THE FOUNDATIONS OF LOGICAL ANALYSES OF TENSE

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ABSTRACT

This essay investigates the interconnection of tense, existence and generality and its representation in first order theories. The central chapter deals with first order theories which employ tensed quantifiers; that is, where '$(\exists x)(-x-)$' represents the tensed 'There is...' locution. An attempt is made to draw together into one system the advantages of Prior's system Q and Kripke type systems without their disadvantages, where these advantages are first two valuedness (a characteristic of Kripke type systems but not of Q) and secondly having as valid certain mixing formulas (a characteristic of Q but not the Kripke type systems). This is attempted by introducing a predicate modifier, the * operator which is a negation operator, into Kripke type systems which involve a more complex syntax. It is claimed that systems involving * are not only useful for founding tensed quantification theory but also, in the context of modal systems, for representing essentialist claims. It is also argued that systems involving * are equivalent to systems without * but which also contain a tensed existence predicate.

The discussion of * systems is within that framework which treats tensing to be adverbial in character and consequently as representable using sentence operators after Prior. Besides model-theoretic semantics homophonic semantics are offered for this representation of tense. An alternative approach is to treat tensing as indexical in character. Two such analyses are examined, namely the Russellian and

the Quinean. We attempt to provide semantic reasons why it is that Quine puts forward his analysis rather than the Russellian analysis. The main reason, we argue, is that the Russellian approach is inadequate for representing certain tensed existence claims.

The essay also contains a chapter on existence, one on Aristotle and tense and a chapter on the Medieval theory of appellation. There are brief discussions of 'tenselessness', aspect and a tentative attempt to place doubt on the indexical analysis of tense on the basis that 'now' unlike 'here' is not indexical because it is not rigid relative to an occasion of use.

INTRODUCTION

The central theme of this essay is the interconnection of tense, existence and generality and its representation in first order theories. The essay is split into three parts where the first two parts are mainly stage settings for the final lengthier part.

In the first part of the essay, consisting of the first two chapters, we look at certain aspects of the Ancient and Medieval accounts of tense. These chapters are very preliminary in which various bits of material are assembled, as it were, into a pile. The framework of the machine in which they function comes afterwards, particularly in part three. Chapter one is centrally concerned with comments by Aristotle about tense, existence and generality and in Chapter two we look at the Medieval's account of the same subject matter.

The second part of the essay consists of Chapters three and four. In Chapter three we look at the bases of two standard frameworks for the logical analyses of tense. One framework treats tense modification as being adverbial making central use of Prior's tense operators. This analysis we call the Lp analysis. The other framework analyses uses of tensed sentences on the basis that tense is an indexical feature of language. This analysis we call the L^D analysis. In order to set up these frameworks at the end of Chapter three some preliminary discussion is required. This includes an account of 'tenseless' verbs where we make a distinction between what we call 'tenseless' verbs proper and what we call 'detensed verbs', a distinction we utilize in part three. There is als in this chapter some discussion of the relationship between sentences involving tenses only and those which also involve dat expressions together with a very tentative discussion of tensing in English.

In Chapter four we offer semantics for the sentential Lp analysis. Two sorts of semantics are offered; first model theoretic and secondly homophonic semantics. In order to set up homophonic semantics certain problems have to be overcome. We claim that these are best overcome via a consideration of the two-dimensionality of 'now'. This property of 'now' occurs when it is treated as a sentence operator within the Lp framework. So we ask if 'now' has an analogous property if treated as an indexical and answer this using rigidity. We then consider two arguments against the L^D approach. One of these which purports to show that there is a logical deficiency in the L^D analysis we claim is unfounded. On the other hand, the other argument, which we tentatively put forward, attempts to show that 'now' is not an indexical like 'here' on the basis that 'now', unlike 'here', is not rigid relative to an occasion of use.

The final part of the essay consists of the last three chapters. The main theme here is first order tensed theories with particular emphasis on questions about existence. It is in this part of the essay where the various issues discussed in the opening two chapters re-emerge in full. Chapter five examines the L^D analysis; Chapter six first order Lp analyses and finally Chapter seven is devoted to a discussion about tensed existence.

In Chapter five two structural analyses of the uses of tensed sentences are considered; first a Russellian analysis, the L_A^D analysis, which analyses uses of tensed predicates into tenseless predicates with an extra place for time variables or constants and secondly, a Quinean analysis, the L_B^D analysis, which treats temporal expressions as belonging to the logical subject. The L_B^D approach therefore takes as values of variables not ordinary continuants but their four-dimensional counterparts. Our main concern in this Chapter is to offer reasons other than those based upon scientific grounds why Quine claims that application of standard first order theory to temporal matters must be done within the L_B^D framework and not the more intuitive L_A^D framework which preserves our ordinary ontology. We attempt to show that the following three claims can not be held together within an L_A^D framework unlike an L_B^D framework; first, that the existential quantifier must always be read tenselessly, secondly that there is only one sense of 'exists', a tenseless sense represented by this quantifier and finally that a canonical notation should be able to represent uses of tensed sentences involving tensed existence claims. Our attempts to show this are based upon arguments which show that the L_A^D theorist can not represent certain sorts of sentences including certain sentences involving tensed existence claims without introducing extra machinery.

Chapter six, the central and longest chapter of the essay, looks at first order Lp theories. Two sorts of systems are considered; first, standard systems which make use of tenseless quantifiers and secondly systems employing tensed quantifiers - that is, where '$(\exists x)(-x-)$' represents the tensed

'There is...' locution. Homophonic semantics are given for some of the systems discussed and model-theoretic semantics for all. Although the majority of this Chapter is concerned with tensed quantifier systems our discussion is expanded to include quantified modal logics.

Two sorts of system employing tensed quantifiers are on the one hand Prior's system Q and on the other hand those based upon Kripkean modal systems whose semantics allow for possible worlds to differ in what exists with respect to them and where the quantifiers are restricted. Prior's system Q, unlike Kripkean systems, has the disadvantage that it is not two valued whereas Kripkean systems have the disadvantage, unlike Q, that no mixing formulas are valid. So we attempt to construct a first order theory which is on the one hand two valued yet on the other hand has as theorem s intuitively acceptable mixing formulas. In order to do this we introduce a negation operator, the * operator, which we claim is a predicate modifier. We attempt to show that there is a close connection between the problem of founding adequate tense and modal first order systems employing restricted quantifiers and the representation of weak and essentialist necessity (and tensed analogues). We offer then as extensions to Kripkean type systems systems which involve the * operator.

However, we show that systems employing the * operator are equivalent to systems involving an existence predicate without this operator. This point about 'reducibility' of systems (to systems without *) brings us into the final chapter. Here our aim is to draw together points made about existence in Chapters five and six (and earlier chapters) into a single discussion. An attempt is made to show that the first two theses of that trilogy mentioned in connection with Chapter five above are not correct. That is, that the existential quantifier must be read tenselessly and the other claim that there is only one sense of 'exists', a tenseless one, represented by the quantifier. Instead, we attempt to argue that there is more than one sense of 'exists' and that these senses are of first level predicates. Consequently, we claim that the existential quantifier should not be understood as representing existence; instead it should be read as 'Some x such that it ...' which does not have that extra predicative 'There is ...' element as in 'There is an x such that it...'.

PART ONE

TENSE AND GENERALITY IN ANCIENT AND MEDIEVAL LOGIC

CHAPTER ONE   TENSE AND ANCIENT LOGIC

1.1 Singular Statements and Tense; Genesis of a Tradition.

In the chapter on substance in the 'Categories' Aristotle discusses a puzzle. He wants to claim that what is most distinctive about primary substances is their ability to receive contraries; an individual man hot at one time may be cold at another. But, someone may object, statements and beliefs are also like this.

> "For the same statement seems to be both true and false. Suppose, for example, that the statement that somebody is sitting is true; after he has got up this same statement will be false. Similarly with beliefs."
>
> (Cat. $4^{a}23$ - $4^{a}26$)

In attempting to solve this problem Aristotle does not relinquish the view expressed here that a singular statement, at least, may vary in truth value over time. Very briefly, his solution is that primary substances receive contraries only because they can change whereas statements and beliefs, strictly speaking, do not because they are completely unchangeable. Their variation in truth value over time, a variation which is not to be seen as a variation of properties is completely attributable to change in substances.

Perhaps this puzzle arises for Aristotle because he treats a statement here in one important respect like a primary substance; they both appear to have individual histories. A person who was a child becomes an adult while a statement which was true becomes false. This idea that a statement has a history is part of the Stoic notion of an 'axioma' considered below. Now, one way to avoid this resemblance between statements and primary substances is to clearly distinguish the type statement from its token instances.

This presumption that statements can and do vary in truth value over time is not unique to Aristotle; it is maintained throughout the logical tradition which he founded until, at least, the end of the Medieval era. He was not, however, the only founder of systematic logic. Mention must also be made of the Megarian and Stoic logicans who held this view of a statement, such as Chrysippus, Diodorus Cronus and Philo whose logical ideas became engulfed within the Aristotelian tradition in the early centuries A.D. It is this feature, truth variability, which is central for understanding their debate about the nature of modality. Diodorus argued in the now very obscure but then highly thought of 'Master Argument' that the possible is that which either is or will be. An alternative account, the Stoic theory, connects the possible with what is true at some time. Accordingly a necessary statement is one which does not admit of being false at any time or admits of being false but is prevented by external circumstance. (Mates 1953 p. 41)

Exemplified, then, in this discussion is a belief common to Ancient and Medieval logicians alike that there is a close connection between the content of certain modal and temporal notions (as opposed to only accepting that there are formal similarities between them - see chapter 6). Besides these Megarian and Stoic views Aristotle claims that, in one sense, the capable is that which is realized ('De Interpretatione' 23^a7 - 10) and the Medieval logician William of Sherwood writes

> "Notice, however, that 'impossible' is used in two ways. It is used in one way of whatever cannot be true now or in the future or in the past ... It is used in the other way of

whatever cannot be true now or in the future although it could have been true in the past ..."

(1966 p. 41)

It is not just statements as linguistic items which can vary in truth value over time but also, according to the Ancients, what they express. This comes out clearest in the case of the Stoics but there is no reason to doubt that Aristotle also held this view. For he believed that statements are symbols for thoughts in the soul ('De Interpretatione' 16[a]19) and, moreover, as noted, he allows beliefs to vary in truth value. The Stoics developed quite a complex theory about what it is that statements express, namely 'axioma' which differ from Fregean thoughts in two important ways. In the first place they are tensed and secondly they can, like individuals, cease to exist by being destroyed and, presumably, though this is not mentioned, come into existence as well. (Kneale and Kneale 1962 p. 154). In order to appreciate what is involved in this idea that a thought can be destroyed Stoic views about inferential connections between statements need to be taken into account. Sextus Empiricus mentions that there are three types of atomic Stoic statement; 'definite' such as 'This man walks', 'indefinite' like 'Someone walks' and, finally, 'intermediate' which involve proper names such as 'Socrates walks' (Mates ibid p. 30). Now, neither an indefinite nor an intermediate statement can be true unless a related definite statement is. Consequently, Chrysippus argued that while Dion is alive the axioma expressed by 'This man is dead' (where 'this man' indicates Dion) is impossible because it is self-contradictory and when Dion is dead the axioma ceases to exist so the question of its truth or falsity does not even arise. (Kneale and Kneale ibid p. 154). An axioma, then, is destroyed when it ceases to be expressible. This view, as far as we know, is not explicitly held by any logician after the Stoic period until very recently when Prior in 1957 argues for a variant of it. (This is discussed in Sections 6.5 and 6.6).

Logic's concern is with principles of valid inference. Central to and underlying this is the notion of truth-preservingness which, of course, in turn depends upon an account of truth. Now, given the Ancient account of a singular statement one should expect that it would be truth at a time (and, hence, truth-preservingness at a time) which is to be fundamental to logic. For, clearly, a consequence of the Ancient view of a singular statement is that tense distinctions are a proper subject for logical reflection. This consequence does hold in the Megarian and Stoic case but, on the other hand, tense considerations do not figure very much in Aristotle's systematic logical writings for reasons which are noted in the next Section.

Unlike the Medievals the Ancients also discussed the 'impure' tenses, that is, tenses other than the simple past, present and future (see Section 3.5). According to Diogenes, Chrysippus wrote works on tense; one entitled 'On Temporal Expressions' and another 'On Axiomata in the Perfect Tense'. And Priscian, the grammarian, gives the Stoics credit for distinguishing a use of the present tense which expresses (what is expressed in English by) the continuous present (Kneale and Kneale ibid p. 153). Aristotle does not mention these tenses in his logical writings but he does discuss certain aspects of them in connection with action. (See Kenny 1963 Chapter VIII and Potts 1965).

1.2 Aristotle on General Statements and Tense.

Because logic's concern is with principles of valid inference an important question is which areas of discourse, if any, are to be taken as central when one attempts to provide a systematic account of these principles? Both Quine (see Section 5.1) and Aristotle answer this question in terms of scientific and mathematical discourse since they both base systematized logic primarily on these disciplines where, of course, tense does not figure much. In this Section attention is paid to what little involvement tense has in Aristotle's account of generality.

Aristotle, according to Ross (1949 p. 33), undertook the study of the syllogism as a stage on the way to the study of scientific method. This comes out very clearly in the opening section of the 'Prior Analytics' the work in which he systematizes syllogistic logic.

> "Our first duty is to state the scope of our inquiry and to what science it pertains: that it is connected with demonstration and pertains to a demonstrative science".
> ($24^{a}10$ - $24^{a}12$)

Which sciences, then, are demonstrative? In Chapter 12 of the 'Categories' where Aristotle discussed different senses of priority he says that in the demonstrative sciences there is a prior and a posterior in order because the elements are prior in order to the diagrams. This shows that he, at least, took geometry to be one of these sciences. Ross believes that he did have that discipline in mind as a paradigm example, inevitably so, because the mathematical sciences were the only ones which had been to any degree developed by the Greeks at the time Aristotle wrote (1949 p. 52).

In the 'Posterior Analytics', especially in the first few chapters, Aristotle expounds his theory of demonstrative science. This concerns the logical form scientific theories do or should exhibit (Barnes 1975A p. 65). They should be axiomatized deductive systems consisting of a set of demonstrations which are a type of syllogism. Because he sets up his account of a syllogism with the organisation and presentation of the results of scientific research in mind one should expect him to pay special attention to definitions and to general statements of the sort that concern relations between kinds of things (Kneale and Kneale 1962 p. 5). This turns out to be the case. In Chapter 2 of the 'Prior Analytics' the only premises he mentions for a syllogism arethe universal and the particular as though singular statements are not a proper part of syllogistic reasoning. And other than two examples of (invalid) syllogisms involving singular statements Aristotle concentrates exclusively on syllogisms which involve general statements. At one point he actually refuses to countenance such premises.

> "We must select consequents not of some part but of the whole of the subject e.g. not those of some individual man but those of every man; for it is from universal premise that the syllogism proceeds."
>
> (Prior Analytics 43^b12 - 15)

This refusal is based upon Aristotle's view of scientific knowledge. In the 'Prior Analytics' (32^b18) he claims that there is no scientific knowledge of what is indeterminate; that is, of that which is capable of happening both in a given way and otherwise like the walking of an animal.

Similarly, in the 'Posterior Analytics' (Chapter 8) he claims that there can not be a demonstration of perishable facts. Instead, scientific knowledge is to consist in knowledge of general facts about permanent relations between kinds of things. Obvious candidates here are those which concern the subordination of a species to a genus.

The fundamental scientific principles upon which demonstration rests are not only general but also necessary.

> "Since it is impossible for that of which there is understanding simpliciter to be otherwise what is understandable in virtue of demonstrative understanding will be necessary, (it is demonstrative if we have it by having a demonstration). Demonstration, therefore, is deduction from what is necessary."
> (Posterior Analytics $73^{a}21$ - 4).

Roughly speaking, this is based upon the following line of thought. The existence of logic depends upon there being sciences and sciences, in their turn, depend upon there being determinate natures. And it is the grasp of these natures via sense perception which is the basis for scientific knowledge.

So far emphasis has been put on the connection between the syllogism and demonstrative science. Aristotle, however, does allow the formal machinery a wider use. For besides demonstrative syllogisms there are also dialectical and modal ones. Dialectical syllogisms are closely connected with argumentation, particularly of a philosophical sort. A premise is posited in order to draw out, syllogistically, untenable consequences so as to show whether it or its contradictory is sound. Premises of dialectical and modal syllogisms which are valid need not be necessary. For in the case of the modal syllogism Aristotle writes

"Now every premise is of the form that some attribute applies, or necessarily applies, or may possibly apply, to some subject."

(Prior Analytics $25^a1 - 3$).

Despite this, however, it is important to realize that the primary function of the syllogism for Aristotle is connected with demonstration. Up to and including the second century A.D. Aristotle's categorical syllogism was associated with geometrical demonstration whereas Stoic logic was associated with dialectic. It was thought then, that these two systematized logics have different fields of application. (Kneale and Kneale 1962 p. 182).

When discussing general statements Aristotle introduces some technical terminology, namely that of 'term' and that of '...contained in...' or'...belongs to...' or '...applies to...' These terms of art have, of course, a central place in the subsequent history of logic. By a 'term' he means

"...that into which the premise can be analysed viz. the predicate and the subject with the addition or removal of the verb to be or not to be."

(Prior Analytics $24^a16 - 24^a18$).

And for one term to be wholly contained in another

"...is the same as for the latter to be predicated of all the former. We say that one term is predicated of all of another when no examples of the subject can be found of which the other term can not be asserted."

(Prior Analytics $24^b27 - 24^b31$).

This terminology is geared towards a particular sort of statement containing the copula; namely, instances of (1).

(1) Things of sort A are cases of sort B.

Basing logic upon this kind of statement which may be called a 'classifier statement' for obvious reasons, has general repercussions for tense's involvement in logic. One repercussion is that even when tensing is taken into account the continuous tenses are neglected because instances of (1) are not open for such tensing.

From a logical point of view, this introduced terminology provides a perspicuous framework within which to represent classifier statements. In the following valid argument

All animals are living beings

All persons are animals

∴ All persons are living beings

the occurrence of 'animals' not only occupies a subject (or part of a subject) position but also a predicate (or part of a predicate) position. Unless these roles have something in common it is unclear why the inference is valid. By utilizing the technical terminology the argument is re-written as

Animal belong to living being

Person belongs to animal

∴ Person belongs to living being.

This is a more perspicusus representation for its validity is merely a consequence of the transitivity of the '...belongs to...' relation. In one sense, then, the notion of a term is intended to transcend the subject/ predicate distinction. But this does not mean that that distinction is unimportant. Alexander of Aphrodisias when considering why it was that Aristotle introduced the term '...belongs to...' suggests three reasons, one of which is that it brings out clearly which term is the subject and which the predicate (Patzig 1968 p. 11).

Patzig, himself, fills this out by suggesting that while 'A is B' is not very perspicuous since it does not bring out that it is A which is subordinated to B. 'A belongs to B' does. The introduction of a common element which can be used in both subject and predicate positions also allows for the introduction of variables, namely term variables. And this adds weight to the point that Aristotle's main concern was with general statements because a singular term can, at best, only occur as part of a predicate term.

Because the interesting true classifier statements are true unrestrictedly Aristotle takes the view that a universal statement is one in which a term is purported to be wholly contained in another in such a way as to be temporally unrestricted.

> "Now I say that something <holds> of every case if it does not hold in some cases and not others, nor at sometimes and not at others."
>
> (Posterior Analytics 73^a28 - 30).

This is hardly surprising given the connection between the syllogism and scientific methodology. Aristotle, however, also argues for this in connection with a syllogism which consists of a mixture of assertoric and problematic premises.

> "We must understand the expression 'applying to all' not as qualified in respect of time e.g., 'now' or 'at such-and-such a time' but in an absolute sense; for it is by means of premises taken in this latter way that we affect our syllogisms. If the premise is taken as relating to the present moment there will be no syllogism. For presumably there is no reason why at some time 'man' should not apply to everything that is in motion: i.e. if nothing else were then in motion; but the term 'in motion' may apply to all horses and 'man' cannot apply to any horse."
>
> (Prior Analytics 34^b7 - $^b16$).

Here he is claiming that if one wants a valid mixed syllogism whose conclusion is problematic from universal assertoric and problematic premises then the former must state something which is permanently true. The example he gives offends this rule and produces a false conclusion, 'All men are possibly horses', from true premises.

Because a universal statement is, in Aristotle's eyes, temporally unrestricted it can not change in truth value over time. Its time of use is irrelevant to assessment of its truth value. In contrast Aristotle also held the view, noted in the first section, that singular statements can and do vary in truth value over time. These different views naturally fit into two different logical frameworks. On the one hand, a framework which takes as basic statements that can vary in truth value over time and on the other hand one in which they can not. These frameworks we develop in Chapter three.

One difficulty then for Aristotle's views is how both these accounts of a statement fit together within a single framework. Now, one aspect here of this difficulty is whether or not a particular statement such as 'Some person runs' can change in truth value over time. If it can, then what is its inferential connection with the temporally unrestricted universal statement 'All people run' given Aristotle's square of opposition? On the other hand, if it can not then what is its connection with singular statements which can? We shall briefly return to this point in Section 1.5. Before then in the next two sections we look at Aristotle's comments about tense and the verb as contained in his 'De Interpretatione', comments which expand upon his view that singular statements can vary in truth value over time.

1.3 Aristotle on the Verb; Tense and the Verb.

Aristotle expressed the view that singular statements can change in truth value over time and this is because they contain tense elements. In 'De Interpretatione' there are comments which bear upon the question of how tenses structurally contribute to a singular statement. Central here is his distinction between names and verbs. Names, he says, are spoken sounds significant by convention without time (19^a19 - 20) whereas

> "A verb is what additionally signifies time..., and it is a sign of things said of something else. It additionally signifies time: 'recovery' is a name but 'recovers' is a verb because it additionally signifies something's holding now. And it is always a sign of what holds, that is, holds of a subject."
>
> (16^b6 - $^b11$)

Two important distinctions between names and verbs are mentioned here. First, verbs are signs of 'what is said' of subjects (that is, they are predicative) and secondly, they also signify time.

Ackrill points out in his notes to the translation (1963 p. 119) that it is unclear from the text how Aristotle would analyse such sentences as 'Socrates is a man' or 'Socrates is white'. The problem is, what is to count as the verb in these sentences? The two specific features which come together in 'runs' are divided between the 'is' and 'a man' in 'Socrates is a man' and between 'is' and 'white' in 'Socrates is white'. While the copula introduces time determinations the expressions 'a man' and 'white' introduce the 'what is said'. The available evidence in the text does not point to a unique answer as Ackrill shows. Four points of reference are the following.

(a) In Chapter One (16^a15) he gives 'white' as an example of a verb and in Chapter 10 (20^a32) he gives 'not-just' as an example of an indefinite verb (see 1.4). But because the copula can be omitted in Greek Ackrill points out that these examples can be thought of as 'is white' or 'is not-just'.

(b) In Chapter Ten (20^b2) name and verb are said to be transposed in sentences whose word-by-word translation Ackrill gives as 'is white man' and 'is man white'. Here it is 'white' which is treated as the verb.

(c) In Chapter Ten (19^b21) Aristotle, it seems, is uncertain of how to characterize the 'is' in 'a man is just'. He says

> "But when 'is' is predicated additionally as a third thing...(I mean, for example, 'a man is just'; here I say that the 'is' is a third component - whether name or verb - in the affirmation."

(d) In Chapter Twelve (21^b9) Aristotle claims that to say that a man walks is no different from saying that a man is walking. And this appears to support the view that 'is white' is the verb.

Two different analyses of a singular statement can be discerned here. One analysis which bears upon Aristotle's account of a statement as offered in his syllogistic writings centres on point (c) and utilizes a general form of point (d). He analyses 'All persons are animals' in his syllogistic writings as 'Person belongs to Animal' (see the last Section) where the copula has become a third item relating terms. Therefore the statement 'Socrates is white' can be taken as 'Socrates now belongs to white things', a form suitable for a syllogism (which allows for tensed singular statements). But what about statements which do not contain a copula like 'Socrates knows' or 'Socrates kicks Callias'? It appears that a generalized version of (d) was implicitly invoked. Care, however, must be taken in expressing point (d). In English, it is better not to say that 'Socrates runs' is no different from 'Socrates is running' because of the change in tense (and anyway, 'Socrates knows' cannot be dealt with in this way - See 3.5). Moreover, 'Socrates is running' unlike 'Socrates is a person' is not in the form of a classifier statement. Instead then, it is better to say that 'Socrates is a runner' is no different from 'Socrates runs' where both are to be understood non-frequentatively. And somewhat more awkwardly, 'Socrates kicks Callias' becomes 'Socrates is a kicker of Callias'. One consequence of this view is that in a fully analysed Aristotelian logical language there are no verbs in his sense (except, perhaps, the existential 'is' as used in 'Socrates no longer is').

An alternative analysis of a singular statement in which the copula belongs with the predicate can also be discerned. Because '... is (now) a runner' performs the same role in 'Socrates is (now) a runner' as 'runs' does in 'Socrates runs' it can be taken as a verb in Aristotle's sense. This analysis has much more in common with twentieth century analyses than does the term analysis. For now in order to explain the validity of the inference

All animals are living beings

All persons are animals

∴ All persons are living beings

appeal has to be made to the predicative role of 'is an animal' in both the (grammatical) subject and predicate positions.

This discussion brings out a problem which Ackrill remarks upon (ibid p. 121). What precisely is the force of Aristotle's discussion of the verb? Is his distinction between names and verbs with its two facets grammatical or logical or even a mixture of the two? (This last case occurs when his distinction concerning the 'what is said' is understood logically whereas reference to time is understood grammatically only - see Section 3.2 on this). As we shall note in Section 2.4 there is tension between the term analysis of a statement and Aristotle's account of tense and the verb. This tension can be overcome if his comments about tense are taken grammatically only. However, in what follows we assume his comments have logical force since that is where their interest for us lies. There is no clear answer to this question of import from what Aristotle, himself, says. And to make matter worse he mentions at 16^a18 a contrast between two senses of 'is' which lets it be understood 'either simply or with reference to time'. But in no place that we know of does he elucidate what he means here by 'simply'.

1.4 Aristotle on the Verb; Inflected and Indefinite Verbs.

Aristotle distinguishes between names proper, indefinite names and inflections of names; labels which he also uses to mark distinctions amongst verbs. A difficulty here is that it is unclear whether or not the motivation for these distinctions amongst verbs is the same as that for the corresponding ones amongst names.

He writes concerning inflected names.

> "'Philo's' 'to Philo' and the like are not names but inflections of names. The same account holds for them as names except that an inflection when combined with 'is' 'was' or 'will be' is not true or false whereas a name always is. Take, for example, 'Philo's is' so far there is nothing either true or false."
> ('De Interpretatione' 16^a32-16^b6).

Unlike names proper, then, they do not form a complete sentence when combined with existential uses of 'is'. In the case of verbs he writes

> "...'recovered' and 'will-recover' are not verbs but inflections of verbs. They differ from the verb in that it additionally signifies the present time they the time outside the present."
> (ibid 16^b16 - 16^b19).

There is a different contrast here. Ackrill (1963 p. 121) comments that if Aristotle had wished to draw a similar distinction for verbs he could have done so, a distinction which would mark the indicative mood from other moods. But this depends upon emphasizing only that an inflected name together with a verb does not produce anything which is true or false rather than emphasizing that they do not produce a complete sentence. So, perhaps inflection is to be given no more weight in Aristotle's thought than that of a grammatical notion which would then give a unity to both name and verb inflection. But there is a problem with this interpretation; verbs were not only inflected for tense but also for mood and person which are not mentioned here by Aristotle. Let us not pursue this point.

What is certainly true is that tense inflection takes on a logical significance given Aristotle's account of a singular statement together with his view that verbs introduce time determinations. Sentences such as 'Socrates sits' and 'Socrates sat' are to be distinguished in such a way as to be systematically connected. The latter is true now if and only if the former was true. Accordingly, one can interpret Aristotle to be making the claim that a sentence like 'Socrates sat' is to be analysed as 'Socrates sits + past inflection' where 'sits' is the verb proper. This line of thought closely connects tense and mood. For instance, we may compare 'Socrates sits + imperative' as an analysis of the imperative 'Sit Socrates!' Not only this but there is also a closer analogy to be made given Aristotle's account of modal statements. He distinguished between 'A belongs to B' and 'A necessarily belongs to B' as noted in Section 1.2. Consequently, if the verb proper is taken as basic to predication distinctions can be made between 'A now belongs to B', 'A belonged to B' and 'A will belong to B'. This similarity between tense and modality is fundamental to Prior's analysis of tense as revealed in his tense logics. This adverbial treatment of tense we develop in Chapter three.

Another aspect of Aristotle's discussion of the verb is what he calls an 'indefinite verb'. In the case of general names he writes

" 'Not man' is not a name nor is there any correct name for it. It is neither a phrase nor a negation. Let us call it an indefinite name."
(De Interpretatione 16^a29 - 32).

Ackrill suggests (p. 117 - 8) that he calls them indefinite because they 'stand for' no definite kind of thing in the sense that they 'stand for' things which may belong to any species or category. In the 'Prior Analytics' (24^a 20 - 23) Aristotle also introduces 'indefinite statements' which are general but indefinite as to whether they are to be understood as universal or particular; his example is 'Pleasure is not good'. In the case of indefinite verbs he says

" 'Does not recover' and 'does not ail' I do not call verbs. For though they additionally signify time and always hold of something, yet there is a difference - for which there is no name. Let us call them indefinite verbs because they hold indifferently of anything whether existent or non existent."
(De Interpretatione 16^b11 - 16^b16).

Ackrill comments (p. 120 ibid) that it does not seem helpful to call 'does not recover' an indefinite verb because it is not a sign that something indefinite holds but rather that something definite does not hold.

Can sense, however, be made of Aristotle's suggestion that 'does not recover' is called 'indefinite' because it 'holds indifferently of anything whether existent or non existent'? Relevant to this is his discussion of opposition between 'things' in the 'Categories' Chapter 10. There he distinguishes between four kinds of opposition; relatives, contraries, privation and possession and finally, affirmation and negation (11^b17). It is only the last three which are relevant here. There are, Aristotle says, two types of contraries; first, those for which it is necessary that one or the other of the pair belong to the things they naturally occur in or are predicated of (11^b38 - 40). Examples, he gives, of these are the pairs sickness, health and odd, even. In contrast, there are pairs of contraries for which there is something intermediate (12^b 10 - 11). These include the pairs black, white and just, unjust. Possessions and privations like sight and blindness, Aristotle says, are only spoken of in connection with whatever is naturally a possessor. Moreover, he believes that a possession is only absent from something at a time when it is natural then to possess it.

Unlike pairs of statements involving contraries or possession and privation it is necessary that one or other of an affirmation and its negation be true. And this is so, he writes, despite there being contraries without intermediates. For it is not necessary that 'Socrates is sick' or 'Socrates is well' be true.

> "For if Socrates exists one will be true and one false, but if he does not both will be false; neither 'Socrates is sick' nor 'Socrates is well' will be true if Socrates himself does not exist at all."
>
> (Categories 13^b17 - 13^b18).

Similarly in the case of possession and privation. If Socrates does not exist then neither 'Socrates has sight' nor 'Socrates is blind' is true. In contrast, it is necessary, for one of the pair 'Socrates is sick' and 'Socrates is not sick' to be true. If he does not exist then 'Socrates is sick' is false and so 'Socrates is not sick' is true (18^b31). It is important to note that Aristotle is here appealing to the sentential nature of the negation in 'Socrates is not sick'. Is he maintaining that the non-existence of the subject makes an affirmative present tensed singular statement false and consequently a negative one true? And is it this which underlies his account of an indefinite verb; in particular, on the point that sentences which involve the verb proper can only be true if the subject exists whereas those which contain indefinite verbs can be true even if the subject does not exist? If so, as Ackrill points out (ibid p. 111), how is this to be reconciled with Aristotle's contention that 'Homer is a poet' does not entail 'Homer is' (De Interpretatione $21^{a}25$)?

Aristotle's discussion centres on the interrelation between tense, prediction, negation and existence in connection with singular statements. An important question here and one which is relevant to understanding the Medieval philosophy of language is what is the relationship, if any, between the tensed existential 'is' and the tensed copula? Or, alternatively expressed, what is the connection between present tensed predication and the tensed existential predicate 'is'? The simplest connection maintained is that an affirmative present tensed predicate can only be true of present existents. This involves two claims. First, to be something now is to be now and secondly, its converse, to be now is to be something now. The latter appears to be unexceptionable for to assert that something is without being anything requires there to be a totally indiscernible existent. The Stoics appear to have believed that there is this simple connection. For, as noted in 1.1, they held the view that atomic sentences such as 'Some person runs' or 'Socrates runs' can only be true if a relevant sentence of the form 'This person runs' is true. But, as they realized, this creates difficulties one of which centres on the sentence 'Dion is dead'.

Neither Aristotle nor the Medievals held that there is such a simple connection. As noted Aristotle remarked that 'Homer is something (say, a poet)' does not entail 'Homer is'. Moreover in 'De Sophisticis Elenchis' he writes that it is an error to believe in that simple connection, an error which can give rise to fallacies.

> "Those [fallacies] that depend on whether an expression is used absolutely or in a certain respect and not strictly, occur whenever an expression used in a particular sense is taken as though is were used absolutely e.g., in the argument 'if what is not is the object of an opinion that what is not is': for it is not the same thing 'to be x' and 'to be' absolutely."
>
> (166^b38 - 167^a3).

The Medievals expanded upon this as we note in Section 2.3. They distinguished between two types of affirmative present tensed predicate. First, those which can only be true at a time of existents then and secondly those like '... is an object of an opinion' which can also be true of non-existents. The latter sort were said to have ampliating force because their extensions at a time are ampliated (extended) beyond the then existing objects. Consequently, when $\emptyset$ is a non-ampliating predicate in the present tense 'a is $\emptyset$' implies 'a is (now)'. And, the predicates Aristotle employs in Chapter 10 of the 'Categories' namely 'is sick' 'is well' 'is blind' and 'has sight' are all non-ampliating.

The full force of Aristotle's discussion of indefinite verbs has not yet been brought out. As a conjecture what may lie behind his discussion is that 'not' in 'Socrates does not recover' can be treated either as a sentential operator or as a predicate modifier. A sentence like 'Socrates does not recover' can then be analysed either as 'It is not the case that Socrates recovers' or into subject 'Socrates' and predicate '... does not recover' where the predicate is ampliating. In this latter analysis the particle 'not' maps non-ampliating (and ampliating) predicates into ampliating prediates. So it may be this, then, which underlies his suggestion that an indefinite verb holds indifferently of existents and non-existents.

This suggests that there might be room for two kinds of negation which have different semantic force. One kind affects the existential presupposition and the other does not. Fundamental to this latter kind is that it maps non-ampliating into non-ampliating predicates. This fits in with Aristotle's account of contraries which have no intermediates. Also of relevance here is the Stoic distinction between negation and privation.

> "A negation... is formed from a proposition by prefixing the negative 'not'. A privation is an atomic proposition obtained from another atomic proposition by reversing the predicate: 'This man is unkind'."
>
> (Mates 1953 p. 31).

Involved in this is the suggestion that a distinction between two such kinds of negation may in the final analysis depend upon distinguishing sentential from predicate negation. It is this suggestion which we consider in Chapter six in some detail. We introduce as a negation operator the * operator which we claim is a predicate modifier and which has the property of mapping non-ampliating predicates into non-ampliating predicates. We attempt to show that this operator has some interesting properties, properties which are useful for setting up tensed quantification theory.

The fact that certain affirmative predicates have ampliating force may create difficulties for Aristotle's account of 'thing' as contained in Chapter two of the 'Categories' (and hence also for the Medieval theory of predication as inherence - see Section 2.2). Very briefly, Aristotle held that certain things are either 'said of' individuals or are 'in' them. The former are those which yield definitions of types of individual substances as in 'Socrates is a person' where person is said of Socrates. On the other hand it is accidents which are 'in' individuals. In 'Socrates is white' it is whiteness which is claimed to be 'in' Socrates. (The Medieval theory of inherence extended Aristotle's notion of 'in' to cover all predication). But what is to be made of sentences which involve ampliating predicates like 'Socrates is famous'? To claim that fame is now 'in' Socrates is to claim that accidents can be 'in' non-existing objects. And this is to accept that non-existents at a time may then have properties. We shall note in Chapter six that Prior rejected this view since he believed that to exist now is just to have properties now. In contrast, we note, in Chapter seven, that both Meinong and MacColl did hold the view that non-existents can have properties.

1.5 Generality and Tense; Some Remarks.

In this Section attention is paid to the interconnection of tense and generality within the framework of the term analysis. As a point of entry we first consider the question of how Aristotle's account of a singular statement fits in with his account of a universal statement, a question we noted in Section 1.2.

Aristotle distinguished between the temporally unrestricted (1) and the temporally restricted (2).

(1) All As are Bs.

(2) All As are now Bs.

He also claimed that verbs differ from names in two ways; first by introducing the 'what is said' and secondly by signifying time. Now, as we noted, under the term analysis these aspects are split up; the former is introduced by the predicate term whereas the latter by the copula. Given this then what difference is there in the tensing of the two uses of the copula in (1) and (2)? For, unlike instances of (1) those of (2) can vary in truth value over time.

One reaction here is to claim that not all uses of the copula are tensed uses. For instance, in (3) the copula may be said to be 'tenseless'. (Tenseless verbs are discussed in Section 3.2).

(3) All numbers are prime.

Even accepting this, can it be maintained that the contrast between (1) and (2) resides in a difference between tenseless and tensed copula uses? One source of doubt is that if (3) is understood 'tenselessly' then it is not clear that there is a statement 'All numbers are now prime' which is to be contrasted with it. For otherwise there would be two fundamentally different predicates '... is (tenseless) prime' and '... is now prime' applying to the same sorts of things. Moreover, if it is this distinction between tensed and tenseless which underlies the difference between (1) and (2) then it is a total mystery why (1) implies (2). This suggests, then, that in cases where there is a contrast to be made between statements like (1) and those like (2) then the former type do contain tensed copula uses albeit not simply tensed but complexly tensed. An alternative reaction here is to claim that both (1) and (2) are present tensed. For the period of time known as the present varies appreciably according to context. For instance, consider the following set of sentences

(4) He wins

(5) He is hungry

(6) He is ill

(7) He loves her

(8) He is called 'Johnny'

(9) The sun rises in the East

(10) Gold is heavier than silver.

Here, there is a gradual transition from what is more or less momentary to what is 'eternally' true (Jespersen 1961 p. 17). Hence, it may be claimed that both (1) and (2) are present tensed. Jespersen, for instance, writes

> "... one might feel tempted to speak of an 'omnipresent' time or tense ... but no special term is needed ... If the present tense is used, it is because they are valid now; the linguistic tense expression says nothing about the length of duration before or after the zero-point..."
>
> (ibid).

But for logical purposes there is a need for distinguishing between (1) and (2) in order to account for inferential connections. Unlike (2), (1) implies 'All As will be Bs'. Now, fundamental to tense logic is the notion of truth at a time. Consequently, it is useful to distinguish between the point present or the order present and the present understood as a period of time which may vary from context to context in length. This difference results in similar differences between two notions of past and future, a difference between order and period notions. And it is assumed here that the pure tenses express temporal order only. A more detailed account is given in Sections 3.4 and 3.5.

The question of distinguishing (1) from (2) occurs also at the particular level. 'All persons are moving beings', 'No person is a 'moving being', 'Some person is a moving being' and 'Some person is not a moving being' have to be in the same tense unless understood to involve multiple generality for otherwise the right logical connections according to the square of opposition would not hold between them. So, in one sense, the particular statement 'Some person is a moving being' can be taken to be temporally unrestricted. Yet, in another sense it can be understood to be variable in truth value over time. This looks to be the case in the inference from 'Socrates is a moving being' to 'Some person is a moving being' given that 'Socrates is a person'.

Without a formal network within which to consider the matter it is difficult to distinguish between temporally restricted and unrestricted statements. In later chapters ways of distinguishing such kinds of statements are considered, ways which depend upon an account of the contribution tenses can make to the truth conditions of statements containing them. In the rest of this Section a number of informal remarks about the term analysis of a statement are made.

One suggestion for distinguishing between (1) and (2) is that the unrestricted 'All As are Bs' is true iff the temporally restricted 'All As are now Bs' is always true. But, there is a problem with this under the traditional analysis of a statement because 'All As are now Bs' presupposes that there are now both As and Bs. Consequently, the temporally unrestricted statement would not only presuppose that there are As and Bs at some time but more strongly that there As and Bs at all times.

So far attention has been paid to the unrestrictedness of Aristotle's universal statements. However, in the context of a demonstrative syllogism a universal statement is also necessary. This notion of necessity may be taken to be physical necessity, a feature of scientific laws. Whereas a universal statement 'All As are Bs' is true (given the Boolean understanding of 'All') iff for any x, if x is an A then x is a B, a law is true iff also if γ which is not in fact an A were to be an A then it would also be a B. Although laws are standardly expressed in the universal form 'All As are Bs' it is assumed here that their correct form of expression is 'Necessarily, All As are Bs'.

Under the term analysis (1) and (2) are represented by

(1') and (2').

(1') A belongs to B

(2') A belongs now to B.

Are the terms 'A' and 'B' in these the same in the two cases? That is, is 'Person' ('Moving Being') the same term in 'Person belongs to moving being' as it is in 'Person belongs now to moving being'? It may, for instance, be thought that the semantic role of 'Person' in these statements differ in that in the former case it 'specifies' persons at any time whereas in the latter it only 'specifies' presently existing people. Closely related to this is the question is 'Person' the same term in 'Person belongs now to moving being' as it is in 'Person belonged to moving being'? Discernible here is a line of thought, fundamental to the Medieval theory of terms, that the interpretation of a term is determined by its use in the context of a sentence. For De Rijk (1967 p. 569) points out that the Medieval theory of supposition is to be understood as an answer to problems which appear in the investigation of the logical import of the copula in categorical affirmative statements. Consequently the Medievals were interested in how tensed copulation affects the interpretation of terms and it is this aspect of their theory of language which is discussed in the next chapter.

After the Medieval era many theorists took the copula to be 'tenseless' in all uses. But this creates the difficulty of being able to distinguish between 'A belongs to B', 'A belongs now to B' and 'A belonged to B'. One way these were distinguished was by making the terms themselves carry the tense distinctions. For example according to Whatley

> "The Copula, as such, has no relation to time but expresses merely the agreement or disagreement of two given terms: hence if any other tense of the substantive verb besides the [timeless] present is used... if the circumstances of time do really modify the sense of the whole proposition ... then this circumstance is to be regarded as part of one of the terms: ... as 'this man was honest'; i.e. 'he is one formerly-honest'."
>
> (quoted from Prior 1957 p. 105).

This has the consequence that tense distinctions are to be taken adjectivally; just as 'tall' restricts 'man' in 'tall man' so 'formerly' restricts 'honest' in 'formerly-honest'.

