Modelling Hydrological, Ecological and Economic Interactions in River Floodplains

A Case Study of the Ouse Catchment (North Yorkshire, England)

Thesis submitted for the degree of

Ph.D

in

Environmental Economics and Environmental Management

by

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Abstract

This thesis answers the following questions. (i) What are the relationships between the values and functions of natural river floodplains? (ii) Under the complicated trade-offs among direct-use and indirect-use values, how should we use river floodplains? (iii) What are the institutions and incentives that are necessary for the optimal management?

In this thesis, we (1) define the appropriate social optimisation problem for floodplain management, (2) provide theoretical models for the static and dynamic problems, (3) develop an applied model and calibrate parameter values from data on the Ouse catchment, and (4) carry out simulations in the context of the Ouse catchment in order to evaluate several policy scenarios. The thesis attempts to make three main contributions. First, it has tried to improve understanding the essential problems of floodplain management (two types of environmental externalities). Second, it has tried to clarify the policy options for the optimal floodplain management. Third, it has explored methods for integrating the hydrology, ecology and economics of floodplains.

The crucial point is that we must take account of environmental externalities. There are two types of externalities. First, the development of floodplains has opportunity costs in terms of lost ecosystem services. Second, the development of floodplains increases flood risks to people downstream (imposes a unidirectional spatial externality). In policy simulations, we obtain the three main results. First, the impact of floodplain development on the expected cost of flood risk is substantial as compared with prices of developed lands, which implies the importance of relevant floodplain management. Second, based on an empirical analysis, floodplains in upstream zones currently tend to be overdeveloped because of unidirectional spatial externalities. Third, price policies relatively function well to internalise external costs and achieve the optimal path, and are robust to irreversibility and uncertainty.

List of Contents

Chapter 3. Static Decision Model of River Floodplains

Chapter 4. Dynamic Decision Model of River Floodplains

Chapter 5. Applied Simulation Model and Calibration: Hydrological Modelling on HEC-HMS

Chapter 6. Policy Simulation

Chapter 7. Conclusion

List of Tables

Chapter 6. Policy Simulation

Appendix C Appendices of Chapter 5

List of Figures

Chapter 4. Dynamic Decision Model of River Floodplains

Chapter 5. Applied Simulation Model and Calibration: Hydrological Modelling on HEC-HMS

Chapter 6. Policy Simulation

Appendix C Appendices of Chapter 5

List of Mathematical Notation

a^i : area potentially protected by averting behaviour (scale of averting behaviour).

a_t^i or $a^i(t)$: change in the scale of averting behaviour in time t.

A_t^i or $A^i(t)$: scale of averting behaviour in time t.

$B(\cdot)$: benefit function of ecosystem services.

$C^i(x^i, a^i, \mathbf{q}(x^j, a^j))$: expected cost function of flood risk.

$C\{X^i, A^i, \mathbf{h}(X^j, A^j)\}$: expected cost function of flood risk in zone i.

$D(\cdot)$: direct cost function of floodplain development and restoration.

$f(\cdot)$: direct net benefit function of floodplain development.

$F(\cdot)$: benefit function of developed floodplains.

$g(\cdot)$: cost function of averting behaviour.

$G(\cdot)$: cost function of averting behaviour.

$h(X^j, A^j)$: function of flood risk that upstream zones j impose on zone i.

i : zone number.

L_F^i : size of floodplains which include natural and developed floodplains.

m : total number of zones.

$M(\cdot)$: cost function of operation and maintenance.

$q(x^j, a^j)$: function of flood risk that upstream zones j impose on zone i.

x^i : size of developed floodplains (the size of development in floodplains).

X_t^i or $X^i(t)$: size of developed floodplains in time t.

$U(\cdot)$: utility function.

y_t^i or $y^i(t)$: change in the size of developed floodplains in time t.

$\pi(\cdot)$: aggregate net benefit function.

δ : discount rate.

ρ : discount factor

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Author's Declaration

I declare that the work included in this thesis is my own, although I did it under the supervision of Professor Charles Perrings. It has not been admitted for any other degree or award. All errors and omissions are my responsibility.

Koichiro Mori

Dedication

To my wife

Aki Mori

with gratitude

Chapter 1

Introduction

1.1 Research Issue

"All our activities are dependent ultimately on resources found in Nature" (Dasgupta, 1996). However, we are often faced with the problems of misallocation of natural resources (environmental and ecological goods or services) due to market and institutional failures. In the words of Pearce and Barbier (2000), "the source of most environmental problems lies in the failure of the economic system to take account of the valuable services which natural environments provide for us".

This research project seeks to understand the interdependent relations between natural (ecological and hydrological) and economic processes in river floodplains (henceforth, simply 'floodplains'). It takes an approach which may broadly be defined as 'ecological economics'. "Ecological economics is a system-oriented field that considers the interdependence of biophysical and economic systems (broadly defined) (Martinez-Alier 1999)" (qtd. in Barnett et al., 2003). This approach makes it possible to understand the essence of many environmental problems and to identify their relevant solutions. To develop appropriate policies, we must consider natural and economic systems, and the practical relationships.

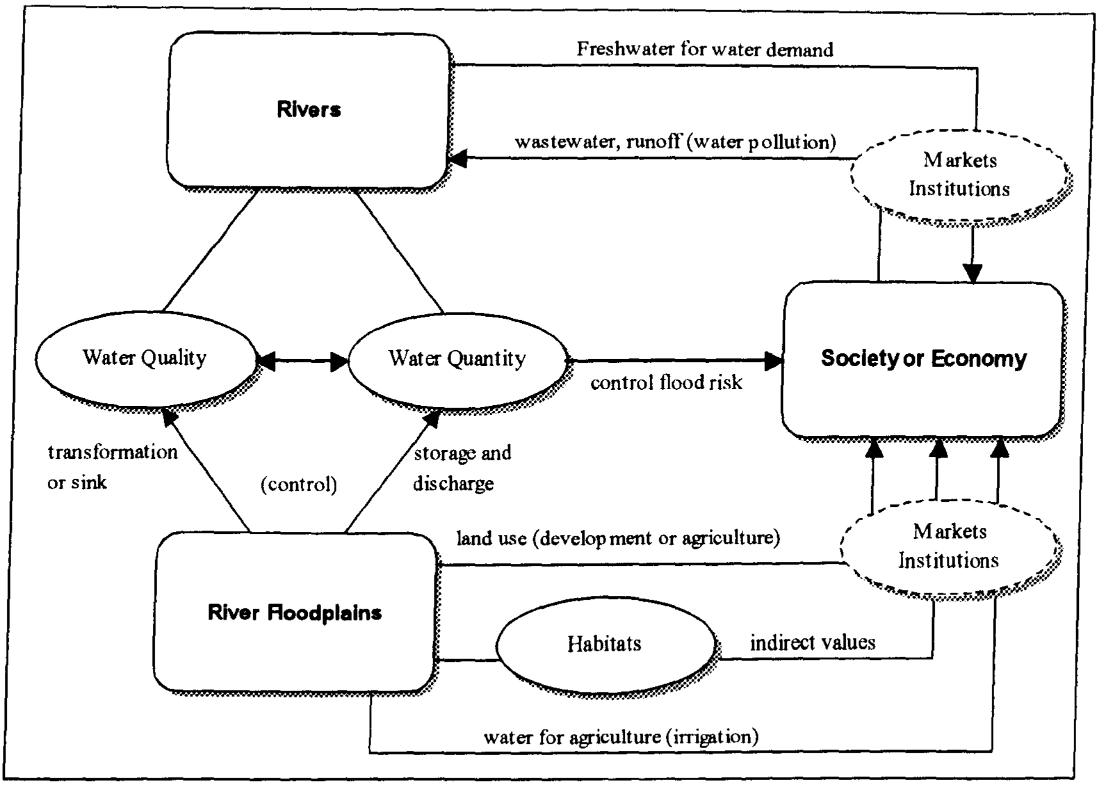
The main research question in this project is "What is the optimal management strategy of floodplains?". The optimum is determined by taking all the values of ecological, hydrological and economic services of floodplains into consideration. This also depends on the discounting factor. The current situation is not optimal because our economic system does not have the way of recognizing and evaluating the indirect use values of ecological services. Without the proper institutions and incentives, the current environmental condition of natural floodplains continues to deteriorate.

Floodplains are important in that they provide us with important environmental, ecological and hydrological services. They are multi-functional resources. Natural floodplains enhance biological productivity. They provide habitats for various species, which have both direct and indirect use values. "Wetland-dependent fish, shellfish, fur animals, waterfowl, and timber provide important and valuable harvests and millions of days of recreational fishing and hunting" (Mitsch and Gosselink, 2000b). In addition, the mix of various species determines what kind of ecological function or services it supplies, and it gives ecosystem resilience (stability) at the same time (Holling et al., 1995). Hydrologically, natural floodplains play an important role in mitigating floods because they can control water discharge volume. Natural floodplains recharge ground water (Dister et al., 1990; Gren et al., 1995; Acharya, 2000; Acharya and Barbier, 2000). Furthermore, natural floodplains improve water quality. They act as nutrient sinks for runoff from uplands but as nutrient transformers for upstream-downstream flow (Mitsch and Gosselink, 2000b). Because natural floodplains contain fertile and nutrient soil, they can be directly used as arable and grazing lands. Finally, they can supply the lands that can be easily developed into residential areas, industrial areas, roads or so because floodplains are flat.

The fact that natural floodplains provide various ecological services poses the following three problems from the ecological, hydrological and economic point of view. In line with the three problems, we can divide the main research question into the following subsidiary questions.

First, the ecosystem functions of natural floodplains are complex and interdependent (see Figure 1-1). What are the relationships between the values and functions of natural floodplains? Sure, it is difficult to understand their values and the interrelationships. It is difficult to understand how to manage floodplains in our economic system. Therefore, the first problem to be solved is to elucidate the values of floodplains and their relationships particularly between natural system (rivers and floodplains) and economic system (economic activities like land use and flood control). In particular, we should note that their values depend on different management regimes. What and how much values do they provide? How are they related?

Figure 1-1. *Interrelationships of Values and Functions*



Second, there are trade-offs between ecological-hydrological functions and economic activities on floodplains. Floodplains are easily developed into residential areas, roads, industrial areas and so on, or directly used as arable farmland. In fact, "[i]n England and Wales, nearly 6 million people live in flood plains, which cover some 10% of the land. The construction of roads and railways and commercial and domestic development is often done in floodplains because it is relatively easier and cheaper to build there. In some areas, the rate of development on flood plains has more than doubled in the past 50 years"

(Environment Agency[1]). Parker (1995) shows that floodplains are substantially developed by the data from six urban locations in England and Wales. The development of floodplains is frequently attracted. However, it leads to the loss of capacity as a nutrient sink and a nutrient transformer, of habitats for many species, and of flood control functions. In other words, there is a trade-off between the direct and indirect use values of floodplains. In addition to the development, a bund, a levee, or a dike is built because people want to protect their properties and farmlands from flooding water. However, this activity mitigates the flood control function that natural floodplains have, and increases the risk of flood especially in the urban areas downstream. At the same time, agriculture in the farmlands outside of bunds depends on artificial chemical fertilizers because it cannot access the fertile soils that flooding water deposits. It leads to the deterioration of water quality in rivers. In many cases, the values of ecological and hydrological functions exceed the economic benefits of direct use. Under these complicated trade-offs, how should we use floodplains? What is the optimal management strategy of floodplains from the ecological, hydrological and economic point of view?

Third, related to the second problem, we need to identify the institutions and incentives that are necessary to implement the optimal floodplain management strategy. The second problem is related to the problem of externality (no property rights) in the context of economics. Private land owners of floodplains have no incentives to take account of the public interests such as the values of ecological and flood mitigation functions that natural floodplains provide. The development and construction of bunds and dykes confer private benefits, but often have a negative effect on the public interest. This problem is attributable to the lack of well-defined property rights. In other words, although the rights to floodplains as lands are defined, rights to the ecological services are not. Natural floodplains are multi-functional resources, and the lack of well-defined rights causes market failure. Therefore, we need institutions and incentive mechanisms for the optimal management of floodplains. What are these necessary institutions and incentives?

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[1] www.environment-agency.gov.uk

What is the best form of property rights?

1.2 Purpose of Research

The thesis has three main objectives. First, we develop a hydrological, ecological and economic model of the interdependence between natural and economic processes in floodplains. In a model, we can describe the essential characteristics of natural and socio-economic processes of floodplains, and understand these interdependences. Second, we estimate the values of the services provided by floodplains in the Ouse catchment under different management regimes and to evaluate the trade-offs between the values. Third, we simulate a measure of social welfare under different floodplain management regimes, test the sensitivity of the outcome to environmental variables such as precipitation and to economic variables such as land prices, and identify the institutions and incentives necessary to implement an optimal floodplains management strategy.

The main rationales for this thesis are the following. First, this thesis contributes to interdisciplinary research on environmental problems by providing an integrated ecological, hydrological and economic analysis. Second, it contributes to the economics of externality, property rights and market failure on ecological services. This can be applied to other environmental and ecological goods and services. Third, this thesis helps policy makers (a local government or governmental agencies) to optimally manage floodplains in the Ouse catchment. In particular, we attempt to provide a simulation model by which we can evaluate the real management options.

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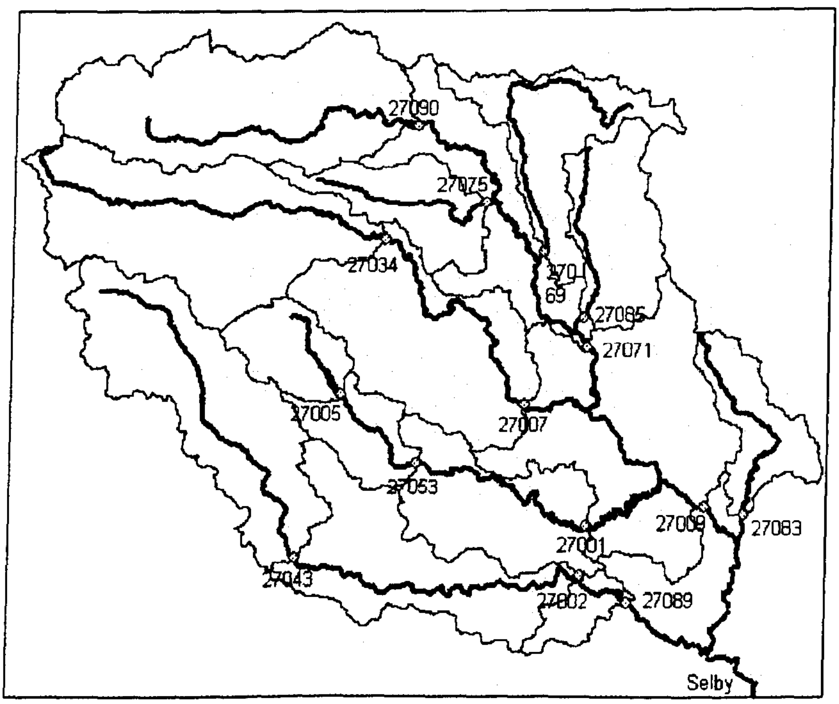
1.3 Scope of Research

1.3.1 Research Area

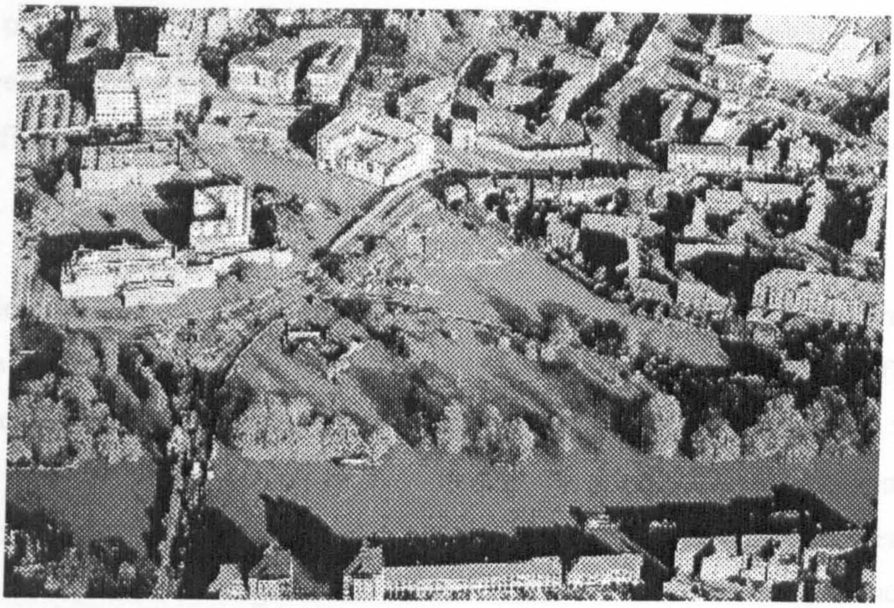
This research project focuses on the Ouse catchment, which includes the watershed of River Ouse and the tributaries (see Figure 1-2). There are three reasons why we choose this catchment. First, the River Ouse is notorious for flood events. For example, a serious flood occurs in York (Ouse) in 2000 (See Figure 1-3). Flood events seem to be related to floodplain development and averting behaviour. Second, it offers considerable scope for collaboration with user communities and the regional governmental agency (the Environment Agency). If possible, we would like to help the regional government and the regional Environment Agency to optimally manage floodplains of River Ouse. In particular, we provide a simulation model by which we can evaluate real management options. Third, the rivers are easily accessible and small enough that the research project is tractable.

Figure 1-2. *Map of the Ouse catchment*



Source: We create this map from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) and OS Strategi® [1:250,000] (EDINA Digimap) by use of ArcGIS.

Figure 1-3. *Flood Event in River Ouse (York, 2000)*



Source: York City Council. www.york.gov.uk

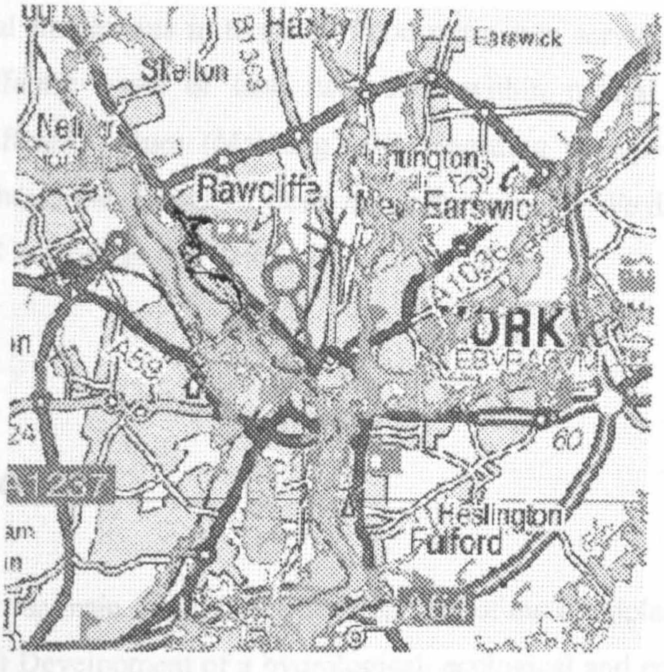
1.3.2 Definition of Floodplain

Floodplains are a form of wetlands. Mitsch and Gosselink (2000b) discuss the definition of wetlands and conclude that there is no absolute definition of wetlands, although legal definitions have been becoming increasingly comprehensive. The definition of wetlands has been ambiguously treated through individual names such as swamp, fen, bog, marsh and so on. All reflect the fact that wetlands have many distinguishing features such as the presence of standing water, soil conditions, organisms, vegetations and so on (Mitsch and Gosselink, 2000b). According to Smith and Smith (2001), the following is a widely accepted definition: "Wetlands are transitional between terrestrial and aquatic systems where the water table is usually at or near the surface or the land is covered by shallow water. . . . Wetlands must have one or more of the following three attributes: (1) at least periodically, the land supports predominantly hydrophytes [plants adapted to the wet conditions]; (2) the substrate is predominantly undrained hydric soil [soils that formed under conditions of saturation, flooding, or ponding long enough during the growing season to develop anaerobic conditions in the upper part]; and (3) the substrate is nonsoil and is saturated with water or covered by shallow water at some time during the growing season of each year" (Cowardin, et al., 1979).[2] We share Mitsch and Gosselink's view that the definition of wetlands depends on the objectives and the field of interest of users.

Floodplains are often called riverine or riparian wetlands. Riverine wetlands are the wetlands that have developed along shallow and periodically flooded banks of rivers and streams (Mitsch and Gosselink, 2000b; Smith and Smith, 2001). Floodplains are low lands adjoining a channel, river, stream and watercourse that have been or may be inundated by flood water (Bedient and Huber, 2002). It is ambiguous to what extent or how often they should be inundated by flood water, but we normally define this point by the notion of the flood frequency or the return period. We can statistically define the flood frequency or the return period. For example, the 100-year flood implies that they happen once per 100 years on an average (statistically). In other words, the 100-year flood has a probability of 0.01 (1%) of being equalled or exceeded in any single year. We can clearly define floodplains by this notion although it is still ambiguous what return period we should use. The 100-year flood is often used. The Environment agency also adopts this and provides maps of floodplains based on this notion (see Figure 1-4).[3] Thus, we adopt this notion in this research project. If low lands adjoining a channel, river, stream and watercourse have been or may be inundated by the 100-year flood water, they are floodplains.[4]

[2] Cowardin, et al. (1979). p.103.
[3] "Special consideration is now given to flood plains in planning urban development, and to this end up-to-date and consistent maps of flood plains have been produced. The Environment Agency produces these maps for England and Wales and they are available on their website alongside other river-related environmental data" (Petts et al., 2002).
[4] In this thesis, we simply use the word 'floodplains' instead of 'river floodplains'.

Figure 1-4. *Map of Floodplain*



Source: Environment Agency (downloaded on 15 Feb 2005)
http://maps.environment-agency.gov.uk/wiyby/mapController
NB: Dark shaded areas could be flooded due to a flood that has a 1% (1 in 100) or greater chance of happening each year.

Finally, it is of great importance that we should distinguish between natural and developed floodplains. Floodplains include both natural and developed floodplains. Natural floodplains provide a range of ecosystem services in addition to the absorption of floodwaters while developed floodplains do not. Developed floodplains provide us with a stream of economic direct-use benefits as resident areas, industrial areas, public infrastructures and so on. This research project focuses on the combination of natural and developed floodplains.

Then, let us concisely define natural and developed floodplains in this thesis. Natural floodplains are composed of various types of land uses such as grass moors, marshes, shrub heaths, woodlands and so on. Hence, once we define developed floodplains as some specific types of land uses, natural floodplains are treated as floodplains excluding developed floodplains. Developed floodplains are defined as the two types of land uses in the category of LCM (land cover map 1990): (1) suburban, rural development and (2) urban development. Note that pastures, meadows and agricultural lands (tilled, arable crops) are categorized into natural floodplains in this thesis because they function in a similar way to other types of natural floodplains in terms of flood mitigation service.[5] However, we distinguish different types of land uses even within natural floodplains by referring to different values (Manning's N) according to types of land uses. Manning's N shows the amount of flood mitigation service. We discuss this issue in Section 5.3.8 in Chapter 5 in detail.

1.4 Approach

There are three main stages in the development of the floodplain models used in this thesis: (1) Development of a hydrological, ecological and economic model for a normative analysis; (2) Calibration of parameters in an applied model; and (3) Quantitative policy simulations.

In the first stage, we develop static and dynamic decision models, in which we attempt to maximise social utility and a stream of social utilities over time respectively. We use both static and dynamic optimisation techniques for a normative analysis (what it should be).

In the second stage, we attempt to provide specific functional forms and calibrate the parameters for an applied model. In our problem, we need to calibrate many parameter values. We need to create the necessary data as well. The hydrological sub-model exploits the hydrological software programs 'HEC-HMS', which uses several techniques for modelling water flows. We use GIS techniques in a GIS software program 'ArcGIS' for obtaining physical parameter values. Frequency analysis techniques are used for obtaining precipitation with a certain exceedance probability. We use relevant econometric techniques and benefit transfer methods for calibrating environmental and

[5] In general, it appears that they are closer to natural floodplains than to developed floodplains. The extent of similarity between types of land uses might depend on the types of ecosystem services which we focus on.

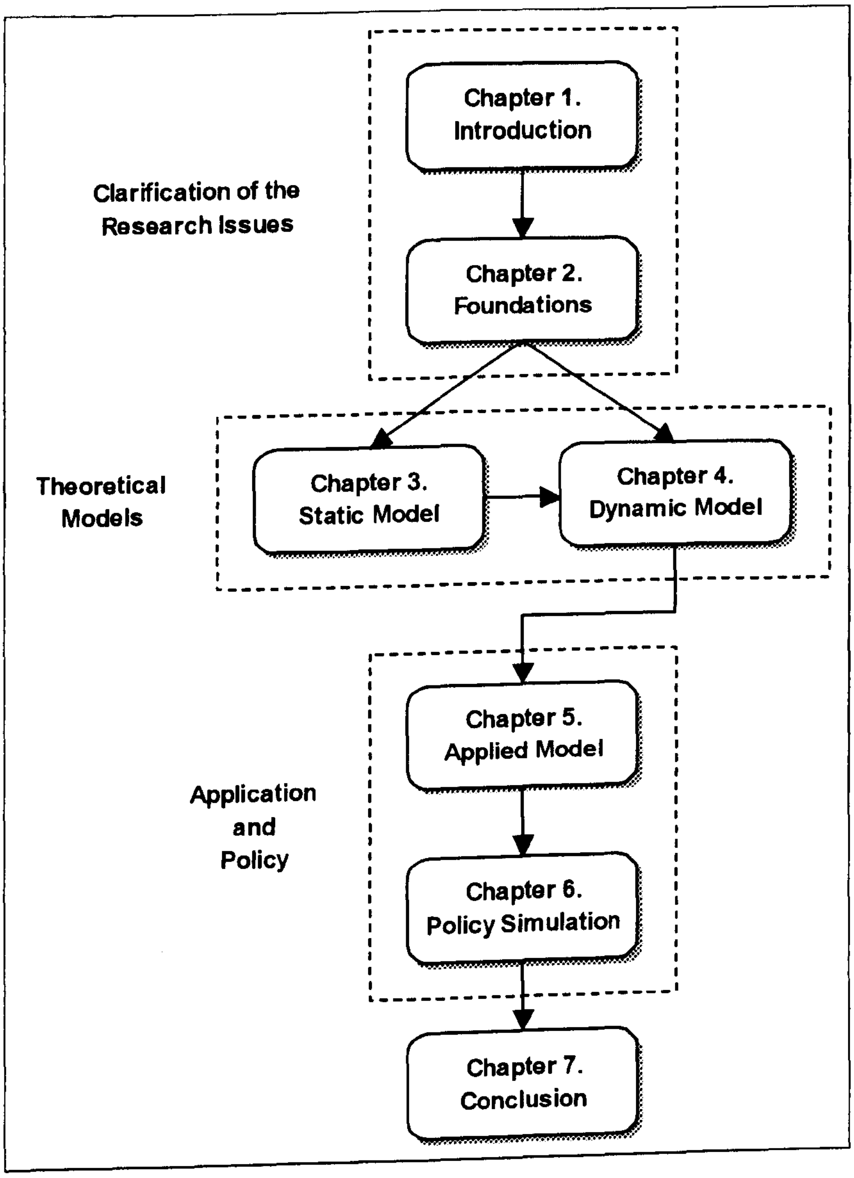
economic parameters.

In the third stage, we attempt to execute policy simulations in which we simulate the effects of alternative floodplain management strategies on a measure of social welfare (sum of discounted social utilities over time). In so doing, we use a mathematical software program 'GAMS' for the numerical solution of dynamic optimisation problems. The simulations explore the hydrological, ecological and economic consequences of different policy options, and estimate the sensitivity of the model to changes of either/both environmental or/and economic conditions.

1.5 Structure of Thesis

In Chapter 2, we provide a literature review in order to clarify why our research is significant, to discuss what has been solved and to describe approaches or methodologies. In Chapter 3, we develop a static decision model for the normative analysis of floodplain management (what it should be). We set up two problems, the social optimisation problem and the private optimisation problem. In Chapter 4, we develop a dynamic decision model. We discuss a steady-state equilibrium and its local stability. In Chapter 5, we provide an applied model with concrete functional forms and calibrate parameters. We set up a hydrological sub-model for the expected cost function of flood risk by use of HEC software programs. In Chapter 6, we implement a number of policy simulations in order to evaluate outcomes under different policy scenarios and to evaluate the robustness of model projections to uncertainty about model parameters and structure. In Chapter 7, we provide conclusions.

Figure 1-5. *Thesis structure*



Chapter 2

Foundations of the Thesis: A Review of the Early Literature

2.1 Introduction

This chapter reviews the relevant literature in order to establish three things: the nature and importance of the problem to be studied, the gaps in knowledge addressed in this research, and the approaches and methods used to address them.

In Section 2.2, we discuss the market failures that are a common feature of environmental goods and services. We take up incomplete or missing markets, externalities, non-excludability and open access, public goods, non-convexities, and asymmetric information. In particular, we consider the externalities that affect the management of river floodplains. In Section 2.3, we discuss ecosystem services of wetlands. We attempt to make the ecosystem services that we should focus on obvious through the review. In addition, we discuss the relationships among biodiversity, physio-chemical environment, hydrology, and ecosystem functions and services. In Section 2.4, we discuss problems in the valuation of multiple ecosystem services. There are obviously limitations to existing valuation methods, about which we should be careful. In Section 2.5, we discuss ecological economic models including hydrological models and indicate the challenges involved in the integration of hydrological, ecological and economic models.

2.2 Market Failure

We are confronted with a common problem when we consider the allocation of environmental and ecological resources: market failure. We cannot simply rely on the market mechanism without any remedial actions. Under such situations, we need to know what is the optimal situation (normative analysis) and how we should reach it (policy analysis plus empirical research).

Floodplains are multi-functional resources. They provide us with various important ecosystem services. The services give several causes of market failure: (1) there is no market for them (missing market); (2) they are sources of environmental externalities; (3) they are often non-rival and/or non-exclusive; (4) they provide complex non-linear relationships between the amount and value of services, which are denoted by non-linear and non-convex functions (non-convexity); and (5) the information on the services is not perfect and the information is often asymmetric between individual landowners and policy makers (asymmetric information). The problem of market failure seems to have been established in theory, but we still need to identify the causes of market failure in real environmental problems and to find appropriate policy options for overcoming the problems in the practical context. The literature has recently been produced from the perspective. This thesis clarifies two types of environmental externalities related to ecosystem services of floodplains and evaluates a number of policy options to resolve the problem by computer simulations in the practical context of the Ouse catchment.

2.2.1 Market Mechanism and Coase Theorem

The pure market mechanism is, in principle, able to solve all allocation problems. In theory, the market is always efficient if several conditions are satisfied: (1) markets exist for all goods and services; (2) all markets are perfectly competitive; (3) all economic agents have perfect information; (4) property rights are well-defined and fully assigned; (5) no externalities exist; (6) all goods and services are private goods that are rival and exclusive; and (7) long-run average costs are non-decreasing. These conditions are important in that a problem of resource allocation occurs unless they are satisfied.

However, the market mechanism does not necessarily work well in practice. In particular, market failure is a rule in problems of environmental and ecological resources. In the words of Pearce and Barbier (2000), "the source of most environmental problems lies in the failure of the economic system to take account of the valuable services which natural environments provide for us". Hanley et al. (1997) provide six main causes of market failure concisely, considering environmental and ecological resources: incomplete markets; externalities; non-exclusion and the commons; non-rivalry and public goods; non-convexities; and asymmetric information. These causes impair at least one of the conditions for market efficiency. Under such situations, we need some other ingenious organizational, institutional, or political systems instead of market mechanism for efficiently allocating environmental and ecological resources.

The Coase theorem established that the optimal allocation, which minimises the social cost, is always possible through the market mechanism without regard to the initial assignment of property rights as long as the property rights are well-defined and allocated even if externalities exist (Coase, 1960). Nevertheless, we should notice that there are some crucial assumptions in the Coase theorem: perfect information; all economic agents are price-takers; a costless court system for enforcing agreements; all economic agents obey their maximisation rules; no income or wealth effects; no transaction costs (Kolstad, 2000). As is the case with the conditions for market efficiency, the validity of the Coase theorem depends on these conditions above. It is usually rare that real situations satisfy all the conditions above. In fact, Coase (1937, 1960, 1993) recognized that transaction costs are of great importance in the real economic world and argued that the existence of transaction costs explains why we need economic organizations such as firms, although market mechanism can coordinate all the transactions theoretically (Coase, 1937 and 1993).[1] Moreover, the assumptions in the Coase theorem are strong, especially in the context of environmental goods and services. It is better not to think that markets will normally solve the allocation problem of environmental and ecological resources. We usually need some organizational, institutional or political solutions such as tax, quota, regulation, law, direct intervention and so on to complement markets as Coase and Williamson have considered in a different context.

2.2.2 Incomplete or Missing Markets

We discuss each type of market failure for the time being, paying attention to environmental problems. Incomplete markets or missing markets are often observed. It is easy to give some examples: (clean) air; tropospheric ozone; freshwater in rivers; biodiversity; services provided by pollinators; natural pest control services; water quality improvement service by wetlands; flood mitigation service by floodplains; and so on. We have no markets for these goods and services, and it is difficult to create the markets because of difficulty in defining property rights under the current economic system, their indivisibility, non-excludability, externalities and so forth.

To address the problem we need at least information on values of such goods and services. In fact, we have to devise several methods in order to evaluate the values of the environment under incomplete or missing markets (Mitchell and Carson, 1989; Freeman, 1993; Garrod and Willis, 1999; Haab and McConnell, 2002). We can use direct hypothetical methods such as bidding games and willingness-to-pay questions, and indirect hypothetical methods such as contingent ranking, activity and referendum methods for evaluating the values of

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[1] Based on the notion 'transaction costs', Williamson has discussed the choice among alternative modes of economic organizations including institutions (Williamson, 1975, 1980 and 1985).

the goods and services for which we have no market. In addition, we can use indirect observed behaviour methods such as travel cost, hedonic property values, avoidance expenditures and referendum voting for evaluating the values of the goods and services for which we have only incomplete markets. However, there are limitations on the methods. We have not solved the problems yet. Daily (1997) describes several challenges to the valuation of ecosystem services: the lack of information on (1) the role and value of biodiversity; (2) the marginal value of ecosystems; (3) the economic values of context-dependent ecosystem services; (4) quantifying ecosystem services such as stability; (5) interdependencies and indivisibility of ecosystem services.

2.2.3 Externalities

An externality happens when the activities of an agent affect those of other agents and this is not considered in direct or indirect market transactions between them. The reason why externalities are serous economic problems is that they violate Pareto efficiency, which implies the inability of market to allocate goods and services efficiently. According to the second theorem of welfare economics, we can always achieve any desired allocation by using market mechanism, but externalities preclude this. However, there is confusion about the notion 'externality'. Baumol and Oats (1988) points out that it is important to distinguish between technological and pecuniary externalities (pseudo-externalities). Pecuniary externalities do not produce market inefficiency. Thus, we do not deal with this type of externalities.

Externality is one of the most important problems in environmental goods and services. "From the innocent parable of the bees to the poisonous gas clouds of Bhopal, environmental external effects are evidence of the price system's inability to signal the true significance of the interdependence of human activities undertaken within a common environment" (Perrings, 1987). The problem of environmental external effects is one of the most important issues in our research. As mentioned in Chapter 1, the external effects frequently occur in the private use of floodplains. The actions of landowners in floodplains upstream frequently increase the flood risk to people downstream. Private land owners of river floodplains have no incentives to take account of the impact of their behaviours on the ecological and hydrological services that natural river floodplains provide. While property rights to land in river floodplains as lands can be easily created and defined, it is difficult to create and define the property rights for each ecological or hydrological service of river floodplains.

Externalities are not new problems in economics.[2] Marshall (1920) discusses external economies and diseconomies. Meade (1952) analyses two distinctive cases of external economies: one is internal to the individual industry; and the other is external to the individual industry. Certainly, Meade deals with examples of externalities, but Meade's problems seem to be on increasing returns to scale. We can see a similar discussion in Marshall (1920). In this respect, Baumol and Oats maintain that the analysis of increasing returns problem is quite different from that of more conventional externalities which constitute the primary threat to the environment (Baumol and Oats, 1988).[3] We share the same perspective here. In the sense that conventional technological externalities are analysed, Buchanan and Stubblebine (1962) provide a clear definition of externalities, which coincides with the above-mentioned definition. They distinguish between marginal (incremental) and infra-marginal (discrete) externalities; potentially relevant (motivated and possible to modify actions of the externality generator through any bargaining actions) and irrelevant (not motivated and possible to do that) externalities; and Pareto-relevant (possible to make one economic agent better off without hurting others in the modification) and Pareto-irrelevant (not possible to do that) externalities. They conclude that Pareto equilibrium cannot be attained in the case of marginal externalities without any remedial actions.

The Pigouvian tax (or subsidy) is one of the most famous remedial actions

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[2] Mishan (1971) provides a concise and good review on literature on externalities.
[3] Bator (1958) and Ellis and Fellner (1943) treat increasing returns in a similar way.

for externalities (Pigou, 1932). The decision determining whether we compensate the victims or not does not depend on whether the problem is a private goods (depletable) externality or a public goods (undepletable) externality, but whether the victims can costlessly control the amount of the damages or not (Freeman III, 1984; Baumol and Oats, 1988). More concretely, compensating victims is difficult if the number of victims is large. Pigouvian tax brings about further inefficiency in the situation where Coasian bargaining is possible because of the small number of victims. In this respect, Turvey (1963) had long argued that *a priori* prescription of the tax is unwise especially when negotiations are possible between externality generators and victims. The basic policy prescription is the same for private and public externalities, which is a Pigouvian tax equal to marginal social damage levied on the producer of the externality (Baumol and Oats, 1988; Freeman III, 1984). However, Papandreou (1998) concludes that we have to better recognise the role of institutions in removing externalities, evaluating the attempts to incorporate institutions into economic models endogenously. Likewise, Davis and Whinston (1967) provide simple models for discussing the provision of public goods and governmental public goods respectively, and conclude that both public goods can be provided in market by use of appropriate institutional arrangements.

However, there are a number of policy options for overcoming the problem of externalities, which we should evaluate depending on real externalities. Let us give some recent discussions. Gustafsson (1998) mentions that the role of market mechanisms in solving problems of environmental externalities is overestimated because it is too simple to consider the complexity of environmental functions. He suggests that the choice of relevant environmental policy instrument should be conditioned by many considerations. Loehman and Randhir (1999) discuss several policy options for internalising two temporal externalities due to agriculture: soil erosion and related pollution. In terms of traditional welfare economics concepts, they come up with three efficient policy options such as a Pigouvian policy, a bargaining solution and 'government as co-producer of environmental goods' policy, but they require the government to play different roles. Owen (2006) estimates the external costs (environmental externality) of combustion of fossil fuels in electric power generation and concludes that a number of renewable energy technologies will be competitive if the external costs are financially internalised by tariffs. Gottfried et al. (1996) provide a viewpoint of land use and land values in terms of the multiproduct nature of ecosystems. Interestingly, they discuss that markets do not work well even when traditional economic instruments to internalise externalities are applied. They mention that we need the institutions that enable us to overcome landscape-scale market failure, but it is not obvious what institutions are required.

We need to clearly identify environmental externalities in the context of real environmental problems. For example, Parker and Munroe (2006) categorize the conflicts of farm incompatible productions between neighboring farms as "edge-effect externalities" and test the hypothesis that the externalities have influenced the location and production patterns of organic farms by the data on California Central Valley organic farmers. Ihalanfeldt and Taylor (2004) estimate the size of externality effect of hazardous waste sites on other land-uses such as commercial and industrial properties and show that it is substantial in a case study.

The problems of externalities are related to property rights to a large extent. Property rights are the rights to use, control and exchange resources (Alchian and Demsetz, 1973; Bromley, 1991). The creation and protection of property rights are important parts of the internalisation of externality. "It is clear, then that property rights specify how persons may be benefited and harmed, and, therefore, who must pay whom to modify the actions taken by persons. . . . A primary function of property rights is that of guiding incentives to achieve a greater internalization of externalities" (Demsetz, 1967). The Coase theorem shows that an economic allocation will be efficient (Pareto efficient) providing that property rights are clearly defined (Coase, 1960). Disregarding the other assumptions needed for the result, the Coase theorem shows that property rights give the correct economic incentives for the optimal use of resources. However, there are many forms of property rights (See Table 2-1). Thus, we still need to consider which type of property rights is the most appropriate and effective for solving problems. The answer may differ in different cases.

Table 2-1. *Types of property rights*

Type	Description
Open Access (no property rights)	Access for resource use is effectively unrestricted, in other words, it is free and open to all.
Private Property	The owner (individual, a group of individuals, coporations and so on) has the right to exclude others from use of the resource and to regulate its use.
Common Property	Resource ownership and management is in the hands of an identifiable community of individuals, who can exclude others and regulate use of the resource.
State Property	Resource ownership is vested exclusively in the government, which determines and controls access, and regulates use.

Source: Barbier et al. (1995), pp.77-78.

Adger and Luttrell (2000) examine the relationship between property rights and wetland resources, giving two case studies. They mention that well-defined property rights are fundamental to sustainable resource use and that the situations are worsened by the lack of clear formal or informal rights interacting with historic under-valuation. However, their conclusions about the relationship between the resultant resource allocation and the type of property rights are ambiguous, although it seems that common property rights are more suitable to the resources where the nature of the habitat makes definition of boundaries difficult. Hodge and McNally (2000) state that collective action may be important for wetland restoration because the control of the wetness of private fields has externalities and the benefits of restoration can be achieved at a lager scale. Likewise, as river floodplains provide multiple ecosystem services, it is difficult and challenging to set up appropriate property rights on them. At this point, we need more case studies and analyses in order to develop management policies. There are cases that distribution of property rights is discussed in the intergenerational context. For example, Pasqual and Souto (2003) analyse the reasons why traditional solutions for externalities do not work in the intergenerational context and suggests that redistributing property rights between generations should be effective for achieving sustainability about environmental resources.

There is a considerable literature on externalities in the context of wetlands. Kohn (1994) develops an interesting two-sector model in which wetlands increase the productivity of fishing industry or alternatively are converted into agricultural lands. In the model, Kohn compares between a tax on converted wetlands and a subsidy on preserved wetlands, considering who has the property rights of wetland externalities. Interestingly, Kohn concludes that a tax is more important than a subsidy to the extent that wetland property owners possess the rights whereas a subsidy is more important than a tax to the extent that the public possess the rights. Ficklin et al. (1996) mention that forest ecosystem management system policies should consider total economic and social costs and benefits of forest cutting, paying a particular attention to an appropriate balance between many uses and values of forest ecosystems (non-commercial uses and values produce externalities). Niskanen (1998) analyses environmental external impacts of reforestation such as carbon sequestration, increased erosion control and so forth in Thailand. Niskanen argues that the free-rider problem is important in carbon sequestration, in which the costs are incurred locally but benefits accrue on the global level. Cataneo et al. (2001) analyse an interesting situation in which livestock, rice recreational fisheries, tourism and possibly forestry are all dependent on the quality, the level and the extent of the wetland, and the quality, the level and the extent of the wetland is itself affected by certain activities-principally hydroelectric power generation and rice production. They argue that if some activity brings about ecological changes that cause the damage of other activities, it may threaten the sustainability of the provincial economy. Cacho (2001) develops a dynamic economic model of agroforestry in the presence of the positive externality that is the enhancement of land productivity, where the conditions of optimal land-use allocation are derived, a measure of externality is identified, and policy analysis is undertaken in the form of subsidies to achieve the optimal allocation. Bhat and Bhatta (2004) find that the optimal land (including coastal floodplains) conversion depends not only on the economics of shrimp production but also on the renewable components of the ecosystems, applying existing literature to the problem of commercial aquaculture in India that cause negative agricultural and environmental impacts.

2.2.4 Non-exclusion and Open Access

A second source of inefficiency is that the market mechanism fails to efficiently allocate resources when resources are non-exclusive, which implies that it is impossible or too costly to deny access to resources. If the resources are rival but non-exclusive, each economic agent has an incentive to take them as many and soon as possible. That leads to over-exploitation or over-consumption of resources.

This happens in open access situations such as exist in many fisheries. Gordon (1954) developed an economic model for the overexploitation problem of the fishery, and Schaefer (1954) analysed the fundamental laws of population growth and the management of the oceanic fisheries. The key idea is that fishers continue to catch fish up to the point at which economic rents are fully exploited under the conditions of open-access common-property, and over-exploitation occurs as a result.[4] Gordon suggested that the "Overexploitation Problem" was applicable to all natural resources that were owned in common (were open-access) and exploited under individualistic competition. He drew two main implications: first we need the policies (agreement, regulation, tax and so on) that limit the fish-catch such that the fish population recovers and the catch-per-unit-effort increases; second, the problem about land animals is more severe because the biotic potential of land animals is much lower than that of fish. The second point is related to habitat provision service that floodplains provide. Furthermore, Clark (1973) shows the possibility of extinction in addition to over-exploitation, based on the Gordon-Schaefer model.[5] Bjorndal and Conrad (1987) provide an empirical application to the North Sea herring fishery, with special reference to the question of stock extinction under open access. In the same context, Brander and Taylor (1998) provide an interesting insight into the rise and fall of the Easter

[4] Strictly speaking, we have to distinguish between open-access and common-property. In open-access, there is no property rights. In common-property, however, there is well-defined property rights that some collective body or a collective set of economic agents possess. In this sense, Gordon's 'common-property' implies open-access although Gordon implies individuals' free access to resources that they commonly possess.

[5] e.g. Clark (1990).

Island Civilization, using the same notion of open-access renewable resources. The point is that the negative effect of population growth on resources would be worse in the absence of established property rights (open-access).

In a more general context, Hardin's seminal paper on the tragedy of the commons argues that the commons are justifiable only under conditions of low-population density if they are justifiable at all (Hardin, 1968).[6] His arguments was as follows. First, each rational herdsman is forced to continue to increase his herd without limit because the herdsman receives all the proceeds from the sale of the additional animal and the negative effects of overgrazing due to an additional increase in animal are shared by all the herdsmen. In this case, the additional benefit that accrues to each herdsman always exceeds the additional cost that accrues to him.[7] It results in the situation that "freedom in a commons brings ruin to all". Second, the tragedy of the commons holds true for the problems of pollution in a reverse way. The rational person understands that his share of the cost of the wastes he discharges into the commons is less than the cost of purifying them by himself. Consequently, we are faced with a serious pollution problem.

2.2.5 Public Goods

A good is non-rival if a person's consumption of one more unit of the good does not decrease the consumption of other people. If goods and services are non-rival and non-exclusive, they are called public goods. In this case, market failure occurs because it implies that the marginal social cost of supplying goods

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[6] Hardin argues the same point in 30 years after this seminal paper: "The more the population exceeds the carrying capacity of the environment, the more freedoms must be given up" (Hardin, 1998).

[7] Hardin provides ad hoc values of the additional benefit and cost to explain the tragedy of the commons in a pasture. However, real situations depend on the values of the additional benefit and cost. In addition, the additional cost that is shared by all the herdsmen seems to go up as the number of cattle increases. There is a threshold in which each herdsman has no incentive to increase his herd.

and services is zero. Then, their prices should be zero from the perspective of efficiency. The result is a Nash-Cournot equilibrium that is neither Pareto-efficient nor unique.

Another problem with public goods is free-riding. Economic agents can enjoy benefits without paying for the costs of supplying public goods. As a result, the market provides public goods less than socially desired level. Many environmental and ecological resources are global or local public goods. For example, tropical forests provide local public goods that are their capacity of managing water flow, soil erosion and nutrient recycling and global ones that are benefits of biodiversity, ecosystem linkages and carbon sequestration (Hanley et al., 1997). Likewise, River floodplains provide local (and global) public goods (services): flood mitigation service, habitat provision service, water quality improvement service, and so on. We are faced with the same problem of public goods.

2.2.6 Non-convexities

The convexity assumption is necessary for the existence of an equilibrium allocation in markets since the assumption of convex preference is used to assure that the demand function is well-defined (Varian, 1992). The second theorem of welfare economics is violated without the assumption.

Cornes and Sandler (1996) provide a simple example in which privately unprofitable courses of action may be socially profitable from the presence of non-convexities in agents' feasible sets or preferences. Furthermore, they imply that such non-convexities become a cause of a genuine externality even if property rights are well-defined. Non-convexities are an important ingredient of externalities models. In this context, Baumol and Bradford (1972) show that severe detrimental externalities induce non-convexity of social production possibility set. Under this condition, we cannot rely on prices to give us the right signal, and tax instruments will be helpful for guiding the economy (Starrett, 1972).

In environmental problems, non-convexity is more important because researches by ecosystem and population ecologists have shown that the processes in the environment and ecology are so often non-convex (Dasgupta and Mäler, 2003). Under non-convex systems, there are multiple basins of attraction (multiple equilibria) (Holling, 1973). Then, the system would flip from one basin to another if a condition exceeds a threshold, and the flip could be irreversible or could show hysteresis. Therefore, a mistake in environmental policy management might be more costly than we envisaged under convex systems. Dasgupta and Mäler insist that economists should pay more attention to the properties of non-convex ecosystems, based on findings about ecosystems and population ecology. If environmental damages (damage function) are non-convex, we have two fundamental issues on economic policies: (1) there is the possibility that regulatory policy sometimes should pursue an all-or-nothing policy; and (2) there is the possibility that decentralized market incentives such as Pigouvian taxes might not lead to an efficient allocation of resources (Repetto, 1987). Tahvonen and Salo (1996) study a model where the decay function of pollution stock is increasing and concave for low stock levels but decreasing and convex for moderate stock levels (entirely non-convex) instead of a function with an increasing rate of decay (convex). It turns out that under the non-convex decay function multiple equilibria and multiple locally optimal solutions may exist. Brock and Starrett (2003) derive a similar implication from a model of the optimal management of dynamic ecological systems such as a shallow lake with non-convex positive feedback. They show that there may be multiple local optima and associated basins of attraction in which the optimal path may depend on initial conditions such as phosphorous stock.

2.2.7 Asymmetric Information

The condition of perfect information is necessary for market mechanism to work efficiently. All relevant information should be concentrated on prices, and all economic agents in markets should access the information of prices. The existence of asymmetric information is a source of transaction costs such as costs of opportunistic behaviours and monitoring costs (Williamson, 1985).

We have two types of asymmetric information problems: moral hazard and adverse selection. Moral hazard is the ex-post problem of asymmetric information. Moral hazard happens when the behaviours of agents cannot be observed by their principals. Helpman and Laffont (1975) analyse the existence and efficiency properties of competitive equilibria under moral hazard situation in which probability distributions are functions of individual actions. They show that a competitive stationary equilibrium is not necessarily Pareto-efficient. The most famous example of moral hazard is an insurance market. The accident-prevention effort tends to decrease under the protection of insurance because individual agents are risk-averse and it is costly to monitor the effort. Arnott and Stiglitz (1988) develop a basic model of insurance markets which shows that insurance indifference curves and feasibility sets are not convex, although they assume standard convex utility functions and technology. It implies that such markets do not function efficiently.

Adverse selection is related to the problem of asymmetric information. Adverse selection happens when the characteristics or types of agents cannot be observed by principals. Akerlof (1970) provides a simple but clear model for explaining adverse selection in the lemons market: bad cars are sold at the same price of good ones since buyers cannot observe the quality of used cars. Bad cars drive out good ones, and only bad cars remain in the market, but no one can trade them at positive prices. Rothschild and Stiglitz (1976) show that a competitive equilibrium may not exist when the characteristics of goods transacted are not fully known to at least one of the economic agents in the transactions. More concretely, they argue that high-risk individuals drive out low-risk ones in insurance markets and the markets crash.

The problems of asymmetric information are pervasive in the economy (Akerlof, 1970; Rothschild and Stiglitz, 1976; Arnott and Stiglitz, 1988). The field of environmental and ecological economics is not an exception. In particular, perfect information is not possible about the environment and ecology. We always have to deal with uncertainties due to imperfect information. The famous example is asymmetric information between polluters and regulators (or pollutees). Cabe (1992) states that information availability and the cost of acquiring information play a crucial role in determining the best structure and parameters of a regulatory mechanism. Based on this idea, he develops a control mechanism that firms pay a tax depending on the ambient concentration of a pollutant about non-point-source pollutants. Thomas (1995) points out a similar notion about the choice between Pigouvian tax and contract-based regulations. If a pollution tax is imperfect due to limited information on abatement to regulators, the role of contracts as pollution regulations is essential.[8]

2.3 Ecosystem Services of Wetlands

2.3.1 Multiple Ecosystem Services

River floodplains are important in that they provide us with important environmental, ecological and hydrological services. They are multi-functional wetlands (Turner, 1991). Natural river floodplains enhance biological productivity. They provide habitats for various species, which have both direct and indirect use values. "Wetland-dependent fish, shellfish, fur animals, waterfowl, and timber provide important and valuable harvests and millions of days of recreational

[8] e.g. Rollins and Briggs III (1996), Huber and Wirl (1998), Gottinger (2001), Yates and Cronshaw (2001), Aggarwal and Lichtenberg (2004).

fishing and hunting" (Mitsch and Gosselink, 2000b). In addition, the mix of various species determines what kind of ecological function or services it supplies, and it gives ecosystem resilience (stability) at the same time (Holling et al., 1995). Hydrologically, natural river floodplains play an important role in mitigating floods because they can control water discharge volume. Furthermore, natural river floodplains improve water quality. They act as nutrient sinks for runoff from uplands but as nutrient transformers for upstream-downstream flow (Mitsch and Gosselink, 2000b). Because natural river floodplains contain fertile and nutrient soil, they can be directly used as arable and grazing lands. Finally, they can supply the lands that can be developed into residential areas, industrial areas, roads or so because floodplains are flat. Turner (1991) comprehensively enumerate functions/services of wetlands: flood storage, flood protection, important wildlife habitats, nutrient cycling/storage and related pollution control, landscape and amenity services, recreational services, non-use existence value benefits, agricultural output, other commercial output, shoreline protection and storm buffer zones, and extended food web control.

Gren et al. (1995) estimate the values of multiple ecosystem services of Danube floodplains. These include: Values of (1) flood mitigation service; (2) water self-purification service; (3) groundwater regeneration service; (4) refuge provision service (for river-related organisms); (5) habitat provision service (for plant and animal species); (6) recreational sites provided; (7) service as nutrient sinks. Dister et al. (1990) discuss the impacts of hydrological engineering policies on multiple functions of river floodplains of the upper Rhine from the historical point of view. They discuss several ecosystem functions: reducing the effect of floods; contributing to the self-purification of water; regeneration of ground water; repopulation of the river; having richly structured habitats and refuges; offering highly productive sites; and having an enormous merit as recreational areas offering unique experience. Referring to ecosystem functions, they argue that the importance of intact Rhine floodplains has been underestimated for a long time and that development, industrialization and structural hydrological engineering have attenuated their functions historically. Crooks et al. (2001)

provide a good survey on the environmental problems of floodplains management in terms of the whole catchment or coast, focusing on policies. They indicate the importance of ecosystem services such as flood attenuation, drinking water storage, water quality amelioration, nutrient recycling, carbon storage and ecological support. They point out the same things as Dister et al. (1990). In addition, they argue the following point about policies. "However, managing floodplains is not just balancing development against biodiversity, it is about optimising a whole range of functions, many of which are not obvious but which nevertheless underpin environmental quality and flood vulnerability status. There is a need for a more overt water and wetlands policy based on a comprehensive assessment of floodplain functions" (Crooks et al., 2001). It is the perspective that we take up in our research for understanding the hydrological, ecological and economic interactions in river floodplains.

It seems difficult to treat all or many ecosystem services and economic development at the same time, but it is crucial to deal with complicated interactions among them. Barbier (1994) focuses on hydrological functions and habitats creation or maintenance functions. He mentions that the total economic value of wetland's ecological functions, its services and its resources may exceed the economic gains of converting the area to an alternative use if they are appropriately valued. Costanza et al. (1989) mention "the economic value of ecosystems is connected to their physical, chemical, and biological role in the overall system, whether the public fully recognizes that role or not. . . . This yields appropriate values only if the current public is fully informed (among a host of other provisions)". Barbier and Thompson (1996) focus on the value of water recharge function for irrigated agriculture in Hadejia-Jama'are floodplain in Northern Nigeria by using a combined hydrological and economic model. They conclude that the conservation of the floodplain is more valuable than building a dam for irrigated agriculture at the expense of the floodplain. Furthermore, they point out that the benefits of floodplains are ignored in many cases and that we should correctly evaluate the values of floodplains. Acharya (2000) values the Hadejia-Nguru wetlands in northern Nigeria by using the production function approach. Acharya's case study focuses on the aquifer recharge function, which has an indirect use value. Barbier (1993) focuses on the groundwater recharge function of floodplain wetlands that may have indirect-use value through its replenishment of aquifer systems. The groundwater recharged by floodplain supplies water for domestic use and agriculture. Acharya and Barbier (2000) study the economic values of groundwater recharge functions from the viewpoint of agricultural irrigation by use of an economic model of production function approach. Cardoch et al. (2000) focus on the wastewater treatment function and conclude that using wetlands for wastewater treatment is more efficient than using the traditional method in a case study of the treatment of the wastewater from a shrimp processor in Dulac, LA.

Some papers have attempted to link the ecological services of wetlands to the economy more directly. Cattaneo et al. (2001) evaluate wetlands in Ibera (Argentine) using a production function approach. Cattaneo et al.'s model includes the interdependence between wetlands-dependent industrial sectors through the wetlands, and contains the elements of interdependencies between natural and economic processes. Rosegrant et al. (2000) similarly develop a hydrologic-economic model for analysing water resource allocation among several sectors such as agriculture, industry, and municipal. Emerton et al. (1998) point out the significance of Nakivubo wetland (East Africa) for a range of small-scale income-generating activities. They refer to wetland resources and services as important inputs of the local economy. "Wetland resources: include the water, land, soils, plants and animals contained within wetlands, all of which provide goods which can be used to generate subsistence, income and employment. In Nakivubo, the use of wetland resources for crop cultivation, papyrus harvesting, brick-making and fish farming are of particular economic importance to surrounding communities . . . because they act as a sink for wastes and residues and protect human and natural production systems. In Nakivubo, the most important wetland service is the purification and treatment of wastewaters. This provides economic benefits which accrue throughout Kampala [capital city of Uganda]" (Emerton et al., 1998). They mention that the economic value of wetland goods and services is poorly understood, which is a serious problem.

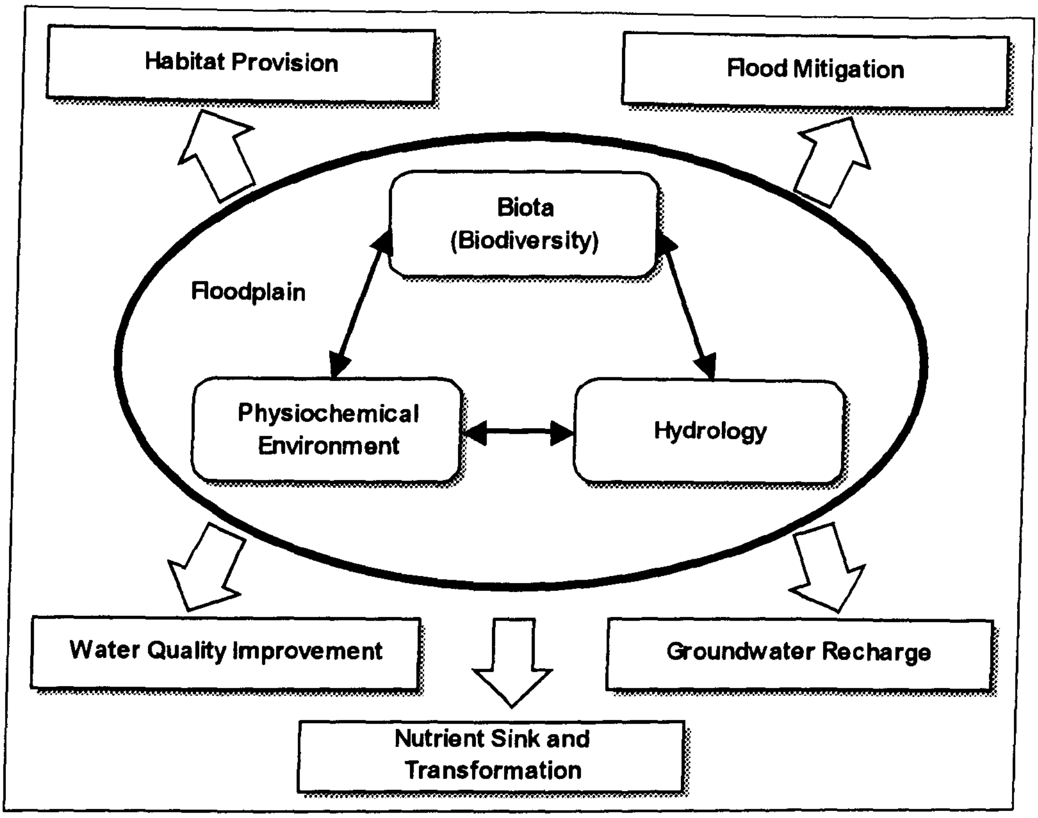
2.3.2 Biodiversity and Ecosystem Functions/Services

In this section, we discuss the interactions between biodiversity, the physio-chemical environment, hydrology, and ecosystem functions and services.

Natural floodplains enhance biological productivity. Floodplains have a high level of spatio-temporal heterogeneity. They are frequently ecotones (transition zones between adjacent patches) and sources of connectivity (the strength of interactions across ecotones) (Ward et al., 1999). The connectivity plays an important role in providing access to complementary habitats for species that require more than one habitat type during their life cycles (Amoros and Bornette, 2002). In addition, differences in the nature and intensity of hydrological connectivity contribute to the spatial heterogeneity of floodplains, which results in high alpha, beta and gamma diversity (Amoros and Bornette, 2002). In general, they provide habitats or corridors (connections between habitats) for various species, which have both direct and indirect use values.

There are five main ecosystem functions of natural floodplains as we mentioned. They are produced by the interactions of three key factors of floodplains: biota (biodiversity), physiochemical environment and hydrology in natural floodplains (See Figure 2-1). Floodplain biota is the combination of vegetation, animals, insects and microbes. Physiochemical environment means the characteristics of sediments, soil chemistry, water chemistry and so on. Hydrology implies water level, flow, frequency and so on.

Figure 2-1. *Structure of ecosystem functions*



When natural floodplains are inundated, fertile soil is deposited on them by flood water, which depends on several hydrological elements such as precipitation and river flow dynamics. This is important for agriculture historically. In addition, it is important for habitats in natural floodplains. The hydrology also determines anaerobic or aerobic conditions, redox (reduction and oxidation) conditions and so on in soil (Mitsch and Gosselink, 2000b). Floodplain biodiversity is determined by hydrology and the physiochemical environment in that it depends on nutrient availability and water accessibility, although it is important to note that the level of inundation and nutrient availability has both a positive and negative impact on biodiversity. Changes in flood pulse cause a change in community structure (vegetation composition) which has a dramatic impact on ecosystem functioning, and an eventual loss of biodiversity (Capon, 2003). "Species richness, at least in the vegetation community, increases as flow through increases. Flowing water can be thought of as a stimulus to diversity, probably caused by its ability to renew minerals and reduce anaerobic conditions" (Mitsch and Gosselink, 2000b). Existing studies tell that species richness declines as various indicators of nutrient availability increase beyond some threshold, but for many community types, the threshold beyond which richness declines has not been verified, and high or low diversity may occur below that threshold (Bedford et al., 1999).

In the interactive processes, ecosystem services are produced. They are determined by the complicated interactions of the three key factors. In these, biodiversity plays a core role in providing ecosystem services although we cannot say which of the three key factors is the most important. Species (biodiversity) play at least two major roles in ecosystems. "First, they mediate energy and material flows, and so give ecosystems their functional properties. Second, they provide the system with the resilience to respond to events or surprises" (Perrings et al., 1995). Certainly, Schmid et al. (2002) mention "a general tendency toward increased ecosystem functioning — and the absence of contrasting evidence of reduced ecosystem functioning — with increasing plant diversity" based on the review of relevant research papers.[9] "If each species performs well under a unique set of environmental conditions with little overlap among species and if environmental conditions also vary in space or time, then ecosystem productivity and stability should increase with species richness (McNaughton 1993)" (qtd. in Wright, 1996).

Biodiversity plays a principal role in providing ecosystem services, depending on the interactions within the biota in itself and between biodiversity and the other two key factors of natural floodplains.

First, species richness is due to the interactions with other key factors. Depending on the conditions of hydrology and physiochemical environment, some plants flourish, and they provide preconditions for growth of other species of vegetation and habitats of animals. Differences in the nature and intensity of hydrological connectivity in floodplains contribute to the spatial heterogeneity which brings about high alpha, beta and gamma diversity, and differences in

[9] According to their review, the results of many researches show that more diverse plant communities had greater resistance to perturbation than less diverse communities. However, they have found that there is no clear pattern for the effects of plant diversity on resilience.

connectivity also provide complementary habitats for the parts of life cycles and life-cycles of some species (Amoros and Bornette, 2002). For example, riparian vegetation in western North America provides habitats for a locally diverse avifauna. It functions as the corridor for bird movements that facilitates faunal mixing on a broader scale, which influences regional diversity within landscapes (Knopf and Samson, 1994). Furthermore, the settlement of some species of animals might induce that of other species of animals via the food chain. Therefore, the conservation of natural floodplains directly implies the conservation of floodplain biodiversity and its growth, because we lose the corridors or habitats for species if we develop natural floodplains.

Next, biodiversity influences the capacity of flood mitigation function. Riparian vegetation plays a crucial role in attenuating floods (Fischenich and Copeland, 2001). The capacity of flood mitigation function of natural floodplains is determined by the condition of vegetation as well as the characteristics of soil and hydrological conditions such as moisture. For example, as compared with bare floodplains, floodplains with vegetation have relatively bigger capacity to mitigate the fierceness of floods because the canopy of plants intercepts precipitation (interception) and vascular plants absorb moisture and exhale to the atmosphere (transpiration). "The area that vegetation presents to flow is proportional to resistance (measured as Manning's n) and effectiveness at reducing flow velocity. This presented vegetational area of vegetation increases directly with increased stem size and density. . . . Plant species differ in their tolerance thresholds to flow above which they completely fail and are torn out of the ground. As with resistance, plant failure thresholds to flow are highly variable depending on the age and size of the plant" (Fischenich and Copeland, 2001). This might imply that flood mitigation service increases with more diverse vegetation because the effects of vegetation on the function differ in types of vegetation.

Another example is that water quality improves due to nutrient uptake function of vegetation as well as percolation function of soil in floodplains. Riparian vegetation plays a vital role in the water quality improvement and nutrient cycling functions of riverine systems (Fischenich and Copeland, 2001). For example, nitrogen can be removed through volatilisation as NH_3, fixation as N_2 and denitrification (drained into rivers) as N_2O and N_2 in soil chemistry. In addition, plants in floodplains can take up nitrogen in the form of NO_3^- after the process of nitrification and downward diffusion in soil. Similar processes of removal holds true about sulfur and phosphorous. It seems that the uptake capacity is enhanced if the flora becomes increasingly abundant, because of differences in rooting depth and seasonal activity among species. Ecosystem nitrogen retention increases with increasing functional group richness of vegetation (Hooper and Vitousek, 1998).[10]

In addition, the diversity of vegetation plays a role in supplying carbon to downstream aquatic habitats (Brinson 1980). Carbon is assimilated from the atmosphere by plants and made available as food to other organisms in the basic form of sugars. Animals eat the plants or microbes decompose the litter, transferring the energy contained in the sugars up the food chain. Litter and leachates from riparian vegetation are flushed into downstream aquatic ecosystems by floodwater and groundwater, thereby supplying energy and supporting the organisms in those areas" (Fischenich and Copeland, 2001). This also supports that the diversity of vegetation in floodplains is a source of the diversity of animals, birds, etc.

Finally, the diversity of vegetation contributes to stabilisation of morphology of rivers and floodplains, which also stabilises ecosystem functions of floodplains. "Riparian vegetation affects hydraulic and hydrologic functions of streams and rivers in several ways. Maintenance of stream morphology is improved by the bank stabilization afforded by riparian vegetation" (Fischenich and Copeland, 2001). In other words, erosion and deposition are both prevented by the diversity of vegetation (less erosion means less deposition). As a result, floodplains are kept physically stable, which implies that ecosystem functions are also stable.

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[10] However, nitrogen-leaching losses do not necessarily decrease with increasing functional group richness (Hooper and Vitousek, 1998).

2.4 Valuation Methods for Ecosystem Services

As we mention in the last section, river floodplains are multiple resources that provide several ecosystem services. In order to evaluate multiple resources in the monetary terms, we need appropriate methods for evaluating multiple ecosystem services respectively (de Groot, 1994). Moreover, as these ecosystem services are global or local public goods, we cannot use market prices in order to value them. Furthermore, the valuation of ecosystem services has not necessarily been established and remains a challenging topic.

2.4.1 Willingness-To-Pay (WTP) Approach

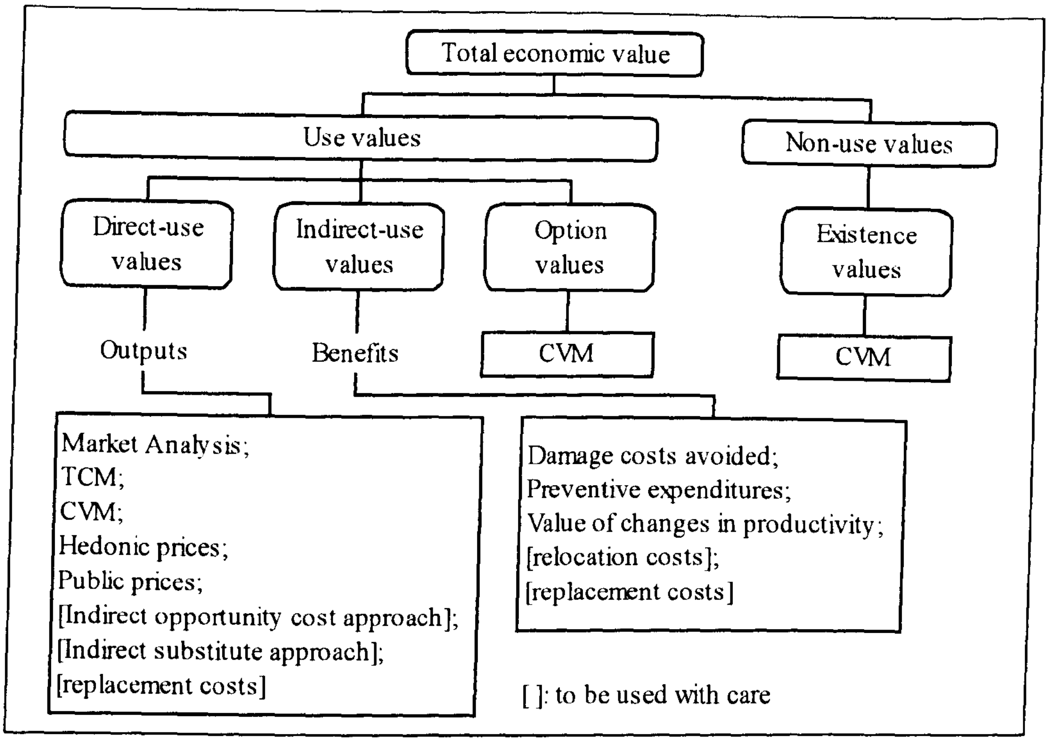
In general, the values of ecosystem services are divided into three main categories: direct-use, indirect-use and non-use value (including option value) (Aylward and Barbier, 1992; Barbier, 1993; Freeman, 1993; Mäler et al., 1994). Aylward and Barbier (1992) provide a useful table that shows the comparison between ecological and economic concepts (See Table 2-2). Based on the concepts, they indicate the relationship between the characteristics (concepts) of ecosystem and types of values (direct-use, indirect-use and non-use) in the example of multiple uses of wetlands. Their way of capturing the relationship between them is applicable to river floodplains.

Table 2-2. *Ecological and economic concepts*

System concepts	Ecosystem concepts	Ecological variables	Economic concepts
Stocks	Structural components	Matter, space	Goods
Flows	Environmental functions	Time, energy	Services
Organization	Biological and cultural diversity	Diversity	Attributes

Source: Aylward and Barbier (1992). Table 1.

Figure 2-2. *Types of values and methods*



Source: Barbier (1993). Fig. 1.

The relation between types of values and methods of valuing them is often provided (Freeman, 1991; Aylward and Barbier, 1992; Barbier, 1993; Goulder and Kennedy, 1997) (See Figure 2-2). We should note that we could not necessarily utilise these methods for the purpose of evaluating values of ecosystem services. Aylward and Barbier (1992) mention that WTP may be directly estimated through contingent valuation methods, hedonic prices and so forth, but that these techniques might be less applicable to most tropical wetlands in developing countries. Moreover, we do not believe that we can completely calculate total economic value by using all the methods available in the figure. On the theoretical side, acquisition of reliable and complete information is always possible, but observing NOAA guidelines strictly makes contingent valuation methods expensive and time consuming on the practical side (Portney 1994). It is often pointed out that the contingent valuation method focuses on preferences of particular people but does not necessarily reflect the ecological importance of ecosystem goods and services (Costanza and Farber, 1984; Gatto and de Leo, 2000). Peoples' preferences are changeable, but economists have no theoretical basis for analysing how preferences change. However, even if the valuation is not complete, it is much better than no valuation. Chavas (2000) remarks that rejecting valuation would severely limit our ability to assess environmental decisions.

de Groot (1994) provides an ambitious overview of the relationship between ecosystem functions and valuation methods from the perspective that it is difficult to solve current environmental problems unless the information on ecosystem functions is structurally integrated into the planning and decision-making processes. We should note that de Groot focuses on ecosystem 'functions' rather than ecosystem 'services'. de Groot defines ecosystem functions as the capacity of natural processes and components to provide goods and services that satisfy human needs directly and/or indirectly. He distinguishes four categories of ecosystem functions: (1) "Regulation functions: this group of functions relate to the capacity of natural and semi-natural ecosystems to regulate essential ecological processes and life support systems which, in turn, contributes to the maintenance of a healthy environment by providing clean air, water, and soil"; (2) "Carrier functions: natural and semi-natural ecosystems provide space and a suitable substrate or medium for many human activities such as habitation, cultivation and recreation"; (3) "Production functions: nature provides many resources, ranging from food and raw materials for industrial use to energy resources and genetic material"; and (4) "Information functions: natural ecosystems contribute to the maintenance of mental health by providing opportunities for reflection, spiritual enrichment, cognitive development, and aesthetic experience" (de Groot, 1994).

Furthermore, de Groot provides a table that indicates the relation between types of socio-economic values and valuation methods (see Table 2-3). According to the table, the relation seems to be clear, but it is complicated. de Groot mentions that it is impossible to add the seven values in order to obtain one total monetary value because the seven values are not comparable and that we should use each value independently in the decision-making process.

Table 2-3. *Types of socio-economic values and valuation methods*

Types of socio-economic value	Monetary valuation methods						
		Shadow price					
	Market price	Cost of environmental damage	Maintenance costs	Mitigation costs	WTP/WTA	Property pricing	Travel cost
Conservation value		X	X	X	X		
Existence value			(X)#		(X)#		
Health			X		X		
Option value			X		X		
Consumptive use value	(X)*						
Productive use value	X					X	X
Employment	X						

Source: de Groot (1994). Figure 9.4.
Note:
#: The existence value could be quantified by these techniques, but it is argued that it is principally wrong to put a monetary price on this value.
*: Is usually derived from a surrogate market price.

It is often observed that WTP approaches convert all the values of ecosystem functions or services into one monetary value. Since some environmental goods and services are too difficult to convert the monetary value, they are not sufficiently considered in cost-benefit analyses (Gatto and de Leo, 2000). Gatto and de Leo insist that multi-criteria analysis is much better than cost-benefit analysis because multi-criteria analysis can treat the value of non-market goods and services without converting the value to the monetary value. However, the valuation in multi-criteria analysis also depends on preferences. If so, we have to measure or stipulate the shape of indifference curve practically and theoretically for using the method. In addition, the trade-off relationships are still ambiguous if we cannot compare the values between market and non-market goods on the practical side. Therefore, multi-criteria analysis is not necessarily better than cost-benefit analysis and contingent valuation.

There is another way of dividing total economic value: primary and secondary values (Gren et al., 1994; Turner et al. 1995; Crooks et al., 2001). Primary value refers to the development and maintenance of ecosystems or their self-organizing capacity. Secondary values are defined as the outputs, life-support functions and services, generated by wetlands. Then, referring to the two values, Gren et al. (1994) provide three case studies and conclude that only part of the total wetland value can be captured in monetary terms. Based on Crooks et al. (2001), primary value is treated as total systems value, which is not a component of total economic value. The total value of ecosystem is composed of total economic value and total systems value. The notion of primary value is interesting, but it is ambiguous in terms of valuation especially in a practical context. In particular, it is vague how we should distinguish between primary and secondary values practically. We also worry about the problem of double-counting. How should we evaluate primary value separately? Furthermore, it is not obvious why the classification between primary and secondary values is advantageous for valuing ecosystems.

The relation between valuation and social goals is of importance (Costanza and Folke, 1997; Costanza, 2000) (see Table 2-4). The point is that valuation of ecosystems should be based on the following three goals: (1) ecological sustainability, (2) social fairness, and (3) economic efficiency. Costanza and Folke (1997) discuss a conceptual model that incorporates the three criteria as goals of policies and mention that we cannot give a value without choosing the goal.

Table 2-4. *Social goals and valuation*

Goal or Value Basis	Who Votes	Preference Basis	Level of Discussion Required	Level of Scientific Input Required	Specific Methods
Efficiency	*Homo economicus*	Current individual preferences	Low	Low	Willingness to pay
Fairness	*Homo communicus*	Community preferences	High	Medium	Veil of ignorance
Sustainability	*Homo naturalis*	Whole system preferences	Medium	High	Modeling with precaution

Source: Costanza and Folke (1997). Table 4.1., and Costanza (2000). Table 1.

Let us discuss the production function approach. Ellis and Fisher (1987) show the processes of using a production function approach, in which environmental goods are treated as an input of the production of marketed or marketable goods. They mention that the approach is firmly rooted in the welfare theory accepted by most economists and that this approach has the advantage of relying primarily on production or cost data. However, there is disadvantage on this approach. On the practical side, we need a demand function of marketed goods. We are required to estimate a compensated demand function in order to get rid of income effects, but we cannot directly estimate a compensated demand function because we cannot directly observe utility functions (Willig, 1976). Mäler et al. (1994) provide production function approach to put monetary value on multifunctional ecosystems. They enumerate four advantages of production function approach: "[1] [The production function is more stable than the utility function.] Even if preferences for different goods change over time because of various exogenous factors, it is less likely that the production function will change, except for changes in the technical knowledge. [2] Furthermore, one could reasonably assume that the production function is constant across different individuals, while preferences are not. [3] Next, information on the production function may be obtained through observing technologies which in general will be simpler than by observing preferences. [4] Finally, all established approaches to value resources through observations on market behavior — artificial markets, travel cost methods, hedonic methods, etc. — can be characterized through *à priori* assumptions on the production function". Barbier (2000) gives a concise summary about two types of production approaches (static and dynamic approaches). He explains the general approach in the following. "The general approach consists of a two-step procedure. First, the physical effects of changes in a biological resource or ecological function on an economic activity are determined. Second, the impact of these environmental changes is valued in terms of the corresponding activity. In other words, the biological resource or ecological function is treated as an 'input' into the economic activity, and like any other input, its value can be equated with its impact on the productivity of any marketed output" (Barbier, 2000).

2.4.2 Energy-based Approach

We have an energy-based (EA) approach that is an alternative to WTP approaches. WTP approaches are based on human demand while EA approaches are based on natural system supply (Costanza and Farber, 1984).

Costanza and Farber (1984) provide a good survey of energy-based approaches as compared with WTP approaches. There are three major assumptions on energy-based approach: (1) Sunlight is only a primary input to the system; (2) Energy-accounting is possible based on the primary input; and (3) More importantly, the energy embodied in both natural and human products would correlate with their economic value if all market imperfections were removed. Biological productivity or gross photosynthesis of the whole ecosystem has been used as an index of its embodied energy, which assumes independence of ecosystems. However, the assumption is so often invalid, and we need the complete description of ecosystem interconnections to calculate the energy costs (Costanza and Farber, 1984). Not to mention, it is (nearly) impossible to get complete, quantitative information on ecosystem interactions. The most critical weakness of energy-based approach lies in the hypothesis that energy embodied in both natural and human products would correlate with their economic value. In addition, it is difficult to find the exact correlation between them. More concretely, we have little information on what variable we should choose as an energy measure and a converter into economic value.

Folke (1991) indicates the usefulness of energy-based approach as a complementary method. "An energy analysis is one approach for comparing the work of "nature" with the work of "the economy", and which makes it possible to consider environmental functions which seldom have a market and on which the general public seldom have perfect information. . . . The approach is useful as a complement to economic analysis . . ." (Folke, 1991).

Farber and Costanza (1987) apply both energy-based and WTP approaches to the valuation of wetlands. They find that WTP approach underestimates the value while energy-based approach overestimates them.

Odum and Odum (2000) insist that we should value the economy on the same basis as the work of the environment although many economists try to measure the value of the environment in the economic terms. Then, they provide an interesting idea: "emergy (not energy)". Emergy expresses all numbers in one kind of energy (for example, solar energy) required to produce designated goods and services. They provide EMDOLLAR as an example. "EMDOLLARS, the economic equivalent of emergy, is defined as the GNP equivalent to emergy contributions (conversion: 1.16 trillion solar emjouls per 1997 US dollar). A recent calculation by Brown and Ulgiati (1999) found two-thirds of the global wealth produced each year to come from emergy of fuel use and one-third from the emergy of renewable energy of nature" (Odum and Odum, 2000). However, we cannot clearly capture how much value they are by using the unit emergy. In addition, it is ambiguous why emergy is better than WTP methods.

2.4.3 Marginal Value vs. Total Value

How much value do ecosystems provide us globally? Costanza et al. (1997) famously tried to calculate the total economic value of ecosystem services on the earth by using the results of over 100 existing studies. They concluded that the value was in the range of US$ 16-54 trillion per year, with an average of US$ 33 trillion per year. This is controversial as well as sensational, and provoked several criticisms.

First, several researchers have stressed the importance of the infrastructure of the ecosystem itself as a contributor to its total value. Costanza et al (1997) themselves mention that this component of the value is not included in their analysis. This value is called 'primary value' in some papers (Gren et al., 1994;

Turner et al., 1995). However, the reason why the distinction between primary and secondary values is important is ambiguous because we cannot estimate primary value of ecosystems directly and separately. The classification is not practical but conceptual.

Second, there is a problem on consumer surplus of ecosystem services. "Here, the demand [marginal willingness-to-pay] approaches infinity as the quantity available approaches zero (or some minimum necessary level of services), and the consumer surplus (as well as the total economic value) approaches infinity. Demand curves for ecosystem services are very difficult, if not impossible, to estimate in practice" (Costanza et al., 1997). If so, the shape of the demand curve means that it is of no use to estimate the value of ecosystem services because it assumes that the value of ecosystem services is infinite and that the value is always larger than that of other goods and services. In addition, "It may well be true that the 'consumer surplus' of ecosystem services is far more than all other economic goods and services put together. However, paradoxically, this cannot be true of the PQP [price-quantity product] proxy. In an evolutionary equilibrium the shadow price of most ecological services, supplied at optimum levels, would be zero. For practical purposes, the cost of control and maintenance of environmental capital is the best proxy for the value of the service flows derived therefrom" (Ayres, 1998). Furthermore, Hueting et al. (1998) insist that supply and demand curves in Costanza et al's paper are both irrelevant. The supply curve that is perpendicular to abscissa means that "the services of the function can be supplied without costs up to the perpendicular". However, this is contradictory to economic theory and empirical fact on the environment. Supply curve must reflect on opportunity costs because provision of ecosystem functions incurs costs of restoring or maintaining at least.

Third, more importantly, the total value of ecosystems in itself does not make sense (Goulder and Kennedy 1997; Opschoor, 1998; Serafy, 1998; Dasgupta et al., 2000; Heal, 2000). Costanza et al. (1997) themselves mention that it is instructive and meaningful to estimate the 'incremental' or 'marginal' value of ecosystem services. Considering life-supporting ecosystem services, human beings cannot exist without them. Their total (absolute) value must be infinite. Thus, the total value is non-sense. We should consider how and how much the change in quantity or quality of ecosystem services influences on the economic welfare. Dasgupta et al. (2000) provide critical comments. "However, the latter estimate [Costanza et al., 1997] should cause us to balk because if crucial environmental services were to cease, life would not exist. However, who would be there to receive $33 trillion of annual benefits if humanity were to exchange its very existence for them? Almost paradoxically, perhaps, the total value of the world's ecosystem services has no meaning and, therefore, is of no use, even though the value of incremental changes to those ecosystems not only has meaning — it also has use" (Dasgupta et al., 2000). Heal (2000) insists that economic valuation should focus on only the marginal value of ecosystem service and it cannot evaluate the total value of ecosystem service although we know how to do it. Heal provides another reason that market-based prices tell us only the value to society of a small amount more or less of a service based on its scarcity and do not indicate the overall contribution of the service. Heal also mentions that economic valuation is more concerned with prices than with values or importance by using the paradox of water and diamonds.[11] Then, Heal mentions that it is much more important to provide incentives to conserve ecosystems than to carry out economic valuations. We understand and agree on the point that incentives are critical for conservation. We need economic valuation in order to provide appropriate incentives to conserve ecosystems. Some economic activities that have a negative effect on the environment are also necessary for our economic welfare. That is why we should balance our economic activities and conservation. For the purpose, we need to know the marginal value of ecosystem services.

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[11] c.f. Smith (1993).

2.4.4 Valuation of Wetlands

In this section, let us review several papers on valuation of wetlands. Farber and Costanza (1987) estimate the social value of a wetland system in Louisiana coastal zone. They use the WTP measurement methods such as CVM and travel cost method, and furthermore use energy analysis method for comparison. WTP measurement tends to underestimate the value because some values such as flood protection value, option value and existence value are not evaluated. On the contrary, energy analysis tends to overestimate value because it includes things that are of little use to the society or the economy.

Costanza et al. (1989) have tried to evaluate multi-functional values of wetlands, but the point of 'multi-functional resource' is ambiguous. The difficulties of valuation of wetlands, especially as compared with other ecological goods and services, lie in the multi-functionality. Externalities and tradeoffs are the source of the difficulties.

Folke (1991) analyses the societal value of a Swedish wetland system with respect to the various economic functions by using two valuation methods: replacement costs approach and GPP (gross primary production) approach. He acknowledges that both valuation techniques consider only a part of values of life-supporting functions. He concludes that we need more development of valuation methods that especially can catch the values of life-supporting functions.

About tropical wetlands, Barbier (1993) emphasizes the importance of indirect-use value derived from supporting economic activities that have directly measurable values.

de Groot (1994) provides meaningful implication for wetland functions, which is useful for our research on river floodplains. "The most important wetland functions for which a monetary value can be calculated are (1) flood prevention,

(2) storage and recycling of human waste, (3) nursery value, (4) aquaculture and recreation, (5) food production and (6) education and science uses" (de Groot, 1994).

Barbier (1994) uses the production function approach for the total valuation of tropical wetlands. He tries to value various regulatory ecological functions of tropical wetlands that have important indirect use values such as the groundwater recharge function.

Turner et al. (1995) present three case studies on the valuation of wetlands functions. The first focuses on the support function value of a Swedish wetland. The second is an application of the contingent valuation method to the Broadland wetlands in the UK. The third focuses on the biophysical and economic measures of the island of Gotland in Sweden.

Gren et al. (1995) try to estimate the values of multiple ecosystem services of Danube floodplains. However, they have faced the serious problems of data availability and model availability. Then, they try to estimate them by using relevant results of other researches. An important finding is that the value of the land as a nutrient sink accounts for about one-half of the total value.

Mitsch and Gosselink (2000a) consider some hypotheses about the relationship between the value of wetlands and the importance of scale and landscape settings. In so doing, they maintain that we should capture the values of wetlands at three levels of ecological hierarchy: population, ecosystem and biosphere (See Table 2-5). They conclude that a wetland with moderate economic development will have the largest value because the healthy functions of wetlands and human population can co-exist in balance.

Table 2-5. *Values of wetlands*

Ecological Scale	Value
Population	Animal harvested for pelts
	Waterfowl and other birds
	Fish and shellfish
	Timber and other vegetation harvest
	Endangered/threatened species
Ecosystem	Flood mitigation
	Storm abatement
	Aquifer recharge
	Water quality improvement
	Aesthetics
	Subsistence use
Biosphere	Nitrogen cycle
	Sulfur cycle
	Carbon cycle
	Phosphorus cycle

Source: Mitsch and Gosselink (2000a). Table 1.

2.4.5 Some Problems of Valuing Multiple Resources

We have some common problems of valuing multiple resources. To begin with, we should be careful about two major difficulties when we evaluate multiple resources in particular in the case of applying the production function approach: double-counting and trade-offs (Aylward and Barbier, 1992; Barbier, 1994; Barbier, 2000).

Aylward and Barbier (1992) provide an example of coastal wetlands' nutrient retention function to explain the problem of double-counting. Their nutrient retention service supports shrimp production within the wetland area. Thus, if the full value of the shrimp production is already accounted for as a direct-use value of the wetland's resources, adding the share of the value of the nutrient retention service as an indirect-use value and aggregating to obtain total economic value will double count this indirect-use value. The point is that the values of multiple uses that are counted should be mutually exclusive and collectively exhaustible.

They also give an example of forest ecosystems to explain the problem of trade-offs. The forest provides direct-use and indirect-use values that consist of timber, non-timber products and off-site watershed protection service. If the full value of timber benefits can only be obtained through clean-cutting the forest land, simply adding other values of non-timber products and off-site watershed protection service to the value of timber to obtain the total economic value will over-estimate the value of forest ecosystem. The reason is that getting all the timber products implies the loss of values from non-timber products and watershed protection service. There is a trade-off between them. The essence is that we should precisely focus on the marginal benefits and costs related to economic decisions or activities.

