

Essays on Price Stickiness and its
Monetary Policy Implications in Developing Economies

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Abstract

In Chapter 2, we empirically assess the existence and degree of price stickiness of a developing country using a store-level price dataset from Rwanda. Using the data, we establish five key findings. First, price changes are consistent across different months and no seasonality is observed in the data. Second, price changes are more frequent in Rwanda, compared to developed countries. However, item weights play a significant role to induce the high weighted frequency of price changes for Rwanda. Third, we observe that the duration of price spells in Rwanda is lower than in developed countries, though, there is a lot of heterogeneity among the commodity groups. Fourth, price decreases are almost as frequently observed as price increases, which is similar to findings from other studies. Fifth, the magnitude of price changes in Rwanda is higher compared to developed countries. Overall, we conclude that, due to the large role played by item weights, price stickiness in Rwanda, in general, is similar to what is observed in developed countries.

In Chapter 3, we study staggered prices and monetary non-neutrality in a model of a developing economy. We develop a dynamic general equilibrium model of a developing economy with money but without bonds or private insurance. Such asset market imperfections are likely to affect the price-setting process, as it involves an intertemporal decision, under staggered prices. Using the model, we establish two key findings. First, following a monetary policy shock, aggregate output may pass through an oscillatory phase on the path towards the steady state. Second, there are heterogeneous effects on sectors leading to a persistent asymmetry in the economy. However, despite the greater intersectoral heterogeneity in the time paths of the variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes in a model of a developing economy in comparison to a model of an advanced economy.

In Chapter 4, we study the effects of implementing an inflation targeting (IT) framework, which involves an interest rate rule, in a developing economy (DE). We develop a dynamic general equilibrium model of a DE by incorporating two sectors: a flexible-price or food sector, where households hold money but have no access to bonds or insurance, and a sticky-price or nonfood sector, where households hold money and have access to bonds and insurance. Following a monetary policy shock, either through an IT framework or a monetary aggregate targeting (MT) framework, we find that the increase in consumption levels is higher in the sticky-price sector compared to the flexible-price sector. However, the difference in consumption levels is higher under an IT framework, which suggests that, in terms of distributional implications, a MT framework may be more desirable than an IT framework in a DE.

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This thesis is dedicated to my mother, who has left us all too early.

Declaration

I hereby declare that this thesis is my own work, and the material contained in this thesis has not been submitted for a degree at any other university. The work contained in this thesis is original and all sources are acknowledged as References.

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Chapter 1

Introduction

Monetary policy is a useful tool used by central bankers and policymakers around the world to control important macroeconomic variables such as output, inflation, and unemployment. In recent decades, there has been remarkable progress in monetary theory and analysis, which have enriched our understanding and study of how central banks use monetary policy instruments, such as interest rates and money supply, to achieve aggregate outcomes. In particular, New Keynesian ideas, such as price stickiness, have been instrumental in demonstrating how monetary policy can have real effects in macroeconomic variables.

The importance of price stickiness has led many researchers to try to understand why prices and wages remain sticky. For instance, Taylor (1979) states that wages are sticky because they are fixed in a staggered form: all firms in an economy do not change wages at the same point in time. Calvo (1983), on the other hand, states that there is a constant probability with which firms in the economy change prices at a given point in time, which introduces a form of staggering in the price-setting process. While these are ‘time-dependent’ reasons for prices and wages to be rigid, Mankiw (1985) and Blanchard and Kiyotaki (1987), state that there are ‘state-dependent’ reasons, such as menu costs, that firms incur to change prices, which result in lower adjustment of prices. In addition, Rotemberg (1982) states that firms are unwilling to change prices due to the fear of losing customers, which leads to sticky prices.

While these models explain why prices are rigid theoretically and conceptually, many applied economists have tried to assess whether prices and wages remain rigid empirically. The seminal paper that studied nominal rigidity in prices is Cecchetti (1986), who tracks annual magazine data over time and finds that prices are sticky. This was then followed by Kashyap (1995) who uses price data from three supermarkets in the United States to demonstrate the stickiness of prices. While these studies focus on small-scale data, Bils and Klenow (2004) use a monthly Consumer Price Index (CPI)-representative dataset for the United States and find that prices are empirically sticky; Nakamura and Steinsson (2008) and Klenow and Kryvtsov (2008) provide

additional evidence of price stickiness in the United States. Álvarez et al. (2006) and Dhyne et al. (2009) find similar results using CPI-representative datasets for European countries. Moreover, Bunn and Ellis (2012) find that prices are relatively more sticky in the United Kingdom in comparison to the United States and European countries.

The advantage of these empirical studies is that they allow macroeconomists, in particular, central bankers, to employ a ‘frequency of price change’, which is an estimate of the mean percentage of prices that are changed during a given period, to accurately calibrate dynamic stochastic general equilibrium (DSGE) models. DSGE models, which have now become the ‘workhorse’ for the analysis of monetary policy, incorporate various structural features, including the degree of price stickiness, to study the impact of monetary policy on macroeconomic variables. Many studies, such as Christiano et al. (2005), Golosov and Lucas (2007) and Nakamura and Steinsson (2010), use the frequency of price change, along with other macroeconomic data, to show that DSGE models can replicate and match the data for the United States.

While many studies have been conducted to study price stickiness and monetary policy for advanced economies, developing countries, in particular, low-income countries, have not received much attention. Given that these countries are structurally quite different, it might be expected that price stickiness and the mechanics of monetary policy would be dissimilar in these countries in comparison to advanced economies. Agénor and Montiel (2008) document some of these structural features, which include weak institutions, high reliance on trade, etc., and study the transmission mechanism of monetary policy. However, most of their analyses focus on emerging economies, such as Mexico, Brazil and India, which are more open and are exposed to global shocks. Low-income countries, such as those in Sub-Saharan Africa, are less open and are influenced primarily by internal or domestic features, which suggest that these features may play a key role in the transmission mechanism of monetary policy in these countries. Moreover, Mishra and Montiel (2013) and Davoodi et al. (2013), who focus specifically on low-income countries, use mostly econometric analyses, and the structural features are not incorporated in their analyses of the transmission mechanism of monetary policy in low-income countries. In addition, these studies place no emphasis on price stickiness and implicitly assume its role in these countries is similar to that in advanced or emerging economies.

The essays in this thesis are devoted to the study of price stickiness and its monetary policy implications in developing economies, or more specifically, low-income countries. In the second chapter, we investigate whether prices are empirically sticky in low-income countries, since very few studies exist which explore the behaviour of prices in these countries. We obtain and employ micro-level disaggregated monthly price data from Rwanda for the period January 2013–December 2014, to assess the degree of price stickiness in a developing economy. Using the data from Rwanda, we estimate that the weighted mean frequency of price changes in Rwanda is 43.62

percent, which is higher than most developed economies but similar to other developing economies. It is observed, in Rwanda, that large frequencies of price change occur mostly in food items, such as fruits and vegetables, where frequencies are close to 90 percent. As food and beverages constitute a large proportion of consumption expenditures in Rwanda, the higher weights assigned to these items contribute significantly to the overall weighted mean frequency of price change. We conclude that item weights play a key role to influence the frequency of price change for Rwanda, as an unweighted mean frequency of price change of Rwanda shows that it is very close to most countries in Europe and the United States. Therefore, we conclude that price stickiness in Rwanda, in general, is similar to what is observed in advanced and emerging economies.

The findings of the second chapter establish the importance of price stickiness in a developing economy. However, the implications of price stickiness in a developing economy are likely to be dissimilar from those in an advanced economy, as their structural features are different and may impact the nature of the firms' price-setting. In order to study the price-setting mechanism in developing economies, we observe that, one of the key features that differentiates Rwanda and other developing economies from advanced economies, is the presence of imperfections in the credit markets, which prevents a significant proportion of households and businesses in the economy to borrow and lend and trade in insurance. Then, it may be expected that imperfections in the credit markets will affect the price-setting decision, as it involves an intertemporal tradeoff, under staggered prices. This is likely to create heterogeneous effects among price-setters, and, in turn, create a potential role for borrowing and lending and insurance.

In the third chapter, in order to study the price-setting process and, thereby, the extent to which monetary policy affects real variables, we develop a dynamic general equilibrium model with money but without bonds and insurance to characterise a developing economy and examine the role it plays in the price-setting decision and, consequently, in the dynamics of aggregate output and other macroeconomic variables. In addition, we explore the heterogeneous effects in the economy, as the nature and timing of price-setting make it likely that there will be distributional implications, in the presence of imperfect credit markets (ICM). Moreover, we develop a model of an economy with perfect credit markets (PCM) to characterise an advanced economy, and compare the effects of monetary policy on the two types of economy. It is worth noting that we use Taylor (1979) price staggering as the sectoral effects of monetary policy are easier to explore under this price-setting mechanism, compared to other price-setting models such as Calvo (1983).

There are two key findings in this chapter. First, under ICM, following a monetary policy shock, aggregate output may pass through an oscillatory phase on the path towards the steady state, although the magnitudes of these oscillations are small. This demonstrates that while ICM create fluctuations in aggregate output in the transition path, these are not expected to cause an issue from a policy perspective. Under PCM, on the other hand, following a monetary policy

shock, aggregate output exhibits a monotonic path towards the steady state. This leads us to conclude that the aggregate effects of a monetary policy shock are not strongly dissimilar across ICM and PCM. Second, although the aggregate dynamics are similar under ICM and PCM, there are notable heterogeneous effects in the behaviour of sectoral variables under ICM. Following a money supply shock, the price-setting sector in the impact period has lower levels of output and nominal revenue than before the shock, as it sets a new higher price. Despite the lower output levels, it has higher consumption levels than before the shock, as it runs down its money holdings to smooth out consumption fluctuations. The non price-setting sector, on the other hand, has higher output levels and nominal revenue than before the shock, and also higher than in the price-setting sector. This results in higher consumption levels and money holdings both relative to the pre-shock situation, and to the other sector. The heterogeneous effects on sectors lead to a persistent asymmetry in the economy, as these effects continue for most of the transition path. As a monetary policy shock leads to heterogeneous effects under ICM, we conduct numerical welfare calculations to determine the overall effect on sectoral utility. We find that, following the shock, the price-setting sector enjoys higher lifetime utility in comparison to the non price-setting sector under ICM. Then, to compare whether these welfare effects exist under PCM, we conduct similar welfare analysis for PCM, and find that this result also holds for PCM, despite the identical consumption levels and money holdings across sectors under PCM. A comparison of the sectoral welfare gains under ICM and PCM leads us to a crucial finding: despite the greater intersectoral heterogeneity in the time paths of the variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes under ICM than PCM. We proceed to suggest that an implication of this result is that a monetary authority concerned about equality can be more vigorous in its monetary policy actions in a developing economy in comparison to an advanced economy.

In the fourth chapter, we continue our analysis by investigating different types of monetary policy regimes in developing economies. We begin by noting that in a majority of developing economies, monetary policy is still conducted using a monetary aggregate targeting (MT) framework, whereas most advanced and emerging economies now follow an inflation targeting (IT) framework using interest rate rules. An IT framework has qualitative advantages, such as more transparency and credibility of central banks, and, in addition, Lin and Ye (2009) find that its adoption has resulted in lower inflation levels and volatility in emerging economies. This makes it a worthwhile question to compare these frameworks in the context of a developing economy. In order to model our developing economy, we incorporate two key findings from the second and third chapters. First, we include two sectors following the finding in the second chapter that, in Rwanda, prices are relatively flexible in the agriculture or food sector, whereas they are sticky in the manufacturing and services sector. Second, we assume that households in the flexible-price

or food sector, which consists mostly of farmers, hold money but are unable to trade in insurance or borrow to smooth out individual fluctuations. On the other hand, households in the non-food or sticky-price sector hold money and are also able to trade in insurance and borrow, since these households are more creditworthy and are likely to be able to repay their loans. The advantage of the model setup in this chapter is that households in the sectors hold money and, in addition, there is an interest rate in the economy through the bond market, which allows us to study both a MT and an IT framework. However, as the model is more complicated than that used in Chapter 3, the results are derived using numerical simulations in this chapter, unlike in Chapter 3 where we were able to obtain some analytical results. For the monetary policy rules, it is noteworthy that we use a money supply rule (MSR) in the form of an exogenous money supply as a MT framework and an interest rate rule (IRR) in the form of a Taylor rule as an IT framework.

To evaluate the performance of the monetary policy rules in our economy, we focus on the sectoral effects resulting from the monetary policy shocks. The key finding of the chapter is that, in a developing economy, a MSR, in contrast to an IRR, leads to a more even distribution of consumption across the flexible- and sticky-price sectors. We find that, following an expansionary monetary policy shock, under a MSR, the change in money supply affects both the sectors directly, as money is injected equally through the budget constraints of the households, leading to higher consumption levels for both the sectors. On the other hand, an IRR leads households in the sticky-price sector to enjoy higher consumption levels, and while general equilibrium effects imply that households in the flexible-price sector have higher consumption levels as well under an IRR, the increase in consumption levels for the flexible-price sector under an IRR is lower in comparison to the increase under a MSR. This is because an expansionary monetary policy under an IRR leads to a direct effect only on the sticky-price sector, where households are able to borrow, and, hence, are positively affected by a decrease in the interest rate, which results in higher consumption levels. A policymaker in a developing economy, particularly interested in social welfare, then can generate more equal outcomes across the sectors under a MSR in comparison to an IRR. Hence, a MT framework, as currently practiced by most developing economies, in terms of distributional consequences, may be more desirable than an IT framework.

Chapter 2

Are Prices Sticky in Developing Countries? Evidence from Rwanda

2.1 Introduction

Central banks use monetary policy to control the behaviour of important macroeconomic variables, such as output, inflation, and unemployment. Quantitative macroeconomic models, such as the Wharton Econometric model and Brookings model, were popular among monetary policymakers prior to the 1980s. However, they fell out of favour following the failure of the models to predict the stagflation in the 1970s. Consequently, ‘New Keynesian’ ideas, such as nominal price stickiness and non-neutrality of money, were utilised to develop New Keynesian models.¹ According to New Keynesian Models, prices are sticky in the short-run, due to wage contracts (Taylor (1979)) and/or menu costs (Mankiw (1985)). These constraints prevent firms to change wages and prices in response to a monetary policy shock, and induce firms to respond, instead, by adjusting consumption and output. The presence of nominal rigidity, therefore, ensures the non-neutrality of money (Galí (2009)).

The importance of nominal stickiness for monetary policy has led to many studies that investigate the price- and wage-setting strategy and behaviour of firms. Time-dependent models, such as Taylor (1979), state that wages are rigid because they are fixed in the form of a staggered setting: all firms in an economy do not change wages at the same point in time.² Calvo (1983), in his price-setting model, assumes that there is a constant probability with which firms in the economy change prices at any point in time. Proponents of state-dependent models, such as Mankiw (1985) and Blanchard and Kiyotaki (1987), contend that sticky prices are observed in the economy due to ‘menu costs’ that firms incur to adjust prices. These small menu costs are

¹For a review of the New Keynesian Models, please see Gordon (1990) and Galí and Gertler (2007).

²Taylor (1999) has extended this model to a staggered price-setting framework to determine the behaviour of firms changing prices of goods, rather than wages.

sufficient to generate aggregate nominal rigidity, and, hence, monetary non-neutrality (Levy et al. (1997)). While these time- and state-dependent models have tried to explain the reasons behind price and wage stickiness, many attempts have also been made to explore empirically the degree of rigidity in prices and/or wages.

These studies on nominal rigidity employ micro-level disaggregated price data and estimate the mean percentage of prices that are changed during a given period as an indicator of price rigidity. In addition, they compute the duration of price spells to find the mean length of time for which prices remain constant.³ Some studies, such as Kashyap (1995) and Nakamura and Steinsson (2008), also estimate the direction and magnitude of these price changes. Further, these empirical studies have been utilised to assess the effectiveness of monetary policy to generate persistence in output (Ascari (2000); Chari et al. (2000); Christiano et al. (2005)). Although ample evidence on price stickiness and its importance in generating output persistence exist for developed countries, studies investigating evidence of nominal rigidity in developing countries are scarce.

There are several reasons that suggest why prices may not be very rigid in developing countries. First, developing countries are generally characterized by high inflation and high uncertainty, which may lead firms to change prices more frequently, and make it less likely to observe the nominal rigidity that is existent in developed countries (Kovanen (2007)). Consumer Price Index (CPI), which is an aggregate price index to measure inflation, is a weighted sum of the individual prices in the CPI basket. Hence, with higher inflation, it is expected that individual prices may be frequently revised (Bunn and Ellis (2012)). Second, developing countries are characterised by high reliance on agriculture and manufacturing sectors (in comparison to service sectors), which are exposed to exogenous shocks and, thereby, high fluctuations in output and prices in the economy. For the second chapter of the thesis, a study has been conducted using micro-level data from a small developing country in Africa to examine and provide evidence on developing country price rigidity (or lack thereof) and its implications on price-setting behaviour of firms.

The study uses store-level price data for Rwanda over the period January 2013-December 2014, that has been obtained from the National Institute of Statistics of Rwanda (NISR) and contains 23,138 price quotes from 100 stores in the capital city, Kigali.⁴ The dataset provides data for two years for Kigali and it contains 86.04 percent of all consumption expenditures from the region in the Consumer Price Index (CPI) basket. Data at such a disaggregated level is generally not collected in developing countries, which may explain the insufficient number of studies on nominal rigidity in these countries. While developing countries are difficult to be categorized under a common definition, Rwanda falls within the definition of a ‘developing country,’ classified by the

³Some studies, such as Blinder (1994), Hall et al. (2000) and Álvarez et al. (2006) conduct surveys of firms to determine how often firms change prices and explore the reasons for the changes.

⁴Many thanks to Yusuf Murangwa, Sébastien Manzi, and Oscar Sibomana at the NISR for assisting to acquire this dataset.

International Monetary Fund, World Bank and United Nations, which makes this an interesting country in which to study price-setting behaviour of firms.⁵ In addition, Rwanda is one of the only few developing countries that collects such disaggregated-level price data.

The estimates using the Rwanda data generated five key results. First, we look at the frequency of price change and find that price changes are relatively stable over time and demonstrate no seasonality. There is a sharp spike in December 2013-January 2014, which was primarily due to price changes that took place in the Health sector during this period. This may have been the result of regular price changes in this sector at the end of the year. Unfortunately, the dataset that we have been provided from the NISR contains information for only two years. Therefore, it is not possible to observe if large changes occur, in general, at the beginning or end of a year in Rwanda.

Second, we estimate that the weighted mean frequency of price changes in Rwanda is 43.62 percent. It is observed that large frequencies of price change occur mostly in food items, such as fruits and vegetables, where frequencies are close to 90 percent. As food and beverages constitute a large proportion of consumption expenditures in Rwanda, the higher weights assigned to these items contribute significantly to the overall weighted mean frequency of price change. The weighted mean frequency of price change for Rwanda is substantially high compared to developed countries such as the United States and United Kingdom, where the weighted mean frequencies are 26.5 percent and 18.8 percent respectively. The only countries where similar high frequencies are observed are in Chile, Brazil and Mexico where the weighted frequency of price changes are 46.1 percent, 40.27 percent, and 30.4-36.6 percent respectively.⁶ The weighted median frequency of price change is 28.68 percent, which is lower than the weighted mean frequency of price change: a feature that is common in the literature. In addition, we find that the frequency of price change is positively skewed, implying that the right tail is longer than the left tail of the distribution of the price changes. Next, we disaggregate the frequency of price change by 12 major CPI groups and observe that a lot of heterogeneity exists among the major groups: the mean frequency of price change for food and non-alcoholic beverage group of the CPI consumption basket is close to 50 percent, whereas, mean frequencies lower than 10 percent are observed for more than half of the major groups. Moreover, we find that there is a lot of heterogeneity within the groups, as demonstrated by large standard deviation for most of the major groups. Furthermore, we observe that, though, heterogeneous prices are observed in most countries, the major groups in which extreme frequencies are observed differ between countries, reflecting the intrinsic differ-

⁵International organizations categorize countries using different indicators and term these countries differently as well, for details, see Nielsen (2011).

⁶Estimates for frequency of price changes for United States, United Kingdom, Chile, Brazil and Mexico are taken from Bils and Klenow (2004), Bunn and Ellis (2012), Medina et al. (2007), Gouvea (2007) and Gagnon (2009) respectively.

ences in these groups in various countries. We also categorised these estimates by type of good and found that prices are more rigid in services compared to durable goods in Rwanda. In terms of the broader picture, we find that item weight is a key determinant for the high frequency of price change in Rwanda. In fact, we can test this by estimating the unweighted frequency of price change of Rwanda and find that it is very close to most countries in Europe. In addition, a closer observation of other studies shows that it is quite common for certain sectors in the economy to exhibit higher frequency of price change, as we observed with food and non-alcoholic beverages for Rwanda. For instance, in the United Kingdom and Euro area countries, in general, transport prices fluctuate more in comparison to other major groups (Dhyne et al. (2009)); in the Netherlands, 88% of transport prices change every month. However, the item weights placed in the transport sector is low for these countries, which is why the overall frequency of price change is not as high in these countries compared to Rwanda. Therefore, we conclude that price stickiness in Rwanda, in general, is similar to what is observed in other advanced economies.

Third, we find that the weighted median duration of price spell in Rwanda is 2.96 months, meaning that prices are changed, on average, about 4 times per year.⁷ This estimate is lower than that observed in the United States and United Kingdom, where the weighted median duration is 4.3 months and 7.2 months respectively.⁸ However, the duration estimate for Rwanda is similar to that observed in other developing countries. In addition, there is a lot of heterogeneity in duration of price spell among and within the major groups in Rwanda. For food and non-alcoholic beverage group, the mean duration of vegetable is 0.7 month, whereas for medical services and hairdressing salon, the mean duration is 22.4 months. The large difference in duration of price spell emphasizes the heterogeneity of price-setting behaviour in different sectors and across item categories.

Fourth, we find that the fraction of the change that were mean price increases is 60 percent, meaning that 40 percent of these price changes are decreases. This is similar to developed countries where the mean percentage of changes that are increases is 60 percent in the United Kingdom and weighted fraction of price increases in the United States is 57.1 percent.⁹ However, this result is unexpected for Rwanda as sales and discounts are, in general, uncommon in Rwanda and other developing countries. The estimates of direction of price changes by major group show that price decreases are mostly observed in food and non-alcoholic beverages and clothing and footwear. This may be attributed to the fact that sellers and vendors of these products may be more vulnerable to exogenous shocks; for instance, a drought in agriculture production may lead to price increases in a relatively short time span. Another reason might be that since food items

⁷Due to methodological issues with estimating weighted duration (which are explained in detail in Section 2.5), we limit our calculation to weighted median duration only.

⁸Estimates for the United States and United Kingdom are taken from Bils and Klenow (2004) and Bunn and Ellis (2012) respectively.

⁹Estimates for direction of price changes for the United Kingdom and United States are taken from Bunn and Ellis (2012) and Nakamura and Steinsson (2008) respectively.

such as fruits and vegetables are mostly non-durable, vendors may be tempted to charge lower prices for these products close to their expiry date. We also find that there is a strong correlation between price increases and decreases in a group, meaning that any major group that exhibits price increase are just as likely to experience price decrease.

Fifth, we find that the weighted mean and median magnitude of price changes in Rwanda are 16.67 percent and 14.19 percent respectively, which is higher compared to the weighted median estimate of 11.3 percent observed in the United States (Klenow and Kryvtsov (2008)). This means that price changes are more frequent and larger in Rwanda. This is unexpected as higher frequency of price change is expected to result in smaller magnitude of price change, when price changes occur. We find that the mean magnitude of price increase and decrease are 18.8 percent and 11.43 percent respectively. We do not observe a strong relationship between the mean frequency and magnitude of price change and a simple regression analysis confirms this. There is a slight variation in the magnitude of price change when we look at major groups. The mean magnitude of price change for any major group is over 10 percent, with the largest change of 50.16 percent observed in the Education group. Since prices changed only 2.17 percent in this group over the sample, the magnitude with which prices changed are large. This also explains why we find large magnitude change in prices for services sector compared to durable goods sector.

The chapter is organized as follows. In the next section, we provide a review of the literature and summarize the results from existing empirical studies on price stickiness. This will be useful later for comparison with results from this study. In Section 2.3, we provide some relevant macroeconomic features of Rwanda that give an overview of the economy and its recent macroeconomic trends. In Section 2.4, we describe the dataset and provide more information on the sample and the cleaning procedure of the data. In Section 2.5, we explain the methodology for constructing the relevant price statistics. In Section 2.6, we use these methods to analyse the data and present the main empirical results, which include a section that provides comparisons between the results obtained from Rwanda with other countries. Section 2.7 concludes the second chapter with possible research avenues for subsequent chapters.

2.2 Previous Literature

The rise of the New Keynesian paradigm in the 1980s has resulted in a surge of empirical studies on evidence of nominal rigidity. The first few studies used supermarket data or data from a few companies on a few specific items to estimate how often these companies change prices. More recently, studies have employed large datasets, typically collected for the purposes of computing CPI, to determine the existence and degree of nominal rigidity. An estimator that has been fairly popular to indicate the extent of nominal rigidity is the frequency of price change, which indicates

the percentage of prices that have changed over the periods in the data (this variable may be estimated annually, monthly or weekly, depending on the frequency at which data is available). In addition, the frequency of price change is then used to estimate the duration of time between these price changes. The literature review that follows summarizes the results of these estimates along with some other price indicators to explain the degree of price stickiness.

In the first part of the review, we summarize the existing evidence of nominal rigidity in prices on developed countries using price and survey data. Although the studies using survey data differed in the questions that were investigated during the surveys, we explore in detail the price statistics that are relevant to this study and present the results. In addition, we try to capture the evolution of these studies by presenting them, for the most part, in chronological order (with overlap as these studies are often ongoing for long periods of time). This will help to understand the various methods that have been attempted so far to explain price stickiness.

Cecchetti (1986) is one of the first studies to explore empirically the existence and extent of nominal rigidity in prices. He analyses the differences in frequency of price adjustment due to changes in general price inflation. He collects newsstand prices data of 38 American magazines from 1953-1979 to determine the annual frequency of nominal price change. He finds that firms do not change prices frequently even during periods of high inflation. He contends that this is due to high costs associated with fixed price changes: a firm believes that if it changes price first, then it will increase its product's relative price and lose its sales to other firms. Over the entire sample period, he finds that there is an increase in the relative annual frequency of price change and a decrease in the average length of a spell. He states that the increase in the annual frequency is due to its positive relationship with the overall price inflation (which increased over time in the sample period) and the adjustment costs, which fell with frequent price changes.

Kashyap (1995) studies the movement of semi-annual catalogue prices of twelve selected retail goods (mostly apparel) for three companies in the United States from 1953-1987.¹⁰ There are four major findings of the paper. First, nominal prices remain constant for more than one year (about fifteen months) and there is no regularity in the timing of when these changes occur. He finds that the average number of months between price change lies between 11.2 months and 30.4 months, irrespective of the length of time that individual products are studied.¹¹ Second, there is no common trend in the size of price changes. However, there is a lot of heterogeneity in the size of the price changes, across products and time. The mean change for the different items varies between 4 and 18 percent, with a lot of variations within the individual items, while the average size of price change over all items is 8 percent. Third, the average size of the price increases

¹⁰The study outlines the advantages and disadvantages of using catalogue prices over retail store prices, the details of which are not covered as part of this study.

¹¹Not all products are studied for the same length of time, due to unavailability of data.

and decreases are not very different: the average size of the price increases are 8.2 percent, while the average size of the price decreases is 7.4 percent. Fourth, frequent price changes are observed during periods of higher average inflation, though, the size of the individual changes are independent of the level of inflation. The frequency of price change is independent of the stage of the business cycle - 30 percent of the price changes occur during a business cycle expansion and 34 percent of the price changes occur during a business cycle contraction - and explains why product price synchronisation is not observed in the data. He concludes, comparing between the different types of price-setting models, that neither time-dependent nor state-dependent models can explain the results completely.

Bils and Klenow (2002, 2004) use a large dataset of 388 categories of consumer goods and services that constitute 68 percent of consumer expenditures collected from 22,000 outlets for the United States from 1995-1997 and 1998-2001 by the Bureau of Labor Statistics (BLS). This is the first study where a dataset has been used that is large enough to represent most of the CPI sectors in the economy. They find that the weighted monthly mean and median frequency of price changes are 26.1 and 20.6 percent respectively and the median duration of price spells is 4.3 months (excluding temporary sales).¹² This is substantially lower than the results found by Kashyap (1995). They also find a lot of heterogeneity in the frequency of price changes; more than 70 percent of prices of gasoline and airfares change every month, while less than 5 percent of prices of newspapers and taxi fares change. They also look at the frequency of price changes by type of goods and find that 30 percent of prices for durable and nondurable goods change every month compared to 21 percent for services. Moreover, they look at the relationship between frequency of price change and the absolute size of the price of the item. They find that items that cost, on average, less than \$2 have more frequent price change compared to items that cost between \$2 and \$100. They use the overall mean frequency of price change and find that time-dependent models, such as Taylor (1979) and Calvo (1983), are not able to explain the observed inflation for a sample period: the models display more persistent and less volatile inflation than is observed in the data.

Baharad and Eden (2004) use a large dataset representing CPI price data to determine the extent of nominal rigidity. They use a dataset containing prices of 381 different products collected by the Israeli Bureau of Statistics from 458 stores during 1991-1992 to compute the CPI. They find that, on average, 24 percent, of the stores change prices per month. This shows that the mean duration between price changes is 4.1 months. However, one of the main focuses of this paper is to demonstrate that the conventional method of estimating duration of price spell is incorrect. Using the correct method of computing duration, they find that the average length that prices

¹²For duration of price spell, they do not report the mean duration of price spell, but instead focuses on the median duration.

remain constant is actually 7.9 months. They show that the reasons for the difference is due to Jensen’s inequality.¹³ In addition, they find that store characteristics and product characteristics are both equally important to estimate the length of the price spell.

Nakamura and Steinsson (2008) study the price-setting behaviour of firms by employing a dataset of 270-360 unique items that constitute 70 percent of CPI consumer expenditures, and a analogous Producer Price Index (PPI) dataset from 1988-2005 for the United States.¹⁴ In comparison to Bils and Klenow (2004), the study by Nakamura and Steinsson (2008), in addition to estimating frequency and magnitude of price changes for a larger dataset, provides additional results by looking at these price changes for regular price changes (excluding sales) and sales price changes. They are motivated to separate the effects as sales have become more common in the US and occur regularly, leading to a high frequency of price decreases (Pesendorfer (2002)). We report three major findings of the study that are relevant to our study. First, for consumer prices, the weighted mean and median frequency of regular price change (all price changes, including sales) is 21.1 percent and 8.7 percent respectively. They find that the median implied duration of regular prices (including product substitutions) is 11 months. There is a lot of heterogeneity in the results. Heterogeneity occurs due to distribution of sales by major group: 87.1 percent of price changes in apparel is due to sales, whereas almost no price changes occurred for vehicle fuel and utilities due to sales. Heterogeneity also occurs at the level of the major group: vehicle fuel has a median frequency of regular price change of 87.6 percent per month, whereas in the recreation goods sector, prices change only 6.0 percent per month. Second, Nakamura and Steinsson (2008) find that one-third (weighted median fraction of price decreases is 35.2 percent) of regular price changes that occur are price decreases, excluding sales and finished-goods producer prices, emphasizing that a substantial portion of the price changes are regular price decreases. Third, Nakamura and Steinsson (2008) find that the median absolute size of regular price change is 8.5 percent. They also look separately at the difference between the magnitude of price increases and decreases and find that the median size of price decreases is larger than the median size of price increases. They use these three results to determine their relationships with the inflation rate. They find that the frequency of price changes varies strongly with the inflation rate, whereas the frequency of price decreases and magnitude of price changes do not. They conclude that the frequency of price change and increases is positively related as the price level is drifting upward.

Klenow and Kryvtsov (2008) use the same dataset as Nakamura and Steinsson (2008) (though Klenow and Kryvtsov (2008) state that their dataset contains observations from 1988-2004, in contrast to 1988-2005, as in Nakamura and Steinsson (2008)) and report similar statistics re-

¹³This is explained in detail in Section 2.5.2.1.

¹⁴Nakamura and Steinsson (2008) analyse price data for 1988-1997 and 1998-2005 separately as item structure was changed in 1998. We present results for the entire period of 1998-2005.

garding the frequency of regular price change, implied duration between price changes, and the relationship between the size and frequency of price adjustments and inflation rate. Klenow and Kryvtsov (2008) find that most of the variation of aggregate inflation comes from variation in the average size of price changes, which is higher in Klenow and Kryvtsov (2008) compared to Nakamura and Steinsson (2008), with median estimates of 9.7 percent and 8.5 percent respectively.

The studies reviewed so far have been conducted with price data from the United States. Now, we present the results of a few studies that have been conducted to provide evidence of nominal rigidity in Europe. A large amount of work on nominal rigidity for the Euro area countries has been carried out by the Inflation Persistence Network (IPN) at the European Central Bank (ECB), while work on United Kingdom have been conducted by the Bank of England.

Dhyne et al. (2009) summarize and present results from large CPI databases covering various length of time for ten countries in Europe: Austria, Belgium, Germany, Finland, France, Italy, Luxembourg, Netherlands, Portugal and Spain. We report results on frequency of price change, proportion that are price increases and decreases, and magnitude of the price increases and decreases. First, in the Euro area, using country weights, the monthly frequency of price changes is 15.1 percent, while the duration of these price changes is about one year. The frequency of price change is slightly lower than the United States estimates for frequency of price change - this may be explained by sales, which are ubiquitous in the United States. It may also be because of higher inflation in the United States (Álvarez et al. (2006)). A lot of heterogeneity is observed for price changes among major groups. In the Euro area, a monthly average frequency of 78 percent is observed for energy products, whereas for non-energy industrial goods, it is only 9.2 percent. Heterogeneity is also present between countries: the mean monthly frequency of price change for Luxembourg is 25 percent compared to 10 percent for Italy with many countries that have frequencies of price changes that are in between these estimates. Second, they find that 42 percent of the price changes are decreases with similar estimates between the groups. This is similar to the estimates for the United States for proportion of price increases and decreases. However, in the Euro area, price decreases are less frequent in services compared to goods. Third, they report that the size of the price increases and decreases are 8 percent and 10 percent respectively. This is similar to the United States where the magnitude of price decreases were found to be larger than price increases. In general, there is heterogeneity in the size of these changes: for lettuce, price changes are in the magnitude of 30 percent compared to under 3 percent for fuels in the sample. In addition, they looked at the price synchronisation and found that price changes are not synchronised within or across countries. They find, using the Fisher and Konieczny (2000) index, that the value is smaller than one, indicating that price changes tend to be more staggered than synchronised. Lastly, they find that frequency of price changes increases with inflation.

Álvarez et al. (2006), in addition to summarizing the results from Dhyne et al. (2009) and

comparing the results with the United States, provide survey results from Fabiani et al. (2005) that cover 11,000 firms to explain price-setting behaviour of firms in the Euro area and the United States. They find that the most important reasons for price increases or decreases are input costs and competitors' price. Prices remain sticky since firms have implicit contracts with the customers (Okun (1981)) - 70 percent of firms have long-term relationships with customers - which is consistent with results found from the United States (Blinder (1994)) and United Kingdom (Hall et al. (2000)).

Bunn and Ellis (2012) use micro-level price data collected by the Office for National Statistics (ONS) covering the period 1996-2006 and supermarket (scanner data) price data, which comprise 280 distinct products from 2005-2008 collected by the research consultancy firm, Nielsen. They find that for the CPI price data, 19 percent of prices change every month, on average, with an implied mean duration of 5.3 months. From the supermarket data, they find that 60 percent of prices change every week on average with an implied mean duration of 1.7 weeks, compared to 33 percent per week in the United States (Kehoe and Midrigan (2007)). Heterogeneity is observed at two levels in the CPI microdata. First, there is a lot of heterogeneity among the major groups. 60 percent of energy prices change every month, on average, compared to housing, transport and recreational services, which change, on average, less than 10 percent. Second, consumer goods prices change on average, 24 percent per month, compared to only 9 percent a month for services. They find heterogeneity in the supermarket price data as well. In addition, they find that, across the product groups, 45 percent of the price changes are decreases. These results are comparable with studies for the Euro area and the United States. While they look at the magnitude of price changes, they look at non-absolute magnitude of price changes primarily to look at the distribution of price changes. They find a median magnitude of price change of 1.7 percent, which are not directly comparable to existing studies as they look at absolute magnitude of changes in prices. Hall et al. (2000) had conducted a survey of 654 firms in 1995 to examine the extent and reasons of price stickiness in the United Kingdom. They find that firms changed prices twice per year, which is similar to the results of average duration of price changes of 5.3 months. They find that costs and contractual agreements (both explicit and implicit contracts) are the most important reasons for firms to keep prices constant.

For the second part of the literature review, we explore the studies that have used micro-level price data and survey data to determine the presence and degree of nominal rigidity in the developing countries. Medina et al. (2007) is one of the first studies that attempted to provide empirical evidence on price stickiness by using a dataset for about 500 unique products and services for Chile over the period January 1999-July 2005. They find that on average, 46.1 percent of firms change prices every month, and the median firm changes price 33.3 percent every month, with a mean duration of about three months. In general, the frequencies of price changes

are very high for all groups. They state that this could be due to inclusion of sales in the data. There is substantial heterogeneity in the frequency of price change among groups and the range of these differences is very large. It is found that, on average, prices change twice every month for fuel and energy and once every month for food, whereas for groups such as education and leisure, they change, on average, only once a year. They find that aggregate inflation is positively correlated with frequency of price increases and price decreases (though, not frequency of price change) and size of the price changes. These show that prices in Chile adjust more frequently than developed countries.

Gouvea (2007) presents evidence on price rigidity in Brazil using a large dataset that cover 487 products and services grouped in seven major sectors. This data was collected from 2500 outlets for the period March 1996-April 2006. Using this dataset, he finds that the weighted mean price change frequency is 37 percent for the whole sample, with a median duration of 1.9 months. These results show that prices in Brazil are more flexible, compared to any of the studies conducted for developed countries. There is some heterogeneity in the major groups, and this is not uncommon in the literature, but the overall individual frequencies of price changes for groups are very high. The monthly mean frequency of price change for Food, Housing and Apparel are 42 percent, 43 percent and 58 percent respectively. The corresponding median durations are 1.67 months, 1.59 months and 1.19 months respectively. Second, he finds that 45 percent of the price changes are decreases, which is similar to the estimate for the United Kingdom and the United States. Third, he finds that the magnitudes of price increases and decreases are 15.99 percent and 12.57 percent respectively and that the size of the price increases is never below 10 percent. The size of the price increases are substantially higher in Brazil compared to the United States and Euro area estimates. This shows that there are large differences in the frequency and magnitude of price changes between Brazil and studies that covered the United States, United Kingdom and Euro area countries.

Şahinöz and Saraçoğlu (2008) study survey data collected by the Central Bank of the Republic of Turkey on 999 firms between May-July 2005. They find that the median firm changes prices 4 times per year, while they review prices once every month.¹⁵ This is noticeably higher than comparable estimates with the Euro area, where prices are changed once per year. They state that this is due to the high annual inflation rate of 8 percent during the period. They also investigate why firms do not change prices often and find that the most important reason for keeping prices constant is markup: firms do not change prices until their profit margin declines beyond a threshold. This is similar to the *S,s* theory of price change (Barro (1972); Sheshinski and Weiss (1977)). Finally, they conduct logit and probit regressions and find that the frequency

¹⁵Şahinöz and Saraçoğlu (2011) conduct another study using price data from 1988-2006 and find that the mean duration of price spell is 3.9 months.

of price changes are lower for smaller firms and when prices are not regulated.

Gagnon (2009) uses store-level price data containing 227 product categories representing 54.1 percent of Mexican consumption expenditures for the period January 1995-June 2002 from Mexico to determine the relationship between individual price changes and inflation. He finds a mean monthly frequency of price changes of 27.3-39.2 percent (range reflecting different inflation periods in the study). Since the focus of this paper is on inflation and individual price changes, the study uses time-series graphs to show the relationship between frequency and magnitude of price changes, so accurate estimates of relevant price statistics are not available from the study. In general, it can be observed from the graphs that the average (non-absolute) magnitude of price change over the sample stays between -5 percent to around 12 percent. Overall, he finds that when inflation is low, there is a weak relationship between inflation and frequency of price changes as the price decreases and increases cancel each other but a strong relationship between inflation and magnitude of price change. However, during periods of high inflation, both the average frequency and magnitude of price change are correlated with inflation. He concludes that his results from the low inflation sample of the study are similar to the studies conducted for the United States and Euro area countries.

Choudhary et al. (2011) conduct 1189 structured interviews of formal firms in the manufacturing and services sectors in two provinces in Pakistan between 2009-2011. They find that firms change prices three times a year, which reflects a duration of price spell of 4 months. They find that, on average, 24 percent of firms change their prices within a month. A calibration of these estimates to generate impulse responses show that a one standard-deviation interest rate shock on output gap dies out within three quarters, compared to the United States, where it dies out after the 17th quarter. They state that this shows that monetary policy is not able to generate output persistence in Pakistan.

2.3 Macroeconomic Context

An overview of the economy of Rwanda is provided in this section. We focus on a few key features of the economy, which may have an impact on the price data that we analyse in Section 2.6. It is worth mentioning here that the sample period for the dataset comprises January 2013-December 2014, but we look beyond this period for the first set of indicators to identify the key features of the economy of Rwanda.

Rwanda is a small relatively-closed economy, primarily reliant on its own production for output and growth. It is relevant to look into the openness of the Rwanda economy as this gives a measure of how exposed the economy is due to exogenous shocks or an imported cost-push inflation. Another important feature of Rwanda is its high reliance on agriculture: 33 percent of

GDP in 2013-2014 is derived from the sector. We present the trade openness ratio, which is the sum of exports and imports of goods and services as a share of gross domestic product, and share of agriculture in GDP in Rwanda in the table below:

Table 2.1: Economic Indicators of Rwanda: 2010-2014

Indicator	2010-2011	2011-2012	2012-2013	2013-2014
Trade Openness	39	43	44	45
Agriculture (value added), percent of GDP	32.60	32.40	33.40	33.40

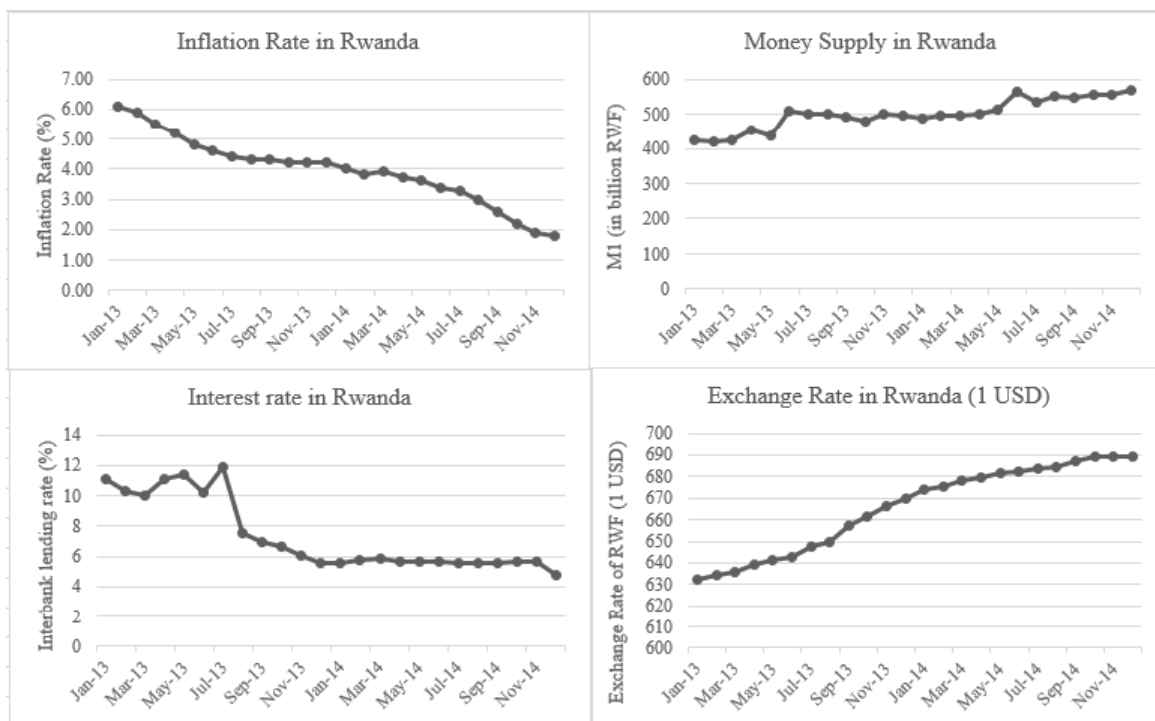
Data Source: World Bank

These two indicators are particularly relevant to this study. Rwanda is less open to trade compared to countries such as United Kingdom and Germany where the trade openness ratio is 62 percent and 85 percent respectively, in 2013-2014. Kenya, a country that shares borders with Rwanda, has a much higher trade openness measure at 51 percent; this may be attributed to the port city of Mombasa, through which large trade takes place in the country. Rwanda's reliance on agriculture is high, particularly compared to developed countries like the United States and United Kingdom where the ratio is 1.30 and 0.70 respectively, in 2013-2014. This will be particularly useful in the analysis that follows later.¹⁶

To explain the macroeconomic characteristics of Rwanda, we look at a few key macroeconomic indicators for Rwanda, that are tied closely with prices in the economy. Specifically, we look at inflation rate (measured as percent increase in the CPI relative to 12 months earlier), exchange rate, interest rate (interbank lending rate), and money supply (M1). Figure 2.1 illustrates the monthly trends of these variables from January 2013-December 2014:

¹⁶Data source: World Bank.

Figure 2.1: Main Macroeconomic Indicators: 2013-2014



Data Source: National Bank of Rwanda

In the sample period under consideration, inflation rate has declined in Rwanda, falling from 6.10 percent in January 2013 to 1.80 percent in December 2014, as is observed from Figure 2.1. This is due to lower imported food and fuel prices, as global commodity prices have remained low during this period (Nishiuchi and Perks (2014)). We also observe in the figure that the local currency (Rwandan Franc) has depreciated against the US Dollar over the period. This is expected to be due to the excess demand of foreign reserves: the season B harvest in 2013 was poor and has led to higher reliance on imported food.¹⁷ There are two spikes in the money supply in July of each year: the National Bank of Rwanda has attempted to increase M1 to allow farmers to produce more domestic food, to help reduce demand on imported food, and, thereby, appreciate the local currency. However, depreciation of the Rwandan franc did not cease. It is worth mentioning that Rwanda follows a ‘declared’ flexible exchange rate regime, though, as is common with developing countries, the monetary policy regime may, given the circumstances, differ in practice. The interbank lending rate has fallen during this period, primarily attributed to decreasing Treasury-bill rate.

¹⁷Rwanda’s agriculture season A runs from September to February and agriculture season B runs from February to June.

2.4 Data

2.4.1 Data Description

The dataset used in the study is acquired from NISR and consists of price data for the period January 2013-December 2014. The dataset consists of price quotes that are collected from 100 stores in the capital city of Rwanda, Kigali, and are primarily used for the computation of the Consumer Price Index (CPI) in Rwanda. The price data in the study comprises 323 unique items and contains a total of over 23,000 price quotes, which encompasses 86.04 percent of items for estimating the CPI of Kigali.

An individual observation in the dataset refers to disaggregated price data of an item from a store in a specific market in Kigali. An item is considered to be identical only when it is the same - by brand, weight and origin. For e.g. a 1 kg bar of soap named Dove, imported from Uganda is not the same if a 1 kg bar of soap named Dove sold in another store is imported from Kenya.

Data is collected by enumerators at the NISR twice per month, once at the beginning and once at the end of the month. The price data that we have been provided is the price quote at the beginning of the month. Price data is collected from the same outlet by the enumerators every month which makes it easy to track outlet and products.

Sales and discounts are not common in Rwanda and any price decrease would be a regular price decrease. This feature of this dataset is similar to Kashyap (1995). Product substitutions, which required adjustments of items contained in the dataset for some studies (for e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)), are not an issue we encountered in this study as the dataset is relatively small. In the two years of data that we have been provided by the NISR, there were no product substitutions.

2.4.2 Data Confidentiality

Price data of this type is collected by the NISR for the purpose of computation of the CPI of Rwanda. The data is collected by enumerators via surveys conducted at the stores. It is stated in the consent form of the questionnaire that individual price quotes will not be shared with any third party or for any other purpose/computation besides the CPI. The confidential nature of this data made it difficult to collect at such a disaggregated level. Initially, the NISR was willing to share average prices of items by market - a dataset that is not available publicly - but the data was not very useful for the purpose of this study since we are keen to track prices of items by stores, not by markets. Eventually, we came to an agreement with the NISR and they provided us with a sample of the data, from which they removed market information where the stores are located. While the additional information - market details, price quotes from other districts - would make

the analysis more complete, the dataset that we have still allows us to compute estimates for the frequency of price change, duration of price spell and supplementary price statistics for Rwanda and compare with existing studies.

2.4.3 Data Sample and Weights

The price data is collected and classified by the NISR under the United Nations Classification of Individual Consumption According to Purpose (COICOP). There are various levels of classifications under the COICOP. The ‘top level,’ as it is termed by COICOP, categorizes the products into twelve major groups: Food and non-alcoholic beverages, Alcoholic beverages, tobacco and narcotics, Clothing and footwear, Housing, water, electricity, gas and other fuels, Furnishings, household equipment and routine household maintenance, Health, Transport, Communication, Recreation and Culture, Education, Restaurants and Hotels, Miscellaneous goods and services. For the computation of the CPI for Rwanda, the products are classified into 62 narrow item categories and individual weights are put on the item categories to calculate the overall CPI for the country. The weights used for the CPI index are derived from the Household Living Conditions Survey 2010-2011, which comprised a total of 14,308 households (National Institute of Statistics of Rwanda (2014)). These individual weights are not published publicly, but the NISR has shared these weights with us for the purpose of this study. The 62 item categories along with individual weights are available in Table A.1 in the Appendix.

The use of the UN COICOP is common among CPI calculation in Europe and Africa. Dhyne et al. (2009) use the UN COICOP as item classification and assign individual country weights to the items to calculate the price statistics of the countries in Europe. However, since these weights depend on consumption expenditures by countries, the weights vary across countries. This is more prominent between countries in Africa and Europe, for e.g., food and non-alcoholic beverages have a weight of 44.11 percent and 11.20 percent in Rwanda and United Kingdom respectively in calculation of the CPI in 2014.¹⁸

2.4.4 Data Cleaning

The dataset that we have received from the NISR was cleaned for calculating relevant price statistics for the study. Most of the items in the dataset were in the local language Kinyarwanda and French and these were translated to classify under the appropriate COICOP category. The original dataset included the COICOP codes and these were verified after the items were translated and any discrepancies cleaned in discussion with the NISR team. In addition, any outlier found for any item in the dataset was cleaned after discussion with the NISR. Finally, individual weights

¹⁸Source: Office for National Statistics (2015)

were inserted manually and assigned to items for calculating weighted price statistics as these were not included in the original dataset.