Both Aristotle and the Medievals travelled some way along this path of attributing certain temporal distinctions to the terms. For instance in 'De Sophisticis Elenchis', a work in which Aristotle discusses and classifies certain paralogisms he gives as examples of equivocation of terms the subject terms 'the sick man' and 'the sitting man' in the statements 'the sick man is healthy' and 'the sitting man stands up'. For it was concluded from these that the same man is both seated and standing or both sick and healthy at the same time. As Aristotle notes

> "... it is he who stood up who is standing and he who is recovering who is in health: but it is the seated man who stood up and the sick man who was recovering."
>
> (165^b38 - 166^a2).

His solution here is to claim that a sentence like 'The sick man does so-and-so' is ambiguous between 'The man who is sick does so-and-so' and 'The man who was sick does so-and-so'. A clearer example of the ambiguity is 'A man sitting in this room fired the fatal shot'. The ambiguity involved here is considered in more detail in the next chapter, particularly in Section 2.4.

One final point in this chapter concerns a particular interpretation of universal statements. Earlier in this Section the view that the distinction between (1) and (2) resides in the tenseless | tensed difference was summarily rejected. There is, however, one line of thought for which this account of their difference is appropriate. Basically, this is that (1) and (2) differ in what they are about. Whereas (2) concerns individual As and Bs, if there are any, (1) is to do with kinds of things. Now, this distinction is of importance for understanding certain aspects of Medieval metaphysics, namely their long debate concerning universals. This distinction is met again in chapter seven in connection with existence claims.

2.1 Introductory Comments on the Medievals.

Chapter One dealt with Aristotle's account of singular statements together with certain aspects of his discussion of general statements. In the case of universal statements his discussion is tied up closely with his philosophy of science as expounded in the 'Posterior Analytics'. On the other hand his views on singular statements are expounded in the 'Categories' and 'De Interpretatione'. And it was these latter two works which were the only generally available translated works of the Organum in the Latin West until the first half of the twelfth century. High Medieval logic came to fruition after the discovery of the remaining works of the Organum. This logic was not only a continuation of Aristotle's but also contains certain important extensions, in particular, the theory of terms within which tense considerations loom large.

Aristotle's vision of demonstrative science plays no part at all in the early development of Medieval logic. It was 'De Sophisticis Elenchis' and not the two Analytics which made the biggest impression upon twelfth century logicians. Later writers, however, and, in particular, Ockham who wrote in the fourteenth century were influenced by Aristotle's theory of science. (See Ockham 1964 p. 3 - 17). Moody notes that Ockham viewed logic very much in the spirit of Aristotle

> "...[Logic's] ulterior purpose is that for the sake of which logic exists - namely, the science of nature. It is worth noting that Ockham here speaks as if logic were the instrument only of the philosophy of nature and not of

... metaphvsics."

(1935 p. 32).

In contrast, Sherwood, a thirteenth century logician, looked upon logic along with his comtemporaries as one part of the 'trivium' concerned with the science of discourse.

"This science [of discourse] has three parts: grammar which teaches one how to speak correctly; rhetoric which teaches one how to speak elegantly; and logic which teaches one how to speak truly."

(1966 p. 21).

De Rijk in his work 'Logica Modernorum' (1962 and 1967) argues that there are two factors central to the rise of terminist logic out of it s Aristotelian origins, namely the discovery of 'De Sophistici Elenchi' in about 1130 in Paris and much more importantly, a renewed interest in grammar. It is then, the interconnection of these factors which go some way to explain the subtleties of the Medieval philosophy of language. For to show why certain arguments are fallacious depends upon a sensitivity for language and in particular for the semantic roles of various words.

In Ancient times grammar and philosophy were closely connected but after about 200 B.C. under the Alexandrians they became detached and grammar came within the province of literary studies (Robins 1967 p. 22). Dionysius Thrax who lived about 100 B.C. and was taught by an Alexandrian wrote in an important grammatical work that the grammarian's chief concern is with practical knowledge of the general usages of poets and prose writers (rather than with the formal working out of regularities in language - Robins ibid p. 31). This trend continued under the Latin grammarians until the eleventh century when the divorce between philosophy and grammar ended as dialectic became more prominent.

After about 1150 grammarians became much more interested in syntax. In particular they approached it in a contextual manner concerning themselves with ascertaining the function of a word according to its use in a statement. This interest laid the basis for a fruitful interplay between grammar and philosophy (De Rijk 1967 p. 115). For one important consequence of this syntactic interest was a fundamental shift in semantics. In the eleventh century and earlier the meaning of a term was judged not by any criterion of its use in some actual construction but by its original imposition - the purpose for which it was principally or properly invented (De Rijk ibid p. 110 - 111). This shift away from meaning as original imposition to an approach in which the force and function of words are determined by the sentential role in which they occur resulted in terminist logic. Unlike the Ancient and early Medieval grammarians, then, the terminists had a clear idea of the fundamental importance of the statement as the verbal context within which the meaning of a term is to be determined. And eventually, the word 'supposition' was used to characterize the semantic role of a term within a sentential context. This (extensional) property of a term was distinguished from its 'signification' which was its meaning by itself (and did have some connection still with imposition in its earlier history).

The upshot of this was that treatises on the modes of signifying were written works which Robins calls 'speculative grammars' (1967 p. 74). From the grammarian's point of view the discussion of tenses in these 'grammars' is disappointing because of their total concentration upon the pure tenses. (This may connect up with the point that the paradigm Aristotelian statement is a classifier statement which is not open for continuous tensing). From the logician's point of view these Medieval works are of interest especially in connection with their discussion of the contribution pure tenses make to the truth conditions of statements containing them. The main feature here of this discussion is the thesis that the semantic role of a general term may vary in the context of statements which only differ in their tensing. In order to bring out informally this feature as clearly as possible a brief outline of the contribution tense consideration made to the rise of terminist logic is given.

2.2 Tense and the Rise of Terminist Logic.

Boethius speaks of fallacies in connection with the opposition of statements. The most important passage where he does this is his second commentary on Aristotle's 'De Interpretatione' (De Rijk 1962 p. 24). Aristotle makes a passing reference to 'De Sophisticis Elenchis' in Chapter Six of that work.

> "I speak of statements as opposites when they affirm and deny the same thing of the same thing - not homonymously together with all other such conditions that add to counter the troublesome objections of sophists."
>
> (17^a34 - 17^a38).

De Rijk points out that as late as the twelfth century little attention was given to Boethius's commentary on this passage. However, once 'De Sophisticis Elenchis' became known interest increased considerably. Boethius mentions six cases in which the opposition of statements is frustrated by a fallacy. One of these is the fallacy of different times as in 'Socrates sits and does not sit' where 'reference' to two different times is involved. However, what is more important for understanding the Medieval theory of tense and terms is not this fallacy but the fallacy of univocation (and its difference from equivocation). Equivocation occurs in 'Cato runs and Cato does not run' where 'Cato' stands in the first place for Cato Censonius and in the second for Cato Marciae. On the other hand univocation occurs in '(A) man runs and man does not run' where 'man' is univocal because it 'refers' to individual men and to the human species. What then is the difference here?

An expression is univocal if when within the context of different statements and arguably because of those contexts it may have different semantic roles without having a differen signification. In contrast an expression is equivocal because it has more than one signification and it is this which explains its varying semantic role in different statements. The full impact of this distinction depended upon that contextual approach to syntax which was a feature of the renewed interest in grammar in the twelth century. Interestingly, univocation was taken to occur within the context of differently tensed statements. Fundamental here is the Medieval acceptance that a general statement as well as a singular statement can vary in truth value over time. Two technical notions, 'supposition' and 'appellation' play an important role in this acceptance.

Originally, it appears, the term 'supposition' had only grammatical content. In Priscian's writings it probably means 'grammatical subject' (De Rijk 1967 p. 516). On the other hand, 'appellation' had from early days semantic content. The proper function of a name (in Aristotle's sense of 'name') was to 'call' or 'make present' an individual thing in a talk or discussion by means of a symbol or sign. (De Rijk ibid p. 556). This feature of 'calling' or 'making present' (appellation) became somewhat tightened in such a way that the appellation of name was its present application only. Now, this semantic feature of a name connects up with or is derived from Priscian's account of the canonical or basic form of names and verbs. In the case of names the basic form was the nominative singular and in verbs the first person singular present indicative active (Robins 1967 p. 58). Both syntax and semantics, then, were geared towards statements which can vary in truth value over time.

The appellative name conveys also a signification which was taken as resulting from the original imposition even though it was the conceptual presentation of a universal nature. The two terms 'appellation' and 'signification' became the central terms of the twelfth century theory of meaning. However, such a theory looks to be totally inadequate unless provision is made for sentences other than in the present tense. And it is here that the notion of 'univocation' comes into play. For it was assumed that the extension of a general term varies according to the tensing of the statement within which it occurs. For instance, the term 'person' in 'A person sits' was taken to have a different extension from its use in 'A person sat'.

In the 'Fallacie Parvipontane' Adam of Balsham mentions three kinds of univocation. First, when a word is used to denote either itself or what it signifies, secondly when a word is used to denote something of a certain kind or the kind itself and finally when an appellative name is ampliated or restricted in accordance with its connection with a verb in the past, present or future tense (De Rijk 1967 p. 528). It is this last kind which is discussed in the next two sections. Now, a very interesting point here is that the twelfth century theory of terms was intended only to take account of the last of the three kinds of univocation. The first two were put to one side as being non-significative.

That is, in this period, tense considerations were absolutely central to the general theory of meaning.

In the late twelth century there was a changeover in the theory when 'supposition' and 'signification' became the focal terms. This was due to taking into account those non-significative kinds of univoation as well. The central position of appellation was then infringed upon and the whole theory widened out into a supposition theory which took account of sentences like 'Man is a name' and 'Man is a species' which do not depend upon tense considerations. The former use of 'Man' was said to have 'material supposition' while the latter has 'formal supposition'. In the early supposition theory as found, for instance in Sherwood, only the subject term has supposition. Thus, 'supposition' changed in meaning from grammatical to semantical subject. However, in the later theory as found, for instance, in Buridan and Ockham it is not only the subject term but also the predicate term which supposits.

This latter change in supposition theory is connected with the changeover from an inherence account of predication to an 'identity' account. According to the inherence theory a statement like 'Socrates is a person' says that the universal nature or form signified by the predicate inheres in Socrates.

> "Aristotle however, defines a statement as follows: a statement is an expression signifying something of something else or something apart from something else. When he says 'of something else' he means the inherence of the predicate in the subject, and when he says 'apart from something else' he means the separation of the predicate from the subject."
> (Sherwood 1966 p. 26)

And that it is a form which is predicated

> "... the predicate does predicate a form only."
> (ibid p. 113).

Therefore, a statement like 'Socrates is not ill' asserts that illness does not inhere now in Socrates. In the case of general statements supposition theory needs to be invoked. For example, in 'a person is ill' where 'person' has personal determinate supposition (and then is understood as 'a person, namely ..., is ill') the form is said to inhere in the suppositia.

In contrast to this account of predication was the so-called 'identity' theory. This theory only makes use of extensions of terms where both subject and predicate terms supposit.

> "The same sort of account holds in the case of the predicate for by the proposition 'Socrates is white' it is asserted that Socrates is the thing which has whiteness; therefore the predicate supposits for the thing which has whiteness."
> (Ockham 1974 p. 189)

And in Buridan's words

> "... for the truth of an affirmative categorical proposition it is required that the terms, namely, the subject and the predicate stand for the same thing or things."
> (1966 p. 90)

This equi-extension account is based upon a tensed notion of equi-extensionality.

Sherwood in his logical writings makes use of the technical terms, 'supposition' 'appellation' 'copulation' and 'signification'. 'Copulation' he took to be a property of the adjectival name and in contrast, he took supposition to be the property of a substantive name and substantivized adjective. Ockham, on the other hand, just makes use of supposition both as a property of a subject and a predicate term in a statement. Instead of making use of 'appellation' he lets it be a special case of supposition (1974 p. 188). His objection to 'signification' as the connotation of a universal nature is closely bound up with his remarks on universals which are not considered here. On the other hand, Ockham retains a notion of a term having a particular semantic property independent of context, a property which 'signification' covered. But he retains this by distinguishing senses of 'supposition', a point we look at in Section 2.5.

2.3 The Theory of Appellation

Appellation like signification is a property of a term independent of sentential context.

> "Appellation... is the present correct application of a term - i.e. the property with respect to which what the term signifies can be [truly] said of something through the use of the verb 'is'."
>
> (Sherwood 1966 p. 106)

In contrast, the supposition of a term varies according to sentential context. For instance, in the statement 'Some person ran' 'person' was taken to supposit for past as well as present people. Of some interest, then, is the question in which sentential contexts is the supposition of a (subject) term coincident with its appellation? Sherwood answers by giving a rule, the essential clauses of which are

> "An unrestricted common term... suppositing in connection with a present tense verb that has no amplicating force supposits for those [things] that do exist."
>
> (ibid p. 123).

This rule then gives the circumstances in which a term's supposition is equivalent to its appellation. However, the rule does not take into account negation. For although the supposition of 'person' in'Some person runs' coincides with its appellation it is not clear whether this is also the case in 'Some person does not run'. This is discussed later.

There are two criteria for the appellation of a term. On the other hand there is the idea that appellation is the present extension of a term and on the other the idea that it consist only of present existents. Now, these two aspects normally go together but in certain cases they conflict as in 'famous person' whose present extension need not just consist of present existents. (Buridan calls such terms 'ampliative' - 1966 p. 113). It is for this reason that Sherwood stipulates that a term is to be unrestricted. Adjectives and relative clauses were taken either to restrict a term's supposition as in 'A white rose is in the garden' where the supposition of 'rose' is restricted to white roses or to ampliate it as in 'A famous person runs' where the supposition of 'person' is ampliated. The example of restriction Sherwood gives is 'A man who has been...' which supposits for things which do not exist now in the context of a statement whose main verb is in the present tense. But the reason for the ampliation here is the subordinate verb in the past tense.

The condition of suppositing in connection with a present tense verb is, of course, the central condition of Sherwood's rule. What is of interest is that Sherwood takes a common term to supposit for present as well as past|future things in connection with a past|future tensed verb (ibid p. 126 and Buridan 1966 p. 100). The reason for this is that a statement like 'A person ran' was taken to be ambiguous in the same way that Aristotle took 'The sick man is healthy' to be (see 1.5). However, Sherwood subsumes this ambiguity under the compounded|divided distinction. In the compounded sense 'A person ran' is equivalent to 'It was the case that a person ran' whereas in the divided sense it means 'There is now person who ran'.

> "If compounded... the supposition must be stricly indicated by the predicate. In that case 'man' supposits for past men and not for present [men] except in so far as they are past."
> (p. 127 ibid)

and

> "If divided... the supposition [of the subject] is not strictly indicated by the predicate."
> (ibid).

In order to allow for this ambiguity - which is discussed in the next section - a subject term in a past|future tensed statement was said to have its supposition extended from present things to include past|future things.

This account of supposition in past|future contexts creates two kinds of difficulty. First, it creates a problem for the Aristotelian account of conversion. The statement 'Some A is B' was said by Aristotle to be equivalent to 'Some B is A'. But given this ambiguity 'Socrates saw something white' does not follow from 'Something white was seen by Socrates'.

> "... if a shield is white now but was black when it was seen by Socrates I maintain that it does not follow if the premise is divided. It does follow if the premise is compounded."
> (Sherwood 1966 p. 120).

This problem is more clearly seen in the context of Ockham's theory of terms. For he held that both subject and predicate terms supposit. But, he notes, that there is an asymmetry of supposition between these in the context of past and future tensed statements.

> "For what does the predicate in 'Socrates was white' supposit? If it supposits for things that are the proposition is false. The response here is that the predicate supposits for things that were regardless of whether they still are; therefore we have an exception to the rule I stated earlier - that in any proposition in which it occurs a term always does or can supposit for things that now are. I meant that rule to hold only for subjects of propositions."
> (1974 p. 205 - 6).

Thus, in 'Socrates saw something white' what Socrates saw was then white because 'white' is part of the predicate term.

But in 'Something white was seen by Socrates' because 'white' is part of the subject term what he saw need not have then been white.

The second problem this account of supposition in tensed contexts creates concerns semantics for past and future tenses. For it will no longer be the case that a future| past tensed statement is now true iff the corresponding present tensed statement will be|was true. For instance by these standard clauses 'Something white was seen by Socrates' may turn out false even though by the semantics Sherwood, Buridan and Ockham offer it may turn out true. This point was noted by Buridan and he took it to show that the standard general semantic clauses for past and future tenses must be qualified. He writes

> "A proposition of the future is not true if the corresponding proposition of the present will never be true. For example if this is true: 'Antichrist will preach' it follows that at some time this will be true 'Antichrist is preaching."
> (1966 p. 110)

But this then has the consequence that the statement 'The white will be black' can never be true.

> "But it has never been true to say that the white is black nor will it ever be true to say this."
> (ibid p. 110).

The final condition of Sherwood's rule for connecting supposition and appellation is that the verb should not have ampliating force because

> "if the verb is an ampliating verb the subject can supposit for something that does not exist."
> (Sherwood 1966 p. 129)

His example of such a verb is '...is praised' in 'A person is praised' which he says can now be true of Caesar. In such a sentence the term 'person' was said to have its supposition ampliated to include things which no longer exist. The existence of such verbs does create a difficulty for the inherence theory of predication as pointed out in Section 1.4. Sherwood merely labels this difficulty when he says

> "An ampliating verb is one that signifies a condition that can occur in something which does not exist."
> (ibid p. 129).

The notion of ampliation is used by some theorists, for example, Albert the Great (Moody 1953 p. 56) in the context of semantics for past|future tense and for modality. Just as the supposition of 'person' is ampliated to cover past as well as present people in 'A person is praised' this was said to happen also for 'A person ran'. And in the case of modal statements the supposition of 'person' was said to be ampliated to cover possible people as well as actual in 'A person can run'.

According to the theory of appellation then, the supposition of 'person' in 'A person runs' is all presently existing persons. Consequently by Sherwood's rule this is also the case for 'All people now run'. Thus the theory of appellation allows apparent universal statements to be capable of truth value variation over time. But what about 'Nobody runs' and 'Some person does not run'? In his 'Syncategorematica' Sherwood writes

> "Negation takes more than affirmation puts. For example, 'a man is running' only means that someone is running but the negation 'no man is running' extends itself to all... Let us say that in a non-ampliating affirmative proposition about the present there is reference to present things alone while in a negative to non existents as well."
> (1968 p. 98).

It appears then that the supposition of 'person' in 'No person runs' and in 'Some person does not run' is extended to include people at all times. But, the trouble with this suggestion is that 'All people (now) run' and 'Some person does not now run' would then no longer be contradictories. For Socrates who no longer exists can not now run even if all presently existing people are. What this brings out is a particular problem for tensed existential generalization. From 'Socrates does not run' should it follow that there is someone who does not run? This is a problem we shall look at in some detail in Chapters five and six.

2.4 Tense and Scope.

According to the Medievals a statement like 'A person ran' is ambiguous between 'It was the case that a person ran' and 'There is something which is a person now and who ran'. Sherwood remarks that this ambiguity is an example of the compounded/divided distinction. Aristotle, himself, noted that modal statements are open to this ambiguity

> "A man can walk while sitting and can write while not writing."
> (De Sophisticis Elenchis 166[a]23).

The distinction here is between the statements 'A person who is sitting can walk' and 'A person can walk while sitting'. In modern terminology the ambiguity is one of scope and the simplest way to distinguish them is given by the difference between (1) and (2)

(1) $(\exists x : x \text{ is a person})(\text{Sits } x \wedge \text{Possibly walks } x)$

(2) $(\exists x : x \text{ is a person})\text{ Possibly }(\text{Sits } x \wedge \text{Walks } x)$.

Similarly in the tense example the two senses of 'A person ran' can be distinguished by (3) and (4) where the quantifier ranges over all objects past present and future.

(3) $(\exists x)(\text{Person } x \wedge \text{Past Runs } x)$

(4) $(\exists x)\text{ Past }(\text{Person } x \wedge \text{Runs } x)$.

(3) is a representation of the divided sense which unlike (4), a representation of the compounded sense, carries the implication that whoever ran is now a person and so exists now, (since 'is a person' is non-ampliating).

The central feature of the supposition theory is that the same term may vary in semantic role according to the sentential context in which it occurs. Since this variation in semantic role is not dependent upon equivocality of a term (see 2.2) it would appear that it should be dependent upon structural ambiguity rather than lexical ambiguity. However, it is lexical ambiguity according to Aristotle, which accounts for the ambiguity of 'The sick man is healthy' as we noted in section 1.5. But what about the compounded|divided distinction? Is this lexically or structurally based?

The fact that in more recent times this ambiguity is accountable for in terms of scope differences and so dependent upon structure does not mean that the Medievals saw it in this way. Sherwood writes

> "If compounded it must be pronounced with continuity and the continuity of the subject with the predicate signifies that the supposition must be strictly indicated by the predicate... If divided, it must be pronounced with discontinuity... and the discontinuity of the expression signifies that the supposition is not strictly indicated by the predicate."
>
> (Sherwood 1966 p. 127).

That is, Sherwood views 'compounded' and 'divided' merely as labels for the two senses involved, labels given because phonetic composition (continuity) and division (pauses) allegedly disambiguate an utterance of the expression. Consequently, in order to represent the distinction involved a pause marker may be introduced. In the divided sense 'A person ran' is to be represented as 'A person [pause] ran'. The terms 'compounded' and 'divided' then relate to disambiguation using phonetic marking without commenting on the source of the ambiguity. Thus, Sherwood writes

> " 'Composition' indicates on act of discourse and 'Division' another, both acts, to be sure being based on a single substance of utterance."
>
> (1966 p. 141).

As to the source of the ambiguity Sherwood claims it is due to the principle that to the same expression there corresponds a diversity in reality (ibid p. 140-1). Thus it seems that he saw the ambiguity as lexically rather than structurally based. Although this conclusion appears to be correct in the tense examples there is a suggestion that the difference involved in modal examples is due to the scope of 'possible' (ibid p. 141-2). Two closely connected difficulties occur for this lexical account.

First, if 'person' is ambiguous in the context of a past|future tensed sentence then why is it not ambiguous in the case of a present tensed sentence? For in 'A person runs' the supposition of 'person' is present existents only, and hence is strictly indicated by the predicate. Or turned around, why is it not the case that in the divided sense the supposition of 'person' does not depend strictly upon the tensing of the predicate? An account to explain this should surely make use of that central feature of supposition theory. But pursuing this line of thought results in the overthrow of the term analysis and this brings us to the second more important difficulty, the nature of this lexical ambiguity.

Surely, the ambiguity depends upon the difference between the open sentence 'x is a person' and the open sentence 'x was a person' which brings out the predicative nature of the term 'person' in 'A person ran'. Moreover, Aristotle's apparent belief that 'The sick man is healthy' is ambiguous in the same sort of way that 'The bank is green' is is unsound according to his own principles.

For unlike the latter the former depends upon differences in temporal signification yet Aristotle claimed

"... a name is a spoken sound significant by convention without time."
(De Interpretatione 16^a19)

So the conclusion to be drawn here is that 'the sick man' contains a hidden verb and that therefore general terms are not names. (We believe that this shows that definite descriptions should not be treated as singular terms).

To accept that there are compounded|divided ambiguities in the case of tensed sentences brings out that the 'A belongs (now) to B' sort of analysis is inadequate because 'A belonged to B' is only ambiguous in this way if 'belonged to' is. That is, unless one is to claim that there are two different kinds of simple past (future) tenses these ambiguities can not be accounted for while treating tensing adverbially. In Chapter five we criticize a Russellian analysis of tense on similar grounds.

This conclusion about terms can be avoided if one treats time determinations adjectivally as later term theorists did - see comments in 1.5. For then the ambiguity involved in 'The sick man is healthy' is given by the difference between (5) and (6)

(5) The formerly sick man belongs to (tenseless) presently healthy beings.

(6) The presently sick man belongs to (tenseless) presently healthy beings.

We believe then that these considerations bring out a fundamental tension between Aristotle's term analysis of general statements (see 1.2) and his account of singular statements and their tensing (see 1.1, 1.3 and 1.4).

In both Buridan and Sherwood there are interesting examples of compounded|divided distinctions involving tense. Sherwood (ibid p. 142-3) gives the following example to show that sentences involving the adverb 'always' may be ambiguous in this way.

(7) whatever lives always exists.

(8) Socrates lives.

(9) ∴ Socrates always exists.

(7) is ambiguous as between (7') and (7")

(7') (x) Always (lives x → Exists x)

(7") (x) (lives x → Always Exists x).

The general form for the compounded|divided distinction using quantifiers and scope distinctions is that between (10') and (10"), where Q is an unrestricted quantifier

(10') (Qx) Operator (ϕx connective ψx)

(10") (Qx) (ϕx connective Operator ψx).

In Buridan, however, there is an unusual example. He claims that in one sense 'An old man will be a boy' is true

> "This is proved since it is the equivalent of the statement that he who is or will be an old man will be a boy. This is true for Antichrist."(1966 p. 111).

But this is not an instance of (10') or (10") but instead is (11)

(11) (∃x) Future (Future Old man x ∧ Boy x).

And there are two senses in which 'An old man will be a boy' is false.

(12) (∃x) Future (Old man x ∧ Boy x).

(13) (∃x) (Old man x ∧ Future Boy x).

(Notice that it can not be represented by (14) in the sense in which it is true, because the subject term is 'Old man' and not 'boy' - this brings out a problem of capturing the medieval account using quantifiers and 'and'.)

(14) $(\exists x)$ Future (Boy x $\wedge$ Future Old man x).

Besides scope differences on pairs of predicates there is also discussion of sophisms which in more recent terminology depends upon quantifier scope. Buridan (ibid p. 107) distinguishes two senses of 'There has always been some person'. The difference was taken to depend upon different suppositions of 'person'. In the sense in which it is the same person always (represented by (15')) 'person' was said to have determinate supposition and in the other sense (represented by (15")) it was said to have confused supposition.

(15') $(\exists x)$ Always (Person x)

(15") Always $(\exists x)$ (Person x)

More famous is Buridan's example 'I owe you a horse' (ibid p. 137).

2.5 Concluding Remarks to Part One.

Central to supposition theory is that notion of univocality discussed in Section 2.2. However, it seems that the Medievals did not see the ambiguities involved as dependent upon structure. Nevertheless, they did believe that univocality is different from equivocality, as we saw. Perhaps this can be understood to be similar to a view held by Frege. He believed that within certain contexts, oblique contexts, expressions have a different semantic role than in non-oblique contexts. The analogy is made more complete here if the indirect sense is identified with the direct sense of an expression (as proposed by Dummett 1973 p. 268). For then an expression may vary in semantic role (reference) according to context without varying in sense.

Also central to supposition theory is the term analysis. This analysis involves the view that general and singular terms are syntactically and semantically on a par. For instance, in Sherwood's writings singular terms like general terms, supposit for they are said to have 'discrete supposition'. It may even be claimed that it was because of this that there is a need for supposition theory in the first place. Indeed, the introduction of the functional analysis employing quantifiers, predicates and variables does overcome the need for distinguishing between certain kinds of supposition. But what about appellation theory? Does this also become redundant once quantifiers, predicates and variables are introduced? We think not since there are still many questions to be answered. Relevant here to these is an idea found in both Sherwood and Ockham that there is a 'fundamental' or 'basic' notion of supposition, a type of supposition which a term may have independently of sentential context. This may take on one of three forms (A), (B) or (C) for a term 'T'.

(A) The term 'T' supposits for all actual and possible Ts.

(B) The term 'T' supposits for all past, present and future Ts.

(C) The term 'T' supposits for all presently existent Ts.

According to which is taken as basic then a term's supposition is said to be ampliated or restricted according to certain types of sentential context. For instance, if (B) is taken as 'basic' then in the sentence 'A person can walk' 'person' will be said to be ampliated to cover possible people whereas, in contrast 'person' will be said to be restricted in 'A person (now) walks'.

Evidence that there is this idea in both these authors is provided by the following quotes. First for (A) from Ockham

> "... a term supposits personally when it supposits for things that are its significata or for things that were, will be or can be it s significata... It is for this reason that I... said that 'to signify' can in one sense be used in this way."
> (1974 p. 204).

And in the case of (B) Sherwood writes

> "A verb may sometimes restrict as in 'a man runs'. The term 'man' can supposit for past, present and future men but here it is confined to present men by the verb in the present tense."
> (1966 p. 124).

Also in Sherwood (C) is found

> "Or putting it another way, if we want to speak strictly we say that a term supposits on its own for present things and if it supposits for other things it will be because of what

is adjoined to it - i.e., an ampliating verb or a past-tense or future - tense verb..."

(1966 p. 130).

What these three accounts yield is a 'basic' way of understanding general statements. For instance, Aristotle's distinction between temporally unrestricted and temporally restricted readings of 'All Ds are Es' (See 1.5) is then, the difference given by (B) and (C) respectively. As a legacy for modern logic (A), (B) and (C) provide three alternative ways of interpreting standard quantification theory. For they offer different ways of considering the range of the quantifiers. Especially interesting is the reading embodied via (C) which suggests that the quantifiers be taken as ranging only over present existents. This provides a basis for tensed quantification theory employing tensed quantifiers.

How do the notions of 'ampliation' and 'restriction' fit in with modern logic? In the case of 'restriction' a natural suggestion is to follow the Medievals. They believed that adjectives and relative clauses restrict terms. Consequently if (A) or (B) is taken as the basic notion of supposition then tensing may be dealt with adjectivally in the manner noted in the last Section. For instance, the sentence (1)

(1) Socrates was a person

may be taken as (1')

(1') Socrates is a former-person

where the 'is' is not present tensed but tenseless and where 'former' restricts the term 'person'. In terms of the quantifier and variable analysis a standard way of dealing with the predicate '... is a former person' is to introduce an extra place into the predicate '... is (tensless) a person' to form '... is-a-person-at...' where the second place is open for a time variable or constant. So (1) becomes (1")

(1") ($\exists$t)(Before (t, now) $\wedge$ Socrates is-a-person-at t).

The restricted general term 'former person' is therefore dealt with in terms of a temporally restricted predicate '... is a-person-at t'. This analysis of tense, the Russellian analysis, is looked at in Chapter five.

In contrast, a natural way of dealing with ampliation when it is dependent upon the occurrence of tense and modal expressions in a sentence is to represent it in terms of a quantified expression within the scope of an operator. For instance, if (A) is taken as basic; that is, in more recent terminology the quantifiers range over all possible objects then (2)

(2) It is possible that someone runs

can be represented either as (2') or as (2")

(2') Poss ($\exists$x)(Person x $\wedge$ Runs x)

(2") ($\exists$x) Poss (Person x $\wedge$ Runs x).

But, if the quantifiers range only over actual objects (so 'person' is ampliated in (2)) then (2) must be represented by (2'). Thus when (C) is taken as basic (3) when understood to be compounded

(3) A person ran

has the form (3')

(3') Past ($\exists$x)(Person x $\wedge$ Runs x).

This has the consequence that proposal (C) is more sensitive to scope distinctions than (B) because (3') will under the former unlike the latter mean something different than (4) does

(4) $(\exists x)$ Past (Person x $\wedge$ Runs x).

As we note below it is (C) which Sherwood takes as basic. That is, he holds that the appellation of a term is its 'fundamental' property independent of context (barring signification). The interest of this for us is that it suggests a development of tense logics as an extension of standard quantification theory when understood to represent present tensed fragments of natural language and where the quantifiers are temporally restricted. And, it is this which is developed in Chapter six of this essay.

Can this account of ampliation be extended to cover ampliating verbs? One suggestion would be that sentence like 'Someone is famous' is to be analysed in terms of a quantifier within the scope of an intentional operator. But, although, this may be true of many ampliating verbs there are others like 'x is taller than y' for which intentionality is not appropriate.

Sherwood unlike Ockham takes (C) to be basic. He says

> "... strictly speaking, we must say that the verb 'can' and others like it ampliate the supposition of a term while the verb 'is running' and others like it do not restrict a term since a term supposits on its own for present things."
> (1966 p. 131).

This difference may be connected with these authors' different attitudes to the relationship between logic and ordinary discourse and between logic and scientific discourse. (We may compare here the contrast between Aristotle's account of tense as contained in 'De Interpretatione' with what little mention there is in his formal writings). In recent times the Kneales appear entirely to discount (C) as a basic account of supposition (and presumably as a 'basic' way of reading the standard quantifiers).

> "But it seems curious that no term is said to have appellatio unless it is applicable to something existing at the time of speaking; for while it may be important to distinguish in logic between terms which have application to something past, present or future and those which have no application at all, it is not so obviously important to draw a line where William of Sherwood and other medieval logicians draw a line between terms which have appellatio and those which do not." (1962 p.247.8)

These authors, then, are putting forward the view that there is no significant point to Medieval appellation theory or more generally to a tensed quantification theory involving tensed quantifiers. Although this attitude may be connected with relating logic exclusively to scientific discourse it is also, if not more importantly, connected with the belief (reinforced by twentieth century physics) that tensing and indexicals like 'here' or 'I' are to be dealt with semantically in the same sort of way. Consequently, temporally restricted quantifiers (and hence the appellation of a term) are taken to be as useful as having spatially restricted quantifiers (or a special spatially restricted property of a term). This condemnation of the Medieval account of tense looked at in twentieth century terms is considered in Part II of this essay to which we now turn.

PART TWO

TWO LOGICAL FRAMEWORKS FOR THE ANALYSIS OF TENSE.

CHAPTER THREE: THE Lp AND L^D ANALYSES OF TENSE.

3.1 Statements and Truth Variability.

There are two important parts to the construction of a semantic theory for fragments of natural language. First and foremost is the semantics themselves which give the meaning of any statement of some regimented language. Secondly, is the working out of sets of translation procedures from the relevant fragments into the regimented language. In the case of semantics for tensed fragments there is a general question as to whether or not the central unit of the semantics, the statement, can vary in truth value over time. For the Ancient and Medieval view that (some) statements may vary in truth value over time has been criticized on the ground that it confuses the nature of truth and falsity. In this Section it is argued that accusations along these lines are based upon a preferred sense of 'statement' or rather 'proposition'. Consequently, it needs to be (at least) recognised that there are alternative accounts of what a statement is.

According to Frege it is what (stating) sentences express, namely 'thoughts', which are primarily true or false (1967 p. 19). Strictly speaking, however, it is not ordinary language sentences which express thoughts but uses of them since they often contain both tense and indicator expressions of one sort or another. Frege writes

> "But are there not thoughts which are true today but false in six months time? The thought, for example, that the tree there is covered with green leaves will surely be false in six months time. No, for it is not the same thought at all. The words 'this tree is covered with green leaves' are not sufficient by themselves for the utterance, the time of

utterance is involved as well. Without the time indication this gives we have no complete thought at all. Only a sentence supplemented by a time-indication and complete in every respect expresses a thought."

(1967 p.37).

Consequently, in order to set up a semantic theory which captures this Fregean conception a statement is to be identified with a potential use of a natural language sentence on some occasion. More formally, it may be identified as an ordered pair consisting of a regimented natural language sentence together with an ordered set of indexes which represent a possible occasion of use. Now one very important consequence of this account of a statement is that they like Fregean thoughts can not vary in truth value over time. They are true or false simplicite

Why did Frege hold the view that thoughts can not vary in truth value over time? It seems that this belief is intimately tied up with his doctrine that semantically speaking, thoughts and senses (of a proper name) are on a par in that they both refer. In the case of thoughts what they refer to are the truth values (1960 p. 63). Now, it is natural to hold, as indeed Frege did that two different referents (of proper names) can not be associated with the same sense. (For Frege, the sense of a proper name is 'the mode of presentation' of the referent). Analogously, then, two uses of a sentence like 'Socrates sits' which actually differ in truth value on two occasions can not express the same thought. In order to take account of the difference in sense here time of use needs to be invoked as a systematic factor which contributes towards the sense of a use of a tensed sentence. This is an argument from 'content'. Closely connected to it but independent of the invocation of senses is an argument from extension dependent upon the principle that if two uses of a sentence differ in their extension (their truth value) then they must differ in referential components. Again the difference has to be accounted for systematically in the case of uses of tensed sentences. This is done by invoking (implicit) reference to time via the use of tense. Consequently, Frege claims

> "If we say: 'The sun is in the tropic of Cancer' this would refer to our present time." (1960 p. 72).

In a similar vein Russell criticizes MacColl's distinction between types of statement. MacColl distinguished between statements which are true, false and variable amongst others (1973 pp. 307ff). A variable statement is one which can vary in truth value over time like 'Mrs Brown is not at home'. To this Russell writes

> "Here it is plain that what is variable primarily is the meaning of the form of words. What is expressed by the form of words at any given instant is not itself variable; but at another instant something else, itself equally invariable is expressed by the same form of words. Similarly, in other cases. The statement 'He is a barrister' expresses a truth in some contexts and a falsehood in others... Ordinary language employs, for the sake of convenience many words whose meaning varies with the context or with the time when they are employed; thus statements using such words must be supplemented by further data before they become unambiguous." (Quoted from Prior 1957 p. 110).

It is not, however, always the case that a natural language sentence expresses a different thought on different occasions of use as instanced by 'seven is prime'. So an alternative proposal to identifying a statement with an ordered pair is to hold that a sentence which is to be regimented is that unit which expresses the same statement on all its occasions of use. But what about tensing and indicator words? Russell proposed that because tensing involves implicit time reference one need only make explicit that reference.

> "One of the objects to be aimed at in using symbols is that they should be free from the ambiguities of ordinary language. When we are told 'Mrs Brown is not at home' we know the time at which this is said, and therefore we know what is meant. But in order to express explicitly the whole of what is meant, it is necessary to add the date and then the statement is no longer 'variable' but always true or false... We may say that 'x is a barrister' 'Mrs Brown is not at home at the time x' is true for some values of x and false for others." (ibid p. 110 - 111).

What may generally be called the Frege-Russell account of a statement differs from the Ancient and Medieval account. The central difference is that alternative criteria are invoked for identifying a statement from a use of a tensed sentence like 'Mrs Brown is not at home'. Given this, it is surprising to find the Kneales (1962 p. 48ff) accusing Aristotle of confusing the nature of truth and falsity by allowing them to be relativized to time. He is said to mix up calling a sentence true or false with the truth or falsity of the thought it expresses. It is the latter, they claim, which primarily has a truth value. Curiously, they believe that if Aristotle had realised this he would have conceded that different uses of the same sentence can express different 'propositions'. But, as remarked in Section 1.1, it appears that Aristotle did hold that it is 'thoughts' which are primarily true or false. And clearly this is true of the Stoics who not only believed that 'axioma' primarily have a truth value but also that they may vary in truth value over time. Johnson also criticized truth variability on the grounds that is is based upon a confusion.

> "Certain logicians have, however, deliberately denied the dictum that what is once true is always true and their denial appears to be due to a confusion between the time at which an assertion is made and the time to which an assertion refers."
> (1921 p. 235).

Both these accusations of confusion depend upon a preferred sense of 'proposition'. For unlike a Fregean thought a Stoic thought is to be identified from a tensed sentence independently of its time of use. But how does the Ancient account of a statement stand in the light of the arguments from 'content' and 'extension'?

The argument from content was based upon the principle that semantically thoughts are on a par with senses of proper names together with the idea that different referents can not be associated with the same sense. Certainly, this latter point is at its strongest in the case of senses of proper names although not so strong in the case of definite descriptions. However, the whole Medieval doctrine of univocation is based upon a rejection of that latter point. For they held that the semantic role (extension) of a general term may vary according to sentential context even though its signification does not. This can be understood to be the claim that, for instance, what is meant by two occurrences of the predicate '... is red' in two uses of 'This poker is red' on (significantly) different time occasions is the same despite the possibility, if not certainty, that the extension of the predicate at those times differ. On the basis of this truth variability for statements is acceptable. That is, putting the matter in a nutshell, there is nothing wrong with displaying a semantic in which the extensions of proper names are not relativized to time while those of predicates and statements are. In reply to the other argument it is sufficient to amend the principle that if two uses of a sentence differ in truth value then they must differ in their referential components to the principle that if two uses of a sentence differ in truth value at the same time then they must differ in their referential components. In connection with this it is interesting to note that Johnson who held the view that propositions can not vary in truth value over time accepted the view that predicatescan vary in extension over time (1921 p. 235).

What about two uses of a sentence like 'I am ill' by different people at the same time? Is there a choice open here as to their treatment? Neither the Ancients nor the Medievals, to our knowledge, allowed that statments may vary in truth value from person to person or from place to place. It appears then that truth relativization to time is a more 'natural' notion here. The most modest explanation of this is simply that tense and indexical expressions differ grammatically in natural languages. In English tensing takes on an adverbial construction whereas indexicals like 'I' and 'here' appear to occupy singular term positions. One consequence of this, utilized fully in Section 4.3, is that the truth predicate is itself open for tensing.

Dependent upon what sorts of constraints are to be put on semantic theories for natural languages this modest explanation may be utilized in defence of the Lp analysis of tense, discussed in 3.6, an analysis based upon the works of Prior. A person of a Frege-Russell bent may argue that ordinary language is 'misleading' in its bias towards temporal expressions over spatial expression (see 4.5) and furthermore may claim that tensing ought to be treated on a par semantically with spatial indexicals (in order to avoid certain philosophical pitfalls). It is this semantic ground that tensing and spatial indexicals are semantically on a par which is presupposed by the Frege-Russell account of a statement. Consequently, a full blooded explanation of the naturalness of truth relativization would appeal not just to grammatical differences between tensing and indexicals but also to a semantic difference which would show that it is an error to treat tensing and indexicals semantically on a par. If there is this appeal to be made, an appeal denied by the L^D theorist (see 3.7) then its ground will reside in some belief along the lines that space and time are importantly different. An attempt to give such a full-blooded explanation is contained in Section 4.5.

3.2 Tenseless Verbs.

Many theorists make use of tenseless verbs in their analysis of tensed sentences. However, what is not so clear is precisely what is meant by 'tenseless' verb. And recently Mott has even expressed doubts about their intelligibility. He seems to be of the opinion that they are philosophers' inventions and not that of the grammarian.

> "... when a verb is said to be tenseless what sort of claim is being pressed? It may seem as if some syntactic or grammatical point is being made; as it would be if we said that the verb was in the perfect or imperfect tense. But there is no such grammatical point. In English there are verbs in the present tense, the perfect tense, there are verb phrases for the future, the continuous past and so on. But there are no verbs in the tenseless sense. The syntax of English does not include them."
>
> (1973 p. 74).

Aristotle's distinction between names and verbs in 'De Interpretatione' discussed in Section 1.3 was criticized in the seventeenth century by the Arnauld brothers. These criticisms when understood to be of a grammatical nature are valid. For, they claim that Aristotle was wrong to hold that time signification is essential to the verb. One strand of their argument consists in exhibiting certain complex uses of the copula which they claim do not signify with time. The uses they picked out were in sentences like 'Everybody is divisible' (1964 p. 107). Consequently, if there are such uses then, grammatically speaking, Aristotle was wrong to hold that time signification is essential to the verb.