Aylward and Barbier (1992) point out other problems on valuing ecosystem services. They mention that the most effective second-best method of determining the value of protective environmental functions is to estimate the damage costs that are currently being avoided. However, they point out the defect that we are required to do substantial fieldwork, data analysis and modelling for estimating the costs. In addition, they argue that we should use replacement costs and related methods with scepticism. The problem lies in the implicit assumption of the methods. The assumption is that the benefits of the replacement exceed the costs of providing these benefits. However, this assumption is contradictory to the fact that the benefit-cost ratio for the replacement can be neither equal to nor greater than one because the replacement incurs costs to reproduce the benefits that are freely provided by ecosystems.

We have another problem related to uncertainty and irreversibility. Chavas (2000) mentions that the valuation of ecosystems might produce wrong conclusions about the choice of an optimal policy if we ignore uncertainty and irreversibility that are related to the incompleteness of valuation. We need to identify circumstances leading to irreversible states that have significant adverse effects on human welfare in the long run and implement appropriate policies to avoid such irreversible situations. Moreover, we are often not fully aware of our indirect uses of ecosystems due to lack of information, true uncertainty, ecological knowledge and data (Gren et al., 1994).

2.5 Ecological Economic Modelling

2.5.1 Ecological Economic Models

"The environmental resource base upon which all economic activity ultimately depends includes ecological systems that produce a wide variety of services" (Arrow et al., 1995). Arrow et al. (2000) mention that the management of ecological systems is not beyond the reach of economic analysis, but they insist that we should develop new dimensions of economic theories and we should make much of the interconnection between economics and ecology. On the other hand, they indicate that the simplification of models is important for understanding the interdependencies among economic and natural factors because ecosystems are highly complex. Based on this perspective, we develop a hydrological, ecological, economic model in our research. Bockstael et al. (1995) try to explain the direction of the project of building an ecological economic model including ecology, economics and spatial analysis. They pose a number of questions that should be solved in the project: how ecosystems function; how they are affected by human activity; what determines human uses and human intervention into ecosystems; and how this is affected, among other things, by the ecosystem's characteristics and regulatory paradigms. In this thesis, we develop an interdisciplinary integrated model to clarify hydrological, ecological and economic interactions in floodplains and identify relevant policy options from the viewpoint.

Braat and van Lierop (1986 and 1987) provide an interesting review of economic-ecological models by using questionnaires and interviews, which cover more than 100 models. They discuss the technical structures of the economic-ecological models, the characteristics of environmental and resource problems, technical problems and institutional problems of the modelling, and the ways of dealing with them. They derive three points from the survey: (1) the majority of models in the survey are complex, integrated ecological-economic models; (2) in the class of complex models completely dynamic models dominate (63%); and (3) for ecological policy issues simulation modelling occurs most frequently while for economic issues optimisation techniques are mostly used. Likewise, we use both of them for normative and policy analyses in this research. Recently, the possibility of computer simulation in ecological economic modelling has attracted researchers' attentions due to its flexibility and the advancement of computer technology. In our research, we rely on computer simulations.

Let us review recent integrated models. Based on the Gordon-Schaefer model (Gordon, 1954; Schaefer, 1954), Clark (1990) provides a sophisticated and thorough ecological-economic models, in which economic choices take account of biological growth functions in the dynamic context. Rosegrant et al. (2000) provide a hydrologic-economic model for analysing the water resource among several sectors such as agriculture, industry, and municipal. The model focuses on how we should allocate the water resources to various uses. The interesting point is that both water quantity and water quality in terms of salinity are included in the simulation model, in which the salt concentration in the return flow from irrigated areas is explicitly calculated. That allows the endogenous consideration of the externality between upstream and downstream irrigation districts. Barbier (1994) provides an ecological-economic dynamic optimisation model as a form of production function that considers ecological services of tropical wetlands. Barbier and Strand (1997) provide a similar model, and mention that a mangrove is an important and essential input into the shrimp fishery in the Campeche, Mexico. Krysanova and Kaganovich (1994) develop a model for a watershed's ecological and economic systems analysis, which determines how to manage water and land resources for ecologically sustainable development. They insist that systems analysis and computer simulation are the best tools for understanding the functions and management of integrated ecological economic systems. Eppink et al. (2004) provide a scenario simulation analysis on the relationship between biodiversity and land use on a non-optimisation hybrid model. van den Bergh et al. (2004) provide a model that heuristically integrates a water quantity model, a water quality model and an ecological model. Using the integrated model, they evaluate three scenarios of land-use: the intensification of agriculture, the nature scenario (converting agricultural lands into nature areas) and the recreation scenario (converting lands into nature areas and designing them for recreation). Interestingly, they give three indicators for evaluating and ranking the scenarios: NPV, spatial equity and environmental quality. They conclude that the recreation scenario is the most-preferred one and that Nature scenario becomes the most preferred one only if the weight assigned to NPV is substantially reduced. Deal and Schunk (2004) develop an integrated large-scale computer model to assess the economic impacts of urban land use transformation. Chopra and Adhikari (2004) provide a dynamic simulation model to investigate the linkage between ecological relationships and economic value in the context of a wetland in Northern India. They show that the economic value substantially depends on ecological health indices.

2.5.2 Optimisation

In our research, we set up a static model and a dynamic model, and then we mathematically optimise them respectively. Techniques for static optimisation are familiar in economics. Thus, we do not discuss them here.[12] However, dynamic optimisation is not necessarily familiar although it has been established especially in financial economics, development economics and resource economics.

Most relevant here are contributions by Clark (1973 and 1990) and Conrad and Clark (1987), which show the application of the optimal control theory to renewable and non-renewable resources (mainly, the fishery model). Ehui et al. (1990) provide a two-sector dynamic model that treats the problem of the

[12] c.f. Chiang (1984); Lambert (1985); Dixit (1990); Leonard and Long (1992); Hoy et al. (2001).

trade-off between current and future interests, taking account of the interactions between deforestation and agricultural productivity. We provide a similar dynamic model in the context of river floodplains.

2.5.3 Hydrological Models

Over the last three decades, hydrologic simulation models have advanced with the help of the development of computer hardware and software. Hydrological simulation models have the advantage that "many alternative schemes for development or flood control can be quickly tested and compared with simulation models" (Bedient and Huber, 2000). In addition, "[h]ydrologic models allow for parameter variations in space and time through the use of well-known numerical methods. Complex rainfall patterns and heterogeneous basins can be simulated with relative ease if watershed and hydrologic data are sufficient, and various design and control schemes can be tested with the models" (Bedient and Huber, 2000). Bedient and Huber (2000) survey both hydrological simulation models and GIS based models. They identify six major categories of watershed analysis models: lumped parameter vs. distributed parameter, event vs. continuous, and stochastic vs. deterministic. Lumped parameter models are black box models that transform actual rainfall input data into runoff output data. An example of the lumped parameter model is Snyder or Clark UH (unit hydrograph). Distributed parameter models describe physical processes and mechanism in space. They have been recently connected to the more advanced GIS and digital elevation models. An example of this model is the Kinematic wave model. Event models are simulation models of rainfall-runoff from single storm events. The examples are HEC-1 Flood Hydrograph Package, HEC-HMS, SWMM, SCS TR-20 and so on. HEC-1 and HEC-HMS have been used for most floodplain computations in the United States. The newest version of HEC-RAS can treat the unsteady flow model (one-dimensional). Bedient and Huber (2000) summarise the family of hydrological simulation models (See Table 2-6 and 2-7).

Spatial data are significant for hydrological models. Therefore, hydrological models should be GIS-based if possible. In particular, digital elevation models (DEM) are important. Thus, some models mentioned above are directly connected with some GIS models or software. HEC-GeoHMS and HEC-GeoRAS are well-known examples, which are useful for analysing river floodplains. "HEC-GeoRAS was used to develop digital floodplains that were analyzed in ArcView for the purpose of comparing various flood control options and displaying these results directly onto Digital Orthophoto Quadrangles (DOQs)" (Bedient and Huber, 2000). In addition, they can be connected with Arc/Info, the oldest and still one of the most powerful GIS software applications. In our research, we do not use the hydrologic software program integrated with GIS, but we use HEC-HMS and GIS data with ArcGIS.[13]

Table 2-6. *Model type and example*

Model Type	Example of Model
Lumped parameter	Snyder or Clark UH
Distributed	Kinematic wave
Event	HEC-1, HEC-HMS, SWMM, SCS TR-20
Continuous	Stanford model, SWMM, HSPF, STORM
Physically based	HEC-1, HEC-HMS, SWMM, HSPF
Stochastic	Synthetic streamflows
Numerical	Kinematic or dynamic wave models
Analytical	Rational Method, Nash IUH

Source: Bedient and Huber (2000), p.315

Table 2-7. *Selected simulation models in hydrology*

Model	Author	Date	Description
Stanford Model	Crawford and Linsley	1966	Stanford Watershed Model
HEC-1	HEC	1973, 1981, 1990	Flood hydrograph package
HEC-2	HEC	1976, 1982, 1990	Water surface profiles
HEC-HMS	HEC	1998, 2001	Hydrologic modeling system (replace HEC-1)
HEC-RAS	HEC	1995, 2000	River Analysis system (replace HEC-2)
SCS-TR20	USDA SCS	1984	Hydrologic simulation model
HSPF	Johanson et al.	1984	Hydrological Simulation Program - FORTRAN
SWMM	Huber and Dickinson	1971, 1988	Storm Water Management Model
DWOPER	NWS, Fread	1978	NWS operational dynamic wave model
UNET	Barkau	1992	One-dimensional dynamic wave

Source: Bedient and Huber (2000), p. 318

[13] ArcGIS is the newest GIS software program, which integrates ArcView and Arc/Info.

We review some papers on hydrological models related to our research. Ogawa and Male (1986) provide an important analysis for our research. They evaluate the capacity of flood mitigation function of upstream wetlands and downstream wetlands by using existing computer simulation models (HEC-1 and HEC-2). They provide an interesting conclusion that the flood mitigation function of wetlands is important for downstream flood protected area, but that downstream main-stem wetlands are more effective in reducing downstream flooding than upstream wetlands. In brief, the extent of flood mitigation function depends on the size of wetland, the location of wetland and the relative location of wetland to protected areas (Ogawa and Male, 1986). Ford and Oto (1989) provide the logic and procedures of a branch-and-bound enumeration approach for choosing an optimal (economically the most efficient[14]) management option. They use the HEC software such as HEC-1, HEC-2, SID and HEC-DSS, and the inputs into the programs are the discharge CDF, elevation-discharge, and elevation-damage functions for the location of interest. Kuchment et al. (1996) apply a physically based distributed rainfall-runoff model to the basin of River Ouse (our research area). They create a model for the Ouse basin and evaluate the accuracy of prediction and the possibility of using as a simulating model. They find that the model can simulate catchment outflows satisfactorily with hydrologically meaningful estimates of internal variables given. Olsen et al. (2000) provide a dynamic model for choosing an optimal mix of floodplain management options. The model is formulated as a Markov decision process. The model can evaluate multiple objectives at the same time by use of weighting factors. The uniqueness of the model lies in the point that the model can take account of non-stationary trends such as hydrological conditions and economic development for sequential decision-making although the traditional cost-benefit approach assumes those factors are constant for the life of the project. Lund (2002) provides a model for choosing the optimal mix of floodplain management options within a probabilistic framework. The objective is to choose the options in order to minimize expected annual damages and flood management expenses.

[14] The word "efficient" here is different from that in economics (allocative efficiency). This word here means "the most cost-saving" or "the cheapest", which is similar to technical efficiency in economics.

Lund attempts to provide an answer to the following two questions. "Given a probability distribution of inundation flows or stages for a floodplain, what nonstructural floodplain management options should be undertaken? And, what is the economic value of a changed set of probabilistic inundation levels, as might arise from the operation of a system of levees, reservoirs, and channel improvements?" (Lund, 2002). Al-Sabhan et al. (2003) review the current status of spatial hydrological models, and then mention that we should develop the real-time Web-based flood prediction system linked with GIS and other hydrological models in stead of relying on the stand-alone hydrological models because the stand-alone models are complex, non-user-friendly and time-consuming. White and Howe (2002) discuss policies against floods in England. They have contended that the recent planning policies of development lead to the increasing risks of flooding because they do not adequately take all the factors into consideration. Flooding has occurred due to the interactions among natural, social, economic, political, ecological factors.

Unfortunately, to the best of our knowledge, there is no literature that treats the problem of floodplain management in an integrated way from the hydrological, ecological and economic point of view. This is our research target although it is challenging.

2.6 Conclusion of Chapter 2

Based on the literature review, we have to solve the following three problems in this thesis. First, we have to clarify the real problem of floodplain management and to show how we should consider it. The theory of market failure has been established, but it is still ambiguous what is the essential problem in individual environmental problems practically. We clarify two types of environmental externalities related to ecosystem services of floodplains and derive the optimal conditions by developing a model.

Second, we have to provide an interdisciplinary integrated model that enables us to identify appropriate policy options in the practical context of floodplain management. In each field of ecology, biology and hydrology, our knowledge on ecosystem functions has been accumulating. On the other hand, the researches on valuation of ecosystem services have been carried out, although valuation of ecosystem services is still a challenging topic. However, even if we can understand that ecosystem services are important, we cannot optimally manage environmental and ecological resources in the social and economic contexts without relevant integrated models. In order to solve real environmental problems, we need to understand environmental and economic interactions. Indeed, there are such attempts, but they are not sufficient and studies on individual environmental problems remain to be done. In this respect, this thesis provides a hydrological, ecological and economic model that enables us to implement policy simulations for the optimal floodplain management.

Third, we have to evaluate potential policy options under some real situations and identify appropriate policies for the optimal floodplain management in the practical context. In theory, several market based policies and command-and-control approaches are equivalently effective as long as they fully internalise externalities. However, the real conditions are different from those in theory in individual environmental problems. In fact, the evaluation of potential policy options has been discussed in various cases, but we have not achieved a definite conclusion about them. Therefore, in this thesis we test a number of policy options for the optimal floodplain management by computer simulations in the concrete context of the Ouse catchment.

Chapter 3

Static Decision Model of River Floodplains

3.1 Introduction

The purpose of this chapter is to provide a static decision model of hydrological, ecological and economic interactions in river floodplains. We need a static model to clarify the essential problems in real situations of floodplain development and relevant policies. The problems are: (a) what types of externalities should we manage? (specification and estimation of externalities); (b) what policies are workable under what situations? (uncertainties about measurement and structure, and the effectiveness of alternative policy instruments)

We derive the socially optimal conditions, and clarify the problem of externalities by comparing them with private optimal conditions. The socially optimal conditions become an essential benchmark. The model includes the values of ecosystem services of river floodplains, although the latter are simplified into an aggregate function. Moreover, we focus on flood mitigation service in order to simplify the model and to shed light on the particular problem of the unidirectional spatial externality related to flood mitigation service.

3.2 Model Settings

We have to set several conditions and limitations as presuppositions of the model in advance in order to simplify the model. They are crucial in that they determine whether we can handle the model (tractability) and that they determine the range that the model can explain (limitations of the model).

- There are two control variables. One is the size of the developed floodplains (development of floodplains). The other is the scale of averting behaviour such as building dykes, banks, bunds, floodwalls and so on.
- We can set some other variables as control variables. For example, land use patterns outside floodplains have an impact on flood risk through a change in imperviousness of land. However, we assume them as exogenous variables for the time being in order to focus on floodplain management.
- The total size of floodplain is fixed, and is defined by the 100-year floodplain (1% chance of flooding each year). Strictly speaking, the size of floodplains will change over time in the long term even if it is defined by the same notion, but we assume that it is constant in the model.
- The type of floodplain development has different effects on ecosystem functions that natural floodplains have. For example, the development into industrial areas reduces the capacity of mitigating floods more than that into agricultural areas. However, we do not distinguish among various types of development in the model. We distinguish just between natural and developed floodplains. We assume that the aggregate value of ecosystem services and the capacity of flood mitigation (or the flood risk) depend on the size of development (the size of remaining natural floodplains).[1]
- The cost of floodplain development is not the same as that of floodplain restoration. Therefore, the solution of an optimisation problem depends on initial conditions with respect to the size of developed floodplains and natural

[1] In the process of calibrating parameters of an applied model, we consider different types of development although we adhere to the dichotomy. At least, we must categorize real situations of floodplains into natural or developed floodplains.

floodplains. However, we assume the initial condition that all the floodplains are natural in the static model, in which we can ignore the cost of floodplain restoration. This assumption tells us to what extent we should develop floodplains from the original situation. Nevertheless, initial conditions are significant for solving practical problems. Thus, we are going to consider initial conditions and the cost of floodplain restoration in the dynamic context in the next chapter.

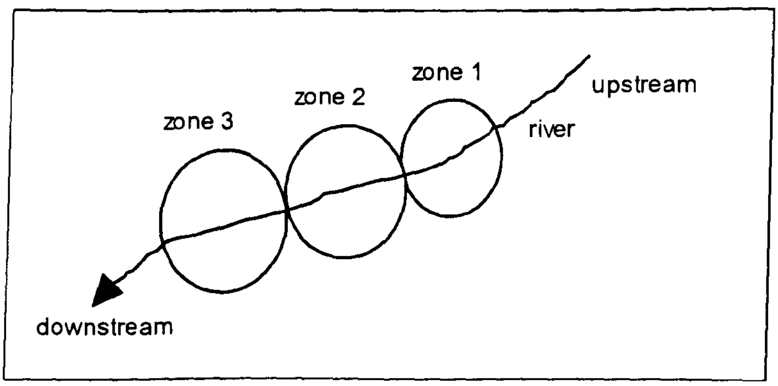
- We assume that the society is risk-neutral. Individuals are risk-averse while organizations including governments and societies are risk-neutral.

3.3 Mathematical Notation

The following notation is used in the model.

i : the zone number. We divide a catchment into several zones (subbasins). The zone numbers are given to each zone from upstream to downstream in ascending order (see figure 3-1).

Figure 3-1. *Concept of zone (subbasin) number*

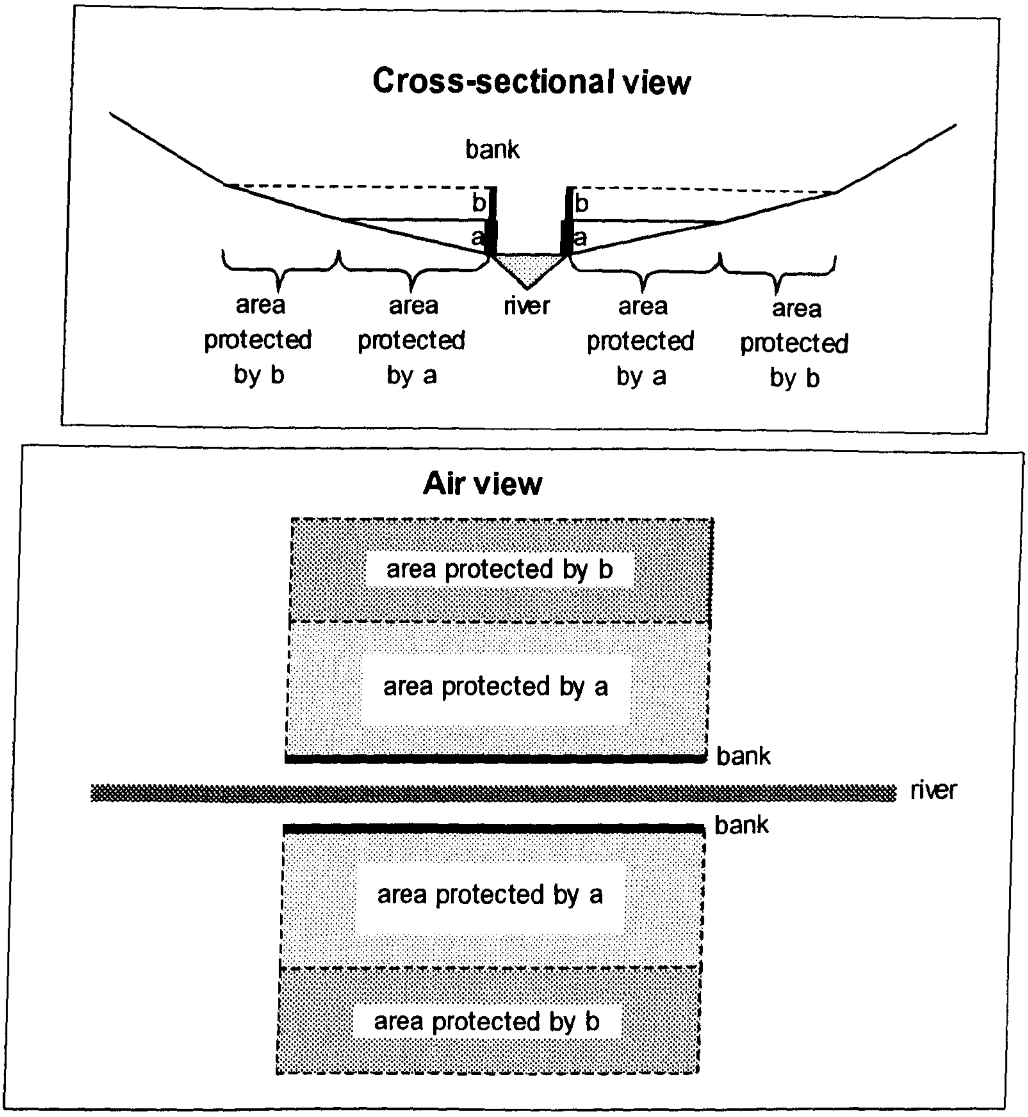


m : the total number of zones.

x^i : the size of developed floodplains (the size of development in floodplains).

L_F^i : the size of floodplains which include natural and developed floodplains. It is assumed constant.

Figure 3-2. *The area potentially protected by averting behaviour*



a^i : the area potentially protected by averting behaviour. The area is determined by the scale of averting behaviour.[2] See figure 3-2. The area potentially protected should be given by a function of height and length of averting behaviour (e.g. floodwalls, banks, bunds or so). In fact, we directly control the scale of averting behaviour, and indirectly control the area potentially

[2] The reason why we indicate, "potentially protected" is that the area protected by averting behaviour is thought of as the area covered by height and length of the averting behaviour. Therefore, the area is potentially protected, but it is not necessarily protected in reality. That is determined by the combination of averting behaviour and other factors such as meteorological variables, patterns of land use etc.

protected through the scale of averting behaviour. Then, we refer to the area potentially protected by averting behaviour as a proxy of the scale of averting behaviour because it is easy and simple to treat only one variable than two or more variables in a theoretical model.

Finally, all the vectors are denoted by bold letters. For example, $\mathbf{q}$, $\mathbf{x}$ and $\mathbf{a}$.

3.4 Functions and Assumptions

In this section, we provide the functions in the model and their assumptions. To begin with, all functions are continuous and twice differentiable with respect to relevant arguments as long as we do not take a special caveat.

$U(\pi(\cdot))$ is the social utility function in which utility is cardinal rather than ordinal.[3] Based on the preceding assumption that society is risk-neutral, the utility level of expected value is equivalent to the expected utility level of each value.[4]

$\pi(\cdot)$ is the aggregate net benefit function. It is evaluated in monetary terms. It is composed of several functions of benefits and costs related to the control variables.

$$U_\pi = \frac{dU}{d\pi} > 0,\ U_{\pi\pi} = \frac{d^2U}{d\pi^2} < 0.$$

The marginal utility with respect to the aggregate net benefit π is positive. The marginal utility is diminishing with respect to the aggregate net benefit.

$B(L_F^i - x^i)$ is the benefit function of ecosystem services of natural floodplains, excluding flood mitigation service. The benefit of ecosystem services is given by an aggregate term of relevant ecosystem services. It is a function of the size of natural floodplains (the size of floodplains minus that of developed

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[3] We take the utilitarian approach.

[4] The aggregate net benefit function includes the expected damage cost of floods, but we do not need to pay a deliberate attention to this point due to the assumption.

floodplains). It is evaluated in monetary terms.[5]

$$B_x = \frac{dB}{dx} = -\frac{dB}{d(L_F - x)} < 0, \quad B_{xx} = \frac{d^2B}{dx^2} = \frac{d^2B}{d(L_F - x)^2} \le 0.$$

The marginal benefit of ecosystem services with respect to floodplain development is negative. The negative marginal benefit (the marginal opportunity cost) is decreasing with respect to the size of developed floodplains.

$f(x^i)$ is the direct net benefit function of floodplain development. It includes only the direct benefits and costs of development. The function reflects the difference between unit land price as a developed land and unit cost of floodplain development. It is evaluated in monetary terms.

$$f_x = \frac{df}{dx} > 0, \quad f_{xx} = \frac{d^2f}{dx^2} \le 0.$$

The marginal direct net benefit with respect to floodplain development is positive. The marginal direct net benefit is decreasing with respect to the size of developed floodplains.

$g(a^i)$ is the cost function of averting behaviour. It is a function of the potentially protected area. It is evaluated in monetary terms.

$$g_a = \frac{dg}{da} > 0, \quad g_{aa} = \frac{d^2g}{da^2} \ge 0.$$

The marginal cost of averting behaviour is positive. The marginal cost is increasing with respect to the potentially protected area which reflects the scale of averting behaviour.

$C^i\{x^i, a^i, \mathbf{q}(x^j, a^j)\}$ $(i \neq j)$ is the expected cost function of flood risk in zone i, in which $q(x^j, a^j)$ is the function of flood risk that upstream zones j impose on zone i. We call this flood risk $q(x^j, a^j)$ "external flood risk". The

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[5] We implicitly assume the same functions for all zones. This assumption is set for other functions in the model except for the expected cost function of flood risk.

external flood risk is given by a function of the size of developed floodplains and the area potentially protected by averting behaviour in zone *j*. We assume that the value of this function is nonnegative. In addition,

$$q_x = \frac{dq}{dx^j} \geq 0, \quad q_a = \frac{dq}{da^j} \geq 0.$$

This implies that the external flood risk increases as we develop more floodplains and/or we enlarge the area potentially protected by increasing the scale of averting behaviour. However, they have no impact on the external flood risk over a certain threshold value. This is shown by the equality sign.

Then, the expected cost of flood risk in zone *i* is given a function of the size of developed floodplains in its own zone *i*, the degree of averting behaviour in its own zone *i*, and the external flood risk that upstream zones *j* impose on zone *i*. It is evaluated in monetary terms. An essential assumption on the location of zones is related to the problem of unidirectional spatial externality. If $i < j$, then $q(x^j, a^j) = 0$. It implies that the choice of control variables in the downstream zones *j* has no effect on the flood risk in zone *i*.

$$C_x = \frac{dC^i}{dx^i} \geq 0, \quad C_a = \frac{dC^i}{da^i} \leq 0.$$

The expected cost of flood risk increases as we develop more floodplains in its own zone *i*. On the contrary, the expected cost decreases as we increase the area protected by averting behaviour in its own zone *i*. In other words, averting behaviour in its own zone *i* is a substitute for the flood mitigation service of natural floodplains in its own zone *i*. However, we should note that floodplain development and averting behaviour have no effect on the expected cost over a certain threshold value respectively. This point is shown by the equality sign in the equations. In addition,

$$C_q = \frac{dC^i}{dq} > 0 \text{ when } i > j \text{ (otherwise } C_q = 0\text{)}.$$

This means that the expected cost of flood risk goes up as the external flood risk increases. We need to take the caveat that the external flood risk has no impact (it is assumed to be zero) in the direction from downstream to upstream. We assume that the expected cost function is convex in all the control variables jointly.[6] In addition, we implicitly assume that there is no external flood risk within each zone i.

3.5 Social Optimisation Problem

The social optimisation problem may be summarised as follows:

$$\max_{\mathbf{x},\mathbf{a}} \; W = U\{\pi(\mathbf{x},\mathbf{a})\}$$

where

$$\pi(\cdot) = \sum_{i=1}^{m} B(L_F^i - x^i) + \sum_{i=1}^{m} f(x^i) - \sum_{i=1}^{m} g(a^i) - \sum_{i=1}^{m} C^i\{x^i, a^i, \mathbf{q}(x^j, a^j)\}$$

subject to

$0 \le x^i \le L_F^i$ (for all i)

$0 \le a^i$ (for all i).

The social (global) optimum provides a benchmark. The social optimisation problem is to choose the values of all the control variables in order to maximise social welfare. Social welfare is composed of the social utility of the aggregate net benefits. In addition, the problem is subject to two constraints on the control variables. The development of floodplains is limited by the size of floodplains, and both control variables are nonnegative.

[6] See Appendix A-1.

3.6 Analytical Solutions

In this section, we get the analytical solutions of the social optimisation problem. First, we derive the first-order Kuhn-Tucker conditions. Second, we categorize them into the cases of interior solution and corner solution. Third, we provide economic interpretations of them.

In the beginning, we set the Lagrange function for solving the problem.

$$Z = U\{\pi(\mathbf{x},\mathbf{a})\} + \sum_{i=1}^{m} \lambda^i (L_F^i - x^i).$$

We differentiate the Lagrange function with respect to relevant variables and set the appropriate conditions to derive the first-order conditions in the following.

The Kuhn-Tucker first-order conditions are the following.

$$\frac{\partial Z}{\partial x^i} = U_\pi \left\{ \frac{dB}{dx^i} + \frac{df}{dx^i} - \frac{\partial C^i}{\partial x^i} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial x^i} \right\} - \lambda^i \le 0,$$

$$x^i \ge 0 \text{ and } x^i \frac{\partial Z}{\partial x^i} = 0 \quad \text{(for all } i\text{)}.$$

$$\text{(3-1)}$$

$$\frac{\partial Z}{\partial a^i} = U_\pi \left\{ -\frac{dg}{da^i} - \frac{\partial C^i}{\partial a^i} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial a^i} \right\} \le 0,$$

$$a^i \ge 0 \text{ and } a^i \frac{\partial Z}{\partial a^i} = 0 \quad \text{(for all } i\text{)}.$$

$$\text{(3-2)}$$

$$\frac{\partial Z}{\partial \lambda^i} = L_F^i - x^i \ge 0,$$

$$\lambda^i \ge 0 \text{ and } \lambda^i \frac{\partial Z}{\partial \lambda^i} = 0 \quad \text{(for all } i\text{)}.$$

$$\text{(3-3)}$$

3.6.1 Interior Solution

We focus on the case where there is an interior solution. Assuming that we have an interior solution, we focus on the ranges $0 < x^i < L_F^i$ and $a^i > 0$ for all i. Within these ranges, we can derive the conditions for the optimal solution from the first-order conditions (3-1), (3-2) and (3-3).

$$\frac{\partial Z}{\partial x^i} = U_\pi \left\{ \frac{dB}{dx^i} + \frac{df}{dx^i} - \frac{\partial C^i}{\partial x^i} - \sum_{j>i} \frac{\partial C^j}{\partial q}\frac{\partial q}{\partial x^i} \right\} - \lambda^i = 0 \quad \text{(for all } i\text{).} \tag{3-4}$$

$$\frac{\partial Z}{\partial a^i} = U_\pi \left\{ -\frac{dg}{da^i} - \frac{\partial C^i}{\partial a^i} - \sum_{j>i} \frac{\partial C^j}{\partial q}\frac{\partial q}{\partial a^i} \right\} = 0 \quad \text{(for all } i\text{).} \tag{3-5}$$

$$\lambda^i = 0 \quad \text{(for all } i\text{)} \tag{3-6}$$

From (3-4), (4-5), (3-6) and $U_\pi > 0$, the optimal choice of control variables (x^{i*}, a^{i*}) must satisfy the following:

$$\frac{df}{dx^i} = -\frac{dB}{dx^i} + \frac{\partial C^i}{\partial x^i} + \sum_{j>i} \frac{\partial C^j}{\partial q}\frac{\partial q}{\partial x^i} \quad \text{(for all } i\text{).} \tag{3-7}$$

$$-\frac{\partial C^i}{\partial a^i} = \frac{dg}{da^i} + \sum_{j>i} \frac{\partial C^j}{\partial q}\frac{\partial q}{\partial a^i} \quad \text{(for all } i\text{).} \tag{3-8}$$

To begin with, let us consider the meaning of the Lagrange multiplier (λ^i). The optimal value of Lagrange multiplier can be interpreted as the marginal utility of the size of floodplains.[7] The Lagrange multiplier implies the imputed value or shadow price of developed floodplains. It is the value of one more unit of floodplains developed. When we have an interior solution, the imputed value (shadow price) of developed floodplains is zero. This means that there is no

[7] See Appendix A-2.

incentive to develop natural floodplains in the equilibrium.

Next, we consider the first condition (3-7). The term on the left hand side is the marginal direct net benefit of floodplain development. The first term on the right hand side is the marginal cost of losing ecosystem services due to floodplain development. The second term on the right hand side is the marginal cost of flood risk due to floodplain development. The third term on the right hand side is the marginal cost of external flood risk that floodplain development in zone i inflicts on the zones downstream. The third term is significant because it measures the marginal external costs (unidirectional spatial externality). In total, the condition (3-7) implies that the marginal benefit of floodplain development must be equal to the marginal cost of floodplain development plus the marginal external cost of floodplain development.

Finally, we consider the second condition (3-8). The term on the left hand side is the marginal benefit of averting behaviour, which means the marginal cost of flood risk that we can save by averting behaviour. The first term on the right hand side is the marginal direct cost of averting behaviour. The second term on the right hand side is the marginal cost of external flood risk that averting behaviour in zone i inflicts on the zones downstream. This term implies the marginal external cost (unidirectional spatial externality). Finally, the condition (3-8) implies that the marginal benefit of averting behaviour should equal the sum of the marginal direct cost of averting behaviour plus the marginal external cost of averting behaviour.

The upshot is that we must take both the marginal cost of losing ecosystem services (externality between private and public people) and the marginal external cost (unidirectional spatial externality) into consideration in order to choose the values of control variables for the social optimisation. The condition is simple, but significant because these costs are normally ignored in our market economy. As a result, floodplains tend to be over-developed, and averting behaviour tends to be overused.

3.6.2 Corner Solution

Next, we focus on the case of corner solution. There are three corner solutions.

The first case is $x^I = 0$. It implies that all the natural floodplains remain undeveloped. From the first-order condition (3-1),

$$U_\pi \left\{ \frac{dB}{dx^I} + \frac{df}{dx^I} - \frac{\partial C^I}{\partial x^I} - \sum_{J>I} \frac{\partial C^J}{\partial q} \frac{\partial q}{\partial x^I} \right\} - \lambda^I < 0.$$

(3-9)

From the first-order condition (3-3),

$$\frac{\partial Z}{\partial \lambda^I} = L_F^I - x^I = L_F^I > 0 \qquad (\because\ x^I = 0).$$

Then, $\lambda^I = 0 \qquad (\because\ \lambda^I \frac{\partial Z}{\partial \lambda^I} = 0)$.

Based on the condition $\lambda^I = 0$, we transform (3-9) into

$$\frac{df}{dx^I} < -\frac{dB}{dx^I} + \frac{\partial C^I}{\partial x^I} + \sum_{J>I} \frac{\partial C^J}{\partial q} \frac{\partial q}{\partial x^I} \qquad (\because\ U_\pi > 0).$$

(3-10)

Hence, $x^I = 0$ happens when the condition (3-10) is true. The condition (3-10) implies that the marginal direct net benefit of floodplain development is smaller than the marginal cost of floodplain development. In this case, there is no incentive to develop natural floodplains from scratch because floodplain development brings about a net loss to the society. All natural floodplains should be conserved. Furthermore, considering the Lagrange multiplier, the imputed value (shadow price) of developed floodplain is zero.

The second case is $x^I = L_F^I$. It implies that all the floodplains are fully developed. From the first-order condition (3-3),

$$\frac{\partial Z}{\partial \lambda^i} = L_F^i - x^i = 0.$$

Then, $\lambda^i > 0 \quad (\because \lambda^i \frac{\partial Z}{\partial \lambda^i} = 0).$

(3-11)

Therefore, $x^i = L_F^i$ occurs when condition (3-11) holds. Thus, the shadow price (imputed value) of developed floodplains is positive in this corner equilibrium. As long as the shadow price of the developed floodplains is positive, there is an incentive to continue to develop them.

The third case is $a^i = 0$. It implies that no area is potentially protected by averting behaviour (no averting behaviour). From the first-order condition (3-2),

$$\frac{dg}{da^i} > -\frac{\partial C^i}{\partial a^i} - \sum_{j>i} \frac{\partial C^j}{\partial q}\frac{\partial q}{\partial a^i} \quad (\because a^i \frac{\partial Z}{\partial a^i} = 0 \text{ and } U_\pi > 0).$$

(3-12)

Thus, $a^i = 0$ happens when condition (3-12) is true. Condition (3-12) implies that the marginal cost of averting behaviour is larger than the marginal benefit of mitigating the flood risk in all the relevant zones. In this case, there is no incentive to carry out averting behaviour at all.

3.7 Sufficiency Conditions

In the previous section, we derive the conditions for the social optimisation based on the first-order Kuhn-Tucker conditions which are just necessary conditions. It is still unknown whether they are not only necessary but also sufficient for the optimality. Thus, we have to verify that they satisfy the sufficiency conditions for the optimality. For this purpose, we have two options for them: the Kuhn-Tucker sufficiency theorem and the Arrow-Enthoven sufficiency theorem. The Kuhn-Tucker sufficiency theorem is stricter than the Arrow-Enthoven one, and we use that.

Chiang (1984) has provided a concise summary of the Kuhn-Tucker sufficiency theorem. The problem given as a nonlinear program is to maximise $\pi = f(x)$ subject to $g^i(x) \le r_i$ $(i = 1,2,...,m)$ and $x \ge 0$. In this case, the three conditions must be satisfied in order that the first-order conditions are not only necessary but also sufficient for deriving a global maximum. First, the objective function is differentiable and concave in the nonnegative orthant. Second, every constraint function is differentiable and convex in the nonnegative orthant. Third, the choice of the values of control variables x* satisfies the Kuhn-Tucker first-order necessary conditions.

Now, we check the three conditions about the social optimisation problem that we set in the previous section. The key things are the concavity of the objective function and the convexity of the constraint functions. Based on the assumptions of functions in section 3.4, all the sub-functions that are components of the objective function are concave.[8] If all the functions are concave, the sum of the functions is also concave. Hence, the objective function is concave.[9] All the constraint functions are linear. Linear functions are convex as well as concave. As a result, the first-order Kuhn-Tucker necessary conditions that we derive satisfy the conditions of the Kuhn-Tucker sufficiency theorem. In addition, if at least one function included in the objective function is strictly concave, the optimal solution derived from the first-order conditions should be unique.[10]

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[8] Some functions are assumed convex. However, if $f(x)$ is a convex function, then $-f(x)$ is a concave function. Hence, all the component functions are concave if we consider minus signs.

[9] Strictly speaking, the objective function is the social utility function that is a function of the aggregate net benefit. It is assumed concave, and the aggregate benefit is composed of concave functions of relevant control variables. In general, if a function is a concave function of a concave function in relevant variables, it is a concave function in them. See Appendix A-3.

[10] We assume that at least one function in the objective function is strictly concave here.

3.8 Problem of Externality

The purpose of this section is to clarify the problem of externalities by considering two types of local optimisation problems. In the social optimisation problem, an economic agent aims to maximise the total social welfare including the benefits of ecosystem services in the whole catchment. The agent might be a regional government or governmental agencies. On the other hand, in the private and local optimisation, individual landowners do not consider costs of losing ecosystem services and external costs that they impose on people downstream.

From the viewpoint of individual landowners, we set the local optimisation problem. Landowners attempt to maximise their net benefits by adjusting their control variables. The local optimisation problem is defined in each zone i by the following.[11]

$$\max_{x^i, a^i} \; W^i = U\{\pi^i(x^i, a^i)\}$$

where

$$\pi^i(\cdot) = f(x^i) - g(a^i) - C^i\{x^i, a^i, \mathbf{q}(x^j, a^j)\} \quad (i \neq j)$$

subject to

$$0 \le x^i \le L_F^i$$

$$0 \le a^i$$

(for all i).

The problem is that landowners aim to maximise their utility (welfare) which depends on their private net benefits. The important thing is that landowners never consider the benefits of ecosystem services and the external flood risk that they

[11] We implicitly set some assumptions. (i) The market for developed lands including developed floodplains is competitive in each zone. (ii) There is no external flood risk from upstream to downstream within each zone. (iii) We set the problem in the unit of zone, based on the competitive market and profit (benefit) maximising private landowners. Thus, this seems to be a problem of a representative zone i. Alternatively, we can assume the representative private landowner who possesses all the floodplains in zone i. In this case, we assume that the private landowner has no monopoly power.

inflict on the zones downstream.[12]

Let us derive the first-order Kuhn-Tucker conditions from the relevant Lagrange function F with a Lagrange multiplier μ.

$$F = U\{\pi^i(\cdot)\} + \mu(L_F^i - x^i)$$

$$\frac{\partial F}{\partial x^i} = U_\pi \left\{ \frac{df}{dx^i} - \frac{dC^i}{dx^i} \right\} - \mu \le 0,$$

$$x^i \ge 0 \text{ and } x^i \frac{\partial F}{\partial x^i} = 0 \quad \text{(for all } i\text{)}.$$

(3-13)

$$\frac{\partial F}{\partial a^i} = U_\pi \left\{ -\frac{dg}{da^i} - \frac{dC^i}{da^i} \right\} \le 0,$$

$$a^i \ge 0 \text{ and } a^i \frac{\partial F}{\partial a^i} = 0 \quad \text{(for all } i\text{)}.$$

(3-14)

$$\frac{\partial F}{\partial \mu} = L_F^i - x^i \ge 0,$$

$$\mu \ge 0 \text{ and } \mu \frac{\partial F}{\partial \mu} = 0 \quad \text{(for all } i\text{)}.$$

(3-15)

Assuming that we have an interior solution, we can derive the following conditions from conditions (3-13), (3-14) and (3-15).

$$\frac{df}{dx^i} = \frac{dC^i}{dx^i} \quad \text{(for all } i\text{)}.$$

(3-16)

[12] Individual landowners also enjoy the benefit of ecosystem services that natural floodplains provide. However, they do not recognize the benefit as the value in monetary terms. The benefit is trivial to each private landowner although it is enormous to the public in total.

$$-\frac{dC^i}{da^i} = \frac{dg}{da^i} \quad \text{(for all } i\text{)}.$$

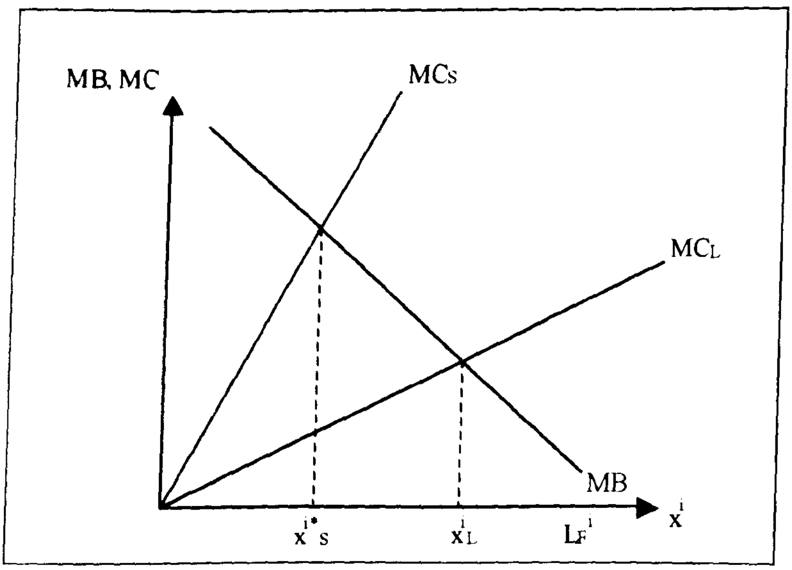
(3-17)

These conditions imply that the marginal benefit equals the marginal cost with respect to both the control variables although the benefit and cost are related to only individual landowners' interests. As a result, landowners underestimate the marginal cost, and then they tend to over-develop floodplains and over-invest in averting behaviour. Table 3-1 and Figure 3-3 provide the summary about the analysis of overdevelopment and over-investment due to ignoring externalities.

Table 3-1. *Summary: problems of externalities*

	Social Optimisation		Local Optimisation
Viewpoint	whole catchment		zone (private lands)
Condition	MB = MC + MEC		MB = MC
Development	x_s^{i*}	<	x_L^i
Averting Behaviour	a_s^{i*}	<	a_L^i

Figure 3-3. *Summary: overdevelopment problem of externalities*



3.9 Policy Implications from Static Model

In this section, we discuss possible policies for the optimal management of floodplains, based on the static model. Now, let us focus on floodplain development rather than averting behaviour. The political target is to realize the optimal amount of floodplain development (developed floodplains) derived from the social optimisation. Thus, we consider possible policies that enable the state to reach the optimal amount of floodplain development (x_S^{i*}) by internalising the marginal external costs ($-\frac{dB}{dx^i}+\sum_{j>i}\frac{\partial C^j}{\partial q}\frac{\partial q}{\partial x^i}$). We can come up with four possible policies in general: (1) Pigouvian tax; (2) marketable permits; (3) subsidy; and (4) command and control.[13]

3.9.1 Pigouvian Tax

Policies of economic incentives such as Pigouvian tax, marketable permits and subsidies aim to accomplish the optimal target at the least costs as long as several conditions are satisfied. Several conditions include information availability, administrative capacity, existence of competitive markets, policy feasibility and so on. In this section, we will focus on uncertainty (information availability) related to effectiveness. In the beginning, we focus on Pigouvian tax. We set a tax rate that coincides with the marginal external costs on the unit of floodplain development. Consider the following problem. *t* is a tax rate.

$$\max_{x^i,a^i} \quad W^i = U\{\pi^i(x^i,a^i)\}$$

where

[13] Command and control policy is a quantity rationing policy. It is similar to marketable permits except that requirements are not tradable under command and control policies. Thus, we do not treat command and control policies in this chapter. We explicitly analyse them in Chapter 6 (Policy Simulation).

$$\pi(\cdot) = f(x^i) - g(a^i) - C^i\{x^i, a^i, \mathbf{q}(x^j, a^j)\} - tx^i \quad (i \neq j)$$

subject to

$$0 \leq x^i \leq L_F^i$$

$$0 \leq a^i$$

(for all i).

We can derive the optimal condition for floodplain development, assuming that we have an interior solution.

$$\frac{df}{dx^i} = \frac{\partial C^i}{\partial x^i} + t \quad \text{(for all } i\text{)}.$$

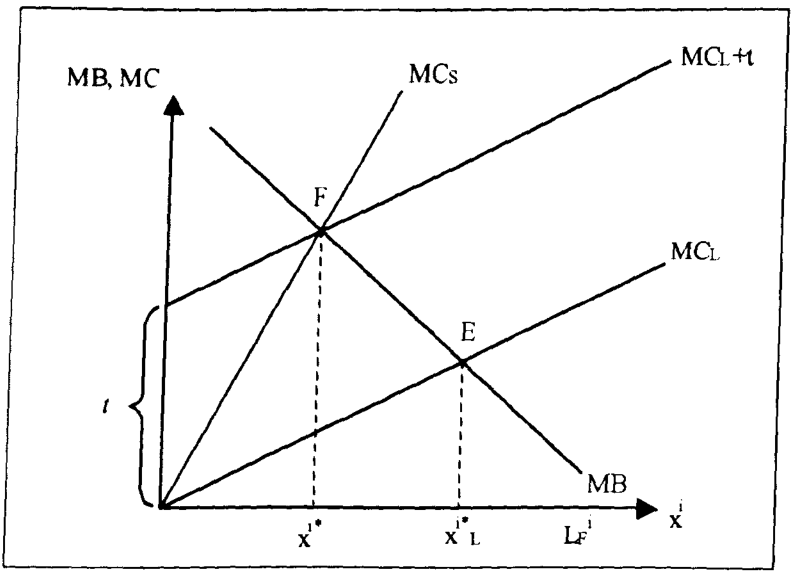
(3-18)

Then, comparing condition (3-18) with condition (3-7) of the social optimisation, the optimal tax rate (Pigouvian tax rate) is determined by the following.

$$t = -\frac{dB}{dx^i} + \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial x^i} \quad \text{(for all } i\text{)}.$$

(3-19)

Figure 3-4. *Pigouvian tax*



As a result, we can attain the optimal floodplain development in each zone if we set the optimal tax rate on floodplain development. Figure 3-4 shows the tax rate t. Landowners are faced with the new marginal cost curve including the tax rate ($MC_L + t$). They choose the amount of development by the intersection of the new marginal cost curve and the marginal benefit curve (point F). It results in the optimal amount of developed floodplains from the social point of view.

However, we should notice that the optimal tax rate differs among zones because the marginal costs are different among zones. The floodplain development upstream has more impact on the external flood risk than that downstream. Therefore, we should set higher tax rates on zones upstream as compared with zones downstream.

3.9.2 Marketable Permits

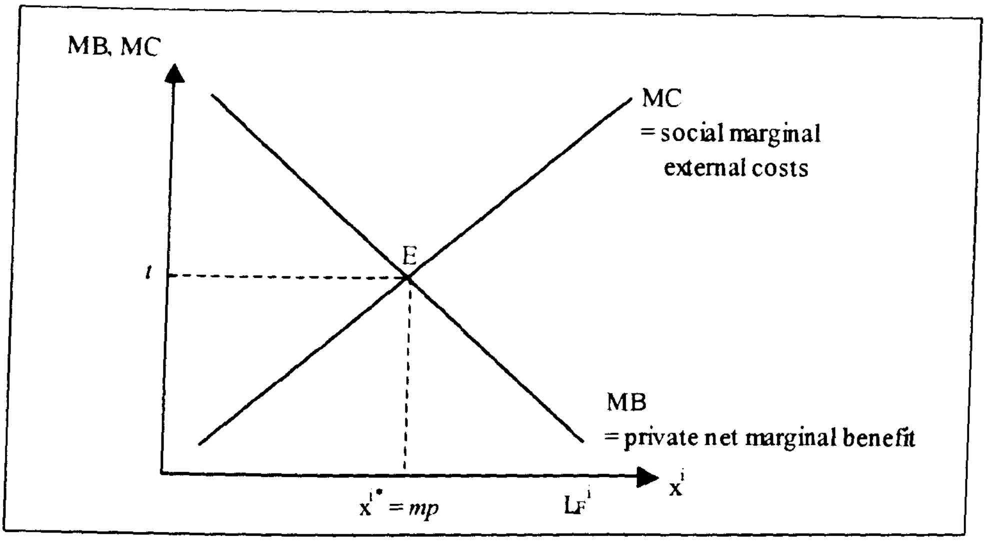
Alternatively, we can achieve the same result by marketable permits. Policy-makers issue the amount of marketable permits for developing floodplains that is equivalent to the optimal amount of floodplain development (x_S^{i*} in Figure 3-3). The marketable permits are the rights to develop a fixed amount of floodplains. Then, if the markets are competitive and the economic agents (individual landowners) to transact the permits are minimising costs and keeping to the amount of floodplain development regulated by the permits, the optimal amount of developed floodplains is attained at the least costs as well. Like the case of Pigouvian tax, we should set different amounts of the marketable permits among zones for the same reason. Each zone should be so large that the market for permits can be efficient. It will be costly as the number of zones increases.

3.9.3 Pigouvian Tax vs. Marketable Permits

In theory, the Pigouvian tax and marketable permits are equivalent. However, in reality there is uncertainty about the relevant marginal benefits, marginal costs and marginal external costs, but it is not possible to get the precise information.

Here, we compare the two instruments under uncertainty (problems on the information availability) about the marginal benefit curve and the marginal cost curve.[14]

Figure 3-5. *A modified diagram on MB and MC*



For the analysis, let us change the figure. Rearranging equation (3-7),

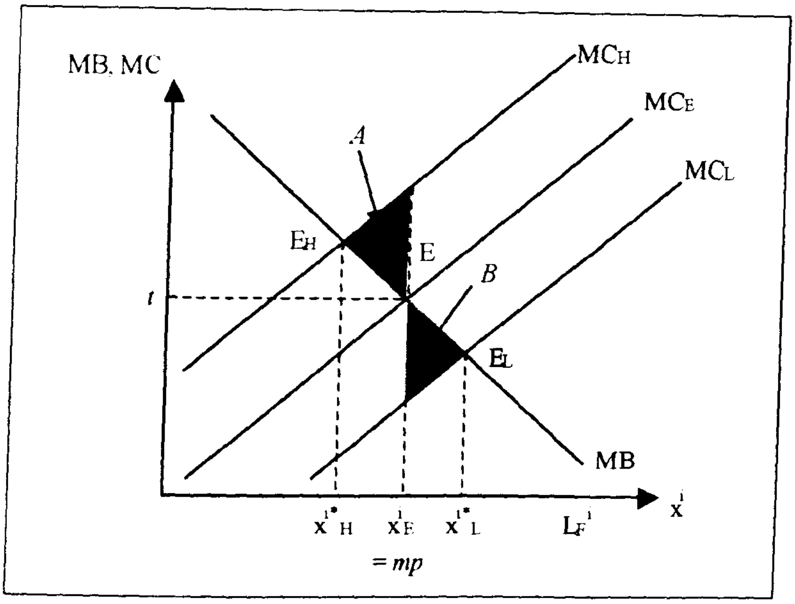
$$\frac{df}{dx^i} - \frac{\partial C^i}{\partial x^i} = -\frac{dB}{dx^i} + \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial x^i} \qquad \text{(for all } i\text{)}$$

(3-20)

The left hand side of equation (3-20) is the marginal net private benefit. We simply call it the marginal benefit (MB curve in figure 3-5). The right hand side is the marginal external costs. We simply call them the marginal cost (MC curve in figure 3-5). Then, we set the appropriate tax rate (t) or the relevant amount of marketable permits (mp) by the intersection of the marginal benefit curve and the marginal cost curve.

[14] The discussion is based on Weitzman (1974); Baumol and Oats (1988); and Hanley et al. (1997). However, these focus on the problem of pollutions. It is not necessarily possible to directly apply the fundamental theorem into the arguments on floodplain management.

Figure 3-6. *Uncertainty about the marginal external costs*

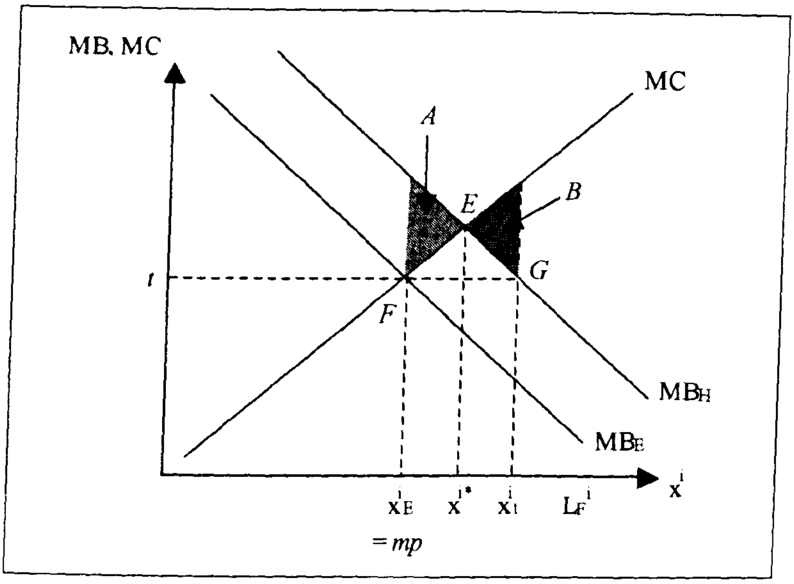


Using this type of diagram, let us analyse under uncertainties on the marginal cost. That is, we consider the situation that we have precise information on the marginal benefit while the marginal cost function (curve) is just an expected one. In this case, the Pigouvian tax and marketable permits yield the same result, which implies both approaches produce the same social dead-weight loss. Figure 3-6 shows this.[15] MC_E curve is an expected marginal cost curve. We set a tax rate or the amount of marketable permits by the intersection of the expected marginal cost curve (MC_E) and the marginal benefit curve (MB). The tax rate is *t*, and the amount of marketable permits is *mp*. In the case that the true marginal cost curve is higher than the expected one, the optimal point is truly at E_H. The optimal amount of floodplain development is x_H^{i*}. However, the resultant amount of floodplain development due to marketable permits is x_E^i, which is larger than the optimal one. Thus, the shaded area *A* is the social dead-weight loss. Likewise, the tax rate *t* makes private landowners choose the amount of floodplain development x_E^i. This yields the same social dead-weight loss as the marketable permits do. In the case that the true marginal cost curve is lower than the expected one, both political measures result in the same social dead-weight loss that is a shaded area

[15] The figure depicts the linear marginal cost and benefit curves, but the results hold true about non-linear curves.

B.

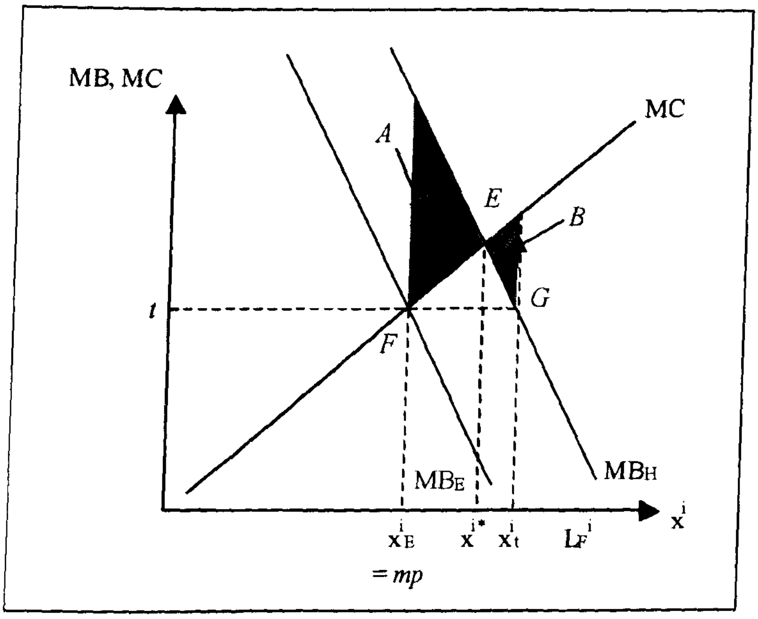
Figure 3-7. *Uncertainty on MB: a case of the same absolute values of the slopes*



Next, we compare them under uncertainties on the marginal benefit. We assume that the marginal cost and benefit curves are both linear. The relative slopes of the marginal cost and benefit curves provide guidance on the choice of policy. To begin with, we analyse the case that the absolute values of slopes of the two curves are the same. In this case, the social dead-weight losses are the same. We show this by Figure 3-7, which analyses the case that the true marginal benefit is higher than the expected one. Based on the marginal cost curve (MC) and the expected marginal benefit curve (MB_E), policy-makers can set the tax rate (*t*) or issue the amount of marketable permits (*mp*). However, the true marginal benefit (MB_H) is higher than the expected one. The optimal amount of floodplain development is x^{i*}. In the case of the tax, landowners determine the total amount of floodplain development by the point which equates the tax rate with the true marginal benefit. Then, the resultant amount of floodplain development is x_t. This brings about the social dead-weight loss that is equivalent to the shaded area *B*. On the other hand, marketable permits cause the social dead-weight loss that is equivalent to the shaded area *A*. In this case, the two social dead-weight losses are equal.[16]

Next, let us consider the case that the absolute value of the slope of the marginal benefit curve is larger than that of the marginal cost curve. The situation is the same as the analysis in Figure 3-7. See Figure 3-8. Likewise, the shaded area *A* is the social-weight loss in the case of marketable permits, and the shaded area *B* is the social-weight loss in the case of the Pigouvian tax. The social dead-weight loss caused by marketable permits is larger than the Pigouvian tax. If the marginal cost curve is steeper than the marginal benefit curve, the results are just the opposite.

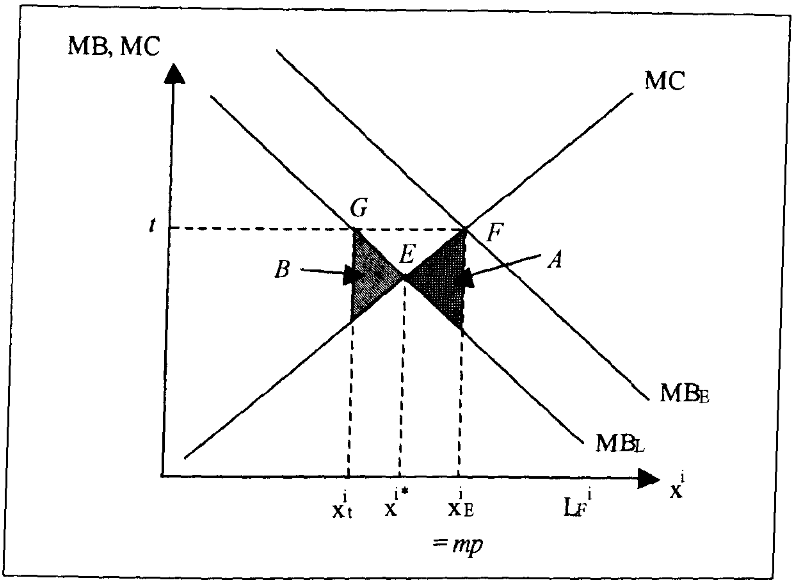
Figure 3-8. *Uncertainty on MB: a case of a steeper slope of MB*



We can get the same results when the true marginal benefit is lower than the expected one. However, we should notice that the order of the amount of floodplain development is opposite. In this case, the resultant amount of floodplain development under the Pigouvian tax is smaller than the optimal one while that under marketable permits is larger than the optimal one (see Figure 3-9).

[16] In this respect, a formal mathematical proof is given in Weitzman (1974), Adar and Griffin (1976), and Baumol and Oats (1988). We can easily apply it to the context of floodplains.

Figure 3-9. *Overestimation of MB*



The results tell us that we are faced with the same risk under the two policies if we do not have appropriate information on the marginal cost and benefit curves. We cannot conclude which is better based on the analysis without relevant information.

Furthermore, as is often the case with environmental problems of development, the problem of overdevelopment is more serious than that of under-development especially under irreversibility. In real situations, it is often difficult and costly or sometimes impossible to restore floodplains (Mitsch and Gosselink, 2000b).[17] Thus, we should consider the risk of overdevelopment. First, it is problematical to underestimate the marginal cost (the marginal external costs). If we underestimate the marginal cost, both the approaches result in overdevelopment (see Figure 3-6). In this case, it is helpful to take the perspective of the precautionary principle or the safe minimum standards. Perrings (1991) mentions with a formal model that it is rational for policy makers to make current decisions to safeguard against the potentially catastrophic future effects of current activity. The safe minimum standards are essential, given the uncertainty related

[17] The difficulties lie in the establishment of appropriate natural hydrological conditions that are supported by appropriate vegetation communities (Mitsch and Gosselink, 2000b).

to the future environmental impacts of current economic activities, the ecological or environmental threshold (discontinuity of environmental costs) and irreversibility of environmental or ecological goods and services (Perrings, Folke and Mäler, 1992; Perrings and Pearce, 1994). In the context here, it turns out to be reasonable for policy makers to take a precautionary margin between the tax rate set and the tax rate calculated from the estimated benefits and costs.

Second, the Pigouvian tax brings about overdevelopment if the marginal benefit is underestimated (see Figure 3-7). On the contrary, marketable permits cause overdevelopment if the marginal benefit is overestimated (see Figure 3-9). Thus, we should consider which risk is higher, the risk of overestimating or underestimating the marginal benefit. This depends on real situations. However, the problem of overdevelopment is unavoidable if the information on the marginal net private benefit is asymmetric between landowners and policy makers. Let us consider the case that only landowners have the precise information on it that policy makers cannot obtain. Thus, policy makers must rely on the information landowners provide. Under the Pigouvian tax, landowners have the incentive to tell lower net benefit than the real one because they can make the tax rate lower. As a result, the marginal benefit is underestimated, and the Pigouvian tax causes overdevelopment. Under marketable permits, landowners have the incentive to tell higher net benefit than the real one because they can make the amount of marketable permits larger. As a result, the marginal benefit is overestimated, and marketable permits cause overdevelopment. The bottom line is that the problem of overdevelopment cannot be avoided under such asymmetric information.

Finally, the target on the amount of floodplain development fluctuates under the tax system while it is fixed under marketable permits. If we keep to the precautionary principles, marketable permits might be preferable because the target is clear and fixed. However, it is often pointed out that it is difficult to establish the competitive market for the permits. In reality, feasibility might be problematical.

3.9.4 Subsidy

We set the problem in the following with the rate of subsidy s and a standard of floodplain development $\overline{x}$.

$$\max_{x^i, a^i} \quad W^i = U\{\pi^i(x^i, a^i)\}$$

where

$$\pi(\cdot) = f(x^i) - g(a^i) - C^i\{x^i, a^i, \mathbf{q}(x^j, a^j)\} + s(\overline{x}^i - x^i) \qquad (i \neq j)$$

subject to

$$0 \leq x^i \leq L_F^i$$

$$0 \leq a^i$$

(for all i).

Then, we derive the optimal conditions on floodplain development, assuming that we have an interior solution.

$$\frac{df}{dx^i} - \frac{\partial C^i}{\partial x^i} = s \qquad \text{(for all } i\text{)}.$$

(3-21)

Condition (3-21) is the same as condition (3-18). Thus, we set the same rate as the Pigouvian tax.

$$\frac{df}{dx^i} - \frac{\partial C^i}{\partial x^i} = s = t = -\frac{dB}{dx^i} + \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial x^i}$$

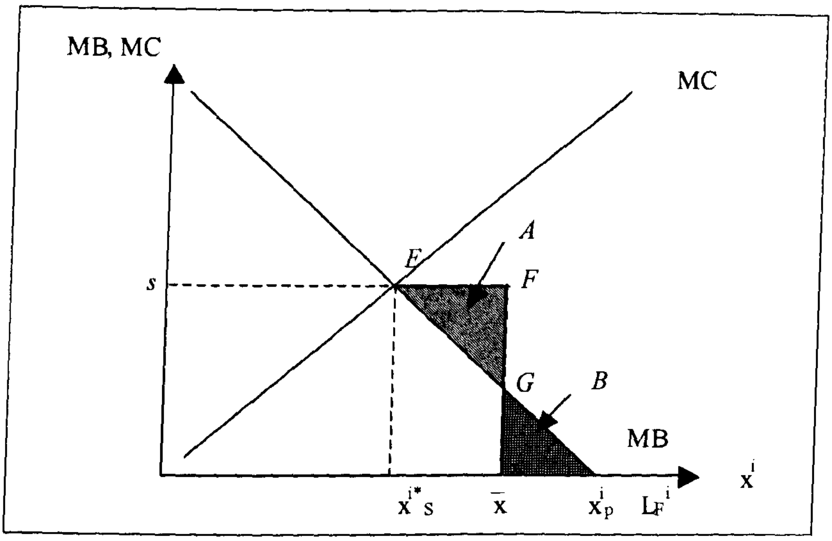
(3-22)

Based on condition (3-22), we can attain the social optimum by the subsidy. As the condition is common between the tax and the subsidy, basic characteristics of the subsidy coincide with those of the tax. However, there are several points that we should point out.

To begin with, it is difficult but necessary to determine the standard of subsidy $\overline{x}^i$. Assuming that the marginal (net private) benefit function is linear, the following condition must be satisfied (the mathematical notations coincide with those in Figure 3-10).

$$\bar{x}^i > \frac{1}{2}\left(x_s^{i*} + x_p^i\right) \tag{3-23}$$

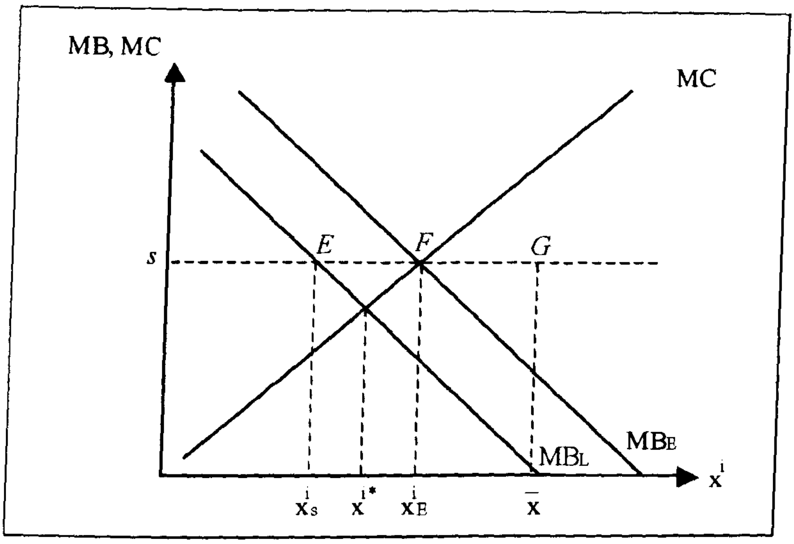
Figure 3-10. *Standard for the subsidy*



Unless the condition is satisfied, landowners have no incentive to choose the optimal amount of floodplain development because they can earn positive benefits by developing floodplains up to x_p^i. The gain from the subsidy is the rectangle $EF\bar{x}x_s^{i*}$, and the benefit from floodplain development is the triangle $Ex_p^i x_x^{i*}$. Thus, comparing the shaded areas *A* and *B*, the subsidy works if the shaded area *A* is larger than *B*. In the case of non-linear functions, it is more difficult to set the standard. In both cases, we need precise information on the marginal benefit curve in order to set a relevant standard. When we have no precise information, we need to set a sufficiently large standard such that condition (3-23) can be satisfied. It is, however, costly. Policy-makers need a lot of money for providing subsidies. Obviously, it is the best to choose the minimum value for the standard that satisfies condition (3-23).

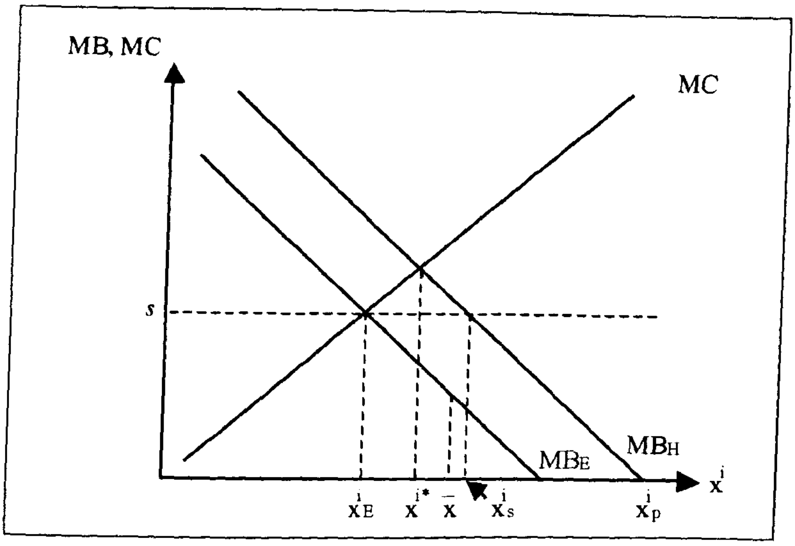
Furthermore, if we overestimate the marginal (net private) benefit, we need more financial resources for the subsidy than we expected. See Figure 3-11. We set the rate of subsidy s and the standard of the subsidy $\bar{x}$. Then, we expect that the total amount of subsidy is the rectangle area $FG\bar{x}x_E^i$. However, landowners choose the amount of floodplain development x_s^i, based on the true lower marginal benefit curve. As a result, we need to provide them with the total subsidy that is equivalent to the rectangle $EG\bar{x}x_s^i$.

Figure 3-11. *Subsidy: overestimation of MB*



Subsidies completely fail if we underestimate the marginal benefit and choose the 'minimum' standard for the subsidy that satisfies condition (3-23). See Figure 3-12. Likewise, we set the rate of subsidy s and the standard for the subsidy. However, landowners' behaviour depends on the true higher marginal benefit curve. Based on the true marginal benefit curve, the standard cannot satisfy condition (3-23). Therefore, they choose x_p^i. Subsidies do not function well.

Figure 3-12. *Case of malfunction of subsidy*



Finally, the subsidy increases the amount of floodplain development in the competitive market in the long run. This is often referred to as a serious problem on the subsidy system.[18] This is analysed in a polluting industry, in which emissions are proportional to the output. Thus, the context which we analyse is different from it, but the same thing still can be applied. Assume that there are large number of landowners in each zone, which satisfies the assumption of the competitive market. They are initially endowed with floodplains as private lands, and they decide whether to develop floodplains. The amount of floodplain development in each landowner is just determined by the rule P (price) = MC, and the price comes from the competitive market in which no private landowner can earn positive economic profits. Thus, they choose x_p^{h*} without the subsidy (see Figure 3-13a).[19] The subsidy shifts the marginal cost curve upward like Pigouvian tax, but shifts the average cost curve downward and to the left. As a result, each landowner chooses the smaller amount of floodplain development, but the total amount of developed floodplain in zone *i* increases because more landowners decide to develop floodplains due to the subsidy which makes the price go down (see Figure 3-13b).[20] Subsidies make the situation worse.

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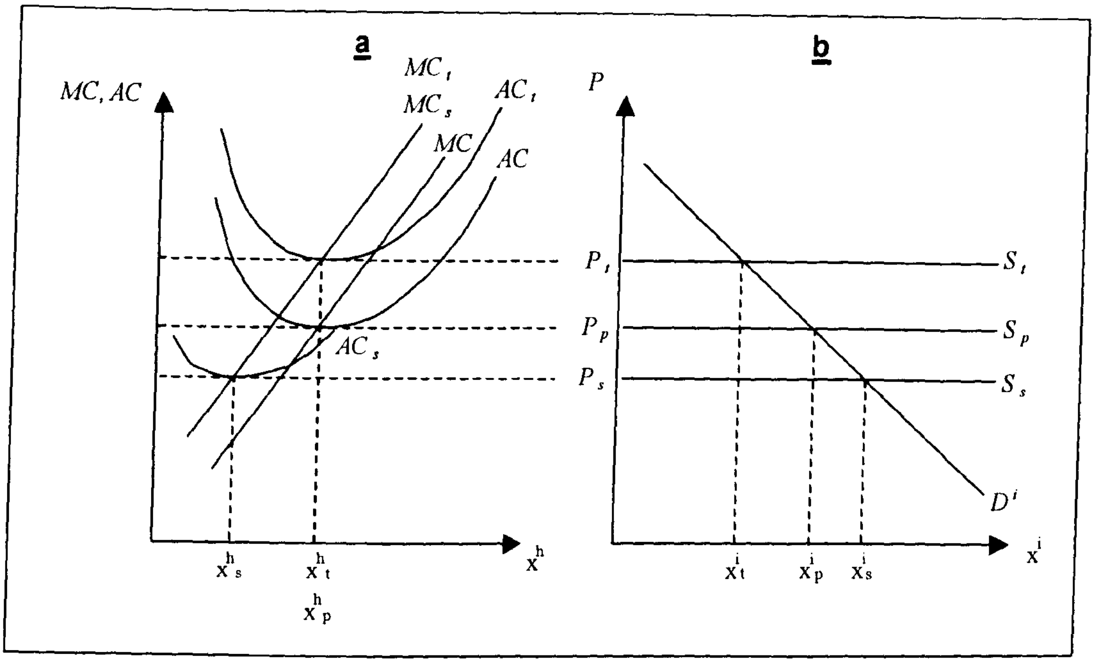
[18] For example, Kohn (1985), Baumol and Oats (1988), and Hanley et al. (1997).
[19] Figure 3-13a shows the marginal cost curves and the average cost curves in the representative landowner in zone *i*. This is different from the previous figures that we have used. The subscript *s* shows the case of subsidy, and *t* shows the case of tax.
[20] D^i is a demand function.

Figure 3-13. *Subsidy in competitive markets in the long run*



Considering the problems we discussed above, subsidies seem to be more difficult to choose than the Pigouvian tax and marketable permits although it depends on real conditions including problems of feasibility and equity.

3.10 Conclusion of Chapter 3

In this chapter, we develop a static decision model of hydrological, ecological and economic interactions on floodplains. The social optimal conditions tell that we should choose the values of control variables such that the marginal benefits are equal to the marginal costs plus the marginal external costs. There are two external costs: the opportunity costs of lost ecosystem services that landowners inflict on the public at large; and the unidirectional spatial external costs of flood risk that floodplain development and/or enhancement of averting behaviour in upstream zones impose on downstream zones. The bottom line is that it is crucial to take the externalities into consideration and to internalise the external costs for avoiding the overdevelopment of floodplains.

The general policy problem, considered in more detail in later chapters, is how to internalise the external costs. This chapter analyses the Pigouvian tax, marketable permits and subsidies in the context of a simple static model. We cannot conclude which instrument is the best although subsidies are generally problematic. The choice of policies depends on real conditions. We will analyse this respect by policy simulations in a dynamic context in Chapter 6.

Chapter 4

Dynamic Decision Model of River Floodplains

4.1 Introduction

The purpose of this chapter is to provide a dynamic decision model, based on the static model in Chapter 3. To begin with, we discuss the significance of extending a model into the dynamic context. There are several reasons why we need a dynamic decision model.

First, we can analyse the time paths of variables in a dynamic model. In the case of convergence, we can observe the time paths of variables through which they converge to certain values such as equilibria or static optimal conditions. On the contrary, in the case of divergence, we can understand how they diverge by observing the time paths. The analysis of time paths is important especially when static optimal conditions are impaired or not necessarily stable.

Second, our decisions are in reality made in the dynamic and continuous context rather than the static and discrete context. Then, it is natural that we consider the economic phenomena in the dynamic context from this point of view.

Third, as we mentioned in Chapter 3, initial conditions are of importance for the optimal strategy of floodplain management because there is normally difference between the costs of floodplain development and floodplain restoration. In other words, the outcomes are path-dependent. In the static decision model, we implicitly assume that all floodplains are natural as an initial condition and then choose the size of development in floodplain. Although we might be able to treat different initial conditions even in a static model, we can treat them more easily, clearly and systematically in the dynamic context because we can explicitly introduce the initial conditions into the model. In addition, we can analyse the irreversibility related to floodplain development more obviously.

Fourth, related to the second reason, we can treat the process of adjustment in a dynamic decision model in the case that we are faced with stricter constraints on control variables. In fact, we can neither develop all the remained natural floodplains nor restore all the existing developed floodplains to natural ones at a moment in time. It takes time to do these. We cannot immediately choose the optimal values of control variables by any policies. The adjustment processes affect our welfare over time. It is possible to analyse the adjustment processes and their consequences in a dynamic model.[1]

Fifth, in a dynamic model we can cope with natural self-organizing processes. For example, natural floodplains are being naturally recovered depending on their conditions, and the functions of natural floodplains are being strengthened or weakened depending on their own conditions. We can denote such self-organizing processes by functions in the equations of motion of state variables. The most famous example related to this point is the logistic growth function in the equation of motion of the biomass in the fishery model.[2] Here, we do not consider such self-organizing processes because such a process is not necessarily clear for the moment although each ecosystem function might have such a process in view of biogeochemistry. Nevertheless, this point will be important when we enrich the model in the future.

Finally, there are two kinds of costs on averting behaviour, fixed and variable costs. Fixed costs imply the initial investments and variable costs imply on-going maintenance costs. We should notice that the variable costs do not depend on economic outputs but on the size of averting behaviour and time in this context. In

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[1] We set this point in Chapter 5 and analyse it by policy simulations in Chapter 6.
[2] e.g. Schaefer (1954), Clark (1973), Clark (1990), Conrad and Clark (1987).

this respect, the notion of variable costs here is different from those in the usual terms of economics. In the real economic world, these kinds of costs are of much importance because the time-dependent variable costs perpetuate once we invest in averting behaviour in order to compensate for the lost flood mitigation service of natural floodplains. In other words, such costs imply a part of the values of ecosystem services that natural floodplains provide. It is, however, complicated and difficult to treat such variable costs in the static context because the costs depend on the size of averting behaviour (kind of accumulated investment) and they perpetuate over time. That is why we need to extend a static model to a dynamic one.

In this chapter, we develop a dynamic decision model from the hydrological, ecological and economic points of view. A dynamic decision model is a basis for an applied model used in policy simulations.

4.2 Model Settings

We set several conditions as presuppositions of the model in advance. They are important in that they determine whether the model can yield analytical solutions (tractability) and that they determine the range that the model can explain (limitations of the model). When we enrich the model, these will become significant. The following conditions are similar to those of the static model in Chapter 3 since the dynamic model is an extension of the static model.[3] We, however, repeat assumptions of the static model in the following in order to make the settings of a dynamic model clear. We want to avoid providing any misunderstandings because they are similar but different in essence.

- There are two kinds of control variables. One is a change in the size of developed floodplains in time t. We can control it by developing or restoring

[3] c.f. Section 3.2 in Chapter 3.

floodplains. The other is a change in the scale of averting behaviour in time t.[4] Likewise, we can control it by enlarging or reducing the scale of averting behaviour.

- There are two kinds of state variables. One is the size of developed floodplains. The other is the scale of averting behaviour.
- Land use patterns outside floodplains have an impact on flood risk through a change in imperviousness of land. However, we assume them to be exogenous for the time being. In the stage of simulations, we treat the exogenous variables as control variables in a hydrological software program.
- The total size of floodplain is fixed, and is defined by the 100-year floodplain (1% chance of flooding each year). Strictly speaking, the size of floodplains will change over time in the long term even if it is defined by the same notion, but we assume that it is constant over time here.
- The type of floodplain development has different effects on floodplains' capacity of mitigating floods and other ecosystem functions. For example, the development into industrial areas presumably alleviates the capacity more than that into agricultural areas. However, we distinguish only between natural and developed floodplains for simplification of the model in this chapter. Then, we assume that the capacity or the flood risk depends just on the size of development (the size of remaining natural floodplains). When we apply a model to simulations, we have to take different land use patterns into consideration in a hydrological software program.
- We assume that the society is risk-neutral. Individuals are risk-averse while organizations including governments and societies are risk-neutral.

[4] In the model, the scale of averting behaviour is treated as the area potentially protected by averting behaviour like in the static model.

4.3 Mathematical Notation

We continue to use the same mathematical notations as in the static model.[5] However, we have to distinguish between control and state variables in the dynamic context.

y_t^i or $y^i(t)$: Change in the size of developed floodplains in time t.

a_t^i or $a^i(t)$: Change in the scale of averting behaviour in time t.

X_t^i or $X^i(t)$: Size of developed floodplains in time t.

A_t^i or $A^i(t)$: Scale of averting behaviour in time t. This is the same as a^i in the static model.[6]

δ : Discount rate. We assume a positive discount rate ($\delta > 0$).

4.4 Functions and Assumptions

We are using the same functions and assumptions as those in the static model.[7] However, we should note that all are functions of time t although they are autonomous.[8] We add explanations of new functions and their assumptions. In the beginning, we assume that all functions are continuous and twice differentiable with respect to relevant arguments as long as we do not indicate a specific assumption explicitly.

$F(X_t^i)$ is the benefit function of developed floodplains. This function is

[5] c.f. Section 3.3 in Chapter 3.

[6] We use the area potentially protected by averting behaviour as a proxy of this variable. However, we abstractly treat this state variable as the scale of averting behaviour in a dynamic decision model.

[7] c.f. Section 3.4 in Chapter 3.

[8] For example, the utility function is $U_t(\bullet)$ or $U(\pi_t)$ rather than $U(\bullet)$ and $U(\pi_t, t)$.

similar to $f(x^i)$ in the static model. However, we should note one difference. This function is inside the net benefit function at time t. Thus, this function cannot be treated as a unit land price. Land prices are determined by the sum of discounted rent that we earn from the land if the market is complete and efficient. Here, this function is referred to as the rent per land that we earn from the developed land in time t.[9] The assumptions on the benefit function of developed floodplains are the same as those of $f(x^i)$.

$D(y_t^i)$ is the direct cost function of floodplain development and restoration. It implies the costs that we incur when we develop or restore floodplains in time t. It is evaluated in monetary terms. We assume the cost function is strictly convex for the relevant range. The following three assumptions imply that we incur the direct cost when we both develop and restore floodplains. When y^i is negative, it implies floodplain restoration because developed floodplains are reduced. In this case, the direct costs increase as y^i gets smaller (the absolute value of y^i gets larger). The final condition means the convexities in both of the ranges of control variable $y^i > 0$ and $y^i < 0$. The marginal cost increases as we develop or restore more floodplains at time t. Figure 4-1 shows the shape of the cost function as an example (see the next page). The shape of the function may be different between the ranges $y^i > 0$ and $y^i < 0$, which reflects the difference between the costs of development and restoration.

$$D_y = \frac{dD}{dy} \geq 0 \text{ if } y^i \geq 0.$$

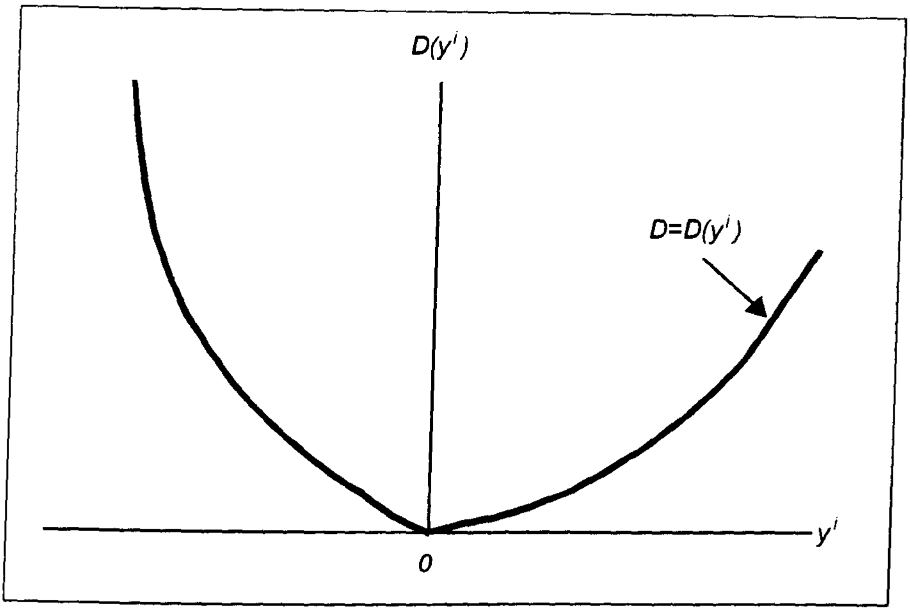
$$D_y = \frac{dD}{dy} < 0 \text{ if } y^i < 0.$$

$$D(0) = 0 \text{ when } y^i = 0.$$

$$D_{yy} = \frac{d^2D}{dy^2} > 0 \text{ when } y^i \neq 0.$$

[9] If we earn the rent from natural floodplains, this function implies the net value between the rent from natural floodplains and that from developed lands.

Figure 4-1. *Shape of the cost function (an example)*



$G(a_t^i)$ is the cost function of averting behaviour. It implies the direct costs that we incur when we enhance or alleviate the scale of averting behaviour in time t. It is evaluated in monetary terms. This function is similar to $g(a^i)$ in the static model. We assume the cost function is strictly convex for the relevant range. The following three conditions imply that we incur costs when we both enhance and reduce the scale of averting behaviour. When a^i is negative, it implies the reduction of averting behaviour. In this case, costs increase as a^i gets smaller (the absolute value of a^i gets larger). The final condition implies the convexity of the cost function in both of the ranges of control variable $a^i > 0$ and $a^i < 0$. Marginal cost increases as we enhance or alleviate the degree of averting behaviour more in time t. The shape of this function is the same as the function $D(y_t^i)$ (c.f. Figure 4-1.)

$$G_a = \frac{dG}{da} \geq 0 \text{ if } a^i \geq 0.$$

$$G_a = \frac{dG}{da} < 0 \text{ if } a^i < 0.$$

$$G(0) = 0 \text{ when } a^i = 0.$$

$$G_{aa} = \frac{d^2G}{da^2} > 0 \text{ when } a^i \neq 0.$$

$M\left(A_t^i\right)$ is the cost function of operation and maintenance, which is assumed to be a function of the scale of averting behaviour. The larger the scale is, the larger the costs of operation and maintenance are. The costs are incurred at time t, and are evaluated in monetary terms. We assume the cost function is strictly convex. The marginal operating cost increases as the scale of averting behaviour increases.

$$M_A = \frac{dM}{dA} > 0$$

$$M_{AA} = \frac{d^2M}{dA^2} > 0$$

$h\left(X^j, A^j\right)$ is the function of external flood risk that the upstream zones j impose on zone i. This function is the same as $q\left(x^j, a^j\right)$ in the static model except that we distinguish between control and state variables. We set the same assumptions. This implies "external flood risk".

$C\left\{X^i, A^i, \mathbf{h}\left(X^j, A^j\right)\right\}$ ($i \neq j$) is the expected cost function of flood risk in zone i. This function is the same as $C^i\left\{x^i, a^i, \mathbf{q}\left(x^j, a^j\right)\right\}$ in the static model except that we distinguish between control and state variables. Then, we set the same assumptions.

4.5 Social Optimisation Problem

The social optimisation problem has the following form:

$$\underset{\mathbf{y}_t,\mathbf{a}_t}{Max} \quad W = \int_0^\infty e^{-\delta t} U\{\pi(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t)\} dt$$

where

$$\pi(\cdot) = \sum_{i=1}^{m} B(L_F^i - X_t^i) + \sum_{i=1}^{m} F(X_t^i) - \sum_{i=1}^{m} D(y_t^i) - \sum_{i=1}^{m} G(a_t^i)$$
$$- \sum_{i=1}^{m} M(A_t^i) - \sum_{i=1}^{m} C^i\{X_t^i, A_t^i, \mathbf{h}(X_t^j, A_t^j)\}$$

subject to

$\dot{X}_t^i = y_t^i$ for all i [10]

$\dot{A}_t^i = a_t^i$ for all i

$-X_t^i \le y_t^i \le L_F^i - X_t^i$ for all i

$-A_t^i \le a_t^i$ for all i

$0 \le X^i(0) = \overline{X}_0^i \le L_F^i$ for all i

$A^i(0) = \overline{A}_0^i \ge 0$ for all i

The social optimisation problem is to choose the paths of control variables, subject to equations of motion, constraints on control variables and initial conditions, in order to maximize social welfare over time. Social welfare is composed of the sum of discounted social utilities over time. This is a problem in optimal control theory.

