Optimal Policy Before, During and After the Crisis

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Doctor of Philosophy

University of York
Economics

July 2015
Abstract

This thesis contributes to the debate on optimal policy, using New Keynesian dynamic stochastic general equilibrium (DSGE) models that contain a variety of monetary, fiscal and credit policy tools.

First, we examine optimal monetary policy in an open economy, utilizing Gali and Monacelli’s (2005) DSGE model. We find that the utility based loss function in the open economy depends on the variance of the terms of trade, in addition to the variance of consumption and inflation. As a result, optimal policy in the open economy is not isomorphic to the one in the closed economy and does not require strict domestic inflation targeting. In the open economy, interest rate rules which react to the movements in inflation and the terms of trade are preferred to the domestic inflation based Taylor rule.

Second, we examine the effectiveness of macroprudential tools and their interaction with monetary policy. Using a Gertler and Karadi (2011) type DSGE model with financial frictions, we present a formal comparative analysis of three macroprudential instruments: (i) reserve requirements, (ii) capital requirements and (iii) a regulation premium. We find that capital requirements are the most effective macroprudential tool in mitigating the negative effects of the financial accelerator mechanism built in banks’ borrowing constraints. Irrespective of the type of shock affecting the economy, use of capital requirements generates the highest welfare gains.

Finally, we analyze unconventional monetary and fiscal policy measures that can be used to counteract the unfavorable consequences of a financial crisis. Adding distortionary taxation to Gertler and Karadi’s (2011) framework, we provide a comprehensive assessment of credit easing and bank capital injections. We find that the use of both policies mitigates the negative effects of financial shocks to the economy. Credit easing results in a lower stabilizing effect on aggregate output and a decrease in tax rates. Bank capital injections, on the other hand, lead to a rise in government expenditures and hence, taxes. As the relative importance of distortions in financial markets is higher than the distortions caused by variable tax rates, use of bank capital injections generates the highest welfare gains, under both distortionary and lump-sum taxation.
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Acknowledgements

I would like to express my deepest appreciation and gratitude to my supervisors Professor F. Gulcin Ozkan and Professor Mike R. Wickens for mentoring me during my studies. I would like to thank them for guiding me to improve my skills as a researcher and supporting me to develop my career after the PhD. Working with them has been a priceless experience for me.

I would also like to thank my Thesis Advisory Panel (TAP) member Dr. Paulo Santos Monteiro for his valuable comments and suggestions.

I am grateful to the Department of Economics and Related Studies at the University of York for providing an outstanding work environment, with excellent administrative and academic staff members. As a holder of a Departmental Studentship, I also thank them for funding my studies and giving me the opportunity to study in the UK.

I would also like to thank all the friends that I have met in York. They have always brought joy to my life and supported me during my studies.

I finally wish to dedicate this thesis to my family and thank Bahriye Gonul Tavman, Irmak Tavman and Emrah Gazioglu for their unconditional love, sacrifice and patience.

Without the support of all these people, this thesis would have never been completed.
Declaration

I declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research. All sources are acknowledged as References.

Some of the material presented within this thesis has been published as:

Chapter 1

Introduction

In recent years, the use of dynamic stochastic general equilibrium (DSGE) models in optimal policy analysis has increased significantly. These models make use of microfoundations to explain the behaviour of the aggregate economy. Hence, they are not subject to the Lucas’ Critique (1976). In addition, as the microfoundations in DSGE models are based on economic agents’ preferences, DSGE models serve as suitable frameworks for evaluating the welfare effects of alternative policies (Woodford, 2003).

One of the main areas of macroeconomic research, where DSGE models have been extensively used, is the determination of optimal monetary policy. Before the global financial crisis, exploring the differences between optimal monetary policies in closed and open economies has been a central area of interest in policy analysis. Prominent examples of models developed for this purpose include Gali and Monacelli (2005), Sutherland (2005) and De Paoli (2009). Examining the various studies in the literature, it can be seen that domestic inflation targeting is the broadly accepted optimal monetary policy in the closed economy. However, there has been no clear agreement on the optimal policy in open economies.
The 2007-09 crisis has clearly shown that a disruption in the financial sector can lead to a sharp downturn in the economy. In the first quarter of 2009, the real U.S. GDP and the world GDP contracted by an annual rate of 6.4% and 7.3%, respectively. By the fourth quarter of 2009, the unemployment rate in the U.S. topped 10% (Mishkin, 2010). Hence, following the recent crisis, the literature on macroeconomic models and policies has been reshaped considerably. Macroeconomists agreed upon the necessity to incorporate financial frictions into macroeconomic models and to examine the significance of financial shocks. As a result, the variety of DSGE models used in policy analysis expanded. Two relevant strands of the literature, based on frameworks that introduce financial intermediation into DSGE models, have emerged. The first includes financial frictions associated with the constraints of non-financial borrowers, making use of the financial accelerator mechanism designed by Bernanke et al (1999). The second set of studies contain financial frictions linked to financial intermediaries. The framework developed by Gertler and Karadi (2011) is one of the leading examples.

The recent financial crisis has also highlighted the need to expand the research on optimal policy. As a result, the new stream of DSGE models have been commonly used in the analysis of alternative policy tools. First, it became evident that price stability, to be obtained with the use of the interest rate, is not sufficient to guarantee the stability of the financial system. As the "Tinbergen principle" states, the number of independent instruments should at least be equal to the number of policy objectives. Consequently, to reduce systemic risks and ensure the stability of the financial system, the monetary policy instrument should be supported by additional tools, which are referred to as macroprudential policy instruments (Bank for International Settlements, 2010). Counter-cyclical capital requirements introduced by the Basel III reform package
is one of the prominent examples of macroprudential tools. A large number of countries, including the U.S. and the EU, have completed the adoption of this instrument in the past few years. Other well-known examples of macroprudential policy tools include counter-cyclical loan-to-value (LTV) ratios and reserve requirements. As the analysis in Claessens (2014) demonstrates, LTV ratio is the most commonly used macroprudential tool in advanced and emerging economies and reserve requirements are highly used in emerging markets. Following the crisis, the effectiveness of these three macroprudential tools and their interaction with monetary policy have been examined in various studies. Galati and Moessner (2014) provide a comprehensive review of the recent progress in theoretical research on the effectiveness of macroprudential policies.

In terms of policy, the global financial crisis has also shown that conventional policies need to be supplemented with unconventional policy measures in times of financial crises. During the 2007-09 episode, the Fed, the Bank of England (BoE) and the Bank of Japan have established several lending and asset purchase programmes to provide liquidity to financial markets. The BoE operated the Asset Purchase Facility (APF) and purchased 3 billion pounds of private assets. The Bank of Japan has also announced purchases of 3 trillion yens in commercial paper (Fawley and Neely, 2013). In addition, the U.S. Treasury has used the Capital Purchase Program (CPP) to inject 205 billion dollars of equity into financial institutions. The U.K. Treasury has also injected 50 billion pounds of equity into British banks (Mishkin, 2010). In line with these developments, the use of unconventional policy measures has also been explored in numerous DSGE models. The papers by Curdia and Woodford (2010) and Gertler and Karadi (2011) are among the first that attempt to examine the effects of unconventional policies, such as credit and quantitative easing.

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This thesis aims to present a comprehensive analysis of diverse policy instruments, using a variety of New Keynesian DSGE models. In Chapter 2, we analyze whether there are any differences between the optimal monetary policy in the open and the closed economy. To do so, we generalize the special setting used in Gali and Monacelli’s (2005) small open economy framework, where the coefficient of relative risk aversion, the elasticity of substitution between home and foreign goods, and the elasticity of substitution between foreign goods from different origins are set equal to 1. Moreover, we examine the implications of using a wider set of simple policy rules in the open economy. These include a domestic inflation based Taylor rule (DITR), a CPI inflation based Taylor rule (CITR), an exchange rate peg (PEG), and two domestic inflation and terms of trade based Taylor rules (DITTR and DITTR(-1)).

Under the special case used by Gali and Monacelli (2005), it is possible to determine the size of the subsidy which will exactly offset the mixed effects of the distortions caused by the market power of firms and the central bank’s incentive to influence the terms of trade. Hence, as in the closed economy, the flexible price equilibrium allocation is optimal and requires domestic prices to be stabilized under the optimal policy. However, under the general case in the open economy, we show that it is not possible to determine the size of this subsidy. As a result, the existence of the distortions pushes optimal policy away from domestic inflation targeting. In addition, we find that the utility based welfare loss function under the general case is quite different from the one obtained under the special case. Under the general setting, welfare losses in the open economy depend on the variance of the terms of trade, consumption and inflation. As a result, monetary policy for the open economy is not isomorphic to the one for the closed economy.
We then analyze the equilibrium properties of various macroeconomic variables under different policy rules. We find that when the general case is used, the volatility of domestic inflation achieved under the PEG is lower than the volatility obtained under the DITR. However, under the special case, the opposite is observed. With the special case, the shock to world output has an effect on domestic inflation under the PEG, while this effect is not present under the DITR. On the contrary, under the general case, domestic inflation is affected by both shocks under both policy rules. As a result, with the general case, the variance of domestic inflation increases due to the shock to world output under the DITR. On the other hand, the lower variance in the terms of trade results in a lower variance in inflation under the PEG, as the exchange rate remains unchanged. Hence, in contrast to the special case, the volatility of inflation obtained under the PEG is lower than the volatility obtained under the DITR. Moreover, among the aforementioned policy rules, the lowest volatility of inflation is achieved under the DITTR(-1).

Finally, we compute the welfare losses achieved under each policy rule. We find that the welfare losses obtained under different policy rules depend on the source of the exogenous shock affecting the small open economy. When both domestic and foreign shocks are present, welfare levels associated with DITTR and DITTR(-1) are higher than the level associated with the DITR. In addition, the lowest welfare loss is obtained under the DITTR(-1). As a result, in the small open economy, interest rate rules that react to the movements in the terms of trade and domestic inflation are preferred to the domestic inflation based Taylor rule.

Next, in the third chapter, we compare the effectiveness of alternative macroprudential policies and determine if there is a leading macroprudential tool for targeting a
specific market externality. In our analysis, we utilize a New Keynesian DSGE model containing financial frictions as described in Gertler and Karadi (2011). The tools that we include in our study are (i) reserve requirements, (ii) capital requirements and (iii) a regulation premium, whose formulation is based on the assumption that macroprudential policies in general increase the costs of financial intermediaries, who in turn pass these costs onto borrowers (Unsal, 2013). In our framework, the financial accelerator mechanism built in banks’ borrowing constraints involves a pecuniary externality, where bankers do not consider the fact that if they issued more equity, they would decrease the risk of the banking sector. As a result, the banking sector’s inefficiently high level of leverage amplifies the negative effects of exogenous shocks to the economy and urges the need for macroprudential regulation (Gertler et al, 2012). Reserve requirements and capital requirements increase the cost of deposits to banks, inducing them to replace deposit financing by equity financing. A rise in the regulation premium is reflected in the increase in cost of borrowing to firms. To ensure comparability, all macroprudential instruments in our study respond to the same financial variable, which is the total nominal credit growth in the economy, with the same intensity.

When the economy is hit by a productivity or a financial shock, the use of all the aforementioned macroprudential tools mitigates the negative effects of the given shocks to the economy. These shocks bring about a decline in asset prices, which triggers the financial accelerator mechanism. As banks are leveraged, the decline in asset prices results in an amplified decrease in their net worth, which causes a downturn in their balance sheets and hence, an increase in their leverage ratios. The increase in the leverage ratio increases the spread, and in turn, results in a rise in the cost of capital, which causes a further decline in investment and asset prices. Finally, the decrease
in investment leads to a decline in aggregate output. When counter-cyclical reserve requirements or capital requirements are used in combination with monetary policy, the decline in banks’ net worth, and hence the increase in the leverage ratios and the spread is smaller. Consequently, the decrease in assets prices, investment and output are lower. Counter-cyclical use of the regulation premium directly lowers the increase in the cost of capital. As a result, the decrease in aggregate output is again lowered. Comparing the dynamics of both shocks under alternative macroprudential policies, it can be seen that capital requirements are the most effective macroprudential tool in lowering the negative effects of the given shocks to the spread, asset prices and investment. As a result, they mitigate the negative effects of the financial accelerator mechanism built in banks’ balance sheet constraints the most.

We also use welfare-maximizing monetary and macroprudential policy rules to calculate the welfare losses obtained under each policy alternative. Using the case where the economy is affected by a productivity or a financial shock only, we see that the use of each macroprudential policy results in a decrease in the welfare loss, under both shocks. The macroprudential policy tool that generates the lowest welfare gains is the regulation premium under the productivity shock, while it is the reserve requirements under the financial shock. The use of capital requirements generates the highest welfare gains, irrespective of the type of the shock affecting the economy.

Finally, in Chapter 4, we present a comprehensive assessment of two types of unconventional policies and compare their fiscal implications. The policy measures that we include in our study are credit easing by the central bank and bank capital injections by the government. When the first policy is used, the central bank increases the total credit in the economy with the supply of loans to non-financial firms. The use of the
second policy directly increases the capital of banks. We assume that the central bank or the government face efficiency costs when one of the given policies is pursued.

We compare both the costs and the benefits of using these policy measures, utilizing the framework that we develop in the third chapter. As the returns and the costs from the use of unconventional policies are both reflected in the government’s budget constraint, fiscal policy tools need to adjust to the changes in the fiscal balance, that are induced by the use of these policies. To examine the consequences of using alternative fiscal instruments, we add two separate distortionary taxes, which are consumption and labour income taxes, to our framework. Hence, we can examine the fiscal implications of alternative unconventional policy measures, under three scenarios: (i) the government adopts lump-sum taxes and uses government spending to respond to the changes in its budget, (ii) the government adopts distortionary (consumption or labour income) taxes and uses the tax rate to adjust to the changes in the government’s budget constraint and (iii) the government adopts distortionary taxes and uses government spending to accommodate the changes in the fiscal balance.

When the economy is affected by a financial shock, the use of both types of unconventional policy measures mitigates its negative effects. As explained in the previous chapter, the financial shock results in a decrease in asset prices, which triggers the financial accelerator mechanism. This decrease corresponds to an increase in the leverage ratio, the credit spread and the cost of capital. As a result, asset prices, investment and aggregate output decrease. When the monetary authority uses credit easing or the government injects equity into banks, the rise in the credit spread is dampened. Hence, the increase in the cost of capital, and the decrease in investment and aggregate output are lower. As bank capital injections induce a straightforward increase in banks’ net
worth, they result in a significantly lower increase in the leverage ratio and the spread, compared to credit easing. Hence, their positive effect on investment and aggregate output is higher. Since the return to government equity is the same as the return to government bonds, there are no excess returns to government equity, under bank capital injections. In contrast, when credit easing is used, the central bank can utilize the excess return on assets present in times of the financial crisis. Consequently, the presence of the efficiency costs of unconventional policies results in an increase in fiscal costs, with the use of bank capital injections. However, utilization of credit easing results in an increase in fiscal revenues, as the excess return to government intermediated assets is higher than the efficiency costs. As a result, under scenarios (i) and (iii), bank capital injections result in a decrease in government sending, while the use of credit easing increases the same variable. Under scenario (ii), there is a decrease (increase) in the tax rates when credit easing (bank capital injections) is used. Under all scenarios, use of bank capital injections still induces the lowest decline in economic activity.

We also compare the welfare implications of credit easing and bank capital injections under scenarios (i), (ii) and (iii). We again utilize welfare-maximizing monetary and unconventional policy rules in our welfare analysis. Our results indicate that when lump-sum taxes instead of distortionary taxes are in place, use of credit policies generates higher welfare gains, only when the government uses the tax rates to respond to the changes in its budget. However, when government spending is used to accommodate these changes, the existence of distortionary taxes does not cause a decline in welfare. The use of unconventional policies still generates welfare gains, when variable tax rates are in place. Moreover, utilization of bank capital injections results in the highest welfare gains, under all three scenarios.
Chapter 2

Optimal Monetary Policy for the Small Open Economy Revisited

2.1 Introduction

The question of how to conduct monetary policy has always been an area of interest in macroeconomic research. When economists first started focusing on non-monetary factors in the real business cycle, the importance of monetary policy was undermined for a period. However, in the late 1980s, there has been a rebirth of interest in this area, as a stream of empirical work has highlighted the non-neutrality of money. During this period, economists have also started evaluating monetary policy rules using dynamic general equilibrium theory. This development has supported the resurgence of interest in the area. Both closed and open economy versions of the dynamic stochastic general equilibrium (DSGE) models have featured in policy analyses. In contrast to the studies that examine the closed economy and suggest domestic inflation targeting as the optimal policy, no clear agreement on optimal monetary policy for the open economy has
emerged.

In this paper, utilizing Gali and Monacelli (GM)'s (2005) small open economy framework with monopolistic competition and sticky prices, we revisit the question of "which monetary policy should central banks implement in an open economy". In their study, using a special case where the coefficient of relative risk aversion ($\sigma$), the elasticity of substitution between home and foreign goods ($\eta$), and the elasticity of substitution between foreign goods from different origins ($\gamma$) are equal to 1, the authors find that the optimal monetary policy for a small open economy is isomorphic to the one for a closed economy.

However, different studies which examine the optimal monetary policy in an open economy setting emphasize that a country might benefit from influencing its terms of trade. To start with, Ball (1999) finds that inflation targeting can be suboptimal in open economies. The optimal policy rule in an open economy has some differences from the Taylor rule of the closed economy. First, the policy instrument of the central bank is not just the interest rate but it is a weighted sum of the interest rate and the exchange rate. Second, in the policy rule, this index responds to "long-run" inflation, which is a measure of inflation altered according to the temporary effects of exchange rate movements. According to Svensson (2000), including the movements in the exchange rate in monetary policy analysis has several important consequences. First, the exchange rate adds channels to the transmission mechanism of monetary policy. Second, the exchange rate is a forward-looking variable determined by expectations. This helps in making forward-looking behaviour and the role of expectations vital in the analysis of monetary policy. Third, some disturbances in the rest of the world, such as the fluctuations in foreign inflation and foreign interest rates are transmitted through the exchange rate.
With strict inflation targeting, policymakers are expected to have a considerable activism in monetary policy, which may result in substantial volatility of macrovariables other than inflation. On the contrary, flexible CPI inflation targeting stabilizes CPI inflation and also results in the stabilization of real exchange rates and other variables. Using a general equilibrium model among interdependent countries, Corsetti and Pesenti (2001) also find that policies just concentrating on stabilizing domestic prices and the output gap might cause the economy to end up with inefficiently high prices of imports, and hence with suboptimal welfare levels. In open economies, policymakers can increase welfare by sacrificing output gap stability for lower consumer prices. Finally, Sutherland (2005) and De Paoli (2009) support these findings in a two-country DSGE model with monopolistic competition and sticky prices. Using an analytical derivation of the representative household’s welfare, they find that for high values of the elasticity of substitution between domestic and foreign goods, an exchange rate peg results in a higher welfare level than the one achieved with inflation targeting.

Inspired by these studies, the purpose of this paper is two-fold. First, we generalize the setting used by GM (2005), without restricting \( \sigma, \eta \) and \( \gamma \) to 1 and examine whether the optimal policy for the open economy is still isomorphic to the one for the closed economy. In addition, we study the implications of a wider set of simple policy rules in the open economy. More specifically, a domestic inflation based Taylor rule (DITR), a CPI inflation based Taylor rule (CITR), an exchange rate peg (PEG), and two domestic inflation and terms of trade based Taylor rules (DITTR and DITTR(-1)), that have not been previously considered in the aforementioned studies, are used in our comparison.

We start our analysis with the derivation of the optimal allocation in the open economy. Under the special case, it is possible to determine the subsidy, which will exactly
offset the mixed effects of the distortions caused by the market power of firms and the central bank’s incentive to influence the terms of trade. This, in turn, guarantees the optimality of the flexible price equilibrium allocation and requires domestic prices to be stabilized under the optimal policy, as in the closed economy. However, under the general case in the open economy, it is not possible to determine this subsidy. Hence, the presence of a monopolistic distortion and a terms of trade externality drives optimal policy away from domestic inflation targeting. To have a precise qualitative analysis of monetary policy in the small open economy, we also derive the utility based welfare loss function under the general case. We find that welfare losses in the open economy depend on the variance of the terms of trade, in addition to the variance of consumption and inflation. As a result, the monetary policy for the open economy is not isomorphic to the one for the closed economy.

We continue our analysis with the examination of the impulse responses to a domestic and a foreign shock. Moreover, we analyze the volatilities of different macroeconomic variables obtained under each simple policy rule, when the economy experiences both shocks. Our results show that with the general setting, the volatility of domestic inflation achieved under the PEG is lower than the one achieved under the DITR, while the opposite is observed with the special case. When the special parametrization is used, the shock to world output has an effect on domestic inflation under the PEG, while this effect is not present under the DITR. On the contrary, under the general case, domestic inflation is affected by both shocks under both policy rules. As a result, when the general case is used, the variance of domestic inflation rises because of the shock to world output under the DITR. On the other hand, the lower variance in the terms of trade results in a lower variance in inflation under the PEG, since the exchange rate
does not fluctuate. In addition, the lowest volatility of inflation is achieved under the DITTR(-1).

We conclude our analysis with the computation of the welfare losses achieved under each simple policy rule. We find that the welfare levels obtained under different monetary policy rules depend on the exogenous shock affecting the small open economy. Under the presence of productivity and world output shocks, welfare levels associated with domestic inflation and terms of trade based Taylor rules are higher than the level associated with the DITR. Moreover, the lowest welfare loss is obtained under the DITTR(-1). Hence, in the small open economy, interest rate rules that react to the movements in the terms of trade and domestic inflation are preferred to the DITR.

Our paper proceeds as follows. Section 2.2 presents the small open economy model and its dynamics. Section 2.3 shows the optimal allocation in the open economy and derives the welfare loss function used in comparing different monetary policy rules. Section 2.4 explains the values of the parameters used in the updated calibration. Section 2.5 analyzes the dynamic effects of two different shocks on various macroeconomic variables, under different monetary policy rules. Section 2.6 presents the volatilities and the welfare losses obtained under each policy rule. Finally, Section 2.7 concludes.

2.2 The Model

The model used in our analysis is an open economy general equilibrium model, with imperfect competition and nominal rigidities. The world economy in the model consists of infinitesimally small open economies. Hence, the decisions of the each small economy regarding its domestic policy do not affect the rest of the world.
2.2.1 Households

The analysis of our model starts with the examination of the representative household’s utility maximization in the small open economy. Accordingly, the representative household maximizes expected discounted utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \]  

(2.1)

The utility function is assumed to be separable in consumption and labour and the period utility is assumed to have the following form\(^1\),

\[ U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \]

Here, \( N_t \) defines hours worked, and \( C_t \) denotes a composite consumption index given by

\[ C_t = \left[ (1 - \alpha)^{1/\eta}(C_{H,t})^{\frac{n-1}{n}} + \alpha^{1/\eta}(C_{F,t})^{\frac{n-1}{n}} \right]^{\frac{n}{n-1}} \]  

(2.2)

The consumption index gives the consumption preferences of the household among domestic and imported bundles of goods, \( C_{H,t} \) and \( C_{F,t} \), respectively. Each bundle consists of imperfectly substitutable varieties, with an elasticity of substitution given by \( \epsilon \) and \( \gamma \). Hence, the bundles are given by

\[ C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{1}{1+\gamma}} dj \right)^{\frac{1+\gamma}{1-\gamma}} \quad C_{F,t} = \left( \int_0^1 C_{F,t}(i)^{\frac{1}{1+\gamma}} di \right)^{\frac{1+\gamma}{1-\gamma}} \]

\(^1\)For \( \sigma = 1 \), utility of consumption is denoted by log-utility.
where \( j \in [0, 1] \) refers to the variety of goods and \( C_{i,t} \) refers to the index of the quantity of goods imported from country \( i \) by domestic consumers. It has the same functional form as \( C_{H,t} \). Following these definitions, it should be noticed that \( \epsilon \) gives the elasticity of substitution between goods produced in the same country, while \( \gamma \) shows the elasticity of substitution between foreign goods produced in different countries.

Other parameters that are important in the analysis of the household’s behaviour are: \( \alpha \in [0, 1] \) which gives the degree of openness and \( \eta \) that gives the elasticity of substitution between domestic and foreign goods.

The household maximizes utility subject to her budget constraint,

\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j) dj di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t + T_t \quad (2.3)
\]

Here, \( P_{i,t}(j) \) is the price of good \( j \) imported from country \( i \). \( D_{t+1} \) is the nominal gain in period \( t + 1 \) of the asset purchased at the end of period \( t \), \( W_t \) is the nominal wage and \( T_t \) denotes lump-sum taxes/transfers. These variables are denoted in units of domestic currency and \( Q_{t,t+1} \) is the stochastic discount factor for one-period ahead nominal gains of the consumer. We assume that asset markets are complete.

Before looking at the optimal allocation of consumption and labour as a result of the period utility maximization, we analyze the optimal allocation of the household’s consumption among different goods from different countries. Firstly, we look at the optimal division of any given expenditure within each kind of goods. This optimization results in
\[ C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t}; \quad C_{i,t}(j) = \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} C_{i,t} \]  

(2.4)  

for all \( i, j \in [0, 1] \).

Secondly, we look at the optimal division of expenditure among imported goods from different countries, which results in

\[ C_{i,t} = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \]

(2.5)  

for all \( i \in [0, 1] \).

