dispersion Measure of Ambiguity: Evidence from The UK and US Stock Market

5.1. Introduction

Ambiguity is believed to have contributed to the 2008 financial crisis. The empirical study of Boyarchenko (2012) shows that an increase in ambiguity could statistically explain the increase in the credit default swap (CDS) spreads, which leads to the 2008 financial crisis. Dimmock et al. (2016) investigate market participation of households under ambiguity. They find that investors under-diversified their portfolios due to ambiguous information and the 2008 financial crisis could be contributed to ambiguity aversion. However, the impact of ambiguity in the equity market before and after the 2008 financial remains unclear. As such, this chapter attempts to investigate the effect of ambiguity on the UK and US stock markets as well as its role in the 2008 financial crisis.

The study develops a new approach of measuring ambiguity based on the gap between highest intraday price and lowest intraday price, which is one of the main contributions of this chapter. As such, it also uncovers whether the empirical results from the previous chapter are consistent with empirical evidence obtained by applying a new approach. The US data are added to check whether the empirical evidence is applicable to other stock markets. The econometric method used in this chapter is the vector autoregressive (VAR) model, which is the same as the model applied in the previous chapter. Following the same logic of the previous chapter, the interaction between ambiguity and market return, ambiguity and implied volatility and ambiguity and two term structure measures are investigated. The empirical evidence verifies the results from the previous chapter. With evidence from both the UK and US stock markets, the ambiguity measure does not seem to interact with market return. Instead, it interacts with investors' expectation on future market volatility, which is measured by the volatility index. In addition, expectations of future economic conditions and future default risks can affect the degree of ambiguity.

After the full sample is investigated, the sample is divided into pre-crisis sub-sample and post-crisis sub-sample. The post-crisis results are quite consistent with the full-sample results while the pre-crisis results show differences. However, the main differences lie in the interactions between the ambiguity measure and the two term structure measures. The results suggest that investors were unaware of ambiguity before the crisis and existence of ambiguity contributed to the economic bubble. The increasing degree of ambiguity contributed to the collapse of the financial markets in the end. As such, consistent to existing literature, ambiguity plays an important role in the 2008 financial crisis and one of the major contributions is that a detailed explanation of how ambiguity contributed to the crisis is provided.

The rest of the chapter is structure as follows. Section 5.2 shows the development of the new empirical ambiguity measure. Section 5.3 depicts the data used in the empirical analyses. Section 5.4 briefly describes the methodology. Section 5.5 – 5.7 illustrate the empirical results from the full-sample period, the pre-crisis period and the post-crisis period respectively. In the end, a discussion of the findings and result and a short conclusion are provided in Section 5.8.

5.2. Ambiguity Measure

The ambiguity measure is based on the multiple-prior preference model, under which investors make decisions based on the worst-case scenario. The worst cases of investors who are in long positions are different from the worst cases of those who are in short positions. Therefore, deviations from the reference model can be two-sided because of position differences. Now let's turn to the derivation of our empirical ambiguity measure. We can start with the simplest case where there is no price fluctuation during the day. Then the intraday highest price, denoted by p_t^H, would be the higher price between the closing price yesterday and the closing price today, which can be expressed as:

$$p_t^H = Max\{p_{t-1}, p_t\} \quad 5.1$$

where p_t^H represents the intraday highest price; and p_t represents the closing price at day t.

With similar logic, the intraday lowest price would be the lower price between the closing price yesterday and the closing price today, which can be expressed as:

$$p_t^L = Min\{p_{t-1}, p_t\} \quad 5.2$$

where p_t^L represents the intraday lowest price; and p_t represents the closing price at day t.

As such, the gap between the intraday highest price and intraday lowest price, denoted by p^{HL}, could be characterised by:

$$p_t^{HL} = |p_t - p_{t-1}| \quad 5.3$$

where p_t^{HL} represents the gap between the intraday highest and lowest prices at day t; and p_t represents the closing price at day t.

Assume that price changes follow a random walk, then p_t^{HL} should also follow a random process according to Equation 5.3, and hence it should not show any patterns over time. Assuming the reference model of stock price does not change within a short period, say a day, p_t^{HL} should represent the deviation from the reference model, namely a result of ambiguity and ambiguity aversion, if there is no volatility. Now suppose that there is volatility, then patterns of p^{HL} can arise from both ambiguity and expectations of volatility. As such, p_t^{HL} in excess of implied volatility can be a good potential measure of ambigutiy. This empirical measure is also theoretically motivated by Epstein and Schneider (2008), who show that price fluctuation is a reflection of ambiguity aversion. From this perspective, p^{HL} can be used to develop an empirical measure of ambiguity. In short, the intuition behind the empirical measure of ambiguity in this chapter is that assume investors' preference models do not change for a short period, which is assumed to be at least one day in this study, then the intraday high price indicates, to some extent, the highest price that an investor can accept as a result of a deviation from her reference model during the day, and the intraday lowest price indicates, to some extent, the lowest price that an investor can accept as a result of a deviation from her reference model. Hence, change in p^{HL} can reflect distortions from the reference model due to ambiguity and ambiguity aversion. However, since the multiple-prior model does not differentiate between level of ambiguity and ambiguity aversion, this empirical measure is also a combination of level of ambiguity and ambiguity aversion. For illustration purposes, level of ambiguity and ambiguity aversion will be named as degree of ambiguity and it is necessary to emphasise that when degree of ambiguity is mentioned in this study, it refers to both the level of ambiguity and the extent of ambiguity aversion.

Furthermore, it is also worth to mention that p^{HL} is not yet the ambigutiy measure that is used in this study. It is further developed to accommodate statistical issues and implied volatility, which is explained in the methodology section.

5.3. Data

Adjusted closing prices of FTSE100 and S&P500 are collected in daily frequency, which are used as proxies for the market return of the UK and US stock markets. The closing prices are converted into daily returns using the following formula:

$$r_t = ln\frac{P_t}{P_{t-1}} \quad 5.4$$

where r_t represents the daily return at time t; and P_t is the closing price at time t.

In addition, intraday low prices and intraday high prices of the indices are collected to construct the ambiguity measures of the UK and US stock markets. As is mentioned in the previous section, the volatility indices of FTSE100 and S&P500 are used as measures of implied volatilities of the UK and US stock markets.

Similar to the previous chapter, the interaction between ambiguity and macroeconomic conditions and the interaction between ambiguity and default risks are also investigated. Following Fama and French (1993), investors' expectations on future macroeconomic conditions, denoted by *Term*, are measured by the difference between returns of short-term government bond (with maturity of 1 month) and long-term government bond (with maturity of over 10 years), and investors' perceptions about future default risks, denoted by *Def*, are measured by the difference between returns of long-term government bond (with maturity of over 10 years) and long-term corporate bond (with maturity of over 10 years).

The sample period is from March 1st, 2004 to June 30th, 2016 for the UK and from November 14th, 2007 to February 28th, 2018 for the US. All the data are collected in daily frequency from DataStream. As such, there are 3,117 observations in the UK sample and 2,590 observations in the US sample. Table 5.1 shows the summary statistics of the variables. The summary statistics indicate that the market return, return of FTSE100, is symmetrically distributed and shows some negative skewness. The UK ambiguity measure has an approximately symmetric distribution centred around zero. On the other hand, the S&P500 index return also has a zero-mean with negative skewness, and the ambiguity measure has a zero-mean with positive skewness, which is quite similar to the statistics of the US ambiguity measure that is calculated using a different method and different data in Antoniou et al. (2015). In comparison, the returns, ambiguity measures, volatilities and two term structure measures of the two stock markets are quite similar. In addition, although for the two markets the summary statistics of p_t^{HL} and $\ln p_t^{HL}$ are different, the summary statistics of the ambiguity measures are quite similar.

Table 5.1 Summary Statistics. This table shows the summary statistics. *Return* represents the daily market return; $\boldsymbol{p_t^{HL}}$ represents the difference between the intraday highest and lowest prices of the indices; $\boldsymbol{lnp_t^{HL}}$ represents the natural logarithm of $\boldsymbol{lnp_t^{HL}}$; *AM* represents the ambiguity measure; *Volatility* represents the daily market volatility index; and *Term* and *Def* represent the two term structure measures of daily frequencies.

	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
UK Stock Market						
Return	0.000	0.013	-0.095	0.110	-0.354	10.838
$\mathbf{p_t^{HL}}$	18.694	12.646	3.680	125.220	2.711	12.079
$\mathbf{lnp_t^{HL}}$	2.764	0.556	1.303	4.830	0.317	0.113
AM	0.000	0.493	-1.529	1.868	0.202	0.021
Volatility	21.585	8.504	11.850	69.240	1.942	4.706
Term	0.023	0.007	0.005	0.038	0.271	-0.749
Def	0.028	0.007	0.015	0.058	1.994	4.808
US Stock Market						
Return	0.000	0.013	-0.095	0.110	-0.354	10.838
$\mathbf{p_t^{HL}}$	18.694	12.646	3.680	125.220	2.711	12.079
$\mathbf{lnp_t^{HL}}$	2.764	0.556	1.303	4.830	0.317	0.113
AM	0.000	0.493	-1.529	1.868	0.202	0.021
Volatility	21.585	8.504	11.850	69.240	1.942	4.706
Term	0.023	0.007	0.005	0.038	0.271	-0.749
Def	0.028	0.007	0.015	0.058	1.994	4.808

To investigate the roll of ambiguity before and after the 2008 financial crisis, the full-sample period is further divided into pre-crisis period and post-crisis period. However, the exact time of when the impact of ambiguity started to take effect on the crisis is not clear and hence data before 2009 are treated as data of the pre-crisis period and the rest are treated as data of the post-crisis period.

5.4. Methodology

As is mentioned before, a possible issue with the p^{HL} measure is that it can be affected by investors' expectations of future volatility levels, which is also recognised as a measure of investor sentiment (Whaley, 2000). Corrado and Truong (2007) use a squared measure of the gap between intraday highest and lowest prices to estimate stock market volatility. They compare the volatility forecasts estimated from this measure with the squared value of volatility index. The implication for this paper from their work is that the squared value of the gap between intraday highest and lowest prices can be used as an estimate of volatility, which might potentially make the effect of implied volatility interfere with the impact of the ambiguity measure. Therefore, the effect of implied volatility should be removed from p_t^{HL}. Epstein and Schneider (2010) show that ambigutiy aversion has a first-order effect on asset returns and hence power one form of the gap between intraday highest and lowest prices is used in this study instead of power two. Natural logarithm conversion is applied to the gap between intraday highest and lowest prices to adjust for the positive skewness of p^{HL}. Then the following linear regression is used to remove the effect of implied volatility from lnp_t^{HL}, where volatility indices developed by stock exchanges, namely implied volatility measures, are used as proxies for investors' expectations on future volatility. Thus, the ambiguity measure used in this study is the residual from the following regression model:

$$lnp_t^{HL} = \beta_0 + \beta_1 Volatility_t + \varepsilon_t \tag{5.5}$$

where lnp_t^{HL} is the natural logarithm of p^{HL} at day t; and $Volatility_t$ is the volatility index at day t. Since time-series data normally have problems of heteroscedasticity, the linear regression is corrected for heteroscedasticity using Newey-West heteroskedasticity and autocorrelation (HAC) standard errors.

Since the purpose of this paper is to uncover the interactions between ambiguity and the variables of interest and hence the VAR regression model is applied, which is characterised by the following equation:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \epsilon_t \tag{5.6}$$

where y_t is a vector of stationary time-series of interest; β's are vectors of coefficients in the regression model; p is the order of the lags included in the model, which is determined by information criteria; and ϵ_t is a vector of error terms. The Augmented Dickey-Fuller (ADF) test is used to ensure the time-series data are stationary before the VAR analyses. If the ADF test indicates that the time series is non-stationary, difference will be taken until the series is stationarity. Information criteria that are used to select lags for the VAR model include the Akaike information criterion (AIC), the Hannan-Quinn information criterion (HQIC), the Schwarz's Bayesian information criterion (SC or SBIC) and the final prediction error (FPE). In order to prevent any significant results from being neglected due to model selection, the maximum number of lags of the four criteria will be included in the VAR model if they indicate different numbers of lags.

Similar to the previous chapter, both Granger-Causality test and orthogonalised impulse response function (OIRF) are used to investigate the interactions between the ambiguity measure and variables of interest. The 95%

confidence interval of an OIRF is constructed using the wild bootstrap method, developed by Wu (1986), to accommodate any heteroskedasticity that might occur in time-series data.

5.5. Empirical Results of Full-Sample Period

This section illustrates the empirical findings of the full samples. The development of the analyses begins with the findings of the UK stock market followed by the US stock market.

5.5.1. Ambiguity and Market Returns

Table 5.2 shows the regression result of FTSE100 and S&P500 returns on the ambiguity measures *AM*. The result of the UK stock market shows that the coefficient of *AM* is significant at the 10% significance level. The negative coefficient indicates that an increase in the ambiguity measure is associated with a decrease in the market return. However, this linear relationship is not statistically significant at the 5% level. The result of the US stock market indicates that market return has a negative linear relationship with the ambiguity measure at the 1% significance level.

Table 5.2 Regression of Market Returns on Ambiguity Measure (*AM*) with Newey-West Standard Errors. This table shows the regression results of the market returns on the ambiguity measure with Newey-West standard errors. *AM* represents the ambiguity measures; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

	Estimate	t-Value
UK Stock Market		
AM	-0.001	-1.830*
Intercept	0.000	0.702
US Stock Market		
AM	-0.001	-2.689***
Intercept	0.000	1.109

5.5.1.1. Evidence from UK Stock Market

The interaction between market return and the ambiguity measure of the UK stock market is displayed in Table 5.3. The result of the return equation in Panel A indicates that market return is autoregressive in lags 2 and 5 at the 1% significance level, lags 1 and 3, the 5% level, and lag 4, the 10% level. Lag 3 of the ambiguity measure is significant at the 5% level, which has a positive impact on the future value of market return. Lag 1 leaves a negative impact, but the significance of the effect is weak, which is the 10% level.

Panel B shows that the ambiguity measure is autoregressive in all the lags at the 1% significance level except that lag 7 is significant at the 5% level. It is noticeable that all the coefficients of the lags are positive, which suggests that past ambiguous information can affect future level of ambiguity and ambiguity aversion. This accumulative characteristic of the ambiguity measure also implies that ambiguous signals and poor-quality information can affect the confidence of investors, which makes them assign a higher weightage on the worst-case scenario for the next few periods and in turn results in a higher degree of ambiguity in the future. In addition, the positive trend of the ambiguity measure equation, which is significant at the 1% level, also confirms the accumulative characteristic of ambiguity and implies that this accumulation process can last for a long time until a market crash occurs. On the other hand, lag 1, lag 2 and lag 3 of market return have negative impacts on the future value of the ambiguity measure, which are significant at the 1% level. The negative coefficients suggest that decreases in past returns result in an increase in the future value of the ambiguity measure. This seems to imply that bad news that leads to a bear stock market can contribute to a higher degree of ambiguity. However, this result has to be verified by the sub-sample analyses because stock market "freezes" when a financial crisis occurs (Epstein and Schneider, 2010) and hence market behaviour near a financial crisis could be quite different from that of non-crisis periods.