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[10] The notation of the dot above variables implies the derivative of the variable with respect to time t. For example, $\dot{X}_t^i = \frac{dX_t^i}{dt}$.

4.6 The Maximum Principle

In the beginning, we need to derive the conditions of the Maximum Principle for this problem. We set the current value Hamiltonian function. λ and μ are costate variables.

$$\tilde{H} = U\{\pi_t(\cdot)\} + \sum_{i=1}^{m} \lambda_t^i y_t^i + \sum_{i=1}^{m} \mu_t^i a_t^i$$

Then, we augment the current value Hamiltonian function into the current value Lagrangian function in order to consider the constraints on control variables. θ_1 θ_2 and θ_3 are the Lagrange multipliers.[11]

$$L = U\{\pi_t(\cdot)\} + \sum_{i=1}^{m} \lambda_t^i y_t^i + \sum_{i=1}^{m} \mu_t^i a_t^i$$

$$+ \sum_{i=1}^{m} \theta_{1,t}^i \left(y_t^i + X_t^i\right) + \sum_{i=1}^{m} \theta_{2,t}^i \left(L_F^i - X_t^i - y_t^i\right) + \sum_{i=1}^{m} \theta_{3,t}^i \left(a_t^i + A_t^i\right)$$

We can derive the conditions of the Maximum Principle from the Lagrangian function. The conditions are the first-order necessary conditions for optimal outcomes.

$$\frac{\partial L}{\partial y^i} = U_\pi\left(-D_y\right) + \lambda_t^i + \theta_{1,t}^i - \theta_{2,t}^i = 0 \qquad \text{(for all } i\text{)}$$

(4-1)

$$\frac{\partial L}{\partial a^i} = U_\pi\left(-G_a\right) + \mu_t^i + \theta_{3,t}^i = 0 \qquad \text{(for all } i\text{)}$$

(4-2)

$$\dot{\lambda}_t^i - \delta\lambda_t^i = -\frac{\partial L}{\partial X_t^i} = -U_\pi\left\{B_X + F_X - \frac{\partial C^i}{\partial X_t^i} - \sum_{j>i} \frac{\partial C^j}{\partial h}\frac{\partial h}{\partial X_t^i}\right\} - \theta_{1,t}^i + \theta_{2,t}^i$$

(for all *i*)

(4-3)

[11] Refer to Chiang (1992) chapter 10 about the current value Lagrangian function.

$$\dot{\mu}_t^i - \delta\mu_t^i = -\frac{\partial L}{\partial A_t^i} = -U_\pi\left\{-M_A - \frac{\partial C^i}{\partial A_t^i} - \sum_{j>i}\frac{\partial C^j}{\partial h}\frac{\partial h}{\partial A_t^i}\right\} - \theta_{3,t}^i \qquad \text{(for all } i\text{)}$$

(4-4)

$$\dot{X}_t^i = \frac{\partial L}{\partial \lambda_t^i} = y_t^i \qquad \text{(for all } i\text{)}$$

(4-5)

$$\dot{A}_t^i = \frac{\partial L}{\partial \mu_t^i} = a_t^i \qquad \text{(for all } i\text{)}$$

(4-6)

$$\frac{\partial L}{\partial \theta_{1,t}^i} = y_t^i + X_t^i \geq 0,\ \theta_{1,t}^i \geq 0 \text{ and } \theta_{1,t}^i \frac{\partial L}{\partial \theta_{1,t}^i} = 0 \qquad \text{(for all } i\text{)}$$

(4-7)

$$\frac{\partial L}{\partial \theta_{2,t}^i} = L_F^i - X_t^i - y_t^i \geq 0,\ \theta_{2,t}^i \geq 0 \text{ and } \theta_2^i \frac{\partial L}{\partial \theta_2^i} = 0 \qquad \text{(for all } i\text{)}$$

(4-8)

$$\frac{\partial L}{\partial \theta_{3,t}^i} = a_t^i + A_t^i \geq 0,\ \theta_{3,t}^i \geq 0 \text{ and } \theta_{3,t}^i \frac{\partial L}{\partial \theta_{3,t}^i} = 0 \qquad \text{(for all } i\text{)}$$

(4-9)

$$0 \leq X^i(0) = \overline{X}_0^i \leq L_F^i \qquad \text{(for all } i\text{)}$$

(4-10)

$$A^i(0) = \overline{A}_0^i \geq 0 \qquad \text{(for all } i\text{)}$$

(4-11)

$$\lim_{t\to\infty} \lambda^i(t) = 0 \qquad \text{(for all } i\text{)}$$

(4-12)

$$\lim_{t\to\infty} \mu^i(t) = 0 \qquad \text{(for all } i\text{)}$$

(4-13)

To begin with, we interpret the economic meaning of these first-order necessary conditions, assuming that we have an interior solution.[12] Based on the assumption, the constraints on control variables are not binding. Therefore, we assume $\theta_{1,t}^{i} = \theta_{2,t}^{i} = \theta_{3,t}^{i} = 0$.

From (4-1),

$$U_{\pi} D_{y} = \lambda_{t}^{i} \tag{4-14}$$

From (4-2),

$$U_{\pi} G_{a} = \mu_{t}^{i} \tag{4-15}$$

The equation (4-14) implies that the marginal cost of floodplain development (or restoration) is equal to the imputed value (or shadow price) of developed floodplains (or natural floodplains in the case of restoration) in time t. Equation (4-15) implies that the marginal cost of averting behaviour is equal to the imputed value (or shadow price) of the area potentially protected by averting behaviour (the scale of averting behaviour) in time t.

From (4-3) and (4-14),

$$\dot{\lambda}_{t}^{i} = U_{\pi} \left\{ \delta D_{y} - B_{X} + \frac{\partial C^{i}}{\partial X_{t}^{i}} + \sum_{j>i} \frac{\partial C^{j}}{\partial h} \frac{\partial h}{\partial X_{t}^{i}} - F_{X} \right\} \tag{4-16}$$

The first term in braces is the interest received if floodplains are not developed (we do not incur the direct cost). This is an opportunity cost. It implies that we would earn the interest if we invested the resource that we could save. The second term in braces implies the marginal cost of losing ecosystem services due to floodplain development. The third term is the marginal cost of increased flood risk due to floodplain development. The fourth term implies the marginal cost of the external flood risk that floodplain development in zone i inflicts on other zones downstream. The fifth term is the marginal direct benefit of floodplain

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[12] We focus on the case of floodplain development for the explanation of the economic meanings. Based on it, it is easy to interpret them in the case of floodplain restoration.

development. We have to compare the marginal cost (the sum of the first four terms) with the marginal benefit (the fifth term) in order to interpret this condition. If the marginal cost exceeds the marginal benefit, the right hand side of equation (4-16) is positive. Thus, $\dot{\lambda}_t^i > 0$. It implies that the imputed value (shadow price) of developed floodplains must increase in order to compensate for the loss due to floodplain development. Vice versa.[13]

From (4-4) and (4-15),

$$\dot{\mu}_t^i = U_\pi \left\{ \delta G_a + M_A + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial A_t^i} + \frac{\partial C^i}{\partial A_t^i} \right\} \tag{4-17}$$

Likewise, we can interpret the economic meaning of equation (4-17). The first term in braces implies the interest received if we do not enhance the scale of averting behaviour (we do not incur the cost). This is an opportunity cost. It implies that we would earn the interest if we invested the resource that we could save. The second term is the marginal maintenance and operation cost of averting behaviour. The third term is the marginal cost of the external flood risk that enhancing the scale of averting behaviour in zone *i* imposes on other zones downstream. The fourth term implies the marginal cost that we can avoid by enhancing the scale of averting behaviour. It is the marginal benefit of averting behaviour. We compare the marginal cost (the sum of the first three terms) with the marginal benefit (the fourth term). If the marginal cost exceeds the marginal benefit, the right hand side of equation (4-17) is positive. Thus, $\dot{\mu}_t^i > 0$. This implies that the imputed value (shadow price) of the area potentially protected by averting behaviour (the scale of averting behaviour) must increase in order to compensate for the loss due to enhancing the scale of averting behaviour. Vice versa.

Conditions (4-5) and (4-6) are equivalent to equations of motion given in the optimisation problem. Conditions (4-10) and (4-11) are initial conditions given.

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[13] In the case of floodplain restoration, the similar interpretation is possible.

Conditions (4-12) and (4-13) are transversality conditions. In this problem, the values of the state variables at the end point are free. Thus, we need the conditions (4-12) and (4-13) in order to get the optimal solution.[14] They imply that the imputed value (shadow price) approaches zero if time goes to infinity. From economic point of view, we have to continue to develop or restore floodplains perpetually if condition (4-12) is violated. Likewise, we have to continue to enhance or reduce averting behaviour perpetually if condition (4-13) is violated. The situation cannot be optimal.

4.7 Steady-state Solution

In this section, we derive the steady-state solution in order to analyse its stability later. We continue to focus on an interior solution.

In the beginning, we leave out the costate variables λ^i and μ^i from the equations by use of the conditions (4-14), (4-15), (4-16) and (4-17). Then, we derive the differential equations without the costate variables. Differentiate equation (4-14) with respect to time t, and arrange them by use of the condition (4-5).

$$\dot{\lambda}_t^i = D_y U_{\pi\pi} \left\{ \sum_{i=1}^{m} B_X y_t^i + \sum_{i=1}^{m} F_X y_t^i - \sum_{i=1}^{m} D_y \dot{y}_t^i - \sum_{i=1}^{m} G_a \dot{a}_t^i - \sum_{i=1}^{m} M_A a_t^i - \sum_{i=1}^{m} \frac{\partial C^i}{\partial X_t^i} y_t^i \right.$$
$$\left. - \sum_{i=1}^{m} \frac{\partial C^i}{\partial A_t^i} a_t^i - \sum_{i=2}^{m} \sum_{j<i} \left(\frac{\partial C^i}{\partial h} \frac{\partial h}{\partial X_t^j} y_t^j + \frac{\partial C^i}{\partial h} \frac{\partial h}{\partial A_t^j} a_t^j \right) \right\} + U_\pi D_{yy} \dot{y}_t^i \quad \text{(for all } i\text{)}$$

(4-18)

Likewise, differentiate equation (4-15) with respect to time t, and arrange them by use of the condition (4-6).

[14] Chiang (1992) provides a mathematical explanation (proof) in this respect (chapter 9).

$$\dot{\mu}_t^i = G_a U_{\pi\pi}\left\{\sum_{i=1}^{m} B_X y_t^i + \sum_{i=1}^{m} F_X y_t^i - \sum_{i=1}^{m} D_y \dot{y}_t^i - \sum_{i}^{m} G_a \dot{a}_t^i - \sum_{i=1}^{m} M_A a_t^i - \sum_{i=1}^{m} \frac{\partial C^i}{\partial X_t^i} y_t^i \right.$$
$$\left. - \sum_{i=1}^{m} \frac{\partial C^i}{\partial A_t^i} a_t^i - \sum_{i=2}^{m} \sum_{j<i} \left(\frac{\partial C^i}{\partial h} \frac{\partial h}{\partial X_t^j} y_t^j + \frac{\partial C^i}{\partial h} \frac{\partial h}{\partial A_t^j} a_t^j \right) \right\} + U_\pi G_{aa} \dot{a}_t^i \quad \text{(for all } i\text{)}$$

(4-19)

Substitute equations (4-18) into the condition (4-16).

$$D_y U_{\pi\pi}\left\{\sum_{i=1}^{m} B_X y_t^i + \sum_{i=1}^{m} F_X y_t^i - \sum_{i=1}^{m} D_y \dot{y}_t^i - \sum_{i=1}^{m} G_a \dot{a}_t^i - \sum_{i=1}^{m} M_A a_t^i \right.$$
$$\left. - \sum_{i=1}^{m} \frac{\partial C^i}{\partial X_t^i} y_t^i - \sum_{i=2}^{m} \frac{\partial C^i}{\partial A_t^i} a_t^i - \sum_{i=1}^{m} \sum_{j<i} \left(\frac{\partial C^i}{\partial h} \frac{\partial h}{\partial X_t^j} y_t^j + \frac{\partial C^i}{\partial h} \frac{\partial h}{\partial A_t^j} a_t^j \right) \right\}$$
$$= U_\pi \left\{ \delta D_y - D_{yy} \dot{y}_t^i - B_X - F_X + \frac{\partial C^i}{\partial X_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial X_t^i} \right\} \quad \text{(for all } i\text{)}$$

(4-20)

Substitute equations (4-19) into the condition (4-17).

$$G_a U_{\pi\pi}\left\{\sum_{i=1}^{m} B_X y_t^i + \sum_{i=1}^{m} F_X y_t^i - \sum_{i=1}^{m} D_y \dot{y}_t^i - \sum_{i=1}^{m} G_a \dot{a}_t^i - \sum_{i=1}^{m} M_A a_t^i \right.$$
$$\left. - \sum_{i=1}^{m} \frac{\partial C^i}{\partial X_t^i} y_t^i - \sum_{i=1}^{m} \frac{\partial C^i}{\partial A_t^i} a_t^i - \sum_{i=2}^{m} \sum_{j<i} \left(\frac{\partial C^i}{\partial h} \frac{\partial h}{\partial X_t^j} y_t^j + \frac{\partial C^i}{\partial h} \frac{\partial h}{\partial A_t^j} a_t^j \right) \right\}$$
$$= U_\pi \left\{ \delta G_a - G_{aa} \dot{a}_t^i + M_A + \frac{\partial C^i}{\partial A_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial A_t^i} \right\} \quad \text{(for all } i\text{)}$$

(4-21)

The number of equations in (4-20) and (4-21) is $2m$ and the number of control variables is $2m$. Then, we can solve the equations for each control variable conceptually. In other words, we can get the following differential equations for each control variable from (4-20) and (4-21) conceptually.

$\dot{y}_t^i = f^i(\mathbf{y}, \mathbf{a}, \mathbf{X}, \mathbf{A})$ (for all i)

$\dot{a}_t^i = g^i(\mathbf{y}, \mathbf{a}, \mathbf{X}, \mathbf{A})$ (for all i)

Adding conditions (4-5) and (4-6) to the differential equations above, we obtain the relevant system of simultaneous differential equations of the problem. Using the system of simultaneous differential equations, we can derive analytical solutions and analyse the steady-state solution and its local stability. Here, we try to derive the steady-state solution. In the steady-state, all the control and state variables do not change. Then,

$\dot{y}_t^i = 0$ (for all i)

$\dot{a}_t^i = 0$ (for all i)

$\dot{X}_t^i = y_t^i = 0$ (for all i)

$\dot{A}_t^i = a_t^i = 0$ (for all i)

We can derive the steady-state solution by substituting these conditions into equation (4-20) and (4-21).

$$U_\pi \left\{ -B_X - F_X + \frac{\partial C^i}{\partial X_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial X_t^i} \right\} = 0$$

$$U_\pi \left\{ M_A + \frac{\partial C^i}{\partial A_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial A_t^i} \right\} = 0$$

According to the assumption, $U_\pi > 0$. Thus,

$$F_X = -B_X + \frac{\partial C^i}{\partial X_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial X_t^i} \quad \text{(for all } i\text{)} \tag{4-22}$$

$$-\frac{\partial C^i}{\partial A_t^i} = M_A + \sum_{j>i} \frac{\partial C^j}{\partial h} \frac{\partial h}{\partial A_t^i} \quad \text{(for all } i\text{)} \tag{4-23}$$

Therefore, the steady-state solution must satisfy equations (4-22) and (4-23).

Equations (4-22) and (4-23) provide the steady-state conditions, which have an important economic meaning. The essential point is that the marginal benefit must be equal to the marginal cost plus the marginal external cost about floodplain development and averting behaviour respectively. We interpret equations (4-22) and (4-23) in more detail.

Equation (4-22) is on floodplain development. The term on the left hand side implies the marginal direct benefit of floodplain development. The first term on the right hand side implies the marginal cost of losing ecosystem services due to floodplain development. The second term on the right hand side implies the marginal cost of increased flood risk due to floodplain development. These first three terms on the right hand side are totally the marginal cost of floodplain development. The third term on the right hand side implies the marginal cost of the external flood risk that floodplain development in zone *i* inflicts on other zones downstream. It is the marginal external cost of floodplain development. Hence, the condition (4-22) implies that the marginal benefit of floodplain development must be equal to the marginal cost plus the marginal external cost of floodplain development.

Equation (4-23) is on averting behaviour. We can acquire a similar economic interpretation. The term on the left hand side implies the marginal cost that we can avoid by enhancing the scale of averting behaviour. This is the marginal benefit of averting behaviour. The first term on the right hand side implies the marginal operation and maintenance cost due to the increase in the scale of averting behaviour. The second term on the right hand side implies the marginal cost of the external flood risk that enhancing the scale of averting behaviour in zone *i* imposes on other zones downstream. This is the marginal external cost of averting behaviour. Therefore, equation (4-23) implies that the marginal benefit of averting behaviour must be equal to the marginal cost plus the marginal external cost of averting behaviour.

4.8 Analytical Solution

In this section, we obtain an analytical solution in the dynamic optimisation problem. To begin with, we obtain a general form of analytical solution, in which there are m zones. However, this merely complicates matters. In order to discuss the essence of the model, it is better to make the model simpler and more tractable. This does not undermine the generality of the model. Thus, we assume that there are two zones. Furthermore, we focus on development of floodplains by omitting the variables related to averting behaviour. This enables us to get more concrete results.

4.8.1 General Form of Analytical Solution

In the beginning, we show the general form of analytical solution. We derive a system of simultaneous differential equations relating all the control and state variables from equations (4-20) and (4-21).

$$\begin{cases} \dot{y}_t^i = f^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) \\ \dot{a}_t^i = g^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) \\ \dot{X}_t^i = k^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) = y_t^i \\ \dot{A}_t^i = q^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) = a_t^i \end{cases} \qquad \text{(for all } i\text{)}$$

(4-24)

If an initial condition is given, we observe the optimal trajectories of control, state and costate variables by using this system.[15] This system satisfies the first-order necessary conditions (the maximum principle). On the theoretical side, we can obtain the differential equations in the system. Nevertheless, on the practical side, we cannot explicitly show them without providing concrete specific

[15] We need equations (4-14) and (4-15) in order to obtain the optimal trajectories of costate variables.

functional forms in the optimisation problem because of its complicacy.

4.8.2 Model of Two Zones with Two Control Variables

We confine the general form to a two-zones case (an example), say $m = 2$. We still treat two control variables and two state variables in this case. Thus, the system of differential equations contains eight differential equations. Based on (4-20) and (4-21), we can obtain four equations. We can solve the system of these four equations for the four time derivatives of control variables, and we add up the four equations of motion to get the system of differential equations.

$$\begin{cases} \dot{y}_t^{i=1} = f^1\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) \\ \dot{y}_t^{i=2} = f^2\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) \\ \dot{a}_t^{i=1} = g^1\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) \\ \dot{a}_t^{i=2} = g^2\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) \\ \dot{X}_t^{i=1} = k^1\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) = y_t^{i=1} \\ \dot{X}_t^{i=2} = k^2\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) = y_t^{i=2} \\ \dot{A}_t^{i=1} = q^1\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) = a_t^{i=1} \\ \dot{A}_t^{i=2} = q^2\left(y_t^{i=1}, y_t^{i=2}, a_t^{i=1}, a_t^{i=2}, X_t^{i=1}, X_t^{i=2}, A_t^{i=1}, A_t^{i=2}\right) = a_t^{i=2} \end{cases}$$

(4-25)[16]

This system of differential equations provides the analytical solution of the dynamic optimisation problem. The optimal paths of four control and four state variables are derived by the system of differential equations. However, they are too complicated to treat abstractly. If we give specific functional forms to the model with relevant parameter values, we can easily derive optimal paths from initial conditions given with the help of a computer software program such as GAMS.

[16] The first four differential equations are given by complicated functions. See Appendix B-1.

4.8.3 Model of Two Zones with One Control Variable

The model above is complex. One of the causes of the complexity is the number of control and state variables. Hence, we consider a reduced model with no option of averting behaviour here. Referring to equations (4-1), (4-2), (4-3) and (4-4) in the first-order necessary conditions (the maximum principle) or equations (4-22) and (4-23) in the steady-state optimal solution, it is possible that we initially ignore the option of averting behaviour in the model because of the symmetric relationship between floodplain development and averting behaviour. Averting behaviour in zones upstream has the same external effects on zones downstream as floodplain development in zones upstream. Averting behaviour is an alternative to floodplain conservation or restoration in terms of flood mitigation of its own zones. Therefore, we can still maintain the essential points even if we focus on floodplain development with no option of averting behaviour.[17] To begin with, we set up a social optimisation problem in a reduced model of two zones.

$$\max_{y_t} \; W = \int_0^{\infty} e^{-\delta t} U\{\pi(\cdot)\} dt$$

where

$$\pi = \sum_i B\left(L_F^i - X_t^i\right) + \sum_i F\left(X_t^i\right) - \sum_i D\left(y_t^i\right) - \sum_i C^i\left(X_t^i, X_t^j\right)$$

subject to

$$\dot{X}_t^i = y_t^i \qquad (i = 1, 2)$$

$$-X_t^i \le y_t^i \le L_F^i - X_t^i \qquad (i = 1, 2)$$

Mathematical notation and assumptions are the same as before (except that averting behaviour is omitted). We should, however, notice that we omit the function of flood risk (h) because it is a function of only the size of developed floodplains.

[17] We should note that averting behaviour is still important as a matter of fact.

We derive the first-order necessary conditions for the dynamic social optimisation problem in the reduced model in the same procedure as section 4.6. The first order necessary conditions yield:[18]

$$
\begin{aligned}
&U_{\pi}\frac{d^2D}{d\left(y_t^{i=1}\right)^2}\dot{y}_t^1+U_{\pi\pi}\frac{dD}{dy_t^{i=1}}\left\{\frac{dB}{dX_t^{i=1}}\dot{X}_t^{i=1}+\frac{dB}{dX_t^{i=2}}\dot{X}_t^2+\frac{dF}{dX_t^{i=1}}\dot{X}_t^{i=1}\right.\\
&\quad+\frac{dF}{dX_t^{i=2}}\dot{X}_t^{i=2}-\frac{dD}{dy_t^{i=1}}\dot{y}_t^{i=1}-\frac{dD}{dy_t^{i=2}}\dot{y}_t^{i=2}-\frac{\partial C^{i=1}}{\partial X_t^{i=1}}\dot{X}_t^{i=1}-\frac{\partial C^{i=2}}{\partial X_t^{i=2}}\dot{X}_t^{i=2}\\
&\quad\left.-\frac{\partial C^{i=2}}{\partial X_t^{i=1}}\dot{X}_t^{i=1}\right\}-\delta U_{\pi}\frac{dD}{dy_t^{i=1}}=-U_{\pi}\left\{\frac{dB}{dX_t^{i=1}}+\frac{dF}{dX_t^{i=1}}-\frac{\partial C^{i=1}}{\partial X_t^{i=1}}-\frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right\}
\end{aligned}
$$

(4-26)

$$
\begin{aligned}
&U_{\pi}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\dot{y}_t^2+U_{\pi\pi}\frac{dD}{dy_t^{i=2}}\left\{\frac{dB}{dX_t^{i=1}}\dot{X}_t^{i=1}+\frac{dB}{dX_t^{i=2}}\dot{X}_t^2+\frac{dF}{dX_t^{i=1}}\dot{X}_t^{i=1}\right.\\
&\quad+\frac{dF}{dX_t^{i=2}}\dot{X}_t^{i=2}-\frac{dD}{dy_t^{i=1}}\dot{y}_t^{i=1}-\frac{dD}{dy_t^{i=2}}\dot{y}_t^{i=2}-\frac{\partial C^{i=1}}{\partial X_t^{i=1}}\dot{X}_t^{i=1}-\frac{\partial C^{i=2}}{\partial X_t^{i=2}}\dot{X}_t^{i=2}\\
&\quad\left.-\frac{\partial C^{i=2}}{\partial X_t^{i=1}}\dot{X}_t^{i=1}\right\}-\delta U_{\pi}\frac{dD}{dy_t^{i=2}}=-U_{\pi}\left\{\frac{dB}{dX_t^{i=2}}+\frac{dF}{dX_t^{i=2}}-\frac{\partial C^{i=2}}{\partial X_t^{i=2}}\right\}
\end{aligned}
$$

(4-27)

$\dot{X}_t^{i=1}=y_t^{i=1}$ and $\dot{X}_t^{i=2}=y_t^{i=2}$

(4-28)

From (4-26), (4-27) and (4-28), we can derive the system of differential equations.

[18] Equations (4-26) and (4-27) are equivalent to equations (4-20) and (4-21).

$$\begin{cases} \dot{y}_t^{i=1} = f^1\left(y_t^{i=1}, y_t^{i=2}, X_t^{i=1}, X_t^{i=2}\right) = \dfrac{NR_1}{DN} \\ \dot{y}_t^{i=2} = f^2\left(y_t^{i=1}, y_t^{i=2}, X_t^{i=1}, X_t^{i=2}\right) = \dfrac{NR_2}{DN} \\ \dot{X}_t^{i=1} = k^1\left(y_t^{i=1}, y_t^{i=2}, X_t^{i=1}, X_t^{i=2}\right) = y_t^{i=1} \\ \dot{X}_t^{i=2} = k^2\left(y_t^{i=1}, y_t^{i=2}, X_t^{i=1}, X_t^{i=2}\right) = y_t^{i=2} \end{cases} \tag{4-29}$$

where

$$DN = U_\pi \frac{d^2D}{d\left(y_t^{i=1}\right)^2}\frac{d^2D}{d\left(y_t^{i=2}\right)^2} - U_{\pi\pi}\left\{\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\left(\frac{dD}{dy_t^{i=1}}\right)^2 + \frac{d^2D}{d\left(y_t^{i=1}\right)^2}\left(\frac{dD}{dy_t^{i=2}}\right)^2\right\}$$

$$NR_1 = U_\pi \frac{d^2D}{d\left(y_t^{i=2}\right)^2}\left\{-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \delta\frac{dD}{dy_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right\}$$

$$+ U_{\pi\pi}\left\{ y_t^{i=1}\left(\frac{dD}{dy_t^{i=1}}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\right)\left(-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right)\right.$$

$$+ y_t^{i=2}\left(\frac{dD}{dy_t^{i=1}}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\right)\left(-\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}}\right)$$

$$+ \frac{dD}{dy_t^{i=1}}\frac{dD}{dy_t^{i=2}}\left(-\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}}\right)$$

$$\left. + \left(\frac{dD}{dy_t^{i=2}}\right)^2\left(\frac{dF}{dX_t^{i=1}} + \frac{dB}{dX_t^{i=1}} - \frac{\partial C^{i=1}}{\partial X_t^{i=1}} - \frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right)\right\}$$

$$NR_2 = U_{\pi} \frac{d^2 D}{d\left(y_t^{i=1}\right)^2} \left\{ -\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \delta \frac{dD}{dy_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}} \right\}$$

$$+ U_{\pi\pi} \left\{ y_t^{i=1} \left(\frac{dD}{dy_t^{i=2}} \frac{d^2 D}{d\left(y_t^{i=1}\right)^2} \right) \left(-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}} \right) \right.$$

$$+ y_t^{i=2} \left(\frac{dD}{dy_t^{i=2}} \frac{d^2 D}{d\left(y_t^{i=1}\right)^2} \right) \left(-\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}} \right)$$

$$+ \frac{dD}{dy_t^{i=1}} \frac{dD}{dy_t^{i=2}} \left(-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}} \right)$$

$$\left. + \left(\frac{dD}{dy_t^{i=1}} \right)^2 \left(\frac{dF}{dX_t^{i=2}} + \frac{dB}{dX_t^{i=2}} - \frac{\partial C^{i=2}}{\partial X_t^{i=2}} \right) \right\}$$

The system of differential equations (4-28) is the analytical solution of the reduced dynamic optimisation problem. The optimal paths of two control and two state variables are provided by the system of differential equations. If an initial condition is given, we can obtain an optimal trajectory of each control, state and costate variable according to the differential equations.

4.9 Sufficiency Conditions

Our discussion has been based on the first-order necessary conditions (maximum principle) up to now. However, we need to check the sufficient conditions in order to verify that the situations derived from the first-order necessary conditions are optimal. We have two options: the Mangasarian sufficiency theorem and the Arrow sufficiency theorem. The Arrow sufficiency theorem uses a weaker condition than Mangasarian's theorem, but we check the sufficient conditions, based on the Mangasarian sufficiency theorem.

Chiang (1992) has provided a concise summary of the theorem in the case of an infinite horizon problem. The problem is to maximise $V = \int_0^\infty F(t, y, u)dt$ subject to $\dot{y} = f(t, y, u)$ and $y(0) = y_0$ (given).[19] In this case, the two conditions must be satisfied in order that the first-order conditions are not only necessary but also sufficient for deriving an optimal path. The first condition is that Hamiltonian function $H \equiv F(t, y, u) + \lambda f(t, y, u)$ is concave in (y, u) for all $t \in [0, T]$. The second condition is: $\lim_{t \to \infty} \lambda(t)\{y(t) - y^*(t)\} \geq 0$.

Now, we check the two conditions in our model. The first key is the concavity of Hamiltonian function. Based on the assumptions in Section 4.4, the integrand function is concave, and the equations of motion are linear in all the control and state variables. The second key is that the limit above should be equal or larger than zero. In our model, there is a positive discounting factor. It always converges to zero as time *t* goes to infinity. As a result, the first-order necessary conditions (the maximum principle) satisfy the conditions of the Mangasarian sufficiency theorem.

4.10 Stability Analysis

In this section, we check the local stability of steady-state equilibrium by using the method of linearization of higher-dimensional non-linear systems. In the beginning, we show the general form of Jacobian matrix in the method of linearization. Then, based on the general form, we analyse the local stability of steady-state equilibrium in the reduced model that we provided in the previous section.

[19] u is a control variable and y is a state variable.

4.10.1 Stability Analysis in the General Form

Based on (4-24), we depict the system of differential equations in the following.

$$\begin{cases} \dot{y}_t^i = f^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) \\ \dot{a}_t^i = g^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) \\ \dot{X}_t^i = k^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) = y_t^i \\ \dot{A}_t^i = q^i(\mathbf{y}_t, \mathbf{a}_t, \mathbf{X}_t, \mathbf{A}_t) = a_t^i \end{cases} \quad \text{(for all } i\text{)}$$

Differentiate the right hand side of these equations with respect to the relevant arguments, and set them as a factor of the Jacobian matrix. We evaluate the Jacobian matrix in the steady-state equilibrium in the following. $(\bar{\mathbf{y}}, \bar{\mathbf{a}}, \bar{\mathbf{X}}, \bar{\mathbf{A}})$ is the set of values in the optimal steady-state solution.

$$J = \begin{bmatrix} \left[\frac{\partial f}{\partial y}\right] & \left[\frac{\partial f}{\partial a}\right] & \left[\frac{\partial f}{\partial X}\right] & \left[\frac{\partial f}{\partial A}\right] \\ \left[\frac{\partial g}{\partial y}\right] & \left[\frac{\partial g}{\partial a}\right] & \left[\frac{\partial g}{\partial X}\right] & \left[\frac{\partial g}{\partial A}\right] \\ \left[\frac{\partial k}{\partial y}\right] & \left[\frac{\partial k}{\partial a}\right] & \left[\frac{\partial k}{\partial X}\right] & \left[\frac{\partial k}{\partial A}\right] \\ \left[\frac{\partial q}{\partial y}\right] & \left[\frac{\partial q}{\partial a}\right] & \left[\frac{\partial q}{\partial X}\right] & \left[\frac{\partial q}{\partial A}\right] \end{bmatrix}_{\substack{\mathbf{y}=\bar{\mathbf{y}} \\ \mathbf{a}=\bar{\mathbf{a}} \\ \mathbf{X}=\bar{\mathbf{X}} \\ \mathbf{A}=\bar{\mathbf{A}}}}$$

where

$$\left[\frac{\partial f}{\partial y}\right] = \begin{bmatrix} \frac{\partial f^{i=1}}{\partial y^{i=1}} & \cdots & \frac{\partial f^{i=1}}{\partial y^{i=m}} \\ \vdots & & \vdots \\ \frac{\partial f^{i=m}}{\partial y^{i=1}} & \cdots & \frac{\partial f^{i=m}}{\partial y^{i=m}} \end{bmatrix}$$

, and the same applied to the rest.

(4-30)

Then, the judgement on the feature of local stability obeys the following theorem (Polking et al., 2002).[20] (1) If the real part of every eigenvalue of the Jacobian matrix is negative, the steady-state equilibrium is an asymptotically stable equilibrium point. (2) If the Jacobian matrix has at least one eigenvalue with positive real part, the steady-state equilibrium is an unstable equilibrium point.

4.10.2 Stability in the Model of Two Zones with One Control Variable

Based on the general form above, we analyse the local stability of the optimal steady-state solution in the reduced model that we discussed in the previous section. In the first place, we check the optimal steady-state solution. In the steady-state equilibrium, all the control and state variables remain unchanged. Thus, we put the condition, $\dot{X}_t^{i=1} = \dot{X}_t^{i=2} = \dot{y}_t^{i=1} = \dot{y}_t^{i=2} = 0$, into the first-order necessary conditions (4-26), (4-27) and (4-28). Then, we obtain the followings. The optimal steady-state solution $\left(\overline{y}_t^{i=1}, \overline{y}_t^{i=2}, \overline{X}_t^{i=1}, \overline{X}_t^{i=2}\right)$ must satisfy these.

$$\frac{dB}{dX_t^{i=1}} + \frac{dF}{dX_t^{i=1}} - \frac{\partial C^{i=1}}{\partial X_t^{i=1}} - \frac{\partial C^{i=2}}{\partial X_t^{i=1}} - \delta \frac{dD}{dy_t^{i=1}} = 0 \tag{4-31}$$

$$\frac{dB}{dX_t^{i=2}} + \frac{dF}{dX_t^{i=2}} - \frac{\partial C^{i=2}}{\partial X_t^{i=2}} - \delta \frac{dD}{dy_t^{i=2}} = 0 \tag{4-32}$$

$$\dot{X}_t^{i=1} = \overline{y}^{i=1} = 0 \tag{4-33}$$

$$\dot{X}_t^{i=2} = \overline{y}^{i=2} = 0 \tag{4-34}$$

[20] In the case of 2×2 Jacobian matrix, we have more categories (c.f. Brock and Malliaris (1989), pp. 77-84).

Based on the general form (4-30), we can provide the 4×4 Jacobian matrix that is evaluated in the optimal steady-state equilibrium.

$$J=\begin{bmatrix} \dfrac{\partial f^1}{\partial y_t^{i=1}} & \dfrac{\partial f^1}{\partial y_t^{i=2}} & \dfrac{\partial f^1}{\partial X_t^{i=1}} & \dfrac{\partial f^1}{\partial X_t^{i=2}} \\ \dfrac{\partial f^2}{\partial y_t^{i=1}} & \dfrac{\partial f^2}{\partial y_t^{i=2}} & \dfrac{\partial f^2}{\partial X_t^{i=1}} & \dfrac{\partial f^2}{\partial X_t^{i=2}} \\ \dfrac{\partial k^1}{\partial y_t^{i=1}} & \dfrac{\partial k^1}{\partial y_t^{i=2}} & \dfrac{\partial k^1}{\partial X_t^{i=1}} & \dfrac{\partial k^1}{\partial X_t^{i=2}} \\ \dfrac{\partial k^2}{\partial y_t^{i=1}} & \dfrac{\partial k^2}{\partial y_t^{i=2}} & \dfrac{\partial k^2}{\partial X_t^{i=1}} & \dfrac{\partial k^2}{\partial X_t^{i=2}} \end{bmatrix}_{\substack{y_t^{i=1}=\bar{y}^{i=1}=0\\ y_t^{i=2}=\bar{y}^{i=2}=0\\ X_t^{i=1}=\bar{X}^{i=1}\\ X_t^{i-2}=\bar{X}^{i-2}}}$$

Then, we can calculate each factor of the Jacobian matrix by using equations (4-29) with the steady-state conditions (4-31), (4-32), (4-33) and (4-34).[21]

$$J=\begin{bmatrix} \alpha & 0 & \beta & \gamma \\ 0 & \alpha & \varepsilon & \phi \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where

$$\alpha=\frac{\delta U_\pi}{DN}\frac{d^2D}{d\left(y_t^{i=1}\right)^2}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}>0$$

$$\beta=\frac{U_\pi}{DN}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\left\{-\frac{d^2F}{d\left(X_t^{i=1}\right)^2}-\frac{d^2B}{d\left(X_t^{i=1}\right)^2}+\frac{\partial^2C^{i=1}}{\partial\left(X_t^{i=1}\right)^2}+\frac{\partial^2C^{i=2}}{\partial\left(X_t^{i=1}\right)^2}\right\}>0$$

$$\gamma=\frac{U_\pi}{DN}\frac{d^2D}{d\left(y_t^{i=2}\right)^2}\frac{\partial^2C^{i=2}}{\partial X_t^{i=1}\partial X_t^{i=2}}$$ [22]

---

[21] See Appendix B-2 about the process of mathematical derivation of the Jacobian matrix.

[22] The inequality sign of this term depends on the term $\frac{\partial^2C^{i=2}}{\partial X_t^{i=1}\partial X_t^{i=2}}=\frac{\partial^2C^{i=2}}{\partial X_t^{i=2}\partial X_t^{i=1}}$. However, this does not affect the result of the local stability analysis.

$$\varepsilon = \frac{U_\pi}{DN} \frac{d^2 D}{d(y_t^{i=1})^2} \frac{\partial^2 C^{i=2}}{\partial X_t^{i=1} \partial X_t^{i=2}}$$

$$\phi = \frac{U_\pi}{DN} \frac{d^2 D}{d(y_t^{i=1})^2} \left\{ -\frac{d^2 F}{d(X_t^{i=2})^2} - \frac{d^2 B}{d(X_t^{i=2})^2} + \frac{\partial^2 C^{i=2}}{\partial (X_t^{i=2})^2} \right\} > 0$$

(4-35)

Based on the Jacobian matrix, we can obtain the following four eigenvalues (r).

$$r_1, r_2, r_3, r_4 = \frac{\alpha}{2} \pm \frac{\sqrt{\alpha^2 + 2\beta + 2\phi + 2\sqrt{\beta^2 - 2\beta\phi + \phi^2 + 4\gamma\varepsilon}}}{2} \quad \text{and}$$

$$\frac{\alpha}{2} \pm \frac{\sqrt{\alpha^2 + 2\beta + 2\phi - 2\sqrt{\beta^2 - 2\beta\phi + \phi^2 + 4\gamma\varepsilon}}}{2}$$

(4-36)

Based on the eigenvalues (4-36), we can check whether the optimal steady-state equilibrium is stable or not. If the second term of the eigenvalues is an imaginary number, the real part of the eigenvalues is obviously positive because $\frac{\alpha}{2} > 0$. If the second term is a real number, at least two eigenvalues are positive because the second term is common in each pair of the eigenvalues. Therefore, we can conclude that the steady-state equilibrium is unstable, based on the judgement rule we mentioned. Notice that we cannot judge whether it is a saddle point in three or more dimensional models (here, four dimensions). If it is a saddle, it is possible to control it.

4.11 Policy Implications

As we have discussed, the system of floodplain management that is derived in this chapter is complex in that all the control and state variables in zones of the catchment interact one another. In addition, the system is unstable, based on the result of local stability analysis derived from the reduced model of two zones with one control variable.[23] In this case, the state can never attain the steady-state equilibrium unless the initial point is on an optimal steady-state equilibrium. It is, however, unknown where initial states go, based on the result of the local stability. If several initial states diverged into extreme situations in which all the floodplains are developed or natural, they would be non-optimal for the infinite optimisation problem because they are not sustainable.[24]

Under such an unstable situation, we have difficulties setting relevant policies because we have to force the current situation to directly move to a steady-state equilibrium with pinpoint accuracy. In addition, we do not know whether it is optimal to do so. On the contrary, in the case of stable equilibria, any initial states approach a steady-state equilibrium without any policies as long as each zone obeys the optimal management strategy although the adjustment process might be long.[25] In the case of saddle equilibria, we can take a feedback or closed-loop control policy (Conrad and Clark, 1987). If we can calculate separatrix curves (or saddle paths), we can reach the optimal paths that lead to a steady-state equilibrium along these curves.

To deal with the difficulties posed by the instability of the reduced model, and to explore the implications for policy, the next step is to develop an applied model based on the theoretical model in Chapter 5 and carry out policy

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[23] Our discussion is limited to the reduced model hereafter for a while.

[24] In the mathematical terms, such states do not satisfy transversality conditions derived from the social optimisation problem. Intuitively speaking, no ecosystem services and full economic activities or no economic activities and full ecosystem services are both non-optimal for our society.

[25] Even if the equilibria are stable, we need a policy for considering the externalities we mentioned.

simulations concretely in Chapter 6.

4.12 Conclusion of Chapter 4

In this chapter, we develop a theoretical continuous dynamic model for considering an optimal floodplain management. Unlike the static model, the dynamic model can provide optimal trajectories as a system of differential equations. The dynamic model is the basis of the applied models used in policy simulations.

The steady-state solutions imply that the marginal benefit should be equal to the sum of the marginal cost and the marginal external cost in terms of floodplain development and the scale of averting behaviour respectively. Interestingly, the conditions are the same as the optimal conditions derived from the static model in Chapter 3. In a reduced model, however, the steady-state equilibrium is locally unstable. Therefore, any initial states cannot achieve the steady-state equilibrium. That is, the optimal conditions derived from the static model cannot be satisfied in the dynamic context. There is interestingly a gap between static and dynamic models based on the same assumptions. Notice, however, that the results of the local stability analysis depend on the assumptions of functions on concavity and convexity. If the assumptions are not satisfied in real situations, the results may change. In any case, it is difficult to provide clear guidance for policies except that externalities should be internalised, based on the analysis of the theoretical dynamic model. Thus, we will develop an applied model, calibrate parameters and conduct policy simulations in the concrete context of the Ouse catchment in the following chapters.

Chapter 5

Applied Simulation Model and Calibration: Hydrological Modelling on HEC-HMS

5.1 Introduction

The purpose of this chapter is to develop an applied model in discrete time based on the theoretical continuous model we developed in Chapter 4, to determine functional forms and a hydrological sub-model (related to HEC-HMS) and to calibrate parameter values. The applied model with relevant calibrated parameter values is used for policy simulation in the next chapter.

The models are calibrated using several types of data such as economic, physical, GIS, and hydrological data. Some data derive from other researches in order to obtain a concrete function of benefits of ecosystem services. Furthermore, related to the hydrological sub-model, we often create data in order to calibrate parameters. For example, we create required data from GIS digital elevation models in order to obtain physical data on subbasins and rivers for the hydraulic model. We try to associate them with observed data as much as possible. However, we acknowledge that we set some assumptions on parameters and sub-models because of limited availability of necessary data. Needless to say, we make the assumptions clear in relevant parts.

In Section 5.2, we develop an applied simulation model in discrete time. We need the model in discrete time steps for policy simulation. In Section 5.3, we develop a hydrological sub-model (related to HEC-HMS) for the expected cost function of flood risk. This determines the impact of the land-use in floodplains as control variables in terms of hydrology. To begin with, we select an appropriate model for the Ouse catchment. Then, we specify measured parameters and fitted parameters based on data generated using the optimisation (calibration) tool of HEC-HMS. In addition, we specify the functional relationships between required variables, using a frequency analysis technique. The hydrological sub-model requires many parameters to be specified and large amounts of data for carrying out simulations. In Section 5.4, we specify the forms and parameter values of the functions included in the model by use of a value transfer method and relevant data. We set constraints on control variables as well. Section 5.5 concludes the chapter.

5.2 Simulation Model in Discrete Time

In Chapter 4, we developed a theoretical model in continuous time in order to analyse the optimal conditions and derive some policy implications. Now, we develop an applied model in discrete time. This is a reduced model with one control variable, floodplain development. That is, we omit the factor of averting behaviour as a control variable.

There are several reasons why we focus on a reduced model. First, even in discrete time, the model of multiple zones with two control variables is intractable. We prefer a simpler model in order to derive essential implications for policy making, as long as we adhere to our main research questions. Our main questions focus on floodplain management, not averting behaviour. Furthermore, essential points will not be lost even if our model does not include averting behaviour explicitly for the following reason. Second, the enhancement of averting behaviour in zone i has the same effect on the expected cost of flood risk in the zone i as the conservation of natural floodplains in zone i. In this respect, averting behaviour is a substitute for natural floodplains. Averting behaviour and floodplain development are complementary. In fact, floodplain development is often accompanied by averting behaviour. In addition, the enhancement of averting behaviour in zone i has the same effect on the expected cost of flood risk in zones j, downstream as the development of floodplains in zone i, which implies unidirectional spatial externality. Thus, only if we check the characteristics of floodplain development in the model, we can understand policy implications. Third, it is too difficult to indicate averting behaviour in a hydrological sub-model because of a limitation of the model and data availability. The types of averting behaviour are various: embankment, dykes, floodwalls, pumping system, and so on. We can control the height of banks in a hydrological sub-model, but we cannot directly address other types of averting behaviour.[1] Nor do we have data on other forms of averting behaviour. Furthermore, the range of averting behaviour is often limited to a particular area in a zone (subbasin). In this case, we have to divide the target watershed (catchment) into so many zones (subbasins) for considering averting behaviour, which is complicated and may not help policy analysis.

Averting behaviour (flood protection) is treated as one of the initial conditions (as an exogenous variable). As we mention in Section 5.3.13, we focus on precipitation volume with 2%, 1% and lower exceedance probability in simulations, given that the current flood protection water level is close to the 1% flood water level in York and Selby urban areas.[2]

The optimisation problem is the following. The interpretation of the problem is the same as in Chapter 4. However, it is a finite discrete time problem. We suppose that decision makers or policy makers attempt to make their decisions in a fixed duration such as 30, 50 and 100 years.

$$\underset{\mathbf{y}_t^i, \mathbf{X}_t^i}{Max} \quad W = \sum_{t=0}^{T-1} \rho^t U\left[\pi\left(\mathbf{y}_t^i, \mathbf{X}_t^i\right)\right] + \rho^T V\left(\mathbf{X}_t^i\right)$$

where

[1] There is a possibility for expanding a model in order to include averting behaviour as far as the embankment is concerned.

[2] This might be a strong assumption for the whole catchment. As the assumption is applied to the whole catchment, we choose precipitation volume with 2%, 1% and lower exceedance probability.

$\rho = \dfrac{1}{1+\delta}$ (ρ is the discount factor, and δ is the periodic discount rate.)

$$\pi(\cdot) = \sum_i B(L_F^i - X_t^i) + \sum_i F(X_t^i) - \sum_i D(y_t^i) - \sum_i C^i(X_t^i, X_t^j) \quad (i \neq j)$$

$V(\cdot)$ denotes a final function that indicates the value of the state variable at terminal time T.

subject to

$X_{t+1}^i - X_t^i = y_t^i$ (for all i)

$-X_t^i \leq y_t^i \leq L_F^i - X_t^i$ (for all i)

$X_0^i = X^i(0)$ given (for all i)

Let us derive the first-order necessary conditions as we did in Chapter 4. We use the Hamiltonian. To begin with, we set the current value Hamiltonian function H with the costate variable λ.

$$H = U(\pi(\cdot)) + \rho \sum_i \lambda_{t+1}^i y_t^i$$

The first-order conditions include:

$$\frac{\partial H}{\partial y_t^i} = -U_\pi \frac{dD}{dy_t^i} + \rho \lambda_{t+1}^i = 0 \quad \text{(for all } i\text{)}$$

(5-1)

$$\rho \lambda_{t+1}^i - \lambda_t^i = -\frac{\partial H}{\partial X_t^i} = -\left\{ U_\pi \left[\frac{dB}{dX_t^i} + \frac{dF}{dX_t^i} - \frac{\partial C^i}{\partial X_t^i} - \sum_{j>i} \frac{\partial C^j}{\partial X_t^i} \right] \right\} \quad \text{(for all } i\text{)}$$

(5-2)

$$X_{t+1}^i - X_t^i = \frac{\partial H}{\partial (\rho \lambda_{t+1}^i)} = y_t^i \quad \text{(for all } i\text{)}$$

(5-3)

$$\lambda_T^i = \frac{\partial V}{\partial X_T^i} \text{ and } X_0^i = X^i(0) \quad \text{(for all } i\text{)}$$

(5-4)

Arranging conditions (5-1), (5-2), (5-3) and (5-4),

$$\rho\lambda_{t+1}^{i} = U_{\pi}\frac{dD}{dy_{t}^{i}} \quad \text{(for all } i\text{)}$$

(5-5)

$$\lambda_{t}^{i} = U_{\pi}\left(\frac{dB}{dX_{t}^{i}} + \frac{dF}{dX_{t}^{i}} - \frac{\partial C^{i}}{\partial X_{t}^{i}} - \sum_{j>i}\frac{\partial C^{j}}{\partial X_{t}^{i}}\right) + \rho\lambda_{t+1}^{i} \quad \text{(for all } i\text{)}$$

(5-6)

$$X_{t+1}^{i} - X_{t}^{i} = y_{t}^{i} \quad \text{(for all } i\text{)}$$

(5-7)

$$\lambda_{T}^{i} = \frac{\partial V}{\partial X_{T}^{i}} \text{ and } X_{0}^{i} = X^{i}(0) \quad \text{(for all } i\text{)}$$

(5-8)

The system of difference equations is given by conditions (5-5), (5-6), (5-7) and (5-8).[3]

In addition, we want to consider the steady-state in which **y**, **X** and **λ** are unchanging. In the steady-state, the following conditions hold:

$$\lambda^{i} = \lambda_{t}^{i} = \lambda_{t+1}^{i} \quad \text{(for all } i\text{)}$$

$$X^{i} = X_{t}^{i} = X_{t+1}^{i} \quad \text{(for all } i\text{)}$$

$$y^{i} = y_{t}^{i} = 0 \;\; \left(\Rightarrow \frac{dD}{dy^{i}} = 0\right) \quad \text{(for all } i\text{)}$$

Then,

$$U_{\pi}\left(\frac{dB}{dX_{t}^{i}} + \frac{dF}{dX_{t}^{i}} - \frac{\partial C^{i}}{\partial X_{t}^{i}} - \sum_{j>i}\frac{\partial C^{j}}{\partial X_{t}^{i}}\right) = 0$$

$$\Rightarrow \frac{dF}{dX_{t}^{i}} = -\frac{dB}{dX_{t}^{i}} + \frac{\partial C^{i}}{\partial X_{t}^{i}} + \sum_{j>i}\frac{\partial C^{j}}{\partial X_{t}^{i}} \quad \text{(for all } i\text{)}$$

The conditions are the same as the steady-state conditions derived in Chapter 4. This implies that the marginal benefit of floodplain development must be equal

[3] We try to numerically solve this dynamic optimisation problem by GAMS software program in the next chapter.

to the marginal cost plus the marginal external cost of floodplain development.

Finally, we should note that the unit of the value is 1990 UK £ and the unit of area is a hectare (ha). In addition, we need to set a specific functional form of the utility function for simulations. A utility that satisfies the assumptions in Chapter 3 is [4]

$$U(\pi) = 2(\pi + c)^{\frac{1}{2}}$$

$$\left(U'(\pi) = (\pi + c)^{-\frac{1}{2}} > 0, \quad U''(\pi) = -\frac{1}{2}(\pi + c)^{-\frac{3}{2}} < 0 \right)$$

where c is a constant that is sufficiently large in order that the term $(\pi+c)$ is always positive.[5]

5.3 Hydrological Sub-model

We give the expected cost of flood risk from a hydrological sub-model (hydrological simulation model). Then, based on a hydrological sub-model, we derive a concrete functional form for the expected cost function of flood risk in Section 5.4.1. To begin with, we define a hydrological model on HEC-HMS (Hydrologic Modeling System) and calibrate relevant parameters. This gives the relationship between control variable (floodplain development) and discharge volume (peak flow). Next, we implement frequency analysis on precipitation in order to acquire the relationship between precipitation and its exceedance probability. Then, we set the function that shows the relation between discharge volume and estimated flood costs in each subbasin. Finally, we calculate the expected cost of flood risk linked with a value of control variable.

[4] We can choose any functional forms as long as they satisfy the assumptions.

[5] The utility function is ordinal. We are not interested in the absolute value but the relative value.

5.3.1 HEC-HMS

HEC-HMS is a GUI (Graphical User Interface) software program of hydrologic modelling simulation. HEC-HMS is freely provided by the US Army Corps of Engineers, Hydrologic Engineering Center.[6] The US Army Corps of Engineers continuously upgrade the software program.[7] These are advantages to using HEC-HMS. This point is related to reproducibility of research work.[8]

HEC-HMS provides mathematical simulations of the precipitation-runoff processes of dendritic watershed systems, based on several theoretical or empirical and hydrological or hydraulic models. It is applicable in a wide range of geographic areas for solving the widest possible range of problems including floodplain regulation (USACE, 2001). Simulation models on HEC-HMS are composed of three sub-models: a basin model, a meteorologic model and a control specification model. The basin model stipulates river system connectivity and physical data describing a watershed. The meteorologic model stores the data on precipitation which is used in simulations. The meteorologic model also deals with virtual or hypothetical precipitation for simulations. The control specification model determines the time duration and time interval in which we execute simulations.

There are, however, some limitations of HEC-HMS (USACE, 2000 and 2001). First, HEC-HMS cannot deal with backwater effects except through Modified Puls which can partly treat them. Tidal fluctuations, significant tributary inflows, dams, bridges, culverts, and channel constructions may cause backwater effects. We focus on the upstream area in the total watershed of River Ouse that is not close to Humber estuary. Thus, we do not need to worry about tidal fluctuations. However, we are not sure about other possible causes. If backwater effects were serious, we should switch to other hydrological modelling programs. Second, some of the routing models in HEC-HMS cannot take flood mitigation

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[6] The software program, HEC-HMS, is downloadable from the website. www.hec.usace.army.mil
[7] We use HEC-HMS version 2.2.2. (28 May 2003).
[8] We provide a review on hydrological software programs in Chapter 2.

function of floodplains into consideration. If flood flows exceed the channel's carrying capacity, water flows into floodplains. Depending on the characteristics of the floodplains, the flows in floodplains can be slowed greatly because of their flood mitigation function. This can be significant in terms of the translation and attenuation of a flood wave. It is what we want to analyse. In this case, we can use a two-dimensional flow model in order to simulate the physical processes. Based on USACE (2000), the Modified Puls model and Muskingum Cunge 8-point model are used for the problem. Third, some of the routing models in HEC-HMS are not suitable if the channel slopes are small. For example, the Kinematic Wave model is not appropriate if the channel slope is less than 0.002. However, Muskingum Cunge model can be used for relatively flat slopes of channels. We can avoid this problem because we choose Muskingum Cunge 8-point model. Fourth, if the shifts between subcritical and supercritical flows are frequent and unpredictable, none of the routing models in HEC-HMS is suitable.[9] In this respect, we cannot judge what is happening in reality because of only limited availability of data. If this problem was serious, we should use other hydrological software programs. Finally, the models in HEC-HMS cannot treat snowfall and snowmelt.[10] Fortunately, as we have only little snow in our target River Ouse catchment, this problem does not seem to be serious.

5.3.2 Modelling on HEC-HMS

We have to set the three sub-models in order to implement simulations on HEC-HMS. The basin model is a core of modelling on HEC-HMS. River system connectivity is given by a schematic map that is composed of several elements: subbasin, reach (river), reservoir and so on. Each element should be provided by a model for calculating runoff or discharge volumes. There are several options of

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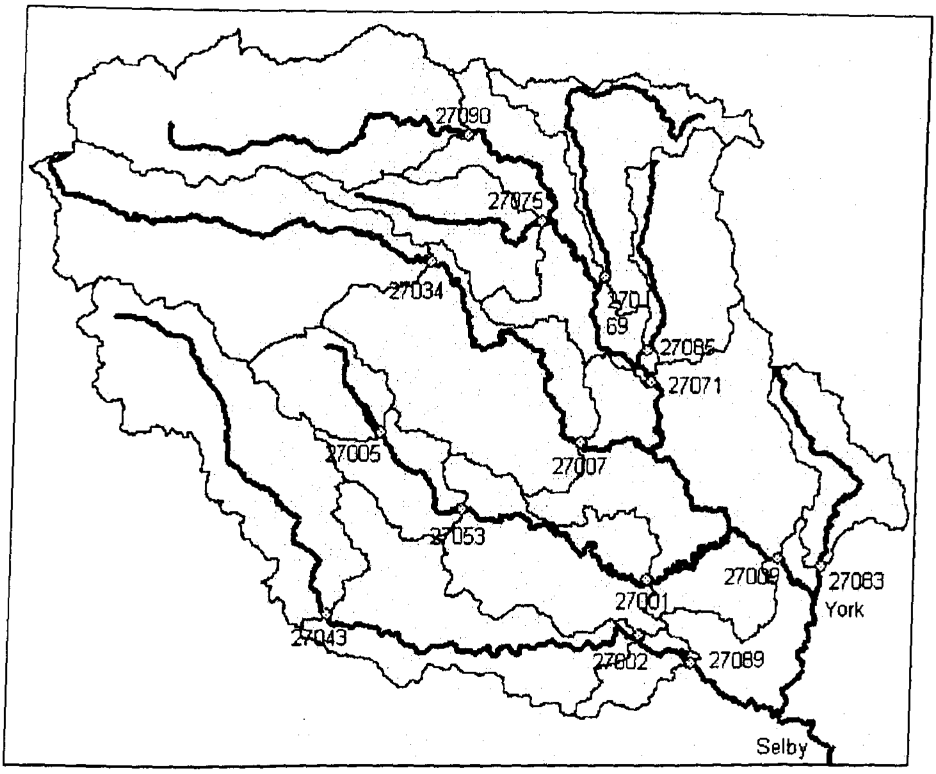
[9] The subcritical flow is the flow of water at a velocity less than critical or tranquil flow while the supercritical flow is the flow of water at a velocity greater than critical or rapid flow (Bedient and Huber, 2002).

[10] HEC-HMS will be able to treat snowfall and snowmelt in the future according to USACE.

the model on each element on HEC-HMS. Thus, we do the following procedures to create a basin model.

1. We set a schematic map as a representative abstract model, based on the geographical map.
2. We choose relevant models (called methods in HEC-HMS) for each element.
3. We specify necessary parameter values by GIS, physical and observed data.

Figure 5-1. *Geographical map of River Ouse catchment*



Source: This geographical map is created from GIS data of EDINA Digimap (OS Strategi and Land-Form Panorama [DEM]).

To begin with, let us set a schematic map. We need a geographic map of rivers and gauging stations. It is obtained from National River Flow Archive or CEH (2003).[11] However, we set a geographical map of River Ouse catchment (watershed) from GIS data by using ArcGIS.[12] Figure 5-1 shows this. The points and numbers indicate the gauging stations. The boundaries show subbasins

[11] National River Flow Archive: www.nerc-wallingford.ac.uk/ih/nwa/index.htm
[12] We explain how to obtain this map in the following sections.

defined by gauging stations.[13] These subbasins are treated as zones in the theoretical and applied model. Table 5-1 is the list of gauging stations.

Table 5-1. *List of gauging stations*

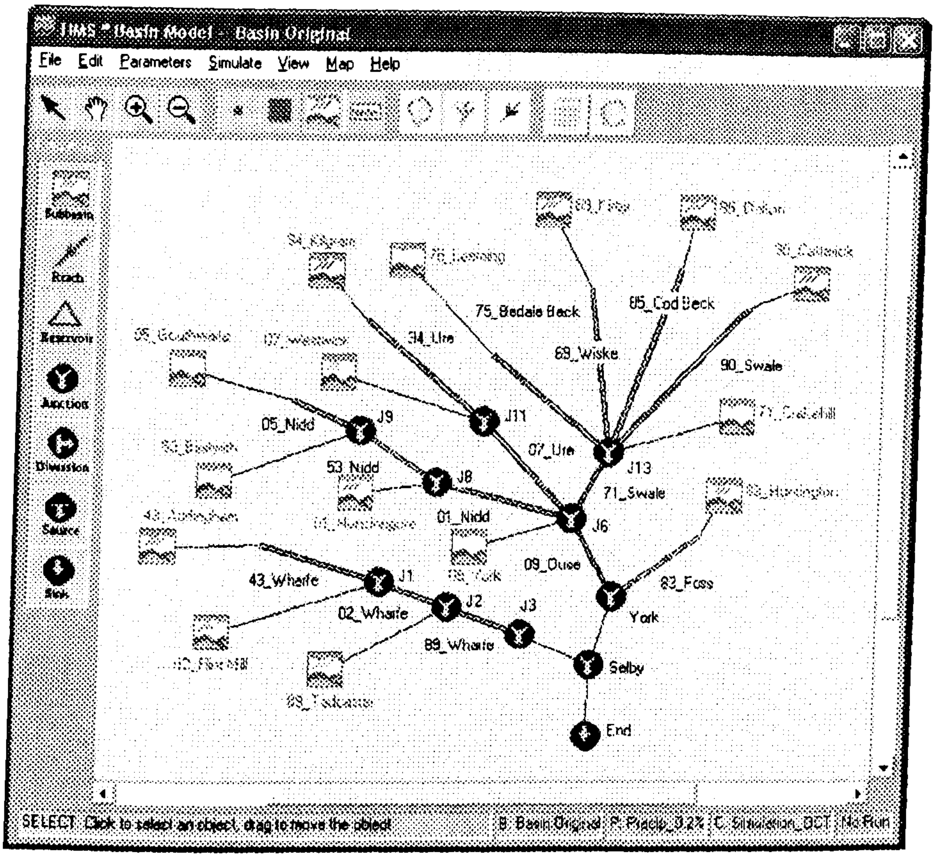
Number	Place	River
27001	Hunsingore Weir	Nidd
27002	Flint Mill Weir	Wharfe
27005	Gouthwaite Reservoir	Nidd
27007	Westwick Lock	Ure
27009	Skelton	Ouse
27034	Kilgram Bridge	Ure
27043	Addingham	Wharfe
27053	Birstwith	Nidd
27069	Kirby Wiske	Wiske
27071	Crakehill	Swale
27075	Leeming	Bedale Beck
27083	Huntington	Foss
27085	Dalton Bridge	Cod Beck
27089	Tadcaster	Wharfe
27090	Catterick Bridge	Swale

Based on the geographical map, we can set a schematic map considering the availability of relevant data and the tractability of our applied simulation model. Figure 5-2 shows the schematic map in HEC-HMS. It goes without saying that we simplify the real river physical connectivity and subbasins in the process of creating the schematic map on HEC-HMS. Thus, we implicitly set some assumptions by simplification. Crucially, we set one subbasin element and one reach (river) element as a representative on each subbasin based on a gauging station. Related to this, we assume that the characteristics of subbasins and rivers are similar in each subbasin although they are locally variable in reality. We do not think that the assumption is problematical, but the assumption does affect the degree of precision of the results of simulations, the tractability of simulation model, several difficult points related to calibration of parameters, and so on. In this respect, the simplification of the model has both advantages and disadvantages.[14]

[13] We distinguish subbasins based on the gauging stations. Thus, we name the subbasins by the same number of gauging stations.

[14] The issue of this model simplification is ascribed to the problem: how many zones we should separate the watershed into when considering policies? However, it depends on the number of

Figure 5-2. *Schematic map of River Ouse catchment on HEC-HMS*



We need to select relevant models for the elements in the schematic map. Two elements are used: subbasins and reaches (rivers). We have to choose relevant models for the loss rate, transform and baseflow of subbasins. We need to pick out a relevant model for reaches (rivers). However, it is difficult to select appropriate models for the elements. A very wide variety of models are generally available to any rainfall-runoff modelling application without any clear basis for their choice (Beven, 2002). Beven (2002) provides practical criteria for choosing models:

- Is a model readily available, or could it made available if the investment of time (and money!) appeared to be worthwhile?
- Does the model predict the variables required by the aims of a particular project?
- Are the assumptions made by the model likely to be limiting in terms of what you know about the response of the catchment you are

gauging stations when we carry out simulations based on observed data. Furthermore, "the ungauged catchment problem is, as yet, not properly resolved due to the fact that it is very difficult to generalize about the nature of catchment responses in any quantitative way" (Beven, 2002).

interested in?
- Can all the inputs required by the model, for specification of the low domain, for the specification of the boundary and initial conditions and for the specification of the parameters values, be provided within the time and cost constraints of a project?

Here, we have no a priori information on hydrological and hydraulic processes in River Ouse catchment. We use the following criteria, considering Beven's practical ones.

1. We choose a model out of the models that HEC-HMS provides.
2. We choose a simple model with only a few parameters that need to be calibrated by observed data, because the calibration (called "optimization" in HEC-HMS) algorithm is not so powerful to adjust multiple variables in order to make the fit of the model optimal.[15]
3. In terms of data availability, we choose lumped models. We need to choose models, considering data availability. If we cannot specify parameter values, it will be meaningless.
4. We fundamentally avoid models that need physical parameters and data on chemical and biological processes because the data on physical parameter values are limited. Thus, we prefer empirical (system theoretic) models to conceptual models.
5. More importantly, we must select the model in which we deal with control variables (floodplain development).

Let us discuss which model we choose for the loss rate, the transform and the baseflow of subbasins, and the reaches (rivers) in order, considering the criteria above. First, we discuss models for loss rate of subbasins. HEC-HMS calculates the volume of water that is intercepted, infiltrated, stored, evaporated, or transpired as losses in order to compute runoff volume. Thus, we need a model for loss rate of subbasins. Table 5-2 shows the list of models. First, we avoid setting Green and Ampt, Gridded SCS Curve No and Gridded SMA because they are distributed models. Second, SCS Curve No is not suitable to our catchment

[15] In this respect, "[i]n many ways, hydrologic modeling is more an art than a science, and it is likely to remain so (Loague and Freeze, 1985)" (qtd. in USACE, 2000). The issue of choosing models is subtle. Ultimately, the problem lies in the extent to which the chosen models with calibrated parameters can reproduce real hydrologic situations.

because it is developed with data from small agricultural watersheds in mid US. Third, we cannot select SMA in terms of data availability. Finally, Initial/Constant rate and Deficit/Constant rate can be good candidates in our applied model although they might be too simple. We choose Initial/Constant rate as the first option for loss rate of subbasins because the number of parameters in it is less than that in Deficit/Constant rate. If we are faced with serious problems in the process of calibration, we will test Deficit/Constant rate as the second option. In addition, one of the important things to note is that the parameter of impervious (%) denotes the area of developed lands in each subbasin. This indicates development of lands in subbasin.

Table 5-2. *Models for loss rate of subbasins*

Model	Type of Model	Parameter (unit)	Characteristics	Choose?
Green and Ampt	Distributed Empirical Fitted parameter	- Initial loss (mm) - Conductivity (mm/hr) - Vol. moisture deficit - Impervious (%) - Wet. front suct. (mm)	Distributed model. Not widely used, so less mature. Less parsimonious than simple empirical models.	No
Initial/Constant rate	Lumped Empirical Fitted parameter	- Initial loss (mm) - Constant rate (mm/hr) - Impervious (%)	May be too simple to predict losses within event even if it predict total losses well.	Yes (1)
SCS Curve No.	Lumped Empirical Fitted parameter	- Initial loss (mm) - SCS curve No. - Impervious (%)	Developed wit data from small agricultural watersheds in mid US, so applicability elsewhere is uncertain. Rainfall intensity not considered.	No
Deficit/Constant rate	Lumped Empirical Fitted parameter	- Initial deficit (mm) - Loss rate (mm/hr) - Max. deficit (mm) - Impervious (%)	Similar to initial/constant rate.	Yes (2)
SMA (Soil Moisture Accounting)	Lumped Empirical Fitted parameter	- SMA unit - Initial storage (%) - Storage capacity (mm) - Impervious (%)	Parameters required are complicated. Difficult in terms of data availability.	No
Gridded SCS Curve No.	Distributed Empirical Fitted parameter	- Initial abstraction ratio - Potential retention scale	Distributed model.	No
Gridded SMA	Distributed Empirical Fitted parameter	- Initial storage (%) [5 ground levels]	Distributed model.	No

Source: Arranged from USACE (2000, 2001), and HEC-HMS Help.

Next, we discuss models for transform of subbasins. We need a model for simulating the process of direct runoff of excess precipitation on the watershed. We may be able to use the empirical parameter prediction equations to specify parameter values of transform models, but the optimal source of the parameters is calibration (USACE, 2000). Furthermore, we often need several measured physical parameter values for using the equations, but they are not available in our target. Table 5-3 shows the list of models for transform of subbasins. First, we cannot choose ModClark because it is a distributed model. Second, we do not choose Kinematic Wave and Snyder's UH because they require physical parameter values that it is difficult to obtain in the Ouse catchment. Finally, SCS UH and Clark's UH can be good candidates in that their parameters can be calibrated. We prefer SCS UH to Clark's UH. We should notice that SCS UH assumes only one peak flow on event level. However, we have no observed data on events although we have daily data. Thus, we cannot check this point. This is an implicit assumption in this applied work.

Table 5-3. *Models for transform of subbasins*

Model	Type of Model	Parameter (unit)	Characteristics	Choose?
Clark's UH (Unit Hydrograph)	Lumped Empirical Fitted parameter	- Time of concentration (hr) - Storage coefficient (hr)	It is a quasi-conceptual model, but parameters can be calibrated by observed precipitation and flow data.	Yes(2)
Kinematic Wave	Lumped Conceptual Measured parameter	- Choice of channel routing method - Detailed information on planes - Detailed information on channels	It is a conceptual model. Required parameters are measurable or observable watershed properties. Data requirement is enormous.	No
ModClark	Distributed Empirical Fitted parameter	- Time of concentration (hr) - Storage coefficient (hr)	It is a quasi-conceptual distributed model.	No
Snyder's UH	Lumped Empirical Fitted parameter	- Snyder standard lag (hr) - Snyder peaking coefficient	Snyder standard lag can be calculated by empirical equations, but they need physical parameters that are difficult to obtain.	No
SCS UH	Lumped Empirical Fitted parameter	- SCS Lag (min or min)	Can be calibrated. Assume only one peak on event level.	Yes(1)

Source: Arranged from USACE (2000, 2001), and HEC-HMS Help.

Next, we discuss models for baseflow of subbasins. Baseflow is the sustained flow in rivers due to soil moisture, ground water or prior precipitation that was stored temporarily in the watershed (Bedient and Huber, 2002; USACE, 2000). The parameters in baseflow models cannot be specified in the process of calibration. We definitely need appropriate data to derive parameter values of base flow models. Table 5-4 shows the list of models for baseflow of subbasins. First, we cannot use Linear Reservoir because this model is suitable only with SMA Loss and Gridded SMA Loss models. Second, we do not use Recession and Bounded Recession because it seems to be difficult to get the data for specifying their parameter values due to data availability. Finally, we choose Constant Monthly in that we can specify the constant monthly baseflow by use of Base Flow Index provided by CEH (Gustard et al., 1992). Alternatively, we can calculate them from the data of gauged daily flow.

Table 5-4. *Models for baseflow of subbasins*

Model	Type of Model	Parameter (unit)	Characteristics	Choose?
Recession	Lumped Empirical Fitted parameter	- Initial Q (cms) - Recession constant - Threshold Q (cms)	Difficult to estimate recession constant and other parameters in terms of data availability.	No
Constant Monthly	Lumped Empirical Fitted parameter	- Constant baseflow (cms) [each month]	Can use the data of BFI (Base Flow Index). Can derive parameters from gauged daily flow data.	Yes
Linear Reservoir	Lumped Empirical Fitted parameter	- Storage coefficient (hr) - Number of reservoir	Compatible only with SMA loss method and gridded SMA loss method.	No
Bounded Recession	Lumped Empirical Fitted parameter	- Initial baseflow (cms or cm/sq km) - Recession ratio - Max. baseflow (cms) [each month]	Difficult to estimate recession ratio and other parameters in terms of data availability.	No

Source: Arranged from USACE (2000, 2001), and HEC-HMS Help.

Finally, we discuss models for reaches (rivers). We need a hydrologic or hydraulic routing model to compute the volume of water that flows through rivers. Fundamentally, routing models solve Saint Venant equations which are composed of the momentum and continuity equations. More importantly, we must deal with control variables such as floodplain development and averting behaviour although we here focus on floodplain development. Table 5-5 shows the list of models for reaches (rivers). This condition should be definitely satisfied although we are faced with some difficult situations for specifying parameter values. In terms of this, we need to choose Muskingum Cunge 8 Point model for rivers. In addition, this model is suitable to the situation that flood wave goes out of bank and into floodplains (USACE, 2000). In fact, we will use knowledge on Muskingum

Cunge Standard model for specifying parameter values because of limited data availability and model simplification.

Table 5-5. *Models for reaches (rivers)*

Model	Type of Model	Parameter (unit)	Characteristics	Choose?
Lag	Lumped Empirical Fitted parameter	- Lag (min or hr)	Cannot deal with control variables.	No
Muskingum	Lumped Empirical Fitted parameter	- Muskingum K (hr) - Muskingum X - Number of subreaches	Cannot deal with control variables.	no
Modified Puls	Lumped Empirical Fitted parameter	- Number of subreaches - Initial conditions - Storage-outflow (cms)	Cannot deal with control variables.	No
Muskingum Cunge Standard	Lumped Quasi-conceptual Measured parameter	- Cross section shape - Reach length (m) - Energy slope (m/m) - Bottom width or diameter (m) - Side slope (xH:1V) - Manning's N	Cannot deal with control variables.	No
Muskingum Cunge 8 Point	Lumped Quasi-conceptual Measured parameter	- Reach length (m) - Energy slope (m/m) - Manning's N [overbank and channel] - Cross section coordinates	Can deal with control variables although we have to set some assumptions to specify several parameter values.	Yes
Kinematic Wave	Lumped Conceptual Measured parameter	- Cross section shape - Reach length (m) - Energy slope (m/m) - Bottom of width or diameter (m) - Side slope (xH:1V) - Manning's N - Maximum number of routing increments	Cannot deal with control variables.	No
Straddle Stagger	Lumped Empirical Fitted parameter	- Lag (hr) - Straddle duration (hr)	Cannot deal with control variables.	No

Source: Arranged from USACE (2000, 2001), and HEC-HMS Help.

5.3.3 Parameters

We have chosen the models for each element. Let us specify parameter values in the models. We have two types of parameters: measured and fitted parameters. Measured parameters are estimated from relevant data while fitted parameters are calibrated by comparing simulated discharge flow data with observed discharge flow data. We can calibrate fitted parameters by using the optimisation tool of HEC-HMS. The procedures for specifying parameter values are:

1. We specify the values of measured parameters by using relevant data such as GIS, hydrological and physical data.
2. We input the values of all the measured parameters into HEC-HMS, and we input observed precipitation and discharge flow data into HEC-HMS.
3. We calibrate the values of fitted parameters by using the optimisation tool in HEC-HMS.

Table 5-6 shows the summary of parameters. In the next section, we specify the parameter values.

Table 5-6. *Parameters*

Type of parameter	Element	Model	Parameter
Measured parameter	Subbasin/Loss rate	Initial/Constance	Impervious
	Subbasin/Baseflow	Constant Monthly	Constant Baseflow [each month]
	Reach (river)	Muskingum Cunge 8 Point	Reach length
			Energy slope
			Manning's N [overbank and channel]
			Cross section coordinates
Fitted parameter	Subbasin/Loss rate	Initial/Constance	Initial loss
			Constant rate
	Subbasin/Transform	SCS UH	SCS lag

5.3.4 Derivation of Constant Monthly Baseflow

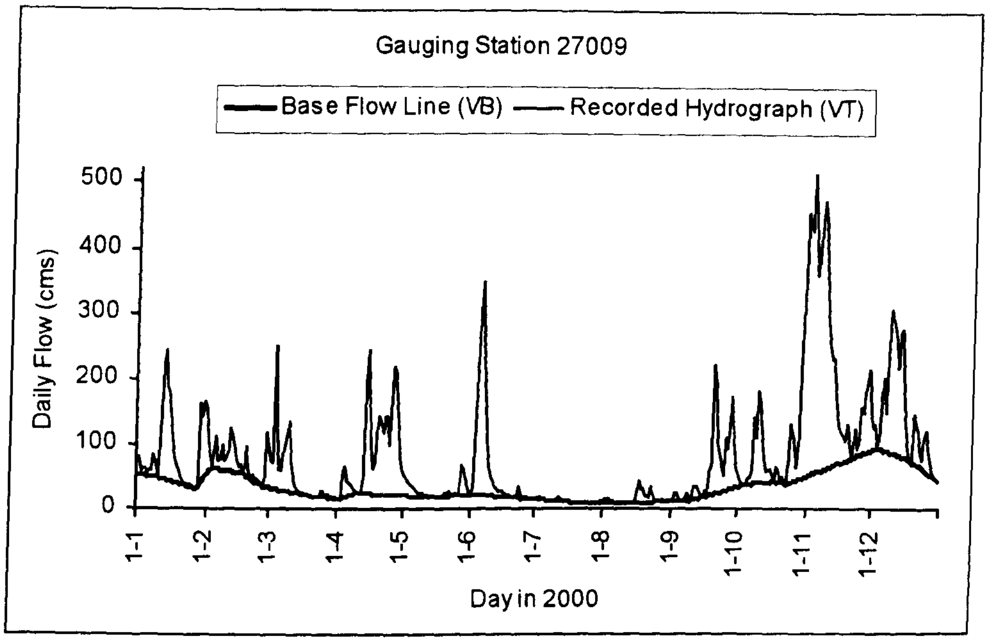
We need data on constant monthly baseflow for the subbasin element in the model on HEC-HMS. In order to derive the values of monthly constant baseflow in each subbasin, we obtain a Base Flow Index (BFI) in each subbasin. Gustard et al. (1992) give the BFI values in gauging stations. The data cover several years before 1992, but the data are missing in some target gauging stations and in any case are old. CEH (2003) also gives data on BFI, but the duration of data on which the calculation is based is not necessarily clear. We obtained gauged daily flow data from National Water Flow archives (CEH [Centre for Ecology and Hydrology]). Thus, we calculate the values of BFI based on the data we have, but it is a good idea to use the definition of the BFI and the method of calculating the BFI that Gustard et al. (1992) provide because we can compare our calculated BFI values with their BFI values.[16] [17] Based on Gustard et al. (1992), the BFI is referred to as the proportion of the river's runoff that is derived from stored sources, and the values of BFI can be calculated as the ratio of the flow under the separated hydrograph (baseflow) to the flow under the total hydrograph (recorded flow) by a smoothing and separation rule. Figure 5-3 shows the relationship between the recorded hydrograph (VT) and the baseflow line (VB). The mathematical expression is given by the following.

$$BFI = \frac{VB}{VT} < 1$$

[16] In general, there is no certain correct method of deriving baseflow or BFI although the abstract notion of baseflow is well defined.

[17] The values of BFI are relatively stable because they mainly depend on the soil type. Thus, the comparison is meaningful.

Figure 5-3. *Relation between recorded hydrograph and baseflow line*



NB. This graph shows the results of the actual calculation in gauging station 27009 (Ouse at Skelton).

More concretely, the procedures for calculating the BFI values are the following:[18]

1. We have the data on gauged daily flow (GDF) (m^3/s or cms), Q_1, Q_2, . . , Q_i, . . , Q_n. We divide this data into non-overlapping blocks (m) of five days, and derive the minima for each block, $\mathbf{M}_{ij}$, . . . , M_{im}. $\mathbf{M}_{ij}$ is a vector. 'i' in $\mathbf{M}_{ij}$ is given by discontinuous integer numbers.
2. Consider the block (M_{i1}, M_{i2}, M_{i3}), (M_{i2}, M_{i3}, M_{i4}), . . . , ($M_{i,j-1}$, $M_{i,j}$, $M_{i,j+1}$), . . . , ($M_{i,m-2}$, $M_{i,m-1}$, M_{im}). In each case, if 0.9 × the central value is less than outer values, we pick out the central value as an ordinate for the baseflow line, $\mathbf{B}_{ij}$. $\mathbf{B}_{ij}$ is a vector. We should note that both 'i' and 'j' in $\mathbf{B}_{ij}$ are given by discontinuous integer numbers.
3. We estimate each daily value of $\mathbf{B}_i$ (i = 1, . . . , n) by linear interpolation between $\mathbf{B}_{ij}$.
4. If $B_i > Q_i$, we replace the value of B_i with Q_i.
5. We calculate the total volume of baseflow below the baseflow line.

$$VB = \sum_{i=1}^{n} B_i$$

[18] The procedures of calculation seem to be easy, but it is not so simple. Thus, we code some programs by Visual Basic on Microsoft Excel for the calculation. The codes are provided in Appendix C-1.

6. Likewise, we calculate the total volume of recorded daily flow under the recorded hydrograph.

$$VT = \sum_{i=1}^{n} Q_i$$

7. Then, we can calculate the BFI value by $BFI = \frac{VB}{VT}$.

Table 5-7. *Calculated BFI*

Gauging Station	Name of Station	Our Calculation		Gustard et al. (1992)		CEH (2003)
		BFI	Year	BFI	Number of Years	BFI
27001	Nidd at Hunsingore Weir	0.471	1991-2002	-	-	0.49
27002	Wharfe at Flint Mill Weir	0.386	1991-2003	0.376	10	0.39
27005	Nidd at Gouthwaite Reservoir	0.506	1991-2002*1	-	-	0.50
27007	Ure at Westwick Lock	0.400	1991-2002	0.395	32	0.40
27009	Ouse at Skelton	0.461	1991-2002*2	0.425	20	0.46
27034	Ure at Kilgram Bridge	0.325	1991-2004	0.321	23	0.33
27043	Wharfe at Addingham	0.340	1991-2002	0.325	16	0.33
27053	Nidd at Birstwith	0.448	1991-2002	-	-	0.46
27069	Wiske at Kirby Wiske	0.146	1991-2002*3	0.170	5	0.16
27071	Swale at Crakehill	0.439	1991-2002	0.504	10	0.47
27075	Bedale Beck at Leeming	0.383	1991-2002*4	0.428	4	0.41
27083	Foss at Huntington	0.445	1991-1995*5, 1997-2002	0.449	3	0.44
27085	Cod Beck at Dalton Bridge	0.487	1991-2002*6	-	-	0.48
27089	Wharfe at Tadcaster	0.410	1991-2002*7	-	-	0.41
27090	Swale at Catterick Bridge	0.377	1992-2004*8	-	-	0.40

*1: The data of Mar in 1999 are missing in GDF.
*2: The data of Jul - Dec in 1991 are missing in GDF.
*3: The data of 29 and 30 Dec in 2002 are missing in GDF.
*4: The data of 25 Feb - 15 Mar in 1999 are missing in GDF.
*5: The data of 12 Nov - 31 Dec in 1995 are missing in GDF.
*6: The data of 2 Aug - 31 Oct in 1992 and 16 - 17 Aug in 2002 are missing in GDF.
*7: The data of 1 Jan - 26 Jun in 1991 are missing in GDF.
*8: The data of 1 Jan - 16 Dec in 1992 are missing in GDF.

Table 5-7 shows the result of the calculation of BFI values in the gauging stations together with the data in Gustard et al. (1992) and CEH (2003). It turns out that there is no big difference between the calculated BFI values and those in Gustard et al. (1992) and CEH (2003). The calculated BFI values seem to be valid.

Finally, using the calculated BFI and the data on gauged monthly flow

(GMF) (m^3/s or cms) that is provided by National Water Flow archives (CEH [Centre for Ecology and Hydrology]), we can calculate constant monthly baseflow in each subbasin (gauging station). The procedures are: (1) calculate the average of gauged monthly flow in each month during 1991 - the most recent year that is available; and (2) multiply the average by the BFI we have obtained in order to derive constant monthly baseflow. Table 5-8 shows the results, which are inputs to constant monthly model of baseflow in subbasins in HEC-HMS.

Table 5-8. *Estimated Monthly Baseflow (m^3/s or cms)*

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
27001	6.60	6.58	4.57	3.68	1.93	2.07	1.12	1.44	2.00	4.08	5.82	7.00
27002	10.93	10.62	8.06	5.72	3.67	3.09	1.97	2.79	3.90	6.69	9.39	11.20
27005	2.46	2.64	1.68	1.30	0.76	0.76	0.39	0.48	0.65	1.65	2.12	2.63
27007	16.07	17.10	11.32	8.16	4.96	4.79	2.45	3.73	5.25	9.30	12.72	16.04
27009	43.10	43.21	29.97	23.44	12.56	12.83	6.32	8.67	12.49	22.10	34.13	43.00
27034	9.70	10.03	6.62	4.32	2.70	2.51	1.26	2.07	3.36	5.92	7.67	9.75
27043	7.68	7.85	5.73	3.93	2.43	2.28	1.46	2.16	3.27	5.31	6.70	8.02
27053	3.88	3.89	2.54	1.96	1.00	1.10	0.52	0.68	0.99	2.26	3.14	4.13
27069	1.35	1.45	0.60	0.66	0.18	0.33	0.10	0.25	0.24	0.51	1.01	1.42
27071	17.39	17.93	11.49	9.74	5.21	5.53	2.65	3.57	5.19	9.14	14.07	17.56
27075	1.89	2.41	1.03	0.86	0.52	0.62	0.23	0.39	0.49	1.01	1.50	2.08
27083*1	0.76	0.60	0.43	0.44	0.14	0.17	0.07	0.08	0.10	0.27	0.67	0.78
27085*2	1.45	1.27	0.93	1.10	0.42	0.41	0.21	0.29	0.32	0.60	1.17	1.45
27089*3	11.46	11.83	8.77	6.50	3.98	3.76	2.27	3.15	4.42	7.54	10.38	12.23
27090*4	8.84	8.56	5.24	3.92	2.86	2.51	1.35	2.13	3.12	5.49	6.01	7.91

NB. The duration of the data is the same as Table 5-7.
*1: The data of Nov in 1995 are missing in GMF.
*2: The data of Aug - Oct in 1992 are missing in GMF.
*3: The data of Jan - Jun in 1991 are missing in GMF.
*4: The data of Jan 1991 - Nov 1992 are missing in GMF.

5.3.5 *Calculation of Impervious*

The percentage of impervious land in each subbasin is a measured parameter, which reflects development of lands (land-use). This plays an important role in controlling the volume of runoff. The excessive development of lands tends to bring about flood events. The impervious (%) is defined as the percentage of the developed area in each subbasin. National River Flow Archive (NWA) provides spatial information on each subbasin, by which we can obtain the information on the percentage of the developed area. However, it includes some missing data, and a GIS formatted file of subbasins in River Ouse catchment is unavailable. Thus, we need a GIS map of subbasins and a GIS map of land-use. In addition, we definitely need the GIS map of subbasins for specifying other measured parameter values.

Therefore, let us derive a GIS map of subbasins by using ArcGIS.[19] We can derive a GIS map of subbasins in the River Ouse catchment from DEM (digital elevation model). One restriction is that we use a depressionless DEM to avoid the problem of pits or sinks. The summarised procedures are (see Figure 5-4):

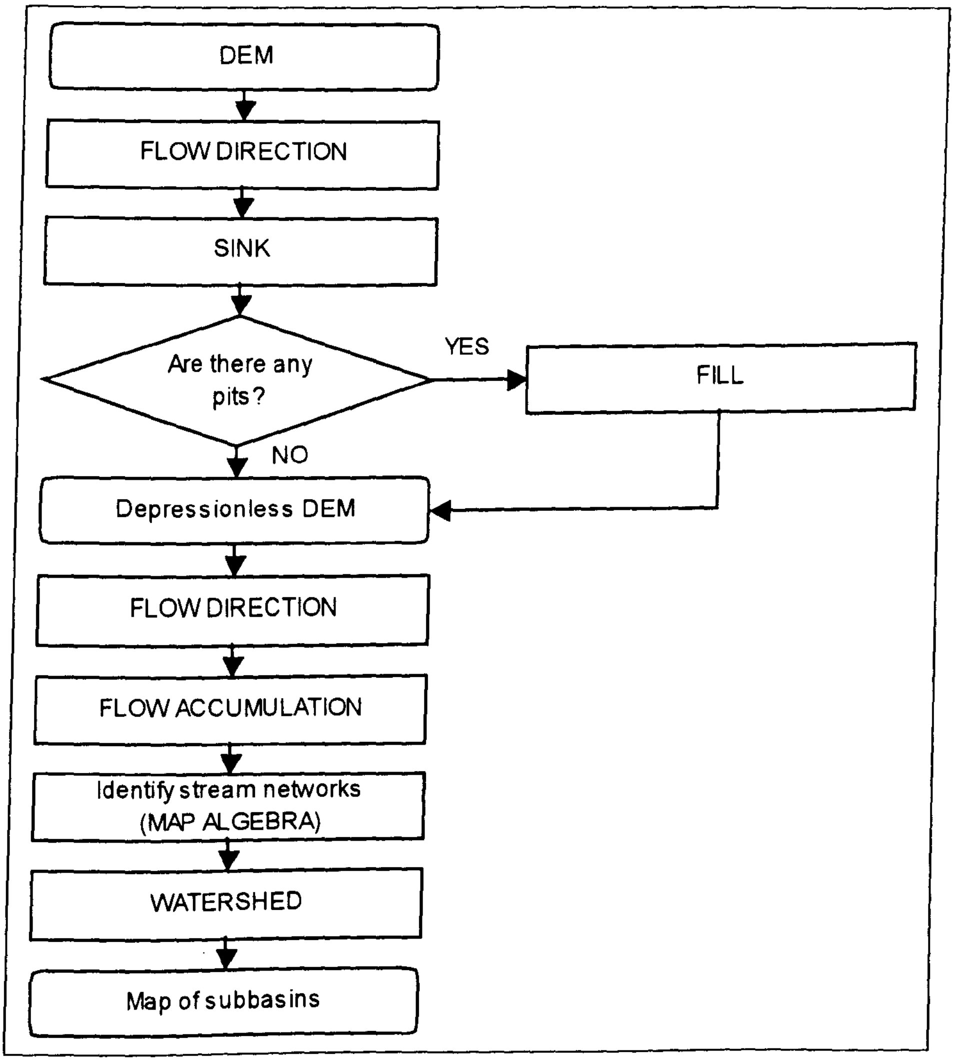
1. Obtain DEM in the River Ouse catchment.
2. Calculate flow direction.
3. Check the existence of pits or sinks.
4. Fill pits or sinks if we detect the existence of pits or sinks, and obtain the depressionless DEM.
5. Detect water flows, and then identify stream networks.
6. Adjust the location of gauging stations in order that they are on stream lines in the depressionless DEM.
7. Specify each subbasin.