There is a much simpler and far less controversial route to the Arnaulds' grammatical thesis based upon the fact that notall languages inflect verbs in order to express tense distinctions. W. Bull writes

> "It is traditional to assume that the structure of a tense system is revealed only by the morphemes affixed to the verb stem. There are nevertheless languages such as Zulu and Haitian French in which order morphenes are bound to the subject and still others such as Yoruba Hawaiin or Mandarin Chinese in which some order morphemes are free forms."
> (1960 p. 20).

This grammatically interesting fact which provides the basis for a vindication of the Arnaulds' position is of doubtful philosophical importance. It does not show as it used to be sometimes claimed that members of these societies have a different conception of time to us because of their wholly or partially different grammatical representation of temporal order. (A point made by Gale 1968 p. 44ff).

A verb need not introduce time determinations then and this may be utilized by constructing languages within which verbs do not signify with time but instead other expressions do. Adopting Quine's proposal (1953) of placing square brackets around a verb to indicate that it is tenseless then English sentences involving the simple pure tenses like 'Socrates ran' 'Socrates runs' and 'Socrates will run' can be represented either by analogy with Zulu as 'Socrates-PAST [runs]', 'Socrates-FUT [runs]' or by analogy with Yoruba as 'PAST Socrates [runs]' 'PRES Socrates [runs]' and 'FUT Socrates [runs]' where 'PAST', 'PRES' and 'FUT' are the simple order morphemes. Introducing these as free forms is preferable for the purposes undertaken here because of their neutrality between attachment to the verb and to the subject. Consequently, Mott's doubts about a syntactic or grammatical sense of tenseless verb depend upon paying too much attention to a single natural language.

The introduction of regimented languages constructed out of tenseless verbs (predicates) provides a useful background from which to assess the merits of certain analyses of tense. First, it allows one to look at tense independently of its attachment to the verb. For it has been claimed by philosophers who reject the doctrine of 'temporal becoming' - that is, the doctrine that there is content to the notion of 'the passage of time' - that this doctrine has a basis in the grammatical fact that tense forms in English are adverbial. A second point is that natural language tense forms serve other purposes than mere expression of temporal order. These include the expression of volition and of modality. On the basis of this grammarians have even expressed doubts that a systematic account of ordinary language tense forms can be given (for example, Robins 1967 p. 30). Thus, grammatical tensing need not coincide with tensing in a regimented language. This is so even in cases where the ordinary language tense forms do express temporal order. For instance, in 'He runs tomorrow' the present tense expresses futurity and in 'It is time he went to bed' the past tense expresses presentness. Consequently, because we want an account of how the morphemes 'PAST' 'PRES' 'FUT', expressing temporal order only, contribute to the truth conditions of sentences containing them these morphemes are not be taken as automatic translations of grammatically past, present and future tenses.

Although a clear sense has been provided to the notion of 'tenseless verb', a sense which is semantically orient ated little has really been done. For afterall the problem is how sentences of natural language are to be translated into a language involving such verbs. One natural question to ask is whether or not there are occurrences of tenseless verbs in English. The Arnaulds' method of isolating tenseless verbs was to seek them in the context of sentences which eschew temporal reference all together. However, this method, although suitable for their project, is of no use to theorists who wish to employ tenseless verbs in the analyses of sentences which do involve temporal reference.

A distinction is sometimes made between 'tenseless' sentences and tensed sentences (For example, see Braude 1973 p. 188). The former class of sentences is intended to include not only sentences like 'Two and two is four' but also those like 'There is a time at which Socrates sits'. The thought here seems to be that a tensed sentence contains tensed occurrences of verbs whereas a tenseless sentence contains tenseless occurrences. Consequently, if one can provide suitable criteria for distinguishing between these kinds of sentence one then also has a way of distinguishing tensed from tenseless uses of verbs. However, it is not clear that such criteria can be found which does not presuppose the tensed|tenseless verb distinction. Braude's paper is a testimonial to this. After criticizing various versions of this distinction he gives as his own account

"I shall say that a sentence S is tensed if and only if it is necessary that for any two moments of time M and M' (where $M \neq M'$) replicas of S produced at those times have different truth conditions. A sentence S will be tenseless if and only if it is not tensed."

(1973 p. 206 - 7)

But, because he allows two replicas of a sentence to have different truth conditions even though it is (logically) impossible that they differ in truth value (ibid p. 209) his distinction here presupposes some means of identifying tensed components within a sentence. And, this is no easy matter. For instance, laws are often expressed as conditionals in which the consequent is in a grammatical future tense. So how is it to be determined whether or not it is necessary that two replicas of 'If copper is heated it will expand' have different truth conditions?

One suggestion for isolating tenseless uses of 'is' is given by Gale.

"The test for determining whether 'is' in some particular sentential context must function as tenseless rather than a tensed copula is as follows: if the context permits the substitution of 'was' or 'will be' for 'is' the copula is not of necessity tenseless; if not it is of necessity tenseless."

(1968 p. 198 - 9).

This test is dependent upon a rather unfortunate notion in the circumstances, namely correct usage (see comments above). For how else is '... if the context permits the substitution of ... for 'is'...' to be understood? Appeal to correct usage is not only likely to be unhelpful but also question begging in controversial cases. For instance, in a dispute as to whether or not the uses of 'is' in the sentences 'Socrates is identical to Socrates' or 'Socrates is the same person as Socrates' are tenseless one side will claim that it is legitimate to substitute 'was' here while the other side will merely deny this.

It has been argued by Mott (1973 p. 75) that Gale's test fails. One general class of sentence which is said to place doubt on it are those sentences in which there has to be compatibility between the tensing and other lexical items. For instance, in 'Socrates ran last week' the tense of 'runs' has to be compatible with the item 'last week' since it involves reference to the past. Buridan found such examples puzzling; 'Socrates will be running tomorrow' he claims is a sophism.

> "The sophism is clear positing that he will run tomorrow... [But] the opposite is argued, because if a proposition of the future is true, it is necessary... that in the future it should correspond to a true proposition of the present tense... But it will never be true to say 'Socrates is running tomorrow'... It is similar concerning the sophism 'Socrates argued last year' because it was never true to say 'Socrates is arguing last year!"
>
> (1966 p. 111).

Are there not instances where the present tense has to be used because of compatibility? It is for this reason that Mott claims that 'I'm ready now' is a counterexample since it appears that by Gale's test 'am' is necessarily tenseless. However, in defence of Gale, Buridan pointed out that his sophism is true if 'tomorrow' is understood as belonging with 'will be' but false if it belongs with 'run' (ibid p. 120). That is, the copula is there to be understood as 'will tomorrow' which can be substituted for by 'was' or 'is'. Likewise for the Mott counterexample if the copula is taken as 'is now' then it is substitutable for by 'was' or 'will be'. But Gale uses his test to claim that the copula in 'X is simultaneous with this token' is tenseless where 'this token' is token-reflexive. It is arguable, however, that the 'is' is not substitutable here because of the occurrence of 'this token' which implicitly refers to the present time.

It is assumed here that the question whether or not a verb use in English is tenseless or tensed is to be answered in the light of a certain amount of theory. In particular, in terms of whether any purported tensing does or does not contribute to truth conditions. In connection with this is Braude's claim that every sentence is either tensed or tenseless. (ibid p. 191). It isn't clear that this is true of English sentences. For instance, in the next Section attention is given to translating 'Caesar invaded Britain in 55 B.C.' into a regimented language. It is there claimed that there is not one correct translation only. Furthermore, there is a straight division in opinion as to whether or not sentences like 'Seven is prime' involve tensed or tenseless uses of verbs For instance, Jespersen (1961 p. 17) and Reichenbach (1966 p. 292) claim that the 'is' is a tensed use whereas Strawson would claim it to be a tenseless use (1952 p. 150). This difference in opinion may depend upon whether or not tensing is treated from a grammatical or semantic point of view. In this essay the latter viewpoint is taken. Consequently, whether or not 'Seven is prime' is tensed depends upon what semantics are given for statements expressing number theory. Since standard semantics here do not involve temporal reference it may be claimed that the 'is' is tenseless.

One essential condition, we believe, on any account of tenseless verbs is that a distinction must be made between a tenseless verb and what is here called a 'detensed verb'. Copi writes

"Here we shall follow the custom of ignoring the time factor and will use the verb 'is' in the tenseless sense of 'is or will be or has been'."

(1968 p. 317).

This version of 'tenseless verb' is not unique to Copi, a point we note on later occasions. Gale does not distinguish between these two senses. He says that the 'is' in 'Two plus two is four' is tenseless (1968 p. 50 and p. 198) yet he also claims that the 'exists' in 'There is a tiger' is tenseless when this sentence means 'Either there did exist a tiger or there now exists a tiger or there will exist a tiger' (ibid p. 40). In order to distinguish these senses of 'tenseless verb' Copi's sense is called 'detensed' and represented by placing square brackets subscripted with a 'd' around the verb [ ]d. Unlike a sentence containing a tenseless verb, one containing a detensed verb, like 'Plato [walks]d is equivalent to a disjunction of simple tensed sentences, namely, 'PRES Plato [walks] or PAST Plato [walks] or FUT Plato [walks]'. So really the difference here is between a genuinely tenseless notion of verb like [walks] and a complexly tensed verb like [walks]d. Because of this we believe it to be a little misleading to call 'detensed' verbs 'tenseless' as Copi and others do. For although they are not specifically present or past or future tensed they are nevertheless still tensed occurrences. Intuitively speaking, then, it appears that 'Plato [walks]' is not a complete sentence unlike 'Seven [is] prime'. On the other hand, 'Plato [walks]d' is acceptable whereas 'Seven [is]d prime' looks unacceptable. We believe that it is especially important to distinguish between these senses in connection with the existential quantifier, a point we note in Chapter five (see 5.2ff).

On the basis of the sense of 'tenseless verb' introduced in this Section an artificial language L is introduced whose primitive expressions consist of a set of n-place tenseless verbs $V_i^n(\ldots)$ where $(\ldots)$ marks the n-places together with a set a_i of proper names (understood so as to include neither definite descriptions nor indexical expressions). By a verb what is really meant is a predicate. The only formation rule for L is given by Definition L.

<u>Definition L</u>.

if $V_j^n(\ldots) \in V_i^n(\ldots)$ then $V_j^n(a_1 \ldots a_n)$ is a formula of L where $a_1 \ldots a_n$ are any n occurrences of any m, $m \leq n$, members of a_i.

Any formula of a language L is abbreviated to ϕ ψ etc.

3.3 Preliminary Remarks on Tensed and Dated Sentences.

Philosophical importance has been attached this century to two kinds of sentence which introduce time determinations. First, there are those sentences which only involve tenses like 'Caesar invaded Britain' and secondly there are those which also involve dates. One instance here, noted in 3.1, was Russell's proposal that the implicit time reference in a use of 'Mrs Brown is not at home' be made explicit by introducing dating devices. In this Section some preliminary remarks are made concerning the relationship between these two kinds of sentence.

Given the account of a tenseless verb introduced in the last Section tensed singular sentences involving pure tenses can be translated into a language which involves tenseless verbs. For instance, 'Caesar invaded Britain' becomes 'PAST Caesar [invades] Britain'. Hence in order to take account of these sentences some language L needs to be extended to also include the tense morphemes in its primitive expressions. So let L_1 be $L \cup \{PAST, PRES, FUT\}$ where the only information rule for L_1 is given by Definition L_1.

Definition L_1

Suppose L is some tenseless language given by Definition L so that ϕ is any formula of it. Then 'PAST ϕ' 'PRES ϕ' and 'FUT ϕ' are formulas of L_1.

Language L_1 is, then, ideally suited for capturing Aristotelian singular statements.

But what about singular sentences involving dates? Let it be assumed that 'At t_n' expressions where 't_n' is a date are free forms like the order morphemes. Consequently, is 'In 55 B.C. Caesar invaded Britain' to be translated as 'In 55 B.C. PAST Caesar [invades] Britain' (that is, in terms of a language which is an extension of L_1), 'In 55 B.C. Casesar [invades] Britain' (that is, in terms of a language which is an extension of L but is neither contained in nor contains L_1) or as something else? Now, because the interest in setting up this translation is not for the purpose of looking at grammatical niceties but rather for assessing logical issues there appears to be a fairly clear criterion for deciding which is the most suitable translation in this particular case.

The only difference between L_1 sentences 'PRES Socrates [runs]' and 'PAST Socrates [runs]' which differ in their truth-at-a-time condition is in their tense morphemes and it is this which accounts for that difference. Consequently the translation of 'In 55 B.C. Caesar invaded Britain' into a language involving tenseless verbs depends upon whether or not occurrences of tense morphemes result in difference in truth conditions. If 'In 55 B.C. Caesar invades Britain' differs in truth condition from 'In 55 B.C. Caesar invaded Britain' then its translation will be into a language which is an extension of L_1. Appeal to ordinary usage in this case is not of much help. However, hardly any philosopher goes along with the proposal that there is a difference in truth-conditions in these examples, one exception being Braude 1973 p. 209.

One reason for this is implicit reference to a distinction made by Rescher (1966) and Rescher and Urquhart (1971).

These authors contrast sentences like 'It rained in London yesterday' with one like 'It's raining on January 1st 3000 A.D. is a fact'. The former are, they say, temporally indefinite whereas the latter are temporally definite (1971 p.25). The force of this is that the truth value of the former type of sentence unlike that of the latter is essentially bound up with their time of use. (This is then a particular subdivision of the tensed| tenseless sentence distinction mentioned in the last Section). Thus when a dated sentence is taken to be temporally definite its time of use is deemed irrelevant to the assessment of its truth value. On this basis then if someone now utters 'Caesar invades Britain in 55 B.C.' that person has not uttered something false (but has uttered something which is not 'correct' English). So for logical purposes the three sentences 'Caesar invade|invaded| will invade Britain in 55 B.C.' are to be construed as having the same truth conditions. Alternatively expressed, the occurrence of a date in a sentence containing the simple tenses semantically overrides these tenses in such a way as to make them redundant. Let us call this 'The Semantic Primacy of Dates Assumption' which almost has the force of a convention amongst philosophers.

Given this assumption then the sentence 'In 55 B.C. Caesar invaded Britain' can not be translated as 'In 55 B.C. PAST Caesar [invades] Britain' because it would have the wrong truth conditions. A natural suggestion, then, is to translate it as 'In 55 B.C. Caesar [invades] Britain'. Let us then introduce a language L_1 which is $L \cup \{At\ldots t_i\}$ where the set t_i consists of a (possibly) infinite set of date constants which represents a dating system based upon some unique event occurring at t_o. The only formation rule for L^1 is given by Definition L^1.

Definition L^1

Suppose L is some tenseless language then if ϕ is any formula of L At t_n ϕ is a formula of L^1 for any $t_n \varepsilon t_i$. Consequently L^1 which is neither contained in nor contains L_1 consists of temporally definite sentences.

Although sentences of L^1 and L_1 have been constructed from underlying tenseless languages (which may be the same language) no account has been given of connecting them. It might be thought that this can easily be done by relating uses of L_1 sentences on particular time occasions to those of L^1. But, account needs also to be taken of sentences which not only involve dates but also tenses which do contribute to truth conditions. For instance, the English sentence 'In 56 B.C. Caesar was about to invade Britain' (Mott 1973 p. 78). Now such sentences have the characteristic of involving complex tensing as well as dates. But The Semantic Primacy Assumption has only been considered in connection with those sentences which involve the simple tenses. What needs to be looked at is the interaction of dates with complex tenses and this, in turn, depends upon some account of complex tensing.

3.4 Tenses, Dates and Vectors.

W. Bull in 1960 produced a framework within which to examine natural language tense systems. (His chief concern in that work was with the Spanish tense system). In this Section certain points are abstracted from his account of tense and dating systems. Fundamental to both a dating and a pure tense system are the occurrence of certain events which define what Bull calls 'axes of orientation'. In the case of tense systems the public event which is central is the act of speaking and it is this which defines the prime axis for all tense systems (1960 p. 7 - 8). Whereas Bull takes an axis to be an event we shall, instead, adopt the terminology that it is a moment of time for the sake of simplicity and generality. It is, however, important to realize that this change in terminology is not intended to be substantive in the sense that we are claiming that tense expressions 'refer' to time, a point which is of importance in connection with tense semantics. (For instance see 3.7).

Bull claims that tense morphemes are 'vectorial' expressions. In order to understand what this means consider a use of an L_1 sentence, 'PAST ϕ'; here the morpheme 'PAST' has the semantic role of directing us away from the prime axis (the time of use or the present time) towards the past. And it is this idea of 'directing towards the past' which underlies the suggestion that tenses are 'vectorial'. The, (indefinite) end point of this vector gives rise to a new axis which (in our terminology) is the time at which the purported event reported in 'ϕ' takes place. The expression

'PRES' is a 0-vector which has the semantic role of directing from an axis back to it.

Besides these vector expressions there are also 'scalar' expressions. Between any two events or points of time there is a definable 'distance' a time interval like 'one minute', 'ten days','three centuries' etc... The scalar expressions - so called because they are order indifferent - can be added to vector formulas to yield the time between the prime axis and the subordinate axis. For instance, in 'Socrates ran three days ago' the scalar is three days and the direction is towards the past.

A dating system like a tense system involves axes and vectorial expressions. The prime axis of such a system is the time of some unique event. Such a system, however, also includes scalars. Take, for instance, the standard Gregorian Calendar; in the expression 'In 55 B.C.' the prime axis is indicated by 'C' the order relation by 'B' and the scalar is '55 years'. (See Bull ibid p.12).
In an abstract dating system like that introduced for language L^1 by the set 't_i' the prime axis is indicated by 't_O', the order relation by the relation of the subscript 'n' in 't_n' to 0 and the central scalar unit by the distance between any two consecutive dates. One central difference between the prime axis of a dating system and that of a tense system is that in the latter the axis 'moves' constantly through a series of intervals. Today will be yesterday tomorrow. Consequently, for human purposes a dating system is useless unless at any time we can know at which point we are on it. As Bull remarks

"Until we know the Gregorian calendar label for 'today' the public calendar is useless." (ibid p. 10).

The Semantic Primacy of Dates Assumption can now be considered in more detail with respect to this framework. This assumes that the occurrence of a date in a sentence like 'World War II began in 1939 A.D.' semantically over-rides the tensing of the verb 'begins' (see 3.3). That is, one merely drops out of consideration the tensing system whose prime axis is the time of use and concentrates on the dating system when the sentence involved is to be assessed for its truth value. Such sentences are examples of those which introduce multiple axes. But they are special cases in that they involve two prime axes and it is because of this property that the assumption may gain a foothold here.
In many sentences the introduced multiple axes are subord-inate to the prime axis and, thus, depend on it for their existence. For instance, in 'John did it before Mary (did)' one can claim that there are three introduced axes, two of which are subordinate. First, the primary axis, the time of use, secondly an axis simultaneous with the purported event of Mary's doing it and finally one simultaneous with John's doing it. The two subordinate axes are subordinate here because they arise as end points of vectors introduced by uses of 'did' and 'before'. The second axis arises as the endpoint of the vector directed towards the past whose origin is the time of use and third as the end point of a vector whose origin is the second axis. This iteration of vectors and axes is most naturally represented by iterations of tense morphemes. The above example can then be re-presented by (1).

(1) PAST (Mary [did] it $\wedge$ PAST John [did] it)

where the latter occurrence of 'PAST' is within the scope of the former occurrence. On this basis iteration has no logical limit.

Not only are there iterations of axes in the case of tensed sentences but also in dated sentences. If the tensing in 'In 55 B.C. Caesar invaded Britain' is neglected two axes are still involved. First, the origin given by O B.C. and secondly the axis 55 B.C. which arises from the former. Here the subordinate axis unlike those in tensing examples is a definite time. However, the convention may be adopted that in such sentences the prime axis is 55 B.C. thus leaving as implicit the claim that the origin is O B.C. Given this convention then a dated sentence like 'After 1933 A.D. Germany went to war' involves two axes given by the dating system. First, the axis 1933 A.D. and secondly an axis which is the (indefinite) end point of the vector introduced by 'after'. A similar situation is to be found in the example 'In 56 B.C. Caesar was about to invade Britain'. In this case the purported event Caesar's invading Britain is said to be later than the axis 56 B.C. Assuming the Semantic Primacy Assumption the natural way to represent this sentence is as 'In 56 B.C. FUT Caesar [invades] Britain' where the morpheme 'FUT' introduces a vector whose origin is not the time or use but instead 'In 56 B.C.' (That is, tense morphemes may express temporal order in both tense and dating systems). So, formally the Semantic Primacy Assumption may be represented by (1).

(1) PAST AT $t_n\phi$iffFUT At $t_n\phi$iff PRES At $t_n\phi$iff At $t_n\phi$.

If this assumption is rejected then all that holds is (2).

(2) PRES At $t_n\phi$ iff At $t_n\phi$.

The framework here can be extended to take account of interval expressions. Also, it can be used to take account of a number of natural language expressions such as 'After the flood...', 'Before the last waltz...', 'At the time of the famine...' etc., expressions which Mott calls 'pseudo-dates' (1973 p. 82) after Rescher. But how does this framework fit in with the continuous and perfect tenses?

3.5 Tense and Natural Language.

The tense morphemes 'PAST', 'PRES' and 'FUT introduced in Section 3.2 express temporal order only. And within the framework introduced in the last section this expression of temporal order is neatly captured via that notion of a 'vectorial expression'. Moreover, it is because these morphemes express temporal order only that we have called them 'pure tense forms'. The idea of a 'pure tense' is intended to be contrasted with that of an 'impure tense'. What then is an impure tense? At the level of natural language the impure tenses are the perfect tenses instanced in 'She had run', 'She has run' and 'She will have run', the continuous tenses 'She was running', 'She is running' and 'She will be running' together with combinations of these as in 'She will have been running'. Clearly, there are important differences between these impure tenses and those which involve only the pure morphemes. The question we want to briefly look at in this Section is how these differences should be accounted for. We shall start by considering the suggestion that the impure tenses should not be semantically distinguished from the pure tenses. Instead differences should be accounted for on pragmatic or conversational grounds.

There is a close similarity in truth conditions between the present perfect and the past tense. (1) is generally true

(1) Tr ($\overline{\text{a has } \phi\text{en}}$ English) iff Tr ($\overline{\text{a } \phi\text{ed}}$ English).

This viewpoint is strengthened when backed up by the observation that in English differences between uses of the perfect tense as opposed to the simple past are generally based upon pragmatic or conversational grounds. Jespersen remarks that the perfect

> "... looks upon the present state as a result of what has happened in the past."
>
> (1961 p. 47).

whereas, in contrast, the simple past tense directs us to the past without saying anything about the connection with the present. This idea of connection with the present can be expanded upon for there is a systematic use of the perfect tense where the grammatical subject refers to or applies to something of 'current relevance' within the speech situation. Now, this could be tightened in such a way as to produce a straightforwardly semantic notion of 'perfect tense' whereby the subject term must apply to or refer to what exists at the time of use. (See 6.10 for a formalization of this). But this does not fit ordinary usage as is shown by the Jespersen example (2)

(2) Newton has explained the movements of the planets

as used in response to an assertion that no one has.
As a backup to the argument here are the following two points. First, if the perfect tense were to be distinguishe semantically from the simple past then the pluperfect sentence 'John had run' should be, at least, ambiguous between (3) and (4), if not also between (5) and (6) as well.

(3) PAST PERF John [runs]

(4) PAST PAST John [runs]

(5) PERF PAST John [runs]

(6) PERF PERF John [runs].

Secondly, a point about simplicity and generality: if the past and present perfect tenses are not semantically distinguished then the regimented form 'PASTϕ' covers a wider range of natural language sentences.

Because the perfect tense, unlike the past tense, is used in contexts where tne present state of affairs is to be seen as resulting from what is being said to have occurred, there are uses of this tense which appear to be semantically distinguishable from corresponding past tense uses. For instance, there appears to be a difference in truth conditions between (7) and (8).

(7) John has lived in York for three years.

(8) John lived in York for three years.

Unlike (8), (7) appears to carry the implication that John still lives in York. One could, of course, attempt to argue that the implication here is not semantic but instead a conventional conversational implicature. But the difference between (7) and (8) is connected with the point that (9) unlike (10) makes sense whereas (11) unlike (12) doesn't make sense.

(9) I haven't seen her so far/this week.

(10) * I didn't see her so far/this week.

(11) * I haven't seen her yesterday/three weeks ago.

(12) I didn't see her yesterday/three weeks ago.

So, this way of distinguishing the past and the present perfect using the semantic/pragmatic distinction doesn't look very hopeful. What about the difference between the pure and the continuous tenses?

Central to tense logics is that a formula have a truth value at a time instant (whatever it is that an instant is). Both 'Socrates is running' and 'Socrates runs' (when understood to be non-frequentative) have truth values at an instant. What, then, in general, is the difference between the continuous present and the simple present tense? Jespersen remarks that we won't go too far wrong here if the question is looked at in the light of a simple example like 'John is hunting' (1961 p. 1979).

> "The hunting is felt to be a kind of frame round something else; it is represented as lasting some time before and possibly (or probably) also some time after something else which may or may not be expressly indicated."
>
> (ibid).

And more generally

> "The essential thing is that the action or state denoted by the expanded [continuous] tense is thought of as a temporal frame encompassing something else which as often is to be understood from the whole situation. The expanded tenses therefore call the attention more specifically to time than the simple tenses which speak of nothing but the action or state itself."
>
> (ibid p. 180).

So a suggestion arising from this is that the difference between the present and the continuous present is not a semantic difference but a pragmatic or conversational one. And that this difference resides in the point that the continuous present carries with it the presumption of truth at a time within an interval. Although, this presumption is in general clear it is not so clear how big an interval has to be, nor is it clear whether or not that at every moment within that interval 'a is ϕing' is claimed to be true - for a distinction is needed between 'a is continually ϕing' and 'a is continuously ϕing' throughout an interval I.

This way of dealing with the continuous tenses, however, is objectionable. For we have left out that element which is central to their use, as Jesperson points out, namely incompletion. This is tied up with a general point about tensing in that it may be used to classify types of verbs. (Potts (1965 p. 75) says that this point was first made and used by Aristotle). Vendler, for instance in (1967), distinguishes between two sorts of verb phrases which do not admit of continuous tensing and two which do. In the former group are 'achievement' terms like '... reaches the hilltop' or '... wins the race' which are true momentarily and 'state' terms like '...loves Jane' or '... knows the plumber' which may be true of someone for a period of time. In the latter group are 'activity' terms like 'running' or '...pushing a cart' and 'accomplishment' terms (which express what Kenny calls 'performances' (1963 p. 172)) like '...running a mile', '...drawing a circle' and '...building a house'.

Because of the difference between 'accomplishment' and 'activity' terms it may be claimed that the present and continuous present must be semantically distinguished. An argument along these lines is based upon Kenny (1963 p. 174). In standard tense logics it is assumed that (13) is valid

(13) $\text{PRES } \phi \rightarrow \text{FUT PAST } \phi$

But one important difference between accomplishments and activities resides in connections between the continuous present and other tenses. When 'ϕing' is an activity term (14) holds

(14) a is ϕing $\rightarrow$ FUT PAST a ϕs.

But when 'ϕing' is an accomplishment term this does not hold. 'Alf is walking to the Rose and Crown' does not imply 'Alf will have walked to the Rose and Crown'. Consequently, if the present and continuous present are not semantically distinguished (13) would be false. On the other hand, if they are distinguished then both (13) and (15) hold

(15) CONT ϕ $\rightarrow$ FUT PAST CONT ϕ

where 'CONT' is taken to be the continuous present morpheme. So on the basis of this it may be claimed that 'PRES ϕ' and 'CONT ϕ' must be semantically distinguished since ' 'PRES ϕ' was true', unlike ' 'CONT ϕ' was true', implies completion.

The semantic|pragmatic or conversational distinction is, then, not too helpful for distinguishing between the pure and impure tenses. Thus, the contrast between 'pure' and 'impure' is not simply accountable for in terms of semantic 'purity'. However, despite this, it is reasonable to claim that the perfect tense, the past continuous and the past tense all do have the same tense in common namely the past. How can this be so? The answer lies in distinguishing tense from aspect. So, one interpretation of the points made above is that the tense|aspect distinction can not be accounted simply by the semantics|pragmatics distinction. It appears that aspect does contribute to truth conditions of sentences involving it.

Aspect relates to the manner in which the events or actions talked about are regarded by providing a particular viewpoint upon those actions or events. It is because the perfect tense is used in contexts where the present time is seen as a result of what has happened, unlike the past tense, that the differences between (7) and (8), (9) and (10), and (11) and (12) arise. And, it is because the continuous tense indicates temporariness that an action or event is occurring that the difference between the validity of (13) and (14) arises. Consequently, the pure|impure distinction relates to tense purity in contrast with a mixture of tense and aspect. The impure tense morphemes 'PERF' and 'CONT' can therefore be introduced. But, what about their syntax and their semantics? In what follows we offer at best a tentative discussion.

In the next Section, following our discussion in the last two Sections, it is claimed the 'PAST' is a sentence operator. Consequently, the sentence 'John ran' is best represented as 'PAST PRES John [runs]' rather than 'PAST John [runs]'. In the case of 'PERF' this can be introduced as an operator which only acts upon present (or present continuous) tensed sentences. Because the characteristic use of the perfect is in the context of looking at the present as a result of what happened it is difficult to see how the forms 'PERF PERF α','PAST PERF α', 'FUT PERF α', 'PERF FUT α' and 'PERF PAST α' can be different from 'PAST PAST α', 'PAST PAST α', 'FUT PAST α', 'PAST FUT α' and 'PAST PAST α' respectively. (In standard discussions of tense logics it is assumed that 'John will have run a mile' is to be represented on 'FUT PAST PRES John [runs] a mile'.)

In this essay two types of semantics for logics employing tense operators are offered; first model theoretic and secondly homophonic semantics. Now, provided there are no objections to (16) or (17)

(16) It is always the case that $(\text{PRES}\phi \rightarrow \text{PRES}\psi) \rightarrow (\text{PERF PRES}\phi \rightarrow \text{PERF PRES}\psi)$

(17) PERF Tr ($\overline{\alpha}$ L) iff Tr ($\overline{\text{PERF}\alpha}$ L).

(where 'Tr' is the tensed truth predicate) then homophonic semantics for 'PERF' fit in with the account given in Section 4.3. Much more difficult is the question of model-theoretic semantics. However, one suggestion would be to impose upon the framework introduced into the last section an aspectual dimension. And in the case of the present perfect one could add an aspectual point upon this dimension. This point would be the time of use of the sentence involving 'PERF'. This idea is based upon Reichenbach's notion of reference time (1966). He distinguished the present perfect from the past tense by claiming that the reference time in the case of the past is (in our terms) the endpoint of the past vector whereas it is the present time or time of use in the case of the present perfect. Our account on the other hand does not introduce an aspectual point in the case of the past tense since it is representable using pure morphemes. This aspectual point can be straightforwardly utilized to account for the difference in truth conditions between (7) and (8). In the case of (7) one may claim that although the expression of pastness points to a three year period this period must contain the aspectual point whereas in the case of (8) there is simply a pointing to a three year period in the past. The hope is that the aspectual point will also be able to account for the differences between (9) and (10), and (11) and (12) although we can not see how this can be done in a generalizable way. The introduction of an aspectual point can be added to the select time semantics introduced in Section 4.1.

It is not clear what the morpheme 'CONT' acts upon. One point, perhaps no more than an oddity, is that if 'CONT' is taken to be a sentence operator upon present tensed sentences then homophonic semantics for it should contain the clause, (18), where 'ϕ' contains a verb open for continuous tensing,

(18) $\text{CONT Tr } (\overline{\text{PRES}\phi}\text{ L}) \text{ iff Tr } (\overline{\text{CONT PRES}\phi}\text{ L})$.

(See section 4.3 for the standard tense clauses in a homophonic semantics). The problem here is that it is not clear what is to be made of 'CONT Tr (...)'. In Vendler's terminology the tensed truth predicate is a 'state' term, not open for continuous tensing. (This is a feature of semantic predicates generally. However, one point to note is that although a singular term may refer to something where the use of 'refer' is not open for continuous tensing a person can be said to be referring to something in a particular situation). This point raises a general question about homophonic semantics. Should every line in a derivation of a T-sentence make sense? If so then much clearer examples than 'CONT' are going to cause trouble if one treats certain adverbs, or uses of certain adverbs, as sentence operators (rather than, say, event predicates); for instance sense will have to be made of (19)

(19) $\text{Carefully Tr } (\overline{\alpha}\text{ L}) \text{ iff Tr } (\overline{\text{Carefully } \alpha}\text{ L})$.

On the other hand, if the steps in the derivation don't have to make sense then what is the proof of a T-sentence a proof of? For it seems difficult to claim that the proof shows how the sense of the whole depends upon the senses of the parts. In the case of 'CONT', one way out of having to propose (18) is to simply claim that there are two present morphemes, a pure and an impure one which are in contrast with each other. Consequently, only (20) is required and not (18)

(20) Tr ($\overline{\text{CONT}\phi}$ L) iff CONTϕ

One is, then, accepting here that there are two basic verb forms, present tensed and present continuously tensed neither of which is syntactically derivable from the other. This proposal has the consequence that 'John is writing' is to be represented as 'CONT John [writes]'. This point can be backed up by a consideration of the completion| incompletion distinction. If ϕ contains an accomplishment term like '...[builds] a house' then it appears that 'PRES ϕ' itself involves completion. This is so because if we represent 'John was building a house' as 'PAST CONT John [builds] a house' and 'John built a house' as 'PAST PRES John [builds] a house' then it can not be the morpheme 'PAST' which introduces completion since this is common to both representations. Moreover 'John built a house' is true iff 'John builds a house' was true which seems to indicate that 'PRES John [builds] a house' does involve completion. One could claim that 'John is building a house' should be represented as 'CONT PRES John [builds] a house' but this means that somehow or other 'CONT' 'undos' the completion element. It seems much more clear to claim that there are two basic or fundamental forms 'John builds a house' and 'John is building a house'.

Model-theoretic semantics in the case of 'CONT' are more straightforward; the aspectual point becomes an interval within which what is said to be occurring continues after the present moment (and was occurring before the present moment) no matter how small this interval is. Thus one can speak of the present vector pointing to within that aspectual interval. As in the 'PERF' case, such an account can be an extension to the select time semantics proposed in 4.1.

Some such framework as this can be extended to cover sentences like 'John has been building a house' which have the form 'PERF CONT John [builds] a house'. In the rest of this essay, except for odd moments, our discussion is of the pure tenses only. This, in part, reflects the point that our discussion in this Section is very tentative indeed.

3.6 The Lp Analysis.

We are now in a position to look at two frameworks within which tense can be analysed. These naturally arise out of the alternative stipulations of what a statement is as contained in Section 3.1. In this Section attention is given to the analysis based upon an Aristotelian singular statement, an analysis which Prior was the first to formalize in his 1957 work.

Although language L_1 is suitable for representing singular statements which involve the simple pure tenses the formatio rules need to be extended in order to take account of sentences involving iterations of tense. On the basis of what was said in the last two Sections 'John will have done it' can be represented as 'FUT PAST John [does] it' where 'FUT' can be taken as an operator upon the sentence 'PAST John [does] it'. In a sentence 'FUT John [does] it' Prior contends that its role is also a tense operator on sentences. This sentence is to be represented as 'FUT PRES John [does] it'. One reason for this is that 'FUTϕ' is now true iff 'PRESϕ' will be true. Asserting 'FUTϕ' then is tantamount to asserting 'It will be the case that PRESϕ' where 'FUT' can be said to be a sentence operator. This account is also justifiable on the basis of the discussion in Section 3.4. 'FUT John [does] it' introduces a vector defined from the prime axis to some indefinite time in the future which is the time purported to be simultaneous with John's doing it. And here the expression '... is simultaneous with...' introduces the 0-vector.

Although the morpheme 'PRES' is redundant in instances like

'PRES FUTϕ' this is not the case in instances like 'FUT PRESϕ', 'PAST PRESϕ' and 'PRESϕ'. For otherwise 'ϕ' and 'PRESϕ' would be equated and clearly from what has been said in this chapter that equivalence is unacceptable because 'ϕ' is a formula of some tenseless language. For if there are sentences which do eschew time reference altogether then these need to be distinguished from present tensed sentences proper whose truth conditions do depend upon the morpheme involved.

In natural languages especially in the case of indirect discourse iterations of tense are not as logically hygenic as proposed to be the case for regimented languages. Prior points our that we say 'He said he was sick' and not 'He said he is sick' but what he said was 'I am sick' and not 'I was sick' thus hiding

> "...the fact that it is the past presentness of his being ill not its past pastness." (1967 p. 14).

L_1 then needs to be given up in favour of a language whose atomic sentences consist of present tensed sentences like 'PRESϕ' together with tensed sentence operators 'FUT' and 'PAST' - which are Prior's F and P. Let us call this analysis of tensed statements or sentences the Lp analysis. Formally, the language Lp is set up as follows. A set of individual constants together with the morpheme 'PRES' and a set of n-place tenseless predicates are assumed.

<u>Definition Lp 1</u>.

An atomic Lp sentence consists of the morpheme 'PRES' any n-place tenseless predicate and n occurrences of any m constants, $m \leq n$.

Expressions and formulas for Lp are now defined where the notation is based upon Kamp (1971).

Definition Lp 2.

The set Σ of expressions for sentential Lp of the first level consists of the symbols (,), the set Q of atomic sentences and for some n ε ω, C_o^n C_1^n... which are n-place connectives.

The connectives C_o^1 C_o^2 C_2^1 C_3^1 are referred to as $\sim$, $\wedge$, G, H respectively.

Definition Lp 3.

Formulas of Lp are defined as follows

(i) q_i ε Q is a formula for any i

(ii) if C_i^n ε Σ and $\alpha_1 \ldots \alpha_n$ are formulae then C_i^n $(\alpha_1 \ldots \alpha_n)$ is a formula.

Instead of writing $\wedge(\alpha\beta)$ $(\alpha\wedge\beta)$ is written. Moreover $(\alpha\vee\beta)$ is written for $\sim(\sim \alpha \wedge \sim \beta)$, $(\alpha \rightarrow \beta)$ for $\sim(\alpha \wedge \sim \beta)$, $P\alpha$ for $\sim H \sim \alpha$, $F\alpha$ for $\sim G \sim \alpha$, $A\alpha$ for $\alpha \wedge G\alpha \wedge H\alpha$ and finally $S\alpha$ for $\sim A \sim \alpha$. Parentheses are omitted wherever no confusion is possible. The intended readings of '$G\alpha$' and '$H\alpha$' are 'It will always be the case that α' and 'It was always the case that α' respectively. '$A\alpha$' represents 'It is always the case that α'.

Attention is only given to the simplest standard sentential tense logics in this essay. Two such systems are Lemmon's minimal system K_t and a system K_L for linear time which are axiomatizable as follows.

<u>System K_t.</u>

Axioms (1) Any instance of an S.C. (sentential calculus) tautology.

(2a) $G(\alpha \rightarrow \beta) \rightarrow (F\alpha \rightarrow F\beta)$

(2b) $H(\alpha \rightarrow \beta) \rightarrow (P\alpha \rightarrow P\beta)$

(3a) $PG\alpha \rightarrow \alpha$

(3b) $FH\alpha \rightarrow \alpha$

<u>Rules</u>

M.P. if $\vdash \alpha$ and $\vdash \alpha \rightarrow \beta$ then $\vdash \beta$.

R.G. if $\vdash \alpha$ then $\vdash G\alpha$

R.H. if $\vdash \alpha$ then $\vdash H\alpha$.

K_t is a minimal system because it leaves open the possibility that time may or may not be linear, dense, or infinite in both directions. For dense linear time the system K_L is introduced which is $K_t \cup \{4a, 4b, 5a, 5b, 6a, 6b\}$ (See Prior 1967 p. 176ff for a variety of tense systems).

(4a) $G\alpha \rightarrow GG\alpha$

(4b) $H\alpha \rightarrow HH\alpha$

(5a) $F\alpha \wedge F\beta \rightarrow F(\alpha \wedge \beta) \vee F(\alpha \wedge F\beta) \vee F(F\alpha \wedge \beta)$

(5b) $P\alpha \wedge P\beta \rightarrow P(\alpha \wedge \beta) \vee P(\alpha \wedge \beta) \vee P(P\alpha \wedge \beta)$

(6a) $F\alpha \rightarrow FF\alpha$

(6b) $P\alpha \rightarrow PP\alpha$.

In Section 4.1 model theoretic semantics are given for L_p. Formally, they are very similar to modal semantics. One difference, however, is that unlike the set of possible worlds the set of times is ordered and it is this order which is central to the semantics of the tense constants.

However, tense systems only employing 'A' and 'S' do not make use of this order. Prior call these 'modal fragments' of tense systems. These are isomorphic to standard modal systems containing the operators L and M. For instance, the 'modal fragment' of K_t is the system B while that of K_L is S_5.

How should sentences which involve dates be construed on the Lp analysis? Given the Semantic Primacy Assumption it was contended in Section 3.4 that 'In 56 B.C. Caesar was about to invade Britain' is to be translated as 'In 56 B.C. FUT Caesar [invades] Britain'. Now, it was here argued that 'FUT Caesar [invades] Britain' is to be taken as 'FUT PRES Caesar [invades] Britain' having the form Fq_m where q_m is a present tensed sentence.'In 55 B.C. Caesar invaded Britain' then is best represented by 'In 55 B.C. PRES Caesar [invades] Britain'.

So one way of dealing with dated sentences under the L_p analysis is to extend language L_p to L_p^+ which is $L_p \cup \{At..t_i\}$ where 't_i' is a dating system and 'At...' is a two place operator whose first place is only open for members of 't_i' and whose second place is only open for L_p sentences. Formulas of L_p^+ are defined as follows.

<u>Definition L_p^+</u>

(i) if α is any L_p formula then it is an L_p^+ formula.

(ii) if α is any L_p formula and t_n any member of t_i then At t_n α is an L_p^+ formula.

(iii) if α, β are L_p^+ formulas then so are $\sim\alpha$, $\alpha\wedge\beta$, $H\alpha$, $G\alpha$.

Defined in this way expressions like 'At t_n At t_m α' are not

L_p^+ formulas. System K_L^+ is K_L plus the following axioms. (Because K_t need not involve linear time a more complex dating system is required in order to allow for the possibility of branching time, a possibility not considered in this essay).

(7) $\sim$ At t_n α iff At t_n $\sim \alpha$.

(8) At t_n $(\alpha \wedge \beta)$ iff At t_n α $\wedge$ At t_n β.

(9) $A\alpha \rightarrow$ At t_n α provided $t_n \varepsilon t_i$ and $\alpha \varepsilon L_p$.

The Semantic Primacy Assumption is enclosed in (10) and (11).

(10) At t_n α iff G At t_n α, for any t_n ε t_i

(11) At t_n α iff H At t_n α, for any t_n ε t_i.