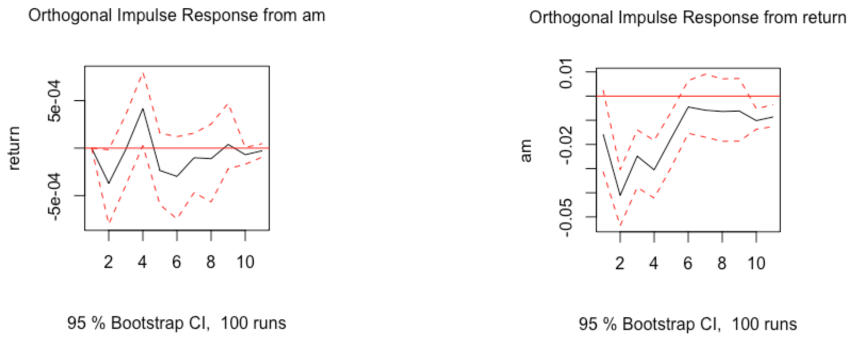
Results of the Granger-causality test in Panel C indicate that the market return can Granger-cause the ambiguity measure at the 1% significance level while the ambiguity measure does not seem to Granger-cause the market return. This is consistently shown in the orthogonalised impulse response function (OIRF) plots in Figure 5.1. In the plots, the zero line falls within the 95% confidence interval in almost all the periods, indicating that return does not tend to respond to a shock in the ambiguity measure at the 5% significance level. This implies that investors may not necessarily be compensated for bearing ambiguity, which suggests that empirical evidence is in favour of the heterogeneous agent models of ambiguity. On the other hand, the ambiguity measure firstly decreases in response to a positive shock in the market return and then increases at the 5% significance level, followed by another decrease starting from period 10. This result can be related to the negative lags of market return and the positive trend in the ambiguity measure equation, which together implies that bad news that leads to a bear market can result in a higher degree of ambiguity and such results can accumulate and may not have an immediate effect on the stock market return until a market crash occurs.

Table 5.3 Interaction between FTSE100 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between FTSE100 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). Return represents the daily return of FTSE100; AM represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.044	-2.453**
	2	-0.054	-2.972***
	3	-0.045	-2.468**
	4	0.032	1.793*
	5	-0.059	-3.215***
	6	-0.029	-1.587
	7	0.006	0.334
	8	0.009	0.480
AM	1	-0.001	-1.757*
	2	0.000	0.017
	3	0.001	2.087**
	4	-0.001	-1.076
	5	-0.001	-1.209
	6	0.000	-0.403
	7	0.000	-0.499
	8	0.000	0.564
Constant		0.000	-0.061
Trend		0.000	0.390
Adjusted R-Squared		0.949%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-3.372	-6.053***
	2	-1.788	-3.183***
	3	-2.114	-3.748***
	4	-0.829	-1.466
	5	0.642	1.134
	6	0.354	0.626
	7	0.394	0.698
	8	0.361	0.642
AM	1	0.091	5.037***
	2	0.113	6.131***
	3	0.100	5.528***
	4	0.104	5.730***
	5	0.077	4.260***
	6	0.085	4.722***
	7	0.041	2.291**
	8	0.063	3.534***
Constant		-0.065	-4.535***
Trend		0.00004	5.275***
Adjusted R-Squared		24.110%	

Panel C	**Granger Causality Test**	
	Return (Predictor)	**AM (Predictor)**
Return	-	1.388
AM	7.307***	-

Figure 5.1 Plots of Orthogonalised Impulse Response Function (OIRF) of FTSE100 Return (*return*) and Ambiguity Measure (*am*)



Notes: *return* represents the daily return of FTSE100; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

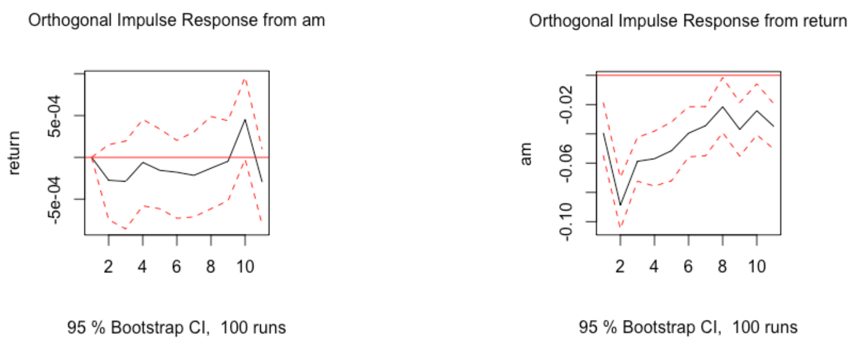
5.5.1.2. Evidence from US Stock Market

The interaction between market return and the ambiguity measure of the US stock market is shown in Table 5.4 and Figure 5.2. The results are similar to those of the UK market. Past values of the ambiguity measure do not seem to have a significantly strong impact on the US market return, suggesting that investors are not compensated for bearing ambiguity. The impact of past market returns on the future ambiguity measure of the US seems stronger than that of the UK. However, the behaviours are similar, implying that increased degree of ambiguity due to bad news that leads to downward movements of the market can accumulate and do not take effect until a stock market crash happens.

Table 5.4 Interaction between S&P500 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between S&P500 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). Return represents the daily return of S&P500; AM represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.104	-5.244***
	2	-0.081	-3.992***
	3	0.009	0.427
	4	-0.034	-1.626
	5	-0.063	-3.066***
	6	-0.002	-0.108
	7	-0.037	-1.787*
	8	0.015	0.743
	9	-0.023	-1.139
	10	0.031	1.542
AM	1	-0.001	-1.089
	2	-0.001	-1.097
	3	0.000	-0.081
	4	0.000	-0.310
	5	0.000	-0.401
	6	0.000	-0.632
	7	0.000	-0.207
	8	0.000	0.184
	9	0.001	2.178**
	10	-0.001	-0.920
Constant		-0.001	-1.262
Trend		0.000001	2.102**
Adjusted R-Squared		1.786%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-6.540	-10.484***
	2	-3.791	-5.920***
	3	-3.116	-4.815***
	4	-2.005	-3.088***
	5	-1.102	-1.693*
	6	-0.893	-1.372
	7	0.387	0.596
	8	-1.115	-1.719*
	9	0.160	0.248
	10	-0.721	-1.136
AM	1	0.139	6.999***
	2	0.170	8.478***
	3	0.110	5.440***
	4	0.071	3.464***
	5	0.043	2.089**
	6	0.050	2.466**
	7	0.029	1.421
	8	0.033	1.618
	9	0.044	2.202**
	10	0.051	2.633***
Constant		-0.048	-2.927***
Trend		0.00004	3.659***
Adjusted R-Squared		33.980%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**AM (Predictor)**
Return		-	0.862
AM		14.348***	-

Figure 5.2 Plots of Orthogonalised Impulse Response Function (OIRF) of S&P500 Return (*return*) and Ambiguity Measure (*am*)



Notes: *return* represents the daily return of S&P500; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.5.2. Ambiguity and Implied Volatilities

This section explains the interactions between the ambiguity measure and investors' expectations on volatilities of the UK and US stock markets.

5.5.2.1. Evidence from UK Stock Market

Table 5.5 shows the interaction between the ambiguity measure and the implied volatility of the UK stock market. Panel A shows that the volatility index is autoregressive in lags 1, 2 and 6 at the 1% significance level, and lags 7 and 8, the 5% level. Lag 1 of the ambiguity measure has a positive impact on the implied volatility of the next period at the 5% significance level and lag 3 has a negative impact at the 10% level. If the 5% significance level is used as the criterion for statistical inference, past ambiguity measure seems to have a positive impact on the future implied volatility of the UK stock market.

Panel B shows that past values of the ambiguity measure are accumulative with statistically significant lags and a positive trend, which is similar to the result shown in the ambiguity measure equation of the previous section. Lag 1 of the volatility index has a positive impact and lag 4, a negative one, on the future ambiguity measure at the 1% significance level. However, the overall effect is positive, as is evident from the OIRF plots in Figure 5.3.

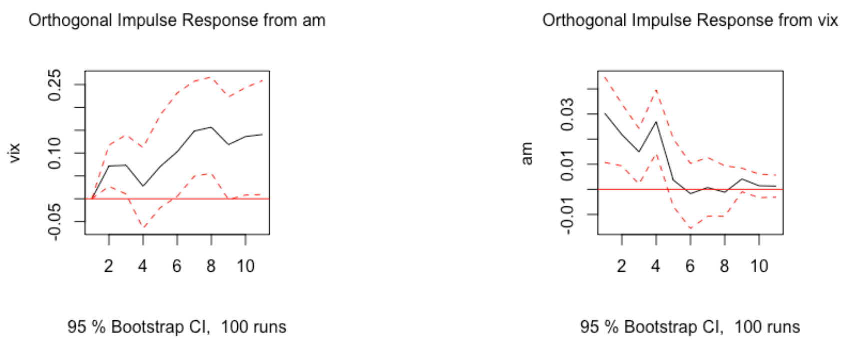
The results of the Granger-causality test in Panel C indicate that the ambiguity measure can Granger-cause the volatility index at the 5% significance level and the volatility index can Granger-cause the ambiguity measure at the 1% significance level. The OIRF plots indicate that the volatility index increases in response to a positive shock in the ambiguity measure from period 2 to 3 and 7 to 9 at the 5% significance level. This period-by-period increase in volatility index in response to a shock in the ambiguity measure can be interpreted as a result of heterogeneous investors and timing. An increase in ambiguity changes the prior beliefs of investors, making them assign a higher weight to the worst case, which results in distortions from the reference model. The distance of the distorted model from the reference model depends on the extent of ambiguity aversion. Since investors take actions at different points of time and their extents of ambiguity aversion are also different, expectations on volatility would respond to the shock across different periods. As is illustrated in the previous chapter, since variations in volatility index is found to be related to stock returns and premiums (Kim et al., 2004), a shock to the degree of ambiguity can indirectly have an impact on stock returns, leading to excess volatility in the short run and a high equity premium in the long run. The OIRF plots also show that the ambiguity measure increases immediately in response to a positive shock in the volatility index at the 5% significance level, which implies that the ambiguity measure is an effective way of capturing ambiguity because it adjusts to investors' expectations. In addition, this result also suggests that volatility can also be ambiguous since changes in investors' expectations on volatility contribute to the degree of ambiguity.

Table 5.5 Interaction between UK Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between UK market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). *Volatility* represents the daily volatility index of FTSE100; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.863	47.835***
	2	0.071	2.987***
	3	0.009	0.396
	4	-0.001	-0.034
	5	0.037	1.560
	6	-0.089	-3.753***
	7	0.054	2.262**
	8	0.043	2.343**
AM	1	0.197	2.429**
	2	0.013	0.157
	3	-0.139	-1.704*
	4	0.100	1.218
	5	0.089	1.083
	6	0.111	1.359
	7	0.021	0.264
	8	-0.120	-1.490
Constant		0.314	3.391***
Trend		0.000	-0.952
Adjusted R-Squared		96.220%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.011	2.866***
	2	-0.004	-0.818
	3	0.006	1.229
	4	-0.015	-2.753***
	5	-0.005	-0.878
	6	0.001	0.144
	7	0.000	0.084
	8	0.003	0.698
AM	1	0.096	5.342***
	2	0.117	6.473***
	3	0.098	5.399***
	4	0.099	5.462***
	5	0.077	4.213***
	6	0.087	4.779***
	7	0.040	2.228**
	8	0.065	3.598***
Constant		-0.037	-1.809*
Trend		0.00004	5.277***
Adjusted R-Squared		23.390%	

Panel C	**Granger Causality Test**	
	Volatility (Predictor)	**AM (Predictor)**
Volatility	-	2.031**
AM	3.578***	-

Figure 5.3 Plots of Orthogonalised Impulse Response Function (OIRF) of UK Market Volatility Index (*vix*) and Ambiguity Measure (*am*)



Notes: *vix* represents the daily volatility index of FTSE100; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.5.2.2. Evidence from US Stock Market

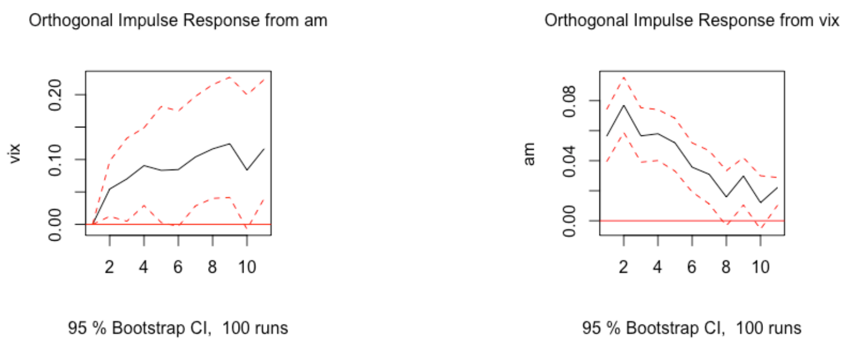
The interaction between the ambiguity measure and the US volatility index is shown in Table 5.6. The results in Panel A are similar to those of the UK market. However, as is show in Panel B, the positive trend of the ambiguity equation is no longer significant. This can be attributed to the complexity of the US stock market. Besides the 2008 financial crisis, several market crashes also happened in the full-sample period. As such, the trend might be flatted by the crashes. Thus, since the ambiguity measure is still autoregressive, the implication behind is the same as that of the UK market, which is that the degree of ambiguity can accumulate and does not have an impact on the market movement until a crash occurs.

Results of the Granger-causality test are different. The volatility index can Granger-cause the ambiguity measure at the 1% significance level while the ambiguity measure does not seem to Granger-cause the volatility index in the US case. This could arise due to complexity of the US stock market. Nevertheless, the results of the OIRF plots in Figure 5.4 are similar to those of the UK market. As such, overall, the findings from the US market are similar to those from the UK market. Qadan et al. (2018) find that an increase in the volatility index is accompanied by a negative relationship between idiosyncratic volatility and future stock returns in the US market. They interpret their findings as a result of risk aversion. However, evidence from this study suggests that this is associated with ambiguity. An increase in the volatility index either is followed by or results in increased degree of ambiguity. According to Epstein and Schneider (2010), ambiguity can result in selective participation and under-diversification because diversification cannot remove idiosyncratic ambiguity. As such, an increase in volatility index can accompany with negative relationship between future stock returns and idiosyncratic volatility, which also incorporates idiosyncratic ambiguity, because of selective participation due to ambiguity.