Let us explain each step in the procedures. First, we derive the depressionless DEM. To begin with, we can obtain DEM (digital elevation model) in the River Ouse catchment from OS Land-Form PROFILE™ DTM [1:10,000] or OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap). The former gives DEM as 20m raster data while the latter gives DEM as 80m raster data. 20m raster data is more accurate than 80m raster data, but 80m raster data is sufficiently good here. Thus, we choose the latter data. The second step is to determine what cells flow into which cells, say the flow direction. In the third step, we have to check whether there are pits on DEM in the target area or not because we cannot derive correct water flows if there are. In more detail, if there are some

[19] We use ESRI® ArcMap™ 9.1.

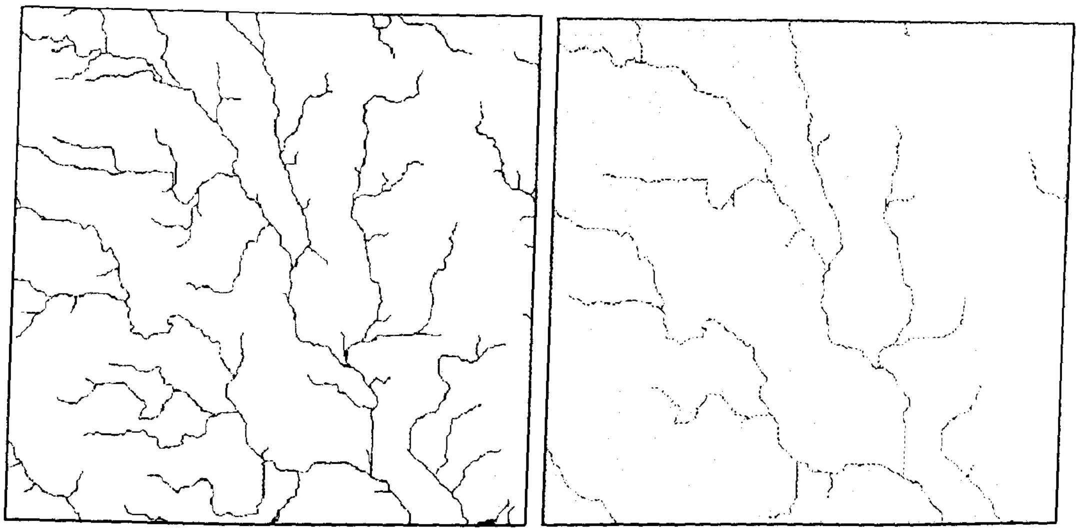
cells that are lower than all the surrounding cells, all water travelling into the cell will not travel out (ArcGIS Desktop Help). Having found pits in the target area we complete a fourth step by which we fill pits in order to obtain the depressionless DEM. Once we obtain the depressionless DEM, we can derive stream flow networks without the problem of pits. Then, based on the depressionless DEM, we re-calculate the flow direction.

Figure 5-4. *Procedures for GIS map of subbasins*



In a fifth step, we calculate accumulated flow, using the depressionless DEM. "The output from the Flow Accumulation function would then represent the amount of rain that would flow into each cell, assuming that all rain became runoff and there was no interceptions, evapotranspiration, or loss to groundwater" (ArcGIS Desktop Help). This provides cells of concentrated flow (a high accumulated flow) with large numbers. Therefore, Cells with a high-accumulated flow are used to identify stream flow networks. Using the output GIS map of flow accumulation, we can identify stream networks (Step 6). We can delineate stream networks by giving a threshold value to the output of flow accumulation. If we give a small value as a threshold, we can detect complicated rivers including small rivers and channels. On the contrary, if we give a larger value as a threshold, we can detect main rivers (see Figure 5-5).

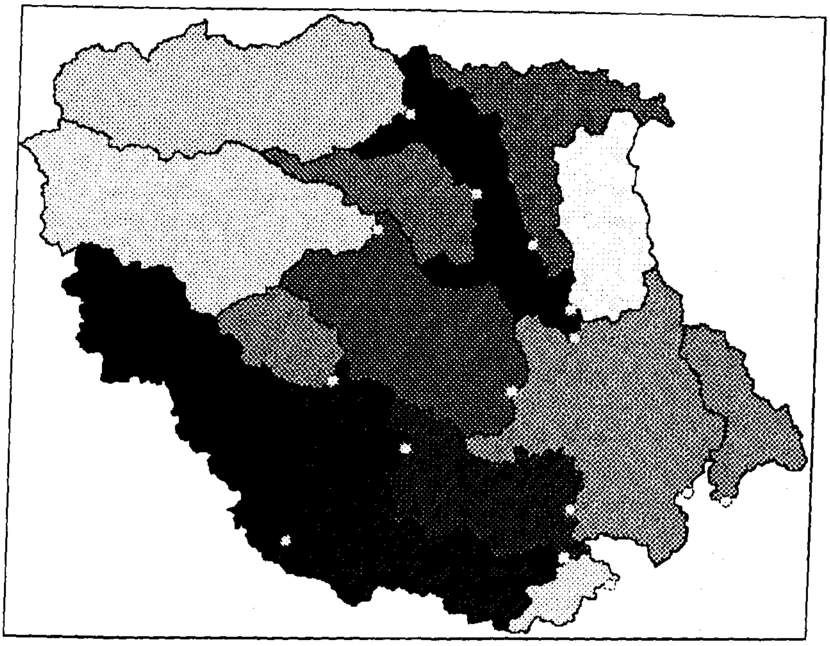
Figure 5-5. *Identifying stream flow networks*



NOTE: These maps are the same part of River Ouse catchment. The map on the left is given by a small threshold value (= 1000) while the one on the right is given by a large threshold value (=5000).
Source: We create these maps from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) by use of ArcGIS.

In the final two steps, we adjust the locations of gauging stations in order that they are on the nearest stream lines, and then specify subbasins linked with gauging stations as a GIS map. The information on the locations of gauging stations is given by the OS grid, but the unit is *km*. The locations are not so precise. In addition, we are using 80m raster DEM data. Thus, the gauging stations are not necessarily on stream lines. We need to adjust the locations of gauging stations manually. Based on the map of the adjusted locations of gauging stations and the map (output) of flow direction on the depressionless DEM, we can specify each subbasin on the GIS map. Figure 5-6 shows the output. The point features indicate the locations of gauging stations.

Figure 5-6. *Map of Subbasins*



Source: We create this map from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) by use of ArcGIS.

We can use the Land Cover Map of Great Britain 1990 (LCM 1990) in order to obtain the information on land-use.[20] The LCM 1990 is 25m raster data, and shows land-use patterns by 25-class system. Using the LCM 1990 and the GIS map of subbasins derived, we can calculate the impervious area in each subbasin by following the procedures:

1. Convert the raster data of subbasins into the feature (vector/polygon) data. Calculate the area of each subbasin.
2. Abstract developed areas from LCM 1990. Developed areas are given by the two categories: Suburban/Rural Development (category number : 20) and Continuous Urban (category number : 21).
3. Clip the developed areas in each subbasin, and convert the raster data into feature (vector/polygon) data. Calculate the area of developed lands in each subbasin.
4. Divide the area of developed lands by the area of subbasin in each subbasin to get the impervious area. Table 5-9 shows the results. The

[20] Land Cover Map of GB 1990 is provided by Centre for Ecology and Hydrology in the form of an academic license.

data are inputs to Initial/Constant rate model of loss rate in subbasins in HEC-HMS.

Table 5-9. *Impervious in each subbasin*

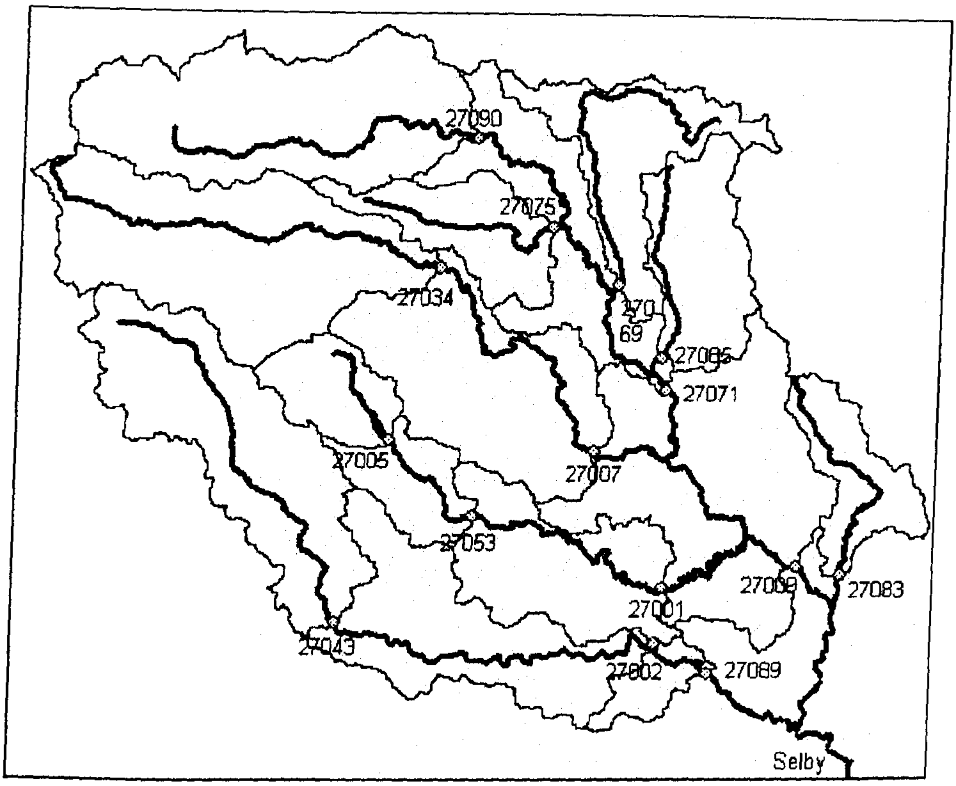
Subbasin	Name of Stations	% impervious
27001	Nidd at Hunsingore Weir	11.722
27002	Wharfe at Flint Mill Weir	6.028
27005	Nidd at Gouthwaite Reservoir	0.720
27007	Ure at Westwick Lock	4.043
27009	Ouse at Skelton	6.813
27034	Ure at Kilgram Bridge	1.042
27043	Wharfe at Addingham	1.043
27053	Nidd at Birstwith	1.421
27069	Wiske at Kirby Wiske	8.162
27071	Swale at Crakehill	9.019
27075	Bedale Beck at Leeming	5.684
27083	Foss at Huntington	6.681
27085	Cod Beck at Dalton Bridge	6.094
27089	Wharfe at Tadcaster	12.148
27090	Swale at Catterick Bridge	3.070

5.3.6 Calculation of River Length

The river length of each subbasin is a measured parameter, which is an input to Muskingum Cunge 8 point model of reach in HEC-HMS. There are no appropriate data on river lengths in that river lines are not distinguished by subbasins we derive. Then, we have to calculate them.

There is a dendritic river network in each subbasin, but we model a main river as a reach in each subbasin for simplification. To begin with, we have to define a main river in each subbasin. For this purpose, we use the map of stream flow networks derived in the previous section and a vector data of rivers from OS Strategi® [1:250,000] (EDINA Digimap). In the case that primary or only one secondary river flows through a subbasin, we clip the river lines in the subbasin. However, it is difficult to choose the river lines if two or more secondary rivers or only small rivers flow on a subbasin. In this case, we compare the river lines from the vector data with the map of stream flow networks derived from a high threshold value in order to determine a main river line in each subbasin. In so doing, we define and obtain the lines of main rivers in subbasins. Figure 5-7 shows the map.

Figure 5-7. *River networks in subbasins in the model*



Source: We create this map from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) and OS Strategi® [1:250,000] (EDINA Digimap) by use of ArcGIS.

Then, we can calculate the river length in each subbasin by following the procedures:

1. Clip the river lines in each subbasin, and convert the feature (vector) data into coverage data. This conversion is necessary because the information on reach length on the river lines that passes over the boundaries of subbasins is not correct (the information reflects the old data).
2. Open the attribute table of river lines, and sum the river lengths in each subbasin. We can obtain the data on river lengths in each subbasin. Table 5-10 shows the results.

Table 5-10. *River length in each subbasin*

Subbasin	Name of Stations	River Length (m)
27001	Nidd at Hunsingore Weir	31820.15
27002	Wharfe at Flint Mill Weir	44410.14
27005	Nidd at Gouthwaite Reservoir	12376.29
27007	Ure at Westwick Lock	37385.34
27009	Ouse at Skelton	31788.14
27034	Ure at Kilgram Bridge	54699.36
27043	Wharfe at Addingham	48433.24
27053	Nidd at Birstwith	14840.30
27069	Wiske at Kirby Wiske	42431.67
27071	Swale at Crakehill	49224.49
27075	Bedale Beck at Leeming	25263.05
27083	Foss at Huntington	29379.04
27085	Cod Beck at Dalton Bridge	22605.91
27089	Wharfe at Tadcaster	9226.85
27090	Swale at Catterick Bridge	41877.08

5.3.7 Calculation of Energy Slope

The energy slope of a river is a measured parameter that is needed for the Muskingum Cunge 8 point reach model in HEC-HMS. However, the data on it are not available. Thus, we have to calculate them by using GIS data such as the map of river lines and the DEM. The acquired data on energy slope might be rough because we are using 80m raster data, but the precision is enough good for our model analysis. The procedures are:

1. Using the determined river lines, specify the two grid cells of DEM on both edges of the river line in each subbasin.
2. Read the information on their heights.
3. Calculate the energy slope by using the information together with the data on river length we have derived. The results are shown by Table 5-11.

Table 5-11. *Energy slope of river in each subbasin*

Subbasin	Name of Stations	Energy Slope (m/m)
27001	Nidd at Hunsingore Weir	0.00153990
27002	Wharfe at Flint Mill Weir	0.00137356
27005	Nidd at Gouthwaite Reservoir	0.01115035
27007	Ure at Westwick Lock	0.00192589
27009	Ouse at Skelton	0.00028312
27034	Ure at Kilgram Bridge	0.00998184
27043	Wharfe at Addingham	0.00538886
27053	Nidd at Birstwith	0.00417781
27069	Wiske at Kirby Wiske	0.00143761
27071	Swale at Crakehill	0.00093449
27075	Bedale Beck at Leeming	0.00823337
27083	Foss at Huntington	0.00405051
27085	Cod Beck at Dalton Bridge	0.00225605
27089	Wharfe at Tadcaster	0.00151731
27090	Swale at Catterick Bridge	0.00487140

5.3.8 Derivation of Manning's N

Manning's N (roughness coefficient) is a measured parameter that plays an important role in reflecting floodplain management in the model. Manning's N represents the resistance to water flows in rivers (channels) and overbank areas (floodplains) (Barnes, 1967; Arcement and Schneider, 2000; Bedient and Huber, 2002). Manning's N for channels is determined by morphological conditions, vegetation and soil conditions of the bottom of rivers.[21] Manning's N for overbank areas is determined mainly by land-use (vegetation).[22] Therefore, land-use patterns on floodplains should be concentrated on the Manning's N for overbank areas. The change in the control variable of floodplain development influences the expected cost of flood risk through the Manning's N value. Thus, this parameter value is significant. However, it is difficult to calculate and

[21] Based on Arcement and Schneider (2000) and Coon (1998), the base Manning's N value for channels should be modified by cross-section irregularities, channel variation, obstruction, vegetation and degree of meandering. However, Manning's N for channels is often given by types of channel shown by Tables 5-12, 5-13 and 5-14.

[22] Based on Arcement and Schneider (2000), Manning's N for overbank areas should be additively calculated by surface irregularities, obstruction and vegetation (land-use). However, Manning's N for overbank areas is often given by categories of land-use (vegetation) as we show in Tables 5-16, 5-17 and 5-18.

determine the Manning's N partly because there are ranges on the value even in the same category of physical characteristics, partly because the linkage of 25 classes of land-use categories with the categories given with Manning's N is ambiguous and partly because the information on physical characteristics of rivers and floodplains is limited. Hence, using available data, we need some assumptions to derive the Manning's N values in subbasins.

To begin with, we discuss the specification of Manning's N for channels in our hydrological model. In order to derive the exact roughness coefficients for rivers (channels) in the model, we need precise information on cross-section irregularities, channel variation, obstruction, vegetation and degree of meandering. Unfortunately, we have no such information on them. Then, we have to assume plausible values of Manning's roughness coefficients based on several tables on Manning's roughness coefficients that are often used for channels. Fortunately, the value for channels lies in a small range. Thus, even if we roughly assume the values, they are not so different from real ones. For example, Barnes (1967) shows that the range of Manning's roughness coefficient for channels is between 0.024 and 0.075. Let us check some tables on it. Table 5-12, 5-13 and 5-14 show Manning's N based on types of streams from various sources. Table 5-12 provides simple data, Table 5-13 provides more detailed data, and Table 5-14 provides complicated data. However, there seems to be no big difference between the tables. If we have sufficient information on the targeted rivers, we can use the detailed data. As we mention, we have no relevant data on rivers, but we can estimate the width of rivers by GIS data. By obtaining the information on the width of rivers, we can refer to the part of the data in Table 5-14 in order to assume plausible values of Manning's N for channels.

Table 5-12. *Manning's N for channels (1)*

Type of Stream	Manning's N
Natural Streams	
Clean and straight	0.030
Major rivers	0.035
Sluggish with deep pools	0.040

Source: LMNO Engineering, Research, and Software, Ltd. (www.lmnoeng.com/manningn.htm)

Table 5-13. *Manning's N for channels (2)*

Types of Channel	Minimum	Normal	Maximum
Natural Streams			
Clean, straight, full stage	0.025	0.030	0.033
Clean, winding, some pools and shoals	0.033	0.040	0.045
Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
Mountain stream steepbanks; gravel and cobbles	0.030	0.040	0.050
Mountain stream steepbanks: cobbles with large boulders	0.040	0.050	0.070

Source: Bedient and Huber (2002), Table 7.1, p. 460.

Table 5-14. *Manning's N for channels (3)*

Type of Channel and Description	Minimum	Normal	Maximum
A. Minor Stream (top width at flood stage less than 100 ft)			
1. Streams on plain:			
a. Clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.033
b. Same as above, but more stones and weeds	0.030	0.035	0.040
c. Clean, winding, some pools and shoals	0.033	0.040	0.045
d. Same as above, but some weeds and stones	0.035	0.045	0.050
e. Same as above, lower stages more ineffective slopes and sections	0.040	0.048	0.055
f. Same as type d, but more stones	0.045	0.050	0.060
g. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
h. Very weedy reaches, deep pools or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150
2. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages:			
a. Bottom: gravels, cobbles, and few boulders	0.030	0.040	0.050
b. Bottom: cobbles and large boulders	0.040	0.050	0.070
B. Major streams (top width at flood stage greater than 100 ft). The n value is less than that for minor streams of similar description because banks offer less effective resistance			
1. Regular section with no boulders or brush	0.025	-	0.050
2. Irregular and rough section	0.035	-	0.100

Source: Coon (1998), Table 2, p. 17.

We can use the vector data of OS Land-Line.Plus® [urban 1:1,250, rural 1:2,500 and moorland 1:10,000] (EDINA Digimap) for estimating the width of rivers. As we will discuss cross sections of rivers in the next section, the width of river is variable even locally. It is very difficult and subtle to determine the width as a representative in each subbasin. Then, we try to measure the width of river at the middle point of the river line in each subbasin on the GIS map by using a measuring tool of ArcGIS.[23] The measured values are treated as the representatives in subbasins. Based on the measured width, we divide two categories by following Table 5-14. Basically, the Manning's N value for large rivers is less than that for small rivers because banks of large rivers offer less effective resistance (Coon, 1998). We assume the value 0.035 as Manning's N for channel if the width is larger than 100 *feet* (30.48 *m*). We assume 0.04 if the width is smaller than 100 *feet* (30.48 *m*). Table 5-15 shows the results.

Table 5-15. *Manning's N for river in each subbasin*

Subbasin	Name of Stations	Width of River (m)	Manning's N for Channel
27001	Nidd at Hunsingore Weir	21.41	0.040
27002	Wharfe at Flint Mill Weir	28.69	0.040
27005	Nidd at Gouthwaite Reservoir	12.63	0.040
27007	Ure at Westwick Lock	39.15	0.035
27009	Ouse at Skelton	41.91	0.035
27034	Ure at Kilgram Bridge	18.59	0.040
27043	Wharfe at Addingham	23.25	0.040
27053	Nidd at Birstwith	22.93	0.040
27069	Wiske at Kirby Wiske	4.30	0.040
27071	Swale at Crakehill	22.01	0.040
27075	Bedale Beck at Leeming	6.06	0.040
27083	Foss at Huntington	4.40	0.040
27085	Cod Beck at Dalton Bridge	9.11	0.040
27089	Wharfe at Tadcaster	32.06	0.035
27090	Swale at Catterick Bridge	24.99	0.040

Next, let us calculate Manning's N for overbank areas in each subbasin, following the procedures:

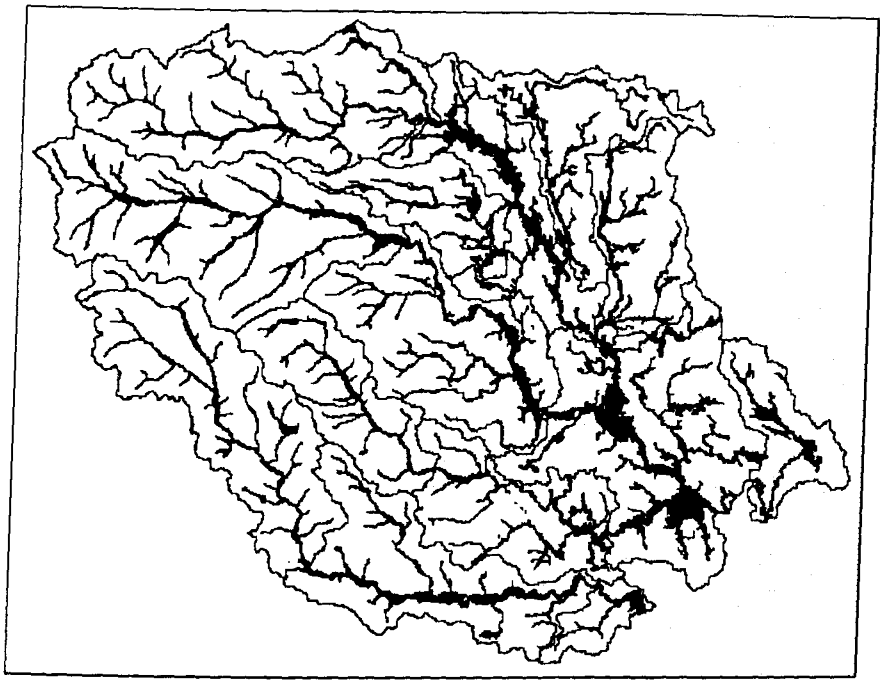
1. Calculate the areas and percentages of each land-use category on floodplains in each subbasin by using relevant GIS data and ArcGIS.
2. Link the categories of land-use provided by tables of Manning's N with the 25 categories of LCM 1990.
3. Calculate Manning's N values in each subbasin.

In the first step, we need a GIS map of floodplains, the map of subbasins we have derived and LCM 1990. We can use Indicative Floodplain Map 2001

[23] The locations of the measurement sites are given in Appendix C-2.

[1:10,000] (© Environment Agency).[24] The GIS map provides the location of 1% floodplains as mentioned in the definition of floodplains in Chapter 1. Figure 5-8 shows floodplains in River Ouse catchment.

Figure 5-8. *Floodplain Map*



Source: Map created from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) and Indicative Floodplain Map 2001 [1:10,000] (© Environment Agency) by use of ArcGIS.

To begin with, we clip the raster data map of LCM 1990 in the area of floodplains of each subbasin. We convert the raster data into the feature (vector/polygon) data, then calculate each area of the 25 categories to obtain their percentages.[25] There is an important assumption that we should notice. Although we define a main river line in each subbasin, we calculate the areas of categories on the basis of the whole floodplains along all the streams in each subbasin. The reason is that it is too difficult to separate the floodplains along the defined main river. Thus, we assume that land-use patterns are even within the floodplains in each subbasin. Related to this, we have another important thing to notice. We distinguish between developed and undeveloped (natural) floodplains, but we do not distinguish between different types of undeveloped floodplains although we

[24] The GIS data is provided by Environment Agency, thanks to the staffs' (York and Leeds offices) special consideration.
[25] The results of calculated areas from GIS are given in Appendix C-3.

consider them in the process of calculation of Manning's N. In reality, what type of natural floodplains (e.g. forests, turf area, bare ground. etc.) affects the flood mitigation function of floodplains. However, if we were to distinguish between them, we would have to consider different incentive mechanism for each area. Thus, for tractability, we abstract from differences in the type of natural floodplains.

Next, we check the table of Manning's N for overbank areas in order to link the categories of LCM 1990 with those of the Manning's N table. Table 5-16, 5-17 and 5-18 show Manning's N values in the categories of ground cover (land-use). These three tables seem to be consistent, although some small differences are observed. Thus, there is no serious problem about which table we should refer to, but Table 5-18 is much more elaborate than the others. We mainly refer to Table 5-18. It is difficult to link the categories between LCM 1990 and Manning's N table, but we need to do that.

Table 5-16. *Manning's N for overbank areas (1)*

Ground Cover	Manning's N
Smooth asphalt	0.012
Asphalt of concrete paving	0.014
Packed clay	0.030
Light turf	0.200
Dense turf	0.350
Dense shrubbery and forest litter	0.400

Source: Bedient and Huber (2002), p. 277, Table 4-2. [Original source: Crawford and Linsley (1966)]

Table 5-17. *Manning's N for overbank areas (2)*

Ground Cover	Manning's N	Range
Concrete or asphalt	0.011	0.01-0.013
Bare sand	0.01	0.01-0.016
Graveled surface	0.02	0.012-0.03
Bare clay-loam (eroded)	0.02	0.012-0.033
Range (natural)	0.13	0.01-0.32
Bluegrass sod	0.45	0.39-0.63
Short-grass prairie	0.15	0.10-0.20
Bermuda grass	0.41	0.30-0.48

Source: Bedient and Huber (2002), p. 277, Table 4-2. [Original source: Engman (1986)]

Table 5-18. *Manning's N for overbank areas (3)*

Cover and Treatment	Residue Rate (ton/acre)	Value recommended	Range
Concrete or asphalt		0.011	0.01 - 0.013
Bare sand		0.010	0.01 - 0.016
Graveled surface		0.020	0.012 - 0.03
Bare clay-loam (eroded)		0.020	0.012 - 0.033
Fallow - no residue		0.050	0.006 - 0.16
Chisel plow	1/4	0.070	0.006 - 0.17
	1/4 - 1	0.180	0.07 - 0.34
	1 -	0.300	0.19 - 0.47
	3	0.400	0.34 - 0.46
Disk/harrow	1/4	0.080	0.008 - 0.41
	1/4 - 1	0.160	0.1 - 0.41
	1 - 3	0.250	0.14 - 0.53
	3	0.300	-
No - till	1/4	0.040	0.03 - 0.07
	1/4 - 1	0.070	0.01 - 0.13
	1 - 3	0.300	0.16 - 0.47
Moldboard Plow (Fall)		0.060	0.02 - 0.1
Coulter		0.100	0.05 - 0.13
Range (natural)		0.130	0.01 - 0.32
Range (clipped)		0.100	0.02 - 0.24
Grass (bluegrass sod)		0.450	0.39 - 0.63
Short grass prairie		0.150	0.1 - 0.2
Dense grass		0.240	0.17 - 0.3
Bermuda grass		0.410	0.3 - 0.48
Woods-Light underbrush		0.400	-
Woods-Dense underbrush		0.800	-

Source: Thomas (1986), Table 1, p. 6.

Table 5-19 indicates the linkage of land-use categories between the tables. LCM 1990 provides 17 land cover categories related to the 25 target classes. If we cannot find the same or similar category in the 25 target classes, we put the same Manning's N value within the same category in the 17 land cover categories. In addition, we assume the value of Manning's N for a few categories of land use that are not included in the tables of Manning's N. For example, we put the Manning's N value of deep grass for the category of bracken.

Table 5-19. *Linkages of land-use categories between tables*

Category of LCM 1990			N	Bedient and Huber (2000)	Thomas (1986
Rough Pasture, Dune Grass, Grass Moor	5	Grass Heath	0.150	Short-grass Prairie	Short Grass Prairie
	9	Moorland Grass	0.150	Short-grass Prairie	Short Grass Prairie
Pasture, Meadow, Amenity Grass	6	Mown, Grazed Turf	0.200	Light Turf	
	7	Meadow, Verge, Semi-natural	0.200	Light Turf	
Marsh, Rough Grass	19	Ruderal Weed	0.400		
	23	Felled Forest	0.400	Dense Shurbbery and Forest Litter	
	8	Rough, Marsh Grass	0.400	Bermuda Grass Bluegrass Sod	Bermuda Grass Bluegrass Sod
Grass Shrub Heath	25	Open Shrub Heath	0.350		
	10	Open Shrub Moor	0.350	Dense Turf	Dense Grass
Shrub Heath	13	Dense Shrub Heath	0.400	Dense Shurbbery and Forest Litter	
	11	Dense Shrub Moor	0.400		
Bracken	12	Bracken	0.400		
Deciduous, Mixed Wood	14	Scrub, Orchard	0.400		Woods-Light
	15	Deciduous Woodland	0.600		Woods-Dense
Coniferous, Evergreen Woodland	16	Coniferous Woodland	0.600		Woods-Dense
Bog (Herbaceous)	24	Lowland Bog	0.800		
	17	Upland Bog	0.800		
Tilled (Arable Crops)	18	Tilled Land	0.250		Chisel Plow
Suburban, Rural Development	20	Suburban, Rural Development	0.011	Concrete or Asphalt	Concrete or asphalt
Urban Development	21	Continuous Urban	0.011	Concrete or Asphalt	Concrete or asphalt
Inland Bare Ground	22	Inland Bare Ground	0.020	Bare Sand Bare Clay-loam	Bare Sand Graveled Surface Bare Clay-loam

Finally, we can calculate Manning's N values for overbank areas in each subbasin, multiplying the Manning's N allocated to the categories by percentages of areas of land-cover categories.[26] Table 5-20 shows the results. These values are treated as initial values in the model, which are also the basis for calibrating fitted parameter values. We again should note that Manning's N values change with the

[26] In HEC-HMS, Muskingum Cunge 8 point model can distinguish between the right and left overbank areas, in which we can put different values into the Manning's N values of the right and left overbank areas. However, we do not distinguish between them. We provide those of the right and left overbank areas with the same value.

change in control variables over time.

Table 5-20. *Manning's N (initial condition)*

Subbasin	Name of Stations	N
27001	Nidd at Hunsingore Weir	0.234
27002	Wharfe at Flint Mill Weir	0.249
27005	Nidd at Gouthwaite Reservoir	0.248
27007	Ure at Westwick Lock	0.263
27009	Ouse at Skelton	0.231
27034	Ure at Kilgram Bridge	0.253
27043	Wharfe at Addingham	0.241
27053	Nidd at Birstwith	0.250
27069	Wiske at Kirby Wiske	0.217
27071	Swale at Crakehill	0.228
27075	Bedale Beck at Leeming	0.227
27083	Foss at Huntington	0.213
27085	Cod Beck at Dalton Bridge	0.239
27089	Wharfe at Tadcaster	0.259
27090	Swale at Catterick Bridge	0.244

5.3.9 Setting of Cross Section Coordinates

Cross section coordinates are necessary for Muskingum Cunge 8 point model of each river in subbasins. They are used for calculating water volume of flows in the hydraulic model. In the model, we need to indicate a cross sectional form of river as 8-point-coordinates.

However, obtaining the data of cross section coordinates is not straightforward. First, the cross-sectional form is variable even locally. The cross-sectional form of natural channels (rivers) is characteristically irregular in outline and locally variable, and width and depth do not uniquely define cross-sectional shapes (Knighton, 1998). Thus, hydrologists choose several points to measure cross-sections of rivers including width, depth and side slopes by dividing a target river into short river lines. Even if we divide a target river into short river lines, the problem remains although the severity of the problem might be alleviated.[27] Nonetheless, we have another problem in this case. The model will become overcomplicated, which may be inappropriate for policy analyses. Thus, we maintain the number of rivers in the model, and choose the appropriate (the best one out of possible choices) data as a representative. Second, we have no data on cross sections of rivers except for a part of River Ouse. Hence, we try to estimate cross section coordinates by using available data including GIS data, together with empirical and theoretical models of cross-sectional forms, and some assumptions. Since we cannot know the relevant shape of cross section, we determine a symmetrical cross sectional form as in the Muskingum Cunge standard model.

Therefore, we set some assumptions for estimating cross section coordinates of rivers. First, we assume a symmetrical trapezoidal shape as in the Muskingum Cunge standard model on the ground that the figures of cross sections of a part of River Ouse (78 sites between Skelton gauging station and Bishopthorpe Bridge) show various 'trapezoidal' shapes.[28] Generally, cross sections should cover the entire floodplain and should be perpendicular to the main flow line (Bedient and Huber, 2002). Figure 5-9 denotes 8-point cross section coordinates with an abstract air view of the river. The station points 3 and 6 are treated as left and right bank stations respectively. The station points 2 and 7 cannot be measured because the degree of precision of DEM (even 20m raster) is not enough high to measure locally.

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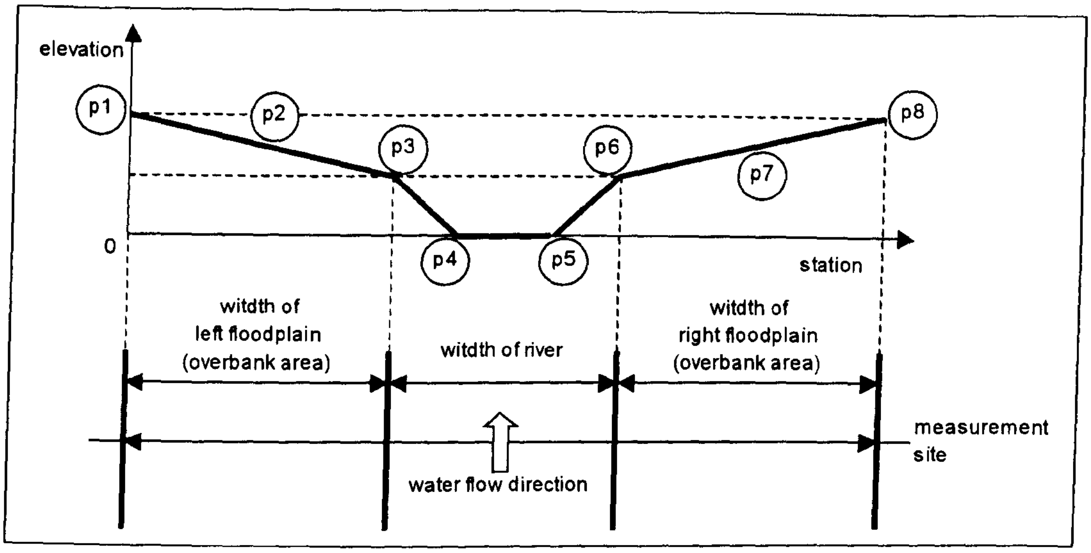
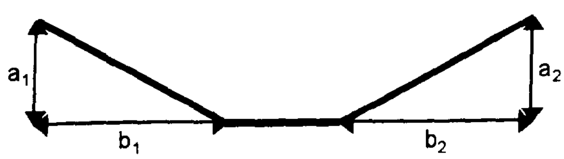
[27] Apart from the severity of the problem, hydrologists choose data on cross-sections as a representative for a river (channel) in a hydrological model. Fundamentally, we do the same thing.
[28] The data of cross sections is provided by Environment Agency (2003).

Figure 5-9. *8-point cross section coordinates*



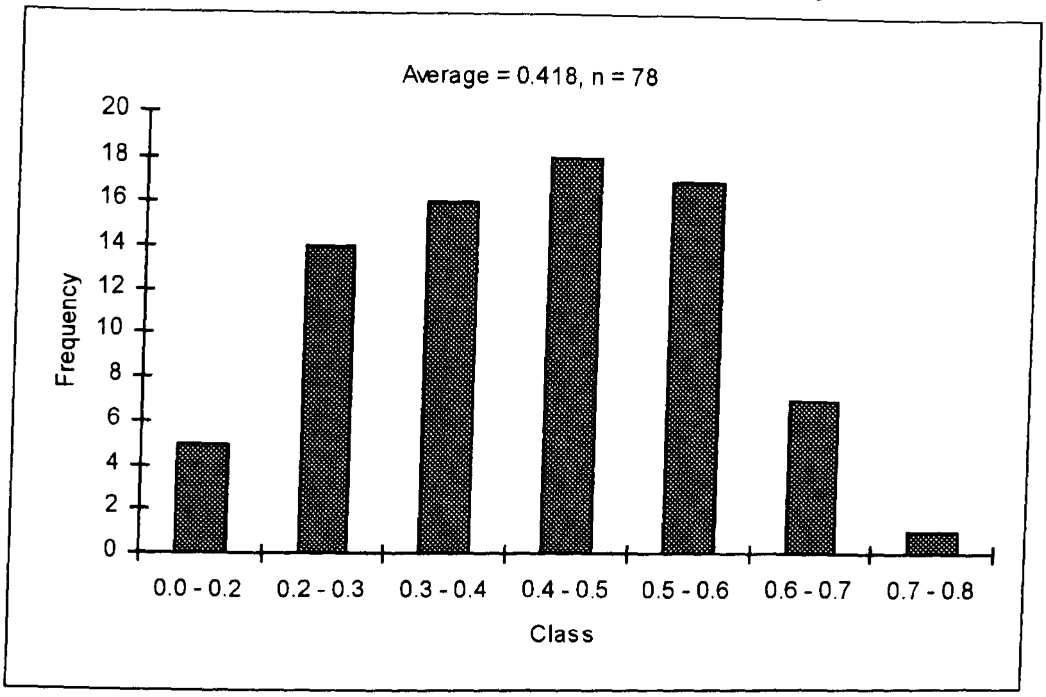
Second, we have to assume the angle of the slopes of both river banks in order to complete 8-point cross section coordinates. However, we have not found an appropriate theoretical or empirical basis for this. Base on data on cross-sections of a part of River Ouse (Environment Agency, 2003), we define the angle of the river banks at one site as $\left(\frac{a_1}{b_1}+\frac{a_2}{b_2}\right)\Big/2$ in Figure 5-10. We can calculate the values from the data of cross sections of a part of River Ouse. Figure 5-11 shows the results as a histogram. The number of data is 78. We set seven classes of intervals in the histogram.[29] Thus, we use the average, 0.418, as the value of angle, considering the shape of the histogram.

Figure 5-10. *Angle of river banks' slopes*



[29] Panofsky and Brier (1968) suggest that we can use $k = 5\log_{10} n$ (*k*: number of classes; *n*: number of data) for determining the number of class intervals (qtd. in Bedient and Huber, 2002). It provides $k = 9.46$. An alternative index (Sturges, H.A.) is $k = 1 + 3.32\log_{10} n$ (Miyakawa, 1991). It provides $k = 7.28$. However, these are not the ultimate rule and are not necessarily based on scientific grounds.

Figure 5-11. *Angles of river banks in the case of a part of River Ouse*



Source: We calculate and arrange from the data of Environment Agency (2003).

Third, we need to estimate the depth of the rivers in order to complete the 8-point cross section coordinates. We estimate the depth based on the measured width of rivers from GIS data, using a model of cross sectional form of natural open rivers. We use the technique of hydraulic geometry, which assumes that discharge (here, bankfull discharge) is the dominant independent variable and that dependent variables are related to it in the form of simple power functions (Knighton, 1998).

$$w = aQ^{\alpha},\ d = cQ^{\beta},\ v = eQ^{\gamma},\ s = gQ^{\delta},\ n = hQ^{\varepsilon},\ ff = kQ^{\theta},\ Q_{susp} = mQ^{\lambda}$$

where Q = discharge volume; w = width of river; d = mean depth of river; v = mean velocity; s = slope; n = Manning's N (resistance); ff = Darcy-Weisbach ff (resistance); Q_{susp} = suspended sediment load; and a, c, e, g, h, k, m, α, β, γ, δ, ε, θ and λ are parameters.

We should note some assumptions of the model above. First, the empirical estimates are equilibrium values. In other words, they are based on the situation in which cross sectional form is maintained by a local balance between erosion and deposition (Knighton, 1998). We consider the problem in the model within instantaneous time (< 10^{-1} years) for calibration and within short timescale (10^1-10^2 years) for policy analyses. We do not need to consider the change in the structure of cross sectional forms. Second, the empirical estimates are valid for natural open channels, streams or rivers. In this respect, there is no big problem in the River Ouse catchment. Third, the choice of an appropriate discharge volume is a crucial issue because exponential values that are empirically calibrated are not independent of the selected discharge (Knighton, 1998). "Bankfull discharge (Q_b) is an obvious candidate but it cannot always be defined and is not necessarily of constant frequency (Williams, 1978)" (qtd. in Knighton, 1998). At this point, we use the results of Hey and Thorne (1986), assuming that bankfull discharge is well defined and constantly frequent in the River Ouse and tributaries.[30] Hey and Thorne (1986) have done an empirical analysis of the hydraulic geometry for gravel-bed rivers (62 sites) in the UK. The resultant calibrated parameter values in the hydraulic geometry equations are:

$$w = 3.67Q_b^{0.45} \quad (5\text{-}9)$$

$$d = 0.33Q_b^{0.35} \quad (5\text{-}10)$$

$$v = 0.83Q_b^{0.20} \quad (5\text{-}11)$$

$$s = 0.008Q_b^{-0.20}$$

We can calculate the depth of rivers in the subbasins from the equations above and the measured width of rivers. The calculated depth of rivers is shown in Table 5-21.

Finally, we measure the difference in height at station points 1, 3, 6 and 8 by use of OS Land-Form PROFILE™ DTM [1:10,000] (EDINA Digimap) which provides DEM as 20m raster data) to determine the elevation and station of the points 1, 2, 3, 6, 7 and 8. We use the average value of the right and left overbank areas (floodplains) for specifying these coordinates. Even in the 20m raster data, we are faced with the problem of the data precision. Thus, we calculate the station and elevation proportionally based on the data on the station points. In addition, we measure the width of floodplains in the same way as we measure the width of

[30] In the process of calculation here, we do not need to distinguish among various types of discharge indexes.

rivers.[31] [32] [33]

Table 5-21. *Depth of rivers*

Subbasin	Name of Stations	Depth of River (m)	Width of River (m)
27001	Nidd at Hunsingore Weir	1.30	21.41
27002	Wharfe at Flint Mill Weir	1.63	28.69
27005	Nidd at Gouthwaite Reservoir	0.86	12.63
27007	Ure at Westwick Lock	2.08	39.15
27009	Ouse at Skelton	2.19	41.91
27034	Ure at Kilgram Bridge	1.16	18.59
27043	Wharfe at Addingham	1.38	23.25
27053	Nidd at Birstwith	1.37	22.93
27069	Wiske at Kirby Wiske	0.37	4.30
27071	Swale at Crakehill	1.33	22.01
27075	Bedale Beck at Leeming	0.49	6.06
27083	Foss at Huntington	0.38	4.40
27085	Cod Beck at Dalton Bridge	0.67	9.11
27089	Wharfe at Tadcaster	1.78	32.06
27090	Swale at Catterick Bridge	1.47	24.99

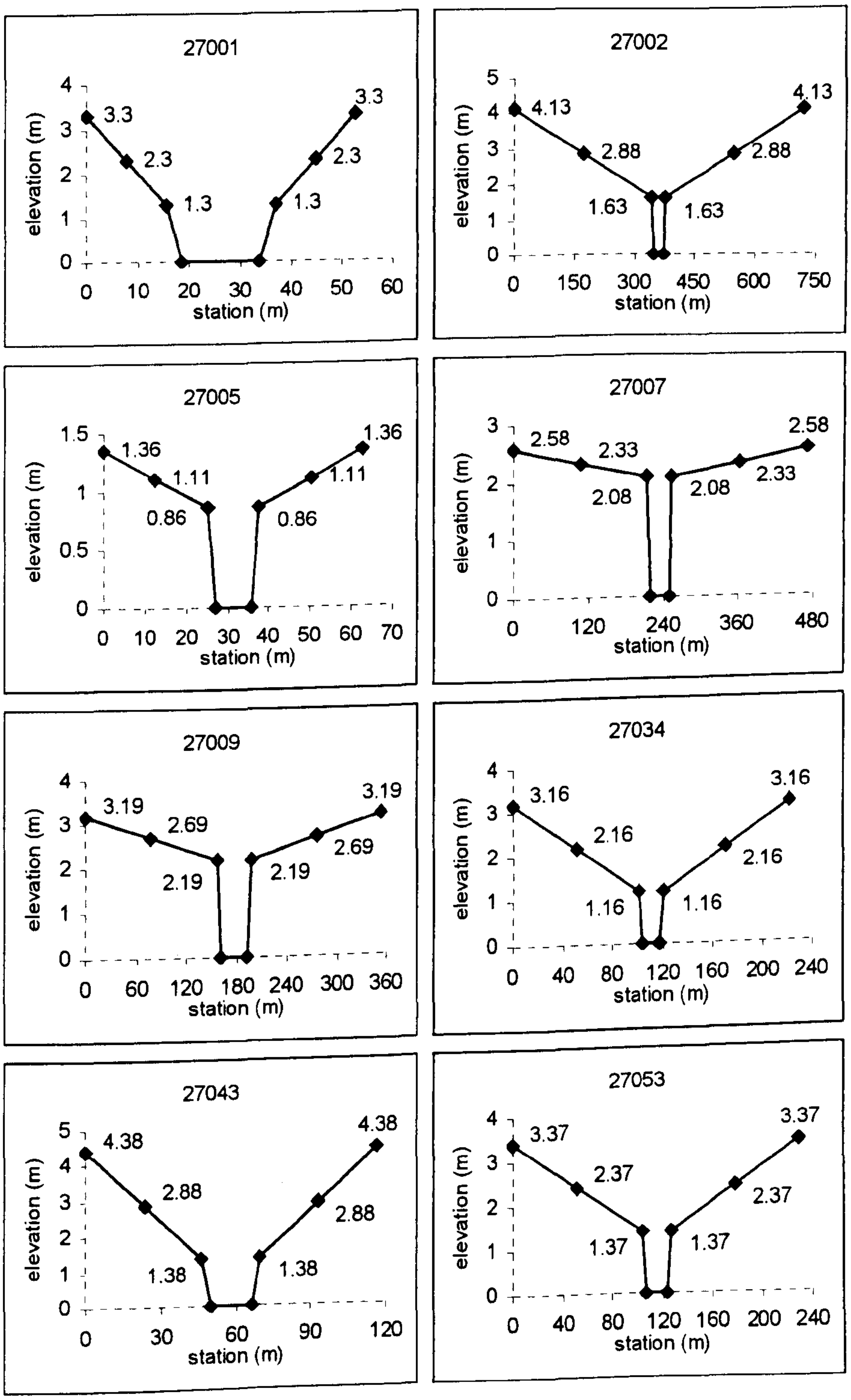
We can calculate and determine the 8-point cross section coordinates in the rivers in the subbasins respectively, using the data derived above. The results of the 8-point cross section coordinates are indicated in Figure 5-12. In addition, we also calculate the cross section coordinates at the points of York and Selby urban area respectively. These areas are not included in the subbasins that we have determined based on the hydrological gauging stations. They are not used in the hydrological model because we have no hydrological data (no gauging stations) in these areas, but they will be used in the calculation of the expected cost of flood risk later. Both urban areas are crucial for evaluating the expected cost of flood risk.

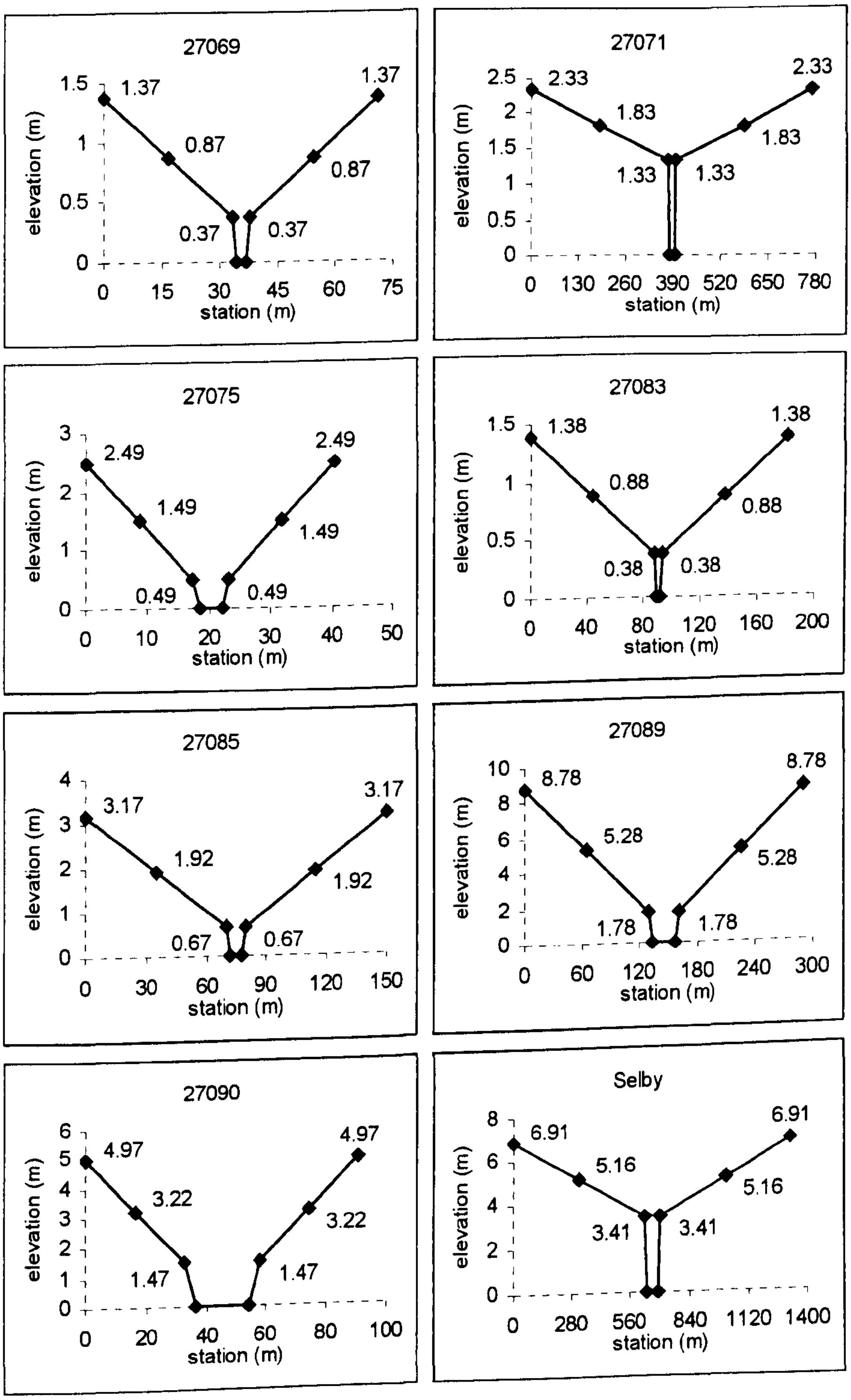
[31] As we mentioned, we try to measure the width of river at the middle point of the river line in each subbasin. To be more precise, we try to choose the measurement site around the middle line in each subbasin in order to avoid extremely narrow and wide floodplains.

[32] We use OS Land-Line.Plus® [urban 1:1,250, rural 1:2,500 and moorland 1:10,000] (EDINA Digimap) and Indicative Floodplain Map 2001 [1:10,000] (© Environment Agency).

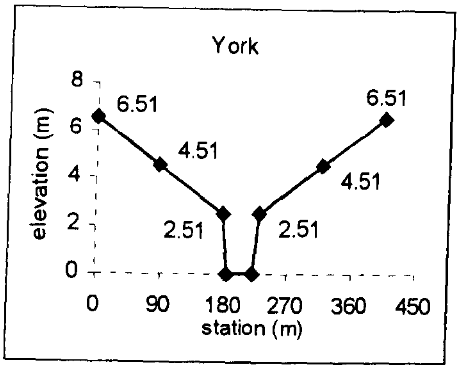
[33] The data measured in this respect is shown in Appendix C-4.

Figure 5-12. *8-point cross section coordinates of rivers*





27069
elevation (m)
station (m)
1.37 0.87 0.37 0.37 0.87 1.37
0 0.5 1 1.5
0 15 30 45 60 75
27071
elevation (m)
station (m)
2.33 1.83 1.33 1.33 1.83 2.33
0 0.5 1 1.5 2 2.5
0 130 260 390 520 650 780
27075
elevation (m)
station (m)
2.49 1.49 0.49 0.49 1.49 2.49
0 1 2 3
0 10 20 30 40 50
27083
elevation (m)
station (m)
1.38 0.88 0.38 0.38 0.88 1.38
0 0.5 1 1.5
0 40 80 120 160 200
27085
elevation (m)
station (m)
3.17 1.92 0.67 0.67 1.92 3.17
0 1 2 3 4
0 30 60 90 120 150
27089
elevation (m)
station (m)
8.78 5.28 1.78 1.78 5.28 8.78
0 2 4 6 8 10
0 60 120 180 240 300
27090
elevation (m)
station (m)
4.97 3.22 1.47 1.47 3.22 4.97
0 1 2 3 4 5 6
0 20 40 60 80 100
Selby
elevation (m)
station (m)
6.91 5.16 3.41 3.41 5.16 6.91
0 2 4 6 8
0 280 560 840 1120 1400



5.3.10 Calibration of Parameters

The three fitted parameter values (initial loss, constant rate and SCS lag) are calibrated by using observed data such as precipitation and discharge flow. Using the optimisation (calibration) tool in HEC-HMS, we can calibrate the fitted parameter values in order that the data of discharge flows that the model in HEC-HMS produces based on the input data of precipitation can be fitted with the observed data of discharge flows as much as possible.

To begin with, let us describe the observed data used for the calibration. The shorter the time interval of the observed data is, the better the precision of the calibration is. If we could obtain the results of frequency analysis of precipitation on an event basis, hourly data should be ideal for analyses. However, they are not available. We can obtain daily gauged (discharge) flow data and monthly catchment rainfall (precipitation) data on the gauging stations during 1990 - the most recent available year from National Water Archive (NWA) (or National River Flow Archive) administrated by CEH.[34] According to NWA, daily gauged flows are calculated by the conversion of the record of stage, or water level, using a stage-discharge relation, in which stage is measured and recorded over time by instruments usually actuated by a float in a stilling well. The precipitation data is derived from a one kilometre square grid of rainfall values generated from all

[34] The information on the availability of the observed data is given in Appendix C-5.

available daily and monthly rainfall data, which are provided by individual rain gauging stations administrated by the Met Office.[35]

About the observed data, there are a few problems. First, there are some missing data as is often the case. Since we need a complete data set for calibration, this excludes time periods that contain missing data. This is one of the constraints. Second, the time interval is set to daily because it is the minimum within the available data. However, we have only monthly data on precipitation in the unit of catchment. We need to convert the monthly data into daily data. For the conversion of data, we can use Met Office - UK Land Surface Stations data (1900-present) that is obtainable from the British Atmospheric Data Centre (BADC). The data provides daily data on precipitation based on rain gauging stations. The problem with this is that data are missing. The locations of hydrological gauging stations do not coincide with that of rain gauging stations. Therefore, we carry out the conversion of data by the following procedures:

1. Find the rain gauging station that is the nearest to the hydrological gauging station, and check the data availability (the existence of missing data).
2. Choose the best daily data of the rain gauging station on a monthly basis as consistently for the whole duration of the data as possible.[36]
3. Convert the monthly precipitation data into the daily data on the pro rata basis, based on the daily precipitation data of the rain gauging station.

Third, we need the observed data for a continuous duration of time in common with every subbasin (hydrological gauging station). During 1990 - 2004, the condition holds true only in 1993, 1994 and 1997. Thus, we use 1993 data for the calibration and 1994 data for the verification.

We use the optimisation tool in HEC-HMS for the calibration. Let us explain

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[35] The detailed information on the data is available on the site of NWA: www.nerc-wallingford.ac.uk/ih/nwa/index.htm

[36] We show the table of linkage between rain and hydrological gauging stations and its data availability in Appendix C-6.

the procedures. The fundamental purpose is to fit simulated discharge flow data with the observed discharge flow based on the precipitation data as an input. Thus, we need an index of goodness-of-fit as the objective function that we try to optimise (minimise). The optimisation tool provides several goodness-of-fit indices: sum of absolute errors; sum of squared residuals; percent error in peak; and peak-weighted root mean square error (see Table 5-22). In our research, we focus on the expected cost of flood risk. Therefore, we want to consider the goodness-of-fit on peak flows more than that on other ordinates. That is why we choose the peak-weighted root mean square error objective function as the objective function that we try to optimise (minimise).

Table 5-22. *HEC-HMS objective function for calibration*

Objective Function	Equation	Description
Sum of absolute errors	$\sum \lvert q_o(i) - q_s(i) \rvert$	Difference in the ordinates, weighting each equally.
Sum of squared residuals	$\sum [q_o(i) - q_s(i)]^2$	Squared differences, weighting each equally.
Percent error in peak	$100 \left\lvert \frac{q_s(prak) - q_o(peak)}{q_o(peak)} \right\rvert$	Compare only between simulated and observed peak flows.
Peak-weighted root mean square error objective function	$\left[\frac{1}{N} \left(\sum (q_o(i) - q_s(i))^2 \left(\frac{q_o(i) + q_o(mean)}{2q_o(mean)} \right) \right) \right]^{\frac{1}{2}}$	Squared differences, weighting each in proportion with the magnitude of the ordinate.

Source: Arranged from USACE (2000, 2001).

In the optimisation tool of HEC-HMS, there are two algorithms as a searching method that tries to minimise the value of the chosen objective function.[37] One is the univariate gradient method that evaluates and adjusts one parameter at a time keeping other parameters constant in order. This process continues until the point that an additional adjustment cannot decrease the value of the objective function by at least 1%. Another is the Nelder and Mead method that uses a downhill simplex to evaluate all the parameters at the same time and determine which parameter to adjust. We use both methods. However, the univariate gradient method rarely provided good results in our case. Hence, the

[37] See USACE (2000) for further details of the two searching methods.

results of the calibration are mainly derived from the Nelder and Mead method.

In the process of calibrating parameters for each subbasin, we carry out several trials with initial values changed, because the results are occasionally sensitive to initial values. Fundamentally, we try to search the parameter values that give the smallest value of objective function. In so doing, we check several results for choosing the parameter values: parameter sensitivity to the value of objective function; difference in total discharge volume; difference in peak flow; flow comparison graph; scatter graph; residual graph; and objective function graph. Importantly, we do not necessarily adopt the parameter values that give the smallest value of the objective function although the value of the objective function is one of the most important factors. If we get values of objective function that are similar among several trials, we should compare percentage difference in volume and peak flow among the results of the trials. The differences in volume and peak flow are important because our interest should be in the expected cost of flood risk. Considering our main purpose of the analysis, we want to avoid underestimating the volume and the peak flow in the process of calibration. Table 5-23 shows the results of the calibration (optimisation).[38]

Table 5-23. *Results of calibration*

Subbasin	Initial loss (mm)	Constant loss (mm)	SCS lag (hr)	Value of objective function	% diff in volume	% diff in peak flow	Method
27001	6.5000	0.0672	0.8	11.266	53	-4.15	Nelder Mead
27002	1.5000	1.6000	1.600	26.922	64	7.43	Nelder Mead
27005	0.4069	0.5641	0.4	7.397	-9	56.35	Nelder Mead
27007	2.4000	3.5190	0.83	32.145	28	0.05	Nelder Mead
27009	1.5000	1.4240	0.63	79.791	105	14.80	Nelder Mead
27034	2.2600	0.1020	2.9	33.741	26	-10.38	Nelder Mead
27043	0.3570	0.0630	0.567	22.099	53	-0.03	Nelder Mead
27053	1.5000	2.4000	1.5	8.251	-6	-0.82	Nelder Mead
27069	6.1327	0.0010	0.1	14.774	20	1.20	Univariant
27071	0.8000	1.8000	1.26	47.483	92	43.97	Nelder Mead
27075	1.1000	0.0157	1.6	9.465	64	-36.64	Nelder Mead
27083	0.4069	0.5641	0.4	1.115	-26	-52.36	Nelder Mead
27085	0.3022	0.5641	0.51	3.991	-12	98.72	Nelder Mead
27089	0.9700	0.7210	1.5	31.008	118	0.09	Nelder Mead
27090	0.9400	0.0300	1.8	29.321	60	-20.69	Nelder Mead

[38] Appendix C-7 shows the flow comparison graphs in the subbasins in 1993.

Finally, we use data in 1994 to validate the parameter values calibrated on data in 1993. We carry out two procedures for the verification. First, we check the goodness-of-fit in 1994 on the visual basis. Appendix C-7 shows the flow comparison graphs in the subbasins in 1994. This is not quantitative, but they seem to be reasonable. Second, we check the value of objective function in 1994 about the same calibrated parameter values and then compare the value in 1994 with that in 1993.[39] If the value in 1994 is much larger than that in 1993, the goodness-of-fit is dubious. Table 5-24 shows the results. There are only two cases that the value in 1994 is larger than that in 1993 (Subbasin 27069 and 27083). Moreover, there is no big difference in the value in these two cases. Therefore, the calibrated parameter values are judged to be valid, based on the verification analysis.

Table 5-24. *Comparison of the value of objective function*

Subbasin	Value of objective function in 1994	Value of objective function in 1993	1994 < 1993
27001	8.2	11.3	Yes
27002	19.8	26.9	Yes
27005	3.2	7.4	Yes
27007	25.8	32.1	Yes
27009	70.9	79.8	Yes
27034	27.1	33.7	Yes
27043	19.4	22.1	Yes
27053	5.7	8.3	Yes
27069	16.1	14.8	No
27071	40.3	47.5	Yes
27075	6.5	9.5	Yes
27083	1.9	1.1	No
27085	3.1	4.0	Yes
27089	25.5	31.0	Yes
27090	23.3	29.3	Yes

Note: The values are calculated by use of the optimisation tool in HEC-HMS.

[39] This value shows the goodness-of-fit. The smaller the better. The number of days in 1994 is the same as that in 1993. Thus, we can compare the values between them.

5.3.11 Relation Between Discharge and Elevation

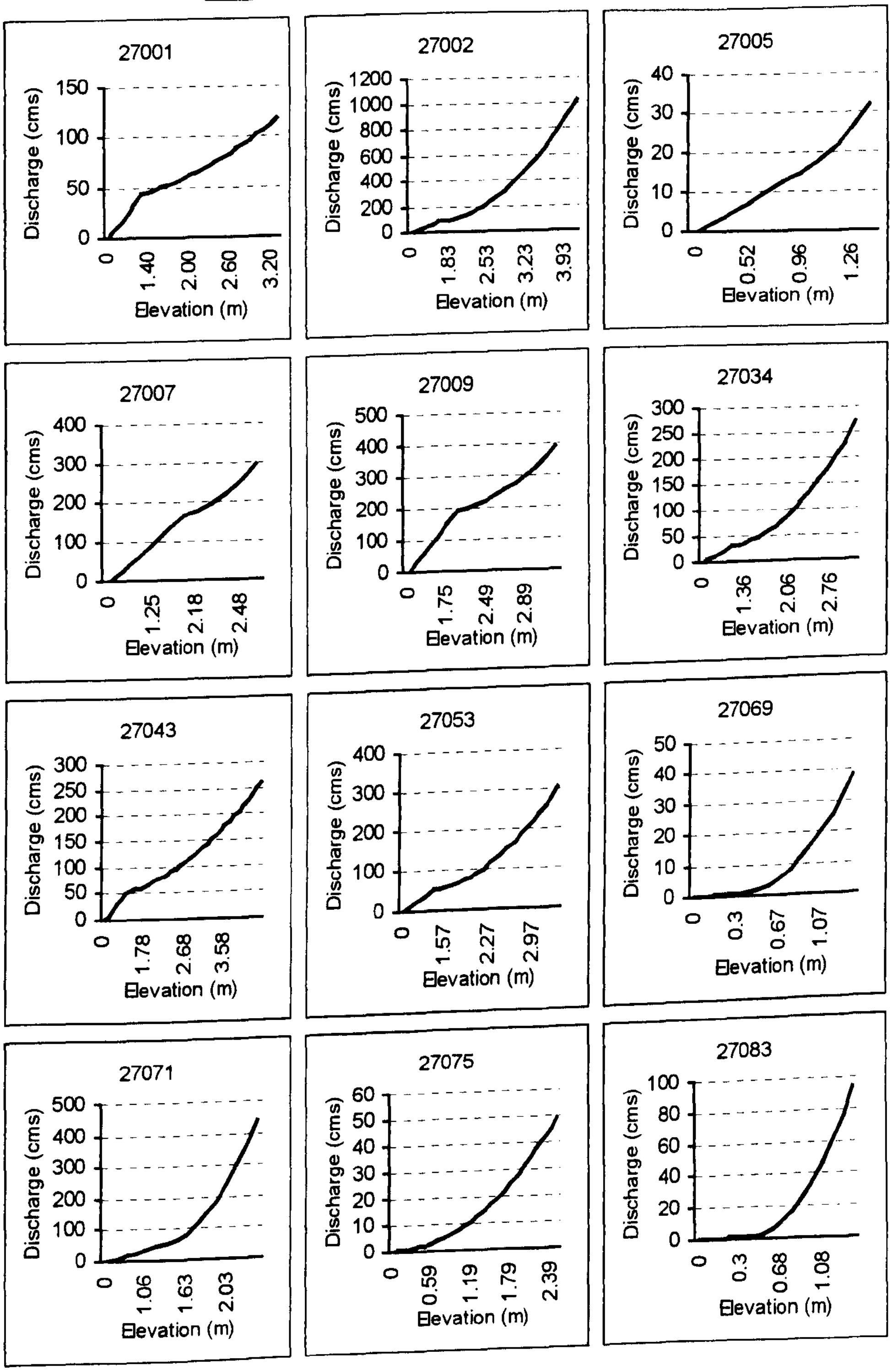
We can derive the relationship between discharge flow and elevation in each subbasin from the cross section coordinates we have derived and an equation of the hydraulic geometry (5-9), although the relationship is dependent on the assumptions of them. To begin with, let us calculate mean velocity of discharge flow in each subbasin (see Table 5-25).

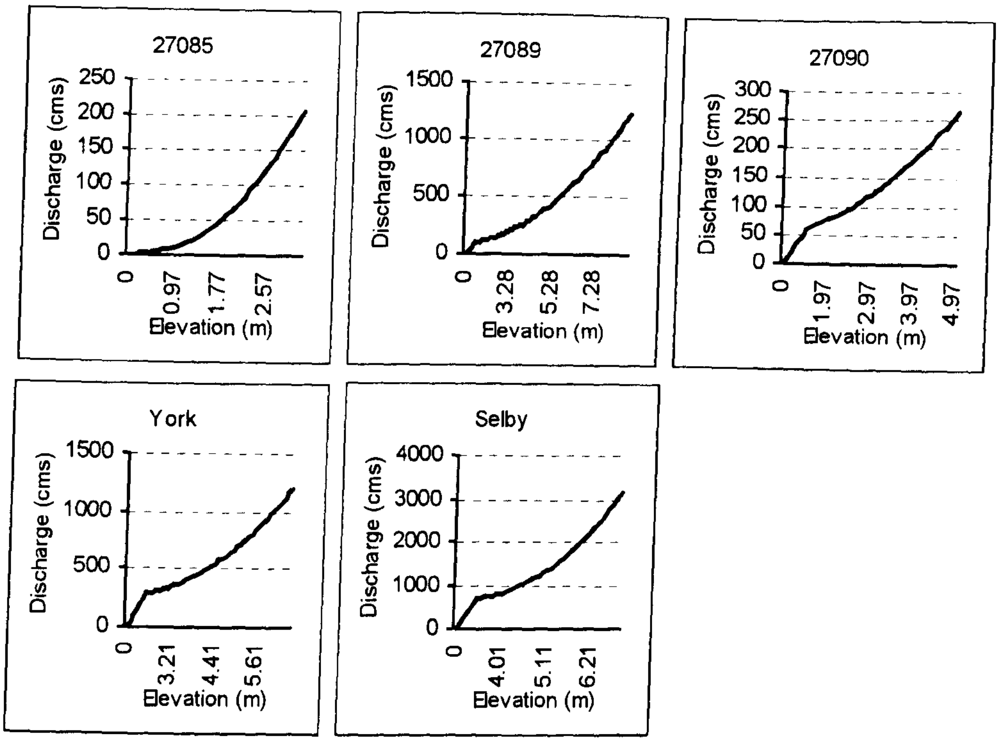
We calculate the area of cross section from the cross section coordinates, and derive the relationship between area and elevation. Then, we obtain the relationship between discharge volume (*cms*) and elevation (*m*). In so doing, we can show the discharge-elevation curve in each subbasin in Figure 5-13. As we mentioned before, we also derive the curves for York and Selby urban areas respectively here.

Table 5-25. *Calculated mean velocity*

Subbasin	Name of Stations	Velocity (m/s)
27001	Nidd at Hunsingore Weir	1.818
27002	Wharfe at Flint Mill Weir	2.070
27005	Nidd at Gouthwaite Reservoir	1.438
27007	Ure at Westwick Lock	2.377
27009	Ouse at Skelton	2.450
27034	Ure at Kilgram Bridge	1.707
27043	Wharfe at Addingham	1.886
27053	Nidd at Birstwith	1.874
27069	Wiske at Kirby Wiske	0.891
27071	Swale at Crakehill	1.840
27075	Bedale Beck at Leeming	1.037
27083	Foss at Huntington	0.900
27085	Cod Beck at Dalton Bridge	1.244
27089	Wharfe at Tadcaster	2.175
27090	Swale at Catterick Bridge	1.947

Figure 5-13. *Discharge-Elevation Curve*





5.3.12 Relation Between Elevation and Damage

We have to estimate the relationship between the elevation and the predicted damage value in order to calculate the annual expected cost of flood risk. It is difficult to derive the precise predicted damage values in the target areas because of both the data requirements and the number of other factors involved.

Let us discuss the difficulties. First, we cannot obtain a precise map of flooded areas in each flood event in simulations partly because the digital elevation model provides only rough data, partly because the hydrological model of river routings is also rough and set simply with relevant assumptions and manly because we cannot obtain the precise and thorough data on cross-sections of rivers and floodplains as we mentioned. Second, developed lands are mainly composed of residential and non-residential areas. In the case of residential areas, we have to consider house types, house age, social class, inventory items, fabric items and so on. Likewise, in the case of non-residential areas, we should consider types of properties, types of shops, moveable equipments included in them, fixtures and fittings included in them, stocks included in them and so on. Furthermore, we should take account of the secondary damage such as sales losses. Such data in the research target area (the Ouse catchment) are not available, and it is too costly to collect them. Third, we have to take other flood losses such as roads, railways and emergency costs into consideration in each flood event. However, it is also too difficult to estimate the values in each flood event in simulations. Fourth, agricultural lands are also damaged by flood events although it depends on farming methods and seasons. If land is fallow, there is little damage. Furthermore, if the farmers do not rely on chemical fertilizers, flood events will be welcome because the flood water supplies fertile soils. This is not a damage but a benefit. We have to distinguish between farming methods. In addition, we have to consider the values of crops (both types of crops and their market prices) that are damaged or lost due to floods. It is also difficult to do so locally depending on changeable flooded areas.

Nevertheless, we need the relationship between the elevation and the predicted damage value for carrying out policy simulations although it is a rough sketch. Taking a similar perspective, Penning-Rowsell et al. (2003, 2005a, 2005b) provide standard data on the relationship between flood depth and damages. They do not recommend the use of detailed surveys for the cost-benefit analysis of flood risk management. Rather, they recommend the use of the standard data that they provide based on the data of flood damages in the UK. We use the data provided by Penning-Rowsell et al. (2005a, 2005b) to set the predicted damage values per hectare in our model.[40] The data include flood damage values as standard data. In addition, they have tried to cover all the relevant sectors of the economy including residential property, non-residential property, agriculture, communications, emergency services and other costs. Furthermore, they consider economic damage values and not financial damage values. That is, they consider

[40] The CD-ROM of the detailed data is attached to Penning-Rowsell et al. (2003), but it is not available for us. It seems to be difficult to obtain it now. However, Penning-Rowsell et al. (2005a, 2005b) give the CD-ROM of the modified version of detailed data. These document have been in general available from Middlesex University Press since February 2006.

opportunity costs and take the standpoint of the nation as a whole.[41]

Hence, we set the unit damage values in developed lands and agricultural lands respectively based on the data of Penning-Rowsell et al. (2005a, 2005b). Then, we calculate the predicted damage values according to the elevation on a pro rata basis (a rough calculation). The concrete procedures are the followings:

1. We set flood damage values of developed lands and agricultural lands per hectare respectively based on the available data.
2. Based on the relationship between discharge volumes and elevations, we calculate the percentage of flooded area in each subbasin in each flood event in simulations.
3. Using the percentage of flooded area, we calculate the areas (hectare) of flooded developed lands and flooded agricultural lands respectively on the 'pro rata' basis.[42] [43]
4. Using the flood damage values per hectare, we can calculate the predicted damage values in subbasins in each flood event in simulations.

Thus, we have to set the flood damage values of developed lands and agricultural lands per hectare in the rest of this section. To begin with, Penning-Rowsell et al. (2005a, 2005b) provide the data on standard depth-damage relationship in residential areas (residential sector average) (see Figure 5-14).[44] [45]

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[41] The standpoint of the nation as a whole implies that the value will be offset if one person's loss can be another person's gain.

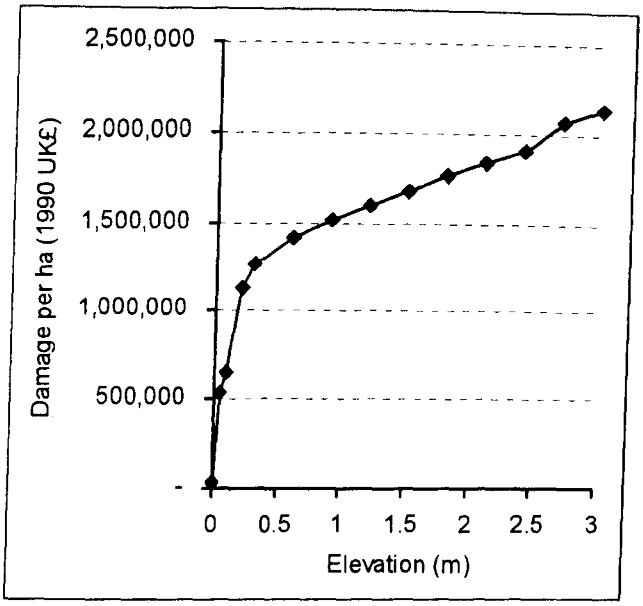
[42] If developed lands are concentrated on the areas close to a river, we will tend to underestimate the damage value. However, we cannot consider this point because the model is simplified mainly because the data are not sufficiently available.

[43] For York and Selby urban areas, we can calculate the flooded developed areas on the same pro rata basis by using the urban areas in floodplains that are calculated by ArcGIS from the sama GIS data (we mentioned before).

[44] The elevation in Figure 5-14 is measured from the location p3 or p6 (overbank) in Figure 5-9.

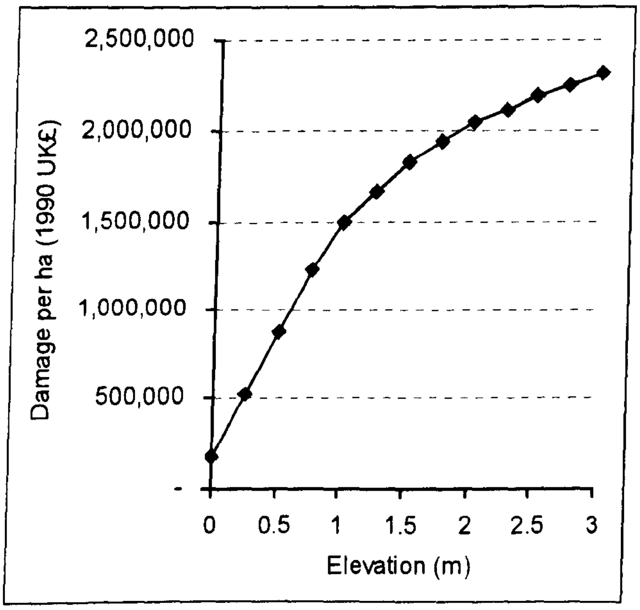
[45] We use the data of short duration because we will evaluate the expected cost of flood risk by peak stages. They do not tend to last over 12 hours.

Figure 5-14. *Elevation-damage relationship in residential areas (UK)*



Source: Penning-Rowsell et al. (2005a, 2005b), Appendix 4.1 in the attached CD-ROM.
Note: We converted the unit from 2005£ per m^2 into 1990£ per ha, using RPI data from UK National Statistics. In the calculation, we assume that the ratio of building area to the total developed area is 40%.

Figure 5-15. *Elevation-damage relationship in non-residential areas (UK)*



Source: Penning-Rowsell et al. (2005a, 2005b), Appendix 5.6 in the attached CD-ROM.
Note: We converted the unit from 2005£ per m^2 into 1990£ per ha, using RPI data from UK National Statistics. In the calculation, we assume that the ratio of building area to the total developed area is 40%.

Next, Penning-Rowsell et al. (2005a, 2005b) provide a standard depth-damage curve as weighted mean values in each small category of non-residential area type in the case of a basic scenario (river flood, no warning and duration less than 12 hours).[46] In addition, they provide the one as weighted mean value in non-residential areas. We refer to this data. Figure 5-15 shows the graph.[47]

Table 5-26. *Emergency costs of Autumn 2000 Floods in North Yorkshire*

	Total	York	Selby	Harrogate	Richmondshire	Others
Care Related Services	104,447	-	101,790	0	0	2,657
Nursing home evacuation	104,447	-	101,790	0	0	2,657
Flood Alleviation	1,033,209	-	499,451	227,508	55,389	250,861
Sandbagging	713,566	-	471,379	53,508	35,045	153,634
Emergency costs	319,643	-	28,072	174,000	20,344	97,227
Highways and Bridges	572,161	-	7,686	174,142	39,591	350,742
Emergency repairs-roads	401,090	-	1,877	111,381	18,434	269,398
Emergency repairs-bridges	171,071	-	5,809	62,761	21,157	81,344
Emergency Planning	0	0	12,700	0	0	12,700
Evacuation	0	0	12,700	0	0	12,700
Education related	0	0	10,015	0	0	10,015
Evacuation-Sherbum School	0	0	10,015	0	0	10,015
Fire Services	414,074	108,268	143,128	34,019	15,601	113,058
North Yorkshire Police	681,398	216,403	328,102	1,788	978	134,127
Total	2,805,289	324,671	1,102,872	437,457	111,559	874,160

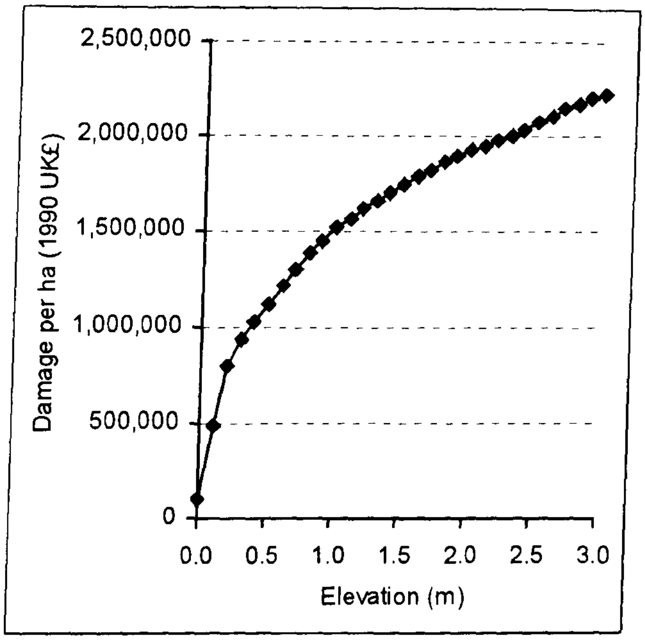
Source: Penning-Rowsell et al. (2005a), Table 6.14, p. 132.

In addition, we obtain the data on emergency costs associated with Autumn 2000 floods from Penning-Rowsell et al. (2003, 2005a) (see Table 5-26). Likewise, we cannot use the data in general in each flood event in simulations although we know that we definitely incur such emergency costs. They include fixed costs in proportion to the size of flood events, but they are still related to the size of flood events. They are probably related in an discontinuous function.

[46] The categories of non-residential area types are: shop/store, vehicle services, retail services, office, distribution/logistics. In the categories of leisure, public building, and industry, the weighted mean values are not provided because no reasonable data are available.
[47] The elevation in Figure 5-15 is measured from the location p3 or p6 (overbank) in Figure 5-9.

Therefore, we set the predicted flood damage values per hectare in developed lands according to their elevation based on Figure 5-14 and 5-15, although there is possibility that we underestimate the damage value. In the calculation, we give even weight to residential and non-residential areas.[48] This gives the results reported in Figure 5-16.[49]

Figure 5-16. *Elevation-damage relationship in developed areas (UK)*



Let us check the data of flood damage values in agricultural lands. Penning-Rowsell et al. (2005) provide the data on financial and economic output and gross margins, fixed costs and net margins for selected crops and livestock enterprises. Let us assume that the crops are completely destroyed after all the inputs are used.[50] Then, we abstract the output data from Penning-Rowsell et al. (2005) and apply the calculated average value in Table 5-27 assuming that the annual probability of the damage is 50% where agricultural lands lie fallow in

[48] Based on the land-use data (LCM 1990), we cannot tell residential areas from non-residential ones. Presumably, residential areas are larger than non-residential ones. Thus, the even weight tends to overestimate the value, but the tendency of overestimation may be offset by the tendency of the underestimation that we mentioned (we do not consider other costs such as emergency costs).
[49] The elevation in Figure 5-16 is measured from the location p3 or p6 (overbank) in Figure 5-9.
[50] In addition, we implicitly assume that all the fixed costs are sunk costs, which implies that all the inputs are irretrievable or depletable.

winter.

Table 5-27. *Output of selected crops and livestock enterprises (UK)*

	Output (1990£ per ha)
Crops	
Winter wheat	391
Oil seed rape	257
Peas	246
Beans	246
Sugar beet	1,156
Potatoes	2,660
Average (crops) [a]	826
Livestock Enterprise	
Dairy cows	1,511
Beef cows	279
Beef cattle	473
Sheep fat lambs	347
Average (livestock) [b]	652
Average (a & b)	739

Source: Penning-Rowsell et al. (2005a), Table 9.4 and 9.5, pp. 194-195.
Note: We converted the unit from 2005 UK£ per ha into 1990 UK£ per ha, using RPI data from UK National Statistics.

5.3.13 Relation Between Flood Risk and Precipitation

Virtual rainfall (hypothetical frequency of precipitation) with an annual probability (1%, 2%, 4%, 10% and 20% [51]) is an important input for calculating the expected cost of flood risk in simulations on HEC-HMS. It derives from precipitation frequency analysis. This section conducts a precipitation frequency analysis in order to obtain the relation between precipitation and the probability of flood damage. We use the data on daily precipitation in subbasins, analysing cumulative precipitation amounts for two days.[52] [53]

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[51] These probabilities are 'annual' probabilities (probabilities based on the unit of year). For example, 1% annual probability implies that the return period is 100 years. In this section, we are dealing with daily data (2-day rainfalls). Thus, we have to convert the probability based on the unit of 2 days into the one based on the unit of year later. Finally, we obtain the 5-year, 10-year, 25-year, 50-year and 100-year return period 2-day rainfalls respectively.

[52] "In order to predict 50-year or 100-year events, we need the data for 50 or 100 years respectively" (Gordon et al., 2004). It implies that the more accurate the prediction becomes the more data we have. However, the data is limitedly available as we mentioned. The analysis is

We have to find a probability distribution function that fits the observed data of 2-day rainfalls. In so doing, we have three methods available to us: graphical method; method of moments; and method of maximum likelihood. The method of maximum likelihood is superior to the method of moments by some statistical measures, but it is computationally much more complicated because it requires an iterative procedure (Bedient and Huber, 2002; Gordon et al., 2004). In addition, an efficient estimate by the method of maximum likelihood is not necessarily found (Gordon et al., 2004). Thus, we use the graphical method and the method of moments. The graphical method enables us to find a candidate probability distribution. We then obtain the location, scale and shape of the distribution from the observed data series by the method of moments.

We put the observed data into a probability diagram by use of a plotting position formula such as Weibull, GEV and Cunnane. We use Cunnane's plotting position formula here because it is the most sophisticated and flexible about probability distribution functions. The probability diagram is designed to produce a straight-line plot when the observed data is completely fitted with the chosen theoretical probability distribution. Then, we judge the goodness-of-fit of the chosen probability distribution (to what extent the drawn line is straight) by eye, or apply the chi-square statistic and Kolmogorov-Smirnov test for assessing the goodness-of-fit. Since the latter are seldom helpful in distinguishing among different distributions because their confidence intervals are so large that the hypothesis that the distribution is fitted with the data tends to be usually accepted (Bedient and Huber, 2002). Furthermore, they are insensitive in the tails of distributions: the areas are normally important for evaluating extreme events (Gordon et al., 2004). Hence, we use the graphical method for judging the

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based on the data for 10 years in 1990s (see Appendix C-5 and C-6). Thus, we do not treat the flood events beyond 100-year return period.

[53] "Cumulative precipitation amounts for specified durations are commonly analyzed, mostly for durations of less than 3 or 4 days" (USACE, 1993). Considering that HEC-HMS gives 1, 2 or 4 days as the options for the duration of a frequency storm, we choose 2 days duration for the analysis.

goodness-of-fit here.[54]

In the precipitation frequency analysis, we follow the following procedures:

1. We obtain the 2-day precipitation data from the observed daily precipitation data in subbasins by adding the data of precipitation on the previous day to the one on the current day, say $D_{i,2-day} = D_{i,1-day} + D_{i-1,1-day}$.
2. There are many zero values included in the data. Thus, we set a mixed frequency distribution that is composed of a discrete probability mass (the data value is zero) and a continuous probability density function (in the rest of the range). $P(X=0)+P(X>0)=1$ (for all $X \geq 0$). If there are several zero values in the data, the total area of the continuous probability density function provides the value that is smaller than one.
3. We exclude the zero values, and focus on the rest of data to find a candidate continuous probability density function. We draw a histogram by using the function $k = 5\log_{10} n$ (k: number of classes; n: number of observations).[55] Based on the shape of the histogram, we find candidate probability distribution functions.
4. We rank the data values from 1 to N (the total number of observations). The largest value should be given a rank of 1, the second largest one a rank of 2, and so on until the smallest one is given a rank of N.
5. We determine the plotting positions of the data values by using the Cunnane's plotting position formula, $\frac{m-\alpha}{(N+1)-2\alpha}$ where m is a rank, α is a constant that is specific to probability distributions and N is the total number of data. The formula gives the probability of exceedance based on the data values.

[54] "In the end, the decision is often subjective and based on a preference for the underlying mechanism of one distribution versus another." (Bedient and Huber, 2002, p. 218).
[55] It is helpful for determining the number of class intervals (Bedient and Huber, 2002).

6. We plot the observed data values on the probability diagram, in which we can compare the observed data with the one derived from the chosen candidate cumulative probability distribution.[56] In so doing, we use the method of moments in order to specify the measures for the distribution.[57] If the observed data coincide with those, they provide a straight line in the diagram.
7. Finally, we obtain the precipitation volume with an 'annual' exceedance probability such as 1%, 2%, 4%, 10% and 20% from the chosen cumulative probability distribution in each subbasin. These data are outputs here.

We carried out the procedures above in fifteen subbasins. We concretely show the procedures here.[58] Take subbasin 27053 as an example. Table 5-28 shows the fundamental statistics on 2-day precipitation data. The number of zero values is 863, which implies that the probability is 0.2839 (= 863/3040). Then, the total area of a continuous probability density function should be 0.7161 (= 1 - 0.2839).

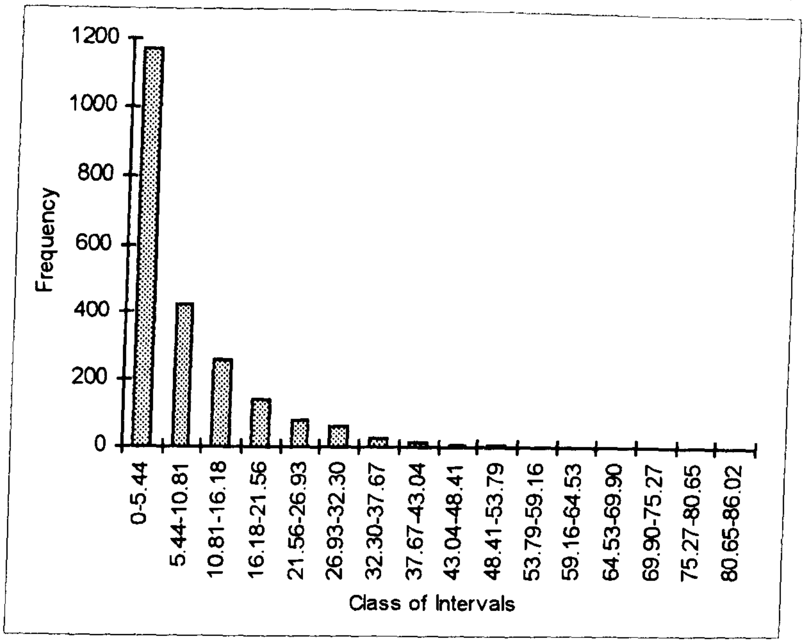
Table 5-28. *Statistics on 2-day precipitation data (27053)*

Data	
N	3,040
Year [a]	8.33
Zeros	
Numer of zeros	863
P(0)	0.2839
Data excluding zeros	
mean	8.111
st.d	9.358

Source: Derived from monthly catchment rainfall (precipitation) data (NWA, CEH) and Met Office - UK Land Surface Stations data (BADC).
Note: a. The range (availability) of the data is shown in Appendix C-5.