Finally, it can be seen that the optimal division of expenditure among domestic and imported goods gives

\[ C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_{t}} \right)^{-\eta} C_{t}; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_{t}} \right)^{-\eta} C_{t} \]

(2.6)  

The price indexes used in these equalities are as follows,

\[ P_{t} = [(1 - \alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}} \] is the CPI (consumer price index),

\[ P_{H,t} = \left( \int_{0}^{1} P_{H,t}(j)^{1-\epsilon}dj \right)^{\frac{1}{1-\epsilon}} \] is the domestic price index,

\[ P_{i,t} = \left( \int_{0}^{1} P_{i,t}(j)^{1-\epsilon}dj \right)^{\frac{1}{1-\epsilon}} \] is the price index for goods imported from country \( i \), for all \( i \in [0, 1] \),

\[ P_{F,t} = \left( \int_{0}^{1} P_{i,t}^{1-\gamma}di \right)^{\frac{1}{1-\gamma}} \] is the price index for imported goods, given in domestic currency.
Using equation (2.4), one can arrive at the conclusion that

\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) dj = P_{H,t}C_{H,t}; \quad \int_0^1 P_{i,t}(j)C_{i,t}(j) dj = P_{i,t}C_{i,t}
\]

Furthermore, using equation (2.5) it can be seen that \( \int_0^1 P_{i,t}C_{i,t} di = P_{F,t}C_{F,t} \).

Finally, utilizing equation (2.6), it can be noticed that the total expenditure on consumption will be given by \( P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t \).

Making use of all these facts, the period budget constraint can be expressed as

\[
P_tC_t + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \quad (2.7)
\]

Accordingly, the following optimality conditions for the household’s maximization problem should hold,

\[
\frac{-U_{c,t}}{U_{n,t}} = \frac{W_t}{P_t} \quad (2.8)
\]

which shows that the marginal rate of substitution between consumption and hours worked should be equal to the real CPI based wage, and

\[
\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \quad (2.9)
\]

which indicates a positive relationship between the stochastic discount factor, \( Q_{t,t+1} \) and the ratio of the current consumption to the one in future, \( \frac{C_t}{C_{t+1}} \), everything else being equal.
Taking conditional expectations on both sides of equation (2.9) and using the fact that $R_t = 1/E_t\{Q_{t,t+1}\}$, the consumption Euler equation can be obtained as follows,

$$\beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1$$  \hspace{1cm} (2.10)

Here, $R_t$ is the gross riskless return on a one-period discount bond that yields one unit of domestic currency and hence, $E_t\{Q_{t,t+1}\}$ is its price.

After examining the optimization by households, to present an analysis of the equilibrium, it is necessary to make some assumptions, give definitions, and compute some identities:

1) **Terms of Trade**

To start with, the bilateral terms of trade between the domestic economy and country $i$ is defined as follows,

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$$

The effective terms of trade is then defined by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \int_0^1 (S_{i,t}^{1-\gamma} di)^{\frac{1}{1-\gamma}}$$

$S_t$ gives us the relative price of imported goods and can be approximated by the following log-linearization,

$$s_t = \int_0^1 s_{i,t} di$$  \hspace{1cm} (2.11)
2) **Law of One Price**

After defining $\epsilon_{i,t}$ as the bilateral nominal exchange rate (the price of country $i$’s currency in terms of home currency), with the assumption of law of one price, we have

$$P_{i,t}(j) = \epsilon_{i,t}P_{i,t}^i(j)$$

for all $i, j \in [0, 1]$. Here, $P_{i,t}^i(j)$ denotes the price of country $i$’s good $j$ expressed in country $i$’s currency.

3) **The Relationship between CPI Inflation, Domestic Inflation and the Terms of Trade**

Log-linearizing the CPI formula around a symmetric steady state that satisfies the purchasing power parity (PPP), $P_{H,t} = P_{F,t}$ yields

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t$$

(2.12)

Then, domestic inflation, $\pi_{H,t}$ and CPI inflation, $\pi_t$ can be defined as $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_t = p_t - p_{t-1}$, respectively and using equation (2.12), it can be seen that they are related as,

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t$$

(2.13)

Accordingly, it can be noticed that the CPI and domestic inflation are linked to each other by the percent change in the terms of trade, and the coefficient of this change is $\alpha$. 
4) The Relationship between the Exchange Rate and Terms of Trade

When the implication of law of one price that \( P_{i,t}(j) = \epsilon_{i,t} P_{i,t}^i(j) \) is inserted into the definition of \( P_{i,t} \), we have

\[
P_{i,t} = \epsilon_{i,t} P_{i,t}^i
\]

where \( P_{i,t}^i = \left( \int_0^1 P_{i,t}^i(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \)

Afterwards, defining \( e_t = \int_0^1 e_{i,t} di \) as the (log) nominal effective exchange rate, and \( p_t^* = \int_0^1 p_{i,t}^i di \) as the (log) world price index, plugging the expression of \( P_{i,t} \) into the definition of \( P_{F,t} \) and log-linearizing around the symmetric steady state, the following expression is obtained,

\[
p_{F,t} = e_t + p_t^* \]

Merging the previous result and the definition of the terms of trade gives the following formula,

\[
s_t = e_t + p_t^* - p_{H,t} \tag{2.14}
\]

Then using the definition of the bilateral real exchange rate with country \( i \), \( s_{i,t} = \epsilon_i \frac{P_t^i}{P_t} \), (log) real effective exchange rate can be defined as follows,

\[
q_t = \int_0^1 (\epsilon_{i,t} + p_{i,t}^j - p_t) di = e_t + p_t^* - p_t = (1 - \alpha)s_t
\]
where \( q_{i,t} = \log s_{i,t} \).

As a result, we end up with a relationship between the terms of trade and the real exchange rate.

5) The Relationship between Domestic Consumption, World Consumption and the Terms of Trade

With the assumption of complete asset markets, we can obtain a relation between domestic consumption, world consumption and the terms of trade. Complete asset markets eliminates any differences in the marginal utilities of income across periods and states. Hence, the efficiency condition for holdings of bonds for a representative household must hold in any given country. Accordingly, for country \( i \), the following expression can be obtained,

\[
\beta \left( \frac{C_{i,t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\epsilon_t^i}{\epsilon_{t+1}^i} \right) = Q_{t,t+1}
\]

Merging (2.9) with the relation above and the definition of the real exchange rate gives

\[
C_t = \nu_i C_t^i \frac{1}{s_{i,t}}
\]  

(2.15)

with \( \nu_i \) defined as a constant which will change according to the initial conditions regarding relative net asset positions. When log on both sides of (2.15) are taken and integrated over \( i \), the following expression is obtained,

\[
c_t = c_t^* + \frac{1 - \alpha}{\sigma} s_t
\]  

(2.16)
where $c^*_t$ is the (log) world consumption index.

6) Uncovered interest parity

Under the same assumption of completeness, we can obtain a version of the uncovered interest parity as

$$E_t \left\{ Q_{t,t+1} \left[ R_t - R^i_t \left( \frac{\epsilon_{i,t+1}}{\epsilon_{i,t}} \right) \right] \right\} = 0$$

When the following equation is log-linearized around a symmetric perfect foresight steady state, and aggregated over $i$, the following expression is obtained\(^2\),

$$r_t - r^*_t = E_t \{ \Delta e_{t+1} \} \quad (2.17)$$

2.2.2 Firms

After the analysis of the household’s behaviour and the explanation of the identities and assumptions related to the international economy, an analysis of the supply side follows. As in any other New Keynesian (NK) model, our model incorporates imperfect competition and nominal rigidities into a dynamic general equilibrium framework.

Each monopolistic firm in the home economy produces a differentiated good using the following production function,

$$Y_t(j) = A_t N_t(j)$$

\(^2\text{In the symmetric perfect foresight steady state, } C = C^*, S = 1 \text{ and no risk premium exists (Gali and Monacelli, 2005).}
where \( j \in [0, 1] \) is a firm-specific index and \( A_t \) represents productivity. In the model, \( a_t = \log A_t \) is assumed to be determined by a first order autoregressive process given by

\[
a_t = \rho_a a_{t-1} + \epsilon_t^a
\]

Cost minimization is common across monopolistic firms and implies

\[
\frac{MC_t A_t}{P_{H,t}} = \frac{W_t}{P_{H,t}}
\]

Hence, the real marginal cost can be expressed as

\[
mc_t = w_t - p_{H,t} - a_t
\]

(2.18)

The other Keynesian element in the model is the staggered price-setting à la Calvo (1983). Accordingly, it is assumed that \( 1 - \theta \) percent of firms changes their prices each period, with an individual firm’s probability of adjusting its price in any given period being random. Profit maximization by the typical firm adjusting its price in period \( t \) yields the optimal price-setting behaviour, which can be approximated by the following rule

\[
p^a_{H,t} = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{mc_{t+k} + p_{H,t}\}
\]

(2.19)

Here, \( p^a_{H,t} \) refers to the (log of) adjusted domestic price, and \( \mu = \log \left( \frac{1}{\epsilon-1} \right) \) denotes the (log of) mark-up in the steady state.

It can be seen that firms select their prices as a mark-up over a weighted average of
expected future marginal costs. This assumption regarding the price adjustment also results in the forward looking nature of inflation, which is an important property of the NK models. As explained above, firms have constraints on the frequency with which they can change their prices. Hence, firms adjusting their prices today are aware of the fact that the prices they choose are likely to remain constant for more than one period and they take this into account in their pricing decisions. As changes in the aggregate price level are a result of these current decisions, inflation needs to have a forward looking component (Gali, 2002).

Consequently, the dynamics of domestic inflation in terms of the real marginal cost are given by

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda (mc_t - mc)$$

(2.20)

where $\lambda = \frac{(1-\theta)(1-\theta)}{\theta}$, and $mc$ is the (log) marginal cost in the steady state.

### 2.2.3 The Equilibrium

After the analysis of the demand and the supply side of the economy, the equilibrium dynamics are explained. In equilibrium, goods market in the small open economy should clear,

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) di$$
\[ Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\epsilon_{i,t}P_{F,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_i^t di \right] \] (2.21)

for all \( j \in [0,1] \) and all \( t \). Here, \( C^t_{H,t}(j) \) refers to country \( i \)'s demand for good \( j \) produced in the home economy and equation (2.21) can be obtained with the assumption of symmetric preferences between countries.

Defining the index for aggregate domestic output, identical to the one for consumption, as

\[ Y_t = \left( \int_0^1 Y_t(j)^{-\frac{1}{\eta}} \, dj \right)^{\frac{1}{\epsilon}} \]

and using this definition in equation (2.21) gives

\[ Y_t = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\epsilon_{i,t}P_{F,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_i^t di \]

\[ = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \left[ (1 - \alpha) + \alpha \int_0^1 (S_i^{i,t}S_{i,t})^{-\eta} \psi_{i,t}^{i,t} di \right] \] (2.22)

where the last equality makes use of equation (2.15). Here,

\[ S_i^{i,t} = \epsilon_{i,t} \frac{P_{F,t}^i}{P_{i,t}} \]

refers to the effective terms of trade of country \( i \),

\[ S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \]

refers to the bilateral terms of trade between the home economy and foreign country \( i \).
Under the general case, the following first order log-linear approximation to (2.22) around the symmetric steady state can be obtained as,

\[
y_t = c_t + \alpha \gamma s_t + \alpha (\eta - \frac{1}{\sigma}) q_t
\]

\[
y_t = c_t + \frac{\alpha \omega}{\sigma} s_t
\]  

(2.23)

Here, \( \omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1) \) and we have made use of the fact \( \int_0^1 s_i^t di = 0^3 \).

By aggregating this condition over all countries, a world market clearing condition is obtained as,

\[
y_t^* = \int_0^1 y_i^t di = \int_0^1 c_i^t di = c_t^*
\]

(2.24)

where \( y_t^* \) is (log of) the index for world output and \( c_t^* \) is (log of) the index for world consumption.

Merging (2.23) with (2.16) and (2.24) results in the following expression,

\[
y_t = y_t^* + \frac{(1 - \alpha)}{\sigma} + \frac{\alpha \omega}{\sigma} s_t
\]

(2.25)

Then log-linearizing equation (2.10), we get

\[
c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(\tau_t - E_t\{\pi_{t+1}\} - \rho)
\]

(2.26)

\(^3\)The proof of the fact that \( \int_0^1 s_i^t di = 0 \) can be found in Gali and Monacelli (2005).
Here, lower case letters refer to the logs of the respective variables, \( \rho = \beta^{-1} - 1 \) refers to the discount rate.

Making use of (2.26) and (2.23), we can obtain

\[
y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma}E_t\{\Delta s_{t+1}\}
\]

Using the relationship between CPI inflation, domestic inflation and the terms of trade given in Section 2.2.1, we can also get

\[
y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma}E_t\{\Delta s_{t+1}\}
\]

Finally, utilizing the relationship between domestic output, world output and the terms of trade, we can obtain the relation between domestic output and the interest rate,

\[
y_t = E_t\{y_{t+1}\} - \frac{(1 - \alpha)}{\sigma} + \frac{\alpha\omega}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta E_t\{\Delta y^*_t\} \tag{2.27}
\]

where \( \Theta = (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = \omega - 1 \).

Following the analysis of the market clearing in the home country, examination of the trade balance gives

\[
nx_t = \frac{1}{Y} \left( Y_t - \frac{P_t}{P_{H,t}} C_t \right)
\]

\( nx_t \) is used to refer to the net exports denoted as a fraction of steady state output, \( Y \). A first order approximation gives \( nx_t = y_t - c_t - \alpha s_t \). Merging this expression with
(2.23), we obtain

\[ nx_t = \alpha \left( \frac{\omega}{\sigma} - 1 \right) s_t \]  \hspace{1cm} (2.28)

As a result, whether there is a positive or a negative relationship between the terms of trade and the net exports depends on the relative size of \( \sigma, \gamma, \) and \( \eta. \)

We conclude the analysis of the equilibrium in the small open economy with the specification of the real marginal cost as a function of domestic output. As given in equation (2.18), the marginal cost in the small open economy can be expressed as

\[ mc_t = w_t - p_{H,t} - a_t \]

and rewritten as

\[ mc_t = w_t - p_t + p_t - p_{H,t} - a_t \]
\[ = \sigma c_t + \varphi n_t + \alpha s_t - a_t \]

An approximate aggregate production function showing the relationship between aggregate domestic output and aggregate employment is given by,

\[ N_t = \int_0^1 N_t(j) dj = \frac{Y_t Z_t}{A_t} \text{ and } Z_t = \int_0^1 \frac{Y_t(j)}{Y_t} dj \]

As shown in GM (2005), equilibrium changes in \( z_t = \log Z_t \) around the perfect foresight steady state are of second order. Hence, the following aggregate relationship, up to a first order approximation is obtained as
Making use of (2.29), (2.16) and the fact that \( c_t^* = y_t^* \), the marginal cost as a function of domestic and world output, terms of trade and productivity, is given by

\[
mc_t = \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t
\]  

Accordingly, marginal cost increases as the terms of trade or world output increases. When there is an increase in world output or terms of trade, domestic output increases. As this increase is higher than that of terms of trade, consumption also rises. The rise in consumption, in turn, causes households to perceive themselves wealthier and hence supply less labour. Consequently, the decrease in the labour supply results in an increase in the consumption wage, \( w_t - p_t \), increasing the marginal cost. The increase in the terms of trade also causes a straight increase in the product wage, \( w_t - p_{H,t} \), for any given real wage. Hence, it increases the marginal cost. The effect of an increase in labour productivity, for any given output, is a decrease in real wages, resulting in a decrease in the marginal cost.

Using (2.25) to express \( s_t \), the previous expression for the real marginal cost in terms of domestic output, productivity, and world output can be defined as

\[
mc_t = (\sigma_\alpha + \varphi)y_t + (\sigma - \sigma_\alpha)y_t^* - (1 + \varphi) a_t
\]  

where \( \sigma_\alpha = \frac{\sigma}{1 - \alpha + \alpha \omega} \).
2.2.4 The New Keynesian Phillips Curve and the Dynamic IS Equation

Before concluding section 2.2, we show that the equilibrium dynamics in this economy can be expressed in terms of a New Keynesian Phillips Curve (NKPC) and a dynamic IS equation, as in any other NK model.

Firstly, to reach at the NKPC, the domestic output gap, \( x_t \) is determined as the difference between the (log) domestic output \( y_t \) and its natural level, \( y^n_t \), which is defined as the equilibrium level of output when nominal rigidities do not exist,

\[
x_t = y_t - y^n_t
\]

The natural level of domestic output can be obtained using the assumption that \( mc_t = -\mu \) for all \( t \),

\[
y^n_t = \Omega + \Gamma a_t + \alpha \psi y^*_t
\]

where \( \Omega = -\frac{\mu}{\sigma_{\alpha+\varphi}} \), \( \Gamma = \frac{1 + \varphi}{\sigma_{\alpha+\varphi}} \) and \( \psi = -\frac{\Theta \sigma_{\alpha}}{\sigma_{\alpha+\varphi}} \).

Using equation (2.31), it can be seen the domestic real marginal cost and output gap are linked as

\[
mc_t - mc = (\sigma_{\alpha} + \varphi)x_t
\]

This expression can be merged with equation (2.20) to get the NKPC for the small open economy as a function of the output gap,
\[ \pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha x_t \]  
(2.33)

where \( \kappa_\alpha = \lambda(\sigma_\alpha + \varphi) \).

The degree of openness, \( \alpha \) has an impact on the magnitude of the slope of the Phillips curve, \( \kappa_\alpha \). In the open economy, an increase in domestic output affects marginal cost through its affect on employment, \( \varphi \) and the terms of trade, \( \sigma_\alpha \).

To obtain the dynamic IS equation for the open economy as a function of the output gap, we make use of equation (2.27),

\[ x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{H,t+1}\} - r^n_t) \]  
(2.34)

where \( r^n_t = \rho - \sigma_\alpha \Gamma(1 - \rho_\alpha)\alpha_t + \alpha \sigma_\alpha(\Theta + \psi)E_t\{\Delta y^*_t\} \) is the natural rate of interest in the small open economy.

The degree of openness has an effect on the responsiveness of the output gap to interest rate movements. When \( \omega > 1 \), higher openness decreases \( \sigma_\alpha \) and hence, increases the responsiveness. In addition, openness causes the natural interest rate to be affected by the world output growth.

### 2.3 Optimal Monetary Policy and the Welfare Loss Function

Next, we explain the analytical derivation of the optimal allocation and the welfare loss function in the small open economy. GM (2005) present this analysis under the
special case. We derive the second order approximation to the welfare using generalized parameters and present a comparison between the two welfare functions.

When it is assumed that a constant employment subsidy \( \tau \), which eliminates the rigidities regarding firms’ market power exists in the closed economy, the only effective distortion that remains is sticky prices. In this case, when mark-ups are stabilized at their steady state level, nominal rigidities are no longer binding, as firms do not need to change prices. As a result, the equilibrium allocation is efficient, and the price level stays the same. Hence, the optimal monetary policy is the one that reproduces the flexible price equilibrium allocation (Rotemberg and Woodford, 1999). Unlike the closed economy, there is an additional rigidity in the open economy, which is the central bank’s incentive to affect the terms of trade. This is possible as a result of the imperfect substitutability between domestic and foreign goods. Hence, in an open economy the use of an employment subsidy that exactly eliminates monopolistic distortions is not enough to make the flexible price equilibrium allocation optimal (Benigno and Benigno, 2003). Under the special case where \( \eta = \sigma = \gamma = 1 \), the employment subsidy that exactly offset the mixed effects of the distortions caused by the market power of firms and the central bank’s incentive to influence the terms of trade can be computed, which in turn results in the flexible price equilibrium allocation to be optimal.

The social planner’s objective is to maximize utility, \( U(C_t, N_t) \) subject to the constraints on resources. These are given by the technological constraint, the international risk sharing condition and the market clearing condition (GM, 2005).

In the special case of \( \eta = \sigma = \gamma = 1 \), the optimal allocation (when world output and consumption are taken as given), needs to satisfy
\[- \frac{\partial U(C_t, N_t)/\partial N}{\partial U(C_t, N_t)/\partial C} = (1 - \alpha) \frac{C_t}{N_t} \]

and with \( \sigma = 1 \), this results in constant employment, \( N = (1 - \alpha)^{1/\sigma} \).

In our analysis, \( \sigma, \eta \) and \( \gamma \) are specified to be different from 1. The optimal allocation then satisfies

\[- \frac{\partial U(C_t, N_t)/\partial N}{\partial U(C_t, N_t)/\partial C} = \frac{1 - \alpha}{1 - \alpha + \alpha \omega} \frac{C_t}{N_t} \]

and employment is obtained as \( N_t^{1+\phi} = \frac{1 - \alpha}{1 - \alpha + \alpha \omega} C_t^{1-\sigma} \).

Under the special case, the flexible price equilibrium in the small open economy satisfies

\[
1 - \frac{1}{\epsilon} = MC_t^n = (1 - \tau)(N_t^n)^{1+\phi}
\]

where variables in the flexible price equilibrium have an \( n \) superscript, \( \tau \) is the employment subsidy and \( N_t^n = N = (1 - \alpha)^{1/\sigma} \).

By specifying \( \tau \) such that \( (1 - \tau)(1 - \alpha) = 1 - \frac{1}{\epsilon} \) is satisfied, it is guaranteed that the flexible price equilibrium allocation is optimal. Consequently, as in the closed economy, the optimal monetary policy demands stabilization of the output gap and the domestic prices. Hence, under the special case, strict domestic inflation targeting is optimal in the small open economy.
On the other hand, under the general case, it is not possible to determine the subsidy that will guarantee the optimality of the flexible price equilibrium. As a result, the monopolistic distortion and the terms of trade externality remain present and we cannot directly conclude that strict domestic inflation targeting is the optimal monetary policy.

Following the exploration of the optimal allocation, we derive the welfare loss function in the open economy. Under the special case, it is easy to compute a second order approximation to the utility losses of the domestic representative consumer. These losses, under the special case, given as a fraction of steady state consumption, can be expressed as:

$$W = -\frac{1 - \alpha}{2} \sum_{t=0}^{\infty} \beta^t \left( \frac{\epsilon}{\lambda} \pi_{H,t}^2 + (1 + \varphi) x_t^2 \right)$$

On the other hand, as shown in Appendix A, under the new specification, the welfare losses will be given by

$$W' = -\frac{1}{2} \left[ (1 + \varphi) \zeta - (1 - \sigma) \right] \sum_{t=0}^{\infty} \beta^t \hat{c}_t^2 - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{1 + \varphi}{2} \sum_{t=0}^{\infty} \beta^t \hat{s}_t^2 - \frac{\epsilon \zeta}{2 \lambda} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2$$

$$+(1 - \zeta) \left[ \sum_{t=0}^{\infty} \beta^t \hat{c}_t - \frac{(1 - \alpha)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{s}_t - \frac{(1 - \alpha)(1 + \varphi)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{c}_t \hat{s}_t \right]$$

where \( \zeta = \frac{1 - \alpha}{1 - \alpha + \alpha \omega} = 1 - \Upsilon \) and \( \Upsilon \) denotes the size of the steady state distortion. \( \hat{c}_t \) and \( \hat{s}_t \) are the deviation of consumption and the terms of trade from their respective steady state values.

The derivation of the welfare loss function under the special case is shown in Gali and Monacelli (2005), Appendix D.
Using the second order approximation to the utility losses of the domestic consumer, the expected welfare losses under the special case are obtained as,

$$L = - \frac{1 - \alpha}{2} \left[ \epsilon \varphi \varpi(\pi_{H,t}) + (1 + \varphi)\varphi(x_t) \right]$$

while the expected welfare losses under a generic case are approximated by (see Appendix A for details),

$$L' = - \frac{1}{2} \left[ (1 + \varphi) \zeta - (1 - \sigma) \right] \varphi(c_t) - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{(1 + \varphi)\zeta}{2} \varphi(s_t) - \frac{\epsilon \zeta}{2\lambda} \varphi(\pi_{H,t}) \quad (2.36)$$

As is clear from above, there are important differences between the two welfare losses. Under the special case, the variation in output is only proportional to the variation in consumption. Hence, the welfare losses in the closed and the open economy have the same functional form. However, under the generic case, the variation in output is proportional to the variation in consumption, in addition to the variation in the terms of trade. Moreover, the presence of distortions in the steady state alter the relative weights of these variations.

The new approximation to the welfare can be used to compare the welfare implications of alternative monetary policy rules and to rank those rules based on their welfare outcomes.

### 2.3.1 Simple Monetary Policy Rules

When comparing the implications of a set of simple monetary policy rules for the small open economy, we add two alternatives to the rules used by GM (2005). As the

\footnote{Under the special case, $c_t = (1 - \alpha)y_t + \alpha y^*_t$, Gali and Monacelli (2005), Appendix D.}
fluctuations in the terms of trade (TOT) are also important in the open economy as is seen in (2.36), interest rate rules that react to the movements in both the terms of trade and domestic inflation are added to our comparison. Table 2.1 presents the rules used in our analysis.

<table>
<thead>
<tr>
<th>Rule Description</th>
<th>Rule Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic inflation (DI) based Taylor rule (DITR)</td>
<td>( r_t = \rho + \phi_H \pi_{H,t} )</td>
</tr>
<tr>
<td>CPI inflation based Taylor rule (CITR)</td>
<td>( r_t = \rho + \phi_x \pi_t )</td>
</tr>
<tr>
<td>Exchange rate peg (PEG)</td>
<td>( e_t = 0 )</td>
</tr>
<tr>
<td>DI &amp; TOT based Taylor rule (DITTR)</td>
<td>( r_t = \rho + \phi_H \pi_{H,t} + \phi_{s,t} s_t )</td>
</tr>
<tr>
<td>DI &amp; lagged TOT based Taylor rule (DITTR (-1))</td>
<td>( r_t = \rho + \phi_H \pi_{H,t} + \phi_{s,t} s_t + \phi_{s,t-1} s_{t-1} )</td>
</tr>
</tbody>
</table>

### 2.4 Calibration

Existing empirical work on international monetary policy reveals that the case where \( \eta = \sigma = \gamma = 1 \) does not receive much empirical support. Moreover, when the special parameterization is in place, some open economy properties in the model disappear. In this case, the domestic real marginal cost is unaffected by the changes in world output. In addition, the slope coefficients of the Phillips Curve and the IS Equation are identical to the ones in the closed economy. For the given reasons, this study uses a more general calibration and analyzes whether the results obtained under the special case still hold.