L_p^+ may be extended to L_p^{++} to include time quantifiers and variables and where also it is assumed that the set 't_i' is a set of date constants which refer to time moments. In K_L^{++} (12) is valid

(12) $A\alpha$ iff $(t)(\text{At } t \ \alpha)$ provided $\alpha \varepsilon$ L_p.

An alternative way of dealing with dated sentences is to claim that they are really disguised metalanguage sentences for some matalanguage ML_p which does not contain L_p - that is, a metalanguage suitable for a non-homophonic semantics. In brief this proposal amounts to treating 'In 55 B.C. Caesar was about to invade Britain' as '$\overline{\text{FUT PAST Caesar [invades] Britain}}$ is true in 55 B.C.'

3.7 The L^D Analysis.

In this Section analyses of tense are given which are based upon comments made in connection with the Frege-Russell account of a statement (see 3.1). There are two important theses involved in that discussion, (A) and (B).

(A) A statement cannot vary in truth value over time.

(B) Tense expressions are indexical.

Let us first consider (B). This can be captured by the following three equivalences, where '>' is the 'later than' relation and 'now' is an indexical.

(1) PRES ϕ iff At now ϕ.

(2) PAST PRES ϕ iff $(\exists t)(\text{now} > t \wedge \text{At } t\ \phi)$

(3) FUT PRES ϕ iff $(\exists t)(t > \text{now} \wedge \text{At } t\ \phi)$.

Thus to accept these is to accept the view that the variation in truth value over time of a tensed sentence is accountable simply in terms of the indexical element 'now'.

One can hold the view that tense expressions are indexical without also holding the view that statements cannot vary in truth value over time. That is, one can accept (B) and, yet, reject (A). But, this is somewhat perverse, unless one also holds the view that statements can vary in truth value from place to place, or from person to person, or from object to object. For (B) places tense expressions on a par syntactically with 'here' and 'there', 'I' and 'you', and with 'this' and 'that' etc..,. The L_P analysts on the other hand, because they reject (B), can hold the view that a statement (proposition) can vary in truth value over time without also holding the view that a statement can vary in truth value from place to place. This is because they analyse tense expressions adverbially, unlike place expressions. Moreover, if one does accept (B) then this makes (A) much easier to formulate, as we show below.

A Frege-Russell account of a statement is based upon (A). In 3.1 it was suggested that such a statement can be identified as an ordered pair whose first place is filled by a sentence and whose second place is filled by a set of indexes which represent a possible occasion of use. Now, it is very important to bear in mind that a statement is here being treated in terms of a (potential) use of some sentence upon some occasion (see 4.4). Because L_P^+ sentences introduced in the last Section contain no indexicals like 'I' or 'here' etc., the statement expressed by a use of a tensed sentence at some time may be identified with the pair given by (4).

(4) $\langle$Sentence $\in L_P^+$, time of use$\rangle$

In the discussion above we noted that one could, but only with perversity, accept (B) but reject (A). Similarly, here, one can hold (A) without also holding (B). In this case it would mean that the ordered pair given in (4) is essential to this representation of a statement. Two uses of the same tensed sentence, such as 'It is raining', at different times would then express different statements because their time of use would differ. However, if besides holding (A) one also accepts (B) then one can give a representation of a statement which is not an ordered pair, but instead is more akin to an English sentence.

Before we consider this it should be pointed out that the Semantic Primacy of Dates Assumption (see 3.3) is independent of both (A) and (B). For this assumption is concerned with whether or not, for instance, we should say that a present use of the sentence 'Caesar will invade Britain in 55 B.C.' is false because 55 B.C. is not in the future. Rejecting this assumption means that our example can be false in two ways; either because 55 B.C. is not a year when Caesar invaded Britain or because 55 B.C. is not a year which is future to the time of use. Acceptance of the assumption removes the latter as a condition for falsity; thus, in this case 'Caesar invaded/will invade/invades Britain in 55 B.C.' have the same falsity conditions. Hence, rejection of the assumption merely complicates the analysis of dated sentences; for then one has to treat them as tensed statements together with dating. For instance, if one accepts (B), and hence (1) to (3) above, and yet rejects the Primacy Assumption, then a present use of 'Caesar will invade Britain in 55 B.C.' is analysed by (5).

(5) $(\exists t)(t>now \wedge t = 55\ B.C. \wedge$ At t Caesar[invades]Britain$)$

If one holds the assumption, on the other hand, (5') is an analysis

(5') At 55 B.C. Caesar[invades]Britain.

Those theorists who argue for (A) are more likely to hold the Primacy Assumption. One important reason is that acceptance of the Assumption adds force to (A), in the sense that dated sentences can be treated as a paradigm case of sentences which do not differ in truth value over time, yet do deal with temporal realities. This then suggests that an analysis of tense should 'reduce' tensed sentences to dated sentences. Moreover, acceptance of the Assumption makes analyses of dated sentences simpler as the difference between (5) and (5') shows.

So, strictly speaking, the question of whether or not one should allow a statement or a proposition to vary in truth value over time is independent of whether or not tense is treated indexically. And both of these issues are independent of the Semantic Primacy Assumption. But, as we noted, a theorist who holds (B) is more likely to hold (A). And, similarly, if one holds (A) then one is likely to hold (B). A theorist who analyses uses of tensed sentences into a language for which any sentence expresses the same statement irrespective of its time of use and who also holds that tenses are indexical is here called an L^D theorist.

Given the Semantic Primacy Assumption then the simplest L^D theory is that which represents all uses of L_p sentences in terms of dated sentences. Using the ordered pair notation (4), then (6) appears to be valid.

(6) $(t_i)(t_j)(\alpha \varepsilon L_p)(<\alpha\ t_i> \text{ iff } <\text{At } t_i \alpha\ t_j>)$.

This amounts to the claim that any use of a tensed sentence expresses a statement which is also expressed by a dated sentence. Now because the time of use is irrelevant to the statement made by a use of a dated sentence, (6) can be amended to (6'), where the right hand side is no longer an ordered pair.

(6') $(\alpha \varepsilon L_p)(t_i)(<\alpha\ t_i> \text{ iff At } t_i \alpha)$.

In 3.4 we noted that all occurrences of tense morphemes within a dated formula take as their primary axis not the time of use but the date. Consequently, the following three hold, which are similar to (1) to (3).

(7) $(t_i)(\phi \varepsilon L)(\text{At } t_i \text{ PRES } \phi \text{ iff At } t_i\ \phi)$

(8) $(t_i)(\phi \varepsilon L)(\text{At } t_i \text{ PAST PRES } \phi \text{ iff } (\exists t)(t_i > t \wedge \text{At } t\ \phi))$

(9) $(t_i)(\phi \varepsilon L)(\text{At } t_i \text{ FUT PRES } \phi \text{ iff } (\exists t)(t > t_i \wedge \text{At } t\ \phi))$

How should 'At' be construed in 'At t_n ϕ'? The simplest method of dealing with it here is to treat 'At...ϕ' as an open sentence true of times which may be represented by a monadic predicate ψ. Consequently, ψt_n is then a representation of a dated sentence. What we have here is Russell's suggestion, noted in 3.1. For a use of the English tensed sentence 'Mrs. Brown will be at home' at t_n is to be represented by the dated sentence $(\exists t)(t>t_n \wedge$ At t Mrs. Brown [is] at home), which has the form $(\exists t)(t>t_n \wedge \psi t)$. Where Q then, is a set of atomic present tensed L_P sentences and where q_n is PRES ϕ_n then there is a corresponding set ψ of monadic predicates such that ψ_n is At...ϕ_n. This analysis of tense is here called 'the L^D analysis'. The language L^D suitable for taking account of Russellian statements is a monadic predicate calculus with one dyadic operator '>', a set of time variables and time constants and quantifiers. In order to set up systems which express assumptions about time non-logical axioms have to be introduced. If identity axioms are assumed than (10) expresses linearity whilst (11) expresses denseness.

(10) $(t_i)(t_j)(t_i>t_j \vee t_j>t_i \vee t_i = t_j)$

(11) $(t_i)(t_j)(\exists t_k)(t_i>t_j \rightarrow t_i>t_k>t_j)$.

One criticism which is often put against the Russellian analysis of tense is that it presupposes an ontology of times. In response to this it must be realized that the introduction of an ontology of times is not something which first occurs in the analysis of the tense morphemes as it would do if one held (B) without (A) and took (1) to (3) as the analysis of those morphemes. Instead the ontology is presupposed in the very data which is to be analysed, namely the use of a tensed sentence at some time. And once times are assumed then the analysis of tense morphemes in terms of quantifiers and the 'later than' relation is hardly objectionable (especially so given the framework of 3.4).

3.8 An Argument for L^D: The Universality of Logic.

In 3.1 an argument against truth variability over time of a statement was rejected. In this Section an alternative argument is mentioned. Quine writes

> "Logical analysis is facilitated by requiring that each statement be true once and for all independently of time."
> (1965 p.6).

How is logical analysis 'facilitated' by only having truth and falsity simpliciter? In terms of complexity there is not much to choose between the L_p and the L^D analyses. Certainly, the L^D analysis has the advantage of keeping the form of uses of tensed sentences within standard first order theory but this is at the expense of having to analyse uses of ordinary language sentences rather than sentences and so presumes an ontology of times unlike the L_p analysis. Moreover, the L_p analysis may be treated as a sentential analysis quite independently of accounts of what a statement is.

However, an important reason which may lie behind Quine's stricture here is a preservation of the universality of the standard statement calculus and, in particular, of providing a semantics for it. In a homophonic tensed semantics as given in Section 4.3 use is made of the tensed truth predicate. On the other hand in a Fregean semantics only one truth predicate, a tenseless one, is required.

> "Only a sentence supplemented by a time-indication and complete in every respect expresses a thought... [Thus a thought] if it is true is true not only today or tomorrow but timelessly. Thus the present tense in 'is true' does not refer to the speaker's present but is, if the expression be permitted, a tense of timelessness."
> (Frege 1967 p.37).

So how does this affect the universality of the statement calculus? Consider the statement calculus theorem (1).

(1) $(\alpha \wedge (\alpha \rightarrow \beta)) \rightarrow \beta$.

and its metalanguage reading according to an L^D and an L_p analysis respectively where 'Tr($\overline{\alpha}$ L)' says ''α' [is] true in L and 'PRES Tr($\overline{\alpha}$ L)' says ''α' is (now) true in L'.

(2) if Tr ($\overline{\alpha}$ L) and Tr ($\overline{\alpha \rightarrow \beta}$ L) then Tr ($\overline{\beta}$ L)

(3) if PRES Tr ($\overline{\alpha}$ L) and PRES Tr ($\overline{\alpha \rightarrow \beta}$ L) then PRES Tr ($\overline{\beta}$ L).

But now consider the following apparent instance of (1), namely (4), as used on a time occasion t_n.

(4) (2+2=4 ∧ (2+2=4 → PRES Socrates [sits]))→PRES Socrates [sits].

But is this an instance of (1)? According to the L^D account it is to be treated as (4')

(4') (2+2=4 ∧ (2+2=4 → At t_n Socrates [sits])) → At t_n Socrates [sits].

which is then an instance of (1) and whose semantic clause is given by (2). On the other hand can (4) be an instance of (1) for the L_p theorist? It seems not unless sense can be made of the expression (5).

(5) PRES Tr ($\overline{2+2=4}$ English)

Consequently, logical analysis is facilitated here simply because under the L^D analysis a single tenseless truth predicate only is required. Note, that this problem is not a problem about the universal applicability, the context independence or topic neutrality, of the logical constants '∼' and '∧' (This is pointed out because in first order theories the topic neutrality of the quantifiers may be encroached upon - but see 7.5).

4.1 Model Theoretic and Select Time Semantics for L_p.

Two sorts of semantics are given for the L_p analysis in this essay, namely model theoretic and homophonic semantics. In this Section the former is dealt with whereas the latter is the subject matter of 4.3. The language L_p given in Section 3.6 is assumed. First, a set J is defined after Kamp (1971) p. 233.

Definition 1.

Specify a non-empty set T and a partial ordering (transitive and asymmetric) '>' on T. Let J = <T,>, >where T is the set of times and '>' is the 'later than' relation.

Other conditions on J depend upon further assumptions made about time. These can include density and infinity conditions Because L_p sentences are true at a time it is natural to consider semantic structures which specify a truth value to each formula at each time. Therefore, an interpretation is defined as follows.

Definition 2.

An interpretation I for L_p is a set <Jf> that is <T,>,f> where f is a valuation function such that

(i) the domain of f consists of $q_1 q_2 \ldots \varepsilon Q$.

(ii) for each i>o $f(q_i)$ is a function with domain T and range {0,1} (the truth values).

Let I be a model for L_p. Then for any formula $\alpha \varepsilon L_p$ the truth value of α at t under I (written as $I(\alpha)(t)$) is given as follows.

Definition 3

(i) if α is atomic $I(\alpha)(t) = 1$ iff $f(\alpha)(t) = 1$

(ii) $I(\sim\alpha)(t) = 1$ iff $I(\alpha)(t) = 0$

(iii) $I(\alpha\wedge\beta)(t) = 1$ iff $I(\alpha)(t) = 1$ and $I(\beta)(t) = 1$.

(iv) $I(H\alpha)(t) = 1$ iff $I(\alpha)(t') = 1$, for all t' s.t. $t>t'$.

(v) $I(G\alpha)(t) = 1$ iff $I(\alpha)(t') = 1$, for all t' s.t. $t'>t$.

Because different formulas are validated according to assumptions made about the set J the notion of validity relative to J, after Kamp (ibid p.235) is appropriate here.

Definition 4.

A formula $\alpha \in L_p$ is valid relative to J iff for every interpretation I of the form $<J,f>$ and all $t \in T$ $I(\alpha)(t) = 1$.
Any system validated by these caluses is called an AL_p system.
Of course dependence upon J results in there being a variety of such systems. A more general notion of validity can therefore be introduced, validity relative to K (after Kamp 1971 p.235).

Definition 5.

Let K be a non-empty set of partial orderings then a formula α is K valid iff α is valid relative to every $J\varepsilon K$.

Definitions of semantic consequence relative to J (K) follow straightforwardly.

An alternative semantics which has proved to be of great use (see next Section) can be introduced based upon Kripke's modal semantics (1963). Instead of defining an interpretation I as $<T,>,f>$ one defines it as $<T,>,f,t_n>$ where t_n is a select member of the set of times T. This interpretation is written as 'I_{t_n}' and is called 'a select time interpretation'. The semantic clauses are those given in Definition 3 except I is replaced by I_{t_n} throughout. What differs, however, is the definition of validity.

Instead of Definition 4, Definition 6 is introduced where $t_n \in T$.

Definition 6.

A formula $\alpha \in L_P$ is t_n valid relative to J iff for every interpretation I_{t_n} of the form $\langle J,f,t_n\rangle$ $I_{t_n}(\alpha)(t_n) = 1$.

That is, validity is not defined in terms of truth at all times but instead in terms of truth at the select time t_n. Any system for which these clauses hold is called a BL_P system. In Kripke's modal semantics his select member of the set of possible worlds is the actual world. Naturally, then, the select member, t_n, can be understood as the present moment.

For some specific J what is the relationship between these two notions of validity? (Or assuming completeness what is the connection between AL_P and BL_P systems?) There are two ways of considering coincidence here which are of relevance for the next Section. First, it may be noted that under an interpretation I_{t_n} the existence of $t_n \varepsilon I_{t_n}$ makes no difference to the valuation functions f' nor to the semantic clauses for $\sim, \wedge$, H and G. So letting I_{t_n} be $I \cup \{t_n\}$, (1) holds.

(1) $I_{t_n}(\alpha)(t_n) = 1$ iff $I(\alpha)(t_n) = 1$

And furthermore (2) holds

(2) $I_{t_n}(\alpha)(t) = 1$ iff $I(\alpha)(t) = 1$ for any t.

Consequently coincidence occurs here iff (3) holds

(3) $(I_{t_n})(I_{t_n}(\alpha)(t_n) = 1)$ iff $(I_{t_n})(t)(I_{t_n}(\alpha)(t) = 1)$

Let this be called the strong sentential A = B thesis. Given (1) and (2), (3) is equivalent to (4).

(4) $(I)(I(\alpha)(t_n) = 1)$ iff $(I)(t)(I(\alpha)(t) = 1)$

Here the difference between Definition 4 and 6 is being treated as that between every time validity (relative to J) and t_n validity (relative to J). Now since right to left holds of the biconditionals (3) and (4) by quantification theory AL_p systems are (at least) contained in corresponding BL_p systems. So the interesting thesis is left to right namely (5).

(5) if $(I)(I(\alpha)(t_n) = 1)$ then $(I)(t)(I(\alpha)(t) = 1)$.

Acceptance of (5) amounts to rejection of any principle along the lines that tense logically speaking, times must be differentiated because what is tense logically valid at one time may not be so at another. (The rejection of any such principle is analogous to the acceptance by most scientists that scientifically speaking all times are the 'same' in the sense that laws are invariant with respect to them.)

The other way of considering coincidence is independent of (1) and (2) above. What t_n validity (relative to J) may be taken to amount to is that of validity of a sentence at a time of (potential) use t_n. So an alternative way of saying that tense logical truth is invariant with respect to time is to say that if a sentence is tense logically true when (potentially) used at some time it is tense logically true when (potentially) used at any time. Thus, '$I_{t_n}(\alpha)(t_n) = 1$' is read as 'α as (potentially) used at t_n is then true under the interpretation $\langle T,>,f,t_n\rangle$'. Given that $I^{t_k}/_{t_n}$ is the select time interpretation that results from substituting t_k for t_n in I_{t_n} (that is, $\langle T,>,f,t_k\rangle$) then we may capture the idea that sentences are tense logically true (relative to J) at every time of potential use by (6)

(6) $(I_{t_n})(t)(I^{t}/t_n(\alpha)(t) = 1)$

So coincidence may alternatively be captured by (7).

(7) $(I_{t_n})(I_{t_n}(\alpha)(t_n) = 1)$ iff $(I_{t_n})(t)(I^{t}/t_n(\alpha)(t) = 1)$.

Let us call this the weak sentential A = B thesis. Its acceptance amounts to the claim that the time it is makes no difference to what is then tense logically true. In the next Section we shall note that there are situations in which the weak thesis may hold without the strong thesis also holding. In this Section our discussion centres on the strong thesis.

It is plain that standard accounts do not envisage the possibility that this thesis might fail to hold. One reason why is based upon the structure of the valuation functions f. For any atomic formula q_i f is a total function from the set T into $\{0,1\}$. Consider now the partial functions f_{t_n} which are the functions from $t_n \varepsilon T$ into $\{0,1\}$ only and the partial functions f_{t_m}. Clearly the sets f_{t_n} and f_{t_m} are isomorphic. Consequently, the valuation functions themselves do not differentiate one time from another. And for standard systems neither do the semantic clauses. So, it may be thought that this lies behind the acceptability of the A = B thesis. Let us pursue this line of thought by spelling out select time partial interpretations, f_{t_n}.

Definition 7.

A partial interpretation at a select time t_n (written as f_{t_n}) is the pair $\langle t_n f \rangle$.

Definition 8.

Let f_{t_n} be a partial interpretation for L_P. For some formulas $\alpha \varepsilon L_P$ the truth value of α under f_{t_n} (written as $f_{t_n}(\alpha)$) is given as follows

(i) if α is atomic then $f_{t_n}(\alpha) = 1$ iff $f(\alpha)(t_n) = 1$.

(ii) $f_{t_n}(\sim\alpha) = 1$ iff $f_{t_n}(\alpha) = 0$.

(iii) $f_{t_n}(\alpha \wedge \beta) = 1$ iff $f_{t_n}(\alpha)$ and $f_{t_n}(\beta) = 1$.

f_{t_n} is, then, not defined for formulas of L_P which contain tense constants. The notion of t_n validity which is independent of time's structure (that is, of any J) is given by Definition 9.

Definition 9.

A formula $\alpha \varepsilon L_P$ is t_n valid iff for every f' $f'_{t_n}(\alpha) = 1$.

Partial interpretations in the case of sentential logic are not as interesting as they are in the case of first order logic (see Section 6.2). What is of interest is the set of formulas which are t_n valid. If $p_1 p_2 \ldots \varepsilon P$ is an atomic L language, closed under '$\sim$' and '$\wedge$' such that there is a 1-1 function g from P into Q then there is a 1-1 function from the set C of classical valuations (for L) into the set

f'_{t_n}. Consequently, (8) holds.

(8) If α is classically valid then α' is t_n valid where α' is $g(\alpha)$.

This sets up a connection between a (tenseless) language L and part of a tensed language L_p. If t_n is the present time then f_{t_n} is defined only for present tensed sentences of L_p. What we have here is standard sentential calculus representing present tensed fragments of ordinary language. Consequently, 'α is t_n valid' can be understood as 'α is a t_n tautology'. T_n tautologies, then, are merely the standard tautologies for present tensed sentences and because the set f_{t_n} is isomorphic to f_{t_m} t_n tautologies satisfy the A=B thesis.

It is often claimed that a tensed sentence like $q_n \vee \sim q_n$ (εL_p) is 'timelessly true' but this use of 'timeless' is rather loose. What we have here is a characterization of tense logical truth which is independent of time's structure. On thisbasis a distinction can be made between two types of omnitemporal truth, one type dependent upon time's structure, the other independent.

The partial interpretations f_{t_n} however do not yield all t_n tautologies of L_p since there are substitution instances which involve tense operators like $F\alpha \vee \sim F\alpha$ which are unaccounted for. Although these may be captured by partial interpretations which treat all formulas containing an outermost tense operator to be atomic this tactic will not capture all formulas which are valid independently of any J. That is, t_n tautologyhood and validity independent of time's structure do not co-incide; an example which shows this is

$G(\alpha \vee \sim\alpha)$. In connection with this is a claim made by Chadwick.

> "It will either rain or not rain tomorrow and it will either freeze or not freeze tomorrow' is not one of th contingent propositions belonging to natural science."
> (1927 p. 7).

He goes on to claim that they are tautologous. Suppose Prior's metrical operators 'F_n' for 'It will be the case in n days time that...' are introduced then Chadwick's claim is that (9) is a tautology.

(9) $F1\ (\alpha \vee \sim\alpha)$.

Now, if (11) and (12) are tautologous then (9) is derivable from (10)

(10) $Fn\ \alpha \vee \sim Fn\alpha$

(11) $Fn(\alpha \vee \beta)$ iff $Fn\ \alpha \vee Fn\beta$

(12) $Fn\sim\alpha$ iff $\sim Fn\ \alpha$

However (12) is not a tautology since (13) is true iff there is a tomorrow.

(13) $\sim F1\alpha \rightarrow F1\sim\alpha$.

Ordinary language, it seems, does not distinguish between $\sim F1\alpha$ and $F1\sim\alpha$. What this brings out is a difference between A and B validity. According to the notion of validity given by Definition 4 (14)

(14) $F(\alpha \vee \sim\alpha)$

is valid only if time is future infinite whereas according to validity as given by Definition 6 (14) is valid provided the select time t_n is not the last moment of time. This point connects up with the claim that (15) represents time's future infinity.

(15) $G\alpha \rightarrow F\alpha$.

4.2 Now, Two Dimensionality and Rigidity.

In the last Section a strong A=B thesis, (1), was distinguished from a weaker version, (2).

(1) $(I_{t_n})(I_{t_n}(\alpha)(t_n) = 1)$ iff $(I_{t_n})(t)(I_{t_n}(\alpha)(t) = 1)$

(2) $(I_{t_n})(I_{t_n}(\alpha)(t_n) = 1)$ iff $(I_{t_n})(t)(I^{t}/t_n(\alpha)(t) = 1)$.

For standard systems these theses coincide because the occurrence of the select time, t_n, makes no difference to the valuation functions f' nor to the semantic clauses for ~,∧, H and G. Operators, however, can be introduced for which the select time is essential to their semantic clauses. In the main part of this Section attention is focused upon a single such operator 'now' as introduced by Kamp (1971).

It appears that the function of the word 'now' is to make the clause to which it applies refer to the moment of utterance. Although in a present tensed sentence 'now' is redundant this is not always the case as Kamp's example (4) shows, with its difference from (3).

(3) I learned last week that there would be an earthquake.

(4) I learned last week that there would now be an earthquake.

(4) unlike (3) involves the claim that what one learnt was that an earthquake would occur now. The occurrence of 'now' there is non-vacuous unlike its occurrence in 'it's raining now'. It is because 'now' apparently refers to the moment of utterance that (5) is valid.

(5) If I am now writing then it is always true that I am now writing.

If 'now' is represented by a sentential operator 'N', then, in order to provide semantics for it one should

" 'keep track'... of the moment of utterance of the entire expression. The concept we ought to analyse is not simply 'The truth value of ϕ at t' but rather 'the truth value of ϕ at t' when part of an utterance made at t'."

(Kamp 1971 p.238).

Select time interpretations are ideally suited here. 'I_{t_n} $(\alpha)(t)$' may then be read as 'the truth value of α at t under the interpretation I_{t_n} when part of an utterance made at t_n'. The semantic clause for 'N' namely (6), can be added to the select time semantics given in the previous Section.

(6) $I_{t_n}(N\alpha)(t) = 1$ iff $I_{t_n}(\alpha)(t_n) = 1$.

Because of the dependence of the clause for N upon two times it is an instance of what Segerberg calls a 'two dimensional operator' (1973). The dependence here upon the select time means that systems which involve 'N' may satisfy the weak A = B thesis (2) without satisfying the strong (1). An instance of this is the formula (7).

(7) α iff $N\alpha$.

Other tense words which have this feature include 'last week', 'next year' and 'yesterday'. For instance 'Yesterday will always be remembered' brings this out. Interestingly, D. Lewis notes that the word 'actual' or 'actually' also has this feature in certain contexts.

"We can distinguish primary and secondary senses of "actual" by asking what world "actual" refers to at a world w in a context in which some other world v is under consideration. In the primary sense, it still refers to w, as in 'If Max ate less, he would be thinner than he actually is'. In the secondary sense it shifts its reference to the world v under consideration as in 'If Max ate less, he would actually enjoy himself more."

(1970 p.185).

In the primary sense 'actually' is two dimensional and in the secondary sense one dimensional. If an S_5 modal select world interpretation U_a is <W,f,a> where W is the set of possible worlds, a the select member, the actual world, and f a valuation function then where 'A_p' is the primary and A_s the secondary 'actually' their difference is that between (8) and (9).

(8) $U_a\ (A_p\alpha)(\omega i) = 1$ iff $U_a\ (\alpha)(a) = 1$.

(9) $U_a\ (A_s\alpha)(\omega i) = 1$ iff $U_a\ (\alpha)(\omega i) = 1$.

Unlike Kamp many theorists (especially those who hold to an L^D analysis) are of the opinion that 'now' is an indexical a temporal analogue of 'here' and not a sentence operator. So suppose 'now' is treated as an indexical then in what way does its two dimensionality manifest itself, if at all? This question appears not to be answerable in the context of the standard indexical accounts of 'now' because they go hand in hand, or rather as an integral part of, the analysis of tense in terms of quantifiers and the 'later than' relation - see 3.7. For the interesting feature of 'now' when construed as a sentence operator is when it lies within the scope of other tense operators. In order to consider this aspect when 'now' is construed as an indexical we may introduce it as a 'date' index into the language $L_p^+ \cup \{now\}$ (see 3.6 for L_p^+). Hence, expressions like 'At now α' and 'F At now α' are formulas of this extension to L_p^+.

Kamp represents the difference between (10) and (11) using the 'now' operator in a representation (11') for (11) and (10') for (10)

(10) A child was born that would become ruler of the world.

(11) A child was born that will become ruler of the world.

(10') $P(\exists x)(x \text{ is born} \wedge F(x \text{ is ruler of the world}))$.

(11') $P\ (\exists x)(x \text{ is born} \wedge NF\ (x \text{ is ruler of the world}))$.

In the extension to L_p^+ introduced here (10) is still represented by (10') whereas (11) now becomes (11").

(11") P($\exists$x)(x is born $\wedge$ At now F(x is ruler of the world)).

And just as (12) holds for the sentence operator, (13) holds for the indexical 'now'.

(12) Nα iff GNα

(13) At now α iff G At now α

(13) is very similar to (14) which is L_p^+ valid

(14) At t_n α iff G At t_n α.

The formula 'At now α' is then rather strange. Like a temporally indefinite sentence it may differ in truth value on different occasions of use. Yet on the other hand it is also like a temporally definite sentence of the form At t_n α in that (13) holds for it. Note, furthermore that even if the Semantic Primacy Assumption is given up (and hence (14) would then be false) (13) would still hold. How is this 'strangeness' to be accounted for? We suggest that this is to be done in terms of tense logical rigidity. At any time of use 'now' is tense logically rigid in the sense that no matter which tense operator it occurs within its 'reference' is invariant. The simplest way to represent this is by adding 'now' as a date index to the language $L_p^{++} \cup \{=\}$ which differs from L_p^+ in that it has quantifiers and time variables. The rigidity claim is then representable by (15) where 0 is any tense or iteration of tense operators.

(15) ($\exists$t)(t = now $\wedge$ 0 At t α) iff 0 At now α.

(The Semantic Primacy Assumption is assumed here). The notion of rigidity appealed to here is that of rigidity relative to a time of use, a notion which is expanded upon in 4.5.

4.3 Homophonic Sentential L_p and L_p^+ Semantics.

Davidson has proposed as an adequacy condition on semantic theories for the purpose of interpreting a language L that they be finite and entail all T-sentences of L. Accordingly, model theoretic semantics are not adequate for this purpose. So in this Section homophonic semantics which are adequate are given for sentential L_p and its extension L_p^+. In a homophonic semantics for a language L all instances of (H) must be derivable.

(H) Tr $(\overline{\alpha}\ L)$ iff α

where it is assumed that the metalanguage ML contains the object language L together with resources for forming canonical descriptions of L sentences. In the schema (H) '$\overline{\alpha}$' is assumed to be a canonical description in ML of 'α'εL and 'α' its translation in ML given by the identity function.

In the case of tensed homophonic semantics a doubt may be raised as to the coherence of instances of (H) like (1) for instance

(1) Tr($\overline{\text{John is ill}}$ English) iff John is ill.

Many theorists may claim that there is a suppressed indexical 'the present time' or 'now' which needs to be brought out here. It might even be claimed that (1) is as nonsensical as (2) or (3).

(2) Tr($\overline{\text{He is ill}}$ English) iff He is ill.

(3) Tr($\overline{\text{John is here}}$ English) iff John is here.

But there is an important difference between (1) and the other two. In (1) the semantic predicate 'Tr' is itself understood to be tensed. Hence if (1) is written out using morphemes (see 3.2) (1') is obtained

(1') PRES Tr ($\overline{\text{PRES John [is] ill}}$ English) iff
PRES John [is] ill.

Consequently the standing of (1) is on a par with tense logical formulas of L_p like (4) which is (4')

(4) q_1 iff q_2

(4') PRESϕ_1 iff PRESϕ_2.

Thus, this argument from indexicality in the case of the truth theory has the same standing as a similar argument against the L_p analysis (see 3.1). But, clearly that analysis is at least coherent.

There are two immediate but closely connected problems for tense homophonic semantics. The first is based upon a point made by Buridan. The axiom for '$\sim$' in a homophonic theory for L is given by (5)

(5) $Tr(\overline{\sim\alpha}\ L)$ iff $\sim Tr(\overline{\alpha}\ L)$.

This suggests that for the tense operator F its natural axiom is (6)

(6) $Tr(\overline{F\alpha}\ L)$ iff $F\ Tr(\overline{\alpha}\ L)$.

which says that a future tensed sentence is true in L iff the corresponding present tensed sentence will be true. But as Buridan points out in a sophism 'You will be an ass'

"I prove this since tomorrow this will be true: 'You are an ass'. Therefore, today this is true: 'You will be an ass'... The antecedent of the first consequence is proved, positing the case that you and others voluntarily change your name, and impose on you the name "ass"... The opposite is argued..."
(1966 p. 158).

The problem is then that the sentence 'α' of L may change in meaning tomorrow so that although '$F\alpha$' is now false in L 'α' may well become true in L because of this change. Buridan's solution is to note that

"... it is impossible for the proposition 'A man is an ass'...

to be true, retaining ... just those significations which it now has."
(1966 p. 159).

In accordance with this is the suggestion that sentences of L retain their present meaning in the future.

The second problem is that if (6) is the natural clause for F then (7) should be the derivable clause for the Operator A.

(7) $Tr(\overline{A\alpha}\ L)$ iff $A\ Tr(\overline{\alpha}\ L)$.

What (7) says is that $\overline{A\alpha}$ is true in L iff $\overline{\alpha}$ is always true in L. But this can only be the case if language L is omnitemporal. So in order to overcome these problems taken together it must be assumed that language L exists always and that its sentences always have the meaning they now have. (This will have the consequence that 'B. Russell will be born' had the meaning it now has before Russell was conceived - see 6.6 on this and Buridan (ibid p. 161)). In the circumstances where L is a representation of tensed fragments of natural language this assumption is, to say the least, rather unfortunate.

Surely, though, the only committment that is wanted here is that there is now a language L whose sentences have some meaning now. Suppose, then, '$Tr(\overline{\alpha}\ L)$' is r ad as (8)

(8) $\overline{\alpha}$ is true with the meaning it now has in L.

Then '$P\ Tr(\overline{\alpha}\ L)$' is to be read as (9)

(9) P($\overline{\alpha}$ is true with the meaning it now has in L).

By invoking the two dimensionality (or tense logical rigidity of 'now') in ML (see last Section) (9) may be read as (9').

(9') $\overline{\alpha}$ was true with the meaning it now has in L.

Dealing with the joint problem in this way (see 6.6 for a more interesting proposal) requires there to be only the assumption that sentences of a language L have some meaning now (and whatever meaning they do have is what one wants to find out when Davidsonian semantics go empirical). A similar solution which makes use of Lewis's primary sense of 'actually' (see last Section) can be given for similar problems in modal homophonic semantics.

The simplest way of giving homophonic semantics is to assume that the axioms are always true. One way of doing this is to invoke the A operator in the metalanguage and assume that all axioms are of the form A(...). (This avoids any worries about what may be substituted within tense operators in the metalanguage). But this way of giving homophonic semantics involves a certain loss of generality. As noted in the quantificational case in Section 6.1 it requires the validity of the tensed Barcan formula and more to the point here it requires the validity of (10)

(10) $A\alpha \rightarrow AA\alpha$

in the metalanguage in order to prove T- theorems which involve iterations of tense operators. But (10) is not a K_t theorem since it is derived from some of the linearity. axioms (namely $G\alpha \rightarrow GG\alpha$ and its mirror image). Yet all that is required is that no matter what the structure of time is, at all times a sentence is true iff its truth conditions hold given the meaning it now has. So instead of assuming the tense necessitations of all the axioms it is simpler to assume that if α is a theorem of the truth theory for L now then it is always a theorem of the truth no matter what the structure of time is. And this may be captured by the rule $\vdash_{TL} \alpha \therefore \vdash_{TL} G\alpha$ and its mirror image where '$\vdash_{TL} \alpha$' says that 'α' is a theorem of the truth theory for L no matter what the structure of time is. The homophonic theory for L_p is straightforward.

TL1) $Tr(\overline{q_i}\ L)$ iff q_i

TL2) $Tr(\overline{\sim\alpha}\ L)$ iff $\sim Tr(\overline{\alpha}\ L)$

TL3) $Tr(\overline{\alpha\wedge\beta}\ L)$ iff $Tr(\overline{\alpha}\ L) \wedge Tr(\overline{\beta}\ L)$.

TL4) $Tr(\overline{G\alpha}\ L)$ iff $G\ Tr(\overline{\alpha}\ L)$

TL5) $Tr(\overline{H\alpha}\ L)$ iff $H\ Tr(\overline{\alpha}\ L)$

TL6) $G(\alpha\rightarrow\beta) \rightarrow (F\alpha\rightarrow F\beta)$

TL7) $H(\alpha\rightarrow\beta) \rightarrow (P\alpha\rightarrow P\beta)$

Rule TLG if $\vdash_{TL} \phi$ then $\vdash_{TL} G\ \phi$

Rule TLH if $\vdash_{TL} \phi$ then $\vdash_{TL} H\ \phi$

TL1) is an axiom schema where it is assumed that the set q_i is finite. Rules TLG and TLH license substitution within tense operators in the metalanguage. It is not clear that a solution similar to the introduction of the rules TLG and TLH may be given for similar problems in homophonic modal semantics. One reason for this is that although 'always' is invariant in meaning this is not so for 'Necessarily'. For example, if a metaphysical notion of necessity occurs in the object language it isn't clear that the metalanguage 'necessarily' which obeys the rule if $\vdash_{TL} \alpha$ then $\vdash_{TL} A\alpha$ can be said to be a metaphysical notion. In order to give a semantics for L_p^+ it is assumed that $\overline{t_n}$ is the metalanguage canonical description of the object language 't_n' and 't_n' is its translation in the metalanguage. A finite set of dates only is assumed. The substitutional quantifier (a)(-a-) is introduced into the metalanguage together with variables whose substitution set is the set 't_i' of dates. Then homophonic semantics for L_p^+ are TL1) - 7),TLG,TLH together with the following

TL8) $A\alpha \rightarrow (a)(At\ a\ \alpha)$

TL9) $(a)(At\ a\ \alpha \rightarrow At\ t_n\ \alpha)$ for any t_n ε substitution set.

TL10) $Tr(\overline{At\ t_n\alpha}\ L)$ iff $At\ t_n\ Tr(\overline{\alpha}\ L)$.

TL11) $At\ t_n(\alpha \wedge \beta)$ iff $At\ t_n\ \alpha \wedge At\ t_n\ \beta$.

TL12) $\sim At\ t_n\alpha$ iff $At\ t_n \sim \alpha$.

We now prove (11)

(11) $Tr(\overline{G\ At\ t_n\ F_{q_n}}\ L)$ iff $G\ At\ t_n\ F_{q_n}$.

i) $Tr(\overline{q}_n\ L)$ iff q_n — TL1

ii) $G(Tr(\overline{q}_n\ L)$ iff $q_n)$ — TLG

iii) $F\ Tr(\overline{q}_n\ L)$ iff F_{q_n} — TL6)

iv) $Tr(\overline{Fq}_n\ L)$ iff Fq_n — Derived from TL4) and TL2)

v) $A(Tr(\overline{Fq}\ \ L)$ iff $Fq_n)$ — From TLG and TLH.

vi) $(a)At\ a\ (Tr(\overline{Fq}_n\ L)$ iff $F_{qn})$ — TL8)

vii) $At\ t_n(Tr(\overline{Fq}_n\ L)$ iff $Fq_n)$ provided $t_n\varepsilon$ sub set — TL9)

viii) $At\ t_n\ Tr(\overline{Fq}_n\ L)$ iff $At\ t_n\ Fq_n$ — From TL11) and TL12)

ix) $Tr(\overline{At\ t_n\ Fq_n}\ L)$ iff $At\ t_n\ Fq_n$ — TL10)

x) $G(Tr(\overline{At\ t_n\ Fq_n}\ L)$ iff $At\ t_n\ Fq_n)$ — TLG)

xi) $GTr(\overline{At\ t_n\ Fq_n}\ L)$ iff $G\ At\ t_n\ Fqn$ — From TL6)

xii) $Tr(\overline{G\ At\ t_n\ Fq_n}\ L)$ iff $G\ At\ t_n\ Fq_n$ — TL4)

4.4 A Problem for the L^D Analysis?

According to the L^D analysis a use at t_n of the sentence 'John runs' is represented by the L^D sentence 'At t_n John [runs]'. Now it may be thought that there is something amiss here. For after all a person may assent to a use of 'John runs' at t_n without assenting to the sentence 'At t_n John [runs]' simply because he or she does not know what the presen time is in calendar or clock terms. This problem appears at its most unfortunate in the following situation. Someone asks what the time is and is told at t_n that it is t_m. On the L^D analysis this utterance is represented by an L^D sentence of the form 'At t_n, it (the time) [is] t_m' (which is equivalent to 'At t_n, t_n [is] t_m'). But a use of 'It is t_m' at t_n can inform (or misinform) in a way in which 'At t_n it is t_m' can not.

It is this problem which we take to be the acceptable core - but only the core - of an argument by Gale in Chapter IV of his work (1968). He claims that there is a logical deficiency in what seemsto be the L^D analysis, a deficiency which would show that analysis to be in error. If Gale were correct then his argument would have an important bearing on the distinction made in 3.1 between a modest and a full blooded explanation of the naturalness of truth relativization to time.

Gale considers analyses of tensed 'sentences' (which he calls A-sentences after McTaggart's A-series) into 'sentences' which only express permanent relations between events (called B-sentences after McTaggart's B-series). Now, he objects to the token-reflexive analysis which analyses 'S is now ϕ'

as 'S's being ϕ is simultaneous with θ' where 'θ' is a description of the token-event, the use of 'S is now ϕ' (1968 p. 54). Furthermore, he believes his objection to hold for what appears to be the L^D analysis.

> "The same considerations hold for the B-Theory's attempt to analyse an A-statement into a B-statement through the tenseless ascription of dates e.g., 'S is ϕ at t_7'."
> (ibid p. 55).

His objection to these analyses is that they are not logically equivalent. For he says

> "The B-statements in the analysans of these... analyses do not entail the A-statement in the analysandum: that S's being ϕ is simultaneous with theta... does not entail that S in now ϕ."
> (ibid p.55).

And in the case of the analysis involving dates

> "The latter statement [i.e. that expressed by 'S is ϕ at t_7'] does not entail that S's being ϕ at t_7 is now present."
> (ibid).

Consequently, he holds that there is a logical deficiency here.

In this brief encounter with Gale's argument a move was made from the idea of mapping sentences into sentences to entailments between statements (where the latter terminology is Gale's own). But the problem is that Gale's specification of what an A-statement is leaves open the possibility that he means an L^D or an L_p (or perhaps even some other type of) statement and this is disastrous for his argument. He writes

> "Any statement which is not necessarily true (false) is an A-statement if, and only if, it is made through the use of a sentence for which it is possible that it is now used to make a true (false) statement and some past or future use of it makes a false (true) statement, even if both statements refer to the same things and the same places."
> (ibid p. 49).

Gale does not, however, give identity conditions for an A-statement other than that one is made by a use of a (non-necessary) sentence which can vary in truth value over time. But without these conditions and, in particular, without answering the question whether or not time of use is essential to the identity of an A-statement it is impossible to know what is meant by entailment between an A and a B statement. Clearly the (type) sentence 'S is ϕ at t_7' neither materially implies nor entails the (type) sentence 'S is now ϕ'; that is, in terms of L_p^+ (1) does not hold

(1) At t_7 α iff α.