[56] There is a problem on the plotting position formula that we should note. "There can be a great amount of uncertainty in the plotting positions assigned to the largest events. The extreme events may plot as 'outliers', far off the line defined by the more frequent events" (Gordon et al, 2004).
[57] Several authors have indicated the bias related to the methods of moments due to uncertainties in the estimation of the flood frequency relationship from limited data, but there are possibilities of both overestimation and underestimation (Arnell, 1989).
[58] We show histograms and probability diagrams about the other subbasins in Appendix C-8.

Figure 5-17. *Histogram of 2-day precipitation(mm) (27053)*



We construct a histogram of the data series excluding zeros. Figure 5-17 shows the histogram. From the shape of the histogram, the exponential distribution appears to be suitable because it is obviously skewed to the right. Normal, log-normal, log-Pearson, gamma and extreme value distributions seem to be unsuitable. In addition, the exponential distribution has the property that its mean is equal to its standard deviation. Looking at Table 5-28, there is no big difference between them. Thus, we expect that we can fit the exponential distribution with the observed data.

We then check the goodness-of-fit in the probability diagram. The cumulative distribution function of the exponential distribution is denoted:

$$F(X) = \int_0^X \lambda e^{-\lambda X} dX = 1 - e^{-\lambda X}$$

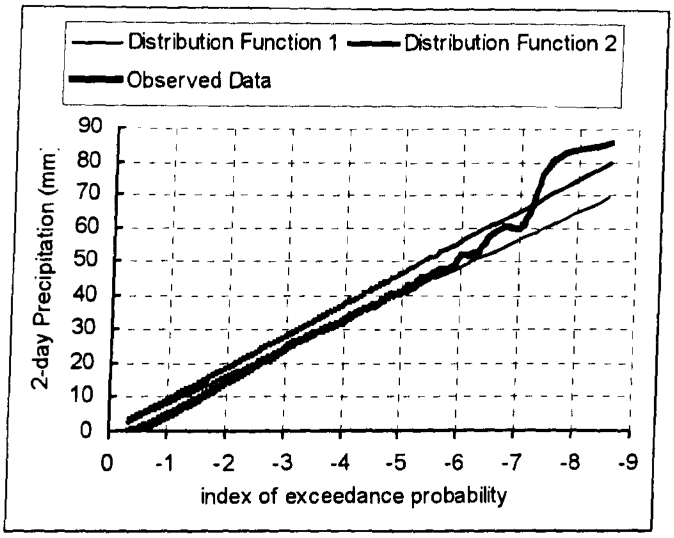
while the exceedance probability is: $G(X) = 1 - F(X)$.

It follows that $G(X) = e^{-\lambda X}$.

Rearranging, $X = \dfrac{-\ln G(X)}{\lambda}$.

Thus, if the left hand side represents the vertical axis and the right hand side represents the horizontal axis in the probability diagram, the plot of the points derived from the distribution function produces a straight line. Hence, we compare the positions of the observed data determined by the plotting position formula with the straight line. Figure 5-18 shows the diagram. As we mentioned, we observe a discrepancy in the range of large values. However, the fit is not bad in this type of analysis. Taking this discrepancy into consideration, we use the distribution function 2 with the value of λ that is estimated by the sample standard deviation for reducing the discrepancy.

Figure 5-18. *Probability diagram (27053) (exponential distribution)*



NOTE: The distribution function 1 is the exponential distribution with the value of λ that is estimated by the standard deviation. The distribution function 2 is the one with the value of λ that is estimated by the mean.

Table 5-29. *2-day precipitation and annual exceedance probability (27053)*

Annual Exceedance probability	Precipitation volume (mm)
20%	60.660
10%	67.147
4%	75.721
2%	82.207
1%	88.694

Based on the chosen exponential distribution with λ = 0.1333, we derive the relationship between 2-day precipitation volumes and an 'annual' exceedance probability. However, we cannot directly derive the 'annual' probabilities from the ones the distribution function provides. We should note that we have to convert the probability based on the unit of 2 days into one based on a year. Let us illustrate this point by an example. If the probability on the yearly basis is 0.01, it implies that the return period is 100 years (1/(100 years) = 0.01). Consider the case that the probability on the 2-day basis is 0.01. This implies that the return period is 200 days (0.01 = 1/(100 2-days)). It is equivalent to only 0.548 year. For example, the annual exceedance probability 0.1 (10%, 10-year return period) is equivalent to the exceedance probability 0.00055 (0.0055%) on the basis of the 2-day unit. The probabilities are quite different if the units are different. Here, let us derive the relationship between 2-day precipitation volumes and the 'annual' exceedance probability, considering the discrete probability mass of zeros. Table 5-29 shows the results.

Table 5-30. *Results of precipitation frequency analysis*

Subbasin	Annual Exceedance Probability					PDF	Measures
	20%	10%	4%	2%	1%		
27001	47.577	52.776	59.647	64.845	70.043	Exponential	λ = 0.133
27002	50.420	55.858	63.048	68.486	73.924	Exponential	λ = 0.127
27005	75.743	83.706	94.232	102.195	110.157	Exponential	λ = 0.087
27007	51.874	57.538	65.025	70.689	76.352	Exponential	λ = 0.122
27009	39.282	43.588	49.281	53.587	57.894	Exponential	λ = 0.161
27034	82.692	91.493	103.127	111.928	120.728	Exponential	λ = 0.079
27043	71.201	78.870	89.007	96.676	104.344	Exponential	λ = 0.090
27053	60.660	67.147	75.721	82.207	88.694	Exponential	λ = 0.107
27069	38.744	43.024	48.682	52.962	57.242	Exponential	λ = 0.162
27071	40.742	45.171	51.026	55.456	59.885	Exponential	λ = 0.156
27075	42.418	46.932	52.899	57.413	61.928	Exponential	λ = 0.154
27083	39.508	43.839	49.564	53.895	58.226	Exponential	λ = 0.160
27085	42.581	47.285	53.503	58.206	62.910	Exponential	λ = 0.147
27089	48.060	53.248	60.106	65.294	70.482	Exponential	λ = 0.134
27090	71.493	79.122	89.206	96.835	104.463	Exponential	λ = 0.091

Likewise, we do the same analysis in each subbasin. Then, we obtain the results shown in Table 5-30. "The two-day, five-year return period rainfall exceeds 150 mm in parts of the north and west of Britain and is less than 50 mm in parts of the south and east (Natural Environment Research Council, 1975)" (qtd. in Güntner et al., 2001). This information is old, but the results in Table 5-30 seem to be valid because the Ouse catchment is located in the north (close to middle) and east of Britain. The data in Table 5-30 can be used as the inputs for frequency precipitation in the meteorologic model of HEC-HMS.[59] Finally, there are three things that we should note about the treatment of the precipitation data in simulations. First, we assume that we have rainfall with equal probability in all subbasins at one time. If the probability functions of precipitation in the subbasins were independent, we would evaluate the probability of precipitation events in the Ouse catchment based on the joint distribution of probability density functions of the subbasins. However, we do not think that precipitation events in the subbasins are completely independent one another. Rather, they are dependent or related one another. The Ouse catchment is not so large (about 4,250 km^2). Observing the daily precipitation data in subbasins, the weather seems to be quite similar in the catchment. Therefore, the assumption might be acceptable.[60] Second, there is a seasonal change that is shown by the volumes of monthly baseflow although they are not drastically different. For simulations, we choose October because the volume of baseflow in October is close to the average (see Section 5.3.4). Third, we use 2-day precipitation volumes with 2%, 1% and lower exceedance probability in simulations. We do not consider averting behaviour as a control variable, but we need to include the scale of the current averting behaviour as part of the initial conditions (i.e. as an exogenous variable). The Environment Agency (2003) provides information on the scale of averting behaviour in part of lower River Ouse (from Skelton gauging station in York to Selby). According to this, the existing flood defence level is close to the estimated 1% annual probability water level. Considering this, we assume that the current scale of averting behaviour protects against the precipitation with 2% and higher exceedance probability.[61]

[59] Strictly speaking, we should evaluate the intensity of rainfalls as well. There are methods for deriving temporal patterns of design rainfalls (Pilgrim, 1969; Güntner et al., 2001), but we still need the base data of rainfalls for shorter durations. Therefore, we assume uniform hypothetical rainfalls in our simulations, and conduct a sensitivity analysis in terms of the intensity of rainfalls in the next chapter.

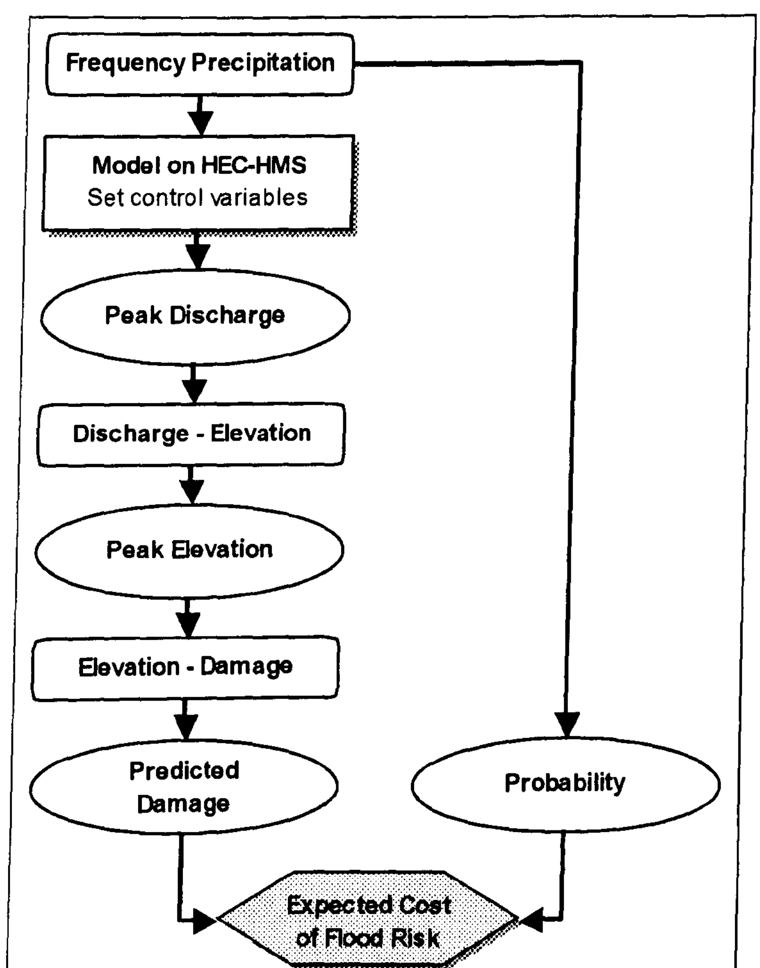
[60] On the other hand, there is a risk that we overestimate the volume of rainfalls.

[61] This might be a strong assumption for the whole catchment. Therefore, we will have to carefully consider the results of simulations in the next chapter.

5.3.14 Calculation of Expected Cost of Flood Risk

This section explains the process of calculating the expected cost of flood risk. Figure 5-19 shows the summary of procedures for calculating the expected cost of flood risk. To begin with, we input the data of hypothetical frequency storms into the meteorologic model of HEC-HMS, and we set relevant values of control variables in HEC-HMS. Next, we derive peak discharge volumes of subbasins from the simulation on HEC-HMS. Using the discharge-elevation and elevation-damage relationships, we obtain the value of predicted damage. Finally, we can calculate the annual expected cost of flood risk from the value of predicted damage and the probability of hypothetical frequency storms.

Figure 5-19. *Process of calculating the expected cost of flood risk*



5.4 Derivation of Specific Functional Forms

5.4.1 Expected Cost Function of Flood Risk

We can now obtain the expected cost of flood risk in subbasins and two city areas from the simulation on the hydrological sub-model (HEC-HMS). Hence, we can analyse some important aspects of the relationships between the expected cost of flood risk and floodplain development. In Section 6.2 (Chapter 6), we try to analyse the impact of some patterns of floodplain development on the expected cost of flood risk in the Ouse catchment.

However, we need specific functional forms for the expected cost function of flood risk in subbasins. This is both because we have to numerically solve the dynamic optimisation problem and because we wish to simulate several scenarios (policies) on the basis of the dynamic optimisation. In addition, we need to know the marginal values in order to find the optimal steady state equilibrium. Since the simulations on the hydrological sub-model cannot provide the 'marginal' expected cost of flood risk, we need to adopt specific functional forms in subbasins. For deriving a specific functional form, we need the data on the relationship between areas of developed floodplains and the expected cost of flood risk, but the historic data on the relationship are not available. Fortunately, the simulations on the hydrological sub-model can produce data on the relationships between the expected cost of flood risk and areas of developed floodplains in subbasins. Thus, using the outputs from the simulations, let us estimate specific functional forms.

To begin with, we have to obtain the data on the relationship between areas of developed floodplains in subbasins (state variables) and the expected cost of flood risk from simulation in the hydrological sub-model. If we set the values of state variables, we can obtain the expected cost of flood risk from the simulation. By this procedure, we obtain 60 observations about the combination of the values of state variables and the expected cost.[62]

Considering the structure of the hydrological sub-model, there are some points that functional forms must satisfy. First, there are unidirectional spatial externalities. The function should include the areas of developed floodplains in its own zone and upstream zones as long as the external impacts are not trivial. Second, the increase in the area of developed floodplains in its own zone and/or upstream zones increases the expected cost of flood risk.

$$\frac{\partial C^i\left(X^i,\mathbf{X}^j\right)}{\partial X^i}>0, \quad \frac{\partial C^i\left(X^i,\mathbf{X}^j\right)}{\partial X^j}>0$$

where zones j are upstream ones relative to the zone i.

Third, the settings of the model structure are common among the subbasins. Only the calibrated parameter values and geographical locations are different. Thus, the functional forms adopted should also be the same. Fourth, we can derive another clue for functional forms from the structure of the hydrological sub-model. The expected cost of flood risk is influenced by peak stages and the area of developed lands that are susceptible to floods in the subbasin. Peak stages are determined by control variables in the reference zone and in upstream zones $\left(X^i,\mathbf{X}^j\right)$. The area of developed lands is the area of developed floodplains in the reference zone X^i. Thus, the impact of X^i is larger than that of $\mathbf{X}^j$. Functional forms should reflect this point.

The criteria for choosing functional forms is: (1) The conditions that we discussed above are satisfied. (2) Adjusted R-squared (coefficient of determination) is sufficiently high. (3) Independent variables are statistically significant (at least at 10% significance level) based on t-test and F-test. (4) Functional forms that satisfy the sufficiency conditions are preferable. (5) They satisfy RESET (regression specification error test) or the Davidson-MacKinnon test if necessary.[63]

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[62] Generally speaking, we need at least 30 observations in order to conduct a regression analysis (Wooldridge, 2000). 60 observations seem to be sufficient.

[63] There are some problems on RESET: (1) There is no optimal choice of the number of

Fortunately, we can find a functional form that satisfies the conditions mentioned above:

$$C^i\left(X^i,\mathbf{X}^j\right)=\alpha\left(X^i\right)^2+\beta\sum_j X^j$$

Zones j are upstream zones on the same river (tributary). If two or more tributaries flow into a zone, the functional form includes two or more second terms (Subbasin 27009 is an example). In addition, there is no term X^i for the two urban areas (York and Selby) because, as we explained, these areas do not contain a gauging station and therefore we cannot use the HEC-HMS model to simulate the link between developed area and flood risk within these zones. Thus, the functional form differs somewhat between subbasins. The equations are estimated by OLS, as follows (see Appendix C-9 for details):

27001 $C^{i=27001}=27.17415\left(X^i\right)^2+23654.05790\sum_j X^j \quad j=27005 \text{ and } 27053$

27002 $C^{i=27002}=3.22602\left(X^i\right)^2+3562.36893\sum_j X^j \quad j=27043$

27005 $C^{i=27005}=137.27578\left(X^i\right)^2$

27007 $C^{i=27007}=6.43880\left(X^i\right)^2+3138.36812\sum_j X^j \quad j=27034$

27009 $C^{i=27009}=4.61088\left(X^i\right)^2+6578.54122\sum_j X^j+7049.52755\sum_k X^k$ [64]

$j=27007 \text{ and } 27034, \quad k=27069, 27071, 27075, 27085 \text{ and } 27090$

27034 $C^{i=27034}=15.50372\left(X^i\right)^2$

27043 $C^{i=27043}=28.96503\left(X^i\right)^2$

27053 $C^{i=27053}=1.08513\left(X^i\right)^2+123.99826\sum_j X^j \quad j=27005$

27069 $C^{i=27069}=24.53457\left(X^i\right)^2$

27071 $C^{i=27071}=4.41719\left(X^i\right)^2+4069.88547\sum_j X^j$

$j = 27069, 27075, 27085 \text{ and } 27090$

---

polynomials (we choose two); (2) The power may be low; and (3) There is no guidance about how to re-specify the model (Wooldridge, 2000). About the Davidson-MacKinnon test, there is a problem that the result of the test is not decisive (Wooldridge, 2000).

[64] Three tributaries (River Nidd, River Ure and River Swale) flow into Subbasin 27009. The function of 27009 should have three terms on X^j, but a term related to River Nidd is obviously statistically insignificant. Therefore, we omit the term. See Appendix C-9 in detail.

27075 $C^{i=27075} = 37.52708(X^i)^2$

27083 $C^{i=27083} = 5.28397(X^i)^2$

27085 $C^{i=27085} = 7.80344(X^i)^2$

27089 $C^{i=27089} = 20.35564(X^i)^2 + 342.83757\sum_j X^j$ j = 27002 and 27043

27090 $C^{i=27090} = 29.55194(X^i)^2$

Selby $C^{i=Selby} = 899343.7842 + 48.69045\sum_j X^j$ j = All 15 subbasins

York $C^{i=York} = 8477662.330 + 290.42594\sum_j X^j$ j = 27001, 27005, 27007, 27009, 27034, 27053, 27069, 27071, 27075, 27083, 27085 and 27090

5.4.2 Benefit Function of Ecosystem Services

As we mentioned before, natural floodplains provide us with several ecosystem services. This point is crucial, and this has been widely recognized recently. Thus, the research on the valuation of wetlands which include floodplains have been accumulating. To the best of our knowledge, however, there has been no valuation of floodplains in the Ouse catchment up to now. In addition, there are few other studies that value river floodplains, although some inland wetlands are implicitly included in floodplains.[65] In the absence of research on the valuation of floodplains in the Ouse catchment, we value ecosystem services of floodplains in the Ouse catchment by the method of benefits transfer.[66]

Fortunately, research on the valuation of wetlands, in general, have accumulated (Woodward and Wui, 2001). Hence, a meta-analysis for value transfer is now possible. Heimlich et al. (1998) provide a good review of 33 studies from the literature on wetland valuation over the last 26 years in order to

[65] e.g. Gren et al. (1995); and Hickman et al. (2001). For example, Gren et al. (1995) try to calculate the total value of ecosystem services that Danube floodplains provide by a method of transferring benefits from the results of other researches that focuses on one or more ecosystem services.

[66] This can become one of the big researches, and it is beyond our target.

derive a range of the values per acre of wetlands. The values derived are standardized using a 6-percent discount rate and a 50-year accounting period, and are indicated in 1992 constant US dollar terms. The review covers the values of marketed goods, non-marketed goods and non-marketed ecosystem services that wetlands provide. Heimlich et al. (1998) have given a summary table (Table 5-31). We want to convert the values of Table 5-31 into the values per year per hectare in 1990 constant GBP (£). Table 5-32 shows the converted data. This is used to provide indicative benefits of ecosystem services.

Table 5-31. *Economic values of wetland functions (per acre, 1992 US dollar)*

Wetland function valued	Number of studies	Median	Mean	Range of means
Marketed goods:				
Fish and shellfish support	8	702	6,132	7 - 43,928
Fur-bearing animals	2	na	137	12 - 261
Nonmarketed goods:				
General-users	12	32,903	83,159	115 - 347,548
General-nonusers	6	623	2,512	105 - 9,859
Fishing-users	7	362	6,571	95 - 28,845
Hunting-users	11	1,031	1,019	18 - 3,101
Recreation-users	8	244	1,139	91 - 4,287
Ecological functions:	17	2,428	32,149	1 - 200,994
Amenity and cultural	4	448	2,722	83 - 9,910

Source: Heimlich et al. (1998), Table 1, p. 15.
Note: It shows the values that are standardized by a 6-percent discount rate and a 50-year accounting period.

Table 5-32. *Economic values of wetland functions (per ha per year, 1990 GBP £)*

Wetland function valued	Median	Mean	Range of means
Marketed goods:			
Fish and shellfish support	55	478	0.5 - 3,424
Fur-bearing animals	na	11	0.9 - 20
Nonmarketed goods:			
General-users	2,564	6,482	9.0 - 27,088
General-nonusers	49	196	8.2 - 768
Fishing-users	28	512	7.4 - 2,248
Hunting-users	80	79	1.4 - 242
Recreation-users	19	89	7.1 - 334
Ecological functions:	189	2,506	0.1 - 15,666
Amenity and cultural	35	212	6.5 - 772
Total	-	-	34.6 - 49,791

Source: Converted and arranged from Heimlich et al. (1998), Table 1, p. 15. GDP deflator is calculated from the data on nominal and real GDP (Source: US Department of Commerce). The exchange rate is £ = 1.7864 US$ in 1990, based on the annual average of spot exchange rate (Source: Bank of England).
Note: The total value is derived by summing the values in the items above. There might be a problem of double-counting (Aylward and Barbier, 1992; Barbier, 1994). Thus, it is a provisional value.

Concretely, we try to use the results of meta-analysis of wetland valuation for setting the function of benefits of ecosystem services. Woodward and Wui (2001) have derived a function that explains the value of wetland by a meta-analysis, using the results of 46 existing researches of wetlands valuation (39 wetlands). They assume that "there exists an unobserved valuation function that determines a wetland's value given its physical, economic and geographic characteristics" (Woodward and Wui, 2001). Based on this, they show two types of results: a simple bivariate meta-analysis with graphical presentation; and a multivariate econometric meta-analysis.[67] We use them to fit results in order to set the benefit function of ecosystem services.

Fundamentally, we need to derive the relationship between the value (or the unit value) of wetland and its area. The model provided by Woodward and Wui (2001) includes many of the system's ecological characteristics and its socio-economic environments as dummy variables. Woodward and Wui (2001) assume the following function:

$$\ln(V) = \alpha + \beta_a \ln(x_a) + \boldsymbol{\beta}_s \mathbf{x}_s + \boldsymbol{\beta}_m \mathbf{x}_m + \boldsymbol{\beta}_0 \mathbf{x}_0 \tag{5-15}$$

where

V = value per year per acre (1990 US$)

α = constant term

$\beta_a, \boldsymbol{\beta}_s, \boldsymbol{\beta}_m, \boldsymbol{\beta}_0$ = coefficient of independent variables

x_a = area of the wetland in acres

$\mathbf{x}_s$ = a function of the services provided [dummy variable (0 or 1)]

$\mathbf{x}_m$ = a methodology [dummy variable (0 or 1)]

$\mathbf{x}_0$ = other variables (year, location and so on) [dummy variable]

The results are shown by Table 5-33. They test the null hypothesis of homoskedasticity in the model, and reject the null hypothesis. Thus, they use

[67] "From our analysis it is clear that the prediction of a wetland's value based on previous studies is, at best, an imprecise science. The need for site-specific studies remains" (Woodward and Wui, 2001).

White's heteroskedasticity-consistent standard errors.[68] Considering the results and our interests we focus on, we use the option A.

Table 5-33. *Estimated models of the wetland valuation function*

Variable	Mean	Option A	Option B	Option C
Intercept	-	7.945[b](1.07)	6.641[b](1.31)	7.872[b](1.74)
Year	14.908	-0.052 (0.03)	-0.004 (0.04)	0.016 (0.04)
Ln acres	9.281	-0.168 (0.10)		-0.286[b](0.11)
Coastal	0.431	-0.523 (0.71)		-0.117 (0.68)
Flood	0.138	-0.358 (1.03)		0.678 (0.77)
Quality	0.2	1.494[c] (0.78)		0.737 (0.75)
Quantity	0.062	0.514 (1.60)		-0.452 (1.54)
Recreation Fish	0.354	0.395 (0.55)		0.582 (0.56)
Commodity Fish	0.277	0.669 (0.79)		1.360 (1.01)
Birdhunt	0.4	-1.311[b](0.49)		-1.055[b](0.52)
Birdwatch	0.277	1.704[b](0.52)		1.804[b](0.59)
Amenity	0.154	-3.352[b](0.92)		-4.303[b](0.95)
Habitat	0.308	0.577 (0.56)		0.427 (0.59)
Storm	0.031	0.310 (2.37)		0.173 (1.66)
Publish or not	0.769		-0.669 (0.72)	-0.154 (0.71)
Data0 [questionable = 1]	0.246		0.302 (0.56)	0.000 (0.60)
Theory0 [questionable = 1]	0.215		-1.020 (0.84)	-1.045 (0.84)
Metric0 [questionable = 1]	0.123		-4.030[b](1.21)	-3.186[b](1.22)
Producer's surplus or not	0.277	-2.416[b](0.83)	-2.034[b](0.72)	-3.140[b](0.86)
Hedonic pricing method	0.031		0.441 (1.02)	5.043[b](1.12)
Net factor income method	0.246		-0.724 (0.82)	0.273 (0.90)
Replacement cost method	0.277		1.376 (0.86)	2.232[b](0.89)
Travel cost method	0.108		-1.196[c](0.64)	-0.341 (1.05)
n	65	65	65	65
R^2	-	0.373	0.364	0.582

Source: Woodward and Wui (2001), Table 2.
a. Standard errors were calculated using White's (1980) correction for heteroskedasticity. All results were obtained using Shazam version 8.0 (White, 1997).
b. Significantly different from zero at the 5% level.
c. Significantly different from zero at the 10% level.

The area of wetland should depend on the control variable in our model. Except for this, all other variables are dummy variables. The problem is to interpret the statistical test for each explanatory variable; say whether each coefficient is statistically different from zero at the 5 or 10% level. Here, we should distinguish between "statistically significant" and "economically significant" (McCloskey, 1985; Goldberger, 1991; McCloskey and Ziliak, 1996).

"In many research reports, the author's conclusions emphasize the

[68] Refer to Wooldridge (2000) in this respect.

> statistical significance, rather than the economic significance, of the coefficient estimates. Yet, a coefficient estimate may be "very significantly different from unity" (by the t-test), while that difference is economically trivial. Or the difference may be "not significantly different from unity" but have an economically substantial magnitude. . . . It may be a good idea to reserve the term "significance" for the statistical concept, adopting "substantial" for the economic concept" (Goldberger, 1991, p.240).

Statistical significance is important, but it is not the whole story. Some explanatory variables are sometimes significant economically although they are not significant statistically.

Hence, we try to use all the explanatory variables except for year, and put zero into 'coastal', 'flood', 'storm' and 'PS' and one into the others because we focus on fluvial floodplains and all the possible ecosystem services except flood mitigation service in the terms of net benefit function.[69] Then, we obtain the following function:

$$\ln(V) = 8.635 - 0.168 \cdot \ln(x_a)$$

The unit of area in this equation is an acre. We convert the unit into hectare in order that we can use the function in our model. Substituting the factor of conversion into the function,

$$\ln(V) = 8.635 - 0.168 \cdot \ln(2.471 \cdot (L_F - X))$$

where

$$x_a = 2.471 \cdot (L_F - X)$$

$L_F - X$ is a notation used in our model whose unit is hectare. Arranging the function above,

$$\ln(V) = 8.48302 - 0.168 \cdot \ln(L_F - X)$$

Solving the equation for V,

$$V = e^{8.48302 - 0.168 \cdot \ln(L_F - X)}$$

The value of this function is still per year per hectare in 1990 "US dollar" terms. We need to convert the unit to per year per hectare in 1990 "GBP (£)" terms for

[69] Even if we put zero into the term of "flood" and "storm", there is a possibility that we double-count the value of flood mitigation service in our model. However, the function has not been a decisive model yet, and our knowledge of ecosystem services is still developing. Hence, there is also a possibility that we underestimate the value of ecosystem services at the same time.

our model.[70]

$$V_{ha,£} = 0.55978 \cdot e^{8.48302-0.168 \cdot \ln(L_F - X)} \tag{5-16}$$

Finally, we convert function (5-16) into the one that is fitted with our model.

$$B\left(L_F^i - X^i\right) = 0.55978\left(L_F^i - X^i\right)e^{8.48302-0.168 \cdot \ln\left(L_F^i - X^i\right)} \tag{5-17}$$

We use the function (5-17) for simulations in the next chapter.

To calculate the marginal benefit of ecosystem services from the function (5-17), differentiate the function with respect to the control variable,

$$\frac{dB}{dX^i} = -0.46573 \cdot e^{8.48302-0.168 \cdot \ln\left(L_F^i - X^i\right)} < 0$$

$$\frac{d^2B}{d\left(X^i\right)^2} = -0.07824 \cdot \left(L_F^i - X^i\right)^{-1} \cdot e^{8.48302-0.168 \cdot \ln\left(L_F^i - X^i\right)} < 0$$

The marginal value is always negative. Likewise, the second derivative of the function with respect to the control variable is also negative. This function is strictly concave. These conditions satisfy the assumptions we set in the model. In terms of the functional form, we should note the following. As we discussed in Chapter 2, the connectivity of floodplains in terms of location seems to play an important role in enhancing the extent of ecosystem services. Considering the connectivity, there should be a range in which the marginal value is increasing. However, we cannot consider such an effect of connectivity because we cannot distinguish the location beyond the demarcation of subbasins (zones) that we have divided in our model.

Using function (5-16), let us check the unit value (per year per hectare) based on the possible maximum area of natural floodplains in each subbasin of the Ouse catchment as compared with Table 5-32. Table 5-34 shows the unit values and the areas.[71] The unit value will be maximised when the term $L_F^i - X^i$ approaches

[70] The exchange rate is £ = 1.7864 US$ in 1990, based on the annual average of spot exchange rate (Source: Bank of England).

[71] The area of floodplains in each subbasin implies the maximum area of natural floodplains.

zero. We cannot define zero in the function. Thus, we put the value 0.0001 ha (1 m^2) to reflect the case of full exploitation. In the end, the range of unit values in the Ouse catchment in our model is 1,544 - 40,551 GBP (1990 £) per year per hectare, which is consistent with Table 5-32.

Table 5-34. *Unit value of ecosystem services in subbasins*

Gauging Stations	Name of Stations	Area of natural floodplains (ha)	Unit value of floodplains (1990 £ y^{-1} ha^{-1})
27001	Nidd at Hunsingore Weir	1,359.7	805
27002	Wharfe at Flint Mill Weir	2,629.7	720
27005	Nidd at Gouthwaite Reservoir	373.4	1,000
27007	Ure at Westwick Lock	2,498.0	727
27009	Ouse at Skelton	6,646.2	617
27034	Ure at Kilgram Bridge	2,476.9	728
27043	Wharfe at Addingham	1,402.5	801
27053	Nidd at Birstwith	348.0	1,012
27069	Wiske at Kirby Wiske	1,338.2	807
27071	Swale at Crakehill	3,621.6	683
27075	Bedale Beck at Leeming	1,029.1	843
27083	Foss at Huntington	1,119.7	832
27085	Cod Beck at Dalton Bridge	1,152.2	828
27089	Wharfe at Tadcaster	443.8	971
27090	Swale at Catterick Bridge	1,632.8	781
	Total	28,071.8	484
	Case of fully exploitation	0.0001	12,710

5.4.3 Benefit Function of Developed Floodplains

Economic benefits will be brought about if we develop floodplains to be used for economic activities. Economic rents will be generated by economic activities on developed lands. The benefits are variable, depending on economic activities. However, if we assume that all economic agents are rational and always try to maximise their profits, current economic activities are efficient in the absence of externalities. The values that they produce are maximal. We use the data on prices of residential lands. If the market for land is complete and efficient, the price of residential land coincides with the present value of the sum of the rents that the land owner can earn in the future (theoretically). This is based on a market equilibrium in which there is no arbitrage that can make any transactions more profitable. In addition, prices are in reality determined by current market conditions which include speculation. However, we want to use the notion as an assumption because we have no relevant data for considering the point. Assuming fixed economic rent from the land, we can show the land price by the following:

$$LP = \sum_{t=1}^{\infty} \frac{ER}{(1+r)^t} = \frac{ER}{r} \qquad (5\text{-}18)$$

where

LP = land price

ER = economic rent

r = discount rate (real interest rate)

This estimates economic benefits from developed areas and ignores externalities, the agglomeration effect of developed areas and the limitation of development.[72] The agglomeration effect is that the unit value (benefit) of developed lands will increase if the total area of developed lands increases and the developed lands are connected with each other in close proximity. This enhances the value of an additional unit of developed land. The agglomeration effect is composed of scale economies (increasing returns to scale) and a location effect (connectivity of economic activities in concentrated areas) that is related to transportation costs and the movement of productive factor (Fujita et al., 1999). On the other hand, there is a possibility that the problem of congestion occurs, which offsets the agglomeration effect. Furthermore, the value of development in peripheral areas will decrease due to the burden of expensive transportation cost and the limitation of population. Therefore, as long as the total area of developed lands is small enough, the marginal value of development may increase, but it may decrease once the total area of developed lands exceeds a certain threshold. Considering the initial conditions, we should assume a decreasing function or a

[72] Geographical aspects had been ignored in neoclassical economics. Not to mention, the perspective is critical in environmental and ecological economics. About geographical aspects in economics, refer to Henderson (1974); Segal (1976); Krugman (1991a, 1991b, 1995); Mori (1998); and Fujita et al. (1999).

linear function. However, we cannot consider the agglomeration effect and the limitation for estimating the benefits here because we have no relevant data.[73] Hence, we assume a linear function for the benefit function of developed floodplains.

$$F(X) = aX$$

We estimate the value of a by calculating the economic rent from the data on land prices, using equation (5-18).

The data on market land prices of residential areas is available from ODPM (Office of the Deputy Prime Minister). We can calculate a real interest rate from a nominal interest rate and an inflation rate. We use the data on annual average UK banks' base rates as the nominal interest rate. In order to calculate the inflation rate, we use RPI. Based on the land prices and the calculated real interest rate, we can derive the economic rent per year. Finally, we convert the value into 1990 UK £ price. Table 5-35 shows the results. We adopt the average value for the first five years in 1990s for the parameter value. We obtain the following function.

$$F(X^i) = 14030.80048X^i$$

Table 5-35. *Economic rent of residential land in Yorkshire and the Humber*

year	Economic rent at 1990 price (£ per ha)	Land price [1] (£ per ha)	Real interest rate (%)	Nominal interest rate [2] (%)	Inflation rate [3] (%)
1990	18,844	355,000	5.31	14.77	9.46
1991	17,833	324,859	5.81	11.68	5.87
1992	15,977	301,786	5.81	9.56	3.75
1993	9,970	251,596	4.42	6.01	1.59
1994	7,530	282,723	3.04	5.46	2.42
1995	9,492	348,517	3.22	6.69	3.47
1996	10,769	367,802	3.55	5.96	2.41
1997	9,191	335,000	3.43	6.57	3.14
1998	10,769	365,000	3.81	7.24	3.43
1999	11,314	390,000	3.81	5.34	1.53

Note

1. Simple average price. Source: Inland Revenue Valuation Office, "Table 561. Housing market: land prices private sector, by region".
2. Source: Bank of England, "Annual average of 4 UK banks' base rates".
3. RPI: All items retail prices index (January 1987=100). Source: UK National Statistics.

[73] This is beyond our scope. It can be one research topic.

5.4.4 Direct Cost Function of Floodplain Development and Restoration

We need the direct cost function of floodplain development and restoration with relevant parameter values in our model. However, there is no appropriate data on costs of floodplain development. The reason might be that this depends on the various site-specific and engineer-specific characteristics. Moreover, in terms of economics, we are always interested in opportunity costs of floodplain development such as costs of lost ecosystem services.

There are some data on wetland restoration, much of it from ecological point of view, but there are only a few pieces of literature that contain relevant data on costs of restoration. The reason may be that we have not sufficiently studied the mechanism of ecological restoration, and that we cannot precisely estimate the costs of restoration without the knowledge of the process of ecological restoration. Furthermore, the costs of restoration depend on the site-specific factors as well. Thus, good and appropriate data on costs of floodplain (wetland) restoration are unavailable about the Ouse catchment. In addition, it seems to be difficult to obtain and choose this type of data because the costs depend on the types of methods for restoration, the types of end uses, the level of restoration (to what extent we can recover biological productivity etc.) and so on. Moreover, data on costs of restoration span a wide range of values. We thus do not aim at deriving a precise and definite function with parameter values here. Instead, setting different parameter values, we analyse some scenarios in this respect for simulations in the next chapter.

1. We assume that the functional form is linear which satisfies the sufficiency conditions in the static and dynamic models in Chapter 4.
2. We assume that the coefficients are different between development and restoration. Since restoration is more difficult than development (Edwards and Abivardi, 1997; Mitsch and Gosselink, 2000b), we assume that the coefficient of restoration is ten times as large as that

of development.[74]

$$D(y^i) = ay^i \text{ if } y^i \geq 0. \quad \text{[development]}$$

$$D(y^i) = 10ay^i \text{ if } y^i < 0. \quad \text{[restoration]}$$

3. The value of coefficient of the function for floodplain restoration is obtained from the wider literature on wetland restoration..

Table 5-36. *Restoration costs (1995 US $1,000 per ha)*[a]

Site	Type of Land	Area (ha)	Restoration cost (1990 UK £)	Reference [original]	Original Unit
Los Angeles, USA	Wetland	120.0	18,264,959[b]	Edwards&Abivardi(1997) [NRC(1992)]	1995 US $
San Diego, USA	Riparian Wetland	3.0	1,038,208	Edwards&Abivardi(1997) [Guinon(1989)]	1995 US $
UK	Wetland (peatland)	50.0	3,298	Edwards&Abivardi(1997) [Wheeler&Shaw(1995)]	1995 US $
Ohio and Colorado, USA	Wetland (marsh)	0.6	39,657	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	0.6	354,266	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	1.9	271,434	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	6.9	475,880	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	5.4	274,953	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	4.2	126,901	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	7.3	37,013	Gutrich&Hitzhusen(2004)	2000 US $
Ohio and Colorado, USA	Wetland (marsh)	25.5	411,761	Gutrich&Hitzhusen(2004)	2000 US $
Kissimmee River, USA	Floodplain	10360.0	198,175,658	Kissimmee River Restoration Project (www.sfwmd.gov)	1997 US $

Note:
a. The unit is converted into 1990 UK £ by use of US GDP deflator (Source: US Department of Commerce) and the annual average of spot exchange rate (Source: Bank of England).
b. The original data is provided by the range of 6,087,328 - 30,442,591 (1990 UK £).

Table 5-36 shows the data that we can obtain on the relationship between areas and restoration costs. Using the data, we can calibrate the parameter values of the linear function above by a simple regression (OLS). We should note that it is difficult to derive statistically significant results because of the small number of observations.

$$D(y^i) = 19146.37353y^i \text{ if } y^i < 0. \qquad (5\text{-}19)$$

($R^2 = 0.993$, t-value = 41.024, p-value = 0.0000)

[74] Edwards and Abivardi (1997) compare the wetland restoration cost with the potential value of the restored land, and indicate that the former is at least around 100 times as large as the latter. Considering this, this assumption might be within reliable prediction even though we take account of direct cost of floodplain development. Fundamentally, it seems to be expensive to restore natural floodplains from urban developed areas as compared with the development.

$$D(y^i) = 1914.63735 y^i \quad \text{if} \quad y^i \geq 0. \tag{5-20}$$

We use functions (5-19) and (5-20) for the direct cost of floodplain restoration and development respectively as a base case in simulations in the next chapter.

5.4.5 *Constraints on Control Variables*

We set the direct cost functions of floodplain development and restoration in the previous section. Note, however, that functions 5-19 and 5-20 do not take the timing of costs into account. Since wetland restoration takes time, this is an important consideration for floodplain restoration. In this respect, Gutrich and Hitzhusen (2004) mention that it is critical to estimate and consider the economic restoration lag costs that are incurred while achieving natural functional equivalency. In fact, society is currently incurring significant lag costs although the lag costs have not been considered historically (Gutrich and Hitzhusen, 2004).

To capture this in the model, we apply capacity constraints on the control variables in simulations. That is, there is a limit to the area that it is possible to restore in any one time period. It is mathematically denoted by the following.

$$-X_t^i \leq \bar{y}_r^i \leq y_t^i \leq \bar{y}_d^i \leq L_F^i - X_t^i \quad \text{(for all } i\text{)}$$

To determine the values of $\bar{y}_r^i$ and $\bar{y}_d^i$, we assume that the time of restoration is ten times as long as that of development and then test the sensitivity of this assumption in the scenarios developed in the next chapter. We introduce the following as a base case in Section 6.6 in the next chapter.

$\bar{y}_r^i = 10$ (for all i) [restoration]

Then, $\bar{y}_d^i = 100$ (for all i) [development]

5.5 Conclusion of Chapter 5

To begin with, we set the discrete dynamic simulation model based on the continuous dynamic theoretical model developed in Chapter 4. Then, we set a hydrological sub-model for the expected cost function of flood risk and specify the other functions in the model. We calibrate the parameter values and specify the functional forms adopted in the sub-model. Thus, we set several assumptions in the process of specification and calibration, which are necessary but might be a source of bias. Our main purpose is not to precisely evaluate the current real situation but to evaluate and identify optimal policies in simulations. In the next chapter, we carry out policy simulations, using the discrete dynamic simulation model.

Chapter 6

Policy Simulation

6.1 Introduction

For the last 30 years, flood events have become more frequent and more serious in the Ouse catchment. The purpose of this chapter is to implement simulations in order to evaluate (a) outcomes under different management regimes and (b) the robustness of model projections to uncertainty about model parameters and structure. We use the applied simulation model specified and calibrated in Chapter 5. In Section 6.2, we show to what extent the change in the area of developed floodplains has an impact on the expected cost of flood risk. In Section 6.3, we use simulations to identify the optimal management strategy, making the problem of externalities clear. In Section 6.4, we discuss the problem of discounting. Discounting often has a large impact on our decisions. We test the sensitivity of the outcomes to the choice of discount rates. In Section 6.5, we attempt to identify the optimal policy, and discuss the outcomes of several potential polices. In Section 6.6, we discuss the problem of irreversibility. In Section 6.7, we test the sensitivity of the outcomes to economic parameters such as the value of ecosystem services and the value of developed lands, and to an environmental variable such as precipitation. In Section 6.8, we conclude the chapter.

6.2 Importance of Floodplain Management

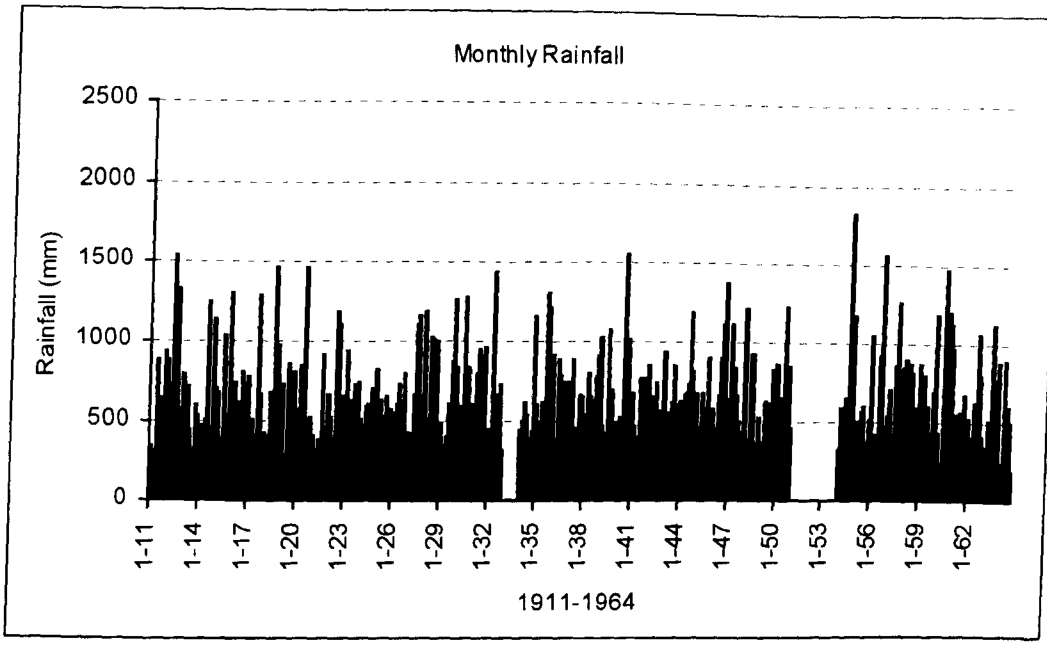
In this section, we analyse the impact of changes in control variables (floodplain development) on the expected cost of flood risk. To be more precise, we clarify to what extent floodplain management influences the expected cost of flood risk (probability of flood occurrence) in simulations. It is often said that the number of floods has increased for the last 30-50 years (we will check the data later). However, there is no agreement about the main causes of this. One of the hypotheses is that the changes in land use including floodplain development have had an impact. Unfortunately, we cannot directly test or verify this hypothesis because the land use data are not available historically (as time series data). Thus, an analysis based on simulations can be treated as an 'indirect' test of this hypothesis.

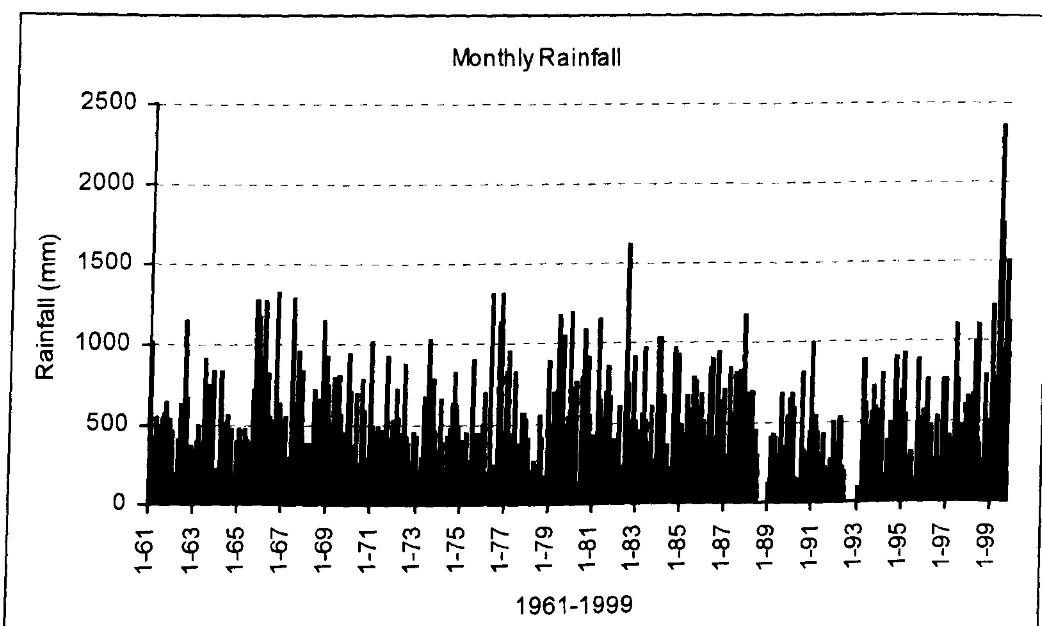
6.2.1 Historical Data on Flood Events

To begin with, let us observe the historical data on flood events and describe the trends. We take York as a sample. Figure 6-1 shows the trend of monthly rainfall for the long period 1911-1999. However, there are no data based on the same MET rainfall gauging station covering the whole duration. Figure 6-1(a) shows the trend of monthly rainfall at the station "York" for the period 1911-1964. Figure 6-1(b) shows it at the station "York Acomb Landing TR.WKS" for the period 1961-1999. They are away from each other by 2 km. We do not think that there is a big difference between them, but we should recognize that the data come from the different stations. Looking at them, it seems that there is neither a big change nor a clear trend.[1]

[1] However, we cannot observe the intensity of rainfalls for a short duration from it.

Figure 6-1 (a) & (b). *Long trend of monthly rainfall in York*



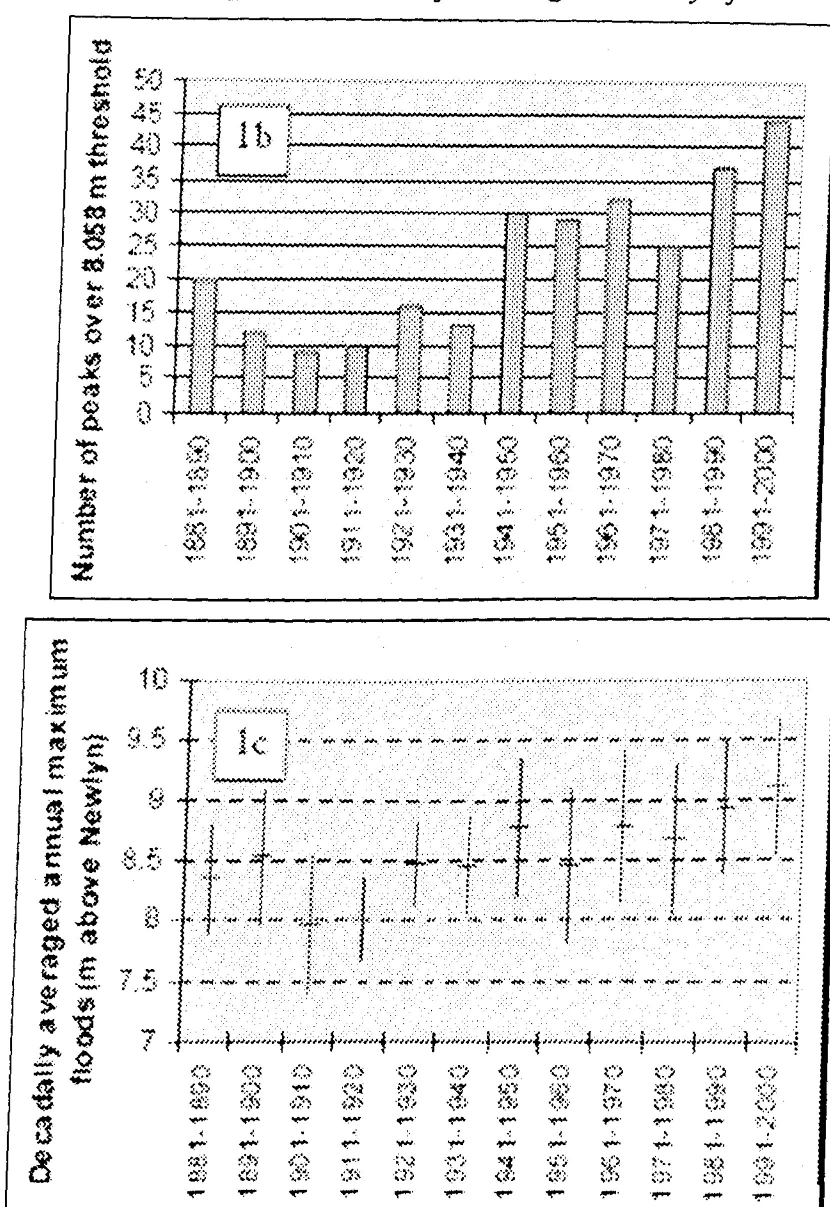


Source: Met Office - UK Land Surface Stations data (1900-present) [obtained from the British Atmospheric Data Centre (BADC)].
Note: (a) York: The data in 1933 and 1951-53 are missing. (b) York Acomb Landing TR.WKS: The missing data are: Jul-Dec 1988, Jul-Dec 1992, Jan and June 1993, Feb 1996, and Nov-Dec 1999.

Interestingly, however, there seems to be a clear increasing trend about flood events. Figure 6-2 shows the flood record for Viking in the City of York. The number of peaks over a threshold value (8.058m) is shown for the long period 1881-2000. An obvious change is observed around 1950. Rather than a trend, flood risk suddenly became high around 1950 and the high flood risk has continued. What has changed?

We hypothesise that floodplain development (changes in land use) plays a crucial role in controlling flood risk. That is, land use changes have altered both run-off and the damage cost of flooding. The evidence suggests that land-use changes in the post-war years marked a threshold.

Figure 6-2. *Flood record for Viking in the City of York*



Source: Lane (2003), Figure 1.

6.2.2 *Impact of Floodplain Development on Expected Cost of Flood Risk*

Let us analyse the impact of changes in the size of developed floodplains on the expected cost of flood risk in simulations, which indirectly tests the hypothesis. Table 6-1 shows the scheme for simulations. First, we set seven initial conditions on floodplain development. They show the percentage of the area of developed floodplain in floodplains, 5, 25, 50, 75 and 95%. In addition, we set the different percentages on upstream and downstream zones as other two initial conditions. In so doing, we define subbasins 27005, 27007, 27034, 27043, 27053, 27069, 27071, 27075, 27085, and 27090 as the upstream zones, and subbasins 27001, 27002, 27009, 27083 and 27089 as the downstream zones (See Figure 6-3). Second, we analyse three scenarios: increase the area of developed floodplains by one ha per subbasin in all zones, in only upstream zones, and in only downstream zones. We analyse 21 cases in simulations in total.

Table 6-1. *Scheme for simulations*

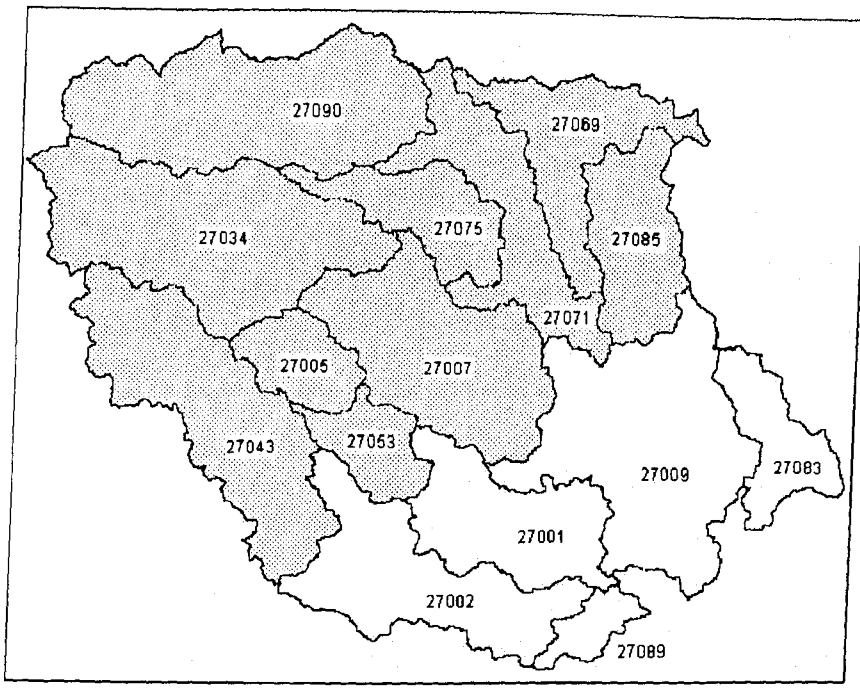
Initial condition: % of developed floodplains in floodplains	Floodplain development in all areas	Floodplain development in upstream areas	Floodplain development in downstream areas
5%	Case 1	Case 2	Case 3
25%	Case 4	Case 5	Case 6
50%	Case 7	Case 8	Case 9
75%	Case 10	Case 11	Case 12
95%	Case 13	Case 14	Case 15
Upstream: 25% Downstream: 75%	Case 16	Case 17	Case 18
Upstream: 75% Downstream: 25%	Case 19	Case 20	Case 21

Then, we calculate the following (IPT) as the measure of the impact of the change in control variables on the expected cost of flood risk.

$$IPT = \frac{C(X+h)-C(X)}{h} \qquad (6\text{-}1)$$

where C is the expected cost function of flood risk in the catchment, X is the initial area of developed floodplains, and h denotes the change in the area of developed floodplains (1 ha per subbasin in this simulation).

Figure 6-3. *Definition of upstream and downstream zones*



Source: We create this map from OS Land-Form PANORAMA™ DTM [1:50,000] (EDINA Digimap) by use of ArcGIS.
Note: The shaded areas denote upstream areas.

Table 6-2 shows the results. Table 6-2 coincides with Table 6-1. There are a few interesting findings. First, the impact of floodplain development (change in control variables) on the expected cost of flood risk is substantial.[2] Comparing the values obtained (the range is 9,017 - 11,053 1990 UK £) with the economic rents on developed lands (the average is 12,168 1990 UK £), they turn out to be large. This shows the importance of floodplain management for alleviating flood risk. Second, there are unidirectional spatial externalities of floodplain development. Comparing the values in each row of the table, the following relationship holds true.

[Development in downstream zones] < [Development in all zones]

< [Development in upstream zones]

The impact of floodplain development in upstream zones is the largest because it has an external effect on the expected cost of flood risk in downstream zones. The

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[2] See Table 5-35 in Chapter 5.

upstream effect, however, is completely unchanged.

Table 6-2. *Impact of flood development on expected cost of flood risk*

Initial condition: % of developed floodplains in floodplains	Floodplain development in all areas	Floodplain development in upstream areas	Floodplain development in downstream areas
5%	10,374	11,053	9,017
25%	10,425	11,053	9,169
50%	10,473	11,053	9,314
75%	10,526	11,053	9,472
95%	10,526	11,053	9,472
Upstream: 25% Downstream: 75%	10,444	11,053	9,227
Upstream: 75% Downstream: 25%	10,481	11,053	9,338

Note: The unit is 1990 UK £. The values show the increase in the expected cost of flood risk per 1-hectare increase in the area of developed floodplains. See function (6-1).

6.3 Optimisation

In this section, we find the optimal steady-state equilibrium from the conditions obtained in theoretical models in previous chapters, and derive the optimal path from the initial conditions. Then, we simulate a measure of social welfare in some scenarios.

6.3.1 Optimal Steady-state Equilibrium

Let us derive the optimal steady-state equilibrium by use of the steady-state conditions derived in previous chapters. In the steady-state equilibrium, the following conditions must be satisfied in all the subbasins.

$$\frac{dF}{dX_t^i} = -\frac{dB}{dX_t^i} + \frac{\partial C^i}{\partial X_t^i} + \sum_{j>i} \frac{\partial C^j}{\partial X_t^i} \quad \text{(for all } i\text{)} \qquad (6\text{-}2)$$

If we can solve the equations for all X^i, we can obtain the optimal size of developed floodplains in all the subbasins in the equilibrium.[3] Notice that these conditions are the same as the optimal conditions in the static decision model in Chapter 3.

Table 6-3. *Optimal steady-state equilibrium*

Subbasin	Size of developed floodplains in equilibrium (ha)	Initial size of developed floodplains (ha)	Size of floodplains (ha)
27001	239.19083	99.39369	1356.91346
27002	1987.28207	217.13234	2402.75427
27005	0 [-39.99124]	14.38793	204.67004
27007	503.39352	155.26437	2447.00171
27009	1426.77342	308.46632	6639.46521
27034	108.47819	51.12944	2451.61496
27043	161.96300	27.69161	1257.75186
27053	0 [-4836.83615]	7.97054	347.13385
27069	38.67005	101.62420	1338.21609
27071	685.14809	306.14877	3595.16936
27075	24.88389	63.81431	1029.10755
27083	1116.83288	106.27285	1119.66767
27085	119.85753	60.06330	1149.58292
27089	318.88009	35.18348	443.79102
27090	32.49644	67.09430	1632.33813

Note: The values in the bracket show the calculated values, based on the conditions obtained from theoretical models.

Table 6-3 shows the results. We obtain interior solutions in all but two subbasins. In the case of corner solutions, no floodplain development is optimal because of the negative values in the table. Based on these results, overdevelopment of floodplains occurs in 5 subbasins, 27005, 27053, 27069, 27075 and 27090. Interestingly, these five subbasins are upstream zones in the Ouse catchment. While this indicates the importance of managing unidirectional spatial externalities, we cannot draw any definite conclusions. As mentioned in Chapter 5, we have to carefully interpret the results because of the assumptions made in the process of setting functional forms and calibrating parameter values for the applied model. It is better to think that the model identifies the characteristics of the problem and implications for policy-making rather than the

[3] We obtain the solutions by help of Maple 8.0 (mathematical software program).

'precise' evaluation of the current situation. Of course, if we could fully access the appropriate and exact data, we could evaluate the current situation by the model.

Table 6-4. *Optimal steady-state equilibrium under different assumptions*

	Base 2% and smaller	4% and smaller	10% and smaller	Initial Conditions
27001	239.190	121.260	47.454	99.393
27002	1987.282	1056.718	456.111	217.132
27005	0 [-39.991]	0 [-66.421]	0 [-78.975]	14.387
27007	503.393	31.840	0 [-174.915]	155.264
27009	1426.773	708.483	263.442	308.466
27034	108.478	0 [-96.051]	0 [-159.282]	51.129
27043	161.963	54.561	0 [-10.483]	27.691
27053	0 [-4836.836]	0 [-14990.79]	0 [-42257.166]	7.970
27069	38.670	0 [-102.279]	0 [-213.736]	101.624
27071	685.148	0 [-85.566]	0 [-621.958]	306.148
27075	24.883	0 [-68.553]	0 [-148.239]	63.814
27083	1116.832	594.245	279.655	106.272
27085	119.857	0 [-351.824]	0 [-831.844]	60.063
27089	318.880	174.985	82.382	35.183
27090	32.496	0 [-86.514]	0 [-187.556]	67.094

Note: The shaded cells imply the overdevelopment. The values in the bracket show the calculated values, based on the conditions obtained from theoretical models.

In particular, the assumption about the scale of averting behaviour is crucial. We assume that the current level of flood protection (flood averting behaviour) can protect against floods with 2% and larger annual exceedance probability, but not floods with 2% and smaller annual exceedance probability.[4] As the theoretical models show, we have to choose the scale of averting behaviour as well, comparing the expected cost of flood risk with the investment costs and maintenance costs of averting behaviour. However, we do not analyse the scale of averting behaviour here. Thus, we cannot offer a definite conclusion on whether floodplains have been overdeveloped unless we analyse the scale of averting behaviour. There is no reason to believe the current scale of averting behaviour is correctly chosen. If this changes, the situation will dramatically change. Then, let us check the same equilibrium under the other assumptions. Table 6-4 shows the results.[5] The evaluation of the current situation changes a lot. For example, if we

[4] We assume that the probability distribution function is continuous.

[5] The parameter values in the expected cost function of flood risk change according to the assumptions. They are provided in Appendix D-1

assume that the current scale of averting behaviour protect against floods with 10% and larger exceedance probability, floodplains are currently overdeveloped in all but three subbasins. Thus, this assumption is critical for evaluating the current situation of floodplains.

In addition, the upstream zones tend to show overdevelopment, based on these results. This is because the development of floodplains in the upstream zones causes unidirectional spatial externalities. It coincides with the fact that many cities are located in downstream zones. The more upstream zones are developed, the greater the need to protect cities in the downstream zones against floods.

6.3.2 Optimal Path

Let us derive the optimal path from the initial conditions in terms of the maximisation of social welfare.[6] The essence of the social optimisation problem in the applied model in Chapter 5 is the same as the theoretical model in Chapter 4. Based on the applied model, we numerically solve the dynamic optimisation problem by using GAMS IDE version 21.3 (a software program for solving large mathematical programming problems).[7] We use GAMS in simulations in the following sections in this chapter.

We set the time duration T = 30 (years). There are three reasons. First, a planning horizon that is shorter than 30 years might be insufficient for policies to be effective. Second, the parameter values in the functions may change due to several shocks for much longer planning horizons. Third, given the rate by which public investments are discounted, a period of thirty years captures the bulk of

[6] We obtain the initial conditions from the GIS land cover map in Chapter 5.
[7] GAMS IDE demo version is downloadable from the web site (www.gams.com). Demo version restricts the number of variables that we can deal with.

future costs.[8] The planning horizon is, however, one of the decision variables for policies.

Table 6-5. *Optimal path*

Control Variable [y] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	137.359	0.117	0	0	0	0
27002	1750.247	1.306	0	0	0	0
27005	-14.388	0.000	0	0	0	0
27007	337.278	1.054	0	0	0	0
27009	1102.745	1.820	0	0	0	0
27034	52.842	0.427	0	0	0	0
27043	131.948	0.147	0	0	0	0
27053	-7.971	0.000	0	0	0	0
27069	-35.588	-1.666	0	0	0	0
27071	363.604	1.104	0	0	0	0
27075	-21.051	-1.075	0	0	0	0
27083	1009.199	0.102	0	0	0	0
27085	51.034	0.699	0	0	0	0
27089	280.532	0.159	0	0	0	0
27090	-11.856	-1.392	0	0	0	0

State Variable [X] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	99.394	236.753	236.870	236.870	236.870	236.870
27002	217.132	1967.379	1968.685	1968.685	1968.685	1968.685
27005	14.388	0.000	0.000	0.000	0.000	0.000
27007	155.264	492.542	493.596	493.596	493.596	493.596
27009	308.466	1411.211	1413.031	1413.031	1413.031	1413.031
27034	51.129	103.971	104.398	104.398	104.398	104.398
27043	27.692	159.640	159.787	159.787	159.787	159.787
27053	7.971	0.000	0.000	0.000	0.000	0.000
27069	101.624	66.036	64.370	64.370	64.370	64.370
27071	306.149	669.753	670.857	670.857	670.857	670.857
27075	63.814	42.763	41.688	41.688	41.688	41.688
27083	106.273	1115.472	1115.574	1115.574	1115.574	1115.574
27085	60.063	111.097	111.796	111.796	111.796	111.796
27089	35.183	315.715	315.874	315.874	315.874	315.874
27090	67.094	55.238	53.846	53.846	53.846	53.846

We can obtain the optimal path from the original initial point by use of GAMS IDE.[9] Table 6-5 shows the optimal path. Given the form of the problem, the state converges to equilibrium from the initial point. The optimal path derives

[8] The final point is related to discounting.

[9] The periodic discount rate is assumed 0.05 as a base case. We show the GAMS code in Appendix D-2.

from the social optimisation problem, in which we consider all the benefits and costs including externalities. Thus, this becomes the criterion for optimum.

There are three interesting findings about the optimal path. First, it is not the most rapid approach path, but the initial state rapidly converges towards a solution. This implies that we should control to a large extent in early stages. However, considering floodplain development and restoration in reality, we cannot control the size of developed floodplains in such a large scale immediately because it normally takes time to develop and restore them. In this respect, we discuss in Section 6.6.1. Second, the equilibrium numerically obtained along the optimal path is different from that theoretically obtained in the previous section. Based on the analysis of the theoretical models, the optimal steady-state equilibrium is unstable. Thus, even though an initial state approaches the optimal steady-state equilibrium, it never converges to the optimal steady-state equilibrium. On the other hand, the initial state will not diverge to corner solutions but converge to the equilibrium that is different from the optimal steady-state equilibrium theoretically obtained. We cannot predict this in the analysis of local stability based on the theoretical model in Chapter 4. We can know only about the local sphere on the equilibrium point. In such a high dimensional system, we cannot know the definite characteristics of the direction of changes unlike two-dimensional models. We have no way to verify the followings, but we can imagine two possibilities. It seems that 'slow' adjustments occur in the sphere near to the equilibrium but the state will 'finally' start to diverge in the far future. Alternatively, it appears that the state will quickly move to the orbit of a vortex near to the equilibrium around which 'slow' adjustments will occur forever thereafter. Third, the optimal path achieves corner solutions (no developed floodplains) in the two subbasins (27005 and 27053) just as the previous section indicates.

6.3.3 Externalities

As we discussed in Chapter 3, there are two kinds of externalities in our problem. First, there is an externality between private and public use. In other words, private landowners of floodplains have no incentive to take account of the public interests such as the values of ecosystem services that natural floodplains provide. In our model, private landowners underestimate the value of the benefit function of ecosystem services. Second, there is a unidirectional spatial externality related to flood mitigation service that natural floodplains provide. In our model, decision makers in upstream zones underestimate the external costs of flood risk that people in the downstream zones incur. About both kinds of externalities, floodplains tend to be overdeveloped. In this section, let us analyse how externalities affect the optimal path from the initial conditions.

Let us derive the path from the initial conditions in the same way when the externalities are not considered (private or local optimisation). Table 6-6 shows the path. The initial state converges to equilibrium. The areas of developed floodplains in the equilibrium are much larger than those in equilibrium in the case of the optimal path (See Table 6-7). Thus, the value of social welfare with externalities ignored is lower than that of the optimal path (See Table 6-8). Therefore, it is required to take account of externalities by appropriate policies. In the next section, we come up with possible policy scenarios.

Table 6-6. *Path when externalities are not considered*

Control Variable [*y*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	155.632	0.811	0	0	0	0
27002	1930.379	7.548	0	0	0	0
27005	36.084	0.171	0	0	0	0
27007	920.992	3.465	0	0	0	0
27009	1192.880	6.406	0	0	0	0
27034	395.848	1.438	0	0	0	0
27043	211.556	0.774	0	0	0	0
27053	339.163	0.000	0	0	0	0
27069	180.827	0.911	0	0	0	0
27071	1262.713	5.041	0	0	0	0
27075	120.845	0.597	0	0	0	0
27083	1013.395	0.000	0	0	0	0
27085	827.985	2.863	0	0	0	0
27089	305.254	1.088	0	0	0	0
27090	167.407	0.751	0	0	0	0

State Variable [*X*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	99.394	255.026	255.837	255.837	255.837	255.837
27002	217.132	2147.511	2155.059	2155.059	2155.059	2155.059
27005	14.388	50.472	50.643	50.643	50.643	50.643
27007	155.264	1076.256	1079.721	1079.721	1079.721	1079.721
27009	308.466	1501.346	1507.752	1507.752	1507.752	1507.752
27034	51.129	446.977	448.415	448.415	448.415	448.415
27043	27.692	239.248	240.022	240.022	240.022	240.022
27053	7.971	347.133	347.133	347.133	347.133	347.133
27069	101.624	282.451	283.362	283.362	283.362	283.362
27071	306.149	1568.862	1573.903	1573.903	1573.903	1573.903
27075	63.814	184.659	185.256	185.256	185.256	185.256
27083	106.273	1119.667	1119.667	1119.667	1119.667	1119.667
27085	60.063	888.048	890.911	890.911	890.911	890.911
27089	35.183	340.437	341.525	341.525	341.525	341.525
27090	67.094	234.501	235.252	235.252	235.252	235.252

Table 6-7. *Areas of developed floodplains in equilibrium (ha)*

Subbasin	Externalities are not considered.		Optimal Path
27001	255.837	>	236.870
27002	2155.059	>	1968.685
27005	50.643	>	0
27007	1079.721	>	493.596
27009	1507.752	>	1413.031
27034	448.415	>	104.398
27043	240.022	>	159.787
27053	347.133	>	0
27069	283.362	>	64.370
27071	1573.903	>	670.857
27075	185.256	>	41.688
27083	1119.667	>	1115.574
27085	890.911	>	111.796
27089	341.525	>	315.874
27090	235.252	>	53.846

Table 6-8. *Values of social welfare*

	Social Welfare
Optimal path	1,059,544
Path when externalities are not considered	1,049,394

6.4 Discounting

In this section, we simulate the paths of social and local optimisations according to several discount rates. Discounting in principle implies how much we should consider social welfare of future generations as compared with that of the current generation. Therefore, this influences our decisions on the choice of the values of control variables over time. We are here interested in how discounting affects them in the problem of floodplain management.

Discounting is one of the critical decision variables for policies. In this respect, there are two extreme ethical notions. Let us cite the followings from Dasgupta (2001).

"A. Low rates of consumption by generations sufficiently far into the future would not be seen to be a bad thing by the current generation if future well-beings were discounted at a positive rate. This suggests we should follow Ramsey in not discounting future well-beings.

B. As there are to be a lot of future generations in a world with an indefinite future, not to discount future well-beings could mean that the present generation would be required to do too much for the future; that is, they would have to save at too high a rate. This suggests we should abandon Ramsey and discount future well-beings at a positive rate" (Dasgupta, 2001).

Dasgupta (2001) mentions that it is necessary to discount the future because the world will not exist forever although we do not know when it will cease to exist. Thus, as long as the world will not exist forever, we have to discount the future by even a tiny discount rate. There are many rationales for discounting such as inflation, physical change over time and so on (Price, 1993). In addition, there are two major approaches to determine the appropriate discount rate: the normative or ethical perspective (the prescriptive approach) and the positive perspective (the descriptive approach) (Arrow et al., 1996). In our analysis, we simulate the model for several discount rates including the zero rates, and discuss the results.

Table 6-9. *Optimal paths for several discount rates*

Control Variable [*y*] (ha)

$\delta = 0.00$	Time (*t*)					
	0	1	2	3	. . .	30
27001	138.602	0	0	0	0	0
27002	1760.548	0	0	0	0	0
27005	-14.388	0	0	0	0	0
27007	343.098	0	0	0	0	0
27009	1111.264	0	0	0	0	0
27034	55.254	0	0	0	0	0
27043	133.150	0	0	0	0	0
27053	-7.971	0	0	0	0	0
27069	-49.718	0	0	0	0	0
27071	371.662	0	0	0	0	0
27075	-30.275	0	0	0	0	0
27083	1009.972	0	0	0	0	0
27085	55.656	0	0	0	0	0
27089	282.150	0	0	0	0	0
27090	-23.602	0	0	0	0	0

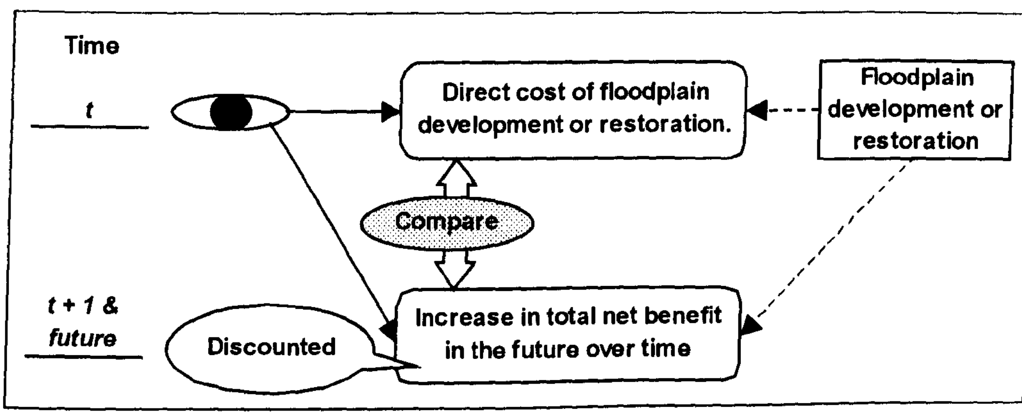
δ = 0.01	Time (t)					
	0	1	2	3	· · ·	30
27001	138.405	0	0	0	0	0
27002	1759.025	0	0	0	0	0
27005	-14.388	0	0	0	0	0
27007	342.252	0	0	0	0	0
27009	1110.054	0	0	0	0	0
27034	54.901	0	0	0	0	0
27043	132.963	0	0	0	0	0
27053	-7.971	0	0	0	0	0
27069	-47.545	0	0	0	0	0
27071	370.431	0	0	0	0	0
27075	-28.852	0	0	0	0	0
27083	1009.863	0	0	0	0	0
27085	54.960	0	0	0	0	0
27089	281.907	0	0	0	0	0
27090	-21.794	0	0	0	0	0

δ = 0.03	Time (t)					
	0	1	2	3	· · ·	30
27001	137.901	0.063	0	0	0	0
27002	1753.413	1.904	0	0	0	0
27005	-14.388	0	0	0	0	0
27007	340.392	0.042	0	0	0	0
27009	1107.472	0.051	0	0	0	0
27034	54.127	0.015	0	0	0	0
27043	132.430	0.122	0	0	0	0
27053	-7.971	0	0	0	0	0
27069	-42.608	-0.075	0	0	0	0
27071	367.715	0.046	0	0	0	0
27075	-25.626	-0.048	0	0	0	0
27083	1009.605	0	0	0	0	0
27085	53.429	0.028	0	0	0	0
27089	281.318	0.011	0	0	0	0
27090	-17.697	-0.060	0	0	0	0

δ = 0.07	Time (t)					
	0	1	2	3	· · ·	30
27001	136.683	0.248	0	0	0	0
27002	1745.216	1.853	0	0	0	0
27005	-14.388	0	0	0	0	0
27007	335.443	0.604	0	0	0	0
27009	1100.805	0.576	0	0	0	0
27034	52.044	0.273	0	0	0	0
27043	131.361	0.237	0	0	0	0
27053	-7.971	0	0	0	0	0
27069	-28.472	-2.711	0	0	0	0
27071	360.506	0.880	0	0	0	0
27075	-16.387	-1.768	0	0	0	0
27083	1008.759	0.181	0	0	0	0
27085	49.341	0.516	0	0	0	0
27089	279.674	0.293	0	0	0	0
27090	-5.955	-2.250	0	0	0	0

Let us derive the optimal paths (social optimisation) for several discount rates. Table 6-9 shows the optimal paths. As discount rates become higher, the amounts of development and restoration both decrease. Higher discount rates provide the incentives to develop or restore them less. In the zones where the optimal level of developed floodplains is larger than the initial level, the amounts of developed floodplains in equilibrium decrease as discount rates go higher. In the zones where the optimal level is smaller than the initial level, the amounts of developed floodplains in equilibrium increase (the amounts of restoration decreases) as discount rates go higher. Why?

Figure 6-4. *Influence of discounting on decisions*



The increase in total net benefit in time *t+1* and beyond is caused by floodplain development or restoration in time *t*. When we determine how much we develop or restore floodplains in time *t*, we compare the direct cost of floodplain development or restoration in time *t* with the increase in total net benefit in time *t+1* and future (see Figure 6-4). Assuming that we stand in time *t*, the increase in total net benefit in time *t+1* and future is discounted. Therefore, if the discount rate goes higher, the value of the increase in total net benefit in time *t+1* will be smaller. The incentive to develop or restore floodplains in time *t* will be alleviated.

Hence, interestingly, the risk of overdevelopment becomes smaller as discount rates rise in the zones where the optimal level is larger than the initial level. On the other hand, the incentive to restore floodplains becomes weaker as we assume higher discount rates in the zones where the optimal level is smaller than the initial level. Therefore, from the normative point of view, the optimal discount rate depends on the evaluation of the current situation. Which is more important, the avoidance of the risk of overdevelopment or the promotion of restoration?

6.5 Analysis of Policy Scenarios under Certainty

In this section, we simulate the effects of a number of different policy options and the paths (adjustment processes) associated with them.[10] We simulate a measure of social welfare under policy scenarios, and discuss the choice of policy scenarios. We suppose that the policy-makers should be (local) governments or governmental agencies.

We take up the Pigouvian tax, subsidies, the mix of tax and subsidy, two types of marketable permits and two direct controls as possible policy scenarios. We will discuss the characteristics of the policy scenarios respectively at first. Then, we will compare the values of social welfare and discuss the choice of them under certainty.

6.5.1 Pigouvian Tax

The Pigouvian tax is one of the famous policies for providing an appropriate economic incentive to consider externalities. As we mentioned in Chapter 3, we impose the unit tax rate on developed floodplains (per ha per year), which is equal to the marginal external costs. The unit tax rate is different among zones because the marginal external costs are different among them. Table 6-10 shows the

[10] GAMS codes for the policy analysis are given in Appendix D-2.

calculated unit tax rates in zones (subbasins). Let us simulate the path under the tax policy. Table 6-11 shows the path from the initial conditions. As externalities are considered through the tax appropriately imposed on developed floodplains, the path is really close to the optimal path. The local optimal path is corrected by the tax.

Table 6-10. *Unit tax rates on developed floodplains*

Subbasin	Unit tax rate (1990 UK£ per ha per year)
27001	1030.94420
27002	1202.78708
27005	25037.64832
27007	7547.76757
27009	873.20209
27034	10666.99101
27043	4648.04262
27053	24835.47271
27069	12135.56239
27071	7977.45707
27075	12165.16123
27083	2115.08188
27085	12159.27972
27089	1044.79636
27090	12111.60695

Table 6-11. *Path under tax*

Control Variable [y] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	137.180	0.300	0	0	0	0
27002	1749.503	2.027	0	0	0	0
27005	-14.388	0.000	0	0	0	0
27007	337.116	1.293	0	0	0	0
27009	1102.948	1.672	0	0	0	0
27034	52.748	0.543	0	0	0	0
27043	131.806	0.291	0	0	0	0
27053	-7.971	0.000	0	0	0	0
27069	-34.020	-3.281	0	0	0	0
27071	362.970	1.779	0	0	0	0
27075	-20.041	-2.111	0	0	0	0
27083	1007.678	1.607	0	0	0	0
27085	50.865	0.906	0	0	0	0
27089	280.294	0.402	0	0	0	0
27090	-10.551	-2.735	0	0	0	0

State Variable [X] (ha)

	Time (t)					
	0	1	2	3	• • •	30
27001	99.394	236.574	236.874	236.874	236.874	236.874
27002	217.132	1966.635	1968.662	1968.662	1968.662	1968.662
27005	14.388	0.000	0.000	0.000	0.000	0.000
27007	155.264	492.380	493.673	493.673	493.673	493.673
27009	308.466	1411.414	1413.086	1413.086	1413.086	1413.086
27034	51.129	103.877	104.420	104.420	104.420	104.420
27043	27.692	159.498	159.789	159.789	159.789	159.789
27053	7.971	0.000	0.000	0.000	0.000	0.000
27069	101.624	67.604	64.323	64.323	64.323	64.323
27071	306.149	669.119	670.898	670.898	670.898	670.898
27075	63.814	43.773	41.662	41.662	41.662	41.662
27083	106.273	1113.951	1115.558	1115.558	1115.558	1115.558
27085	60.063	110.928	111.834	111.834	111.834	111.834
27089	35.183	315.477	315.879	315.879	315.879	315.879
27090	67.094	56.543	53.808	53.808	53.808	53.808

6.5.2 Subsidy

In principle, the function of subsidies is the same as that of Pigouvian tax, but their monetary flows are opposite. As we showed in Chapter 3, we provide individual landowners with the unit subsidy on natural floodplains (per ha per year), which is equal to the marginal external costs. Landowners have the incentive to keep or restore floodplains more because of subsidies. The unit subsidy rates are the same as the unit tax rates (see Table 6-10). Let us simulate the path under the subsidy policy. Table 6-12 shows the derived path from the same initial conditions. As externalities are considered through the subsidy correctly given to natural floodplains, the path is close to the optimal path.

Table 6-12. *Path under subsidy*

Control Variable [*y*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	137.425	0.061	0	0	0	0
27002	1750.917	0.538	0	0	0	0
27005	-14.388	0.000	0	0	0	0
27007	338.118	0.271	0	0	0	0
27009	1104.261	0.361	0	0	0	0
27034	53.183	0.095	0	0	0	0
27043	132.044	0.045	0	0	0	0
27053	-7.971	0.000	0	0	0	0
27069	-36.719	-0.511	0	0	0	0
27071	364.414	0.338	0	0	0	0
27075	-21.746	-0.364	0	0	0	0
27083	1009.029	0.281	0	0	0	0
27085	51.565	0.186	0	0	0	0
27089	280.618	0.073	0	0	0	0
27090	-12.784	-0.445	0	0	0	0

State Variable [*X*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	99.394	236.819	236.880	236.880	236.880	236.880
27002	217.132	1968.049	1968.587	1968.587	1968.587	1968.587
27005	14.388	0.000	0.000	0.000	0.000	0.000
27007	155.264	493.382	493.653	493.653	493.653	493.653
27009	308.466	1412.727	1413.088	1413.088	1413.088	1413.088
27034	51.129	104.312	104.407	104.407	104.407	104.407
27043	27.692	159.736	159.781	159.781	159.781	159.781
27053	7.971	0.000	0.000	0.000	0.000	0.000
27069	101.624	64.905	64.394	64.394	64.394	64.394
27071	306.149	670.563	670.901	670.901	670.901	670.901
27075	63.814	42.068	41.704	41.704	41.704	41.704
27083	106.273	1115.302	1115.583	1115.583	1115.583	1115.583
27085	60.063	111.628	111.814	111.814	111.814	111.814
27089	35.183	315.801	315.874	315.874	315.874	315.874
27090	67.094	54.310	53.865	53.865	53.865	53.865

6.5.3 Mix of Tax and Subsidy

We can use the mix of the Pigouvian tax and subsidies for the same purpose. In essence, we should make individual landowners consider the marginal external costs when they change areas of developed floodplains. We can impose the unit tax (per ha per year) on the amounts of developed floodplains that exceed a reference value and set the unit subsidy (per ha per year) on the amounts of natural floodplains that exceed the same reference value instead of setting the tax (subsidy) on all the developed floodplains (natural floodplains). In this case, the unit tax and subsidy rates are the same as before (see Table 6-10).

We have several options for choosing the reference value. The choice of the reference value does not matter in terms of the effectiveness of policies (social welfare). We can achieve the same result even if we set different reference values. Here, we choose the initial conditions as the reference values. This is one of the possible choices in reality. Private landowners can earn the subsidy (per ha per year) if they restore natural floodplains and they have to pay the tax (per ha per year) if they develop floodplains from the initial conditions.

Let us simulate the path under the mix policy of tax and subsidy. Table 6-13 shows the derived path. Likewise, the local optimisation is appropriately corrected by the mix policy. The path is close to the optimal path.

Table 6-13. *Path under the mix policy of tax and subsidy*

Control Variable [*y*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	136.794	0.000	0.648	0	0	0
27002	1747.989	3.530	0	0	0	0
27005	-14.388	0.000	0	0	0	0
27007	336.408	1.696	0.111	0	0	0
27009	1101.992	0.000	2.438	0	0	0
27034	51.888	1.397	0	0	0	0
27043	131.837	0.263	0	0	0	0
27053	-7.971	0.000	0	0	0	0
27069	-33.167	-4.115	0	0	0	0
27071	362.031	2.686	0	0	0	0
27075	-19.435	-2.707	0	0	0	0
27083	1006.901	0.000	1.97	0.019	0	0
27085	50.495	1.272	0	0	0	0
27089	280.067	0.560	0	0	0	0
27090	-9.807	-3.463	0	0	0	0

State Variable [X] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	99.394	236.188	236.188	236.836	236.836	236.836
27002	217.132	1965.121	1968.651	1968.651	1968.651	1968.651
27005	14.388	0.000	0.000	0.000	0.000	0.000
27007	155.264	491.672	493.368	493.479	493.479	493.479
27009	308.466	1410.458	1410.458	1412.896	1412.896	1412.896
27034	51.129	103.017	104.414	104.414	104.414	104.414
27043	27.692	159.529	159.792	159.792	159.792	159.792
27053	7.971	0.000	0.000	0.000	0.000	0.000
27069	101.624	68.457	64.342	64.342	64.342	64.342
27071	306.149	668.180	670.866	670.866	670.866	670.866
27075	63.814	44.379	41.672	41.672	41.672	41.672
27083	106.273	1113.174	1113.174	1115.144	1115.163	1115.163
27085	60.063	110.558	111.830	111.830	111.830	111.830
27089	35.183	315.250	315.810	315.810	315.810	315.810
27090	67.094	57.287	53.824	53.824	53.824	53.824

6.5.4 *Marketable Permits*

As we discussed in Chapter 3, we can set marketable permits as an alternative policy for price rationing policies such as taxes, subsidies and the mix policy. This policy rations quantities. The strengths of this policy are (1) to directly control the target level as long as individual owners abide by the rules, and (2) to achieve the target level at the least costs because individual landowners can transact the permits in the market in each zone.

We have two options for marketable permits. To begin with, we set marketable permits for development of floodplains (control variables). In the zones where the optimal level of developed floodplains is larger than the initial level, we can issue the amount of marketable permits that is equivalent to the difference between the optimal and initial levels. Through the permits, we can limit the development up to the optimal level. In this type of marketable permits, we cannot issue any permits in the zones where the initial quantity is larger than the optimal one. It implies that floodplain development is forbidden in such zones. However, we cannot provide the incentive to restore floodplains there. Table 6-14 shows the amount of marketable permits in zones. Let us simulate the path under the marketable permits, assuming that individual landowners observe the rules. Table 6-15 shows the path derived from the initial conditions. Controlling the quantity of development, the path is close to the optimal path only in the zones where permits are issued.