2.5 Methodology

The dataset that we have obtained from the NISR is used to estimate price statistics for Rwanda. Since these estimation methods are not very straightforward to compute, we provide details on the construction of these statistics in this section.

The data has been divided into three broad categories for analysis. First, we look at the price changes by 62 narrow item categories, denoted as i , which are used by the NISR to compute the CPI index for Rwanda. Since these item categories are used to calculate the CPI, we use this to compute an accurate and reliable weighted mean and median of price statistics for Rwanda. It is worth emphasizing that the ‘weighted’ price statistics are calculated using the weights of these narrow item categories; the categorizations that follow are mainly for the purpose of observing the heterogeneity in the results. For the second broad categorization, we divide the data into 12 major groups. It is useful to make larger groups as it allows to generalise the results by sector and major groups in the economy. In addition, it allows us to report the heterogeneity that is observed in the data by major groups. This is important to consider since heterogeneity in price changes is observed in developed and emerging countries, as seen in Section 2.2. Finally, we look at the relevant price statistics by type of good, as prices of goods may change differently in comparison to prices of services. For calculation of the weighted price statistics, it is useful to note that it needs to be verified that the weights from the items in the dataset add up to 100 percent. To ensure this, we follow Bils and Klenow (2004) and Nakamura and Steinsson (2008), and estimate the weights or the cumulative distribution of frequency of each item category, by $\omega_i = w_i / \sum_{i=1}^I w_i$, where the ω_i = cumulative distribution of frequency (CDF) of price changes within the share of CPI covered, w_i = individual weights of i , and $i = 1, 2, \dots, I$, where in the data, $I = 62$.

2.5.1 Frequency of Price Changes

The frequency of price changes is defined as the fraction of times prices are changed (independent of store or item) on average over all months in the data. The dataset contains monthly price observation of items, which, we assume to be $p_{i,s,t}$, where s refer to store and t refers to the current month and i as defined before. We create a dummy variable, $f_{i,s,t}$, and compute that a price change has occurred when:

$$f_{i,s,t} = \begin{cases} 1 & \text{if } p_{i,s,t} \neq p_{i,s,t-1} \\ 0 & \text{if } p_{i,s,t} = p_{i,s,t-1} \end{cases}$$

where $f_{i,s,t}$ refers to a price change for a specific item category i at a specific outlet s at time t . The frequency of price change for product i at store s , $f_{i,s}$, is given by:

$$f_{i,s} = (1/T) \sum_{t=1}^T f_{i,s,t}$$

where $T = 0, 1 \dots T$ and in the data $T = 23$. Next, we aggregate all these frequencies of price change at the item category level i by averaging over all the stores that sell the same item, which gives us the mean frequency of price change for that item category i , which we refer to as $\bar{f}_i$:

$$\bar{f}_i = (1/S) \sum_{s=1}^S f_{i,s}$$

where S refers to the number of stores in which a particular item, i , is sold.

The weighted mean frequency of price change, $\bar{f}_i^\omega$ is then estimated by assigning weights (CDF, to be precise) to items and then aggregating at the item category level, as shown:

$$\bar{f}_i^\omega = \sum_{i=1}^I \omega_i \bar{f}_i$$

The weighted median frequency of price changes at the item category level is estimated using the same method as described above, where the weighted median is taken at the item category level to compute the weighted median frequency at the item category level and is denoted by f_i^ω . The estimates $\bar{f}_i^\omega$ and f_i^ω are referred to as the weighted mean and median frequency of price change respectively for the overall dataset.

The mean and median frequency of price change for major group is estimated analogously and are denoted as $\bar{f}_m$ and f_m , respectively, where m refers to major group (and replaces i in the estimation for frequency of price change by major group), with $m = 1, 2 \dots M$, and, in the data $M=12$. The purpose of separating by major group is to observe the heterogeneity of the frequency of price change at the major group level. Similar exercise is carried out for type of good, where the mean and median frequency of price change by type of good are denoted as $\bar{f}_g$ and f_g , respectively, where g refer to type of good, with $g = 1, 2, \dots G$, and, in the data $G=4$ (durable good, semi-durable good, non-durable good and services).

2.5.2 Duration of Price Spells

Duration of a price spell is defined as the length of time between a price change. In reality, prices can change continuously and can change more than once per month. However, to estimate the duration of price spell, due to the constraint of the dataset, we assume that prices have

changed if $p_t \neq p_{t-1}$. If we assume that there is a constant hazard h of a price change, the monthly probability of a price change at any instant is given by $f_i = 1 - e^{-h}$, which implies that $h = -\ln(1 - f_i)$ and duration can then be written as $d_i = 1/h = -1/\ln(1 - f_i)$ (Nakamura and Steinsson (2008)). We calculate mean duration by:

$$\bar{d}_i = -1/\ln(1 - \bar{f}_i)$$

where $\bar{d}_i$ is the mean duration of price spell by item. The weighted mean duration of price spell is estimated by:

$$\bar{d}_i^\omega = \sum_{i=1}^I \omega_i \bar{d}_i$$

For estimating the weighted median duration of price spells, we follow the literature and use a direct approach by using the weighted median frequency of price change, given by:

$$d_i^\omega = -1/\ln(1 - f_i^\omega)$$

2.5.2.1 Estimation of Duration: Existing Issues

Baharad and Eden (2004) found that the conventional method of estimating duration of price spell is incorrect. They show that if the length of the period varies across products and stores, there may be a downward bias in the estimation of duration. They argue that inverting at different stages of aggregation of products/stores lead to different estimates due to Jensen's inequality ($E(1/x) \geq 1/E(x)$). In addition, Bils and Klenow (2004) use the mean and median durations of price spell for a one-sector and multi-sector model and observe the responses due to an aggregate nominal or real shock. They find that the mean duration of price spell is not able to explain the multisector model well - this is the reason why they do not report the mean duration in their study. Therefore, we report only the weighted median duration of price spell for comparison across various studies.

2.5.3 Direction of Price Changes

It is useful to look at whether these price changes have been increases or decreases. Since sales and discounts are not very common in developing countries, the direction of price change provides an understanding of the price-setting behaviour of firms.

For computing the direction of price changes, we simply deduce the number of times prices have increased (decreased) for an item. We developed a dummy variable, $x_{i,s,t}$,

$$x_{i,s,t} = \begin{cases} 1 & \text{if } p_{i,s,t} > p_{i,s,t-1} \\ -1 & \text{if } p_{i,s,t} < p_{i,s,t-1} \\ 0 & \text{otherwise} \end{cases}$$

where $x_{i,s,t} = 1, -1$, which are dummies for instances when we have a price increase and decrease respectively, and refers to the change in direction in price for a specific item i and specific outlet s at time t . The direction of price change for product i at store s , which we refer to as, $x_{i,s}$, is given by:

$$x_{i,s} = (1/T) \sum_{t=1}^T x_{i,s,t}$$

We use the same methodology to calculate weighted mean and median price increase and decrease as we previously have used for estimation of frequency of price change, but with two separate dummies here to reflect price increase and decrease separately. The weighted mean and median direction (increase/decrease) of price changes are denoted by $\bar{x}_i^\omega$ and x_i^ω , which refer to the overall weighted mean and median direction (increase/decrease) of price changes respectively.¹⁹

The mean and median direction of price change for major groups is estimated analogously and is denoted as $\bar{x}_m$ and x_m , respectively. The mean and median direction of price change by type of good are denoted as $\bar{x}_g$ and x_g , respectively.

2.5.4 Magnitude of Price Changes

It's important to look at the magnitude of price changes as it is expected to be different due to differences in frequency of price changes. We compute the absolute magnitude of price increases and decreases separately to look at the differences between the size of the price increase and decrease. As we developed an indicator for instances where prices increase or decrease, we can easily estimate the magnitude of price change by:

$$z_{i,s,t} = |(p_t - p_{t-1})/p_{t-1}|$$

where $z_{i,s,t}$ refers to the absolute magnitude of price increase or decrease for an item, i , at store s , at time t . The weighted mean and median of magnitude of price change is calculated similarly to previous price statistics and we denote them as $\bar{z}_i^\omega$ and z_i^ω . For the major group, we denote the weighted mean and median magnitude of price change as $\bar{z}_m$ and z_m respectively. For type of good, we denote the weighted mean and median magnitude of price as $\bar{z}_g$ and z_g respectively.

¹⁹It may help to also think of direction of mean price increase and decrease as $\bar{x}_i^+$ and $\bar{x}_i^-$.

2.6 Empirical Results

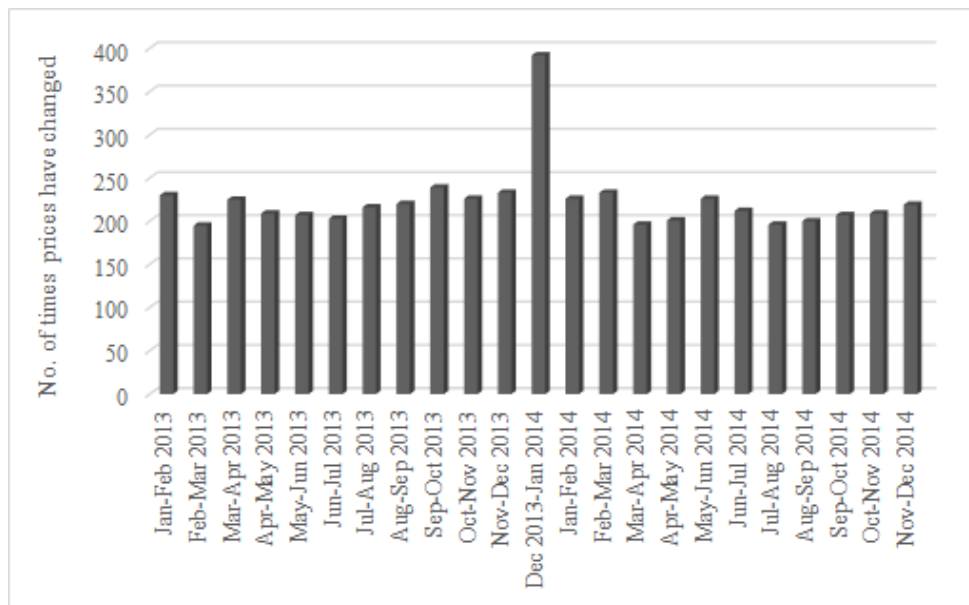
The results based on the dataset are presented in two broad sections. The first section where the overall results are presented take into account of the weights that are used to compute the CPI and estimate the weighted price statistics in this study. This provides the weighted mean and median frequency of price changes, which is the primary indicator to determine the degree of nominal rigidity, and also serves as the main indicator to compare across different studies. Using the estimates of frequency of price change, estimates of duration of price spells are derived, which indicates the length of time for which prices have remained constant. Moreover, we assess the proportion of the price changes that are increases and decreases. This is an important indicator that explain the trend in inflation data in the country and provides an understanding of the price-setting behaviour of firms. This is particularly relevant for a developing country where sales and discounts are not common, and, hence, a price decrease may be motivated by factors such as bumper crop harvest, etc., which are unlikely reasons for price decreases in developed countries. We also compute the weighted mean and median magnitude of the price changes to determine the percentage change in prices of items.

The second set of results that we present are price estimates categorized, first, by major groups, and, second, by type of good. This is particularly important because it demonstrates the heterogeneity of the results. For studies on developed countries, it has been stressed that price changing behaviour and frequency vary between sectors (Bils and Klenow (2004); Millard and O’Grady (2012); Dixon and Kara (2011)). This is expected as some sectors such as agriculture and food, where the products are usually unprocessed and sold directly by the farmers, a change in price of commodity is reflected in a direct change in the price of the product itself. This is in contrast to sectors such as services, where there are different stages of production, and any change in price is absorbed by one of the stages of production, and results in no change in the price of the product (Kovanen (2007)). It is also important to explore the heterogeneity of price changes for direction of price changes and magnitude of price changes. Nakamura and Steinsson (2008) find that travel (excluding services) has a higher median absolute change in magnitude compared to other sectors such as apparel, utilities, etc. This is expected as the travel industry in developed countries is very competitive and many offers and discounts (in addition to regular price changes) are offered to customers in order to attract them to purchase their services.

2.6.1 Results: Overall

The first statistic we report is the absolute number of times price changes have occurred in a month over all the months in the data. The primary purpose of this is to see whether a noticeable trend can be observed in the data.

Figure 2.2: Number of Price Changes by Month in Rwanda: 2013-2014



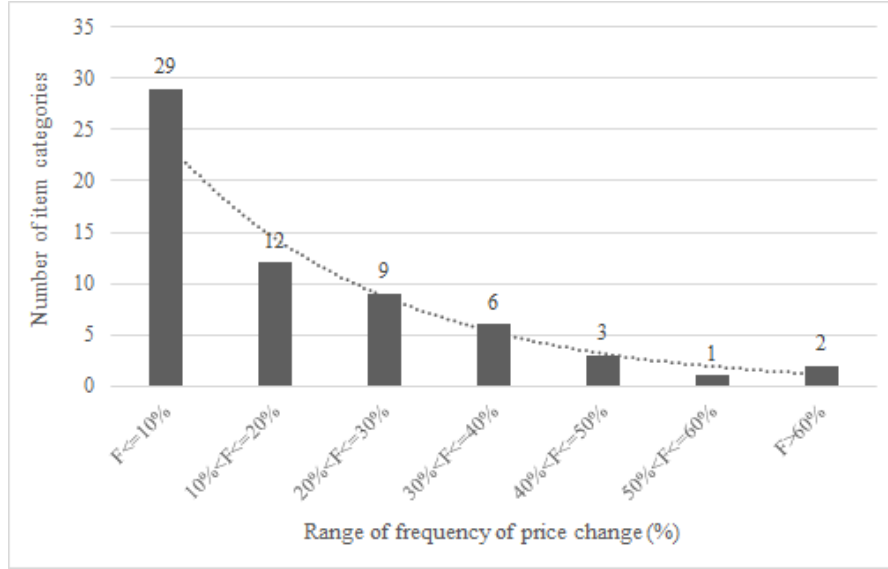
Notes: Frequency refers to the total number of times prices have changed

Figure 2.2 shows that between January 2013 and December 2014, price changes have remained relatively stable, except in December 2013-January 2014. A large price change at the end of the year is not very uncommon and a closer observation at the dataset shows that majority of the changes during this time took place in the Health sector. This may be simple price increases that occurred at the end of the year for the sector. It is unfortunate, though, that we do not have a dataset covering more periods, as that would have allowed us to provide a better understanding of the spike in the frequency of price changes and the trend of the price data.

Table A.1 in the Appendix lists the 62 different item categories in the Rwanda CPI consumption basket and provides the mean frequency by item category. The weighted mean and median frequency of price change in Rwanda for 2013-2014 are 43.62 percent and 28.68 percent respectively. The large difference in mean and median frequency is due to the fact that a large number of item categories have low frequencies of price change and few high frequency items contribute to the high mean frequency of price change.

As seen in Figure 2.3, the frequency of price change is positively skewed, implying that the right tail is longer than the left tail of the distribution of the frequency of price change. Almost half of the item categories (46.77 percent) have price change frequencies at or below 10 percent. Further, the trendline shows that fewer item categories are observed as the frequency of price change increases. There are only three item categories that have a price frequency change of greater than 50 percent.

Figure 2.3: Frequency of Price Changes by Item Categories in Rwanda: 2013-2014



There is, in general, a lot of heterogeneity in the mean frequency of price changes among the item category, which is not very surprising, as factors affect different sectors differently. For instance, the mean frequency of price change for fruit and vegetables are 87.76 percent and 91.82 percent respectively, whereas for restaurants, cafés and likes and stationary are 4.60 percent and 6.39 percent respectively. This shows that prices are very flexible for some items and very rigid for others.²⁰

Another factor that contributed to the high weighted mean frequency is the weight of the individual item categories. Weights play a big role to induce the high weighted mean frequency that is observed in Rwanda. Vegetables, which has a mean frequency of 91.82 percent, has a weight of 22.7 percent in the Rwanda CPI consumption basket, which is 2.6 times the weight of bread and cereal, the second-largest weight after vegetables in the CPI consumption basket. The weighted mean frequency of price change is skewed largely by the high frequency of price change observed among vegetables. As was explored in Section 2.3, agriculture is an important sector for Rwanda, which explains the large weights assigned to components within this sector.

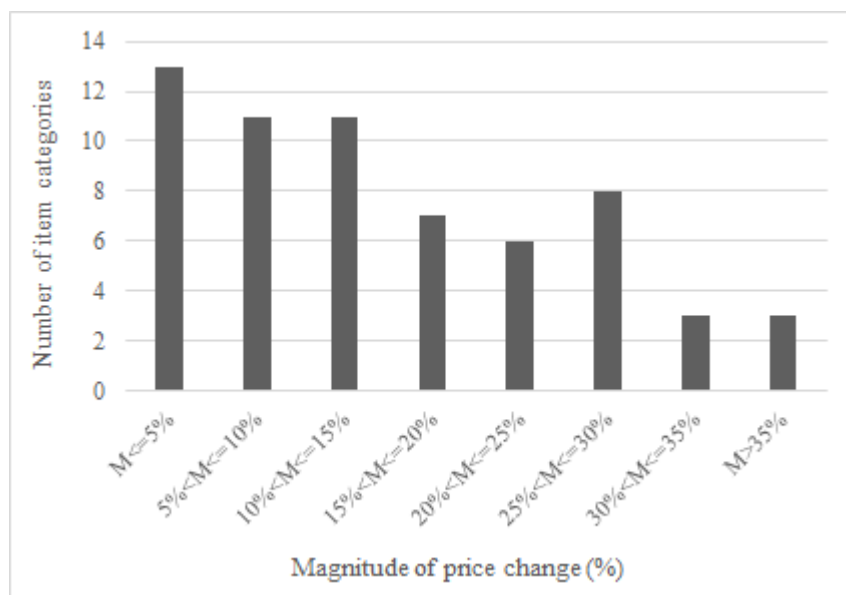
The weighted median duration of price spell is 2.96 months. Table A.1 in the Appendix lists the mean duration of price spell by item category. We observe that the duration of price spell varies considerably among item categories. The mean duration of price spell for vegetables is 0.7 month, whereas for medical services and hairdressing salon, it is 22.4 months. The large difference in duration enforces the heterogeneity of price setting behaviour in different sectors and across

²⁰However, there are not many observations under each item category, which makes it difficult to generalize among item categories and the issue of heterogeneity is tackled in detail in the next section. However, Table A.1 in the Appendix is still useful as it gives us a narrow breakdown of the frequency of price changes, especially for item categories for which many price observations are available.

item categories. Due to the fewer number of observations under each item category, looking at the duration of price spell by item category does not provide a reasonable distribution of duration of price spell. We explore duration in greater detail in the next section when we segregate duration of price spell by major group as there are more observations under each major group.

The (unweighted) mean price increase as a proportion of any price change is around 60 percent, which means that 40 percent of the price changes are decreases. Since sales and discounts are not common marketing strategies in Rwanda, these price decreases can be attributed as regular price decreases. This is consistent with the inflation data that we observed in Section 2.3: higher proportion of price increase compared to price decrease is reflected in the inflation existing in the economy. The weighted mean and median magnitude of price change is 16.67 percent and 14.19 percent respectively.²¹ We further divide these price changes into magnitude of price increases and decreases. The weighted mean and median magnitude of price increase is 18.80 and 16.08 percent respectively. The weighted mean and median magnitude of price decrease is 11.43 percent and 12.33 percent respectively. The figure below summarizes the analysis of the magnitude of price change by item categories.

Figure 2.4: Magnitude of Price Changes by Item Categories in Rwanda: 2013-2014



As Figure 2.4 shows, the distribution of the magnitude of price change is right skewed, though, the right tail is not long. There are item categories with both high and low magnitude of price changes, and explain why the mean and median magnitude of price change is close to each other.

We do not find a strong relationship between the mean frequency and magnitude of price

²¹This is the magnitude of price change, for any period, when there has been a price change. Initially, an estimate was created for total absolute price change, which was averaged over the periods in the data. It is deemed that the former estimate is more accurate for estimating the absolute percentage change in magnitude of prices.

change. A simple correlation and regression results show that, though, frequency and magnitude of price change is negatively correlated, the coefficient is not very significant. We expect that, intuitively, that an inverse relationship may exist between frequency and magnitude of price change, as stores that change prices more frequently may not change prices by a large amount and vice versa. However, we did not observe a significant relationship between the two estimates.

2.6.2 Results: Major Group

(i) *Frequency of Price Changes*

We look at the frequency of price change by major group in this section. The results presented in this section provides us with a more reliable estimate of the observed heterogeneity of price change frequency in comparison to the estimates presented in the previous section. Since there are more observations under each major group (in contrast to item category), we are able to compute more representative estimates of the frequency of price change and its heterogeneity among groups. Table 2.2 shows the mean and median frequency of price change by major group.

Table 2.2: Frequency of Price Changes and Duration in Rwanda: 2013-2014

	No. of obs.	Frequency (% per month)		Standard Deviation	Duration (months)
Major Group	N	Mean	Median	Mean	Mean
Communication	3	0.00%	0.00%	0	-
Education	6	2.17%	0.00%	0.0364	16.74
Health	260	5.32%	4.35%	0.0941	20.48
Restaurant and hotels	45	5.60%	4.35%	0.0605	14.40
Recreation and culture	84	7.66%	0.00%	0.1208	9.57
Alcoholic beverages, tobacco and narcotics	27	7.73%	4.35%	0.0864	14.34
Miscellaneous goods and services	59	11.13%	0.00%	0.1677	8.04
Transport	22	14.03%	15.22%	0.1321	7.78
Furnishings, household equipment and routine household maintenance	55	19.37%	8.70%	0.2273	6.03
Housing, water, electricity, gas and other fuels	44	25.89%	23.91%	0.2262	3.26
Clothing and footwear	140	27.30%	26.09%	0.2454	5.01
Food and non-alcoholic beverages	260	49.11%	47.83%	0.3936	4.54

Table 2.2 shows the mean and median frequency of price change by major group. The first observation is that the median frequency of price change is lower than the mean frequency of price change for all groups except transport. This shows that distribution of frequency of price change is right skewed, which means that many observations exist with low frequency in each of the major groups except transport.

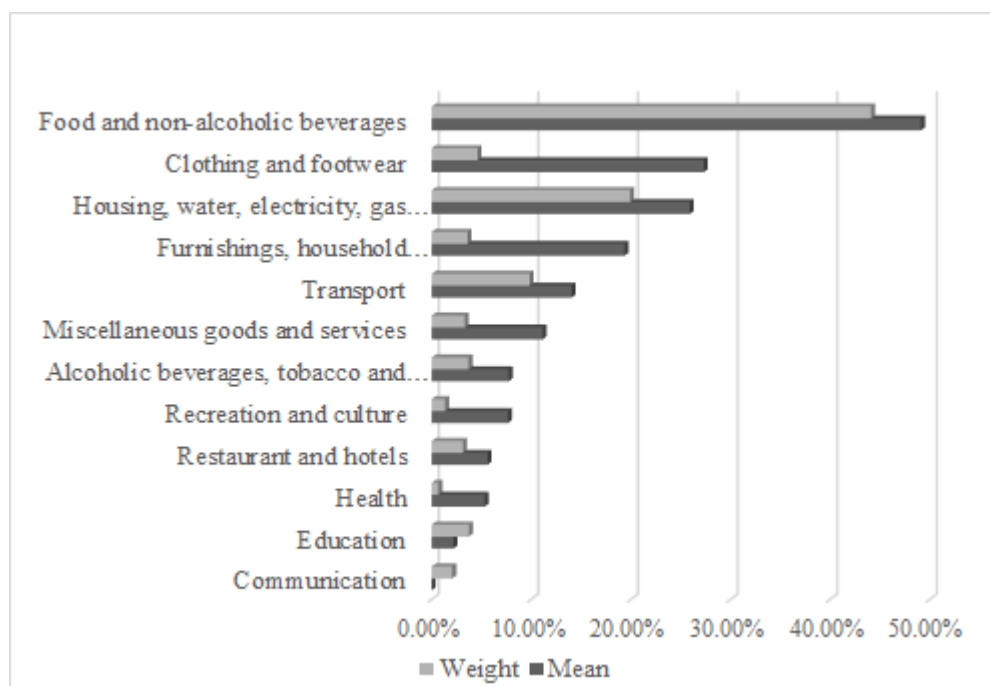
There is notable heterogeneity among the major groups and also within the groups. Food and non-alcoholic beverage has the highest frequency of price change with 49.11 percent of prices changing each month. The standard deviation is also very high demonstrating that there is a lot of variation within the group. It is worth noting that vegetables and fruits are components of this group, which may have largely contributed to the high frequency of price change. Health sector has a mean frequency of price change of 5.32 percent, which includes the large number of changes that were observed in December 2013. It is expected that the mean frequency of price change in this sector is generally lower. Similar to the breakdown of item category, half of the major groups have a frequency of price change of less than 10 percent, re-enforcing the right skewness in the distribution of price change frequency.

(ii) *Duration of Price Spells*

Duration of price spell here is calculated by computing individual item duration and then taking mean of duration of price change by major groups. Therefore, we do not use the direct approach of applying the frequency of price change to estimate the mean duration of price change by major groups. The advantage to this approach is that all of the price spells by item can be taken into account. As Table 2.2 shows, housing, water, electricity, gas and other fuels and food and non-alcoholic beverages have the shortest price spells with mean durations of 3.26 and 4.54 months respectively, on average. Health sector has, on average, a large price spell with a mean duration of 20.48 months. This shows the large heterogeneity in the duration of price spells among major groups.

We look at the effect of weights of major groups on the frequency of price change, which is given by the figure below.

Figure 2.5: Frequency of Price Changes and Weights of Major Group in Rwanda: 2013-2014



It is evident looking at Figure 2.5 that weights contribute significantly to the high mean frequency of price change. Food and non-alcoholic beverages have a weight of 44.11 percent in the Rwanda CPI consumption basket, and given that it also has the highest frequency of price change, it is obvious that this plays a big role in the high weighted mean frequency of price change. Housing, water, electricity, gas and other fuels, which has a weight of 19.92 percent in the consumption basket, has a mean frequency of 25.89 percent. Clothing and footwear is the only exception: it has a mean frequency of price change of 27.30 percent, but its weight is only 4.60 percent in the CPI consumption basket. As a result, it did not contribute substantially to an overall increase in the price change frequency.

(iii) *Direction of Price Changes*

In this section, we look at the direction of the price changes. Since Rwanda does not have ‘sales’ or discounts, any decrease in price can be referred to as a regular price decrease. The table below shows the direction of price changes:²²

²²We exclude the results of the Communication group from our analysis of direction of price change as we did not observe any price changes in the small sample of this group.

Table 2.3: Direction of Price Changes in Rwanda: 2013-2014

Major Group	No. of observations	Price increase (%) per month		Price decrease (%) per month	
		Mean	Median	Mean	Median
Education	6	1.45%	0.00%	0.72%	0.00%
Health	260	4.08%	4.35%	1.24%	0.00%
Recreation and culture	84	4.24%	0.00%	3.42%	0.00%
Restaurant and hotels	45	4.35%	4.35%	1.26%	0.00%
Alcoholic beverages, tobacco and narcotics	27	4.99%	4.35%	2.74%	0.00%
Miscellaneous goods and services	59	6.12%	0.00%	5.01%	0.00%
Transport	22	7.91%	6.52%	6.13%	4.35%
Furnishings, household equipment and routine household maintenance	55	9.72%	4.35%	9.64%	4.35%
Housing, water, electricity, gas and other fuels	44	13.24%	10.87%	12.65%	13.04%
Clothing and footwear	140	14.47%	13.04%	12.83%	13.04%
Food and non-alcoholic beverages	260	25.38%	21.74%	23.73%	21.74%

Table 2.3 shows the mean and median price increases and decreases per month. We can establish that there is a strong correlation between mean price increases and decreases among major groups. In fact, we find that price increases and decreases have a correlation of 0.98. Therefore, it can be concluded that any major group that exhibits price increases are almost just as likely to exhibit price decreases. This, in turn, shows that idiosyncratic shocks are driving most of the price changes, rather than inflation in the economy.

We observe variation in the frequency of price increase and decrease between the major groups. For the food and non-alcoholic beverages group, mean frequency of price increase is 25.38 percent per month, whereas for the health sector, it is only 4.08 percent. Similarly, there is a frequency of price decrease of 23.73 percent per month for the food and non-alcoholic beverage group, but it is only 3.42 percent for the recreation and culture group.

We also investigate the proportion of these price changes that are price increases and decreases. It can be discerned from Table 2.3 that price increases are slightly more likely than

price decreases. For health and restaurant and hotels, the proportion of price changes that are increases are 76.73 percent and 77.59 percent respectively. On average, we find that almost 60 percent of any price change in the data has been increases in price.

(iv) *Magnitude of Price Changes*

We report the size of price changes by major group in this section. We are motivated to examine the magnitude of the price changes because it is expected that more frequent price changes would mean that when price changes occur, the magnitude with which price changes occur will be small. Table 2.4 shows the mean and median magnitude of price change percent by major group.²³

Table 2.4: Magnitude of Price Changes in Rwanda: 2013-2014

Major Group	No. of observations N	Magnitude of price change (%)	
		Mean	Median
Housing, water, electricity, gas and other fuels	44	12.15	13.21
Alcoholic beverages, tobacco and narcotics	27	12.75	11.11
Furnishings, household equipment and routine household maintenance	55	14.03	13.54
Clothing and footwear	140	15.03	12.44
Food and non-alcoholic beverages	260	15.10	12.16
Restaurant and hotels	45	15.28	14.29
Transport	22	21.14	8.60
Health	260	23.26	20.00
Recreation and culture	84	24.82	20.00
Miscellaneous goods and services	59	30.17	13.33
Education	6	50.16	50.16

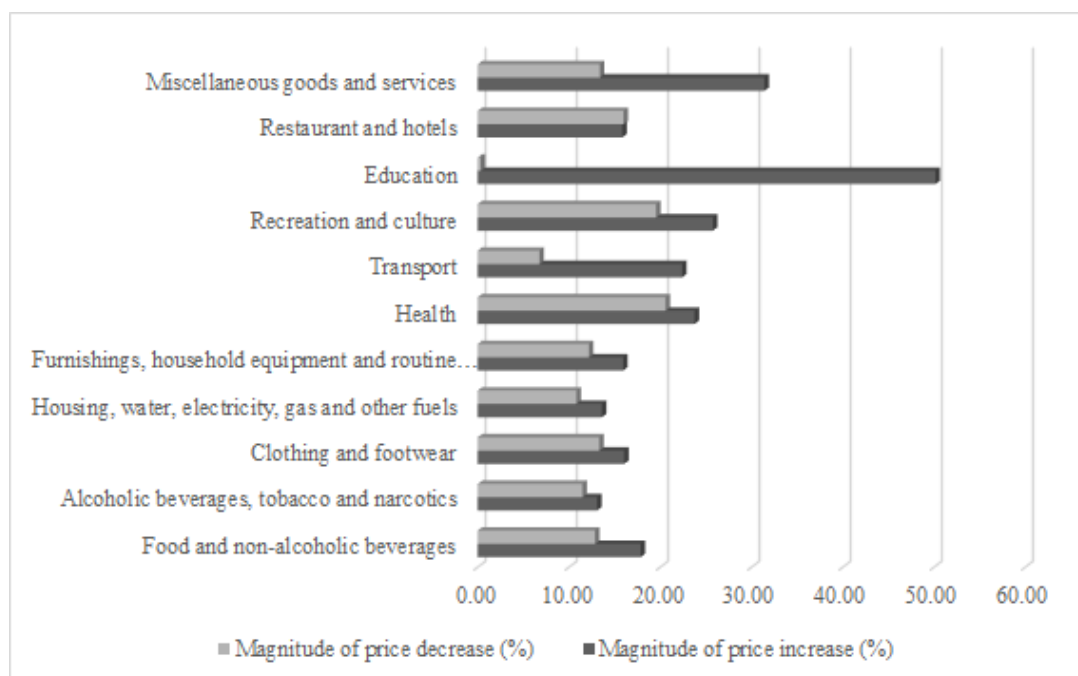
For any price change, it is observed that the magnitude with which these price changes occur are greater than 10 percent for any major group. We do not observe significant variation in the

²³We exclude the results of the Communication group from our analysis of magnitude of price change as we did not observe any price changes in the small sample of this group. This is similar to our analysis of direction of price changes.

magnitude of price changes, with the lowest magnitude in the housing, water, electricity, gas and other fuels group with 12.15 percent and the magnitude lower than 31 percent for all major groups except Education. It is believed the large magnitude is due to the fact that the frequency of price change is low (2.17 percent), so any price change occurring in the group is a relatively large change. On average, the standard deviation of magnitude of price change is 11.19 percent (not shown). The mean and median magnitude of price change are very close to each other for most groups, implying a uniform distribution with the major groups.

Figure 2.6 shows the magnitude of price increases and decreases in percentage in Rwanda for the sample period. It is obvious from the figure that magnitude of price increases is greater than the magnitude of price decreases. In fact, a simple (unweighted) average of the magnitude of price increase is 22.36 percent whereas it is only 12.48 percent for price decreases. We observed in the last section that 60 percent of the price changes in the data are price increases. It is clear from the estimates from this section that these price increases are also large in magnitude.

Figure 2.6: Magnitude of Price Increases and Decreases in Rwanda: 2013-2014



There is a slight variation between the magnitude of these price changes among major groups, as observed in Figure 2.6. For education group, almost all the price changes that occurred were price increases, and, therefore, the magnitudes of price change and increase are almost the same. To measure for variation, simple standard deviations of the magnitude of price change are computed. The standard deviation of magnitude of price increases and decreases are 10.84 percent and 5.61 percent respectively.

2.6.3 Results: Types of Goods

In this section, we explore if any significant differences exist in the frequency at which prices change for different types of goods. The goods are divided into four types: i) Non-durable good, ii) Semi-durable good, iii) Durable good, and iv) Services. We find that prices change the most for non-durable goods and least for services. Table 2.5 summarizes the main results:

Table 2.5: Frequency of Price Changes and Duration by Type of Good in Rwanda: 2013-2014

	No. of obs.	Frequency (% per month)		Duration (months)
Type of good	N	Mean	Median	Mean
Services	346	3.90%	4.35%	20.38
Semi-durable good	169	27.48%	26.09%	4.48
Durable good	14	29.19%	26.09%	6.94
Non-durable good	476	33.32%	19.57%	5.77

Figure 2.7 below illustrates the proportion of price increases and decreases by type of goods. We find that price decreases are not very common for services; most price changes in services are increases. For semi-durable goods, we observe very similar proportion of price increases and decreases. However, for non-durable goods, the proportion of price decreases are almost twice as much as price increases. This result is interesting as it shows that even in Rwanda, stores reduce prices to dispose of non-durable goods, demonstrating that although the term ‘clearance’ is not common in Rwanda, the concept exists and is used by retailers.

Figure 2.7: Proportion of Price Increases and Decreases by Type of Good in Rwanda: 2013-2014

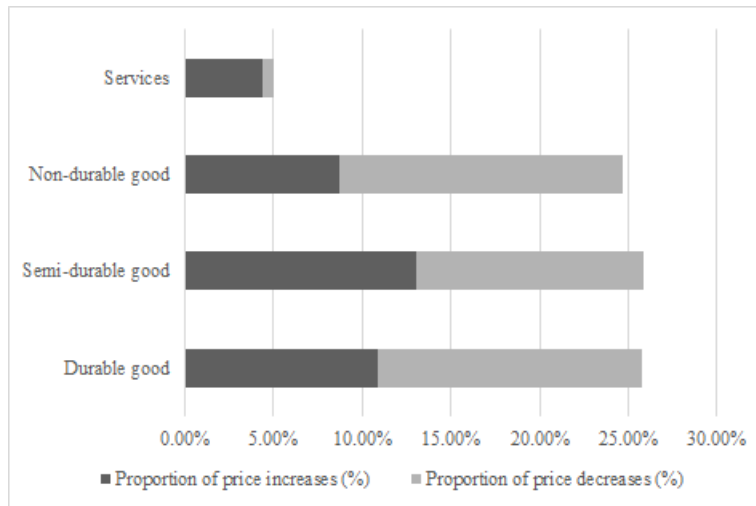
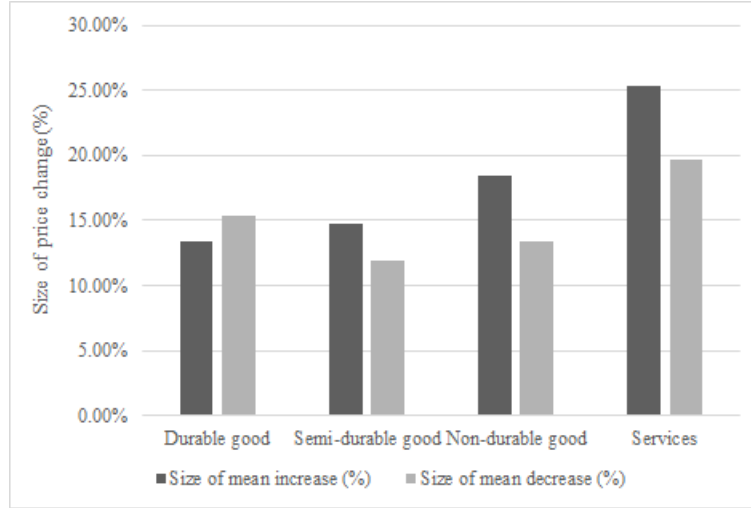


Figure 2.8 below shows that the size of the absolute change in prices is the largest in services.

This is logical and similar to the explanations provided earlier for goods with lower frequency of price change: when prices are changed for services, they are changed by a large amount. Among the goods, non-durable goods increase prices with the largest magnitude and durable goods decrease prices with the largest magnitude.

Figure 2.8: Magnitude of Price Increases and Decreases by Type of Good in Rwanda: 2013-2014



2.6.4 International Comparisons

In this section, we compare the results from this study with the existing results of price change behaviour in other countries. As mentioned before, the price change behaviour in the economy has implications for monetary policy, and, central banks, particularly on the transmission mechanism of monetary policy. We investigate the differences in price rigidity between Rwanda and studies conducted in other countries.

- (i) *Price changes are more frequent in Rwanda*

Table 2.6 below summarizes the main results from other studies. We provide a summary of the frequency of price changes and duration of price spells from other studies that used CPI micro-level data.

Table 2.6: International Evidence of Price Data

Study	Country	Frequency (percent)		Duration (months)	
		Mean	Median	Mean	Median
Aucremanne and Dhyne (2004)	Belgium	16.84	13.25	5.4	7
Dhyne et al. (2009)	Europe	15.1	-	6.1	-
Baudry et al. (2004)	France	18.90	14.90	4.8	6.2
Baharad and Eden (2004)	Israel	24	21	3.6	4.2
Bils and Klenow (2004)	USA	26.1	20.9	3.3	4.3
Nakamura and Steinsson (2008)	USA	26.5	19.40	-	4.6
Bunn and Ellis (2012)	United Kingdom	18.80	-	10.60	7.2
Medina et al. (2007)	Chile	46.1	33.3	3	-
Gouvea (2007)	Brazil	37	-	2.7	-
Gagnon (2009)	Mexico	30.4-36.6	-	2.2-2.8	-
This study (2015)	Rwanda	43.62	28.68	4.48	2.96

Notes: Data is acquired from various studies for comparisons

Table 2.6 shows the frequency and duration of price changes observed around the world. The first observation is the lack of developing country studies on price changes. Chile, Brazil and Mexico are the only emerging countries for which studies are conducted on price statistics and price change behaviour.²⁴ Medina et al. (2007) use micro-level dataset for Chile and find prices change, on average, 46.1 percent per month, which is much higher than the frequencies observed in the developed world. Gouvea (2007), using a micro-level dataset, find that the weighted mean frequency in Brazil is 37 percent with a mean duration (using a direct approach with frequency of price change) of 2.7 months. The results for frequency of price change and duration of price spell in these countries compare similarly to the results for Rwanda. Gagnon (2009), who use a large dataset from Mexico covering 227 products and eight years of data from 1994-2002, finds a weighted mean frequency of price change in the range of 30.4-36.6 percent with a mean duration of price spell of 2.2-2.8 months. These show that prices change more frequently in Rwanda and other developing countries and prices are not very rigid. Mean duration of price spell is a tricky estimate to compare due to its varying estimation technique. Using a direct approach by directly applying frequency of price change to estimate median duration yields a weighted median duration

²⁴Burstein et al. (2003) conducted a study on Argentina, another emerging country, for 58 products and for nine months. Ahlin and Shintani (2007) conducted a study on Mexico with 44 food products for one year of price data. In addition, Choudhary et al. (2011) conducted a study to determine the extent of degree of price rigidity for Pakistan. However, they conducted a survey of firms to determine the frequency of price change. Similarly, Şahinöz and Saraçoğlu (2008) carried out surveys among firms to look at price rigidity for Turkey. We have excluded results from these studies due to their different methods of estimating the price statistics and focused on studies that use similar estimation procedures.

of price spell of 2.96 months, which is similar to findings from developing countries.

The results for Rwanda in comparison with developed countries show large differences in frequency of price change and duration of price spell between the countries. Nakamura and Steinsson (2008) find the weighted mean frequency of price change of 26.5 percent for the United States looking at all prices. Dhyne et al. (2009) look at the entire Euro area and find a weighted mean frequency of price of 15.1 percent, which is much lower compared to Rwanda. Bunn and Ellis (2011) find a weighted mean frequency of price change of 18.8 percent for the United Kingdom. As these show, the weighted mean frequency of price change is much lower in the developed countries compared to Rwanda and other developing countries.

In general, there is a lot of heterogeneity in frequencies of price change between the countries and in the groups in which frequencies are observed for price changes. For the United States, Nakamura and Steinsson (2008) find a frequency of price change of 87.5 percent for vehicle fuel. An equivalent of vehicle fuel for Rwanda is ‘Fuels and lubricants for personal transport equipment,’ where the mean frequency of price change is 21.74 percent.²⁵ For the United Kingdom and the Euro area countries, in general, we find that transport prices fluctuate more in comparison to other major groups (Dhyne et al. (2009)). For Rwanda, food and non-alcoholic beverages had the largest mean frequency of price change and with high weights within its subcomponents, induces an overall high mean frequency of price change. We compare the frequency of price changes for transport and food and non-alcoholic beverage across the Euro area (with results from Dhyne et al. (2009)):

²⁵There are only two observations for this item, however, so this estimate is not very reliable.

Table 2.7: Frequency of Price Changes by Selected Group: International Comparison

	Major Group	
Country	Food and non-alcoholic beverage	Transport
Rwanda	49.11%	14.03%
Austria	18%	35.80%
Belgium	20.40%	46%
Germany	18.40%	34.40%
Finland	20.40%	36.50%
France	19%	36%
Italy	14.60%	24.80%
Luxembourg	19%	21%
Netherlands	23.20%	88%
Portugal	37.20%	25.70%
Spain	32.20%	8.30%
United Kingdom	21.80%	36.70%

Table 2.7 shows that the group ‘Transport’ fluctuates more in the Euro area and United Kingdom (except for Spain), compared to other groups within the countries and in comparison to Rwanda. For the Netherlands, transport has a mean frequency of 88 percent, which means that 88 percent of firms change prices in this sector per month; in Rwanda, transport prices change 14.03 percent per month. Rwanda has a high frequency of price change in food and non-alcoholic beverage. For a country such as Rwanda, which is highly reliant on agriculture and unprocessed food, a slight change in the weather may lead farmers to change prices often. However, in general, this issue is not expected in the Euro area and the data demonstrates that for almost all the countries, except Portugal and Spain, the mean frequency of price change for food and non-alcoholic beverage is under 25 percent. For the United States, a comparable statistic does not exist as they break down food into processed and unprocessed food. In spite of this, we can still observe that the frequency of price change for processed and unprocessed food is 25.5 and 39.5 percent respectively (Nakamura and Steinsson (2008)), which is higher than most Euro area countries, though, still lower than Rwanda. For existing studies on emerging countries, the mean frequency of price changes for food and non-alcoholic beverages is 40 percent for Mexico (Gagnon (2009)). Gouvea (2007) find that the mean frequency of price change for the CPI sector Food is 42 percent in Brazil.²⁶

²⁶This group is not the same as Food and non-alcoholic beverage as in the UN COICOP, so this result is not

Bils and Klenow (2004) and Bunn and Ellis (2012) find that durable good prices fluctuate more than services prices in the United States and United Kingdom, respectively. Dhyne et al. (2009) find that services prices change less frequently than goods in the Euro area countries. However, the estimates for the Euro area countries are much lower than for the United States and United Kingdom. Bunn and Ellis (2012) also report that they observe larger proportion of the price changes in services to be price increases. All the results we found by breaking down goods and services for Rwanda are similar to the findings in the United States, United Kingdom and the Euro area countries.

(ii) *Weights play a significant role in the weighted mean of price statistics in Rwanda*

It is important to note that, in addition to high frequency of price change for the food and non-alcoholic beverages, the weight assigned to this group in Rwanda is substantially higher compared to the weight assigned for this group in the Euro area countries.²⁷ For the Euro area countries, a weight of 16.68 percent is assigned in the CPI consumption basket whereas this is 44.11 percent for Rwanda. This shows that weight contributes notably to the overall weighted mean frequency: a high frequency of price change in this group leads to a high weighted mean frequency of price change for Rwanda. To test whether weights play a large role in the weighted mean of frequency of price change, we look at the unweighted mean price frequency of Rwanda and find it to be only slightly higher compared to the unweighted average of the Euro area countries.

(iii) *Price change directions are similar in Rwanda*

In Rwanda, the fraction of the change that are mean price increases are 60 percent, implying that mean price decreases occur 40 percent of the time when a price change has occurred. This is similar to developed countries where 60 percent of price changes are increases in the United Kingdom (Bunn and Ellis (2011)). Nakamura and Steinsson (2008) find that the weighted fraction of price increases including sales is 57.1 percent. Though, the estimates are similar for Rwanda and these countries, the reasons resulting in lower prices are different. For the United States, a large fraction of these prices decreases are due to sales, as stores try to sell items at cheaper prices to dispose of items that may expire soon. In Rwanda, however, price decreases occur mostly in agriculture commodities and are due to weather conditions, etc.

directly comparable.

²⁷We could not find comparable weights for Gagnon (2009) and Gouvea (2007) in their studies so we are unable to comment on the weight composition in these two developing countries.

(iv) *Price change magnitudes are larger in Rwanda*

We now compare the magnitude of price changes across countries. For the United States, Nakamura and Steinsson (2008) and Klenow and Kryvtsov (2008) find that the weighted median magnitude of price change is 10.7 percent and 10 percent respectively, whereas for Rwanda, we find that the weighted median magnitude of price change is slightly higher at 13 percent. The weighted mean magnitude of price change is 16.67 percent for Rwanda; Klenow and Kryvtsov (2008) find the weighted mean magnitude is 11 percent for the United States. This shows that Rwanda has a higher frequency and magnitude of price change in comparison with the US, which is counter-intuitive, since we expect that each (mean) price change in Rwanda to be smaller as they change prices more often (assuming similar trend inflation rates). However, in contrast to frequency of price change, we find that weights do not play a large role here as the magnitude of price change is relatively more uniform across the item categories.

Further, we compare the size of price increases and decreases with existing studies. The weighted mean magnitude of price increase for Rwanda is higher at 16.67 percent than the Euro area countries in Dhyne et al. (2009). The weighted mean magnitude of price decrease of Rwanda, however, is slightly lower than a few countries in Europe, such as the Netherlands and Austria, where the weighted mean magnitude of price decrease is 13.9 percent and 15.1 percent respectively, compared to a mean magnitude of 11.43 percent in Rwanda.

2.7 Conclusion

The study attempted to provide evidence on nominal rigidity in prices using a CPI micro-level dataset for Rwanda. We found that the weighted mean frequency of price changes is higher compared to developed countries, though, the results are similar to those of other developing countries. We found that weights play a significant role to induce the large weighted mean since items with large frequency have, according to the consumption expenditure in Rwanda, higher weights. Moreover, we found that there is a lot of heterogeneity in price stickiness among the major commodity groups. In addition, we determined the direction of these price changes and found that price decreases are almost as common as price increases, with majority of the price decreases observed in the food and clothing sectors. Next, we explored the magnitude of these price changes. We found that, compared to developed countries, the magnitude of the price changes is larger in Rwanda.

The substantial role played by item weights led us to conclude that, overall, there is no substantial difference in the way prices are set in Rwanda, in comparison to developed countries. In addition, the role played by prices in explaining output fluctuations is not different from what

we expect in a developed country. Hence, theoretically, we may expect that, in response to a monetary policy shock, money will be non-neutral, as we observe in developed countries. However, the structural features of developing countries are dissimilar to what we observe in developed countries, which may play an important role in the transmission mechanism of monetary policy in developing countries. Therefore, we are required to set up a framework that suitably characterises the intrinsic features of a developing country, which will then allow us to accurately assess the effectiveness of monetary policy in the economy.

Chapter 3

Credit Market Imperfections, Staggered Prices and Monetary Non-Neutrality

3.1 Introduction

Monetary policy is used by central banks to control macroeconomic variables such as output, inflation and unemployment. According to the New Keynesian (NK) literature, for monetary policy to be effective in generating real effects, prices (and wages) are required to be sticky in the short run. The stickiness of prices, which can occur due to wage contracts (Taylor (1979)), a fear of losing customers (Rotemberg (1982)), or menu costs (Mankiw (1985)), allows changes in real output. If, instead, prices are flexible, the effect of the monetary policy shock will be eliminated by adjustment in prices and cause real variables to be unchanged. Hence, sticky prices provide a mechanism through which money and interest rates can affect output and ensure the non-neutrality of monetary policy.

The importance of sticky prices for monetary policy has led to many price-setting models that determine why prices remain rigid. Time-dependent models, such as Taylor (1979), state that wages are rigid because they are fixed in the form of a staggered setting: all firms in an economy do not change wages at the same point in time.¹ Calvo (1983), in his price-setting model, assumes that there is a constant probability with which firms in the economy change prices at any point in time, thereby, proposing a form of staggering in the price-setting process. Proponents of state-dependent models, such as Mankiw (1985) and Blanchard and Kiyotaki (1987), contend that prices are sticky due to ‘menu costs’ that firms incur to adjust prices.

¹Taylor (1999) extends this model to a staggered price-setting framework to determine the behaviour of firms changing prices of goods, rather than wages.