So presumably, Gale does not mean by 'A-statement' what we mean by 'Aristotelian statement' (see 3.1) otherwise Gale's argument amounts to the trivial thesis that (1) is not valid which is no way an argument against the L^D theorist since his account of a statement is based upon uses of sentences. On the other hand, if we understand 'A-statement' in the Frege-Russell sense then it is a use of a tensed sentence on a time occasion which expresses an A-statement. That is, it is not the sentence 'S is now ϕ' which expresses an A-statement. That is, it is not the sentence 'S is now ϕ' which expresses a statement but instead a use of that sentence on some occasion. Consequently, treating a statement as an ordered pair then it is difficult to see what 'logical' deficiency there is in (2)

(2) $\langle$S is now ϕ $t_7\rangle$ iff $\langle$At t_7 A [is] ϕ $\rangle$

Moreover consider the following re-construal of Gale's account of an A-statement.

> "Any statement which is not necessarily true (false) is an X-statement/Y-statement if, and only if, it is made through the use of a sentence for which it is possible that it is used by me/here to make a true (false) statement and some

other use of it by someone else/elsewhere makes a false (true) statement even if both statements refer to the same places and the same times/the same times and the same things."

Then, whatever Gale's argument is it could be used to show that the X-statement expressed by 'I am ill' by utterer p or the Y-statement 'The well is here' uttered at place n are not entailed by the statements expressed by 'p is ill' or 'The well is at place n' respectively. And this is unfortunate because Gale uses his argument to show that the A-series is not reducible to the B-series account of time - an issue which appears to depend upon showing that temporal expressions are importantly different from spatial expressions, (and other indexical expressions).

Our account of what we take to be the acceptable core of Gale's argument at the start of this Section is merely an instance of a more general problem which also occurs in the case of other indexicals. This is the problem that although the same statement may be identified from uses of different sentences these sentences inuse may differ in potential information content. Since there is nothing special about this problem for tense analyses and also because we believe it only becomes a problem in intentional contexts we shall not consider it further in this essay.

4.5 Rigidity, Now and Actualality.

In 3.1 a distinction was made between a modest and full-blooded explanation of the naturalness of truth relativization to time. The modest explanation given there was at the grammatical leve whereas a full-blooded explanation, if there is one, is to be based upon semantic considerations. In this Section an attempt is made to show that there is some point to the belief that there is a full-blooded explanation to be had, an attempt which is based upon an argument by Romney (1978).

An argument in favour of the view that full blooded explanation is not forthcoming can be built around Peacocke's cla: (1975) that 'now' like 'I' and 'here' etc., are rigid relative to an occasion of utterance. In 4.2 it was argued that 'now' when treated as a date index is tense logically rigid relative to time of use and this was taken to be the indexical analogue of the two dimensionality of 'now' when treated as a sentence operator. Clearly, both 'I' and 'here' are also tense rigid. Peacocke's claim, however, is much more general since he holds that indexicals are rigid relative to an occasion of utterance with respect to any context including intentional contexts.

In order to represent this rigidity claim we make use of those ordered pairs (see 3.7) which represent a use of a sentence on some occasion. Where O is any operator (or iteration of operators) a is some person, p some place and t_n some time then the rigidity claims are representable by (1) to (3) for 'I', 'here' and 'now' respectively.

(1) $\langle(\exists x)(x=I \wedge O(-x-))\ a\rangle$ iff $\langle O(-I-)\ a\rangle$

(2) $\langle(\exists x)(x=here \wedge O(-x-))\ p\rangle$ iff $\langle O(-here-)\ p\rangle$

(3) $\langle(\exists x)(x=now \wedge O(-x-))\ t_n\rangle$ iff $\langle O(-now-)\ t_n\rangle$

(2) is to be read as follows; a (potential) use of '$(\exists x)(x=here \wedge O(-x-))$' at place p is true iff a (potential) use of '$O(-here-)$' at p is also true. (The 'iff' may be strengthened to '... is the same statement as ...' if so desired). As an example of his rigidity thesis Peacocke claims that (4) and (5) have the same truth conditions (1975 p. 119)

(4) It is possible that : it rained yesterday

(5) Yesterday is a day such that: it is possible that it rained on that day.

If Peacocke's thesis is correct then treating tense expressions semantically on a par with indexicals is to a large extent vindicated. And, this treatment may be backed up by a general philosophical argument which is summarized by Romney.

> "It is because we have a point of view on Space that we can distinguish this place, 'here', from others; we speak of places, and the place (perhaps very roughly) indicated as 'here' is the place where we are speaking. Similarly, it is suggested, we have a point of view on Time, we speak or think at times and tense modifications and temporal adverbs like 'now' and 'long ago' indicate temporal locations in relation to the time of speaking." (1978 p. 237).

This general argument is implicit or explicit in a large number of twentieth century philosophical writings. For surely it is on this basis that Quine says

> "Our ordinary language shows a tiresome bias in the treatment of time. Relations of date are exalted grammatically as relations of position, weight and colour are not. The bias is of itself an inelegance or breach of theoretical simplicity. However, the form that it takes - that of requiring that every verb form show a tense - is peculiarly productive of needless complications since it demands lip service to time even when time is farthest from our thoughts. Hence in fashioning canonical notations it is useful to drop tense distinctions." (1960 p. 170).

This view about time then has consequences concerning the importance of tense within logic. One example already noted in 2.5 is the Kneales' attitute towards the Medieval notion of appellation which they find 'curious'. Another example is Williams (1967) who claims that (6)

(6) A sea fight not present in time nevertheless exists

is no more contradictory than (7)

(7) A sea fight not present in space nevertheless exists.

This appears not to be the case because there happens to be a temporal reference (namely tensing) built into verbs rather than spatial reference (1967 p. 101). Consequently, none of these theorists would offer much weight behind the modest explanation of truth relativization to time.

One particular problem here is that to strike a blow at this general account of 'now' is to strike a blow at the standard interpretation of special relativity. For one consequence of special relativity appear to be that now is relative to a point of view because temporal distances between happenings are not invariant under Lorentz transformations. When the interval given by δs^2, which is invariant (and equal to $\delta x^2 + \delta y^2 + \delta z^2 - c^2\delta t^2$) between two events is space-like (greater than zero), so that there can not be a direct causal connection between the events then there are disagreements between different observers as to whether one of the events occurred before, after or at the same time as the other event. Now, the natural interpretation of this result is not just that now is relative to a point of view which could have been argued for given classical assumptions but also the much stronger point that there is no unique course of cosmological events. Instead there are only courses of events relative to temporal points of view.

Despite this, however, an argument by Romney seems to create a difficulty for the belief that the meaning of 'now' is to be given in terms of (potential) time of use. In the version we put forward of Romney's argument we attempt to show that there are grounds for claiming that 'now' unlike 'here' is not rigid relative to an occasion of use. That is, although (2) is true (3) is false. For it may be maintained that an instance of the right hand of (3), namely (8) is true without the left hand side of this instance, namely (9) also being true

(8) $\langle$I wish that now were t_m where $t_m \neq t_n$ $t_n\rangle$

(9) $\langle(\exists x)(x = \text{now} \wedge$ I wish that x were t_m where $t_m \neq t_n)$ $t_n\rangle$.

Romney points out that there is a sense in which 'I wish that it were next month' can be true without there being an analogous sense of 'I wish that Jerusalem were here instead of York' which is also true unless absolute places are assumed. For one may understand 'I wish that it were next month'

> "... as a wish that present events, including my present wish were past instead of present and later events were present instead where this would involve no difference in the temporal order of events." (p. 24 1).

But there is not a sense in which 'I wish that Jerusalem were here instead of York' can be true without there being also a difference in spatial order of things. She writes

> "What does not seem to make sense is the wish just that another place were here, instead of the place which is in fact here where this would not involve any difference in the spatial order of things (and therefore in the relation of people who distinguish them)." (p. 141-2).

Consequently, unless this wish about next month is illusory then (8) can be true without it also being the case that (9) is true and also without there being an analogous instance for 'here'.

And the reason why 'now' isn't rigid here whereas 'here' is is based upon the point that there are no select places whereas there are select times namely the present which is to be understood in terms of what is actual. A place is distinguishable by someone as here because that person is there whereas a time is distinguishable as the present because that is the actual time.

> "What is the case at present is just what is actually the case: this distinguishes it from what only was (is no longer) or will be (is not yet) the case. No parallel account could be given of the distinction between what is here and elsewhere."
> (p. 240).

Romney's argument then when understood in the way we have taken it namely as an argument which shows that 'now' is not indexical can then be utilized to provide a full-blooded explanation of the naturalness of truth relativization. But this argument requires there to be a notion of actuality which goes beyond 'relative to this temporal perspective'. And this goes against that conclusion based upon special relativity that there is no unique course of events. As a final word on this is Prior's reply here to this difficulty

> "What the relativistic physicists cannot calculate from how the course of events appears from certain points of view is how, in all its details, the course of events actually is. It is not clear to me that there is anything surprising or unacceptable in this conclusion or that we should be driven by it to renounce the use of forms like 'It appears from such-and-such a point of view that p' which assumes that there is also a plain p which is or is not the case."
> (1968 p. 133).

PART THREE

TENSE EXISTENCE AND FIRST - ORDER LOGIC

CHAPTER FIVE; FIRST ORDER L^D THEORIES.

5.1 Quine's Thesis.

In this final part of the essay we examine tensed first order theories. In this chapter our concern is with L^D theories and in the next chapter L_p first order theories. Central to our discussion in these chapters are issues about tensed existence; issues which connect up with the earlier discussion of the Ancients and Medievals. So the final chapter is a more general discussion about existence and in particular, about tensed existence.

Aristotle, as noted in 1.2, undertook the study of the syllogism as a stage on the way to the study of scientific methodology. Because of this tense considerations do not loom very large in his formal writings and when they do appear they occur as a hindrance to valid inference. This view was and still is a common view. For instance, Bosanquet writing at the end of the last century claims that when concerned with the operation of formal logic it is important to

> "... get rid of tenses which do not belong to scientific judgement and are very troublesome in formal inference."
> (1895 p. 98).

Like Aristotle, Quine influenced by science and mathematics, sees his role as a logician being to provide a canonical notation adequate for just these disciplines. Moreover, he also sees this in terms of the philosophical quest for what is ultimately 'real', a quest which shows that his prejudices lie almost entirely with physics. He writes

> "The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate

> categories, a limning of the most general traits of reality.
> (1960 p. 161).

This canonical notation which has little of the expressive power of ordinary language turns out to be suitable for capturing a large variety of inferences. He says

> "When our objective is an austere canonical form for the system of the world we are not to rest with the renunciation of propositional altitudes and the subjective conditional: we must renounce also the indicator words and other sources of truth value fluctuation."
> (1960 p. 228).

Quine intends that his canonical scheme represent the four dimensional framework. Now it may be thought that this is a consequence of his predilections for physics or, more to the point, for the Minkowski interpretation of special relativity. Geach, for instance, questions this aspect of Quine's thought when he writes

> "nor ought any logician to try to accommodate his doctrines to demands made in the name of contemporary physics."
> (1972 p. 304).

However, the impetus for Quine's adherence to the four dimensional framework is not just based upon scientific grounds but also and more importantly, for our purposes, upon logical grounds. For central to Quine's canonical notation is standard first order logic which first arose out of representation of mathematical inference, (unlike Medieval logic which grappled with inference in a more general way). Now, it is the assumption that standard first order logic is not only applicable to mathematics but also to empirical matters which is the important logical factor in Quine's adoption of the four dimensional view. He writes

> "The 4-dimensional view of space-time is part and parcel of the use of modern formal logic and in particular the use

of quantification theory in application to temporal matters."
(1953 p. 442).

That is, he claims that variables are open only for four-dimensional objects when first order theory is used for representing temporal realities. It is this thesis - which to our knowledge Quine no where argues for in detail - which is the central concern of this chapter.

Because this thesis was put forward before the introduction of tense logics it is best assessed within the context of the L^D analysis. In the circumstances it is a surprising claim since Quine is denying the intuitively most plausible way of understanding the structure of L^D sentences. For the most natural parsing of the L^D sentence 'At t_n Socrates [sits]' is as 'Socrates [sits]-at t_n' where '...[sits]-at...' is a two place predicate and where 'Socrates' stands for the continuant Socrates. This analysis, the L^D_A analysis (see 5.3), is here, then, rejected by Quine.

5.2 From Tenseless Truth to Tenseless Existence?

Although Quine's thesis is best taken within the context of L^D analyses he does, it seems,intend his thesis to be wider ranging. As we shall note in 5.5,central to his framework is that quantification be tenseless in all contexts. And, in connection with this he claims

> "It would be hard to exaggerate the importance of recognising the tenselessness of quantification over temporal entities. I see no reason to expect a coherent application of quantification theory to temporal matters on any other basis."
> (1953 p. 442-3).

This claim is disputed in the next chapter, the central chapter of the essay where tensed quantification theories employing tensed quantifiers are looked at.

A motivation behind preserving tenseless quantifiers when they are used in connection with the analysis of tensed discourse rather than mathematical discourse is to preserve a single notion of existence, namely tenseless existence. But is a tenseless reading of the quantifiers forced upon one as a consequence of the L^D analysis? For once it is stipulated that statements are true or false simpliciter (or in Frege's words 'timelessly' true or false - see 3.8) then constraints are placed upon accounts of their structure and their semantics. For instance, there appears to be a straightforward connection between tenseless truth and tenseless satisfaction which is noted below. But is there also such a connection between tenseless truth and tenseless existence?

The move from tenseless truth to tenseless satisfaction is somewhat straightforward. A sentence, say '7 [is] prime' belonging to some language L is true iff the predicate, in this case '...[is] prime' is true of the logical subjects. Here it is clear that the semantic relation '... is true of ...' is to be taken as tenseless not on the grounds that 7 is 'timeless' but simply because of its connection with the tenseless truth predicate. And, it is this semantic relation which is the foundation stone for '... [satisfies]...'.

Any presumed connection between tenseless truth and tenseless existence is mediated by consideration of the 'existential' quantifier '(∃x)(-x-)'. Russell's own introduction of the quantifier in his classic work of 1905 connects it with the 'true of' relation. He there writes that 'C (Something)' means 'C(x)' is true for at least one value of the variable. And in 'Principia' he and Whitehead stay with this account.

> "In addition to the proposition that a function ϕx is 'always true' (i.e. $(x)\phi x$) we need also the proposition that ϕx is 'sometimes true' i.e. true for at least one value of x. This we denote by '$(\exists x)\phi x$'."
>
> (1910 Introduction p. xxi).

Effectively, we have here that '$(\exists x)\phi x$' is true iff something satisfies 'ϕx'. However, the authors equate this with 'There exists an x which satisfies 'ϕx'.

> "The symbol '$(x)\phi x$' may be read 'ϕx always' or 'ϕx is always true' or 'ϕx is true for all possible values of x'. The symbol '$(\exists x)\phi x$' may be read 'there exists an x for which ϕx is true' or 'there exists an x satisfying $\phi \hat{x}$' and this conforms to the natural form of the expression of thought."
>
> (1910 p. 15).

The purported connection between tenseless truth and tenseless existence depends upon the Russellian understanding of the 'some' quantifier to express existence. If this reading is not accepted (as for instance when read substitutionally) then there is not this foothold for the thesis. However, putting this to one side it can still be argued that from the assumption of tenseless truth there is nothing in the brief sketch of Russell's introduction of the quantifier which forces one to treat 'exists' as tenseless unless extra assumptions are added.

Certainly, under the L^D analysis if $(\exists x)\phi x$ is a sentence then it can not be read as 'There (now) exists an x such that it ϕs'. But this does not mean that it 'must' be read tenselessly in all its uses. For instance, one line of thought which is found in Russell is that what distinguishes apples, say, from numbers is that the former unlike the latter are time bound and that this distinction is fundamental. (see Russell 1912 p. 57). Now one way this distinction can be represented is by introducing two sorts of existential quantifier, a tenseless one 'There [exists] an x such that it...' and a detensed one 'There $[\text{exists}]_d$ an x such that it...' (see 3.2 for the distinction between tenseless and detensed).

On what grounds could this duality of quantifiers be rejected? It might be contended that the quantifiers ought to be unrestricted and thus range only over a single set of objects on grounds of simplicity. Two quantifiers here demand two types of variables. (See 6.1 for a brief account of two sortal satisfaction). But this contention needs to be backed up either with an argument that despite appearances using only a single tenseless existential quantifier one can still bring out the Russellian distinction (see next Section) or alternatively it may be backed up by an argument which rejects that there is a fundamental distinction here.

A simpler argument to show that there isn't a connection between tenseless truth and tenseless existence unless extra assumptions are added depends upon treating a Frege - Russell statement as an ordered pair (1) (see the discussion 3.7).

(1) <Sentence ε L_p time of use>

Satisfaction is tenseless under this proposal when treated in terms of possible uses of a predicate at a time. Now, in the next chapter (see 6.3ff), under the L_p analysis, consideration is given to tensed quantification theory employing tensed quantifiers. Consequently although (2)

(2) $\langle (\exists x)\phi x \quad t_n \rangle$

is tenselessly true or false the quantifier may here be read to be tensed.

5.3 The L^D_A Analysis.

Atomic sentences of L^D like 'At t_n Socrates [walks]' were treated as predicates with an argument place for time and written as 'ψt_n' (see 3.7). ψ is a predicate of the form At... $\phi^n(a_1..a_n)$ where $\phi^n(a_1...a_n)$ may be seen as some sentence of a tenseless language L. Given this, the natural analysis of ψt_n is ϕ^{n+1} $(t_n a_1 ... a_n)$ where the t_n occupies an argument place in the same way that a_1 ... a_n do. This analysis which appears to be the natural interpretation of Russell's analysis of tense (see 3.1) has the consequence that the statement expressed by an English tensed sentence is to be represented as having one more argument place than surface structure reveals. For instance, in 'Socrates walks' 'walks' appears to be a monadic predicate. However, in this analysis corresponding to this predicate is '...[walks]-at...'.

This analysis of tensed discourse involves what is probably the least amount of 'theoretical effort' in that it is assumed that the resources of standard quantification theory (with the addition of non-logical axioms for the later than relation '>') is adequate for representing uses of tensed sentences. We call this analysis the L^D_A analysis and the language L^D_A is then standard first order logic.

One important feature of L^D_A is that a tenseless language L can be taken as a proper subset of it. Consequently L^D_A can both represent tenseless sentences like those of arithmetic as well as uses of tensed sentences. This feature is connected with the thesis that in some sense, logic is universal (see 3.8 for instance). Given this then the natural interpretation of the single existential quantifier of L_A^D is the tenseless reading. In the last Section, however, a certain worry was expressed. Apples, it was said, do not [exist] but $[\text{exist}]_d$ unlike numbers. So the worry is that the tenseless quantifier does not connect up with existence in the way required in the case of time dependent individuals.

However, the L_A^D theorist may claim that this worry is averted here. The existential quantifier is existential because by asserting '$(\exists x)\phi x$' one is asserting that the predicate 'ϕ' [has] instances. But, it is notthe predicate '... [is] an apple' which [has] instances; instead, it is, for instance, the predicate '...[is]-an-apple-at t_n'. Consequently, it may be claimed that the thought that there is a distinction to be made between apples and numbers is well taken under this analysis. For they are distinguishable in terms of the non-logical predicates which [are] true of them. Only predicates with an argument place for a time can be true of time bound objects whereas in the case of 'abstract' objects it is predicates which have no place for time variables. Thus the general statements (1) and (2) are representable by (1') and (2') respectively.

(1) There [are] prime numbers

(2) There $[\text{are}]_d$ people

(1') $(\exists x)$(x [is] a prime number)

(2') $(\exists x)(\exists t)$(x [is] a person-at t)

If Quine's thesis is correct (see 5.1) then this analysis is not a going concern. Let us then look at certain difficulties with it concerning existence.

5.4 Predicates and Tense.

The existential quantifier is a second level predicate and it is this which is taken to underly the claim that 'exists' is second-order. (See 7.3 for a criticism of this view). This prompts the following general question; although the L_A^D analysis is only described in terms of a first order theory how should second order predicates in the context of tensed sentences be considered in the spirit of this analysis?

The 'eternalized' sentences (1) and (2)

(1) At t_n diamonds [are] rare

(2) At t_n pigeons [are] numerous

are second order since they contain the predicates '...is rare' and '...is numerous' respectively. Now, given the L_A^D analysis one may claim that the time determination belongs either to the first order or to the second order or to both predicates. Does it matter which?

Suppose we allow it to belong to the second order predicate (1) is then taken as (1')

(1') Diamonds [are] rare-at t_n.

(This will mean that '...[is]...' takes first and zero order arguments). Taking (1) in the style of (1') has the consequence that first order predicates are tenseless. But this will then mean that there are two different accounts of first order predicates which can be true of time bound objects. For under the L_A^D analysis (3) becomes (3') while under the present proposal (4) becomes (4').

(3) At t_n Socrates [is] a philosopher.

(4) At t_n Philosophers [are] rare.

(3') Socrates [is]-a-philosopher-at t_n

(4') Philosophers [are]-rare-at t_n.

And in (3') unlike (4') the first order predicate carries a time argument. This unsatisfactory situation of requiring two accounts of a first order predicate is repeatable at every order if it is the higher order predicate which carries the time determination. This view also results in a certain difficulty of how to construe the statement made by a use of (5).

(5) Socrates was a philosopher at a time when they were rare.

Not only this but also there is a certain loss of generality in analysing (1) as (1') while treating (6) as (6').

(6) At t_n diamonds [exist].

(6') Diamonds-at t_n [exist].

This stimulates the question what is so special about 'exists', if understood to be second order, which prevents it from being treated here like other second order predicates'

The view that all second order predicates are tenseless therefore dispels the oddity of claiming that 'exists' is. Moreover, this view means that only a single account of first order predicates true of time bound objects has to be given. Consequently, (1) is taken as (1")

(1") Diamonds-at t_n [are] rare.

But this prompts the question of what is so special about first order predicates that only they take time determination: Anyway, this view is objectionable if an account is to be given of tensed sentences which involve a second order predicate falling within a third order predicate. Besides this there is also the difficulty of dealing with sentences involving more than one time determination as in (7).

(7) Men who fought in World War I are now rare.

What this suggests is that a predicate of any order dealing with temporal realities is open for tensing. But, it is this which is denied by the L_A^D theorist who claims that there is one second order predicate, namely 'exists' used in connection with temporal matters but which is not open for tensing. This does not just result in mere eccentricity on the part of the L_A^D theorist for it has consequences and, in particular, it results in there being general difficulties about representation of existence claims.

In the last Section it was noted that the L_A^D theorist can distinguish between certain claims about the existence of time bound and abstract objects through the difference of the predicates which fall under the second order 'exists'. In the case of time bound objects they satisfy predicates which involve an argument place for a time. Hence, a use of 'There were dinosaurs' at t_n is represented by (8)

(8) $(\exists x)(\exists t)(t_n > t \wedge x$ [is]a-dinosaur-at $t)$

Tensed existence, then, is here being treated as parastic upon the tensing of the predicates that are within the scope of the quantifier. For what (8) really states is a use of (8') at t_n

(8') There [is] something which was a dinosaur.

But this parasitic account of tensed general existence comes into difficulty in certain cases.

The first problem occurs if the predicate which lies within the scope of the quantifier is ampliating (see 1.5 and 2.3), for it may be argued that using the parasitic account yields incorrect representations. Take, for instance, a use of

(9) at t_n and its representation (9')

(9) There is (now) something which is famous.

(9') $(\exists x)$(x[is]famous-at t_n)

But, arguably unlike (9), (9') can be true if nothing existent at t_n is then famous. This is because the extension of '...[is]-famous-at t_n' need not include existents at t_n.

This problem is generalized when contexts involving negation are considered. For instance (10)

(10) There are things which are not living

or more colloqually 'Not everything is alive' as used at t_n is misrepresented by (10')

(10') $(\exists x)\sim$(x [is]-alive-at t_n)

And this is because (10') can be true if everything is living at t_n although some things which lived, like Socrates, are no longer alive at t_n.

The third type of problem occurs in cases where the 'there is' tensing does not coincide with the tensing of the predicates that follow it. An example is that of (11) and its contrast with (12) as used at t_n

(11) There is something which will fly to Pluto

(12) There will be something which flies to Pluto.

The difference here appears to be an instance of the divided|compounded distinction discussed in Sections 2.3 and 2.4. But this difference as it stands can not be accounted for in terms of the parasitic tensing account unless an extra predicate is added to (11), a predicate which agrees in tense with the quantifier expression.

These problems of representation bring out that the parasitic tensing view is not only inadequate as it stands but also an unnatural account of the 'There is...' locution. More generally the arguments put forward in this Section support the view that like any other second order predicate which is used in connection with temporal matters 'There is...' , understood as expressing existence, is tensed in its own right.

5.5 L^D_A and Individual Existence.

Closely connected to difficulties for the L^D_A theorist of representing certain general existence claims are difficulties of representation of individual existence claims. The first point is that just as there may be doubts about whether or not 'People [exist]' as opposed to 'Numbers[exist]' is a sentence (see 5.2) then similarly there are doubts as to whether 'Socrates [exists]' as opposed to '7 [exists]' is a sentence. For in 3.2 it was questioned whether or not 'Plato [thinks]' is a sentence. (If no distinction is made between tenseless and detensed verbs then there is no problem here. For instance, Woods suggests that 'x exists (tenseless) = df (∃t)(x exists at t)' may be utilized for time bound individuals - 1976 p.251).

Secondly, and more to the point here is the question of how singular tensed attributions are to be represented, sentences like (1)

(1) Geach exists but Prior no longer does.

For it is not only general existence claims which are open for tensing but also individual existence claims. Now if the latter problem is solved then problems connected with general existence representation will also be solved under the L^D_A analysis. The converse of this can also be true as is now shown.

The arguments of the last Section may be taken as a defence of the view that the 'There is...' locution should be represented by a quantifier which makes allowance for the fact that it is open for tensing. So suppose we introduce '(Ex)(-x-)' which is to be read as the tensed 'There is (now) an x such that it...'. Suppose, furthermore, we identify an L^D sentence in terms of an ordered pair (see 3.7). Then a use of (2) at t_n

(2) There are people

may be represented by (2')

(2') <(Ex)(x-[is]-a-person-at t_n) t_n>

This use of the ordered pair notation utilizes the place for the time of use as the time at which the 'There is...' locution applies. Consequently, uses of (3) and (4) at t_n may be represented by (3') and (4') respectively.

(3) There is (now) something which will fly to Pluto.

(4) There will be something which flies to Pluto.

(3') <(Ex)(∃t)(t>t_n ∧ x [flies]-to-Pluto-at t) t_n>

(4') (∃t)<(t>t_n ∧ (Ex)(x[flies]-to-Pluto-at t)) t>

According to this L^D account because it introduces more than one place for a time the problematic general existence claims can be represented. Not only this, however, but also the individual existence problem is overcome.

In first order theory the standard way of representing the tenseless individual existence claim (5)

(5) 7 [exists]

is by (5')

(5') (∃x)(x=7)

which is to be understood as 'There [is] an x such that it [is] identical to 7', where both the quantifier expression and the identity predicate are tenseless. Thus, given the tensed quantifier a use of (1) at t_n is representable by (1').

(1') $<(Ex)(x = \text{Geach}) \wedge \sim (Ex)(x = \text{Prior}) \quad t_n>$

Although this is an L^D solution here the use of the ordered pair notation is essential to it. But the L^D_A analysis is intended to be given in terms of sentences which on all occasions of use express the same statement. And the way this was done in the case of tensed first order predicates was by introducing an extra place for time into those predicates. Consequently, it would be natural to do this for the second order tensed 'There is...' predicate. This may be done by introducing the expression '($\exists$x at t) (-x-)' where t is a free variable which may be bound from the outside. So (6)

(6) $(\exists t)(\exists x \text{ at } t)(-x-)$

is a representation of the detensed quantifier 'There [is]$_d$ an x such that it...' which is to be contrasted with the tenseless quantifier.

Consequently, this L^D_A solution to the problems of representation consists in a total rejection of the parasitic tensing view. For now uses of sentences (1) (2) (3) and (4) at t_n are represented by (1") (2") (3") and (4"), respectively.

(1") $(\exists x \text{ at } t_n)(x = \text{Geach}) \wedge \sim (\exists x \text{ at } t_n)(x = \text{Prior})$

(2") $(\exists x \text{ at } t_n)(x \text{ [is]-a-person-at } t_n)$

(3") $(\exists t)(\exists x \text{ at } t_n)(t>t_n \wedge x \text{ [flies]-to-Pluto-at } t)$

(4") $(\exists t)(t>t_n \wedge (\exists x \text{ at } t)(x \text{ [flies]-to-Pluto-at } t))$

But how does this L^D_A analysis stand in the light of Quine's Thesis (see 5.1)? The main point is that we no longer have standard quantification theory for representing temporal realities; instead, we have something more awkward. Moreover, Quine takes great exception to the belief that

'There is...' is ambiguous between temporal and non-temporal realities as instanced in 1960 p. 241-2. (This point may be connected with the belief that 'There is...' should be universally applicable, a logical constant - see 3.8).

The alternative solutions briefly mentioned here to the representation problems tackle the individual problem and then utilize that solution for solving the general problems. The first of these starts from the point that '7 [exists]' is represented in first order theory using the tenseless identity predicate. Now, if a tensed identity predicate were to be introduced then (1) can be represented. For instance, if '...=t_n=...' is introduced to mean '...[is] identical-at t_n to...' then a use of (1) at t_n is represented by (1''')

(1''') $(\exists x)(x =t_n= \text{Geach}) \wedge \sim (\exists x)(x =t_n= \text{Prior})$

where the tenseless quantifier is used. Given this, then, the general problems are solved. For instance uses of (3) and (4) at t_n are represented by (3''') and 4''').

(3''') $(\exists t)(\exists x)(t>t_n \wedge x=t_n=x \wedge x$ [flies]-to-Pluto-at $t)$

(4''') $(\exists t)(\exists x)(t>t_n \wedge x=t=x \wedge x$ [flies]-to-Pluto-at $t)$

One particular worry about this solution is voiced by Woods.

> "...it is doubtful how far such a two-term relation, variable over time deserves to be called identity, and in so far as it is not rightly so regarded its capacity for illuminating tensed assertions and denials of existence is limited."
> (1976 p. 251).

Finally, a solution is forthcoming if the L_A^D analysis is couched within a first order language containing a two place existence predicate for representing tensed existence. A use of (1) at t_n is then represented by (1'''') and uses of (3) and (4) by (3'''') and (4'''')

(1'''') $E!\ \text{Geach}\ t_n \wedge \sim E!\ \text{Prior}\ t_n$

(3'''') $(\exists t)(\exists x)(\ t>t_n \wedge x\ [\text{flies}]\text{-to-Pluto-at}\ t \wedge E!xt_n)$

(4'''') $(\exists t)(\exists x)(\ t>t_n \wedge E!xt \wedge x\ [\text{flies}]\text{-to-Pluto-at}\ t)$

Both these latter solutions, the introduction of tensed identity or an existence predicate are compatible with or instances of the parasitic tensing account. For under these analyses 'There will be an x s.t. it...' is to be treated as 'There [is] an x which will exist and such that it...'. And in the case of the latter of these two it must be maintained that there are at least two senses of 'exists', a second order sense given by the quantifier and a first-order sense given by the existence predicate, a view which is argued against in Chapter 7. Moreover, if a use of 'Socrates exists' at t_n is to be represented by 'E! Socrates t_n' then surely '7 [exists]' should be represented as 'E!7' which seems to imply that there are three accounts of 'exist' to be given.

5.6 The L_B^D Analysis.

Under the L_A^D analysis (1)

(1) At t_n Socrates [walks]

becomes (1')

(1') Socrates [walks]-at t_n

An alternative structural analysis here is (1")

(1") Socrates-at-t_n [walks]

where the logical subject is 'Socrates-at-t_n' which is to be understood as referring to a temporal slice of a four dimensional object which will be written here as 'Socrates$_4$-at-t_n'. This analysis referred to here as the L_B^D analysis features in Quine's writings. He writes

> "A drastic departure from English is required in the matter of tense. The view to adopt is the Minkowski one which sees time as a 4-th dimension on a par with 3-dimensions of space."
>
> (1952 p. 166).

This analysis of uses of tensed sentences is not just motivated by scientific principles according to Quine. For as noted in 5.1, he claims that the application of standard quantification theory to temporal matters depends upon assuming that four-dimensional objects are values of the variables. So the best way of looking at the matter

> "...is to recognise both in the four dimensional approach with its notable technical advantages and in quantification theory with its notable technical advantages two interrelated contributions to scientific method."
>
> (1953 p. 442).

The four dimensional view requires us to indulge in some intellectual gymnastics. Time is just another dimension in which bodies$_4$ are extended and it is this which here makes it space like. Photographs of a person taken at different times represent three-dimensional cross sections. Thus, the concepts of object and property are displaced by that of a process-thing; for now, my walking to the Rose and Crown last night is just a part of me_4.

Under this conception all predicates are tenseless. Some of these will only be true of the four-dimensional object as a whole like, perhaps, '...[is] a $person_4$'. Others may be true of an $object_4$ as a sum of its temporal parts or of the parts themselves, namely temporal slices like '...[is] heavy' (We may compare here '... is wide' said of a road). And certain predicates may just be true of temporal slices of objects; for instance, the predicate '...is old' in three dimensional talk becomes something like '...[is] a long way down it_4s world line'. For many predicates, especially intentional predicates (see Geach 1965), it is not clear that they can go over into a Quinean language. Now, as noted in 5.1 this does not bother Quine since his canonical scheme is inspired not only by a very limited part of our ordinary conceptual scheme but also by a very limited part of twentieth century science. And the sorts of predicates which are relevant here, particularly classificatory predicates (see 1.2) do go over into a four-dimensional language.

What then is the logical basis for Quine's Thesis? Consider the following three theses about canonical notations built upon first order theories.

(A) The existential quantifier must always be read tenselessly

(B) Thereis only one sense of 'exists', a tenseless sense represented by this quantifier.

(C) The canonical notation should be able to represent uses of tensed sentences involving tensed existence claims. (see 5.4 and 5.5).

Under the L^D_A analysis not all three of these theses can be maintained together as we saw in the last two Sections. (C) doesn't hold if (A) and (B) are held. And the strategies utilized to bring about (C) we have looked at amount either to a rejection of (A) (defended by Quine - see, for instance, 1952 p. 166) or a rejection of (B) (again defended by Quine - see, for instance, 1960 p. 131).

Despite the very limited aims of L^D_B representability the three theses (A) to (C) do hold for this analysis. Consider first individual objects$_4$. Socrates$_4$, a process-thing, is unchangeable having atemporal existence. Thus just as '7 [exists]' is acceptable then so is 'Socrates$_4$ [exists]'. The difference between 7 and Socrates$_4$ resides in the point that the latter unlike the former has spatio-temporal parts.

But what about representations of tensed existence claims; for instance, a representation of 'Socrates no longer exists' as used at t_n? According to the L^D_B theorist this claim must be understood relative to a particular dating system which encompasses a particular temporal point of view. For what he takes as real is just the four dimensional manifold. Relative to our dating system it is now true to say that Socrates no longer exists. This claim can be coped with on the L^D_B analysis by introducing the operator '...at...' which is the intersection of a world line with a date belonging to some dating system. Consequently, when t_n is the present time 'Socrates$_4$-at-t_n' is the present temporal slice of Socrates$_4$ which is the null element. Thus, introducing the null element Λ, a use of 'Socrates no longer exists' at t_n is representable by (2)

(2) $\text{Socrates}_4\text{-at-}t_n = \Lambda$.

Alternatively, we can introduce the predicate '...R...' for '...[is] a (proper) part of...' then instead of (2) we have (2')

(2') $\sim(\exists x)(x\text{-at-}t_n \ R \ \text{Socrates}_4)$

What we are doing is making use of set theory which as we shall note below may be used to overcome problems of higher-order predication.

General tensed existence claims can now be dealt with. A use of (3) at t_n

(3) 'There were dinosaurs'

is represented by (3')

(3') $(\exists t)(\exists x)(t<t_n \wedge x\text{-at-}t \neq \Lambda \wedge x \text{ [is] a dinosaur}_4)$

And uses of (4) and (5) at t_n

(4) There is something now which will fly.

(5) There will be something which flies.

becomes representable by (4') and (5') where the predicate [flies] unlike '...[is]a dinosaur_4' is taken to be true of temporal parts

(4') $(\exists t)(\exists x)(x\text{-at-}t_n \neq \Lambda \wedge t>t_n \wedge x\text{-at-}t \text{ [flies]})$

(5') $(\exists t)(\exists x)(t>t_n \wedge x\text{-at-}t \text{ [flies]})$

Because all predicates are tenseless on this approach then it looks to be problematic how a use of a sentence like (6) can be represented

(6) Poverty is less rare today than 10 years ago.

However, Quine lets the 'at...' operator be extended for sets

"We easily extend 'at' to classes. Where z is mankind, z at t may be explained as the class $\hat{y}\ (\exists x)(y=(x \text{ at } t)$ and

$x \varepsilon z$) of appropriate man stages."
(1960 p. 173).

So when (6) is used at t_n and where t_m is 10 years before and z is the class of poor people (6) is represented by (6') according to this proposal.

(6') $\hat{y}(\exists x)(y=(x\text{-at-}t_n) \wedge x \varepsilon z)$ is larger than $\hat{y}(\exists x)(y=(x\text{-at-}t_m) \wedge x \varepsilon z)$

But Quine's proposal here not only allows temporal slices to be values of a variable (rather than the complete 4-dimensional object) but also presupposes that sets can only be defined for non-ampliating predicates. For instance, if z is the set of famous people_4 then z at t would be, according to Quine (7)

(7) $\hat{y}(\exists x)(y=(x \text{ at } t)$ and $x \varepsilon$ the set of famous people_4)

but Socrates_4 may be a member of the set of famous people_4 at t_n without there being a temporal slice $\text{Socrates}_4\text{-at-}t_n$. The L_B^D analysis requires not only a large amount of unnatural paraphrase to fit fragments of our ordinary tensed discourse into the four-dimensional framework but also a certain amount of set theory. Moreover, in the next two chapters we shall argue that both (A) and (B) are to be rejected.

CHAPTER SIX FIRST ORDER Lp THEORIES

6.1 The Language Qp and the Systems QK.

This chapter is about first-order Lp systems. Because similar problems occur for quantified modal logic as they do for quantified tense logic more attention than in previous chapters is given to modal questions. First, the language Qp is introduced. In Section 3.6, it was assumed that an atomic Lp sentence is formed out of an n-place tenseless predicate together with n-occurrences of any m individual constants, $m \leq n$ and the morpheme 'PRES'. In the case of Qp it is taken that atomic predicates are present tensed. That is, the Lp atomic sentence 'PRES Socrates [sits]' is analysed as 'Socrates PRES-[sits]'. It is being assumed, then, that logically speaking tensed predication is fundamental to tensed languages however it may be grammatically, (see 1.3 and 3.2). Definition Qp1 introduces atomic Qp predicates.

Definition Qp1.

An n-place atomic predicate is given by the ordered pair <PRES n-place tenseless predicate>. All atomic predicates of Qp are assumed to be of this form, and the resulting set is Q_i^n.

Definition Qp2.

The set E_Q of expressions for Qp consists of the symbols (,), the set C_i^n of connectives, a denumerable set of symbols $v_0 v_1 \ldots$ called individual variables, a set a_i of individual constants, the set Q_i^n of atomic predicates and finally the quantifier $(\exists v_i)(-v_i-)$ for any v_i.

Formulas of Qp are now defined.

Definition Qp3.

Atomic Qp formulas are first defined

(i) if $Q_i^n \in Q_i^n$ and $u_1 \ldots u_n$ are each either a variable or a constant then $Q_i^n(u_1 \ldots u_n)$ is an atomic formula.

formulas are now defined

(ii) Any atomic formula is a formula.

(iii) If $C_j^n \in C_i^n$ and $\alpha_1 \ldots \alpha_n$ are formulas then $C_j^n(\alpha_1 \ldots \alpha_n)$ is a formula.

(iv) If α is a formula containing v_i free then so is $(\exists v_i)\alpha$

The connectives C_0^1 C_1^1 C_2^1 C_3^2 are $\sim$, G, H $\wedge$ respectively. And it is assumed that the standard rewrite rules apply to Qp formulas together with the standard definitions of closed and open formulas. The language $Q_p^=$ is Qpu {=} where '=' is intended to be identity. However '=' is an unusual Qp predicate in that it is tenseless, (see 5.5).

In this chapter two sorts of Lp system are considered. In one type, the quantifier $(\exists v)(-v-)$ is taken to be the standard tenseless (or, perhaps, the detensed) quantifier whereas in the other type it is understood to be tensed. On the latter reading '$(\exists v)(-v-)$' says 'There is (now) something s.t.it...'. Besides this divergence in quantificational tense systems there is still, of course, that divergence which is due to different assumptionsbeing made about time's structure. In this Section attention is given to the former more standard type of tense system involving the standard quantifiers.

The systems which result from the addition of standard quantification theory to K_t and K_L (see 3.5) are called

QK_t and QK_L respectively. QK_L, then, is quantified tense logic for linear dense time. The interesting theorems of these systems are those which involve the interconnection of tense operators with the quantifiers. In both systems the following six theorems together with their minor images are provable.

(1) $G(x)\alpha \rightarrow (x)G\alpha$

(2) $(\exists x)F\alpha \rightarrow F(\exists x)\alpha$

(3) $(\exists x)G\alpha \rightarrow G(\exists x)\alpha$

(4) $F(x)\alpha \rightarrow (x)F\alpha$

(5) $(x)G\alpha \rightarrow G(x)\alpha$

(6) $F(\exists x)\alpha \rightarrow (\exists x)F\alpha$

(2) is equivalent to (1), (3) to (4) and (5) to (6) in standard systems. The modal fragments of QK_t and QK_L are standard quantified B and S_5 respectively. Hence, corresponding to (1) to (6) (and their mirror images) are the modal formulas (1') to (6').

(1') $A(x)\alpha \rightarrow (x)A\alpha$

(2') $(\exists x)S\alpha \rightarrow S(\exists x)\alpha$

(3') $(\exists x)A\alpha \rightarrow A(\exists x)\alpha$

(4') $S(x)\alpha \rightarrow (x)S\alpha$

(5') $(x)A\alpha \rightarrow A(x)\alpha$

(6') $S(\exists x)\alpha \rightarrow (\exists x)S\alpha$

(5') and (6') are the Barcan formulas whereas (1') and (2') are their converses. Because of this (5) and (6) are here called the tensed versions of the Barcan formula. (4') and it s equivalent are called the 'Buridan formulas' after Prior's observation that Buridan objected to a modal reading of (4') (Prior 1967 p. 138).

Language Q_p^+ is Qp together with

(i) set of time variables $u_0 u_1 \ldots$

(ii) set t_i of individual time constants.

(iii) the At.... operator

(iv) the quantifier $(\exists u_i)(-u_i-)$, for any u_i, where u_i is a set of time variables.

Formation rules for Q_p^+ are given by Definition Q_p^+.

<u>Definition Q_p^+</u>.

(i) if α is a Q_p formula then it is a Q_p^+ formula.

(ii) if α is any Q_p formula and s_k is t_m or u_n for any m,n then At s_k α is a Q_p^+ formula.