Table 6-14. *Quantity of marketable permits for development*

Subbasin	Permits for development (ha)
27001	137.47631
27002	1751.55266
27005	0
27007	338.33163
27009	1104.56468
27034	53.26856
27043	132.09539
27053	0
27069	0
27071	364.70823
27075	0
27083	1009.30115
27085	51.73270
27089	280.69052
27090	0

Table 6-15. *Path under marketable permits for development*

Control Variable [*y*] (ha)

	Time (*t*)					
	0	1	2	3	• • •	30
27001	137.473	0	0	0	0	0
27002	1751.550	0	0	0	0	0
27005	0.000	0	0	0	0	0
27007	338.329	0	0	0	0	0
27009	1104.562	0	0	0	0	0
27034	53.266	0	0	0	0	0
27043	132.092	0	0	0	0	0
27053	0.000	0	0	0	0	0
27069	0.000	0	0	0	0	0
27071	364.705	0	0	0	0	0
27075	0.000	0	0	0	0	0
27083	1009.298	0	0	0	0	0
27085	51.730	0	0	0	0	0
27089	280.688	0	0	0	0	0
27090	0.000	0	0	0	0	0

State Variable [X] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	99.394	236.867	236.867	236.867	236.867	236.867
27002	217.132	1968.682	1968.682	1968.682	1968.682	1968.682
27005	14.388	14.388	14.388	14.388	14.388	14.388
27007	155.264	493.593	493.593	493.593	493.593	493.593
27009	308.466	1413.028	1413.028	1413.028	1413.028	1413.028
27034	51.129	104.395	104.395	104.395	104.395	104.395
27043	27.692	159.784	159.784	159.784	159.784	159.784
27053	7.971	7.971	7.971	7.971	7.971	7.971
27069	101.624	101.624	101.624	101.624	101.624	101.624
27071	306.149	670.854	670.854	670.854	670.854	670.854
27075	63.814	63.814	63.814	63.814	63.814	63.814
27083	106.273	1115.571	1115.571	1115.571	1115.571	1115.571
27085	60.063	111.793	111.793	111.793	111.793	111.793
27089	35.183	315.871	315.871	315.871	315.871	315.871
27090	67.094	67.094	67.094	67.094	67.094	67.094

Table 6-16. *Quantity of marketable permits for developed floodplains*

Subbasin	Permits for developed floodplains (ha)
27001	236.870
27002	1968.685
27005	0
27007	493.596
27009	1413.031
27034	104.398
27043	159.787
27053	0
27069	64.370
27071	670.857
27075	41.688
27083	1115.574
27085	111.796
27089	315.874
27090	53.846

We can also set marketable permits for developed floodplains that landowners can possess (state variables). In this case, unlike the former case, we can force landowners to restore natural floodplains appropriately. If the amount of permits is smaller than the initial level of developed floodplains, landowners have to buy permits in the market in the same zone or to restore natural floodplains. In the zones where the total amount of permits is smaller than the initial total area of developed floodplains, landowners have to restore floodplains in total. Notice that we need a rule that landowners should obey the permits from time $t = 1$ because they have no incentive to restore floodplains in early stages without any regulations. If marketable permits are set in an appropriate way, the optimal level can be achieved at the lowest costs because less efficient landowners can buy permits from more efficient ones in the market in each zone. Table 6-16 shows the amounts of marketable permits. Likewise, let us simulate the path under this marketable permits policy, assuming that landowners abide by the rules. Table 6-17 shows the derived path. As we directly ration the quantities of developed floodplains, the path is close to the optimal path. This type of marketable permits is better than the previous type in that it can give the right incentive to restore floodplains.

Table 6-17. *Path under marketable permits for developed floodplains*

Control Variable [y] (ha)

	Time (t)					
	0	1	2	3	. . .	30
27001	137.476	0	0	0	0	0
27002	1751.553	0	0	0	0	0
27005	-14.388	0	0	0	0	0
27007	338.332	0	0	0	0	0
27009	1104.565	0	0	0	0	0
27034	53.269	0	0	0	0	0
27043	132.095	0	0	0	0	0
27053	-7.971	0	0	0	0	0
27069	-37.254	0	0	0	0	0
27071	364.708	0	0	0	0	0
27075	-22.126	0	0	0	0	0
27083	1009.301	0	0	0	0	0
27085	51.733	0	0	0	0	0
27089	280.691	0	0	0	0	0
27090	-13.248	0	0	0	0	0

State Variable [*X*] (ha)

	Time (*t*)					
	0	1	2	3	. . .	30
27001	99.394	236.870	236.870	236.870	236.870	236.870
27002	217.132	1968.685	1968.685	1968.685	1968.685	1968.685
27005	14.388	0.000	0.000	0.000	0.000	0.000
27007	155.264	493.596	493.596	493.596	493.596	493.596
27009	308.466	1413.031	1413.031	1413.031	1413.031	1413.031
27034	51.129	104.398	104.398	104.398	104.398	104.398
27043	27.692	159.787	159.787	159.787	159.787	159.787
27053	7.971	0.000	0.000	0.000	0.000	0.000
27069	101.624	64.370	64.370	64.370	64.370	64.370
27071	306.149	670.857	670.857	670.857	670.857	670.857
27075	63.814	41.688	41.688	41.688	41.688	41.688
27083	106.273	1115.574	1115.574	1115.574	1115.574	1115.574
27085	60.063	111.796	111.796	111.796	111.796	111.796
27089	35.183	315.874	315.874	315.874	315.874	315.874
27090	67.094	53.846	53.846	53.846	53.846	53.846

6.5.5 Direct Controls

We can directly control the level of developed floodplains by coercive regulations. Let us consider two options. First, like marketable permits for development, we impose quota (constraints) on development in the zones where the optimal level is larger than the initial level. However, landowners cannot transact them because they are not provided marketable permits but compulsory regulations. On the other hand, we force landowners to restore floodplains by a coercive regulation in the zones where the optimal level is smaller than the initial level. Assuming that landowners follow such regulations, we can obtain the same path as marketable permits for developed floodplains. Under this type of direct control, however, it is dubious that it can be achieved at the lowest costs. The policy of marketable permits is preferable to this regulation policy in this respect if the path is the same in reality.

Second, policy makers can centrally control the level of developed floodplains in the catchment. In the beginning, policy makers buy out all natural floodplains in the zones where the optimal level is larger than the initial level, and buy out the areas of developed floodplains that should be converted into the natural ones in the zones where the optimal level is smaller than the initial level. Then, policy makers can control floodplain development, and restore floodplains. In this case, policy makers can realize the optimal path. This policy is here called 'buy out and central control'.

6.5.6 Evaluation of Policy Scenarios

We derived the paths under policy scenarios above. In this section, we discuss the choice of policy scenarios in terms of effectiveness, efficiency and equity.[11]

To begin with, we discuss the effectiveness of policies under certainty. We basically focus on the effectiveness in our analysis. The effectiveness implies to what extent policies achieve the policy-maker's objective. That is to what extent they maximise social welfare over time. Thus, let us simulate and compare the values of social welfare among the policy scenarios. Table 6-18 shows the values of social welfare respectively. Under certainty, the policy scenarios except for marketable permits for development are equivalent. As we mentioned, the policy of marketable permits for development does not provide the right incentive to restore floodplains in the zones where the optimal level is smaller than the initial level. However, it is still better than no policies (local optimisation when externalities are not considered).

[11] We will analyse and discuss the flexibility of policies in the context of irreversibility and uncertainty in Sections 6.6 and 6.7.

Table 6-18. *Social welfare under certainty*

Scenario	Social Welfare	Rank
Social Optimisation	1,059,544	1
Buy out and central control	1,059,544	1
Marketable permits for developed floodplains	1,059,544	1
Regulation	1,059,544	1
Tax	1,059,544	1
Subsidy	1,059,544	1
Mix of tax and subsidy	1,059,544	1
Marketable permits for development	1,059,401	8

Next, we discuss the efficiency of policies briefly. The efficiency of policies implies that the policy-maker's objective (maximisation of social welfare) can be attained 'at the lowest possible costs'. This depends on real situations, but we make some comments on this. The 'buy out and central control' policy seems to be costly because we need funds for buying out floodplains and it incurs management costs to control the development and restoration in the centralized system over time. Furthermore, we may need a legal coercive regulation to buy the needed amounts of floodplains at a reasonable price. As this is related to individual property rights, it seems to be politically difficult. Hence, considering the feasibility of this policy in reality, this policy looks too difficult to be carried out although it can realize the optimal path on the theoretical side. As for marketable permits, we need to design the efficient markets for them in each zone. In order to consider unidirectional spatial externalities, we need to divide the catchment into zones. The degree of taking account of externalities increases as the number of zones increases. However, if the areas of zones are not large enough to create efficient markets, the policy of marketable permits will be ineffective. We also need a legal foundation to define the property rights to transact permits and to ensure that the rights are enforceable. In addition, monitoring and penalties are critical for making landowners abide by permits. However, marketable permits enable landowners to achieve the optimal level at the least costs by the transactions of permits in the markets. Unlike marketable permits, this kind of cost-saving cannot be achieved in the regulation policy of quota on development and restoration. In addition, monitoring and providing appropriate penalties are also crucial in regulations. On the other hand, the policies of tax and subsidy depend on decentralized system. The costs of implementing policies seem to be lower than marketable permits and regulations. However, notice that we have to set different unit tax (subsidy) rates among zones in the catchment (zonal taxes).

Finally, let us discuss the equity of policies shortly. The equity of policies implies that the policy-makers' objective (maximisation of social welfare) can be achieved without aggravating income distribution among members of society. There are a few comments. First, landowners should compensate for the external costs due to floodplain development. In this respect, the policies of tax, marketable permits and regulations function well because they impose costs on landowners. Second, however, landowners are compensated for opportunity costs of avoiding development under the policy of subsidy. In addition, landowners do not pay under the 'buy out and central control' policy. Third, it will be preferable if rich groups directly or indirectly incur the external costs. Landowners are not poor groups in that they possess lands. It is desirable that they should pay for the external costs. The policy of subsidy may be unacceptable at this point.

6.6 Irreversibility and Constraints on Control Variables

In this section, we discuss constraints on control variables in general and irreversibility of floodplain development. The irreversibility is shown by constraints on control variables, but it is shown by prohibitively high direct costs of floodplain restoration as well. We re-evaluate the policy scenarios under the constraints and irreversibility.

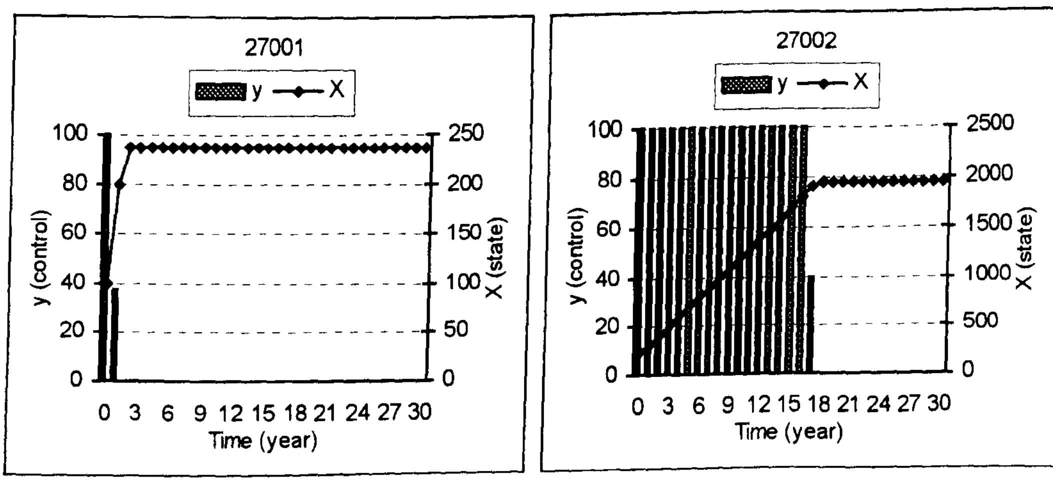
6.6.1 Constraints on Control Variables

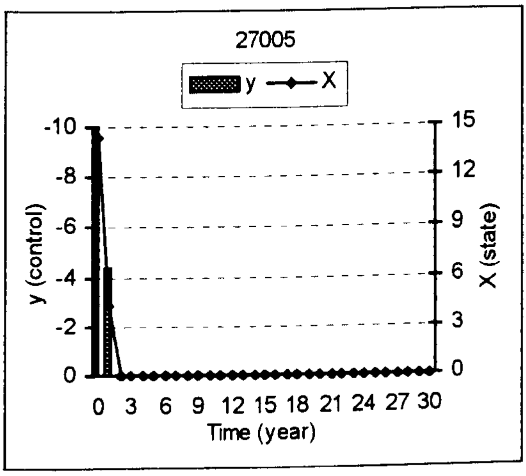
As we mentioned in Chapter 5, it takes time to develop and restore floodplains, which is referred to as the costs of time lag. We introduce the constraints assumed in Chapter 5 into the model. The constraints are:

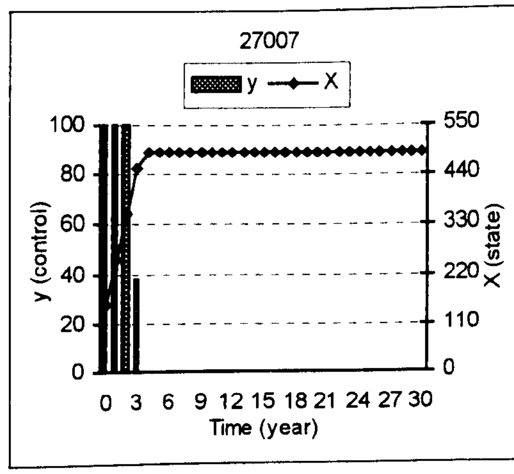
$\bar{y}_r^i = 10(ha) \le y_t^i \le \bar{y}_d^i = 100(ha)$ (for all i)

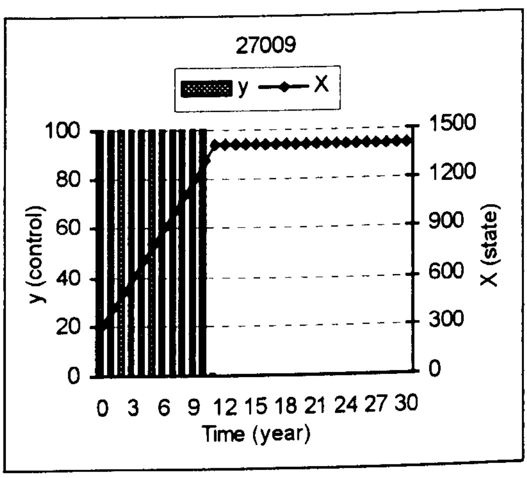
To begin with, let us derive the optimal path (social optimisation) under the constraints. Figure 6-5 shows the path. Due to the constraints, it takes more time steps to converge to equilibrium. The level of the equilibrium is slightly different from that under no constraints, because the adjustment processes (paths) are different. The time lag costs change the optimal path. Comparing the values of social welfare, it turns out that the constraints are costs. Table 6-19 shows them. Obviously, the value of social welfare under no constraints is lager than that under the constraints.

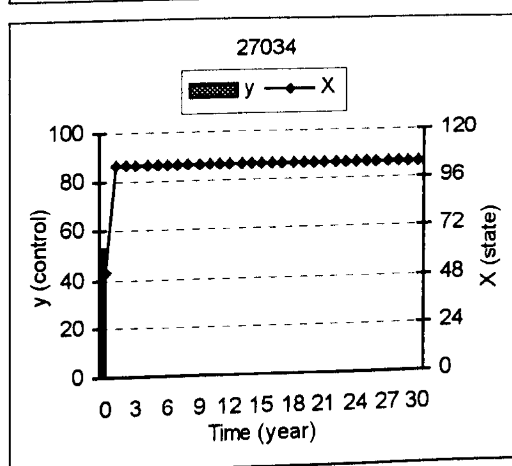
Figure 6-5. *Optimal path under constraints on control variables*

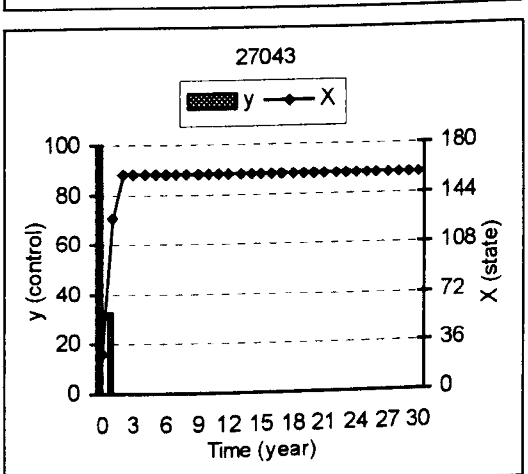


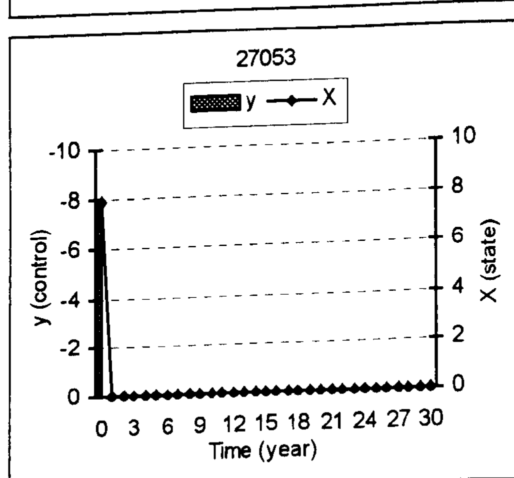

27005
y
X
-10
-8
-6
-4
-2
0
15
12
9
6
3
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

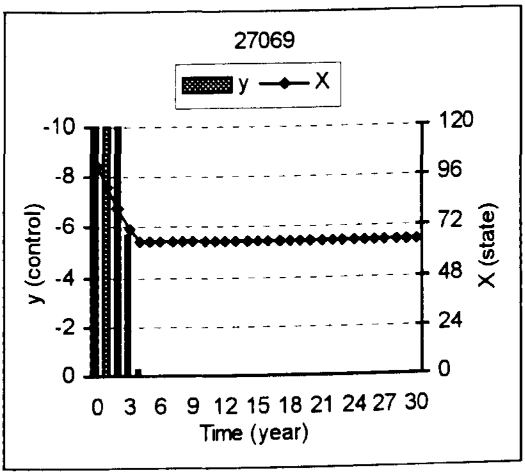

27007
y
X
100
80
60
40
20
0
550
440
330
220
110
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

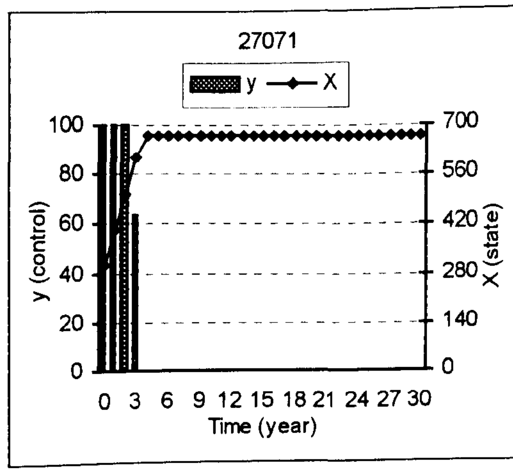

27009
y
X
100
80
60
40
20
0
1500
1200
900
600
300
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

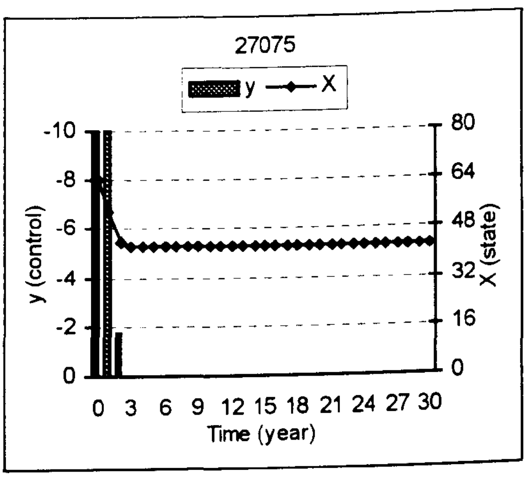

27034
y
X
100
80
60
40
20
0
120
96
72
48
24
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

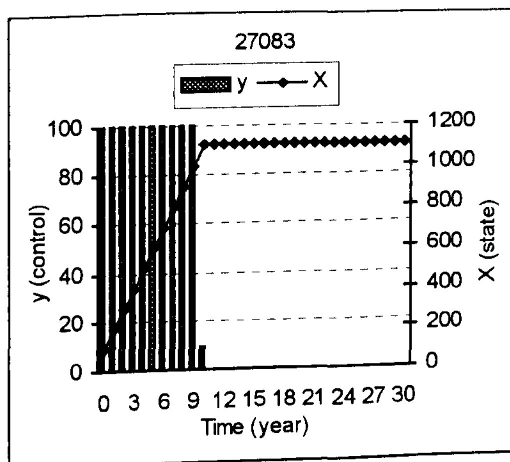

27043
y
X
100
80
60
40
20
0
180
144
108
72
36
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

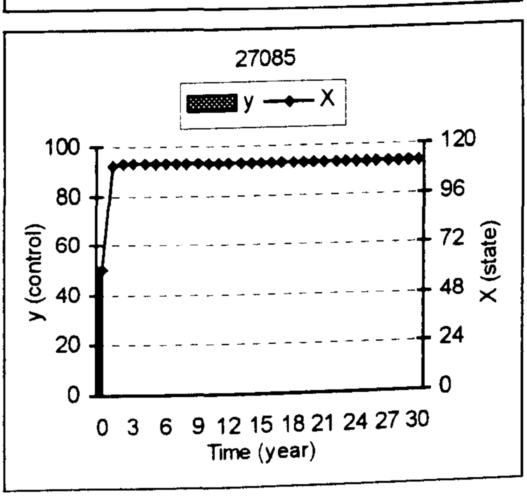

27053
y
X
-10
-8
-6
-4
-2
0
10
8
6
4
2
0
y (control)
X (state)
0 3 6 9 12 15 18 21 24 27 30
Time (year)

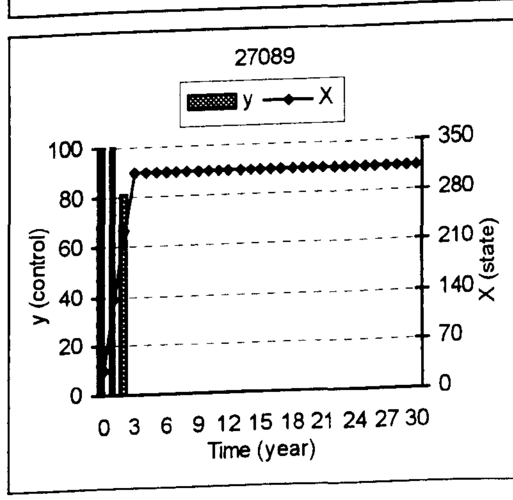

27069
y X
y (control)
X (state)
Time (year)


27071
y X
y (control)
X (state)
Time (year)


27075
y X
y (control)
X (state)
Time (year)


27083
y X
y (control)
X (state)
Time (year)


27085
y X
y (control)
X (state)
Time (year)


27089
y X
y (control)
X (state)
Time (year)

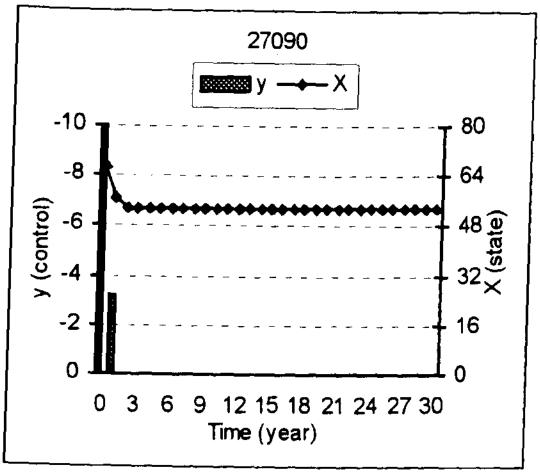


Table 6-19. *Values of social welfare under constraints*

	Social Welfare
Optimal path under no constraints (base case)	1,059,544
Optimal path under the constraints	1,057,121

6.6.2 Analysis of Policy Scenarios under Constraints

We discuss and evaluate the policy scenarios under the constraints. In so doing, we distinguish two cases. The first case is that policy-makers do not (exactly) know the constraints. The second case is that policy-makers exactly know the constraints. We qualitatively discuss policy scenarios respectively at first, and compare them in terms of social welfare.

To begin with, we consider the tax policy. If policy-makers do not (exactly) know the constraints, they impose the same tax rates (see Table 6-10). Therefore, the tax rates are not optimal under such a situation. If they exactly know the constraints, they set correct new tax rates. Likewise, under the policies of subsidy and the mix of tax and subsidy, policy-makers set the same rates, which are not appropriate, if they do not exactly know the constraints.

Let us discuss policies of marketable permits. In the beginning, we discuss the policy of marketable permits for development (quantity of control variables). If policy-makers do not know the constraints, they cannot issue correct amounts of permits. If they exactly know the constraints, they set the relevant quantity of permits. Next, we discuss the policy of marketable permits for developed floodplains (quantity of state variables). The problem is more complicated if policy-makers do not know the constraints. The problem that policy-makers cannot set the appropriate amounts of permits is the same. In addition, policy-makers immediately give penalties to landowners in the zones where the optimal level of developed floodplains is smaller than the initial level because landowners cannot immediately satisfy the requirements of permits due to the constraints. If landowners accept this marketable permits policy, the pressure of penalties make them restore floodplains as fast as possible. This process is close to the optimal path under the constraints. However, landowners try to refuse such a policy, and they have no incentive to tell policy-makers the correct value of constraints, because they can earn private net benefits if they tell the value of constraints that are smaller than the true ones. Hence, marketable permits for developed floodplains without relevant information might be politically infeasible. If policy-makers precisely know the constraints, they can set the appropriate deadlines for satisfying the requirements of permits. However, as the deadlines can be set only discretely in the unit of year, landowners still can earn private net benefits by decreasing the amounts of restoration in the early stages (first year) as long as they abide by the deadlines. Thus, social welfare is not maximised under this policy even if policy-makers have relevant information on the constraints.

Let us discuss policy scenarios of direct controls. First, we discuss the regulation of quota on development and restoration. This policy has the same problems as marketable permits for developed floodplains if policy-makers do not know the constraints. Second, we discuss the 'buy out and central control' policy. Under this policy, it does not matter whether policy-makers know the constraints in advance or not. Eventually, they get the information on them after they buy out floodplains. If the policy is in reality feasible, it can realize the optimal path.

Finally, let us compare the values of social welfare among the policy scenarios in the two cases. Table 6-20 shows the values of social welfare when policy-makers do not know the constraints. Like the results of the analysis under certainty, the 'buy out and central control' policy is the best. The policies of tax, subsidy and the mix of them provide almost the same value as the social optimum although the rates are not exactly correct.[12] These policies are robust to the constraints (existence of time lag costs). The policy of marketable permits for development provides the lowest value of social welfare, but it is still better than the local optimisation (not considering externalities at all).

Table 6-20. *Social welfare when policy-makers do not know constraints*

Scenario	Social Welfare	Rank
Social Optimisation	1,057,121	1
Buy out and central control	1,057,121	1
Tax	1,057,121	1
Subsidy	1,057,121	1
Mix of tax and subsidy	1,057,121	1
Marketable permits for development	1,056,978	6
Marketable permits for developed floodplains	politically infeasible	-
Regulation	politically infeasible	-

Table 6-21. *Social welfare when policy-makers know constraints*

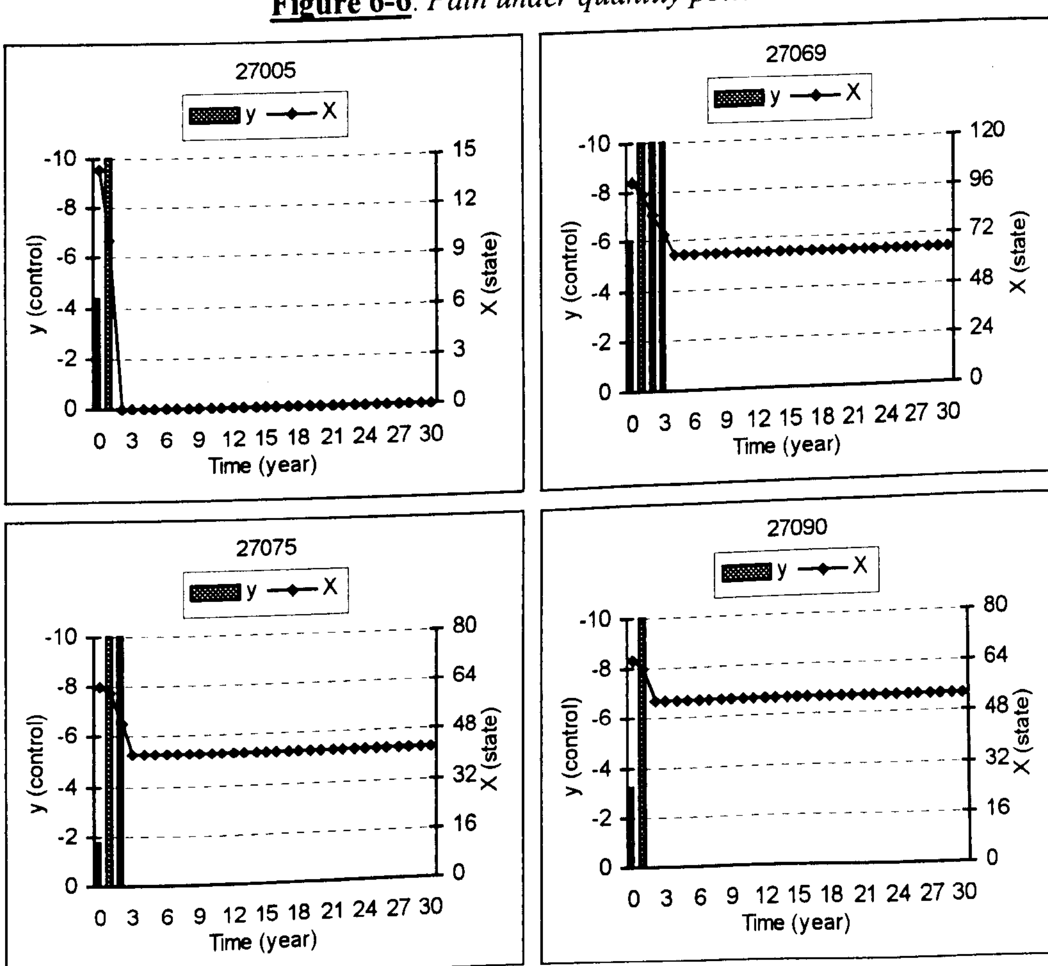
Scenario	Social Welfare	Rank
Social Optimisation	1,057,121	1
Buy out and central control	1,057,121	1
Tax	1,057,121	1
Subsidy	1,057,121	1
Mix of tax and subsidy	1,057,121	1
Marketable permits for developed floodplains	1,057,118	6
Regulation	1,057,118	6
Marketable permits for development	1,056,980	8

Table 6-21 shows the values of social welfare when policy-makers completely know the constraints. Likewise, the 'buy out and central control'

[12] The values are slightly smaller than the social optimum.

policy provides the largest value. However, interestingly, the policies of tax, subsidy and the mix of them (price rationing policies) are clearly preferable to the policies of marketable permits for developed floodplains and the regulation (quantity rationing policies). As we indicated, under the quantity rationing policies, there is a room for landowners to exploit private net benefits by decreasing the amounts of restoration in the first year even though policy-makers set the correct deadlines for satisfying the requirements of permits. Figure 6-6 shows the path under the policies of marketable permits for developed floodplains and the regulation in the zones where the optimal level is smaller than the initial level (restoration is needed). Unlike the optimal path, the amounts of restoration in the first year are not at the maximum level (the constraints are not binding).

Figure 6-6. *Path under quantity policies*



6.6.3 Irreversibility

The irreversibility is shown by constraints on control variables and/or prohibitively high direct costs of floodplain restoration. Under the irreversible situation that it is impossible or too expensive to restore natural floodplains, the overdevelopment of floodplains may prove costly. Once floodplains are overdeveloped, the non-optimal state perpetuates forever.

To begin with, we concretely show this by an example in the case that the irreversibility is shown by constraints on floodplain restoration in the following.

$\bar{y}_r^i = 0(ha) \le y_t^i \le \bar{y}_d^i = 100(ha)$ (for all i)

Table 6-22 shows the values of social welfare. Obviously, social welfare deteriorates in the case of irreversibility because the non-optimal situations cannot be improved in the zones where the optimal level is smaller than the initial level. Based on the results, it is important to avoid risks of overdevelopment for choosing policies under irreversibility. This is related to the analysis of policy scenarios under uncertainty in Section 6.7.

Table 6-22. *Social welfare under irreversibility*

	Social Welfare
Optimal path under no constraints (base case)	1,059,544
Optimal path under the constraints	1,057,121
Optimal path under irreversibility	1,056,980

Next, we discuss the case where irreversibility is shown by prohibitively high direct costs of floodplain restoration. We still have no precise information on the costs of floodplain restoration. Thus, let us assume that they become ten times as high as the base case in the following.

$$D(y^i) = 191463.7353 y^i \text{ if } y^i < 0. \quad (6\text{-}4)$$

Let us derive the optimal path under the situation, and calculate the value of social welfare. Table 6-23 shows the optimal path under this type of irreversibility. In the optimal path as the base case, floodplains are restored in five zones while they are restored in only one zone under this type of irreversibility. Table 6-24 shows the value of social welfare. The value of social welfare under irreversibility is smaller than that under no irreversibility.

Table 6-23. *Optimal path under irreversibility (high costs)*

	Time (*t*)					
	0	1	2	3	· · ·	30
27001	137.358	0.116	0	0	0	0
27002	1750.549	0.930	0	0	0	0
27005	-6.141	-2.285	0	0	0	0
27007	337.862	0.487	0	0	0	0
27009	1103.936	0.681	0	0	0	0
27034	53.073	0.203	0	0	0	0
27043	131.983	0.109	0	0	0	0
27053	0	0	0	0	0	0
27069	0	0	0	0	0	0
27071	364.027	0.710	0	0	0	0
27075	0	0	0	0	0	0
27083	1009.224	0.076	0	0	0	0
27085	51.349	0.400	0	0	0	0
27089	280.539	0.149	0	0	0	0
27090	0	0	0	0	0	0

	Time (*t*)					
	0	1	2	3	· · ·	30
27001	99.394	236.752	236.868	236.868	236.868	236.868
27002	217.132	1967.681	1968.611	1968.611	1968.611	1968.611
27005	14.388	8.247	5.962	5.962	5.962	5.962
27007	155.264	493.126	493.613	493.613	493.613	493.613
27009	308.466	1412.402	1413.083	1413.083	1413.083	1413.083
27034	51.129	104.202	104.405	104.405	104.405	104.405
27043	27.692	159.675	159.784	159.784	159.784	159.784
27053	7.971	7.971	7.971	7.971	7.971	7.971
27069	101.624	101.624	101.624	101.624	101.624	101.624
27071	306.149	670.176	670.886	670.886	670.886	670.886
27075	63.814	63.814	63.814	63.814	63.814	63.814
27083	106.273	1115.497	1115.573	1115.573	1115.573	1115.573
27085	60.063	111.412	111.812	111.812	111.812	111.812
27089	35.183	315.722	315.871	315.871	315.871	315.871
27090	67.094	67.094	67.094	67.094	67.094	67.094

Table 6-24. *Value of social welfare in case of irreversibility*

	Social Welfare
Optimal path (base case)	1,059,544
Optimal path under irreversibility	1,059,405

When we are faced with irreversibility, precautionary principles are important. It may be appropriate to introduce a safe minimum standard into the processes of floodplain development. How much we should put the margin for a safe minimum standard depends on the size and type of uncertainty. In this respect, we conduct sensitivity analyses and evaluate the policy scenarios under uncertainty in Section 6.7.

6.6.4 Analysis of Policy Scenarios under Irreversibility

Likewise, let us calculate the values of social welfare under the policy scenarios respectively and evaluate them. Under the first type of irreversibility shown by the constraints, the evaluation of the policy scenarios is quite similar to that under certainty (see Table 6-18). There are a few noticeable things. Under the policies of marketable permits and the regulation, landowners have no chance to exploit private net benefits by decreasing the amounts of restoration in the first year. Thus, social welfare under these policies does not deteriorate. In addition, the policy of marketable permits for development is equivalent to the policies of marketable permits for developed floodplains and the regulation of quota on development because of no restoration. Table 6-25 shows the values of social welfare. As a result, the policies are substitutable one another in terms of effectiveness (social welfare).

Table 6-25. *Social welfare under irreversibility (no restoration)*

Scenario	Social Welfare	Rank
Social Optimisation	1,056,980	1
Buy out and central control	1,056,980	1
Marketable permits for developed floodplains	1,056,980	1
Regulation	1,056,980	1
Marketable permits for development	1,056,980	1
Tax	1,056,980	1
Subsidy	1,056,980	1
Mix of tax and subsidy	1,056,980	1

Table 6-26. *Social welfare under irreversibility (high costs) with no information*

Scenario	Social Welfare	Rank
Social Optimisation	1,059,405	1
Buy out and central control	1,059,405	1
Tax	1,059,405	1
Subsidy	1,059,405	1
Mix of tax and subsidy	1,059,405	1
Marketable permits for development	1,059,398	6
Marketable permits for developed floodplains	1,059,027	7
Regulation	1,059,027	7

Next, we discuss the second type of irreversibility shown by prohibitively high costs of restoration. We distinguish the two cases. The evaluation of the policy scenarios is the same as that under certainty (see Table 6-18) if policy-makers have the information on the irreversibility. If not, the policies of tax, subsidy and the mix of them are better than the policies of marketable permits and the regulation (quantity rationing policies) because the optimal tax rates remain the same. The costs of restoration do not affect the unit tax rates (the marginal external costs). Under the quantity rationing policies, policy-makers will set inappropriate amounts of permits or quota if they cannot obtain relevant information. Moreover, interestingly, the policy of marketable permits for development is better than the policies of marketable permits for developed floodplains and the regulation. As policy makers do not care about restoration under the policy of marketable permits for development, the degree of incorrectness about the amounts of permits is relatively small. Table 6-26 shows the values of social welfare.

6.7 Uncertainty and Sensitivity Analysis

In this section, we conduct the sensitivity of the policy outcomes to uncertainty. Practically, we calculate the sensitivity of the equilibrium to which the initial state converges, assuming that parameters or exogenous variables change by a certain percentage. We define the sensitivity (sv) as the following.

$$sv = \frac{d\overline{X}/\overline{X}}{d\alpha/\alpha} = \frac{\%\ change\ of\ \overline{X}}{\%\ change\ of\ \alpha} \tag{6-5}$$

where $\overline{X}$ is the size of developed floodplains in the equilibrium and α is one of the parameters or exogenous variables.

Then, let us simulate changes in social welfare under the policy scenarios, assuming that parameters or exogenous variables suddenly change by a certain percentage and that policy-makers cannot detect the changes or cannot change the values of policy variables immediately. We can interpret the assumption in a different way. That is, policy-makers cannot correctly know the values of parameters or exogenous variables, and they overestimate or underestimate them by a certain percentage when they set policies. Which policy scenario is more flexible and robust under these uncertain situations?

6.7.1 Sensitivity to Value of Ecosystem Services

The value of ecosystem services cannot be given in real markets because of missing markets. Thus, the value is in itself a kind of decision variable. Practically, this relies on environmental valuations. The estimated value might be uncertain. In particular, we use a result of meta analysis of environmental valuation in our applied model. Hence, it is important to check the sensitivity to the value and evaluate the flexibility of policy scenarios.

To begin with, it is difficult to treat the functional form (5-17) for the benefit function of ecosystem services in Chapter 5, because it contains three parameter values and the economic meaning of them is not clear. Then, let us mathematically simplify the functional form. Leaving out the log functions, we obtain the following simplified functional form.

$$B\left(L_F^i - X^i\right) = 2704.868484\left(L_F^i - X^i\right)^{0.832} \tag{6-6}$$

Function (6-6) is equivalent to function (5-17). Thus, we can use function (6-6). This function fortunately includes only two parameter values, which are deeply related to the marginal opportunity cost of lost ecosystem services with respect to floodplain development (the marginal value of ecosystem services with respect to the size of natural floodplains). Here, we do the sensitivity analysis on the first parameter value (2704.868484).

We are interested in the risk of overdevelopment. Thus, we assume that this parameter value increases by 50%. This implies that the marginal value of ecosystem services increases by 50%.[13] As the knowledge of ecosystem services has been still limited, this amount of error seems to be plausible. Let us calculate the sensitivity values under the assumption. Table 6-27 shows the results. The sensitivity values are small. Even at the maximum, the optimal size of developed floodplains decreases by only 0.397% when the marginal value of ecosystem

[13] The marginal value is not constant with respect to the size of developed floodplains. Thus, to be precise, it implies that the marginal value for the same size of developed floodplains increases by 50%.

services increases by 1 %.

Table 6-27. *Sensitivity to the value of ecosystem services*

Subbasin	Sensitivity value	% change	New Equilibrium	Equilibrium (base)
27001	-0.054	-2.679	230.524	236.870
27002	-0.060	-2.992	1909.776	1968.685
27005	-	-	0.000	0.000
27007	-0.098	-4.918	469.319	493.596
27009	-0.041	-2.037	1384.243	1413.031
27034	-0.188	-9.400	94.585	104.398
27043	-0.075	-3.739	153.813	159.787
27053	-	-	0.000	0.000
27069	-0.214	-10.682	57.494	64.370
27071	-0.099	-4.931	637.778	670.857
27075	-0.225	-11.248	36.999	41.688
27083	-0.035	-1.758	1095.961	1115.574
27085	-0.397	-19.833	89.624	111.796
27089	-0.074	-3.706	304.167	315.874
27090	-0.204	-10.224	48.341	53.846

6.7.2 Analysis of Policy Scenarios under Uncertainty about Value of Ecosystem Services

Let us derive paths under the policy scenarios and evaluate them in terms of social welfare under the uncertain situation that policy makers underestimate the value of ecosystem services by 50% (the same situation as Section 6.7.1). In this case, the paths under the policy scenarios are respectively the same as those under certainty because individual landowners do not care the value of ecosystem services and policy makers do not precisely know it. However, the value of ecosystem services changes in reality. Thus, let us evaluate the paths under the new value of ecosystem services.

As the optimal size of developed floodplains changes, the amounts of marketable permits and the quantity of development and restoration that policy-makers set as regulations are not appropriate. As the marginal external value changes, the unit tax rates that policy makers set are not appropriate as well.

In this case, the 'buy out and central control' policy is also inappropriate because policy makers manage development and restoration based on the underestimated value. Which is more inappropriate under such a situation? Table 6-28 shows the values of social welfare. The results are the same as those under certainty. The policies are equivalent except for marketable permits for development.

Table 6-28. *Social welfare under uncertainty of the value of ecosystem services*

Scenario	Social Welfare	Rank
Social Optimisation	1,063,566	1
Tax	1,063,545	2
Subsidy	1,063,545	2
Mix of tax and subsidy	1,063,545	2
Buy out and central control	1,063,545	2
Marketable permits for developed floodplains	1,063,545	2
Regulation	1,063,545	2
Marketable permits for development	1,063,385	8

6.7.3 Sensitivity to Benefits of Developed Floodplains

The benefit function of developed floodplains is crucial because it is only the function that positively evaluates floodplain development in the model. The sensitivity analysis to the parameter value in the function is important. The parameter value is estimated from economic rents of developed lands (land prices), but they fluctuate largely over time (see Table 5-35 in Chapter 5). As in the previous section, we focus on the risk of overdevelopment. We assume that the marginal (unit) benefits of developed floodplains decreases by 25%. This assumption is plausible because the average value during the five years 1990-1994 decreases to the average value during the five years 1995-1999 by 26.5% (see Table 5-35 in Chapter 5). Under the situation, let us calculate the sensitivity values. Table 6-29 shows the results. The sensitivity values are much higher than those in Section 6.7.1. In some zones, no development is optimal under the new situation. Even at the minimum, the optimal size of developed floodplains decreases by about 1% if the benefits of developed floodplains decreases by 1%. Thus, it is important to evaluate the flexibility of policies under uncertainty of the benefits of developed floodplains.

Table 6-29. *Sensitivity to the benefits of developed floodplains*

Subbasin	Sensitivity value	% change	New Equilibrium	Equilibrium (base)
27001	1.089	-27.220	172.393	236.870
27002	1.072	-26.791	1441.264	1968.685
27005	-	-	0.000	0.000
27007	2.201	-55.016	222.040	493.596
27009	1.076	-26.901	1032.914	1413.031
27034	2.616	-65.389	36.133	104.398
27043	1.513	-37.820	99.355	159.787
27053	-	-	0.000	0.000
27069	4.000	-100.000	0.000	64.370
27071	2.175	-54.364	306.149	670.857
27075	4.000	-100.000	0.000	41.688
27083	0.889	-22.219	867.704	1115.574
27085	4.000	-100.000	0.000	111.796
27089	1.065	-26.623	231.779	315.874
27090	4.000	-100.000	0.000	53.846

6.7.4 Analysis of Policy Scenarios under Uncertainty about Benefits of Developed Floodplains

In this section we consider changes in social welfare under the assumption that policy makers overestimate the benefits of developed floodplains by 25% (the same situation as Section 6.7.3). This case is slightly different from the case in Section 6.7.2 in that individual landowners consider the change in the benefits of developed floodplains and change their economic behaviour.

The situations of the policy scenarios here are the same as those under uncertainty of the value of ecosystem services. The unit tax rates are underestimated while the amount of permits and quota are overestimated. Table 6-30 shows the values of social welfare. Interestingly, the 'buy out and central control' policy is the worst. This policy is a kind of planned economy system, and is susceptible to uncertainty. It is completely inflexible. The policies of marketable permits for developed floodplains and the regulation (quantity policies) are worse than the policies of tax, subsidy and the mix of them (price policies). These findings imply that quantity-setting policies are relatively weak under uncertainty because these policies never use decentralized decisions by individual landowners.

Table 6-30. *Social welfare under uncertainty of the benefits of developed floodplains*

Scenario	Social Welfare	Rank
Social Optimisation	1,050,054	1
Tax	1,050,035	2
Subsidy	1,050,035	2
Mix of tax and subsidy	1,050,035	2
Marketable permits for developed floodplains	1,049,014	5
Regulation	1,049,014	5
Marketable permits for development	1,048,643	7
Buy out and central control	1,048,065	8

6.7.5 Sensitivity to Precipitation

Precipitation is an important exogenous variable for the expected cost function of flood risk. We have obtained the observed data on precipitation, but we have to estimate its exceedance probability. It is not necessarily certain and exact. In addition, it is said that we have the risk of climate change in the future. The volume and intensity of rainfalls may increase in the future. Therefore, tt is important to conduct the sensitivity analysis to precipitation.

Hulme et al. (2002) provide a comprehensive analysis and prediction with some scenarios about climate change in the future in the UK. They predict that winter precipitation increases while summer precipitation decreases by the 2080s and that the total volume has little change. We are interested in wetter winters in the future because our interests are in flood risk. "Winter precipitation increases for all periods and scenarios, although these increases by the 2080s range from 5 to 15 per cent for the Low Emissions scenario, to more than 30 per cent for some regions for the Medium-High Emissions and High Emissions scenarios" (Hulme et al., 2002). Based on the information, we assume that precipitation increases by 20%.

As precipitation is an exogenous variable, we have to calibrate the parameter values of the expected cost function of flood risk again. The followings are the calibrated functions when precipitation increases by 20% (see Appendix D-3 for details).

27001 $C^{i=27001} = 30.04626(X^i)^2 + 26152.53996\sum_j X^j \quad j = 27005 \text{ and } 27053$

27002 $C^{i=27002} = 4.35800(X^i)^2 + 3961.07442\sum_j X^j \quad j = 27043$

27005 $C^{i=27005} = 151.77984(X^i)^2$

27007 $C^{i=27007} = 8.11519(X^i)^2 + 5240.60216\sum_j X^j \quad j = 27034$

27009 $C^{i=27009} = 4.90695(X^i)^2 + 8033.70837\sum_j X^j + 6816.17875\sum_k X^k$

$j = 27007 \text{ and } 27034, \quad k = 27069, 27071, 27075, 27085 \text{ and } 27090$

27034 $C^{i=27034} = 16.59264(X^i)^2$

27043 $C^{i=27043} = 34.94831(X^i)^2$

27053 $C^{i=27053} = 9.04461(X^i)^2 + 868.37807\sum_j X^j \quad j = 27005$

27069 $C^{i=27069} = 26.48690(X^i)^2$

27071 $C^{i=27071} = 6.22730(X^i)^2 + 5233.00159\sum_j X^j$

$j = 27069, 27075, 27085 \text{ and } 27090$

27075 $C^{i=27075} = 40.64391(X^i)^2$

27083 $C^{i=27083} = 7.63154(X^i)^2$

27085 $C^{i=27085} = 10.40179(X^i)^2$

27089 $C^{i=27089} = 27.57896(X^i)^2 + 463.50249\sum_j X^j$ $j = 27002$ and 27043

27090 $C^{i=27090} = 32.55725(X^i)^2$

Selby $C^{i=Selby} = 1665552.34 + 47.74673\sum_j X^j$ $j =$ All 15 subbasins

York $C^{i=York} = 11661443.1 + 259.0927\sum_j X^j$ $j = 27001, 27005, 27007,$ $27009, 27034, 27053, 27069, 27071, 27075, 27083, 27085$ and 27090

Table 6-31. *Sensitivity to precipitation*

Subbasin	Sensitivity value	% change	New Equilibrium	Equilibrium (base)
27001	-0.465	-9.303	214.835	236.870
27002	-1.305	-26.091	1455.036	1968.685
27005	-	-	0.000	0.000
27007	-1.915	-38.301	304.546	493.596
27009	-0.290	-5.796	1331.136	1413.031
27034	-3.409	-68.172	33.228	104.398
27043	-1.086	-21.728	125.068	159.787
27053	-	-	0.000	0.000
27069	-1.686	-33.713	42.669	64.370
27071	-1.291	-25.826	497.599	670.857
27075	-1.709	-34.178	27.440	41.688
27083	-1.263	-25.250	833.889	1115.574
27085	-2.314	-46.274	60.063	111.796
27089	-1.287	-25.731	234.595	315.874
27090	-1.743	-34.866	35.072	53.846

Based on the functions above, let us calculate the sensitivity values. Table 6-31 shows the results. The size of developed floodplains in the equilibrium is sensitive to precipitation although they are variable among subbasins. At the maximum, the optimal size of developed floodplains decreases by 3.49% if precipitation volume increases by 1%. Like the benefits of developed floodplains, uncertainty about precipitation is important for evaluating the effectiveness and flexibility of policy scenarios.

6.7.6 Analysis of Policy Scenarios under Uncertainty about Precipitation

Finally, let us derive paths under the policy scenarios and evaluate them in terms of social welfare under the uncertain situation that precipitation increases by 20% (the same situation as Section 6.7.5).

Likewise, let us evaluate the policy scenarios. Table 6-32 shows the values of social welfare. The 'buy out and central control' policy is again the worst because of its inflexibility. The policies of tax, subsidy and the mix of them are preferable to the others. They have the advantages that they use decentralized decisions (adjustments) by individual landowners.

Table 6-32. *Social welfare under uncertainty of precipitation*

Scenario	Social Welfare	Rank
Social Optimisation	1,053,328	1
Tax	1,053,202	2
Subsidy	1,053,202	2
Mix of tax and subsidy	1,053,202	2
Marketable permits for developed floodplains	1,052,878	5
Regulation	1,052,878	5
Marketable permits for development	1,052,610	7
Buy out and central control	1,051,984	8

6.8 Linkage with Policy Implication from Static Decision Model

The essential finding is that price policies are more robust to irreversibility and uncertainty than quantity policies. There are two reasons. First, price polices can utilise decentralized adjustment processes (decisions) of individual landowners (decision makers). Second, the appropriate unit tax (subsidy) rates do not change so much under uncertainty as compared with the optimal size of developed floodplains (the quantity set by marketable permits or direct controls).

The second point is related to the policy implication derived from the static decision model in Chapter 3. It tells that taxes are better than marketable permits if the absolute value of the slope of marginal private net benefit (MB) curve is larger than that of marginal external cost (MC) curve, and vice versa. The analysis in Chapter 3 assumes linear functions of MB and MC curves, which is slightly different from the situations in Chapter 6. We have a non-linear function for MC curve. Nonetheless, the essential proposition remains true for non-linear functions around the equilibrium.

Table 6-33. *Comparison of slopes between MB and MC curves*

	Absolute value of slope of MB (marginal private net benefit)		Absolute value of slope of MC (marginal external costs)
27001	53.348	>	0.104
27002	6.452	>	0.314
27005	274.552	>	0.756
27007	12.878	>	0.054
27009	9.222	>	0.017
27034	31.007	>	0.044
27043	57.930	>	0.106
27053	2.170	>	0.408
27069	49.069	>	0.089
27071	8.834	>	0.034
27075	75.054	>	0.120
27083	10.568	<	72.884
27085	15.607	>	0.113
27089	40.711	>	1.308
27090	59.104	>	0.070

Note: The Absolute value of the slope of MC curve is evaluated at the empirical equilibrium.

Let us verify this under non-linear functions in the applied context. We can expect that MB curves are steeper than MC curves in most of subbasins because we obtain the result that taxes are better than marketable permits under uncertainty. We calculate the absolute values of the slopes of MB and MC curves. As MC curves are non-linear functions, the absolute values of the slopes of MC curves are evaluated at the empirical equilibrium to which the initial state converges. Table 6-33 compares the slopes of MB and MC curves. In all but Subbasin 27083, MB curves are steeper than MC curves.

This comparison of the slopes of the MB and MC functions is a consequence of the functional forms estimated for the individual equations which comprise MB and MC (expected cost of flood risk, etc.). Estimations of these functional equations are statistically strong and robust (see Chapter 5), adding evidence to our overall evaluation. Then, tax policies rather than quantity policies offer a more robust mechanism for dealing with the uncertainty surrounding floodplain development.

6.9 Conclusion of Chapter 6

Floodplain management is crucial for reducing the costs of flood risk. It will be costly if external costs are not appropriately considered. In real situations, there is the tendency that floodplains are overdeveloped in upstream zone in the Ouse catchment. Under such a situation, it is important to set an appropriate policy for floodplain management.

Under certain situations, several policies such as tax, subsidy, the mix of them, marketable permits, a regulation and a central control work well. Under uncertainty and irreversibility, however, price rationing policies such as tax, subsidy and the mix of them are preferable to the other policies because the appropriate unit tax (subsidy) rates do not change a lot (as compared with the optimal size of developed floodplains) and because these policies use decentralized decisions by individual landowners (adjustments are possible). That is why they are more flexible and robust against uncertainty.

Chapter 7

Conclusion

7.1 Concluding Remarks

In this thesis, we (1) define the appropriate social optimisation problem for floodplain management, (2) provide theoretical models for the static and dynamic problems, (3) develop an applied model and calibrate parameter values from data on the Ouse catchment, and (4) carry out simulations in the context of the Ouse catchment in order to evaluate several policy scenarios. The thesis attempts to make three main contributions. First, it has tried to improve understanding the essential problems of floodplain management (two types of environmental externalities). Second, it has tried to clarify the policy options for the optimal floodplain management. Third, it has explored methods for integrating the hydrology, ecology and economics of floodplains.

Floodplains are multi-functional resources. They offer direct and indirect use values. They provide us with various ecosystem services. However, the way they are used generates environmental externalities. Our economic system does not have the way of recognizing and evaluating the indirect use values of many of the ecosystem services affected by decisions to develop floodplains in various ways. Without the appropriate policy interventions, the current condition of many floodplains will continue to deteriorate. Therefore, we develop an interdisciplinary integrated model that includes both direct and indirect use values in the optimal management of floodplains. The implications of this for different policy options are then tested through simulations.

It is crucial to understand how ecosystem functions and human economic activities interact, and how human economic activities are affected by the ecosystem's characteristics and policy instruments. It is often pointed out that interdisciplinary integrated models are needed for solving this type of problem (Bockstael et al., 1995; Costanza and Farber, 2002). The models developed in the thesis make such an attempt. The models have two important features. First, they integrate a hydrological sub-model into an economic model on the choice between flood mitigation and the economic development of floodplains. Second, they make it possible to conduct an environmental and economic analysis of the flood risk implications of alternative floodplain development options.

This thesis is intended to help local decision makers to improve floodplain management in the following respects. First, it shows how and what we should consider for the optimal management of floodplains. Second, it provides a simulation model by which we can evaluate alternative management policies. We can test them under uncertainty and irreversibility. Third, it offers guidance on the choice of policy options under currently available data in the context of the Ouse catchment. Fourth, it indicates what data we need for make better decisions (environmental, physical and economic data).

Let us summarize the main findings. In the static model, the optimal conditions show that we should choose the size of developed floodplains so that the marginal benefits are equal to the sum of the marginal costs and the marginal external costs. The crucial point is that we must take account of environmental externalities. There are two types of externalities. First, the development of floodplains has opportunity costs in terms of lost ecosystem services. Second, the development of floodplains increases flood risks to people downstream (imposes a unidirectional spatial externality). Just as many environmental problems are related to externalities, the overdevelopment of floodplains is also the problem of externalities. Interestingly, the conditions of the optimal steady-state equilibrium in the dynamic model are the same as the optimal conditions in the static decision model. However, the local stability analysis indicates that the optimal steady-state equilibrium is unstable. This implies that the optimal conditions derived from the static model cannot be satisfied in the dynamic context. Related to the instability, there are a few noticeable things to keep in mind. First, if the equilibrium is a saddle, it is still possible to control the dynamical system. Unfortunately, however, it is unknown whether it is a saddle in such a three or more dimensional model. Second, more importantly, the results of the local stability analysis depend on the assumptions made about the concavity and convexity of the underlying functions, although there is interestingly a gap between static and dynamic models under the same assumptions. That is why it is important to implement policy simulations in the applied model in the concrete context.

Certainly, we cannot derive concrete and practical guidance for policy from the theoretical models except that we should internalise externalities. What we have been able to do is to use an applied model to evaluate a number of policy options for the Ouse catchment. These lead to three general conclusions. First, the impact of floodplain development on the expected cost of flood risk is large. Second, floodplains in upstream zones tend to be overdeveloped currently because of unidirectional spatial externalities. Third, price policies (e.g. the Pigouvian tax policy) function well to internalise external costs and attain the optimal path, and are robust to irreversibility and uncertainties.

Related to the third point, let us give several interesting findings in more detail.

1. Under uncertainty, both price and quantity instruments have the potential to internalise externalities and accomplish the optimal path.
2. If there are constraints on floodplain development and restoration, quantity policies such as marketable permits and regulations do not work well. Individual landowners have the incentive to delay the necessary restoration as long as possible because they can exploit their private net benefits.
3. Under irreversibility, price policies such as the Pigouvian tax and subsidies are relatively robust because the optimal tax and subsidy

rates remain the same.

4. Under uncertainty about the value of ecosystem services, price and quantity instruments are equivalent, but they cannot always achieve an efficient outcome.
5. Under uncertainty about the benefits of developed floodplains, price policies get better results than quantity restrictions although they cannot completely achieve the optimal path.
6. Under uncertainty about precipitation, price policies are better than other policies although they cannot completely accomplish the optimal path.

In brief, price policies such as Pigouvian taxes are preferable in that they allow decentralized decisions (individual flexible adjustment processes) by landowners.

7.2 Future Research

In this section, we mention the limitations of this research and raise future research topics based on them.

7.2.1 Interactions with Averting Behaviour

We cannot cope with averting behaviour as a control variable in the applied model and simulations although we introduce it into the theoretical models. Indeed, averting behaviour plays almost the same role as floodplain development, but there are complex interactions between floodplain functions and averting behaviour. Ecosystem functions are not independent of averting behaviour. Averting behaviour may, for example, change hydrologic conditions and the physiochemical environment of floodplains. If so, the services offered by floodplains will change depending on the nature of averting behaviour. In this case, we need more complex models including the interactions between floodplains' ecosystem functions and averting behaviour. In order to investigate this, we need to expand the models to include interactions between ecosystem services with averting behaviour.

7.2.2 *Ecological Economic Modelling*

In this research, we use various modelling techniques - mathematical optimisation, GIS, econometrics, hydrological modelling, and frequency analysis - to carry out policy simulations using an applied model. In our model, we can treat environmental or physical parameters and variables. However, it is not really an integrated model of the hydrology, ecology and economics in that we cannot 'directly' control relevant hydrological and ecological parameters and variables in the main model. We mainly use an economic model, which indirectly employs the outputs from hydrological and ecological sub-models. Ideally, we should integrate them evenly so that we can directly control both environmental and economic variables in the main model. If we could create interdisciplinary integrated models, it would be easier to discuss ecosystem processes in terms of economic values.

7.2.3 *Environmental Valuation*

Amongst the most important data for estimating ecological economic models are the values of ecological or environmental goods and services. In our research, we use the results of meta analysis on the values of ecosystem services of wetlands, but they are neither certain nor precise. We need more environmental valuation researches. This requires a method that is practical and easy to execute. Without appropriate valuations, we are unable to make integrated models more significant and meaningful.

Environmental or ecological restoration may, for example be an important option. Unless situations are irreversible, restoration is potentially one of the most effective options for managing environmental and ecological resources. However, if we have no relevant information on costs of restoration, we cannot make appropriate decisions from the economic point of view. Valuation of environmental or ecological restoration is an important research challenge for the future.

Appendix A

Appendices of Chapter 3

A-1 Convexity of Expected Cost Function of Flood Risk

We show the conditions for the convexity of the expected cost function with respect to all the control variables.

$$z = C^i\{x^i, a^i, q(x^j, x^j)\}$$

The second-order differential, d^2z, should be everywhere positive semidefinite for its convexity. The second-order differential can be expressed by a 2m×2m array. The coefficients of the array can be shown by the symmetric Hessian that is properly arranged. The Hessian is the following.

$$|H| = \begin{vmatrix} C^i_{x_i x_i} & C^i_{x_i a_i} & C^i_{x_i x_{j1}} & \cdots & C^i_{x_i x_{j_{m-1}}} & C^i_{x_i a_{j1}} & \cdots & C^i_{x_i a_{j_{m-1}}} \\ C^i_{a_i x_i} & C^i_{a_i a_i} & C^i_{a_i x_{j1}} & \cdots & C^i_{a_i x_{j_{m-1}}} & C^i_{a_i a_{j1}} & \cdots & C^i_{a_i a_{j_{m-1}}} \\ C^i_{x_{j1} x_i} & C^i_{x_{j1} a_i} & C^i_{x_{j1} x_{j1}} & \cdots & C^i_{x_{j1} x_{j_{m-1}}} & C^i_{x_{j1} a_{j1}} & \cdots & C^i_{x_{j1} a_{j_{m-1}}} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ C^i_{x_{j_{m-1}} x_i} & C^i_{x_{j_{m-1}} x_i} & C^i_{x_{j_{m-1}} x_{j1}} & \cdots & C^i_{x_{j_{m-1}} x_{j_{m-1}}} & C^i_{x_{j_{m-1}} a_{j1}} & \cdots & C^i_{x_{j_{m-1}} a_{j_{m-1}}} \\ C^i_{a_{j1} x_i} & C^i_{a_{j1} a_i} & C^i_{a_{j1} x_{j1}} & \cdots & C^i_{a_{j1} x_{j_{m-1}}} & C^i_{a_{j1} a_{j1}} & \cdots & C^i_{a_{j1} a_{j_{m-1}}} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ C^i_{a_{j_{m-1}} x_i} & C^i_{a_{j_{m-1}} a_i} & C^i_{a_{j_{m-1}} x_{j1}} & \cdots & C^i_{a_{j_{m-1}} x_{j_{m-1}}} & C^i_{a_{j_{m-1}} a_{j1}} & \cdots & C^i_{a_{j_{m-1}} a_{j_{m-1}}} \end{vmatrix}$$

$|H_1|, |H_2|, \cdots, |H_{2m}|$ are its principal minors.

Then, we need the condition that all the determinants of 2m principal minors are non-negative for the convexity.

$$|H_1|, |H_2|, \cdots, |H_{2m}| \geq 0$$

A-2 Meaning of Lagrange Multiplier

The optimal value of Lagrange multiplier can be interpreted as the marginal utility of the size of floodplain. The Lagrange multiplier implies the imputed value or shadow price of the natural floodplain that can be developed. We mathematically show the reason why we can interpret the meaning of the Lagrange multiplier as such.

To begin with, we assume that $\overline{\lambda}^i$, $\overline{x}^i$ and $\overline{a}^i$ are the optimal values for all i. They can be expressed as a function of parameters $\mathbf{L}_F$ $\left(=\left(L_F^1, L_F^2, \cdots, L_F^m\right)\right)$ (exogenous variables): $\overline{\lambda}^i(\mathbf{L}_F)$, $\overline{x}^i(\mathbf{L}_F)$ and $\overline{a}^i(\mathbf{L}_F)$. These optimal values satisfy the first-order necessary conditions (3-4), (3-5) and the constraint.

$$U_\pi \left\{ \frac{dB}{d\overline{x}^i} + \frac{df}{d\overline{x}^i} - \frac{\partial C^i}{\partial \overline{x}^i} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{x}^i} \right\} - \overline{\lambda}^i = 0 \quad \text{(for all } i\text{)}$$

(A-2-1)

$$U_\pi \left\{ -\frac{dg}{d\overline{a}^i} - \frac{\partial C^i}{\partial \overline{a}^i} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{a}^i} \right\} = 0 \quad \text{(for all } i\text{)}$$

(A-2-2)

$$L_F^i - \overline{x}^i = 0 \quad \text{(for all } i\text{)}$$

(A-2-3)

Next, $\overline{\lambda}^i(\mathbf{L}_F)$, $\overline{x}^i(\mathbf{L}_F)$ and $\overline{a}^i(\mathbf{L}_F)$ provide the optimal value $\overline{Z}$.

$$\overline{Z} = U\{\pi(\overline{\mathbf{x}}, \overline{\mathbf{a}})\} + \sum_{i=1}^{m} \overline{\lambda}^i \left(L_F^i - \overline{x}^i\right)$$

The function $\overline{Z}$ is a function of the parameters $\mathbf{L}_F$. Differentiating the function $\overline{Z}$ with respect to L_F^i as a representative,

$$\frac{\partial \overline{Z}}{\partial L_F^i} = U_\pi \sum_{k=1}^{m} \left(\frac{dB}{d\overline{x}^k} + \frac{df}{d\overline{x}^k} - \frac{\partial C^k}{\partial \overline{x}^k} - \sum_{j>k} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{x}^k} \right) \frac{\partial \overline{x}^k}{\partial L_F^i}$$

$$+ U_\pi \sum_{k=1}^{m} \left(-\frac{dg}{d\overline{a}^k} - \frac{\partial C^k}{\partial \overline{a}^k} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{a}^k} \right) \frac{\partial \overline{a}^k}{\partial L_F^i}$$

$$+ \overline{\lambda}^i - \sum_{k=1}^{m} \overline{\lambda}^k \frac{d\overline{x}^k}{dL_F^i} + \sum_{k=1}^{m} \frac{\partial \overline{\lambda}^k}{\partial L_F^i} \left(L_F^k - \overline{x}^k \right)$$

$$= U_\pi \sum_{k=1}^{m} \left(\frac{dB}{d\overline{x}^k} + \frac{df}{d\overline{x}^k} - \frac{\partial C^k}{\partial \overline{x}^k} - \sum_{j>k} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{x}^k} - \overline{\lambda}^k \right) \frac{\partial \overline{x}^k}{\partial L_F^i}$$

$$+ U_\pi \sum_{k=1}^{m} \left(-\frac{dg}{d\overline{a}^k} - \frac{\partial C^k}{\partial \overline{a}^k} - \sum_{j>i} \frac{\partial C^j}{\partial q} \frac{\partial q}{\partial \overline{a}^k} \right) \frac{\partial \overline{a}^k}{\partial L_F^i}$$

$$+ \overline{\lambda}^i + \sum_{k=1}^{m} \frac{\partial \overline{\lambda}^k}{\partial L_F^i} \left(L_F^k - \overline{x}^k \right)$$

$= \overline{\lambda}^i$ (because of (A-2-1), (A-2-2) and (A-2-3))[1]

Then,

$$\overline{\lambda}^i = \frac{\partial \overline{Z}}{\partial L_F^i} \left(= \frac{\partial \overline{U}}{\partial L_F^i} \right) \quad \text{(for all } i\text{)}$$

This implies that the Lagrange multiplier is the marginal utility of the size of floodplain.

[1] The values of the parentheses are equal to zero.

A-3 Composite Concave Function

If a function is a concave function of a concave function in relevant variables, it is a concave function in them. We will show the proof of the statement in the case that fits with the model we developed. Here, we focus on the case of two variables.

Set the functions below. Both of them are continuous and twice differentiable with respect to relevant variables.

$W = U(\pi)$

where $\pi = f(x_1, x_2)$

We assume that $U(\pi)$ is concave in π and that $f(x_1, x_2)$ is concave in (x_1, x_2).

$$U_\pi = \frac{dU}{d\pi} > 0 \tag{A-3-1}$$

$$U_{\pi\pi} = \frac{d^2U}{d\pi^2} < 0 \tag{A-3-2}$$

$$f_{11} = \frac{\partial^2 f}{\partial x_1^2} < 0 \tag{A-3-3}$$

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} = f_{11}f_{22} - f_{12}^2 > 0 \tag{A-3-4}$$

Then, under the assumptions we show that $U(\pi)$ is concave in (x_1, x_2) jointly.

$$U_{11} = \frac{\partial^2 U}{\partial x_1^2} = U_{\pi\pi} f_1^2 + U_\pi f_{11} < 0 \qquad \text{(from A-3-1, A-3-2 and A-3-3)}$$

$$\begin{vmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{vmatrix} = U_{11}U_{22} - U_{12}^2$$

$$= \left(U_{\pi\pi} f_1^2 + U_\pi f_{11}\right)\left(U_{\pi\pi} f_2^2 + U_\pi f_{22}\right) - \left(U_{\pi\pi} f_1 f_2 + U_\pi f_{12}\right)^2$$

$$= U_\pi^2\left(f_{11}f_{22} - f_{12}^2\right) + U_\pi U_{\pi\pi} f_{11}\left\{\left(f_2 - f_1\frac{f_{12}}{f_{11}}\right)^2 + \left(\frac{f_1}{f_{11}}\right)^2\left(f_{11}f_{22} - f_{12}^2\right)\right\} > 0$$

(from A-3-1, A-3-2, A-3-3 and A-3-4)

Hence, $U(\pi)$ is concave in (x_1, x_2) jointly.

■

Appendix B

Appendices of Chapter 4

B-1 General Form of System of Differential Equations

We show a general form of the system of differential equations in case of two zones (4-25) with the process of deriving it. We use a mathematics software program, 'Maple version 8.00' because it is too complicated to calculate and arrange the differential equations by hand. Thus, we provide Maple codes as the process of derivation. We omit the final results for the purpose of saving many sheets of papers because they provide four long equations. You can obtain the results if you copy and paste the following codes into Maple and execute the worksheet.

```
> with(student):
> Dy1:=diff(D(y1),y1):
> Dyy1:=diff(Dy1,y1):
> Dy2:=diff(D(y2),y2):
> Dyy2:=diff(Dy2,y2):
> Up:=diff(U(pi),pi):
> Upp:=diff(Up,pi):
> BX1:=diff(B(X1),X1):
> BX2:=diff(B(X2),X2):
> FX1:=diff(F(X1),X1):
> FX2:=diff(F(X2),X2):
> Ga1:=diff(G(a1),a1):
> Gaa1:=diff(Ga1,a1):
```

```
> Ga2:=diff(G(a2),a2):
> Gaa2:=diff(Ga2,a2):
> MA1:=diff(M(A1),A1):
> MA2:=diff(M(A2),A2):
> dC1dX1:=diff(C1(X1,A1),X1):
> dC1dA1:=diff(C1(X1,A1),A1):
> dC2dX2:=diff(C2(X2,A2,h),X2):
> dC2dA2:=diff(C2(X2,A2,h),A2):
> dC2dh:=diff(C2(X2,A2,h),h):
> dhdX1:=diff(h(X1,A1),X1):
> dhdA1:=diff(h(X1,A1),A1):
> y1t:=diff(y1(t),t):
> y2t:=diff(y2(t),t):
> a1t:=diff(a1(t),t):
> a2t:=diff(a2(t),t):
>
eq1:=Dy1*Upp*(BX1*y1+BX2*y2+FX1*y1+FX2*y2-Dy1*y1t-Dy2*y2t-Ga1*a
1t-Ga2*a2t-MA1*a1-MA2*a2-dC1dX1*y1-dC2dX2*y2-dC1dA1*a1-dC2dA2*a2
-dC2dh*dhdX1*y1-dC2dh*dhdA1*a1)=Up*(delta*Dy1-Dyy1*y1t-BX1-FX1+dC
1dX1+dC2dh*dhdX1):
>
eq2:=Dy2*Upp*(BX1*y1+BX2*y2+FX1*y1+FX2*y2-Dy1*y1t-Dy2*y2t-Ga1*a
1t-Ga2*a2t-MA1*a1-MA2*a2-dC1dX1*y1-dC2dX2*y2-dC1dA1*a1-dC2dA2*a2
-dC2dh*dhdX1*y1-dC2dh*dhdA1*a1)=Up*(delta*Dy2-Dyy2*y2t-BX2-FX2+dC
2dX2):
>
eq3:=Ga1*Upp*(BX1*y1+BX2*y2+FX1*y1+FX2*y2-Dy1*y1t-Dy2*y2t-Ga1*a
1t-Ga2*a2t-MA1*a1-MA2*a2-dC1dX1*y1-dC2dX2*y2-dC1dA1*a1-dC2dA2*a2
-dC2dh*dhdX1*y1-dC2dh*dhdA1*a1)=Up*(delta*Ga1-Gaa1*a1t+MA1+dC1dA
1+dC2dh*dhdA1):
```

```
>
eq4:=Ga2*Upp*(BX1*y1+BX2*y2+FX1*y1+FX2*y2-Dy1*y1t-Dy2*y2t-Ga1*a
1t-Ga2*a2t-MA1*a1-MA2*a2-dC1dX1*y1-dC2dX2*y2-dC1dA1*a1-dC2dA2*a2
-dC2dh*dhdX1*y1-dC2dh*dhdA1*a1)=Up*(delta*Ga2-Gaa2*a2t+MA2+dC2dA
2):
> solve({eq1,eq2,eq3,eq4},{y1t,y2t,a1t,a2t});
```

B-2 Derivation of Jacobian Matrix

Let us show the process of deriving Jacobian matrix (4-35) in this appendix.

To begin with, we consider the first row of the matrix. We focus on the first differential equation in the system of differential equations (4-29).

$$\dot{y}_t^{i=1} = f^1\left(y_t^{i=1}, y_t^{i=2}, X_t^{i=1}, X_t^{i=2}\right) = \frac{NR_1}{DN}$$

We have to differentiate this with respect to relevant arguments.

In the beginning, let us differentiate this in a general form and evaluate the derivative in the optimal steady-state solution $\left(\bar{y}_t^{i=1}, \bar{y}_t^{i=2}, \bar{X}_t^{i=1}, \bar{X}_t^{i=2}\right)$ in order to make the calculation easy and simple. Substituting conditions (4-31), (4-32), (4-33) and (4-34) into NR_1, we get $NR_1 = 0$. Utilising $NR_1 = 0$, we can obtain the following.

$$\left(\dot{y}_t^{i=1}\right)'\Big|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \left(\frac{NR_1}{DN}\right)'\Bigg|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{\left(NR_1\right)' \cdot DN - NR_1 \cdot \left(DN\right)'}{\left(DN\right)^2}\Bigg|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{\left(NR_1\right)'}{DN}\Bigg|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} \tag{B-2-1}$$

We set the followings.

$$NR_{11} = \frac{d^2 D}{d\left(y_t^{i=2}\right)^2}\left\{-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \delta\frac{dD}{dy_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right\}$$

$$NR_{12} = y_t^{i=1}\left(\frac{dD}{dy_t^{i=1}}\frac{d^2 D}{d\left(y_t^{i=2}\right)^2}\right)\left(-\frac{dF}{dX_t^{i=1}} - \frac{dB}{dX_t^{i=1}} + \frac{\partial C^{i=1}}{\partial X_t^{i=1}} + \frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right)$$

$$+ y_t^{i=2}\left(\frac{dD}{dy_t^{i=1}}\frac{d^2 D}{d\left(y_t^{i=2}\right)^2}\right)\left(-\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}}\right)$$

$$+ \frac{dD}{dy_t^{i=1}}\frac{dD}{dy_t^{i=2}}\left(-\frac{dF}{dX_t^{i=2}} - \frac{dB}{dX_t^{i=2}} + \frac{\partial C^{i=2}}{\partial X_t^{i=2}}\right)$$

$$+\left(\frac{dD}{dy_t^{i=2}}\right)^2\left(\frac{dF}{dX_t^{i=1}}+\frac{dB}{dX_t^{i=1}}-\frac{\partial C^{i=1}}{\partial X_t^{i=1}}-\frac{\partial C^{i=2}}{\partial X_t^{i=1}}\right)$$

Evaluating the derivative in $\left(\overline{y}_t^{i=1},\overline{y}_t^{i=2},\overline{X}_t^{i=1},\overline{X}_t^{i=2}\right)$, $NR_{11}=NR_{12}=0$. Then, we can further the calculation (B-2-1) by using these.

$$\left(\dot{y}_t^{i=1}\right)'\Big|_{\overline{y},\overline{X}}=\frac{(NR_1)'}{DN}\Big|_{\overline{y},\overline{X}}$$

$$=\frac{(U_\pi)'\cdot NR_{11}+U_\pi\cdot(NR_{11})'+(U_{\pi\pi})'\cdot NR_{12}+U_{\pi\pi}\cdot(NR_{12})'}{DN}\Bigg|_{\overline{y},\overline{X}}$$

$$=\frac{U_\pi\cdot(NR_{11})'+U_{\pi\pi}\cdot(NR_{12})'}{DN}\Bigg|_{\overline{y},\overline{X}}$$

(B-2-2)

Using equation (B-2-2) and the optimal steady-state solution (4-31), (4-32), (4-33) and (4-34), we can calculate the factors of the first row in the Jacobian matrix.[1]

$$\frac{\partial f^1}{\partial y_t^{i=1}}\Bigg|_{\overline{y},\overline{X}}=\frac{U_\pi\cdot\frac{\partial NR_{11}}{\partial y_t^{i=1}}+U_{\pi\pi}\cdot\frac{\partial NR_{12}}{\partial y_t^{i=1}}}{DN}=\frac{\delta U_\pi\frac{\partial^2 D}{\partial\left(y_t^{i=1}\right)^2}\frac{\partial^2 D}{\partial\left(y_t^{i=2}\right)^2}}{DN}$$

(B-2-3)

$$\left(\because\ \frac{\partial NR_{12}}{\partial y_t^{i=1}}\Bigg|_{\overline{y},\overline{X}}=0\right)$$

$$\frac{\partial f^1}{\partial y_t^{i=2}}\Bigg|_{\overline{y},\overline{X}}=\frac{U_\pi\cdot\frac{\partial NR_{11}}{\partial y_t^{i=2}}+U_{\pi\pi}\cdot\frac{\partial NR_{12}}{\partial y_t^{i=2}}}{DN}=0$$

(B-2-4)

---

[1] In addition, we can use $\frac{dD}{dy_t^{i=1}}\Big|_{y_t^{i=1}=0}=\frac{dD}{dy_t^{i=2}}\Big|_{y_t^{i=2}=0}=0$, based on the assumption.

$$(\because \left.\frac{\partial NR_{11}}{\partial y_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \left.\frac{\partial NR_{12}}{\partial y_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

$$\left.\frac{\partial f^1}{\partial X_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \frac{\partial NR_{11}}{\partial X_t^{i=1}} + U_{\pi\pi} \cdot \frac{\partial NR_{12}}{\partial X_t^{i=1}}}{DN}$$

$$= \frac{U\pi}{DN} \cdot \frac{d^2 D}{d\left(y_t^{i=2}\right)^2} \cdot \left\{ -\frac{d^2 F}{d\left(X_t^{i=1}\right)^2} - \frac{d^2 B}{d\left(X_t^{i=1}\right)^2} + \frac{\partial^2 C^{i=1}}{\partial\left(X_t^{i=1}\right)^2} + \frac{\partial^2 C^{i=2}}{\partial\left(X_t^{i=1}\right)^2} \right\}$$

(B-2-5)

$$(\because \left.\frac{\partial NR_{12}}{\partial X_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

$$\left.\frac{\partial f^1}{\partial X_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \frac{\partial NR_{11}}{\partial X_t^{i=2}} + U_{\pi\pi} \cdot \frac{\partial NR_{12}}{\partial X_t^{i=2}}}{DN} = \frac{U_\pi}{DN} \frac{d^2 D}{d\left(y_t^{i=2}\right)^2} \frac{\partial^2 C^{i=2}}{\partial X_t^{i=1} \partial X_t^{i=2}}$$

(B-2-6)

$$(\because \left.\frac{\partial NR_{12}}{\partial X_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

Likewise, we can treat the second differential equation in the system of differential equations (4-29).

$$\left.\left(\dot{y}_t^{i=2}\right)'\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \left.\frac{\left(NR_1\right)'}{DN}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}}$$

$$= \left.\frac{U_\pi \cdot \left(NR_{21}\right)' + U_{\pi\pi} \cdot \left(NR_{22}\right)'}{DN}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}}$$

(B-2-7)

Using equation (B-2-7) and the optimal steady-state solution (4-31), (4-32), (4-33) and (4-34), we can calculate the factors of the second row in the Jacobian matrix.[2]

$$\left.\frac{\partial f^2}{\partial y_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \dfrac{\partial NR_{21}}{\partial y_t^{i=1}} + U_{\pi\pi} \cdot \dfrac{\partial NR_{22}}{\partial y_t^{i=1}}}{DN} = 0$$

(B-2-8)

$$(\because \left.\frac{\partial NR_{21}}{\partial y_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \left.\frac{\partial NR_{22}}{\partial y_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

$$\left.\frac{\partial f^2}{\partial y_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \dfrac{\partial NR_{21}}{\partial y_t^{i=2}} + U_{\pi\pi} \cdot \dfrac{\partial NR_{22}}{\partial y_t^{i=2}}}{DN} = \frac{\delta U_\pi \dfrac{\partial^2 D}{\partial (y_t^{i=1})^2} \dfrac{\partial^2 D}{\partial (y_t^{i=2})^2}}{DN}$$

(B-2-9)

$$(\because \left.\frac{\partial NR_{22}}{\partial y_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

$$\left.\frac{\partial f^2}{\partial X_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \dfrac{\partial NR_{21}}{\partial X_t^{i=1}} + U_{\pi\pi} \cdot \dfrac{\partial NR_{22}}{\partial X_t^{i=1}}}{DN} = \frac{U_\pi}{DN} \frac{d^2 D}{d(y_t^{i=1})^2} \frac{\partial^2 C^{i=2}}{\partial X_t^{i=1} \partial X_t^{i=2}}$$

(B-2-10)

$$(\because \left.\frac{\partial NR_{22}}{\partial X_t^{i=1}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = 0)$$

$$\left.\frac{\partial f^2}{\partial X_t^{i=2}}\right|_{\bar{\mathbf{y}},\bar{\mathbf{X}}} = \frac{U_\pi \cdot \dfrac{\partial NR_{21}}{\partial X_t^{i=2}} + U_{\pi\pi} \cdot \dfrac{\partial NR_{22}}{\partial X_t^{i=2}}}{DN}$$

$$= \frac{U\pi}{DN} \cdot \frac{d^2 D}{d(y_t^{i=1})^2} \cdot \left\{ -\frac{d^2 F}{d(X_t^{i=2})^2} - \frac{d^2 B}{d(X_t^{i=2})^2} + \frac{\partial^2 C^{i=2}}{\partial (X_t^{i=2})^2} \right\}$$

(B-2-11)

---

[2] In addition, we can use $\left.\frac{dD}{dy_t^{i=1}}\right|_{y_t^{i=1}=0} = \left.\frac{dD}{dy_t^{i=2}}\right|_{y_t^{i=2}=0} = 0$, based on the assumption.

$$(\because \left.\frac{\partial NR_{22}}{\partial X_t^{i=2}}\right|_{\overline{\mathbf{y}},\overline{\mathbf{X}}} = 0)$$

It is easy to derive the factors of the third and fourth row in the Jacobian matrix. Hence, we can obtain the Jacobian matrix (4-35) from (B-2-3) ~ (B-2-6) and (B-2-8) ~ (B-2-11).

Appendix C

Appendices of Chapter 5

C-1 Visual Basic Code for Calculation of Baseflow

Let us show the visual basic code on Microsoft Excel for the calculation of baseflow in this appendix. The raw data of daily discharge flow should be put from C1 cell in the C column on a spread sheet of Excel. The following procedures are allocated to command buttons respectively. You can carry out all the code at a time as a general procedure if you adjust the variables appropriately.

Procedure 1. Abstracting the minima

The result will be shown in the column of 'D'.

```
Private Sub CommandButton1_Click()

Dim i As Integer
Dim a As Double
Dim b As Double
Dim c As Double
Dim d As Double
Dim e As Double

For i = 1 To 1600
   If Cells(5 * i, 3).Value > 0 Then
         If Cells(5 * i - 4, 3).Value < Cells(5 * i - 3, 3).Value Then
            If Cells(5 * i - 4, 3).Value < Cells(5 * i - 2, 3).Value Then
               If Cells(5 * i - 4, 3).Value < Cells(5 * i - 1, 3).Value Then
                  If Cells(5 * i - 4, 3).Value < Cells(5 * i, 3).Value Then
                     Cells(5 * i - 4, 4).Value = Cells(5 * i - 4, 3).Value
                  Else
                     Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                  End If
               Else
                  If Cells(5 * i - 1, 3).Value < Cells(5 * i, 3).Value Then
                     Cells(5 * i - 1, 4).Value = Cells(5 * i - 1, 3).Value
                  Else
                     Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                  End If
               End If
```

```
                    Else
                        If Cells(5 * i - 2, 3).Value < Cells(5 * i - 1, 3).Value Then
                            If Cells(5 * i - 2, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 2, 4).Value = Cells(5 * i - 2, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        Else
                            If Cells(5 * i - 1, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 1, 4).Value = Cells(5 * i - 1, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        End If
                    End If
                Else
                    If Cells(5 * i - 3, 3).Value < Cells(5 * i - 2, 3).Value Then
                        If Cells(5 * i - 3, 3).Value < Cells(5 * i - 1, 3).Value Then
                            If Cells(5 * i - 3, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 3, 4).Value = Cells(5 * i - 3, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        Else
                            If Cells(5 * i - 1, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 1, 4).Value = Cells(5 * i - 1, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        End If
                    Else
                        If Cells(5 * i - 2, 3).Value < Cells(5 * i - 1, 3).Value Then
                            If Cells(5 * i - 2, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 2, 4).Value = Cells(5 * i - 2, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        Else
                            If Cells(5 * i - 1, 3).Value < Cells(5 * i, 3).Value Then
                                Cells(5 * i - 1, 4).Value = Cells(5 * i - 1, 3).Value
                            Else
                                Cells(5 * i, 4).Value = Cells(5 * i, 3).Value
                            End If
                        End If
                    End If
                End If
        Else
            MsgBox ("This procedure is over")
            Exit Sub
        End If
    Next

    End Sub
```

Procedure 2. Choosing the ordinates in blocks

The result will be shown in the column of 'E'.

```
Private Sub CommandButton2_Click()

Dim i As Integer
Dim j As Integer
Dim k As Integer

Dim a As Double
Dim b As Double
Dim c As Double

j = 1

Do While i <= 6000 And j <= 6000 And k <= 6000

    i = j

    Do While i <= 6000
        If Cells(i, 4).Value > 0 Then
            a = Cells(i, 4).Value
            Exit Do
        Else
            i = i + 1
        End If
    Loop

    j = i + 1

    Do While j <= 6000
        If Cells(j, 4).Value > 0 Then
            b = Cells(j, 4).Value
            Exit Do
        Else
            j = j + 1
        End If
    Loop

    k = j + 1

    Do While k <= 6000
        If Cells(k, 4).Value > 0 Then
            c = Cells(k, 4).Value
            Exit Do
        Else
            k = k + 1
        End If
    Loop

    If 0.9 * b < a And 0.9 * b < c Then
        Cells(j, 5).Value = Cells(j, 4).Value
    End If

Loop

MsgBox ("This procedure is over.")

End Sub
```

Procedure 3. Linear interpolation

The result will be shown in the column of 'F'.

```
Private Sub CommandButton3_Click()

Dim i As Integer
Dim j As Integer
Dim k As Integer

Dim a As Double
Dim b As Double
Dim c As Double

j = 1

Do While i <= 6000 And j <= 6000

    i = j

    Do While i <= 6000
        If Cells(i, 5).Value > 0 Then
            Cells(i, 6).Value = Cells(i, 5).Value
            a = Cells(i, 5).Value
            Exit Do
        Else
            i = i + 1
        End If
    Loop

    j = i + 1

    Do While j <= 6000
        If Cells(j, 5).Value > 0 Then
            Cells(j, 6).Value = Cells(j, 5).Value
            b = Cells(j, 5).Value
            Exit Do
        Else
            j = j + 1
        End If
    Loop

    c = (b - a) / (j - i)

    If i <= 6000 And j <= 6000 Then
        For k = i + 1 To j - 1
            Cells(k, 6).Value = a + c * (k - i)
        Next
    End If

Loop

MsgBox ("This procedure is over.")

End Sub
```

Procedure 4. Modification of exceptional values

The result will be shown in the column of 'G'.

```
Private Sub CommandButton6_Click()

Dim i As Integer
Dim j As Integer

For i = 1 To 6000
    If Cells(i, 6).Value > 0 Then
        If Cells(i, 6).Value > Cells(i, 3).Value Then
            Cells(i, 7).Value = Cells(i, 3).Value
        Else
            Cells(i, 7).Value = Cells(i, 6).Value
        End If
    End If
Next

MsgBox ("This procedure is over.")

End Sub
```

Procedure 5. Calculating BFI

The result will be shown in the cell 'A4'.

```
Private Sub CommandButton4_Click()

Dim i As Integer

Dim a As Double
Dim b As Double
Dim c As Double

a = 0
b = 0

For i = 1 To 6000
    If Cells(i, 7).Value > 0 Then
        a = a + Cells(i, 7).Value
        b = b + Cells(i, 3).Value
    End If
Next

c = a / b

Cells(4, 1).Value = c

MsgBox ("This procedure is over")

End Sub
```

C-2 Location of Measurement Sites

The locations of the measurement sites are provided by OS grid system (m). They are collected on the relevant maps by ArcGIS software program. Table C-1 shows them.

Table C-1. *Location of the measurement sites*

Subbasin	Name of Stations	Location	
27001	Nidd at Hunsingore Weir	435059	456443
27002	Wharfe at Flint Mill Weir	425979	445960
27005	Nidd at Gouthwaite Reservoir	410745	472690
27007	Ure at Westwick Lock	427383	478870
27009	Ouse at Skelton	446140	463487
27034	Ure at Kilgram Bridge	397973	489536
27043	Wharfe at Addingham	398779	464658
27053	Nidd at Birstwith	419019	463349
27069	Wiske at Kirby Wiske	432914	502574
27071	Swale at Crakehill	434084	486579
27075	Bedale Beck at Leeming	422791	490287
27083	Foss at Huntington	463470	464529
27085	Cod Beck at Dalton Bridge	441747	486083
27089	Wharfe at Tadcaster	444830	445527
27090	Swale at Catterick Bridge	408154	497446
	York	460543	450684
	Selby	461610	432726

C-3 Data of Areas in Subbasins

We calculate the areas of floodplains and the areas in each category of land-use in each subbasin by ArcGIS. Table C-2 shows the areas of subbasins and floodplains that we calculate.[1] Table C-3 shows the areas of categories of land-use in each subbasin that we calculate from LCM 1990 by ArcGIS.