While these time- and state-dependent models have tried to explain the theoretical reasons behind price and wage stickiness, many attempts have also been made to explore empirically the existence and degree of rigidity in prices and/or wages. These studies on sticky prices employ micro-level disaggregated price data and estimate the mean percentage of prices that are changed during a given period as an indicator of price rigidity. Bils and Klenow (2004) use a large dataset that is representative of the CPI sectors in the United States to determine the frequency and magnitude with which firms change prices. More recently, studies have been conducted for the United Kingdom (Bunn and Ellis (2012)), Europe (Álvarez et al. (2006); Dhyne et al. (2009)), and other advanced economies. These studies find and confirm the existence of sticky prices in these countries and have been, subsequently, utilised to assess the effectiveness of monetary policy to generate persistent effects in output, using calibration and simulation methods (Christiano et al. (2005); Golosov and Lucas (2007)) and econometric methods (Olivei and Tenreyro (2010)).² It is noteworthy that most of the existing evidence on price stickiness and output persistence is for advanced economies; studies investigating nominal rigidity and output persistence for developing economies, in particular, low-income countries (LICs), are scarce.

In the second chapter, we investigated the existence and degree of nominal rigidity in a developing economy by employing a monthly store-level price dataset from Rwanda for the period January 2013-December 2014. Using the data, it was established that weighted price changes are more frequent in Rwanda, compared to advanced economies, though, similar to the frequency observed in other developing economies. However, an unweighted frequency of price change indicated that price changes in Rwanda are very similar to what are observed in advanced economies, which demonstrated that item weights play a significant role to induce the high weighted frequency of price changes for Rwanda. Hence, we concluded that, overall, there is no substantial difference in the frequency of price changes in Rwanda, in comparison to advanced economies.

While this established that price stickiness is an important feature of a developing economy, the implications of price stickiness in a developing economy are likely to be dissimilar from those in an advanced economy, as their structural features are different. Agénor and Montiel (2008) document some of these structural features, which include weak institutions, high openness to trade in goods and services, varying exchange rate regimes, etc., and study their implications on the transmission mechanism of monetary policy. Most of these analyses relate to emerging market countries, such as Mexico and India, which are highly reliant on trade and foreign markets, and, therefore, more exposed to global shocks. They assume that price stickiness plays a similar role

²In the early 2000s, Chari et al. (2000), among others, developed models with sticky prices to generate persistent effects in output in response to a monetary policy shock. However, for plausible parameter values, these models were, surprisingly, not able to generate much output persistence and gave rise to what was termed the ‘persistence puzzle.’ Recently, however, most DSGE models with sticky prices, such as Christiano et al. (2005), have been able to calibrate and demonstrate output persistence and match the empirical data observed for the United States.

in these economies to that in an advanced economy, without an emphasis on how the structural differences in a developing economy may impact the nature of the firms' price-setting. However, countries such as Rwanda, and other countries in Sub-Saharan Africa, are not as open as emerging market economies, which suggests that the structural features within the economy are likely to play a key role in the transmission mechanism of monetary policy. For instance, in Rwanda, monetary policy is conducted using a monetary aggregate targeting rule, in contrast to a monetary policy following an inflation targeting framework or exchange rate peg, which are observed in many emerging market economies.

In order to study the price-setting mechanism in developing economies, we observe that, one of the key features that differentiates Rwanda and other developing economies from advanced economies, is the presence of imperfections in the credit markets, which prevents a significant proportion of households in the economy to borrow and lend or trade in insurance.³ In the existing literature on staggered prices and monetary policy, credit markets are assumed to be perfect (Rotemberg and Woodford (1997); Woodford (2003)), which is not expected to hold for developing economies.⁴ Then, it may be expected that imperfections in the credit markets, which include the absence of the ability to trade in insurance, will affect the price-setting decision, as it involves an intertemporal tradeoff, under staggered prices. Staggered prices create heterogeneity among price-setters, and, in turn, create a potential role for borrowing and lending and trading in insurance. Thus, to study the price-setting process and, thereby, the extent to which monetary policy affects real variables, we incorporate credit market imperfections in a standard staggered price model to characterise a developing economy and examine the role it plays in the price-setting decision and, consequently, in the dynamics of aggregate output and other macroeconomic variables. In addition, we explore the heterogeneous effects in the economy, as the nature and timing of price-setting make it likely that there will be distributional implications, in the presence of imperfect credit markets.⁵

We develop a dynamic general equilibrium (DGE) model with money but without bonds or private insurance, as a characteristic of a representative developing economy. We are primarily interested in analytical solutions, in order to obtain an insight and understand the mechanism via which credit market imperfections affect the price-setting process and macroeconomic dynamics. To assess the differences and obtain comparative statics, we also develop an economy with perfect credit markets, as a representative advanced economy. Then, we derive the consumption and

³Section 3.3 contains detailed statistics on the extent of imperfections in the credit markets in developing and advanced economies.

⁴Bernanke et al. (1999) incorporate credit market frictions in their model, but their analysis does not explicitly address the effects of these frictions on the price-setting mechanism.

⁵The analyses in this chapter focus on time-dependent pricing, and not state-dependent pricing. The implications of state-dependent pricing in the context of imperfect credit markets are likely to be very different, and are beyond the scope of this chapter.

output dynamics and compare the findings to determine the effects of the credit and insurance constraints and their implications for monetary non-neutrality, which enable us to explore the effectiveness of monetary policy in generating real effects in developing economies. Moreover, we explore and compare the heterogeneous effects in developing and advanced economies.

The DGE model allows us to analytically deduce that, in an economy with imperfect credit markets (ICM), the price-setting process is directly affected by individual money holdings, which, thereby, determine the aggregate output in the economy. Following a monetary policy shock, there are heterogeneous effects on consumption levels and money holdings of the sectors in the economy since the absence of borrowing and lending and trading in insurance prevents agents from smoothing out individual fluctuations. The extent to which individual consumption and income can be smoothed out, in the absence of credit markets, is determined solely by individual money holdings. This is in contrast to an economy with perfect credit markets (PCM), where agents in the sectors are able to borrow and trade in insurance to smooth out individual fluctuations.

There are two main findings in the chapter. First, following a monetary policy shock, aggregate output exhibits a monotonic path initially, followed by an oscillatory path that slowly converges towards the steady state, although the magnitudes of these oscillations are small. This demonstrates that while we observe oscillations in aggregate output in the transition path under ICM, which is intrinsically undesirable, these are not expected to cause an issue from a policy perspective. This is due to the role of individual money holdings, which help to partially smooth out consumption fluctuations, in the absence of credit markets. In addition, the speed of convergence, as determined by the half-life, is slightly more rapid under ICM than PCM. This is possibly linked to the oscillations in the time path of the aggregate output under ICM, which overshoots the steady state values in some finite interval of time. Thus, in comparison with PCM, the persistence of aggregate output is lower with ICM.

Second, although the aggregate dynamics are quite similar under ICM and PCM, we find notable heterogeneous effects in the behaviour of sectoral variables under ICM. Following a money supply shock, the price-setting sector in the impact period has lower levels of output and nominal revenue than before the shock, which is due to the higher price set by the sector in the impact period. The price-setting sector has higher consumption levels than before the shock, in spite of the lower output, as it runs down its money holdings to smooth out consumption fluctuations. The non price-setting sector, on the other hand, has higher output levels and nominal revenue than before the shock, and also higher than in the price-setting sector. This leads to higher consumption levels and money holdings both relative to the pre-shock situation, and to the other sector. These differences lead to a persistent asymmetry between the sectors in the economy, as these effects persist in the transition paths.

As a result of these heterogeneous effects of monetary policy, we examine sectoral welfare

effects: the price-setting sector derives higher utility than before the shock from higher consumption and lower output levels, but lower utility from lower money holdings, and so the effect on overall utility of the sector is ambiguous. The non price-setting sector has higher consumption and output levels, and money balances, but as higher output levels lead to lower leisure, the overall effect of monetary policy on utility of the non price-setting sector is ambiguous as well. In order to gain a thorough understanding of the sectoral utility, we conduct numerical welfare calculations. We find that, following the shock, the price-setting sector enjoys higher welfare in comparison to the non price-setting sector. Moreover, we conduct a similar welfare analysis for PCM, and find that this result holds for PCM as well, inspite of the identical consumption levels and money holdings across sectors under PCM. Then, we make a comparison of the sectoral welfare gains under ICM and PCM, which leads us to a crucial finding: despite the greater intersectoral heterogeneity in the time paths of the variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes under ICM than PCM. We proceed to suggest that an implication of this result is that a monetary authority concerned about equality can be more vigorous in its monetary policy actions under ICM in comparison to PCM.

We organise the chapter as follows. In Section 3.2, we provide an extended literature review on the existing research, which highlights a gap in the literature and provides motivation for this study. In Section 3.3, we present stylised facts on developing and advanced economies to provide microeconomic evidence and justify the features that are included in the DGE models. In Section 3.4, we develop the benchmark model with credit market imperfections and staggered prices and provide a strategy for how the model can be solved. We then establish the equilibrium conditions and present the analytical results. In Section 3.5, we present our numerical simulation results for the economy with imperfect credit markets and compare the findings with perfect credit markets. In Section 3.6, we conduct welfare analysis to evaluate the heterogeneous effects in response to monetary policy shocks. In Section 3.7, we conclude with a brief discussion on the contribution of the chapter towards monetary policy in developing economies.

3.2 Previous Literature

The impact of monetary shocks on output in an economy with staggered prices has been studied quite extensively, particularly in the NK literature. Many approaches have been taken to study this relationship, which includes analytical models, econometric techniques, calibration techniques or a combination of these approaches. Most of the models on monetary policy and output in the 1980s, 1990s and early 2000s focus on the models' ability to show persistence in output in response to a monetary policy shock. The models are developed to determine the mechanism through which sticky prices affect output and whether this is able to generate persistence in output, as is seen

in the time-series data. Though a consensus on this issue was never reached, research in this area since the early 2000s moved on to large general equilibrium models, which incorporated many aggregate variables, and aimed to match the data. However, except for a handful of papers, most of the models have been for advanced economies and data has subsequently been used to demonstrate that the models can match the data in these countries. In this section, we review the literature in this topic and show the evolution of these models, with emphasis on the key results derived for sticky prices and output fluctuations via the effects of monetary shocks.

Taylor (1980) develops an *ad hoc* linear model with staggered wages of the type in Taylor (1979) and rational expectations to study the aggregate dynamics of the economy. The model shows persistence in output in the real effects of money shocks, which is found to depend on two key parameters: degree of forward-looking variables and on the sensitivity of money wage to conditions in the business cycles. This work is supplemented by West (1988), who uses a feedback policy rule in money supply and Phaneuf (1990), who uses a monetary policy rule in interest rate and another in money supply. They both demonstrate that using reasonable values for Taylor (1980)'s key parameters, an economy with staggered wage contracts can exhibit persistence in output through the effects of money supply, which was in refutation to Nelson and Plosser (1982)'s empirical findings that money supply did not have an impact on real variables.⁶

In the late 1990s, it was acknowledged that the previous models on sticky prices and output fluctuations lacked microfoundations, which weaken the findings of these models. Ascari (2000) and Chari et al. (2000) are two papers developed contemporaneously, which study the impact of monetary shocks on output in the presence of staggered contracts in a dynamic general equilibrium (DGE) framework.⁷ We review these papers in turn.

Ascari (2000) incorporates Taylor (1979)'s staggered wage setting into an optimising DGE framework to study whether persistent real effects of money shocks are likely, such as those found in Taylor (1980), West (1988) and Phaneuf (1990). The paper uses both analytical and calibration methods but the key result is that a staggered wage model is unlikely to generate realistic levels of persistence in output. The model shows that a high degree of persistence in output is possible only from low income effects on labour supply and a high intertemporal elasticity of substitution of labour. Moreover, calibration results show that while implausible parameter values can generate persistence in output, non-zero steady state inflation, which is quite likely in advanced economies, diminishes persistence sharply. This is in sharp contrast to the results found in previous studies, though, consistent with the empirical findings of Nelson and Plosser (1982).

⁶Narayan and Narayan (2010), similarly, in an empirical study, find unit root in GDP for 40 out of 79 developing countries.

⁷Yun (1996) is one of the first papers to incorporate nominal price rigidity and explore the implication on persistence in output following a monetary policy shock. Although a dynamic general equilibrium model is developed in the paper, it is in a standard real business cycle framework, which is why the findings are not covered in detail in this literature review.

Chari et al. (2000) develop a similar DGE model, which is stochastic, to study whether staggered contracts can generate output persistence. In contrast to Ascari (2000), this paper uses staggering in prices, rather than wages, and allows accumulation of capital. The paper uses the notion of contract multiplier - ratio of half-life of output deviations after a monetary shock with staggering in prices to corresponding half-life in prices that are synchronised - to determine whether staggered contracts can generate output persistence. The main conclusion is that, similar to Ascari (2000), staggered contracts, by itself, is unable to generate output persistence through the effects of monetary shocks. Since this paper incorporates more variables than Ascari (2000), it relies more on calibration techniques. However, no plausible level of price stickiness is able to generate persistence in output. Moreover, the paper uses robustness checks, which includes i) consumption and leisure being close to perfect substitutes in preferences so costs are less sensitive to output, ii) convex demand in intermediate goods so prices are less sensitive to cost changes, and iii) upward-sloping marginal cost curves so prices and costs are less sensitive to wage changes, and a model which is a combination of these features, but the result remains the same: realistic levels of output persistence is unlikely to be explained by staggered prices. Taylor (1999) argues that incorporating monopolistic competition is not sufficient as a microfoundation to show the levels of persistence that is observed in Taylor (1980).

The two papers reviewed so far include staggering in prices or wages but not both. Huang and Liu (2002) include both staggering in prices and wages and show that the implication on persistence is different. They construct a DSGE model which includes both the goods and labour market, with firms setting nominal prices for their products and households setting nominal wages for their labour. The main finding is that staggered price, by itself, does not produce persistence, whereas staggered wage may be able to generate persistence in monetary shocks. The key mechanism is the elasticity of relative wage/price with respect to aggregate demand in wage/price equation, with a higher value of this corresponding to lower persistence in output. When wages are staggered, the elasticity of relative wage with respect to aggregate demand is less than one under plausible parameter values, whereas when prices are staggered, the elasticity of relative price with respect to aggregate demand is typically greater than one. Hence, staggered wages tend to generate persistence but staggered prices do not. Further, they show that incorporating capital into the model does not alter their key findings.

Edge (2002), however, refutes this result and shows that staggered price is able to generate as much persistence as staggered wages if firm-specific factor inputs are considered, as seen in Kimball (1995) and Chari et al. (2000). A stripped-down analytical model is developed in the paper which shows that staggered prices and wages can produce equivalent level of persistence. This is then followed by a detailed model which shows, using calibration techniques, that staggered prices and wages can produce similar level of persistence.

Ascari (2003), in an attempt to reconcile the differences in the results obtained for output persistence due to monetary shocks, looks at various staggered wage and price models to determine the parameters important for output persistence. He develops four types of economy: i) staggered price and perfect labour mobility, ii) staggered price with no labour mobility, iii) staggered wage and perfect labour mobility, and, iv) staggered wage with no labour mobility. He finds that, irrespective of whether staggering is in prices or wages, persistence is lower in case of perfectly mobile labour. He supports this quantitatively, and concludes that persistence can only be generated if there is some form of labour immobility. If labour is mobile, high demand for labour by the sectors which cannot adjust prices or wages attracts labour away from sectors which can, which will push up the costs of the latter, and, thereby, encourage them to set higher prices or wages. On the other hand, if labour is immobile, following a monetary policy shock, firms will not have an incentive to change wages, leading to greater stickiness and output persistence.

Wang and Wen (2006) investigate if price rigidity, along with some additional parameter, can demonstrate output persistence. They show that reasonable price stickiness can generate highly persistent, hump-shaped movements in output only if investment is subject to a cash-in-advance (CIA) constraint. Intuitively, this is because the CIA constraint prevents consumption from rising too much in the period of impact since consumption can only increase by the amount of the current cash injections. This limits the increase in aggregate demand, leading to higher output persistence.

Daros and Rankin (2009) explore analytically whether persistence is observed when monetary policy is governed by a Taylor rule. They develop DGE models using both Taylor (1979)-type staggering and Calvo (1983) staggering in both prices and wages. They find that under Taylor (1979)-type staggering in either prices or wages, output oscillates in response to a monetary policy shock, confirming previous results that output is not persistent following monetary shocks. Under Calvo (1983)-type staggering in wages, it is found that output can be persistent if there are decreasing returns to labour; though, persistence is not observed in case of staggered prices as the model is completely forward-looking.

Dixon and Kara (2011) develop a DSGE model with Taylor (1979) pricing to allow for an economy with many different contract lengths, which they term the generalised Taylor economy (GTE), unlike the standard approach where a single contract length is assumed in the economy. They focus on how a monetary shock can generate changes in output through time, and, in particular, the degree of persistence of deviations of output from steady state. They find that due to presence of contracts of varying length, the degree of persistence in output following a monetary shock is higher. Intuitively, this is due to a spillover effect or strategic complementarity in terms of wage or price-setting through the price level - a reasoning that has also been put forth by Nakamura and Steinsson (2010) in their menu-cost model. In addition, they construct a GTE

which has the same distribution of completed contract lengths as Calvo (1983) pricing and find similar persistence in output following a monetary shock.

Söderberg (2013) assumes, given empirical evidence, that prices are non-uniformly staggered, meaning that the fraction of firms changing prices is not constant over time. He explores the implication of partial synchronisation, using the Fisher and Konieczny (2000) index, on the persistence of output due to a monetary shock. The paper first focuses on a simple analytical model using Taylor (1979) pricing and finds that the effect on output of a monetary policy shock is non-linear: small deviations from uniform staggering have negligible effects on output persistence, whereas small deviations from perfect synchronization have large effects. This is then extended to a microfounded model which results in similar findings. Intuitively, this is explained by strategic complements: a small number of non-adjusters have a disproportionately large influence on the behaviour of the aggregate economy.

Another strand of literature on sticky prices focuses on large micro-datasets in prices to measure empirically the extent of nominal rigidity. These datasets are usually representative of the datasets used in computing the CPI. The purpose of these studies is to show how frequently prices are changed and this is then used to calibrate a DSGE model to match the data. Since the primary focus of this chapter is to understand the relationship between sticky prices and output, a review of the literature on these studies will explore the extent of price stickiness and its role in influencing output fluctuations.

While Bills and Klenow (2004) is one of the first papers to empirically determine, using large datasets, the frequency at which prices change, they sought mainly to use this to explore whether Calvo (1983) or Taylor (1979) sticky-price models can explain the inflation data observed in the United States. They find that such models are not able to explain the observed inflation for a sample period: the models display more persistent and less volatile inflation than is observed in the data. They do not provide any results for the model's ability to exhibit persistence in output in response to a monetary policy shock.

Christiano et al. (2005), on the other hand, determine the frequency of price changes, develop a large DSGE model that incorporates staggered price and wage contracts, and replicate inflation and output data for the United States. The model incorporates Calvo (1983) price-staggering and indexation and includes habit formation in consumption preferences, investment adjustment costs, variable capital utilisation, and working capital. They find that wage contracts alone can produce persistence in output, though, a critical role is played by variable capital utilisation to generate both output persistence and inflation inertia. For robustness, in addition to a money supply shock, (an approximation of a) Taylor rule is utilised to observe whether output responds similarly. They find that although the rise in output is smaller, output exhibits persistence. However, the Taylor rule used in the paper contains a lagged variable in the interest rate and a

forward-looking inflation term, which may influence the sluggishness in output.

Golosov and Lucas (2007) develop a model of a monetary economy in which firms pay a fixed cost - menu cost - in order to change nominal prices, which is unlike previous studies where time-dependent models are used. The model includes idiosyncratic shocks in addition to general inflation, along with other standard features as commonly observed in other New Keynesian models. They use individual U.S. prices to calibrate the menu cost and the variance and autocorrelation of the idiosyncratic shocks. Next, they calculate the responses of output, employment and prices to an unanticipated increase in money. The predicted output response were small and temporary, which is in contrast to other time-dependent models such as Christiano et al. (2005), but is similar to findings in papers such as Chari et al. (2000) and Ascari (2000) where output responses to monetary shocks are not found to be persistent.

Nakamura and Steinsson (2010), in an attempt to review the results found by Golosov and Lucas (2007), develop a menu cost model to determine the response in output due to monetary policy shocks. In addition to the characteristics of the model in Golosov and Lucas (2007), they add two features, i) heterogeneity across sectors in the frequency and size of price changes, and ii) intermediate inputs. They find that half of the differences in the monetary non-neutrality observed in Golosov and Lucas (2007) can be explained by heterogeneity across sectors while the other half is explained by intermediate inputs. Heterogeneity amplifies the degree of monetary non- neutrality three times for the multisector menu cost model compared to the single sector model calibrated to the mean frequency and size of price changes. They attribute this to, i) low levels of inflation in the U.S., ii) large average size of price changes and negligible correlation between the size and frequency of price change across sectors, and iii) fairly low average price change frequency in the U.S. Intermediate inputs amplify the degree of monetary non-neutrality because intermediate inputs cause the pricing decisions of firms to be strategic complements (Basu (1995), most notably, among others). Intuitively, firms that change their price after a shock to nominal aggregate demand adjust by a smaller amount than they otherwise would because many of their input prices have not yet responded to the shock, which leads to sluggish prices and, hence, persistence in output.

Olivei and Tenreyro (2010) use a slightly different approach to show how systematic differences in the timing of wage setting decisions among industrialised countries can explain the importance of wage rigidity in the transmission of monetary policy. They explore, using an econometric model, whether the economy responds differently to monetary policy shocks according to the time of year in which the policy innovation takes place. For instance, in the U.S., a large fraction of firms adjusts wages in the last quarter of the calendar year. In contrast, wage agreements in Germany are well-spread within the year, implying a relatively uniform degree of rigidity. Theories of the transmission of monetary policy would hence predict that, other things equal,

monetary policy changes in the U.S. should have a larger effect if the shock occurs in the first half of the year. However, in Germany, where there appears to be little bunching in wage setting decisions within the year, the effect should not vary with the quarter in which the shock takes place. Their empirical findings support these and show that wage-rigidity plays a key role in the transmission of monetary policy.

Davoodi et al. (2013) is one of the only studies in the literature to investigate the impact of monetary policy on aggregate variables for developing countries. They examine empirically using structural VAR (SVAR), among other VAR models, the impact of monetary policy on output and other macroeconomic variables for five East African Community (EAC) countries. Data for most of these countries is not available for many periods and, hence, they rely on interpolation and proxies to generate sufficient data to run the VAR. They find that an expansionary monetary policy, in the form of a positive shock to reserve money, increases output significantly in Burundi, Rwanda and Uganda but generates no significant effect on prices in the EAC countries. These shocks also generate long lags in Burundi and Rwanda - significant output effects for 6 to 15 months - which is substantially different from advanced economies. An expansionary monetary policy in the form of a negative shock to the policy rate increases prices in Kenya and Uganda, though, not Burundi and Rwanda. It is evident that policy rate plays a greater role in countries that are bigger and have deeper financial markets and a comparatively stronger banking system, like Kenya and Uganda.

Anand et al. (2015) is the only study that has tried to explore the macroeconomic dynamics in the context of a DGE model developed specifically for a developing country. The primary purpose of the paper is to study the type of inflation that the central bank of a developing country should target, using welfare analysis. They characterise a developing country by incorporating, in an otherwise standard NK model, incomplete financial markets, which prevents households from borrowing to smooth out consumption. The households invest a high share of expenditure on food in households' overall consumption expenditure. In addition, there are three sectors in the economy, i) food sector: where prices are flexible, households live hand-to-mouth and have no access to credit, ii) sticky price sector, with access to credit markets, and iii) sector open to trade where prices are flexible, and are subject to large external shocks. The key result of the paper is that targeting headline CPI inflation improves aggregate welfare compared to targeting core inflation, which is derived using calibration techniques. The intuition provided is as follows: from the impulse response functions, it is found that following a negative food productivity shock, central bank increases interest rates. This affects only the unconstrained households, who decrease their consumption as a result. The credit-constrained households benefit from higher wage income, and, hence, consumption, due to higher prices as a consequence of the food productivity shock. Hence, the sectors react in opposite directions. The mechanism works similarly for both types of

inflation targeting, however, the food price increase is lower in case of headline inflation targeting, emphasising its ability to control aggregate demand, in contrast to core inflation targeting.

3.3 Stylised Facts

We establish some key facts about developing economies in this section that justify the features we include in our theoretical models in the next section. It is important to note that since we are interested to obtain closed-form solutions, we are constrained and are able to include only some features that are likely to impact the model and its findings.

A key feature of a developing economy is that households are credit constrained. The first column in Table 3.1 shows the percentage of adults in different income groups of countries, on average, who have an account with a financial institution (FI), which is used as an indicator of access to financial intermediaries.⁸ The second column shows the percentage of adults who have borrowed from a FI. The table shows that, households in LICs, which include many countries in Sub-Saharan Africa, are severely credit constrained: less than one-third of the population in these countries have an account with a FI. Compared to advanced economies, adults in LICs also borrow substantially less, which poses a significant problem, especially given their volatile income sources and consumption. Therefore, we model a developing economy by incorporating an imperfection in the credit markets that prevents households from borrowing or lending.⁹

Table 3.1: Access to Credit, by Income Groups of Countries, as of 2014

Income Group	Account (% age 15+)	Borrowed from a FI (% age 15+)
Low income	27.54	8.57
Lower middle income	42.73	7.54
Middle income	57.57	9.08
Upper middle income	70.47	10.42
High income	90.63	17.27

Source: Global Findex, World Bank

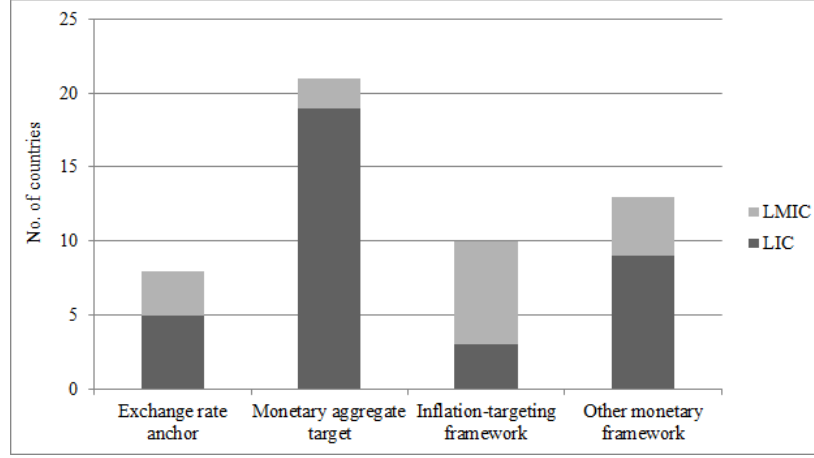
The second feature that we incorporate in our model for developing economies is a monetary aggregate targeting rule. While advanced economies have almost unanimously adopted a monetary policy which is some form of a Taylor rule, developing economies still follow monetary aggregate or exchange rate targeting. IMF (2015) provides a compilation of the monetary policy

⁸The categorisation of countries along with the data is obtained from the World Bank.

⁹It is important to note that we are interested to develop a model for an economy that, according to Table 3.1, is classified as a low income country, such as Rwanda. There is a broad literature already on monetary policy for advanced economies and, more recently, for emerging market countries (see Frankel et al. (2010) for a review).

regimes followed by low income and lower middle income countries, which is shown in Figure 3.1.

Figure 3.1: Monetary Policy Regimes in LIC and LMIC, 2013



Source: International Monetary Fund (2015)

As we can observe in Figure 3.1, a significant number of low income countries, which include but are not limited to Congo Rep., Sierra Leone, Afghanistan, Guinea, Rwanda, Gambia, are still using monetary aggregate targeting to achieve the country's macroeconomic outcomes. Only a few LICs have adopted inflation targeting, like Uganda and Kenya, though, it is unclear if the declared policy is the same as what is practiced, which provides further justification to assume a traditional monetary targeting rule for developing economies.¹⁰ Moreover, Davoodi et al. (2013) find that shocks to the policy rate has an insignificant effect on output in Rwanda, Burundi and Kenya and with short lags. This may be due to the fact that the pass-through from policy rate to the interest rate is low for developing economies (Gigineishvili (2011), Mishra and Montiel (2013)), which results in a lower effect on macroeconomic aggregates following a change in policy rate. For advanced economies, we develop a model with perfect credit markets, and a money supply rule, to compare the dynamics of aggregate output under both types of credit markets.

3.4 A Model of Imperfect Credit Markets

3.4.1 The Basic Framework

The economy is composed of yeoman farmers, as in Blanchard and Fischer (1989), with a continuum of goods which are imperfect substitutes, and money. The economy is closed with infinitely-lived farmers who have identical preferences over goods and money. Each good is produced by a monopolistic competitor, who chooses the nominal price and the level of production of the

¹⁰In fact, this has given rise to classifications of regimes known as 'de facto' and 'de jure' regimes (Calvo and Reinhart (2002)), although, these are primarily to describe exchange rate regimes.

good given the demand function. Each producer is also a consumer, who derives utility from the consumption of all goods and real money holdings. Farmer i maximises the following objective function:

$$\max_{\{C_{it}, M_{it}/P_t, Y_{it}\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta^t \left(\gamma \ln C_{it} + (1 - \gamma) \ln(M_{it}/P_t) - \frac{d}{\varphi} Y_{it}^{\varphi} \right) \right], \quad d > 0, \varphi \geq 1 \quad (3.1)$$

where

$$C_{it} \equiv \left[\int_0^1 C_{jit}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[\int_0^1 P_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (3.2)$$

Farmer i 's utility depends positively on consumption, C_{it} , and real money balances, M_{it}/P_t , and negatively on the level of production of good i , Y_{it} , at time t . C_{it} is a Dixit-Stiglitz aggregate of a continuum of goods in the interval $[0, 1]$, which is a function of the consumption of each good, j . This shows that all goods enter the utility function symmetrically and implies a constant elasticity of substitution between goods, which is given by θ , and is assumed to be $\theta > 1$. Real money balances provide convenience in buying goods and affect the utility of the farmer directly, which is deflated by the nominal price index, P_t . Moreover, we assign weights on consumption, γ , and real money balances, $1 - \gamma$. Production of good i requires effort, which decreases leisure, thereby, negatively affecting the farmer. φ refers to the elasticity of marginal disutility with respect to output, which can be interpreted as the product of the elasticity of disutility with respect to work and the elasticity of work with respect to output. The maximisation of the utility function, (3.1), is subject to the constraints below (and to price-setting constraints, described subsequently):

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} \quad t = 0, \dots, \infty \quad (3.3)$$

$$\int_0^1 P_{jt} C_{jit} dj + M_{it} = P_{it} Y_{it} + M_{it-1} + G_t \quad t = 0, \dots, \infty \quad (3.4)$$

where (3.3) shows that farmer i faces a demand function, Y_{it} , which is proportional to the aggregate demand in the economy and is a function of the relative price, P_{it}/P_t , with a negative elasticity, $-\theta$. (3.4), which is the budget constraint, shows that nominal expenditures on consumption and the demand for nominal money balances will equal the farmer's nominal income from the sale of the produced good, the initial money balances and government transfers, where the government budget constraint is given by $\int_0^1 G_{it} di = G_t = M_t - M_{t-1}$. It can be observed that, unlike in standard NK models, such as Ascari (2000) and Galí (2009), we do not assume a bond market in our budget constraint or allow transfers between farmers under private insurance

agreements.¹¹ In a developing economy, the effective interest rates charged to households are usually very high, which disincentivises households to borrow. In order to meet their income fluctuations, households are more inclined to save and utilise their money holdings, instead of borrowing at high interest rates (Mullainathan and Shafir (2013)). To model this imperfection in the credit market, we assume that bond markets do not exist in our economy, which prevents households from borrowing or lending.

We can solve the first order conditions to derive an intertemporal relationship between consumption and real money balances:

$$\frac{\zeta}{M_{it}} = \frac{1}{P_t C_{it}} - \frac{\beta}{P_{t+1} C_{it+1}} \quad (3.5)$$

where $\zeta = \frac{1-\gamma}{\gamma}$. It is noteworthy that a relationship from the bond market does not exist in our model, which reinforces the notion that only individual money holdings can help to smooth out income and consumption fluctuations, in the absence of borrowing. We can now rewrite the budget constraint, (3.4):

$$P_t C_{it} = P_{it} Y_{it} + M_{it-1} - M_{it} + G_t \quad (3.6)$$

This can be inserted in (3.5) and written as:

$$\frac{\zeta}{M_{it}} = \frac{1}{P_{it} Y_{it} + M_{it-1} + G_t - M_{it}} - \frac{\beta}{P_{it+1} Y_{it+1} + M_{it} + G_{t+1} - M_{it+1}} \quad (3.7)$$

which is an intertemporal relationship in money balances, government transfers and expenditure on nominal output.

3.4.2 Staggered Prices

We introduce staggered prices in the economy as in Taylor (1979). Households are divided into two sectors of equal size, with households in the sectors setting prices in a staggered manner. Households in a particular sector set prices in time t for t and $t + 1$, which means that prices are fixed for two consecutive periods. As there are only two sectors, Taylor (1979) price staggering makes it intuitively easier to understand and derive the implications of imperfect credit markets on agents and macroeconomic variables, compared to other forms of price staggering, such as that of Calvo (1983).

We formally set up two sectors which makes it convenient to derive the aggregate dynamics. We assume: sector A , indexed by $i \in [0, \frac{1}{2}]$, refers to the sector in which farmers reset prices in

¹¹If farmers perceive a monetary policy shock, ex ante, as a random event (and, for simplicity, we may think of it as an event which may only happen once), then they will have an incentive to agree to make an ex post transfer in order to offset consumption differences which would otherwise occur due to price staggering.

even-numbered periods, and sector B , indexed by $i \in [\frac{1}{2}, 1]$, refers to the sector in which farmers reset prices in odd-numbered periods. We define X_t as the ‘reset’ price, that is, the price that farmer i would set in period t , if allowed to reset price in that period (and which would also remain in force in period $t+1$).¹² It follows then, in even-numbered periods, for instance, at time t : $P_{it} = P_{it+1} = X_t$, where $i = A$, and sector B is locked into the price they set one period before so $P_{it} = P_{it-1} = X_{t-1}$, where $i = B$. Therefore, $X_t, X_{t+2}, \dots$ are prices fixed by sector A , and $X_{t-1}, X_{t+1}, \dots$ are prices set by sector B .

To derive the reset price, a farmer maximises (3.1) with respect to X_t , and subject to the aforementioned demand functions and price-setting constraints, which yields:¹³

$$X_t = \left[\frac{1}{\gamma} \left(\frac{d\theta}{\theta - 1} \right) \left(\frac{Y_t^\varphi P_t^{\varphi\theta} + \beta Y_{t+1}^\varphi P_{t+1}^{\varphi\theta}}{\frac{Y_t P_t^{\theta-1}}{C_{it}} + \frac{\beta Y_{t+1} P_{t+1}^{\theta-1}}{C_{it+1}}} \right) \right]^{\frac{1}{1+\theta(\varphi-1)}} \quad (3.8)$$

This shows that, in the presence of staggered prices, the optimal price for a price-setting household is determined by a markup on the ratio of the marginal disutility of output supply to the marginal utility of consumption averaged over the time periods. Log-linearising (3.8) around the zero-inflation steady state (ZISS) leads to:^{14,15}

$$\tilde{x}_t = \varepsilon [(\varphi - 1)(\tilde{y}_t + \beta \tilde{y}_{t+1}) + (1 + \theta(\varphi - 1))(\tilde{p}_t + \beta \tilde{p}_{t+1}) + \tilde{c}_{it} + \beta \tilde{c}_{it+1}] \quad (3.9)$$

where $\varepsilon = \frac{1}{(1+\beta)(1+\theta(\varphi-1))}$, and is an equation for reset price as a function of aggregate output, price levels and sectoral consumption levels.¹⁶

3.4.3 Credit Market Imperfections and Staggered Prices

Equation (3.9) shows the optimal price a farmer wants to set in the period that s/he is allowed to change the nominal price. It is commonly assumed in the NK literature that the farmers in the sector who are unable to change price in a given period will borrow from or lend to the price-setting sector to smooth out income fluctuations. Rotemberg and Woodford (1997) assume that there is a complete set of state-contingent financial claims that allows a sector to insure itself against idiosyncratic income fluctuations that households suffer since they change prices in different periods. This is like assuming that there is an insurance scheme that is agreed on beforehand that will allow them to insure against losses resulting from exogenous shocks in the periods that they are not able to set prices optimally, such as in Ascari (1998) and Aoki

¹²Due to the assumption of symmetry, all farmers will choose the same reset price, so we drop the ‘i’ subscript.

¹³A detailed derivation of the method used to obtain the reset price is available in B.1 in the Appendix.

¹⁴A tilde over a variable refers to its percentage deviation from the steady state.

¹⁵Henceforth, unless otherwise stated, we always log-linearise around the ZISS.

¹⁶The terms ‘individual’ and ‘sectoral’ are used interchangeably.

(2001). Without an insurance contract, an unanticipated shock will cause an asymmetry in the economy since it will affect the sectors in a different way. As households are infinitely-lived and have identical preferences, these differences in the sectors will persist and result in the households of the sectors to have different lifetime wealth levels. For our model, we drop these assumptions, since in a developing economy, we assume that there are no credit markets or insurance contracts. This complicates the aggregation of the sectors as it implies that the marginal utility of consumption of the households in each period are not equal. To avoid this complication in the literature on price-setting models, it is always assumed that $C_{it} = C_t$ since in equilibrium, insurance contracts and perfect credit markets ensure that consumption across the households are equal, and, consequently, that $C_{it} = C_t = Y_t$.¹⁷ However, for our economy, this does not hold true, which necessitates an important role for individual money holdings. In the absence of credit markets, households can potentially protect themselves against shocks with money holdings. It seems likely that individual money holdings can only partially help to smooth out income and consumption fluctuations, and that full smoothing is not possible without a credit market or insurance contract. The asymmetry in the sectors, therefore, plays an important role for money balances: we cannot now simply aggregate money holdings in the sectors, i.e. $M_{it} \neq M_t$. This means that sectoral money holdings will play an important role to offset losses occurring in the event of a shock. In the literature on developing economy monetary policy, this is a key distinction in our model compared to, for instance, Anand et al. (2015), where money is not included in the utility function. In fact, they state that a possible extension to their model is to explicitly include money in the utility function since it serves as a savings mechanism for households, which is what we explore in this chapter and contribute to the literature.

To determine the role of sectoral money holdings on the reset price, we can incorporate them in our equation for reset price, (3.9). We can use the budget constraint, (3.6), at t and $t + 1$, to eliminate $\tilde{c}_{it}$ and $\tilde{c}_{it+1}$, and introduce money holdings in (3.9). To make progress with this, we log-linearise (3.4) to yield an equation for consumption in terms of sectoral output, money holdings, government transfers and price levels:¹⁸

$$\tilde{c}_{it} = \tilde{y}_{it} + \tilde{p}_{it} + \sigma(\tilde{m}_{it-1} - \tilde{m}_{it}) - \tilde{p}_t + \sigma\tilde{g}_t \quad (3.10)$$

where $\sigma = \frac{\zeta}{1-\beta}$. To substitute out the sectoral outputs, we use the demand function in (3.10), and using the resulting equations in (3.9), we can derive the reset price in terms of sectoral money

¹⁷Consequently, in the literature, it is assumed that $\tilde{c}_{it} = \tilde{c}_t$ and $\tilde{c}_t = \tilde{y}_t$.

¹⁸For $\tilde{g}_t$, we take the actual deviation of the variable from zero, or the ‘level-deviation’, and then scale it by the ZISS level of nominal output, so $\tilde{g}_t = \frac{G_t}{P_t Y_t}$, as $G_t = 0$ in the ZISS.

holdings, government transfers, aggregate output and price levels:¹⁹

$$\tilde{x}_t = \varepsilon[\varphi(\tilde{y}_t + \beta\tilde{y}_{t+1}) + \theta\varphi(\tilde{p}_t + \beta\tilde{p}_{t+1}) + (1-\theta)(\tilde{p}_{it} + \beta\tilde{p}_{it+1}) + \sigma(\tilde{m}_{it-1} - \beta\tilde{m}_{it+1} - (1-\beta)\tilde{m}_{it}) + \sigma(\tilde{g}_t + \beta\tilde{g}_{t+1})] \quad (3.11)$$

It is important to note here that, due to imperfections in the credit markets, farmers in the sector who will not reset prices in t will not be able to borrow or trade in insurance to smooth out income and consumption fluctuations, so we have separate sectoral Euler equations, which is reflected in the sectoral money holdings in (3.5). In order to proceed, a piece of notation that we introduce is to denote the price-setting sector in any period as S and the non price-setting sector as N . The advantage of this is that the difference equations will hold, irrespective of whether we are considering even- or odd-numbered periods. Then using this notation and using $\tilde{p}_{it} = \tilde{p}_{it+1} = \tilde{x}_t$, (3.11) can be written as:

$$\tilde{x}_t = \frac{1}{\theta\varphi(1+\beta)}[\varphi(\tilde{y}_t + \beta\tilde{y}_{t+1}) + \theta\varphi(\tilde{p}_t + \beta\tilde{p}_{t+1}) + \sigma(\tilde{m}_{Nt-1} - \beta\tilde{m}_{Nt+1} - (1-\beta)\tilde{m}_{St}) + \sigma(\tilde{g}_t + \beta\tilde{g}_{t+1})] \quad (3.12)$$

which is a simplified expression for reset price in terms of sectoral money holdings, government transfers, aggregate output and price levels.

We can derive another equation in reset price and sectoral money holdings using the Euler equation. To simplify our analysis, we determine a relationship between aggregate output and price levels, where we show in B.2 in the Appendix that:

$$\tilde{p}_{t+1} + \tilde{y}_{t+1} - \frac{1}{\beta}(\tilde{p}_t + \tilde{y}_t) = -\frac{1-\beta}{\beta}\tilde{m} \quad (3.13)$$

which is a first-order difference equation in aggregate nominal output. Since $\beta < 1$, (3.13) shows that aggregate nominal income is unstable in the forward dynamics. In addition, $\tilde{p}_t$ and $\tilde{y}_t$ are both non-predetermined variables, which allows us to conclude, using the usual “saddlepath” argument, that aggregate nominal output jumps to its steady state solution. Therefore, the path of aggregate nominal output is determined by the time-invariant money supply.²⁰

We can now use this simplification to derive a relationship between reset price and sectoral money holdings using the Euler equation. We use the demand function in (3.10), and insert in a log-linearised version of (3.7), to obtain:

$$\begin{aligned} & [\sigma^2(1+\beta) + \zeta]\tilde{m}_{it} - \sigma^2\tilde{m}_{it-1} - \beta\sigma^2\tilde{m}_{it+1} + \sigma(\theta-1)(\tilde{p}_{it} - \beta\tilde{p}_{it+1}) \\ & - \sigma(\tilde{y}_t + \theta\tilde{p}_t) + \beta\sigma(\tilde{y}_{t+1} + \theta\tilde{p}_{t+1}) - \sigma(\tilde{g}_t - \beta\tilde{g}_{t+1}) = 0 \end{aligned} \quad (3.14)$$

¹⁹Here ‘i’ is the sector which is allowed to set prices in period t .

²⁰We are interested to introduce a time-invariant money supply shock in the economy.

We can introduce the effect of staggered prices: for sector S , at t , $\tilde{p}_{it} = \tilde{p}_{it+1} = \tilde{x}_t$, where we use the saddle path solution of (3.13) to eliminate the aggregate price levels in (3.14) and obtain:

$$\begin{aligned} & [\sigma^2(1 + \beta) + \zeta]\tilde{m}_{St} - \sigma^2\tilde{m}_{Nt-1} - \beta\sigma^2\tilde{m}_{Nt+1} + \theta\sigma(\beta - 1)\tilde{m} \\ & + [\sigma(\theta - 1)(1 - \beta)]\tilde{x}_t - \sigma(\tilde{g}_t - \beta\tilde{g}_{t+1}) = \beta\sigma(\theta - 1)\tilde{y}_{t+1} - \sigma(\theta - 1)\tilde{y}_t \end{aligned} \quad (3.15)$$

where (3.15) is an equation in reset price, sectoral money holdings, government transfers, aggregate output and exogenous money supply, derived using the Euler equation.

3.4.4 General Equilibrium

The equilibrium conditions are not straightforward to derive since we are unable to equate the money holdings in the two sectors in our model of imperfect credit markets. To deal with this issue, we now provide a strategy to look at the time paths of the macroeconomic variables in this economy. This entails developing a system of linear difference equations in reset price and individual money holdings, which, in turn, is used to derive the dynamics of aggregate output.

3.4.4.1 Dynamics of Reset Prices and Individual Money Holdings

In (3.13), since we have shown that aggregate nominal output is determined by time-invariant money supply, i.e. $\tilde{p}_t + \tilde{y}_t = \tilde{m}$ for all t , we can use $\tilde{y}_t = \tilde{m} - \tilde{p}_t$ to eliminate all instances of $\tilde{y}_t$. Moreover, we can log-linearise the aggregate price index and obtain:²¹

$$\tilde{p}_t = \frac{1}{2}\tilde{x}_t + \frac{1}{2}\tilde{x}_{t-1} \quad (3.17)$$

which can be replaced in (3.12) to obtain an equation in reset price and sectoral money holdings:

$$\begin{aligned} \tilde{x}_t = \frac{1}{\theta\varphi(1 + \beta)} & \left\{ \varphi(\theta - 1) \left[\left(\frac{1}{2}\tilde{x}_t + \frac{1}{2}\tilde{x}_{t-1} \right) + \beta \left(\frac{1}{2}\tilde{x}_{t+1} + \frac{1}{2}\tilde{x}_t \right) \right] \right. \\ & \left. + \varphi(1 + \beta)\tilde{m} + \sigma(\tilde{g}_t + \beta\tilde{g}_{t+1}) + \sigma[\tilde{m}_{Nt-1} - \beta\tilde{m}_{Nt+1} - (1 - \beta)\tilde{m}_{St}] \right\} \end{aligned} \quad (3.18)$$

²¹The aggregate price index can be derived as:

$$P_t = \left(\frac{1}{2}P_{St}^{1-\theta} + \frac{1}{2}P_{Nt}^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad (3.16)$$

which shows that the aggregate price index is a function of the prices in sectors S and N . In period t , the price set in sector S is X_t and in sector N , the price set in the previous period, X_{t-1} , still remains in place in period t .

Moreover, we can replace $\tilde{y}_t = \tilde{m} - \frac{1}{2}(\tilde{x}_t + \tilde{x}_{t-1})$, so (3.15) becomes:

$$\begin{aligned} & [\sigma^2(1 + \beta) + \zeta]\tilde{m}_{St} - \sigma^2\tilde{m}_{Nt-1} - \beta\sigma^2\tilde{m}_{Nt+1} + \sigma(\beta - 1)\tilde{m} + \sigma(\theta - 1)(1 - \beta)\tilde{x}_t \\ & - \sigma(\tilde{g}_t - \beta\tilde{g}_{t+1}) = \beta\sigma(\theta - 1)\left(\frac{1}{2}\tilde{x}_{t+1} + \frac{1}{2}\tilde{x}_t\right) + \sigma(\theta - 1)\left(\frac{1}{2}\tilde{x}_t + \frac{1}{2}\tilde{x}_{t-1}\right) \end{aligned} \quad (3.19)$$

In (3.19), sectoral money holdings, $\tilde{m}_{it}$, and its leads and lags, enter as an exogenous ‘forcing’ variable in a second-order difference equation in $\tilde{x}_t$.²² Hence, if we knew the time path for $\tilde{m}_{it}$, we could use this to solve for the time path of $\tilde{x}_t$. This is the novelty of this staggered-price model: it is via the effects of $\tilde{m}_{it}$ on $\tilde{x}_t$ that credit market imperfections affect the price-setting process. Now, to determine the time path of $\tilde{m}_{it}$, we can, for now, assume that the time path of $\tilde{x}_t$ is given, i.e. consider $\tilde{x}_t$ to be an exogenous forcing variable and use (3.19) to solve for this. Then, we can solve the system of difference equations simultaneously, thereby, jointly determining the time paths of $\tilde{x}_t$ and $\tilde{m}_{it}$.

In pursuit of this strategy, we now use the money market equilibrium, where $\frac{1}{2}\tilde{m}_{St} + \frac{1}{2}\tilde{m}_{Nt} = \tilde{m}$, in all t , in (3.18) and (3.19), and after some rearrangement, we obtain two second-order difference equations in reset prices, money holdings of the price-setting sector, S , exogenous money supply and government transfers:²³

$$\begin{aligned} & -\beta\varphi(\theta - 1)(1 - \beta)\tilde{x}_{t+1} + \varphi(\theta + 1)(1 - \beta^2)\tilde{x}_t - \varphi(\theta - 1)(1 - \beta)\tilde{x}_{t-1} = 2(1 - \beta)(\tilde{g}_t + \beta\tilde{g}_{t+1}) \\ & + 2\beta\zeta\tilde{m}_{St+1} - 2\zeta(1 - \beta)\tilde{m}_{St} - 2\zeta\tilde{m}_{St-1} + 2(1 - \beta)[(\varphi(1 + \beta) + 2\zeta)]\tilde{m} \end{aligned} \quad (3.20)$$

$$\begin{aligned} & -\beta\zeta(\theta - 1)(1 - \beta)\tilde{x}_{t+1} - \zeta(\theta - 1)(1 - \beta)^2\tilde{x}_t + \zeta(\theta - 1)(1 - \beta)\tilde{x}_{t-1} = -2\zeta(1 - \beta)(\tilde{g}_t - \beta\tilde{g}_{t+1}) \\ & 2\beta\zeta^2\tilde{m}_{St+1} + 2[\zeta^2(1 + \beta) + \zeta(1 - \beta)^2]\tilde{m}_{St} + 2\zeta^2\tilde{m}_{St-1} + 2\zeta[(1 - \beta)(\beta - 1) - 2\zeta(\beta + 1)]\tilde{m} \end{aligned} \quad (3.21)$$

which is, essentially, a fourth-order system in reset price and sectoral money holdings.

Now, for the saddlepoint stability condition to hold in the fourth-order system, we need two eigenvalues to lie outside the unit circle and two eigenvalues to lie inside the unit circle, following the Blanchard and Kahn (1980) (B-K) conditions, since there are two predetermined variables, $\tilde{x}_{t-1}$ and $\tilde{m}_{St-1}$. An analysis of the fourth-order system, which is available in details in B.3 in

²²Intuitively, (3.18) is the equation that determines the time path of $\tilde{x}_t$, and (3.19) is the equation that determines the time path of $\tilde{m}_{St}$ (or $\tilde{m}_{Nt}$).