The model calibration is summarized in Table 2.2.

In our calibration, to set the values for \( \sigma, \eta \) and \( \gamma \), we use the behavioural parameters given in the Global Trade Analysis Project Database. The chosen value of \( \eta \) is also in

---

6Global Trade Analysis Project is a global network of researchers and policy makers conducting quantitative analysis of international policy issues. The elasticities in their database are computed econometrically.
Table 2.2 Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>6</td>
<td>Elasticity of substitution between goods produced in the same country</td>
</tr>
<tr>
<td>$\eta$</td>
<td>3</td>
<td>Elasticity of substitution between domestic and imported goods</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>6</td>
<td>Elasticity of substitution between imported goods from different origins</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.1</td>
<td>Coefficient of relative risk aversion</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1.7</td>
<td>Inverse elasticity of labour supply</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>Degree of openness</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.7</td>
<td>Degree of price stickiness</td>
</tr>
<tr>
<td>$\phi_H$</td>
<td>1.5</td>
<td>Inflation coefficient of the Taylor rule</td>
</tr>
<tr>
<td>$\phi_{s,t}$</td>
<td>0.5</td>
<td>Terms of trade coefficient of DITTR</td>
</tr>
<tr>
<td>$\phi_{s,t-1}$</td>
<td>-0.2</td>
<td>Lagged terms of trade coefficient of DITTR(-1)</td>
</tr>
</tbody>
</table>

In line with Obstfeld and Rogoff (2007), while the value of $\gamma$ is also consistent with Jomini et al. (1999) and Liu et al. (2003).

For the calibration of the other parameters, the values used in Bank of Canada’s Terms-of-Trade Economic Model (ToTEM) are taken as reference, as Canada is considered as a prototype small open economy. ToTEM is an open-economy, dynamic stochastic general-equilibrium model used for principal projection and policy-analysis. Accordingly, $\varphi$ is set to 1.7, which implies a labour supply elasticity of 0.6. This value is in the range given by Bargain et al. (2012) and McClelland and Mok (2012) who present a review of the research on labour supply elasticity. The elasticity of substitution between differentiated goods (of the same origin), $\epsilon$ is set as 6. Parameter $\theta$ is set equal to 0.7, which implies that prices are adjusted on average every three quarters and the chosen value is also consistent with the study by Blinder et al. (1998). We set $\beta = 0.99$, in line with much of the literature, implying an annual interest rate of 0.04. The value for $\alpha$ (the degree of openness) is taken as 0.4, the import/GDP ratio in Canada.

In the calibration of the interest rate rules, the original Taylor estimate is used and
\( \phi_H \) and \( \phi_\pi \) are set equal to 1.5. Since fluctuations in the exchange rate should be taken into account for a small open economy, the terms of trade in the DITTR and DITTR (-1) should have a non-zero coefficient. More specifically, we need to have \( \phi_{s,t} > 0 \) and \( \phi_{s,t-1} \leq 0 \). This is needed as an appreciation of the domestic currency should result in a decrease in the domestic interest rate, i.e., currency appreciation tends to be deflationary. The coefficients for the terms of trade in the interest rate rules, \( \phi_{s,t} \) and \( \phi_{s,t-1} \) are set to 0.5 and -0.2, respectively, to induce partial adjustment (Cavoli and Rajan, 2006).

In order to allow a comparison of our results to the ones obtained by GM (2005), the parameters estimated by the authors for productivity and world output shocks are used:

\[
a_t = 0.66a_{t-1} + \epsilon^a_t \quad \sigma_a = 0.0071
\]

\[
y^*_t = 0.86y^*_{t-1} + \epsilon^*_t \quad \sigma_{y^*} = 0.0078
\]

with \( \text{corr}(\epsilon^a_t, \epsilon^*_t) = 0.3 \).

### 2.5 Model Dynamics

#### 2.5.1 Technology Shocks

The impulse responses to a 100 basis point increase in domestic productivity under different policy rules are presented in Figure 2.1. Under all the specified rules, the increase in domestic productivity results in an increase in the natural level of output, which in turn decreases the output gap and domestic inflation. As the world output
and world consumption remain unchanged, the dynamics of domestic output and consumption follow that of terms of trade. The movements in the interest rate depend on the variables that the interest rate reacts to under each simple rule.

Figure 2.1. Impulse Responses to Technology Shocks

Most of the macroeconomic variables have qualitatively similar dynamics under the DITR, DITTR and DITTR(-1). Under the additional policies, the initial decrease in
domestic inflation is magnified, while the initial increase in terms of trade is dampened. This is a consequence of the nature of these policy rules, where the interest rate not only reacts to the movements in the domestic inflation but also to the changes in the terms of trade. Under the CITR, the increase in domestic productivity demands a real depreciation, which results in a rise in CPI inflation. Since stabilization of CPI inflation is required under the CITR, this results in a smaller reaction of the terms of trade and a hump-shaped pattern. Under the PEG, it is not possible to decrease the nominal rate and let the currency depreciate. As a result, compared to the other rules, the increase in consumption, output and the terms of trade are lower, leading to a greater fall in the output gap. In addition, the stationarity of the terms of trade is reflected in the movements of domestic and CPI inflation, as the exchange rate remains constant.

2.5.2 World Output Shocks

Figure 2.2 presents the impulse responses to a 100 basis point increase in world output under different monetary rules. It should be mentioned that, with the special case used by GM (2005), domestic inflation and the interest rate are not affected by the movements in the world output under the DITR. As a result, the dynamics under the alternative policy rules cannot be examined rigorously.

An increase in the world output generates a decline in the terms of trade. As a result, exports of the domestic economy and domestic consumption decline, leading to a decrease in domestic output. On the other hand, the increase in the world output, given the terms of trade, results in an increase in the exports of the domestic economy and domestic consumption. Under the generic calibration, the contractionary effects on domestic output are higher than the contractionary effects on domestic consumption.
The dynamics of domestic inflation under the DITTR and DITTR(-1) are qualitatively different from the dynamics under the DITR. Under the two additional rules, the increase in the world output leads to a smaller decrease in the terms of trade and domestic output, resulting in an increase in domestic inflation. Under the CITR, the increase in world output results in an appreciation, and hence, a decline in CPI inflation. Since CPI inflation should be stabilized under the CITR, this causes a smaller decrease in

Figure 2.2 Impulse Responses to World Output Shocks
the terms of trade and a hump-shaped pattern. As the nominal exchange rate remains unchanged under the PEG, the initial decrease in the terms of trade is the smallest.

2.6 Volatilities and Welfare Losses

Table 2.3 presents the standard deviations of some macrovariables obtained under alternative policy rules, when the economy is affected by domestic and foreign shocks. It can be seen that the volatility of the terms of trade is the lowest under the PEG, as expected. With the use of the special case in GM (2005), the lower volatility of the terms of trade achieved under a policy rule is reflected in the higher volatility of domestic inflation obtained under the same rule.

<table>
<thead>
<tr>
<th></th>
<th>DITR</th>
<th>CITR</th>
<th>PEG</th>
<th>DITTR</th>
<th>DITTR(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>1.11</td>
<td>1.14</td>
<td>1.20</td>
<td>1.15</td>
<td>1.14</td>
</tr>
<tr>
<td>Domestic Inflation</td>
<td>0.26</td>
<td>0.30</td>
<td>0.19</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.32</td>
<td>0.31</td>
<td>0.11</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>0.64</td>
<td>0.61</td>
<td>0.52</td>
<td>0.58</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Note: Entries correspond to standard deviations in percentages.

However, under the general setting, this result does not hold, as the volatility of inflation achieved under the PEG is also quite low. With the special case, while world output shocks have an effect on domestic inflation under the PEG, this effect is not present under the DITR. On the other hand, under the general case, domestic inflation is affected by both shocks under both policy rules. Consequently, when the generic case
is used, the variance of domestic inflation increases under the DITR. However, under the PEG, the lower variance of the terms of trade is reflected in the lower variance of inflation, since the exchange rate remains constant. The volatility of domestic inflation is the lowest under DITTR(-1). Moreover, under the additional rules, the volatility of both the terms of trade and domestic inflation are lower than the volatility obtained under the DITR.

Finally, we present the welfare losses obtained under each policy rule in Table 2.4. These losses are given for three cases, where the economy is affected by: (i) domestic shocks only, (ii) foreign shocks only or (iii) both shocks. As the existence of foreign shocks increases the relative importance of the volatility of the terms of trade, the preferred policy rule for the small open economy changes under the considered scenarios. For the first case, the smallest welfare loss is obtained under the CITR, while it is obtained under the DITTR(-1) for the second and the third case. Hence, the source of the fluctuations in economic activity is a determinant of the preferred policy rule in the small open economy.

<table>
<thead>
<tr>
<th></th>
<th>Productivity Shocks</th>
<th>World Output Shocks</th>
<th>Both Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>DITR</td>
<td>-0.001266</td>
<td>-0.012541</td>
<td>-0.012822</td>
</tr>
<tr>
<td>CITR</td>
<td>-0.001045</td>
<td>-0.012692</td>
<td>-0.013227</td>
</tr>
<tr>
<td>PEG</td>
<td>-0.001111</td>
<td>-0.009958</td>
<td>-0.009861</td>
</tr>
<tr>
<td>DITTR</td>
<td>-0.001443</td>
<td>-0.010006</td>
<td>-0.010248</td>
</tr>
<tr>
<td>DITTR (-1)</td>
<td>-0.001254</td>
<td>-0.009542</td>
<td>-0.009790</td>
</tr>
</tbody>
</table>

Note: Results are given in percentage units of steady state consumption.

In contrast to the results achieved by GM (2005), where the DITR generates the smallest welfare losses in the presence of both shocks, the smallest welfare loss is
achieved under the DITTR(-1). In addition, use of the DITTR and DITTR(-1) results in a lower welfare loss than the use of the DITR. Hence, in the small open economy, interest rate rules that react to the movements in the terms of trade and domestic inflation are preferred to the DITR.

2.6.1 Sensitivity Analysis

As presented in the previous section, the calibration of the coefficient of relative risk aversion ($\sigma$), the elasticity of substitution between home and foreign goods ($\eta$), and the elasticity of substitution between foreign goods from different origins ($\gamma$) is quite important for determining the preferred monetary policy rule in the small open economy. Hence, in this section, we examine how the preferred policy rule changes with different values of these parameters. We use the case where the economy experiences both domestic and foreign shocks. We change the value of one of these parameters, keeping the others equal to 1 (1.1 for $\sigma$) and report the welfare losses obtained under each policy. Table 2.5 presents our results.

First, we analyze the results for different values of $\sigma$. It can be seen that the DITR is the preferred policy rule, for values of $\sigma$ below 5. For higher values, use of the DITTR(-1) generates the lowest welfare losses. In addition, as the value of $\sigma$ increases, use of the PEG results in lower welfare losses than the use of the CITR. As the value of $\sigma$ increases, the coefficients for the variance of the terms of trade and the variance of domestic inflation in the welfare loss function decreases. However, the relative decline in the coefficient of inflation is much higher. As a result, the relative importance of the

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7Since we do not want to restrict ourselves to the case of log-utility obtained under $\sigma = 1$, we keep the value of $\sigma$ very close to, but not equal to 1, as in our generic calibration.
variation in the terms of trade increases and results in a change in the ranking of the policy rules.

Examining the welfare losses obtained for different values of $\eta$, we observe that the DITR is the preferred policy rule when $\eta$ is lower than 4. For the remaining values, the lowest welfare loss is obtained under the DITTR(-1). In addition, use of the PEG results in lower welfare losses than the use of the DITR, for values of $\eta$ above 4. This is as a result of the increase in the coefficient of the terms of trade and the decrease in the coefficient of domestic inflation in the welfare loss function, as the value of $\eta$ rises.

| Table 2.5. Welfare Losses with Different Values of $\sigma$, $\eta$ and $\gamma$ |
|-----------------|---|---|---|---|---|
| $\sigma$       | 2 | 3 | 4 | 5 | 6 |
| DITR           | 0.0221 | 0.0314 | 0.0426 | 0.0551 | 0.0440 |
| CITR           | 0.0304 | 0.0461 | 0.0645 | 0.0829 | 0.0779 |
| PEG            | 0.0349 | 0.0466 | 0.0575 | 0.0716 | 0.0632 |
| DITTR          | 0.0583 | 0.0702 | 0.1070 | 0.0972 | 0.0869 |
| DITTR(-1)      | 0.0299 | 0.0363 | 0.0441 | 0.0528 | 0.0367 |

| $\eta$       | 2 | 3 | 4 | 5 | 6 |
| DITR           | 0.0166 | 0.0161 | 0.0157 | 0.0154 | 0.0153 |
| CITR           | 0.0178 | 0.0172 | 0.0167 | 0.0166 | 0.0160 |
| PEG            | 0.0213 | 0.0184 | 0.0163 | 0.0147 | 0.0139 |
| DITTR          | 0.0357 | 0.0263 | 0.0209 | 0.0179 | 0.0157 |
| DITTR(-1)      | 0.0206 | 0.0169 | 0.0150 | 0.0134 | 0.0129 |

| $\gamma$       | 2 | 3 | 4 | 5 | 6 |
| DITR           | 0.0162 | 0.0155 | 0.0165 | 0.0150 | 0.0140 |
| CITR           | 0.0174 | 0.0166 | 0.0160 | 0.0150 | 0.0143 |
| PEG            | 0.0193 | 0.0161 | 0.0139 | 0.0123 | 0.0113 |
| DITTR          | 0.0279 | 0.0214 | 0.0157 | 0.0136 | 0.0120 |
| DITTR(-1)      | 0.0192 | 0.0142 | 0.0129 | 0.0119 | 0.0111 |
Finally, we examine the results for different values of $\gamma$. DITR is the preferred policy rule only for values of $\gamma$ below 3. For the remaining values, use of the DITTR(-1) generates the lowest welfare losses. In addition, for values of $\gamma$ above 3, the highest welfare losses are obtained under the DITR and the CITR. This result is again due to the increase in the coefficient of the terms of trade and the decrease in the coefficient of domestic inflation in the welfare loss function, as the value of $\gamma$ increases.

To summarize, we observe that the relative importance of the variation in the terms of trade increases, while that of domestic inflation decreases, with an increase in the values of the aforementioned parameters. As a result, the DITTR(-1) becomes the preferred policy rule in the small open economy. Examining the change in the welfare losses obtained with an increase in the value of each parameter, it can be seen that the DITTR(-1) turns into the preferred policy rule with a slight and a moderate increase in $\gamma$ and $\eta$, respectively. For the same result to hold for different values of $\sigma$, a significant increase is required.

2.7 Conclusions

Generalizing the special setting used in GM’s (2005) framework, we explore the optimal monetary policy in the New Keynesian small open economy. With the special case, it is possible to determine the subsidy which will exactly balance the combined impacts of the market power and the terms of trade rigidities, and guarantee the optimality of the flexible price equilibrium allocation. Consequently, as in the closed economy, the optimal monetary policy demands stabilization of the output gap, which then requires domestic prices to be stabilized under the optimal policy. Under the general case in the
open economy, we show that it is not possible to determine this subsidy. As a result, the presence of a monopolistic distortion and a terms of trade externality pushes optimal policy away from domestic inflation targeting. Deriving the welfare loss function in the open economy, we also find that there are important differences between the welfare losses obtained under the general and the special case. Under the general setting, welfare losses depend on the variance of the terms of trade, consumption and inflation. Hence, monetary policy for the open economy is not isomorphic to the one for the closed economy.

Analyzing the equilibrium properties of various macroeconomic variables under different policy rules, we find that the main difference between our results and those obtained with the special case is regarding the volatility of inflation achieved under the PEG and under the inflation based Taylor rule (DITR). With the special case, while the shock to world output has an effect on domestic inflation under the PEG, this effect is not present under the DITR. As a result, when the special case is used, the variance of domestic inflation is only due to the shock to domestic productivity under the DITR. On the other hand, under the general case, domestic inflation is influenced by both shocks under both simple rules. Consequently, when the general case is used, the variance of domestic inflation increases because of the shock to world output under the DITR. In contrast, the lower variance in the terms of trade results in a lower variance in inflation under the PEG, as the exchange rate remains constant. Hence, the volatility of inflation obtained under the PEG is lower than the volatility obtained under the DITR, in contrast to the special case. Adding two alternative domestic inflation and terms of trade based Taylor rules (DITTR and DITTR(-1)) to our analysis, we also observe that the lowest volatility of inflation is achieved under the DITTR(-1).
Finally, computing the welfare losses obtained under each policy rule in the open economy, we find that the welfare levels associated with different monetary policy rules depend on the exogenous shock affecting the small open economy. When both domestic and foreign shocks are present, the use of domestic inflation and terms of trade based Taylor rules are preferred to the use of the DITR. Moreover, the lowest welfare losses are obtained under the DITTR(-1).
Chapter 3

A Comparative Analysis of Macroprudential Policies

3.1 Introduction

The global financial crisis has recast the literature on macroeconomic models and policies. The mainstream dynamic stochastic general equilibrium (DSGE) models used by macroeconomists before the crisis did not contain the role of financial frictions in generating or propagating business cycle fluctuations. Subsequent to the 2007-09 episode, there has been a growing consensus among macroeconomists about the necessity to incorporate financial frictions into macroeconomic models and to examine the significance of financial shocks. Two relevant strands of the literature, based on DSGE models that attempt to overcome these shortcomings, have emerged. The first analyzes monetary policy using models that include financial frictions associated with the constraints of non-financial borrowers, making use of the financial accelerator mechanism designed by Bernanke et al (1999). The second studies financial frictions linked to financial in-
termediaries and analyzes the function of bank capital in the monetary transmission mechanism. The framework developed by Gertler and Karadi (2011) is one of the leading examples.

The recent financial crisis has also shown that a single policy objective, inflation stability, to be achieved with the use of a single policy instrument, the interest rate, is not sufficient to guarantee the stability of the financial system. As stated by the well-known "Tinbergen principle", the number of independent instruments should at least be equal to the number of policy objectives. Hence, following the recent experience, the financial accelerator mechanism has been increasingly employed in macroeconomic studies that include supplementary policy instruments and a common finding emerges from these: to reduce systemic risks and ensure the stability of the financial system, the main monetary policy instrument needs to be supported by additional tools, which are referred to as macroprudential policy instruments (Bank for International Settlements, 2010).

There are, by now, relatively well-defined proposals for macroprudential policy tools. Counter-cyclical capital requirements introduced by the Basel III reform package is one of the prominent examples. A large number of countries, including the U.S. and the EU, has completed the adoption of this instrument in the past few years\(^1\). Another well-known example of macroprudential tools is the counter-cyclical reserve requirements. In recent years, they have been increasingly used by central banks with the purpose of accommodating credit in booms and relaxing liquidity constraints in contractions. Among others, China, Brazil, Malaysia, Peru, Columbia and Turkey are some of the

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\(^1\)Progress Report on the Implementation of the Basel Regulatory Framework (as of October 2014). The other countries are: Australia, Brazil, Canada, China, Hong Kong, India, Japan, Korea, Mexico, Singapore, South Africa, Switzerland and Turkey.
countries that have been using reserve requirements for this purpose (Montoro and Moreno, 2011).

In this paper, using a New Keynesian general equilibrium model that incorporates a banking sector, we compare the effectiveness of three macroprudential policies and their interaction with monetary policy. In our study, we include the aforementioned widely-used macroprudential policies; reserve requirements and capital requirements, and a third macroprudential policy tool, a regulation premium, whose formulation is based on the assumption that macroprudential policies in general increase the costs of financial intermediaries, who in turn pass these costs onto borrowers (Unsal, 2013). Our motivation is threefold. First, we complement the studies that analyze the use of reserve requirements as a macroprudential policy tool. To examine the effectiveness of reserve requirements, Glocke and Towbin (2012) use a small open economy model with financial frictions, while Mimir et al (2013) use a model that includes financial frictions in the banking sector, but does not incorporate a monetary policy rule. In both studies, welfare losses in the presence of reserve requirements are computed using ad-hoc loss functions, whereas we use welfare maximizing monetary and macroprudential policies in our analysis. Second, we also contribute to the literature on bank capital and capital requirements. Even though there are various studies that analyze the use capital requirements employing models with a banking sector\(^2\), most of them lack a rigorous welfare analysis. The two studies where optimized monetary and macroprudential policy rules are used as in our analysis are by Angeloni and Faia (2013) and by Christensen et al (2011). However, the modeling of the banking sector in these studies is different from ours. Last but not least, our paper is the first to present a detailed comparative

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analysis of the given alternative macroprudential policies, in contrast to much of the existing literature where the implications of the use of a single macroprudential policy are analyzed.

To conduct our analysis, we build a monetary DSGE model in which the frictions in the financial intermediation process are as described in Gertler and Karadi (2011). The financial accelerator mechanism built in banks’ balance sheet constraints features a pecuniary externality, where bankers do not consider the fact that if they issued more equity, they would decrease the risk of the banking sector. Hence, they accumulate high levels of leverage, which amplifies the negative effects of exogenous shocks to the economy. In other words, bankers’ inability to internalize the benefits of equity financing results in a decline in welfare and induces the need for macroprudential regulation (Gertler et al, 2012). In our framework, reserve requirements and capital requirements both increase the cost of deposits to banks, encouraging them to replace external financing by equity financing. An increase in the regulation premium is reflected in the increase in cost of borrowing to firms. In accordance with the literature, the macroprudential policy tools in our model are assumed to be counter-cyclical. To establish comparability, all three instruments respond to the same financial variable, which is the total nominal credit growth in the economy, with the same intensity.

Our simulation results indicate that when the economy experiences a productivity (TFP) or a financial shock, the use of all the aforementioned macroprudential tools mitigates the negative effects of the given shocks to the economy. Each shock results in a decrease in asset prices, which triggers the financial accelerator mechanism. Since banks are leveraged, the decrease in asset prices results in an amplified decline in their net worth and a downturn in their balance sheets that increases their leverage ratios.
The rise in the leverage ratios increases the spread, which is defined as the difference between the gross return to risky loans and the gross riskless return. The increase in the spread, in turn, results in an increase in the cost of capital, which causes a further decline in investment and asset prices. Finally, the decline in investment leads to a decrease in aggregate output. When counter-cyclical reserve requirements or capital requirements are used in combination with monetary policy, the decrease in banks’ net worth and hence their leverage ratios is smaller, and so is the increase in the spread. As a result, the negative effects of the shocks on asset prices, investment and output are lower. Counter-cyclical use of the regulation premium, on the other hand, directly results in a smaller increase in the cost of capital. Consequently, the negative effects of the shocks on aggregate output are again lowered.

Comparing the dynamics of both shocks under alternative macroprudential policies, we find that irrespective of the cause of the decline in economy activity, capital requirements perform the best in lowering the negative effects of the given shocks to the spread, asset prices and investment. As a result, they are the most effective macroprudential tool in mitigating the negative effects of the financial accelerator mechanism built in banks’ endogenous capital constraints.

Using welfare-maximizing monetary and macroprudential policy rules, we also compute welfare losses and consumption equivalents under each policy alternative. Analyzing productivity and financial shocks separately, we observe that under both shocks, the adoption of each macroprudential policy results in a decrease in the welfare loss. The least effective macroprudential policy tool is the regulation premium under the TFP shock, while it is the reserve requirements under the financial shock. The most effective macroprudential tool is, however, the same under both shocks; capital require-
ments generate the highest positive effect on welfare, regardless of the type of the shock affecting the economy.

The remainder of this paper proceeds as follows. Section 3.2 sets out the structure of our model by giving a detailed description of the economic agents, the monetary policy and the macroprudential policies. Section 3.3 presents our quantitative results, including the discussion of impulse responses, the analysis of macroprudential tools’ impact on volatilities and the computation of welfare losses. Finally, Section 3.4 concludes.

3.2 The Model

Our framework is a monetary DSGE model with nominal rigidities. It contains a banking sector that is characterized by credit frictions à la Gertler and Karadi (2011). The model economy is populated by households, banks, capital goods producers, wholesale firms, retail firms, the fiscal authority and the central bank. We now proceed with a detailed description of the economic agents in the economy.

3.2.1 Households

The population consists of a continuum of identical households. Within the household, there are 1-p "workers" and p "bankers" who perfectly insure each other. Workers supply labour and earn wages while bankers manage financial intermediaries, i.e., banks and transfer dividends back to households. Households deposit their savings in the banks. Deposits are assumed to be riskless one period securities.

A representative household maximizes expected discounted utility which is a function of consumption, $C_t$, $C_{t-1}$ and leisure, $L_t$, $C_t$, $C_{t-1}$ and leisure, $L_t$. $C_t$, $C_{t-1}$ and leisure, $L_t$. $C_t$.
subject to the following flow of funds constraint,

\[ C_t = W_t h_t + \Pi_t - T_t + R_t D_{t-1} - D_t \] (3.2)

where 0 < \beta < 1 is the subjective discount factor and E is the expectation operator. \( W_t \) is the wage rate, \( h_t (= 1 - L_t) \) denotes hours worked, \( D_t \) bank deposits and \( R_t \) the gross risk free deposits rate, set in period \( t - 1 \) to pay out interest in period \( t \). \( T_t \) is the lump sum taxes remitted by the government and \( \Pi_t \) is the profits earned from the ownership of banks and firms.