Table 5.6 Interaction between US Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between US market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C). Volatility represents the daily volatility index of S&P500; AM represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.859	43.038***
	2	0.049	1.887*
	3	0.051	1.932*
	4	-0.048	-1.828*
	5	0.008	0.296
	6	0.041	1.584
	7	-0.032	-1.225
	8	0.065	2.489**
	9	-0.017	-0.634
	10	0.010	0.478
AM	1	0.137	2.240**
	2	0.040	0.643
	3	0.038	0.612
	4	-0.037	-0.594
	5	-0.010	-0.156
	6	0.043	0.685
	7	0.023	0.364
	8	0.010	0.170
	9	-0.125	-2.051**
	10	0.064	1.069
Constant		0.460	3.441***
Trend		-0.0001	-2.698***
Adjusted R-Squared		97.900%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.056	8.615***
	2	-0.019	-2.198**
	3	-0.003	-0.387
	4	-0.008	-0.984
	5	-0.012	-1.392
	6	-0.004	-0.506
	7	-0.013	-1.568
	8	0.013	1.577
	9	-0.017	-2.011**
	10	0.007	1.001
AM	1	0.138	6.913***
	2	0.164	8.171***
	3	0.109	5.344***
	4	0.069	3.370***
	5	0.048	2.359**
	6	0.058	2.836***
	7	0.038	1.867*
	8	0.038	1.886*
	9	0.046	2.323**
	10	0.049	2.484*
Constant		-0.004	-0.080
Trend		0.000	1.485
Adjusted R-Squared		33.200%	
Panel C		**Granger Causality Test**	
		Volatility (Predictor)	**AM (Predictor)**
Volatility		-	1.226
AM		12.353***	-

Figure 5.4 Plots of Orthogonalised Impulse Response Function (OIRF) of US Market Volatility Index (*vix*) and Ambiguity Measure (*am*)



Notes: *vix* represents the daily volatility index of S&P500; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.5.3. Ambiguity and Macroeconomic Conditions

This section illustrates the interactions between the ambiguity measure and the expectations on future macroeconomic conditions of the UK and the US respectively.

5.5.3.1. Evidence from UK Stock Market

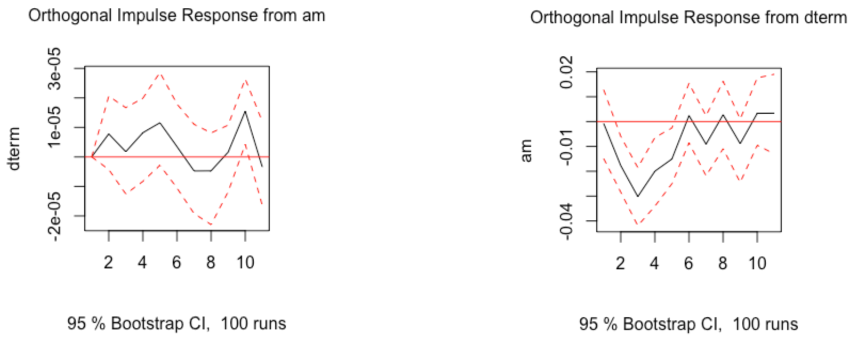
The interaction between the ambiguity measure and the first-order difference of the term spread of the UK is shown in Table 5.7. Panel A suggests that *dTerm* is autoregressive in lags 2 and 7 at the 1% significance level, and lags 1 and 3, the 10% level. Lags of the ambiguity measure do not have a significant impact on the future value of *dTerm*. Results in Panel B indicate the ambiguity measure is still accumulative and follows a positive trend over time. Lag 1 and lag 2 of *dTerm* have a negative impact on the ambiguity measure at the 1% significance level, and lag 3, the 5% level.

The result of the Granger-causality test in Panel C shows that *dTerm* can Granger-cause the ambiguity measure at the 1% significance level while the ambiguity measure cannot Granger-cause *dTerm*. The OIRF plots in Figure 5.5 suggests that the ambiguity measure decreases in response to a positive shock in *dTerm* from period 3 to 5 at the 5% significance level and *dTerm* increases in response to a positive shock in the ambiguity measure around period 10. As such, the findings imply that expectation of the future macroeconomic conditions has an inverse relationship with the future value of the ambiguity measure. A shock to the UK economy that makes investors more optimistic about the future macroeconomic conditions can lead to a decrease in the degree of ambiguity of the equity market in the following periods. This finding is similar to that of the previous chapter where a different ambiguity measure is applied.

Table 5.7 Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	0.032	1.764*
	2	-0.081	-4.493***
	3	-0.031	-1.696*
	4	0.006	0.307
	5	-0.013	-0.712
	6	-0.011	-0.594
	7	-0.059	-3.222***
	8	-0.030	-1.624
	9	0.028	1.543
	10	0.018	1.007
AM	1	0.00002	1.015
	2	0.00000	0.101
	3	0.00002	0.995
	4	0.00003	1.271
	5	0.00000	0.096
	6	-0.00002	-0.886
	7	-0.00002	-0.860
	8	0.00000	0.044
	9	0.00003	1.781*
	10	-0.00002	-0.785
Constant		0.00002	0.932
Trend		-0.00000002	-1.775*
Adjusted R-Squared		1.269%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-41.260	-2.693***
	2	-65.180	-4.249***
	3	-35.190	-2.345**
	4	-23.500	-1.521
	5	21.740	1.401
	6	-6.842	-0.441
	7	24.510	1.581
	8	-9.622	-0.620
	9	17.890	1.156
	10	16.380	1.060
AM	1	0.095	5.303***
	2	0.119	6.616***
	3	0.105	5.784***
	4	0.094	5.188***
	5	0.072	3.923***
	6	0.080	4.406***
	7	0.037	2.005**
	8	0.056	3.079***
	9	0.005	0.283
	10	0.054	2.999***
Constant		-0.060	-4.118***
Trend		0.00004	4.640***
Adjusted R-Squared		24.050%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	0.851
AM		4.341***	-

Figure 5.5 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm*) and Ambiguity Measure (*am*) – UK



Notes: *dterm* represents the first-differenced value of the term structure measure *Term*; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

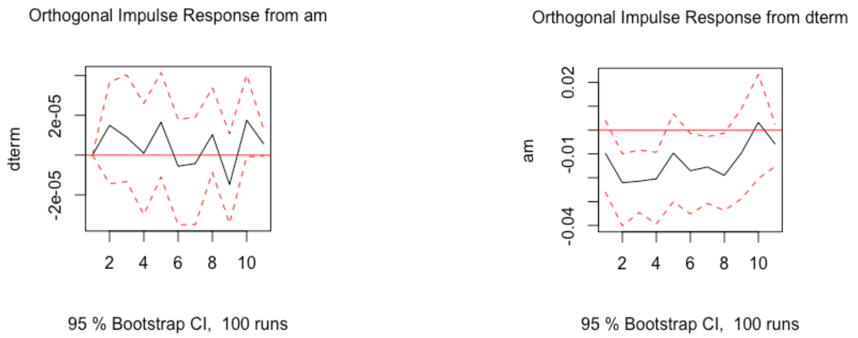
5.5.3.2. Evidence from US Stock Market

Empirical evidence from the US market is similar to that of the UK market, as can be observed by comparing the results of the UK market with the results shown in Table 5.8 and Figure 5.6. As such, it seems a general situation that a shock that makes investors more optimistic about the future economic conditions can result in a subsequent decrease in the degree of ambiguity of the equity market.

Table 5.8 Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	0.084	4.259***
	2	-0.081	-4.087***
	3	-0.028	-1.424
	4	-0.047	-2.351**
	5	-0.012	-0.622
	6	-0.009	-0.471
	7	-0.006	-0.291
	8	-0.025	-1.269
	9	-0.021	-1.089
AM	1	0.0000	1.093
	2	0.0000	0.358
	3	0.0000	-0.205
	4	0.0000	0.985
	5	0.0000	-0.800
	6	0.0000	-0.380
	7	0.0000	0.655
	8	0.0000	-1.318
	9	0.0000	1.414
Constant		0.000	1.440
Trend		0.000	-1.578
Adjusted R-Squared		1.384%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-29.250	-2.546**
	2	-19.890	-1.618
	3	-16.450	-1.410
	4	0.979	0.078
	5	-13.330	-1.192
	6	-8.422	-0.728
	7	-13.030	-1.076
	8	3.274	0.289
	9	18.980	1.410
AM	1	0.182	9.229***
	2	0.196	9.761***
	3	0.122	5.973***
	4	0.070	3.412***
	5	0.039	1.886*
	6	0.048	2.318**
	7	0.036	1.747*
	8	0.042	2.103**
	9	0.048	2.411**
Constant		-0.032	-1.949*
Trend		0.00003	2.268**
Adjusted R-Squared		30.520%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	0.799
AM		2.041**	-

Figure 5.6 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm*) and Ambiguity Measure (*am*) – US



Notes: *dterm* represents the first-differenced value of the term structure measure *Term*; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.5.4. Ambiguity and Default Risks

This section illustrates the interaction between the ambiguity measure and investors' perceptions of default risks, with evidence from the UK and US stock markets respectively.

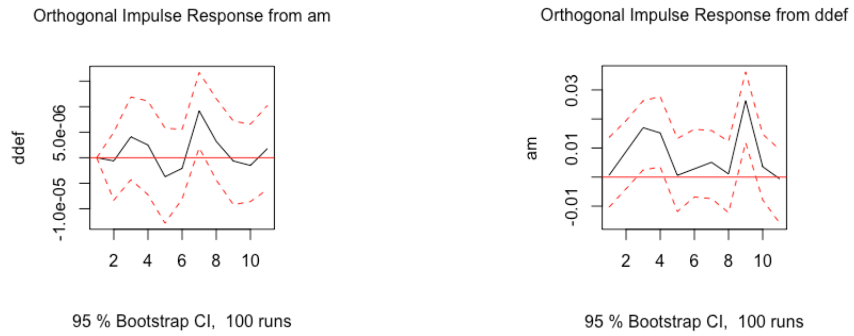
5.5.4.1. Evidence from UK Stock Market

The interaction between the ambiguity measure and the first-order difference of the default risk measure of the UK is presented in Table 5.9. Results in Panel A indicate that *dDef* is autoregressive in lags 1, 2, 6, 9 and 10 at the 1% significance level. Lag 6 of the ambiguity measure has a positive impact on *dDef* at the 5% significance level. In Panel B, lag 8 of *dDef* has a positive impact on the future value of the ambiguity measure at the 1% significance level, and lags 2 and 3, the 5% level. The ambiguity measure is again accumulative and trending upwards over time. Panel C shows that *dDef* can Granger-cause the ambiguity measure at the 1% significance level while the ambiguity measure cannot Granger-cause *dDef*. The OIRF plots in Figure 5.7 indicate that *dDef* increases in response to a positive shock in the ambiguity measure around period 7 at the 5% significance level while the ambiguity measure increases around period 4 and increases again around period 9 in response to a positive shock in *dDef*. The implication behind is that a shock to the UK economy that makes investors more worried about the default risk can lead to a subsequent increase in the degree of ambiguity of the stock market, and this is consistent with the findings from the previous chapter.

Table 5.9 Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	-0.068	-3.775***
	2	-0.049	-2.743***
	3	-0.005	-0.292
	4	0.027	1.484
	5	0.021	1.137
	6	-0.049	-2.731***
	7	-0.029	-1.591
	8	0.009	0.514
	9	0.089	4.923***
	10	0.049	2.727***
AM	1	0.00000	-0.146
	2	0.00001	0.962
	3	0.00001	0.558
	4	-0.00001	-0.942
	5	-0.00001	-0.584
	6	0.00002	2.034**
	7	0.00001	0.641
	8	0.00000	-0.258
	9	-0.00001	-0.514
	10	0.00000	0.084
Constant		0.00001	0.773
Trend		0.00000	-0.633
Adjusted R-Squared		1.779%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	36.780	1.331
	2	70.050	2.540**
	3	58.270	2.108**
	4	-9.616	-0.348
	5	-6.089	-0.219
	6	0.807	0.029
	7	-13.610	-0.490
	8	98.760	3.557***
	9	1.075	0.039
	10	-27.080	-0.973
AM	1	0.101	5.623***
	2	0.123	6.794***
	3	0.100	5.544***
	4	0.090	4.947***
	5	0.064	3.488***
	6	0.075	4.102***
	7	0.034	1.872*
	8	0.057	3.121***
	9	0.006	0.354
	10	0.055	3.086***
Constant		-0.063	-4.353***
Trend		0.00004	4.957***
Adjusted R-Squared		23.660%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	0.726
AM		2.417***	-

Figure 5.7 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef*) and Ambiguity Measure (*am*) – UK



Notes: *ddef* represents the first-differenced value of the term structure measure *Def*; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.5.4.2. Evidence from US Stock Market

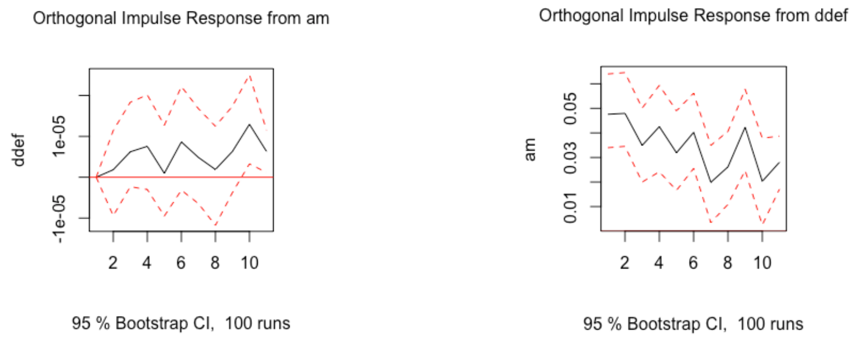
Results of the US market shown in Table 5.10 and Figure 5.8 also have the similar implication as the result of the UK market. However, it is noticeable that the response of the ambiguity measure from a shock to *dDef* of the US market is much stronger than that of the UK market. This could also be due to the complexity of the US market. As an example, the 2008 financial crisis started with the default of the US subprime mortgage market. Mortgages were securitised for sale, which made information about the asset ambiguous. A lesson from the crisis as well as the regression results would be that US regulators can prevent market crashes by closely monitoring default risks and setting restrictions on financial innovations that make information ambiguous.

Table 5.10 Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	0.107	5.367***
	2	0.058	2.870***
	3	0.104	5.197***
	4	0.030	1.493
	5	0.031	1.560
	6	0.051	2.538**
	7	0.040	2.008**
	8	-0.012	-0.585
	9	0.034	1.709*
AM	1	0.0000	0.310
	2	0.0000	0.954
	3	0.0000	0.894
	4	0.0000	-0.469
	5	0.0000	0.883
	6	0.0000	0.068
	7	0.0000	-0.385
	8	0.0000	0.380
	9	0.0000	1.466
Constant		0.0000	1.350
Trend		-0.00000001	-1.681*
Adjusted R-Squared		6.426%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	133.400	4.922***
	2	45.890	1.678*
	3	60.600	2.214**
	4	6.553	0.238
	5	41.760	1.519
	6	-36.250	-1.319
	7	-4.321	-0.158
	8	50.970	1.860*
	9	-32.100	-1.175
AM	1	0.168	8.450***
	2	0.185	9.214***
	3	0.111	5.439***
	4	0.066	3.204***
	5	0.037	1.796*
	6	0.048	2.316**
	7	0.030	1.474
	8	0.037	1.831*
	9	0.050	2.563**
Constant		-0.046	-2.738***
Trend		0.00004	3.193***
Adjusted R-Squared		31.380%	

Panel C	**Granger Causality Test**	
	dDef (Predictor)	**AM (Predictor)**
dDef	-	1.026
AM	5.027***	-

Figure 5.8 Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef*) and Ambiguity Measure (*am*) – US



Notes: *ddef* represents the first-differenced value of the term structure measure *Def*; *am* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.6. Empirical Results of Pre-Crisis Period

This section explains the empirical results of the pre-crisis period. The illustration is mainly based on comparison with the empirical results of the full sample instead of explaining into great details. It is necessary to emphasise that the point is to compare the pre-crisis results with the full-sample results for each market instead of comparing the results between the two markets because the sample period for the two markets are different. The pre-crisis period of the UK market starts from 2004 while that of the US market starts from 2007, and hence differences in the pre-crisis results between the two markets can be expected.