²³It is worth noting that, except in the first period of a permanent change in money supply, $\tilde{g}_t$ and $\tilde{g}_{t+1}$ will be zero.

the Appendix, requires that we derive the Hurwitz polynomial:

$$\begin{aligned}
& z^4 - \underbrace{\frac{2\zeta(\beta+1)[\theta(\varphi-1)+1] + \varphi(1-\theta)(1-\beta)^2}{\theta\varphi(\beta^2-1)}}_{\hat{p}_1} z^3 \\
& - \underbrace{\frac{2\zeta[(\beta^2+1)(2\theta(\varphi-1)+2-\varphi)+2\beta\varphi] + \varphi(\theta+1)(\beta^2-1)(\beta-1)}{\theta\varphi(\beta^2-1)(\beta-1)}}_{\hat{p}_2} z^2 \\
& - \underbrace{\frac{2\zeta(\beta+1)[\theta(\varphi-1)+1-2\varphi] + \varphi(\theta-1)(1-\beta)^2}{\theta\varphi(\beta^2-1)}}_{\hat{p}_3} z \\
& + \underbrace{\frac{2\zeta(\beta+1) + (1-\beta)^2}{\theta(1-\beta)^2}}_{\hat{p}_4}
\end{aligned} \tag{3.22}$$

To determine the number of stable and unstable eigenvalues in the fourth-order system, where the eigenvalues are the solutions for z obtained by setting (3.22) to zero, we can make use of Routh's stability criterion, which requires that we know the signs for $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$, and $\hat{p}_4$ in (3.22). As shown in B.3, we can unambiguously establish that $\hat{p}_4 > 0$ and $\hat{p}_2 < 0$. For $\hat{p}_3$ and $\hat{p}_1$, the two terms in the numerators have opposite signs, and it is unclear which is larger in magnitude. Hence, we cannot establish unambiguously the signs for $\hat{p}_3$ and $\hat{p}_1$, and are unable to use Routh's stability criterion to determine whether the stability condition holds.

However, an alternative method of determining the number of eigenvalues is Descartes' rule of signs (Curtiss (1918); Anderson et al. (1998)). If we assume real eigenvalues,²⁴ we can use the aforementioned method, which does not require that we unambiguously know the signs for $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$, and $\hat{p}_4$, and derive the following result.

Proposition 1. *Assuming real eigenvalues, under imperfect credit markets, for a constant money supply, a unique bounded perfect-foresight equilibrium exists, where reset price and sectoral money holdings are jointly determined.*

Proof: See Appendix B.3.

Using Descartes' rule of signs, we find that two eigenvalues lie outside and two eigenvalues lie inside the unit circle, which, as we mentioned before, satisfies the B-K conditions as there are two predetermined variables. This allows us to conclude that a saddlepath exists where, along the perfect-foresight path, reset price and sectoral money holdings are jointly determined.

In the next step of our analysis, we make an attempt to understand the time paths of the

²⁴We find numerically, using empirically plausible parameter values, that complex eigenvalues do not exist.

variables. To study the dynamics of the model, we can determine the pattern of convergence to the steady state. This requires that we study the signs of the two stable eigenvalues in our model, as these will help to characterise the transition paths of the variables, with a positive stable eigenvalue referring to a monotonic path and a negative stable eigenvalue referring to an oscillatory path. To determine the range in which the stable eigenvalues lie, we can evaluate (3.22) at $z = -1$ and $z = 0$, with a sign change between the two cases indicating that a stable eigenvalue lies in the interval $(-1,0)$. The evaluation of (3.22) at $z = -1$ and $z = 0$ leads to the following result.

Proposition 2. *Assuming real eigenvalues, under imperfect credit markets, a negative stable eigenvalue exists, which suggests that there are oscillations in the time paths of the variables, following a money supply shock.*

Proof: See Appendix B.4.

In Appendix B.4, we show that $(3.22) < 0$ when $z = -1$ and $(3.22) > 0$ when $z = 0$, which establishes that a stable eigenvalue lies in the interval $(-1,0)$. A corollary of this result is that, since we know that there are exactly two stable eigenvalues, and we know that one lies in the interval $(-1,0)$, the second eigenvalue must lie in the interval $(0,1)$.²⁵ It is a novel finding that there is a negative stable eigenvalue, as this shows that, under imperfect credit markets, there is the potential for oscillations in the transition paths of the variables, following a money supply shock. In Chari et al. (2000), and as we show later in Appendix B.6, under perfect credit markets, the variables follow a monotonic path following a money supply shock. This is a crucial difference between ICM and PCM as it shows that, under ICM, the variables are likely to produce oscillations as well as persistence along the transition paths. We can also show this result numerically, which is presented in the table below:

Table 3.2: Reset Price and Sectoral Money Holdings: Eigenvalues

Parameter values				Eigenvalues			
θ	φ	β	ζ	λ_1	λ_2	λ_3	λ_4
5	1.05	0.995	0.001	22.59	0.04	-1.38	-0.72
5	1.05	0.99	0.001	21.40	0.05	-1.95	-0.52
3	1.05	0.995	0.001	43.63	0.02	-1.30	-0.77
5	4.5	0.995	0.001	2.95	0.34	-1.19	-0.84
5	1.05	0.995	0.01	22.97	0.04	-1.11	-0.90

²⁵For details, please see Appendix B.4.

Table 3.2 shows the values of the stable and unstable eigenvalues for some parameter values, where the positive and negative stable eigenvalues are given by λ_2 and λ_4 respectively. From the table, it is noteworthy that the value of the stable eigenvalue, λ_4 , is robustly negative for various parameter values.²⁶ This numerically confirms the potential for oscillations in the transition paths, and is consistent with our analytical results.

Hence, using analytical methods, we can partially characterise the time paths of the variables, under ICM. In the next section, we present quantitative results, where we numerically simulate the time paths of the macroeconomic and sectoral variables under ICM. This will help us to understand the effects stemming from the negative and positive stable eigenvalues in the system, as these will be reflected in the time paths of the variables.

3.4.4.2 Dynamics of Aggregate Output

The dynamics of aggregate output can be derived if we know the path of the reset price and sectoral money holdings. From (3.13), we know that aggregate nominal output is determined by the time-invariant money supply. Since we know that the aggregate price index is an average of the reset prices in the sectors, we can simply replace it in (3.13), which yields:

$$\tilde{y}_t + \frac{1}{2}(\tilde{x}_t + \tilde{x}_{t-1}) = \tilde{m} \quad (3.23)$$

This shows that under imperfect credit markets, following a monetary policy shock, the adjustment path of aggregate output is entirely determined by the time path of reset price, which is jointly determined with sectoral money holdings.

The analytical results allow us to understand some aspects of the dynamics of reset price, sectoral money holdings and aggregate output under ICM. We can gain further insights into the time path of the variables by calibrating and numerically simulating the model.

3.5 Quantitative Results

We have now set up the model with imperfect credit markets and derived the qualitative results. The analytical results demonstrate that, in the absence of credit markets, reset price and individual money holdings are jointly determined, which, in turn, determine the path of aggregate output. In order to understand the behaviour of these macroeconomic variables following a monetary policy shock, we are required to calibrate and simulate the model as it is difficult to progress further using analytical methods. We present our quantitative findings in this section, along with sensitivity analysis.

²⁶The values chosen here of the various parameters are discussed in details below.

We are interested to generate simulations for the aggregate and sectoral variables. We introduce an unanticipated, one-off, shock with a 5% increase in aggregate money supply, $\tilde{m}$, after which $\tilde{m}$ stays at this higher level perpetually. We introduce a money supply shock in our model, and present the numerical simulation results, where the parameter values are chosen taking into consideration that we are interested to model a developing economy.²⁷ By setting $\theta = 5$, θ is kept sufficiently low, unlike in Basu (1996) and Anand et al. (2015) where it is assumed to be 11, as we expect there to be less substitutability between goods in a developing economy. The parameter value for $\beta = 0.995$ is standard in the literature. $\gamma = 0.999$ is chosen to be quite high, compared to the literature, such as Ascari (2000), because in a developing economy, a large emphasis is placed to smooth out consumption and income fluctuations, which imply that they choose to have higher consumption and hold lower money balances. In addition, the quantitative results are very sensitive to the value of φ . We use a very low value of $\varphi = 1.05$ to demonstrate the effects of the negative stable eigenvalue as found in the previous section. At the same time, it is likely that in a developing economy, where households often live hand-to-mouth and the willingness to work long hours is high, the elasticity of disutility of work with respect to output will be low.

3.5.1 Shocks to Aggregate Variables

In this section, we report the responses of aggregate variables to a money supply shock. Figure 3.2 shows the response of reset price to a shock, where we find that the reset price increases following a shock and then it oscillates for a few periods before converging to the steady state. This is consistent with the results in the previous section, where we found a negative stable eigenvalue. Since aggregate output is jointly determined with the reset price, aggregate output also increases following a shock, and then oscillates for a few periods before converging towards the steady state, as we find in Figure 3.3. To understand these oscillations better, we can derive the time path for the overall price index, as it is very closely related to output. As we observe from Figure 3.4, the price level also oscillates before converging to the steady state.

²⁷We have conducted our numerical simulations in Dynare for 1000 periods, but we report graphs for 20 periods.

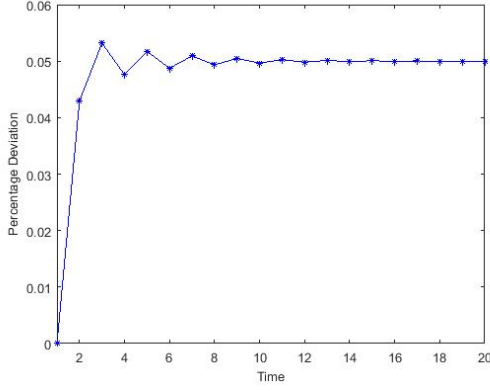


Figure 3.2: Response of Reset Price

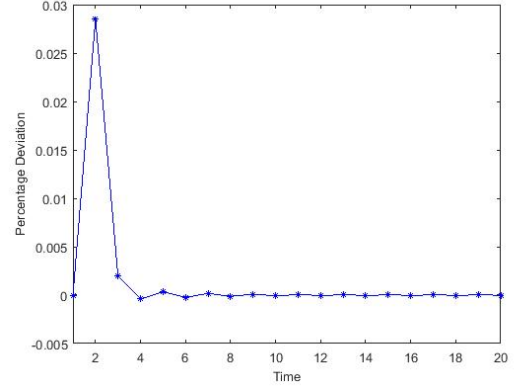


Figure 3.3: Response of Aggregate Output

We can also examine this theoretically. We can write $\tilde{x}_t - \tilde{m} = A_1(\lambda_1)^t + A_2(\lambda_2)^t$, where $\tilde{m}$ is the new steady state value of $\tilde{x}_t$, $0 < \lambda_1 < 1$ and $-1 < \lambda_2 < 0$. In the first two periods, $\tilde{x}_t - \tilde{m}$ does not change sign. However, as time progresses, $\tilde{x}_t - \tilde{m}$ enters a phase in which it flips between positive and negative every period, which implies that the absolute value of λ_2 should be greater than the absolute value of λ_1 . In fact, if we numerically solve for the eigenvalues, as in Table 3.2, we find that the absolute value for negative stable eigenvalue is greater than the absolute value of the positive stable eigenvalue. This means the oscillatory component of $\tilde{x}_t$ will go to zero more slowly than the monotonic component, so the former will eventually dominate the latter. Indeed, our simulation shows that this condition is satisfied. Applying the same logic to $\tilde{p}_t$, we can show that if $\lambda_2 > \lambda_1$, $\tilde{p}_t - \tilde{m}$, and, equivalently, $\tilde{y}_t$ must oscillate as well. In fact, our numerical simulations confirm this as we see that $\tilde{y}_t$ does oscillate, and so does $\tilde{p}_t$. In general, it is undesirable for a money supply shock to lead to large oscillations in aggregate output as it would then trigger a high frequency business cycle.

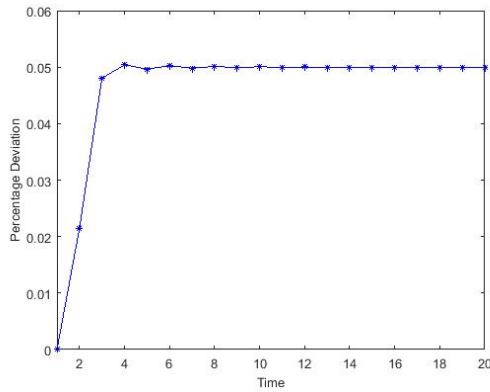


Figure 3.4: Response of Aggregate Price

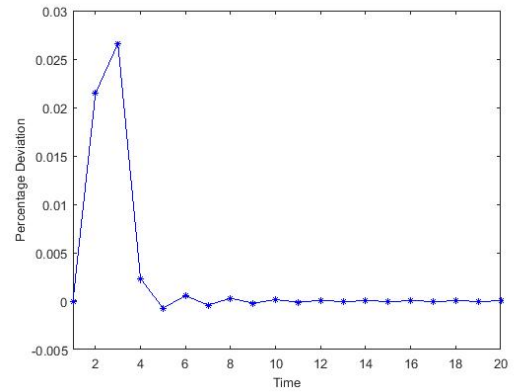


Figure 3.5: Response of Aggregate Inflation

The response of aggregate inflation following a shock can be seen in Figure 3.5, which shows that aggregate inflation increases following the shock, oscillates similarly to the price level, before

converging towards the steady state. This result is intuitive: a money supply shock causes reset price to increase, leading to an increase in the price level. As price level oscillates, inflation oscillates before slowly converging to the steady state.

3.5.1.1 Comparisons of Numerical Simulation Results: ICM and PCM

We develop a similar model with perfect credit markets, which is available in B.5 in the Appendix, to compare the responses of macroeconomic variables to a monetary policy shock.²⁸ The analytical results demonstrate that following a shock, aggregate output follows a monotonic path towards the steady state, which is consistent with the literature, such as Chari et al. (2000), among many others.²⁹ To compare the quantitative responses, we introduce the same shock to the economies, using the same parameter values as in the baseline calibration, and the responses of aggregate output and other macroeconomic variables are provided below.

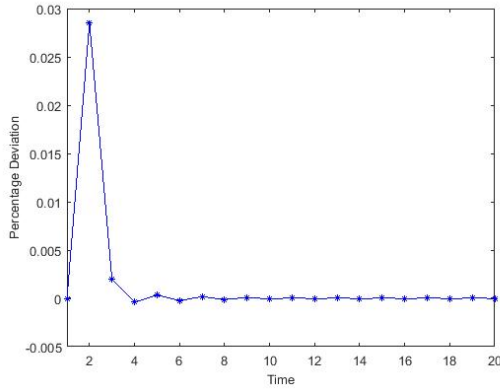


Figure 3.6: Aggregate Output: ICM

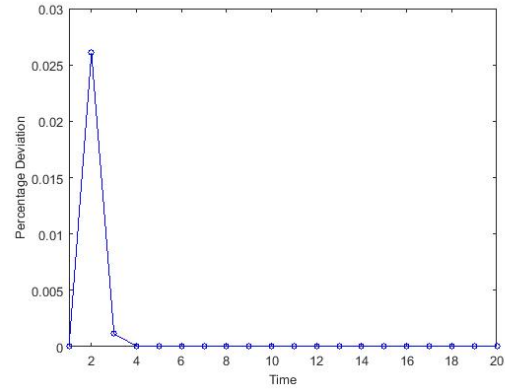


Figure 3.7: Aggregate Output: PCM

We find that the initial impact under ICM is higher in comparison to PCM. Following a shock, aggregate output is higher under ICM, and then follows an oscillatory path before converging towards the steady state. Under PCM, aggregate output increases following a shock and then follows a monotonic path before converging towards the steady state, as in Figures 3.6 and 3.7. These oscillations for ICM are, however, not large, and, although the oscillations are larger for certain parameters, these are not large enough to cause an issue from a policy perspective. This shows that, while an absence of credit markets poses difficulty for the households in their ability to borrow or trade in insurance, in the aggregate, these effects are not large.

²⁸It is important to note that ‘perfect credit markets’ in this chapter is used synonymously with ‘complete asset markets’, which include non-contingent assets, like bonds, as well as contingent assets, like insurance.

²⁹A comparison of the speed of convergence of aggregate output towards the steady state can be found in B.5.2 in the Appendix.

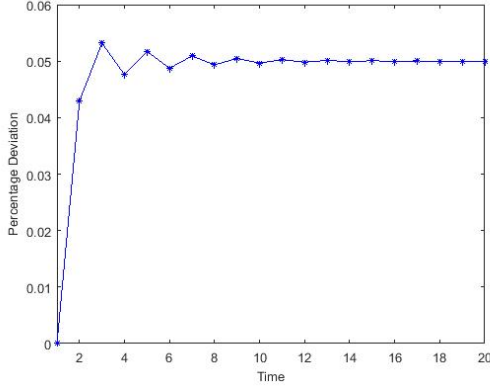


Figure 3.8: Reset Price: ICM

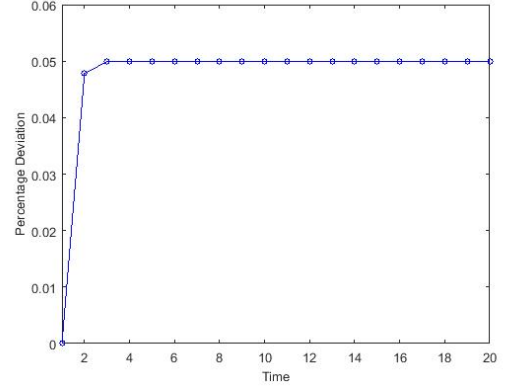


Figure 3.9: Reset Price: PCM

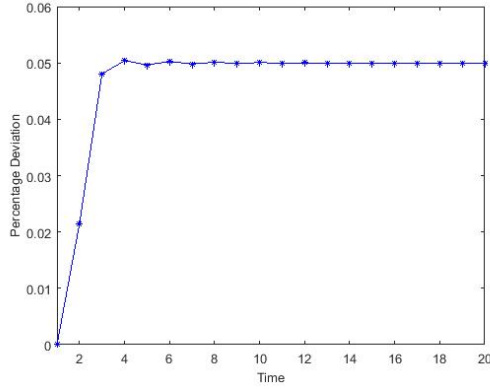


Figure 3.10: Aggregate Price Level: ICM

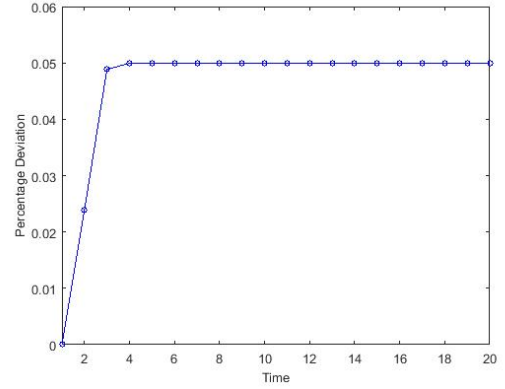


Figure 3.11: Aggregate Price Level: PCM

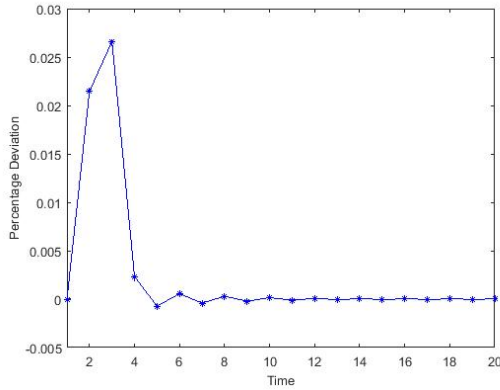


Figure 3.12: Aggregate Inflation: ICM

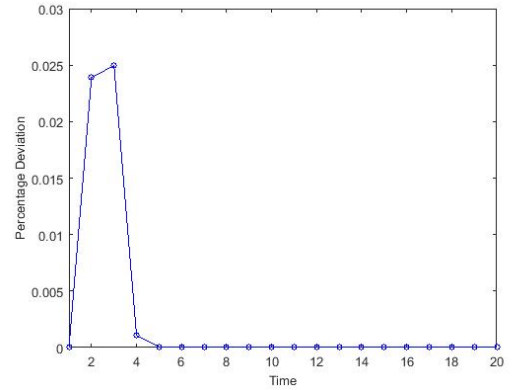


Figure 3.13: Aggregate Inflation: PCM

The results for reset price, aggregate price level and inflation show noticeable differences between ICM and PCM. The oscillations present in reset price are due to the heterogeneity in individual money holdings, and lead to oscillations under ICM. Since aggregate price level and inflation are simply a function of reset price, we observe oscillations in these variables as well. In

the case of PCM, the reset price, aggregate price level and inflation increases following a shock and follows a monotonic path towards the steady state. The oscillations in the model of imperfect credit markets are novel and demonstrate a mechanism through which aggregate variables may be subject to fluctuations following a shock.

3.5.2 Effects on Sectoral Variables

In this section, we look at the impact of the shock on the sectoral variables. The heterogeneity of the sectors implies that the sectors will be affected differently by the money supply shock. Figures 3.14 and 3.15 show the sectoral responses to the money supply shock, where we can see that the consumption levels increase for both sectors, although sector B , the price-setting sector in the impact period, has less money holdings and consumption in the impact period in comparison to the non price-setting sector. These results are at first sight slightly puzzling but the responses to output levels and nominal revenue as seen in Figures 3.16 and 3.17,³⁰ explain why we observe this behaviour. In the impact period, the output levels and nominal revenue for the price-setting sector decrease, as they set a higher price now. On the other hand, the non price-setting sector produces more output and is able to generate higher income, which enables it to have higher money holdings and consumption levels.

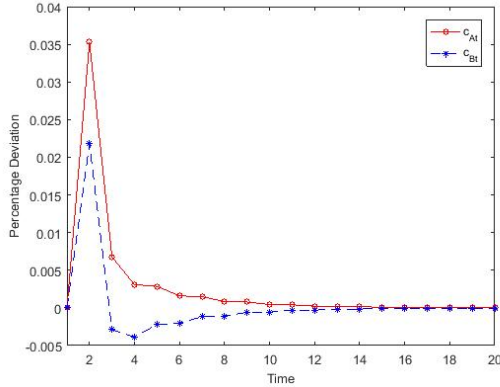


Figure 3.14: Individual Consumption Levels

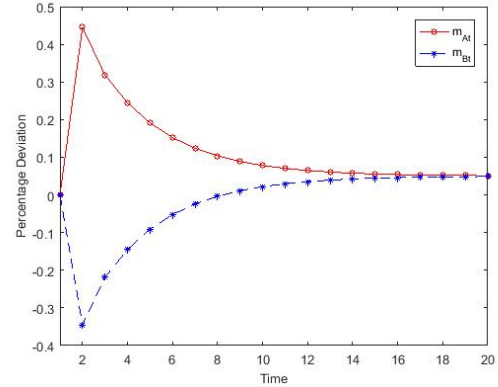


Figure 3.15: Individual Money Holdings

³⁰Nominal revenue is, in log-linearised terms, the addition of price and output levels.

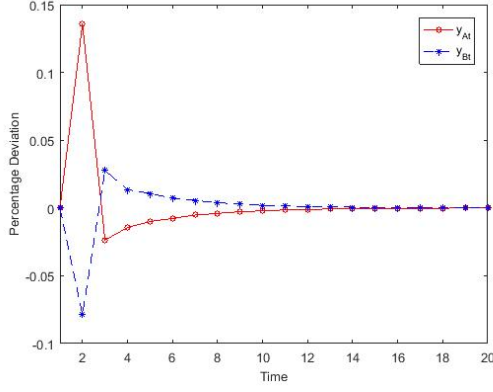


Figure 3.16: Individual Output Levels

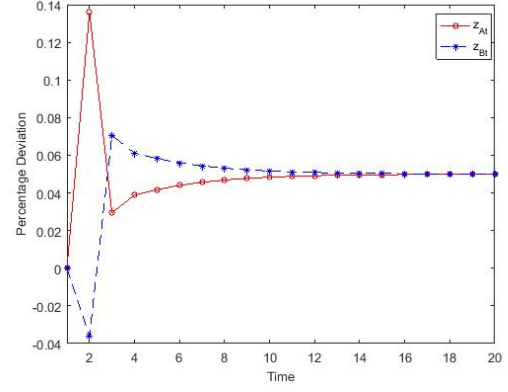


Figure 3.17: Individual Nominal Revenue

In the impact period, the price-setting sector has higher levels of consumption than before the shock, with lower levels of money holdings and output levels. Therefore, the price-setting sector derives higher utility from higher consumption and lower output levels, as the latter leads to higher leisure, but lower utility from lower money holdings, which implies that in terms of overall utility, the effect on the price-setting sector is ambiguous. The non price-setting sector, in the impact period, has higher levels of consumption, money holdings and output, which means that the effect on utility is again ambiguous as higher output leads to lower leisure for the non price-setting sector. The later parts of the transition paths of the sectors can be given similar explanations. Hence, in terms of overall utility, the effect on the sectors is not easy to see and requires us to directly calculate welfare numerically, which is done in the next section.

3.5.3 Role of Individual Money Holdings

An important question that we have not yet explicitly addressed is how the sectors are able to have fairly smooth patterns of levels in consumption, as in Figure 3.14, even with fluctuations in output levels resulting from price-setting. Under PCM, the presence of borrowing and lending and insurance arrangements allows the price-setting sector to smooth out its consumption levels. Under ICM, the price-setting sector has lower income in the impact period as it sets prices, but it is still able to enjoy higher consumption levels, although not as high as the consumption levels in the non price-setting sector. It is likely that, in the absence of borrowing and lending and insurance, individual money holdings are helping to smooth out consumption levels. To understand the role of individual money holdings, we can look at the sectoral variables under ICM and PCM, and observe how they behave in response to a monetary policy shock.

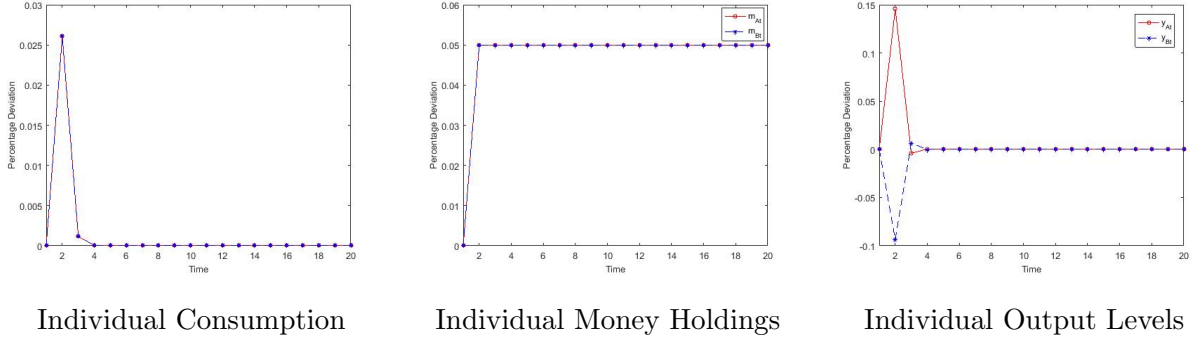


Figure 3.18: Perfect Credit Markets: Shocks to Individual Variables

Under PCM, as in Figure 3.18, individual consumption and money balances are the same for both sectors. Individual money holdings, then, are the same for any parameter values, as they will just change by the amount of the injection of money supply into the economy in any period following the shock. On the other hand, under ICM, as in Figures 3.14 and 3.15, we see that the price-setting sector runs down its holdings of money in the impact period in order to ensure that it can smooth out consumption fluctuations. This shows that individual money holdings are utilised, in the absence of borrowing and lending and insurance, to compensate for the income fluctuations due to price-setting. Further, this can be elaborated by looking at the agent's log-linearised budget constraint, $\tilde{c}_{it} = \tilde{y}_{it} + \tilde{p}_{it} - \tilde{p}_t - \sigma(\tilde{m}_{it} - \tilde{m}_{it-1}) + \sigma\tilde{g}_t$, which shows that a negative value of $\sigma(\tilde{m}_{it} - \tilde{m}_{it-1})$ allows $\tilde{c}_{it} > \tilde{y}_{it} + \tilde{p}_{it} - \tilde{p}_t$, i.e. an agent can use past savings to have higher consumption levels even with lower output levels.

Individual money holdings also play a key role in the price-setting process in this model. The oscillations in reset price and aggregate output are due to the fact that the price-setter changes every period, so $\tilde{m}_{St}$ is shifting between the two time paths, that is $\tilde{m}_{At}$ and $\tilde{m}_{Bt}$, as we see in Figure 3.15. Moreover, as we see algebraically, in (3.11), the oscillations in the individual money holdings lead to the time path of reset price to oscillate, which, in turn, causes aggregate output to oscillate as well.

3.5.4 Sensitivity Analysis

We conduct some sensitivity analyses in this section, to understand the role of the parameters in detail.³¹ In addition, changes to the parameter values will help us to understand which parameters, in particular, amplify the effects of the shock.

(i) Sensitivity Analysis: Aggregate Variables

³¹The graphs for the sensitivity analysis are available in B.7 in the Appendix.

The results for the sensitivity analyses on aggregate variables are presented with emphasis on the results for aggregate output.³² The main finding from the sensitivity analysis is that aggregate output produces oscillations for low values of φ . In other words, monetary policy does not produce destabilising real responses in output in a developing economy, as long as φ is high. In general, we find that oscillations are higher when fluctuations in reset prices or individual money holdings are higher. (3.23) shows why we observe this - aggregate output is determined by the time path of reset prices, which, in turn, are determined by individual money fluctuations. For instance, for low values of φ (and ζ) and high values of γ , we find that reset prices fluctuate, which, in turn, lead to greater fluctuations in aggregate output. We also find oscillations for lower θ . These results are available in Figures B.1-B.4.

The parameter values can be changed to see which parameter causes monetary policy to be less effective, in terms of the magnitude on impact. We find that lower θ lead to lower impact on aggregate output in the impact period, as seen in Figure B.1. Intuitively, lower θ implies lower substitutability of goods and lower competitiveness among farmers. Hence, farmers are less interested in rival farmers' prices, when it is a farmer's turn to reset prices. Farmers then are more inclined to charge a higher reset price, which results in a fall in aggregate output. We make changes to β and ζ as well, but these do not result in notable changes in aggregate output in the impact period.

(ii) Sensitivity Analysis: Sectoral Variables

In this subsection, we discuss the parameters that amplify the heterogeneous effects. Since there are many variables and parameters, we report only the changes that lead to substantial differences in the sectoral outcomes.

When we lower the value of θ , in other words, there is a lower substitutability of goods, the first observation is that the reset price is now higher. Figure B.8 shows that the fall in output for sector B is now much lower even with a higher price since their goods are less substitutable now, while the increase in output in sector A falls. In addition, it can be seen from Figure B.9 that the nominal revenue for sector B no longer falls in the impact period. The money holdings and consumption levels are also slightly higher for sector B in this scenario, which implies that the difference between them is now lower, although the overall increase in consumption levels is lower. This is expected as higher nominal income leads the price-setting sector to be able to enjoy higher consumption levels and money holdings.

A higher φ leads to a lower increase in reset price following a shock, as the marginal cost of

³²The graphical results, with changes in parameter values, are presented side-by-side, instead of being overlaid. The subplots make it difficult to observe the differences in the graphs.

production is now lower, as seen in Figure B.10. This leads the output of sector B to fall less than under lower φ , while the output of sector A increases less. The nominal revenues for sector B now falls less, as seen in Figure B.14.

If the value of β is lower, implying households are less patient, we find that sector A , the non price-setting sector, holds higher money balances and consumption levels, as seen in Figures B.16 and B.17, resulting in higher differences between the sectors. This shows that a lower β leads to higher gains for this sector, in the transition path.

A higher value of ζ leads the price-setting sector to hold more money and have higher consumption levels, as seen in Figures B.21 and B.22. This reinforces the idea that households use individual money holdings, in the absence of borrowing or lending or trading in insurance, to smooth out individual consumption fluctuations. However, this does not significantly affect output levels or nominal revenues.

3.6 Welfare

The quantitative analyses in the previous section show that the sectors are affected differently due to a monetary policy shock. In the period of impact, the price-setting sector derives higher utility than before the shock from higher consumption levels and lower output, but lower utility from lower money holdings, so the effect on the overall utility of the sector is at first sight ambiguous. The non price-setting sector has higher output, consumption and money balances, but as higher output leads to lower leisure, the overall effect of the monetary policy on the utility of the non price-setting sector is unclear as well. In order to understand the overall impact on the sectors due to the monetary policy shocks, we conduct sectoral welfare analysis in this section. We are primarily interested to look at the lifetime utility of the sectors as a monetary policy shock leads to heterogeneous effects in the impact period and in the transition path.

3.6.1 Derivation of the First- and Second-Order Approximations of the Utility Function

In order to explicitly derive the welfare of the sectors following the shock, we can take first- and second-order approximations of the utility function. For the sectoral welfare analysis, we use a combination of first- and second-order approximations, as these allow us to make a thorough analysis of the numerical simulation results. Since the derivation of the first-order approximations of the utility function is simpler and can be derived similarly, we only show how we derive the second-order approximations of the utility function and state the key difference in the first- and second-order approximations below.

The utility function is given by:

$$U_i = \left[\sum_{t=0}^{\infty} \beta^t \left(\gamma \ln C_{it} + (1 - \gamma) \ln(M_{it}/P_t) - \frac{d}{\varphi} Y_{it}^{\varphi} \right) \right] \quad (3.24)$$

The second-order Taylor series approximations for individual utility around the zero inflation steady state, using proportional deviations, can be derived as:

$$\begin{aligned} W_i^F = & \gamma \left[\frac{1}{C_i} C_i \left(\frac{C_{it} - C_i}{C_i} \right) - \frac{1}{2} \frac{1}{C_i^2} C_i^2 \left(\frac{C_{it} - C_i}{C_i} \right)^2 \right] \\ & + (1 - \gamma) \left[\frac{P}{M_i} \frac{M_i}{P} \left(\frac{M_{it}/P_t - M_i/P}{M_i/P} \right) - \frac{1}{2} \frac{P^2}{M_i^2} \frac{M_i^2}{P^2} \left(\frac{M_{it}/P_t - M_i/P}{M_i/P} \right)^2 \right] \\ & - d \left[Y_i^{\varphi-1} Y_i \left(\frac{Y_{it} - Y_i}{Y_i} \right) + \frac{\varphi - 1}{2} Y_i^{\varphi-2} Y_i^2 \left(\frac{Y_{it} - Y_i}{Y_i} \right)^2 \right] \end{aligned} \quad (3.25)$$

where W_i^F is the change in the flow utility. Now, we can write the second-order deviation variables as:

$$\frac{C_{it} - C_i}{C_i} = \tilde{c}_{it} + \frac{1}{2} \tilde{c}_{it}^2 \quad (3.26)$$

which allows us to write (3.25), in terms of log deviations, as:

$$\begin{aligned} W_i^F = & \gamma \left[\tilde{c}_{it} + \frac{1}{2} \tilde{c}_{it}^2 - \frac{1}{2} \tilde{c}_{it}^2 \right] + (1 - \gamma) \left[\tilde{m}_{it} - \tilde{p}_t + \frac{1}{2} (\tilde{m}_{it}^2 - \tilde{p}_t^2) - \frac{1}{2} (\tilde{m}_{it}^2 - \tilde{p}_t^2) \right] \\ & - d \left[Y_i^{\varphi} \left(\tilde{y}_{it} + \frac{1}{2} \tilde{y}_{it}^2 \right) + \frac{\varphi - 1}{2} Y_i^{\varphi} \tilde{y}_{it}^2 \right] + O[2] \end{aligned} \quad (3.27)$$

where $O[2]$ refers to terms that are higher than second-order. (3.27) can be simplified, which allows us to write the change in lifetime utility, W_i^L , as:

$$W_i^L = \sum_{t=0}^{\infty} \beta^t \left[\gamma \tilde{c}_{it} + (1 - \gamma)(\tilde{m}_{it} - \tilde{p}_t) - d Y_i^{\varphi} \left(\tilde{y}_{it} + \frac{\varphi}{2} \tilde{y}_{it}^2 \right) \right] + O[2] \quad (3.28)$$

As shown in B.8 in the Appendix, we can derive an analytical solution for the steady state value of Y_i as $Y_i^{\varphi} = \frac{\gamma(\theta-1)}{d\theta}$, which allows us to write the second-order approximations of the individual utility function as:

$$W_i^L = \sum_{t=0}^{\infty} \beta^t \left[\gamma \tilde{c}_{it} + (1 - \gamma)(\tilde{m}_{it} - \tilde{p}_t) - \frac{\gamma(\theta-1)}{\theta} \left(\tilde{y}_{it} + \frac{\varphi}{2} \tilde{y}_{it}^2 \right) \right] + O[2] \quad (3.29)$$

(3.29) is the second-order approximation of the individual utility function, where the only difference between (3.29) and the first-order approximation is that the squared output deviation term above does not appear in the first-order approximation of the individual utility function,

the latter then can be written as:

$$W_i^L = \sum_{t=0}^{\infty} \beta^t \left[\gamma \tilde{c}_{it} + (1 - \gamma)(\tilde{m}_{it} - \tilde{p}_t) - \frac{\gamma(\theta - 1)}{\theta} \tilde{y}_{it} \right] \quad (3.30)$$

Now that we have derived first- and second-order approximations of the individual utility function, we are interested to look at and compare the lifetime utility of the sectors numerically. However, the utility function, on its own, is not useful numerically as it is not cardinal. A common measure used in the literature is the consumption-equivalent of the welfare gain, which measures the constant fraction that households would need to gain in terms of consumption in each period in the starting steady state, to equate the lifetime utility with the value it would take under increased nominal money supply (Anand et al. (2015); Ascari and Ropele (2012)). We can write the individual utility function, without the monetary policy shock, as:

$$W_i^{old} = \left[\sum_{t=0}^{\infty} \beta^t \left(\gamma \ln C_{i,-1} + (1 - \gamma) \ln(M_{i,-1}/P_{-1}) - \frac{d}{\varphi} Y_{i,-1}^{\varphi} \right) \right] \quad (3.31)$$

Once the monetary policy shock is implemented, we write the individual utility function as:

$$W_i^{new} = \left[\sum_{t=0}^{\infty} \beta^t \left(\gamma \ln(1 + \lambda) C_{it} + (1 - \gamma) \ln(M_{it}/P_t) - \frac{d}{\varphi} Y_{it}^{\varphi} \right) \right] \quad (3.32)$$

The parameter λ measures the consumption-equivalent gain of the households, which we can then simply express, in percentage terms, as:

$$\lambda = \left\{ \exp \left[(1 - \beta) \left(\frac{W_i^{new} - W_i^{old}}{\gamma} \right) \right] - 1 \right\} * 100 \quad (3.33)$$

3.6.2 Sectoral Lifetime Utility Gains Calculations

For the sectoral welfare calculations, first, we calculate the lifetime utility gains of the sectors following a monetary policy shock using first-order approximations of the utility functions. We estimate and report the lifetime utility gains of the sectors in Table 3.3, using the consumption-equivalent measure in (3.33), in response to a 5% increase in nominal money supply. The baseline parameter values are $\beta = 0.995$, $\gamma = 0.999$, $\theta = 5$, $\varphi = 1.05$, where the last four rows represent the results under various deviations from the baseline parameter values. Here, we look at the absolute changes in the utility of the sectors, where the price-setting sector in the impact period is B , and the non price-setting sector in the impact period is A :

Table 3.3: Sectoral Lifetime Utility Gains: First-Order Approximations

Parameter Values	W_B	W_A
Baseline	0.003%	0.003%
$\theta=3$	0.0047%	0.0047%
$\varphi=4.5$	0.0051%	0.0051%
$\beta=0.99$	0.0066%	0.0066%
$\zeta=0.01$	0.00296%	0.00296%

There are two main results concerning the sectoral lifetime utility gains calculated using first-order approximations visible in Table 3.3. First, we observe that, under the baseline case and other parameter values, following a positive money supply shock, the lifetime utility of both the sectors increases. Intuitively, we expect this result, as in the pre-shock steady state, monopoly power causes output to be too low relative to the Pareto-efficient level. Second, we observe that the lifetime utility gains of the sectors are identical under first-order approximations. This result is surprising, as we might expect that the lifetime utility of the price-setting sector in the impact period would be higher than the lifetime utility of the non price-setting sector in the impact period. Intuitively, this is because placing a constraint on an agent's price-setting ability should lower the agent's welfare. Moreover, the agent facing the constraint in the impact period is expected to have higher disutility as disturbance to the economy is greatest then and, hence, the need to adjust price is highest. However, we find no differences in sectoral lifetime utility gains under first-order approximations. To investigate further, we can look at this algebraically. As shown in B.9 in the Appendix, we find algebraically as well that the welfare difference between the sectors is zero under first-order approximations. Hence, although there is considerable heterogeneity across sectors in the time paths of those variables which directly affect utility (namely, consumption, real balances and output), which we observed in our quantitative results, the lifetime utility effects are homogeneous across the sectors, according to this method of analysis.

To test whether this result holds under other methods, we can use second-order approximations of the individual utility function, which we have derived in the previous subsection. However, to use this method, it appears that we may be required to derive second-order approximations of the structural equations of the model as well, which are more complicated and difficult to obtain than the first-order approximations. However, as shown in B.10 in the Appendix, under special circumstances, it is valid to use first-order approximations to the structural equations when deriving second-order approximations to the change in utility. More specifically, we can show algebraically that, if we focus only on the 'difference' across sectors in changes in lifetime utility, and not on the 'absolute' values of these changes, then we are not required to

derive second-order approximations of the structural equations of the model in order to calculate a second-order approximation of the relative lifetime utility gain: first-order approximations of the structural equations are enough.

In Table 3.4, we estimate and report the relative lifetime utility gains of the sectors using second-order approximations of the utility function, using the consumption-equivalent measure in (3.33), in response to a 5% increase in nominal money supply. The baseline parameter values are as above, while the last four rows represent the results under various deviations from the baseline parameter values. As mentioned before, we look at the differences in the utility of the sectors, where the second column reports the difference between the welfare of the price-setting sector in the impact period, W_B , and that of the non price-setting sector, W_A :

Table 3.4: Sectoral Lifetime Utility Gains: Second-Order Approximations

Parameter Values	Welfare Difference ($W_B - W_A$)
Baseline	0.002534%
$\theta = 3$	0.001295%
$\varphi = 4.5$	0.008745%
$\beta = 0.99$	0.004835%
$\zeta = 0.01$	0.002595%

Using the second-order approximations method, we now find confirmation of our initial hypothesis: the price-setting sector in the impact period has higher lifetime utility gains compared to the non price-setting sector, as shown in Table 3.4. In the baseline scenario, the price-setting sector has the equivalent of an extra 0.0025% of consumption in each period, in comparison to the non price-setting sector. This shows that a monetary policy shock leads to different outcomes for the sectors, in terms of lifetime utility gains, and creates an asymmetry between the sectors. As seen in the table, these asymmetries are robust to other plausible parameter values. These results show that, although the non price-setting sector has higher consumption levels and money holdings following the shock, it suffers disutility from not being able to reset prices optimally in the impact period, which lead them to produce more than they ideally would. The over-production leads to a decrease in leisure for the non price-setting sector which results in the gain in the lifetime utility of the sector to be lower than the price-setting sector. To put this another way: the non-price-setter is prevented from exploiting its monopoly power, which is the power to restrict output and thereby force up the price. This means that the non price-setting sector is forced to produce higher output than the price-setter, but is able to partially offset this disadvantage by utilising its greater sales revenue to consume more and hold more money. The price-setting sector, on the other hand, derives lower utility from lower money holdings but enjoys higher utility from

higher consumption and lower output levels than before the shock. Therefore, the true benefit of being able to exploit its monopoly power lies in being able to enjoy more leisure, relative to the non-price-setter: the lower consumption and money holdings are worth the higher utility from greater leisure.

In addition, it is worth noting that, while this holds in the impact period, the situation of the sectors is reversed in the following period (and so on in successive periods). Then, it may be expected that this difference would average to zero in the transition path. However, the fact that it does not suggests that the effects arising from the impact period dominate. In general, the fact that the relative utility gain of the price-setting sector only becomes apparent under second-order approximations suggests that the relative utility gain may be small, in comparison with the absolute utility gain of each sector. Nevertheless, if the shock is large, the relative gain may not be particularly small in comparison with the absolute gain. The present case illustrates this, as can be seen by comparing the numbers in Tables 3.3 and 3.4.

3.6.3 Comparisons of Sectoral Lifetime Utility Gains: ICM and PCM

The welfare differences between the non price-setting and price-setting sectors under ICM show the heterogeneous effects caused by staggered prices. The heterogeneous effects hold for PCM as well, as staggered prices still lead to different output fluctuations between the sectors. However, as we saw in Figures 3.14, 3.15 and 3.18, for PCM, there are no sectoral differences in consumption levels and money holdings, in the presence of borrowing and lending and trading in insurance. This suggests that it is likely that the welfare differences between the sectors with perfect credit markets will be larger, as the non price-setting sector in the impact period is not able to offset the disutility from higher output with higher consumption levels and money holdings.

To compare the utility differences between ICM and PCM, we investigate two main areas: (a) the average lifetime utility gains across the two sectors under ICM and PCM, and (b) the difference in lifetime utility gains between the two sectors under ICM and PCM, as mentioned before. For these comparisons, we are unable to use second-order approximations, as that will require us to obtain second-order approximations of the full model under PCM; the method in B.10 is valid only under ICM. Therefore, for these comparisons of lifetime utility gains between ICM and PCM, we use first-order approximations of the structural equations and utility functions.

We estimate and report the average lifetime utility gains of the sectors under ICM and PCM in the following table, where similar to previous tables, the baseline parameter values are as above, and the last four rows represent the results under various deviations from the baseline parameter values:

Table 3.5: Average Lifetime Utility Gains Across Sectors: ICM and PCM

Parameter Values	W^{ICM}	W^{PCM}
Baseline	0.003%	0.0028%
$\theta=3$	0.0047%	0.0044%
$\varphi=4.5$	0.0051%	0.0051%
$\beta=0.99$	0.0066%	0.0055%
$\zeta=0.01$	0.00296%	0.0029%

We see in Table 3.5 that the average lifetime utility gains across the two sectors under ICM and PCM are very similar. In order to check for the robustness of this result, we change the parameter values and find that this result holds under different parameter values. This shows that, in terms of the average lifetime utility gains of the sectors, the difference is minimal between ICM and PCM.

Now, we look at the differences in the lifetime utility gains between the sectors under ICM and PCM. We have already found that, under first-order approximations, there are no differences in lifetime utility gains between the sectors under ICM. For PCM, we investigate whether this holds algebraically in B.11 in the Appendix, where we find that, unlike ICM, there are differences in lifetime utility gains between the sectors under PCM. In addition, we find differences in lifetime utility gains between the sectors numerically as well, which are reported below:

Table 3.6: Sectoral Lifetime Utility Gains: PCM

Parameter Values	Welfare Difference ($W_B - W_A$)
Baseline	0.092%
$\theta = 3$	0.048%
$\varphi = 4.5$	0.049%
$\beta = 0.99$	0.183%
$\zeta = 0.01$	0.092%

Table 3.6 shows that the price-setting sector in the impact period, B , has higher lifetime utility gains compared to the non price-setting sector in the impact period, A , under PCM. This can be contrasted with Table 3.3, where we see that the differences in lifetime utility gains between the sectors are zero under ICM. This shows a key difference in sectoral lifetime utility gains between PCM and ICM: the differences in lifetime utility gains between the sectors as calculated using first-order approximations are larger under PCM than ICM. Hence, despite the greater intersectoral heterogeneity in the time paths of those variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes under ICM in comparison to PCM.

To highlight this point and show more clearly the differences in lifetime utility gains in the sectors, we can use a social welfare function for ICM and PCM, for instance, a Rawlsian social welfare function (SWF), which places a large weight on the equality of utility levels across agents, and is given by:

$$W = \min(W_A, W_B) \quad (3.34)$$

This states that the Rawlsian SWF, W , uses the utility of the worst-off sector of the economy to calculate welfare. This means that, in the following table, for ICM, $W^{ICM} = W_A^{ICM} = W_B^{ICM}$, as these are equal, whereas, for PCM, $W^{PCM} = W_A^{PCM}$ as $W_A^{PCM} < W_B^{PCM}$.

Table 3.7: Lifetime Utility Gains: ICM and PCM

Parameter Values	Welfare Difference ($W^{ICM} - W^{PCM}$)
Baseline	0.0461%
$\theta = 3$	0.0242%
$\varphi = 4.5$	0.0247%
$\beta = 0.99$	0.0928%
$\zeta = 0.01$	0.0459%

The results above confirm that the increase in social welfare is higher for ICM in comparison with PCM. Under PCM, the non price-setting sector in the impact period is unable to compensate for the greater loss of leisure with higher consumption levels and money holdings, as under ICM. This means that, under PCM, the sectoral differences in lifetime utility gains are simply through the differences in utility arising from output fluctuations, which leads to greater disutility for the non price-setting sector, and, consequently to higher utility differences between the sectors. This is in contrast to ICM, where the non price-setting sector is able to partially compensate for higher output levels with higher consumption levels and money holdings. This explains why the social welfare gain under ICM is higher than under PCM.

These results lead to an interesting implication: a monetary authority concerned about equality can be more vigorous in its monetary policy actions under ICM in comparison to PCM. Under ICM, monetary policy actions lead to lower utility differences between the sectors, in comparison with PCM. As the Rawlsian SWF shows, social welfare gains are then higher, in response to a given change in money supply, under ICM than under PCM. Therefore, a monetary authority, particularly concerned about equality, might prefer to make larger money supply changes under ICM than under PCM.

3.7 Conclusion

In this chapter, we examined the implications of staggered prices and assessed the non-neutrality of monetary policy in a developing economy. We developed a DGE model to study the process of price-setting and its effects on the macroeconomic dynamics. We built our model of a developing economy by incorporating credit market imperfections - a feature we empirically establish is a prominent feature in such economies - starting from an otherwise standard DGE model with staggered prices and rational expectations. We then developed an economy without imperfections in the credit markets, as a representative advanced economy, and compared our findings for the dynamics of aggregate output in the economies in response to a monetary policy shock. We also conducted sectoral analyses as it is expected that there may be heterogeneous effects in the sectors due to the absence of borrowing and lending and insurance.