Solution of the utility maximization problem of households gives the following optimality conditions,

\[ U_{C,t} = \beta R_{t+1} E_t [U_{C,t+1}] \] (3.3)

\[ \frac{U_{h,t}}{U_{C,t}} = - \frac{U_{L,t}}{U_{C,t}} = -W_t \] (3.4)

where \( U_t = \frac{(C_t-x_{t-1})^{(1-\phi)(1-\sigma)}(1-h_t)^{\phi(1-\sigma)} - 1}{1-\sigma} \).

As in Schmitt-Grohe and Uribe (2007), we include habit formation and investment adjustment costs in our model, since empirical work has demonstrated that such real frictions improve the ability of macroeconomic models to explain U.S. business cycles. The given form of the utility function is also adopted from Schmitt-Grohe and Uribe.
(2007). Various other studies show that non-separable preferences over consumption and leisure explain the aggregate consumption data well\(^3\).

Equation (3.3) describes the optimal consumption-savings decision. Accordingly, the marginal utility from consuming one unit of income in period \(t\) is equal to the discounted marginal utility from consuming the gross income obtained by saving.

Taking expectations on both sides and defining \(\Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}}\) as the real stochastic discount factor over the interval \([t, t+1]\), we obtain the consumption Euler equation,

\[
1 = R_{t+1} E_t [\Lambda_{t,t+1}] \tag{3.5}
\]

Equation (3.4) shows that the marginal rate of substitution between consumption and leisure should be equal to the real wage.

### 3.2.2 Banks

The modelling of the financial sector closely follows that in Gertler and Karadi (2011). A representative bank supplies credit to wholesale firms and finances this activity by borrowing from households and using its own net worth. As a result, the bank’s balance sheet has the following form,

\[
Q_t s_t = n_t + d_t \tag{3.6}
\]

where \(s_t\) denotes loans to non-financial firms, \(Q_t\) their price, \(n_t\) net worth and \(d_t\) household deposits.

\(^3\)See Kilponen (2009) for a survey of these studies.
The balance sheet of the bank implies an accumulation of net worth according to

\[ n_t = R_{k,t}Q_{t-1}s_{t-1} - R_td_{t-1} \]  \hspace{1cm} (3.7)\]

To eliminate the case where bankers can accumulate sufficient net worth that makes their financial constraints not binding, we assume that with probability 1-\(\gamma\), a banker exits and becomes a worker. The bank pays dividends only when it exists. In addition, we assume that \((1-\gamma)p\) workers randomly become bankers so that the number of both professions stays constant.

Given the fact that the bank only pays dividends when it exists, the banker’s objective at the end of period \(t\) is to maximize expected discounted terminal net worth, given by

\[ V_t = E_t \sum_{i=1}^{\infty} (1 - \gamma)^{i-1} \Lambda_{t,t+i} n_{t+i} \]  \hspace{1cm} (3.8)\]

Substituting for \(d_t\) from Equation (3.6) in Equation (3.7) gives another form of net worth accumulation,

\[ n_t = R_t n_{t-1} + (R_{k,t} - R_t)Q_{t-1}s_{t-1} \]  \hspace{1cm} (3.9)\]

Since the returns and \(Q_t\) are exogenous to the bank, given \(n_{t-1}\) at the beginning of period \(t\), net worth in period \(t\) is given by the choice of \(\{s_{t+i}\}\) subject to the bank’s borrowing constraint.

The financial friction in the banking sector is based on a moral hazard problem between the banks and the households. After a bank obtains funds, the banker’s manager may transfer a fraction, \(\Theta\) of total assets, \(Q_t s_t\) for her own benefit. In this case, the
bank defaults on its debt, shuts down and the creditors can reclaim the remaining 1-\(\Theta\) fraction of funds. As households know this possibility, they limit the funds (deposits) that they lend to banks. As a result, the bankers choice of \(s_t\) at any time \(t\) is subject to the following incentive constraint,

\[ V_t \geq \Theta Q_t s_t \]

To solve the bank’s optimization problem, we start by guessing that the solution has the following form,

\[ V_t = V_t(s_t, d_t) = v_{s,t} s_t - v_{d,t} d_t \]

(3.10)

where \(v_{s,t}\) and \(v_{d,t}\) are time-varying marginal values of the assets at the end of each period. By eliminating \(d_t\) from Equation (3.10) using Equation (3.6), we obtain

\[ V_t = V_t(s_t, n_t) = \mu_{s,t} Q_t s_t + v_{d,t} n_t \]

(3.11)

and \(\mu_{s,t} = \frac{v_{s,t}}{Q_t} - v_{d,t}\) is the excess value of the bank’s assets over its deposits.

Defining \(\phi_t\) as the leverage ratio that satisfies the binding incentive constraint, we have

\[ Q_t s_t = \phi_t n_t \]

(3.12)

where \(\phi_t = \frac{v_{d,t}}{\Theta - \mu_{s,t}}\).

Using the solution to the banker’s optimization problem, we can determine \(v_{s,t}\) and \(v_{d,t}\) as
\[ v_{s,t} = E_t \Lambda_{t,t+1} \eta_{t+1} R_{k,t+1} Q_t \]

\[ v_{d,t} = E_t \Lambda_{t,t+1} \eta_{t+1} R_{t+1} \]

As a result, we also have

\[ \mu_{s,t} = E_t \Lambda_{t,t+1} \eta_{t+1} (R_{k,t+1} - R_{t+1}) \]

where \( \eta_t = (1 - \gamma) + \gamma (\mu_{s,t} \phi_t + v_{d,t}) \) gives the shadow value of a unit of net worth. \( \eta_t \) is a weighted average of the marginal values for exiting and surviving banks. If a surviving bank has an additional net worth, it can save the cost of deposits, \( v_{d,t} \) and increase assets with an excess value of \( \mu_{s,t} \) by \( \phi_t \).

The difference between the gross return to risky loans, \( R_{k,t} \) and the gross riskless return, \( R_t \) is defined as the spread, which is a distortion generated by the frictions in the financial sector.

Since \( \phi_t \) is not dependent on bank specific factors, we can aggregate Equation (3.12) across individual banks to obtain the banking sector balance sheet at the aggregate level,

\[ Q_t S_t = \phi_t N_t \quad (3.13) \]

The evolution of net worth at the aggregate level depends on the net worth of surviving bankers \( (N_{o,t}) \) and that of new entrants \( (N_{e,t}) \). Net worth of surviving bankers is given by the earnings on their assets from the previous period minus the cost of deposits, multiplied by the probability that they will survive \( (\gamma) \).

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\[ N_{o,t} = \gamma (R_{k,t} Q_{t-1} S_{t-1} - R_t D_{t-1}) \]

And net worth of the new bankers is obtained with the assumption that the fraction \( \frac{\varepsilon}{1-\gamma} \) of the total value of the exiting bankers’ assets are transferred to new entrants,

\[ N_{e,t} = \varepsilon (R_{k,t} Q_{t-1} S_{t-1}) \]

where \( \varepsilon \) denotes the proportional transfer to the new bankers. As a result, the evolution of net worth at the aggregate level is given by

\[ N_t = R_{k,t} (\gamma + \varepsilon) Q_{t-1} S_{t-1} - \gamma R_t D_{t-1} \]

### 3.2.3 Wholesale Firms

Wholesale firms combine capital acquired from capital producers and labour supplied by households to produce the wholesale output, \( Y^W_t \) using the production function

\[ Y^W_t = Y^W_t (A_t, h_t, K_{t-1}) = (A_t h_t)^\alpha K_{t-1}^{1-\alpha} = A_t h_t \left( \frac{K_{t-1}}{Y^W_t} \right)^{\frac{1-\alpha}{\alpha}} \]

Here, it should be noted that \( K_t \) is the end-of-period \( t \) capital stock and \( A_t \) denotes factor productivity. Cost minimization by wholesale firms gives the following labour demand function,

\[ \frac{P^W_t}{P_t} Y^W_{h,t} = W_t \]

Equation (3.18) shows that the marginal product of labour, \( Y^W_{h,t} \) is equal to the real
wage. Here $P^W_t$ and $P_t$ are the aggregate price indices in the wholesale and retail sectors, respectively.

To finance its capital purchase each period, the firm obtains funds from banks. The number of claims issued by the firm, $S_t$, is equal to the number of units of capital needed, $K_t$ and hence the price of each claim is also equal to the price of each unit of capital,

$$Q_t S_t = Q_t K_t$$ (3.19)

In obtaining funds from a bank, the wholesale firm does not face any additional financial frictions. However, the credit frictions between the households and the banks have an effect on the amount of funds available to wholesale firms. Because of perfect competition, wholesale firms earn zero profits and hence they completely pay the return on their capital,

$$R_{k,t+1} = \frac{(1 - \alpha) P^W_{t+1} Y^W_{t+1}}{P_{t+1} K_t} + (1 - \delta) Q_{t+1}$$ (3.20)

where $\delta$ is the depreciation rate of capital. It can be seen that the demand for capital, $K_t$ increases with a rise in the production or the price of the wholesale good and decreases with an increase in the cost of capital, $R_{k,t+1}$.

3.2.4 Capital Producing Firms

Incorporation of capital producers enables us to explore the changes in the price of capital and to introduce the capital quality shock, which is the exogenous shock that initiates the financial crisis in our model. We assume that at time $t$, $I_t$ of raw output is
converted into \((1 - S(X_t)) I_t\) of new capital. Here \(S(X_t)\) denotes the investment costs. As a result, capital accumulates according to

\[
K_t = [(1 - \delta) K_{t-1} + (1 - S(X_t)) I_t]
\]  

(3.21)

where \(X_t = \frac{I_t}{I_{t-1}}\). We assume that investment costs have the following form,

\[
S(X_t) = \phi_X X_t^2
\]

Accordingly, capital producing firms maximize expected discounted profits with respect to \(\{I_t\}\),

\[
E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} [Q_{t+k} (1 - S(X_{t+k})) I_{t+k} - I_{t+k}]
\]

(3.22)

The optimality condition that we achieve as a result of this maximization problem is given by

\[
Q_t (1 - S(X_t) - X_t S'(X_t)) + E_t [\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2] = 1
\]

(3.23)

which indicates a positive relationship between investment and asset prices.

### 3.2.5 Retail Firms

We introduce two New Keynesian elements; price stickiness and monopolistic competition into our framework through retail firms. The retail sector uses the homogenous wholesale output to produce a basket of differentiated goods for consumption. Consumers’ expenditure minimization gives the following output demand equation for each
retail firm,

\[ Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} Y_t \]  

(3.24)

where \( \zeta \) is the elasticity of substitution and the aggregate price index, \( P_t \) is given by

\[ P_t = \left( \int_0^1 P_t(f)^{1-\zeta} df \right)^{1/(1-\zeta)}. \]

We include price stickiness in the retail sector with the assumption that firms set their prices à la Calvo (1983). The optimal price-setting behaviour for each firm setting its price in period \( t \) is obtained by the maximization of the retailer’s discounted nominal profits,

\[ E_t \sum_{k=0}^{\infty} \theta^k \gamma_{t,t+k} Y_{t+k}(f) [P_t^a(f) - P_{t+k} MC_{t+k}] \]  

(3.25)

subject to Equation (3.24). Here, \( MC_t \) denotes the real marginal cost, \( P_t^a(f) \) the adjusted price and \( \gamma_{t,t+k} = \beta^k \frac{U_{C,t+k}/P_{t+k}}{U_{C,t}/P_t} \) the nominal stochastic discount factor over the period \( [t, t+k] \). As described in Calvo (1983), \( \theta \) is the probability that a firm cannot adjust its price in a given period, independent from the time passed since the last adjustment.

Under the given price-setting mechanism, the evolution of the price index is given by the weighted average of the previous price level and the newly adjusted price\(^4\),

\[ P_{t+1}^{1-\zeta} = \theta P_t^{1-\zeta} + (1 - \theta) \left( P_{t+1}^a \right)^{1-\zeta} \]  

(3.26)

Combining the solution to the retail firm’s optimization problem and the evolution

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\(^4\)As the costs and the demand equations faced by each retail firm is the same, all the firms adjusting their prices choose the same price. As a result, \( P_t^a(f) = P_t^a \).
of the price index, we can obtain the New Keynesian Phillips Curve (NKPC). The NKPC indicates that inflation in the current period rises as expected inflation or the marginal cost of the retail firm increases.\(^5\)

### 3.2.6 Monetary Policy

The monetary policy instrument is the gross nominal interest rate, \(R_{n,t}\) set in period \(t\) to pay out interest in period \(t + 1\). The relationship between the nominal and real interest rate is given by the following Fisher equation,

\[
R_{n,t-1} = R_t E_t \Pi_t
\]  
(3.27)

We suppose monetary policy is conducted using a simple Taylor rule given by,

\[
\log \left( \frac{R_{n,t}}{R_n} \right) = \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right)
\]  
(3.28)

where \(R_n\) denotes the steady state nominal rate, \(\Pi\) the steady state inflation and \(Y\) the steady state level of output.

### 3.2.7 Macroprudential Policies

In our model we study the implications of using three different macroprudential policies; reserve requirements, capital requirements and a regulation premium. Each policy is characterized by a macroprudential policy rule.

\(^5\)The set of equations that represent the NKPC are included in Appendix B.1, where the model equations that describe the competitive equilibrium are presented.
According to reserve requirements, banks need to hold a portion of their deposits at the central bank, which generally earns zero interest. Hence, such requirements can be regarded as a tax that increases the cost of extending credit. If banks did not need to hold non-interest-bearing reserves, they would probably use the extra funds to supply more loans. This would, in turn, increase their interest income and improve their profitability, as it would result in a larger asset base for them to earn their spread (Hein and Stewart, 2002). As a result, an increase in the central bank’s level of reserve requirements can be considered to increase the return to deposits.

The holdings of reserves by banks beyond the required level are called excess reserves. Before the global financial crisis, reserves held with the Fed did not earn any interest so banks had an inclination to minimize their holdings of excess reserves. In 2007, the excess reserves held by U.S. banks were only about 0.3% above the requirement (Keister and McAndrews, 2009). Since there are no gains from holding excess reserves, it can be assumed that the cost of deposits to banks varies only with the level of the required reserves imposed by the central bank. The change in the cost of deposits, in turn, affects the marginal values of a bank’s assets, and hence, the leverage of the financial sector (Areosa and Coelho, 2013).

Accordingly, we assume that when the central bank demands banks to hold a required ratio \( rr_t \) of their deposits as non-interest-earning reserves, the rise in the cost of deposits is reflected as a change from \( R_t \) to \( \frac{R_t - rr_t}{1 - rr_t} \). As a result, the new accumulation of bank net worth will be given by
\[ n_t = R_{k,t}Q_t s_{t-1} - \left( \frac{R_t - rr_t}{1 - rr_t} \right) d_{t-1} \] (3.29)

It can be seen that when there is an increase in reserve requirements, the return to deposits increases. Hence, banks are encouraged to substitute internal financing \((n_t)\) for external financing \((d_t)\). To reflect the changes in the bank’s maximization problem as a result of the introduction of reserve requirements, we replace \(R_t\), the gross return to deposits with \(\frac{R_t - rr_t}{1 - rr_t}\).

Consequently, in the presence of reserve requirements, the marginal value of the bank’s loans is given by

\[ v_{s,t}^{rr} = E_t \Lambda_{t,t+1} \eta_{t+1}^{rr} R_{k,t+1} Q_t \]

whereas the marginal value of the bank’s deposits and the excess marginal value of the bank’s loans over its deposits are now represented by

\[ v_{d,t}^{rr} = E_t \Lambda_{t,t+1} \eta_{t+1}^{rr} \left[ \frac{R_{t+1} - rr_t}{1 - rr_t} \right] \]

\[ \mu_{s,t}^{rr} = E_t \Lambda_{t,t+1} \eta_{t+1}^{rr} \left[ R_{k,t+1} - \frac{R_{t+1} - rr_t}{1 - rr_t} \right] \]

\( \phi_t^{rr} \) denotes the leverage ratio in the presence of reserve requirements and \( \eta_t^{rr} \), the shadow value of a unit of net worth, is now equal to \( (1 - \gamma) + \gamma (\mu_{s,t}^{rr} \phi_t^{rr} + v_{d,t}^{rr}) \).

Moreover, the evolution of net worth at the aggregate level changes to

\[ N_t = R_{k,t} (\gamma + \varepsilon) Q_t s_{t-1} - \gamma \left[ \frac{R_t - rr_t}{1 - rr_t} \right] D_{t-1} \] (3.30)
We assume that the required reserves ratio follows a rule that reacts to the deviations of the total nominal credit from its steady state value,

\[ rr_t - rr = \rho_{rr} \left( \frac{Q_t S_t - QS}{QS} \right) \] (3.31)

Here, variables without any time subscript denote steady state values and we assume that \( \rho_{rr} > 0 \). Consequently, when the total nominal credit in the economy is increasing, the central bank demands banks to hold higher reserves, which increases the return to deposits and encourages banks to prefer equity financing. Hence, reserve requirements are counter-cyclical. The macroprudential tools in our study respond to the fluctuations in the total nominal credit, since stabilizing the total credit is expected to reduce the deviations in the spread. As the spread is an inter-temporal distortion created by financial frictions, the welfare level is expected to be higher when macroprudential policy rules are used by the central bank.

**Capital Requirements (CR)**

Different from reserve requirements, macroprudential policy in the form of counter-cyclical capital requirements focuses on the size of a bank’s balance sheet instead of the composition of its assets. Capital requirements deal with the leverage of banks, while reserve requirements address liquidity risk. When a bank’s capital ratio is below the capital requirement, the macroprudential authority will enforce corrective measures which can cause serious reputational costs and adverse market reactions. Hence, falling below the capital requirement is extremely costly for a bank. Since capital requirements reduce the ability of banks to supply credit by accepting deposits and limit the percentage of bank assets that can be financed by issuing deposits, the increase in the bank’s
funding cost in the presence of capital requirements can be regarded as an increase in the cost of deposits (Borio and Zhu, 2012).

As reported in the study by Van den Heuvel (2008), capital adequacy ratios are important determinants of the capital structure of U.S. banks. Majority of U.S. banks hold some buffer of equity above the regulatory minimum since they would like to lower the risk of a negative shock resulting in capital inadequacy. Most bank assets are in U.S. banks with a ratio of at least 3% above the capital requirement. As a result, even though both reserve requirements and capital requirements increase the costs to banks, the way they do so is modelled differently. The cost of capital requirements is given by the first-order derivative of a quadratic function of deviations from the required capital/assets ratio. Positive (negative) deviations decrease (increase) the cost of deposits and larger deviations result in higher changes in the bank’s cost. In this case, the banker would like to issue as many loans as possible, increasing leverage and thus profits, with the knowledge of the fact that when leverage increases, the capital/assets ratio can fall below the requirement and the bank pays a cost. Consequently, when capital requirements are in place, the banker will choose the bank’s optimal capital/assets ratio in line with the profit maximization, while the quantity of reserves is determined essentially by the central bank’s decisions.

In line with this interpretation, we formulate the return to deposits in the presence of capital requirements as in Brzoza-Brzezina et al (2013). In this case, the accumulation of bank’s net worth is given by,

\[ n_t = R_{k,t}Q_{t-1}s_{t-1} - \left[ R_t - \left( \frac{1}{\phi_t^{cr}} - cr_t \right) \left( \frac{1}{\phi_t^{cr}} \right)^2 \right] d_{t-1} \]  \hspace{1cm} (3.32)
where $\frac{1}{\phi_t^{cr}}$ is the inverse of the leverage ratio; the ratio of bank’s equity to its loans in the presence of capital requirements. As a result, if a bank’s capital/assets ratio is lower than the required ratio, it needs to pay a higher return to deposits, which induces the bank to substitute internal financing for external financing.

Incorporating capital requirements in the bank’s profit maximization problem is straightforward. This can be done by replacing the gross return to deposits by the new gross return given in Equation (3.32). Accordingly, in the presence of capital requirements, the marginal value of the bank’s loans and deposits are represented by,

$$v_{s,t}^{cr} = E_t \Lambda_{t,t+1} \eta_{k,t+1}^{cr} R_{k,t+1} Q_t$$

$$v_{d,t}^{cr} = E_t \Lambda_{t,t+1} \eta_{k,t+1}^{cr} \left[ R_{t+1} - \left( \frac{1}{\phi_t^{cr}} - cr_t \right) \left( \frac{1}{\phi_t^{cr}} \right)^2 \right]$$

while the excess marginal value of the bank’s loans over its deposits are now given by

$$\mu_{s,t}^{cr} = E_t \Lambda_{t,t+1} \eta_{k,t+1}^{cr} \left[ R_{k,t+1} - R_{t+1} + \left( \frac{1}{\phi_t^{cr}} - cr_t \right) \left( \frac{1}{\phi_t^{cr}} \right)^2 \right]$$

The shadow value of a unit of net worth in the presence of capital requirements is obtained as $\eta_t^{cr} = (1 - \gamma) + \gamma (\mu_{s,t}^{cr} \phi_t^{cr} + v_{d,t}^{cr})$.

In addition, the evolution of net worth at the aggregate level changes to

$$N_t = R_{k,t} (\gamma + \varepsilon) Q_{t-1} S_{t-1} - \gamma \left[ R_t - \left( \frac{1}{\phi_t^{cr}} - cr_t \right) \left( \frac{1}{\phi_t^{cr}} \right)^2 \right] D_{t-1} \quad (3.33)$$

The capital adequacy ratio also follows a rule that reacts to the deviations of total nominal credit from its steady state value,
\[ cr_t - cr = \rho_{cr} \left( \frac{Q_t S_t - QS}{QS} \right) \]  

(3.34)

where \( QS \) is the steady state level of nominal credit and \( cr \) is the steady state level of the capital adequacy ratio. Again, the counter-cyclical nature of capital requirements implies that \( \rho_{cr} > 0 \).

### Regulation Premium (RP)

Finally, we turn to a more general representation of macroprudential policy. If banks were competitive in the deposit market but they had market power in the loan market, the marginal cost of deposits would be fixed, while the demand schedule and the marginal revenue for loans would be downward sloping. In this case an increase in the cost of deposits would shift the marginal cost curve up. As a result, at the equilibrium, the interest rate on loans would be higher and the level of credit would be lower. The increase in the lending rates induced by macroprudential policies is called the "regulation premium" (Unsal, 2013). The regulation premium can be interpreted as a tax that increases the cost of borrowers. In the presence of reserve or capital requirements, the costs relating to macroprudential policies are incurred by banks, while in the presence of the regulation premium, these costs are incurred by borrowing firms.

Accordingly, the spread in the economy is now given by

\[
\text{spread} = \frac{R_{k,t} - R_t}{1 + rp_t} \]

(3.35)

where \( rp_t \) is the regulation premium. To be able to make a comparative analysis of the three macroprudential policies, \( rp_t \) also reacts to the deviations of total nominal
credit from its steady state value, in line with $rr_t$ and $cr_t$,

$$r_{pt} - r_p = \rho_{rp} \left( \frac{Q_t S_t - QS}{QS} \right)$$  \hspace{1cm} (3.36)

where we assume that $\rho_{rp} > 0$. As a result, when the total nominal credit in the economy is lower, the cost of borrowing to firms decreases. Hence, the regulation premium is also counter-cyclical.

### 3.2.8 Government Budget Constraint

We assume that government expenditures, $G_t$, are financed by lump-sum taxes, $T_t^6$.

$$G_t = T_t$$  \hspace{1cm} (3.37)

### 3.2.9 Exogenous Processes

We suppose that the model economy is affected by two exogenous processes, which are total factor productivity (TFP) and capital quality shocks. Both shocks are supposed to follow an AR(1) process,

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \varepsilon_A$$

$$\log \psi_t = \rho_\psi(\log \psi_{t-1}) + \varepsilon_\psi$$

\(^6\text{We also maintain that the proceeds from the use of macroprudential policies are lumped into } T_t.\)

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By incorporating the capital quality shock into the model, we can conduct a financial crisis experiment. Accordingly, the capital accumulation process (3.21) is now given by

\[ K_t = \psi_{t+1} [(1 - \delta) K_{t-1} + (1 - S(X_t)) I_t] \] (3.38)

resulting in the following gross return to capital,

\[ R_{k,t} = \psi_t \frac{(1 - \alpha) \frac{P^W Y^W}{R} + (1 - \delta) Q_t}{Q_{t-1}} \] (3.39)

\[ S_t = [(1 - \delta) K_{t-1} + (1 - S(X_t)) I_t] \] now gives the capital in process which is by (3.38) transformed into capital for next period’s production according to \( K_t = \psi_{t+1} S_t \).

As a result, the capital quality shock causes a wedge between capital and the capital in process, where the evolution of capital in process is given by

\[ S_t = [(1 - \delta) \psi_t S_{t-1} + (1 - S(X_t)) I_t] \] (3.40)

Capital quality shocks in New Keynesian models without any financial sectors only have an effect on the accumulation of and the return to capital. With a banking sector in place, they also have an effect on the evolution of bank’s net worth. A negative capital quality shock reduces the net worth, which results in the tightening of the budget constraint. Accordingly, Equation (3.16) can now be rewritten as

\[ N_t = R_{k,t} \gamma + \varepsilon \psi_t Q_{t-1} S_{t-1} - \gamma R_t D_{t-1} \] (3.41)
3.3 Quantitative Analysis

3.3.1 Calibration

The parameters used in the calibration of our model are given in Table 3.1. We start by calibrating the non-financial parameters. As in Gertler and Karadi (2011), for the labour share $\alpha$, the elasticity of substitution between goods $\zeta$, and the government expenditure share, we choose conventional values. The steady state depreciation rate $\delta$, the habit parameter $\chi$, and the price rigidity parameter $\theta$ are also set in line with the values used by Gertler and Karadi (2011). The parameters that are specific to our model are $\sigma$ in the utility function and $\phi_X$ in the investment cost function. The chosen values for these parameters roughly reflect the empirical literature. For calibrating the discount factor $\beta$, and the preference parameter $\varphi$, we use typical U.S. observations of 0.35 for hours worked and 1.01 for the gross interest rate.