5.6.1. Ambiguity and Market Returns

Table 5.11 shows the linear relationships between the market returns and the ambiguity measures. As the results indicate, the negative linear relationships are not significant in the pre-crisis period whereas they are significant in the full-sample period.

Table 5.11 Pre-Crisis Period Regression of Market Returns on Ambiguity Measure (*AM*) with Newey-West Standard Errors. This table shows the regression results of the market returns on the ambiguity measure with Newey-West standard errors during the pre-crisis period. *AM* represents the ambiguity measures; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

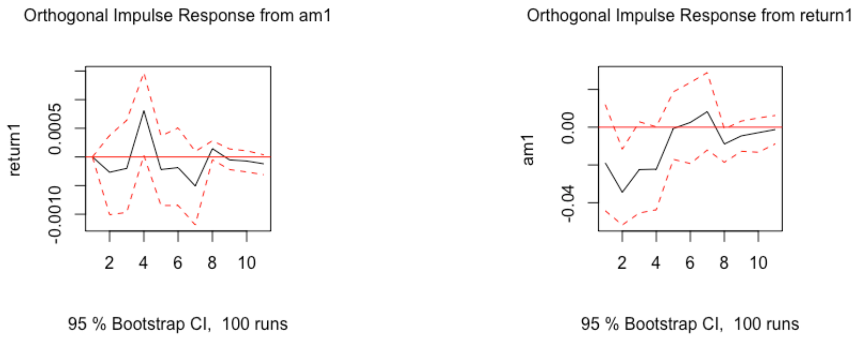
	Estimate	**t-Value**
UK Stock Market		
AM	-0.002	-1.469
Intercept	0.000	-0.037
US Stock Market		
AM	-0.004	-1.079
Intercept	-0.002	-1.730*

The interactions between the market returns and the ambiguity measures of the UK and US stock markets are shown in Table 5.12 and Figure 5.9, and Table 5.13 and Figure 5.10 respectively. The results suggest that the interactions between the market returns and the ambiguity measures during the pre-crisis period are similar to those of the full-sample period. Thus, the ambiguity measures do not seem to have an impact on market returns while decreases in market returns can lead to a subsequent increase in the ambiguity measures.

Table 5.12 Pre-Crisis Period Interaction between FTSE100 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between FTSE100 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the pre-crisis period. *Return* represents the daily return of FTSE100; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.108	-3.744***
	2	-0.095	-3.282***
	3	-0.090	-3.105**
	4	0.091	3.125***
	5	-0.092	-3.178***
	6	-0.077	-2.660***
AM	1	-0.001	-0.516
	2	-0.001	0.017
	3	0.002	2.273**
	4	-0.001	-0.645
	5	0.000	-0.351
	6	-0.001	-1.279
Constant		0.001	1.200
Trend		0.000	1.380
Adjusted R-Squared		4.839 %	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-2.497	-3.001***
	2	-1.451	-1.732*
	3	-1.359	-1.616
	4	0.488	0.582
	5	1.295	1.547
	6	1.533	1.836*
AM	1	0.153	5.340***
	2	0.115	4.004***
	3	0.114	3.947***
	4	0.135	4.676***
	5	0.132	4.553***
	6	0.077	2.669***
Constant		-0.063	-2.667***
Trend		0.0001	2.970***
Adjusted R-Squared		31.400%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**AM (Predictor)**
Return		-	2.277*
AM		3.278***	-

Figure 5.9 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of FTSE100 Return (*return1*) and Ambiguity Measure (*am1*)

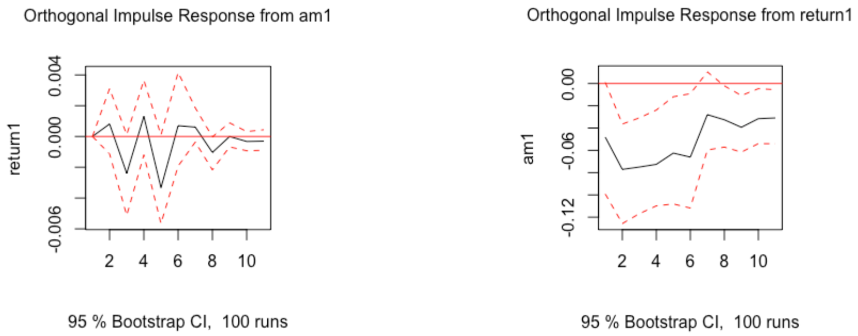


Notes: *return1* represents the daily return of FTSE100; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.13 Pre-Crisis Period Interaction between S&P500 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between S&P500 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the pre-crisis period. *Return* represents the daily return of S&P500; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.147	-2.403**
	2	-0.247	-3.962***
	3	0.047	0.732
	4	-0.144	-2.279**
	5	-0.065	-1.055
AM	1	0.002	0.571
	2	-0.006	-1.671*
	3	0.003	0.932
	4	-0.009	-2.596***
	5	0.003	0.822
Constant		0.000	0.072
Trend		0.000	-1.094
Adjusted R-Squared		9.304%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-3.012	-2.889***
	2	-2.964	-2.798***
	3	-3.160	-2.866***
	4	-1.979	-1.837*
	5	-1.888	-1.801*
AM	1	0.118	1.951*
	2	0.133	2.205**
	3	0.100	1.654*
	4	0.151	2.490**
	5	0.141	2.288**
Constant		0.022	0.436
Trend		0.000	-1.203
Adjusted R-Squared		26.450%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**AM (Predictor)**
Return		-	2.246*
AM		3.571***	-

Figure 5.10 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of S&P500 Return (*return1*) and Ambiguity Measure (*am1*)



Notes: *return1* represents the daily return of S&P500; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

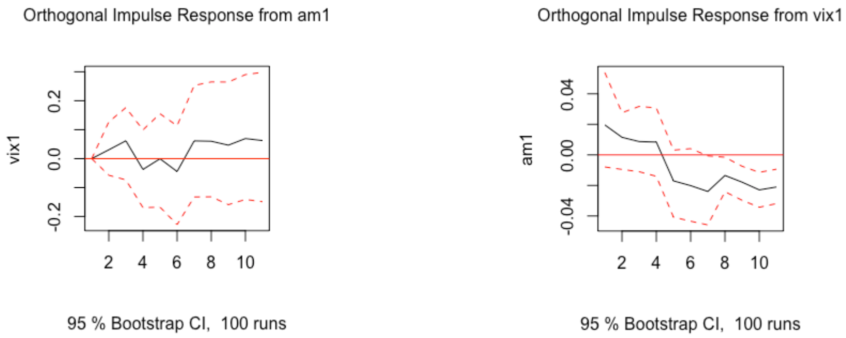
5.6.2. Ambiguity and Implied Volatilities

The interactions between the implied volatilities and the ambiguity measures of the UK and US stock markets are shown in Table 5.14 and Figure 5.11, and Table 5.15 and Figure 5.12 respectively. As is shown in the tables and figures, some results are different from those of the full-sample period. For the UK stock market, the volatility index does not respond to a shock to the ambiguity measure. In addition, instead of increasing, the ambiguity measure decreases in response to a positive shock to the volatility index at the 5% significance level. This suggests that investors did not realise the important role of ambiguity in asset pricing before the crisis. However, the US results are quite similar to those of the full sample. The difference between the two markets arises from the different pre-crisis sample period. Since the US pre-crisis period starts from 2007, the result seems to suggest that investors start to realise the existence of ambiguity from 2007.

Table 5.14 Pre-Crisis Period Interaction between UK Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between UK market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the pre-crisis period. *Volatility* represents the daily volatility index of FTSE100; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.797	27.845***
	2	0.160	4.424***
	3	0.023	0.636
	4	0.008	0.238
	5	0.085	2.332**
	6	-0.232	-6.445***
	7	0.142	4.895***
AM	1	0.084	0.607
	2	0.092	0.661
	3	-0.273	-1.960*
	4	0.068	0.484
	5	-0.112	-0.804
	6	0.273	1.962**
	7	0.040	0.288
Constant		0.131	1.138
Trend		0.000	1.332
Adjusted R-Squared		97.130%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.005	0.868
	2	-0.001	-0.131
	3	-0.001	-0.096
	4	-0.015	-2.027**
	5	-0.004	-0.473
	6	-0.002	-0.237
	7	0.008	1.392
AM	1	0.128	4.414***
	2	0.091	3.118***
	3	0.087	2.984***
	4	0.106	3.648***
	5	0.108	3.740***
	6	0.061	2.094**
	7	0.027	0.937
Constant		-0.038	-1.594
Trend		0.0003	5.935***
Adjusted R-Squared		32.560%	
Panel C		**Granger Causality Test**	
		Volatility (Predictor)	**AM (Predictor)**
Volatility		-	1.945*
AM		2.049**	-

Figure 5.11 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of UK Volatility Index (*vix1*) and Ambiguity Measure (*am1*)



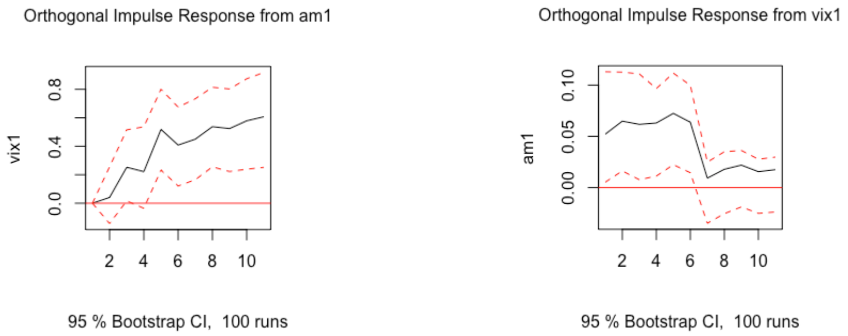
Notes: *vix1* represents the daily volatility index of FTSE100; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.15 Pre-Crisis Period Interaction between US Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between US market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the pre-crisis period. *Volatility* represents the daily volatility index of S&P500; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.838	13.624***
	2	-0.057	-0.713
	3	0.236	3.009***
	4	-0.221	-2.833***
	5	0.096	1.221
	6	0.085	1.364
AM	1	0.101	0.339
	2	0.532	1.800*
	3	-0.047	-0.159
	4	0.756	2.540**
	5	-0.337	-1.112
	6	0.151	0.499
Constant		0.225	0.710
Trend		0.004	1.838*
Adjusted R-Squared		97.510%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.030	2.354**
	2	-0.001	-0.059
	3	0.003	0.175
	4	-0.002	-0.114
	5	-0.005	-0.292
	6	-0.028	-2.219**
AM	1	0.120	1.957*
	2	0.126	2.083**
	3	0.087	1.434
	4	0.132	2.167**
	5	0.135	2.173**
	6	-0.011	-0.173
Constant		0.083	1.283
Trend		0.000	0.047
Adjusted R-Squared		26.320%	

Panel C	**Granger Causality Test**	
	Volatility (Predictor)	**AM (Predictor)**
Volatility	-	2.459**
AM	3.394***	-

Figure 5.12 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of US Volatility Index (*vix1*) and Ambiguity Measure (*am1*)



Notes: *vix1* represents the daily volatility index of S&P500; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

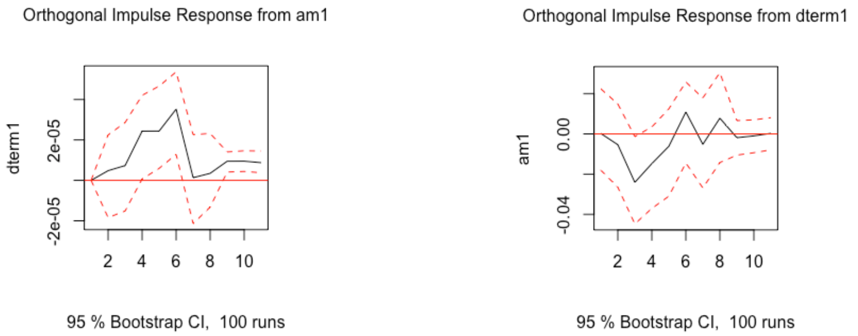
5.6.3. Ambiguity and Macroeconomic Conditions

The interactions between *Term* and the ambiguity measures of the UK and US stock markets are shown in Table 5.16 and Figure 5.13, and Table 5.17 and Figure 5.14 respectively. The results are different from those of the full-sample period. In both markets, *dTerm* increases in response to a positive shock to the ambiguity measure at the 5% significance level, which suggests that a sudden increase in the degree of ambiguity is seen as a signal of better future economic conditions. This in turn implies that ambiguity contributes to the crisis. On the other hand, the ambiguity measure does not respond to a shock to *dTerm* in the UK market and it decreases, instead of increasing, in response to a positive shock to *dTerm* at the 5% level. This suggests that investors were unware of the role of ambiguity before the crisis.