Table C-2. *Areas of subbasins and floodplains*

Subbasin	Name of Stations	Area of Subbasin (ha)	Area of Floodplain (ha)
27001	Nidd at Hunsingore Weir	26,124.8	1,359.7
27002	Wharfe at Flint Mill Weir	32,933.1	2,629.7
27005	Nidd at Gouthwaite Reservoir	11,551.4	373.4
27007	Ure at Westwick Lock	40,069.8	2,498.0
27009	Ouse at Skelton	52,964.5	6,646.2
27034	Ure at Kilgram Bridge	51,308.2	2,476.9
27043	Wharfe at Addingham	42,824.3	1,402.5
27053	Nidd at Birstwith	10,629.8	348.0
27069	Wiske at Kirby Wiske	23,264.6	1,338.2
27071	Swale at Crakehill	26,199.7	3,621.6
27075	Bedale Beck at Leeming	16,854.4	1,029.1
27083	Foss at Huntington	13,870.1	1,119.7
27085	Cod Beck at Dalton Bridge	20,710.4	1,152.2
27089	Wharfe at Tadcaster	5,563.5	443.8
27090	Swale at Catterick Bridge	49,410.6	1,632.8

[1] The areas of subbasins that we calculate are slightly different from those provided by NWA. Calculating values of variables based on the data of areas, we use the one that is a basis of the variables. In addition, we calculate values of variables by using the ratios of relevant areas.

Table C-3. *Areas of each land-use category in floodplains (ha)*

ID	Category	27001	27002	27005	27007	27009	27034	27043	27053
1	Sea / Estuary	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	Inland Water	2.742	226.984	168.737	51.003	6.697	25.327	144.755	0.880
3	Beach & Coastal Bare	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	Saltmarsh	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	Grass Heath	108.962	267.487	5.185	129.793	17.174	70.051	72.748	50.052
6	Mown / Grazed Turf	173.180	725.335	5.908	504.658	1213.976	396.931	117.348	68.514
7	Meadow / Verge / Semi-natural	136.891	253.883	46.541	332.331	1185.977	879.778	512.933	84.171
8	Rough / Marsh Grass	0.006	0.772	0.019	1.873	18.226	0.299	0.628	0.086
9	Moorland Grass	0.000	9.533	37.486	10.187	0.000	106.201	110.398	14.547
10	Open Shrub Moor	0.000	3.816	7.463	14.745	0.000	10.903	24.555	7.936
11	Dense Shrub Moor	0.000	2.739	0.311	7.144	0.000	2.220	1.946	0.196
12	Bracken	6.924	7.009	10.474	41.959	99.820	26.415	28.153	2.338
13	Dense Shrub Heath	0.000	0.000	0.000	0.000	6.153	0.000	0.000	0.000
14	Scrub / Orchard	0.040	2.254	0.000	3.851	6.331	0.000	0.000	0.778
15	Deciduous Woodland	78.138	321.376	18.405	299.385	158.187	221.611	84.782	37.146
16	Coniferous Woodland	3.558	29.491	0.760	35.522	34.990	18.947	7.449	1.498
17	Upland Bog	0.000	0.032	0.709	1.872	0.000	2.884	2.036	0.355
18	Tilled Land	713.951	463.419	37.780	798.162	3279.341	621.813	228.630	66.689
19	Ruderal Weed	0.000	0.000	0.000	1.375	2.477	0.000	0.000	0.000
20	Suburban / Rural Development	79.303	143.444	4.629	122.001	249.684	44.112	17.638	7.846
21	Continuous Urban	20.091	73.689	9.759	33.263	58.782	7.018	10.054	0.125
22	Inland Bare Ground	13.373	29.905	3.490	42.525	112.964	2.422	5.869	0.001
23	Felled Forest	0.008	0.431	1.043	0.612	4.591	0.015	1.343	0.321
24	Lowland Bog	0.000	2.367	2.175	2.493	2.854	17.079	11.774	1.057
25	Open Shrub Heath	17.889	56.809	0.000	37.745	62.767	0.000	0.000	1.749
0	Unclassified	4.599	8.963	12.532	25.505	125.166	22.913	19.467	1.726

The table is continued to the next page.

Table C-3. *Areas of each land-use category in floodplains (ha) [continue]*

ID	Category	27069	27071	27075	27083	27085	27089	27090
1	Sea / Estuary	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	Inland Water	0.000	26.446	0.000	0.000	2.570	0.000	0.459
3	Beach & Coastal Bare	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	Saltmarsh	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	Grass Heath	49.686	194.517	89.174	51.720	9.942	25.434	66.911
6	Mown / Grazed Turf	473.975	707.355	285.256	188.484	346.488	123.492	154.878
7	Meadow / Verge / Semi-natural	226.971	263.449	151.344	74.323	148.184	79.442	469.922
8	Rough / Marsh Grass	0.892	0.125	0.458	0.123	0.000	0.307	1.117
9	Moorland Grass	0.000	0.214	1.113	0.000	1.506	0.000	148.313
10	Open Shrub Moor	0.223	0.000	2.170	0.000	2.016	0.000	29.173
11	Dense Shrub Moor	0.000	0.000	0.172	0.000	0.000	0.000	3.661
12	Bracken	29.704	30.063	0.418	19.379	89.288	0.522	36.518
13	Dense Shrub Heath	0.000	0.000	0.000	0.178	0.026	0.000	0.000
14	Scrub / Orchard	0.500	0.455	1.117	0.125	0.237	0.885	0.000
15	Deciduous Woodland	43.333	159.108	61.920	7.421	30.573	58.911	93.981
16	Coniferous Woodland	0.545	12.314	0.190	0.113	1.210	9.493	4.803
17	Upland Bog	0.000	0.000	0.153	0.000	0.000	0.000	10.581
18	Tilled Land	382.732	1807.957	362.344	614.526	410.460	95.289	509.906
19	Ruderal Weed	0.200	0.000	0.000	0.624	0.000	0.000	0.000
20	Suburban / Rural Development	96.132	272.438	61.708	96.596	43.458	28.297	65.243
21	Continuous Urban	5.493	33.710	2.107	9.677	16.605	6.886	1.851
22	Inland Bare Ground	7.475	34.264	3.470	21.606	5.324	1.604	1.708
23	Felled Forest	0.516	0.250	0.000	0.125	0.064	0.000	0.945
24	Lowland Bog	0.000	3.095	0.739	0.120	0.000	0.000	16.900
25	Open Shrub Heath	17.669	52.907	3.075	23.808	38.455	11.346	0.000
0	Unclassified	2.170	22.945	2.179	10.719	5.744	1.882	15.924

C-4 Width and Height of Floodplains

We measure the width of height of floodplains by using ArcGIS software program based on the relevant vector data and DEM. Table C-4 shows the data.

Table C-4. *Width and height of floodplains*

Subbasin	Width of left floodplain (m)	Width of right floodplain (m)	Average width	Height of left floodplain above riverbank (m)	Elevation of left riverbank (m)	Height of right floodplain above riverbank (m)	Elevation of right riverbank (m)	Average height (m)
27001	13.41	17.75	15.58	2	34	2	34	2
27002	336.81	356.48	346.65	3	43	2	43	2.5
27005	21.96	28.01	24.99	0	150	1	150	0.5
27007	20.73	410.27	215.50	0	45	1	44	0.5
27009	201.58	109.95	155.77	1	14	1	14	1
27034	67.16	136.37	101.77	2	198	2	197	2
27043	64.55	28.23	46.39	0	178	6	178	3
27053	193.68	13.33	103.51	1	98	3	98	2
27069	42.63	23.92	33.28	0	39	2	39	1
27071	250.64	497.03	373.84	-1	26	3	24	1
27075	4.78	29.66	17.22	0	54	4	54	2
27083	19.02	157.57	88.30	0	19	2	19	1
27085	74.41	66.09	70.25	3	39	2	39	2.5
27089	28.23	230.77	129.50	6	9	8	9	7
27090	51.26	14.47	32.87	3	158	4	159	3.5

Note: Refer to Table C-1 about the locations of the measurement sites.

C-5 Availability of Observed Data

The data of gauged daily flow (GDF), catchment monthly rainfall (CMR) and gauged monthly flow (GMF) are provided by National Water Flow Archive (NWA) (managed by CEH). Table C-5 shows the availability of the data.[2]

Table C-5. *Availability of observed hydrological data*

Subbasin	Name of Stations	Gauged Daily Flow	Catchment Monthly Rainfall	Gauged Monthly Flow
27001	Nidd at Hunsingore Weir	1935-2002	1991-2001	1991-2002
27002	Wharfe at Flint Mill Weir	1955-2002	1991-2001	1991-2003
27005	Nidd at Gouthwaite Reservoir	1991-2002 Missing in Mar 1999	1991-2001	1991-2002
27007	Ure at Westwick Lock	1958-2002	1991-2001	1991-2002
27009	Ouse at Skelton	1969-2002 Missing in Jul-Dec 1991	1991-2003	1991-2004 Missing in Aug-Sep 2004
27034	Ure at Kilgram Bridge	1991-2004	1991-2001	1991-2004
27043	Wharfe at Addingham	1991-2002	1991-2001	1991-2002
27053	Nidd at Birstwith	1991-2002	1991-2001	1991-2002
27069	Wiske at Kirby Wiske	1991-2002 Missing in 29-30 Dec 2001	1991-2001	1991-2002
27071	Swale at Crakehill	1991-2002	1991-2001	1991-2002
27075	Bedale Beck at Leeming	1991-2002 Missing in 25 Feb - 8 Mar 12-15 Mar 1999	1991-2001	1991-2002
27083	Foss at Huntington	1991-1995 1997-2002 Missing in 12 Nov 1995	1991-2001	1991-2002
27085	Cod Beck at Dalton Bridge	1991-2002 Missing in 2 Aug-1 Sep 1992 4-31 Oct 1992 16-17 Aug 2002	1991-2001	1991-2002 Missing in Aug-Oct 1992
27089	Wharfe at Tadcaster	27 Jun 1991-2002	1991-2001	Jul 1991-2002
27090	Swale at Catterick Bridge	17 Dec 1992-2004	1991-2001	Dec 92-04

[2] The recent data are being updated by CEH.

C-6 Linkage of Data between Rain and Hydrological Gauging Stations

We convert the data of catchment monthly rainfall into the daily data by using the data of daily rainfall based on the nearest rain gauging stations that are managed by MET Office. Table C-6 shows the linkage of the data between rain and hydrological gauging stations. Table C-7 provides the explanation of the data type that is included in Table C-6.

Table C-6. *Linkage between rain and hydrological gauging stations*

Gauging Station	From	Until	Met Office Rainfall Gauging Station	Approximate Distance (km)	Data Type
27001	JAN,91	JUN,92	Long Marston South Park	8	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	JUN,98	Long Marston South Park	8	WADRAIN
	JUL,98	OCT,98	Missing Data	-	-
	NOV,98	DEC,98	Long Marston South Park	8	WADRAIN
	JAN,99	MAY,99	Missing Data	-	-
	JUN,99	AUG,99	Long Marston South Park	8	WADRAIN
	SEP,99	DEC,99	Missing Data	-	-
27002	JAN,91	JUN,92	Bramham	6	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	DEC,97	Bramham	6	WADRAIN
	JAN,98	OCT,99	Bramham	6	DLY3208
	NOV,99	NOV,99	Missing Data	-	-
	DEC,99	DEC,99	Bramham	6	DLY3208
27005	JAN,91	JUN,92	Gouthwaite RESR	0	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	MAY,98	Gouthwaite RESR	0	WADRAIN
	JUN,98	JUL,98	Grimwith RESR	9	WADRAIN
	AUG,98	OCT,99	Gouthwaite RESR	0	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-
27007	JAN,91	JUN,92	Ripon S.WKS	5	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	FEB,99	Ripon S.WKS	5	WADRAIN
	MAR,99	MAR,99	Lower Dunsforth	8	WADRAIN
	APR,99	MAY,99	Ripon S.WKS	5	WADRAIN
	JUN,99	JUL,99	Lower Dunsforth	8	WADRAIN
	AUG,99	OCT,99	Ripon S.WKS	5	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-

The table is continued to the next page.

Table C-6. *Linkage between rain and hydrological gauging stations (continue)*

Gauging Station	From	Until	Met Office Rainfall Gauging Station	Approximate Distance (km)	Data Type
27009	JAN,91	JUN,92	York, Acomb Landing TR.WKS	3	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	FEB,93	York, The Retreat	7	WADRAIN
	APR,93	MAY,93	York, Acomb Landing TR.WKS	3	WADRAIN
	JUN,93	JUL,93	York, The Retreat	7	WADRAIN
	AUG,93	MAR,94	York, Acomb Landing TR.WKS	3	WADRAIN
	APR,94	MAY,94	York, The Retreat	7	WADRAIN
	JUN,94	JAN,96	York, Acomb Landing TR.WKS	3	WADRAIN
	FEB,96	MAR,96	York, The Retreat	7	WADRAIN
	APR,96	OCT,99	York, Acomb Landing TR.WKS	3	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-
27034	JAN,91	JUN,92	Little Crakehall	6	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	FEB, 99	Little Crakehall	6	WADRAIN
	MAR,99	MAR,99	Missing Data	-	-
	APR,99	OCT,99	Little Crakehall	6	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-
27043	JAN,91	JUN,92	March Ghyll RESR	4	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	MAY,98	March Ghyll RESR	4	WADRAIN
	JUN,98	JUL,98	Bolton Abbey	5	WADRAIN
	AUG,98	AUG,98	Chelker RESR	5	WADRAIN
	SEP,98	OCT,98	Bolton Abbey	5	WADRAIN
	NOV,98	DEC,98	March Ghyll RESR	4	WADRAIN
	JAN,99	FEB,99	Bolton Abbey	5	WADRAIN
	MAR,99	MAR,99	Chelker RESR	5	WADRAIN
	APR,99	JUL,99	March Ghyll RESR	4	WADRAIN
	AUG,99	SEP,99	Bolton Abbey	5	WADRAIN
	OCT,99	OCT,99	Chelker RESR	5	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-
27053	JAN,91	NOV,91	Birstwith Hall	2	WADRAIN
	DEC,91	DEC91	Scargill RESR	7	WADRAIN
	JAN,92	JUN,92	Birstwith Hall	2	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	NOV,97	Birstwith Hall	2	WADRAIN
	DEC,97	JAN,98	Scargill RESR	7	WADRAIN
	FEB,98	MAR,98	Birstwith Hall	2	WADRAIN
	APR,98	MAY,98	Scargill RESR	7	WADRAIN
	JUN,98	SEP,98	Harrogate	8	DLY3208
	OCT,98	OCT,98	Harlow Hill RESR	9	WADRAIN
	NOV,98	FEB,99	Scargill RESR	7	WADRAIN
	MAR,99	MAR,99	Harrogate	8	DLY3208
	APR,99	MAY,99	Birstwith Hall	2	WADRAIN
	JUN,99	JUN,99	Harrogate	8	DLY3208
	JUL,99	JUL,99	Birstwith Hall	2	WADRAIN
	AUG,99	OCT,99	Harrogate	8	DLY3208
	NOV,99	DEC,99	Missing Data	-	-

The table is continued to the next page.

Table C-6. *Linkage between rain and hydrological gauging stations (continue)*

Gauging Station	From	Until	Met Office Rainfall Gauging Station	Approximate Distance (km)	Data Type
27069	JAN,91	JUN,92	Thirsk South Villa	7	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	OCT,94	Thirsk South Villa	7	WADRAIN
	NOV,94	DEC,94	Leeming	8	WADRAIN
	JAN,95	JAN,95	Thirsk South Villa	7	WADRAIN
	FEB,95	MAR,95	Leeming	8	WADRAIN
	APR,95	SEP,99	Thirsk South Villa	7	WADRAIN
	OCT,99	DEC,99	Missing Data	-	-
27071	JAN,91	APR,92	Thirsk South Villa	7	WADRAIN
	MAY,92	JUN,92	Dishforth Airfield (SAMOS)	5	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	JAN,95	Dishforth Airfield (SAMOS)	5	NCM
	FEB,95	NOV,95	Dishforth Airfield (SAMOS)	5	WADRAIN
	DEC,95	DEC,95	Thirsk South Villa	7	WADRAIN
	JAN,96	FEB,97	Dishforth Airfield (SAMOS)	5	WADRAIN
	MAR,97	DEC,97	Thirsk South Villa	7	WADRAIN
	JAN,98	APR,98	Dishforth Airfield (SAMOS)	5	WADRAIN
	MAY,98	DEC,98	Thirsk South Villa	7	WADRAIN
	JAN,99	FEB,99	Topcliffe MET.OFFICE	6	WADRAIN
	MAR,99	SEP,99	Thirsk South Villa	7	WADRAIN
	OCT,99	DEC,99	Missing Data	-	-
27075	JAN,91	JUN,92	Leeming	1	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	DEC,95	Leeming	1	WADRAIN
	JAN,96	APR,96	Little Crakehall	7	WADRAIN
	MAY,96	FEB,97	Leeming	1	WADRAIN
	MAR,97	DEC,97	Little Crakehall	7	WADRAIN
	JAN,98	APR,98	Leeming	1	WADRAIN
	MAY,98	FEB,99	Little Crakehall	7	WADRAIN
	MAR,99	MAR,99	Missing Data	-	-
	APR,99	OCT,99	Little Crakehall	7	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-
27083	JAN,91	JUN,92	York, Acomb Landing TR.WKS	3	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	FEB,93	York, The Retreat	4	WADRAIN
	MAR,93	MAY,93	York, Acomb Landing TR.WKS	3	WADRAIN
	JUN,93	JUL,93	York, The Retreat	4	WADRAIN
	AUG,93	MAR,94	York, Acomb Landing TR.WKS	3	WADRAIN
	APR,94	MAY,94	York, The Retreat	4	WADRAIN
	JUN,94	JAN,96	York, Acomb Landing TR.WKS	3	WADRAIN
	FEB,96	MAR,96	York, The Retreat	4	WADRAIN
	APR,96	OCT,99	York, Acomb Landing TR.WKS	3	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-

The table is continued to the next page.

Table C-6. *Linkage between rain and hydrological gauging stations (continue)*

Gauging Station	From	Until	Met Office Rainfall Gauging Station	Approximate Distance (KM)	Data Type
27085	JAN,91	JUN,92	Thirsk South Villa	5	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	OCT,94	Thirsk South Villa	5	WADRAIN
	NOV,94	DEC,94	Dishforth Airfield (SAMOS)	7	NCM
	JAN,95	JAN,95	Thirsk South Villa	5	WADRAIN
	FEB,95	MAR,95	Dishforth Airfield (SAMOS)	7	WADRAIN
	APR,95	SEP,99	Thirsk South Villa	5	WADRAIN
	OCT,99	DEC,99	Missing Data	-	-
27089	JAN,91	JUN,92	Bramham	5	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	FEB,98	Bramham	5	WADRAIN
	MAR,98	JUN,98	Long Marston, South Park	7	WADRAIN
	JUL,98	AUG,98	Bramham	5	WADRAIN
	SEP,98	NOV,98	Askham Bryan	8	DLY3208
	DEC,98	DEC,98	Long Marston, South Park	7	WADRAIN
	JAN,99	JAN,99	Bramham	5	DLY3208
	FEB,99	OCT,99	Bramham	5	WADRAIN
	NOV,99	NOV,99	Missing Data	-	-
	DEC,99	DEC,99	Bramham	5	DLY3208
27090	JAN,91	JUN,92	Richmond, Green Howard RD	6	WADRAIN
	JUL,92	DEC,92	Missing Data	-	-
	JAN,93	DEC,98	Little Crakehall	9	WADRAIN
	JAN,99	JAN,99	Richmond, Green Howard RD	6	WADRAIN
	FEB,99	MAR,99	Little Crakehall	9	WADRAIN
	APR,99	MAY,99	Richmond, Green Howard RD	6	WADRAIN
	JUN,99	AUG,99	Little Crakehall	9	WADRAIN
	SEP,99	OCT,99	Richmond, Green Howard RD	6	WADRAIN
	NOV,99	DEC,99	Missing Data	-	-

Note: The approximate distance is the distance between the hydrological gauging station and the Met Office rainfall gauging station. It is automatically calculated on the BADC web site when we search for the nearest Met Office rainfall gauging station on the basis of OS grid reference.

Table C-7. *Type of rainfall data*

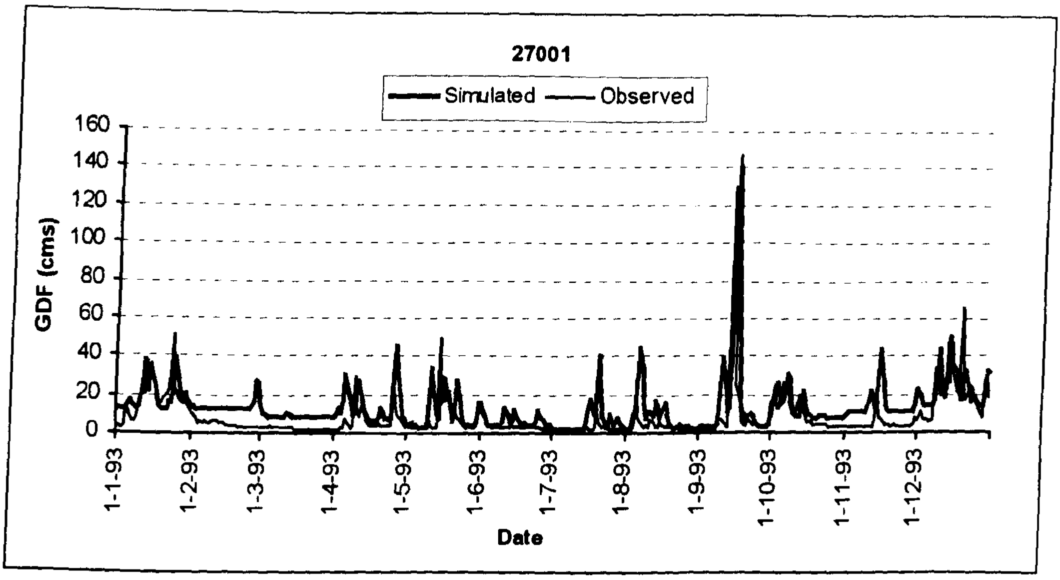
Data Type	Comments	Descripition
WADRAIN	Precipitation amount from daily rainfall station.	Daily rainfall amounts from rainfall network.
DLY3208	Daily precipitation amount from ordinary climatological station.	Elements from Metform 3208 - Monthly Return of Daily Obs.
NCM	Daily precipitation amount from synoptic station (no 12-hour values reported).	Elements from National Climate Messages reports (including climate reports pre 1982) - 12 hourly.

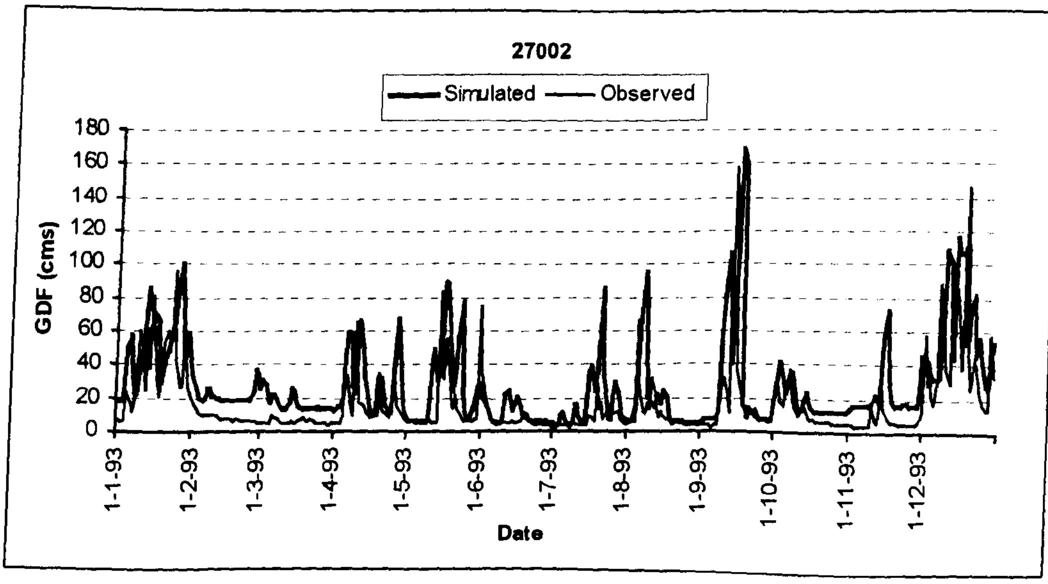
Source: Met Office - UK Land Surface Stations data from the British Atmospheric Data Centre (BADC)

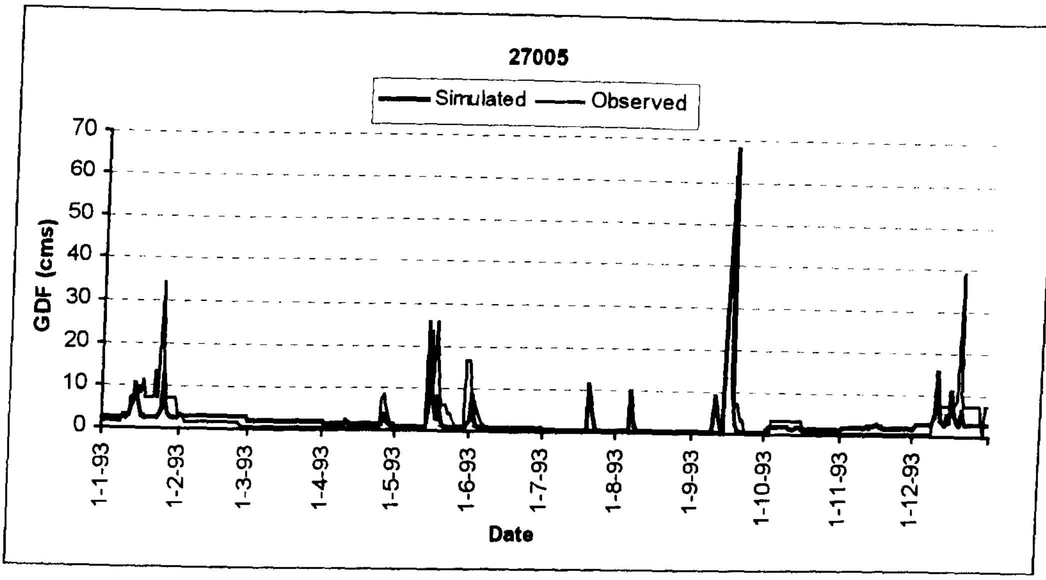
C-7 Flow Comparison Graphs

We calibrate the relevant hydrological parameter values based on the data in 1993. Then, we check flow comparison graphs in 1994 as well. In this appendix, we show the results of the flow comparison graphs in 1993 and 1994. Figure C-1 and C-2 shows them respectively. We use the software program HEC-DSS Microsoft Excel Data Exchange Add-In (USACE) for retrieving the data produced by the simulations on HEC-HMS. Using this, we can treat the data on Microsoft Excel as usual.

Figure C-1. *Flow comparison graphs in 1993*



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Simulated
Observed
GDF (cms)
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10
20
30
40
50
60
70
1-1-93
1-2-93
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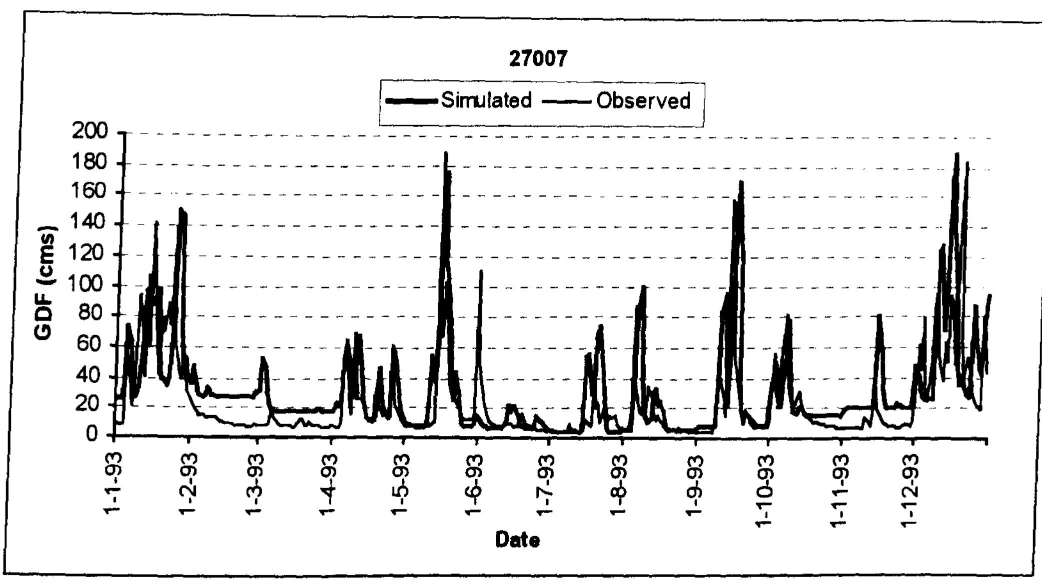
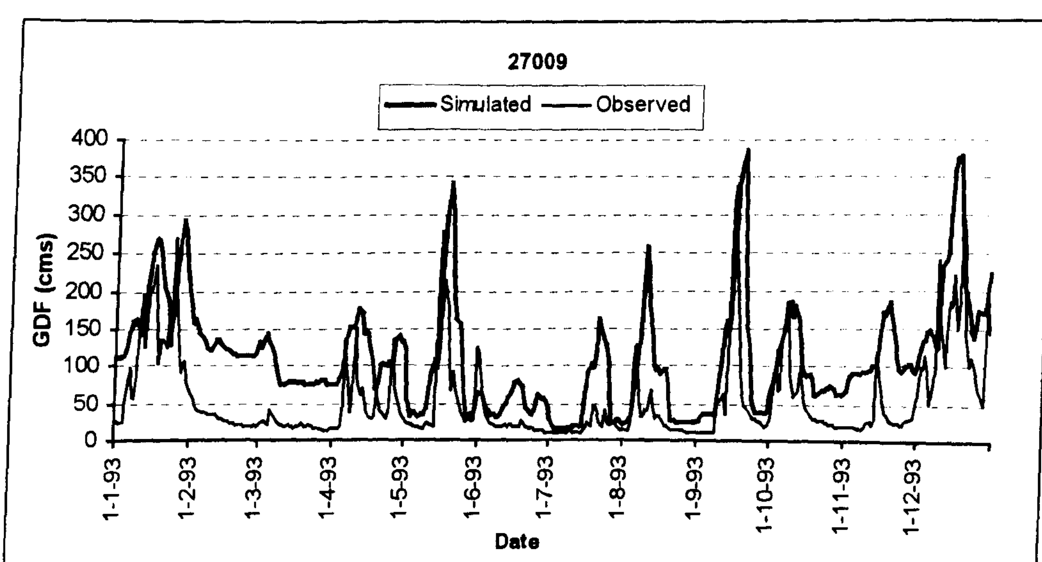
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date


27007
Simulated
Observed
GDF (cms)
0
20
40
60
80
100
120
140
160
180
200
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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50
100
150
200
250
300
350
400
1-1-93
1-2-93
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1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
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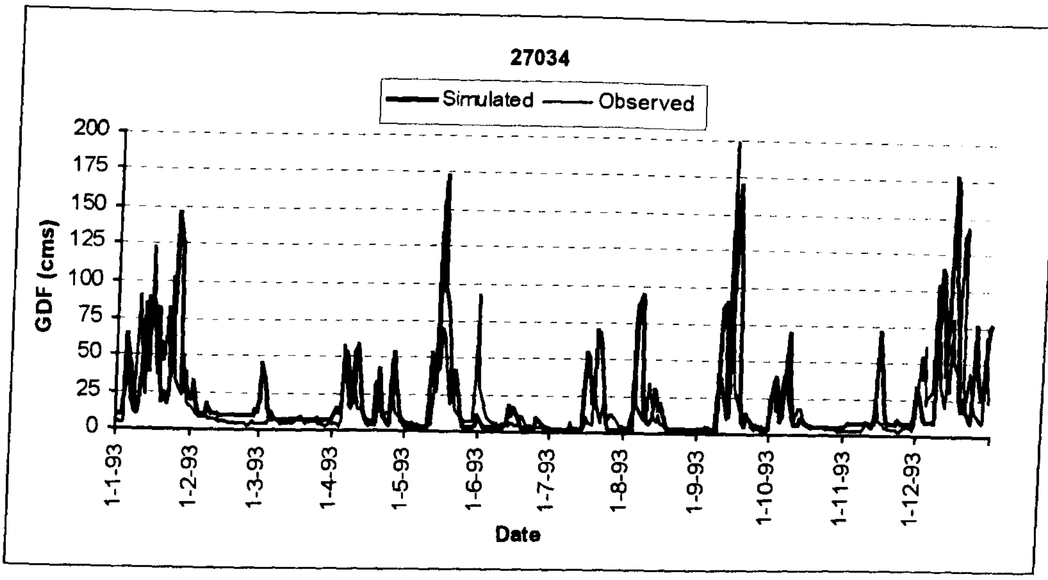
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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25
50
75
100
125
150
175
200
1-1-93
1-2-93
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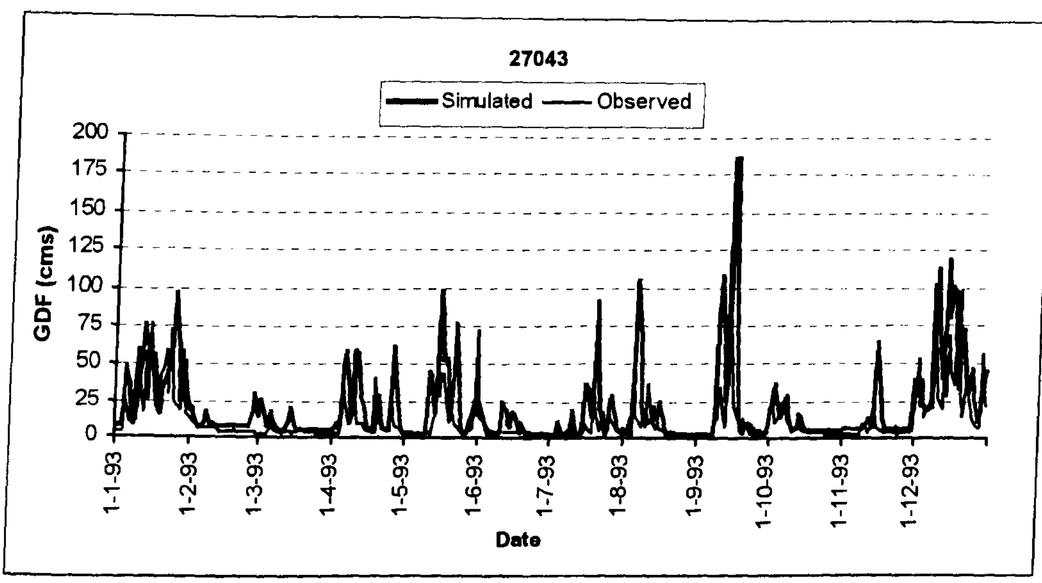
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
0
25
50
75
100
125
150
175
200
1-1-93
1-2-93
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1-4-93
1-5-93
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1-9-93
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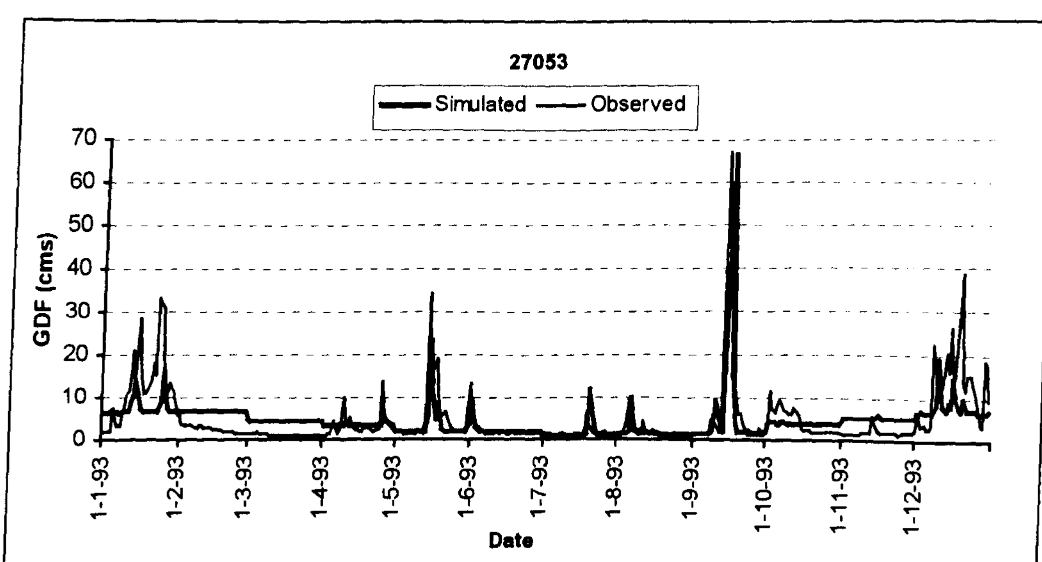
1-12-93
Date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Simulated
Observed
GDF (cms)
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20
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40
50
60
70
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1-4-93
1-5-93
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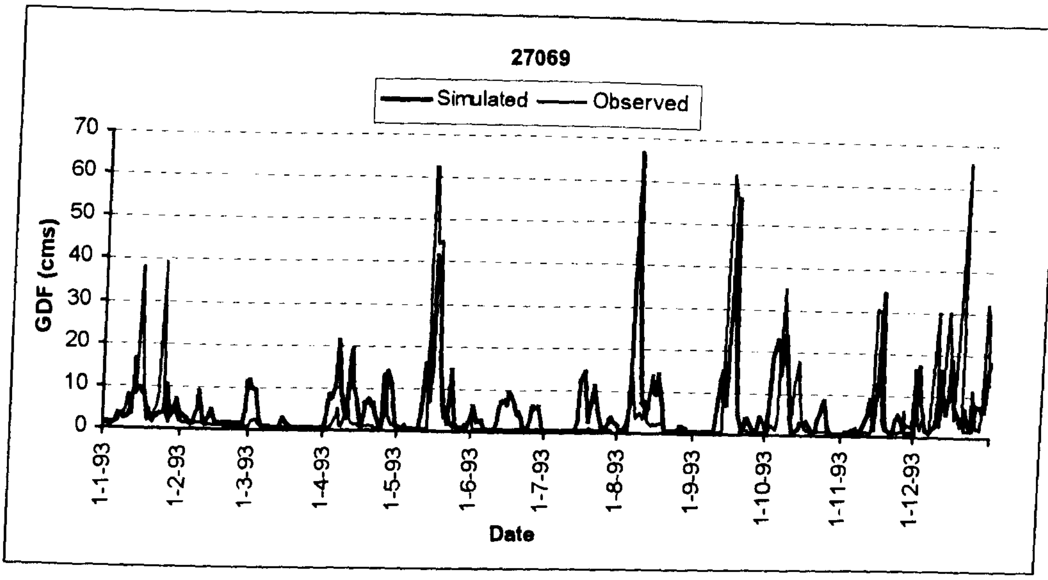
1-9-93
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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20
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40
50
60
70
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1-4-93
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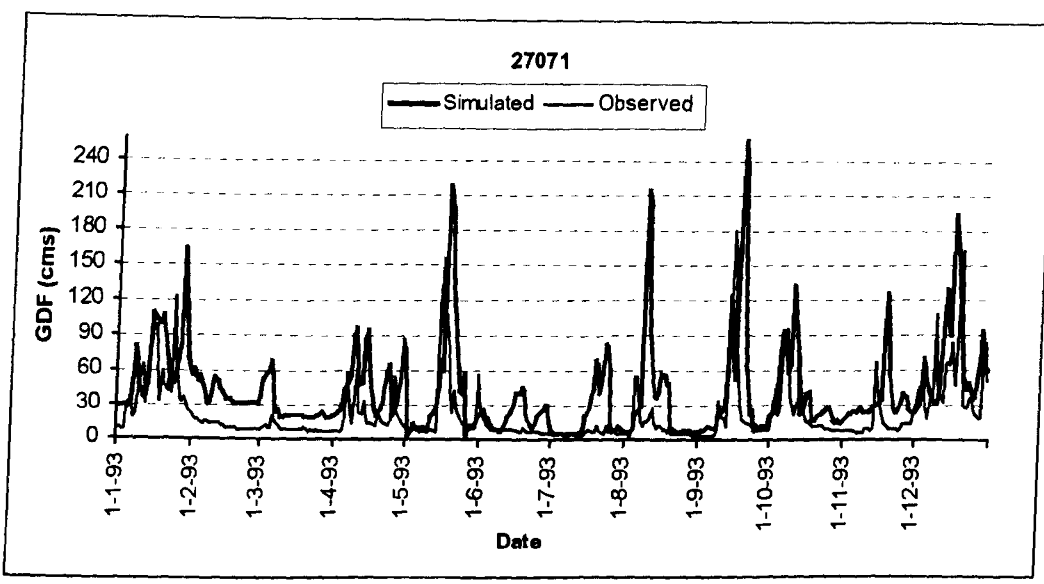
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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60
90
120
150
180
210
240
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1-2-93
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1-4-93
1-5-93
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1-7-93
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1-10-93
1-11-93
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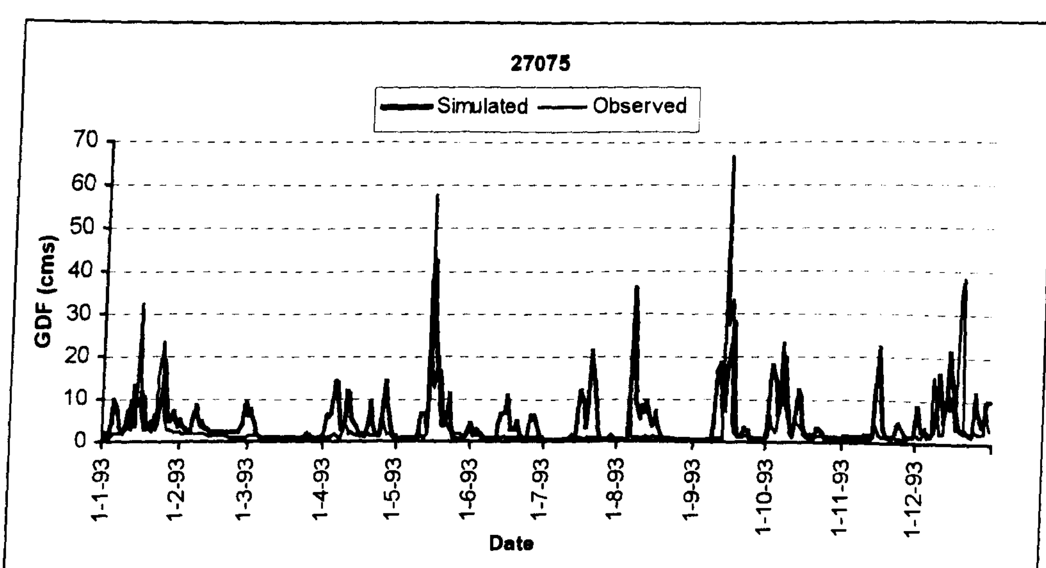
Date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Simulated
Observed
GDF (cms)
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60
70
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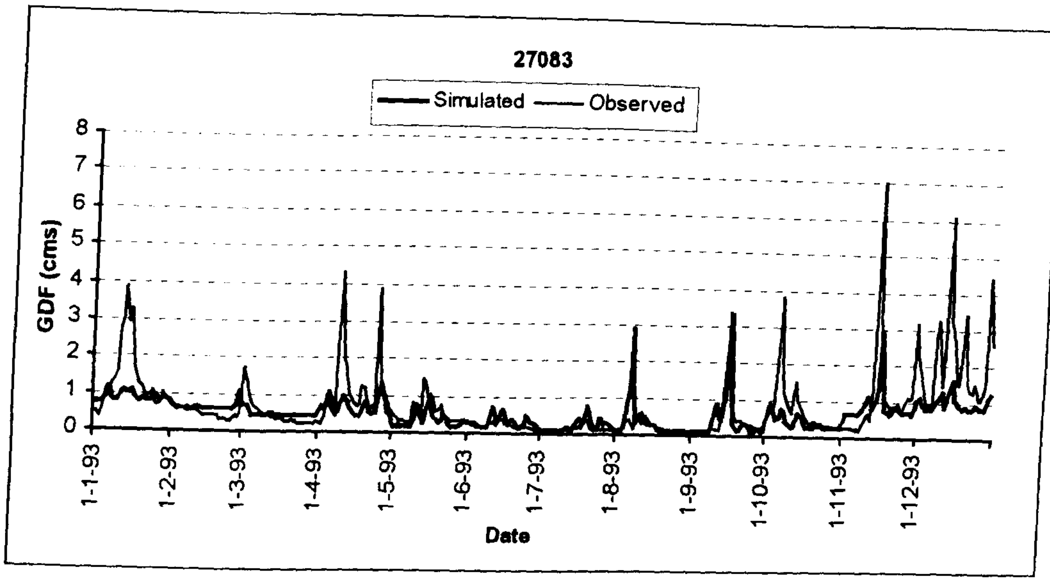
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date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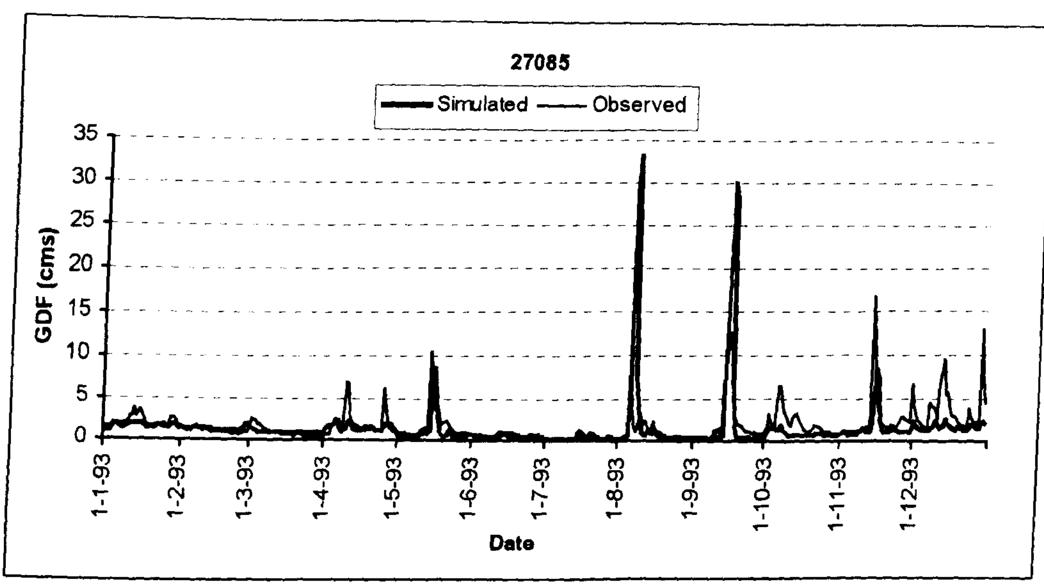
Simulated
Observed
GDF (cms)
Date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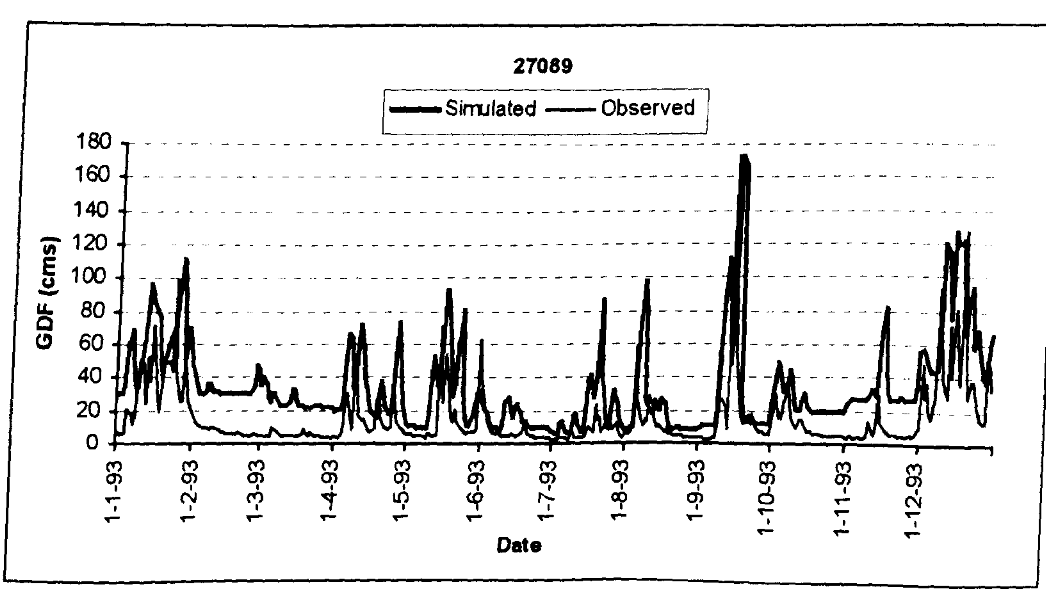
Simulated
Observed
GDF (cms)
Date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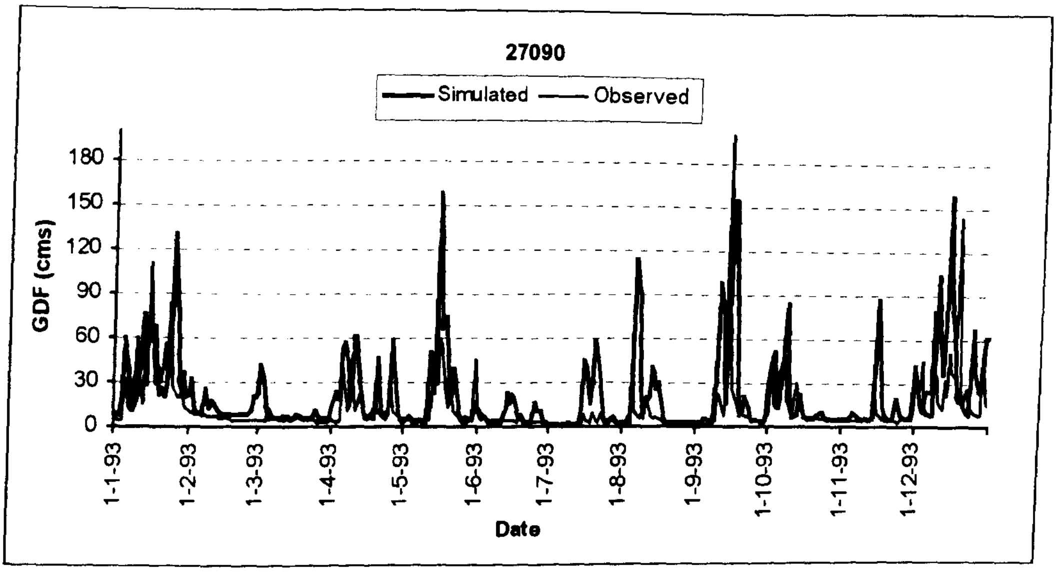
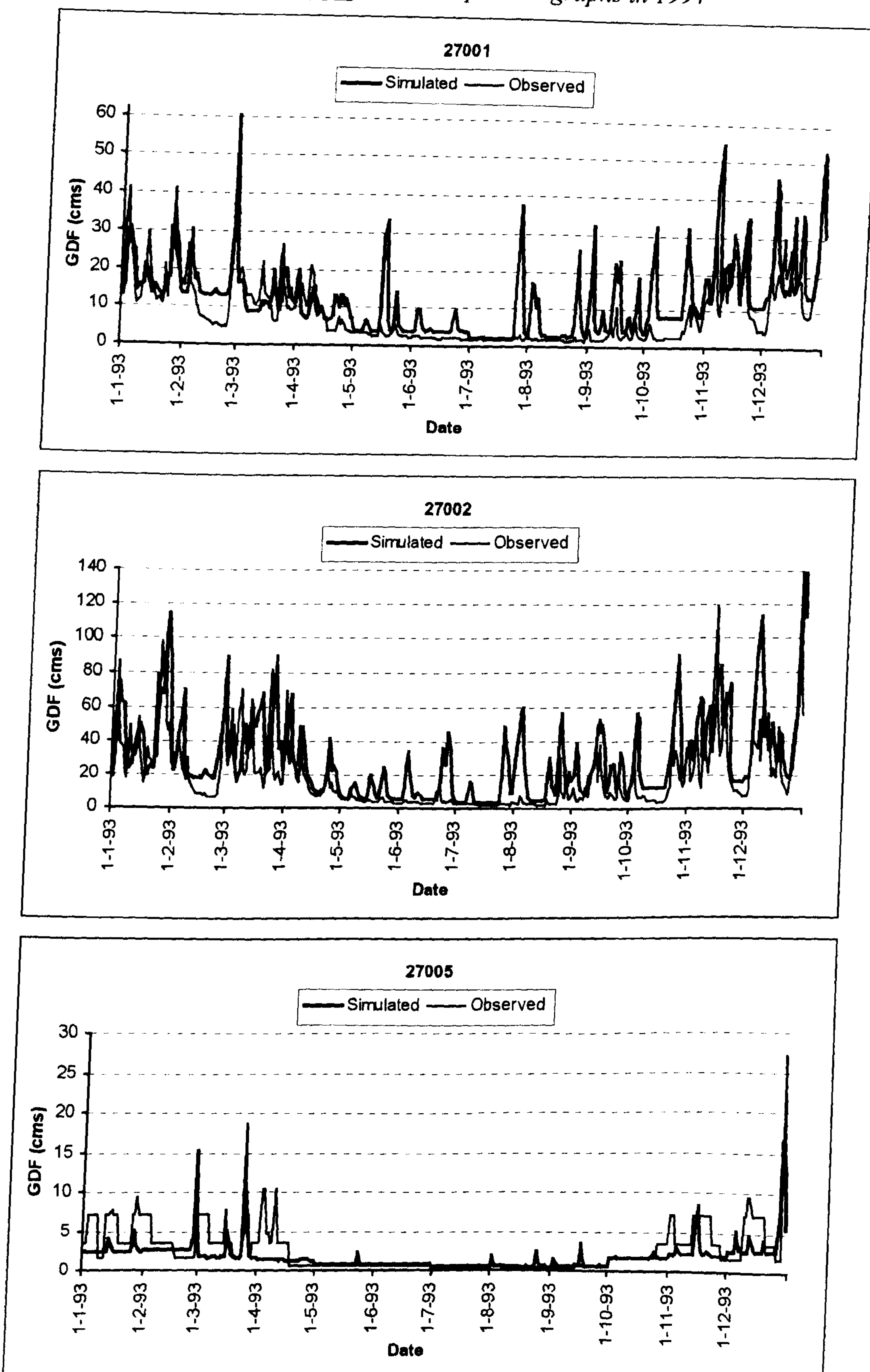
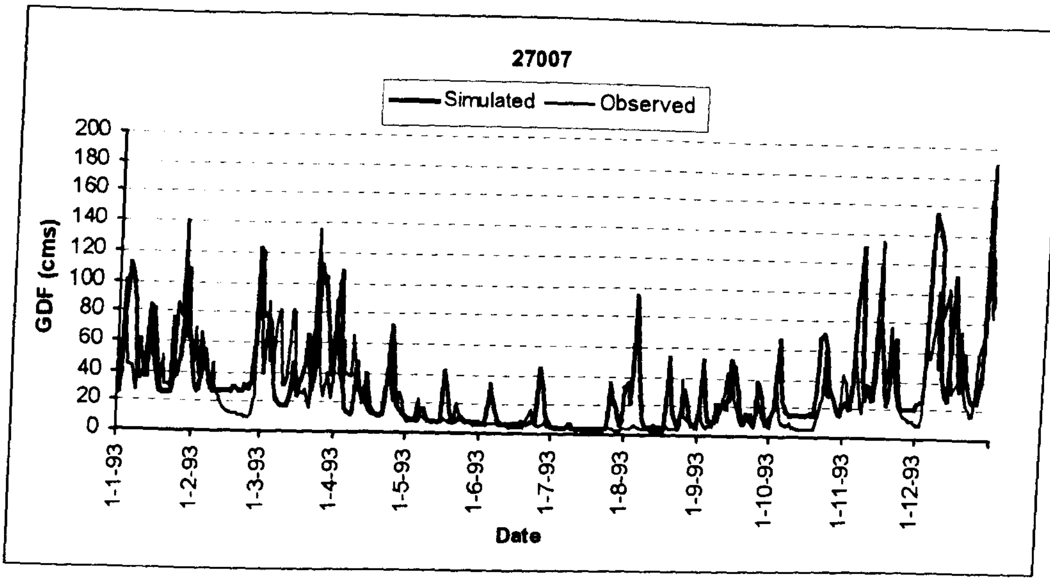
Simulated
Observed
GDF (cms)
Date



27090
Simulated
Observed
GDF (cms)
180
150
120
90
60
30
0
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

Figure C-2. *Flow comparison graphs in 1994*

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Simulated
Observed
GDF (cms)
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40
60
80
100
120
140
160
180
200
1-1-93
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1-3-93
1-4-93
1-5-93
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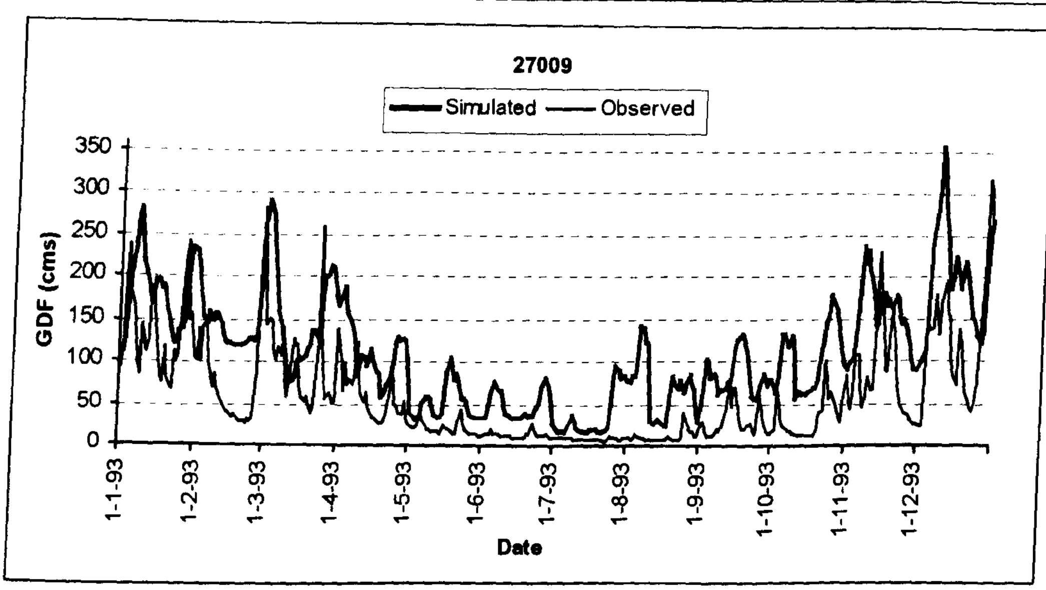
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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50
100
150
200
250
300
350
1-1-93
1-2-93
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1-4-93
1-5-93
1-6-93
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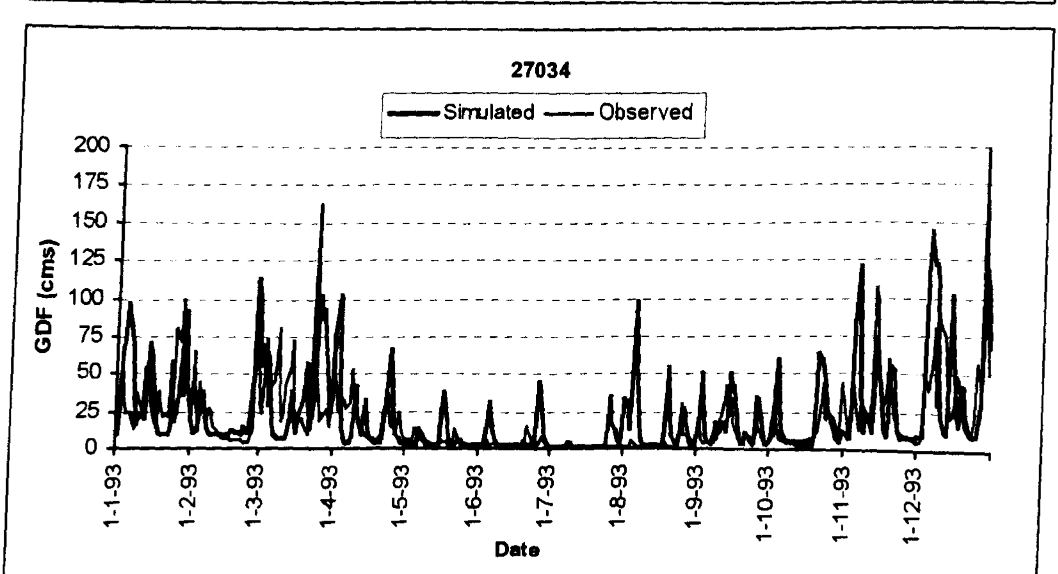
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date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Simulated
Observed
GDF (cms)
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50
75
100
125
150
175
200
1-1-93
1-2-93
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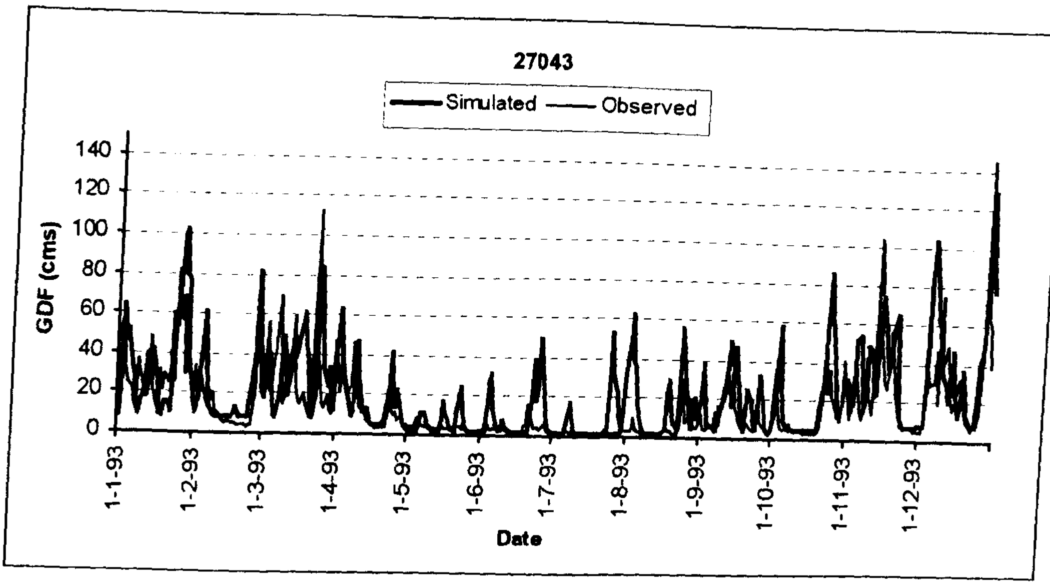
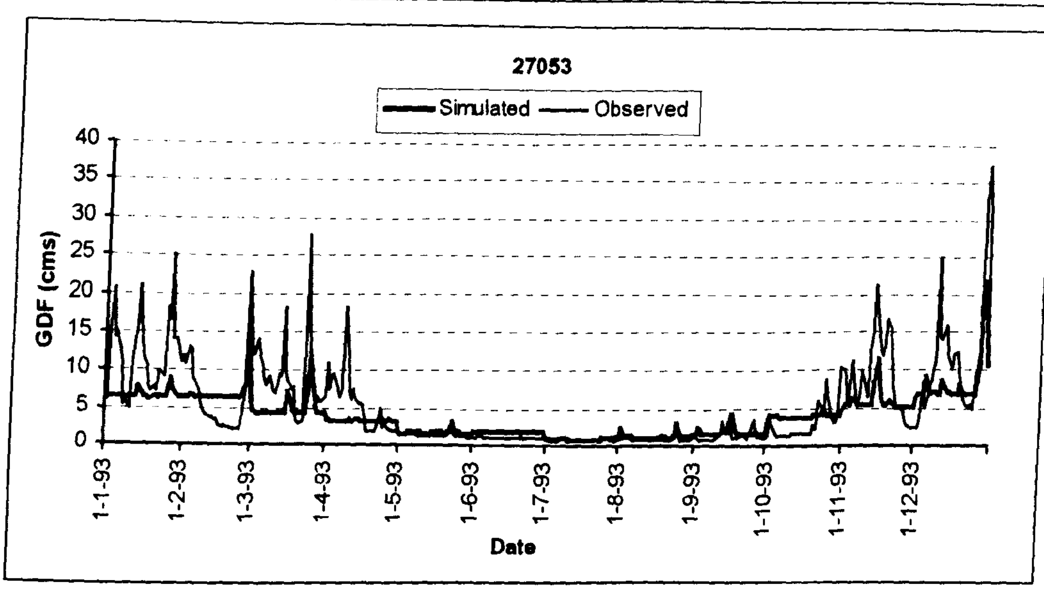
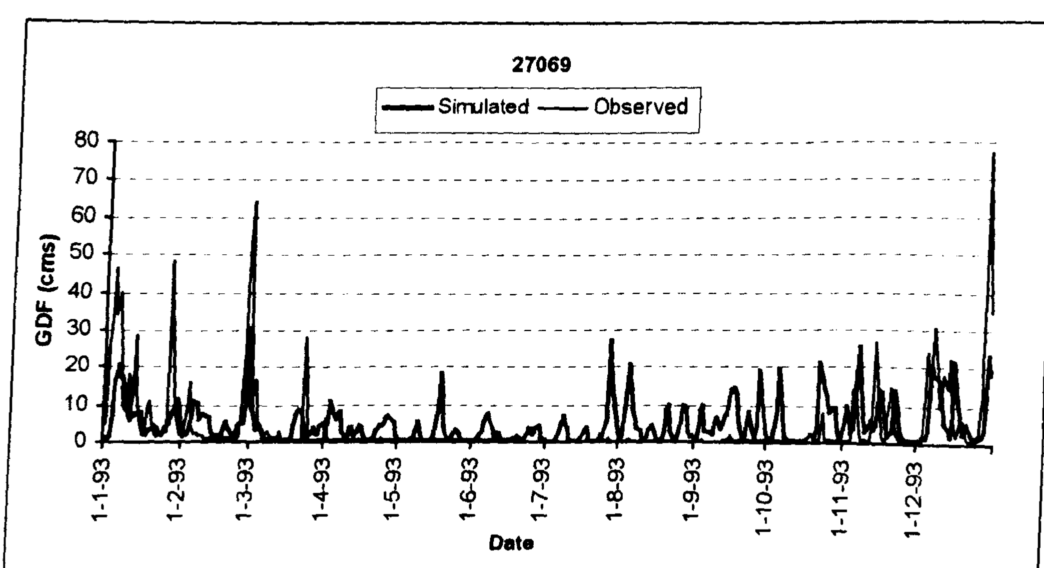
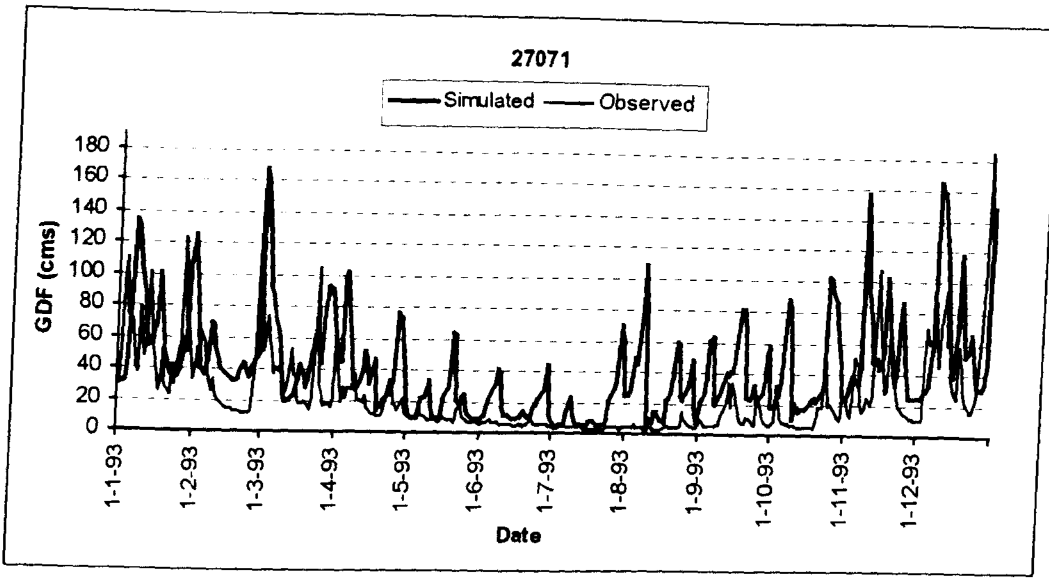
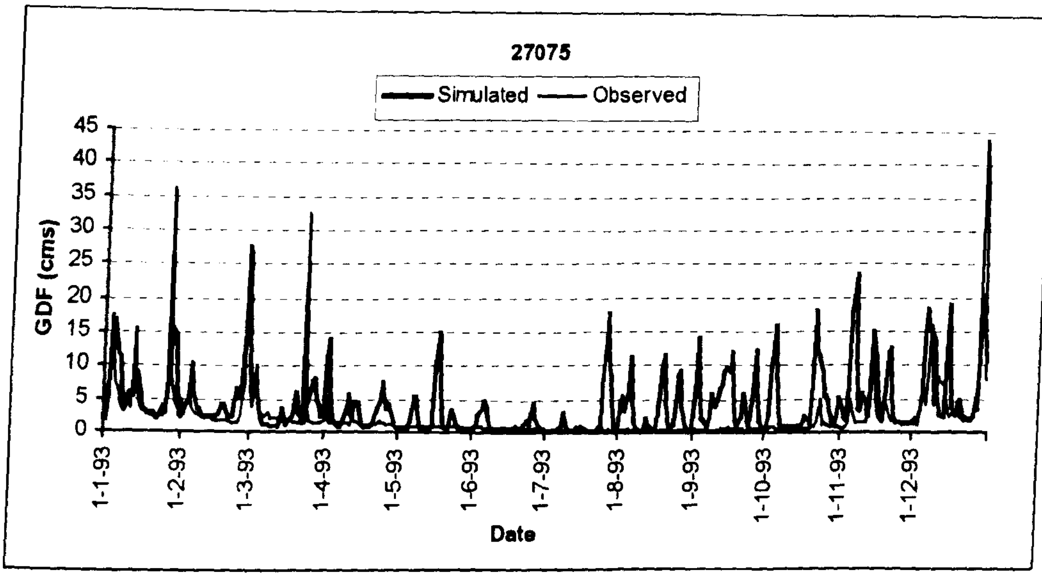
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

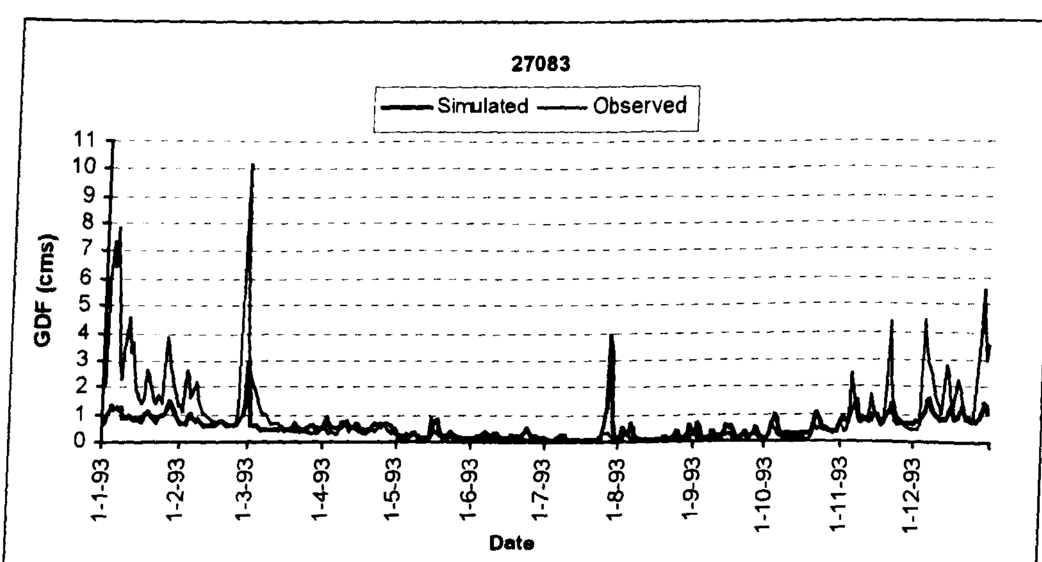

27043
Simulated
Observed
GDF (cms)
0
20
40
60
80
100
120
140
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

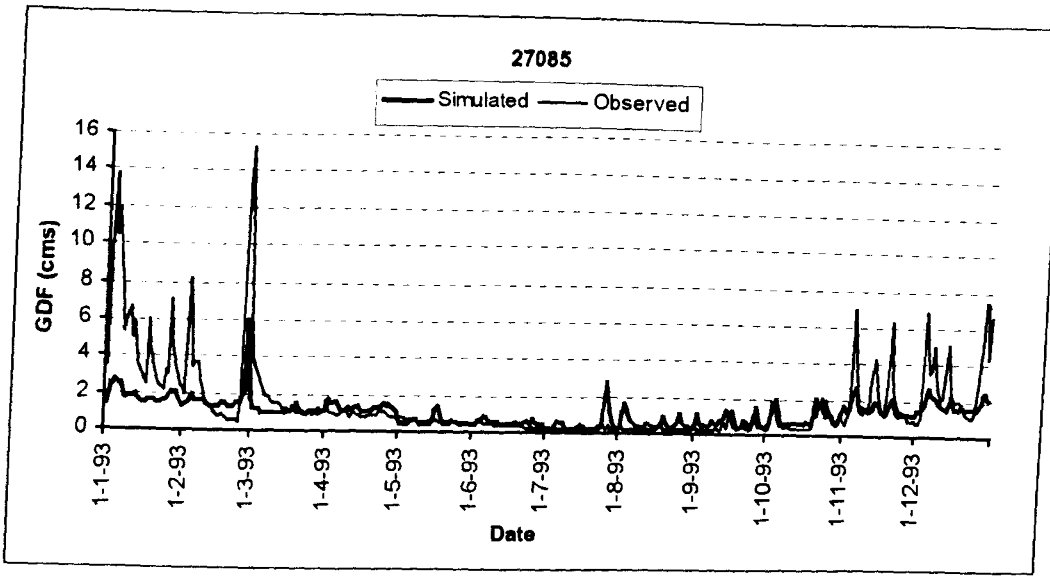

27053
Simulated
Observed
GDF (cms)
0
5
10
15
20
25
30
35
40
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

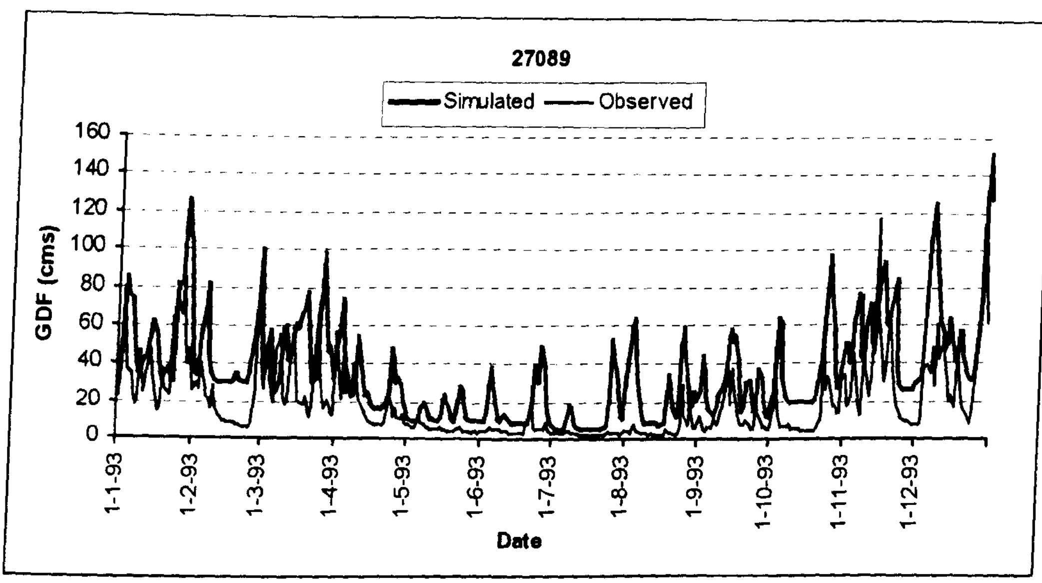

27069
Simulated
Observed
GDF (cms)
0
10
20
30
40
50
60
70
80
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

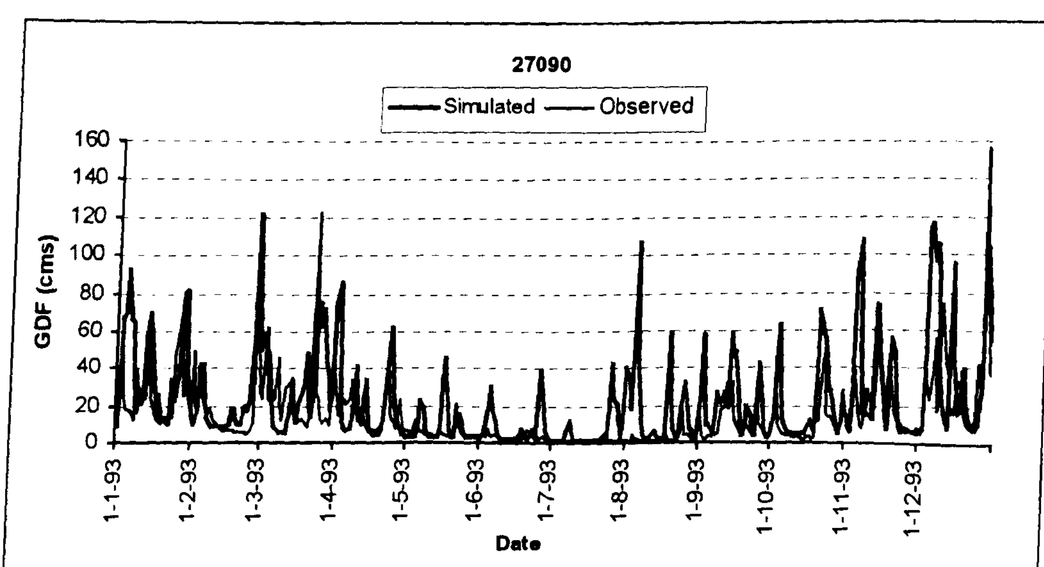

27071
Simulated
Observed
GDF (cms)
0
20
40
60
80
100
120
140
160
180
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date


27075
Simulated
Observed
GDF (cms)
0
5
10
15
20
25
30
35
40
45
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date


27083
Simulated
Observed
GDF (cms)
0
1
2
3
4
5
6
7
8
9
10
11
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date


27085
Simulated
Observed
GDF (cms)
0
2
4
6
8
10
12
14
16
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

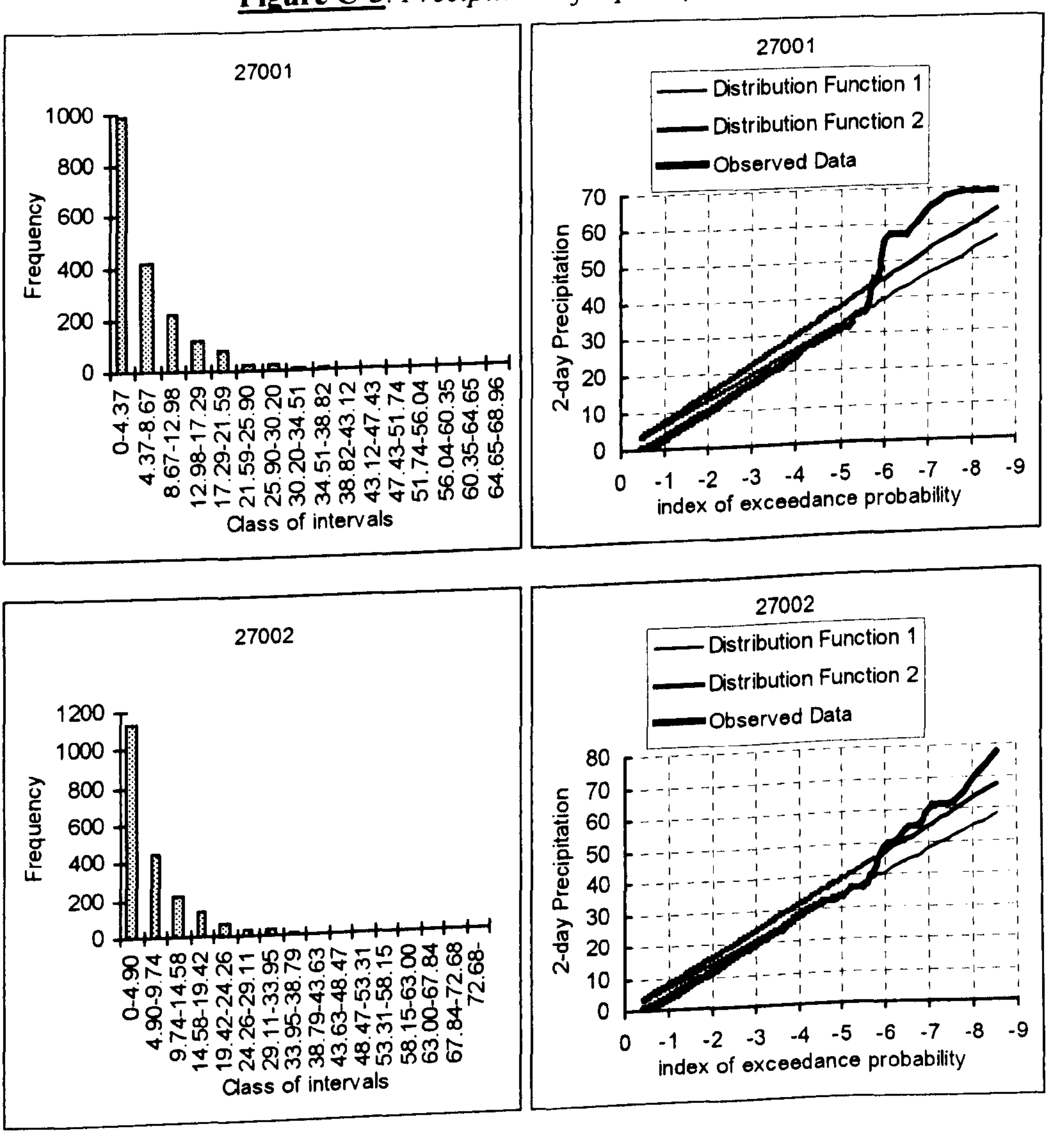

27089
Simulated
Observed
GDF (cms)
0
20
40
60
80
100
120
140
160
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

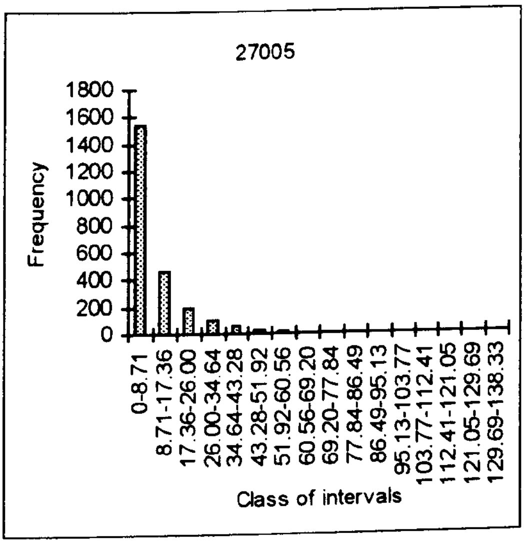

27090
Simulated
Observed
GDF (cms)
0
20
40
60
80
100
120
140
160
1-1-93
1-2-93
1-3-93
1-4-93
1-5-93
1-6-93
1-7-93
1-8-93
1-9-93
1-10-93
1-11-93
1-12-93
Date

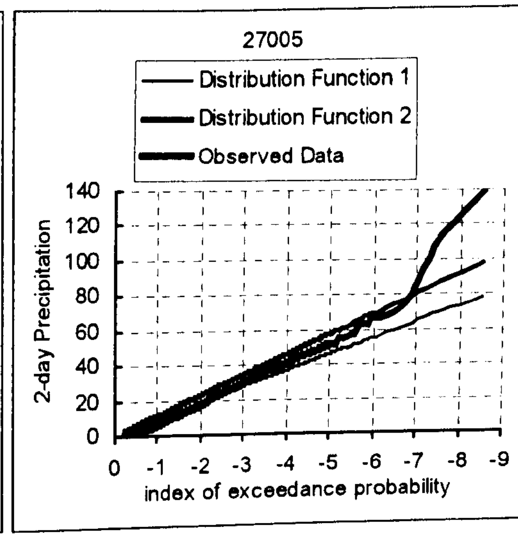
C-8 Data of Precipitation Frequency Analysis

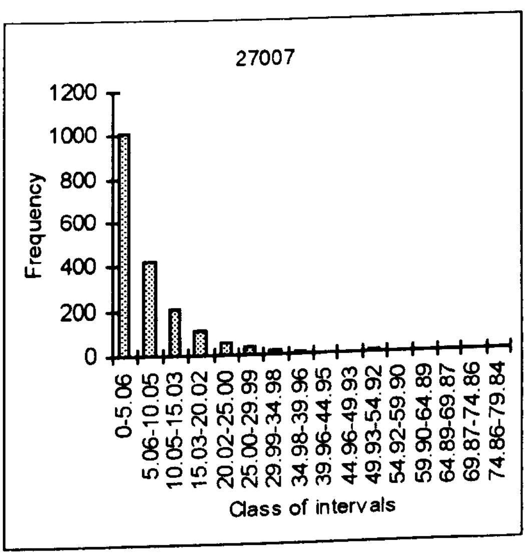
We carry out precipitation frequency analysis in the subbasins. This appendix gives all the results of it as histograms and probability diagrams. Figure C-3 shows the histograms and probability diagrams in each subbasin.