(iii) if α, β are any Q_p^+ formulas then so are $\sim\alpha$, $\alpha \wedge \beta$, $H\alpha$, $G\beta$.

(iv) if α is a Q_p^+ formula of the form (-At u_i-) then so is $(\exists u_i)$(-At u_i-).

(v) if α is a Q_p^+ formula of the form $(-v_i-)$ then so is $(\exists v_i)(-v_i-)$.

System QK_L^+ is QK_L together with (7) to (12).

(7) $\sim$ At u_i α iff At $u_i \sim\alpha$

(8) At u_i $(\alpha \wedge \beta)$ iff At u_i α $\wedge$ At u_i β.

(9) $A\alpha$ iff (u_i)(At u_i α) provided $\alpha \varepsilon Q_p$.

(10) (v_i)At $u_i(-v_i-)$ iff At $u_i(v_i)(-v_i-)$.

(11) (-H-)At u_i α iff (-)At u_i α.

(12) (-G-)At u_i α iff (-)At u_i α.

(11) and (12) are the quantified tense logic versions of the Semantic Primacy of Dates Assumption. (see 3.3 and 3.6).

Homophonic Semantics are now given for Q_p. We assume that the set of atomic Q_p predicates consists of a single two place predicate ϕ and the set of individual constants of a single constant a. The metalanguage contains the object language together with elementary arithmetic and the following

(i) A two place predicate 'Closed' such that 'Closed $(\bar{\alpha}$ L)' says that '$\bar{\alpha}$' is a canonical description of the closed formula α of L.

(ii) A two place function symbol 'Val' such that 'Val (is)' says 'the ith member of the sequence s'.

(iii) A three place predicate 'Sats' such that 'Sats (s $\bar{\alpha}$ L)' says that 's satisfies $\bar{\alpha}$ in L'. (Note 'Sats' is a tensed predicate).

(iv) A two place function 'Ref' such that 'Ref $(\bar{a}_i$ L)' says 'the reference of $\bar{a}_i$ in L'.

The truth predicate and '$s' \underset{i}{\sim} s$' are assumed as usual.

The homophonic system is called **HQ** where it is assumed that the language L is Q_p.

HQ1) Sats (s $\overline{\phi v_i v_j}$ L) iff ϕval(is) val(js)

HQ2) Sats (s $\overline{\phi a\ v_i}$ L) iff ϕref($\bar{a}$L)val(is).

HQ3) Sats (s $\overline{\phi v_i a}$ L) iff ϕval(is)ref($\bar{a}$L).

HQ4) Ref ($\bar{a}$ L) = a.

HQ5) Sats (s $\overline{\sim\alpha}$ L) iff $\sim$Sats(s$\bar{\alpha}$L)

HQ6) Sats (s $\overline{\alpha\wedge\beta}$ L) iff Sats(s$\bar{\alpha}$L)$\wedge$Sats(s$\bar{\beta}$L)

HQ7) Sats (s $\overline{(\exists v_i)\alpha}$ L) iff $(\exists s')(s' \underset{i}{\sim} s \wedge$ Sats(s'$\bar{\alpha}$L)).

HQ8) Sats (s $\overline{G\alpha}$ L) iff G Sats(s$\bar{\alpha}$L)

HQ9) Sats (s $\overline{H\alpha}$ L) iff H Sats(s$\bar{\alpha}$L)

HQ10) $(x)(\exists s')(s' \underset{i}{\sim} s \wedge x =$ val(is'))

HQ11) $G(\alpha\to\beta) \to (F\alpha \to F\beta)$

HQ12) $H(\alpha\to\beta) \to (P\alpha \to P\beta)$

HQ13) Tr($\bar{\alpha}$L) $=_{df}$ (s)(Sats(s$\bar{\alpha}$L) $\wedge$ Closed ($\bar{\alpha}$L))

RHQG $\vdash_{HQ} \phi$ therefore $\vdash_{HQ} G\,\phi$

RHQH $\vdash_{HQ} \phi$ therefore $\vdash_{HQ} H\,\phi$

Alternatively, HQ may be set up without the rules RHQG and RHQH where HQ1) to HQ13) are prefaced by the 'always' operator A. But setting it up in this way not only requires the theorem $A\alpha\to AA\alpha$ (see 4.3) but also the theses $PG\alpha\to\alpha$

and FH$\alpha \rightarrow \alpha$. These latter two are required since they are necessary for the proof of the Barcan formula which is need to prove stra ghtforward T-sentences. This is shown below by giving a proof of (13)

(13) $Tr(\overline{G(\exists x)H\phi xa}\ L)$ iff $G(\exists x)H\phi xa$.

(i)	$Sats(s\ \overline{\phi v_i a}\ L)$	iff	$\phi val(is)ref(\overline{a}\ L)$	HQ3)
(ii)	$Sats(s\ \overline{\phi v_i a}\ L)$	iff	$\phi val(is)a$	HQ4)
(iii)	$H(Sats(s\ \overline{\phi v_i a}\ L)$	iff	$\phi val(is)a)$	RHQH
(iv)	$HSats(s\ \overline{\phi v_i a}\ L)$	iff	$H\phi val(is)a$	from HQ12)
(v)	$Sats(s\ \overline{H\phi v_i a}\ L)$	iff	$H\phi val(is)a$	HQ9)
(vi)	$\sim Sats(s\ \overline{H\phi v_i a}\ L)$	iff	$\sim H\phi val(is)a$	Q-Theory.
(vii)	$(s^1)(s^1 \underset{i}{\sim} s \rightarrow \sim Sats(s\ \overline{H\phi v_i a}\ L)$	iff	$s^1 \underset{i}{\sim} s \rightarrow \sim H\phi val(is^1)a)$	Q-Theory.
(viii)	$(s^1)(s^1 \underset{i}{\sim} s \rightarrow \sim Sats(s^1\ \overline{H\phi v_i a}\ L))$	iff	$(s^1)(s^1 \underset{i}{\sim} s \rightarrow \sim H\phi val(is^1)a)$	Q-Theory.
(ix)	$(\exists s^1)(s^1 \underset{i}{\sim} s \wedge Sats\ (s\ \overline{H\phi v_i a}\ L))$	iff	$(\exists s^1)(s^1 \underset{i}{\sim} s \wedge H\phi val(is^1)a)$	Q-Theory.
(x)	$Sats(s\ \overline{(\exists v_i)H\phi v_i a}\ L)$	iff	$(\exists s^1)(s^1 \underset{i}{\sim} s \wedge H\phi val(is^1)a)$	HQ7)
(xi)	$Sats(s\ \overline{(\exists v_i)H\phi v_i a}\ L)$	iff	$(\exists x)H\phi xa$	HQ10)
(xii)	$G(Sats(s\ \overline{(\exists v_i)H\phi v_i a}\ L)$	iff	$(\exists x)H\phi xa)$	RHQG
(xiii)	$GSats(s\ \overline{(\exists v_i)H\phi v_i a}\ L)$	iff	$G(\exists x)H\phi xa$	from HQ11)
(xiv)	$Sats(s\ \overline{G(\exists v_i)H\phi v_i a}\ L)$	iff	$G(\exists x)H\phi xa$	HQ8)
(xv)	$(s)(Sats(s\ \overline{G(\exists v_i)H\phi v_i a}\ L)$	iff	$G(\exists x)H\phi xa)$	Q-Theory
(xvi)	$Closed(\overline{G(\exists v_i)H\phi v_i a}\ L)$			
(xvii)	$\therefore Tr(\overline{G(\exists v_i)H\phi v_i a}\ L)$	iff	$G(\exists x)H\phi xa$	HQ13)

On the alternative HQ set up this proof will consist in the tense necessitations of i) to vi). However, (vii') will then be of the form $(s^1)A(-s^1-)$ whereas what is wanted is $A(s^1)(-s^1-)$ which requires the theoremhood of the Barcan formula.

Quantifiers of QK_L may be understood either as tenseless or as detensed. Perhaps one way of distinguishing these readings depends upon how one sets up the homophonic semantics of Q_p^+. If read tenselessly then the time variables, 'u_i' and time constants 't_i' may be taken as subsets of 'v_i' and 'a_i' respectively. Thus, the sequence set will involve times as members as well as individuals. The homophonic development is straightforward, utilizing (14).

(14) $(\alpha \varepsilon MQ_p)(A\alpha \rightarrow (t)(At\ t\ \alpha))$.

On the other hand if $(\exists v_i)(-v_i-)$ is understood as detensed and it is assumed that times are atemporal then one can introduce a two sortal satisfaction theory based upon a four place satisfaction predicate SATS such that SATS (s s* α L) where s is a sequence for Q_p and s* a sequence consisting of times only. The development is straightforward. (15) holds for every $\alpha \varepsilon Q_p$.

(15) SATS (s s* $\bar{\alpha}$ L) iff sats (s $\bar{\alpha}$ L).

And truth is defined by (16)

(16) Tr($\bar{\alpha}$ L) iff (s)(s*)(s s* $\bar{\alpha}$ L) $\wedge$ Closed ($\bar{\alpha}$ L) .

Two versions of HQ7) and HQ10) are required together with clauses for 'At' (see 4.3) and reference clauses for the set 't_i'.

6.2 Modal Theoretic Semantics for Q_p and Partial Interpretations.

In this section model theoretic semantics are given for standard first order tense systems (as introduced in the previous Section). The discussion here follows closely Section 4.1 where sentential semantics were given. There discussion centred on semantic structures which specify a truth value to each formula at each time. The natural extension of this for first order theories is to consider semantic structures which specify a truth value to each formula at each time relative to a sequence. So first a quantificational interpretation U is defined. (Language Q_p is assumed - see last Section.)

Definition 1.

A quantificational interpretation is a set $U = \langle T,>,D,g\rangle$ where

(i) $\langle T,>\rangle$ is as J in Definition 1 of 4.1;

(ii) D is a non-empty set (called the domain of individuals of U); and

(iii) g is a valuation function satisfying the conditions:

(iii.a) if u is an individual constant then $g(u) \in D$

(iii.b) if Q^n_j is an n-place predicate constant then $g(Q^n_j) \in (D^n)^T$.

Where D is any set we understand by $\Sigma(D)$ the set of all denumerable sequences of members of D. If U is a quantificational interpretation $\Sigma(U)$ is to denote the set $\Sigma(D_u)$, where D_u is the domain of individuals of U.

Definition 2.

Let $U = \langle J,D,g\rangle$ be a quantificational interpretation and $s \in \Sigma(U)$. The value $g^*_s(u)$ of a singular term u of Q_p in U relative to S is defined by:

$$g^*_s(u) = \begin{cases} g(u) & \text{if u is an individual constant} \\ val(i,s) & \text{if u is the variable } v_i. \end{cases}$$

We now define the truth value $U_s(\alpha)(t)$ of a formula α in an interpretation U at a time $t \in T_u$ and relative to a sequence $s \in \Sigma(U)$.

<u>Definition 3.</u>

(i) if α is $Q^n_j(u_1 \ldots u_n)$ then $U_s(\alpha)(t) = 1$ iff $\langle g^*_s(u_1) \ldots g^*_s(u_n) \rangle \in g(Q^n_j)(t)$

(ii) $U_s(\sim\alpha)(t) = 1$ iff $U_s(\alpha)(t) = 0$

(iii) $U_s(\alpha \wedge \beta)(t) = 1$ iff $U_s(\alpha)(t) = 1$ and $U_s(\beta)(t) = 1$

(iv) $U_s((\exists v_i)\alpha)(t) = 1$ iff $(\exists s' \in \Sigma)(s' \underset{i}{\approx} s \wedge U_{s'}(\alpha)(t) = 1)$

(v) $U_s(H\alpha)(t) = 1$ iff $(t')(t>t' \rightarrow U_s(\alpha)(t') = 1)$

(vi) $U_s(G\alpha)(t) = 1$ iff $(t')(t'>t \rightarrow U_s(\alpha)(t') = 1)$

<u>Definition 4.</u>

A formula α is true at t under $U(U(\alpha)(t) = 1)$ iff for all $s \in \Sigma(U)$ $U_s(\alpha)(t) = 1$.

A formula α is false at t under $U(U(\alpha)(t) = 0)$ iff for no s, $s \in \Sigma(U)$ $U_s(\alpha)(t) = 1$.

That is, unlike the homophonic case we allow open formulas to be true as well as closed formulas. Validity is defined along the lines of Definition 4 of Section 4.1: A formula $\alpha \in Q_p$ is valid relative to J iff for every interpretation U of the form <J,D,g> and all $t \in T$ $U(\alpha)(t) = 1$. Systems validated by the above clauses are called AQ_p systems - compare AL_p systems of 4.1.

Select time interpretations can be introduced for quantificational logic in a similar fashion to their introduction for sentential logic. U_{t_n} is the set $\langle T_1,>,D,g,t_n \rangle$ where $t_n \in T$ is a select time. Validity is then defined for BQ_p systems analogous to Definition 6 of 4.1: A formula $\alpha \in Q_p$ is t_n valid relative to J iff for every interpretation U_{t_n}, of the form $\langle T,D,g,t_n \rangle$, $U_{t_n}(\alpha)(t_n) = 1$.

One can ask what the relationship is between these two notions of validity and arrive at theses for the strong and weak first order A = B thesis, namely (1) and (2) respectively,

(1) $(U_{t_n})(U_{t_n}(\alpha)(t) = 1)$ iff $(U_{t_n})(t)(U_{t_n}(\alpha)(t) = 1)$

(2) $(U_{t_n})(U_{t_n}(\alpha)(t_n) = 1)$ iff $(U_{t_n})(t(U^t/t_n(\alpha)(t) = 1)$

where U^t/t_n is a model of the form $\langle J,D,g,t\rangle$ formed by substituting t for t_n in $\langle J,D,g,t_n\rangle$.

Of more interest in the quantificational than in the sentential case is that of a partial interpretation at a select time. There are two important features of these in the sentential case (see 4.1). First, they exhibit a way of treating standard sentential logic as a representation of present tensed fragments of natural language and secondly the set of partial interpretations at the time t_n are isomorphic with those at t_m. It is this latter feature which accounts for them being somewhat uninteresting. In the quantificational case, however, there are two ways of defining partial interpretations, one of which need not satisfy the isomorphism feature.

The first way of defining them is as the triple $\langle D,t_n,g\rangle$ which we write as M_{t_n}. Consequently M^{t_m}/t_n is the partial interpretation $\langle D,t_m,g\rangle$. The alternative way of defining them provides a basis for a more complex semantics for tense systems. However, this latter way is best considered when a certain assumption is made.
Fundamental to partial interpretation clauses is the following (3).

(3) if α is $Q^n_j \in Q^n_i$ then $M_{s\,t_n}(\alpha) = 1$ iff
$\langle g^*_s(u_1)\ldots g^*_s(u_n)\rangle \in g(Q^n_j)(t_n)$

Suppose the following assumption (4) is made.

(4) Let U_{t_n} be the intended interpretation of Q_p (at t_n) where
U_{t_n} is $\langle T,>,D,g,t_n\rangle$ then for any atomic predicate Q^n_j
if $U_{st}(Q^n_j u_1..u_n)(t) = 1$ then $g^*_s(u_1)\ldots g^*_s(u_n)$ all exist at t.

That is, it is assumed that the atomic predicates given their intended meaning are non-ampliating - for their extension at a time consists in their 'appellation', namely existents then (see 1.5 and 2.3). This assumption may be strengthened is model theoretic semantics.

(5) Let U_{t_n} be any interpretation of Q_P then for any atomic predicate Q^n_j, if $U_{s\,t_n}(Q^n_j u_1 .. u_n)(t) = 1$ then $g^*_s(u_1)..g^*_s(u_n)$ all exist at t relative to U_{t_n}.

What is assumed here is that under any interpretation the atomic predicates are satisfied at a time only by existents then (under that interpretation). This, then, is a much stronger assumption amounting to the claim that, in some sense, it is necessary that the extension of an atomic predicate at a time consist in existents then. Let us call this assumption the 'atomic predicate non-ampliating assumption' or APNA for short.

Given this assumption partial interpretations at a select time may be constructed in a different way than from above. Let M'_{t_n} be $\langle D_n, t_n, g\rangle$ where D_n is an arbitrary set of individuals for which it is assumed that under M'_{t_n} they then exist. Also let M'_{t_m} be $\langle D_m, t_m, g\rangle$. Not only do these partial interpretations represent certain present tensed fragments of natural language but also they exhibit a way of letting the existential quantifier represent the present tensed 'there is' locution. These partial interpretations may be spelt out in such a way as to allow for truth value gaps. We let Σ_n be the set of sequences defined over $D_n \in M'_{t_n}$.

Definition 7.

A partial interpretation at a select time t_n is the triple $\langle D_n, t_n, g\rangle$ where g is the following function

(i) if $u_i \in a_i$ and $g(u_i) \notin D_n$ then $g(u_i)$ is not defined.

(ii) if u_i is v_i then $g(u_i) = val(is)$.

(iii) if $Q^m_j \in Q^n_j$ then $g(Q^m_j) \subseteq D^m_n$.

The semantic clauses are straightforward requiring sub-clauses for undefined formulas.

The important difference, then, between the two kinds of partial interpretation introduced in this Section resides in their interpretation of the quantifiers. In the former type unlike the latter (6) is valid

(6) $\phi a \rightarrow (\exists x)\phi x$

for any predicate ϕ which does not involve a tense operator. For instance, under the intended partial interpretation at the present time (7) is not valid.

(7) Socrates is not a man $\rightarrow$ $(\exists x)$ (x is not a man).

(This connects up with the discussion in 2.4.) The systems QK_t and QK_L are effectively systems which are formed by extension of the first kind of partial interpretation. So the question is what kinds of system are formed by extension of the latter kind of partial interpretation involving tensed quantifiers and are there such systems which are two valued?

6.3 Kripkean Semantics for Tensed Quantifier Systems.

Our aim is to provide tensed first order systems which involve tensed quantifiers. Now, one natural way to provide their semantics is by extending the partial interpretations at a select time given in the last Section. But there is one important disadvantage with these and that is that tensed bivalence doesnot hold for them. However, semantics can be given which not only allow for variable domains over time but also for which tensed bivalence holds. Such semantics are based upon Kripke's modal semantics (1963) where domains differ from possible world to possible world.

Definition 1.

A Kripkean tense model is a set $U_{t_n} = \langle T,>,\psi,D,g,t_n \rangle$ where T,>,D and t_n are as before, and

(i) ψ is a function from T into [illegible] s.t. $\psi(t_m) = D_m \subseteq D$;

(ii) g is a valuation function satisfying the conditions:

(iia) if u is an individual constant then $g(u) \in D$

(iib) if Q_j^n is an n-place predicate then $g(Q_j^n) \in (D^n)^T$

such that $g(Q_j^n)(t_m) \subseteq [\psi(t_m)]^n$.

The definition of $g_s^*(u)$ is as in Definition 2 of the previous section. Semantic clauses are those of Definition 3 of the previous section but with $U_{s\ t_n}$ replacing U_s throughout and more importantly, the quantifier clause differs.

Definition 2.

(i) - (iii) and (v), (vi) as in Definition 3 Section 6.2.

(iv) $U_{s\ t_n}((\exists v_i)\alpha)(t) = 1$ iff $(\exists s')(s' \underset{i}{\sim} s \wedge val(i,s') \in \psi(t) \wedge U_{s'\,t_n}(\alpha)(t) = 1)$

Truth is defined as in Definition 4. 6.2 except that U is replaced by U_{t_n}. Validity is defined as t_n validity, similar to Definition 6 of Section 4.1: A formula $\alpha \in Q_P$ is t_n Kripke valid relative to J iff for every interpretation U_{t_n} of the form <J,D, ,g,t_n> $U_{t_n}(\alpha)(t_n) = 1$.

Kripke does not in his modal semantics explicitly adopt the modal APNA assumption; that is, the assumption that an atomic predicate can only be true with respect to a possible world of objects which exist with respect to it. (He does mention the assumption in footnote (1) p.86 of that work.) Also he does not include clauses for individual constants as we have done. Now, the semantics satisfy bivalence because formulas are assigned a truth value at a time (for

Kripke, with respect to a possible world) relative to a sequence even though the relevant members of that sequence do not then exist (or exist with respect to that world).

The consequence of having these semantics is that mixing formulas which were valid under the semantics of the last Section are not Kripke valid. Consider QK_L, for instance, whose modal fragment is S_5 then according to Kripkean semantics none of (1) to (6), nor their modal counterparts (1') to (6') of 6.1; that is, the tensed Barcan formulas, their converses and the Buridan formulas are valid. As far as we know it was Prior who first pointed out that mixing formulas of standard tense systems are unacceptable when understood to involve tensed quantifiers (see 6.5). But, there is an asymmetry in unacceptability between these formulas. For, clearly, (2) to (5) are intuitively unacceptable

(2) $F(\exists x)\alpha \rightarrow (\exists x)F\alpha$

(3) $(x)G\alpha \rightarrow G(x)\alpha$

(4) $F(x)\alpha \rightarrow (x)F\alpha$

(5) $G(x)\alpha \rightarrow (x)G\alpha$

On the other hand (6) and (7) are not intuitively speaking, as unacceptable even though they are equivalent to (4) and (5) respectively.

(6) $(\exists x)G\alpha \rightarrow G(\exists x)\alpha$

(7) $(\exists x)F\alpha \rightarrow F(\exists x)\alpha$

It is noted in 6.5 that both these formulas are valid in Prior's system Q (although their standard equivalents are not). Moreover, this asymmetry in unacceptability is further noted in the construction of more complex tense systems (see 6.7 to 6.11).

Systems, then, which employ tensed quantifiers are more sensitive to scope distinctions than those employing tenseless (or detensed) quantifiers. One advantage, then, is that the compounded|divided distinction as instanced in (8) and (9) respectively

(8) It will be the case that something flys.

(9) Something now will fly.

is straightforwardly representable by (8') and (9')

(8') $F(\exists x)(x \text{ flys})$

(9') $(\exists x)F(x \text{ flys})$

And tensed individual existence can be represented simply by using the tensed quantifier and the tenseless identity predicate; (10) is represented by (10').

(10) Quine exists but Prior no longer does.

(10') $(\exists x)(x = \text{Quine}) \wedge \sim (\exists x)(x = \text{Prior})$.

Consequently, an analysis of tensed discourse which makes use of tensed quantifiers is not open to the problems of representation of existence claims which beset the L_A^D account (see 5.4) and which are also a feature of those systems introduced in 6.1. That is, by accepting tensed quantifiers the parasitictensing account (see 5.4) is rejected. (This point connects up with the discussion in 2.5 where it was claimed that a historical legacy of appellation theory is the suggestion that representation of tensed discourse should be based upon a tensed quantifier quantification theory).

But this advantage of sensitivity to scope distinctions becomes a problem when confronted with the question of which tense systems are satisfied by the Kripkean clauses. For instance, in the case of the modalized tense systems both the converse Barcan formulas ((1') and (2') of 6.1) and the Buridan formulas ((3') and (4') of 6.1) are provable in the weakest standard modal logic Lemmon's $S_{0.5}$(provided α contains no modal operators) when added to standard quantification theory, yet these formulas are Kripke invalid.

6.4 Kripke Systems.

In this Section attention is given to Kripke's solution to the problem of finding formal systems which are validated by his modal semantics and to how this solution provides us with tense systems. In the proofs of the Barcan formulas, their converses and the Buridan formulas the following step is made.

(1) $(x)\alpha \rightarrow \alpha\, {}^{y}/_{x}$

(2) $A(\ (x)\alpha \rightarrow \alpha\, {}^{y}/_{x})$.

(In their tensed counterparts Rule G or H is used). Now, Kripke objects that this move from (1) to (2), itself, presupposes the Kripke invalid converse Barcan formula.

> "In a formula like (1) we give the free variables the generality interpretation. When (1) is asserted as a theorem it abbreviates assertions of its ordinary universal closure..."
> (1963 p. 88-9).

So (1) is really (1')

(1') $(y)(\ (x)\alpha \rightarrow \alpha\, {}^{y}/_{x})$

which by necessitation gives (3)

(3) $A(y)(\ (x)\alpha \rightarrow \alpha\, {}^{y}/_{x})$

whereas (2) really asserts (2')

(2') $(y)A(\ (x)\alpha \rightarrow \alpha\, {}^{y}/_{x})$.

And the move from (3) to (2') is merely an instance of the Converse Barcan formula. (And using the argument in the tense case, one assumes the move G(y)(-y-) to (y)G(-y-) or its mirror image).

On the basis of this, Kripke suggests that one should seek closed modal systems. In the case of tense systems, following Kripke's modal system (ibid p. 89) we may define the system CQK_L as the closures of the following where the closure of a formula α is the resulting formulas which arise by prefixing universal quantifiers and the tense operators G and H in any order to α.

$CQK_L1)$ Any valid S.C. instance

$CQK_L2)$ $G(\alpha \to \beta) \to (F\alpha \to F\beta)$

$CQK_L3)$ $H(\alpha \to \beta) \to (P\alpha \to P\beta)$

$CQK_L4)$ $G\alpha \to GG\alpha$

$CQK_L5)$ $H\alpha \to HH\alpha$

$CQK_L6)$ $PG\alpha \to \alpha$

$CQK_L7)$ $FH\alpha \to \alpha$

$CQK_L8)$ $F\alpha \wedge F\beta \to F(\alpha \wedge \beta) \vee F(\alpha \wedge F\beta) \vee F(F\alpha \wedge \beta)$

$CQK_L9)$ $P\alpha \wedge P\beta \to P(\alpha \wedge \beta) \vee P(\alpha \ P\beta) \vee P(P\alpha \wedge \beta)$

$CQK_L10)$ $(x)(\alpha \to \beta) \to ((x)\alpha \to (x)\beta)$

$CQK_L11)$ $(y)((x)\alpha \to \alpha \ {}^{y}/_{x})$

Rules M.P.

The system CQK_L then, has no valid mixing formulas since all of (1) to (6) of 6.1 are Kripke invalid. However, if an open formula is valid, say (-x-) then so are (x)G(-x-). G(x)(-x-) and G(x)G(-x-) by the closure condition. For example, $\phi x \to \phi x$ is an instance of $CQK_L1)$; hence all of (4) to (6) hold

(4) $(x)A(\phi x \to \phi x)$

(5) $A(x)(\phi x \to \phi x)$

(6) $A(x)A(\phi x \to \phi x)$.

An alternative approach to finding systems which satisfy the Kripke clauses is to place all the blame as it were upon (1)

(1) $(x)\alpha \to \alpha \ {}^{y}/_{x}$

and not upon the move from (1) to (2). For it can be said that the problem is that (1) is invalid. A sequence may satisfy the antecedent without satisfying the consequent.

For instance if α is ϕv_i where ϕ is an atomic predicate then a sequence with a presently non-existent in the ith place may satisfy the antecedent without satisfying the consequent. On the basis of this, one might suggest that what is needed is an amendment to quantification theory of the sort argued for by free logicians (Kripke notes this solution p. 89 footnote (1)). Hence, instead of (1), (7) will be an axiom

(7) $(x)\alpha \rightarrow (E!y \rightarrow \alpha \, {}^{y}/_{x})$

And the 'closest' one gets to the converse Barcan formula is (8).

(8) $A(x)\alpha \rightarrow (y)A(E!y \rightarrow \alpha \, {}^{y}/_{x})$

Thomason proposed this solution in (1969) which is considered here via a different more complex route.

Date axioms can be added to CQK_L but they are somewhat complex because the Semantic Primacy Assumption in the form of (11) and (12) of Section 6.1 have to be changed (and also (10) has to be rejected). Because of this we shall not consider the date extension to CQK_L.

Homophonic semantics for Kripkean systems follow very closely those given in 4.1 provided certain assumptions are made about sequences. For given the tensed reading of the quantifiers in Kripke systems how is (9) to be read?

(9) $(\exists s)(-s-)$

For it may be read either as 'There is now a sequence such that it...' or if two sorted quantification theory is allowed in the metalanguage as 'There [is] a sequence such that it...' One general point relevant here is that Kripke systems become provably equivalent to BQ_p systems if everything exists at all times. Consequently, if it is assumed that at all times anything can be a member of a sequence whether existent or not then, then it does not matter how (9) is read. This assumption is made here. Consequently, pure sequence formulas satisfy the mixing formulas of 6.1. On the basis of this a homophonic semantic theory can be given for Kripke systems, one which is the Kripke closures of HQ1) to HQ13) of 6.1 except for sequence places which may remain open. One important change though is the quantifier clause. Instead of HQ7) one has HQ7').

HQ7') $\mathrm{Sats}(s\ \overline{(\exists v_i)\alpha}\ L)$ iff $(\exists s')(s' \underset{i}{\sim} s \wedge (\exists x)(x = \mathrm{val}(is')$ $\wedge\ \mathrm{Sats}(s'\ \overline{\alpha}\ L))$.

Thus, in order to prove (13) of 4.1 one proceeds as far as step (vi) but replace steps (vii) to (xi) by the following (vii)' to (xi)'.

vi) $\sim\mathrm{Sats}(s\ \overline{H\phi v_i a}\ L)$ iff $\sim H\phi\,\mathrm{val}(is)a$

vii)' $(s')(x)((s' \underset{i}{\sim} s \wedge x = \mathrm{val}(is') \rightarrow \sim\mathrm{Sats}(s'\ \overline{H\phi v_i a}\ L)$ iff $((s' \underset{i}{\sim} s) \wedge x = \mathrm{val}(is') \rightarrow \sim H\phi\,\mathrm{val}(is')a))$

viii)' $(s')(x)\ (s' \underset{i}{\sim} s \wedge x = \mathrm{val}(is') \rightarrow \sim\mathrm{Sats}(s'\ \overline{H\phi v_i a}\ L)$ iff $(s')(x)\ (s' \underset{i}{\sim} s \wedge x = \mathrm{val}(is') \rightarrow \sim H\phi\,\mathrm{val}(is')a)$.

ix)' $(\exists s')(\exists x)(s' \underset{i}{\sim} s \wedge x = \mathrm{val}(is') \wedge \mathrm{Sats}(s'\ \overline{H\phi v_i a}\ L)$ iff $(\exists s')(\exists x)(s' \underset{i}{\sim} s \wedge x = \mathrm{val}(is') \wedge H\phi\,\mathrm{val}(is')a)$

x)' $(\exists s')(s' \underset{i}{\sim} s \wedge (\exists x)(x = \mathrm{val}(is') \wedge \mathrm{Sats}(s'\ \overline{H\phi v_i a}\ L)$ iff $(\exists x)H\phi xa$

xi)' $\mathrm{Sats}(s\ \overline{(\exists v_i)H\phi v_i a}\ L)$ iff $(\exists x)H\phi x\ a$

And then continue as before.

6.5 Prior's System Q.

Prior's system Q, first discussed in his 'John Locke Lectures' (1957), arose as a result of the problems created by some of those mixing formulas which are Kripke invalid. In this Section the philosophical basis for Q is considered. Now, unlike Kripke systems Q does not satisfy tensed bivalence. However, this disadvantage is somewhat balanced by its containing mixing formulas which are theorems. Prior's rejection of bivalence is essentially bound up with his demonstrative view of free variables. And this, in turn, is bound up with a generalized version of the non-ampliating assumption. Let us look at this first. He writes

> "where x stands for a proper name it seems to me that the form 'x exists' must be logically equivalent to and definable as 'There are facts about x' $(\exists\phi)\phi x$. If there are facts about x I can not see what further fact about x would consist in its existing. And when x no longer exists or does not exist but there are nevertheless facts about x now I do not know what the present facts about x would be."
> (1957 p. 31).

Prior claims that (1) holds.

(1) $(\exists\phi)\phi x$ iff x exists.

That is, it is not only atomic predicates but also any predicate which is to be non-ampliating, a view similar to that held by the Stoics (see 1.4).

Closely connected to this view is that the Russellian quantifiers are to be given a tensed reading in the sort of way outlined in the last three sections. For free variables, he assumes, are only open at a time for what is then demonstrable. In (2)

(2) $(x)\ \alpha \rightarrow \alpha\ {}^{y}/_{x}$

'y' is only replaceable by a logically proper name - that is, a proper name which is replaceable by a demonstrative. Thus, what we have here is the demonstrative (as opposed to the pronoun) account of free variables which connects up intimately with his generalized non-ampliating assumption.

Because this is the only account of quantifiers and free variable Prior considers it becomes somewhat urgent for him to provide a tensed quantification theory. For AQ_p (BQ_p) systems validated by the semantics in 6.2 and their L_A^D equivalents, he believes, are objectionable not on the ground noted in 5.4 but because they presuppose that all individuals are sempiternal. This, he takes as showing that there is a clear dispute between himself and those theorists who accept as valid the Barcan formula and its converse (or their L_A^D equivalents) in the form of (3).

(3) $(x)A\alpha$ iff $A(x)\alpha$.

It would be better, however, to see the difference as that between two different accounts of quantifiers and free variables.

Unlike Kripke or a 'free logican' Prior does not believe that tensed quantification theory employing tensed quantifiers needs to be amended. To the claim that (4)

(4) $\alpha^{y}/_{x} \rightarrow (\exists x)\alpha$

is invalid because substitution instances like (5) appear to be counterexamples

(5) $P\alpha^{y}/_{x} \rightarrow (\exists x)P\alpha$

as in 'If Alexander rode Bucephalus' then 'there exists an object which Alexander rode' he writes

"This objection can be met by saying that since Alexander's horse has ceased to exist the word 'Bucephalus' can no longer count as a logically proper name - that is it can not in principle be replaceable by a demonstrative - and so is not substitutable for 'y' in (4)."
(1957 p. 33).

'Bucephalus' (and 'Alexander'), then, are to be construed as definite descriptions in the Russellian way and, consequently, the purported counter-example is not an instance of (4) at all for it states

"If it has been the case that this α's there is something of which it has been the case that it α's."
(ibid).

This has the consequence that ordinary proper names of past existents are to be construed as definite descriptions a view, not unsurprizingly, Prior is uncomfortable with. (His attitude to 'names' of future existents is noted in the next Section).

Although he believes that standard quantification theory does not need to be amended he claims that the problem of the invalidity of the mixing formulas lies in the assumption that standard sentential logic satisfies tensed bivalence. His argument for this is as follows. The expression '$(\exists\phi)\phi x$' not only states that there are facts about x but also that there are statements about x. Thus, the sentence 'Alexander rode Bucephalus' does not now express a statement which is either about Alexander or about Bucephalus. Prior, then, holds (A)

(A) x exists iff there are statements about x.

Now, fundamental to tense logic is the assumption (B) together with its mirror image

(B) 'Pα' is now true iff 'α' was true.

Now, Prior claims that (A) and (B) are in conflict with (C).

(C) At every time, each statement is true or false. The problem is that a sentence like 'this does not exist' can never express a false statement. Moreover, nor can 'P(this does not exist)' or 'F(this does not exist)'. (We may compare the Stoic notion of 'axioma' here briefly outlined in Section 1.1). For by (B) 'P(this does not exist)' is true if and only if at some moment of past time the statement (then) expressed by 'this does not exist' (where 'this' refers to this) is true which clearly it can not be.

But what happens when one considers, say the truth value of 'Pn (This does not exist)' where 'Pn' is the metric operator 'It was the case 100 years ago'? By (B) this is true iff 'this does not exist' said in 1879 where 'this' demonstrates this, expresses a true statement. But, suppose there was no this at that time then by (A) there were no statements about this. And this includes statements about this which deny its existence then. Thus, (C) is false. For the statement expressed now by 'this does not exist' itself did not exist in 1879.

Although Prior holds that 'Pn $\sim$ (x exists)' can never be true he claims that its equivalent in standard metric logic '$\sim$ Pn (x exists)' can be true.

> "On the other hand a statement of the form $\sim Pn(\exists\phi)\phi x$ 'It was not the case n days ago that x exists' may very well be true, on the assumption that there are non-sempiternal individuals. For example, it was not the case 100 years ago that I existed; there were, I would contend, no facts about me then - not even this fact of there being no facts about me at that time; though it is now a fact that there are no facts about me then." (1957 p. 34).

So although at any time at which a statement exists it is either true or false there are times when it is undefined.

Consequently, in order to preserve (C) one requires the assumption that everything is sempiternal since then there would always be statements about everything. Rejecting tensed bivalence means that a statement may not always be true and yet never be false. For instance (5) is not true

(5) A (this is $\phi \rightarrow$ this is ϕ)

for almost all uses of 'this'. Consequently, (6) holds

(6) $\sim$ A (this is $\phi \rightarrow$ this is ϕ)

On the other hand (7) is never true

(7) S $\sim$ (this is $\phi \rightarrow$ this is ϕ)

So (8) holds

(8) $\sim$ S$\sim$ (this is $\phi \rightarrow$ this is ϕ)

Thus, in a modal or tense system when bivalence does not hold the standard relationship between the operators does not hold. (This feature is considered by Bell and Humberstone in connection with logics for strict presupposition (1977)). What does hold, however, are both (9) and (10), for tense operators

(9) $G\alpha \rightarrow \sim F \sim \alpha$

(10) $F\alpha \rightarrow \sim G \sim \alpha$

The difference in semantic content between '$G\alpha$' and '$\sim F \sim \alpha$' is that the former expresses that 'α' will always be true whereas the latter expresses that 'α' will never be false. Now, if validity is defined in terms of truth then we here have an example of the breakdown of the sentential A=B thesis. For instance, '$\alpha \vee \sim \alpha$' can be t_n valid without being t_m valid. Prior, however, introduces a weaker notion of validity defined in terms of a formula not being false which does satisfy the equivalence thesis. (This weaker notion of validity does create difficulties as to what precisely the rule of detachment is).

Because a valid formula may not always be true the rules RG and RH have to be given up. But because a valid formula is never false instead of RG one may introduce the rule R∿F∿ and its mirror image. Semantics can be given for Q. Although neither of the tensed Barcan formulas turn out to be valid both (11) and (12) do

(11) $(\exists x)F\alpha \rightarrow F(\exists x)\alpha$

(12) $(\exists x)G\alpha \rightarrow G(\exists x)\alpha$

(11) is one version of the tensed converse Barcan formula whereas (12) is one version of the tensed Buridan formula. It is these formulas which in 6.4 were claimed to be intuitively acceptable. In Prior's tensed Q because of the breakdown between the operators their standard equivalents are not provable. Instead both (13) and (14) are

(13) $G(x)\alpha \rightarrow (x){\sim}F{\sim}\alpha$

(14) $F(x)\alpha \rightarrow (x){\sim}G{\sim}\alpha$

In 6.10 formulas which are similar to (11) to (14) turn out to be valid in a semantics for which bivalence holds.

6.6 Time's Asymmetry and System Q.

Prior's system Q, as a tense system, has some rather unfortunate consequences. First, on certain occasions it may be unclear which statement is made by the use of a sentence. This is because a sentence like 'Prior was born in New Zealand' when used in the 1960's expressed a different statement than when now used, not on Frege-Russell ground (see 3.1) but on the grounds that Prior no longer exists. Thus, ordinary proper names of continuants change their status once the bearer passes away. Connected here is the point that predicates like 'v_i will be a Prime Minister', 'v_i no longer exists', 'v_i is not a person', 'v_i invaded Britain' etc., are not now defined for objects which do not now exist. And this is so despite the fact corresponding predicates 'v_i invades Britain', 'v_i is a Prime Minister' etc., were or will be defined for those objects.

On the one hand it seems acceptable to claim that 'v_i is a horse' is not defined for Pegasus but on the other hand to say that 'v_i was a logician of some repute' isn't now defined for Prior appears unacceptable. For the past is not quite as non-existent as Prior make out. Moreover how is it that we can speak about the past on Prior's account? His answer here is that past is spoken about through the use of definite descriptions but this gives it a generality it often lacks. (This generality is manifested in that a formula '$P(\exists x)\phi x$' may now be true without it also being the case that a formula of the form '$P\phi a$' is also true even if everything has an ordinary proper name).

Another consequence of Prior's arguments which underly Q is that the past and the future are equally 'indiscernible'. Yet this fits ill with another view he argues for, namely that one can not name future objects (see 1967 p. 138 ff for a clear statement). But the grounds for this do not hold in the case of past objects.

> "Things that have existed do seem to be individually identifiable and discussable in a way in which things that don't yet exist are not (the dead are metaphysically less frightening than the unborn)."
> (ibid p.171).

That is, the point is that future objects unlike past objects can not be individuated which has the consequence that there is a generality about the future which does not also hold in the case of the past. There are, of course, different versions of this asymmetry dependent upon what force 'can' has in 'one can not name future objects'. Let us not pursue this but merely note that the asymmetry involved is squarely based upon the fact that although there are proper names of past objects (in one good sense of 'proper name') this not true for future objects. Now this asymmetry can not come out in Q because according to that representation of tensed discourse one can neither name past not future existents. Instead since all proper names of past objects are to be construed as descriptions this puts them on a par with future descriptions.

As noted in the last Section Prior distinguishes between '∿Pn(x exists)' and 'Pn∿(x exists)' where the former unlike the latter can be true. The difference is between: 'It now is not the case that x's existence was the case nago' and 's's existence was not the case nago'. Given Prior's asymmetry argument it seems that there is such a distinction to be made even after rejecting Prior's demonstrative account of free variables. For instance, it is now true that it was the case in 1865 that Russell will be born although in 1865 no such sentence as 'Russell will be born' could be true. And this difference is based upon the fact that in 1865 there was no proper name 'Russell' (referring to the philosopher). That is, there is a difference between the way we now can speak about the past and the way it could then be spoken about.

This difference may be captured by utilizing the point that as time passes the set of individual constants increases and at any moment a member of that set refers to a past or to a present existent. This means that (A) and its mirror image have to be rejected.

(A) '$P\alpha$' is now true iff 'α' was true.

What hold instead are (B) and (C)

(B) if 'α' was true then '$P\alpha$' is now true.

(C) if '$F\alpha$' is now true then 'α' will be true.

This amounts to rejection of the A=B theses.

However, as noted in 4.3, (A) is objectionable when understood as (A') (and it is this understanding of A which Prior uses in his argument for Q - see last section).

(A') the statement expressed by '$P\alpha$' is now true iff at some time in the past the statement expressed by 'α' then is true.

For this commits one to very strong assumptions about languages namely that they always exist and that their sentences always have the meaning they now have. In 4.3 then we suggested that (1)

(1) $Tr(\overline{P\alpha}\ L)$ iff $P\ Tr(\overline{\alpha}\ L)$

be understood as ''Pα' is true with the meaning it now has in L iff 'α' was true with the meaning it now has'. However, the discussion here suggests an alternative account. And this is not to invoke the two dimensionality of 'now' but instead to claim that the operators 'P' and 'F' are themselves two-dimensional where the select time is to be treated as a perspective from which to assess truth values. That is, the operator 'P' is to be understood as 'It was the case (relative to this perspective) that...' where 'this perspective' is given by the select time. Consequently where U_{t_n} is a Kripkean select time interpretation (2) may hold without (3)'s holding.