We found that a key difference between economies with perfect and imperfect credit markets is the mechanism via which agents in the sectors manage the consequences of an unanticipated shock. In the absence of credit markets, agents in the sectors are unable to borrow or lend or trade in insurance, which requires them to adjust to shocks with individual money holdings. Due to staggered prices, the price-setting process is an intertemporal decision, which implies that the agents' reset prices in the sectors are influenced by individual money holdings. Further, we found that, following a monetary policy shock, aggregate output is determined by reset price, and, thereby, directly affected by sectoral differences in individual money holdings. To understand the behaviour of aggregate output, we computed some numerical simulations and found that, in an economy with imperfect credit markets, following a monetary policy shock, aggregate output exhibits a monotonic path initially, followed by an oscillatory path that slowly converges towards the steady state, although the magnitudes of these oscillations are small. This led us to conclude that while imperfect credit markets create oscillations in aggregate output, these are unlikely to cause an issue from a policy perspective. Further, we conducted some quantitative analysis at the sectoral level and found heterogeneous effects in response to a monetary policy shock. Following a shock, the money holdings and consumption levels increase for the non price-setting sector in the impact period, but they also produce more output, which decreases leisure. The price-setting sector in the impact period, on the other hand, has less money holdings but higher consumption levels in the impact period than before the shock. This is due to the higher price set by the price-setting sector in the impact period, resulting in lower output levels and nominal revenue, and, thereby, leading the price-setting sector to use money holdings to smooth out consumption levels. In terms of lifetime utility, we found that the price-setting sector in the impact period has higher lifetime utility compared to the non price-setting sector. We found that this holds true for an economy with perfect credit markets as well due to output fluctuations. However,

a comparison of the economies shows that despite the greater intersectoral heterogeneity in the time paths of the variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes in an economy with imperfect than perfect credit markets. Hence, a monetary authority concerned about equality can be more vigorous in its monetary policy actions in a developing economy in comparison to an advanced economy.

The chapter aimed to study the importance of staggered prices and its implications for the transmission mechanism of monetary policy in developing economies. Agénor and Montiel (2008) explore many channels which affect the way monetary policy is conducted in developing economies, and these substantially advance our understanding of the transmission mechanism of monetary policy in developing economies. In fact, they include financial frictions and credit constraints as prominent features of a developing economy. However, they do not explore the sectoral asymmetries which are caused by staggered prices, in the presence of financial frictions and credit constraints, and their effects in response to a monetary policy shock. Other studies on monetary non-neutrality in developing economies, such as Davoodi et al. (2013) and Mishra and Montiel (2013), empirically investigate the transmission mechanism of monetary policy in low income countries, but poor quality of macroeconomic data makes it difficult to obtain reliable estimates for these economies. We have taken an analytical approach, with a focus on staggered prices, to explore the heterogeneous effects of monetary policy in a developing economy. However, the model is fairly restricted as it becomes unwieldy with the addition of other structural features, which then makes it difficult to obtain analytical results. For instance, the inclusion of a labour market will incorporate wages, which may affect the economy by its impact on the labour supply and demand, and, thereby, affect production and output. Further work on this topic would, hence, benefit from including other relevant variables, and then studying their confounding effects on the aggregate and sectoral outcomes.

Chapter 4

Monetary Policy Rules in Developing Economies: Monetary Aggregate or Inflation Targeting Framework?

4.1 Introduction

Since the 1990s, Inflation Targeting (IT) has been adopted by many high- and middle-income countries for the conduct of monetary policy. New Zealand was the first country to adopt an IT framework, which was then followed by other advanced economies, such as Canada, United Kingdom, Sweden, Finland and Australia. More recently, emerging economies, for instance, Latin American countries, which often encounter periods of hyperinflation, have also adopted IT as a monetary policy framework due to its potential for targeting and reducing inflation.

An IT framework, as practiced in these countries, constitutes an explicit quantitative inflation target, as part of inflation-forecast targeting, where a conditional forecast of inflation and output is used as an intermediate target variable, and transparency and accountability of the central banks in their conduct of monetary policy.¹ Svensson (1999a) argues that there is a high degree of commitment associated with an IT framework, compared to other monetary policy regimes. As the numerical inflation target under an IT framework is publicly-announced, it can be monitored and, thereby, ensured that the inflation target is met.

While these theoretical discussions emphasise the mechanism and importance of IT in the conduct of monetary policy, some studies have empirically assessed the performance of IT. Ball and Sheridan (2004), Mishkin and Schmidt-Hebbel (2007) and Lin and Ye (2007), among others,

¹Specifically, Svensson (1997a) defines ‘inflation-forecast targeting’ (I-FT) as a framework for policy decisions, where the inflation forecast is compared to the announced target. Hence, the forecast under an I-FT framework serves as an intermediate target.

using data for a sample of advanced economies, and various econometric methods, find insignificant effects of IT on inflation and inflation volatility. Vega and Winkelried (2005), on the other hand, find that IT leads to lower inflation and inflation volatility using a sample of advanced and emerging economies. Lin and Ye (2009), using a large sample of developing countries over a long period, find that IT leads to lower inflation. Moreover, they find this effect to be quantitatively large and statistically significant. Overall, the findings suggest that IT helps to reduce inflation in emerging economies, while the results are mixed for advanced economies.

In developing or low-income countries, central banks still predominantly use a monetary aggregate targeting (MT) framework in the conduct of monetary policy (please see Figure 3.1 in Chapter 3 to see the disparities in the conduct of monetary policy between developing and advanced economies). In the third chapter, we conducted our analysis of monetary policy in developing economies by noting two key features: imperfect asset markets, and MT framework. While we have incorporated these two key features in our general equilibrium model in the third chapter, we did not explicitly address why developing countries still follow a MT framework, especially since central banks in advanced and emerging economies have unanimously adopted a monetary policy rule following an IT framework.²

In this chapter, we contribute to the literature by studying the effects of money supply (MSR) and interest rate rules (IRR) in developing economies.³ The lack or low adoption of an IT framework in developing economies means that it is difficult, both analytically and empirically, to study the effects of IT monetary policy rules in a developing economy. To date, Armenia, Albania and Ghana are the only developing economies that have adopted IT. Peter et al. (2011) empirically evaluate the impact of adoption of an IT framework in these economies, and find that an IT framework leads to lower inflation and inflation volatility. However, they conclude that the small number of countries and short time span since its adoption means that we cannot rely on these findings regarding the general effects of IT on other developing economies. Moreover, the econometric approach taken in the study means that the structural features of the economies are not incorporated in the analysis. Hence, in this chapter, we develop a general equilibrium model of a developing economy by incorporating relevant structural features and study the effects of monetary policy shocks on the macroeconomic variables in a developing economy.

In order to model our developing economy, we incorporate two key findings from the second and third chapters. First, we include two sectors in our developing economy following the finding

²There is, ex-ante, no reason to assume that an IT framework works better than a MT framework in a developing economy. However, the shift towards IT frameworks by most advanced and emerging economies, particularly with notable performances in these economies, makes it a worthwhile question to investigate in the context of a developing economy.

³Implicitly, we are assuming that in a MT framework, monetary policy is conducted through a money supply rule, and in an IT framework, monetary policy is conducted through a simple nominal interest rate rule with feedback on the inflation rate.

in the second chapter that, in Rwanda, prices are relatively flexible in the agriculture or food sector, whereas they are sticky in the manufacturing and services sector (for details, please see Table 2.2 in Chapter 2). Second, we assume that households in the flexible-price or food sector, which consists mostly of farmers, hold money but are unable to trade in insurance or borrow to smooth out individual fluctuations.⁴ On the other hand, households in the non-food or sticky-price sector hold money and are also able to trade in insurance and borrow, since these households are more creditworthy and are likely to be able to repay their loans. The assumption of imperfect credit markets is similar to that used in Chapter 3, although the imperfections held for all the households in that model.⁵ The advantage of the model setup in this chapter is that households in the sectors hold money and, in addition, there is an interest rate in the economy through the bond market, which allows us to study both a MT and an IT framework in our developing economy in this chapter.

To evaluate the performance of the monetary policy rules in our economy, we focus on the sectoral effects of the monetary policy rules. There are two reasons why we are motivated to assess the monetary policy rules in terms of their effects on the sectoral variables. First, it is generally difficult to compare the aggregate effects of monetary policy rules, since the MSR and IRR are quite different. The rules incorporate different parameters and are implemented differently, and, therefore, the aggregate results are not expected to provide a helpful comparison of the performance of the rules when looking at monetary policy shocks. Second, and more importantly, we find in the third chapter that a monetary policy shock leads to differential effects on the sectors in the economy. Although the differential effects in the third chapter are due to the staggering of prices, we see in this chapter a different sort of heterogeneity of the sectors, namely in terms of the degree of price stickiness and imperfections in credit markets, which leads to the monetary policy rules to affect the sectoral variables differently. In addition, the high prevalence of income inequality in developing economies, which is well documented in Todaro and Smith (2009), demonstrates the importance of looking at the distributional consequences of monetary policy in developing economies.⁶ Hence, we are primarily interested to evaluate and compare the impact of these rules on the flexible- and sticky-price sectors in the economy, rather than to compare the aggregate effects of the rules. However, it is noteworthy that all the results in this chapter are derived using numerical simulations, unlike in Chapter 3, where we were able to obtain some analytical results. The model in this chapter is more complicated compared to the

⁴It is worth mentioning that in a developing economy, households in rural areas, which constitute primarily of farmers, have particularly poor access to finance, compared to urban areas.

⁵It is important to note that, we use credit and asset markets interchangeably, as in Chapter 3. The imperfections in these markets refer to the absence of borrowing and lending, and the inability to trade in insurance.

⁶In fact, in a recent literature, the sectoral effects of monetary policy shocks are empirically assessed for advanced economies: Coibion et al. (2016) and Furceri et al. (2016) find that contractionary monetary policy leads to higher inequality in the United States; Peersman and Smets (2005) find similar results for the Euro area countries. A thorough review of the sectoral effects of monetary policy is provided in Section 4.2.

model in Chapter 3, and, therefore, we solve the model numerically to compare the impact of the monetary policy rules on the sectors in the economy.

The key finding of the chapter is that, in a developing economy, a money supply rule, in contrast to an interest rate rule, leads to a more even distribution of consumption across the flexible- and sticky-price sectors. Following an expansionary monetary policy shock using a MSR or IRR, the increase in price level in the flexible-price sector is higher than that in the sticky-price sector. This leads to the demand for goods in the sticky-price sector to increase, while the demand in the flexible-price sector slightly falls, as the goods in the sectors are substitutes. Consequently, labour demand in the flexible-price sector is slightly lower, whereas the labour demand in the sticky-price sector is higher. While the price level increases in both the sectors, the differential impact on the demand in the sectors means that the increase in real and nominal wages for the flexible-price sector is much lower than the sticky-price sector. The aforementioned effects are similar across the MSR and IRR, but the main difference lies in the consumption levels of the sectors following a monetary policy shock. Under a MSR, the change in money supply affects both the sectors directly, as money is injected equally through the budget constraints of the households in the sectors. This allows the households in the flexible-price sector, which experience a lower increase in real wages and lower labour demand, to smooth out consumption fluctuations by running down their additional money holdings. On the other hand, an IRR leads households in the sticky-price sector to enjoy higher consumption levels, and while general equilibrium effects imply that households in the flexible-price sector have higher consumption levels as well under an IRR as money is injected into both the sectors, the increase in consumption levels for the flexible-price sector under an IRR is lower in comparison to the increase under a MSR. This is because an expansionary monetary policy under an IRR leads to a direct effect only on the sticky-price sector, where households are able to borrow, and, hence, are positively affected by a decrease in the interest rate, which results in higher consumption levels. A policymaker in a developing economy, particularly interested in social welfare, then can generate more equal outcomes across the sectors under a MSR in comparison to an IRR. Hence, a MT framework, as currently practiced by most developing economies, in terms of distributional consequences, may be more desirable than an IT framework.

The chapter is organised as follows. In Section 4.2, we provide a literature review on MT and IT frameworks and their effects in advanced and developing economies. In Section 4.3, we present our general equilibrium model, which includes the heterogeneous features of the sectors, and the monetary policy shocks that we introduce in our economy. In Section 4.4, we present the quantitative results, with monetary policy shocks introduced under the MSR and IRR, along with some sensitivity results. In addition, we develop a model with flexible- and sticky-price sectors and perfect credit markets and compare with our baseline model, in order to understand the

implications of imperfect credit markets in our developing economy. In Section 4.5, we conclude with a brief discussion on the effects of MT and IT frameworks in developing economies.

4.2 Previous Literature

In this chapter, we are primarily interested to evaluate the effects of an IT framework in a developing economy. Since IT was first adopted in the 1990s, numerous studies have been conducted to understand its implications, both theoretically and empirically. In this section, we review the studies which have evaluated the performance of IT in advanced and emerging economies. In addition, as we are interested in the sectoral effects of MT and IT frameworks, we review the studies on the distributional implications of monetary policy.

In the 1990s and early 2000s, Lars Svensson, in several papers, discussed the theoretical underpinnings of an IT framework. In Svensson (1997a), he examines an IT framework and the problems that may arise with its implementation by the monetary authority. He argues that IT helps to achieve low and stable inflation, as there is a specified quantitative target and higher accountability, both of which increase the likelihood of achieving and maintaining inflation expectations and low inflation targets. However, there are potential issues with IT as well, such as the imperfect control of central banks over inflation. Since current inflation is dependent on previous contracts, central banks can only affect future inflation. Moreover, other factors, such as those which central banks are not in control of, may also affect inflation. Hence, central banks may argue that they cannot be held responsible if the realised inflation is different from the target. This makes the monitoring and evaluation of an IT framework by the public difficult. Despite this, he shows that IT is still a more favourable monetary policy rule compared to a MT or an exchange rate targeting framework, as these are inefficient and lead to worse outcomes than IT. In Svensson (1999a), he further argues that IT can be interpreted as a targeting rule, where an explicit loss function is developed and minimised. Due to the explicit nature of the loss function, he claims that IT is a commitment to a systematic and rational monetary policy in contrast to other monetary policy rules, such as money or nominal GDP growth targeting. Further, in Svensson (1999b), some extensions are incorporated to look at the impact of monetary policy shocks in an IT framework, and uncertainty is introduced in the model and the resulting changes in the economy are investigated. Moreover, other prominent works include Svensson (1997b), where he explores inflation targeting with its relation to commitment and discretion in monetary policy, originally analysed by Kydland and Prescott (1977) and Barro and Gordon (1983). In this paper, he investigates various monetary policy regimes, where the central bank faces a trade-off between controlling inflation and employment. As is well known, this may lead to the problem of ‘time inconsistency’: it is optimal for a central bank to announce a policy and then deviate on

its commitment once the public has made its decision based on the announcement. To study this further, he considers a scenario when the central bank is controlled by the government, which ‘commits’ to an inflation rate in each period. Then, he considers a scenario where the central bank is controlled by the government, but it does not commit to an inflation rate, and instead acts under ‘discretion’. The results yield that, if the implicit employment target exceeds the natural employment rate, there is an ‘inflation bias’ under the discretion regime, which means that the average inflation rate exceeds the socially desirable rate. To reduce the inflation bias, he argues that inflation targeting is a superior solution compared to other solutions such as delegation to weight-conservative central banks, etc.

In Svensson (2000), the analysis is extended to an open economy, where foreign variables such as foreign inflation, output, interest rates and exchange rates are now important for conducting monetary policy. In an open economy, the real exchange rate affects the relative price between domestic and foreign goods, which affect the domestic and foreign demand for domestic goods, and, consequently, affect the transmission mechanism of monetary policy via their effects on aggregate demand. Moreover, the exchange rate directly affects the domestic currency prices of imported final goods (as well as imported intermediate inputs), which enter the consumer price index (CPI) and CPI inflation. In this paper, some key issues for the case of an open economy have been addressed, i) differences between the domestic inflation and CPI inflation cases, where domestic inflation is the domestic component of the CPI, which is less affected by the exchange rate in comparison to CPI inflation, ii) differences between strict and flexible inflation targeting, where in a strict inflation targeting, no weight is placed on output stabilisation, and iii) appropriate monetary policy response to domestic and foreign shocks, which includes a comparison with the Taylor rule. The key findings of the paper are that flexible domestic-inflation targeting stabilises domestic inflation and output gap, whereas strict inflation targeting stabilises domestic inflation further but at the cost of higher variability in output gap. In contrast, strict CPI-inflation targeting leads to higher variability in most of the macroeconomic variables, including the exchange rate, but flexible CPI-inflation targeting leads to relatively lower variability in all variables, particularly the real exchange rate, since there is a component of output stabilisation under this targeting case. Finally, it is found that Taylor rule leads to lower variability in all variables, but with lower capacity to stabilise the variables in comparison to the inflation-targeting cases. Further, it is argued that the Taylor rule does not incorporate all relevant information, such as those not captured by current inflation and output gap. Hence, although it appears to be robust, the Taylor rule is not optimal, especially in an open economy.

While the previous papers provide theoretical and conceptual discussions regarding the adoption of IT, other studies have attempted to empirically evaluate the performance of IT. Ball and Sheridan (2004), in the volume ‘The Inflation-Targeting Debate’ by the National Bureau of

Economic Research, measure the effects of IT on macroeconomic performance in twenty Organization of Economic Development (OECD) countries, with seven that adopted IT in the 1990s and thirteen that did not. The advantage of this selection of these countries is that it allows them to evaluate the performance of the targeters and non-targeters and understand more clearly the implications of adopting an IT framework. Using a ‘difference-in-difference’ approach, they find that average inflation falls by 2.2 percentage points more for the targeters than the non-targeters. However, they argue that this is because the targeters had high initial inflation and that there is regression to the mean. In fact, when they control for this effect, they find that the estimated effect of targeting is much lower with weak statistical significance. In addition, they find that targeting has no effects on inflation variability or inflation persistence as the behaviour of these variables are similar across the targeting and non-targeting countries in the targeting period. Moreover, they find that IT does not lead to any significant effects on output growth and variability. The results from this paper contradict that of Neumann and Von Hagen (2002), who state that IT results in lower inflation levels; however, they do not control for regression to the mean. Moreover, the results here contradict that of Vega and Winkelried (2005) who find, using propensity score matching (PSM) methods, that IT reduces the level and volatility of inflation. In conclusion, Ball and Sheridan (2004) state that their results on inflation targeting should not be interpreted as harming macroeconomic performance, but that there may be other benefits to adopting it, such as more transparent and open policy making.

Lin and Ye (2007), in an attempt to eliminate the self-selection problem of policy adoption, use PSM methods with a comprehensive dataset covering 22 industrial countries, to study the performance of IT. Similar to Ball and Sheridan (2004), seven of these countries had adopted IT during the sample period, and fifteen had not. They use a variety of PSM methods to estimate the average treatment effect for the treated (ATT) and find that the results are strong and robust: the estimated ATTs are quantitatively small and statistically insignificant. Specifically, the evidence suggests that inflation targeting has no significant impact on either inflation or inflation variability, which is in line with Ball and Sheridan (2004). In addition, these results are consistent with more recent work, such as Willard (2012), who finds, using a sample of OECD countries, that inflation variability and uncertainty are unaffected by the adoption of IT. Lin and Ye (2007) conclude that it is possible that the non-targeters are following policies similar to the targeters, even if they do not publicly announce the targets, which may explain why the targeters and non-targeters have similar macroeconomic performances. These authors continue their study of the performances of IT in Lin and Ye (2009); however, in this paper, they focus on emerging and transition economies. It is expected that IT will benefit these economies more as a public announcement of inflation targets will make the central banks in these economies more credible. Using the same method as in Lin and Ye (2007), for a sample of 52 countries

(out of which thirteen had adopted an IT framework), they find strong and robust evidence that the average treatment effects of targeting on inflation and inflation variability are quantitatively large and statistically significant. On average, they find that adoption of IT leads to a reduction in inflation levels by nearly 3 percentage points in targeting countries. In addition, they find that the performance of a given IT regime depends on country factors such as the time since the policy adoption, government's fiscal discipline, etc. The main findings from this study on the effects of inflation targeting on inflation levels and variability are consistent with Gonçalves and Salles (2008), who look at the performance of IT in 36 emerging economies using the method in Ball and Sheridan (2004). However, these results are refuted in Brito and Bystedt (2010), who use a larger sample of countries and periods, and find that the impact of IT on inflation levels and variability is much lower than previously found. A crucial finding of this paper is that inflation targeting countries experienced lower output growth during IT adoption, which is a key aspect of the IT regime which other studies ignored. Overall, these results suggest that it is still not certain if IT leads to strong and positive macroeconomic effects, but that it potentially leads to better performances in emerging economies in comparison to advanced economies.

Another area of research that this chapter is related to is the sectoral effects of monetary policy. Due to the heterogeneity of sectors in an economy, it is imperative to investigate the distributional implications of monetary policy shocks. One of the pioneer works in this area is Carlino and DeFina (1998), who argue that monetary policy shocks lead to regional effects due to the industry composition of the various US states.⁷ They identify two key channels for the transmission mechanism of monetary policy: interest rate and broad credit channel. For example, if a state contains a large share of manufacturing firms, it will be more sensitive to changes in interest rates, as the manufacturing sector has higher interest rate elasticity. On the other hand, as identified earlier by Gertler and Gilchrist (1993), if a state has a higher proportion of small firms relative to large firms, it will be more affected by FED's policy changes as small firms are required to borrow more to meet their credit needs. They use quarterly data for the periods 1958-1992 and estimate a structural vector autoregression (VAR) model and find from impulse response functions (IRFs) that regions respond differently to monetary policy shocks. An investigation of the state-level IRFs shows that manufacturing-intensive states are more responsive to changes in monetary policy shocks than the more industrially diverse states, providing evidence for the interest rate channel. However, there is weak evidence for the credit channel, as they find that states with higher proportion of small firms tend to be more responsive to changes in monetary policy, but this effect is marginally significant.

⁷Bernanke and Gertler (1995) is the first paper that looks at monetary transmission at a disaggregated level. However, the work focuses more on the components of final expenditures, and is not as detailed as other works that followed.

Another paper published around the same time is Ganley and Salmon (1997), who look at the industry effects of monetary policy shocks for 24 sectors of the UK. They introduce an unexpected contractionary monetary policy and estimate IRFs which reveal that aggregate output is depressed in the first four to eight quarters after the shock. In terms of the sectors, they find that the largest responses are in the construction and distribution sectors, followed closely by the manufacturing sector. House purchases are highly interest rate sensitive which may explain the swift reaction in the construction sector. The smallest output contraction is in agriculture, possibly referring to the fact that the output of this sector is primarily staple products, which are not expected to abruptly contract. They also study the credit channel of monetary policy in the UK, and find some evidence that within the sectors, a higher proportion of small firms leads to higher contraction in output. However, data at such a disaggregated level is not available for all the UK industries, and, hence, this result is not conclusive.

Peersman and Smets (2005) investigate the sectoral effects of monetary policy in the Euro area countries, namely Austria, Belgium, France, Germany, The Netherlands, Italy and Spain. They use a similar methodology to other studies in the literature, however, they place a large emphasis on investigating and separating the sectoral effects of monetary policy shocks during booms and recessions. They use data for 74 industries over the periods 1980-1998 and using a VAR model, find a significant degree of asymmetry in booms versus recessions, which shows that an investigation into booms and recessions is worthwhile. In terms of country effects, the overall output effects of the common monetary policy shock do not differ significantly from the average effect in the Euro area. In terms of the industry effects, the overall effects are smallest in the food, beverages and tobacco sector, which is consistent with the finding in Ganley and Salmon (1997) who find the smallest response of monetary policy in agriculture. They attribute this to the fact that demand for food, which is a non-durable good, is not that significantly affected by a rise in the interest rate, in comparison to durable goods, such as investment goods or manufacturing products. In fact, they use a ‘durability dummy’ and find it to be significant, which lends weight to the interest rate channel. In addition, they find that the impact of monetary policy on industries producing durable goods is almost three times as high than the impact on non-durable goods. Moreover, they find some evidence that small firms are more adversely affected due to monetary policy shocks, demonstrating the relevance of the credit channel. However, they are unable to conclude why their results vary over business cycles.

To our knowledge, there are only two studies which look at the sectoral effects of monetary policy in emerging economies. Ibrahim (2005) analyses the sectoral effects of monetary policy shocks for eight sectoral outputs for Malaysia using quarterly data from 1978:1-1999:4. The main findings of the paper are similar to the findings in other studies: in response to a tightening of monetary policy, he finds that manufacturing, construction, finance, insurance, real estate and

business services decline more than the average decline in aggregate production. The effects are much smaller for other sectors such as agriculture, forestry and fishing, mining and quarrying, etc., which show that in Malaysia, the interest rate channel is strong. Alam and Waheed (2006) use quarterly data from 1973:1-2003:4 for Pakistan and investigate the sectoral effects of monetary policy shocks. They investigate these effects for seven key sectors and, similar to other authors, use VAR models to generate sector-level IRFs. Their results are very similar to Ibrahim (2005) and other studies: in response to a positive change in the interest rate, output declines more in manufacturing, finance and insurance sectors in comparison to the agriculture sector. The results from these studies show that the interest rate channel is strong in emerging economies, and, perhaps why the sectoral effects of monetary policy are similar in advanced and emerging economies. It should be noted that the financial institutions in emerging economies are relatively developed, which may explain why the interest rate channel is found to be quite important in these economies.

The paper that is most similar to ours is Anand et al. (2015), which looks at the type of inflation-targeting framework that the central bank of a developing country should adopt. Since this paper concerns only IT regimes, they do not explicitly model money in their economy, and is the crucial difference between their paper and this chapter. They develop a DSGE model with a flexible- and sticky-price sector, and, in addition, a sector which takes the world price as given and exports to foreign countries. The flexible-price sector is credit-constrained, whereas the sticky-price sector, similar to our model, is able to borrow to meet credit needs. They evaluate the welfare under the different policy rules to determine which policy leads to the highest level of lifetime utility, following Schmitt-Grohé and Uribe (2004). They find that headline inflation targeting leads to higher welfare and does better to stabilise output than core inflation targeting. Following a negative food productivity shock, the central bank raises the interest rate, but it only affects the sticky-price sector where the consumption demand now falls. In the flexible-price sector, the food productivity shock leads to an increase in the relative price of food, which increases the wage income and consumption demand of flexible-price sector households. The policy regime determines which of the demand changes dominates: under core inflation targeting (headline or total inflation minus food and energy price inflation) the increase in food prices is higher than under headline inflation targeting, which leads to higher demand for the flexible-price sector and, in the process, offsets the lower demand in the sticky-price sector. This leads to an overall increase in aggregate demand. However, when the central bank targets headline inflation, the price increases in the food sector are lower, leading to a lower increase in consumption in the food sector. Therefore, if the objective of the central bank is to stabilise output, they conclude that headline inflation targeting is a better monetary policy measure.

4.3 The Model

In this section, we set up our model of a developing economy. In comparison to the third chapter, the model in this chapter includes heterogeneous sectors and labour markets. The addition of structural features leads to the drawback that it is now difficult to derive analytical results. However, we numerically simulate the model and obtain quantitative results in the next section, which help us to understand the implications of money supply and interest rate rules in a developing economy.

There are two sectors in the economy, flexible- and sticky-price sectors. The prices of goods in the flexible-price sector adjust frequently, such as in the food sector, as we found in the second chapter. In the sticky-price sector, prices are adjusted slowly, which in the context of the second chapter, represents the non-food sectors in the economy. Moreover, within the sticky-price sectors, there are two sub-sectors, A and B . There is a continuum of flexible-price goods, and a continuum of sticky-price goods.

The model is similar to Aoki (2001) and Anand et al. (2015) with three variations: i) households in this economy have money, which does not exist in either of the aforementioned papers, ii) labour markets are segmented in this economy (more details below), and iii) Taylor (1979) price-staggering is used in this chapter.

4.3.1 Households

The economy, which is divided into the flexible- and sticky-price sectors in equal size, is composed of infinitely-lived households, where $i \in [0, \frac{1}{2}]$, is the continuum of households in the flexible-price sector, and $i \in [\frac{1}{2}, 1]$ is the continuum of households in the sticky-price sector. The two sub-sectors in this sector are equal, so sector A households are $i \in [\frac{1}{2}, \frac{3}{4}]$ and sector B households are $i \in [\frac{3}{4}, 1]$. Households hold equal shares in the firms in their own sectors, and they provide labour to their respective sectors and sub-sectors and it is assumed that labour is immobile across the flexible- and sticky-price sectors, and within the sub-sectors in the sticky-price sector.⁸ This is because of the different skills that are required in the sectors in the economy.

Asset markets are incomplete in this economy: households in the flexible- and sticky-price sectors may have non-identical initial wealth, and households in the flexible-price sector are unable to borrow or lend or trade in insurance, which implies that households in the two sectors also have non-identical consumption plans. Households in the sticky-price sector, which includes the sub-sectors, are able to borrow and lend and trade in insurance, which means that households in the sticky-price sector have identical consumption plans. To capture the sectoral asymmetry, the representative household in a sector, denoted by i , is indexed by F (flexible-price sector) and

⁸We assume segmentation of labour markets in a developing economy following Agénor and Montiel (2008).

S (sticky-price sector), where S can be either A or B . Household i maximises the discounted stream of utility:

$$\max_{\{C_t^i, M_t^i/P_t, N_t^i\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^i)^{1-\sigma}}{1-\sigma} + d \ln(M_t^i/P_t) - \frac{(N_t^i)^{1+\chi}}{1+\chi} \right) \right] \quad (4.1)$$

where σ is the risk aversion parameter, χ is the inverse of the Frisch elasticity of labour supply, and d is the weight placed on real money balances. C_t^i is an index of household i 's consumption of a continuum of flexible-price goods, $C_{F,t}^i$ and sticky-price goods, $C_{S,t}^i$, in period t , defined respectively as:⁹

$$C_t^i = \left[\gamma^{\frac{1}{\zeta}} (C_{F,t}^i)^{1-\frac{1}{\zeta}} + (1-\gamma)^{\frac{1}{\zeta}} (C_{S,t}^i)^{1-\frac{1}{\zeta}} \right]^{\frac{1}{1-\frac{1}{\zeta}}} \quad (4.2)$$

where

$$C_{F,t}^i = \left[\int_0^1 (C_{Fk,t}^i)^{\frac{\epsilon-1}{\epsilon}} dk \right]^{\frac{\epsilon}{\epsilon-1}} \quad (4.3)$$

$$C_{S,t}^i = \left[\int_0^1 (C_{Sj,t}^i)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (4.4)$$

and $\gamma \in [0, 1]$ is the weight on food in the consumption index, $\zeta > 1$ is the elasticity of substitution between flexible- and sticky-price goods, $\epsilon > 1$ is the elasticity of substitution between flexible-price goods and $\theta > 1$ is the elasticity of substitution between sticky-price goods. Moreover, we assume that $\epsilon = \theta$. The aggregate price level, P_t , is defined as:

$$P_t = \left[\gamma (P_{F,t})^{1-\zeta} + (1-\gamma) (P_{S,t})^{1-\zeta} \right]^{\frac{1}{1-\zeta}} \quad (4.5)$$

where $P_{F,t}$ is the Dixit-Stiglitz price index of flexible-price goods, given by:

$$P_{F,t} = \left[\int_0^1 (P_{Fk,t})^{1-\epsilon} dk \right]^{\frac{1}{1-\epsilon}} \quad (4.6)$$

and $P_{S,t}$ is the Dixit-Stiglitz price index of sticky-price goods, given by:

$$P_{S,t} = \left[\int_0^1 (P_{Sj,t})^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (4.7)$$

where $P_{Fk,t}$ and $P_{Sj,t}$ are the prices of flexible- and sticky-price goods indexed on k and j respectively.

⁹A subscript refers to the supply side, whereas a superscript refers to the demand side of the economy.

4.3.1.1 Flexible-Price Sector

Households in the flexible-price sector face the following budget constraint:

$$P_t C_t^F + M_t^F = W_t^F N_t^F + M_{t-1}^F + \Pi_t^F + G_t, \quad t = 0, \dots, \infty \quad (4.8)$$

where W_t^F is the wage set in the flexible-price sector, N_t^F is the labour supply in the sector and Π_t^F is the profits received by the households in the sector. G_t is the lump-sum transfer by the government, which is defined below. This sector is unable to borrow which is reflected in the budget constraint by the absence of bonds. Households maximise (4.1) subject to (4.8), and using the first-order conditions, we can derive the Euler equation, in the absence of bond markets, as:

$$\frac{d}{M_t^F} = \frac{1}{P_t (C_t^F)^\sigma} - \frac{\beta}{P_{t+1} (C_{t+1}^F)^\sigma} \quad (4.9)$$

Similarly, we can derive the labour-consumption trade-off in the flexible-price sector as:

$$\frac{W_t^F}{P_t} = \frac{(N_t^F)^\chi}{(C_t^F)^{-\sigma}} \quad (4.10)$$

The consumption decisions faced by this sector are as follows. Demand for the composite flexible-price good is given by:

$$C_{F,t}^F = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\zeta} C_t^F \quad (4.11)$$

and demand for the composite sticky-price good is given by:

$$C_{S,t}^F = (1 - \gamma) \left(\frac{P_{S,t}}{P_t} \right)^{-\zeta} C_t^F \quad (4.12)$$

4.3.1.2 Sticky-Price Sector

Households in the sticky-price sector face the following budget constraint:

$$P_t C_t^S + M_t^S + Q_t B_t^S = W_{z,t}^S N_{z,t}^S + M_{t-1}^S + B_{t-1}^S + H_t^S + G_t + \Pi_{z,t}^S, \quad t = 0, \dots, \infty \quad (4.13)$$

where $z = A, B$, and $Q_t = \frac{1}{I_t}$. Here, households have access to a bond market and an insurance market, and, hence, are able to borrow or lend or trade in a one-off insurance payment, given by H_t^S , to smooth out consumption fluctuations. $W_{z,t}^S$ and $N_{z,t}^S$ are the wages set and labour demand in the sub-sectors. Households in this sector maximise (4.1) subject to (4.13), which allows us to

derive the Euler equation faced by households in this sector:

$$\frac{1}{I_t} = \beta \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \quad (4.14)$$

Moreover, we can use (4.14) and (4.9) for the sticky-price sector to derive an expression for the money demand function:

$$\frac{M_t^S}{P_t C_t^S} = d \frac{I_t}{I_t - 1} \quad (4.15)$$

We can also use the first-order conditions to derive the equation for the labour-consumption trade-off in this sector, as given below:

$$\frac{W_{z,t}^S}{P_t} = \frac{(N_{z,t}^S)^\chi}{(C_t^S)^{-\sigma}} \quad (4.16)$$

The consumption decisions faced by this sector are as follows. Demand for the composite flexible-price good is given by:

$$C_{F,t}^S = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\zeta} C_t^S \quad (4.17)$$

and demand for the composite sticky-price good is given by:

$$C_{S,t}^S = (1 - \gamma) \left(\frac{P_{S,t}}{P_t} \right)^{-\zeta} C_t^S \quad (4.18)$$

4.3.2 Firms

4.3.2.1 Flexible-Price Sector

Firms in the flexible-price sector use only labour for production:¹⁰

$$Y_{F,t} = (N_t^F)^\alpha \quad (4.19)$$

where α determines the degree of diminishing returns to labour and $0 < \alpha \leq 1$. It is assumed that wages are set in a flexible-manner in the sectors in a developing economy.¹¹ Firms in this sector maximise profits by setting the price of its good at a fixed markup over marginal cost, where the price, $P_{F,t}$, set by firms in this sector is derived as:

$$P_{F,t} = \mu MC_{F,t} \quad (4.20)$$

¹⁰Since there is a continuum of firms in this sector, variables such as $Y_{F,t}$, $N_{F,t}$ should be indexed by k , where k denotes a firm. However, since we assume that the firms are perfectly symmetric, we can drop the 'k' subscript.

¹¹There is limited evidence on how wages are set in a developing economy. However, some evidence provided in Horton et al. (1994), using labour force surveys, show that real wages are set flexibly in developing countries.

where $\mu = \frac{\epsilon}{\epsilon-1}$ is the mark-up over marginal cost, $MC_{F,t}$. We can derive the latter as:

$$MC_{F,t} = \frac{1}{\alpha} \frac{W_t^F}{(N_t^F)^{\alpha-1}} \quad (4.21)$$

The market clearing condition for the composite flexible-price good can be written as:

$$Y_{F,t} = C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\zeta} C_t \quad (4.22)$$

where the measure of households in the flexible- and sticky-price sectors is the same, so we can write:

$$\frac{1}{2} C_t^F + \frac{1}{2} C_t^S = C_t = Y_t \quad (4.23)$$

Hence, in equilibrium, total composite demand equals total supply.

4.3.2.2 Sticky-Price Sector

Firms in the sticky-price sector use labour for production:

$$Y_{S,z,t} = (N_{z,t}^S)^\alpha \quad (4.24)$$

where, as before, $z = A, B$, and refers to the sub-sectors within the sticky-price sector. The marginal cost for the firms can be derived as:

$$MC_{S,z,t} = \frac{1}{\alpha} \frac{W_{z,t}^S}{(N_{z,t}^S)^{\alpha-1}} \quad (4.25)$$

The market clearing condition for the composite sticky-price good can be written as:¹²

$$Y_{S,t} = C_{S,t} = (1 - \gamma) \left(\frac{P_{S,t}}{P_t} \right)^{-\zeta} C_t \quad (4.26)$$

We introduce staggered prices in this sector following Taylor (1979). The two sub-sectors, A and B , are of equal size, with firms in the sub-sectors setting prices in a staggered manner. Firms in a particular sub-sector set prices at time t for t and $t + 1$, which means that prices are fixed for two consecutive periods.

Formally, we assume: sub-sector A , refers to the sub-sector in which firms reset prices in even-numbered periods, and sub-sector B , refers to the sub-sector in which firms reset prices in odd-numbered periods. We define X_t as the ‘reset’ price, that is, the price that firm i would

¹²It is important to note that we are deliberately setting up the sectors such that their outputs, employment levels and prices are the same, in the pre-shock steady state. This is done so we can make a comparison later with an economy with perfect credit markets.

set in period t , if allowed to reset price in that period (and which would also remain in force in period $t + 1$).¹³ It follows then, in even-numbered periods, for instance, at time t : $P_{S,t}^i = P_{S,t+1}^i = X_t$, where $i = A$, and sub-sector B is locked into the price they set one period before so $P_{S,t}^i = P_{S,t-1}^i = X_{t-1}$, where $i = B$. Therefore, $X_t, X_{t+2}, \dots$ are prices fixed by sub-sector A , and $X_{t-1}, X_{t+1}, \dots$ are prices set by sub-sector B . In Section 3.4.2 of Chapter 3, we explain in depth the intertemporal price-setting decision of the sectors, and here, similarly, for the price-setting firm, we can maximise the discounted profit stream subject to its demand function, and derive the reset price as:

$$X_t = \left(\frac{\theta}{\theta - 1} \right) \left(\frac{P_{S,t}^{\theta-\zeta} P_t^{\zeta+1} Y_t MC_{Sj,t}^r + \beta P_{S,t+1}^{\theta-\zeta} P_{t+1}^{\zeta+1} Y_{t+1} MC_{Sj,t+1}^r}{P_{S,t}^{\theta-\zeta} P_t^{\zeta} Y_t + \beta P_{S,t+1}^{\theta-\zeta} P_{t+1}^{\zeta} Y_{t+1}} \right) \quad (4.27)$$

where $MC_{Sj,t}^r = \frac{MC_{Sj,t}}{P_t}$ is the real marginal cost, and $\frac{\theta}{\theta-1}$ is the constant mark-up over marginal cost. Further, the price index in the sticky-price sector can be written as:

$$P_{S,t} = \left(\frac{1}{2} X_t^{1-\theta} + \frac{1}{2} X_{t-1}^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad (4.28)$$

which shows that the price level in the sticky-price sector in the current period is a function of the reset price set in the current period and in the previous period.

4.3.3 Monetary Authority

The central bank in this economy sets a simple Taylor rule of the form:¹⁴

$$\log \left(\frac{I_t}{I} \right) = \varphi_\pi \log \left(\frac{\pi_t}{\pi} \right) + \varphi_y \log \left(\frac{Y_t}{Y} \right) - \log Z_t \quad (4.30)$$

where I , π , and Y are the zero inflation steady state (ZISS) values of nominal interest rate, inflation and output. φ_π and φ_y are the weights on inflation and output assigned by the central bank, and $\log Z_t$ is an exogenous shock term. Instead, the central bank can inject money into the economy with a money supply shock, which is a once-and-for-all injection of money supply. The government budget constraint is given by: $G_t = M_t - M_{t-1}$, which shows that the government equally divides a lump-sum amount among households in the flexible-price sector and the sub-sectors in the sticky-price sector.

¹³Due to the assumption of symmetry, all firms will choose the same reset price, so there is no 'i' subscript.

¹⁴Instead, we can use a Taylor rule of the form used in Christiano et al. (2005) and Anand et al. (2015):

$$\log \left(\frac{I_t}{I} \right) = \varphi_r \log \left(\frac{I_{t-1}}{I} \right) + (1 - \varphi_r) \left(\varphi_\pi \log \left(\frac{\pi_t}{\pi} \right) + \varphi_y \log \left(\frac{Y_t}{Y} \right) \right) - \log Z_t \quad (4.29)$$

where φ_r is an interest rate smoothing parameter. Using this rule, we find that the results are similar across the Taylor rules, and which is included in the sensitivity analysis.

4.3.4 General Equilibrium (GE)

The GE in this model is a sequence of prices $[X_t, P_{F,t}, P_{S,t}]_{t=0}^{\infty}$, and a set of processes and allocations $[C_t^F, C_t^S, M_t^F, M_t^S, N_t^F, N_t^S, W_t^F, W_t^S, Y_{F,t}, Y_{S,t}]_{t=0}^{\infty}$. The log-linearised equations for the macroeconomic and sectoral identities are available in C.1 in the Appendix, where we take a log-linear approximation around the ZISS. In the numerical simulation results, the variables are represented by lower-case letters, which are log-deviations of variables from the ZISS.

4.4 Baseline Results

In this section, we present the baseline results of our numerical simulation. As mentioned earlier, the advantage of our model setup is that we are able to introduce a nominal shock in terms of either money supply or interest rate. This will allow us to understand the effects of the monetary policy rules on the sectoral (and aggregate) variables in our economy.

4.4.1 Calibration

In order to proceed with the numerical simulation, we are required to assume certain parameter values, taking into account that we are setting up a model for a developing economy. d is the weight placed on real money balances and we assume it is 0.002, which is fairly low, which imply that households choose to have higher consumption and hold lower money balances. β , which is the discount factor, is 0.987, which is slightly lower, as the annual real interest rate in developing economies is generally higher, than that in advanced economies. σ , the coefficient of relative risk aversion parameter, is assumed to be 1. We choose log utility in consumption, mainly to keep the model simple.¹⁵ χ , the inverse of the Frisch elasticity of labour supply, is chosen to be 3, following Anand et al. (2015), among many others, and α , which is the degree of diminishing returns to labour, is 0.8. ϵ and θ , the elasticities of substitution between the goods in the flexible- and sticky-price sectors respectively, is 11, a common value that is assumed in the literature. ζ , on the other hand, which is the elasticity of substitution between flexible- and sticky-price goods, is chosen to be 7, which is lower than 11, because we assume that households are not very willing to substitute between food and non-food goods. However, following an expansionary monetary policy shock, it is expected that households will increase their overall consumption of both food and non-food goods.

The parameters relevant for the monetary policy shocks are chosen according to the literature, as that will allow us to compare our results to those found in other studies. For the interest rate rule, we take an approximation of a Taylor rule, so we assume $\varphi_{\pi} = 2$ and $\varphi_y = 0.5$. We include

¹⁵More specifically, the log-linearisations, under imperfect credit markets, are difficult without the simplifying assumption $\sigma = 1$.

an interest rate smoothing parameter in the section on Sensitivity Analysis, which is $\varphi_r = 0.7$. The parameter values are summarised in the following table.

Table 4.1: Benchmark Calibration

Parameter	Definition	Value
d	Weight on real money balances	0.002
β	Discount factor	0.987
σ	Coefficient of relative risk aversion	1
χ	Inverse of the Frisch elasticity of labour supply	3
α	Returns to labour	0.8
ϵ	Elasticity of substitution between flexible-price goods	11
θ	Elasticity of substitution between sticky-price goods	11
ζ	Elasticity of substitution between flexible- and sticky-price goods	7
φ_r	Interest rate smoothing parameter	0.7
φ_π	Weight on inflation in Taylor Rule	2
φ_y	Weight on output in Taylor Rule	0.5

4.4.2 Numerical Simulation

The simulations are performed in Dynare, after the structural equations are log-linearised.¹⁶ For a money supply rule, we inject the same level of money to the flexible- and sticky-price sectors, which is a step-change in aggregate money supply, after which it stays at this level perpetually. For an interest rate rule, we follow a Taylor rule of the form in (4.30). The effect of the shock on the interest rate is then transmitted to the sectors.

4.4.2.1 Money Supply Rule

We first look at the numerical simulation results following a 5% increase in money supply, which is a once-and-for-all injection of money into the economy. We find that an expansionary monetary policy leads to an increase in the aggregate price level, but the price level in the flexible-price sector rises by more than that in the sticky-price sector, as seen in Figure 4.1. Households now have higher money balances, so the overall demand in the sectors is higher. However, as the prices of food are flexible and increase by more than in the sticky-price sector, households slightly decrease their demand for food, as seen in Figure 4.2. Conversely, households increase their demand for goods in the sticky-price sector.

¹⁶The simulations are conducted for 1000 periods, and then the results of 30 periods are reported.

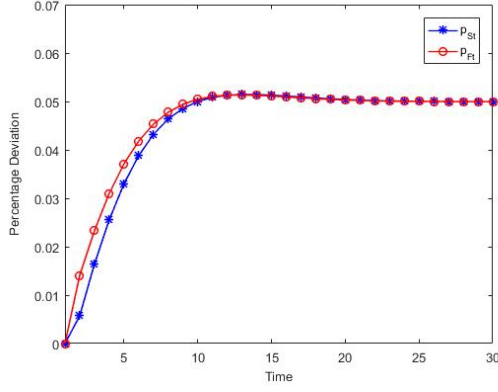


Figure 4.1: Sectoral Price Levels: MSR

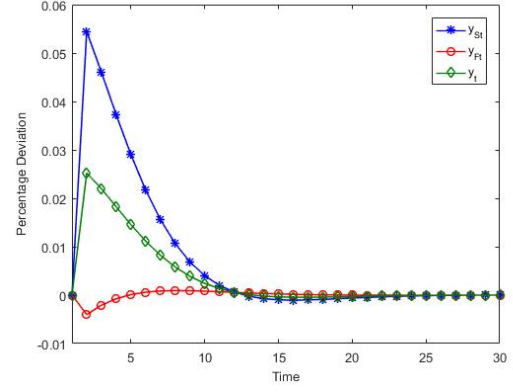


Figure 4.2: Sectoral Output Levels: MSR

The higher demand and price level lead to an increase in sectoral wages. However, as the demand for goods in the sticky-price sectors is now higher in comparison to the demand for food in the flexible-price sector, we see in Figure 4.3 that the real wage (which is deflated by the aggregate price level) is higher in the sticky-price sector than that in the flexible-price sector. The sectoral labour demand, in Figure 4.4, is determined by the time paths of the output levels: more labour is demanded in the sticky-price sector, whereas the labour demand slightly falls in the flexible-price sector.

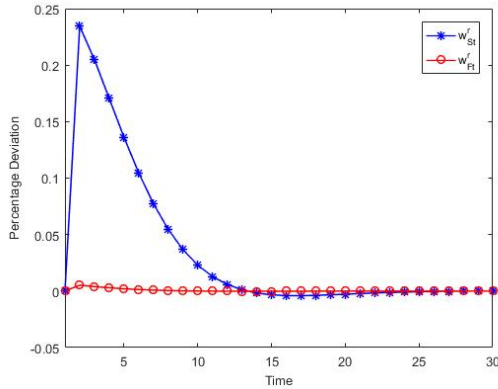


Figure 4.3: Sectoral Real Wage: MSR

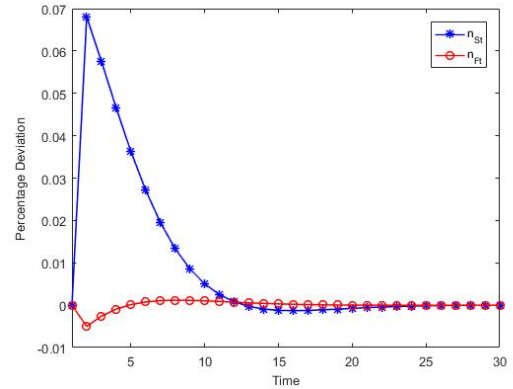


Figure 4.4: Sectoral Labour Demand: MSR

The sectoral effects on the consumption levels are presented in Figure 4.5.¹⁷ The higher price level in the flexible-price sector results in a lower increase in sectoral real wage (in comparison to the sticky-price sector) and lower labour demand and output levels. However, the consumption level of the flexible-price sector still increases following a money supply shock, although not as much as in the sticky-price sector. For $d = 0.002$, we see in Figure 4.6 that households in the flexible-price sector smooth out their consumption fluctuations by running down their money

¹⁷The consumption levels presented in Figure 4.5 are a composite of food and non-food goods for households in the flexible- and sticky-price sectors.

holdings. For higher values of d , flexible-price households are not required to run down their money holdings. However, they still have higher consumption levels (in comparison to pre-shock level) and money balances, but lower than the consumption levels and money balances in the sticky-price sector. This is explained in details in the Sensitivity Analysis below.

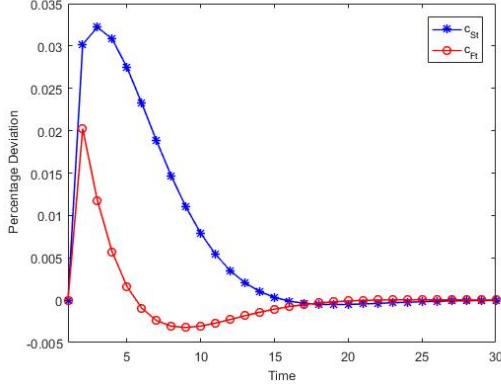


Figure 4.5: Sectoral Consumption: MSR

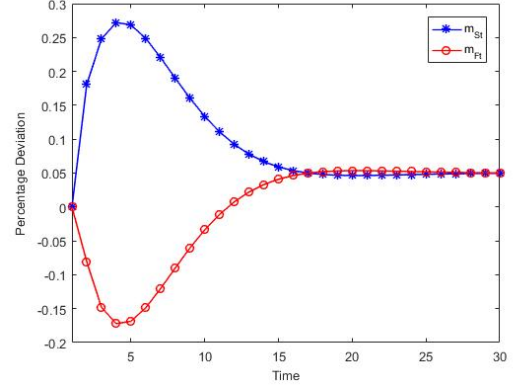
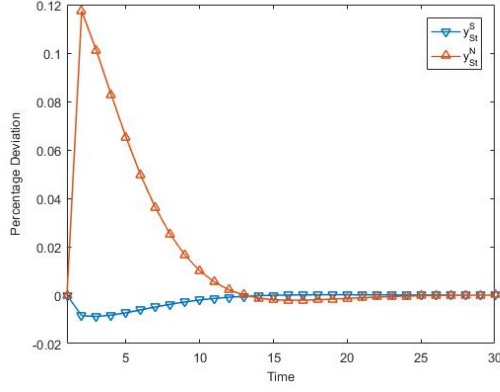


Figure 4.6: Sectoral Money Holdings: MSR

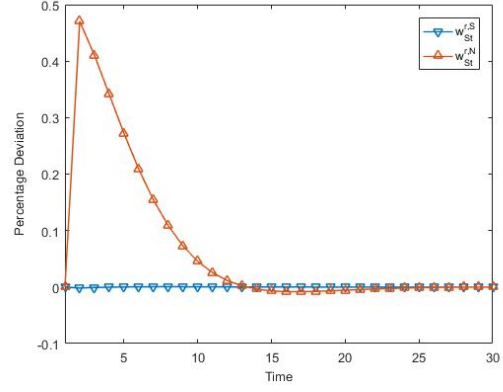
Within the sticky-price sector, the effects on sub-sectoral output levels, labour demand and real wage are presented in Figure 4.7.¹⁸ These results are similar to our findings in the third chapter, with the exception that consumption levels and money holdings are assumed to be the same across the sub-sectors as they can borrow and lend and trade in insurance to smooth out sub-sectoral fluctuations. Following a monetary policy shock, the non price-setting sector has higher output levels, real wage and labour demand (not reported), whereas the output levels and labour demand fall in the price-setting sector. Real wage in the non price-setting sector increases, with a marginal increase in the price-setting sector following a shock.¹⁹

¹⁸It is worth mentioning that, as in Chapter 3, these variables are ‘oscillating’ between the two sub-sectors due to the price staggering. In Figure 4.7, we do not observe these oscillations because the graphs here are for the price- (blue inverted-triangle line) and non-price-setting sectors (orange triangle line) in any period, whose identities alternate between periods (see definition in Chapter 3).