For calibrating the financial parameters, we again follow values similar to those used by Gertler and Karadi (2011). We choose the value of $\gamma$ so that the bankers survive 10 years on average. The values of $\varepsilon$ and $\Theta$ are calibrated so that we will have an economy wide leverage ratio of 4, which will roughly capture the aggregate data and an average credit spread of 100 basis points per year, which is based on pre-2007 spreads between BAA corporate versus government bonds. Using conventional values, we set the persistence parameter for the total factor productivity and the capital quality shock, $\rho_A$ and $\rho_\psi$, as 0.95 and 0.75, respectively. Finally, the coefficients of the Taylor rule and the macroprudential policy rules are also presented in Table 3.1. To make our three macroprudential experiments comparable, we assume that the coefficient of the macroprudential policy rule under each macroprudential instrument is the same. At
<table>
<thead>
<tr>
<th>Table 3.1. Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$\chi$</td>
</tr>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$\vartheta$</td>
</tr>
<tr>
<td><strong>Wholesale Firms</strong></td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td><strong>Capital Producing Firms</strong></td>
</tr>
<tr>
<td>$\phi_X$</td>
</tr>
<tr>
<td><strong>Retail Firms</strong></td>
</tr>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td><strong>Banks</strong></td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$\Theta$</td>
</tr>
<tr>
<td><strong>Government</strong></td>
</tr>
<tr>
<td>$\frac{G}{Y}$</td>
</tr>
<tr>
<td><strong>Monetary and Macroprudential Policy Rules</strong></td>
</tr>
<tr>
<td>$\rho_\pi$</td>
</tr>
<tr>
<td>$\rho_y$</td>
</tr>
<tr>
<td>$\rho_{rr} = \rho_{cr} = \rho_{rp}$</td>
</tr>
</tbody>
</table>

the steady state, required reserves ratio is determined as 0.06, while the capital
adequacy ratio is set equal to 0.08, in line with the average values employed by the U.S. Federal Reserve System.\footnote{http://www.federalreserve.gov/econresdata/default.htm, last accessed in January 2014.}

### 3.3.2 Model Dynamics

In the following subsections, we start by comparing the dynamics of negative TFP shocks under alternative policy rules. First, we look at the behaviour of certain macroeconomic variables when only the monetary policy rule is used by the central bank. We then analyze the behaviour of these variables when the monetary policy instrument is used in combination with one of the macroprudential policy tools. Lastly, we conduct a financial crisis experiment, one that is triggered by a negative capital quality shock, and compare the dynamics under the same alternatives.

**Impulse Responses to TFP Shocks**

Figure 3.1 illustrates the impulse responses under different policy rules when there is a negative one percent change in domestic productivity. The unanticipated decline in domestic productivity decreases investment and reduces asset prices, which triggers the financial accelerator mechanism. Since banks are leveraged, the decrease in asset prices results in a decline in their net worth, which is multiplied by a factor equal to their leverage ratio. As a result, banks experience a downturn in their balance sheets that increases the leverage ratio and pushes up the spread. The rise in the spread increases the cost of capital, which adds on to the decrease in investment and asset prices. The overall decline in investment, in turn, decreases aggregate output.
The unanticipated decline in productivity also results in an increase in hours worked. Due to the costs of adjustment in consumption and investment, neither of the two variables move much on impact. As a result, the decrease in productivity must be accompanied with a decrease in leisure large enough to ensure that output does not decrease too much on impact. The increase in hours worked, in turn, results in an increase in marginal cost and hence inflation. With the use of the monetary policy
rule only, since the weight of the movements in inflation is higher than the weight of the fluctuations in output, the interest rate increases.

When macroprudential policy rules are used in combination with the monetary policy rule, it can be seen that the negative effects of the financial accelerator mechanism in the economy dampens. According to the reserve requirements rule, the fall in the total nominal credit induced by the decline in productivity results in a decrease in the required reserves. Hence, cost of extending loans for banks declines. As a result, banks’ net worth decreases less, leading to a smaller increase in the leverage ratio and the spread. Consequently, the negative effects of the TFP shock on investment & output are lower. In the presence of the capital requirements rule, the decrease in the total nominal credit results in a decrease in the target capital adequacy ratio. Similar to the case under the reserve requirements, this decline lowers the decrease in bank’s net worth, which results in the negative effects of the financial accelerator mechanism to be reduced. Finally, when the regulation premium is used, the decrease in the total nominal credit lowers the premium. As a result, the cost of borrowing increases less, leading to the depression of the productivity shock’s negative effects. When there is a decrease in domestic productivity, it can be observed that counter-cyclical capital requirements are the most effective macroprudential tool in stabilizing output, since their positive effect on the spread, asset prices and investment is the largest.

The Financial Crisis Experiment

In our model, we postulate the negative capital quality (CQ) shock as the origin of the financial crisis as in Gertler and Karadi (2011). The aim is to find a shock that affects the quality of the financial intermediaries’ assets, which will cause an amplified
decrease in their net worth, because of their high level of leverage. In this way, we can broadly mimic the dynamics of the sub-prime crisis. Figure 3.2 demonstrates the impulse responses under alternative policy rules when the economy is affected by a negative one percent change in capital quality. As suggested by Equation (3.38), the shock results in a decline in capital, which in turn reduces asset prices. In addition to this negative effect, the capital quality shock also causes a decline in banks’ net worth, as given by Equation (3.41). As a result, banks’ leverage ratios increase and so does the spread and the cost of borrowing. The increase in the cost of borrowing results in a further reduction in asset prices and investment. The fall in investment in turn, leads to a decrease in aggregate output and hours worked. The decline in factor demands due to the contraction in production reduces $P^w_t$, the retail sector’s marginal cost. Hence, inflation decreases. As a response to the decrease in inflation and aggregate output, the central bank lowers the interest rate.

When used in combination with the monetary policy, all counter-cyclical macroprudential policies dampen the negative effects of the financial accelerator mechanism. They achieve this by lowering the decline in banks’ net worth, asset prices and investment. Capital requirements once again, mitigate the negative effects of the financial shock on output the most, since they perform the best in lowering the negative effects to the spread, asset prices and investment. When compared with the TFP shock, the capital quality shock results in a higher reduction in asset prices. As a result, all three macroprudential instruments are required to decrease more when the economy experiences a financial crisis.
3.3.3 Volatility Analysis

Following the analysis of the impulse responses to two different exogenous shocks, we first compare the real and financial statistics in the data and the model. Our aim is to analyze the performance of the model by its ability to mimic the cyclical properties of
real and financial variables. In our analysis, we use HP-filtered (smoothing factor: 1600) quarterly U.S. data for the period 1980-2010. To obtain the statistics in the model, we simulate the model 500 times for 100 quarters, with the assumption that both productivity and capital quality shocks affect the model economy. We then compute the business cycle statistics using the cyclical components of the HP-filtered series. In Table 3.2, we report the relative standard deviations of real and financial variables with respect to output and their cross-correlations with output.

Table 3.2. Business Cycle Properties of Real and Financial Variables

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>Investment</td>
<td>3.74</td>
<td>3.92</td>
</tr>
<tr>
<td>Employment</td>
<td>0.84</td>
<td>0.34</td>
</tr>
<tr>
<td>Bank assets</td>
<td>1.34</td>
<td>1.81</td>
</tr>
<tr>
<td>Net Worth</td>
<td>7.08</td>
<td>13.50</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>5.68</td>
<td>10.05</td>
</tr>
<tr>
<td>Spread</td>
<td>0.18</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Examining the real and financial statistics in the data, it can be seen that consumption and employment are less volatile than output, while investment volatility is much higher. In addition, consumption, investment and employment are highly pro-cyclical. These are known as standard business cycle facts (King and Rebelo, 1999). Except the spread, all financial variables are more volatile than output. It can also be noticed that bank assets and net worth are pro-cyclical, while the spread and the leverage ratio are counter-cyclical. These business cycle properties of real and financial variables broadly match the data statistics found in Angeloni and Faia (2013) and Mimir (2013). The

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8Data sources are presented in Appendix B.2 with details.
pro-cyclicality of bank capital is also reported in these studies. On the other hand, Meh and Moran (2010) and Rannenberg (2013) find that bank net worth and bank capital ratio are counter-cyclical, i.e., bank leverage is pro-cyclical.

We see that the model is able to reproduce the key business cycle facts in the U.S. data and it is able to replicate most of the facts related to financial variables. It nearly matches the relative volatility of consumption and produces pro-cyclical real variables as in the data. However, it underestimates the employment statistics\(^9\). In addition, net worth and leverage ratio have relatively high volatilities in the model. The higher volatility of bank net worth and leverage ratio within the model is as a result of the direct effect of the changes in asset prices on banks’ net worth and leverage. Since the fluctuations in asset prices have a direct and pro-cyclical effect on bank net worth, bank capital is also pro-cyclical. Moreover, when output declines, the greater decrease in bank capital indicates a significant rise in bank leverage, which results in a highly counter-cyclical leverage ratio.

In this section, we also study each macroprudential policy tool’s impact on the volatilities of different macroeconomic variables. In doing so, we employ the methodology used in obtaining the model statistics reported in Table 3.2. Our results are presented in Table 3.3.

\(^9\)We believe that the performance of the model would improve with the introduction of wage stickiness. Moreover, the relative volatility of employment depends on the preference parameter, \(g\), in the utility function. A higher value of \(g\) implies a higher relative volatility.
| Table 3.3. Volatilities under Different Policy Rules: Standard Deviations(%) |
|---------------------------------|---------------|---------------|---------------|---------------|
|                                 | Taylor Rule   | Taylor + RR   | Taylor + CR   | Taylor + RP   |
| Real Variables                  |               |               |               |               |
| Output                          | 1.37          | 1.15          | 1.03          | 1.23          |
| Consumption                     | 0.93          | 1.15          | 1.33          | 1.02          |
| Investment                      | 5.36          | 3.68          | 2.90          | 4.09          |
| Employment                      | 0.46          | 0.38          | 0.52          | 0.36          |
| Financial Variables             |               |               |               |               |
| Net Worth                       | 18.47         | 12.60         | 14.55         | 16.30         |
| Spread                          | 0.38          | 0.28          | 0.19          | 0.25          |
| Asset Prices                    | 3.59          | 2.90          | 2.30          | 3.02          |
| Monetary & Macroprudential Variables |         |               |               |               |
| Inflation                       | 0.09          | 0.09          | 0.10          | 0.09          |
| Interest Rate                   | 0.13          | 0.16          | 0.24          | 0.12          |
| Macropru. Tool                  | -             | 6.43          | 5.71          | 6.58          |

To start with, we examine the differences in the volatilities of certain real variables. It can be mentioned that all three macroprudential tools are effective in decreasing output volatility, while the adoption of these tools increases the volatility of consumption. The lowest volatility of output and investment are obtained under the capital requirements. When we analyze the volatility of the financial variables, it can be seen that all three macroprudential alternatives are effective in decreasing the volatility of net worth, the spread and asset prices. The lowest volatility of the spread and asset prices are also obtained in the presence of capital requirements. When inflation stability is the main concern, it can be suggested that there is no trade-off between the use of alternative macroprudential tools. Since all three macroprudential tools respond to the fluctuations in total nominal credit, the order of the volatilities of asset prices is...
reflected in the order of the volatilities of these tools.

### 3.3.4 Macroprudential Policies and Welfare

Following Faia and Monacelli (2007) and Gertler and Karadi (2011), we begin our welfare analysis by writing the household’s utility function recursively,

\[
\Gamma_t = U_t(C_t, C_{t-1}, L_t) + \beta E_t \Gamma_{t+1}
\]

(3.42)

We then take a second order approximation of \( \Gamma_t \) around the steady state, under each policy alternative. Using the second order solution of the model, we compute the value of \( \Gamma_t \), which corresponds to the welfare loss under each alternative. In this computation, we use the values of the monetary and macroprudential policy parameters (\( \rho_\pi, \rho_y, \rho_{rr}, \rho_{cr} \) and \( \rho_{rp} \)) that optimize \( \Gamma_t \) in response to productivity or financial shocks.\(^{10}\) By taking the difference of the values of \( \Gamma_t \) obtained under the monetary policy rule only and each macroprudential policy alternative, we can find the welfare gains from using each macroprudential tool. To convert these gains to consumption equivalents (CEs), we then compute the fraction of the steady state consumption required to equate welfare under the monetary policy rule, to the one under each macroprudential alternative. In Table 3.4, we present the optimized values for the policy parameters and the welfare gain obtained under each macroprudential alternative in terms of the CE. Under both shocks, we find that the optimal parameter for the output gap in the Taylor Rule is equal to zero, as in Schmitt-Grohe and Uribe (2007). As a result, in our analysis we set this parameter equal to zero and find the optimal parameters for inflation and total

\(^{10}\)Schmitt-Grohe and Uribe (2007) provide a detailed discussion on the calculation of the welfare loss in New Keynesian DSGE models.
nominal credit in the monetary and macroprudential policy rules, respectively.

Table 3.4. Optimal Parameters & Welfare Gains under Different Policy Rules

<table>
<thead>
<tr>
<th></th>
<th>In response to TFP shocks</th>
<th>In response to financial shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE</td>
<td>Optimal Parameters (%)</td>
<td>CE</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi$ $\rho_{rr}/\rho_{cr}/\rho_{rp}$</td>
<td></td>
</tr>
<tr>
<td>Taylor (TR)</td>
<td>-</td>
<td>3.82</td>
</tr>
<tr>
<td>TR + RR</td>
<td>0.0019</td>
<td>5.00</td>
</tr>
<tr>
<td>TR + CR</td>
<td>0.0429</td>
<td>5.00</td>
</tr>
<tr>
<td>TR + RP</td>
<td>0.0001</td>
<td>3.82</td>
</tr>
</tbody>
</table>

Table 3.4 shows that the degree of the counter-cyclicality of each macroprudential tool depends on the origin of the shock affecting the economy. However, the adoption of all macroprudential policies results in a decrease in the welfare loss when the economy experiences a TFP or a financial shock. The least effective macroprudential policy tool is the regulation premium under the TFP shock, while it is the reserve requirements under the financial shock. It should be noted that under each shock, the macroprudential tool that has the smallest positive effect on welfare is the one with an optimized macroprudential policy parameter that is closer to zero. When the economy experiences a TFP shock, macroprudential policies improve welfare, but the change is quantitatively small. Under the financial shock, the utilization of the capital requirements and the regulation premium has a higher positive effect on welfare.

It is important to notice that the use of capital requirements has the highest positive effect on welfare irrespective of the type of the shock affecting the economy. This finding is in line with the impulse responses presented in Section 3.3.2, where it is seen that counter-cyclical capital requirements are the most effective macroprudential tool in mitigating the negative effects of both shocks to the spread, asset prices and
investment. As previously mentioned, the financial accelerator mechanism used in our model features a pecuniary externality, where bankers do not consider the fact that if they issued more equity, they would decrease the risk of the banking sector. Consequently, they accumulate high levels of leverage, which amplifies the negative effects of exogenous shocks to the economy and results in a decline in welfare. Since capital requirements directly target banks’ leverage (or capital ratio), it is not counterintuitive to find that they are the most effective macroprudential tool in mitigating the negative effects of the financial accelerator mechanism.

Before concluding our welfare analysis, we also consider the scenario where both monetary and macroprudential policy instruments respond to the fluctuations in the total nominal credit in the economy. In this case, the optimized values for the policy parameters and the value of the CE obtained under each macroprudential alternative are reported in Table 3.5.

| Table 3.5 Optimal Parameters & Welfare Gains with Credit Growth in the TR |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
|                             | In response to TFP shocks   | In response to financial shocks |
| CE (%                       | Optimal Parameters         | CE (%)                      | Optimal Parameters          |
| TR + RR 0.0019              | ρπ 5.00 ρQ×S 0.00 ρrr/ρcr/ρrp 1.26 | TR + RR 0.0006              | ρπ 5.00 ρQ×S 0.01 ρrr/ρcr/ρrp 0.19 |
| TR + CR 0.0429              | ρπ 5.00 ρQ×S 0.00 ρrr/ρcr/ρrp 1.40 | TR + CR 0.1867              | ρπ 4.98 ρQ×S 0.00 ρrr/ρcr/ρrp 0.96 |
| TR + RP 0.0001              | ρπ 3.82 ρQ×S 0.00 ρrr/ρcr/ρrp 0.00 | TR + RP 0.0036              | ρπ 5.00 ρQ×S 0.01 ρrr/ρcr/ρrp 1.80 |

Under financial shocks, there are little welfare gains from including financial market developments in the Taylor rule, when the reserve requirements or the regulation premium are already in place. The optimized coefficient for the total nominal credit in the monetary policy rule is close to zero. In the presence of capital requirements, which are
the most effective macroprudential tool in mitigating the negative effects of the financial accelerator mechanism, the monetary authority cannot generate additional welfare gains by responding to the fluctuations in the total nominal credit. When the economy experiences a productivity shock and one of the aforementioned macroprudential tools is in place, the optimal coefficient of the total nominal credit in the Taylor rule is equal to zero. As a result, we conclude that our analysis suggests the use of two different policy instruments, to achieve two distinct but related objectives, namely financial and macroeconomic stability.

In our welfare analysis, we have assumed that the use of the interest rate and the macroprudential tools is assigned to the central bank, or put differently, the monetary and the macroprudential authorities cooperate. In case of non-cooperation, each authority would minimize its own loss function, taking the other’s policy rule as given. In this case, we would need to use an exogenously determined loss function for each authority.\textsuperscript{11}

\subsection*{3.3.5 Robustness Checks}

When presenting the welfare gains obtained under the three macroprudential policies (as in Table 3.4), we use the values given in our baseline calibration. To check the robustness of our results, we re-calculate the welfare gains obtained under each macroprudential policy, changing the values of different parameters. Table 3.6 reports the ordering of macroprudential policies according to their welfare gains when the low, baseline or the high value for a parameter is used. We display our results for the case where the econ-

\textsuperscript{11}See Angelini et. al (2011) for a discussion on the topic.
Table 3.6. Ordering of Macroprudential Policies for Different Parameter Values

<table>
<thead>
<tr>
<th>Low</th>
<th>Baseline</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 2$</td>
<td>$\phi = 4$</td>
<td>$\phi = 6$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>CR&gt;RP=RR</td>
<td>CR&gt;RP&gt;RR</td>
</tr>
<tr>
<td>$\theta = 0.60$</td>
<td>$\theta = 0.75$</td>
<td>$\theta = 0.90$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>CR&gt;RP&gt;RR</td>
<td>CR&gt;RP&gt;RR</td>
</tr>
<tr>
<td>$\chi = 0.4$</td>
<td>$\chi = 0.7$</td>
<td>$\chi = 0.9$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>CR&gt;RP=RR</td>
<td>CR&gt;RP&gt;RR</td>
</tr>
<tr>
<td>$\phi_X = 1$</td>
<td>$\phi_X = 2$</td>
<td>$\phi_X = 4$</td>
</tr>
<tr>
<td>$\phi_X$</td>
<td>CR&gt;RP&gt;RR</td>
<td>CR&gt;RP&gt;RR</td>
</tr>
<tr>
<td>$\rho_\psi = 0.60$</td>
<td>$\rho_\psi = 0.75$</td>
<td>$\rho_\psi = 0.90$</td>
</tr>
<tr>
<td>$\rho_\psi$</td>
<td>CR&gt;RR&gt;RP</td>
<td>CR&gt;RP&gt;RR</td>
</tr>
</tbody>
</table>

Note: RR = reserve requirements, CR = capital requirements, RP = regulation premium.

The economy is affected by a negative capital quality shock. The parameters that we include in our robustness analysis are: bank leverage at the steady state ($\phi$), degree of price stickiness ($\theta$), habit persistence ($\chi$), degree of investment adjustment costs ($\phi_X$) and persistence of the capital quality shock ($\rho_\psi$).

As with the baseline calibration, use of the regulation premium results in higher welfare gains than the utilization of reserve requirements under most of the calibrations. The exceptions to this result are present for high values of $\chi$ and $\rho_\psi$. Moreover, for low values of the steady state leverage ratio and habit persistence, use of both of these policies does not result in any welfare gains. Hence, their ordering is the same. On the other hand, utilization of capital requirements generates the highest welfare gains under
all the parameter values considered. As a result, the finding that the capital requirements are the most effective macroprudential tool in mitigating the negative effects of the financial accelerator mechanism, is robust to different parameter specifications.

3.4 Conclusions

In this paper, utilizing a New Keynesian DSGE model with financial frictions à la Gertler and Karadi (2011), we present a comparative analysis of three macroprudential policy tools; reserve requirements, capital requirements and a regulation premium. Our analysis is motivated by the lack of studies in the macroprudential policy literature that make a comparison of alternative policies, using a unified framework.

Running a number of simulations, we find that all of the aforementioned macroprudential tools are successful in lowering the negative effects of exogenous shocks to the economy. They do so by mitigating the negative effects of the financial accelerator mechanism, which is triggered by the decrease in asset prices. As a result of this decrease, banks experience a downturn in their balance sheets, which increases their leverage ratios and raises the spread. The rise in the spread increases the cost of capital, which results in a further decline in investment and asset prices. Finally, the decline in investment lowers the aggregate output. Irrespective of the source of the decline in economic activity, capital requirements are the most effective macroprudential tool in lowering the negative effects of the given shocks to the spread, asset prices and investment. As a result, they perform the best in mitigating the negative effects of the financial accelerator mechanism built in banks’ balance sheet constraints.

Computing the welfare loss and the corresponding consumption equivalent under
each policy alternative, we can also identify the macroprudential tool that generates the highest positive effect on welfare. Examining the case where the economy experiences a productivity or a financial shock only, we find that all three macroprudential policies are successful in decreasing the welfare loss. Consistent with the results of the simulations, use of capital requirements generates the highest welfare gains, under each shock.
Chapter 4

Evaluating the Net Benefits of
Unconventional Policies

4.1 Introduction

The recent global financial crisis is regarded by many economists as the worst since
the Great Depression. In a large number of countries, central banks and governments
have used a variety of unconventional policy measures to mitigate its adverse effects.
Central banks have mainly provided liquidity to the economy as a whole. Governments
have assisted central banks with the implementation of programs that provide direct
support to financial institutions.

As a prominent example, the Fed in the U.S. has used a variety of policy tools
to provide direct support to credit markets and hence to the economy. These can be
classified into three sets. The first set of tools, which fall under the central bank’s
role as the lender of last resort, include the provision of short-term liquidity to financial
institutions. The second set, referred to as credit easing, involves the supply of liquidity
directly to borrowers and investors in the financial markets. As part of these, the Fed has introduced facilities to purchase highly rated commercial paper and to provide liquidity for money market mutual funds. The final set of tools includes the purchase of longer-term securities\textsuperscript{1}. The U.S. Treasury, on the other hand, has used direct support to banks under its Troubled Asset Relief Program (TARP), as its main policy instrument. Even though the Treasury has first considered buying troubled assets, through TARP, their main support has refocused on injecting equity to financial institutions, using the Capital Purchase Program (CPP) included in TARP (Contessi and El-Ghazaly, 2011).

Most of these programs are often justified by the aim of reducing the fall in lending and recapitalizing financial institutions. In this way, they are expected to relax the constraints in financial markets and restore their functioning. However, taxpayers are concerned about the costs of the support programs, which might more than offset the benefits and in the end, lead to higher taxes. Economists are worried about the effects of these programs on increasing moral hazard problems. Hence, the debate among economists and policy makers on the optimal policy response to financial crises is still ongoing.

In this paper, motivated by the lack of studies that focus on the fiscal implications of unconventional policies, we present a comprehensive assessment of credit easing and bank capital injections. Our study is related to two strands of the literature. The first includes the recent works that examine the effects of capital purchase or lending by the central bank. Gertler and Karadi (2011) construct a model where banks are financially constrained due to credit market frictions. Unlike financial intermediaries, the monetary authority does not face any constraints. Hence, during a crisis direct lending by the

\textsuperscript{1}http://www.federalreserve.gov/newsevents/speech/bernanke20090113a.htm
central bank to private sector can be beneficial. Examining various unconventional monetary policies, Curdia and Woodford (2010) arrive at similar conclusions regarding credit easing by the central bank. Del Negro et al (2010) also explore the outcomes of government purchase of private assets in a DSGE model that contains financial frictions à la Kiyotaki and Moore (2008). They find that the policy intervention eases the constraints in the financial markets, which in turn reduces the decline in investment and consumption. The positive effect of this unconventional policy can be substantial.

The second strand contains the studies that analyze the implications of bank capital injections. Using an estimated New Keynesian model with a global bank, Kollmann et al (2012) study the effects of various fiscal stimulus packages employed in Europe. They formulate the government support to banks as a public transfer. Their results suggest that this type of policy has a stabilizing effect on output, investment and consumption. Hirakata et al (2013) also examine the use of bank capital injections, in addition to spread-adjusted Taylor rules. The model that they use contains the financial accelerator mechanism developed by Bernanke et al (1999). They find similar results regarding the government support to banks.