Table 5.16 Pre-Crisis Period Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market during the pre-crisis period. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	0.003	0.110
	2	-0.041	-1.423
	3	-0.022	-0.780
	4	-0.042	-1.458
	5	-0.007	-0.260
	6	-0.023	-0.788
	7	-0.058	-2.042**
AM	1	0.00001	0.444
	2	0.00002	0.601
	3	0.00006	2.101**
	4	0.00005	1.790*
	5	0.00008	2.601***
	6	-0.00003	-0.873
	7	-0.00001	-0.444
Constant		0.00006	2.317**
Trend		-0.0000001	-2.974***
Adjusted R-Squared		1.783%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-14.410	-0.511
	2	-62.090	-2.205**
	3	-28.130	-0.999
	4	-4.107	-0.146
	5	41.700	1.489
	6	-6.389	-0.228
	7	34.750	1.244
AM	1	0.157	5.461***
	2	0.115	3.969***
	3	0.109	3.759***
	4	0.122	4.187***
	5	0.126	4.302***
	6	0.072	2.463**
	7	0.045	1.557
Constant		-0.058	-2.399**
Trend		0.0001	2.654***
Adjusted R-Squared		30.830%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	2.994***
AM		1.644	-

Figure 5.13 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm1*) and Ambiguity Measure (*am1*) – UK

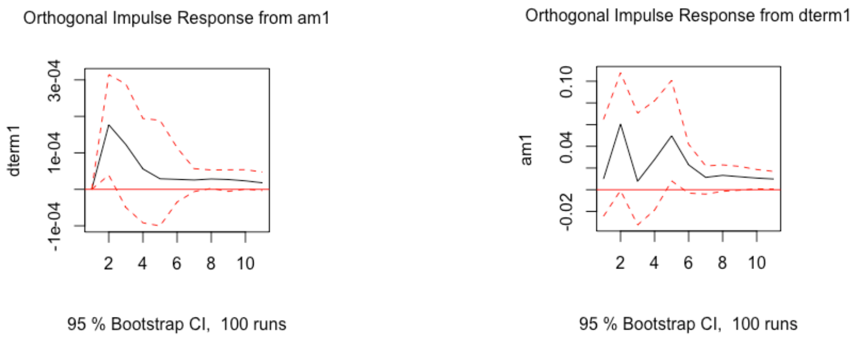


Notes: *dterm1* represents the first-differenced value of the term structure measure *Term*; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.17 Pre-Crisis Period Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market during the pre-crisis period. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**			
	Lag	**Estimate**	**t-Value**	
dTerm	1	0.241	3.982***	
	2	-0.215	-3.430***	
	3	-0.027	-0.428	
	4	-0.129	-2.114**	
AM	1	0.0004	2.183**	
	2	0.0001	0.544	
	3	0.0000	0.215	
	4	0.0000	-0.038	
Constant		0.000	1.264	
Trend		0.000	-1.075	
Adjusted R-Squared		9.507%		
Panel B	**Ambiguity Measure Equation**			
	Lag	**Estimate**	**t-Value**	
dTerm	1	43.330	2.345**	
	2	-14.930	-0.780	
	3	21.140	1.105	
	4	22.270	1.196	
AM	1	0.200	3.340***	
	2	0.155	2.561**	
	3	0.126	2.084**	
	4	0.150	2.476**	
Constant		0.005	0.108	
Trend		0.000	-0.293	
Adjusted R-Squared		22.610%		
Panel C		**Granger Causality Test**		
		dTerm (Predictor)		**AM (Predictor)**
dTerm		-		1.928
AM		1.979*		-

Figure 5.14 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm1*) and Ambiguity Measure (*am1*) – US



Notes: *dterm1* represents the first-differenced value of the term structure measure *Term*; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

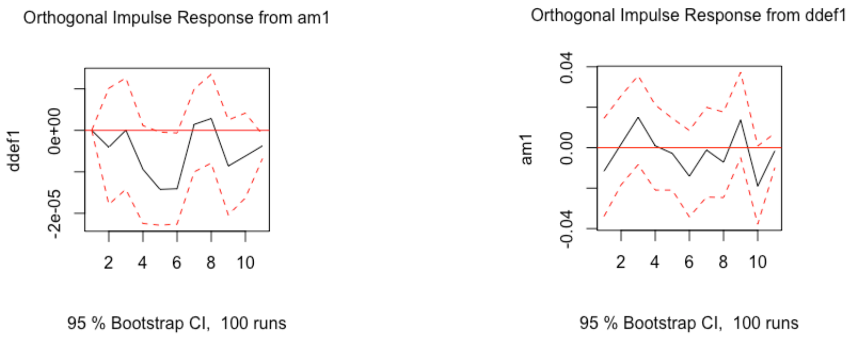
5.6.4. Ambiguity and Default Risks

Results between the ambiguity measure and *dDef* during the pre-crisis period are also different from those of the full-sample period, as can be seen from Tables 5.18 and 5.19 and Figures 5.15 and 5.16. For the UK market, default risk does not seem to play an important role in contributing to the degree of ambiguity before the crisis. However, it plays an equally important role in the US market before the crisis and for the full sample. The difference is that the ambiguity measure can Granger-cause *dDef* at the 1% significance level before the crisis, suggesting that investors started to realise that the US market did not look right from 2007.

Table 5.18 Pre-Crisis Period Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market during the pre-crisis period. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	-0.046	-1.595
	2	-0.047	-1.633
	3	-0.062	-2.139**
	4	-0.073	-2.503**
	5	-0.031	-1.075
	6	-0.070	-2.421**
	7	-0.042	-1.461
	8	0.033	1.123
	9	0.077	2.648***
AM	1	-0.00001	-0.666
	2	0.00000	0.081
	3	-0.00002	-1.467
	4	-0.00004	-2.079**
	5	-0.00003	-1.850*
	6	0.00001	0.807
	7	0.00001	0.818
	8	-0.00002	-1.023
	9	-0.00001	-0.435
Constant		-0.00004	-3.120***
Trend		0.0000001	4.600***
Adjusted R-Squared		3.260%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	16.970	0.341
	2	76.600	1.539
	3	2.140	0.043
	4	-12.670	-0.253
	5	-62.140	-1.240
	6	3.116	0.062
	7	-31.680	-0.635
	8	74.890	1.498
	9	-88.180	-1.749*
AM	1	0.160	5.564***
	2	0.114	3.903***
	3	0.107	3.642***
	4	0.106	3.622***
	5	0.111	3.785***
	6	0.062	2.091**
	7	0.034	1.169
	8	0.023	0.792
	9	0.045	1.540
Constant		-0.057	-2.276**
Trend		0.0001	2.413**
Adjusted R-Squared		31.120%	
Panel C		**Granger Causality Test**	
		dDef (Predictor)	**AM (Predictor)**
dDef		-	1.276
AM		1.201	-

Figure 5.15 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef1*) and Ambiguity Measure (*am1*) – UK



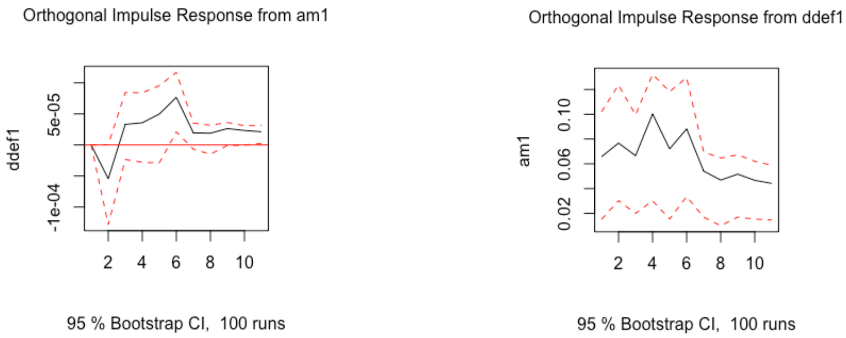
Notes: *ddef1* represents the first-differenced value of the term structure measure *Def*; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.19 Pre-Crisis Period Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market during the pre-crisis period. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	0.123	1.998**
	2	-0.080	-1.277
	3	0.131	2.075**
	4	-0.004	-0.061
	5	-0.007	-0.114
AM	1	-0.0001	-1.645
	2	0.0001	1.336
	3	0.0001	0.873
	4	0.0001	1.636
	5	0.0002	2.005**
Constant		0.000	0.491
Trend		0.000	0.836
Adjusted R-Squared		7.115%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	130.400	2.924***
	2	84.300	1.855*
	3	151.600	3.303***
	4	55.140	1.187
	5	83.490	1.815*
AM	1	0.080	1.304
	2	0.100	1.635
	3	0.068	1.124
	4	0.118	1.946*
	5	0.107	1.765*
Constant		-0.001	-0.030
Trend		0.000	-1.435
Adjusted R-Squared		28.640%	

Panel C	**Granger Causality Test**	
	dDef (Predictor)	**AM (Predictor)**
dDef	-	3.187***
AM	5.201***	-

Figure 5.16 Pre-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef1*) and Ambiguity Measure (*am1*) – US



Notes: *ddef1* represents the first-differenced value of the term structure measure *Def*; *am1* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

5.7. Empirical Results of Post-Crisis Period

The purpose of this section is to illustrate the empirical results of the post-crisis period. Again, the key point of this section is to figure out any differences between the post-crisis results and the full-sample results. Since the post-crisis results are quite similar to the full-sample results for both the UK and the US stock markets, the results are not further illustrated. Hence, tables and figures that show the results of the post-crisis period are put in the appendices for reference purposes.

5.8. Discussion and Conclusion

Overall, results of the ambiguity measure are quite consistent with the results of the previous chapter. Empirical findings of the post-crisis period are similar to those of the full-sample period. The pre-crisis results are different from the full-sample results and post-crisis results. However, the interactions between the ambiguity measure and market returns are similar. As such, ambiguity does not seem to have an impact on market returns, which can be rationalised under heterogeneous agent models of ambiguity. Thus, investors who are more ambiguity-averse can simply choose not to participate the market, leaving those who are less ambiguity-averse continue (Epstein and Schneider, 2010). Hence, investors are not necessarily compensated for bearing ambiguity, which is the reason why ambiguity does not have a direct impact on market returns.

In terms of the relationships between ambiguity measures and volatility indices, both full-sample results and post-crisis results suggest that the two variables increase in response to a shock to each other. This suggests that investors perceive ambiguous volatilities. The volatility index reflects investors' expectation on future volatility movement, which also implies their prior beliefs on volatility. As such, the interaction between the ambiguity measure and the volatility index reflects the interaction between the degree of ambiguity and investors' prior beliefs on volatility. Thus, the findings seem to suggest that volatility is also ambiguous and the prior beliefs on volatility can be affected by the degree of ambiguity. It is noticeable that for the UK stock market, the ambiguity measure does not affect the volatility index during the pre-crisis period from 2004 to 2008. This implies that investors did not pay attention to the degree of ambiguity before crisis. The US result suggests that investors started to realise the existence of ambiguity from 2007 since the US pre-crisis period starts from 2007. The two findings imply that ambiguity plays a role in the 2008 financial crisis and this can be further confirmed with the results between the ambiguity measure and two term structure measures.

According to the full-sample results and post-crisis results, the ambiguity measure does not have an impact on the two term structure measures while the term structure measures can affect the ambiguity measure. A shock to the economy that makes investors more optimistic about the future macroeconomic conditions can lead to a decrease in the future degree of ambiguity of the stock market. On the other hand, a shock to the economy that makes investors more worried about the future default risks can lead to an increase in the future degree of ambiguity. These findings apply to both the UK and the US markets. Furthermore, the pre-crisis results suggest that unawareness of ambiguity contributes to the 2008 financial crisis. Before the crisis, investors viewed ambiguous information and signals as signs of better economic conditions although they did not know what was going on and were shocked by the "crazy" market. This contributes to the bubble. In addition, investors did not realise the default risks were high until 2007 when the crisis was about to happen. The two situations together led to the collapse of the financial markets and investors started to become aware of the importance of ambiguity.

This chapter provides a new empirical measure of ambiguity. The empirical evidence is consistent with the results from the previous chapter. In addition, the evidence also suggests that ambiguity plays an important role in the 2008 financial crisis, which is consistent with existing literature (Boyarchenko, 2012; Dimmock et al., 2016).

5.9. Appendices

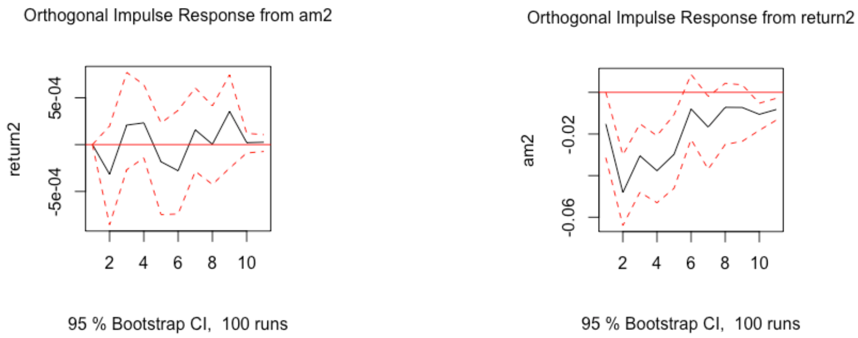
Table 5.20 Post-Crisis Period Regression of Market Returns on Ambiguity Measure (*AM*) with Newey-West Standard Errors. This table shows the regression results of the market returns on the ambiguity measure with Newey-West standard errors during the post-crisis period. *AM* represents the ambiguity measures; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

	Estimate	**t-Value**
UK Stock Market		
AM	-0.001	-1.211
Intercept	0.000	0.928
US Stock Market		
AM	-0.001	-2.627***
Intercept	0.0005	2.622***

Table 5.21 Post-Crisis Period Interaction between FTSE100 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between FTSE100 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the post-crisis period. *Return* represents the daily return of FTSE100; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	0.009	0.393
	2	-0.029	-1.228
	3	-0.011	-0.460
	4	-0.037	-1.573
	5	-0.024	-1.004
	6	0.017	0.735
	7	-0.035	-1.472
	8	-0.016	-0.690
AM	1	-0.001	-1.245
	2	0.001	0.883
	3	0.001	0.965
	4	-0.001	-0.730
	5	-0.001	-1.133
	6	0.000	0.646
	7	0.000	0.202
	8	0.001	1.380
Constant		0.001	1.204
Trend		0.000	-0.886
Adjusted R-Squared		-0.055%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-4.319	-5.683***
	2	-2.406	-3.132***
	3	-2.831	-3.672***
	4	-1.880	-2.429**
	5	0.122	0.158
	6	-0.701	-0.905
	7	0.402	0.520
	8	0.161	0.209
AM	1	0.041	1.795**
	2	0.101	4.363***
	3	0.088	3.806***
	4	0.078	3.378***
	5	0.035	1.522
	6	0.080	3.456***
	7	0.037	1.592
	8	0.075	3.272***
Constant		-0.068	-3.757***
Trend		0.0001	4.470***
Adjusted R-Squared		15.200%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**AM (Predictor)**
Return		-	0.858
AM		7.492***	-

Figure 5.17 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of FTSE100 Return (*return2*) and Ambiguity Measure (*am2*)