(2) $U_{s\ t_n}\ (F\alpha)(t) = 1$

(3) $U_{s}\ {}^{t}/_{t_n}\ (F\alpha)(t) = 1.$

For instance, suppose U_{t_n} is the intended select time interpretation and 'Fα' is 'Russell will be born', t_n is the present time and t is 1865. Then (2) is true whereas (3) is false. For (2) is (2') and (3) is (3')

(2') 'It will be the case (relative to this perspective) that Russell is born' is true in 1865.

(3') 'It will be the case (relative to a perspective in 1865) that Russell is born' is true in 1865.

For in 1865 there was no proper name 'Russell' referring to the philosopher. Provided it is t_n validity which is invoked then Kripkean systems will be valid under this conception. However, time's asymmetry is to be explained in terms of the fact that a formula which is t_m valid need not be t_n valid. More, specifically, time's asymmetry is expressible by (4) and (5).

(4) where $t<t_n$ $U_{s\ t_n}\ (\alpha)(t) = 1 \not\rightarrow U_{s}\ {}^{t}/_{t_n}\ (\alpha)(t) = 1.$

(5) where $t>t_n$ $U_{s\ t_n}(\alpha)(t) = 1 \rightarrow U_s{}^{t}/_{t_n}(\alpha)(t) = 1$.

We also have the basis here for an argument which claims that our temporal perspective is essential to our description of Reality. And that there can not be a complete description of Reality (except at the last moment of time). Thus, this amounts to an objection to the four dimensional 'model' of Reality.

(An alternative approach to a conclusion along these lines is found in Dummett (1960)).

6.7 Scope Distinctions and Predicate Modifers.

Prior's system Q fares better as a modal system than as a tense system. For it is more acceptable to hold that predicates are not defined with respect to a possible world for objects which do not exist there than it is to hold that they are not defined at a time for objects which no longer exist then. (There is an asymmetry here between the acceptability of this for past and for future objects). On the basis of this, it may be said that Kripke systems fare better as tense systems than as modal systems. Moreover we shall now attempt to argue that Kripke modal systems are defective in a more complex way.

In Prior's modal system Q both (1) and (2) are provable even though their standard equivalents (1') and (2') are not because of the breakdown between A and S.

(1) $(\exists x)S\phi x \rightarrow S(\exists x)\phi x$

(2) $(\exists x)A\phi x \rightarrow A(\exists x)\phi x$

(1') $A(x)\phi x \rightarrow (x)A\phi x$

(2') $S(x)\phi x \rightarrow (x)S\phi x$

Understood modally (1) states that 'It is possible that something is ϕ' follows from 'There is actually something which possibly is ϕ'. This formula is Kripke invalid which seems to be counter intuitive. For instance, from the claim that something actual, say Tony Benn, possibly is prime minister it does seem to follow, in possible world talk, that there is a possible world in which he and hence something is prime minister. Why then is this formula Kripke invalid?

On the surface it appears to be a result of two points; first, not all objects are necessary beings and secondly Kripke systems satisfy bivalence. Consequently, a sentence like 'There is something (say, Socrates) who is possibly not a person' can be true simply because Socrates doesn't exist in every possible world. For in a world in which he does not exist the sentence 'Socrates is not a person' is true and this is so even though in that world the sentence 'Something is not a person' may well be false.

But beneath the surface here of this counter-example to (1) there lurks an essentialist claim. For the situation depended upon the truth of the claim that Socrates is a person in all worlds in which he figures. But this is tantamount to the claim that Socrates is necessarily a person; where 'necessarily' is to be understood as expressing that notion of necessity which applies to non-necessary beings. This is a philosophically well entrenched although somewhat controversial sense of 'necessarily' which is more or less captured by Kripke's later notion of weak necessity.

> "Let us interpret necessity here weakly. We count statements as necessary if whenever the objects mentioned therein exist the statement would be true." (1971 p. 137).

If one wanted to formally capture this notion then surely Kripke style semantics which allow for domains to vary from possible world to possible world is the obvious candidate. But the only necessary qua essentialist type truths capturable in Kripke systems are 'negative' ones like 'Necessarily, Socrates is not a mouse'. For in no possible world whether Socrates figures there or not is the sentence 'Socrates is not a mouse' false. But the reason one would maintain that essentialist claim is because it is true that Socrates is necessarily a person, a truth which is uncapturable here unless Socrates exists in every possible world (in which case formulas like (1) and (2) would turn out to be valid if all objects were necessary).

Is there an analogous sense of 'always' which applies to non-sempiternal beings? Jespersen points out that 'always' is often used with the import of 'at all times we are just now concerned with' (1961 p. 191). Consequently, we may introduce tensed analogues of weak necessity. For instance, it may be claimed that there is a sense in which (3) is true

(3) Socrates was always a person.

In what follows an attempt is made to represent these senses of necessity and omnitemporality partly as an end in itself but much more importantly, for the purposes undertaken here, as a purported contribution to the discussion of the problem of founding tensed or modal quantification systems which employ quantifiers in the Kripkean manner. What is wanted are systems for which bivalence holds yet in which formulas like (1) and (2) are provable, that is systems which have the advantages of Prior's Q and Kripke systems without having their disadvantages. Very briefly, our strategy for the rest of this chapter is to argue that an operator when understood as a predicate modifier may have different logical powers than when construed as a sentence operator. In order to make good this claim we must, of course, distinguish between predicate modifiers and sentence operators. As a point of entry into some of the problems involved we shall briefly look at Wiggins' proposal (1976 A 1976 B) that λ abstraction may be utilized in distinguishing between sentence operators and predicate modifiers.

Stalnacker and Thomason (1968 A) use λ abstraction to mark scope differences in order to distinguish between sentences like (4) and (5).

(4) Necessarily, the President of the U.S. is a citizen of the U.S.

(5) The President of the U.S. is necessarily a citizen of the U.S.

In more standard discussion these are distinguished via the use of Russellian description theory, as in Smullyan (1948). However, both authors reject this method on the grounds that definite descriptions are primitive singular terms. Instead they distinguish between (4) and (5) using λ abstraction as in (4') and (5').

(4') $A\ (\lambda y)[\phi y](\ (\iota x)(\psi x)\ )$

(5') $(\lambda y)A\ [\phi y](\ (\iota x)(\psi x)\ )$

Semantically, the difference is that (4') is true iff whatever is the President of the U.S. with respect to any possible world is also a citizen there whereas (5') is true iff whatever is the President in the actual world is a citizen of the U.S. in every possible world.

This use of λ abstraction is not one with which we are concerned since we believe the Smullyan analysis to be more perspicuous. What does interest us, however, is a more general claim made by these authors in (1968 B). They say there that λ abstraction may be used to distinguish de re from de dicto claims when any singular term, including proper names, is involved. If D is an operator then (6) is a de dicto claim

(6) $D(\lambda y)[\phi y](a)$

because it is equivalent to (7).

(7) $D\phi a$

where a is the singular term involved. On the other hand

"...$(\lambda y)D[\phi y](a)$ is a de re formula since in it the modal operator is used to construct a predicate $(\lambda y)D[\phi y](...)$ which is applied to a singular term; such a formula represents the ascription of a modal property to a thing."

(1968 B p.364).

Expressed here is the view that an operator can map predicates into predicates which intuitively appears to be the view that in de re formulas operators act as predicate modifiers (as opposed to sentence operators). Furthermore, the authors believe that in the de re case substitutivity holds; that is (8) holds

(8) $a = b \rightarrow ((\lambda y)D[\phi y](a) \text{ iff } (\lambda y)D[\phi y](b))$

However if the only operators we are dealing with are modal or tense operators and individual terms are both tense and modally rigid then this distinction using λ abstraction has no semantic content as it stands. (If D is, say, an epistemic operator then it may have content.)

Wiggins, however, in (1976 A,B) believes that there is a de re/de dicto distinction to be captured using λ abstraction. In the case of necessity he claims, that this distinction is between weak and strong Kripkean necessity

"I am not sure but the change [Kripke's change from strong to weak necessity in 1971] may have been made in order to distinguish clearly the de re from the de dicto occurrence of 'necessarily'..."

(1967 A p.108).

He takes weak necessity to be represented by a predicate modifier which is to be distinguished from sentential necessity using λ abstraction. His use of λ abstraction, which we shall follow, is technically different from Stalnacker and Thomason's.

Provided not all objects necessarily exist then Wiggins claims that there is a semantic difference between (9), which is true, and (10) which is false.

(9) $(A[(\lambda x)[x \text{ is a person}]], [\text{Socrates}])$

(10) $A([(\lambda x)[x \text{ is a person}]], [\text{Socrates}])$

In (9) A is to be understood as a predicate modifier modifying the predicate construct $[(\lambda x)[x \text{ is a person}]]$. The use of ',' and scope distinguishes this example from (10) where A operates upon a sentence, albeit one containing a predicate construct. This notation separates a predicate from its arguments, by treating a sentence as a complex ordered pair. Thus, it allows us to distinguish clearly between open sentence operators and predicate modifiers. Unlike Wiggins, Stalnacker and Thomason introduce λ distinctions using quantifier sentences as their paradigm. Thus their distinction is based upon that between, for example, $(\exists x)D\phi x$ and $D(\exists x)\phi x$. Wiggins, on the other hand, introduces λ abstraction as a way of filtering out predicate expressions unambiguously. (In the next Section we provide full syntax rules for λ abstraction.) However, Wiggins does not to our knowledge, clearly argue for either of the two theses

(A) Weak necessity should be represented by a predicate modifier.

(B) Necessity when represented by a predicate modifier is weak necessity.

For suppose (A) is true but (B) is false. That would then mean that although weak necessity is to be represented by a predicate modifier this is not the only sense of necessity which can be represented in this way. (It is more or less this which is argued for in 6.9.) But what about (A)? Davies (1978), partly because he finds the de re/de dicto distinction confusing, has claimed that (A) is false; instead he holds that weak necessity is to be captured using a sentence operator. On the other hand, it seems to us that, at least intuitively speaking (A), is true (although in one good sense of 'predicate modifier' (B) is false).

Certainly if (11) is a weak necessity claim and (12) a strong necessity claim

(11) Socrates is necessarily a person

(12) Necessarily Socrates is a person

then not only is 'necessarily' a semantic unit of both these sentences but so also are 'Socrates' and 'is a person', as is brought out in (9) and (10). And, clearly, the semantics for both these sentences are tied up with that for (13)

(13) Socrates is a person.

But there is a difference. Semantics for sentences of type (11) (unlike those for (12)) must make an appeal to the fact that sentences of type (13) have particular kinds of semantic units. That is, in giving semantic clauses for weak 'necessarily' one must have recourse to the structure of the sentence containing it in a way which is not necessarily required when semantics for strong 'necessarily' are given. (Because this difference is rejected by Davies it means that for him '$A(\exists x)\phi x$', where ϕ is monadic, is ambiguous as between A's being a weak or strong necessity even though semantically, unlike the case of '$A\phi a$', there is no difference. On the other hand if weak 'necessarily' is a predicate modifier and hence syntactically like 'large' then '$A(\exists x)\phi x$' is not ambiguous.)

An attempt to make good the claim that (A) is true, namely that weak necessity should be represented by a predicate modifier, is based upon the claim that there are properties. A distinction can then be made between 'real' and 'apparent' predicates. For, unlike the latter, the former are taken to express properties, and one may use λ abstraction to represent 'real' predicates. And, it can be argued, as Prior does, that to exist now is to have properties now. This can be expressed by (14).

(14) y exists (now iff $([(\lambda x)[\phi x]],[y])$.

Now, given (14) even though a sentence like 'Socrates is not a person' may now be true it is counter intuitive to claim that this expresses that Socrates now has or lacks properties. (And this is really the point that underlies Prior's criticisms of AQp (BQp) systems - see 6.5). Similarly, with respect to a possible world in which Socrates does not exist, it is counter intuitive to claim that Socrates has or lacks properties there. (Consider Cerberus and the actual world.) On the basis of this, it is useful to distinguish in tense and modal logics between senses of negation which may be represented by λ abstraction as in (15) and (16)

(15) $(\sim[(\lambda x)[\phi x]],[a])$

(16) $\sim([(\lambda x)[\phi x]],[a])$

By appealing to that use of λ abstraction for representing a real predicate then (15) says that an existent a has a certain negative property where the occurrence of the negation sign is essential to the expression of that property - and so is essentially predicative. On the other hand (16) merely denies that a has a certain property. Thus these differ semantically in tense and modal systems provided objects are not all omnitemporal or necessary respectively. (What we have here is that idea suggested in 1.4 that a predicate negation may map non-ampliating predicates into non-ampliating predicates.) And as a reply then to Davies (1978 p.415-6) we here have that 'Socrates is not a person' is ambiguous as to the role of 'not' without denying that in both its readings 'Socrates' and 'is a person' are semantic constituents.

Similarly one may distinguish (17) and (18)

(17) $(A[(\lambda x)[\phi x]],[a])$

(18) $A([(\lambda x)[\phi x]],[a])$

(18) can be understood as claiming that in all possible worlds a has the property expressed by $(\lambda y)[\phi y]$ whereas (17) can be understood as claiming that a has a necessary property. But because it has been assumed that an object can only have properties when it exists a natural way of understanding 'a has the necessary property expressed by $A[(\lambda x)[\phi x]]$' is as 'whenever a exists, a has the property expressed by $[(\lambda x)[\phi x]]$'. Thus (17) and (18) would then differ semantically provided that not all objects are necessary or omnitemporal.

This argument for the claim that weak necessity or rather essentialist necessity (see 6.9 for their difference) is to be represented by a predicate modifier depends upon two points. First, that essentialist necessity claims are to be construed in terms of objects having necessary properties and secondly that for something to have a necessary property is for it to have a property whenever it exists. The first point is open to the objection that an ontology of properties is presupposed and the second point is open to the objection that one is not compelled to understand 'a has a necessary property' in the way suggested. However, the distinction between the formulas containing negation ((15 and (16)) is only open to the first kind of objection. And, as we shall attempt to argue it is distinctions amongst negations which hold the key to a representation of weak necessity.

6.8 The Language λQp.

In this Section we introduce a language, λQp, an extension of Qp, containing λ abstraction and predicate modifiers. The central feature of Wiggins' notation is that it treats a λ formula as an ordered pair. And this allows one to distinguish clearly between predicate modifiers and (open) sentence operators.

λQp is then to be an extension of Qp, the standard first order language introduced in Section 6.1. The vocabulary of λQp is that of Qp together with

i) a denumerable set of (new) variables $x_1,..x_n,..$

ii) the symbol λ

iii) a set Δ of predicate modifiers.

The set $x_1..x_n..$ is for exclusive use within λ constructs.
We assume that λ-formulas arise as the result of transformations on formulas. That is, we do not assume that a λ-construct is separable syntactically from a λ-formula. The rule for λ-formulas is now given.

A) Suppose α is a formula of λQp, that $z_1,\ldots,z_n$ are all its free variables, that O is a (possibly empty) finite string of members of Δ, and that $u_1\ldots u_n$ are singular terms (variables or individual constants) of Qp. Then

$$(O[(\lambda x_1)\ldots(\lambda x_n)[\alpha\,{}^{x_1}/z_1\ldots{}^{x_n}/z_n]],[u_1,\ldots,u_n])$$

is a formula of λQp.

The part $O[\lambda x_1)\ldots(\lambda x_n)[\alpha\,{}^{x_1}/z_1\ldots{}^{x_n}/z_n]]$ of this formula will be called a λ-construct, and in that part, as well as in the formula containing the part, $[\alpha\,{}^{x_1}/z_1\ldots{}^{x_n}/z_n]$ will be the scope of each of the variable binders (λx_i) $1\le i\le n$.

The definition of the formulas of λQp is obtained by adding A) to the formation rules given in Definition Qp3, 6.1.

We have followed Wiggins' restriction on λ-constructs, that they can not contain free variables. Thus one cannot quantify into the scope of a λ operator. So for example $(y)((\lambda x_1)(\lambda x_2)[\phi x_1 x_2 y]],[u_1,u_2])$ is not a formula. The restriction on individual constants disallows λ abstraction upon a constant within the scope of a λ operator. So, $((\lambda x_3)[([(\lambda x_1)(\lambda x_2)[\phi x_1 x_2 x_3]],[u_1 u_2])],[a])$ is not a formula. On the other hand, one can iterate λ abstraction. Thus the following formula is a λQp formula

1) $([(\lambda x_4)(\lambda x_5)[(y)([(\lambda x_1)(\lambda x_2)(\lambda x_3)[\phi x_1 x_2 x_3]],[x_4 x_5 y])]],[u_1 a])$

A) allows us, for instance, to immediately transform a λQp formula $\phi v_1 ab$ into the λ-formulas

2) $([(\lambda x_1)[\phi x_1 ab]],[v_1])$

3) $([(\lambda x_1)(\lambda x_2)[\phi x_1 x_2 b]],[v_1 a])$

4) $([(\lambda x_1)(\lambda x_2)[\phi x_1 a x_2]],[v_1 b])$

5) $([(\lambda x_1)(\lambda x_2)(\lambda x_3)[\phi x_1 x_2 x_3]],[v_1 ab])$

The interest, for us, in having such a rich syntax arises when we consider predicate modification. For one can introduce modifiers such that the semantics for the same modification of the four above formulas turns out to be different. (The * operator introduced in the next Section is an example of this.

Thus, our aim in introducing λ abstraction in the style of Wiggins is not to commit ourselves to an ontology of properties. (For instance, one might introduce λ abstraction to capture syntactically a traditional semantic view that a sentence is true iff the subject (or rather arguments) 'participate in' the predicate). Instead, it is to be able to have a sufficiently rich language for distinguishing between predicate modifiers and sentence operators.

In the next Section we offer semantics for a λQp language. We assume that a formula which is transformed into a λ formula without the addition of predicate modifiers has equivalent clauses to its transform. (Thus, formulas 2), 3), 4), 5) and $\phi v_1 ab$ are always true under the same conditions.) We assume that a λ-construct is a semantic unit in the same way that standard semantics assumes that atomic predicates are, viz. we give extension functions for both. However, in the λ-construct case, the extension is determined from the satisfaction conditions of the reverse transform.

Let us assume from now on that the set Δ of predicate modifiers consists of the five operators $\neg, *, H, G, A$. P, F, S will be abbreviations for $\neg H \neg$, $\neg G \neg$, $\neg A \neg$ respectively. The models for λQp are simply the Kripkean models $\langle T, >, \psi, D, g, t_n \rangle$ defined in Section 6.3.

Definition 1.

The value of a singular term of λQp in a model U relative to a sequence $s \in \Sigma(U)$, $g_s^*(u)$, is defined as before:

$$g_s^*(u) = \begin{cases} g(u) \text{ if u is a constant} \\ val(i,s) \text{ if u is the variable } v_i. \end{cases}$$

To state the definition of truth for λQp it will be convenient to define by simultaneous recursion the truth values of formulas and the extensions of λ-constructs. To assign extensions to these λ-terms is, it should be kept in mind, only a matter of technical convenience and should not be construed as a slightly dishonest way of retroactively conferring upon these terms the independent status which we took pains to deny them in the formulation of the syntax. It is easy enough to eliminate the reference to extensions of λ-constructs from the truth definition, but the definition thereby becomes a great deal more clumsy and less perspicuous. Where U_{tn} is a model for λQp we shall denote the extension of the λ-construct $\gamma = O[(\lambda x_1)\ldots(\lambda x_n)[\alpha\, {}^{x_1}/z_1 \ldots {}^{x_n}/z_n]]$ in U_{tn} at t as $h_{U_{tn}}(\gamma)(t)$. (We use this notation rather than that used so far to denote truth values, i.e. $U_{s\ tn}(\gamma)(t)$, to convey that the extension function is an auxiliary to the truth value assignment, which applies to genuine syntactic units.) Where no danger of ambiguity exists we shall drop the subscript U_{tn}, thus writing $h(\gamma)(t)$. (Note that since λ-constructs do not contain free variables their extensions never depend upon assignment sequences.)

Definition 2.

Let U_{tn} be a model for λQp, $s \in \Sigma(U_{tn})$, $t, t_n \in T_u$, the truth value of a formula α of λQp in U_{tn} at t relative to s, $U_{s\ tn}(\alpha)(t)$, and the extension of a λ-construct γ of λQp in U_{tn} at t, $h(\gamma)(t)$, are defined by simultaneous recursion via the following clauses:

i) $U_{s\ tn}(Q_i^n u_1 .. u_n)(t) = 1$ iff $\langle g_s^*(u_1) \ldots g_s^*(u_n)\rangle \in g(Q_i^n)(t) \subseteq [\psi(t)]^n$

ii) $U_{s\ tn}(\sim\alpha)(t) = 1$ iff $U_{s\ tn}(\alpha)(t) = 0$

iii) $U_{s\ tn}(\alpha \wedge \beta)(t) = 1$ iff $U_{s\ tn}(\alpha)(t) = 1$ and $U_{s\ tn}(\beta)(t) = 1$

iv) $U_{s\ tn}((\exists v_i)\alpha)(t) = 1$ iff

$(\exists s' \in \Sigma(U_{tn}))(s' \underset{i}{\approx} s \wedge val(i,s) \in \psi(t) \wedge U_{s'\ tn}(\alpha)(t) = 1)$

v) if α is of the form $(\gamma, [u_1 .. u_n])$ where γ is a λ-construct then

$U_{s\ tn}(\alpha)(t) = 1$ iff $\langle g_s^*(u_1) .. g_s^*(u_n)\rangle \in h(\gamma)(t)$

vi) Suppose γ is a λ-construct of the form $[(\lambda x_1) .. (\lambda x_n)[\alpha^{x_1}/z_1 .. {}^{x_n}/z_n]]$

Then $h(\gamma)(t) = \{\langle e_1 .. e_n\rangle : (\exists v_{j_1} \ldots v_{j_n})(\exists s' \in \Sigma(U))(v_{j_1} .. v_{j_n}$ do not occur in α for $1 \leq k \leq n$, $val(j_k, s') = e_k$ and $U_{s'\ tn}(\alpha^{v_{j_1}}/z_1 \ldots {}^{v_{j_n}}/z_n)(t) = 1\}$

vii) Suppose γ is a λ-construct of the form $O[\lambda x_1 .. \lambda x_n [\alpha^{x_1}/z_1 \ldots {}^{x_n}/z_n]]$

then $h(\neg\gamma)(t) = D^n_{U_{tn}} - h_{U_{tn}}(\gamma)(t)$

Similarly

viii) $h(*\gamma)(t) = [\psi(t)]^n - h(\gamma)(t)$

ix) $h(G\gamma)(t) = \{\langle e_1 .. e_n\rangle : (t')(t' > t \rightarrow \langle e_1 .. e_n\rangle \in h(\gamma)(t'))\}$

x) $h(H\gamma)(t) = \{\langle e_1 .. e_n\rangle : (t')(t' < t \rightarrow \langle e_1 .. e_n\rangle \in h(\gamma)(t'))\}$

xi) $h(A\gamma)(t) = \{\langle e_1 .. e_n\rangle : (t')(\langle e_1 .. e_n\rangle \in h(\gamma)(t'))\}$

xii) $U_{s\ tn}(G\alpha)(t) = 1$ iff $(t')(t' > t \rightarrow U_{s\ tn}(\alpha)(t') = 1)$

xiii) $U_s(H\alpha)(t) = 1$ iff $(t')(t' < t \rightarrow U_{s\ tn}(\alpha)(t') = 1)$.

The definition of truth is that of Definition 4 of Section 6.2 and validity is t_n validity as in Section 6.3.

$h(\neg H \neg \lambda\beta)(t)$, viz. $h(P\lambda\beta)(t)$, is as expected

$$h(\neg H\neg\lambda\beta)(t) = D^n - h(H\neg\lambda\beta)(t)$$

$$= D^n - Y^n \text{ where } e^n \in Y^n \text{ iff}$$

$$(t')(t>t' \rightarrow e^n \in h(\neg\lambda\beta)(t'))$$

$$= D^n - Y^n \text{ where } e^n \in Y \text{ iff}$$

$$(t')(t>t' \rightarrow e^n \in (D^n - h(\lambda\beta)(t')))$$

and so

$$e^n \in h(\neg H\neg\lambda\beta)(t) \text{ iff } (\exists t')(t>t' \wedge e^n \notin (D^n - h(\lambda\beta)(t')))$$

$$\text{iff } (\exists t')(t>t' \wedge e^n \in h(\lambda\beta)(t')).$$

Although syntactically and semantically $\neg, H, G, A$ are predicate modifiers on the basis of criteria given in the last Section they nevertheless do perform very much like ~,G,H,A. For instance the λ-sentence $(\neg[(\lambda x_1)[\emptyset x_1]],[a])$ can only be true in the same circumstances as ~Øa. Thus, the distinction between $\neg$ and ~ connects up with the earlier discussion of Aristotle's account of indefinite verbs in 1.4. For it was there conjectured that Aristotle may have held that a sentence like 'Socrates does not recover' may be analysed both as 'It is not the case that Socrates recovers' and as 'Socrates does not recover' where the verb is '...does not recover'. It is the latter analysis which is relevant to his notion of indefinite verb. A second point to note is that although A is a predicate modifier it does not when understood modally express weak necessity. This means then that the distinction between predicate modifier and sentence operator, in the case of necessity does not correspond simply to the de re/de dicto distinction. In connection with this latter point it is interesting to note that none of the modifiers $\neg, H, G, A$ exclusively map atomic predicates into non-ampliating predicates (unlike *). (Consequently, our use of λ abstraction is not intended to underly that view expressed in Section 6.7 that there are properties. For instance, $(\neg[(\lambda x_1)[\emptyset x_1]],[a])$ can be true at a time even if a does not then exist.)

Because of clauses (v) and (vi) of Definition 2, any λ formula which does not contain predicate modifiers is equivalent to a λ free formula. Suppose α is the λ formula $([(\lambda x_1)..(\lambda x_n)[\emptyset x_1...x_n]],[u_1..u_n])$ then (1) holds

(1) α iff $\emptyset u_1...u_n$

And clearly (1) holds for α when it is a subformula of a formula also containing quantifiers and connectives.

Some terminology is useful here. We call a formula which contains no predicate modifiers a normal formula, and if a formula containing predicate modifiers is equivalent to a normal formula we say that it is reducible to normal form. Now, the semantics for λQp are an extension of Kripke's, given in Section 6.3. For λ-free formulas the semantics are identical. Moreover, the semantics for a λ-normal formula are equivalent to those for λ-free formulas, given (1) above. So, it is the addition of predicate modifiers which makes λQp more than a syntactic variant of a Kripkean language.

However, λ-formulas which do not contain * are reducible to normal form. Suppose α is of the form $(O\lambda\beta,[u_1...u_n])$, where λβ is a λ-construct, and O is any set of predicate modifiers (including *) then the following hold

(2) $(\neg O\lambda\beta,[u_1...u_n])$ iff $\sim\alpha$

(3) $(HO\lambda\beta,[u_1...u_n])$ iff $H\alpha$

(4) $(GO\lambda\beta,[u_1..u_n]$ iff $G\alpha$

(5) $(AO\lambda\beta,[u_1...u_n])$ iff $A\alpha$.

(See (vii), (ix), (x), (xi) and (ii), (xii), (xiii) of Definition 2 and the definition of A. For instance, the proof of (2) is

$U_{s\ tn}(\sim\alpha)(t) = 1$ iff $U_{s\ tn}(\alpha)(t) = 0$

iff $\langle g(u_1)\ldots g(u_n)\rangle \notin h(0\lambda\beta)(t)$

iff $\langle g(u_1)\ldots g(u_n)\rangle \in D^n - h(0\lambda\beta)(t)$

iff $\langle g(u_1)\ldots g(u_n)\rangle \in h(\neg 0\lambda\beta)(t)$

iff $U_{s\ tn}(\neg 0\lambda\beta, [u_1..u_n])$).

The interesting modifier then is *. If $\lambda\beta$ is $0[(\lambda x_1)..(\lambda x_n)[\emptyset x_1..x_n]]$ then (6) holds, where 0 is any set of predicate modifiers,

(6) $h(*\lambda\beta)(t) \subseteq [\psi(t)]^n$.

If ψ is an atomic predicate and 0 is empty then (7) holds

(7) $h(*\lambda\beta)(t) \cup h(\lambda\beta)(t) = [\psi(t)]^n$.

This is a consequence of (8) and the APNA assumption (see Definition 2 (i))

(8) $h(\lambda\beta)(t) = g(\emptyset)(t)$.

Moreover in this case (9) holds

(9) $h(**\lambda\beta)(t) = h(\lambda\beta)(t) = g(\emptyset)(t)$.

We also have that (10) and (11) hold for any predicate construct $\lambda\beta$ which is $0[(\lambda x_1)\ldots(\lambda x_n)[\emptyset x_1..x_n]]$, and variables $v_1\ldots v_n$.

(10) $(v_1)\ldots(v_n)(*\lambda\beta, [v_1\ldots v_n])$ iff $(v_1)\ldots(v_n) \sim (\lambda\beta, [v_1\ldots v_n])$

(11) $(\exists v_1)\ldots(\exists v_n)(*\lambda\beta, [v_1\ldots v_n])$ iff $(\exists v_1)\ldots(\exists v_n) \sim (\lambda\beta, [v_1..v_n])$

These hold because of the clauses for the restricted quantifiers. (Note that these are closed formulas where the term set $[v_1\ldots v_n]$ only contains variables and not individual constants as well.)

We now argue that * is useful for representing weak necessity. From (10) and (11) we know that (12) and (13) hold

(12) $(z)(*[(\lambda x)[\emptyset x]], [z])$ iff $\sim(\exists z)([(\lambda x)[\emptyset x]], [z])$

(13) $(\exists z)(*[(\lambda x)[\emptyset x]], [z])$ iff $\sim(z)\ ([(\lambda x)[\emptyset x]], [z])$.

We show that (14) does not hold.

(14) $(z)A([(\lambda x)[\emptyset x]], [z])$ iff $(z) \sim S(*[(\lambda x)[\emptyset x]], [z])$.

That is, * does not perform with the tense operators (or modifiers) in the same sort of way that it does with the quantifiers in (12) and (13). The way we show (14) to be invalid is to show that it does not hold for atomic $\emptyset$.

Because of the APNA assumption (15) is valid for any monadic atomic predicate $\emptyset$.

(15) $(\exists y)S([(\lambda x)[\emptyset x]],[y]) \rightarrow S(\exists y)([(\lambda x)[\emptyset x]],[y])$

Furthermore because of the clause for * (16) holds

(16) $(\exists y)S(*[(\lambda x)[\emptyset x]],[y]) \rightarrow A(\exists y)(*[(\lambda x)[\emptyset x]],[y])$

(16) is equivalent to (17), as we have seen.

(17) $(\exists y)S(*[(\lambda x)[\emptyset x]],[y]) \rightarrow S(\exists y) \sim ([(\lambda x)[\emptyset x]],[y])$.

From (17) we deduce (19) as follows

(18) $\sim S(\exists y) \sim ([(\lambda x)[\emptyset x]],[y]) \rightarrow \sim(\exists y)S(*[(\lambda x)[\emptyset x]],[y])$

by S.C. on 11.

(19) $A(y)([(\lambda x)[\emptyset x]],[y]) \rightarrow (y)\sim S(*[(\lambda x)[\emptyset x]],[y])$

by Definition of A and (y)

which is reducible to the normal form, (20) if (14) is valid.

(20) $A(y)([(\lambda x)[\emptyset x]],[y]) \rightarrow (y)A([(\lambda x)[\emptyset x]],[y])$.

But (20) only holds if everything is omnitemporal. Thus (14) is not valid.

What then are the truth conditions of $(y)\sim S(*[(\lambda x)[\emptyset x]],[y])$ when understood as a modal formula? Let us take an instance of this formula, namely (21),

(21) $\sim S(*[(\lambda x)[x \text{ is a person}]],[\text{Socrates}])$

to bring out the semantic force of the iteration of modifiers. This formula is true iff it is not possible that (*[(λx)[x is a person]],[Socrates]). That is, in no possible world is (*[(λx)[x is a person]],[Socrates]) true. But *[(λx)[x is a person]] only holds of Socrates with respect to a possible iff Socrates exists there and is not a person there. Thus, (21) is true iff in no world in which Socrates exists is he not a person. That is (21) is a weak necessity claim. Or, alternatively, the iteration, $\neg S^*$, of predicate modifiers appears to express weak necessity.

Up to now we have not distinguished between weak necessity claims and essentialist claims. Accordingly to Wiggins' account an essentialist claim can only be true of an object which actually exists. Hence (22)

(22) $(\neg S^*[(\lambda x)[x \text{ is a dog}]],[\text{Cerberus}])$

is not a true essentialist claim, even though it might be claimed to be a true weak necessity claim. However, the complex predicate modifier $*S*$ is adequate for representing essentialist claims. The simplest way of showing this is by noting that * formulas are reducible to normal form in a language $\lambda Qp \cup \{E!\}$ where E! is the existence predicate. For (23) holds

(23) $(*[(\lambda x_1)....(\lambda x_n)[\emptyset x_1..x_n]],[u_1...u_n])$ iff

$E!u_1 \wedge E!u_2 \wedge ... \wedge E!u_n \wedge\sim ([(\lambda x_1)...(\lambda x_n)[\emptyset x_1..x_n]],[u_1...u_n])$

Hence, $(\neg S^*[(\lambda x)[\emptyset x]],[u])$ is equivalent to (24), whereas $(*S^*[(\lambda x)[\emptyset x]],[u])$ is equivalent to (25).

(24) $A(E!u \rightarrow ([(\lambda x)[\emptyset x]],[u]))$

(25) $E!u \wedge A(E!u \rightarrow ([(\lambda x)[\emptyset x]],[u]))$.

These reduction theses are discussed in Section 7.1. (Clearly, in the case of $\lambda Qp \cup \{E!\}$ the introduction of λ abstraction merely adds to the syntactic richness of the languages without adding to the semantics). However, we shall use the reduction thesis (23) to help unravel what a * formula means.

In the case of weak omnitemporality 'Socrates was always a person' is representable by (26).

(26) $P(*S^*[(\lambda x)[x \text{ is a person}]],[\text{Socrates}])$

Representing it by (27) is objectionable

(27) $(\neg P^*[(\lambda x)[x \text{ is a person}]],[\text{Sorates}])$

because it is reducible to (28)

(28) $H(E!\text{Socrates} \rightarrow ([(\lambda x)[x \text{ is a person}]],[\text{Socrates}]))$

which would mean that 'Socrates will always be a mouse' is true, because Socrates will not exist in the future.

Wiggins uses the λ abstraction notation to distinguish between purported asymmetric modal properties. He says

> "Kripke maintains that it is a de re necessity for Elizabeth II to be the child of George VI even if George VI did not need to beget Queen Elizabeth."
>
> (1976 A p.310).

According to this claim we need to distinguish between (29) and (30)

(29) 'x necessarily begat Elizabeth II' is true of George VI

(30) 'George VI necessarily begat x' is true of Elizabeth II

It is claimed that (29) is false because there are possible worlds in which George VI exists but where he did not begat Elizabeth II because no-one did, for she doesn't exist there. On the other hand it is claimed that (30) is true because in whatever world Elizabeth II exists, she was begat by George VI. And clearly one needs a sufficiently rich language to distinguish between these claims. Using * (29) and (30) are represented by (29') and (30').

(29') (*$\mathcal{S}$*[(λx)[x begat Elizabeth II]],[George VI])

(30') (*$\mathcal{S}$*[(λx)[George VI begat x]],[Elizabeth II])

In reduced form these are (29") and (30").

(29") (E!George VI$\wedge$ A(E!George VI -> ([(λx)[x begat Elizabeth II]], [George VI])))

(30") (E!Elizabeth II $\wedge$ A(E!Elizabeth II -> ([(λx)[George VI begat x]]. [Elizabeth II])))

Buw what about (31)?

(31) 'x necessarily begat y' is true of George VI and Elizabeth II respectively.

This is open to more than one reading. It may be understood as saying that in all possible worlds in which both George VI and Elizabeth II exist then that predicate is true of them. Alternatively it may say that both George VI and Elizabeth II exist in the same possible worlds and that that predicate is true of them there. However, Kripkean weak necessity seems to fit the former alternative when he says

> "We count statements as (weakly) necessary if whenever the objects mentioned therein exist the statement would be true."
>
> (1971 p.137).

(And this prevents a disjunctive reading here of (31) along the lines that if George VI or Elizabeth II exists then that predicate applies to them). It is the former alternative which is captured in (31')

(31') $(*\mathcal{S}*[(\lambda x_1)(\lambda x_2)[x_1 \text{ begat } x_2]], [\text{George VI}, \text{Elizabeth II}])$

The use of λ abstraction here, however, does prevent a generalization of the distinction between (21) and (30). For instance (32) is not representable using *.

(32) $(E!y \wedge A(E!y \wedge \emptyset xy))$

The problem here is that one cannot have free variables within the scope of a λ operator. (Alternative notations can be given which preclude this problem.) We comment upon this in Section 7.1 when we consider the relationship between the λ system (given in the next Section) and the reduced version.

Is essentialist necessity a predicate modifier? Well, clearly in one good sense it is. Relative to λQp $*\mathcal{S}*$ is such a modifier. But given the reducibility of * in a language with an existence predicate then it is no longer so clear.

6.10 * Formulas and the System λQK_L

In the last Section semantic clauses were given for the language λQA. And as noted these clauses are an extension of those given for Kripke systems (see 6.3). Because of this the system which satisfies these clauses when linear time is assumed, called λQK_L, is an extension of CQK_L introduced in Section 6.4. The extension involved depends upon there being * formulas which are not reducible to normal form. In the last Section we saw how * adds to the expressibility of Kripke systems when considering representation of certain claims. In this Section we further consider * formulas, and provide axioms for λQpK_L.

In Section 3.5 it was suggested that one could make the past and perfect tense semantically distinguishable by demanding that (1) hold for the perfect tense

(1) a has ϕen $\rightarrow$ a exists

which does fit in with ordinary usage to a large extent. So if distinguished in this way and where ϕ is monadic the difference between the simple past and the perfect is that between (2) and (3)

(2) $P([(\lambda x_1)[\emptyset x_1]],[a])$

(3) $(*\neg P[(\lambda x_1)[\emptyset x_1]],[a])$

In reduced form, in the language $\lambda Qp \cup \{E!\}$ (3) is equivalent to (3')

(3') $E!a \wedge P([(\lambda x_1([\emptyset x_1]],[a])$

For dealing with cases where a predicate is polyadic one needs to distinguish the subject place (relative to a context). For example, 'Mary has loved John' is represented by (4).

(4) $(*\ P[(\lambda x_1)[x_1 \text{ loves John}]],[\text{Mary}])$

In 6.7 it was said that our aim was to bring into one two valued system both the advantages of Prior's tensed Q and that of the Kripkean CQK_L. Now, so far we have noted that Kripke's system is contained in λQK_L. In the case of Q, however, valid formulas need to be re-expressed because of Prior's demonstrative view of free variables. And as we shall note this may be done in terms of * formulas. First, using the language of λQp we note why there is a breakdown between operators and their standard duals in System Q (see Section 6.5).

In Prior's system Q the semantic force of 'PØy' may be seen as a result of a combination of two sources; first from that of the operator P and secondly from that of his demonstrative view of free variables. In terms of λQpK_L, which assumes a non-demonstrative account of variables, 'PØy' (i.e. (PØthis') is true in Q iff (5) is true in λQpK_L.

(5) $(*\neg P*\neg[(\lambda x)[\emptyset x]],[y])$

In reduced form (5) is equivalent to (5').

(5') $E!y \wedge P(E!y \wedge ([(\lambda x_1)[\emptyset x_1]],[y]))$.

On the other hand 'HØy' is true in Q iff (6) holds in λQpK_L (and where (6') is the reduced form).

(6) $(*\neg H*\neg[(\lambda x_1)[\emptyset x_1]],[y])$

(6') $E!y \wedge H(E!y \wedge ([(\lambda x_1)[\emptyset x_1]],[y]))$

But (6) is not the dual of (5) in λQpK_L. For the dual of the predicate modifier *¬P*¬ is not *¬H*¬ but ¬*¬P*, thus giving us (7), and in reduced form (7').

(7) $(\neg*\neg P*[(\lambda x_1)[\emptyset x_1]],[y])$

(7') $\sim(E!y \wedge P(E!y \wedge\sim([(\lambda x_1)[\emptyset x_1]],[y])))$

We turn our attention to axiomatizing λQK_L. Besides the usual tense axioms for H and G we need axioms for the predicate modifiers. For these, except *, we use the reduction thees mentioned in the last Section. Suppose λα is a λ-construct of the form $O[(\lambda x_1)..(\lambda x_1)[\emptyset x_1..x_n]]$, where O is any series of modifiers then

(8) to (10) hold.

(8) $\sim(\lambda\alpha, [u_1..u_n])$ iff $(\neg\lambda\alpha[u_1..u_n])$

(9) $G(\lambda\alpha, [u_1..u_n])$ iff $(G\lambda\alpha, [u_1..u_n])$

(10) $H(\lambda\alpha, [u_1..u_n])$ iff $(H\lambda\alpha, [u_1..u_n])$

We also need an axiom to connect λ-formulas and non-λ formulas. (11) is this axiom

(11) $(\lambda\alpha, [u_1..u_n])$ iff $\emptyset u_1..u_n$ provided 0 is empty.

This leaves us with providing axioms for * and the quantifiers. We use the following three as axioms for *.

(12) $(*\lambda\alpha, [u_1..u_n])$ -> $(\neg\lambda\alpha, [u_1..u_n])$

(13) $(**\ \lambda\alpha[u_1..u_n])$ iff $(*\neg\lambda\alpha, [u_1..u_n])$

(14) $(*\lambda\alpha, [u_1..u_n])$ -> $(***\lambda\alpha, [u_1..u_n])$.

These are valid according to the semantics given in the last Section. (12) is so because $h(*\lambda\alpha)(t) \subseteq h(\neg\lambda\alpha)(t)$, for the former is $[\psi(t)]^n - h(\lambda\alpha)(t)$ and the latter $D^n - h(\lambda\alpha)(t)$. We now show that (13) holds.

$$h(**\lambda\alpha)(t) = [\psi(t)]^n - ([\psi(t)]^n - h(\lambda\alpha)(t))$$
$$= [\psi(t)]^n - (D^n - h(\lambda\alpha)(t))$$
$$= h(*\neg\lambda\alpha)(t).$$

And (14) is shown

$$h(*\lambda\alpha)(t) = [\psi(t)]^n - h(\lambda\alpha)(t)$$
$$= [\psi(t)]^n - ([\psi(t)]^n \cap h(\lambda\alpha)(t))$$
$$= [\psi(t)]^n - ([\psi(t)]^n - ([\psi(t)]^n - h(\lambda\alpha)(t)))$$
$$= h(***\lambda\alpha)(t).$$

(We have also proved the converse of (14) here which is derivable from (13) and (12)).

In the last Section we noted a converse connection between * and ¬ than that given in (12) but within a quantified formula. (See (10) and (11) of Section 6.9.) Thus, we also have (15).

(15) $(y_1)\ldots(y_n)((\neg\lambda\alpha,[y_1..y_n]) \rightarrow (*\lambda\alpha,[y_1..y_n]))$

Here, we may note that (15) is a closed formula.