¹⁹The simulation results of the sub-sectors within the sticky-price sector following a monetary policy shock are identical across the MSR and IRR, so we do not present these results under an IRR.



Output Levels



Real Wage

Figure 4.7: Effects on Sticky-Price Sectors

4.4.2.2 Interest Rate Rule

For the IRR, we introduce a monetary policy shock by a temporary 5% decrease in the interest rate, a graph of which is provided in Figure C.1.²⁰ Unlike the money supply shock, the interest rate only directly affects the households in the sticky-price sector, as seen in (4.14). A Taylor rule shock will still affect the flexible-price sector, but the effect will be indirect, and will be through an induced change in money supply corresponding to a change in interest rate.

Following a monetary policy shock, we observe less persistence in the time paths of the sectoral variables under an IRR compared to a MSR. In terms of the dynamics under an IRR, of the form used in (4.30), which does not include an interest rate smoothing parameter, the aggregate and sectoral variables converge quite rapidly to the steady state as we see in Figures 4.8-4.13. Moreover, under a Taylor rule, we find a negative stable eigenvalue, but the ‘oscillations’ are very small and not readily observed in the time paths of the aggregate and sectoral variables.

In response to a monetary policy shock under an IRR, the price level increases more in the flexible-price sector than in the sticky-price sector (in fact, it overshoots under an IRR), which leads to marginally lower output levels in the flexible-price sector and higher output levels in the sticky-price sector. The sectoral wage increases more in the sticky-price sector compared to the flexible-price sector, whereas labour demand increases in the sticky-price sector and falls marginally in the flexible-price sector. The low elasticity between flexible- and sticky-price goods leads to the marginal decrease in the flexible-price sector, as households are unwilling to shift away from food in the flexible-price sector.

²⁰More specifically, we temporarily decrease interest rate by 5% of its pre-shock level, where the latter was 1.3% (quarter-on-quarter). Hence, the shock is to cut this quarterly rate to 1.235%, i.e. to cut it by 6.5 ‘basis points’.

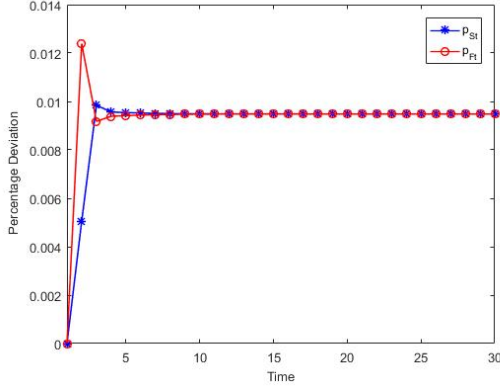


Figure 4.8: Sectoral Price Levels: IRR

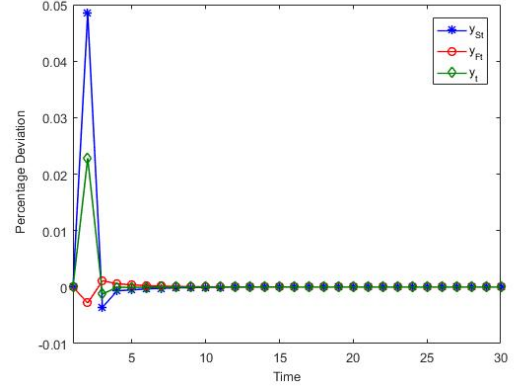


Figure 4.9: Sectoral Output Levels: IRR

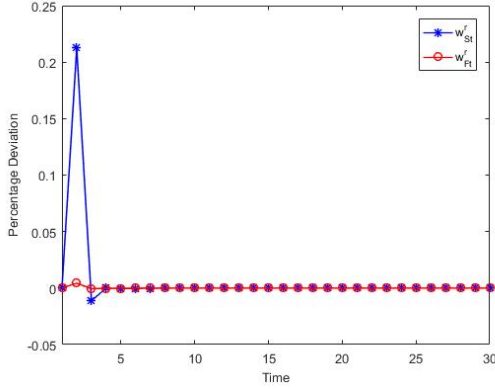


Figure 4.10: Sectoral Real Wage: IRR

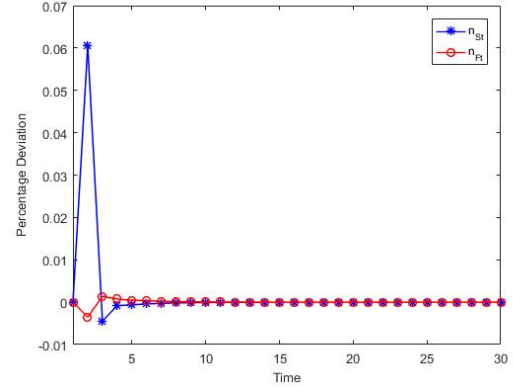


Figure 4.11: Sectoral Labour Demand: IRR

The effects of monetary policy shocks on the consumption levels are provided in Figure 4.14.²¹ It is evident from Figure 4.14 that IRR leads to a lower increase in consumption levels in the impact period for the flexible-price sector (as a proportion of the increase in the sticky-price sector), in comparison to a MSR. Therefore, although we are not able to draw a useful conclusion regarding the relative aggregate effects of the shocks, due to the shocks affecting the economy differently, we can see that MSR leads to a relatively more even impact on the sectors, in comparison to the IRR. In fact, the numerical series confirms this result: in the impact period, the increase in the consumption level of the flexible-price sector is 48.62% of the increase in the consumption level of the sticky-price sector under an IRR, whereas it is 67.03% under a MSR.²² As mentioned earlier, this is due to the fact that interest rate affects the sticky-price sector directly, whereas it affects the flexible-price sector only indirectly through general equilibrium effects.²³ The implication of

²¹The consumption levels under a MSR is replicated to show the differential impact on consumption under different monetary policy rules.

²²It is worth mentioning that the more even impact under the MSR is observed in the impact period, as well as the transition paths. From $t = 2$ onwards, C_t^F and C_t^S have opposite signs under the IRR, whereas, although the same eventually also happens under the MSR, they keep the same sign until $t = 6$.

²³The sectoral money holdings under an IRR show that corresponding to a change in the interest rate, there are

this is that an IRR leads to more uneven consumption levels between the sectors, in comparison with a MSR.

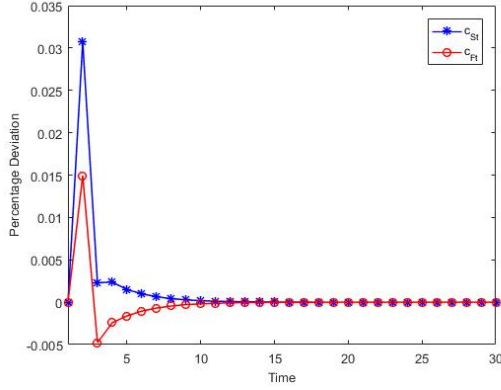


Figure 4.12: Sectoral Consumption: IRR

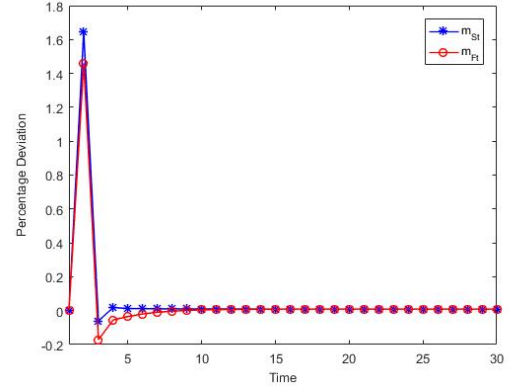
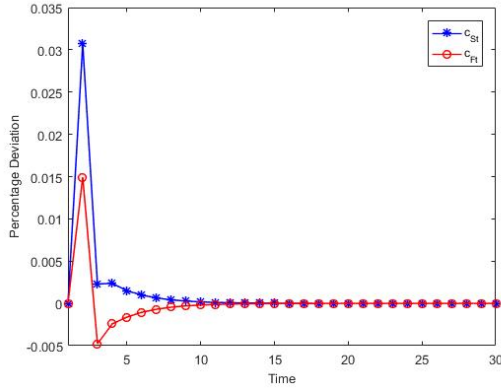
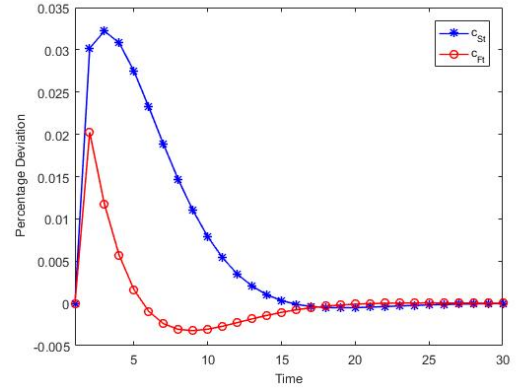


Figure 4.13: Sectoral Money Holdings: IRR



Sectoral Consumption Levels: IRR



Sectoral Consumption Levels: MSR

Figure 4.14: Effects on Sectoral Consumption Levels: IRR and MSR

The findings in this section highlight an important point: IRR may lead to a worse outcome than MSR, if a policymaker in a developing economy is interested in social welfare. MSR is able to directly affect the flexible- and sticky-price sectors in an economy, whereas an IRR is only able to directly affect the sticky-price sector. An IRR leads to a more unequal impact on the consumption levels of the sectors. We have noted that we are unable to comment on the relative aggregate effects of the monetary policy rules in a meaningful way given that the monetary policy rules are very different. It is possible that an IRR may lead to better aggregate outcomes in comparison to a MSR, but the (more) uneven sectoral effects of the IRR will still hold, in that case.

monetary injections into the flexible- and sticky-price sectors. As expected, following an expansionary monetary policy, we find that the money holdings of the households in the sticky-price sector increase by more than those in the flexible-price sector, as in Figure 4.13.

4.4.3 Sensitivity Analysis

In this subsection, we conduct some sensitivity analysis to see whether our simulation results are robust. In general, we find that our main findings are robust across different parameter values and monetary policy rules. We separate and present our sensitivity analysis in two parts. First, we make changes to parameter values and report the changes in sectoral output and consumption levels, as these are our primary variables of interest. Second, we use other money supply and interest rate rules and see if they result in any changes in our time paths for sectoral consumption and output levels.²⁴

(i) *Changes in Parameter Values*

We make changes to all the parameters in our model, but we only report the results for the parameters which lead to substantial changes in the time paths of the variables. d , which is the weight on real money balances, is an important parameter, as it is used to indicate households' preferences between consumption, real money balances and leisure. If we increase d to 0.01, we see in Figure C.2 that, following a money supply shock, households in the flexible-price sector run down their money holdings less now, in regards to the magnitude, in comparison to the baseline scenario. Households place a higher emphasis on holding on to money balances, and, in fact, households in the flexible-price sector now increase their money holdings compared to their pre-shock levels in the initial period before running down their money holdings. As households are interested, in general, to smooth out consumption fluctuations, an increase in money supply leads to higher consumption levels for both sectors. It is interesting to note that, under $d = 0.01$, the consumption levels of the sectors are now more equal than in the baseline scenario. In fact, the consumption levels are almost identical in the impact period, followed by a drop for the flexible-price sector and a gradual decrease to the steady state for the sticky-price sector. A change in d does not lead to significant changes in the response of sectoral output levels.

ζ is the elasticity of substitution between goods in the flexible- and sticky-price sectors. It is assumed that ζ is quite low since households are not very willing to substitute between food and non-food goods, as these goods are not close substitutes. If we decrease ζ to 3, we see under an IRR and MSR, that output of the flexible-price sector falls less now. These results are presented in Figure C.3. The simulation results for consumption levels are presented in Figure C.4: a change in ζ does not result in significant changes in the sectoral consumption levels under an IRR, but the consumption levels are more equal under a MSR.

²⁴The simulation results for the sensitivity analysis are presented in C.2 in the Appendix.

(ii) *Changes in Monetary Policy Rules*

We use some variants of our monetary policy rules to see if the results are similar across different money supply and interest rate rules. We introduce a persistence parameter for the money supply rule. Using the notation from our baseline model, we can write:

$$\log \left(\frac{M_t}{M} \right) = \varphi_m \log \left(\frac{M_{t-1}}{M} \right) + \log \varepsilon_t^m \quad (4.31)$$

ε_t^m is an exogenous shock term, where $\varphi_m = 0.6$ is a persistence parameter with our MSR. As we see in Figure C.5, with a persistence parameter, the time paths for the consumption and output levels are very similar to our baseline scenario.

For the interest rate rules, we use two variants. The first change we make is similar to the change we make to our money supply rule, where we introduce a persistence parameter. Following from (4.30), it means that Z_t follows an AR(1) process of the form:

$$\log \left(\frac{Z_t}{Z} \right) = \varphi_r \log \left(\frac{Z_{t-1}}{Z} \right) + \log \varepsilon_t^r \quad (4.32)$$

where ε_t^r is the exogenous shock term now, with a persistence parameter given by φ_r . The simulation results are presented in Figure C.6, where the time paths for the consumption and output levels are now more persistent, under an IRR due to φ_r .

For a second variant of IRR, we use a Taylor rule of the form:

$$\log \left(\frac{I_t}{I} \right) = \varphi_r \log \left(\frac{I_{t-1}}{I} \right) + (1 - \varphi_r) \left(\varphi_\pi \log \left(\frac{\pi_t}{\pi} \right) + \varphi_y \log \left(\frac{Y_t}{Y} \right) \right) - \log Z_t^{TR} \quad (4.33)$$

where Z_t^{TR} is now the exogenous shock term. Under this Taylor rule, we see the effect on the sectoral consumption levels. As we see in Figure C.7, the increase in consumption levels of the flexible-price sector is lower compared to the increase in consumption levels for the sticky-price sector, similar to our finding under the Taylor rule without the interest rate smoothing parameter. The difference in sectoral consumption levels are slightly lower now in comparison with the IRR with no smoothing parameter: in the impact period, the increase in the consumption level of the flexible-price sector is 53.49% of the increase in the consumption level of the sticky-price sector, whereas it is 67.03% under a MSR. Moreover, as expected, we find that the consumption levels of the sectors are more persistent under this Taylor rule, due to the exogenous persistence induced by the interest rate smoothing parameter, φ_r . It is worth noting that a persistence parameter seems to lessen the inequality in consumption levels between the sectors. This can perhaps be due to the fact that money holdings are able to compensate better for the imperfections in the credit markets, when the macroeconomic effects are more persistent.

4.4.4 Comparisons: Imperfect Credit Markets and Perfect Credit Markets

As noted earlier, one of the key features in a developing economy is the imperfections that exist in the credit markets. In this subsection, we develop an economy where, similarly to in our baseline model, there are flexible- and sticky-price sectors, but we assume that the credit markets in this economy are perfect. We compare the key results of this economy with our baseline model, which helps us to isolate and highlight the role played by the imperfections in the credit markets.

The complete model of an economy with perfect credit markets (PCM) is available in C.3 in the Appendix, but the key distinction between economies with imperfect credit markets (ICM) and PCM is that households in both the flexible- and sticky-price sectors under PCM have access to bond markets and can trade in insurance, which means that they have similar budget constraint:

$$P_t C_t^i + M_t^i + Q_t B_t^i = W_{z,t}^i N_{z,t}^i + B_{t-1}^i + M_{t-1}^i + G_t + H_t^i + \Pi_t^i, \quad t = 0 \dots, \infty \quad (4.34)$$

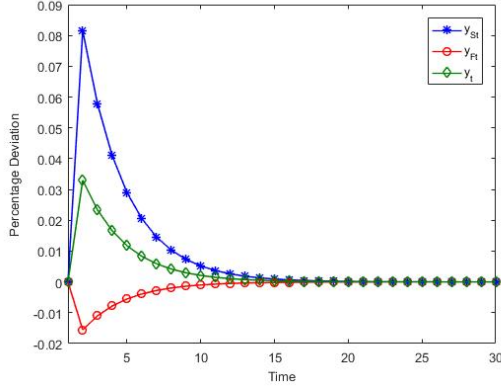
Under PCM, sectoral consumption levels and money holdings are equal, as households in the sectors can borrow or lend or trade in insurance to smooth out individual fluctuations. We assume that, in the pre-shock steady state, the consumption levels of the households in the flexible- and sticky-price sectors are equal. Then, the households will agree on an insurance arrangement which ensures that the consumption levels remain equal after an unexpected monetary policy shock.²⁵ Assuming equal consumption levels then, we can derive the Euler equation, which similarly holds for both the sectors:

$$\frac{1}{I_t} = \beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \quad (4.35)$$

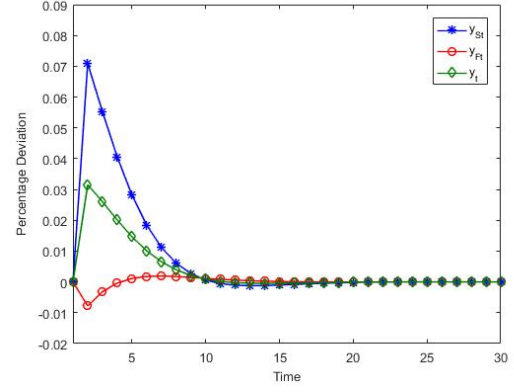
The numerical simulation results under PCM are provided below. These results are generated using the same parameter values as under ICM. The results are similar across the MSR and IRR so we provide comparisons for ICM and PCM only under the MSR.

Figure 4.15 shows that the IRFs following a monetary policy shock under PCM and ICM are similar: the price level rises further in the flexible-price sector in comparison to the sticky-price sector, leading to the output in the flexible-price sector to fall, whereas output in the sticky-price sector increases. It can be seen in the graphs that output in the sticky-price sector increases by slightly more, and the output in the flexible-price sector decreases by slightly more, under PCM in comparison to ICM. This shows that the differences in output between the sectors are higher under PCM than under ICM. As we see in Figure 4.16, the numerical simulation results for the sectoral labour supply are similar to those for sectoral output levels.

²⁵This requires that we assume that the utility functions and the wealth and income levels across the households in the sectors are equal. To ensure this, we assume that the monopoly power between the sectors is equal. These assumptions to equate sectoral consumption levels are quite strong, but we make them as these allow us to set up an economy with perfect credit markets and make a comparison with imperfect credit markets.

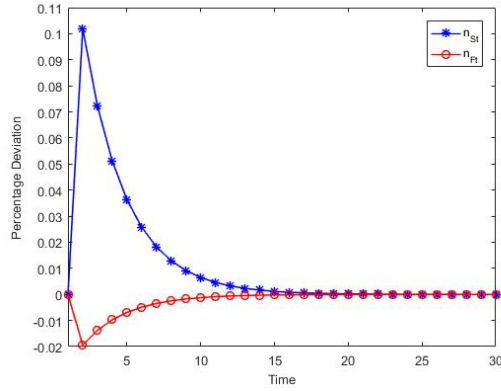


Perfect Credit Markets

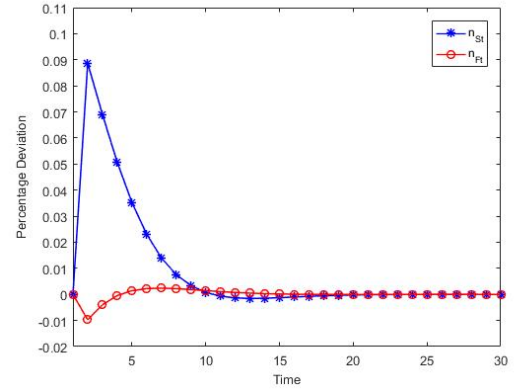


Imperfect Credit Markets

Figure 4.15: Sectoral Output Levels: PCM and ICM



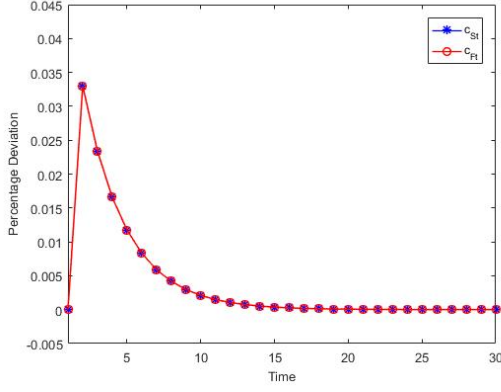
Perfect Credit Markets



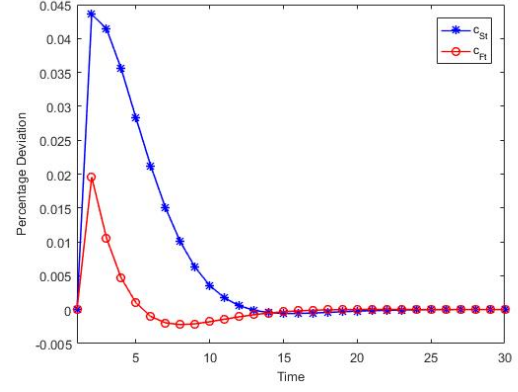
Imperfect Credit Markets

Figure 4.16: Sectoral Labour Supply: PCM and ICM

A comparison of the sectoral consumption levels and money holdings shows the key differences under ICM and PCM. When credit markets are perfect, households in the flexible-price sector are able to borrow or lend or trade in insurance to equate the consumption levels and money holdings with the sticky-price sector, as in the left-hand panel of Figures 4.17 and 4.18. In comparison, under ICM, the consumption levels, following an expansionary monetary policy shock, increase less for the flexible-price sector in comparison to the sticky-price sector, as in the right-hand panel of Figure 4.17. In Figure 4.18, we see the key role played by money holdings under ICM: households in the flexible-price sector run down their money holdings to smooth out consumption fluctuations, whereas under PCM, money holdings are the same in both the sectors (these are equal, by construction).

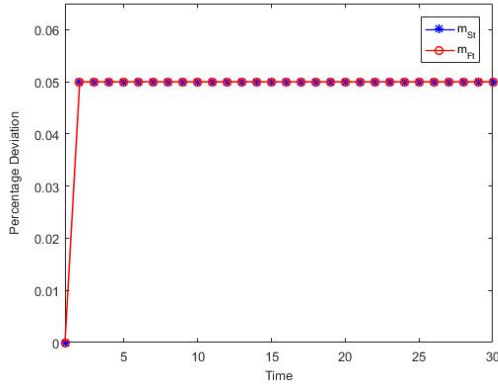


Perfect Credit Markets

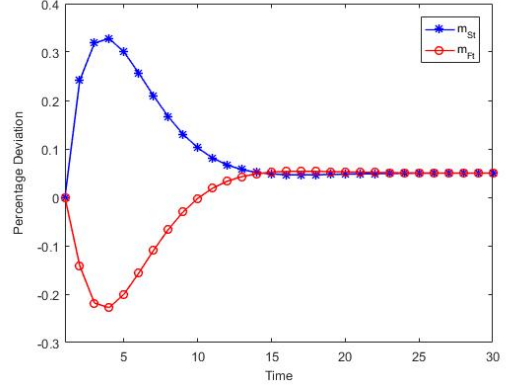


Imperfect Credit Markets

Figure 4.17: Sectoral Consumption Levels: PCM and ICM



Perfect Credit Markets



Imperfect Credit Markets

Figure 4.18: Sectoral Money Holdings: PCM and ICM

As we highlighted before, the differences in output between the sectors are higher under PCM than under ICM. Intuitively, this may due to the income effects on labour supply. As we see in Figures 4.16 and 4.17, under PCM, the labour supply in the flexible-price sector falls slightly more, with higher consumption levels, than under ICM, which leads to lower output for this sector under PCM.²⁶ On the other hand, the labour supply for the sticky-price sector is higher, with lower consumption levels, than under ICM, which leads to higher output for this sector. Overall, we can conclude that different credit market conditions across sectors do not result in any major differences in sectors' output responses to monetary policy.

²⁶The labour-consumption trade-off for the sectors are given by the FOCs, (4.10) and (4.16).

4.5 Conclusion

In this chapter, we studied the sectoral effects of monetary policy in a developing economy. We modelled a developing economy with a flexible- and sticky-price sector, where the flexible-price sector, which is comprised of farmers and who produce food, held money but had no access to bonds or insurance. On the other hand, the sticky-price sector, which referred to the non-food sectors, held money and, in addition, had access to bonds and insurance. We then introduced monetary policy shocks either through a money supply rule or an interest rate rule and studied the effects of the shocks on the households in the flexible- and sticky-price sectors.

In response to an expansionary monetary policy shock, either through the money supply or the interest rate rule, we found that the price level in the flexible-price sector increases more than that in the sticky-price sector. Since food and non-food goods are substitutes, the demand for food decreases while the demand for non-food goods increases, although the decrease in the demand for food is small as the demand for food is relatively inelastic. This leads to the output levels and labour demand to slightly fall in the flexible-price sector while these increase in the sticky-price sector. The consumption levels increase in both the sectors following an expansionary monetary policy; however, the increase is higher in the sticky-price sector in comparison to the increase in the flexible-price sector. While these effects are similar across the monetary policy rules, the key difference lies in the effects on consumption levels and money holdings in the sectors. When a monetary policy shock is introduced through a money supply rule, it affects both the sectors directly as money is equally injected through the budget constraints of the households in the sectors. However, when a shock is introduced through a change in the interest rate, it affects the sticky-price sector directly as only households in this sector have access to bonds, and, hence, are affected by a change in the interest rate. Households in the flexible-price sector are affected as well indirectly through general equilibrium effects, as corresponding to a change in interest rate, there is an induced change in money supply, leading to monetary injections for both the sectors. Hence, a shock via a money supply rule leads to a more equal distribution of consumption levels between the sectors, whereas, an interest rate rule affects the sticky-price sector directly and the flexible-price sector only indirectly, resulting in a larger difference in consumption levels between the sectors. Therefore, a policymaker or central bank interested in social welfare can achieve a more equal outcome through a money supply rule in comparison to an interest rate rule in a developing economy.

The central banks of developing economies still predominantly use money supply rules to affect macroeconomic outcomes. The findings of this chapter highlight that a money supply rule may be a better policy tool as it leads to a more equal distribution of consumption across the sectors in a developing economy, in comparison to an interest rate rule. However, there are other

benefits of an inflation targeting framework, such as higher accountability and transparency of inflation targets, which can be adopted and which can greatly influence the effective functioning and monitoring of central banks in developing economies.

Chapter 5

Conclusion

The essays in this thesis are an attempt to understand the mechanics of monetary policy in developing economies, particularly low-income countries. Although many studies exist for how monetary policy is conducted in advanced and, more recently, emerging economies, very few studies exist on monetary policy in developing economies. As these economies are structurally quite different, it might be expected that the mechanics of monetary policy in these economies would be dissimilar from advanced and emerging economies. There are several aspects that are important for the transmission mechanism of monetary policy, but in this thesis, we emphasise the implications of price stickiness for monetary policy in developing economies, due to its crucial role in rendering money to be non-neutral.

In the second chapter, we began our study of price stickiness with an empirical assessment of price stickiness in developing economies. We used a disaggregated monthly level price dataset from Rwanda for the period January 2013-December 2014, to determine the degree of price stickiness in developing economies, such as Rwanda. We found that prices, on average, are sticky in Rwanda, where prices are more sticky in the non-food sector, such as clothing, and flexible in the food sector, such as vegetables. Hence, price stickiness, in general, is observed, and it is important to understand its implications for monetary policy in developing economies.

In the third chapter, we continued our study of price stickiness and monetary policy in developing economies, by noting that a key difference in the effects of monetary policy is due to imperfections that exist in the credit markets in these economies. This is likely to affect the price-setting mechanism, as it involves an intertemporal decision, under staggered prices. Thus, to study the price-setting mechanism and its role on monetary policy, we developed a general equilibrium model of a developing economy with money but without bonds and insurance. We found that, following a monetary policy shock, aggregate output may pass through an oscillatory phase on the path towards the steady state, although the magnitudes of these oscillations are small. This demonstrates that while imperfect credit markets (ICM) create fluctuations in

aggregate output in the transition path, these are not expected to cause an issue from a policy perspective. However, we found notable heterogeneous effects in the behaviour of sectoral variables under ICM, which lead to a persistent asymmetry between the sectors. To investigate this further, we conducted numerical welfare calculations where we found that the price-setting sector in the impact period derives higher lifetime utility than the non price-setting sector. Moreover, we conducted a similar welfare analysis for an economy with perfect credit markets (PCM), and found that this result holds for PCM as well. A comparison of the sectoral welfare gains under ICM and PCM led us to a crucial finding: despite the greater intersectoral heterogeneity in the time paths of the variables which directly affect utility, there is greater intersectoral homogeneity of lifetime utility changes under ICM than PCM. We proceeded to suggest that an implication of this result is that a monetary authority concerned about equality can be more vigorous in its monetary policy actions in a developing economy in comparison to an advanced economy.

In the fourth chapter, we studied why many developing economies still conduct monetary policy using a monetary aggregate targeting (MT) framework, such as a money supply rule, whereas most advanced and emerging economies follow an inflation targeting (IT) framework, such as an interest rate rule. We developed a slightly different model of a developing economy in this chapter as we require an interest rate in the economy, which did not exist in Chapter 3. We included two sectors in this model: a flexible-price sector, where households held money but had no access to bonds or insurance, and a sticky-price sector, where households held money and also had access to bonds or insurance, through which we had an interest rate in the economy. The main finding in this chapter was that, following a monetary policy shock, a money supply rule leads to a more even distribution of consumption across the sectors, as a monetary policy shock through money supply leads to a direct effect on both the flexible- and sticky-price sectors. In comparison, an interest rate rule leads to a direct effect only on the sticky-price sector as households in the flexible-price sector do not have access to bonds, and, hence, are not directly affected by a change in interest rates. Therefore, we concluded that, in terms of distributional implications, a MT framework may be more desirable in a developing economy in comparison to an IT framework.

The essays in this thesis aim to provide a better understanding of monetary policy in developing economies. In this thesis, we have kept the models simple, as our aim has been to provide a clear explanation of the different forces at work in these economies. However, there are other aspects as well, which may be incorporated in a model of a developing economy, such as informal sectors, etc., which are important for the transmission mechanism of monetary policy. A policymaker interested to develop a more realistic model of a developing economy may incorporate these features and then study their implications for monetary policy. In addition, it may be worthwhile to use these models to look at optimal monetary policy in developing economies.

Appendix A

Appendix to Chapter 2

A.1 Supplementary Tables and Figures

		Frequency (% per month)		Duration (months)		
Item Category	N	Mean	Median	Median	Weights	CDF
Bread and cereals (ND)	47	28.68%	26.09%	3.31	6.33	0.0736
Meat (ND)	28	33.39%	17.39%	5.23	1.83	0.0213
Milk, cheese and eggs (ND)	13	19.40%	21.74%	4.08	1.4	0.0163
Oils and fats (ND)	19	21.97%	17.39%	5.23	2.1	0.0244
Fruit (ND)	27	87.76%	100.00%		5.83	0.0678
Vegetables (ND)	68	91.82%	100.00%		22.77	0.2646
Sugar, jam, honey, chocolate and confectionery (ND)	21	10.77%	0.00%		0.93	0.0108
Food products n.e.c. (ND)	21	40.79%	43.48%	1.75	0.96	0.0112
Coffee, tea and cocoa (ND)	1	13.04%	13.04%	7.16	0.07	0.0008
Mineral waters, soft drinks, fruit and vegetable juices (ND)	15	7.25%	0.00%		1.01	0.0117
Spirits (ND)	1	8.70%	8.70%	10.99	0.09	0.0010
Beer (ND)	24	7.61%	4.35%	22.50	2.44	0.0284
Tobacco (ND)	2	8.70%	8.70%	10.99	1.14	0.0132
Clothing materials (SD)	11	17.00%	13.04%	7.16	0.19	0.0022
Garments (SD)	71	35.64%	39.13%	2.01	2.39	0.0278
Other articles of clothing and clothing accessories (SD)	19	14.42%	8.70%	10.99	0.57	0.0066

Cleaning, repair and hire of clothing (S)	17	4.86%	0.00%		0.33	0.0038
Shoes and other footwear (SD)	12	44.57%	39.13%	2.01	1.04	0.0121
Repair and hire of footwear (S)	10	21.30%	28.26%	3.01	0.07	0.0008
Actual rentals paid by tenants (S)	1	13.04%	13.04%	7.16	1.36	0.0158
Materials for the maintenance and repair of the dwelling (ND)	38	27.23%	26.09%	3.31	2.02	0.0235
Services for the maintenance and repair of the dwelling (S)	3	0.00%	0.00%		2.29	0.0266
Solid fuels (ND)	2	45.65%	45.65%	1.64	6.16	0.0716
Household textiles (SD)	2	39.13%	39.13%	2.01	0.26	0.0030
Major household appliances whether electric or not (D)	4	57.61%	54.35%	1.28	0.12	0.0014
Repair of household appliances (S)	1	4.35%	4.35%	22.50	0	0.0000
Glassware, tableware and household utensils (SD)	25	20.52%	8.70%	10.99	0.13	0.0015
Small tools and miscellaneous accessories (SD)	1	0.00%	0.00%		0.35	0.0041
Non-durable household goods (ND)	22	10.87%	4.35%	22.50	1.16	0.0135
Pharmaceutical products (ND)	32	15.90%	4.35%	22.50	0.24	0.0028
Other medical products (ND)	9	16.43%	13.04%	7.16	0.01	0.0001
Medical services (S)	159	3.31%	4.35%	22.50	0.19	0.0022
Paramedical services (S)	40	3.70%	4.35%	22.50	0.06	0.0007
Hospital services (S)	20	2.61%	4.35%	22.50	0.12	0.0014
Motor cars (D)	2	28.26%	28.26%	3.01	2.72	0.0316
Motor cycles (D)	1	30.43%	30.43%	2.76	0.24	0.0028
Bicycles (D)	1	39.13%	39.13%	2.01	0.2	0.0023
Spare parts and accessories for personal transport equipment (SD)	9	14.98%	21.74%	4.08	0.88	0.0102
Fuels and lubricants for personal transport equipment (ND)	2	21.74%	21.74%	4.08	1.22	0.0142
Maintenance and repair of personal transport equipment (S)	4	1.09%	0.00%		0.4	0.0046
Passenger transport by road (S)	3	0.00%	0.00%		4.12	0.0479
Postal services (S)	3	0.00%	0.00%		0.01	0.0001

Equipment for the reception, recording and reproduction of sound and pictures (D)	3	11.59%	13.04%	7.16	0.11	0.0013
Photographic and cinematographic equipment and optical instruments (D)	1	4.35%	4.35%	22.50	0.04	0.0005
Information processing equipment (D)	2	6.52%	6.52%	14.83	0.07	0.0008
Recording media (SD)	3	0.00%	0.00%		0.2	0.0023
Games, toys and hobbies (SD)	1	8.70%	8.70%	10.99	0.02	0.0002
Equipment for sport, camping and open-air recreation (SD)	9	21.26%	21.74%	4.08	0.01	0.0001
Recreational and sporting services (S)	4	0.00%	0.00%		0.02	0.0002
Cultural services (S)	4	2.17%	0.00%		0.11	0.0013
Books (SD)	2	26.09%	26.09%	3.31	0.61	0.0071
Newspapers and periodicals (ND)	4	1.09%	0.00%		0.02	0.0002
Stationery and drawing materials (ND)	51	6.39%	0.00%		0.06	0.0007
Pre-primary and primary education (S)	2	0.00%	0.00%		0.27	0.0031
Secondary education (S)	4	3.26%	2.17%	45.50	2.77	0.0322
Restaurants, cafés and the like (S)	35	4.60%	4.35%	22.50	2.76	0.0321
Accommodation services (S)	10	9.13%	8.70%	10.99	0.37	0.0043
Hairdressing salons and personal grooming establishments (S)	10	1.74%	0.00%		1.04	0.0121
Other appliances, articles and products for personal care (ND)	29	16.79%	8.70%	10.99	0.67	0.0078
Other personal effects (SD)	4	34.78%	36.96%	2.17	0.61	0.0071
Insurance connected with transport (S)	3	0.00%	0.00%		0.15	0.0017
Other services n.e.c. (S)	13	1.00%	0.00%		0.38	0.0044

Table A.1: Frequency of Price Changes and Duration by Item Categories in Rwanda: 2013-2014

A.2 Notes on Data

Total weight add up to 86.04% as per Rwanda CPI expenditure weights. Missing items (in Rwanda CPI expenditure but not in dataset) and their weights are listed below:

- 01.1.3: Fish and seafood – 0.88%
- 02.1.2: Wine – 0.02%

- 04.2.1: Imputed rentals of owner-occupiers – 6.91%
- 04.4.1: Water Supply – 0.49%
- 04.4.2: Rubbish collection services – 0.05%
- 04.5.1: Electricity – 0.41%
- 05.2.2: Gas – 0.25%
- 05.1.1: Furniture and furnishings – 0.91%
- 05.1.2: Carpets and other floor coverings – 0.06%
- 05.1.3: Repair of furniture, furnishings and floor coverings – 0.01%
- 05.3.2: Small electric household appliances – 0.13%
- 05.6.2: Domestic services and household services – 0.47%
- 06.1.3: Spectacle/eye lenses – 0.03%
- 07.2.4: Other services in respect of personal transport equipment – 0.02%
- 07.3.3: Passenger transport by air – 0.02%
- 08.2.1: Telephone and telefax equipment – 0.25%
- 08.3.1: Telephone and telefax services – 1.81%
- 09.2.2: Musical instrument and majors durables for indoor recreation – 0.05%
- 09.3.3: Natural & artificial flowers, foliage, and plants – 0.03%
- 10.3.1: Post-secondary non-tertiary education – 0.12%
- 10.4.1: Tertiary education – 0.59%
- 12.3.1: Jewellery, clocks and watches – 0.06%
- 12.5.3: Health insurance (Mutuelle, RAMA, MMI, etc.) – 0.33%
- 12.7.0: Security services – 0.08%

A.3 Key Definitions

Classification of Individual Consumption According to Purpose (COICOP): A classification system developed by the United Nations for countries to calculate and compare Consumer Price Index (CPI) and Gross Domestic Products (GDP) and its component expenditures.

Type of good:

- D – Durable good
- ND – Non-durable good
- SD – Semi-durable good
- S – Services

Weights: The weights are the CPI expenditure weights used by NISR to compute the CPI. These are obtained from NISR and not available publicly.

Frequency: Number of times prices are changed each month.

Duration: Number of months that prices remain constant.

Mean frequency: (Weighted) Mean frequency of price changes using CPI expenditure weights.

Mean implied duration: (Weighted) Mean implied duration of price changes. It is calculated in two stages: i) Implied duration for each items within a group is calculated by $-1/\ln(1 - \lambda)$, where λ is the frequency of price changes for a particular group, ii) A weighted mean is then taken across groups using CPI expenditure weights.

Median implied frequency: (Weighted) Median frequency of price changes. It is calculated in two stages: i) Mean frequency of price change for each items within a group is calculated, ii) A weighted median across groups using CPI expenditure weights.

Median implied duration: $-1/\ln(1 - \lambda)$, where λ is the frequency of price change.

Appendix B

Appendix to Chapter 3

B.1 Derivation of the Reset Price

We introduce staggered prices in the economy as in Taylor (1979). Households are divided into two sectors of equal size, with households in the sectors setting prices in a staggered manner. Households in a particular sector set prices in time t for t and $t + 1$, which means that prices are fixed for two consecutive periods. To incorporate this formally into a dynamic optimisation problem, we assume $f_{it} \equiv$ an index of firm i 's price flexibility in period t , such that:

$$f_{it} = \begin{cases} 0 & \text{in periods where firm } i \text{ is not allowed to adjust its price} \\ 1 & \text{in periods where firm } i \text{ is allowed to adjust its price} \end{cases} \quad (\text{B.1})$$

where f_{it} is a simple function of t . However, depending on which sector firm i is in, f_{it} will be a different function of t . To specify, if firm i is in the sector which is allowed to adjust its price in even-numbered periods, then:

$$f_{it} = [(-1)^t + 1]/2 \quad (\text{B.2})$$

and, if firm i is in the sector which is allowed to adjust its price in odd-numbered periods, then:

$$f_{it} = [(-1)^{t+1} + 1]/2 \quad (\text{B.3})$$

This allows us to introduce a price staggering constraint in terms of f_{it} . We define X_t as the ‘reset’ price, that is, the price that farmer i would optimally set in period t , if allowed to reset price in that period (and which would also remain in force in period $t + 1$).¹ The price staggering

¹Due to the assumption of symmetry, all farmers will choose the same reset price, so we drop the ‘i’ subscript.

constraint can, therefore, be expressed as:

$$P_{it} = f_{it}X_t + (1 - f_{it})X_{t-1} \quad (\text{B.4})$$

Given this, it can be seen that:

$$P_{it} = \begin{cases} X_t & \text{if } t \text{ is even} \\ X_{t-1} & \text{if } t \text{ is odd} \end{cases} \quad (\text{B.5})$$

It follows then, in even-numbered periods, for instance, at time t : $P_{it} = P_{it+1} = X_t$. Additionally, to enable us to keep the time periods simple, we will assume that $t, t + 2, \dots$ are even-numbered periods and $t - 1, t + 1, \dots$ are odd-numbered periods.

To derive the optimal price, a firm maximises (3.1) with respect to X_t , subject to the demand function and price-setting constraints. The reset price can then be derived as:

$$X_t = \left[\frac{1}{\gamma} \left(\frac{d\theta}{\theta - 1} \right) \left(\frac{Y_t^\varphi P_t^{\varphi\theta} + \beta Y_{t+1}^\varphi P_{t+1}^{\varphi\theta}}{\frac{Y_t P_t^{\theta-1}}{C_{it}} + \frac{\beta Y_{t+1} P_{t+1}^{\theta-1}}{C_{it+1}}} \right) \right]^{\frac{1}{1+\theta(\varphi-1)}} \quad (\text{B.6})$$

B.2 Aggregate Nominal Output is Determined by Money Supply

To determine the time path for aggregate nominal output, we can use (3.5), which can be log-linearised to obtain:

$$\frac{\beta}{1 - \beta}(\tilde{p}_{t+1} + \tilde{c}_{it+1}) + \tilde{m}_{it} - \frac{1}{1 - \beta}(\tilde{p}_t + \tilde{c}_{it}) = 0 \quad (\text{B.7})$$

To obtain the aggregate dynamics, we can now expand this to both sectors, S and N , which can be written as:

$$\frac{\beta}{1 - \beta}(\tilde{p}_{t+1} + \tilde{c}_{St+1}) + \tilde{m}_{St} - \frac{1}{1 - \beta}(\tilde{p}_t + \tilde{c}_{St}) = 0 \quad (\text{B.8})$$

$$\frac{\beta}{1 - \beta}(\tilde{p}_{t+1} + \tilde{c}_{Nt+1}) + \tilde{m}_{Nt} - \frac{1}{1 - \beta}(\tilde{p}_t + \tilde{c}_{Nt}) = 0 \quad (\text{B.9})$$

Now, we can aggregate, where we use $\frac{1}{2}\tilde{c}_{St} + \frac{1}{2}\tilde{c}_{Nt} = \tilde{c}_t$, $\frac{1}{2}\tilde{c}_{St+1} + \frac{1}{2}\tilde{c}_{Nt+1} = \tilde{c}_{t+1}$, and the money market equilibrium, $\frac{1}{2}\tilde{m}_{St} + \frac{1}{2}\tilde{m}_{Nt} = \tilde{m}$, where $\tilde{m}$ is time-invariant money supply, via which we obtain the intertemporal relationship between aggregate consumption and price levels, or the aggregate nominal expenditures on consumption:

$$\frac{\beta}{1 - \beta}(\tilde{p}_{t+1} + \tilde{c}_{t+1}) - \frac{1}{1 - \beta}(\tilde{p}_t + \tilde{c}_t) = -\tilde{m} \quad (\text{B.10})$$

Using the resource constraint, we can derive (3.13) in the chapter:

$$\tilde{p}_{t+1} + \tilde{y}_{t+1} - \frac{1}{\beta}(\tilde{p}_t + \tilde{y}_t) = -\frac{1-\beta}{\beta}\tilde{m} \quad (\text{B.11})$$

which is a first-order difference equation in aggregate nominal output.

B.3 Proof of Proposition 1

Solving the fourth-order (FO) system in reset price and sectoral money holdings

The fourth-order system is difficult to solve but conveniently, we can convert the system of second-order difference equations into first-order difference equations in the two variables. To do this, we assume:

$$\tilde{f}_{t+1} = \tilde{x}_t \quad (\text{B.12})$$

$$\tilde{z}_{t+1} = \tilde{m}_{St} \quad (\text{B.13})$$

and then we can convert (3.20) and (3.21), assuming $\tilde{g}_{t+1} = 0$, to derive the following system of first-order linear difference equations:

$$\begin{aligned} \beta\varphi(\theta-1)(1-\beta)\tilde{x}_{t+1} + 2\beta\zeta\tilde{m}_{St+1} &= \varphi(\theta+1)(1-\beta^2)\tilde{x}_t + 2\zeta(1-\beta)\tilde{m}_{St} \\ -\varphi(\theta-1)(1-\beta)\tilde{f}_t + 2\zeta\tilde{z}_t - 2(1-\beta)[\varphi(1+\beta) + 2\zeta]m + 2(1-\beta)\tilde{g}_t & \end{aligned} \quad (\text{B.14})$$

$$\begin{aligned} -\beta\zeta(\theta-1)(1-\beta)\tilde{x}_{t+1} - 2\beta\zeta^2\tilde{m}_{St+1} &= \zeta(\theta-1)(1-\beta)^2\tilde{x}_t + 2[\zeta^2(1+\beta) + \zeta(1-\beta)^2]\tilde{m}_{St} + \zeta(\theta-1)(1-\beta)\tilde{f}_t \\ + 2\zeta^2\tilde{z}_t + 2\zeta[(1-\beta)(\beta-1) - 2\zeta(\beta+1)]m - 2\zeta(1-\beta)\tilde{g}_t & \end{aligned} \quad (\text{B.15})$$

$$\tilde{f}_{t+1} = \tilde{x}_t \quad (\text{B.16})$$

$$\tilde{z}_{t+1} = \tilde{m}_{St} \quad (\text{B.17})$$

We can arrange the system of equations in the form of matrices:

$$\begin{bmatrix} a_0 & a_2 & 0 & 0 \\ a_1 & a_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{m}_{St+1} \\ \tilde{f}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix} = \begin{bmatrix} b_0 & b_2 & b_4 & b_6 \\ b_1 & b_3 & b_5 & b_7 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{m}_{St} \\ \tilde{f}_t \\ \tilde{z}_t \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} m \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{g}_t \end{bmatrix} \quad (\text{B.18})$$

where $a_0 = \beta\varphi(\theta-1)(1-\beta)$, $a_1 = -\beta\zeta(\theta-1)(1-\beta)$, $a_2 = 2\beta\zeta$, $a_3 = -2\beta\zeta^2$, $b_0 = \varphi(\theta+1)(1-$

β^2), $b_1 = \zeta(\theta - 1)(1 - \beta)$, $b_2 = 2\zeta(1 - \beta)$, $b_3 = 2\zeta(\zeta(1 + \beta) + (1 - \beta)^2)$, $b_4 = -\varphi(\theta - 1)(1 - \beta)$, $b_5 = -\zeta(\theta - 1)(1 - \beta)$, $b_6 = 2\zeta$, $b_7 = 2\zeta^2$, $c_1 = -2(1 - \beta)[\varphi(1 + \beta) + 2\zeta]$, $c_2 = 2\zeta[(1 - \beta)(\beta - 1) - 2\zeta(1 + \beta)]$, $d_1 = 2(1 - \beta)$ and $d_2 = -2\zeta(1 - \beta)$. Since money supply, m and government transfers, $\tilde{g}_t$, are exogenous terms, we can ignore them. Taking the inverse and multiplying out the matrices allow us to write in state-space form:

$$\begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{m}_{St+1} \\ \tilde{f}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix} = \begin{bmatrix} p_0 & p_2 & p_4 & p_6 \\ p_1 & p_3 & p_5 & p_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{m}_{St} \\ \tilde{f}_t \\ \tilde{z}_t \end{bmatrix} \quad (\text{B.19})$$

where $p_0 = \frac{\varphi(\theta+1)(\beta+1) - (\theta-1)(\beta-1)}{\beta(\theta-1)(\varphi-1)}$, $p_1 = \frac{\varphi(\beta-1)(\beta+\theta)}{\zeta\beta(\varphi-1)}$, $p_2 = -\frac{2(1-\beta)^2 + 4\zeta}{\beta(\varphi-1)(\theta-1)(\beta-1)}$, $p_3 = \frac{\zeta(\beta-1) - \varphi[\zeta(\beta+1) + (\beta-1)^2]}{\beta\zeta(\varphi-1)}$, $p_4 = -\frac{\varphi+1}{\beta(\varphi-1)}$, $p_5 = -\frac{\varphi(\theta-1)(\beta-1)}{\beta\zeta(\varphi-1)}$, $p_6 = \frac{4\zeta}{\beta(\varphi-1)(\theta-1)(\beta-1)}$. Hence $X_{t+1} = AX_t$, where:

$$X_{t+1} = \begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{m}_{St+1} \\ \tilde{f}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix}, \quad A = \begin{bmatrix} p_0 & p_2 & p_4 & p_6 \\ p_1 & p_3 & p_5 & p_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \text{and } X_t = \begin{bmatrix} \tilde{x}_t \\ \tilde{m}_{St} \\ \tilde{f}_t \\ \tilde{z}_t \end{bmatrix} \quad (\text{B.20})$$

The characteristic polynomial of a fourth-order system is:

$$c(w) = w^4 + p_1w^3 + p_2w^2 + p_3w + p_4 = 0 \quad (\text{B.21})$$

where the coefficient of the characteristic polynomial are:

$p_1 = -\text{trace}(A) = -\text{sum of the principal first-order minors of } A$

$p_2 = \text{sum of the principal second-order minors of } A$

$p_3 = -\text{sum of the principal third-order minors of } A$

$p_4 = \text{determinant of } A = \text{principal of fourth-order.}$

The characteristic polynomial for our system of equation then is:

$$\begin{aligned}
c(w) = & w^4 - \underbrace{\frac{\varphi[2\zeta(1+\beta) + (1-\theta)(1-\beta)^2]}{\beta\zeta(\varphi-1)(\theta-1)}}_{p_1} w^3 \\
& - \underbrace{\frac{(\beta^2+1)[\zeta\theta(\varphi-1) + \varphi(\zeta+1)] + \varphi(\theta+1)(\beta^2-1)(\beta-1) + 4\zeta\beta\varphi}{\zeta\beta^2(\varphi-1)(\theta-1)}}_{p_2} w^2 \\
& - \underbrace{\frac{\varphi[2\zeta(1+\beta) + (1-\theta)(1-\beta)^2]}{\beta^2\zeta(\varphi-1)(\theta-1)}}_{p_3} w \\
& + \underbrace{\frac{1}{\beta^2}}_{p_4}
\end{aligned} \tag{B.22}$$

To determine whether saddlepoint stability holds, we need to deduce whether the eigenvalues have a negative real part. Since our model is in discrete time, the eigenvalues refer to whether they lie outside or inside the unit circle, unlike the continuous-time case where the eigenvalues refer to a negative or a positive real part. We can apply a bilinear transformation here, which requires that we transform the eigenvalue w into a new variable z , where z is defined as:

$$z \equiv \frac{w-1}{w+1} \tag{B.23}$$

It can be proven that this transformation allows us to show that w lies in the unit circle if and only if z has a negative real part. This tells us that the number of transformed eigenvalues, z , that have negative real parts is equivalent to determining the number of original eigenvalues, w that lie in the unit circle. We obtain, by inverting the definition of z :

$$w = \frac{1+z}{1-z} \tag{B.24}$$

which can be replaced in the characteristic equation to express in terms of z , rather than w :

$$\begin{aligned}
z^4 + \underbrace{\frac{4-2p_1+2p_3-4p_4}{1-p_1+p_2-p_3+p_4}}_{\hat{p}_1} z^3 + \underbrace{\frac{6-2p_2+6p_4}{1-p_1+p_2-p_3+p_4}}_{\hat{p}_2} z^2 + \underbrace{\frac{4+2p_1-2p_3-4p_4}{1-p_1+p_2-p_3+p_4}}_{\hat{p}_3} z + \underbrace{\frac{1+p_1+p_2+p_3+p_4}{1-p_1+p_2-p_3+p_4}}_{\hat{p}_4} = 0
\end{aligned} \tag{B.25}$$

Applying this transformation to get the Hurwitz polynomial yields:

$$\begin{aligned}
& z^4 - \underbrace{\frac{2\zeta(\beta+1)[\theta(\varphi-1)+1] + \varphi(1-\theta)(1-\beta)^2}{\theta\varphi(\beta^2-1)}}_{\hat{p}_1} z^3 \\
& - \underbrace{\frac{2\zeta[(\beta^2+1)(\varphi(2\theta-1)+2(1-\theta)+2\beta\varphi)] + \varphi(\theta+1)(\beta^2-1)(\beta-1)}{\theta\varphi(\beta^2-1)(\beta-1)}}_{\hat{p}_2} z^2 \\
& - \underbrace{\frac{2\zeta(\beta+1)[\theta(\varphi-1)+1-2\varphi] + \varphi(\theta-1)(1-\beta)^2}{\theta\varphi(\beta^2-1)}}_{\hat{p}_3} z \\
& + \underbrace{\frac{2\zeta(\beta+1) + (1-\beta)^2}{\theta(1-\beta)^2}}_{\hat{p}_4}
\end{aligned} \tag{B.26}$$

where we can establish unambiguously that $\hat{p}_4 > 0$ and $\hat{p}_2 < 0$; however, $\hat{p}_3$ and $\hat{p}_1$ cannot be determined unambiguously.