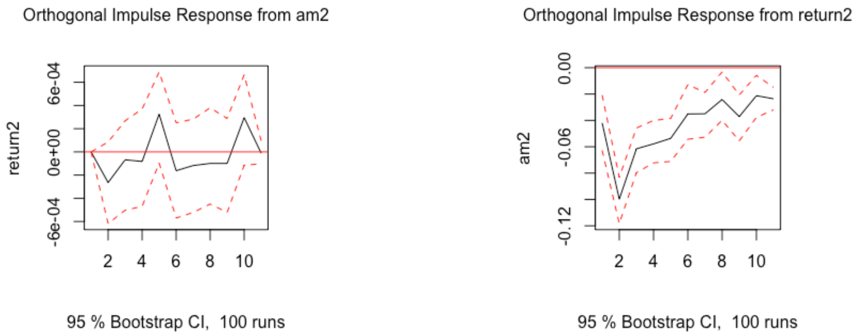


Notes: *return2* represents the daily return of FTSE100; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.22 Post-Crisis Period Interaction between S&P500 Return (*Return*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between S&P500 return and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the post-crisis period. *Return* represents the daily return of S&P500; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Return Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-0.068	-3.231***
	2	-0.004	-0.187
	3	-0.046	-2.106**
	4	0.019	0.881
	5	-0.055	-2.505**
	6	-0.019	-0.893
	7	0.002	0.090
	8	-0.015	-0.695
	9	-0.046	-2.139**
AM	1	-0.001	-1.214
	2	0.000	-0.233
	3	0.000	-0.136
	4	0.001	1.635
	5	0.000	-0.604
	6	0.000	-0.697
	7	0.000	-0.303
	8	0.000	-0.351
	9	0.001	1.739*
Constant		0.001	1.608
Trend		0.000	-0.319
Adjusted R-Squared		0.716%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Return	1	-9.068	-11.729***
	2	-4.556	-5.493***
	3	-2.918	-3.500***
	4	-2.594	-3.102***
	5	-0.367	-0.439
	6	-0.935	-1.123
	7	0.166	0.199
	8	-1.002	-1.213
	9	0.457	0.560
AM	1	0.132	6.253***
	2	0.171	8.081***
	3	0.112	5.189***
	4	0.067	3.120***
	5	0.041	1.877*
	6	0.061	2.848***
	7	0.028	1.302
	8	0.047	2.238**
	9	0.040	1.967**
Constant		-0.060	-3.325***
Trend		0.0001	4.477***
Adjusted R-Squared		33.700%	
Panel C		**Granger Causality Test**	
		Return (Predictor)	**AM (Predictor)**
Return		-	0.814
AM		17.737***	-

Figure 5.18 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of S&P500 Return (*return2*) and Ambiguity Measure (*am2*)

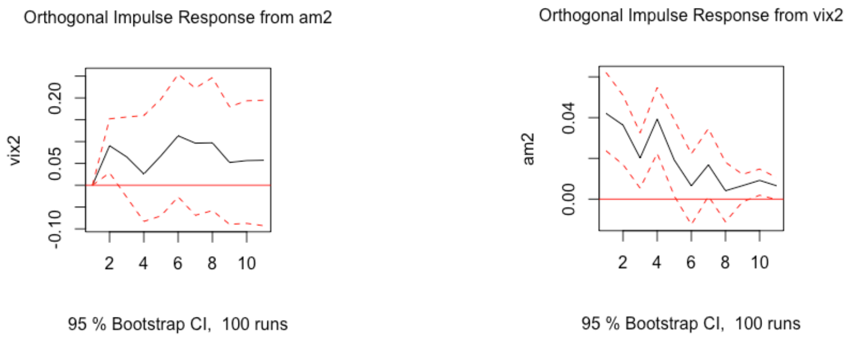


Notes: *return2* represents the daily return of S&P500; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.23 Post-Crisis Period Interaction between UK Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between UK market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the post-crisis period. *Volatility* represents the daily volatility index of FTSE100; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.950	40.953***
	2	-0.071	-2.214**
	3	0.058	1.180*
	4	-0.028	-0.884
	5	-0.008	-0.257
	6	0.051	1.632
	7	-0.048	-1.515
	8	0.069	2.987***
AM	1	0.251	2.579***
	2	-0.070	-0.722
	3	-0.106	-1.086
	4	0.107	1.087
	5	0.127	1.295
	6	-0.032	-0.327
	7	-0.019	-0.191
	8	-0.123	-1.265
Constant		0.640	3.458***
Trend		-0.0001	-1.673*
Adjusted R-Squared		95.190%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.023	4.079***
	2	-0.012	-1.618
	3	0.013	1.735*
	4	-0.014	-1.886*
	5	-0.007	-0.950
	6	0.006	0.780
	7	-0.009	-1.181
	8	0.003	0.465
AM	1	0.046	1.999**
	2	0.104	4.454***
	3	0.084	3.590***
	4	0.073	3.129***
	5	0.035	1.501
	6	0.079	3.376***
	7	0.034	1.482
	8	0.073	3.143***
Constant		-0.120	-2.706***
Trend		0.0001	4.399***
Adjusted R-Squared		14.080%	
Panel C		**Granger Causality Test**	
		Volatility (Predictor)	**AM (Predictor)**
Volatility		-	1.494
AM		4.418***	-

Figure 5.19 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of UK Market Volatility Index (*vix2*) and Ambiguity Measure (*am2*)

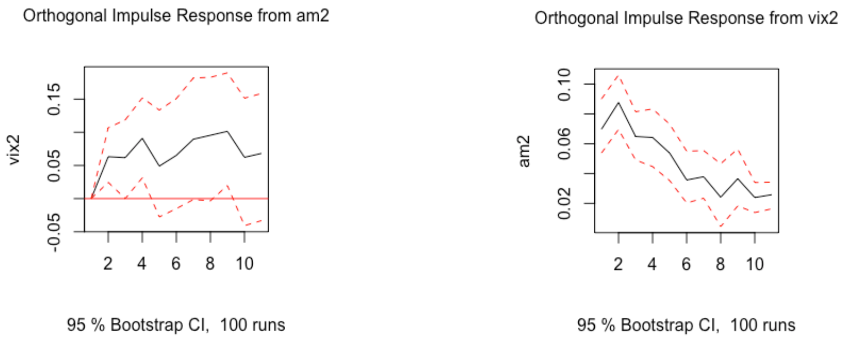


Notes: *vix2* represents the daily volatility index of FTSE100; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.24 Post-Crisis Period Interaction between US Market Volatility Index (*Volatility*) and Ambiguity Measure (*AM*). This table shows the VAR regression results between US market volatility index and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) during the post-crisis period. *Volatility* represents the daily volatility index of S&P500; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Volatility Index Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.859	40.541***
	2	0.067	2.428**
	3	-0.017	-0.613
	4	0.023	0.820
	5	-0.017	-0.609
	6	0.015	0.549
	7	-0.004	-0.158
	8	0.026	0.954
	9	0.026	1.203
AM	1	0.160	2.769***
	2	-0.001	-0.015
	3	0.055	0.944
	4	-0.114	-1.927*
	5	0.024	0.411
	6	0.072	1.216
	7	0.008	0.142
	8	0.010	0.168
	9	-0.117	-2.046**
Constant		0.602	3.749***
Trend		-0.0001	-2.415**
Adjusted R-Squared		97.600%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
Volatility	1	0.072	9.294***
	2	-0.024	-2.314**
	3	-0.008	-0.737
	4	-0.009	-0.914
	5	-0.017	-1.647*
	6	0.003	0.252
	7	-0.012	-1.144
	8	0.012	1.136
	9	-0.013	-1.653*
AM	1	0.124	5.854***
	2	0.160	7.487***
	3	0.107	4.967***
	4	0.062	2.872***
	5	0.040	1.829*
	6	0.063	2.896***
	7	0.030	1.412
	8	0.045	2.141**
	9	0.039	1.867*
Constant		-0.200	-3.386***
Trend		0.0001	4.542***
Adjusted R-Squared		32.650%	
Panel C		**Granger Causality Test**	
		Volatility (Predictor)	**AM (Predictor)**
Volatility		-	1.836*
AM		13.466***	-

Figure 5.20 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of US Market Volatility Index (*vix2*) and Ambiguity Measure (*am2*)

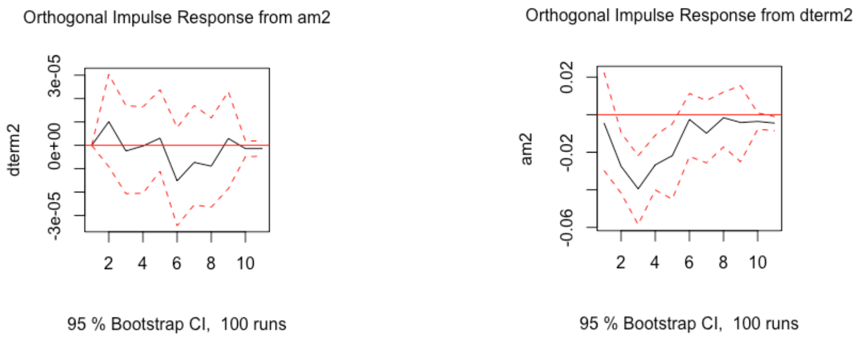


Notes: *vix2* represents the daily volatility index of S&P500; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.25 Post-Crisis Period Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market during the post-crisis period. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	0.036	1.546
	2	-0.105	-4.553***
	3	-0.036	-1.548
	4	0.022	0.948
	5	-0.026	-1.106
	6	-0.011	-0.451
	7	-0.070	-2.999***
	8	-0.050	-2.116**
AM	1	0.0000	0.963
	2	0.0000	-0.311
	3	0.0000	-0.013
	4	0.0000	0.237
	5	0.0000	-1.544
	6	0.0000	-0.564
	7	0.0000	-0.865
	8	0.0000	0.459
Constant		0.000	-0.081
Trend		0.000	-0.680
Adjusted R-Squared		1.605%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-59.900	-3.255***
	2	-80.560	-4.368***
	3	-50.110	-2.690***
	4	-38.050	-2.035**
	5	10.940	0.581
	6	-5.981	-0.319
	7	14.380	0.772
	8	3.712	0.199
AM	1	0.049	2.121**
	2	0.114	4.947***
	3	0.096	4.149***
	4	0.076	3.285***
	5	0.037	1.599
	6	0.077	3.323***
	7	0.029	1.254
	8	0.067	2.902***
Constant		-0.071	-3.916***
Trend		0.0001	4.374***
Adjusted R-Squared		14.350%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	0.713
AM		5.187***	-

Figure 5.21 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm2*) and Ambiguity Measure (*am2*) – UK

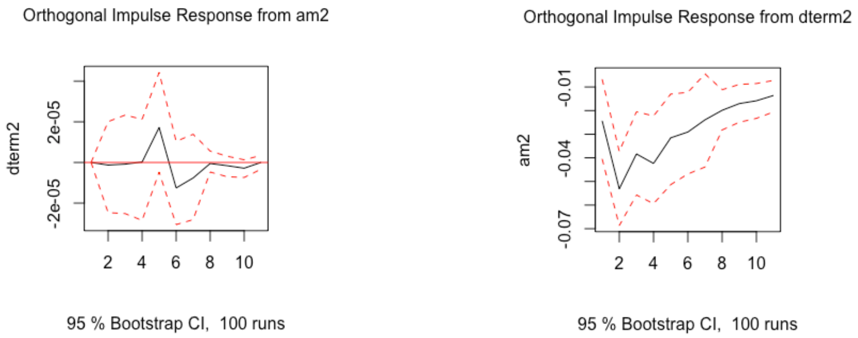


Notes: *dterm2* represents the first-differenced value of the term structure measure *Term*; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.26 Post-Crisis Period Interaction between First-Differenced Term Structure Measure *Term* (*dTerm*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the term spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market during the post-crisis period. *dTerm* represents the first-differenced value of the term structure measure *Term*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Term Spread Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-0.045	-2.131**
	2	-0.043	-2.059**
	3	0.006	0.261
	4	0.007	0.325
	5	-0.064	-3.042***
	6	-0.017	-0.811
AM	1	0.0000	-0.112
	2	0.0000	-0.057
	3	0.0000	0.045
	4	0.0000	1.486
	5	0.0000	-1.255
	6	0.0000	-0.766
Constant		0.000	0.168
Trend		0.000	-0.432
Adjusted R-Squared		0.478%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dTerm	1	-89.100	-5.721***
	2	-48.560	-3.099***
	3	-46.810	-2.980***
	4	-18.790	-1.195
	5	-12.270	-0.782
	6	-7.678	-0.491
AM	1	0.172	8.239***
	2	0.193	9.121***
	3	0.123	5.705***
	4	0.072	3.357***
	5	0.046	2.187**
	6	0.074	3.580***
Constant		-0.075	-4.086***
Trend		0.0001	4.646***
Adjusted R-Squared		30.290%	
Panel C		**Granger Causality Test**	
		dTerm (Predictor)	**AM (Predictor)**
dTerm		-	0.698
AM		7.914***	-

Figure 5.22 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Term* (*dterm2*) and Ambiguity Measure (*am2*) – US



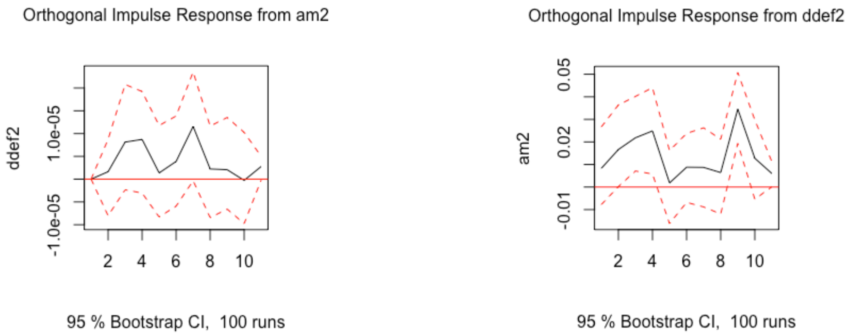
Notes: *dterm2* represents the first-differenced value of the term structure measure *Term*; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.27 Post-Crisis Period Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – UK. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the UK stock market during the post-crisis period. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	-0.100	-4.340***
	2	-0.066	-2.829***
	3	0.100	0.432
	4	0.058	2.498**
	5	0.027	1.149
	6	-0.047	-2.030**
	7	-0.047	-2.026**
	8	-0.031	-1.323
	9	0.060	2.608***
AM	1	0.0000	0.285
	2	0.0000	1.434
	3	0.0000	1.556
	4	0.0000	0.178
	5	0.0000	0.382
	6	0.0000	1.609
	7	0.0000	0.080
	8	0.0000	0.011
	9	0.0000	-0.444
Constant		0.000	-1.327
Trend		0.000	1.011
Adjusted R-Squared		2.504%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	65.080	1.918*
	2	86.580	2.542**
	3	96.210	2.818***
	4	-4.904	-0.144
	5	8.183	0.238
	6	4.555	0.133
	7	1.177	0.034
	8	122.600	3.601***
	9	41.000	1.208
AM	1	0.061	2.619***
	2	0.119	5.171***
	3	0.091	3.921***
	4	0.068	2.926***
	5	0.024	1.045
	6	0.072	3.077***
	7	0.029	1.263
	8	0.069	2.961***
	9	-0.024	-1.027
Constant		-0.068	-3.728***
Trend		0.0001	4.428***
Adjusted R-Squared		13.800%	

Panel C	**Granger Causality Test**	
	dDef (Predictor)	**AM (Predictor)**
dDef	-	1.647*
AM	3.248***	-

Figure 5.23 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef2*) and Ambiguity Measure (*am2*) – UK

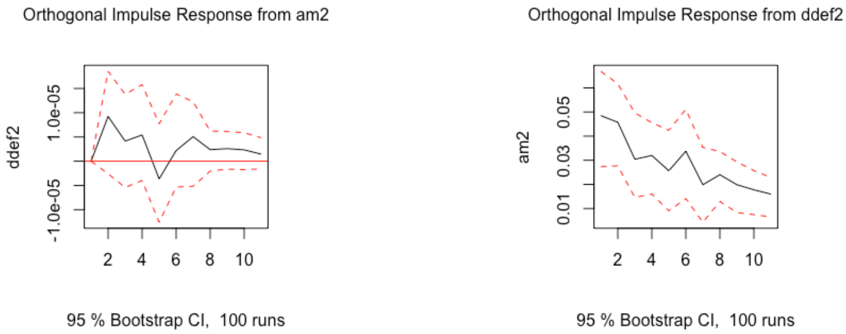


Notes: *ddef2* represents the first-differenced value of the term structure measure *Def*; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Table 5.28 Post-Crisis Period Interaction between First-Differenced Term Structure Measure *Def* (*dDef*) and Ambiguity Measure (*AM*) – US. This table shows the VAR regression results between the default spread and the ambiguity measure (Panel A and Panel B) and the Granger causality test result (Panel C) of the US stock market during the post-crisis period. *dDef* represents the first-differenced value of the term structure measure *Def*; *AM* represents the ambiguity measure; t-statistics are robust to heteroskedasticity; *** represents 1% significance level; ** represents 5% significance level; and * represents 10% significance level.