We distinguish our paper from the aforementioned studies in two aspects. First, in contrast to the given studies that examine the effects of using unconventional monetary or fiscal policies one at a time, we include both measures in our study and conduct a comparison of the two. More importantly, we examine the costs, as well as the benefits, of utilizing these credit policies. Since the returns and the costs from the use of unconventional policies are both reflected in the government’s budget constraint, fiscal policy tools need to adjust to the net gains/losses. To examine the use of alternative fiscal instruments to respond to these changes in the fiscal balance, we add two sep-
arate distortionary taxes to our analysis. These are consumption and labour income taxes. As a result, we can analyze the fiscal implications of alternative unconventional policy measures under different scenarios. We include three scenarios in our analysis:

(i) the government adopts lump-sum taxes and uses government spending to adjust to the changes in the government’s budget constraint, (ii) the government adopts distortionary (consumption or labour income) taxes and uses the tax rate to respond to the changes in its budget and finally (iii) the government adopts distortionary taxes and uses government spending as its variable fiscal instrument. Unlike our paper, the aforementioned studies that analyze the positive effects of using credit policies only include lump-sum taxes. We also use distortionary taxation in our analysis for another reason. It enables us to bring our experiments closer to the experience of policy makers, that mainly raise their revenues through distortionary taxes. Lump-sum taxes are rarely used by fiscal authorities in practice. The main rationale behind this method is the government’s inefficiency in observing the differences among taxpayers’ ability. Since actual governments cannot observe ability, models with lump-sum taxes do not suggest useful and realistic prescriptions (Mankiw et al, 2009).

To carry out our analysis, we build a New Keynesian DSGE model where the frictions in the financial intermediation process are as described in Gertler and Karadi (2011). These frictions create an inter-temporal distortion in the economy; the credit spread, which is defined as the difference between the gross return to risky assets and the gross riskless return. We characterize credit easing in our model as a policy tool where the central bank increases the total credit in the economy with the supply of loans to non-financial firms. Bank equity injections, on the other hand, directly increase the capital of banks. We assume that the central bank or the government face efficiency
costs when one of the given policies is used.

The main finding in our paper is that when the economy experiences a financial shock, the use of both types of unconventional policy measures mitigates its negative effects. The financial shock results in a decrease in asset prices, which triggers the financial accelerator mechanism. As a result of this decrease, banks experience a downturn in their balance sheets, that increases their leverage ratios and hence, the credit spread. The rise in the spread increases the cost of capital, which in turn, decreases investment and asset prices further. Finally, the fall in investment lowers the aggregate output. When the central bank pursues credit easing or the government injects capital into banks, the increase in the credit spread is lowered. As a result, the rise in the cost of capital dampens. With the use of both credit policies, the lower increase in the cost of capital corresponds to a smaller decrease in investment and hence, aggregate output. Since bank capital injections result in a direct increase in banks’ net worth, they induce a much lower increase in the leverage ratio and the spread compared to credit easing. Consequently, their positive effect on investment and aggregate output is higher. When credit easing is used, the central bank can utilize the excess return on assets present in times of the financial crisis. However, with bank capital injections, there are no excess returns to equity, since the return to government equity is the same as the return to government bonds. Hence, due to the efficiency costs of credit policies, use of equity injections causes an increase in fiscal costs, while utilization of credit easing results in an increase in fiscal revenues. As a result, under scenarios (i) and (iii), use of credit easing increases government spending, whereas bank capital injections cause a decrease in the same variable. Under scenario (ii), there is a rise (decline) in the tax rates when bank capital injections (credit easing) is used. However, use of bank capital injections
still results in the lowest decrease in economic activity, under all three scenarios.

Since the benefits and the costs from pursuing bank capital injections are both higher than that of credit easing, it is important to analyze the welfare outcomes of using these alternative credit policies. We compare the welfare implications of the given policies under the same scenarios. Our welfare results show that when lump-sum taxes rather than distortionary taxes are in place, use of credit policies generates higher welfare gains, only when tax rates adjust to the changes in the fiscal balance. However, when government spending is used to respond to the changes in government’s budget, the presence of distortionary taxes does not result in a decrease in welfare. Even though the use of variable tax rates cause a decrease in welfare, the utilization of unconventional policies still generates welfare gains. In addition, use of bank capital injections results in the highest welfare gains, regardless of the government’s approach to raising fiscal revenues.

The rest of the paper is organized as follows. Section 4.2 describes our model by presenting a detailed explanation of the economic agents, including the central bank, the government and the unconventional policies that they can use. Section 4.3 presents our quantitative results. The impulse responses to a financial shock are elaborated with the use of bank capital injections and/or credit easing, under three different scenarios regarding the government’s approach to raising fiscal revenues. The presentation of the welfare implications of pursuing credit policies under these scenarios follows the impulse responses. Finally, Section 4.4 concludes.
4.2 The Economy

Our framework is a New Keynesian DSGE model that contains a credit market and goods markets. The model economy includes seven type of agents: households that consist of bankers and workers, banks, capital goods producers, wholesale firms, retail firms, the fiscal and the monetary authority. We now continue with a detailed exploration of the economic agents in the economy.

4.2.1 Banks

The setting for the banking sector in our model follows that in Gertler and Karadi (2011). Bankers intermediate assets between households and wholesale firms. A representative bank supplies loans to non-financial firms, using the deposits acquired from households and its own net worth, which is given by the accumulation of past bank profits. Accordingly, the bank’s balance sheet is given by,

\[ Q_t s_t = n_t + d_t \]  \hspace{1cm} (4.1)

where \( s_t \) denotes loans to non-financial firms, \( Q_t \) their price, \( n_t \) net worth and \( d_t \) deposits.

The bank’s balance sheet implies an accumulation of net worth according to

\[
\begin{align*}
n_t &= R_{k,t} Q_{t-1} s_{t-1} - R_t d_{t-1} \\
&= R_t n_{t-1} + (R_{k,t} - R_t) Q_{t-1} s_{t-1} \\
\end{align*}
\]  \hspace{1cm} (4.2)
where $R_t$ is the gross risk-free return to deposits and $R_{k,t}$ denotes the gross risky return to bank’s assets. Net worth at the end of period $t$ is equal to the gross riskless return plus the excess return on bank’s assets.

The friction in the financial sector is based on a principal-agent problem between the banks and the households. After a bank obtains funds, the banker’s manager may transfer a fraction, $\Theta$ of total assets for her own benefit. In such a case, the creditors can force the bank to default on its debt and reclaim the remaining $1-\Theta$ fraction of funds. As households are aware of this possibility, they limit the funds that they supply to banks.

To exclude the case where bankers accumulate sufficient net worth that makes their financial constraints not binding, we assume that with probability $1-\gamma$, a banker exits and becomes a worker. In addition, the same number of workers randomly become bankers. Only upon exiting the bank pays dividends. As a result, the banker’s objective at the end of period $t$ is to maximize expected discounted terminal net worth,

$$V_t = E_t \sum_{i=1}^{\infty} (1 - \gamma)^{i-1} \Lambda_{t,t+i} n_{t+i}$$  \hspace{1cm} (4.3)

Given $n_{t-1}$ at the beginning of period $t$, net worth in period $t$ is given by the choice of $\{s_{t+i}\}$.

Since households know that the banker may transfer a fraction of total assets, they are willing to supply deposits to banks only if the following financial constraint is satisfied,

$$V_t \geq \Theta Q_t s_t$$
The constraint shows that for depositors to be eager to lend to banks, the banker’s loss from diverting funds should be at least as large as her gain from diverting. As a result, the banker’s objective is to maximize $V_t$ subject to the given incentive constraint.

We solve the banker’s optimization problem using backward induction. Hence, we start by guessing that $V_t$ can be expressed in the following form,

$$V_t = V_t(s_t, d_t) = v_{s,t}s_t - v_{d,t}d_t$$  \hspace{1cm} (4.4)

Eliminating $d_t$ from Equation (4.4) using the bank balance sheet, we can obtain,

$$V_t = V_t(s_t, n_t) = v_{s,t}Q_t s_t + v_{d,t}n_t$$ \hspace{1cm} (4.5)

Here, $v_{s,t}$ and $v_{d,t}$ are time-varying marginal values of loans and deposits, respectively and $\mu_{s,t} = \frac{v_{s,t}}{Q_t} - v_{d,t}$ is the excess value of the bank’s assets over its deposits.

Maximization of $V_t(s_t, n_t)$ subject to the incentive constraint results in the following optimality conditions,

$$(1 + \lambda_t) \mu_{s,t} = \lambda_t \Theta$$

$$\mu_{s,t}Q_t s_t + v_{d,t}n_t \geq \Theta Q_t s_t$$

Defining $\phi_t$ as the leverage ratio, the maximum ratio of a bank’s assets to its net worth that satisfies the incentive constraint, we obtain

$$Q_t s_t = \phi_t n_t$$ \hspace{1cm} (4.6)
where \( \phi_t = \frac{v_{d,t}}{\Theta - \mu_{s,t}}. \)

It can be seen that the leverage ratio of the bank is an increasing function of the excess marginal value of loans, \( \mu_{s,t} \) and a decreasing function of the fraction of assets that can be diverted by the banker, \( \Theta \). An increase in \( \mu_{s,t} \) increases the franchise value of the bank, reducing the banker’s incentive to divert funds and making depositors more willing to lend to the bank. As result, the bank can increase the supply of its loans, and hence, its leverage ratio. In contrast, a rise in \( \Theta \) causes an increase in the banker’s incentive, reducing the deposits supplied to the bank.

Using Equations (4.5) and (4.6) to rewrite Equation (4.3) gives,

\[
V_t(s_t, n_t) = E_t \Lambda_{t,t+1} \left[ (1 - \gamma) + \gamma (\mu_{s,t+1} \phi_{t+1} + v_{d,t+1}) \right] n_{t+1} \\
= E_t \Lambda_{t,t+1} \eta_{t+1} n_{t+1} \\
= E_t \Lambda_{t,t+1} \eta_{t+1} [R_{k,t+1} Q_t s_t - R_{t+1} d_t]
\]

Finally, comparing the above equation with our initial guess, we obtain \( v_{s,t} \), \( v_{d,t} \) and \( \mu_{s,t} \) as

\[
v_{s,t} = E_t \Lambda_{t,t+1} \eta_{t+1} R_{k,t+1} Q_t \\
v_{d,t} = E_t \Lambda_{t,t+1} \eta_{t+1} R_{t+1} \\
\mu_{s,t} = E_t \Lambda_{t,t+1} \eta_{t+1} (R_{k,t+1} - R_{t+1})
\]

where \( \eta_t = (1 - \gamma) + \gamma (\mu_{s,t} \phi_t + v_{d,t}) \) is the shadow value of a unit of net worth.

The difference between the gross return to loans, \( R_{k,t} \) and the gross riskless return, \( R_t \)
is defined as the *spread*. The *spread* is the inter-temporal distortion created by the existence of financial frictions in the banking sector. In a model with no credit market frictions, we would have $E_t [R_{t+1}] = E_t [R_{k,t+1}]$.

As the components of $\phi_t$ are not dependent on bank specific factors, we can sum across individual banks to obtain the aggregate banking sector balance sheet,

$$Q_t S_t = \phi_t N_t$$  \hspace{1cm} (4.7)

The accumulation of aggregate net worth is given by the sum of the net worth of surviving bankers ($N_{o,t}$) and of new entrants ($N_{e,t}$).

Net worth of surviving bankers equals earnings on assets net debt payments from the previous period, multiplied by the portion that survive ($\gamma$),

$$N_{o,t} = \gamma (R_{k,t} Q_{t-1} S_{t-1} - R_t D_{t-1})$$  \hspace{1cm} (4.8)

while net worth of the new entrants is obtained with the assumption that the ratio $\frac{\varepsilon}{1 - \gamma}$ of the total value of the exiting bankers’ assets are transferred to new bankers,

$$N_{e,t} = \varepsilon (R_{k,t} Q_{t-1} S_{t-1})$$  \hspace{1cm} (4.9)

Hence, the accumulation of net worth at the aggregate level is obtained as

$$N_t = R_{k,t} (\gamma + \varepsilon) Q_{t-1} S_{t-1} - \gamma R_t D_{t-1}$$  \hspace{1cm} (4.10)
4.2.2 Unconventional Policies

During the global financial crisis, various central banks have employed different credit policies to cope with the downturn in their economies. In the U.S., the Fed and the Treasury (acting in coordination with the Fed) have pursued two main policy actions, which are credit easing and bank equity injections. In the following subsections, we use the above set out framework to analyze the working mechanism of these policies.

Credit Easing (CE)

As explained in Fed president Bernanke’s speech on February 18, 2009, the use of conventional monetary policies, that work through influencing short-term interest rates, has not been sufficient to mitigate the negative effects of the global financial crisis on credit conditions. Hence, central banks have utilized unconventional policy measures to enhance the functioning of financial markets and to increase the supply of credit to non-financial firms. Although central banks have first provided liquidity to financial institutions, continued credit risk limited the willingness of many financial intermediaries to use this extra liquidity to extend credit. As a result, central banks have directly supplied credit to businesses, to reduce spreads, and in turn improve asset prices and the flow of funds in the economy.²

The Federal Reserve has established several lending programmes to provide liquidity to credit markets during the global financial crisis. It has formed the Commercial Paper Funding Facility (CPFF) to improve the market for high-quality commercial paper. Similar to the Fed, the Bank of England (BoE) operated the Asset Purchase Facility

²http://www.federalreserve.gov/newsevents/speech/bernanke20090218a.htm
(APF), which included private asset purchases to relax credit market conditions and more traditional QE. Within this facility, the BoE purchased 3 billion pounds of private assets. During the current crisis, the Bank of Japan has also announced purchases of 3 trillion yens in commercial paper. The purchases in all three countries have increased asset prices, resulting in an increase in the number of buyers and the resumption of trade (Fawley and Neely, 2013).

To model this type of unconventional policy, referred to as credit easing, we follow Gertler and Karadi (2011) and assume that the central bank can directly supply private securities (loans) to non-financial firms at the market lending rate, $R_{k,t+1}$. To finance this action, it issues government bonds that are perfect substitutes for bank deposits and that pay the gross riskless return, $R_{t+1}$. Households know that the central bank would not default on its debt. As a result, unlike private financial intermediaries, the central bank is not balance sheet constrained and it can utilize the excess return on assets in times of financial distress. However, it faces an efficiency cost, $\tau^s$, per unit of credit supplied to the market.

In case of credit easing, loans to non-financial firms at the aggregate level are now given by the sum of privately intermediated securities, $S^p_t$, and the securities that are intermediated via the central bank, $S^g_t$,

$$Q_tS_t = Q_t(S^p_t + S^g_t) \quad (4.11)$$

As in Gertler and Karadi (2011), we assume that the securities intermediated by the central bank are given by a fraction of total loans,

$$S^g_t = \varsigma_tS_t \quad (4.12)$$
Accordingly, Equation (4.7) must be rewritten as,

\[ Q_t S_t = \phi_t N_t + \varphi_t Q_t S_t \]  

(4.13)

**Equity Injections (BCI)**

Another form of unconventional policy that can be used to limit the unfavorable consequences of the financial crisis is bank capital injections. The focus of this policy action is the stabilization of the banking sector, by the improvement of its safety and soundness through increased capitalization. Increasing lending during a financial crisis forces banks to raise the riskiness of their lending. Hence, government support to banks is expected to ease the financing of projects that private banks would not be willing to finance otherwise, and increase the supply of credit (Black and Hazelwood, 2012).

During the global financial crisis, the U.S. Treasury has used the Capital Purchase Program (CPP) to inject equity into financial institutions, similar to the arrangements used by the Bank of Japan to stabilize Japanese banks in the 1990s. In total, 205 billion dollars of funds has been distributed to 707 institutions. The U.K. Treasury has also injected about 50 billion pounds of equity into British banks (Mishkin, 2010). In addition, bank capital injections had been widely used before the 2007-09 experience. As mentioned by Contessi and El-Ghazaly (2011), governments had used equity injections in 32 out of the 42 banking crises that have occurred between 1970 and 2007.

In line with this experience, we assume that the fiscal authority in our framework can support the central bank by injecting equity into the banking sector and finance these injections by issuance of government bonds. We maintain that a unit of equity injected by the government, \( N_t^g \) has the same return as a unit of private equity, \( N_t \),
that is, the government does not pay a premium for bank equity. In addition, the surviving bankers pay back the return to government equity the next period. As given in Equation (4.2), the return to a unit of private bank equity, \( N_{t-1} \) is \( R_t \). Accordingly, in the presence of bank equity injections, Equation (4.10) needs to be rewritten as

\[
N_t = R_{k,t} (\gamma + \varepsilon) Q_{t-1} S_{t-1} - \gamma R_t D_{t-1} + N_t^g - \gamma R_t N_{t-1}^g
\]  

(4.14)

where the last two terms correspond to the increase in bank net worth with the injection, net of repayments. Since the bank needs to repay the return on the capital injected by the government, it takes time for bank net worth to rebuild and the exit from the credit policy lasts long. As with credit easing, we maintain that government equity is a fraction of total bank equity,

\[
N_t^g = \Upsilon_t N_t \tag{4.15}
\]

and there are efficiency costs relating to government equity injections, which are given by \( \tau^N \) per unit of equity supplied.

### 4.2.3 Households

The representative household consists of a continuum of members of measure unity. The two types of members within the household are workers and bankers. Workers supply labour and earn wages while bankers manage financial intermediaries and transfer dividends back to households. Households hold their savings as deposits, which are assumed to be riskless one period securities.

\[3\text{Even though very few of the Capital Purchase Program beneficiaries have failed in the period between 2008 and 2010, not all have survived. (Contessi and El-Ghazaly, 2011).} \]
A representative household maximizes expected discounted utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, C_{t-1}, L_t)$$  \tag{4.16}$$

subject to the following budget constraint,

$$(1 + t_i^c)C_t = (1 - t_i^h)W_i h_t + \Pi_t + R_t D_{t-1} - D_t$$  \tag{4.17}$$

where $0 < \beta < 1$ is the subjective discount factor, $E$ is the expectation operator, $C_t$ denotes consumption and $L_t$ leisure, $W_t$ the wage rate, $h_t(= 1 - L_t)$ hours worked and $D_t$ bank deposits. In Equation (4.17) $t_i^c$ and $t_i^h$ denote the consumption and the labour income tax rate, respectively. $\Pi_t$ are the profits earned from the ownership of banks and firms.

The household’s first order conditions for the consumption Euler equation and labour supply are given by,

$$1 = R_{t+1} E_t \left[ \Lambda_{t,t+1} \frac{1 + t_i^c}{1 + t_i^{c+1}} \right]$$  \tag{4.18}$$

$$\frac{U_{h,t}}{U_{C,t}} = - \frac{U_{L,t}}{U_{C,t}} = -W_t \frac{1 - t_i^h}{1 + t_i^c}$$  \tag{4.19}$$

where $U_t = \frac{(C_t - \chi C_{t-1})^{(1-\sigma)(1-\sigma)}(1-h_t)^{\sigma(1-\sigma)-1}}{1-\sigma}$ and $\Lambda_{t,t+1} = \frac{U_{C,t+1}}{U_{C,t}}$ is the real stochastic discount factor over the interval $[t, t+1]$. Examining the household’s first order conditions, we can observe the different distortions caused by the two tax instruments. The labour income tax distorts the intra-temporal substitution between consumption and
leisure. The current consumption tax distorts the same margin. Moreover, both the current and next period’s consumption tax enters into the current Euler equation\(^4\).

### 4.2.4 Wholesale Firms

Wholesale firms produce output, \(Y_t^w\), using a constant returns to scale Cobb-Douglas production function that contains labour and capital as factor inputs,

\[
Y_t^w = Y_t^w (A_t, h_t, K_{t-1}) = (A_t h_t)\alpha K_{t-1}^{1-\alpha} \tag{4.20}
\]

\(A_t\) denotes aggregate productivity, which follows an AR(1) process.

\[
\log A_t - \log A = \rho_A (\log A_{t-1} - \log A) + \varepsilon_A
\]

It should be noticed that \(K_t\) is the end-of-period \(t\) capital stock. Firms choose labour to satisfy,

\[
\frac{P_t^w}{P_t} Y_{h,t}^w = W_t \tag{4.21}
\]

\(P_t^w\) and \(P_t\) are the aggregate price indices in the wholesale and retail sectors, respectively. \(Y_{h,t}^w = \alpha \frac{Y_t^w}{h_t}\). Profit maximization by whole firms yields the given labour demand equation, which indicates that the marginal product of labour should be equal to the real wage. It also implies that labour demand increases with an increase in the production or the price of the wholesale output, while it decreases with an increase in wages.

\(^4\)The optimization problem of the household, in the presence of lump-sum rather than distortionary taxes, is presented in Appendix C.1.
A wholesale firm issues new securities to obtain funds from banks, which are then used to buy new capital goods from capital producers. The number of claims issued by the firm, $S_t$, is equal to the number of units of capital needed, $K_t$ and so are their prices,

$$Q_t S_t = Q_t K_t$$  \hfill (4.22)

Through perfect competition, wholesale firms earn zero profits and they fully pay the return to capital to the banks,

$$R_{k,t} = \psi_t \left( (1 - \alpha) \frac{P^W Y^W}{P_t K_{t-1}} + (1 - \delta) \frac{Q_t}{Q_{t-1}} \right)$$  \hfill (4.23)

The return to capital depends on the marginal product of capital and the change in price of capital, net of depreciation, $\delta$.

### 4.2.5 Capital Producers

At time $t$, capital producers convert $I_t$ of raw output into $(1 - S(X_t)) I_t$ of new capital, subject to investment costs, $S(X_t)$. Hence, capital accumulation is given by

$$K_t = \psi_{t+1} [(1 - \delta) K_{t-1} + (1 - S(X_t)) I_t]$$  \hfill (4.24)

where $X_t = \frac{I_t}{I_{t-1}}$ and $\psi_t$ denotes the shock to the quality of capital, which follows an AR(1) process.

---

5The functional form of the investment costs is given by $S(X_t) = \phi X_t^2$, as large investment adjustment costs are needed to match the smoother investment responses observed in U.S. business cycles.

6The shock also has an effect on the evolution of bank net worth. Accordingly, Equation (4.10) should be rewritten as $N_t = R_{k,t} (\gamma + \varepsilon) \psi_t Q_{t-1} S_{t-1} - \gamma R_t D_{t-1}$. 

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\[ \log \psi_t = \rho \psi_{t-1} + \epsilon \psi \]

The maximization of expected discounted profits by capital producers yields\(^7\)

\[ Q_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[ A_{t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2 \right] = 1 \quad (4.25) \]

a positive relationship between investment and asset prices. The given equation is known as the Tobin’s (1969) Q relation.

**4.2.6 Retail Firms**

Retail firms produce a basket of differentiated goods for consumption. The demand for consumption is given by

\[ C_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} C_t \quad (4.26) \]

In aggregate, demand for investment, government expenditures and hence the final/retail output has the same functional form as consumption,

\[ Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} Y_t \quad (4.27) \]

where \( \zeta \) is the elasticity of substitution and the aggregate price index, \( P_t \) is given by

\[ P_t = \left( \int_0^1 P_t(f)^{1-\zeta} df \right)^{1/(1-\zeta)}. \]

\(^7\)A detailed explanation of the capital producer’s maximization problem is presented in Appendix C.2.
We assume that retail firms modify the wholesale output $Y^W_t$ at a cost of $c$. Hence, the relationship between the retail and the wholesale sector is given by\(^8\),
\[
Y_t = \frac{(1 - c)Y^W_t}{\Delta_t}
\]
where $\Delta_t = \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\frac{1}{\zeta}} df$ is a measure of price dispersion across retail firms which set their prices à la Calvo(1983). According to this type of price staggering, 1-$\theta$ percent of firms changes their prices each period, with an individual firm’s probability of adjusting its price in any given period being random. The typical firm adjusting its price in period $t$ maximizes its discounted nominal profits
\[
E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} Y_{t+k}(f) \left[ P^a_t(f) - P_{t+k}MC_{t+k} \right] = 0 \tag{4.28}
\]
subject to Equation (4.27). Here, $MC_t$ is the real marginal cost, $P^a_t(f)$ is the adjusted price and $\Omega_{t,t+k} = \beta^k \frac{U_{C,t+k}/P_{t+k}}{U_{C,t}/P_t}$ is the nominal stochastic discount factor over the period $[t, t+k]$. The solution to the maximization problem is given by
\[
E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} Y_{t+k}(f) \left[ P^a_t(f) - \mu P_{t+k}MC_{t+k} \right] = 0 \tag{4.29}
\]
where $\mu = \frac{1}{1 - \frac{1}{\zeta}}$ is the steady state mark-up. Equation (4.29) can be rewritten as
\[
E_t \sum_{k=0}^{\infty} (\theta\beta)^k U_{C,t+k} \Pi_{t,t+k}^{k-1} Y_{t+k} \left[ \frac{P^a_t(f)}{P_t} - \mu \Pi_{t,t+k}MC_{t+k} \right] = 0 \tag{4.30}
\]
where $\Pi_{t,t+k} = \frac{P_{t+k}}{P_t}$ is $k$ periods ahead inflation and $P_{t+k}(f) = P^a_t(f)$ is the price set at time $t$ that remains the same with probability $\theta^k$.