Since there are two predetermined variables, we need two eigenvalues to lie outside the unit circle and two eigenvalues to lie inside the unit circle for the saddlepoint stability condition to hold. To obtain the necessary and sufficient conditions on w_1, w_2, w_3 and w_4 , we apply Routh's theorem. For a fourth-order system, we can derive Routh's array as follows:

$$\begin{bmatrix}
1 & \hat{p}_2 & \hat{p}_4 \\
\hat{p}_1 & \hat{p}_3 & 0 \\
\hat{p}_2 - \frac{\hat{p}_3}{\hat{p}_1} & \hat{p}_4 & 0 \\
\hat{p}_3 - \frac{\hat{p}_4\hat{p}_1}{\hat{p}_2 - \frac{\hat{p}_3}{\hat{p}_1}} & 0 & 0 \\
\hat{p}_4 & 0 & 0
\end{bmatrix} \tag{B.27}$$

Routh's theorem states the number of solutions of the polynomial with positive real parts is equal to the number of changes of sign in the entries in the first column of array. Since we are interested to find whether saddle-path stability (SPS) holds, we will only state the conditions under which two eigenvalues will lie outside the unit circle. This can be achieved under many different cases, which we outline below:²

²The first and last rows will always be positive since we know with certainty that $\hat{p}_0 > 0$ and $\hat{p}_4 > 0$.

Table B.1: Saddlepath Stability Conditions

$\hat{p}_0$	+	+	+	+	+
$\hat{p}_1$	-	-	-	+	+
$\hat{p}_2 - \frac{\hat{p}_3}{\hat{p}_1}$	+	-	-	-	+
$\hat{p}_3 - \frac{\hat{p}_4\hat{p}_1}{\hat{p}_2 - \frac{\hat{p}_3}{\hat{p}_1}}$	+	+	-	+	-
$\hat{p}_4$	+	+	+	+	+

Table B.1 states the conditions under which there are two sign changes in the array; equivalently, that two eigenvalues lie outside the unit circle. Hence, this can be used to observe whether SPS holds. However, for our system, we cannot unambiguously determine the values of $\hat{p}_1$ and $\hat{p}_3$, which makes it difficult to conclude with certainty whether SPS holds.

Descartes' Rule of Signs

Since we are not able to deduce using Routh's theorem whether SPS holds, we use Descartes' rule of signs. The rule states that to determine the number of positive or negative real roots of a polynomial, we are required to count the number of sign changes in the polynomial's coefficients. The number of positive roots is equal to the number of sign differences between consecutive nonzero coefficients, or is less than it by an even number. As a corollary of the rule, the number of negative roots is determined by multiplying the coefficients of odd-power terms by -1, where the number of sign differences refer to the number of negative roots in the original polynomial, or is fewer than it by an even number (Curtiss (1918); Anderson et al. (1998)).³

It can be seen from (B.26) that $\hat{p}_2 = -\frac{2\zeta[(\beta^2+1)(2\theta(\varphi-1)+2-\varphi)+2\beta\varphi]+\varphi(\theta+1)(\beta^2-1)(\beta-1)}{\theta\varphi(\beta^2-1)(\beta-1)}$, where for plausible moderate values for φ (for e.g., $\varphi = 1.1$, as set often in the literature, such as in Daros and Rankin (2009)), we can unambiguously determine that $\hat{p}_2 < 0$.⁴ Hence, we know with certainty that $\hat{p}_0 > 0, \hat{p}_2 < 0$ and $\hat{p}_4 > 0$. For $\hat{p}_3$ and $\hat{p}_1$, there are four possibilities: i) $\hat{p}_1 > 0, \hat{p}_3 > 0$, ii) $\hat{p}_1 < 0, \hat{p}_3 < 0$, iii) $\hat{p}_1 > 0, \hat{p}_3 < 0$, iv) $\hat{p}_1 < 0, \hat{p}_3 > 0$, which we will look in turn.

i) $\hat{p}_1 > 0, \hat{p}_3 > 0$

³Descartes' Rule of Signs will not provide conclusive results if there are complex eigenvalues in the system. However, we find through numerical results that complex eigenvalues do not exist in our system.

⁴The numerical results show that for $\hat{p}_2 < 0$ to hold, we do not need to assume that $\varphi = 1.1$; as long as $\varphi > 1$, we find that $\hat{p}_2 < 0$.

Table B.2: i) No. of positive roots, $\hat{p}_1 > 0, \hat{p}_3 > 0$

Change of sign			1	2	
Sign	>	>	<	>	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

which shows that there are two or zero positive roots. The corollary is:

Table B.3: i) No. of negative roots, $\hat{p}_1 > 0, \hat{p}_3 > 0$

Change of sign		1			2
Sign	>	<	<	<	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

i.e. there are two or zero negative roots. Since there are four roots in the equation, we can conclude there are two positive roots and two negative roots; equivalently, there are two eigenvalues outside the unit circle and two eigenvalues inside the unit circle.

ii) $\hat{p}_1 < 0, \hat{p}_3 < 0$

Table B.4: ii) No. of positive roots, $\hat{p}_1 < 0, \hat{p}_3 < 0$

Change of sign		1			2
Sign	>	<	<	<	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

which shows that there are two or zero positive roots. The corollary is:

Table B.5: ii) No. of negative roots, $\hat{p}_1 < 0, \hat{p}_3 < 0$

Change of sign			1	2	
Sign	>	>	<	>	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

i.e. there are two or zero negative roots. Since there are four roots in the equation, we can conclude there are two positive roots and two negative roots; equivalently, there are two eigenvalues outside the unit circle and two eigenvalues inside the unit circle.

iii) $\hat{p}_1 > 0, \hat{p}_3 < 0$

Table B.6: iii) No. of positive roots, $\hat{p}_1 > 0, \hat{p}_3 < 0$

Change of sign			1		2
Sign	>	>	<	<	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

which shows that there are two or zero positive roots. The corollary is:

Table B.7: iii) No. of negative roots, $\hat{p}_1 > 0, \hat{p}_3 < 0$

Change of sign		1		2	
Sign	>	<	<	>	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

i.e. there are two or zero negative roots. Since there are four roots in the equation, we can conclude there are two positive roots and two negative roots; equivalently, there are two eigenvalues outside the unit circle and two eigenvalues inside the unit circle.

iv) $\hat{p}_1 < 0, \hat{p}_3 > 0$

Table B.8: iv) No. of positive roots, $\hat{p}_1 < 0, \hat{p}_3 > 0$

Change of sign		1		2	
Sign	>	<	<	>	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

which shows that there are two or zero positive roots. The corollary is:

Table B.9: iv) No. of negative roots, $\hat{p}_1 < 0, \hat{p}_3 > 0$

Change of sign			1		2
Sign	>	>	<	<	>
Coefficients	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$

i.e. there are two or zero negative roots. Since there are four roots in the equation, we can conclude there are two positive roots and two negative roots; equivalently, there are two eigenvalues outside the unit circle and two eigenvalues inside the unit circle.

This shows that, following Descartes' rule of signs, any nonzero values for $\hat{p}_1$ and $\hat{p}_3$ results in two eigenvalues to lie outside the unit circle and two eigenvalues to lie inside the unit circle. Hence, this shows that SPS condition holds for reset price and sectoral money holdings.

B.4 Proof of Proposition 2

We can evaluate (3.22) at $z = -1$ and $z = 0$ to determine whether a stable eigenvalue lies in the interval $(-1,0)$. For a negative stable eigenvalue to exist, we require that the sign of (3.22) changes between the cases $z = -1$ and $z = 0$. Below, we evaluate $z = 0$, where we denote the Hurwitz polynomial as $H(z)$, and which results in:

$$H(0) = \frac{2\zeta(\beta + 1) + (1 - \beta)^2}{\theta(1 - \beta)^2} \quad (\text{B.28})$$

From (B.28), we can establish that $H(0) > 0$. Similarly, we evaluate $z = -1$, which we can simplify and rearrange to write:

$$H(-1) = -\frac{8\zeta(\theta - 1)(\varphi - 1)}{\theta\varphi(\beta - 1)^2(\beta + 1)} \quad (\text{B.29})$$

Since $\theta > 1$ and $\varphi > 1$, we can establish that $H(-1) < 0$. This shows that the sign of (3.22) changes between $z = -1$ and $z = 0$, which demonstrates that there is a stable eigenvalue in the interval $(-1,0)$, or that there is a negative stable eigenvalue.

We have previously showed that there are two stable eigenvalues. Since we have established that there is a stable eigenvalue in the interval $(-1,0)$, the only possibility is that there are either 1 or 3 (or 5 and so on) eigenvalues in the interval $(-1,0)$. However, since there are exactly two stable eigenvalues, the other stable eigenvalue must then be in the interval $(0,1)$.

B.5 A Model of Perfect Credit Markets

B.5.1 The Basic Framework

The model we develop in this section is similar to our economy with imperfect credit markets, except that there are now perfect credit markets, which allows farmers to borrow and lend to smooth out income and consumption fluctuations. The economy is composed of yeoman farmers with differentiated goods and money, where the farmer is both a consumer and a producer. Farmer i maximises utility according to (3.1), where consumption and real money balances affect farmer positively, while production of output leads to lower utility. The presence of bond markets in this economy is reflected in the budget constraint for farmer i , where the bond market allows farmers to borrow and lend in the economy:

$$P_t C_{it} + M_{it} + Q_t B_{it} = P_t Y_{it} + M_{it-1} + B_{it-1} + G_{it} \quad t = 0, \dots, \infty, \text{ and } i = A, B \quad (\text{B.30})$$

In each period, farmer i chooses a level of consumption, C_{it} , quantity of money, M_{it} , and

bonds, B_{it} , which is purchased at price $Q_t = \frac{1}{1+I_t}$, that s/he will transfer to next period, and a lump-sum transfer, G_{it} from the government. Farmer i enters period t with a predetermined level of wealth, given by money balances M_{it-1} and B_{it-1} , which is the value of bonds purchased last period with each bond yielding a payoff of one. The budget constraint is similar to standard NK models such as Galí (2009). We can form a Lagrangian function as in the economy with imperfect credit markets, and derive the first-order conditions. Moreover, due to the presence of bond markets, we now have an additional first-order condition:

$$-\lambda_{it}Q_t + \lambda_{it+1} = 0 \quad (\text{B.31})$$

where Q_t allows us to tie in the macroeconomic variables and the interest rate in the economy. In addition, we can derive a relationship for intertemporal nominal consumption levels in terms of interest rates:

$$\beta C_{it} P_t (1 + I_t) = C_{it+1} P_{t+1} \quad (\text{B.32})$$

which is the standard IS relationship, and states that the intertemporal nominal consumption expenditure is a function of the interest rate. To gain a broader understanding of the mechanism via which interest rate affects the aggregate dynamics in the economy, we log-linearise (B.32) around the ZISS to obtain:

$$\tilde{i}_t = \tilde{c}_{it+1} + \tilde{p}_{t+1} - \tilde{p}_t - \tilde{c}_{it} \quad (\text{B.33})$$

In order to obtain some further results regarding the interest rate, we can explicitly derive the steady state value of interest rate. Otherwise, we will be unable to conclusively state the implications of the interest rate on the economy, since it serves as the primary difference between an economy with imperfect and perfect credit markets. To derive the steady state solution of the interest rate, we extend (B.32) to the money market, for which we insert (B.32) in (3.5) and simplify to yield a relationship between consumption and money balances:

$$\frac{M_{it}}{P_t C_{it}} = \zeta \frac{1 + I_t}{I_t} \quad (\text{B.34})$$

where the presence of credit markets allows agents to borrow and lend to smooth out individual income fluctuations due to staggered prices. In addition, there is an insurance contract, which a household may use to insure itself against any losses in the event of an exogenous shock, which will affect sectors differently. Due to the presence of bond markets, we note from (B.34) that the ratio of money balances and consumption is simply dependent on the interest rates in the economy, and would, hence, be the same across the sectors. This serves as the main difference between an economy with imperfect credit markets: the interest rate, along with the insurance contracts,

allows households to borrow in the event of a shock to smooth out consumption fluctuations and equate the marginal utility of consumption across the sectors. Hence, consumption levels in the sectors are equal in every period, irrespective of which sector is resetting prices.⁵ We additionally assume, following the literature, that the initial wealth of the sectors are equal, which ensures that, in equilibrium, we can equate the households in the sectors. Since we know that:

$$C_t \equiv \int_0^1 C_{it} di \quad \text{and} \quad M_t \equiv \int_0^1 M_{it} di \quad (\text{B.35})$$

and we can rearrange (3.5) to write:

$$\zeta = \frac{M_{it}}{C_{it}P_t} - \frac{\beta M_{it}}{C_{it+1}P_{t+1}} \quad (\text{B.36})$$

where, conveniently, we can insert $C_t = C_{it}$ and $M_t = M_{it}$, and write (B.36) as:

$$\zeta = \frac{M_t}{C_t P_t} - \frac{\beta M_t}{C_{t+1} P_{t+1}} \quad (\text{B.37})$$

We introduce an aggregate money supply growth rate in (B.37), which help us to obtain a difference equation in the ratio of real money balances and consumption levels, as shown below:

$$\zeta = \frac{M_t}{C_t P_t} - \frac{\beta M_{t+1}}{C_{t+1} P_{t+1}} \frac{M_t}{M_{t+1}} \quad (\text{B.38})$$

This can be simplified to write:

$$Z_t = \zeta + \beta \Phi_{t+1} Z_{t+1} \quad (\text{B.39})$$

where $Z_t = \frac{M_t}{P_t C_t}$ (and $Z_{t+1} = \frac{M_{t+1}}{P_{t+1} C_{t+1}}$) and $\Phi_{t+1} = \frac{M_t}{M_{t+1}}$, which is the rate of decrease of money supply. Therefore, in (B.39), we have obtained a first-order difference equation in Z_t , where Z_t is the ratio of real money balances and consumption in period t .

In order to determine the time path, we observe that Z_t is not a predetermined variable as C_t is endogenous to our model. In addition, assuming a constant money growth, $\Phi = 1$, (B.39) is not stable in its forward dynamics. Following the saddlepath argument, we rule out all divergent paths, which implies that our unique solution will be the steady state solution. Hence, this establishes that (B.39) can simply be written as:

$$Z = \zeta + \beta Z \quad (\text{B.40})$$

⁵The common interest rate ensures that $\frac{C_{At+1}}{C_{At}} = \frac{C_{Bt+1}}{C_{Bt}}$, for all t . However, to prove that $C_{At} = C_{Bt}$, for all t will require considerable work, which is why we follow the literature and assume that this condition holds.

from which we derive the the steady state value of Z :

$$Z = \frac{\zeta}{1 - \beta} \quad (\text{B.41})$$

We have now derived the required macroeconomic relationships and set of equations to determine the aggregate dynamics following a monetary policy shock in our economy with perfect credit markets.

B.5.2 Dynamics of Aggregate Output

In the economy with imperfect credit markets, the absence of interest rate implies that the only monetary policy shock that can be introduced in the economy is to the money supply. Although a Taylor rule is more commonly observed in advanced economies, for comparison with the imperfect credit markets, it is useful to derive the dynamics of the aggregate output in the perfect credit markets case, under a money supply rule. A comparison of the economies under different monetary policy rules may include effects that stem from the different types of monetary policy rules, rather than the idiosyncratic features of the economies.

To introduce a money supply shock into our economy with perfect credit markets, we rewrite the reset price, (3.9), as:

$$\tilde{x}_t = \varepsilon[(\varphi - 1)(\tilde{y}_t + \beta\tilde{y}_{t+1}) + (1 + \theta(\varphi - 1))(\tilde{p}_t + \beta\tilde{p}_{t+1}) + \tilde{c}_t + \beta\tilde{c}_{t+1}] \quad (\text{B.42})$$

Under perfect credit markets, we can derive a relationship between money supply, price and consumption as:

$$\tilde{z}_t = \tilde{m}_t - \tilde{p}_t - \tilde{c}_t \quad (\text{B.43})$$

where we have shown that Z_t is time-invariant, i.e. $\tilde{z}_t = 0$. We can rearrange (B.43) and write an expression for aggregate consumption in t :

$$\tilde{c}_t = m - \tilde{p}_t \quad (\text{B.44})$$

which can now be substituted into (B.42) to derive a reset price in aggregate output, price levels and money supply:

$$\tilde{x}_t = \varepsilon[(\varphi - 1)(\tilde{y}_t + \beta\tilde{y}_{t+1}) + \theta(\varphi - 1)(\tilde{p}_t + \beta\tilde{p}_{t+1}) + (1 + \beta)m] \quad (\text{B.45})$$

Therefore, reset price for the previous period, $\tilde{x}_{t-1}$, is:

$$\tilde{x}_{t-1} = \varepsilon[(\varphi - 1)(\tilde{y}_{t-1} + \beta\tilde{y}_t) + (\theta(\varphi - 1))(\tilde{p}_{t-1} + \beta\tilde{p}_t) + (1 + \beta)m] \quad (\text{B.46})$$

Having derived the reset prices, we can plug these into the aggregate price level, given by:

$$\tilde{p}_t = \frac{1}{2}(\tilde{x}_t + \tilde{x}_{t-1}) \quad (\text{B.47})$$

We use (B.44) to eliminate the price level (and its leads and lags), and derive:

$$[\beta\kappa(1-\theta)(\varphi-1)]\tilde{y}_{t+1} + [(1+\beta)(\kappa(1-\theta)(\varphi-1)+2)]\tilde{y}_t + [\kappa(1-\theta)(\varphi-1)]\tilde{y}_{t-1} = 0 \quad (\text{B.48})$$

which is a second-order difference equation in aggregate output. An analysis of the second-order system yields the following result.

Proposition 3. *Under perfect credit markets, following a monetary policy shock under a money supply rule, a unique bounded perfect-foresight equilibrium exists, where output converges along a persistent path.*

Proof. See Appendix B.6.

As shown in Appendix B.6, we can derive the Hurwitz polynomial as:

$$T(z) = -\underbrace{\frac{2(1+\beta)}{\beta\kappa(1-\theta)(\varphi-1)}}_{p_1} z^2 + \underbrace{\frac{2(\beta-1)}{\beta}}_{p_2} z + \underbrace{\frac{2(1+\beta)[\kappa(1-\theta)(\varphi-1)+1]}{\beta\kappa(1-\theta)(\varphi-1)}}_{p_3} \quad (\text{B.49})$$

To determine if the saddlepath stability criterion holds, the necessary and sufficient conditions are that one eigenvalue will lie outside the unit circle (equivalently, one eigenvalue will lie inside the unit circle, since there is one predetermined variable, $\tilde{y}_{t-1}$). Using Routh's theorem, we know that this holds if $\frac{p_3}{p_1} < 0$, which, we determine, for our system, as:

$$\frac{p_3}{p_1} = -\frac{2(1+\beta)[\kappa(1-\theta)(\varphi-1)+1]}{2(1+\beta)} = -\frac{\varphi}{1+\theta(\varphi-1)} < 0 \quad (\text{B.50})$$

This shows that one eigenvalue lies in the unit circle and, hence, one eigenvalue lies outside the unit circle. This proves that the saddlepoint stability condition holds under the money supply rule for the case of perfect credit markets. We can now determine whether the stable eigenvalue lies in (0,1) or in (-1,0). We know that there is a unique eigenvalue in the interval (-1,1). If the convergence is monotonic, $T(z)$ should intersect the horizontal axis in (0,1) only once, so we expect that the function will take opposite signs for $z = 0$ and $z = 1$, i.e. $T(0)/T(1) < 0$. This,

equivalently states that we are required to analytically evaluate $\frac{d}{b+c+d}$, which yields:⁶

$$\frac{T(0)}{T(1)} = \frac{d}{b+c+d} = \frac{(1-\theta)(\varphi-1)}{2\varphi(1+\beta)} < 0 \quad (\text{B.51})$$

which shows that, following a monetary policy shock, output follows a persistent path, under a money supply rule.

The finding that money supply shock causes output to follow a persistent path is consistent with the literature, such as Chari et al. (2000). Following a money supply shock, aggregate output follows a monotonic path as the effect of the money supply persists over time. In contrast, under a Taylor rule, aggregate output follows an oscillatory path as found in Daros and Rankin (2009), among others.⁷ Therefore, it is important to take into account the types of monetary policy rules as they play a role in determining the path of the aggregate output. Moreover, we may also want to be careful of the particular form of the monetary policy rule. For instance, a Taylor rule of the form used in equation (15) of Dixon and Kara (2011), which includes a lagged term for interest rate, may yield different time path implications for macroeconomic variables.

Speed of Convergence

To determine how effective monetary policy is in generating a real impact on aggregate output, we may be interested to find out the speed with which output converges towards the steady state, or the “persistence” of output. In the literature, a measure that has been used to measure persistence is the half-life: the length of time following the shock that the output falls to half its impact value (Chari et al. (2000)). In our comparisons of ICM and PCM, aggregate output converges very rapidly such that, to have a useful comparison of half-lives, we would have to allow the half-lives to take non-integer values, for example, by using linear interpolation of the value of $\tilde{y}_t$ between two successive periods. We find that, with a linear interpolation of the value of $\tilde{y}_t$, aggregate output is slightly less persistent for ICM than PCM. Intuitively, it seems that this is linked to the oscillatory nature of the former time paths. The presence of oscillations means that, at some point, the time path overshoots its steady state value as it converges towards it; whereas when convergence is monotonic, this does not happen. This means that, with oscillatory time paths, output always reaches, and, indeed, goes beyond its steady state value in some finite interval of time, whereas with monotonic time paths, it takes until infinity for output to reach its steady state value. Other things being equal, this makes it likely that the measured persistence of

⁶The terms b, c, d are defined in B.6 in the Appendix.

⁷We have introduced a monetary policy shock following a Taylor rule in our model and found that aggregate output follows an oscillatory path. In this chapter, we are interested to compare the effect of a monetary policy shock in developing and advanced economies, so we have only included the results from the money supply rule as that is the only shock that is possible in a developing economy.

ICM time paths will be lower. However, we acknowledge that while half-life has been commonly used in the literature, for our models, the time paths of aggregate output following the period of half-life are not the same: aggregate output converges along a monotonic path in PCM, and along an oscillatory path in ICM.

B.6 Proof of Proposition 3

The second-order difference equation in aggregate output and money supply is:

$$a_0 \tilde{y}_{t+1} + a_1 \tilde{y}_t + a_2 \tilde{y}_{t-1} = 0 \quad (\text{B.52})$$

where $a_0 = \beta\kappa(1-\theta)(\varphi-1)$, $a_1 = (1+\beta)(\kappa(1-\theta)(\varphi-1)+2)$ and $a_2 = \kappa(1-\theta)(\varphi-1)$. The characteristic equation for a second-order polynomial can be written as:

$$P(w) = bw^2 + cw + d = 0 \quad (\text{B.53})$$

where $b = 1$, $c = \frac{(1+\beta)(\kappa(1-\theta)(\varphi-1)+2)}{\beta\kappa(1-\theta)(\varphi-1)}$, $d = \frac{1}{\beta}$. A bilinear transformation that has been used previously is applied here to obtain:

$$T(z) = p_1 z^2 + p_2 z + p_3 = 0 \quad (\text{B.54})$$

where $p_1 = b - c + d$, $p_2 = 2(b - d)$, $p_3 = b + c + d$, which can be used to derive the characteristic polynomial:

$$T(z) = -\underbrace{\frac{2(1+\beta)}{\beta\kappa(1-\theta)(\varphi-1)}}_{p_1} z^2 + \underbrace{\frac{2(\beta-1)}{\beta}}_{p_2} z + \underbrace{\frac{2(1+\beta)[\kappa(1-\theta)(\varphi-1)+1]}{\beta\kappa(1-\theta)(\varphi-1)}}_{p_3} \quad (\text{B.55})$$

The necessary and sufficient conditions for saddlepoint condition to hold, equivalently, one eigenvalue to lie outside the unit circle and one eigenvalue to lie inside the unit circle are $\frac{p_3}{p_1}$, which is expressed as:

$$\frac{p_3}{p_1} = -\frac{2(1+\beta)[\kappa(1-\theta)(\varphi-1)+1]}{2(1+\beta)} = -\frac{\varphi}{1+\theta(\varphi-1)} < 0 \quad (\text{B.56})$$

which shows that one eigenvalue lies in the unit circle and, hence, one eigenvalue lie outside the unit circle, i.e. saddlepoint stability condition holds. To determine the pattern of convergence towards the saddle path, we are required to analytically evaluate $T(0)/T(1)$. This yields:

$$\frac{d}{b+c+d} = \frac{(1-\theta)(\varphi-1)}{2\varphi(1+\beta)} < 0 \quad (\text{B.57})$$

Hence, we find that, following a monetary policy shock, output follows a persistent, hump-shaped path, under a money supply rule.

B.7 Supplementary Tables and Figures

B.7.1 Aggregate Responses to Money Supply Shock

The responses of the aggregate variables due to a money supply shock are provided below. The baseline parameter values are $\theta = 5$, $\varphi = 1.05$, $\beta = 0.995$ and $\gamma = 0.999$ ($\zeta = 0.001$).

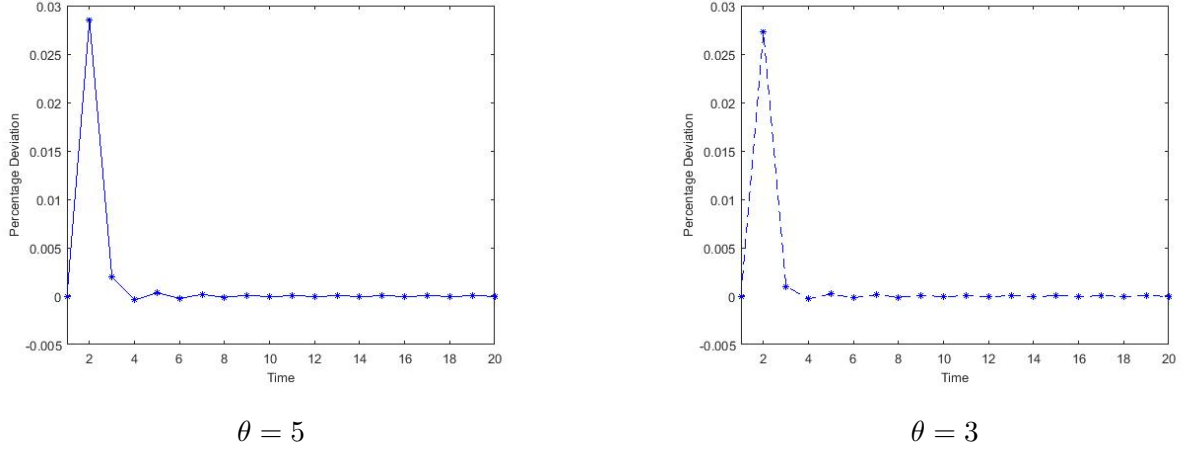


Figure B.1: Aggregate Output, $\theta = 5, 3$

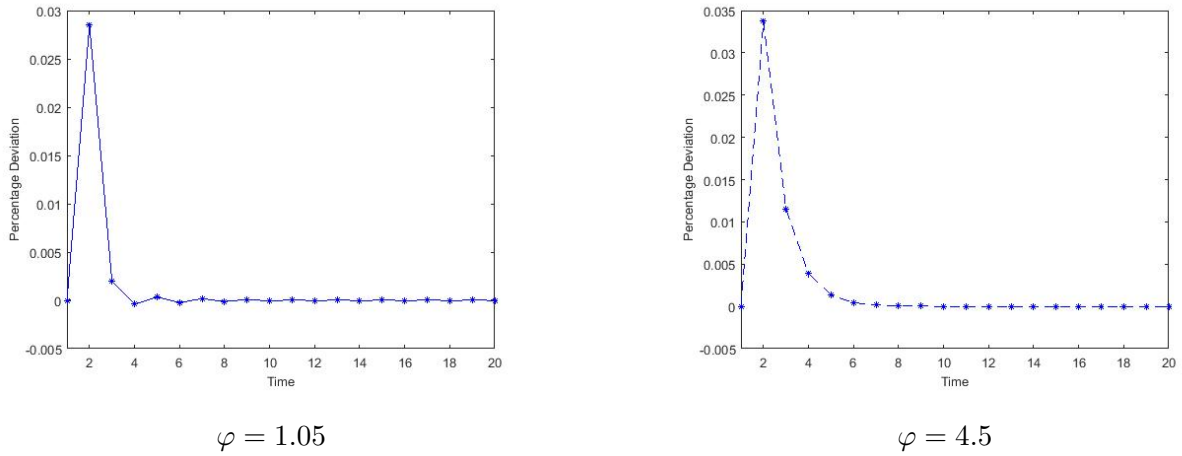
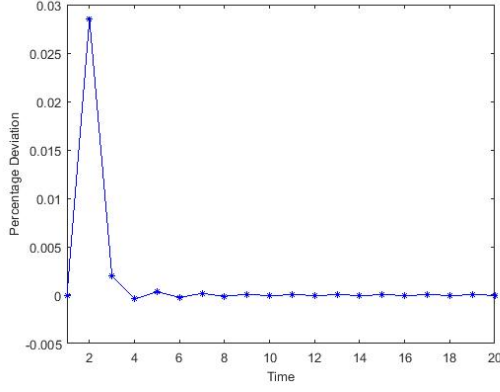
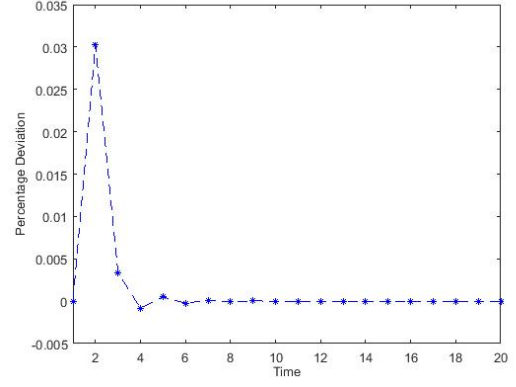


Figure B.2: Aggregate Output, $\varphi = 1.05, 4.5$

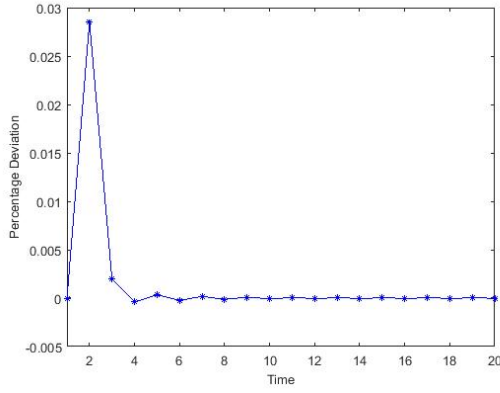


$$\beta = 0.995$$

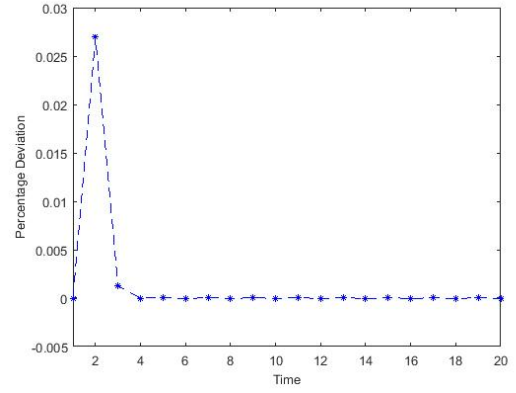


$$\beta = 0.99$$

Figure B.3: Aggregate Output, $\beta = 0.995, 0.99$



$$\zeta = 0.001$$



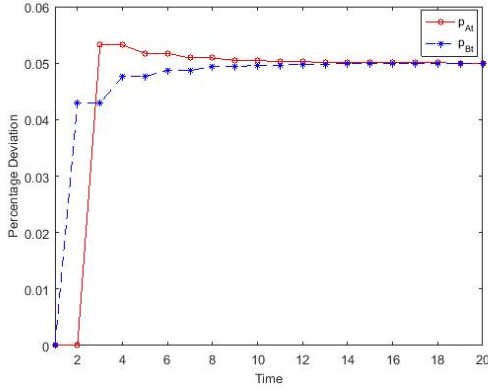
$$\zeta = 0.01$$

Figure B.4: Aggregate Output, $\zeta = 0.001, 0.01$

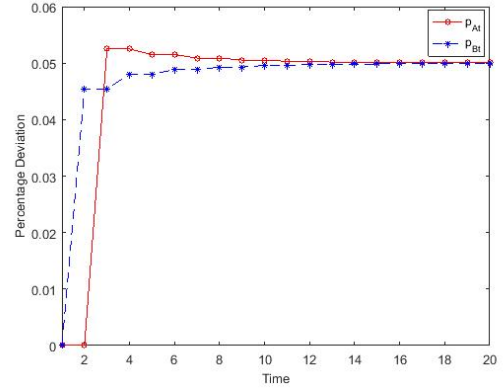
B.7.2 Sectoral Responses to Money Supply Shock

The responses of the sectoral variables due to a money supply shock are provided below. The baseline parameter values are $\theta = 5$, $\varphi = 4.5$, $\beta = 0.99$ and $\zeta = 0.33$.

Changes in values of θ , assuming $\beta = 0.995$, $\varphi = 1.05$, $\zeta = 0.001$

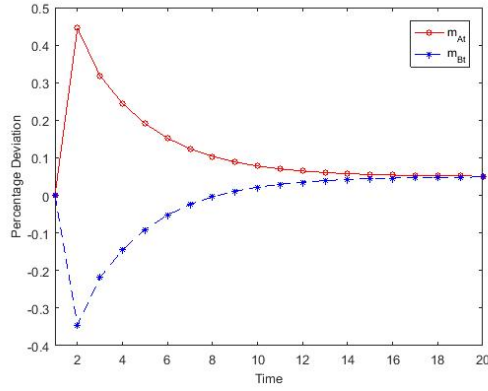


$\theta = 5$

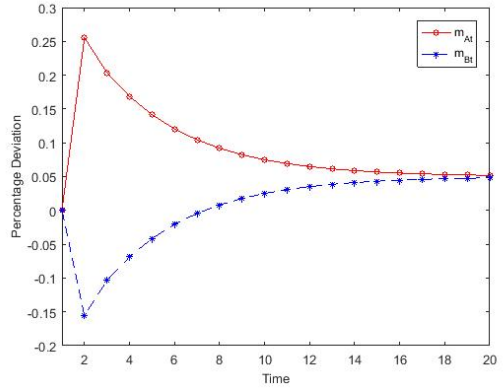


$\theta = 3$

Figure B.5: Sectoral Prices, $\theta = 5, 3$

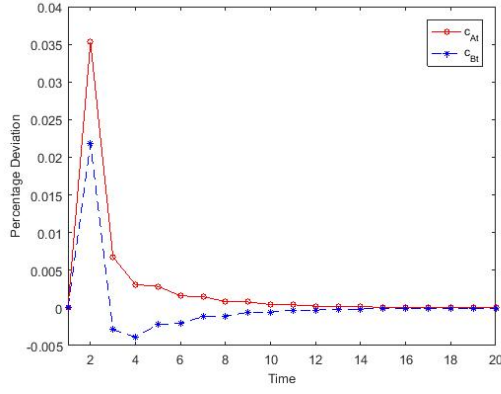


$\theta = 5$

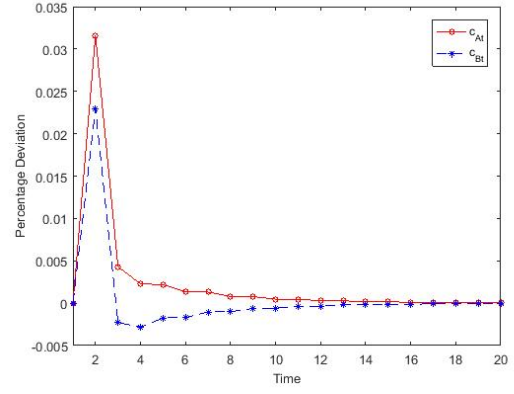


$\theta = 3$

Figure B.6: Sectoral Money Holdings, $\theta = 5, 3$

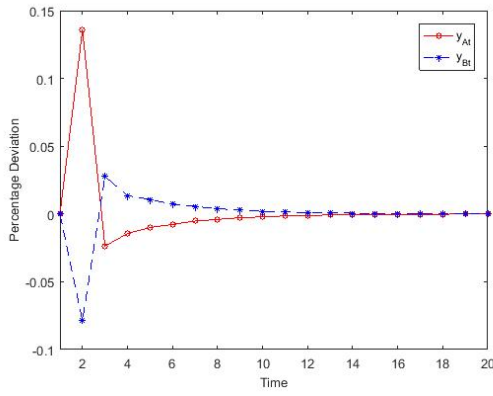


$\theta = 5$

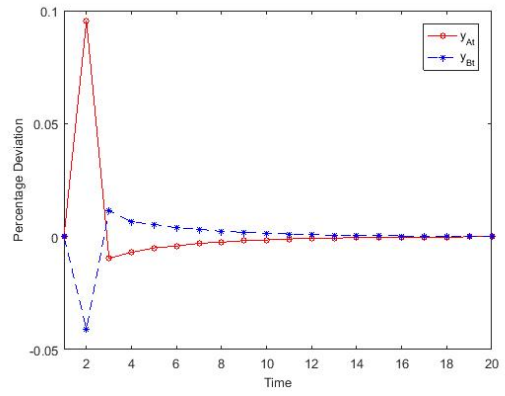


$\theta = 3$

Figure B.7: Sectoral Consumption Levels, $\theta = 5, 3$

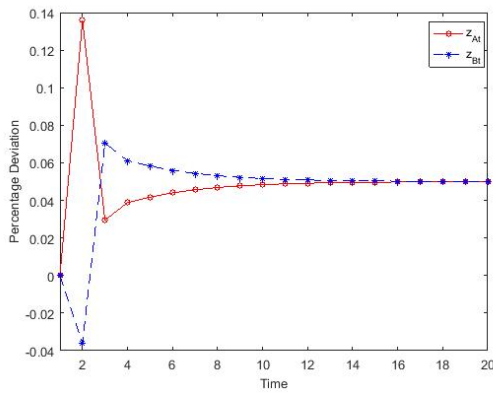


$\theta = 5$

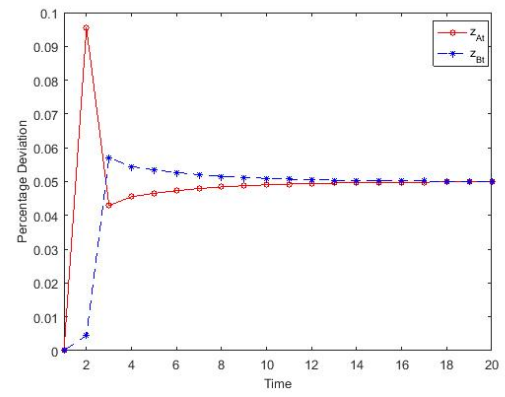


$\theta = 3$

Figure B.8: Sectoral Output, $\theta = 5, 3$



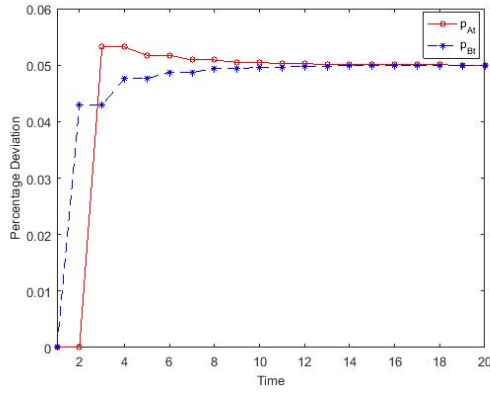
$\theta = 5$



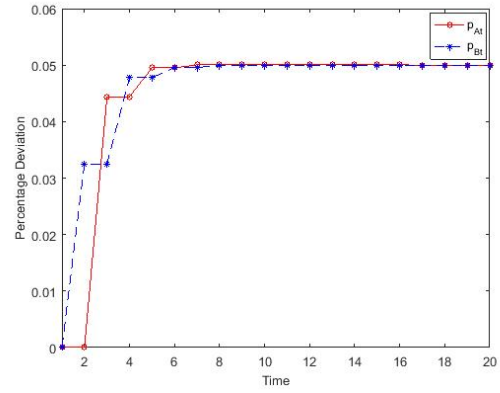
$\theta = 3$

Figure B.9: Sectoral Nominal Revenue, $\theta = 5, 3$

Changes in values of φ , assuming $\beta = 0.995$, $\theta = 5$, $\zeta = 0.001$

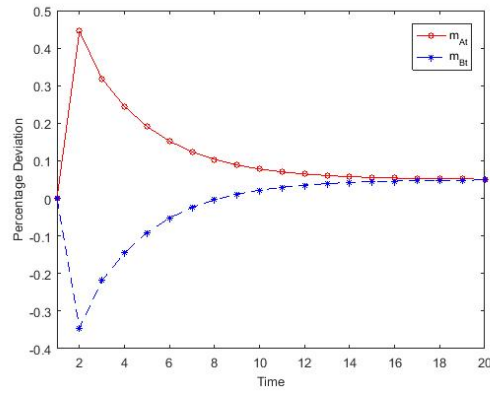


$\varphi = 1.05$

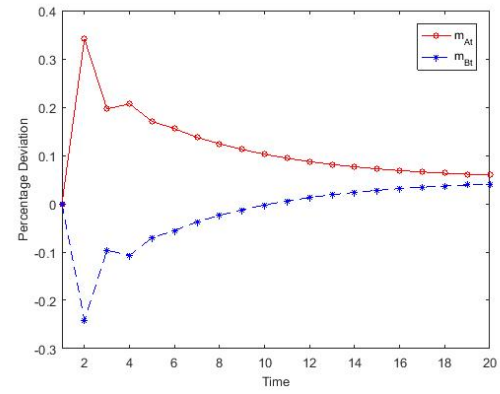


$\varphi = 4.5$

Figure B.10: Sectoral Prices, $\varphi = 1.05, 4.5$

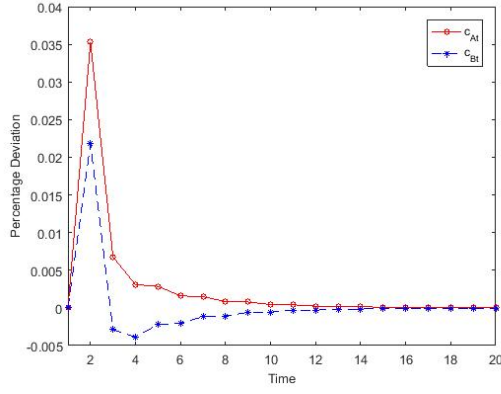


$\varphi = 1.05$

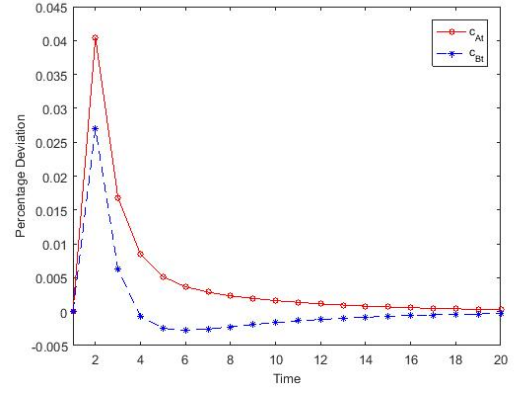


$\varphi = 4.5$

Figure B.11: Sectoral Money Holdings, $\varphi = 1.05, 4.5$

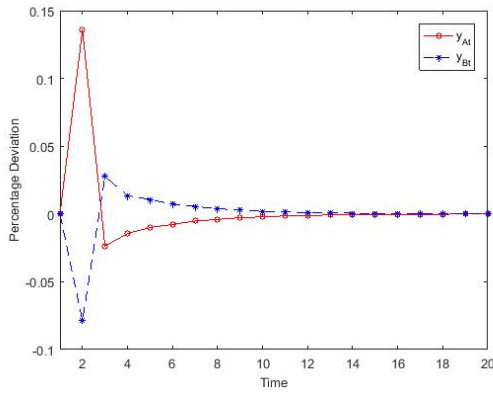


$$\varphi = 1.05$$

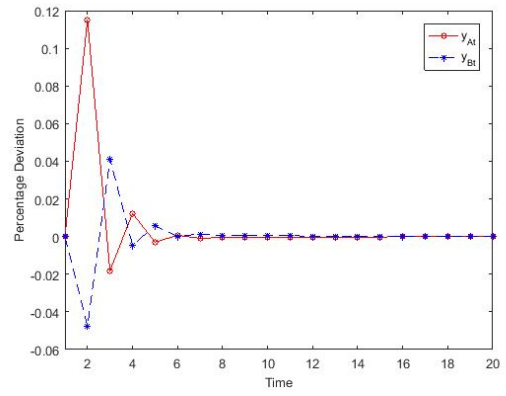


$$\varphi = 4.5$$

Figure B.12: Sectoral Consumption Levels, $\varphi = 1.05, 4.5$

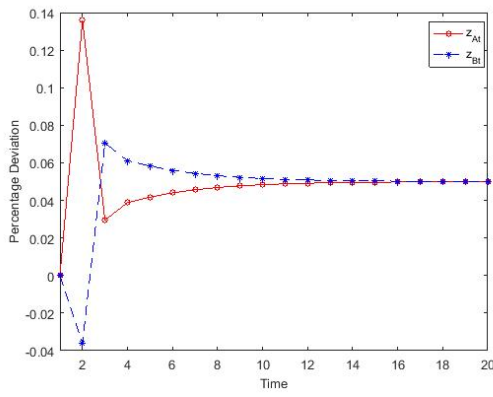


$$\varphi = 1.05$$

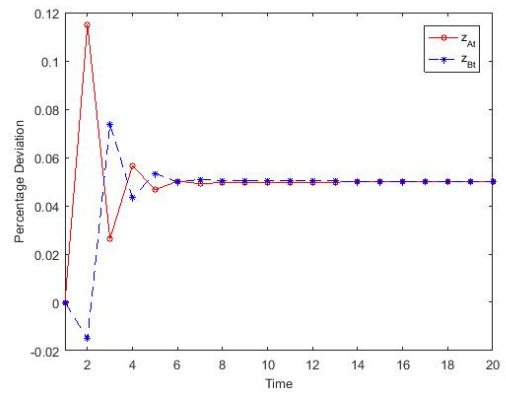


$$\varphi = 4.5$$

Figure B.13: Sectoral Output, $\varphi = 1.05, 4.5$



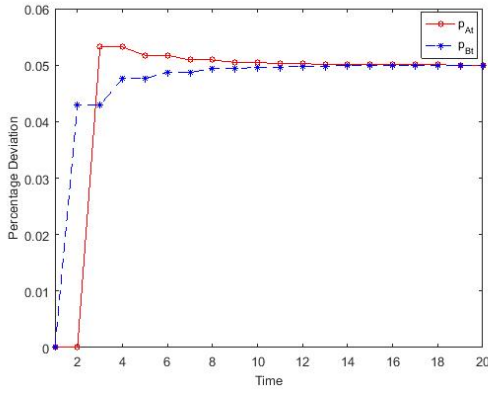
$$\varphi = 1.05$$



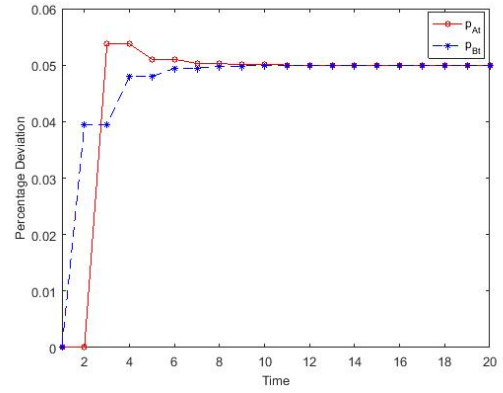
$$\varphi = 4.5$$

Figure B.14: Sectoral Nominal Revenue, $\varphi = 1.05, 4.5$

Changes in values of β , assuming $\theta = 5$, $\varphi = 1.05$, $\zeta = 0.001$