Panel A	**Default Spread Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	0.096	4.579***
	2	0.132	6.230***
	3	0.094	4.408***
	4	0.026	1.233
	5	0.032	1.527
	6	0.028	1.345
AM	1	0.00002	1.773*
	2	0.00000	0.308
	3	0.00000	0.262
	4	-0.00002	-1.497
	5	0.00000	0.075
	6	0.00001	0.824
Constant		0.000	-1.356
Trend		0.000	0.600
Adjusted R-Squared		6.057%	
Panel B	**Ambiguity Measure Equation**		
	Lag	**Estimate**	**t-Value**
dDef	1	149.100	4.306***
	2	37.020	1.061
	3	22.150	0.631
	4	-4.899	-0.140
	5	40.010	1.152
	6	-29.080	-0.840
AM	1	0.176	8.374***
	2	0.194	9.085***
	3	0.121	5.572***
	4	0.067	3.097***
	5	0.041	1.914*
	6	0.075	3.571***
Constant		-0.071	-3.855***
Trend		0.0001	4.648***
Adjusted R-Squared		29.600%	
Panel C		**Granger Causality Test**	
		dDef (Predictor)	**AM (Predictor)**
dDef		-	1.220
AM		4.113***	-

Figure 5.24 Post-Crisis Period Plots of Orthogonalised Impulse Response Function (OIRF) of First-Differenced Term Structure Measure *Def* (*ddef2*) and Ambiguity Measure (*am2*) – US



Notes: *ddef2* represents the first-differenced value of the term structure measure *Def*; *am2* represents the ambiguity measure; the OIRF plot shows the 95% confidence interval of response of one variable from a one standard deviation shock to another variable.

Chapter 6. Conclusion

6.1. Introduction

The thesis attempts to fill in the following research gaps. Firstly, it attempts to uncover whether sample size can affect forecasting accuracy. Secondly, it aims at testing the adaptive market hypothesis (AMH) by applying the high-frequency forecasting method. Thirdly, it attempts to answer whether it is proper to use the STAR model as the reference model in ambiguity literature. Fourthly, it tries to develop new empirical measures of ambiguity to provide empirical evidence on the role of ambiguity in stock markets. Last but not least, it attempts to shed light on the role of ambiguity in the 2008 financial crisis. Results from the three empirical chapters bridge these gaps, which makes the thesis produce original contributions to the field of study.

6.2. Summary of Empirical Results

In Chapter 3, exponential smoothing models and AR based forecasting models are used to investigate their forecasting performances for predicting mean price movements. The findings suggest that nonlinear AR models do not necessarily have superior forecasting performances than the exponential smoothing models and the linear AR based models. This suggests that it is indeed improper for Viale et al. (2014) to use the STAR model as the reference model. Thus, this provides motivations for the following two empirical chapters. This finding also has implication for forecasters, which is that forecasters should not take it for granted that nonlinear models can fit the data better and hence they should generate better forecasts.

The MDM test results indicate that there is weakly significant evidence from the daily and hourly data suggesting that the UK stock market is not in a weak form of efficiency. This conclusion is statistically significant with evidence from minute data. As such, the EMH does not seem to provide a precise description of the UK stock market.

The subsample forecasting results suggest that increasing sample size does not necessarily result in more accurate forecasts. Instead, the continuity of data, increases forecasting accuracy for each subsample, and this also applies to the full sample. In addition, the results also imply that the exponential smoothing models and the AR based models do not suffer from over-fitting problems caused by inactive data. If such a problem exists, forecasts of hourly data should be more accurate than those of minute data in full sample analyses or subsample analyses or both because minute data have more non-changing data points than hourly data. This can be further confirmed by the result that in-sample MAPEs are generally similar to out-of-sample MAPEs regardless of what data frequency is used. Moreover, the subsample results also have implication for the AMH. The empirical evidence seems to support that the AMH characterises the UK stock market better than the EMH based on the high-frequency forecasting method used in the chapter.

In Chapter 4, findings from the analysis between the ambiguity measure and market return suggest that the interaction between the ambiguity measure and market return and the interaction between the ambiguity measure and excess market return are statistically weak. As such, investors may not necessarily be compensated for bearing ambiguity. The implication for investors is that they should not participate the market if they perceive that the information is of poor quality or the financial markets are faced with ambiguity.

The interaction between the ambiguity measure and the volatility index indicates that an unexpected increase in the degree of ambiguity can lead to perceptions of more volatile financial markets, which can in turn affect the equity premium in the long run (Kim et al., 2004). This is consistent with the result of quantitative studies using heterogeneous agent models (Condie, 2008). As such, theories of ambiguity asset pricing can indeed help to explain the equity premium puzzle, as is illustrated by Epstein and Schneider (2010). In addition, empirical evidence suggests that ambiguity can also come from ambiguous volatility.

The interactions between the ambiguity measure and the two term structure measures suggest that macroeconomic conditions can affect the degree of ambiguity of the equity market. When investors are more optimistic about the future economic state, the degree of ambiguity of the equity market tends to decrease, and when they are more worried about future default risks, the degree of ambiguity of the equity market would increase. As such, similar to theoretical evidence, empirical results also suggest that ambiguity literature can help to explain why investors respond differently when they are faced with good news and bad news about macroeconomic conditions.

Last but not least, the results suggest that empirical evidence is in favour of heterogenous agents models of ambiguity.

Full-sample results and post-crisis results of Chapter 5 are quite consistent with the results reported in Chapter 4. However, the pre-crisis results show some differences. Nevertheless, the interactions between the ambiguity measure and market returns are similar. As such, ambiguity does not seem to have an impact on market returns, which can be rationalised under heterogeneous agent models of ambiguity.

Thus, investors who are more ambiguity-averse can simply choose not to participate the market, leaving those who are less ambiguity-averse continue (Epstein and Schneider, 2010). Hence, investors are not necessarily compensated for bearing ambiguity, which is the reason why ambiguity does not have a direct impact on market returns.

One of the differences from the pre-crisis analyses is that for the UK stock market, the ambiguity measure does not affect the volatility index during the pre-crisis period, which starts from 2004 to 2008. This implies that investors did not pay attention to the degree of ambiguity before the 2008 financial crisis. On the other hand, the US result suggests that investors started to realise the existence of ambiguity from 2007, which is the start of the US pre-crisis period. The two findings imply that ambiguity plays an important role in the 2008 financial crisis.

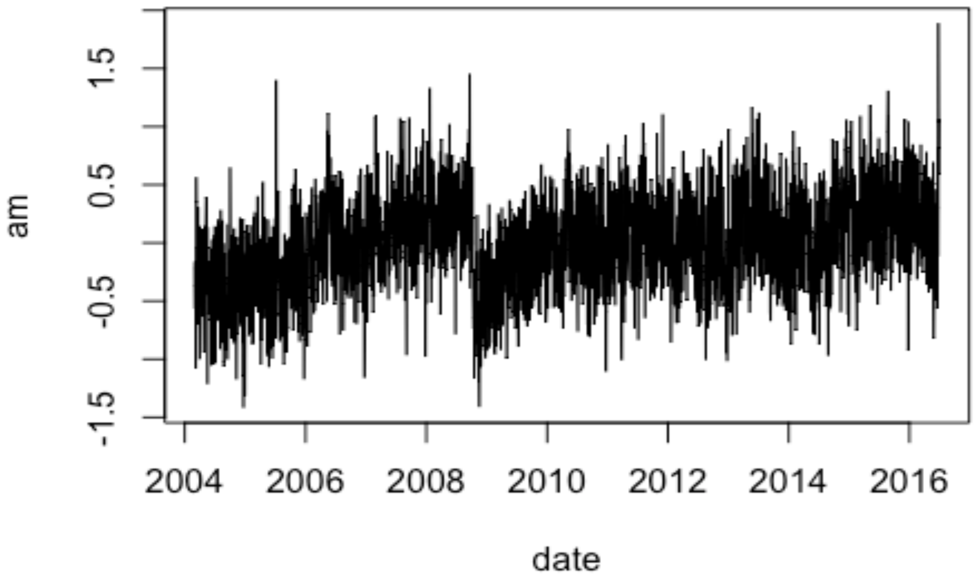
The results between the ambiguity measure and two term structure measures provide further evidence, which suggests that unawareness of the existence of ambiguity contributes to the financial crisis. Before the crisis, investors viewed ambiguous information and signals as signs of better future economic conditions, which contributes to the economic bubble. The pre-crisis results also imply that investors did not realise that the default risks were high until 2007 when the crisis was about to happen. The two situations together led to the collapse of the financial markets, which subsequently gave rise to a global crisis.

6.3. Discussion of Empirical Results

As is mentioned above, the forecasting chapter proves that the STAR model is improper to use as the reference model in ambiguity literature. This provides motivations for conducting the two empirical researches on ambiguity. In comparison, the ambiguity measure of Chapter 5, which is based on the intraday highest and lowest prices, is better than the ambiguity measure of Chapter 4, which is based on the bid and ask prices. One of the reasons is that the ambiguity measure of Chapter 4 is highly skewed, which is not an ideal property for econometric analysis especially when the sample size is small. In addition, as the measure is based on the gap between bid and ask prices, it is necessary to ensure that it is separated from liquidity because bid-ask spread is commonly adopted as a measure of liquidity. Although in Chapter 4, attempt has been made to separate liquidity from it by regressing it against volume by turnover, this method may not be an effective way of removing liquidity from the ambiguity measure. However, the positive side is that this provides a direction for further studies.

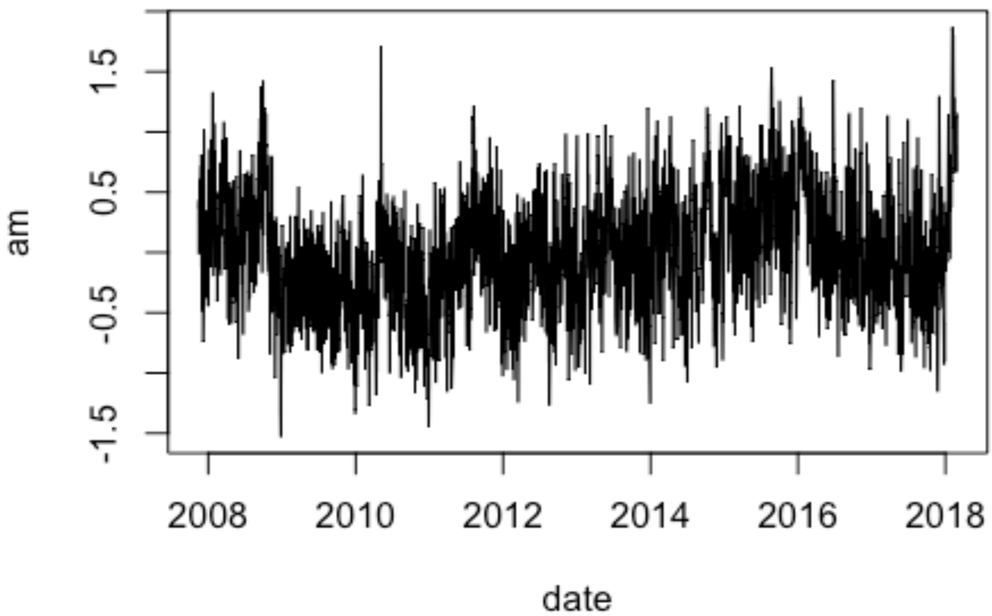
The approximately normal distribution of the ambiguity measure of Chapter 5 makes it more favourable than the ambiguity measure of Chapter 4. Figure 6.1 and Figure 6.2 are time-series plots of the ambiguity measures of Chapter 5 for the UK stock market and the US market respectively. The figures show that there are more stock market crashes in the US market than in the UK market. An up and down movement indicates a stock market crash and large crashes, for instance the 2008 financial crisis, have a more evident pattern. The implication is that regulators and financial practitioners can use the plots of the ambiguity measure in Chapter 5 to monitor market crashes. Once there is a clear upward trend, regulators should be alert of a market crash and they can adjust regulations to prevent the crash, and practitioners should stop taking excess risks and uncertainties unless they clearly understand what is happening in the market.

Figure 6.1 Time-Series Plot of Ambiguity Measure of UK Stock Market



Notes: *am* represents the ambiguity measure that is calculated using the intraday highest and lowest prices of FTSE100.

Figure 6.2 Time-Series Plot of Ambiguity Measure of US Stock Market



Notes: *am* represents the ambiguity measure that is calculated using the intraday highest and lowest prices of S&P500.

6.4. Further Study

The two empirical ambiguity measures developed in the thesis make it possible to conduct further empirical studies on ambiguity. For instance, researchers can investigate stock market participation using the two empirical measures. Since the calculation of the ambiguity measure based on the bid and ask prices only requires the data of bid and ask prices of an asset, this measure can also be applied to other assets that have bid and ask prices. Similarly, the ambiguity measure based on intraday highest and lowest prices can be applied to other assets as long as the intraday highest and lowest prices are available.

Chapter 5 split the full sample according to the 2008 financial crisis. However, there are actually other market crashes within the full-sample period, especially in the US stock market. As such, it would be interesting to conduct even-based studies, which track the role of ambiguity in different stock market crashes. In addition, it would also be interesting to study the stock markets of other countries using the ambiguity measures developed in Chapter 4 and Chapter 5.

On the other hand, efforts can be made to improve the ambiguity measure that is based on the bid and ask prices. Further studies can attempt to separate the ambiguity measure from liquidity in a more precise way.

6.5. Conclusion

The empirical findings summarised in this chapter answers the questions related to the research gaps identified in the thesis. In addition, the discussion section proposes a feasible way for regulators to monitor and take control of stock market crashes. In short, this thesis has made original and material contributions to the field of ambiguity studies as well as the field of high-frequency forecasting.

Reference

Akaike, H., 1969. Fitting autoregressive models for prediction. *Annals of The Institute of Statistical Mathematics, 21*(1), pp.243-247.

Akaike, H., 1969. Power spectrum estimation through autoregressive model fitting. *Annals of The Institute of Statistical Mathematics, 21*(1), pp.407-419.

Akaike, H., 1970. Statistical predictor identification. *Annals of The Institute of Statistical Mathematics, 22*(1), pp.203-217.