\(^8\)The derivation of the relationship between the retail and the wholesale sector is presented in Appendix C.3.
With the given price-setting mechanism, the evolution of the price index is obtained as

$$P_{t+1}^{1-\zeta} = \theta P_t^{1-\zeta} + (1 - \theta) \left( P_{t+1}^a \right)^{1-\zeta}$$  \hspace{1cm} (4.31)$$

As shown by Benigno and Woodford (2005), equations (4.30) and (4.31) can be written recursively defining variables $F_t$ and $H_t$,

$$H_t - \theta \beta E_t \left[ \Pi_{t+1}^{\zeta-1} H_{t+1} \right] = Y_t U_{C,t}$$  \hspace{1cm} (4.32)$$

$$F_t - \theta \beta E_t \left[ \Pi_{t+1}^{\zeta} F_{t+1} \right] = \mu Y_t U_{C,t} M C_t$$  \hspace{1cm} (4.33)$$

$$\theta \Pi_t^{\zeta-1} + (1 - \theta) \left( \frac{F_t}{H_t} \right)^{1-\zeta} = 1$$  \hspace{1cm} (4.34)$$

Here, $\Pi_{t+1} = \Pi_{t,t+1}$ and $\Pi_t$ is the gross inflation rate. The ratio $\frac{F_t}{H_t}$ is equal to $\frac{P_t^a}{P_t}$ which is the optimal relative price. These three equations represent the non-linear Phillips Curve. Using (4.31) and the assumption that the distribution of prices among the firms that do not adjust their prices is the same as the overall distribution in period $t$, the relationship between price dispersion and inflation is obtained in line with Benigno and Woodford (2005) as follows,

$$\Delta_t = \theta \Pi_t^\zeta \Delta_{t-1} + (1 - \theta) \left( \frac{F_t}{H_t} \right)^{-\zeta}$$  \hspace{1cm} (4.35)$$

One should notice that the real marginal cost for the retail sector can be expressed as
\[ MC_t = \frac{P_t^W}{P_t} \]  

(4.36)

since \( P_t^W \) is the cost of purchasing the wholesale good.

### 4.2.7 Monetary Policy

The central bank sets the gross nominal interest rate, \( R_{n,t} \) in period \( t \) to pay out interest in period \( t + 1 \). The following Fisher equation gives the relation between the nominal and the real interest rate,

\[ R_{n,t-1} = R_t \Pi_t \]  

(4.37)

Monetary policy is conducted using a simple Taylor rule,

\[ \log \left( \frac{R_{n,t}}{R_n} \right) = \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \]  

(4.38)

where the steady state nominal rate, inflation and output are given by \( R_n \), \( \Pi \) and \( Y \), respectively.

As previously mentioned, securities intermediated by the central bank and capital injected by the government are determined as a fraction of total loans and total bank net worth, respectively. As presented in Equations (4.12) and (4.15), these fractions are given by \( \varphi_t \) and \( \Upsilon_t \), and they respond to the deviations of the credit spread from its steady state value,

\[ \varphi_t = \rho_\varphi [ (R_{k,t} - R_t) - (R_k - R) ] \]
\[ \Upsilon_t = \rho T [(R_{k,t} - R_t) - (R_k - R)] \]

### 4.2.8 Fiscal Policy

We assume that in normal times government spending, \( G_t \), is financed by tax revenues. In other words, the government keeps a balanced budget and does not issue any bonds. However, when the central bank and/or the government pursues credit policies in times of a crisis, government bonds, \( B_{t-1}^g \), which are perfect substitutes for deposits, finance total government intermediated assets, given by the sum of the fraction of loans intermediated by the central bank, \( \varphi_{t-1}Q_{t-1}S_{t-1} \) and the fraction of bank capital injected by the government, \( \Upsilon_{t-1}N_{t-1} \). In addition, the costs and the returns from the utilization of credit policies should be accounted for in the government’s budget constraint. Accordingly, total government expenditures are financed by tax revenues, (net) issuance of government bonds and net earnings from credit market interventions,

\[
G_t + (1 + \tau^S)\varphi_tQ_tS_t + (1 + \tau^N)\Upsilon_tN_t = T_t + R_{k,t}\varphi_{t-1}Q_{t-1}S_{t-1} \\
+ R_t\Upsilon_{t-1}N_{t-1} + B_{t-1}^g - R_tB_{t-1}^g \tag{4.39}
\]

As can be seen in Equation (4.39), total government expenditures in period \( t \) consist of government spending and costs incurred by the government for supplying loans and injecting equity, \( (1 + \tau^S)\varphi_tQ_tS_t + (1 + \tau^N)\Upsilon_tN_t \). \( B_{t-1}^g \) denotes the bonds issued in period \( t \) to finance the intermediation of assets in the given period. The government earns the
gross risky return, $R_{k,t}$ from the loans supplied and the gross return to net worth, $R_t$ from the equity injected in period $t - 1$. The cost of these intermediations is the gross riskless return, $R_t$ that the government pays on $B^q_{t-1}$. Total government revenues also include the earnings from tax collection, $T_t$. When the fiscal authority adopts lump-sum taxation, $T_t$ directly represents lump-sum taxes. When the government adopts distortionary taxation, $T_t$ is equal to earnings from consumption or labour income taxes, $t^c_t C_t$ and $t^h_t h_t W_t$, respectively. Government spending follows an AR(1) process,

$$
\log G_t - \log G = \rho_G (\log G_{t-1} - \log G) + \varepsilon_G
$$

### 4.3 Financial Crisis Simulations and Policy Experiments

In this section, we present a financial crisis experiment to demonstrate how the model may replicate some features of the recent crisis and how credit policies can be utilized to mitigate its negative effects. In our model, we postulate the negative capital quality shock as the origin of the financial crisis as in Gertler and Karadi (2011). This shock affects the quality of the financial intermediaries’ assets. Since banks are leveraged, the decline in the value of their assets results in an amplified decrease in their net worth. In this way, we can broadly mimic the dynamics of the global financial crisis. We begin by calibrating the model and continue with elaborating the crisis experiment.
4.3.1 Calibration

Table 4.1 presents the parameters used in the calibration of our model. We start our calibration by setting the financial parameters and set their values in line with Gertler and Karadi (2011). We choose the value of $\gamma$ to ensure an average survival of 10 years for bankers. The values of $\varepsilon$ and $\Theta$ are calibrated to match an economy-wide leverage ratio of 4, which approximately captures the aggregate data and an average credit spread of 100 basis points per year, which takes the pre-2007 spreads between BAA corporate versus government bonds as the reference. As in Gertler and Karadi (2011), we choose conventional values for the labour share, $\alpha$ and the elasticity of substitution between goods, $\zeta$. The steady state depreciation rate $\delta$, the habit parameter $\chi$, and the price rigidity parameter $\theta$ are also set in line with the values used by Gertler and Karadi (2011). The specific parameters in our model are $\sigma$ (in the utility function) and $\phi_X$ (in the investment cost function). The values for these parameters are set to roughly reflect the empirical literature. We include habit formation and investment adjustment costs in our analysis, since empirical work has shown that such real frictions improve the ability of macroeconomic models to explain the business cycles in the U.S. (Schmitt-Grohe and Uribe, 2007). For calibrating the discount factor, $\beta$ and the preference parameter, $\rho$, we use typical U.S. observations of 0.35 for hours worked and 1.01 for the gross interest rate. To conclude, we present the coefficients of the Taylor rule and the values for the tax rates in Table 4.1. The steady state levels of the consumption and the labour income tax rates are given by 18% and 10%, respectively. We use the pre-2007 average OECD value-added tax rate\(^9\) for determining the first, while the pre-2007 av-

<table>
<thead>
<tr>
<th>Table 4.1. Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
</tr>
<tr>
<td>$\beta$ 0.987 Discount factor</td>
</tr>
<tr>
<td>$\chi$ 0.7 Habit parameter</td>
</tr>
<tr>
<td>$\sigma$ 2 Preference parameter</td>
</tr>
<tr>
<td>$\varrho$ 0.876 Preference parameter</td>
</tr>
<tr>
<td><strong>Capital Producers</strong></td>
</tr>
<tr>
<td>$\phi_X$ 2 Coefficient of adjustment costs</td>
</tr>
<tr>
<td><strong>Wholesale Firms</strong></td>
</tr>
<tr>
<td>$\alpha$ 0.7 Labour share</td>
</tr>
<tr>
<td>$\delta$ 0.025 Depreciation rate</td>
</tr>
<tr>
<td><strong>Retail Firms</strong></td>
</tr>
<tr>
<td>$\zeta$ 7 Elasticity of substitution</td>
</tr>
<tr>
<td>$\theta$ 0.75 Probability of keeping prices constant</td>
</tr>
<tr>
<td><strong>Banks</strong></td>
</tr>
<tr>
<td>$\gamma$ 0.975 Probability that bankers survive</td>
</tr>
<tr>
<td>$\varepsilon$ 0.001 Proportional transfer to the new entrants</td>
</tr>
<tr>
<td>$\Theta$ 0.410 Fraction of bank assets that can be diverted</td>
</tr>
<tr>
<td><strong>Central Bank</strong></td>
</tr>
<tr>
<td>$\rho_x$ 1.5 Inflation coefficient of the Taylor rule</td>
</tr>
<tr>
<td>$\rho_y$ 0.5/4 Output gap coefficient of the Taylor rule</td>
</tr>
<tr>
<td><strong>Government</strong></td>
</tr>
<tr>
<td>$t^c$ 0.18 Steady state consumption tax rate</td>
</tr>
<tr>
<td>$t^h$ 0.10 Steady state labour income tax rate</td>
</tr>
</tbody>
</table>
average labour income tax rate in the U.S.\textsuperscript{10} is used as a basis to set the second. As in Gertler and Karadi (2011), we assume that the efficiency costs of credit policies are equal to 10 basis points, $\tau^S = t^N = 0.0010$. In addition, we set the persistence parameters for all the shock processes in our model to 0.75, following the conventional business cycle literature. Before concluding the quantitative analysis in our paper, we check the robustness of our results to the changes in the calibration of key parameters.

### 4.3.2 Financial Crisis Experiment

We now present the financial crisis experiment. As in Gertler and Karadi (2011), we assume that the financial crisis is triggered by a negative shock to capital quality. This shock decreases the quality of the financial intermediaries’ assets and causes an amplified decrease in their net worth, due to high leverage. We believe that we can principally obtain the dynamics of the mortgage crisis, using such a shock. We analyze the behaviour of certain macroeconomic variables under four different policy responses. First, we look at the case where only the monetary policy rule is used by the central bank. Then, we examine the behaviour of these variables when one of the credit policies is in place. Finally, we consider the case where the funds obtained by the issuance of government bonds are divided equally between the use of bank capital injections and credit easing. We assume that the total gross fiscal costs of credit policies to the government amount to 10\% of the steady-state output\textsuperscript{11}. However, the net costs of


\textsuperscript{11}Contessi and El-Ghazaly (2011) estimate the gross fiscal cost of bank recapitalizations used during the global financial crisis to be equal to ($700 billion) 5\% of GDP for the U.S. The Commercial Paper Funding Facility established by the Fed during the recent crisis has lent out roughly the same amount before it was closed. Hence, adding up the two results in the gross fiscal costs of credit policies to be equal to 10\% of GDP for the U.S.
the two policies differ since the returns from the use of each credit policy are different. As we would like to compare these net costs under different scenarios, we analyze the model dynamics with the assumption that the fiscal authority can collect taxes using three alternatives; lump-sum, consumption or labour income taxes.

**Policy Responses with Lump-sum Taxes**

First, we use the benchmark scenario to analyze the impulse responses to a negative one percent change in capital quality under different credit policies. Under this scenario, we assume that the fiscal authority adopts lump-sum taxation. Hence, use of government spending or taxes to accommodate the changes in the fiscal balance has the same implications. Figure 4.1 presents the dynamics under the benchmark scenario, where government spending responds to the changes in the government budget constraint caused by the use of unconventional policies. The shock to capital quality causes a decline in banks’ net worth. It also results in a reduction in asset prices, that triggers the financial accelerator mechanism. Since banks are leveraged, the decrease in asset prices results in a decrease in net worth, that is multiplied by a factor equal to the leverage ratio. As a result, banks experience a downturn in their balance sheets that increases the leverage ratio. This deterioration makes it more difficult for banks to obtain funds from households. Hence, it causes a decrease in the supply of credit and a rise in the spread, which in turn increases the cost of capital. As a result, demand for capital decreases resulting in a further decline in investment and asset prices. The overall decline in investment, in turn, decreases aggregate output. The decrease in aggregate output depresses labour demand, marginal cost and hence, inflation. The monetary authority lowers the interest rate in response to the decrease in inflation and
Figure 4.1. Impulse Responses to Financial Shocks with Different Credit Policies

Benchmark Scenario*

*The solid red line denotes the case where only the monetary policy rule is used by the central bank. The dashed blue line shows the use of bank capital injections with the monetary policy rule. The dashed dotted black line presents the use of credit easing with the monetary policy rule. Finally, the green line corresponds to the case where the funds obtained by the issuance of government bonds are divided equally between the use of bank capital injections and credit easing.
output. Finally, the overall decline in aggregate output is also reflected in the reduction in aggregate consumption. The use of credit policies by the central bank and/or the government significantly mitigates the negative effects of the financial shock. The main reason behind this positive outcome is the credit policies’ ability to dampen the increase in the spread. As a result, the increase in the cost of capital, the decline in investment, and in turn, the decrease in aggregate output are lowered. When the central bank pursues credit easing, the supply of credit to non-financial firms increases and the increase in the spread is lowered. As a result, firms are affected less from the disruption in the financial markets caused by the deterioration in banks’ balance sheets. In case of bank capital injections, the government injects the funds obtained to banks, which directly results in a significant increase in banks’ net worth. Hence, the rise in the leverage ratio dampens and bankers find it easier to acquire funds from depositors. This results in an increase in the supply of credit and an important decline in the increase in the spread. It can be seen that the positive effect of pursuing bank capital injections on aggregate output is higher, as it results in a much lower increase in the spread compared to credit easing.

However, as previously mentioned, the exit from the capital injections takes much longer. Hence, the bonds issued by the government remain well above the steady-state level for a longer period. In the absence of unconventional policies, the government keeps a balanced budget and government spending stays constant. When bank capital injections are used, the government needs to decrease its spending. With the assumption that a unit of equity injected by the government has the same payout rule as a unit of private equity, the return to bank capital injections exactly offsets the cost from the issuance of government bonds. As a result, the government earns zero profits from
bank capital injections and their efficiency costs need to be financed with a decrease in government spending. On the other hand, when credit easing is used by the central bank, the net earnings from the utilization of this credit policy, \((R_{k,t} - R_t)\varphi_{t-1}Q_{t-1}S_{t-1}\) are higher than the efficiency costs. Consequently, government spending increases. In other words, use of bank capital injections causes a net increase in government expenditures, while utilization of credit easing results in a net increase in government revenues. Looking at the three alternative cases, it can be seen that the equal division of funds between the use of capital injections and credit easing results in a case where both the benefits and the costs from pursuing these policies stay in between those of using only one of the policies.

**Policy Responses with Distortionary Taxation**

Since the gains and losses from the credit market interventions pursued changes the government’s budget constraint, using distortionary taxation in our analysis enables us to conduct a more detailed comparison of alternative credit policies. Hence, we present the impulse responses to a negative one percent change in capital quality under different credit policies, with the assumption that the fiscal authority adopts consumption or labour income taxes. Under each case, the government can either use government spending or tax rates to respond to the changes in its budget constraint. When distortionary taxes are in place, use of alternative fiscal instruments has different implications.

**Consumption Taxes**

In Figure 4.2, we present the impulse responses to a unit decrease in capital quality, when the government adopts consumption taxes and government spending accommo-
dates the changes in the fiscal balance. Since consumption decreases and the consumption tax rate stays constant, government spending also decreases under all policy alternatives. Moreover, the dynamics of government spending closely follows that of

Figure 4.2. Impulse Responses to Financial Shocks with Different Credit Policies
Consumption Taxes & Use of Government Spending
consumption. The decrease in the spending is higher under bank capital injections, while it is lower under credit easing, compared to the case with no credit intervention. This difference is mainly due to the aforementioned differences in the returns to bank capital injections and credit easing. Under each policy alternative, the decrease in government spending caused by the use of consumption taxes results in a higher decrease in aggregate output, compared to the benchmark scenario.

In Figure 4.3, impulse responses to the same shock are presented with the assumption that the fiscal authority adopts consumption taxes and the tax rate is used to accommodate the changes in the government budget constraint. The tax rate increases under all policies due to the decrease in consumption. Compared to the case with no credit policy, use of bank capital injections mainly results in an increase in the tax rate, while the utilization of credit easing slightly causes a decrease. When consumption taxes adjust to the changes in the fiscal authority’s budget, instead of government spending, the decrease in the level of consumption is higher. In addition, under all policy alternatives, the decrease in aggregate output is higher, compared to the benchmark scenario.

Labour Income Taxes

Figure 4.4 presents the impulse responses to a unit decrease in capital quality, when the government adopts labour income taxes and government spending accommodates the changes in the fiscal balance. As the tax rate stays constant, the decrease in the labour income due to lower wages and hours worked results in a decrease in government
spending under all policy alternatives. As in Figure 4.2, government spending with bank capital injections is mainly lower than the spending under credit easing, as a result of the higher returns to credit easing. Compared to the benchmark scenario, the decrease in the level of aggregate output is higher under each policy alternative, as government spending decreases with the use of labour income taxes.
In Figure 4.5, impulse responses to the same shock are presented with the assumption that the fiscal authority adopts labour income taxes and the tax rate is used to accommodate the changes in the government budget constraint. With labour income taxes, since bank capital injections result in a lower decline in aggregate output, and hence, hours worked, the tax rate does not increase as much as it does under consumption taxes. Still, it is mainly higher than the tax rate obtained with the use of credit
easing. When labour income tax rates, rather than government spending, adjust to the
changes in the government budget constraint, the decrease in the level of hours worked
is higher. Moreover, under all policies, the decrease in aggregate output is higher than
the decline obtained with the benchmark scenario.

Figure 4.5. Impulse Responses to Financial Shocks with Different Credit Policies
Labour Income Taxes & Use of Tax Rate
Most of the aggregate variables under consumption and labour income taxes have a similar outlook, even though the tax/government spending profiles in the two scenarios are quite different. Compared with the benchmark scenario (that assumes lump-sum taxation), existence of consumption taxes results in a decrease in the level of consumption, while it does not cause a significant change in the behaviour of hours worked. On the other hand, in the presence of labour income taxes, the dynamics of both consumption and hours worked are distorted. However, the decrease in the level of consumption is lower than then the decline obtained under consumption taxes.

As previously mentioned, the difference in the returns to bank capital injections and credit easing results in a difference in their contribution to the government’s budget. With the assumption that a unit of equity injected by the government has the same payout rule as a unit of private equity, the costs from the issuance of government bonds are equal to the returns to bank capital injections. As a result, the efficiency costs of bank capital injections need to be financed with an increase in taxes (decrease in government spending). On the contrary, the net earnings from credit easing are higher than its efficiency costs. Hence, use of credit easing results in a decrease in taxes (increase in government spending). In Table 4.2, we aim to quantify the positive effect of the lower tax rates obtained with the use of credit easing in a simple manner. To do so, we present the percentage increase in aggregate output obtained under each unconventional policy alternative with the benchmark scenario (lump-sum taxes) and variable tax rates. The steady-state level of lump-sum taxes in each benchmark scenario is equal to the steady-state tax revenues obtained with the use consumption or labour income taxes. The percentage increase in output under alternative unconventional policies is given relative to the output obtained under no credit policy. The numbers
are presented for the period with the lowest economic activity.

Table 4. 2. Relative Increase in Aggregate Output under Different Credit Policies (%)

<table>
<thead>
<tr>
<th></th>
<th>Benchmark for Cons. Taxes</th>
<th>Variable Cons. Tax Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCI</td>
<td>34.77</td>
<td>29.62</td>
</tr>
<tr>
<td>CE</td>
<td>8.64</td>
<td>9.13</td>
</tr>
<tr>
<td>BCI+CE</td>
<td>14.51</td>
<td>14.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Benchmark for Lab. Inc. Taxes</th>
<th>Variable Labour Inc. Tax Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCI</td>
<td>31.41</td>
<td>30.34</td>
</tr>
<tr>
<td>CE</td>
<td>8.70</td>
<td>9.65</td>
</tr>
<tr>
<td>BCI+CE</td>
<td>14.55</td>
<td>15.33</td>
</tr>
</tbody>
</table>

Compared to the benchmark scenario, the decline in aggregate output due to the higher tax rates caused by bank capital injections is roughly equal to 5% for consumption and 1% for labour income taxes. The increase in aggregate output, due to the lower tax rates induced by credit easing is equal to 0.5% and 1%, when consumption and labour income taxes are adopted, respectively. With distortionary taxes, the level of aggregate output obtained under bank capital injections is roughly 20% higher than the level obtained under credit easing. Under the benchmark scenario, this increase is equal to 26% with consumption and 23% with labour income taxes. Hence, our findings point out the importance of using alternative fiscal policy instruments, when examining and comparing the net benefits of different unconventional policies.

4.3.3 Credit Policies and Welfare

Finally, we present a welfare analysis of alternative credit policies. As explained in Schmitt-Grohe and Uribe (2007), it is now common to calculate the welfare using a
second order approximation to the utility function in New Keynesian DSGE models. We follow their methodology in our welfare calculations. First, we write the household’s utility function recursively,

\[ V_t = U_t(C_t, C_{t-1}, L_t) + \beta E_t V_{t+1} \]  

(4.40)

We then take a second order approximation of \( V_t \) around the steady state and a second order approximation of the whole model. Using the second order solution, we compute the value of \( V_t \), which gives the welfare loss under the set out policy alternatives. While calculating the welfare losses, we use the values of the policy parameters \((\rho_y, \rho_T, \rho_\varphi)\) that optimize \( V_t \) in response to the capital quality shock. By taking the difference of the values of \( V_t \) obtained under the monetary policy rule only and each credit policy alternative, we can find the welfare gains from using each alternative. Then computing the fraction of the steady state consumption required to equate welfare under the monetary policy rule, to the one under each credit policy alternative gives the consumption equivalent (CE) for that policy. We present the CE obtained under each credit policy alternative for two scenarios: lump-sum taxation (benchmark scenario) and distortionary taxation. In Table 4.3, we use consumption taxes, while in Table 4.4 we employ labour income taxes to compare with the benchmark scenario. The second column in each table corresponds to the scenario where the fiscal authority adopts distortionary taxes and uses government spending to respond to the changes in the government budget constraint. The third column displays the scenario where the tax rates are used to accommodate the changes in the fiscal balance. In each table, we assume that the steady-state level of lump-sum taxes is equal to the steady-state tax revenues obtained with the use consumption or labour income taxes.
We first observe that the use of bank capital injections generates higher welfare gains than the use of credit easing, under all scenarios. When both credit policies are in place, the optimal coefficient of the policy rule used for determining the quantity of credit easing equals zero. Hence, the funds obtained by the issuance of government bonds are fully allocated to bank capital injections. We then compare the welfare gains obtained under different scenarios to the ones obtained under lump-sum taxation. Looking at Table 4.3 column 2, we see that when the consumption tax rate remains constant, the existence of distortionary taxes does not cause a change in the welfare gains. When labour income taxes are in place and the tax rate does not change (Table 4.4 Column 2), welfare gains also remain roughly the same under both policies. Finally, looking at the 3rd columns in both tables, we see that the use of tax rates to accommodate the changes in the fiscal balance causes a decrease in the welfare gains obtained under both unconventional policies. Use of a variable consumption tax rate is preferred to a variable labour income tax rate when the government injects equity into banks. However, the opposite holds when the central bank pursues credit easing.
To summarize, four main points emerge from our welfare results: (i) bank capital injections are more effective in mitigating the negative effects of financial shocks to the economy, as they stabilize credit spreads more, (ii) existence of lump-sum taxes, in place of variable tax rates over-estimates the gains from pursuing credit market interventions, (iii) in the presence of distortionary taxes, use of government spending, rather than tax rates, to respond to the changes in the government’s budget are preferred and finally (iv) under distortionary taxation, the benefits from using credit policies still outweigh their costs.

4.3.4 Robustness Checks

When reporting the welfare gains obtained under the two credit policies, we use the baseline calibration that has been presented in Section 4.3.1. To check the robustness of our results, we re-calculate the welfare gains obtained under bank capital injections and credit easing, changing the values of five key parameters: the efficiency costs of government intermediation \( \tau^S = t^N \), bank leverage at the steady state \( \phi \), habit persistence \( \chi \), degree of price stickiness \( \theta \) and the steady state values for the tax rates \( t^c \) and \( t^h \). In each figure, we plot the CE(%) obtained with different values of a given parameter. The figure consists of two graphs. The values of the CE in the first plot are obtained for labour income taxes, while the values in the second are obtained for consumption taxes. In each plot, CEs are given under bank capital injections (BCI) and credit easing (CE), with lump-sum taxes (T) or variable tax rates (t).

Figure 4.6 presents the results for the given alternatives, using a range of values for the efficiency costs. As efficiency costs of credit policies increase, welfare gains
from using them decrease, as expected. Our findings are robust to the changes in the efficiency costs. In general, bank capital injections with lump-sum or distortionary taxes, generate higher welfare gains than credit easing. In addition, use of distortionary taxes decreases the gains from the use both credit policies, compared to lump-sum taxes. When efficiency costs are quite high at the level of 100 basis points, use of credit policies still result in welfare gains, but the gains are quantitatively small.

Figure 4.6. Welfare Gains for Different Values of $\tau^S = t^N$

In Figure 4.7, we demonstrate the welfare gains obtained with different values of the steady state bank leverage, under the given scenarios. An increase in the steady state bank leverage corresponds to a decrease in the steady state fraction of bank assets that can be diverted. A decline in the fraction of assets relaxes the bank’s constraint.
and enables the bank to accumulate a higher level of leverage. As the leverage ratio increases, negative effects of the shocks to the economy are amplified more. Hence, use of credit policies results in higher benefits, as shown in the figure. When labour income taxes are in place, the results that we have obtained in our welfare calculations hold for most values: bank capital injections generate higher welfare gains than credit easing under both lump-sum and distortionary taxes. In addition, introduction of distortionary taxation, reduces the welfare gains from the use of both policies. On the other hand, when distortionary consumption taxes are used, bank capital injections generate lower welfare gains than credit easing, for sum values of $\phi$.