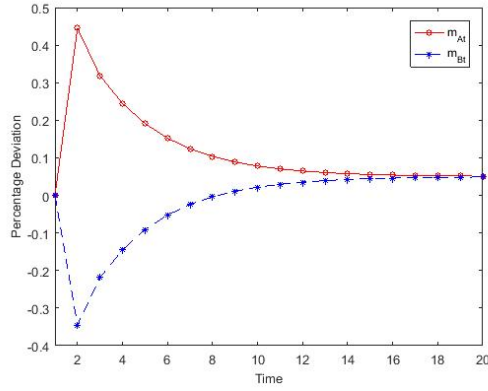


$\beta = 0.995$

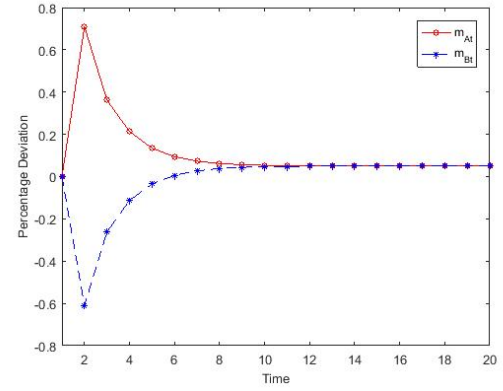


$\beta = 0.99$

Figure B.15: Sectoral Prices, $\beta = 0.995, 0.99$

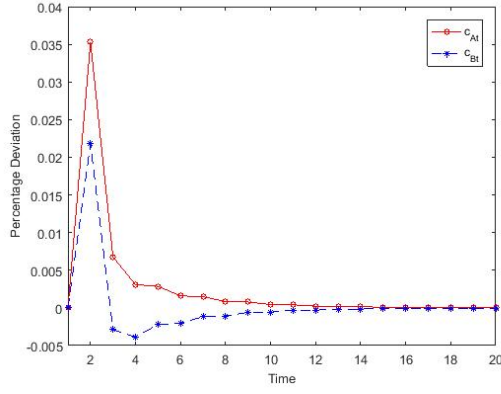


$\beta = 0.995$

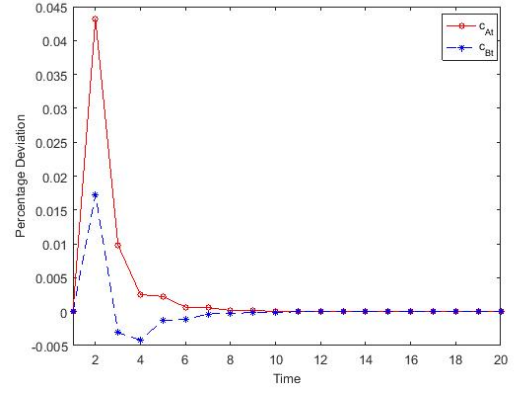


$\beta = 0.99$

Figure B.16: Sectoral Money Holdings, $\beta = 0.995, 0.99$

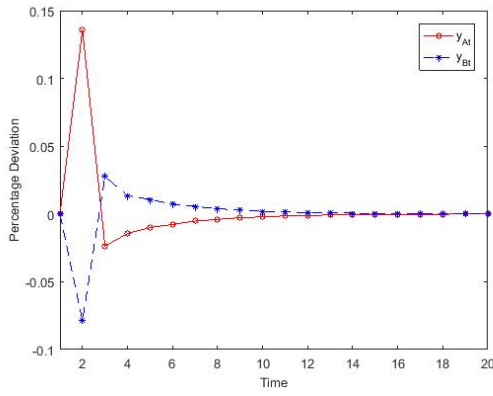


$$\beta = 0.995$$

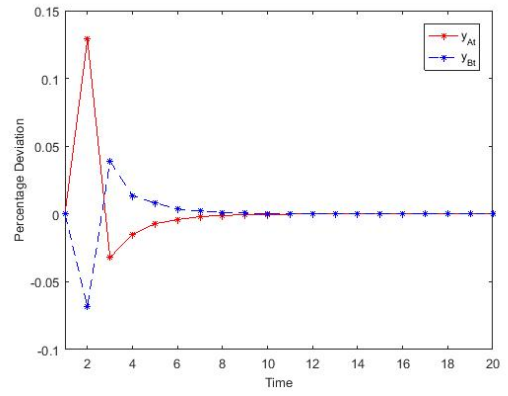


$$\beta = 0.99$$

Figure B.17: Sectoral Consumption Levels, $\beta = 0.995, 0.99$

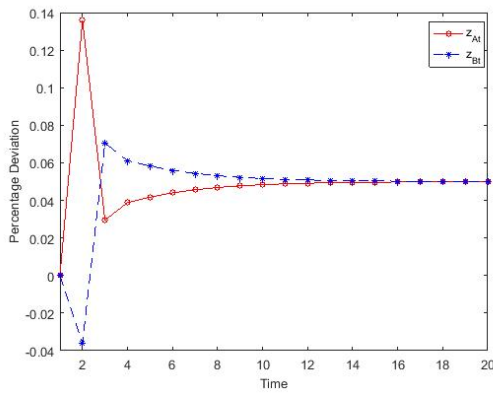


$$\beta = 0.995$$

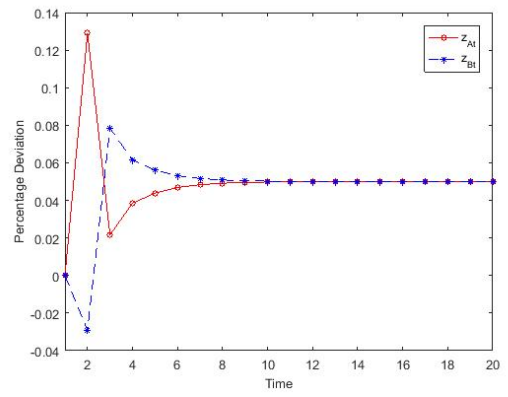


$$\beta = 0.99$$

Figure B.18: Sectoral Output, $\beta = 0.995, 0.99$



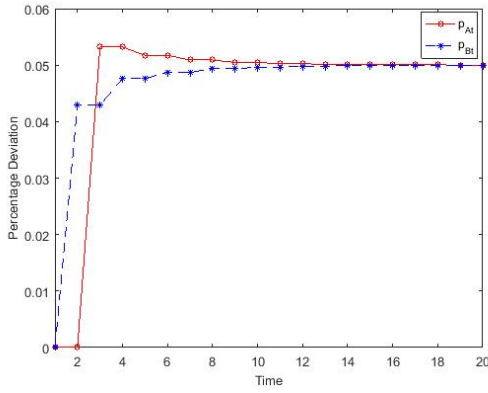
$$\beta = 0.995$$



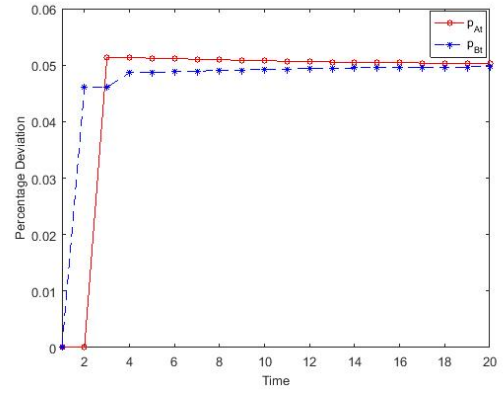
$$\beta = 0.99$$

Figure B.19: Sectoral Nominal Revenue, $\beta = 0.995, 0.99$

Changes in values of ζ , assuming $\theta = 5$, $\varphi = 1.05$, $\beta = 0.995$

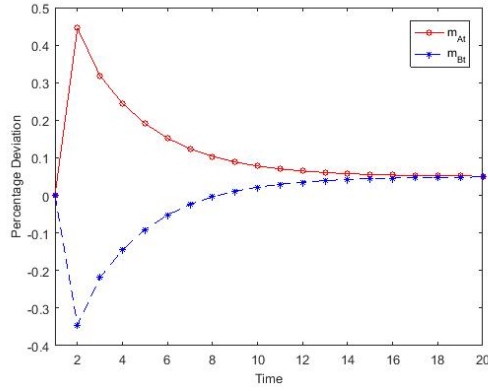


$\zeta = 0.001$

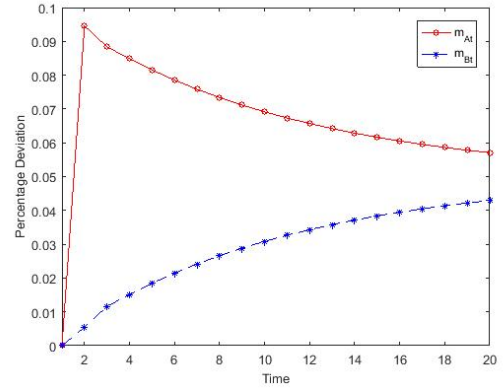


$\zeta = 0.01$

Figure B.20: Sectoral Prices, $\zeta = 0.001, 0.01$

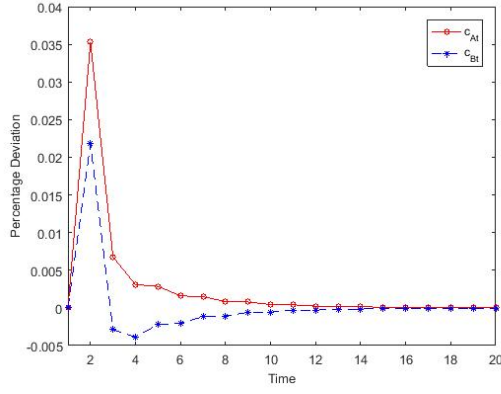


$\zeta = 0.001$

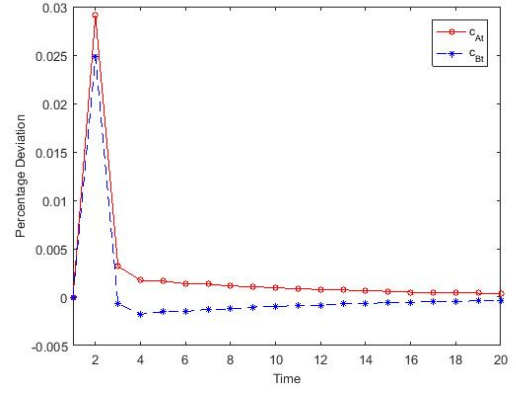


$\zeta = 0.01$

Figure B.21: Sectoral Money Holdings, $\zeta = 0.001, 0.01$

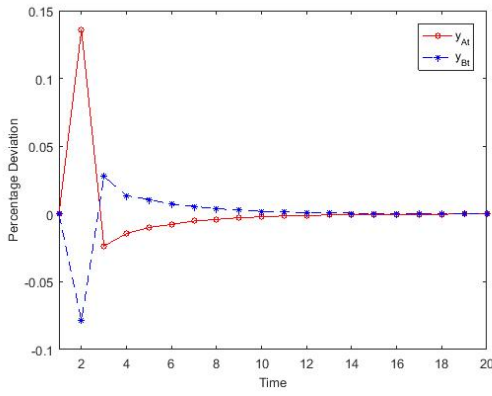


$$\zeta = 0.001$$

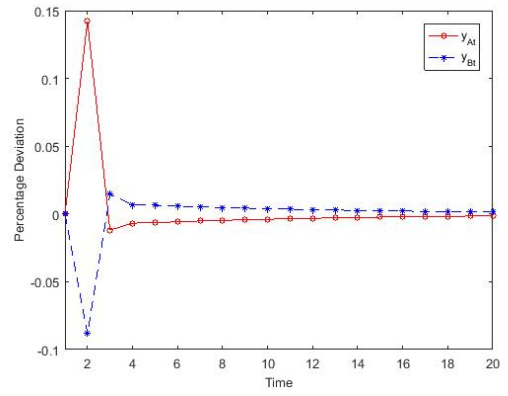


$$\zeta = 0.01$$

Figure B.22: Sectoral Consumption Levels, $\zeta = 0.001, 0.01$

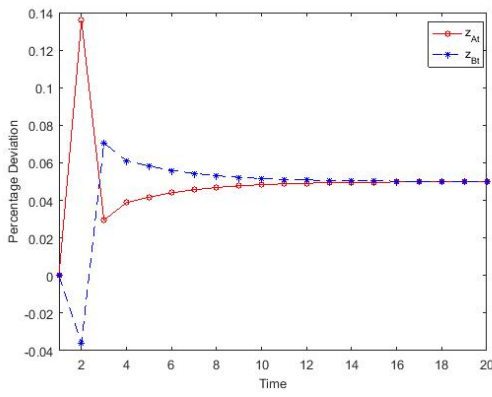


$$\zeta = 0.001$$

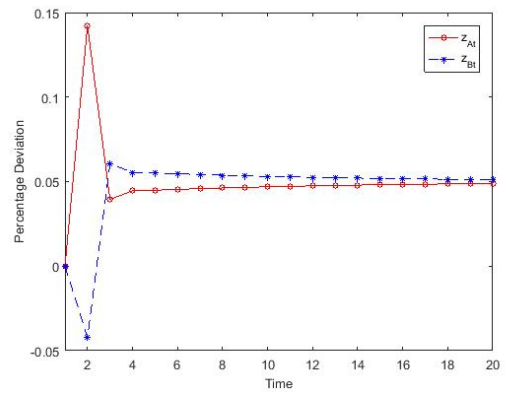


$$\zeta = 0.01$$

Figure B.23: Sectoral Output, $\zeta = 0.001, 0.01$



$$\zeta = 0.001$$



$$\zeta = 0.01$$

Figure B.24: Sectoral Nominal Revenue, $\zeta = 0.001, 0.01$

B.8 Deriving the Steady State Value of Individual Output, Y_i

To find a steady state value of Y_i , we begin by using the reset price:

$$X_t = \left[\frac{1}{\gamma} \left(\frac{d\theta}{\theta - 1} \right) \left(\frac{Y_t^\varphi P_t^{\varphi\theta} + \beta Y_{t+1}^\varphi P_{t+1}^{\varphi\theta}}{\frac{Y_t P_t^{\theta-1}}{C_{it}} + \frac{\beta Y_{t+1} P_{t+1}^{\theta-1}}{C_{it+1}}} \right) \right]^{\frac{1}{1+\theta(\varphi-1)}} \quad (\text{B.58})$$

We can introduce individual output into (B.58) by using the demand function, which yields:

$$X_t = \left[\frac{1}{\gamma} \left(\frac{d\theta}{\theta - 1} \right) \left(\frac{Y_{it}^\varphi P_{it}^{\varphi\theta} + \beta Y_{it+1}^\varphi P_{it+1}^{\varphi\theta}}{\frac{Y_{it} P_{it}^{\theta-1}}{C_{it}} + \frac{\beta Y_{it+1} P_{it+1}^{\theta-1}}{C_{it+1}}} \right) \right]^{\frac{1}{1+\theta(\varphi-1)}} \quad (\text{B.59})$$

For the sake of keeping notations consistent, we replace $X_t = P_{it}$, where, in the steady state, (B.59) reduces to:

$$P_i = \left[\frac{1}{\gamma} \left(\frac{d\theta}{\theta - 1} \right) \left(\frac{Y_i^\varphi P_i^{\varphi\theta} + \beta Y_i^\varphi P_i^{\varphi\theta}}{\frac{Y_i P_i^{\theta-1}}{C_i} + \frac{\beta Y_i P_i^{\theta-1}}{C_i}} \right) \right]^{\frac{1}{1+\theta(\varphi-1)}} \quad (\text{B.60})$$

which, after some simplification, can be written as:

$$P_i^{1+\theta(\varphi-1)} = \frac{1}{\gamma} \frac{d\theta}{\theta - 1} Y_i^{\varphi-1} P_i^{\theta(\varphi-1)} P C_i \quad (\text{B.61})$$

In the steady state, the budget constraint reduces to $P C_i = P_i Y_i$, which can be replaced in (B.61) to derive a steady state value of Y_i :

$$Y_i^\varphi = \frac{\gamma(\theta - 1)}{d\theta} \quad (\text{B.62})$$

B.9 Imperfect Credit Markets: First-Order Analysis of the Sectoral Difference in the Change in Welfare Following a Money Supply Shock

The first-order, log-linear, approximation of agent i 's change in flow utility is:

$$\tilde{u}_{it} = \gamma \tilde{c}_{it} + (1 - \gamma)(\tilde{m}_{it} - \tilde{p}_t) - \gamma \frac{\theta - 1}{\theta} \tilde{y}_{it} \quad (\text{B.63})$$

The first-order, log-linear, approximation to agent i 's change in lifetime utility (discounted to

period 0) is then:

$$\tilde{U}_{i0} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \tilde{u}_{it} \quad (\text{B.64})$$

Now, consider the difference between the utility of an agent in sector A and agent in sector B . The aggregate variable $\tilde{p}_t$ cancels, so:

$$\tilde{u}_{At} - \tilde{u}_{Bt} = \gamma(\tilde{c}_{At} - \tilde{c}_{Bt} + (1 - \gamma)(\tilde{m}_{At} - \tilde{m}_{Bt}) - \gamma \frac{\theta - 1}{\theta}(\tilde{y}_{At} - \tilde{y}_{Bt})) \quad (\text{B.65})$$

We also know that the log-linearised budget constraint of agent i is:

$$\tilde{c}_{it} = \tilde{y}_{it} + \tilde{p}_{it} + \sigma(\tilde{m}_{it-1} - \tilde{m}_{it}) - \tilde{p}_t + \tilde{g}_{it} \quad (\text{B.66})$$

Taking the difference between sectors, aggregate variables cancel, so:

$$\tilde{c}_{At} - \tilde{c}_{Bt} = (\tilde{y}_{At} - \tilde{y}_{Bt}) + (\tilde{p}_{At} - \tilde{p}_{Bt}) + \sigma[(\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) - (\tilde{m}_{At} - \tilde{m}_{Bt})] \quad (\text{B.67})$$

Further, the log-linearised demand function for sector i is:

$$\tilde{y}_{it} = \tilde{y}_t - \theta(\tilde{p}_{it} - \tilde{p}_t) \quad (\text{B.68})$$

Taking the difference between sectors, aggregate variables cancel, so:

$$\tilde{y}_{At} - \tilde{y}_{Bt} = -\theta(\tilde{p}_{At} - \tilde{p}_{Bt}) \quad (\text{B.69})$$

We can use this to eliminate $\tilde{p}_{At} - \tilde{p}_{Bt}$ from the sectorally-differenced budget constraints:

$$\tilde{c}_{At} - \tilde{c}_{Bt} = \frac{\theta - 1}{\theta}(\tilde{y}_{At} - \tilde{y}_{Bt}) + \sigma[(\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) - (\tilde{m}_{At} - \tilde{m}_{Bt})] \quad (\text{B.70})$$

Now, use this to eliminate $\tilde{c}_{At} - \tilde{c}_{Bt}$ from the sectoral flow utility difference, and note that $\tilde{y}_{At} - \tilde{y}_{Bt}$ cancels:

$$\begin{aligned} \tilde{u}_{At} - \tilde{u}_{Bt} &= (1 - \gamma)(\tilde{m}_{At} - \tilde{m}_{Bt}) + \gamma\sigma[(\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) - (\tilde{m}_{At} - \tilde{m}_{Bt})] \\ &= (1 - \gamma - \gamma\sigma)(\tilde{m}_{At} - \tilde{m}_{Bt}) + \gamma\sigma(\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) \end{aligned} \quad (\text{B.71})$$

We have thus shown that sectoral flow utility difference depends only on the current and lagged values of the sectoral money holding differences.

Money market equilibrium, for a constant money supply, implies that:

$$\tilde{m}_{At} - \tilde{m}_{Bt} = 2(\tilde{m}_{At} - \tilde{m}) \quad (\text{B.72})$$

Hence

$$\tilde{u}_{At} - \tilde{u}_{Bt} = (1 - \gamma - \gamma\sigma)2(\tilde{m}_{At} - \tilde{m}) + 2\gamma\sigma(\tilde{m}_{At-1} - \tilde{m}) \quad (\text{B.73})$$

Thus, knowing the time path $\{\tilde{m}_{At}\}_0^\infty$, following a money supply shock is enough to compute the time path of flow utility differences $\{\tilde{u}_{At} - \tilde{u}_{Bt}\}_0^\infty$. Now, we can simplify (B.73):

$$\tilde{u}_{At} - \tilde{u}_{Bt} = (1 - \gamma)(\tilde{m}_{At} - \tilde{m}_{Bt}) - \gamma\sigma[(\tilde{m}_{At} - \tilde{m}_{Bt}) - (\tilde{m}_{At-1} - \tilde{m}_{Bt-1})] \quad (\text{B.74})$$

From this, we can say that there are two influences on the flow utility difference: one is just the difference in current money holdings, which affects utility directly via the presence of real balances in the utility function, and the other is the accumulation of money holdings over the period, which is the ‘saving’ of the agent. More saving leads to lower relative utility in period t .

The numerical simulations of welfare changes, based on first-order approximations, yield the result that the sectoral difference in the changes in lifetime utility is always zero, i.e.

$$\tilde{U}_{A0} - \tilde{U}_{B0} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t (\tilde{u}_{At} - \tilde{u}_{Bt}) = 0 \quad (\text{B.75})$$

We can use (B.74) to show why we observe this. Note that, $\sigma = \frac{\zeta}{1-\beta}$, and $\zeta = \frac{1-\gamma}{\gamma}$. Thus $\sigma\gamma = \frac{1-\gamma}{1-\beta}$. Using this, we get:

$$\begin{aligned} \tilde{u}_{At} - \tilde{u}_{Bt} &= (1 - \gamma) \left\{ (\tilde{m}_{At} - \tilde{m}_{Bt}) - \frac{1}{1 - \beta} [(\tilde{m}_{At} - \tilde{m}_{Bt}) - (\tilde{m}_{At-1} - \tilde{m}_{Bt-1})] \right\} \\ &= (1 - \gamma) \left\{ -\frac{\beta}{1 - \beta} (\tilde{m}_{At} - \tilde{m}_{Bt}) + \frac{1}{1 - \beta} (\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) \right\} \\ &= \frac{1 - \gamma}{1 - \beta} [-\beta(\tilde{m}_{At} - \tilde{m}_{Bt}) + (\tilde{m}_{At-1} - \tilde{m}_{Bt-1})] \end{aligned} \quad (\text{B.76})$$

Now, substitute this into the lifetime utility formula above:

$$\tilde{U}_{A0} - \tilde{U}_{B0} = (1 - \gamma) \left[-\beta \sum_{t=0}^{\infty} \beta^t (\tilde{m}_{At} - \tilde{m}_{Bt}) + \sum_{t=0}^{\infty} \beta^t (\tilde{m}_{At-1} - \tilde{m}_{Bt-1}) \right] \quad (\text{B.77})$$

Note that the second of these discounted sums first uses the same sequence of the $\tilde{m}_{At} - \tilde{m}_{Bt}$ variables as the first, except that it is lagged by one period. To see better the pattern, we can

expand each sum:

$$\begin{aligned} \tilde{U}_{A0} - \tilde{U}_{B0} = (1 - \gamma) \{ & -\beta[(\tilde{m}_{A0} - \tilde{m}_{B0}) + \beta(\tilde{m}_{A1} - \tilde{m}_{B1}) + \beta^2(\tilde{m}_{A2} - \tilde{m}_{B2}) + \dots] \\ & [(\tilde{m}_{A,-1} - \tilde{m}_{B,-1}) + \beta(\tilde{m}_{A0} - \tilde{m}_{B0}) + \beta^2(\tilde{m}_{A1} - \tilde{m}_{B1}) + \dots] \} \end{aligned} \quad (\text{B.78})$$

It is assumed that $t = 0$ is the impact period of the shock. Thus $t = -1$ is the period just before the shock, when the economy is in the reference steady state where money holdings in both sectors are equal. It then follows that $\tilde{m}_{A,-1} - \tilde{m}_{B,-1} = 0$. Hence $\tilde{U}_{A0} - \tilde{U}_{B0} = 0$.

B.10 Validity of Using a Second-Order Approximation of the Utility Function but only a First-Order Approximation of the Structural Model

Let $U = U(\ln C)$, where $U'' \neq 0$ and $\ln C = F(\ln M)$, where $F'' \neq 0$ ('structural model')

A second-order approximation of the utility function is:

$$U \simeq U(\ln \bar{C}) + U'(\ln \bar{C})\tilde{c} + \frac{1}{2}U''(\ln \bar{C})\tilde{c}^2, \text{ where } \tilde{c} \equiv \ln C - \ln \bar{C} \quad (\text{B.79})$$

A second-order approximation of the structural equation is:

$$\tilde{c} \simeq F'(\ln \bar{M})\tilde{m} + \frac{1}{2}F''(\ln \bar{M})\tilde{m}^2, \text{ where } \tilde{m} \equiv \ln M - \ln \bar{M} \quad (\text{B.80})$$

When we substitute the equation for $\tilde{c}$ into the equation for U and simplify, we ignore terms which are of order higher than second in $\tilde{m}$. Thus, the expression for $\tilde{c}^2$ becomes:

$$\tilde{c}^2 = [F'(\ln \bar{M})]^2 \tilde{m}^2 \quad (\text{B.81})$$

i.e. the terms involving $F''(\ln \bar{M})$ drop out. As far as the expression for $\tilde{c}^2$ is concerned, the fact that $F'' \neq 0$ is therefore irrelevant to the second-order approximation of the change in U .

On the other hand, the place where F'' will not drop out is when we substitute out $\tilde{c}$ (as opposed to $\tilde{c}^2$) in the equation for U . We, thus, obtain, as our second-order approximation to the change in U :

$$U - U(\ln \bar{C}) \simeq U'(\ln \bar{C})F'(\ln \bar{M})\tilde{m} + \frac{1}{2} \{ U'(\ln \bar{C})F''(\ln \bar{M}) + U''(\ln \bar{C})[F'(\ln \bar{M})]^2 \} \tilde{m}^2 \quad (\text{B.82})$$

This shows that the full second-order approximation to the change in U does, in general, partly depend on the fact that the approximation to the structural model is second-order, not

first-order, because it depends on F'' , which is absent in a purely first-order approximation of the structural model.

However, we can also see that there is a special case in which the fact that $F'' \neq 0$ does not matter. This is where $U'(\ln \bar{C}) = 0$. If this holds, F'' drops out from the coefficient on $\tilde{m}^2$. It is also clear, however, that if $U'(\ln \bar{C}) = 0$, then the first-order effect of $\ln M$ on U is zero. This is therefore a particular case: only when the policy change has no first-order effect on utility can we calculate the second-order effect by using a first-order approximation to the structural model.

B.11 Perfect Credit Markets: First-Order Analysis of the Sectoral Difference in the Change in Welfare Following a Money Supply Shock

The first-order, log-linear approximation of agent i 's change in flow utility is:

$$\tilde{u}_{it} = \gamma \tilde{c}_{it} + (1 - \gamma)(\tilde{m}_{it} - \tilde{p}_t) - \gamma \frac{\theta - 1}{\theta} \tilde{y}_{it} \quad (\text{B.83})$$

The difference in utility between the two sectors can be written as:

$$\tilde{u}_{At} - \tilde{u}_{Bt} = \gamma(\tilde{c}_{At} - \tilde{c}_{Bt} + (1 - \gamma)(\tilde{m}_{At} - \tilde{m}_{Bt} - \gamma \frac{\theta - 1}{\theta}(\tilde{y}_{At} - \tilde{y}_{Bt})) \quad (\text{B.84})$$

In PCM, agents can borrow or lend and take out insurance to equate consumption and money holdings, which means that the difference in utility in the two sectors is determined by the difference in output between the sectors:

$$\tilde{u}_{At} - \tilde{u}_{Bt} = -\gamma \frac{\theta - 1}{\theta} (\tilde{y}_{At} - \tilde{y}_{Bt}) \quad (\text{B.85})$$

Using the demand functions, the difference in output between the sectors can be written as:

$$\tilde{y}_{At} - \tilde{y}_{Bt} = -\theta(\tilde{p}_{At} - \tilde{p}_{Bt}) \quad (\text{B.86})$$

Then, we can write the difference in utility between the sectors as:

$$\tilde{u}_{At} - \tilde{u}_{Bt} = \gamma(\theta - 1)(\tilde{p}_{At} - \tilde{p}_{Bt}) \quad (\text{B.87})$$

Further, the difference in lifetime utility between the sectors can be written as, and then

expanded:

$$\begin{aligned}
\tilde{U}_{A0} - \tilde{U}_{B0} &= \gamma(\theta - 1) \sum_{t=0}^{\infty} [-\beta]^t (\tilde{x}_{t-1} - \tilde{x}_t) \\
\tilde{U}_{A0} - \tilde{U}_{B0} &= \gamma(\theta - 1) [(\tilde{x}_{-1} - \tilde{x}_0 + \beta(\tilde{x}_1 - \tilde{x}_0) + \beta^2(\tilde{x}_1 - \tilde{x}_2) + \beta^3(\tilde{x}_3 - \tilde{x}_2) + \dots)] \\
&= \gamma(\theta - 1) \{[-(\tilde{x}_0 - \tilde{x}_{-1} + \beta(\tilde{x}_1 - \tilde{x}_0))] + \beta^2[-(\tilde{x}_2 - \tilde{x}_1) + \beta(\tilde{x}_3 - \tilde{x}_2)] + \dots\} \quad (\text{B.88})
\end{aligned}$$

Now, along the transition path, $\tilde{x}_t$ rises monotonically and at an ever-decreasing rate. Thus, $\tilde{x}_t - \tilde{x}_{t-1}$ is always positive, but it is always decreasing. It then follows that such term $[\cdot]$ inside $\{\cdot\}$ above is negative, for e.g. $-(\tilde{x}_0 - \tilde{x}_{-1}) + \beta(\tilde{x}_1 - \tilde{x}_0) < 0$, since $\tilde{x}_0 - \tilde{x}_{-1} > \beta(\tilde{x}_1 - \tilde{x}_0)$ and moreover $\beta < 1$. Therefore, $\tilde{U}_{A0} - \tilde{U}_{B0} < 0$.

Appendix C

Appendix to Chapter 4

C.1 Macroeconomic and Sectoral Identities

To study the behaviour of the macroeconomic variables, we take a log-linear approximation around the zero inflation steady state (ZISS). The log-linearised equations for the macroeconomic and sectoral identities are given below, where lower-case letters with a tilde represent log-deviations of variables from their steady state values, i.e. $\tilde{v}_t \equiv \ln\left(\frac{V_t}{V}\right)$, and V_t denotes any variable and V is its steady state value.

Price Level in Sticky-Price Sector:

$$\tilde{p}_{S,t} = \frac{1}{2}(\tilde{x}_t + \tilde{x}_{t-1}) \quad (\text{C.1})$$

Output Levels in Sticky-Price Sector:

$$\tilde{y}_{S,t} = \frac{1}{2}(\tilde{y}_{S,t} + \tilde{y}_{S,Nt}) \quad (\text{C.2})$$

where:

$$\tilde{y}_{S,t} = -\theta(\tilde{x}_t - \tilde{p}_{S,t}) + \tilde{y}_{S,t}; \quad \tilde{y}_{S,Nt} = -\theta(\tilde{x}_{t-1} - \tilde{p}_{S,t}) + \tilde{y}_{S,t} \quad (\text{C.3})$$

Total Demand of Sticky- and Flexible-Price Goods:

$$\tilde{y}_{S,t} = -\zeta(\tilde{p}_{S,t} - \tilde{p}_t) + \tilde{y}_t; \quad \tilde{y}_{F,t} = -\zeta(\tilde{p}_{F,t} - \tilde{p}_t) + \tilde{y}_t \quad (\text{C.4})$$

Aggregate Price Level:

$$\tilde{p}_t = \frac{1}{2}(\tilde{p}_{F,t} + \tilde{p}_{S,t}) \quad (\text{C.5})$$

Aggregate Output and Consumption Levels:

$$\tilde{y}_t = \frac{1}{2}(\tilde{y}_{F,t} + \tilde{y}_{S,t}); \quad \tilde{c}_t = \frac{1}{2}(\tilde{c}_t^F + \tilde{c}_t^S) \quad (\text{C.6})$$

Aggregate Money Demand:

$$\tilde{m}_t = \frac{1}{2}(\tilde{m}_t^F + \tilde{m}_t^S) \quad (\text{C.7})$$

Aggregate Resource Constraint:

$$\tilde{c}_t = \tilde{y}_t \quad (\text{C.8})$$

C.2 Supplementary Tables and Figures

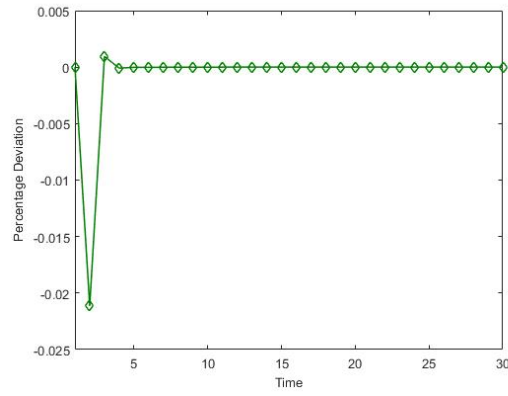
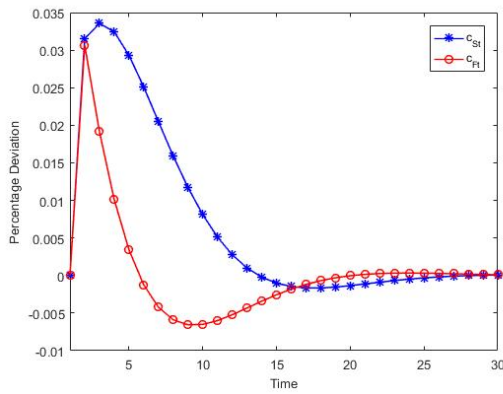
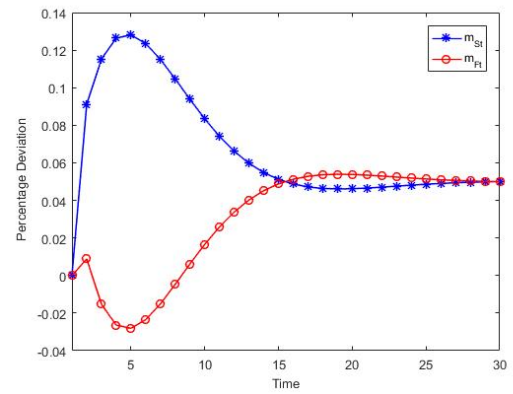


Figure C.1: Nominal Interest Rate: IRR

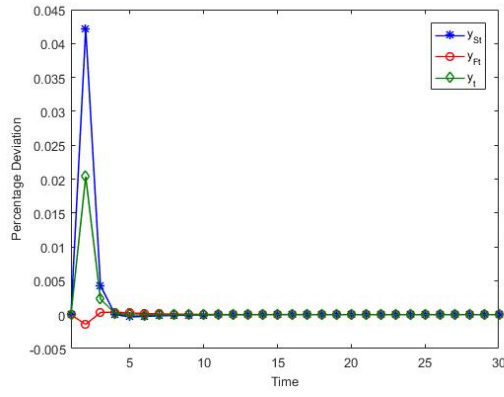


Sectoral Consumption Levels

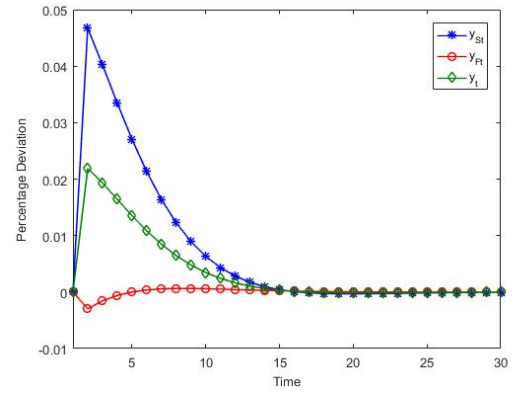


Sectoral Money Holdings

Figure C.2: Change in d to 0.01 under a MSR

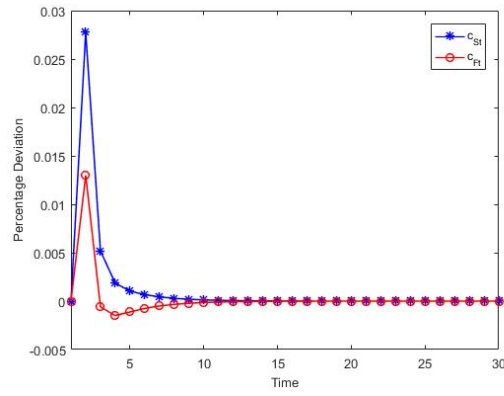


Sectoral Output Levels: IRR

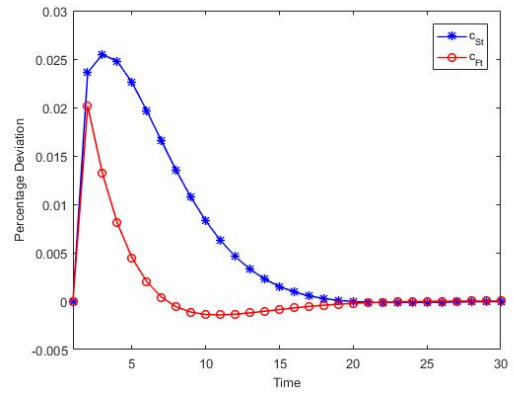


Sectoral Output Levels: MSR

Figure C.3: Change in ζ to 3 under the monetary policy rules

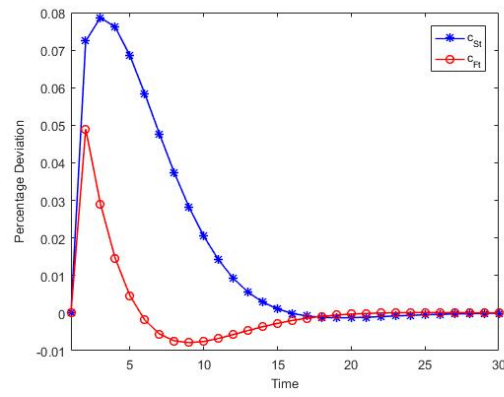


Sectoral Consumption Levels: IRR

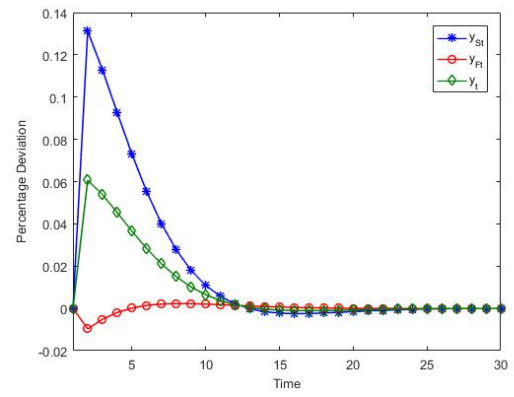


Sectoral Consumption Levels: MSR

Figure C.4: Change in ζ to 3 under the monetary policy rules

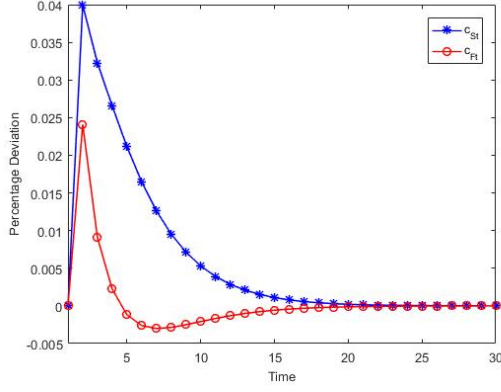


Sectoral Consumption Levels

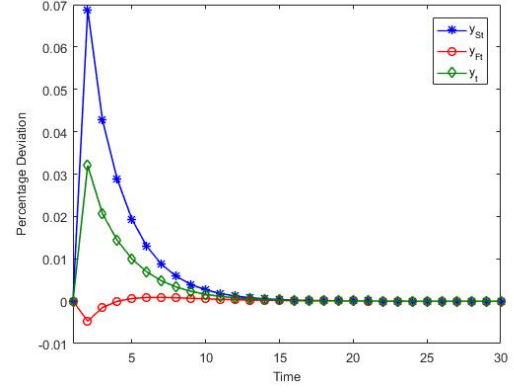


Sectoral Output Levels

Figure C.5: Change in money supply rule, including persistence parameter

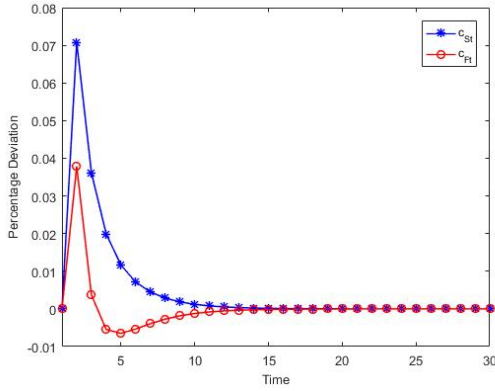


Sectoral Consumption Levels

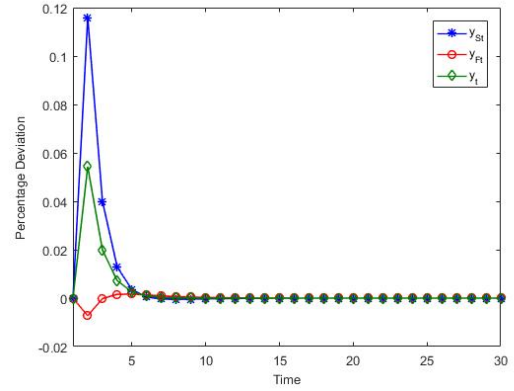


Sectoral Output Levels

Figure C.6: Change in interest rate rule, including persistence parameter



Sectoral Consumption Levels



Sectoral Output Levels

Figure C.7: Change in interest rate rule: Equation (4.33)

C.3 Flexible- and Sticky-Price Sectors with Perfect Credit Markets

There are two sectors in the economy, flexible- and sticky-price sectors. The prices of goods in the flexible-price sector, which are food products, adjust frequently. In the sticky-price sector, which comprise non-food products, prices are adjusted slowly. Moreover, within the sticky-price sectors, there are two sub-sectors, A and B . There is a continuum of flexible-price goods, and of sticky-price goods, which are defined below.

C.3.1 Households

The economy is composed of infinitely-lived households, where households have equal shares in the firms in their respective sectors in the economy. Households provide labour to the respective

sectors and sub-sectors and it is assumed that labour is immobile across the flexible- and sticky-price sectors, and within the sub-sectors in the sticky-price sector. More specifically, there are three labour markets in this economy: households in the flexible-price sector provide labour to this sector, whereas households in sub-sectors A and B provide labour to their respective sub-sectors. This is assumed because of the different skills that are required in the (sub-)sectors in the economy. Households in the economy have identical initial wealth, and as asset markets are complete, households have identical consumption plans. The representative household in a sector, denoted by i is indexed by F (flexible-price sector) and S (sticky-price sector), where S can be either A or B . Household i maximises the discounted stream of utility:

$$\max_{\{C_t^i, M_t^i/P_t, N_t^i\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^i)^{1-\sigma}}{1-\sigma} + d \ln(M_t^i/P_t) - \frac{(N_t^i)^{1+\chi}}{1+\chi} \right) \right] \quad (\text{C.9})$$

where σ is the risk aversion parameter, d is the weight placed by households on real money balances, and χ is the inverse of the Frisch elasticity of labour supply. C_t^i is an index of household i 's consumption of the continuum of flexible-price goods, $C_{F,t}^i$, and of sticky-price goods, $C_{S,t}^i$, in period t , which are defined as, respectively:

$$C_t^i = \left[\gamma^{\frac{1}{\zeta}} (C_{F,t}^i)^{1-\frac{1}{\zeta}} + (1-\gamma)^{\frac{1}{\zeta}} (C_{S,t}^i)^{1-\frac{1}{\zeta}} \right]^{\frac{1}{1-\frac{1}{\zeta}}} \quad (\text{C.10})$$

where

$$C_{F,t}^i = \left[\int_0^1 (C_{Fk,t}^i)^{\frac{\epsilon-1}{\epsilon}} dk \right]^{\frac{\epsilon}{\epsilon-1}} \quad (\text{C.11})$$

$$C_{S,t}^i = \left[\int_0^1 (C_{Sj,t}^i)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (\text{C.12})$$

and $\gamma \in [0, 1]$ is the weight on food in the consumption index, $\zeta > 1$ is the elasticity of substitution between flexible- and sticky-price goods, $\epsilon > 1$ is the elasticity of substitution between flexible-price goods and $\theta > 1$ is the elasticity of substitution between sticky-price goods. Moreover, we assume that $\epsilon = \theta$. The aggregate price level, P_t , is defined as:

$$P_t = \left[\gamma (P_{F,t})^{1-\zeta} + (1-\gamma) (P_{S,t})^{1-\zeta} \right]^{\frac{1}{1-\zeta}} \quad (\text{C.13})$$

where $P_{F,t}$ is the Dixit-Stiglitz price index of flexible-price goods, given by:

$$P_{F,t} = \left[\int_0^1 (P_{Fk,t})^{1-\epsilon} dk \right]^{\frac{1}{1-\epsilon}} \quad (\text{C.14})$$

and $P_{S,t}$ is the Dixit-Stiglitz price index of sticky-price goods, given by:

$$P_{S,t} = \left[\int_0^1 (P_{Sj,t})^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (\text{C.15})$$

where $P_{Fk,t}$ and $P_{Sj,t}$ are the prices of flexible- and sticky-price goods indexed on k and j respectively. Households in the sectors face the budget constraint:

$$P_t C_t^i + M_t^i + Q_t B_t^i = W_{z,t}^i N_{z,t}^i + M_{t-1}^i + B_{t-1}^i + H_t^i + G_t + \Pi_{z,t}^i, \quad t = 0, \dots, \infty \quad (\text{C.16})$$

where $W_{z,t}^i$ is the wage set in the flexible-and sticky-price sectors, N_t^i is the labour supply in the sectors and G_t is government transfer. Households in the sectors maximise the utility function subject to the budget constraint, which can be used to derive the Euler equation:

$$\frac{1}{I_t} = \beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \quad (\text{C.17})$$

where I_t is the gross nominal interest rate. Further, we can derive the consumption-labour trade-off, assuming flexible wages in perfectly competitive labour markets in the flexible- and sticky-price sectors:

$$\frac{W_t^i}{P_t} = \frac{(N_t^i)^\chi}{(C_t^i)^{-\sigma}} \quad (\text{C.18})$$

C.3.2 Firms

C.3.2.1 Flexible-Price Sector

Firms in the flexible-price sector use only labour for production:

$$Y_{F,t} = (N_t^F)^\alpha \quad (\text{C.19})$$

where $0 < \alpha \leq 1$ is the degree of diminishing returns to labour. Firms in this sector set price, $P_{F,t}$, which can be derived as:

$$P_{F,t} = \mu MC_{F,t} \quad (\text{C.20})$$

where $\mu = \frac{\epsilon}{\epsilon-1}$ and

$$MC_{F,t} = \frac{1}{\alpha} \frac{W_t^F}{(N_t^F)^{\alpha-1}} \quad (\text{C.21})$$

C.3.2.2 Sticky-Price Sector

Firms in the sticky-price sector (which includes the sub-sectors A and B) use labour for production:

$$Y_{Sz,t} = (N_{z,t}^S)^\alpha \quad (\text{C.22})$$

where $z = A, B$, and refers to the sub-sectors within the sticky-price sector. The marginal cost, $MC_{Sz,t}$, for firms in this sector can be derived as:

$$MC_{Sz,t} = \frac{1}{\alpha} \frac{W_{z,t}^S}{(N_{z,t}^S)^{\alpha-1}} \quad (\text{C.23})$$

We introduce staggered prices in this sector following Taylor (1979). The two sub-sectors, A and B , are of equal size, with firms in the sub-sectors setting prices in a staggered manner. Firms in a particular sub-sector set prices at time t for t and $t + 1$, which means that prices are fixed for two consecutive periods.

Formally, we assume: sub-sector A , refers to the sub-sector in which firms reset prices in even-numbered periods, and sub-sector B , refers to the sub-sector in which firms reset prices in odd-numbered periods. We define X_t as the ‘reset’ price, that is, the price that firm i would set in period t , if allowed to reset price in that period (and which would also remain in force in period $t + 1$).¹ It follows then, in even-numbered periods, for instance, at time t : $P_{S,t}^i = P_{S,t+1}^i = X_t$, where $i = A$, and sub-sector B is locked into the price they set one period before so $P_{S,t}^i = P_{S,t-1}^i = X_{t-1}$, where $i = B$. Therefore, $X_t, X_{t+2}, \dots$ are prices fixed by sub-sector A , and $X_{t-1}, X_{t+1}, \dots$ are prices set by sub-sector B . For the reset price, we can use the same method as in Chapter 3 and derive the reset price as:

$$X_t = \left(\frac{\theta}{\theta - 1} \right) \left(\frac{P_{S,t}^{\theta-\zeta} P_t^{\zeta+1} Y_t MC_{Sj,t}^r + \beta P_{S,t+1}^{\theta-\zeta} P_{t+1}^{\zeta+1} Y_{t+1} MC_{Sj,t+1}^r}{P_{S,t}^{\theta-\zeta} P_t^\zeta Y_t + P_{S,t+1}^{\theta-\zeta} P_{t+1}^\zeta Y_{t+1}} \right) \quad (\text{C.24})$$

where $MC_{Sj,t}^r = \frac{MC_{Sj,t}}{P_t}$ is the real marginal cost, and $\frac{\theta}{\theta-1}$ is the constant mark-up over marginal cost. Further, the price index in the sticky-price sector can be written as:

$$P_{S,t} = \left(\frac{1}{2} X_t^{1-\theta} + \frac{1}{2} X_{t-1}^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad (\text{C.25})$$

where the price level at time t in the sticky-price sector is a function of the reset prices in periods t and $t - 1$.

¹Due to the assumption of symmetry, all firms will choose the same reset price, so we drop the ‘i’ subscript.

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