What then of quantifier axioms? Our aim is to have an open system. But, the usual quantifier axiom with the usual restriction,

(16) $(y)\emptyset y \rightarrow \emptyset^{z}/y$

would mean that * and ¬ are equivalent. (From (15) one could derive the converse of (12)). Anyway, (16) is objectionable because the quantifier is restricted. However, what is valid is (17).

(17) $(y)([(\lambda x_1)[\emptyset x_1]],[y]) \rightarrow (\neg *[(\lambda x_1)[0x_1]],[^{z}/y])$.

In reduced notation (17) is (17')

(17') $(y)([(\lambda x_1)[\emptyset x_1]],[y]) \rightarrow (E!z \rightarrow ([(\lambda x_1)[\emptyset x_1]],[^{z}/y]))$

which is, in this monadic case, the free logic axiom for quantification theory.

We now give axioms for λQK_L followed by some comments. Soundness and completeness proofs are given in the appendix. We assume that β, Δ are any λQ formulas and that λα is a construct of the form $0[(\lambda x_1)\ldots(\lambda x_n)[\gamma\,^{x_1}/z_1\ldots\,^{x_n}/z_n]]$ where γ is any λQp formula.

λQK_L 1) Any S.C. instance.

λQK_L 2) $G(\beta \rightarrow \Delta) \rightarrow (F\beta \rightarrow F\Delta)$

λQK_L 3) $H(\beta \rightarrow \Delta) \rightarrow (P\beta \rightarrow P\Delta)$

λQK_L 4) $G\beta \rightarrow GG\beta$

λQK_L 5) $H\beta \rightarrow HH\beta$

λQK_L 6) $PG\beta \rightarrow \beta$

λQK_L 7) $FH\beta \rightarrow \beta$

λQK_L 8) $F\beta \wedge F\Delta \rightarrow F(\beta \wedge \Delta) \vee F(\beta \wedge F\Delta) \vee F(F\beta \wedge \Delta)$

λQK_L 9) $P\beta \wedge P\Delta \rightarrow P(\beta \wedge \Delta) \vee P(\beta \wedge P\Delta) \vee P(P\beta \wedge \Delta)$

λQK_L 10) $\sim(\lambda\alpha,[u_1..u_n])$ iff $(\neg\lambda\alpha,[u_1..u_n])$

λQK_L 11) $G(\lambda\alpha,[u_1..u_n])$ iff $(G\lambda\alpha,[u_1..u_n])$

λQK_L 12) $H(\lambda\alpha,[u_1..u_n])$ iff $(H\lambda\alpha,[u_1..u_n])$

λQK_L 13) $(\lambda\alpha,[u_1..u_n])$ iff $\gamma\ {}^{u_1}/z_1..{}^{u_n}/z_n$ provided that 0 is empty

λQK_L 14) $(*\lambda\alpha,[u_1..u_n]) \rightarrow (\neg\lambda\alpha,[u_1..u_n])$

λQK_L 15) $(**\lambda\alpha,[u_1..u_n])$ iff $(*\neg\lambda\alpha,[u_1..u_n])$

λQK_L 16) $(*\lambda\alpha,[u_1..u_n]) \rightarrow (***\lambda\alpha,[u_1..u_n])$

λQK_L 17) $(y_1)...(y_n)((\neg\lambda\alpha,[y_1..y_n]) \rightarrow (*\lambda\alpha,[y_1..y_n]))$

λQK_L 18) $(y)(\beta \rightarrow \Delta) \rightarrow ((y)\beta \rightarrow (y)\Delta)$

λQK_L 19) $(y)(\lambda\alpha,[y,u_2..u_n]) \rightarrow (\neg*\lambda\alpha,[{}^z/y,u_2..u_n])$

The rules are RG,RH, Gen and MP.

Axioms 2) to 9) ensure a system for dense linear time. 10) to 13) are the reduction axioms. 14) to 17) are * axioms and the rest are quantifier axioms. 17) is, perhaps, more 'open' than expected. This is because $(\neg*)^n, n\geq 1$ is equivalent to $\neg*$ (and $(*\neg)^n$ is equivalent to $*\neg$). This is shown.

(18) $(\lambda\alpha,[u_1..u_n]) \rightarrow (\neg*\lambda\alpha,[u_1..u_n])$ from Ax. 14 and 10.

Therefore, an instance of (18) is (19).

(19) $(\neg*\lambda\alpha\ ,[u_1..u_n]) \rightarrow (\neg*\neg*\lambda\alpha,[u_1..u_n])$

For the converse we have (20)

(20) $(*\lambda\alpha,[u_1..u_n]) \rightarrow (*\neg*\lambda\alpha,[u_1..u_n])$ from Ax. 16 + 15.

Therefore

(21) $(\neg*\neg*\lambda\alpha,[u_1..u_n]) \rightarrow (\neg*\lambda\alpha,[u_1..u_n])$.

Our aim was to bring together the advantages of both Kripke's and Prior's systems. We noted in Section 6.5 that within Prior's system Q the following tensed mixing formulas (and their minor images) are valid.

(22) $G(y)\emptyset y \rightarrow (y)\sim F\sim\emptyset y$

(23) $(\exists y)F\emptyset y \rightarrow F(\exists y)\emptyset y$

(24) $F(y)\emptyset y \rightarrow (y)\sim G\sim\emptyset y$

(25) $(\exists y)G\emptyset y \rightarrow G(\exists y)\emptyset y$

In Kripke's system there are no valid mixing formulas analogous to (22) to (25). However, in λQK_L there are such formulas even though the system is two valued,

(26) G(y)([(λx)[Øx]],[y]) -> (y)G(¬*[(λx)[Øx]],[y])

(27) (∃y)F(*¬[(λx)[Øx]],[y]) -> F(∃y)([(λx)[Øx]],[y])

(28) F(y)([(λx)[Øx]],[y]) -> (y)~G(*[(λx)[Øx]],[y])

(29) (∃y)G(*¬[(λx)[Øx]],[y]) -> G(∃y)([(λx)[Øx]],[y])

together with their mirror images. These have the same semantic force as Prior's (22) to (25). (27) and (26) are versions of the tensed converse Barcan Formulas where (29) and (28) are versions of the Buridan formulas. To finish off this Section we prove (26) and (28). ((27) and (29) are straightforwardly derivable from the other two.)

i) (y)([(λx)[Øx]],[y]) -> (¬*[(λx)[Øx]],[^{z}/y]) Ax. 17

ii) G(y)([(λx)[Øx]].[y]) -> G(¬*[(λx)[Øx]],[^{z}/y]) Rule RG and Ax. 2

iii) G(y)([(λx)[Øx]],[y]) -> (z)G(¬*[(λx)[Øx]],[z]) Gen + q-theory

i) (y)([(λx)[Øx]],[y]) -> ~(*[(λx)[Øx]],[^{z}/y]) Ax. 17 + 10

ii) F(y)([(λx)[Øx]],[y]) -> F~(*[(λx)[Øx]],[^{z}/y]) RG and Ax. 2

iii) F(y)([(λx)[Øx]],[y]) -> (z)~G(*[(λx)[Øx]],[z]) Gen + q-theory and defn. of G.

7.1 The Reducibility Theses.

The system λQK_L of Section 6.10 depended upon making the syntax of Kripkean languages more complex. If we also add to the language λQp the tensed existence predicate, E!x, then a * reducibility formula is available.

(1) $(*[(\lambda x_1)..(\lambda x_n)[\emptyset x_1...x_n]],[u_1..u_n])$ iff $E!u_1 \wedge ... \wedge E!u_n \wedge \sim([(\lambda x_1)...(\lambda x_n)[\emptyset x_1..x_n]],[u_1..u_n])$

Here we assume that the semantics for E!y is given by (2).

(2) $g(E!)(t) = [\psi(t)]$

(Here, we are treating E! as an atomic predicate.)

Given the exists predicate, λ abstraction becomes unnecessary. For we can reduce all λ-formulas containing predicate modifiers to ones which do not, and thus re-express these formulas without the use of λ abstraction. In fully reduced form the quantifier axiom of λQK_L becomes (3).

(3) $(y)\emptyset y \rightarrow (E!z \rightarrow \emptyset^z/y)$

This is the 'free logic' axioms for quantification theory. The resulting Kripkean semantics for linear time, without λ abstraction, and with the existence predicate clause, is satisfied by a system FQK_L. Its axiomatization is simply that for standard quantificational tense logic, assuming time's linearity, except instead of the standard quantifioation axiom $(x)\emptyset x \rightarrow \emptyset^z/y$, one has (3).

λQK_L and FQK_L are not equivalent because there are formulas in FQK_L which cannot be expressed in λQK_L. For instance, (4) is a theorem of FQK_L which has no λQK_L counterpart.

(4) $(E!x \wedge F(E! \wedge \emptyset xy)) \vee \sim(E!x \wedge F(E!x \wedge \emptyset xy))$.

The problem here is that the notation we have used for λ abstraction does not allow free variables to occur within a λ-construct. Hence the formula (4') which would be equivalent to (4) without this restriction is not well-formed.

(4') $(*\neg F*\neg[(\lambda x)[\emptyset x_1 y]],[z]) \vee\sim (*\neg F*\neg[(\lambda x_1)[\emptyset x_1 y]],[z])$.

However, if a formula in λQK_L then there is a related formula which is a theorem of FQK_L. This followsstraightforwardly from the reduction theses of λQK_L together with the * reduction thesis and the fact that the reduced quantification axiom of λQK_L is (3). (See theorem below for an explicit method to show how this could be proved.)

As we noted in Section 6.4 the introduction of the free-logic axiom (3) is Thomason's solution to the problem of the invalidity of the mixing formulas. For now the valid modal versions of the λQK_L 'mixing' formulas of Section 6.10 become in FQK_L, (5) to (8).

(5) $A(x)\alpha \to (x)A(E!x \to \alpha)$

(6) $(\exists x)S(E!x \wedge \alpha) \to S(\exists x)\alpha$

(7) $S(x)\alpha \to (x)F(E!x \to \alpha)$

(7) $(\exists x)A(E!x \wedge \alpha) \to A(\exists x)\alpha$.

And (5) is provable in FQK_L as follows.

(i) $(x)\alpha \to (E!y \to \alpha^y/x)$

(ii) $A((x)\alpha \to (E!y \to \alpha^y/x))$

(iii) $A(x)\alpha \to A(E!y \to \alpha^y/x)$

(iv) $A(x)\alpha \to (x)A(E!a \to \alpha)$.

This shows, then, the Kripkean quantifiers are best embedded within what appears to be a free logic. Moreover, this also shows that weak necessity is expressible using open sentence operators together with the existence predicate. But why have tensed quantifiers together with a tensed existence predicate? For after all the tensed existential quantifier represents tensed existence as well as the tensed existence predicate. Where $(\forall x)(-x-)$ and $(Ex)(-x-)$ are the Kripkean quantifiers and $(x)(-x-)$ and $(\exists x)(-x-)$ the standard tenseless quantifiers then both (9) and (10) hold.

(9) $(\forall x)(-x-)$ iff $(x)(E!x \rightarrow -x-)$

(10) $(Ex)(-x-)$ iff $(\exists x)(E!x\ -x-)$.

So why not introduce Kripkean interpretations which contain clause (2) above yet which do not contain the Kripke quantifier clause but instead the standard clause as given in Section 6.2? That is, system FQK_L is reducible to the system QK_L (see 6.1) expressed in a language containing the tensed existence predicate. So, provided the introduction of the free logic axiom is motivated only by tense considerations and not singular term considerations then standard quantification theory together with an existence predicate is adequate. For instance (5') to (8') which are equivalent to (5) to (8) are valid QK_L formulas.

(5') $A(x)(E!x \rightarrow \alpha) \rightarrow (x)(E!x \rightarrow A(E!x \rightarrow \alpha))$

(6') $(\exists x)(E!x \wedge S(E!x \wedge \alpha)) \rightarrow S(\exists x)(E!x \wedge \alpha)$

(7') $S(x)(E!x \rightarrow \alpha) \rightarrow (x)(E!x \rightarrow S(E!x \rightarrow \alpha))$

(8') $(\exists x)(E!x \wedge A(E!x \wedge \alpha)) \rightarrow A(\exists x)(E!x \wedge \alpha)$.

And (5') is provable as follows.

(i) $(x)(E!x \rightarrow \alpha) \rightarrow (E!y \rightarrow \alpha^{y}/x)$

(ii) $A((x)(E!x \rightarrow \alpha) \rightarrow (E!y \rightarrow \alpha^{y}/x))$

(iii) $A(x)(E!x \rightarrow \alpha) \rightarrow A(E!y \rightarrow \alpha^{y}/x)$

(iv) $A(x)(E!x \rightarrow \alpha) \rightarrow (E!y \rightarrow A(E!y \rightarrow \alpha^{y}/x))$ By S.C.

(v) $A(x)(E!x \rightarrow \alpha) \rightarrow (x)(E!x \rightarrow A(E!x \rightarrow \alpha))$.

Consequently, there are two reducibility theses connected with λQK_L. First, the * reducibility thesis (1) where λQK_L is reduced to FQK_L which takes a free logic form because it involves tensed quantifiers.

The second reducibility thesis is given by (9) or (10) and this is the reducibility of FQK_L to QK_L which employs the standard quantifiers but expressed in a language involving the tensed existence predicate.

Consequently, the following theorem holds where E! is the tensed existence predicate.

Theorem

If $\vdash_{\lambda QK_L} \alpha$ then $\vdash_{QK_L \cup \{E!\}} \alpha'$
where α' is a transform of α in QK_L.

First, we specify inductively the transformation of α to α'.

i) If α is of the form (y)β then α' is (y)(E!y -> β')

ii) If α is of the form ~β then α' is ~β'

iii) If α is of the form (β v Δ) then α' is (β' v Δ')

iv) If α is of the form G then α' is G '

v) If α is of the form Hβ then α' is Hβ'

vi) If α is a λ-formula of the form $(G\lambda\alpha, [u_1.u_n])$ then α' is $G(\lambda\alpha, [u_1..u_n])'$.

vii) If α is a λ-formula of the form $(H\lambda\alpha, [u_1..u_n])$ then α' is $H(\lambda\alpha, [u_1..u_n])'$

viii) If α is a λ-formula of the form $(\neg\lambda\alpha, [u_1..u_n])$ then α' is $(\sim(\lambda\alpha, [u_1..u_n]))'$

ix) If α is a λ-formula of the form $(*\lambda\alpha, [u_1..u_n])$ then α' is $E!u_1 \wedge \ldots \wedge E!u_n \wedge (\sim(\lambda\alpha, [u_1..u_n]))'$

x) If α is a λ-formula of the form $([(\lambda x_1)..(\lambda x_n)[\emptyset x_1..x_n]], [u_1..u_n])$ then α' is $\emptyset' u_1 \ldots u_n$

xi) If α is an atomic formula $\emptyset u_1..u_n$ then α' is $\emptyset u_1..u_n$.

It is sufficient to show that all the transforms of the axioms of λQK_L are theorems of $QK_L \cup \{E!\}$ and that the restricted Gen in λQK_L

holds in $QK_L \cup \{E!\}$. First we show that the transforms of λQK_L are theorems. Clearly, this has only to be done for λQK_L 14) to 19), excluding λQK_L 18). (See 6.10)

We have to show the following

A) $\vdash_{K_L} ((*\lambda\alpha, [u_1..u_n]) \text{ -> } (\neg\lambda\alpha, [u_1..u_n]))'$

B) $\vdash_{K_L} ((**\lambda\alpha, [u_1..u_n]) \text{ iff } (*\neg\lambda\alpha, [u_1..u_n]))'$

C) $\vdash_{K_L} ((*\lambda\alpha, [u_1..u_n]) \text{ -> } (**\lambda\alpha, [u_1..u_n]))'$

D) $\vdash_{K_L} ((y_1)..(y_n)((\lambda\alpha, [u_1..u_n]) \text{ -> } (*\lambda\alpha, [y_1..y_n])))'$

E) $\vdash_{K_L} ((y)(\lambda\alpha, [y, u_2..u_n]) \text{ -> } (\neg*\lambda\alpha, [^z/y, u_2..u_n]))'$

Suppose throughout $(\lambda\alpha, [u_1..u_n])'$ is β and $(E!u_1\wedge..\wedge E!u_n)$ is Δ

A) Using ii), iii), viii) and ix) this reduces to the theorem

$\vdash_{K_L} \Delta \wedge\sim\beta \text{ -> } \sim\beta$

B) Similarly this reduces to the theorem

$\vdash_{K_L} \Delta \wedge (\Delta \text{ -> } \beta) \text{ iff } (\Delta \wedge \beta)$

C) This becomes

$\vdash_{K_L} (\Delta \wedge\sim \beta) \text{ -> } (\Delta \wedge (\Delta \text{ -> } (\Delta \wedge\sim \beta)))$

D) This becomes

$\vdash_{K_L} (u_1)..(u_n)(\Delta \text{ -> } (\sim\beta \text{ -> } (\Delta \wedge\sim \beta)))$

E) This again is a theorem

$\vdash_{K_L} (y)(E!y \text{ -> } \beta^y/u_1) \text{ -> } (\Delta^z/u_1 \text{ -> } \beta^z/u_1).$

Since λQK_L has the rule

if $\vdash_{\lambda QK_L} \alpha$ then $\vdash_{\lambda QK_L} (v_i)\alpha$

we must show that for each λQK_L-formula α

if $\vdash_{K_L} \alpha'$ then $\vdash_{K_L} ((v_i)\alpha)'$

That is,

if $\vdash_{K_L} \alpha'$ then $\vdash_{K_L} (v_i)(E!v_i \text{ -> } \alpha')$

But this clearly holds because

$\vdash_{K_L} \alpha' \text{ -> } (E!x \text{ -> } \alpha').$

Hence the theorem is proved.

But what about the modal case? Certainly, the * reducibility thesis is acceptable but the problem is the other reducibility thesis. For this depends upon allowing quantifiers to range over all possible objects. (Scott 1970 p.145 says that he has been persuaded by the UCLA school that quantification over possible objects is acceptable. His acceptance here is based upon formal similarities between modal and tense logics; becau he believes that quantification over past and future objects as well as present is required in tense logics this is taken to show that in modal logic one has to quantify over all possible objects - that is, objects in all possible worlds. We believe that this is not a good argument for quantification over possible objects for reasons which are rather complex and not considered in this essay).

In the tense case then the reducibility theses depend upon the acceptability of a tensed existence predicate in first order languages. And the rest of this chapter is concerned with arguing for this acceptability together with a discussion of existence in general. Furthermore, we try to argue for a particular interpretation of standard quantification theory.

7.2 The Inclusive Ambiguity Thesis.

The difficulties of taking 'exists' to be a first order predicate true of everything appear to be insurmountable. First, there is that paradox of reference; if 'a' is a semantic constituent of 'a exists' then surely it should also be a constituent of 'a does not exist'. But, how could this sentence ever be false (given the standard semantics for singular terms). Secondly, if 'exists' is a first order predicate like '...is red' then it should express a property but what a seemingly odd uninformative property it would be. Furthermore, 'a does not exist' should then express that something lacks a property but what sort of thing may lack this property? These difficulties have been taken to show that 'exists' is not a first order predicate. Consequently, in the sentence 'a exists' the grammatical predicate is not the logical predicate and because subject and predicate are correlative 'a' is not the logical subject. (Geach 1969A p.54). Instead 'exists' is taken to be a second order predicate represented by the existential quantifier, '$(\exists x)(-x-)$' - see 5.2.

Additional to this it may be asked whether this is true of all uses of 'exists' (see Geach ibid). Suppose, for example, there are good arguments for claiming that there are at least two senses of 'exists' one more inclusive than another then it appears to be legitimate to predicate non-existence of things in the more exclusive sense provided it is presupposed that they exist in the less select sense. Now, let us say that an expression is 'inclusively ambiguous' if in one sense it is true of some of the things it is true of in another (closely connected) sense. For instance, the predicate '...is a man' is ambiguous in this way; in one sense, it is true of any member of the human race and in another just of the adult male population. Now, is 'exists' inclusively ambiguous? Belief that it is so was quite popular at the turn of the century where it was defended by both Russell and Moore during their early Realist days. Russell sums it up

> "Being is that which belongs to every conceivable term to every possible object of thought... Existence, on the other hand is the prerogative of some only amongst beings." (1903 p.449).

(This view must be distinguished from Russell's later one mentioned in 4.2). Interestingly, neither Meinong nor the Scottish logician MacColl held that 'exists' or 'being' is inclusively ambiguous - although it is difficult to see how else their position can be understood. This is the source of the view that Russell misunderstood both these philosophers since his criticisms of them after 1904 appear to be directed against inclusive ambiguity theorists. (see 7.4).

What is central to the truth of this ambiguity in the case of 'exists' is the truth of the sentence 'Something does not exist' taken at face value. Recently, this has been accepted by a number of logicians on the basis of modal considerations - see comments on Scott in the previous Section. Rescher, for example, takes (1) (1959 p. 161)

(1) Something is possible which does not exist.

to be representable by (1')

(1') $(\exists x)(SE!x \wedge \sim E!x)$

which has (2) as immediate consequence

(2) $(\exists x)(\sim E!x)$

thus contradicting (3) 'everything exists'

(3) $(x)(E!x)$.

Rescher claims that his position is based upon that of MacColl's but that does depend upon the reading of the quantifiers in (2) and (3) - see comments in Section 7.4.

Although Rescher uses 'E!x' to represent his more select sense of 'exists' he does leave open the question of whether or not it is a primitive. The main point is though that the inclusive ambiguity thesis could be used to show that certain uses of 'exists' may be construed to be first-order because they do not suffer from the problems posed at the beginning of this Section. Geach appears to argue for this using a different version of the ambiguity thesis (1969A). He claims that 'exists' is three ways ambiguous; one sense concerns its use in general sentences whereas the other two concern its use in (apparent) singular sentences. It is these latter two which satisfy the inclusive ambiguity thesis. In examples like (4) and (5)

(4) Prior exists but Sherlock Holmes does not.

(5) Geach exists but Prior no longer does.

Geach claims that 'exists' is first order in (5) unlike (4) because of the paradox of reference.

Geach's basis for the ambiguity is Wittgenstein's remark that although the bearer of a name may pass away the name retains its reference. (That is, the semantic relation 'refers' is here claimed by Wittgenstein to be ampliating, a position we may compare to Prior's - see 6.6). Because of this Geach takes those uses of 'exists' in (4) to be tenseless whereas thosein (5) he says are tensed. Consequently, what appears to be his variant of the ambiguity thesis consists in there being a tenseless sense of 'exists' which is more inclusive than a tensed sense. Although 'exists' is first order in (5), in (4) one has to give a metalinguistic account of its use. For the name 'Prior' is a perfectly good name in that it has a reference whereas 'Sherlock Holmes' is only a seemingly good name because we only make believe its reference.

Geach's third sense of 'exists' is its use in general statements like 'Tigers exist' which he takes to be understood as 'There [are] tigers'; it is this use which he takes to be the fundamental tenseless sense and representable by the quantifier (1969B p.65). Consequently, it is this use which should connect up systematically with individual tenseless existence claims.

It seems then that Geach's position is that argued for in the last Section via the second reducibility thesis, namely that tensed quantifier formulas are equivalent to tenseless quantifier and tensed existence predicate formulas. (And Rescher's position appears to be that via the second reducibility thesis in the case of modal logics). However, we want to claim that the inclusive ambiguity thesis is false even though the reducibility thesis is acceptable.

7.3 'Exists' as a First-Order Predicate.

That 'exists' in a restricted sense is, at least, compatible with its being first-order places doubt on the conclusion that in its most inclusive sense it can not be first-order. For otherwise, the inclusive ambiguity is no longer what it seems. In Fregean terms, it appears to be an instance of a concept subordinate to another but if Geach is correct the ambiguity is that of a concept falling under a higher order concept. And this certainly seems odd if we compare the ambiguity of '...is a man'. Moreover, Geach's claim that tenseless 'exists' is second order whereas tensed 'exists' is first-order leaves us initially somewhat at a loss of how to account for the tensed and detensed quantifiers - however, see next Section.

What this suggests is that those paradoxical difficulties which initiated 7.2 occur not so much as a consequence of the assumption that 'exists' is first-order but instead depend upon the presumption of its universal applicability. Moreover two theorists may appear to agree upon a sense of 'exists' yet treat it differently according to which variant, if any, of the inclusive ambiguity thesis they hold. For instance, it is open for Rescher to treat his less inclusive sense of 'exists' as first-order even though this sense appears to be Geach's more inclusive sense. And here the determining factor is the range of 'exists'.

Moreover, the strategy used to reject the view that 'exists' is first order has recently been attacked by Woods (1976 p.249). If 'exists' (in some sense) were really a second order predicate then it is not clear that one should be able to draw consequences from supposing it to be first order. For if we attempt to do the same for the predicates '...is rare' or '...is numerous' a blank is drawn because it is difficult, if not impossible to make literal sense of 'Socrates is rare' or 'Socrates is numerous'. Furthermore, Woods points out that if 'exists' is taken as a first-order predicate true of everything then, at least, the truth conditions of general existentials turn out correct. For example, 'Some tigers exist' is true iff something exists and is a tiger. And, of course, if everything exists only then does the occurrence of 'exists' in this truth condition become redundant.

But, how does all this fit in with the Russellian claim that '(∃x)(-x-)' is a second order predicate representing existence? One suggestion here is that it is not because '(∃x)(-x-)' is second order that it represent existence but instead because of its peculiar property of binding free variables. And given Quine's dictum this binding must connect systematically the quantifier with individual existence. This latter connection is perspicuously brought out by that representation of individual existence in firt-order theories using quantifier and identity. And it is on this point that Geach's account of individual tenseless existence (see previous Section) comes into difficulty. For it is this sense which is to connect up with the tenseless quantifier But it can not quite do this unless everything has a name. This problem then, is due to Geach's metalinguistic analysis.

If it is accepted that '[exists]' is a first order predicate there is still that question of whether or not it expresses a property. The disquiet felt on this point is brought out by the ninth century Arab logician Al Farabi. He asks whether or not 'exists' in 'Man exists' is a predicate.

> "This is a problem on which both the ancients and moderns disagree; some say that this sentence has no predicate and some say that it has ... To my mind both of these judgements are in a way correct, each in its own way... This is so because when a natural scientist who investigates perishable things considers this sentence... it has no predicate, for the existence of a thing is nothing other than the thing itself, and for the scientist a predicate must furnish information about what exists and what is excluded from being. Regarded from this point of view, this proposition does not have a predicate. But when a logician investigates this proposition he will treat it as composed of two expressions...[which] is liable to truth and falsehood. And so it does have a predicate from this point of view." (Quoted from Rescher (1968) p.71-2).

There is much in common here with Kant's discussion of existence in the 'Transcendental Dialectic' as Rescher points out (ibid). Kant is traditionally associated with the view the 'exists' is not a predicate but his position is somewhat puzzling. On the one hand he claims that existential judgements are synthetic (B626) where a synthetic judgment is one whose predicate adds to the concept of the subject (B11). On the other hand, he claims that 'exists' is not a real predicate (B626) where a real predicate is one which determines a thing (B626). His claim that 'exists' is not a real predicate is based upon a similar point made by Al Farabi, that the existence of a thing is nothing other than the thing itself.

> "By whatever and by however many predicates I may think a thing... nothing is really added to it, if I add that the thing exists. Otherwise, it would not be the same that exists but something more than was contained in the concept and I could not say that the exact object of my concept existed." (Critique of Pure Reason B628).

Campbell argues that Kant is not inconsistent here (1974 and 1976 p.55ff). He claims that when Kant denies that 'exists' is a real predicate he is denying that it is a determining predicate in much the same way that Al Farabi denies that it is a predicate for the scientist. A determining predicate is one like '... is red' which, according to Kant, not only adds to the concept of the subject but also 'enlarges' it. (B626). Consequently, in order to claim that there is compatibility between the point that existential judgments are synthetic and that 'exists' is not a real (that is, determining) predicate sense must be made of the view that 'exists' adds to the concept of the subject without enlarging it. Campbell claims that sense can be made here for he argues that for Kant 'exists' is really relational.

> "Through its existence [the object] is thought as belonging to the context of experience as a whole. In being thus connected with the content of experience as a whole, the concept of the object is not, however, in the least enlarged." (B628-9).

We shall suggest below a way in which this generality of the connection between existence and experience as a whole may be more formally brought out, a way in which 'exists' can be thought to express a property.

One interesting and important point about standard first order theory is its apparent compatibility with either the view that 'exists' is or is not first order. This under-determination may be taken as a virtue of first order theory but it also lends itself to certain confusions. For example, it means that philosophical questions about existence can not be simply answered by appeal to such a theory which shows why Thomson's claim that philosphers who argue that 'exists' is not a predicate just mean that 'exists' is not a predicate of first order logic can not be correct (1967 p.104).

What happens then if we consider 'exists' against the backcloth of a second order theory? From Campbell's discussion of Kant it is natural to conclude that objects exist in virtue of their possession of determining properties. And it is this relation to other properties, we believe, that underlies the oddity of saying that 'exists' expresses a property. More formally expressed this may be taken to show that 'exists' is impredicative, a view argued for by Cocchiarella (1968 and 1969). A member of a set is impredicative if in its analysis appeal must be made to other members of that set. This comes out in a second-order theory in the case of 'exists' (and possibly' is identical to...') because in its analysis quantification over other predicates is required as in (1).

(1) E!x iff $(\exists\phi)\phi x$.

where ϕ is a determining predicate. (This is Prior's view - see 6.5).

The inclusive ambiguity thesis is naturally represented in terms of differing restrictions on quantifiers over predicates. For instance, if Geach's tenseless existence locates an object with respect to experience as a whole then tensed existence locates it in a more specific fashion by specifying, to some extent, when also. Given this, we can get a set of senses of 'exists' as follows. First Rescher's most inclusive sense which we represent as SE!x becomes (2)

(2) SE!x iff $(\exists\phi)\phi x$

where ϕ is any predicate barring contradictory ones. Geach's tenseless existence is given by (3)

(3) E!x iff $(\exists\phi)\phi x$

where 'ϕ' is any extensional predicate and tensed existence is also given by (3) but where 'ϕ' is any non-ampliating predicate. (We may compare here senses of 'exists' with the suggestion in 2.5 concerning a basic notion of supposition).

Seen in this light it seems strange that anyone would claim that existence in (3), when tenseless, is second order whereas when tensed is first order. Note, that we have not talked about definitions of 'exists' in second order theory here. One reason for this is that tensed existence is given in terms of non-ampliating predicates but it is unclear whether or not the notion of non-ampliating predicate can be given in terms independent of tensed existence which means that there is a mutual dependency here between the existence predicate and non-ampliating predicates.

7.4 Quantifiers and Existence.

In 7.2 it was noted that Geach only offers three senses of 'exists'. However, it is initially somewhat strange that anyone should offer a contrast between tensed and tenseless individual existence claims while at the same time only offer a tenseless general sense since it may be claimed that the same inclusive ambiguity occurs at the general as well as the individual level, as instanced by the contrast between (1) and (2).

(1) Elephants exist but mermaids do not.

(2) Elephants exist but dinosaurs do not.

So why does Geach not offer a fourth sense to take account of?

Perhaps, the simplest way of representing (2) is by the use of tensed quantifiers discussed in the last chapter. But, on Geach's account this would be tantamount to actually accepting a fourth sense of 'exists'. However, by the second reducibility thesis of 7.1 (2) is representable simply as (2') where the quantifier is the standard quantifier

(2') $(\exists x)(E!x \wedge x \text{ is an elephant}) \wedge \sim(\exists x)(E!x \wedge x \text{ is a dinosaur})$

That is, general tensed existence is representable using the tensed existence predicate and the standard quantifier (see 5.5).

This bring out an interesting point. A theoretical constraint on quantification theory according to Frege (Russell and Quine) is that the only quantifiers one should appeal to are the unrestricted ones. Consequently, a restricted quantifier like (3)

(3) $(\exists x : x \text{ is a person})(-x-)$

should be seen in terms of the unrestricted quantifier followed by a qualifying predicate as in (4)

(4) $(\exists x)(x \text{ is a person} \wedge -x-)$.

According to this constraint, then, a tensed existential quantifier (5)

(5) $(\exists x)(-x-)$

which is tantamount to (6) where '$(\exists x)(-x-)$' is unrestricted

(6) $(\exists x : x \text{ now exists})(-x-)$

becomes (7)

(7) $(\exists x)(E!x \wedge -x-)$

This is surprising, for although first-order theory underdetermines what sort of predicate 'exists' is in its universal sense this is not so in the case of its restricted sense given this constraint. It appears, then, that the tensed existence predicate somehow or other 'disappears' into the tensed existential quantifier (5) and which is explicitly brought out by (7). But couldn't this also be the case with the standard tenseless quantifier? Kennick writes

> "If a man has disappeared into the woods then (assuming he has not come out again) he must still be there. [Likewise] one may suspect that the predicate existence is still there [in the quantifier] too."
> (1970 p.172).

If this is correct then the standard existential quantifier performs two closely related roles, first that of being a quantifier - that is, representing the syncategorematic 'some...' which belongs to the same class as 'each...' 'all...' 'seven...' etc..., - and secondly, that of introducing existence. Can these two roles be distinguished?

By distinguishing these two roles then we have the basis for the alleged misunderstanding by Russell of both Meinong and MacColl. For it seems that a fair representation of their views must make use of the quantifier without its existential implications. For Meinong held that there are objects which neither exist as Socrates does nor subsist like 7 does. Findlay writes

> "That an object can have definite properties, a definite nature, although there is no such object is the high ly paradoxical principle which these considerations [that is, something is neither existent nor subsistent] imply. This principle is called by E. Mally the independence of so-being (Sosein) from being: an object can still be so, i.e. such and such, even though it has no being of any sort. Meinong admits that the principle is very difficult to stomach but thinks that this is solely due to our prejudice in favour of the actual." (1963 p.144).

The independence of so-being is based upon the acceptance of the discernibility of non-existents (see 1.4). So by bringing out explicitly the existence predicate from the Russellian quantifier Meinong's non-existents could then be quantified over. Now, this position is not an example of the inclusive ambiguity because these non-existents are intended to have no being at all, and hence are to be contrasted with and not include existents and this contrast can be provided by the use of an existence predicate.

Hugh MacColl defended the discernibility of non-existents in a series of papers. In particular, he was against the procedure of defining the null class as containing no members and being a subset of every class. Instead, he took the null class to consist of unreal members and consequently, to be excluded from every real class, (that is, a class containing real members). He writes

"Their convention of universal inclusion leads to awkward and, I think, needless paradoxes, as for example, that 'Every round square is a triangle' because 'round squares' form a null class, which by them is understood to be contained in every class. My convention leads in this case to the directly opposite conclusion, namely, that 'No round square is a triangle' : because I hold that every purely unreal class such as the class of round squares is necessarily excluded from every purely real class, such as the class of figures called 'triangles'."
(1973 p.318).

In defence of Russell, it is clear that he did hold the indiscernibility of non-existents and moreover, it is difficult to understand let alone accept any alternative to this. Nevertheless, alternatives have been proposed and so one could argue that Russell's position is based upon a certain philosophical view which can be explicitly represented if we do distinguish quantification from existential quantification by (8)

(8) $\sim(\exists x)\sim(E!x)$

That is (8) is not intended to be a simple tautology. However, this basis for distinguishing between the two roles of the existential quantifier is not, to say the least, a very strong basis. We shall now argue in this and the next Section that a far less controversial basis can be given.

(7) is the suggested analysis of the tensed quantifier (5). However, (7) imputes two existential claims, a tenseless one given by the existential quantifier and a tensed one given by 'E!x'. On the other hand it seems that (5) only imputes one existential claim, the tensed. That is, in order to express general tensed existence one has to explicitly presume tenseless existence. On the other hand, when expressing individual tensed existence one does not have to presume explicitly tenseless existence.

So general tensed existence claims don't quite stand in the right relationship to individual tensed existence claims. One way in which this oddity may be overcome is to distinguish the quantifier from the existential quantifier. Suppose 'E!x' is the tenseless existence predicate argued for in Section 7.3 then the existential quantifier may be seen as (9)

(9) $(\exists x)(E!x \wedge -x-)$

where the '$(\exists x)(-x-)$' represents 'something is such that it...' and not the existential 'There is something such that it...' which includes the extra predicative 'There is...'

Certainly, here, we have less objectionable grounds than Meinong's for distinguishing the quantifier aspect and the existential aspect of the standard existential quantifier. Nevertheless, it might be felt that this is too fine a point, a point of little importance provided everything exists tenselessly. But, if the inclusive ambiguity thesis is false and in particular, if from the standpoint of logics which take tensing into account the relation between tenseless and tensed is not inclusive but mutually exclusive then there is room here for distinguishing between the two aspects of the existential quantifier in a non-trivial way.

7.5 Concluding Remarks on 'Exists'.

Geach offers a contrast between tensed and tenseless individual existence (see 7.2). But is it correct to claim that the contrast between (1) and (2)

(1) Prior exists but Sherlock Holmes does not.

(2) Geach exists but Prior no longer does

is a contrast between tenseless and tensed existence? Now, as (1) stands to (2), it appears that (3) stands to (4).

(3) Elephants exist but mermaids don't

(4) Elephants exist but dinosaurs no longer do.

But is this a contrast between tenseless and tensed general existence? In a recent paper Miller (1975) defending Geach's view that tensed 'exists' is a first order predicate claims that whereas (3) is about kinds apparent general existentials like (4) are really about individuals. He writes

> "In (4) on the contrary neither do 'elephants' refer to 'being an elephant' nor 'dinosaurs' to 'being a dinosaur'. If they did, the proposition would not only be false but the 'but' would be quite misleading since there would be no point of contrast between the first and second clauses. The only way in which the contrast can be retained is if 'elephants' and 'dinosaurs' refer to individuals." (1975 p.345).

But is this the only way in which the contrast can be retained? Is it even a viable way of accounting for the contrast?

Miller seems to be pushing Geach's position to its logical conclusion. In (3) 'exists' is a second order predicate whereas in (4) it is a first order tensed predicate. But what we seem to get is the inverse of the Russellian position for tensed existence. Instead of construing individual existence claims in terms of general Miller advocates construing general tensed existence claims in terms of individual claims. If we have understood him correctly, he is claiming that 'elephants' in (4) unlike its occurrence in (3) refers to individuals. But this is rather an unhappy position. One might as well claim that the restricted general term 'Indian elephants' also refers to its instances if the temporally restricted term 'elephants now' does.

We shall now examine the claim that (3) is a tenseless existence claim in contrast to (4). One way in which this may be understood is that (3) is representable within a framework which does not take tense considerations into account. That is, (3) may be claimed to involve a tenseless sense of 'exists' because it is representable in standard tenseless quantification theory in terms of predicates having instances. But, if our concern is with the contrast between (1) and (2) then in order to facilitate a fair contrast they must both be looked at in the context of a framework which does make allowance for tense distinctions.

The claim that (3) does involve tenseless 'exists' within frameworks taking account of tense can still be made. For instance, it is so represented by the L^D_A theorist (see 5.3) as (3')

(3') $(\exists t)(\exists x)$(x[is]-an-elephant-at t) $\wedge \sim (\exists t)(\exists x)$(x[is]-a-mermaid-at t)

and by (3") by the L^D_B analyst (see 5.6)

(3") $(\exists x)$(x[is] an elephant$_4$) $\wedge \sim (\exists x)$(x[is] a mermaid$_4$)

and finally by (3'") by the L_p analyst where the quantifier is to be understood as tenseless (see 6.1)

(3''') S(∃x)(x is an elephant) ∧ ∿ S(∃x)(x is a mermaid).

But both the L_A^D and L_p representations used here presuppose that parasitic tensing account which we argued against in 5.5 because it is inadequate for representing certain general and individual existence claims. Within these frameworks when an existence predicate is introduced - for then they become adequate - (3) is to be represented by (5') and (5") respectively.

(5') (∃t)(∃x)(E!xt ∧ x[is]-an-elephant-at t) ∧ ∿ (∃t)(∃x)(E!xt ∧ x[is]-a-mermaid-at t)

(5") S(∃x)(E!x ∧ x is an elephant) ∧ ∿ S(∃x)(E!x ∧ x is a mermaid).

In contrast a use of (4) at t_n is represented by (4') and (4") under these analyses.

(4') (∃x)(E!xt_n ∧ x[is]-an-elephant-at t_n) ∧ ∿ (∃x)(E!xt_n ∧ x[is]-a-dinosaur-at t_n)

(4") (∃x)(E!x ∧ x is an elephant) ∧ ∿ (∃x)(E!x ∧ x is a dinosaur).

That is, neither (3) nor (4) can be said to involve a tenseless sense of 'exists' unless one takes up the L_B^D analysis. This suggests strongly that (1) and (2) is, therefore, not a contrast between tenseless and tensed existence claims. That is, the fact that a name still has a use after the bearer has died is in no way tantamount to declaring that the bearer still exists in some strange sort of way.

What do sentences look like which do involve tenseless 'exists'? The most obvious place to look is arithmetic

(6) Odd numbers exist but transcendental numbers don't.

(7) Seven exists but pi doesn't.

That is, we are here arguing that 'exists' is not inclusively ambiguous. Instead it is ambiguous in a mutually exclusive way. Thus, not only do we disagree with Geach that tenseless 'exists' is primarily a second order predicate whereas tensed 'exists' is first order (see 7.3) but also we disagree with the view that what exists [exists].

In effect then we are arguing that when tense distinctions are taken into account 'exists' is not topic neutral. However, on the other hand, it seems to us that 'some...' is. That is, this is an argument for distinguishing between the two aspects of the existential quantifier. For, as we have already claimed, none of (5'),(5"), (4') or (4") involve a tenseless ascription as well as a tensed ascription of existenc Consequently, what is common to both abstract and spatio-temporal objects is not their existence but their discernibilit - understood as the claim that certain predicates are true and false of them - and, furthermore, if we claim that discernibility of x implies that x can be a value of a variable then there are good grounds for distinguishing those two aspects of the existential quantifier. And this then reinforces the thesis that 'exists' is a first-order predicate in both tensed and tenseless uses.

One point about this view should be noted. When one uses standard tenseless quantification theory which makes no allowance for tense distinctions then it is perfectly acceptabl to only have single sense of 'exists' given by the quantifier and identity predicate. (This is connected with the point that when representing an argument one should only reveal as much structure as is necessary). Moreover, this shows, then, that there is not as is commonly assumed a simple move from standard quantification theory to tensed counterparts. Certainly, syntactically tense logics should be conservative extensions of quantification theory - and this is part of the force behind the second reducibility thesis of 7.1 for otherwise a free logic version of quantification theory is required given only the first reducibility thesis. And for the L_A^D analysis the detensed quantifier (see 5.5) can be expressed using the standard quantifier and the existence predicate thus preserving a syntactic extension. But, we believe that we have shown that tense logics are not interpretationally conservative extensions of standard tenseless quantification theory. And this point touches as deep as the quantifier and identity representation of existence.

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