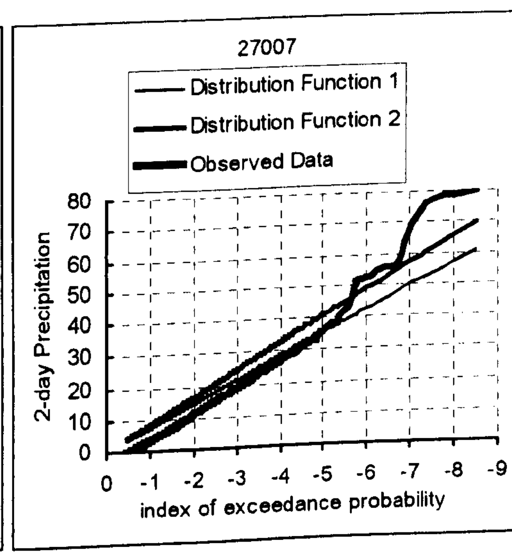
Figure C-3. *Precipitation frequency analysis*

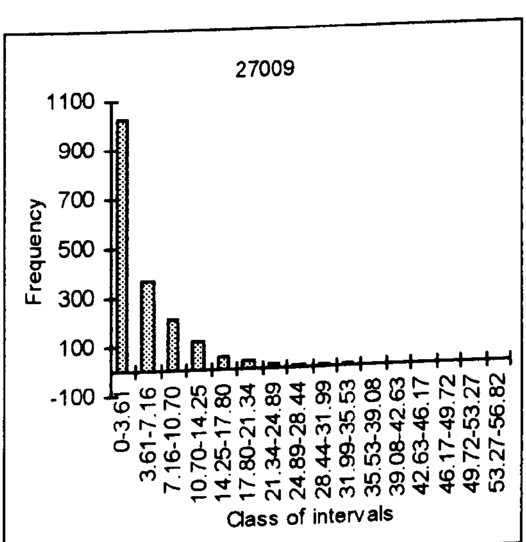


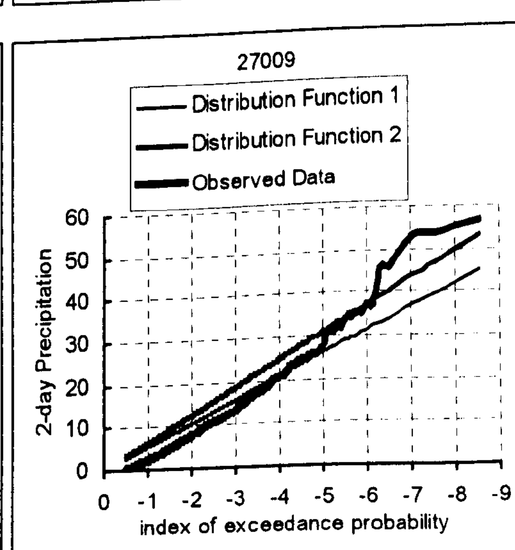

27005
Frequency
1800
1600
1400
1200
1000
800
600
400
200
0
0-8.71
8.71-17.36
17.36-26.00
26.00-34.64
34.64-43.28
43.28-51.92
51.92-60.56
60.56-69.20
69.20-77.84
77.84-86.49
86.49-95.13
95.13-103.77
103.77-112.41
112.41-121.05
121.05-129.69
129.69-138.33
Class of intervals

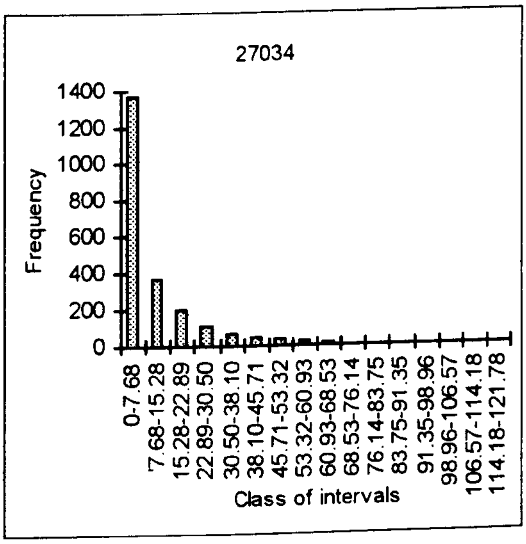

27005
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
140
120
100
80
60
40
20
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

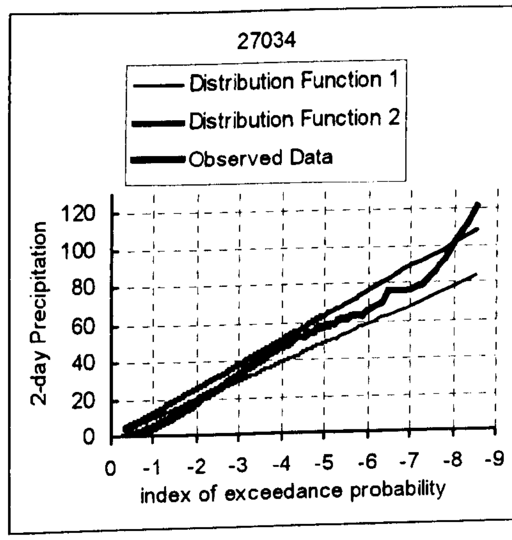

27007
Frequency
1200
1000
800
600
400
200
0
0-5.06
5.06-10.05
10.05-15.03
15.03-20.02
20.02-25.00
25.00-29.99
29.99-34.98
34.98-39.96
39.96-44.95
44.96-49.93
49.93-54.92
54.92-59.90
59.90-64.89
64.89-69.87
69.87-74.86
74.86-79.84
Class of intervals

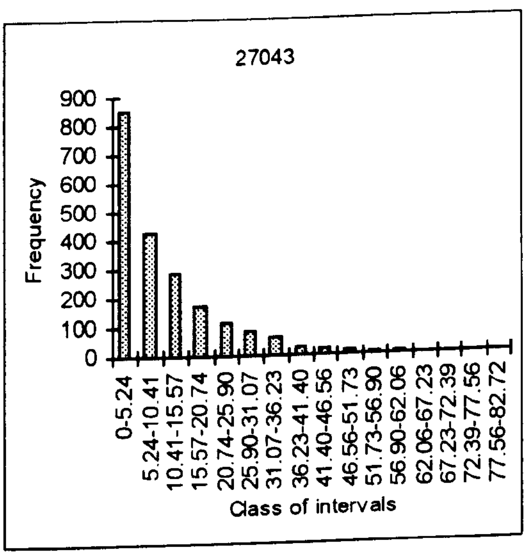

27007
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
80
70
60
50
40
30
20
10
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

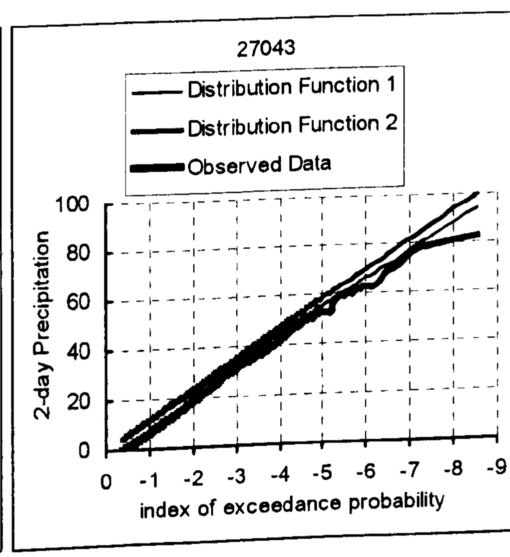

27009
Frequency
1100
900
700
500
300
100
-100
0-3.61
3.61-7.16
7.16-10.70
10.70-14.25
14.25-17.80
17.80-21.34
21.34-24.89
24.89-28.44
28.44-31.99
31.99-35.53
35.53-39.08
39.08-42.63
42.63-46.17
46.17-49.72
49.72-53.27
53.27-56.82
Class of intervals

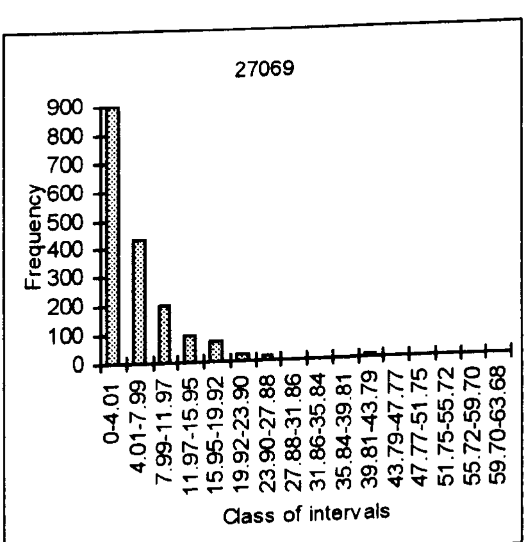

27009
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
60
50
40
30
20
10
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

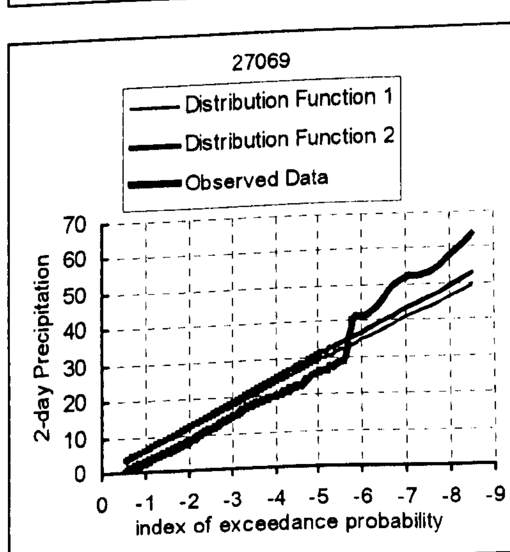

27034
Frequency
1400
1200
1000
800
600
400
200
0
0-7.68
7.68-15.28
15.28-22.89
22.89-30.50
30.50-38.10
38.10-45.71
45.71-53.32
53.32-60.93
60.93-68.53
68.53-76.14
76.14-83.75
83.75-91.35
91.35-98.96
98.96-106.57
106.57-114.18
114.18-121.78
Class of intervals

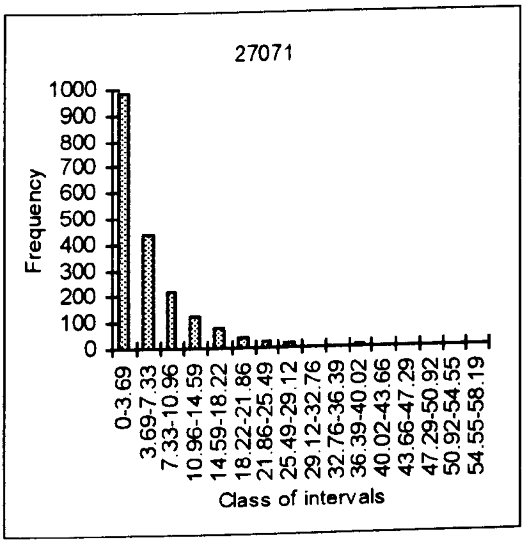

27034
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
120
100
80
60
40
20
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

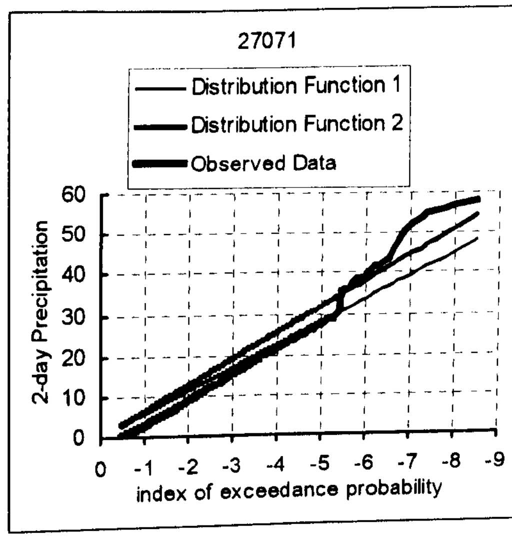

27043
Frequency
900
800
700
600
500
400
300
200
100
0
0-5.24
5.24-10.41
10.41-15.57
15.57-20.74
20.74-25.90
25.90-31.07
31.07-36.23
36.23-41.40
41.40-46.56
46.56-51.73
51.73-56.90
56.90-62.06
62.06-67.23
67.23-72.39
72.39-77.56
77.56-82.72
Class of intervals

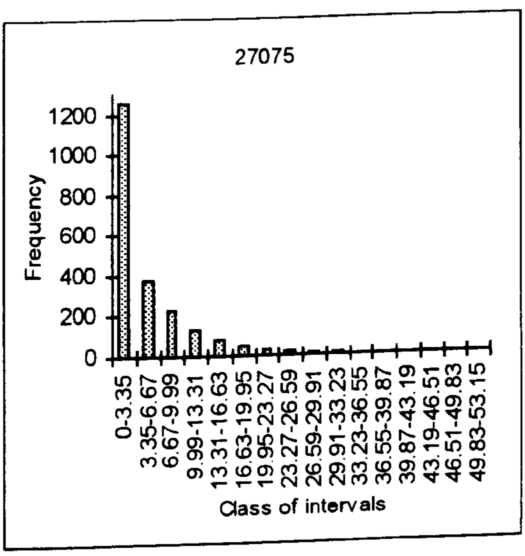

27043
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
100
80
60
40
20
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

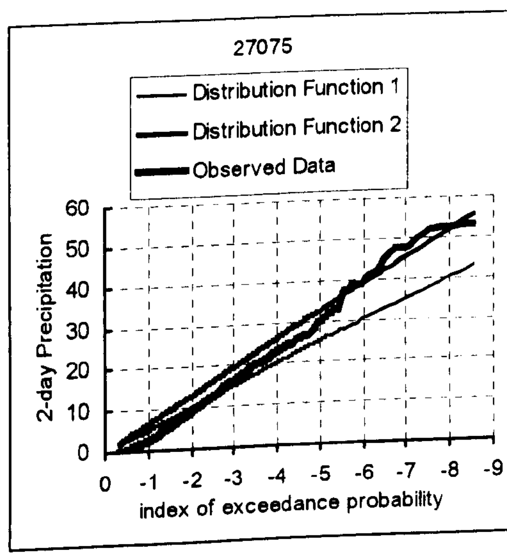

27069
Frequency
900
800
700
600
500
400
300
200
100
0
0-4.01
4.01-7.99
7.99-11.97
11.97-15.95
15.95-19.92
19.92-23.90
23.90-27.88
27.88-31.86
31.86-35.84
35.84-39.81
39.81-43.79
43.79-47.77
47.77-51.75
51.75-55.72
55.72-59.70
59.70-63.68
Class of intervals

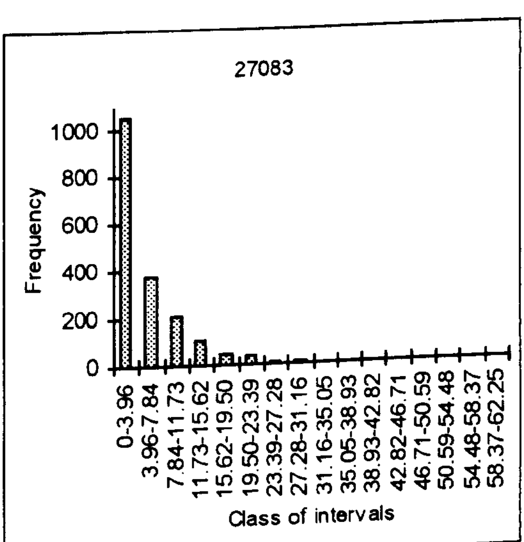

27069
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
70
60
50
40
30
20
10
0
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

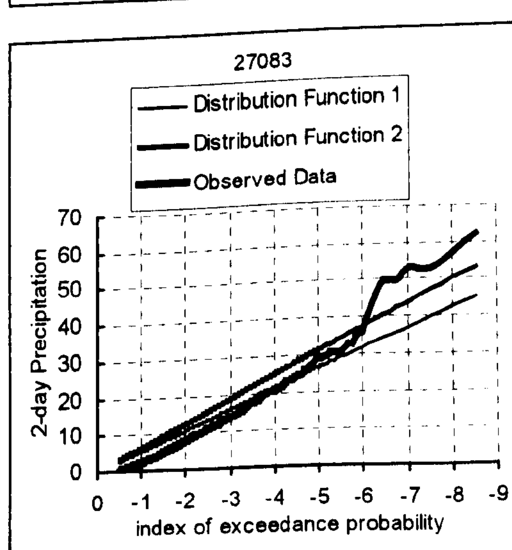

27071
Frequency
0
100
200
300
400
500
600
700
800
900
1000
0-3.69
3.69-7.33
7.33-10.96
10.96-14.59
14.59-18.22
18.22-21.86
21.86-25.49
25.49-29.12
29.12-32.76
32.76-36.39
36.39-40.02
40.02-43.66
43.66-47.29
47.29-50.92
50.92-54.55
54.55-58.19
Class of intervals

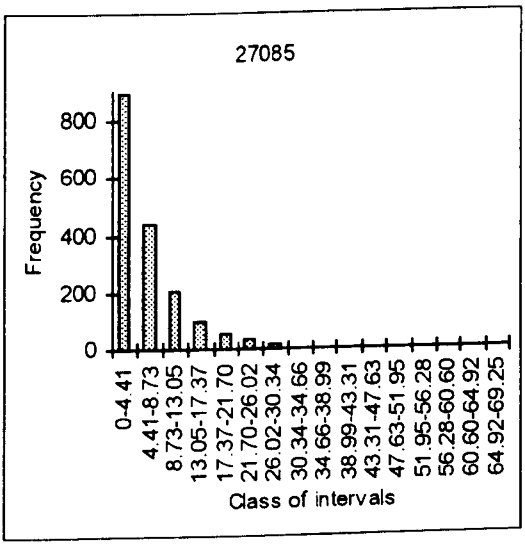

27071
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
10
20
30
40
50
60
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
index of exceedance probability

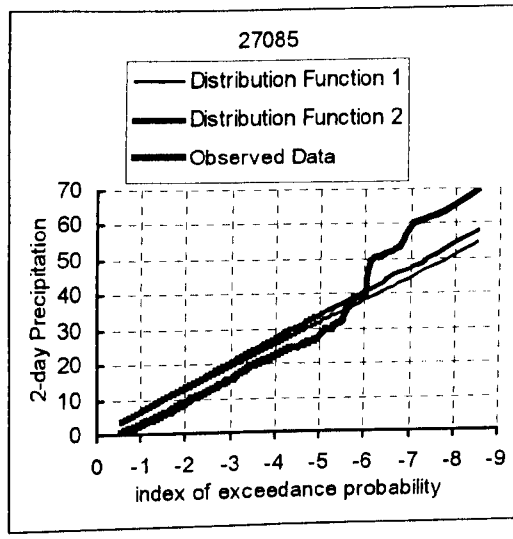

27075
Frequency
0
200
400
600
800
1000
1200
0-3.35
3.35-6.67
6.67-9.99
9.99-13.31
13.31-16.63
16.63-19.95
19.95-23.27
23.27-26.59
26.59-29.91
29.91-33.23
33.23-36.55
36.55-39.87
39.87-43.19
43.19-46.51
46.51-49.83
49.83-53.15
Class of intervals

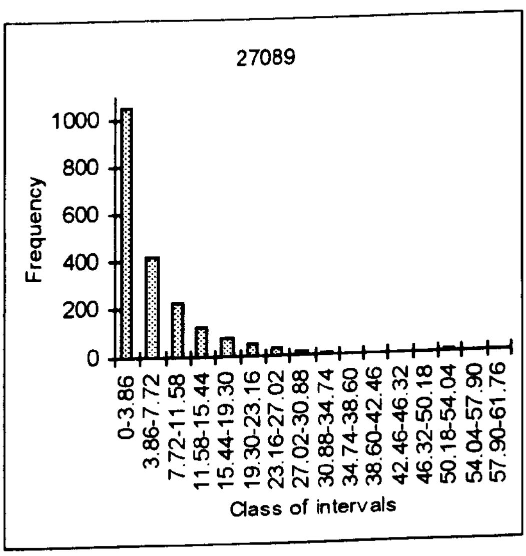

27075
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
10
20
30
40
50
60
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
index of exceedance probability

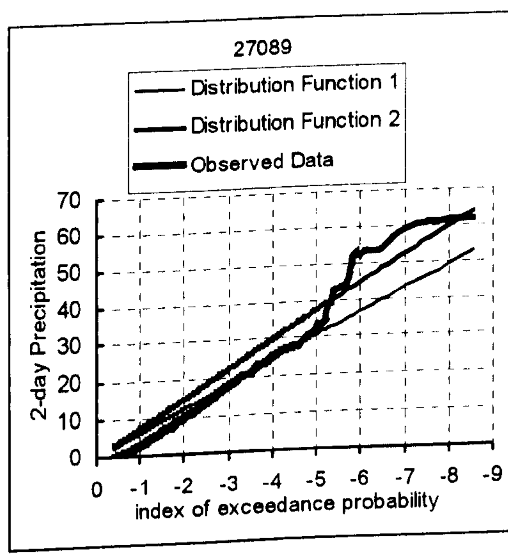

27083
Frequency
0
200
400
600
800
1000
0-3.96
3.96-7.84
7.84-11.73
11.73-15.62
15.62-19.50
19.50-23.39
23.39-27.28
27.28-31.16
31.16-35.05
35.05-38.93
38.93-42.82
42.82-46.71
46.71-50.59
50.59-54.48
54.48-58.37
58.37-62.25
Class of intervals

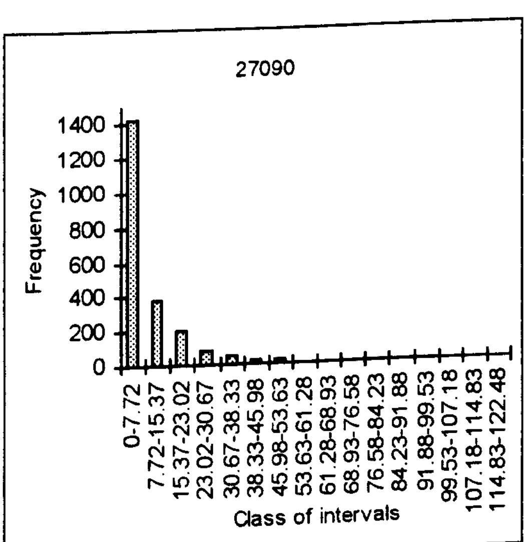

27083
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
10
20
30
40
50
60
70
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
index of exceedance probability

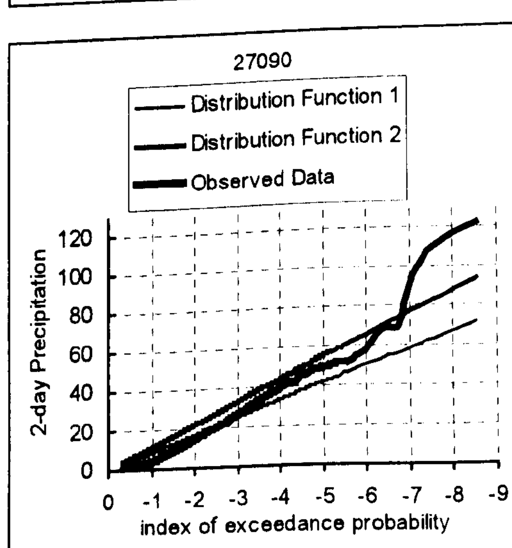

27085
Frequency
0
200
400
600
800
0-4.41
4.41-8.73
8.73-13.05
13.05-17.37
17.37-21.70
21.70-26.02
26.02-30.34
30.34-34.66
34.66-38.99
38.99-43.31
43.31-47.63
47.63-51.95
51.95-56.28
56.28-60.60
60.60-64.92
64.92-69.25
Class of intervals


27085
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
10
20
30
40
50
60
70
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability


27089
Frequency
0
200
400
600
800
1000
0-3.86
3.86-7.72
7.72-11.58
11.58-15.44
15.44-19.30
19.30-23.16
23.16-27.02
27.02-30.88
30.88-34.74
34.74-38.60
38.60-42.46
42.46-46.32
46.32-50.18
50.18-54.04
54.04-57.90
57.90-61.76
Class of intervals


27089
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
10
20
30
40
50
60
70
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability


27090
Frequency
0
200
400
600
800
1000
1200
1400
0-7.72
7.72-15.37
15.37-23.02
23.02-30.67
30.67-38.33
38.33-45.98
45.98-53.63
53.63-61.28
61.28-68.93
68.93-76.58
76.58-84.23
84.23-91.88
91.88-99.53
99.53-107.18
107.18-114.83
114.83-122.48
Class of intervals


27090
Distribution Function 1
Distribution Function 2
Observed Data
2-day Precipitation
0
20
40
60
80
100
120
0 -1 -2 -3 -4 -5 -6 -7 -8 -9
index of exceedance probability

C-9 Results of Regressions

27001

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_01      Mean=   22694374.26    , S.D.=   13822511.61     |
| Model size: Observations =        60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .1227720622E+16, Std.Dev.=  4600825.67880  |
| Fit:        R-squared=  .891089, Adjusted R-squared =          .88921  |
| Model test: F[  1,     58] =  474.54,    Prob value =          .00000  |
| Diagnostic: Log-L =  -1004.6240, Restricted(b=0) Log-L =  -1071.1406   |
|             LogAmemiyaPrCrt.=   30.716, Akaike Info. Crt.=     33.554  |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X01    27.17415683      1.1537273        23.553   .0000  560485.77
 XS        23654.05790      3126.6117         7.565   .0000  253.79385
```

27002

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_02      Mean=   9506357.442    , S.D.=   5625251.480     |
| Model size: Observations =        60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .2360285321E+15, Std.Dev.=  2017289.62974  |
| Fit:        R-squared=  .873576, Adjusted R-squared =          .87140  |
| Model test: F[  1,     58] =  400.77,    Prob value =          .00000  |
| Diagnostic: Log-L =   -955.1552, Restricted(b=0) Log-L =  -1017.1987   |
|             LogAmemiyaPrCrt.=   29.067, Akaike Info. Crt.=     31.905  |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X02    3.226024463      .16633126        19.395   .0000  1962517.3
 XS        3562.368939      538.61536         6.614   .0000  735.86611
```

27005

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_05      Mean=   2161548.172    , S.D.=   1147661.520     |
| Model size: Observations =        60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .2464536573E+14, Std.Dev.=   646311.11948  |
| Fit:        R-squared=  .682857, Adjusted R-squared =          .68286  |
| Model test: F[  1,     59] =  127.04,    Prob value =          .00000  |
| Diagnostic: Log-L =   -887.3743, Restricted(b=0) Log-L =   -921.8263   |
|             LogAmemiyaPrCrt.=   26.775, Akaike Info. Crt.=     29.612  |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X05    137.2757863      4.8590471        28.252   .0000  13007.425
```

27007

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_07     Mean=   18843308.88    , S.D.=    11255499.19    |
| Model size: Observations =      60, Parameters =   2, Deg.Fr.=     58 |
| Residuals:  Sum of squares= .7304338185E+15, Std.Dev.=  3548758.44855 |
| Fit:        R-squared=  .902276, Adjusted R-squared =          .90059 |
| Model test: F[  1.    58] =  535.51,    Prob value =           .00000 |
| Diagnostic: Log-L =   -989.0458, Restricted(b=0) Log-L =  -1058.8141 |
|             LogAmemiyaPrCrt.=  30.197, Akaike Info. Crt.=      33.035 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X07     6.438809849     .28174347        22.853   .0000  2084693.1
XS         3138.368126     512.25298         6.127   .0000  1321.3177
```

27009

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_09     Mean=   121429938.2    , S.D.=    71160439.53    |
| Model size: Observations =      60, Parameters =   3, Deg.Fr.=     57 |
| Residuals:  Sum of squares= .2150059843E+17, Std.Dev.= 19421727.03954 |
| Fit:        R-squared=  .928035, Adjusted R-squared =          .92551 |
| Model test: F[  2.    57] =  367.53,    Prob value =           .00000 |
| Diagnostic: Log-L =  -1090.5117, Restricted(b=0) Log-L =  -1169.4590 |
|             LogAmemiyaPrCrt.=  33.613, Akaike Info. Crt.=      36.450 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X09     4.610883010     .20756005        22.215   .0000  16370091.
XS71       7049.527551     1730.4980         4.074   .0001  3588.9758
XS07       6578.541221     2240.4825         2.936   .0048  2589.8980
```

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_09     Mean=   121429938.2    , S.D.=    71160439.53    |
| Model size: Observations =      60, Parameters =   4, Deg.Fr.=     56 |
| Residuals:  Sum of squares= .2125165434E+17, Std.Dev.= 19480601.31048 |
| Fit:        R-squared=  .928868, Adjusted R-squared =          .92506 |
| Model test: F[  3.    56] =  243.76,    Prob value =           .00000 |
| Diagnostic: Log-L =  -1090.1623, Restricted(b=0) Log-L =  -1169.4590 |
|             LogAmemiyaPrCrt.=  33.634, Akaike Info. Crt.=      36.472 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X09     4.619711475     .20847440        22.160   .0000  16370091.
XS71       6524.761458     1852.7274         3.522   .0009  3588.9758
XS07       5417.338154     2665.6612         2.032   .0469  2589.8980
XS01       5530.407879     6828.2373          .810   .4214  894.59143
```

27034

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_34     Mean=    40857205.41    , S.D.=    22551177.62   |
| Model size: Observations =        60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .6409122951E+16, Std.Dev.= 10422533.40428 |
| Fit:        R-squared=  .786397, Adjusted R-squared =          .78640 |
| Model test: F[  1,     59] =  217.21,    Prob value =          .00000 |
| Diagnostic: Log-L =  -1054.2009, Restricted(b=0) Log-L =  -1100.5100 |
|             LogAmemiyaPrCrt.=   32.335, Akaike Info. Crt.=     35.173 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X34     15.50372215      .45933450       33.753   .0000  2269018.5
```

27043

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_43     Mean=    21235821.35    , S.D.=    9508436.437   |
| Model size: Observations =        60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .1523575579E+16, Std.Dev.= 5081664.57907  |
| Fit:        R-squared=  .714377, Adjusted R-squared =          .71438 |
| Model test: F[  1,     59] =  147.57,    Prob value =          .00000 |
| Diagnostic: Log-L =  -1011.1011, Restricted(b=0) Log-L =  -1048.6935 |
|             LogAmemiyaPrCrt.=   30.899, Akaike Info. Crt.=     33.737 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X43     28.96503911      .83776876       34.574   .0000  648268.17
```

27053

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_53     Mean=    53191.10199    , S.D.=    34241.71579   |
| Model size: Observations =        60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .1205334945E+11, Std.Dev.=   14415.83746  |
| Fit:        R-squared=  .825761, Adjusted R-squared =          .82276 |
| Model test: F[  1,     58] =  274.88,    Prob value =          .00000 |
| Diagnostic: Log-L =   -658.6842, Restricted(b=0) Log-L =   -711.1041 |
|             LogAmemiyaPrCrt.=   19.185, Akaike Info. Crt.=     22.023 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X53     1.085130743      .60256411E-01   18.009   .0000  32913.926
 XS         123.9982608      24.456539        5.070   .0000  100.91424
```

27069

```
+---------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none   |
| Dep. var. = C_69     Mean=   16629034.91    , S.D.=   10224777.88   |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=  59 |
| Residuals:  Sum of squares= .1644808973E+16, Std.Dev.= 5279973.31286 |
| Fit:        R-squared=  .733341, Adjusted R-squared =        .73334 |
| Model test: F[  1,     59] =  162.26,    Prob value =         .00000 |
| Diagnostic: Log-L =  -1013.3980, Restricted(b=0) Log-L =  -1053.0516 |
|             LogAmemiyaPrCrt.=   30.975, Akaike Info. Crt.=    33.813 |
+---------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X69    24.53457742       .89148864       27.521   .0000  545723.23
```

27071

```
+---------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none   |
| Dep. var. = C_71     Mean=   29180477.83    , S.D.=   17541714.73   |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=  58 |
| Residuals:  Sum of squares= .1720480706E+16, Std.Dev.= 5446417.21203 |
| Fit:        R-squared=  .905234, Adjusted R-squared =        .90360 |
| Model test: F[  1,     58] =  554.03,    Prob value =         .00000 |
| Diagnostic: Log-L =  -1014.7474, Restricted(b=0) Log-L =  -1085.4376 |
|             LogAmemiyaPrCrt.=   31.054, Akaike Info. Crt.=    33.892 |
+---------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X71    4.417190730       .22133825       19.957   .0000  4038213.7
 XS        4069.885476       449.32390        9.058   .0000  2493.8689
```

27075

```
+---------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none   |
| Dep. var. = C_75     Mean=   13744305.46    , S.D.=   8466511.286   |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=  59 |
| Residuals:  Sum of squares= .1117603505E+16, Std.Dev.= 4352290.46420 |
| Fit:        R-squared=  .735743, Adjusted R-squared =        .73574 |
| Model test: F[  1,     59] =  164.27,    Prob value =         .00000 |
| Diagnostic: Log-L =  -1001.8049, Restricted(b=0) Log-L =  -1041.7298 |
|             LogAmemiyaPrCrt.=   30.589, Akaike Info. Crt.=    33.427 |
+---------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X75    37.52708497      1.3589053        27.616   .0000  296156.24
```

27083

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_83     Mean=   2426382.209    , S.D.=   1498428.229    |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=   59 |
| Residuals:  Sum of squares= .3094102215E+14, Std.Dev.= 724171.32243 |
| Fit:        R-squared=  .766433, Adjusted R-squared =        .76643 |
| Model test: F[  1,    59] =  193.60,   Prob value =          .00000 |
| Diagnostic: Log-L =   -894.1991, Restricted(b=0) Log-L =  -937.8277 |
|             LogAmemiyaPrCrt.=   27.002, Akaike Info. Crt.=    29.840 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X83     5.283970837       .17943430       29.448   .0000  378024.33
```

27085

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_85     Mean=   3964037.244    , S.D.=   2529879.029    |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=   59 |
| Residuals:  Sum of squares= .8653277780E+14, Std.Dev.= 1211056.25420 |
| Fit:        R-squared=  .770845, Adjusted R-squared =        .77085 |
| Model test: F[  1,    59] =  198.47,   Prob value =          .00000 |
| Diagnostic: Log-L =   -925.0523, Restricted(b=0) Log-L =  -969.2530 |
|             LogAmemiyaPrCrt.=   28.031, Akaike Info. Crt.=    30.868 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X85     7.803441530       .26903611       29.005   .0000  414488.51
```

27089

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_89     Mean=   2100439.905    , S.D.=   1135195.792    |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=   58 |
| Residuals:  Sum of squares= .7973174590E+13, Std.Dev.= 370767.48431 |
| Fit:        R-squared=  .895133, Adjusted R-squared =        .89333 |
| Model test: F[  1,    58] =  495.08,   Prob value =          .00000 |
| Diagnostic: Log-L =   -853.5191, Restricted(b=0) Log-L =  -921.1710 |
|             LogAmemiyaPrCrt.=   25.679, Akaike Info. Crt.=    28.517 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X89     20.35564286       .87812549       23.181   .0000  65359.427
 XS         342.8375726       35.151949        9.753   .0000  1944.2826
```

27090

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_90      Mean=    30832102.50    , S.D.=    14598822.59  |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .4084947920E+16, Std.Dev.= 8320841.62952  |
| Fit:        R-squared=  .675138, Adjusted R-squared =          .67514 |
| Model test: F[  1,      59] =  122.62,     Prob value =         .00000 |
| Diagnostic: Log-L =  -1040.6885, Restricted(b=0) Log-L =  -1074.4192  |
|             LogAmemiyaPrCrt.=   31.885, Akaike Info. Crt.=    34.723  |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X90    29.55194380      .96060440       30.764   .0000   891152.11
```

Selby

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_SELBY  Mean=    1577399.987    , S.D.=    370928.4745   |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .4340093118E+13, Std.Dev.= 273549.24910   |
| Fit:        R-squared=  .465354, Adjusted R-squared =          .45614 |
| Model test: F[  1,      58] =   50.48,     Prob value =         .00000 |
| Diagnostic: Log-L =   -835.2735, Restricted(b=0) Log-L =   -854.0580  |
|             LogAmemiyaPrCrt.=   25.071, Akaike Info. Crt.=    27.909  |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant  899343.7842      101756.57        8.838   .0000
 XS        48.69045100      6.8528564        7.105   .0000   13925.856
```

York

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_YORK   Mean=    11891807.80    , S.D.=    1766124.756   |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .8304948312E+14, Std.Dev.= 1196615.07596  |
| Fit:        R-squared=  .548724, Adjusted R-squared =          .54094 |
| Model test: F[  1,      58] =   70.52,     Prob value =         .00000 |
| Diagnostic: Log-L =   -923.8197, Restricted(b=0) Log-L =   -947.6900  |
|             LogAmemiyaPrCrt.=   28.023, Akaike Info. Crt.=    30.861  |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant  8477662.330      434909.51       19.493   .0000
 XS        290.4259484      34.583225        8.398   .0000   11755.649
```

Appendix D

Appendices of Chapter 6

D-1 Expected Cost Function of Flood Risk under Different Assumptions

Protection against floods with 4% and more exceedance probability

27001 $C^{i=27001} = 52.18253(X^i)^2 + 45424.08264\sum_j X^j$ $j = 27005$ and 27053

27002 $C^{i=27002} = 5.97383(X^i)^2 + 6603.29977\sum_j X^j$ $j = 27043$

27005 $C^{i=27005} = 249.09306(X^i)^2$

27007 $C^{i=27007} = 11.56311(X^i)^2 + 6499.99968\sum_j X^j$ $j = 27034$

27009 $C^{i=27009} = 9.04258(X^i)^2 + 11991.50609\sum_j X^j + 14274.49675\sum_k X^k$

$j = 27007$ and 27034, $k = 27069, 27071, 27075, 27085$ and 27090

27034 $C^{i=27034} = 29.97472(X^i)^2$

27043 $C^{i=27043} = 55.06877(X^i)^2$

27053 $C^{i=27053} = 1.08513(X^i)^2 + 123.99825\sum_j X^j$ $j = 27005$

27069 $C^{i=27069} = 47.48196(X^i)^2$

27071 $C^{i=27071} = 8.79495(X^i)^2 + 8111.08186\sum_j X^j$

$j = 27069, 27075, 27085$ and 27090

27075 $C^{i=27075} = 71.06721(X^i)^2$

27083 $C^{i=27083} = 10.55981(X^i)^2$

27085 $C^{i=27085} = 13.79704(X^i)^2$

27089 $C^{i=27089} = 37.29225(X^i)^2 + 634.35004\sum_j X^j$ $j = 27002$ and 27043

27090 $C^{i=27090} = 56.02235(X^i)^2$

Selby $C^{i=Selby} = 1302679.815 + 100.34526\sum_j X^j$ $j =$ All 15 subbasins

York $C^{i=York} = 14800517.68 + 594.5666162\sum_j X^j$ $j = 27001, 27005, 27007,$
27009, 27034, 27053, 27069, 27071, 27075, 27083, 27085 and 27090

Protection against floods with 10 % and more exceedance probability

27001 $C^{i=27001} = 118.62265(X^i)^2 + 103265.9424\sum_j X^j$ $j = 27005$ and 27053

27002 $C^{i=27002} = 12.85452(X^i)^2 + 14218.25946\sum_j X^j$ $j = 27043$

27005 $C^{i=27005} = 584.54491(X^i)^2$

27007 $C^{i=27007} = 22.55078(X^i)^2 + 13709.86037\sum_j X^j$ $j = 27034$

27009 $C^{i=27009} = 21.66655(X^i)^2 + 19221.56211\sum_j X^j + 38920.03753\sum_k X^k$
$j = 27007$ and 27034, $k = 27069, 27071, 27075, 27085$ and 27090

27034 $C^{i=27034} = 67.80163(X^i)^2$

27043 $C^{i=27043} = 121.10688(X^i)^2$

27053 $C^{i=27053} = 1.08513(X^i)^2 + 123.99825\sum_j X^j$ $j = 27005$

27069 $C^{i=27069} = 110.65511(X^i)^2$

27071 $C^{i=27071} = 22.14088(X^i)^2 + 19659.42123\sum_j X^j$
$j = 27069, 27075, 27085$ and 27090

27075 $C^{i=27075} = 159.65159(X^i)^2$

27083 $C^{i=27083} = 20.03577(X^i)^2$

27085 $C^{i=27085} = 28.41622(X^i)^2$

27089 $C^{i=27089} = 78.28033(X^i)^2 + 1377.77230\sum_j X^j$ $j = 27002$ and 27043

27090 $C^{i=27090} = 126.05450(X^i)^2$

Selby $C^{i=Selby} = 468506.1841 + 296.37153\sum_j X^j$ $j =$ All 15 subbasins

York $C^{i=York} = 23485840.16 + 1802.10147\sum_j X^j$ $j = 27001, 27005, 27007,$
27009, 27034, 27053, 27069, 27071, 27075, 27083, 27085 and 27090

D-2 GAMS Codes

Social Optimisation (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*---------------------------------------------
* Set time t, subbasins i and urban areas j
*---------------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01,s02,s05,s07,s09,s34,s43,s53,
                          s69,s71,s75,s83,s85,s89,s90/
     j urban areas /selby,york/ ;

*---------------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*---------------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*---------------------------------------------
* Set initial values etc.
*---------------------------------------------

Parameters
    lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
               s05 204.67004, s07 2447.00171, s09 6639.46521,
               s34 2451.61496, s43 1257.75186, s53 347.13385,
               s69 1338.21609, s71 3595.16936, s75 1029.10755,
               s83 1119.66767, s85 1149.58292, s89 443.79102,
               s90 1632.33813/
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
               s05 14.38793, s07 155.26437, s09 308.46632,
               s34 51.12944, s43 27.69161, s53 7.97054,
               s69 101.6242, s71 306.14877, s75 63.81431,
               s83 106.27285, s85 60.06330, s89 35.18348,
               s90 67.09430/ ;
```

```
*------------------------------------------------
* Set discount factor and common parameter
* values in the functions.
*------------------------------------------------

Scalars
     adj adjustment factor for utility function /1000000000/
     dis1 periodic discount rate /0.05/
     dis2 discount factor
     b1 parameter 1 of ecosystem benefit function /0.55978/
     b2 parameter 2 of ecosystem benefit function /8.48302/
     b3 parameter 3 of ecosystem benefit function /-0.168/
     f1 parameter of FP development benefit function /14030.80048/
     r1 parameter of restoration cost function /19146.37353/
     d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*------------------------------------------------
* Calculate the discount factor in time t.
*------------------------------------------------

Parameters
     dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*------------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*------------------------------------------------

Scalars
     c01_1 parameter of flood cost function in 27001 /27.17415/
     c01_2 parameter of flood cost function in 27001 /23654.05790/
     c02_1 parameter of flood cost function in 27002 /3.22602/
     c02_2 parameter of flood cost function in 27002 /3562.36893/
     c05   parameter of flood cost function in 27005 /137.27578/
     c07_1 parameter of flood cost function in 27007 /6.43880/
     c07_2 parameter of flood cost function in 27007 /3138.36812/
     c09_1 parameter of flood cost function in 27009 /4.61088/
     c09_2 parameter of flood cost function in 27009 /7049.52755/
     c09_3 parameter of flood cost function in 27009 /6578.54122/
     c34   parameter of flood cost function in 27034 /15.50372/
     c43   parameter of flood cost function in 27043 /28.96503/
     c53_1 parameter of flood cost function in 27053 /1.08513/
     c53_2 parameter of flood cost function in 27053 /123.99826/
     c69   parameter of flood cost function in 27069 /24.53457/
     c71_1 parameter of flood cost function in 27071 /4.41719/
     c71_2 parameter of flood cost function in 27071 /4069.88547/
     c75   parameter of flood cost function in 27075 /37.52708/
     c83   parameter of flood cost function in 27083 /5.28397/
     c85   parameter of flood cost function in 27085 /7.80344/
     c89_1 parameter of flood cost function in 27089 /20.35564/
     c89_2 parameter of flood cost function in 27089 /342.83757/
     c90   parameter of flood cost function in 27090 /29.55194/
```

```
     csb_1 parameter of flood cost function in Selby /899343.7842/
     csb_2 parameter of flood cost function in Selby /48.69045/
     cyk_1 parameter of flood cost function in York /8477662.33/
     cyk_2 parameter of flood cost function in York /290.42594/ ;

*------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*------------------------------------------------

Variables
     x(i,t) area of developed floodplains
     yd(i,t) floodplain development
     yr(i,t) floodplain restoration
     nb(t) net benefit
     disnb(t) discounted utility of net benefit
     z objective function
     eco(i,t) benefit of ecosystem services
     pro(i,t) benefit of developed floodplains
     rest(i,t) cost of floodplain restoration
     deve(i,t) cost of floodplain development
     risk(i,t) expected cost of flood risk in subbasin i
     urisk(j,t) expected cost of flood risk in urban ;

*------------------------------------------------
* Define equations in the model.
*------------------------------------------------

Equations
     ecosystem(i,t) benefit function of ecosystem services
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;
```

```
ecosystem(i,t).. eco(i,t) =e= b1*(lf(i)-x(i,t))*exp(b2+b3*log(lf(i)-x(i,t))) ;
dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2)+
                              c01_2*(x('s05',t)+x('s53',t)) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2)+
                              c02_2*x('s43',t) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2)+
                              c07_2*x('s34',t) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2)+
                              c09_2*(x('s69',t)+x('s71',t)+x('s75',t)+
                                     x('s85',t)+x('s90',t))+
                              c09_3*(x('s07',t)+x('s34',t)) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2)+
                              c53_2*x('s05',t) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2)+
                              c71_2*(x('s69',t)+x('s75',t)+
                              x('s85',t)+x('s90',t)) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2)+
                              c89_2*(x('s02',t)+x('s43',t)) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1+csb_2*(x('s01',t)+x('s02',t)+
                                  x('s05',t)+x('s07',t)+x('s09',t)+x('s34',t)+
                                  x('s43',t)+x('s53',t)+x('s69',t)+x('s71',t)+
                                  x('s75',t)+x('s83',t)+x('s85',t)+x('s89',t)+
                                  x('s90',t)) ;
ufld_yk(t).. urisk('york',t) =e= cyk_1+cyk_2*(x('s01',t)+x('s05',t)+
                                 x('s07',t)+x('s09',t)+x('s34',t)+
                                 x('s53',t)+x('s69',t)+x('s71',t)+
                                 x('s75',t)+x('s83',t)+x('s85',t)+
                                 x('s90',t)) ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
net(t).. nb(t) =e= sum(i,eco(i,t))+sum(i,pro(i,t))-sum(i,rest(i,t))-
                   sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;
```

```
*---------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*---------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*---------------------------------------
* Attempt to solve the problem.
*---------------------------------------

Solve floodplain maximizing z using nlp
```

Tax (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*---------------------------------------
* Set time t, subbasins i and urban areas j
*---------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01, s02, s05, s07, s09, s34, s43, s53,
                          s69, s71, s75, s83, s85, s89, s90/
     j urban areas /selby, york/ ;

*---------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*---------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;
```

```
*----------------------------------------
* Set initial values etc.
*----------------------------------------

Parameters
    lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
                     s05 204.67004, s07 2447.00171, s09 6639.46521,
                     s34 2451.61496, s43 1257.75186, s53 347.13385,
                     s69 1338.21609, s71 3595.16936, s75 1029.10755,
                     s83 1119.66767, s85 1149.58292, s89 443.79102,
                     s90 1632.33813/
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
                     s05 14.38793, s07 155.26437, s09 308.46632,
                     s34 51.12944, s43 27.69161, s53 7.97054,
                     s69 101.6242, s71 306.14877, s75 63.81431,
                     s83 106.27285, s85 60.06330, s89 35.18348,
                     s90 67.09430/
    txr(i) tax rate /s01 1030.9442, s02 1202.78708, s05 25037.64832,
          s07 7547.76757, s09 873.20209, s34 10666.99101,
          s43 4648.04262, s53 24835.47271, s69 12135.56239,
          s71 7977.45707, s75 12165.16123, s83 2115.08188,
          s85 12159.27972, s89 1044.79636, s90 12111.60695/ ;

*----------------------------------------
* Set discount factor and common parameter
* values in the functions.
*----------------------------------------

Scalars
    adj adjustment factor for utility function /1000000000/
    dis1 periodic discount rate /0.05/
    dis2 discount factor
    f1 parameter of FP development benefit function /14030.80048/
    r1 parameter of restoration cost function /19146.37353/
    d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*----------------------------------------
* Calculate the discount factor in time t.
*----------------------------------------

Parameters
    dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*----------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*----------------------------------------

Scalars
    c01_1 parameter of flood cost function in 27001 /27.17415/
    c02_1 parameter of flood cost function in 27002 /3.22602/
    c05   parameter of flood cost function in 27005 /137.27578/
```

```
     c07_1 parameter of flood cost function in 27007 /6.43880/
     c09_1 parameter of flood cost function in 27009 /4.61088/
     c34   parameter of flood cost function in 27034 /15.50372/
     c43   parameter of flood cost function in 27043 /28.96503/
     c53_1 parameter of flood cost function in 27053 /1.08513/
     c69   parameter of flood cost function in 27069 /24.53457/
     c71_1 parameter of flood cost function in 27071 /4.41719/
     c75   parameter of flood cost function in 27075 /37.52708/
     c83   parameter of flood cost function in 27083 /5.28397/
     c85   parameter of flood cost function in 27085 /7.80344/
     c89_1 parameter of flood cost function in 27089 /20.35564/
     c90   parameter of flood cost function in 27090 /29.55194/
     csb_1 parameter of flood cost function in Selby /899343.7842/
     cyk_1 parameter of flood cost function in York /8477662.33/ ;

*-----------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*-----------------------------------------------------

Variables
     x(i,t) area of developed floodplains
     yd(i,t) floodplain development
     yr(i,t) floodplain restoration
     tx(i,t) tax
     nb(t) net benefit
     disnb(t) discounted utility of net benefit
     z objective function
     pro(i,t) benefit of developed floodplains
     rest(i,t) cost of floodplain restoration
     deve(i,t) cost of floodplain development
     risk(i,t) expected cost of flood risk in subbasin i
     urisk(j,t) expected cost of flood risk in urban ;

*-----------------------------------------------------
* Define equations in the model.
*-----------------------------------------------------

Equations
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     tax(i,t) tax function
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
```

```
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;

dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
tax(i,t).. tx(i,t) =e= txr(i)*x(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))-sum(i,tx(i,t))-
                    sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*---------------------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*---------------------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
```

```
yr.lo(i,t) = 0.0001 ;

*------------------------------------------
* Attempt to solve the problem.
*------------------------------------------

Solve floodplain maximizing z using nlp
```

Subsidy (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*------------------------------------------
* Set time t, subbasins i and urban areas j
*------------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01,s02,s05,s07,s09,s34,s43,s53,
                                s69,s71,s75,s83,s85,s89,s90/
     j urban areas /selby,york/ ;

*------------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*------------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*------------------------------------------
* Set initial values etc.
*------------------------------------------

Parameters
     lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
                    s05 204.67004, s07 2447.00171, s09 6639.46521,
                    s34 2451.61496, s43 1257.75186, s53 347.13385,
                    s69 1338.21609, s71 3595.16936, s75 1029.10755,
                    s83 1119.66767, s85 1149.58292, s89 443.79102,
                    s90 1632.33813/
```

```
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
                   s05 14.38793, s07 155.26437, s09 308.46632,
                   s34 51.12944, s43 27.69161, s53 7.97054,
                   s69 101.6242, s71 306.14877, s75 63.81431,
                   s83 106.27285, s85 60.06330, s89 35.18348,
                   s90 67.09430/
    ssr(i) subsidy rate /s01 1030.9442,s02 1202.78708,s05 25037.64832,
          s07 7547.76757, s09 873.20209, s34 10666.99101,
          s43 4648.04262, s53 24835.47271, s69 12135.56239,
          s71 7977.45707, s75 12165.16123, s83 2115.08188,
          s85 12159.27972, s89 1044.79636, s90 12111.60695/ ;

*------------------------------------------------
* Set discount factor and common parameter
* values in the functions.
*------------------------------------------------

Scalars
    adj adjustment factor for utility function /1000000000/
    dis1 periodic discount rate /0.05/
    dis2 discount factor
    f1 parameter of FP development benefit function /14030.80048/
    r1 parameter of restoration cost function /19146.37353/
    d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*------------------------------------------------
* Calculate the discount factor in time t.
*------------------------------------------------

Parameters
    dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*------------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*------------------------------------------------

Scalars
    c01_1 parameter of flood cost function in 27001 /27.17415/
    c02_1 parameter of flood cost function in 27002 /3.22602/
    c05   parameter of flood cost function in 27005 /137.27578/
    c07_1 parameter of flood cost function in 27007 /6.43880/
    c09_1 parameter of flood cost function in 27009 /4.61088/
    c34   parameter of flood cost function in 27034 /15.50372/
    c43   parameter of flood cost function in 27043 /28.96503/
    c53_1 parameter of flood cost function in 27053 /1.08513/
    c69   parameter of flood cost function in 27069 /24.53457/
    c71_1 parameter of flood cost function in 27071 /4.41719/
    c75   parameter of flood cost function in 27075 /37.52708/
    c83   parameter of flood cost function in 27083 /5.28397/
    c85   parameter of flood cost function in 27085 /7.80344/
    c89_1 parameter of flood cost function in 27089 /20.35564/
```

```
    c90   parameter of flood cost function in 27090 /29.55194/
    csb_1 parameter of flood cost function in Selby /899343.7842/
    cyk_1 parameter of flood cost function in York /8477662.33/ ;

*----------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*----------------------------------------------------

Variables
    x(i,t) area of developed floodplains
    yd(i,t) floodplain development
    yr(i,t) floodplain restoration
    ss(i,t) subsidy
    nb(t) net benefit
    disnb(t) discounted utility of net benefit
    z objective function
    pro(i,t) benefit of developed floodplains
    rest(i,t) cost of floodplain restoration
    deve(i,t) cost of floodplain development
    risk(i,t) expected cost of flood risk in subbasin i
    urisk(j,t) expected cost of flood risk in urban ;

*----------------------------------------------------
* Define equations in the model.
*----------------------------------------------------

Equations
    dfp(i,t) benefit function of developed floodplains
    r_cost(i,t) cost function of floodplain restoration
    d_cost(i,t) cost function of floodplain development
    subsidy(i,t) subsidy function
    fld_01(t) expected cost function of flood risk in subbasin
    fld_02(t) expected cost function of flood risk in subbasin
    fld_05(t) expected cost function of flood risk in subbasin
    fld_07(t) expected cost function of flood risk in subbasin
    fld_09(t) expected cost function of flood risk in subbasin
    fld_34(t) expected cost function of flood risk in subbasin
    fld_43(t) expected cost function of flood risk in subbasin
    fld_53(t) expected cost function of flood risk in subbasin
    fld_69(t) expected cost function of flood risk in subbasin
    fld_71(t) expected cost function of flood risk in subbasin
    fld_75(t) expected cost function of flood risk in subbasin
    fld_83(t) expected cost function of flood risk in subbasin
    fld_85(t) expected cost function of flood risk in subbasin
    fld_89(t) expected cost function of flood risk in subbasin
    fld_90(t) expected cost function of flood risk in subbasin
    ufld_sb(t) expected cost function of flood risk in urban
    ufld_yk(t) expected cost function of flood risk in urban
    ini(i,t) provision of intial conditions
    nconst1(i,t) natural constraints on development
    nconst2(i,t) natural constraints on restoration
    net(t) net benefit function in time t
    disnet(t) discounted utility function of net benefit in time t
    motion(i,t) equations of motion
    welfare definition of objective function ;
```

```
dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
subsidy(i,t).. ss(i,t) =e= ssr(i)*(lf(i)-x(i,t)) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))+sum(i,ss(i,t))-
                   sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*-----------------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*-----------------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*-----------------------------------------------
* Attempt to solve the problem.
*-----------------------------------------------

Solve floodplain maximizing z using nlp
```

Mix of Tax and Subsidy (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*---------------------------------------------
* Set time t, subbasins i and urban areas j
*---------------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01, s02, s05, s07, s09, s34, s43, s53,
                              s69, s71, s75, s83, s85, s89, s90/
     j urban areas /selby, york/ ;

*---------------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*---------------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*---------------------------------------------
* Set initial values etc.
*---------------------------------------------

Parameters
     lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
                 s05 204.67004, s07 2447.00171, s09 6639.46521,
                 s34 2451.61496, s43 1257.75186, s53 347.13385,
                 s69 1338.21609, s71 3595.16936, s75 1029.10755,
                 s83 1119.66767, s85 1149.58292, s89 443.79102,
                 s90 1632.33813/
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
                 s05 14.38793, s07 155.26437, s09 308.46632,
                 s34 51.12944, s43 27.69161, s53 7.97054,
                 s69 101.6242, s71 306.14877, s75 63.81431,
                 s83 106.27285, s85 60.06330, s89 35.18348,
                 s90 67.09430/
   rate(i) rate /s01 1030.9442, s02 1202.78708, s05 25037.64832,
         s07 7547.76757, s09 873.20209, s34 10666.99101,
         s43 4648.04262, s53 24835.47271, s69 12135.56239,
         s71 7977.45707, s75 12165.16123, s83 2115.08188,
         s85 12159.27972, s89 1044.79636, s90 12111.60695/
   target(i) standard /s01 99.39369, s02 217.13234,
```

```
                    s05 14.38793, s07 155.26437, s09 308.46632,
                    s34 51.12944, s43 27.69161, s53 7.97054,
                    s69 101.6242, s71 306.14877, s75 63.81431,
                    s83 106.27285, s85 60.06330, s89 35.18348,
                    s90 67.09430/ ;

*----------------------------------------------
* Set discount factor and common parameter
* values in the functions.
*----------------------------------------------

Scalars
    adj adjustment factor for utility function /1000000000/
    dis1 periodic discount rate /0.05/
    dis2 discount factor
    f1 parameter of FP development benefit function /14030.80048/
    r1 parameter of restoration cost function /19146.37353/
    d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*----------------------------------------------
* Calculate the discount factor in time t.
*----------------------------------------------

Parameters
    dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*----------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*----------------------------------------------

Scalars
    c01_1 parameter of flood cost function in 27001 /27.17415/
    c02_1 parameter of flood cost function in 27002 /3.22602/
    c05   parameter of flood cost function in 27005 /137.27578/
    c07_1 parameter of flood cost function in 27007 /6.43880/
    c09_1 parameter of flood cost function in 27009 /4.61088/
    c34   parameter of flood cost function in 27034 /15.50372/
    c43   parameter of flood cost function in 27043 /28.96503/
    c53_1 parameter of flood cost function in 27053 /1.08513/
    c69   parameter of flood cost function in 27069 /24.53457/
    c71_1 parameter of flood cost function in 27071 /4.41719/
    c75   parameter of flood cost function in 27075 /37.52708/
    c83   parameter of flood cost function in 27083 /5.28397/
    c85   parameter of flood cost function in 27085 /7.80344/
    c89_1 parameter of flood cost function in 27089 /20.35564/
    c90   parameter of flood cost function in 27090 /29.55194/
    csb_1 parameter of flood cost function in Selby /899343.7842/
    cyk_1 parameter of flood cost function in York /8477662.33/ ;
```

```
*--------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*--------------------------------------------------

Variables
     x(i,t) area of developed floodplains
     yd(i,t) floodplain development
     yr(i,t) floodplain restoration
     pty1(i,t) penalty
     nb(t) net benefit
     disnb(t) discounted utility of net benefit
     z objective function
     pro(i,t) benefit of developed floodplains
     rest(i,t) cost of floodplain restoration
     deve(i,t) cost of floodplain development
     risk(i,t) expected cost of flood risk in subbasin i
     urisk(j,t) expected cost of flood risk in urban ;

*--------------------------------------------------
* Define equations in the model.
*--------------------------------------------------

Equations
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     penal1(i,t) penalty
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;

dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
```

```
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
penal1(i,t).. pty1(i,t) =e= rate(i)*(x(i,t)-target(i)) ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))-sum(i,pty1(i,t))
                   -sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*-----------------------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*-----------------------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*-----------------------------------------------------
* Attempt to solve the problem.
*-----------------------------------------------------

Solve floodplain maximizing z using nlp
```

Marketable Permits for Development (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*-----------------------------------------------
* Set time t, subbasins i and urban areas j
*-----------------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01,s02,s05,s07,s09,s34,s43,s53,
                         s69,s71,s75,s83,s85,s89,s90/
     j urban areas /selby,york/ ;

*-----------------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*-----------------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*-----------------------------------------------
* Set initial values etc.
*-----------------------------------------------

Parameters
     lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
               s05 204.67004, s07 2447.00171, s09 6639.46521,
               s34 2451.61496, s43 1257.75186, s53 347.13385,
               s69 1338.21609, s71 3595.16936, s75 1029.10755,
               s83 1119.66767, s85 1149.58292, s89 443.79102,
               s90 1632.33813/
     x0(i) initial state variables /s01 99.39369, s02 217.13234,
               s05 14.38793, s07 155.26437, s09 308.46632,
               s34 51.12944, s43 27.69161, s53 7.97054,
               s69 101.6242, s71 306.14877, s75 63.81431,
               s83 106.27285, s85 60.06330, s89 35.18348,
               s90 67.09430/
     mp(i) marketable permits for development /s01 139.79714,
               s02 1770.14972, s05 0.0031, s07 348.12915,
               s09 1118.3071, s34 57.34874, s43 134.27139,
               s53 0.0031, s69 0.0031, s71 378.99932,
               s75 0.0031, s83 1010.56003, s85 59.79422,
               s89 283.6966, s90 0.0031/ ;
```

```
*---------------------------------------------------
* Set discount factor and common parameter
* values in the functions.
*---------------------------------------------------

Scalars
     adj adjustment factor for utility function /1000000000/
     dis1 periodic discount rate /0.05/
     dis2 discount factor
     f1 parameter of FP development benefit function /14030.80048/
     r1 parameter of restoration cost function /19146.37353/
     d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*---------------------------------------------------
* Calculate the discount factor in time t.
*---------------------------------------------------

Parameters
     dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*---------------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*---------------------------------------------------

Scalars
     c01_1 parameter of flood cost function in 27001 /27.17415/
     c02_1 parameter of flood cost function in 27002 /3.22602/
     c05   parameter of flood cost function in 27005 /137.27578/
     c07_1 parameter of flood cost function in 27007 /6.43880/
     c09_1 parameter of flood cost function in 27009 /4.61088/
     c34   parameter of flood cost function in 27034 /15.50372/
     c43   parameter of flood cost function in 27043 /28.96503/
     c53_1 parameter of flood cost function in 27053 /1.08513/
     c69   parameter of flood cost function in 27069 /24.53457/
     c71_1 parameter of flood cost function in 27071 /4.41719/
     c75   parameter of flood cost function in 27075 /37.52708/
     c83   parameter of flood cost function in 27083 /5.28397/
     c85   parameter of flood cost function in 27085 /7.80344/
     c89_1 parameter of flood cost function in 27089 /20.35564/
     c90   parameter of flood cost function in 27090 /29.55194/
     csb_1 parameter of flood cost function in Selby /899343.7842/
     cyk_1 parameter of flood cost function in York /8477662.33/ ;

*---------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*---------------------------------------------------

Variables
     x(i,t) area of developed floodplains
```

```
     yd(i,t) floodplain development
     yr(i,t) floodplain restoration
     nb(t) net benefit
     disnb(t) discounted utility of net benefit
     z objective function
     pro(i,t) benefit of developed floodplains
     rest(i,t) cost of floodplain restoration
     deve(i,t) cost of floodplain development
     risk(i,t) expected cost of flood risk in subbasin i
     urisk(j,t) expected cost of flood risk in urban ;

*----------------------------------------------
* Define equations in the model.
*----------------------------------------------

Equations
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     mpconst(i) marketable permits for development
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;

dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
```

```
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
mpconst(i).. sum(t,yd(i,t)) =l= mp(i) ;
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))-
              sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*----------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*----------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*----------------------------------------
* Attempt to solve the problem.
*----------------------------------------

Solve floodplain maximizing z using nlp
```

Marketable Permits for Developed Floodplains (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext
```

```
*----------------------------------------
* Set time t, subbasins i and urban areas j
*----------------------------------------

Sets
    t time periods /1990*2020/
    tfirst(t) first period
    i subbasins /s01, s02, s05, s07, s09, s34, s43, s53,
                 s69, s71, s75, s83, s85, s89, s90/
    j urban areas /selby, york/ ;

*----------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*----------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*----------------------------------------
* Set initial values etc.
*----------------------------------------

Parameters
    lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
                s05 204.67004, s07 2447.00171, s09 6639.46521,
                s34 2451.61496, s43 1257.75186, s53 347.13385,
                s69 1338.21609, s71 3595.16936, s75 1029.10755,
                s83 1119.66767, s85 1149.58292, s89 443.79102,
                s90 1632.33813/
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
                s05 14.38793, s07 155.26437, s09 308.46632,
                s34 51.12944, s43 27.69161, s53 7.97054,
                s69 101.6242, s71 306.14877, s75 63.81431,
                s83 106.27285, s85 60.06330, s89 35.18348,
                s90 67.09430/
    mp(i) marketable permits for developed floodplains /
                s01 236.87, s02 1968.685, s05 0.0002,
                s07 493.596, s09 1413.031, s34 104.398,
                s43 159.787, s53 0.0002, s69 64.37,
                s71 670.857, s75 41.688, s83 1115.574,
                s85 111.796, s89 315.874, s90 53.846/ ;

*----------------------------------------
* Set discount factor and common parameter
* values in the functions.
*----------------------------------------

Scalars
    adj adjustment factor for utility function /1000000000/
    dis1 periodic discount rate /0.05/
    dis2 discount factor
    f1 parameter of FP development benefit function /14030.80048/
```

```
    r1 parameter of restoration cost function /19146.37353/
    d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;

*--------------------------------------------------
* Calculate the discount factor in time t.
*--------------------------------------------------

Parameters
    dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*--------------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*--------------------------------------------------

Scalars
    c01_1 parameter of flood cost function in 27001 /27.17415/
    c02_1 parameter of flood cost function in 27002 /3.22602/
    c05   parameter of flood cost function in 27005 /137.27578/
    c07_1 parameter of flood cost function in 27007 /6.43880/
    c09_1 parameter of flood cost function in 27009 /4.61088/
    c34   parameter of flood cost function in 27034 /15.50372/
    c43   parameter of flood cost function in 27043 /28.96503/
    c53_1 parameter of flood cost function in 27053 /1.08513/
    c69   parameter of flood cost function in 27069 /24.53457/
    c71_1 parameter of flood cost function in 27071 /4.41719/
    c75   parameter of flood cost function in 27075 /37.52708/
    c83   parameter of flood cost function in 27083 /5.28397/
    c85   parameter of flood cost function in 27085 /7.80344/
    c89_1 parameter of flood cost function in 27089 /20.35564/
    c90   parameter of flood cost function in 27090 /29.55194/
    csb_1 parameter of flood cost function in Selby /899343.7842/
    cyk_1 parameter of flood cost function in York /8477662.33/ ;

*--------------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*--------------------------------------------------

Variables
    x(i,t) area of developed floodplains
    yd(i,t) floodplain development
    yr(i,t) floodplain restoration
    nb(t) net benefit
    disnb(t) discounted utility of net benefit
    z objective function
    pro(i,t) benefit of developed floodplains
    rest(i,t) cost of floodplain restoration
    deve(i,t) cost of floodplain development
    risk(i,t) expected cost of flood risk in subbasin i
    urisk(j,t) expected cost of flood risk in urban ;
```

```
*----------------------------------------------------
* Define equations in the model.
*----------------------------------------------------

Equations
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     mpconst1(i) marketable permits for developed floodplains
     mpconst2(i) marketable permits for developed floodplains
     mpconst3(i) marketable permits for developed floodplains
     mpconst4(i) marketable permits for developed floodplains
     mpconst5(i) marketable permits for developed floodplains
     mpconst6(i) marketable permits for developed floodplains
     mpconst7(i) marketable permits for developed floodplains
     mpconst8(i) marketable permits for developed floodplains
     mpconst9(i) marketable permits for developed floodplains
     mpconst10(i) marketable permits for developed floodplains
     mpconst11(i) marketable permits for developed floodplains
     mpconst12(i) marketable permits for developed floodplains
     mpconst13(i) marketable permits for developed floodplains
     mpconst14(i) marketable permits for developed floodplains
     mpconst15(i) marketable permits for developed floodplains
     mpconst16(i) marketable permits for developed floodplains
     mpconst17(i) marketable permits for developed floodplains
     mpconst18(i) marketable permits for developed floodplains
     mpconst19(i) marketable permits for developed floodplains
     mpconst20(i) marketable permits for developed floodplains
     mpconst21(i) marketable permits for developed floodplains
     mpconst22(i) marketable permits for developed floodplains
     mpconst23(i) marketable permits for developed floodplains
     mpconst24(i) marketable permits for developed floodplains
     mpconst25(i) marketable permits for developed floodplains
     mpconst26(i) marketable permits for developed floodplains
     mpconst27(i) marketable permits for developed floodplains
     mpconst28(i) marketable permits for developed floodplains
     mpconst29(i) marketable permits for developed floodplains
```

```
     mpconst30(i) marketable permits for developed floodplains
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;

dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
mpconst1(i).. x(i,'1991') =l= mp(i) ;
mpconst2(i).. x(i,'1992') =l= mp(i) ;
mpconst3(i).. x(i,'1993') =l= mp(i) ;
mpconst4(i).. x(i,'1994') =l= mp(i) ;
mpconst5(i).. x(i,'1995') =l= mp(i) ;
mpconst6(i).. x(i,'1996') =l= mp(i) ;
mpconst7(i).. x(i,'1997') =l= mp(i) ;
mpconst8(i).. x(i,'1998') =l= mp(i) ;
mpconst9(i).. x(i,'1999') =l= mp(i) ;
mpconst10(i).. x(i,'2000') =l= mp(i) ;
mpconst11(i).. x(i,'2001') =l= mp(i) ;
mpconst12(i).. x(i,'2002') =l= mp(i) ;
mpconst13(i).. x(i,'2003') =l= mp(i) ;
mpconst14(i).. x(i,'2004') =l= mp(i) ;
mpconst15(i).. x(i,'2005') =l= mp(i) ;
mpconst16(i).. x(i,'2006') =l= mp(i) ;
mpconst17(i).. x(i,'2007') =l= mp(i) ;
mpconst18(i).. x(i,'2008') =l= mp(i) ;
mpconst19(i).. x(i,'2009') =l= mp(i) ;
mpconst20(i).. x(i,'2010') =l= mp(i) ;
mpconst21(i).. x(i,'2011') =l= mp(i) ;
mpconst22(i).. x(i,'2012') =l= mp(i) ;
mpconst23(i).. x(i,'2013') =l= mp(i) ;
mpconst24(i).. x(i,'2014') =l= mp(i) ;
mpconst25(i).. x(i,'2015') =l= mp(i) ;
mpconst26(i).. x(i,'2016') =l= mp(i) ;
mpconst27(i).. x(i,'2017') =l= mp(i) ;
mpconst28(i).. x(i,'2018') =l= mp(i) ;
```

```
mpconst29(i).. x(i,'2019') =l= mp(i) ;
mpconst30(i).. x(i,'2020') =l= mp(i) ;
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))-
                   sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*-----------------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*-----------------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*-----------------------------------------------
* Attempt to solve the problem.
*-----------------------------------------------

Solve floodplain maximizing z using nlp
```

Regulation (base)

```
$Title  Model of Optimal Floodplain Management
$Ontext
     Koichiro Mori
     Modelling Hydrological, Ecological and Economic
     Interactions in River Floodplains
     A Case Study of Ouse Catchment
$Offtext

*-----------------------------------------------
* Set time t, subbasins i and urban areas j
*-----------------------------------------------

Sets
     t time periods /1990*2020/
     tfirst(t) first period
     i subbasins /s01,s02,s05,s07,s09,s34,s43,s53,
                           s69,s71,s75,s83,s85,s89,s90/
```

```
    j urban areas /selby,york/ ;

*-------------------------------------------------
* The elements in the sets are strings. Thus,
* we have to convert the strings into the
* numbers so that we can use the time t to
* calculate the discounting factor.
*-------------------------------------------------

tfirst(t) = yes$(ord(t) eq 1) ;
Display tfirst ;
Display t ;

*-------------------------------------------------
* Set initial values etc.
*-------------------------------------------------

Parameters
    lf(i) area of floodplains /s01 1356.91346, s02 2402.75427,
                    s05 204.67004, s07 2447.00171, s09 6639.46521,
                    s34 2451.61496, s43 1257.75186, s53 347.13385,
                    s69 1338.21609, s71 3595.16936, s75 1029.10755,
                    s83 1119.66767, s85 1149.58292, s89 443.79102,
                    s90 1632.33813/
    x0(i) initial state variables /s01 99.39369, s02 217.13234,
                    s05 14.38793, s07 155.26437, s09 308.46632,
                    s34 51.12944, s43 27.69161, s53 7.97054,
                    s69 101.6242, s71 306.14877, s75 63.81431,
                    s83 106.27285, s85 60.06330, s89 35.18348,
                    s90 67.09430/
    mp(i) quota on development / s01 137.47631, s02 1751.55266,
                    s05 0.0031, s07 338.33163, s09 1104.56468,
                    s34 53.26856, s43 132.09539, s53 0.0031,
                    s69 0.0031, s71 364.70823, s75 0.0031,
                    s83 1009.30115, s85 51.7327, s89 280.69052,
                    s90 0.0031/
    md(i) duties on restoration /
                    s01 0.0031, s02 0.0031, s05 14.38793,
                    s07 0.0031, s09 0.0031, s34 0.0031,
                    s43 0.0031, s53 7.97054, s69 37.2542,
                    s71 0.0031, s75 22.12631, s83 0.005,
                    s85 0.0031, s89 0.0031, s90 13.2483/ ;

*-------------------------------------------------
* Set discount factor and common parameter
* values in the functions.
*-------------------------------------------------

Scalars
    adj adjustment factor for utility function /1000000000/
    dis1 periodic discount rate /0.05/
    dis2 discount factor
    f1 parameter of FP development benefit function /14030.80048/
    r1 parameter of restoration cost function /19146.37353/
    d1 parameter of development cost function /1914.63735/ ;

dis2 = 1/(1+dis1) ;
```

```
*----------------------------------------------
* Calculate the discount factor in time t.
*----------------------------------------------

Parameters
    dis3(t) discount factor in time t ;

dis3(t)  = dis2**(ord(t)-1) ;
Display dis3 ;

*----------------------------------------------
* Set parameter values included in
* the expected cost function of flood risk.
*----------------------------------------------

Scalars
    c01_1 parameter of flood cost function in 27001 /27.17415/
    c02_1 parameter of flood cost function in 27002 /3.22602/
    c05   parameter of flood cost function in 27005 /137.27578/
    c07_1 parameter of flood cost function in 27007 /6.43880/
    c09_1 parameter of flood cost function in 27009 /4.61088/
    c34   parameter of flood cost function in 27034 /15.50372/
    c43   parameter of flood cost function in 27043 /28.96503/
    c53_1 parameter of flood cost function in 27053 /1.08513/
    c69   parameter of flood cost function in 27069 /24.53457/
    c71_1 parameter of flood cost function in 27071 /4.41719/
    c75   parameter of flood cost function in 27075 /37.52708/
    c83   parameter of flood cost function in 27083 /5.28397/
    c85   parameter of flood cost function in 27085 /7.80344/
    c89_1 parameter of flood cost function in 27089 /20.35564/
    c90   parameter of flood cost function in 27090 /29.55194/
    csb_1 parameter of flood cost function in Selby /899343.7842/
    cyk_1 parameter of flood cost function in York /8477662.33/ ;

*----------------------------------------------
* Define control and state variables.
* Set other necessary variables.
*----------------------------------------------

Variables
    x(i,t) area of developed floodplains
    yd(i,t) floodplain development
    yr(i,t) floodplain restoration
    nb(t) net benefit
    disnb(t) discounted utility of net benefit
    z objective function
    pro(i,t) benefit of developed floodplains
    rest(i,t) cost of floodplain restoration
    deve(i,t) cost of floodplain development
    risk(i,t) expected cost of flood risk in subbasin i
    urisk(j,t) expected cost of flood risk in urban ;

*----------------------------------------------
* Define equations in the model.
*----------------------------------------------
```

```
Equations
     dfp(i,t) benefit function of developed floodplains
     r_cost(i,t) cost function of floodplain restoration
     d_cost(i,t) cost function of floodplain development
     fld_01(t) expected cost function of flood risk in subbasin
     fld_02(t) expected cost function of flood risk in subbasin
     fld_05(t) expected cost function of flood risk in subbasin
     fld_07(t) expected cost function of flood risk in subbasin
     fld_09(t) expected cost function of flood risk in subbasin
     fld_34(t) expected cost function of flood risk in subbasin
     fld_43(t) expected cost function of flood risk in subbasin
     fld_53(t) expected cost function of flood risk in subbasin
     fld_69(t) expected cost function of flood risk in subbasin
     fld_71(t) expected cost function of flood risk in subbasin
     fld_75(t) expected cost function of flood risk in subbasin
     fld_83(t) expected cost function of flood risk in subbasin
     fld_85(t) expected cost function of flood risk in subbasin
     fld_89(t) expected cost function of flood risk in subbasin
     fld_90(t) expected cost function of flood risk in subbasin
     ufld_sb(t) expected cost function of flood risk in urban
     ufld_yk(t) expected cost function of flood risk in urban
     ini(i,t) provision of intial conditions
     nconst1(i,t) natural constraints on development
     nconst2(i,t) natural constraints on restoration
     mpconst(i) marketable permits
     mdconst(i) marketable duties
     net(t) net benefit function in time t
     disnet(t) discounted utility function of net benefit in time t
     motion(i,t) equations of motion
     welfare definition of objective function ;

dfp(i,t).. pro(i,t) =e= f1*x(i,t) ;
r_cost(i,t).. rest(i,t) =e= r1*yr(i,t) ;
d_cost(i,t).. deve(i,t) =e= d1*yd(i,t) ;
fld_01(t).. risk('s01',t) =e= c01_1*(x('s01',t)**2) ;
fld_02(t).. risk('s02',t) =e= c02_1*(x('s02',t)**2) ;
fld_05(t).. risk('s05',t) =e= c05*(x('s05',t)**2) ;
fld_07(t).. risk('s07',t) =e= c07_1*(x('s07',t)**2) ;
fld_09(t).. risk('s09',t) =e= c09_1*(x('s09',t)**2) ;
fld_34(t).. risk('s34',t) =e= c34*(x('s34',t)**2) ;
fld_43(t).. risk('s43',t) =e= c43*(x('s43',t)**2) ;
fld_53(t).. risk('s53',t) =e= c53_1*(x('s53',t)**2) ;
fld_69(t).. risk('s69',t) =e= c69*(x('s69',t)**2) ;
fld_71(t).. risk('s71',t) =e= c71_1*(x('s71',t)**2) ;
fld_75(t).. risk('s75',t) =e= c75*(x('s75',t)**2) ;
fld_83(t).. risk('s83',t) =e= c83*(x('s83',t)**2) ;
fld_85(t).. risk('s85',t) =e= c85*(x('s85',t)**2) ;
fld_89(t).. risk('s89',t) =e= c89_1*(x('s89',t)**2) ;
fld_90(t).. risk('s90',t) =e= c90*(x('s90',t)**2) ;
ufld_sb(t).. urisk('selby',t) =e= csb_1 ;
ufld_yk(t).. urisk('york',t) =e= cyk_1 ;
ini(i,tfirst).. x(i,tfirst) =e= x0(i) ;
nconst1(i,t).. yd(i,t) =l= lf(i)-x(i,t) ;
nconst2(i,t).. yr(i,t) =l= x(i,t) ;
mpconst(i).. sum(t,yd(i,t)) =l= mp(i) ;
mdconst(i).. yr(i,'1990') =g= md(i);
net(t).. nb(t) =e= sum(i,pro(i,t))-sum(i,rest(i,t))-
```

```
                sum(i,deve(i,t))-sum(i,risk(i,t))-sum(j,urisk(j,t)) ;
disnet(t).. disnb(t) =e= dis3(t)*2*((nb(t)+adj)**(1/2)) ;
motion(i,t+1).. x(i,t+1) =e= x(i,t)+yd(i,t)-yr(i,t) ;
welfare.. z =e= sum(t,disnb(t)) ;

Model floodplain /all/ ;
option domlim = 1000000 ;
option reslim = 5400 ;
floodplain.iterlim = 10000000 ;

*-----------------------------------------------
* Set the constraints on control and state
* variables. We should note that we give
* a small value to the lower bounds so that
* we can avoid function evaluation errors
* in the procedures in GAMS.
*-----------------------------------------------

x.lo(i,t) = 0.0001 ;
x.up(i,t) = lf(i)-0.0001 ;
yd.lo(i,t) = 0.0001 ;
yr.lo(i,t) = 0.0001 ;

*-----------------------------------------------
* Attempt to solve the problem.
*-----------------------------------------------

Solve floodplain maximizing z using nlp
```

D-3 Expected Cost Function of Flood Risk in Case of the Increase in Precipitation

27001

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_01     Mean=   25092499.81    , S.D.=   15283509.78      |
| Model size: Observations =       60, Parameters =  2, Deg.Fr.=     58  |
| Residuals:  Sum of squares= .1500852700E+16, Std.Dev.=  5086921.53300  |
| Fit:        R-squared=  .891097, Adjusted R-squared =          .88922  |
| Model test: F[  1,      58] =  474.58,    Prob value =         .00000  |
| Diagnostic: Log-L =  -1010.6503, Restricted(b=0) Log-L =  -1077.1692   |
|             LogAmemiyaPrCrt.=  30.917, Akaike Info. Crt.=     33.755   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X01    30.04626669     1.2756232        23.554   .0000   560485.77
 XS        26152.53996     3456.9509         7.565   .0000   253.79385
```

27002

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_02     Mean=   12136406.91    , S.D.=   7480309.735      |
| Model size: Observations =       60, Parameters =  2, Deg.Fr.=     58  |
| Residuals:  Sum of squares= .3053043200E+15, Std.Dev.=  2294312.00705  |
| Fit:        R-squared=  .907521, Adjusted R-squared =          .90593  |
| Model test: F[  1,      58] =  569.17,    Prob value =         .00000  |
| Diagnostic: Log-L =   -962.8759, Restricted(b=0) Log-L =  -1034.2992   |
|             LogAmemiyaPrCrt.=  29.325, Akaike Info. Crt.=     32.163   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X02    4.358000294      .18917254       23.037   .0000   1962517.3
 XS        3961.074429     612.58020         6.466   .0000   735.86611
```

27005

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_05     Mean=   2374597.299    , S.D.=   1280844.151      |
| Model size: Observations =       60, Parameters =  1, Deg.Fr.=     59  |
| Residuals:  Sum of squares= .2753929044E+14, Std.Dev.=   683203.94807  |
| Fit:        R-squared=  .715483, Adjusted R-squared =          .71548  |
| Model test: F[  1,      59] =  148.37,    Prob value =         .00000  |
| Diagnostic: Log-L =   -890.7050, Restricted(b=0) Log-L =   -928.4139   |
|             LogAmemiyaPrCrt.=  26.886, Akaike Info. Crt.=     29.724   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X05    151.7798415     5.1364119        29.550   .0000   13007.425
```

27007

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_07      Mean=   25829213.86    , S.D. =   14323356.38  |
| Model size: Observations =       60, Parameters =   2, Deg.Fr. =   58 |
| Residuals:  Sum of squares= .1815307468E+16, Std.Dev. = 5594497.70969 |
| Fit:        R-squared= .850029, Adjusted R-squared =           .84744 |
| Model test: F[  1,    58] =  328.74,    Prob value =           .00000 |
| Diagnostic: Log-L =  -1016.3569, Restricted(b=0) Log-L =   -1073.2762 |
|             LogAmemiyaPrCrt. =   31.107, Akaike Info. Crt. =   33.945 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X07    8.115190155     .44415905       18.271   .0000  2084693.1
XS        5240.602168     807.54950        6.490   .0000  1321.3177
```

27009

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_09      Mean=   129692331.3    , S.D. =   75388077.06  |
| Model size: Observations =       60, Parameters =   3, Deg.Fr. =   57 |
| Residuals:  Sum of squares= .2558954187E+17, Std.Dev. = 21188188.47825 |
| Fit:        R-squared= .923686, Adjusted R-squared =           .92101 |
| Model test: F[  2,    57] =  344.96,    Prob value =           .00000 |
| Diagnostic: Log-L =  -1095.7348, Restricted(b=0) Log-L =   -1172.9217 |
|             LogAmemiyaPrCrt. =   33.787, Akaike Info. Crt. =   36.624 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X09    4.906959076     .22643823       21.670   .0000  16370091.
XS71      6816.178755     1887.8918        3.610   .0006  3588.9758
XS07      8033.708372     2444.2608        3.287   .0017  2589.8980
```

27034

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none    |
| Dep. var. = C_34      Mean=   43726755.04    , S.D. =   24135210.59  |
| Model size: Observations =       60, Parameters =   1, Deg.Fr. =   59 |
| Residuals:  Sum of squares= .7340879717E+16, Std.Dev. = 11154447.10020 |
| Fit:        R-squared= .786404, Adjusted R-squared =           .78640 |
| Model test: F[  1,    59] =  217.22,    Prob value =           .00000 |
| Diagnostic: Log-L =  -1058.2730, Restricted(b=0) Log-L =   -1104.5830 |
|             LogAmemiyaPrCrt. =   32.471, Akaike Info. Crt. =   35.309 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
SQ_X34    16.59264269     .49159088       33.753   .0000  2269018.5
```

27043

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_43      Mean=   25607081.20    , S.D.=   11498825.18    |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .2206266515E+16, Std.Dev.= 6115091.79864 |
| Fit:        R-squared=  .717187, Adjusted R-squared =          .71719 |
| Model test: F[  1,      59] =  149.62,    Prob value =          .00000 |
| Diagnostic: Log-L =  -1022.2083, Restricted(b=0) Log-L =  -1060.0974 |
|             LogAmemiyaPrCrt.=   31.269, Akaike Info. Crt.=     34.107 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X43     34.94831377      1.0081407       34.666   .0000  648268.17
```

27053

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_53      Mean=   421812.7600    , S.D.=   287146.6070    |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=    58 |
| Residuals:  Sum of squares= .7281709573E+12, Std.Dev.=  112047.63129 |
| Fit:        R-squared=  .850316, Adjusted R-squared =          .84774 |
| Model test: F[  1,      58] =  329.48,    Prob value =          .00000 |
| Diagnostic: Log-L =   -781.7200, Restricted(b=0) Log-L =   -838.6970 |
|             LogAmemiyaPrCrt.=   23.286, Akaike Info. Crt.=     26.124 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X53     9.044618811      .46834519       19.312   .0000  32913.926
 XS         868.3780724      190.08936        4.568   .0000  100.91424
```

27069

```
+----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_69      Mean=   17969735.67    , S.D.=   11027903.72    |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=    59 |
| Residuals:  Sum of squares= .1940938810E+16, Std.Dev.= 5735614.00085 |
| Fit:        R-squared=  .729496, Adjusted R-squared =          .72950 |
| Model test: F[  1,      59] =  159.11,    Prob value =          .00000 |
| Diagnostic: Log-L =  -1018.3644, Restricted(b=0) Log-L =  -1057.5885 |
|             LogAmemiyaPrCrt.=   31.141, Akaike Info. Crt.=     33.979 |
+----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X69     26.48690175      .96842057       27.351   .0000  545723.23
```

27071

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_71      Mean=   39718419.90    , S.D.=   24482883.42     |
| Model size: Observations =       60, Parameters =   2, Deg.Fr.=     58 |
| Residuals:  Sum of squares= .2684742564E+16, Std.Dev.=  6803577.36065  |
| Fit:        R-squared=  .924085, Adjusted R-squared =          .92278  |
| Model test: F[  1,     58] =  706.02,    Prob value =          .00000  |
| Diagnostic: Log-L =  -1028.0968, Restricted(b=0) Log-L =  -1105.4412   |
|             LogAmemiyaPrCrt.=  31.499, Akaike Info. Crt.=     34.337   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X71    6.227308270      .27649221       22.523   .0000  4038213.7
 XS        5233.001596      561.28824        9.323   .0000  2493.8689
```

27075

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+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_75      Mean=   14885759.63    , S.D.=   9169794.763     |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=     59 |
| Residuals:  Sum of squares= .1310904754E+16, Std.Dev.=  4713674.21988  |
| Fit:        R-squared=  .735759, Adjusted R-squared =          .73576  |
| Model test: F[  1,     59] =  164.28,    Prob value =          .00000  |
| Diagnostic: Log-L =  -1006.5908, Restricted(b=0) Log-L =  -1046.5176   |
|             LogAmemiyaPrCrt.=  30.748, Akaike Info. Crt.=     33.586   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X75    40.64391775      1.4717393       27.616   .0000  296156.24
```

27083

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_83      Mean=   3504168.820    , S.D.=   2164384.433     |
| Model size: Observations =       60, Parameters =   1, Deg.Fr.=     59 |
| Residuals:  Sum of squares= .6451137581E+14, Std.Dev.=  1045663.97532  |
| Fit:        R-squared=  .766592, Adjusted R-squared =          .76659  |
| Model test: F[  1,     59] =  193.78,    Prob value =          .00000  |
| Diagnostic: Log-L =   -916.2419, Restricted(b=0) Log-L =   -959.8909   |
|             LogAmemiyaPrCrt.=  27.737, Akaike Info. Crt.=     30.575   |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X83    7.631545266      .25909337       29.455   .0000  378024.33
```

27085

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_85     Mean=   5283904.399     , S.D. =   3372330.547   |
| Model size: Observations =       60, Parameters =   1, Deg.Fr. =    59 |
| Residuals:  Sum of squares= .1537404707E+15, Std.Dev. = 1614239.97272 |
| Fit:        R-squared= .770873, Adjusted R-squared =           .77087 |
| Model test: F[  1,      59] =  198.50,    Prob value =         .00000 |
| Diagnostic: Log-L =   -942.2946, Restricted(b=0) Log-L =    -986.4990 |
|             LogAmemiyaPrCrt.=  28.605, Akaike Info. Crt.=      31.443 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X85     10.40179703       .35860336       29.006   .0000  414488.51
```

27089

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_89     Mean=   2847261.318     , S.D. =   1532964.486   |
| Model size: Observations =       60, Parameters =   2, Deg.Fr. =    58 |
| Residuals:  Sum of squares= .1493707985E+14, Std.Dev. =  507479.91044 |
| Fit:        R-squared= .892267, Adjusted R-squared =           .89041 |
| Model test: F[  1,      58] =  480.37,    Prob value =         .00000 |
| Diagnostic: Log-L =   -872.3520, Restricted(b=0) Log-L =    -939.1949 |
|             LogAmemiyaPrCrt.=  26.307, Akaike Info. Crt.=      29.145 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X89     27.57896201      1.2019151        22.946   .0000  65359.427
 XS         463.5024988      48.113464         9.634   .0000  1944.2826
```

27090

```
+-----------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none     |
| Dep. var. = C_90     Mean=   33967493.46     , S.D. =   16083601.46   |
| Model size: Observations =       60, Parameters =   1, Deg.Fr. =    59 |
| Residuals:  Sum of squares= .4957887225E+16, Std.Dev. = 9166896.24996 |
| Fit:        R-squared= .675154, Adjusted R-squared =           .67515 |
| Model test: F[  1,      59] =  122.62,    Prob value =         .00000 |
| Diagnostic: Log-L =  -1046.4987, Restricted(b=0) Log-L =   -1080.2307 |
|             LogAmemiyaPrCrt.=  32.079, Akaike Info. Crt.=      34.917 |
+-----------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 SQ_X90     32.55725064      1.0582777        30.764   .0000  891152.11
```

Selby

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_SELBY  Mean=   2330466.429   , S.D.=   343628.1044       |
| Model size: Observations =       60, Parameters =  2, Deg.Fr.=      58 |
| Residuals:  Sum of squares= .3334157137E+13, Std.Dev.=   239761.27269 |
| Fit:        R-squared=  .521418, Adjusted R-squared =          .51317 |
| Model test: F[  1,     58] =   63.19,     Prob value =          .00000 |
| Diagnostic: Log-L =   -827.3632, Restricted(b=0) Log-L =    -849.4710 |
|             LogAmemiyaPrCrt.=   24.808, Akaike Info. Crt.=     27.645 |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant  1665552.340      89187.903        18.675   .0000
 XS        47.74673062      6.0064123         7.949   .0000  13925.856
```

York

```
+------------------------------------------------------------------------+
| Ordinary    least squares regression    Weighting variable = none      |
| Dep. var. = C_YORK   Mean=   14707246.01   , S.D.=   1551439.865       |
| Model size: Observations =       60, Parameters =  2, Deg.Fr.=      58 |
| Residuals:  Sum of squares= .6164201671E+14, Std.Dev.=  1030918.71242 |
| Fit:        R-squared=  .565935, Adjusted R-squared =          .55845 |
| Model test: F[  1,     58] =   75.62,     Prob value =          .00000 |
| Diagnostic: Log-L =   -914.8769, Restricted(b=0) Log-L =    -939.9137 |
|             LogAmemiyaPrCrt.=   27.725, Akaike Info. Crt.=     30.563 |
+------------------------------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant  11661443.10      374687.20        31.123   .0000
 XS        259.0927084      29.794455         8.696   .0000  11755.649
```

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