Anderson, E. W., Hansen, L. P. and Sargent, T. J., 2003. A quartet of semigroups for model specification, robustness, prices of risk, and model detection. *Journal of The European Economic Association, 1*(1)*,* pp.68-123.

Anderson, E. W., Ghysels, E. and Juergens, J. L., 2009. The impact of risk and uncertainty on expected returns*. Journal of Financial Economics, 94*(2), pp.233-263.

Ang, A. and Bekaert, G., 2007. Stock return predictability: is it there?. *The Review of Financial Studies, 20*(3), pp.651-707.

Antoniou, C., Harris, R. D. and Zhang, R., 2015. Ambiguity aversion and stock market participation: an empirical analysis. *Journal of Banking & Finance, 58*, pp.57-70.

Atchison, M. D., Butler, K. C. and Simonds, R. R., 1987. Nonsynchronous security trading and market index autocorrelation. *The Journal of Finance, 42*(1), pp.111-118.

Avramov, D., Chordia, T. and Goyal, A., 2006. Liquidity and autocorrelations in individual stock returns. *The Journal of Finance, 61*(5), pp.2365-2394.

Barillas, F., Hansen, L. P. and Sargent, T. J., 2009. Doubts or variability? *Journal of Economic Theory*, *144*(6), pp.2388-2418.

Beechey, M., Gruen, D. W. and Vickery, J., 2000. *The efficient market hypothesis: a survey.* Reserve Bank of Australia, Economic Research Department.

Bernoulli, D., 1738. Specimen theoriae novae de mensura sortis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae, 5,* pp.175–192.

Black, F., 1986. Noise. *The Journal of Finance*, *41*(3), pp.528-543.

Blair, B. J., Poon, S. H. and Taylor, S. J., 2001. Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. *Journal of Econometrics, 105*(1), pp.5-26.

Bluhm, H. and Yu, J., 2001. Forecasting volatility: evidence from the German stock market. Working paper, University of Auckland.

Boyarchenko, N., 2012. Ambiguity shifts and the 2007–2008 financial crisis. *Journal of Monetary Economics, 59*(5), pp.493-507.

Campbell, J. Y. and Thompson, S. B., 2008. Predicting excess stock returns out of sample: can anything beat the historical average?. *The Review of Financial Studies, 21*(4), pp.1509-1531.

Cao, H. H., Wang, T. and Zhang, H. H., 2005. Model uncertainty, limited market participation, and asset prices. *The Review of Financial Studies, 18*(4), pp.1219-1251.

Carhart, M. M., 1997. On persistence in mutual fund performance. *The Journal of Finance, 52*(1), pp.57-82.

Chapman, D. A. and Polkovnichenko, V., 2009. First-order risk aversion, heterogeneity, and asset market outcomes. *The Journal of Finance, 64*(4), pp.1863-1887.

Chen, Z. and Epstein, L., 2002. Ambiguity, risk, and asset returns in continuous time. *Econometrica, 70*(4), pp.1403-1443.

Chen, H., Ju, N. and Miao, J., 2014. Dynamic asset allocation with ambiguous return predictability. *Review of Economic Dynamics*, *17*(4), pp.799-823.

Cochrane, J. H., 2009. *Asset Pricing:(Revised Edition)*. Princeton university press.

Collard, F., Mukerji, S., Sheppard, K. and Tallon, J. M., 2018. Ambiguity and the historical equity premium. *Quantitative Economics, 9*(2), pp.945-993.

Condie, S., 2008. Living with ambiguity: prices and survival when investors have heterogeneous preferences for ambiguity. *Economic Theory, 36*(1), pp.81-108.

Conte, A. and Hey, J. D., 2013. Assessing multiple prior models of behaviour under ambiguity. *Journal of Risk and Uncertainty*, *46*(2), pp.113-132.

Connolly, R. A., 1989. An examination of the robustness of the weekend effect. *Journal of Financial and Quantitative Analysis*, *24*(2), pp.133-169.

Corrado, C. and Truong, C., 2007. Forecasting stock index volatility: comparing implied volatility and the intraday high-low price range. *Journal of Financial Research, 30*(2), pp.201-215.

Cromwell, J. B., Labys, W. C. and Terraza, M., 1994. Univariate tests for time series models. *Sage, Thousand Oaks, CA.,* pp.32-36.

Diebold, F. and Mariano, R., 1995. Comparing predictive accuracy. *Journal of Business & Economic Statistics, 13*(3), pp.253-63.

Dimmock, S. G., Kouwenberg, R., Mitchell, O. S. and Peijnenburg, K., 2016. Ambiguity aversion and household portfolio choice puzzles: empirical evidence. *Journal of Financial Economics, 119*(3), pp.559-577.

Dow, J. and Werlang, S. R. da Costa, 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica, 60*(1), pp.197-204.

Duffie, D. and Epstein, L. G., 1992. Stochastic differential utility. *Econometrica, 60*(2), pp.353-394.

Easley, D. and O'Hara, M., 2010a. Liquidity and valuation in an uncertain world. *Journal of Financial Economics, 97*(1), pp.1-11.

Easley, D. and O'Hara, M., 2010b. Microstructure and ambiguity. *The Journal of Finance, 65*(5), pp.1817-1846.

Ellsberg, D., 1961. Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics, 75*(4), pp.643-669.

Epstein, L. G. and Wang, T., 1994. Intertemporal asset pricing under Knightian uncertainty. *Econometrica*, *62*(2), pp.283-322.

Epstein, L. G. and Schneider, M., 2003. Recursive multiple-priors. *Journal of Economic Theory, 113*(1), pp.1-31.

Epstein, L. G. and Schneider, M., 2007. Learning under ambiguity. *The Review of Economic Studies, 74*(4), pp.1275-1303.

Epstein, L. G. and Schneider, M., 2008. Ambiguity, information quality, and asset pricing. *The Journal of Finance*, *63*(1), pp.197-228.

Epstein, L. G. and Schneider, M., 2010. Ambiguity and asset markets. *Annual Review of Financial Economics, 2*, pp.315-346.

Etner, J., Jeleva, M. and Tallon, J. M., 2012. Decision theory under ambiguity. *Journal of Economic Surveys*, *26*(2), pp.234-270.

Fama, E. F., Fisher, L., Jensen, M. C. and Roll, R., 1969. The adjustment of stock prices to new information. *International Economic Review*, *10*(1), pp.1-21.

Fama, E. F. and Malkiel, B. G., 1970. Efficient capital markets: a review of theory and empirical work. *The Journal of Finance*, *25*(2), pp.383-417.

Fama, E. F. and French, K. R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics, 33*(1), pp.3-56.

Fostel, A. and Geanakoplos, J., 2012. Why does bad news increase volatility and decrease leverage?. *Journal of Economic Theory, 147*(2), pp.501-525.

Garlappi, L., Uppal, R. and Wang, T., 2007. Portfolio selection with parameter and model uncertainty: a multi-prior approach. *Review of Financial Studies*, *20*(1), pp.41-81.

Gilboa, I. and Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, *18*(2), pp.141-153.

Goyenko, R., Holden, C.W. and Trzcinka, C., 2009. Do measures of liquidity measure liquidity?. *Journal of financial economics, 92*(2), pp.153-181.

Grossman, S. J. and Miller, M. H., 1988. Liquidity and market structure. *The Journal of Finance, 43*(3), pp.617-637.

Guidolin, M. and Rinaldi, F., 2013. Ambiguity in asset pricing and portfolio choice: a review of the literature. *Theory and Decision*, *74*(2), pp.183-217.

Guidolin, M. and Rinaldi, F., 2014. A simple model of trading and pricing risky assets under ambiguity: any lessons for policy-makers?. *In The Global Financial Crisis* (pp. 109-140). Routledge.

Hansen, L. P. and Sargent, T. J., 2001. Robust control and model uncertainty. *The American Economic Review*, *91*(2), pp.60-66.

Harvey, D., Leybourne, S. and Newbold, P., 1997. Testing the equality of prediction mean squared errors. *International Journal of forecasting, 13*(2), pp.281-291.

Huang, R. D. and Stoll, H. R., 1997. The components of the bid-ask spread: a general approach. *The Review of Financial Studies, 10*(4), pp.995-1034.

Illeditsch, P. K., 2009. Ambiguous information, risk aversion, and asset pricing. Available at SSRN: https://ssrn.com/abstract=1364296 or http://dx.doi.org/10.2139/ssrn.1364296

Jegadeesh, N. and Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *The Journal of Finance, 48*(1), pp.65-91.

Jegadeesh, N. and Titman, S., 2001. Profitability of momentum strategies: An evaluation of alternative explanations. *The Journal of Finance, 56*(2), pp.699-720.

Jeong, D., Kim, H. and Park, J. Y., 2015. Does ambiguity matter? Estimating asset pricing models with a multiple-priors recursive utility. *Journal of Financial Economics*, *115*(2), pp.361-382.

Ju, N. and Miao, J., 2012. Ambiguity, learning, and asset returns. *Econometrica*, *80*(2), pp.559-591.

Jurado, K., Ludvigson, S. C. and Ng, S., 2015. Measuring uncertainty. *American Economic Review, 105*(3), pp.1177-1216.

Kahneman, D. and Tversky, A., 1979. Prospect theory: an analysis of decision under risk. *Econometrica, 47*(2), pp.263-292.

Kendall, M. G. and Hill, A. B., 1953. The analysis of economic time-series-part 1: Prices. *Journal of The Royal Statistical Society, 116*(1), pp.11-34.

Kim, C. J., Morley, J. C. and Nelson, C. R., 2004. Is there a positive relationship between stock market volatility and the equity premium?. *Journal of Money, Credit and Banking*, *36*(3), pp.339-360.

Klibanoff, P., Marinacci, M. and Mukerji, S., 2003. A smooth model of decision making under ambiguity. *Econometrica, 73*(6), pp.1849-1892.

Klibanoff, P., Marinacci, M. and Mukerji, S., 2009. Recursive smooth ambiguity preferences. *Journal of Economic Theory*, *144*(3), pp. 930-976.

Klibanoff, P., Marinacci, M. and Mukerji, S., 2012. On the smooth ambiguity model: a reply. *Econometrica, 80*(3), pp.1303-1321.

Knight, F. H., 1921. Risk, uncertainty and profit. *New York: Hart, Schaffner and Marx.*

Leung, M. T., Daouk, H. and Chen, A. S., 2000. Forecasting stock indices: a comparison of classification and level estimation models. *International Journal of Forecasting, 16*(2), pp.173-190.

Lybek, M. T. and Sarr, M. A., 2002. *Measuring liquidity in financial markets* (No. 2-232). International Monetary Fund.

Liu, H., 2011. Dynamic portfolio choice under ambiguity and regime switching mean returns. *Journal of Economic Dynamics and Control*, *35*(4), pp.623-640.

Lo, A. W., and MacKinlay, A. C., 1988. Stock market prices do not follow random walks: evidence from a simple specification test. *The Review of Financial Studies, 1,* pp.41-66.

Lo, A. W. and MacKinlay, A. C., 2002. *A non-random walk down Wall Street.* Princeton University Press.

Lo, A. W., 2004. The adaptive markets hypothesis. *Journal of Portfolio Management, 30*, pp.15-29.

Lütkepohl, H., 2006. Forecasting with VARMA models. *Handbook of Economic Forecasting, 1*, pp.287-325.

Maccheroni, F., Marinacci, M. and Rustichini, A., 2006. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, *74(6),* pp.1447-1498.

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R., Newton, J., Parzen, E. and Winkler, R., 1984. *The forecasting accuracy of major time series methods.* John Wiley & Sons.

Makridakis, S. G., Wheelwright, S. C. and Hyndman, R. J., 1998. *Forecasting: methods and applications.* 3rd ed. John Wiley & Sons.

Malkiel, B. G., 2003. The efficient market hypothesis and its critics. *The Journal of Economic Perspectives, 17*(1), pp.59-82.

Matías, J. M. and Reboredo, J.C., 2012. Forecasting performance of nonlinear models for intraday stock returns. *Journal of Forecasting, 31*(2), pp.172-188.

Meade, N., 2002. A comparison of the accuracy of short-term foreign exchange forecasting methods. *International Journal of Forecasting*, *18*(1), pp.67-83.

Miao, J., 2009. Ambiguity, risk and portfolio choice under incomplete information. *Annals of Economics and Finance*, *10*(2), pp.257-279.

Mills, A. 2009., Introduction to forecasting methods for actuaries. *Analytics and Futurism,* pp.6-9.

Ozsoylev, H. and Werner, J., 2011. Liquidity and asset prices in rational expectations equilibrium with ambiguous information. *Economic Theory*, *48*(2), pp. 469-491.

Poon, S. H. and Granger, C. W., 2003. Forecasting volatility in financial markets: a review. *Journal of Economic Literature, 41*(2), pp.478-539.

Qadan, M., Kliger, D. and Chen, N., 2018. Idiosyncratic volatility, the VIX and stock returns. *The North American Journal of Economics and Finance,* In Press.

Routledge, B. R. and Zin, S. E., 2009. Model uncertainty and liquidity. *Review of Economic Dynamics*, *12*(4), pp.543-566.

Savage, L. J., 1954, *The foundations of statistics.* Wiley, New York.

Shiller, R. J., 1981. The use of volatility measures in assessing market efficiency. *The Journal of Finance, 36*(2), pp.291-304.

Smith, G., 2012. The changing and relative efficiency of European emerging stock markets. *European Journal of Finance, 18*, pp.689-708.

Strzalecki, T., 2011. Axiomatic foundations of multiplier preferences. *Econometrica*, *79*(1), pp.47-73.

Taylor, J. W., 2004. Volatility forecasting with smooth transition exponential smoothing. *International Journal of Forecasting, 20*(2), pp.273-286.

Teräsvirta, T., 1994. Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of The American Statistical association, 89*(425), pp.208-218.

Uhlig, H., 2010. A model of a systemic bank run. *Journal of Monetary Economics, 57*(1), pp.78-96.

Urquhart, A. and Hudson, R., 2013. Efficient or adaptive markets? Evidence from major stock markets using very long run historic data. *International Review of Financial Analysis, 28*, pp.130-142.

Viale, A. M., Garcia-Feijoo, L. and Giannetti, A., 2014. Safety first, learning under ambiguity, and the cross-section of stock returns. *The Review of Asset Pricing Studies*, *4*(1), pp.118-159.

von Neumann, J. and Morgenstern, O., 1944. *Theory of games and economic behaviour.* Princeton: Princeton University Press.

Welch, I. and Goyal, A., 2007. A comprehensive look at the empirical performance of equity premium prediction. *The Review of Financial Studies, 21*(4), pp.1455-1508.

Whaley, R. E., 2000. The investor fear gauge. *The Journal of Portfolio Management, 26*(3), pp.12-17.

Wu, C. F. J., 1986. Jackknife, bootstrap and other resampling methods in regression analysis. *The Annals of Statistics. 14*(4), pp.1261–1295.

Yen, G. and Lee, C. F., 2008. Efficient market hypothesis (EMH): past, present and future. *Review of Pacific Basin Financial Markets and Policies, 11*(02), pp.305-329.