Figure 4.7. Welfare Gains for Different Values of $\phi$

![Graph showing welfare gains](image)

Figure 4.8 displays the welfare gains for different values of habit persistence. As habit persistence increases, consumption smoothing improves welfare more. Since the
use of the credit policies considered in our study decreases the volatility of consumption, their contributions to the increase in welfare are higher for higher values of the habit persistence parameter. Our results are robust to the changes in the habit parameter. Bank capital injections result in higher welfare gains and the introduction of distortionary taxes reduces welfare, for all the values considered.

Figure 4.8. Welfare Gains for Different Values of $\chi$

In Figure 4.9, we present the welfare gains for different values of the probability that firms will keep prices constant. We see that regardless of the value of $\theta$, bank capital injections generate higher welfare gains than credit easing, under both lump-sum and distortionary taxes. However, with distortionary consumption taxes, the difference between the welfare gains of the two policies is quite small, for relatively low values of the price rigidity parameter.
Finally, Figure 4.10 presents the welfare gains obtained for different values of the steady state consumption or labour income tax rate. Since we assume that the steady-state level of lump-sum taxes is equal to the steady-state tax revenues obtained with the use consumption or labour income taxes, we adjust the steady state lump-sum tax level according to each value of the tax rate considered. When lump-sum taxes are in place, our results are robust to different steady state tax levels. With distortionary consumption taxes, whether the use of credit easing results in lower welfare gains than the use of bank capital injections, is dependant on the steady state tax rate.
Examining the results of our robustness checks, we can conclude that when labour income taxes are in place, our results are robust to different parameter specifications with both lump-sum and distortionary taxes. In general, bank capital injections induce higher welfare gains than credit easing, and introduction of the distortionary labour income tax rate decreases the welfare gains of both unconventional policies. However, in the presence of the distortionary consumption tax rate, the unconventional policy that generates the higher welfare gains varies with different values of the steady state leverage ratio and the steady state tax rate.
4.4 Conclusions

In this paper, we provide a comprehensive evaluation of unconventional monetary and fiscal policies; namely credit easing and bank capital injections. We conduct our analysis utilizing a New Keynesian DSGE model that contains a banking sector with financial frictions and a fiscal authority which can obtain revenues with the use of alternative fiscal policy instruments.

Our simulation results indicate that when the economy experiences a financial shock, the use of both credit policies mitigates the negative effects of the given shock to the economy. The shock causes a decrease in asset prices, which triggers the financial accelerator mechanism. Since banks are leveraged, the decline in asset prices results in an amplified decline in their net worth, which increases their leverage ratios. The rise in the leverage ratios increases the credit spread, which in turn, results in an increase in the cost of capital and a further decline in investment and asset prices. Finally, the decrease in investment leads to a decline in aggregate output. When credit easing or bank capital injections are employed, the rise in the spread is lower. Consequently, the negative effects of the financial shock on assets prices, investment and output are lower. Compared to credit easing, use of bank capital injections has a higher positive effect on aggregate output, since it results in a direct increase in banks’ net worth, which, significantly lowers the increase in the spread. However, use of equity injections causes an increase in the expenditures of the government, as opposed to an increase in the government revenues obtained with credit easing. Hence, when the government injects capital into banks, taxes increase or government spending decreases.

Our quantitative results include the computation of welfare losses and consumption
equivalents under each policy alternative, using welfare-maximizing monetary and credit policy rules. Our first finding is that the utilization of both unconventional policies generates welfare gains. Use of lump-sum taxes, in place of distortionary taxes overestimates the gains from pursuing credit market interventions, only if the tax rate is used to accommodate the changes in the fiscal balance. However, in the presence of distortionary taxes, use of government spending to adjust to the changes in the government budget constraint roughly results in the same welfare gains. Our second finding is that when bank capital injections are used by the government, welfare gains are higher compared to the use of credit easing by the central bank. This result holds under both lump-sum taxes and variable tax rates.

Before we conclude, it should be mentioned that our results rely on the assumption that the government does not default on its debt. If government bonds become subject to default risk, the government would have limited ability to conduct unconventional policy and this would, in turn, affect the valuation of government bonds. We believe that analyzing these issues should be a subject of further research.
Chapter 5

Conclusion

This thesis contributes to a variety of debates on optimal policy, using alternative New Keynesian DSGE frameworks. The research questions that we aim to answer are as follows: (i) are there differences among optimal monetary policies in open and closed economies?, (ii) are macroprudential policy tools effective in ensuring the stability of the financial system?, (iii) is there a leading macroprudential policy tool for addressing a certain financial market externality?, (iv) are unconventional policy measures effective in mitigating the negative effects of financial crises and finally (v) what are the differences between the fiscal implications of alternative unconventional policies?

We aim to answer question (i) in Chapter 2. In our analysis, we utilize the small open economy framework developed by Gali and Monacelli (2005) and generalize their special setting, where the coefficient of relative risk aversion ($\sigma$), the elasticity of substitution between home and foreign goods ($\eta$), and the elasticity of substitution between foreign goods from different origins ($\gamma$) are assumed to be equal to 1. We find that using a generalized parameterization has several important implications. Under the general
setting, welfare losses in the open economy depend on the variance of the terms of trade, in addition to the variance of consumption and inflation. Consequently, monetary policy for the open economy is not isomorphic to the one for the closed economy. As the fluctuations in the terms of trade are also important in the open economy, it is plausible to add domestic inflation and terms of trade based Taylor rules to the comparison of alternative monetary policies. Increasing the value of $\sigma$, $\eta$ or $\gamma$ by one unit and computing the welfare losses obtained under a number of simple policy rules, we observe that the relative importance of the variation in the terms of trade increases, while that of domestic inflation decreases, with an increase in the values of the aforementioned parameters. As a result, the domestic inflation and the terms of trade based Taylor rule turns out to be the preferred policy rule in the small open economy. Exploring the change in the welfare losses obtained with an increase in the value of each parameter, we see that this policy rule becomes the preferred rule with a slight and a moderate increase in $\gamma$ and $\eta$, respectively. The same result holds with a significant increase in $\sigma$. Hence, our results indicate that in open economies with high elasticities of substitution between domestic and foreign goods and among different foreign goods, monetary authorities should react to the movements in both domestic inflation and the terms of trade.

Next, in Chapter 3, we answer questions (ii) and (iii) by utilizing a Gertler and Karadi (2011) type DSGE model with financial frictions. Using this framework, we present a comparative analysis of three macroprudential policy tools, which are reserve requirements, capital requirements and a regulation premium. In our model, the financial accelerator mechanism built in banks’ borrowing constraints contains a pecuniary externality, where bankers do not internalize the benefits of equity financing and ac-
cumulate high levels of leverage. As a result, the negative effects of exogenous shocks to the economy are magnified through the following process: when a shock results in a decline in asset prices, banks' net worth decrease in an amplified manner due to the high leverage ratio. This corresponds to a downturn in banks' balance sheets, which increases their leverage ratios and pushes up the spread. The increase in the spread, in turn, increases the cost of capital and causes a further decrease in investment and asset prices. Finally, the decrease in investment lowers aggregate output. All of the aforementioned macroprudential tools are effective in mitigating the negative effects of exogenous shocks to the economy and capital requirements perform the best in lowering the negative effects of the financial accelerator mechanism built in banks' borrowing constraints. This result is due to their superior performance in lowering the negative effects of the given shocks to the spread, asset prices and investment. Comparing the welfare implications of the macroprudential tools used in our study, we also find that the use of capital requirements generates the highest welfare gains, irrespective of the source of the decline in economic activity. Since the financial accelerator mechanism in our framework works through the banking sector's high level of leverage and capital requirements directly target the capital (inverse of the leverage) ratio of banks, we believe that our findings are not counter-intuitive. In addition, they indicate the importance of determining the externalities in the financial markets and using the most effective macroprudential tool in addressing each externality.

Finally, we aim to answer questions (iv) and (v) in Chapter 4. To do so, we present a detailed evaluation of unconventional monetary and fiscal policies; namely credit easing and bank capital injections. We conduct our analysis by adding a fiscal authority, which can obtain revenues with the use of alternative fiscal policy instruments, to the
framework that we develop in the previous chapter. As explained in Chapter 3, when an exogenous shock affects the model economy and brings about a decrease in asset prices, the financial accelerator mechanism is triggered. As a result, the credit spread and the cost of capital increases, due to the banking sector’s high level of leverage. Consequently, asset prices, investment and aggregate output decrease. The use of credit easing or bank capital injections lowers the rise in the spread and hence, mitigates the negative effects of exogenous shocks to the economy. Utilization of bank capital injections lowers the decrease in aggregate output more, as it results in a direct increase in banks’ net worth, which induces a significant decline in the increase in the spread. On the other hand, use of equity injections causes a rise in government expenditures, in contrast to the increase in government revenues, when credit easing is used. As a result, when the government injects equity into banks, taxes should increase or government spending needs to decrease. Since the costs and the benefits of bank capital injections are both higher than that of credit easing, it is important to compute the welfare losses obtained under each policy alternative. Our welfare results indicate the following findings: (i) the use of both type of unconventional policies generates welfare gains, (ii) the presence of lump-sum taxes, in place of distortionary taxes over-estimates the gains from pursuing unconventional policies, only if the tax rate is used to adjust to the changes in government’s budget. On the contrary, when distortionary taxes exist, use of government spending to respond to the changes in the fiscal balance roughly induces the welfare gains obtained under lump-sum taxes. Finally (iii) when bank capital injections are used, welfare gains are higher compared to the use of credit easing, under both lump-sum taxes and variable tax rates. Consequently, we anticipate that our results indicate a higher relative importance of distortions in financial markets, compared to the distortions caused by variable tax rates.
As we have previously mentioned, the literature on the effectiveness of macroprudential and unconventional policy tools is growing rapidly. However, most of the papers in the literature feature a closed economy model and hence, neglect an important dimension related to macroprudential policies: the regulatory arbitrage that arises when the use of macroprudential tools is not coordinated across borders. Coordination of regulatory standards would prevent a regulatory race-to-the-bottom, where countries free-ride on the soundness of foreign banking systems (Bengui, 2014). Aiyar et al (2014) show that the “regulatory leakage” can be substantial. According to their study, UK-owned banks decrease lending in response to higher capital requirements but this effect is partially offset by an increase in lending from resident foreign branches. The increase in lending is about one-third of the initial response to the regulatory change. National policies that ignore these transmission channels and do not internalize the international effects may result in an insufficient degree of stabilization in the face of adverse financial shocks. For this reason, we believe that combining the frameworks that we used in the second and the third chapter, to build a two-country DSGE model that contains financial frictions as described in Gertler and Karadi (2011), would be quite interesting. Utilizing this framework, we can analyze the implications of introducing domestic macroprudential policies in open economies and examine the consequences of regulatory arbitrage that arises when the use of macroprudential tools is not coordinated across borders. We aim to address these issues in further research.
APPENDIX A. DERIVATION OF THE WELFARE LOSS FUNCTION
UNDER THE GENERAL CASE

The second order Taylor expansion of $U_t$ around the steady state $(C, N)$ gives

$$U_t - U \simeq U_C \left( \frac{C_t - C}{C} \right) + U_N \left( \frac{N_t - N}{N} \right) + \frac{1}{2} U_{CC} \left( \frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{NN} \left( \frac{N_t - N}{N} \right)^2$$

(A.1)

where $U_t \equiv U(C_t, N_t)$ and $U \equiv U(C, N)$.

First, using the following second-order approximation of log deviations,

$$\frac{K_t - K}{K} \simeq \hat{k}_t + \frac{1}{2} \hat{k}_t^2$$

(A.2)

where $\hat{k}_t \equiv k_t - k$ is the log deviation from the steady state for a generic variable $k_t$, gives

$$U_t - U \simeq U_C \left( \hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t \right) + U_N \left( \hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)$$

Here, $\sigma \equiv -\frac{U_{NC}}{U_{CC}}$ and $\varphi \equiv \frac{U_{NN}}{U_{CC}}$.

Second, using the fact that $N_t = \left( \frac{N_t}{\hat{N}_t} \right) \int_0^1 \left( \frac{P_{H,t}(\iota)}{P_{H,t}} \right)^{-\epsilon} \, d\iota$ to obtain

$$\hat{n}_t = \hat{y}_t - a_t + z_t$$

with $z_t = \log \int_0^1 \left( \frac{P_{H,t}(\iota)}{P_{H,t}} \right)^{-\epsilon} \, d\iota$, and combining the following expression with $y_t = c_t + \frac{\alpha}{\sigma} s_t$, yields the relationship between (the log deviation of) labour, consumption and the terms of trade,

$$\hat{n}_t = \hat{c}_t + \frac{\alpha \omega}{\sigma} \hat{s}_t - a_t + z_t$$

(A.3)
Lemma 1 \( z_t = \frac{1}{2} \text{var}_i \{p_{H,t}(i)\} + o(\|a\|^2) \). \textbf{Proof:} Gali and Monacelli, 2005.

As a result, the period utility \( t \) can be rewritten as

\[
U_t - U = U_c C \left( \hat{c}_t + \frac{1 - \sigma}{2} \tilde{c}_t^2 \right) + U_n N \left( \hat{c}_t + \frac{\alpha \omega}{\sigma} \hat{s}_t + \frac{\epsilon}{2} \text{var}_i \{p_{H,t}(i)\} \right) + U_n N \left( \frac{1 + \varphi}{2} \left( \hat{c}_t + \frac{\alpha \omega}{\sigma} \hat{s}_t \right)^2 \right) + \text{t.i.p.} \tag{A.4}
\]

where \( \text{t.i.p.} \) stands for terms independent of policy.

Using the optimality condition for the social planner, \( \frac{U_n}{U_c} = \frac{1 - \alpha}{1 - \alpha + \alpha \omega} \frac{C}{N} \), the expression above can be written as,

\[
\frac{U_t - U}{U_c C} = \hat{c}_t + \frac{1 - \sigma}{2} \tilde{c}_t^2 - \frac{1 - \alpha}{1 - \alpha + \alpha \omega} \left( \hat{c}_t + \frac{\alpha \omega}{\sigma} \hat{s}_t + \frac{\epsilon}{2} \text{var}_i \{p_{H,t}(i)\} \right) - \frac{1 - \alpha}{1 - \alpha + \alpha \omega} \left( \frac{1 + \varphi}{2} \right) \left( \hat{c}_t^2 + \left( \frac{\alpha \omega}{\sigma} \right)^2 \hat{s}_t^2 + 2 \frac{\alpha \omega}{\sigma} \hat{c}_t \hat{s}_t \right) + \text{t.i.p.}
\]

\[
= - \frac{1}{2} \left[ (1 + \varphi) \zeta - (1 - \sigma) \right] \tilde{c}_t^2 - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{(1 + \varphi) \zeta}{2} \hat{s}_t^2 - \frac{\epsilon \zeta}{2} \text{var}_i \{p_{H,t}(i)\} + (1 - \zeta) \hat{c}_t - (1 - \zeta) \frac{1 - \alpha}{\sigma} \hat{s}_t - (1 - \zeta) \frac{(1 - \alpha)(1 + \varphi)}{\sigma} \hat{c}_t \hat{s}_t + \text{t.i.p.} \tag{A.5}
\]

where \( \zeta = \frac{1 - \alpha}{1 - \alpha + \alpha \omega} = 1 - \Upsilon \) and \( \Upsilon \) denotes the size of the steady state distortion.

Accordingly, the second-order approximation to welfare losses of the representative consumer in the small open economy can be expressed as a fraction of steady state consumption (and up to additive t.i.p.) as

\[
W' = \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U}{U_c C} \right) \tag{A.6}
\]

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Lemma 2 $\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_{H,t}(i)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi^2_{H,t}$. \textbf{Proof:} Woodford, 2003, Chapter 6.

Merging Lemma 2 with the expression for the welfare losses above, it can be seen that,

$$W' = -\frac{1}{2} \left[ (1 + \varphi) \zeta - (1 - \sigma) \right] \sum_{t=0}^{\infty} \beta^t \hat{c}^2_t - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{(1 + \varphi) \zeta}{2} \sum_{t=0}^{\infty} \beta^t \hat{s}^2_t - \frac{\epsilon \zeta}{2\lambda} \sum_{t=0}^{\infty} \beta^t \pi^2_{H,t}$$

$$(1 - \zeta) \left[ \sum_{t=0}^{\infty} \beta^t \hat{c}_t - \frac{(1 - \alpha)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{s}_t - \frac{(1 - \alpha)(1 + \varphi)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{c}_t \hat{s}_t \right]$$

The following expression for the welfare losses can be rewritten using the distortion in the steady state, $\Upsilon$ as follows,

$$W' = -\frac{1}{2} \left[ (1 + \varphi) \zeta - (1 - \sigma) \right] \sum_{t=0}^{\infty} \beta^t \hat{c}^2_t - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{(1 + \varphi) \zeta}{2} \sum_{t=0}^{\infty} \beta^t \hat{s}^2_t - \frac{\epsilon \zeta}{2\lambda} \sum_{t=0}^{\infty} \beta^t \pi^2_{H,t}$$

$$+ \Upsilon \left[ \sum_{t=0}^{\infty} \beta^t \hat{c}_t - \frac{(1 - \alpha)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{s}_t - \frac{(1 - \alpha)(1 + \varphi)}{\sigma} \sum_{t=0}^{\infty} \beta^t \hat{c}_t \hat{s}_t \right]$$

(A.7)

Finally, taking unconditional expectations on $W'$, letting $\beta \to 1$ and using the case of small distortions in the steady state, which eliminates the linear terms in $W'$ and gives the central bank’s problem the convenient quadratic format (Gali 2008, Chapter 5), the expected welfare losses under a generic case can be approximated by

$$L' \approx -\frac{1}{2} [(1 + \varphi) \zeta - (1 - \sigma)] \text{var}(c_t) - \left( \frac{\alpha \omega}{\sigma} \right)^2 \frac{(1 + \varphi) \zeta}{2} \text{var}(s_t) - \frac{\epsilon \zeta}{2\lambda} \text{var}(\pi_{H,t})$$

(A.8) corresponds to equation (2.36) in the main text.

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APPENDIX B.1 THE COMPETITIVE EQUILIBRIUM

A competitive equilibrium of the model economy is defined by sequences of allocations, prices, shock processes and the government policy (without the use of any macroprudential tools) that satisfy the following optimality and market clearing conditions,

Consumption Euler Equation
\[ U_{C,t} = \beta R_{t+1} E_t [U_{C,t+1}] \]

Labour Supply
\[ \frac{U_{h,t}}{U_{C,t}} = -W_t \]

Wholesale Output
\[ Y^W_t = (A_t h_t)^\alpha K_{t-1}^{1-\alpha} \]

Labour Demand
\[ \frac{P^W_{t}}{P_t} Y^W_{h,t} = W_t \]

Return to Capital
\[ R_{k,t+1} = (1-\alpha) \frac{P^W_{t+1} Y^W_{t+1}}{P_{t+1} K_{t+1}^{1-\delta}} \]

Capital Accumulation
\[ K_t = [(1-\delta) K_{t-1} + (1-S(X_t)) I_t] \]

Investment & Asset Prices
\[ Q_t (1-S(X_t) - X_t S'(X_t)) + E_t [\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2] = 1 \]

Price Dispersion in the Retail Sector
\[ \Delta t = \int_0^1 \left( \frac{P(f)}{P_t} \right)^{-\zeta} df \]

Retail Output
\[ Y_t = (1-c) \frac{Y^W_t}{\Delta} \]

Optimal Relative Price
\[ \frac{P^a_t}{P_t} = \frac{F_t}{H_t} \]

Price Dispersion & Inflation
\[ \Delta_t = \theta \Pi_t^\zeta \Delta_{t-1} + (1-\theta) \left( \frac{F_t}{H_t} \right)^{-\zeta} \]

Non-linear Phillips Curve
\[ H_t - \theta \beta E_t \left[ \Pi_{t+1}^{\zeta} H_{t+1} \right] = Y_t U_{C,t} \]
\[ F_t - \theta \beta E_t \left[ \Pi_{t+1}^{\zeta} F_{t+1} \right] = \mu Y_t U_{C,t} M C_t \]
\[ \theta \Pi_t^{\zeta-1} + (1-\theta) \left( \frac{F_t}{H_t} \right)^{1-\zeta} = 1 \]

Fisher Equation
\[ R_{n,t-1} = R_t E_t \Pi_t \]

Taylor Rule
\[ \log \left( \frac{R_{n,t}}{R_n} \right) = \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \]
<table>
<thead>
<tr>
<th>Financial Relationship</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Budget Constraint</td>
<td>$G_t = T_t$</td>
</tr>
<tr>
<td>Output Equilibrium</td>
<td>$Y_t = C_t + I_t + G_t$</td>
</tr>
<tr>
<td>Loans to Non-financial Firms &amp; Capital</td>
<td>$S_t = K_t$</td>
</tr>
<tr>
<td>Deposits Held at Banks</td>
<td>$D_t = Q_t S_t - N_t$</td>
</tr>
<tr>
<td>Accumulation of Bank Net Worth</td>
<td>$N_t = R_{k,t} (\gamma + \varepsilon) Q_{t-1} S_{t-1} - \gamma R_t D_{t-1}$</td>
</tr>
<tr>
<td>Marginal Value of Deposits</td>
<td>$v_{d,t} = E_t A_{t,t+1} \eta_{t+1} R_{t+1}$</td>
</tr>
<tr>
<td>Excess Marginal Value of Loans</td>
<td>$\mu_{s,t} = E_t A_{t,t+1} \eta_{t+1} (R_{k,t+1} - R_{t+1})$</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>$\phi_t = \frac{v_{d,t}}{\Theta - \mu_{s,t}}$</td>
</tr>
<tr>
<td>Banking Sector Balance Sheet</td>
<td>$Q_t S_t = \phi_t N_t$</td>
</tr>
<tr>
<td>The Spread</td>
<td>$spread = R_{k,t} - R_t$</td>
</tr>
</tbody>
</table>

$^1 c = \text{cost of converting wholesale output to retail output.}$

$^2 \mu = \text{steady state mark-up.}$

**APPENDIX B.2 DATA SOURCES**

This appendix presents the details of the data sources used to construct Table 3.2 in the main text. All the time series of the nominal macroeconomic and financial variables are deflated using the GDP deflator.

- GDP Deflator: Bureau of Economic Analysis (BEA), NIPA Table 1.1.9. Implicit Price Deflators for Gross Domestic Product.
- Consumption: BEA, NIPA Table 1.1.5. Personal Consumption Expenditures.
• Investment: BEA, NIPA Table 1.1.5. Gross Private Domestic Investment.

• Government Spending: BEA, NIPA Table 1.1.5. Government Consumption Expenditures.

• Gross Domestic Product: BEA, NIPA Table 1.1.5. Sum of Consumption, Investment and Government Spending.


• Bank Assets: Federal Reserve Board (FRB), Data Download Program of Statistical & Historical Database. Bank Credit at the Asset Side of the U.S. Commercial Banks’ Balance Sheet.

• Deposits: FRB, Data Download Program of Statistical & Historical Database. Deposits Held at the U.S. Commercial Banks.

• Bank Net Worth: FRB, Data Download Program of Statistical & Historical Database. Bank Credit minus Deposits.

• Leverage Ratio: FRB, Data Download Program of Statistical & Historical Database. Ratio of Bank Credit to Net Worth.

• Spread: Federal Reserve Bank of St. Louis. Moody’s Seasoned BAA Corporate Bond Yield minus Effective Federal Funds Rate.
APPENDIX C.1 HOUSEHOLD’S OPTIMIZATION PROBLEM WITH LUMP-SUM TAXES

A representative household maximizes expected discounted utility,

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, C_{t-1}, L_t)
\]  

subject to her budget constraint,

\[
C_t = W_t h_t + \Pi_t + R_t D_{t-1} - D_t - T_t
\]  

Here, \(0 < \beta < 1\) is the subjective discount factor, \(E\) is the expectation operator, \(C_t\) denotes consumption and \(L_t\) leisure, \(W_t\) the wage rate, \(h_t(= 1 - L_t)\) hours worked and \(D_t\) bank deposits. \(\Pi_t\) are the profits earned from the ownership of banks and firms and \(T_t\) denotes lump-sum taxes/transfers.

The solution to the maximization problem is given by,

\[
1 = R_{t+1} E_t [A_{t,t+1}]
\]  

\[
\frac{U_{h,t}}{U_{C,t}} = -\frac{U_{L,t}}{U_{C,t}} = -W_t
\]

where \(A_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}}\) is the real stochastic discount factor over the interval \([t, t+1]\).
APPENDIX C.2 OPTIMAL INVESTMENT WITH INVESTMENT COSTS

Capital producing firms maximize their expected discounted profits with respect to $I_t$,

$$E_t \sum_{k=0}^{\infty} \Lambda_{t, t+k} \left[ Q_{t+k} \left( 1 - S \left( I_{t+k} / I_{t+k-1} \right) \right) I_{t+k} - I_{t+k} \right] \quad (C.2.1)$$

where $\Lambda_{t, t+k} = \beta^k \frac{A_{C,t+k}}{A_{C,t}}$ is the real stochastic discount factor over the interval $[t, t+k]$. The first two terms in (C.2.1) are given by

$$E_t [\Lambda_{t, t} [Q_t (1 - S (I_t / I_{t-1})) I_t - I_t] + \Lambda_{t, t+1} [Q_{t+1} (1 - S (I_{t+1} / I_t)) I_{t+1} - I_{t+1}]]$$

Since $\Lambda_{t, t} = 1$, the first order condition with respect to $I_t$ is obtained as

$$E_t [Q_t (1 - S (I_t / I_{t-1})) - 1 - Q_t S' (I_t / I_{t-1}) \frac{I_t}{I_{t-1}} \left( -\frac{I_{t+1}}{I_t^2} I_{t+1} \right)]$$

$$- \Lambda_{t, t+1} Q_{t+1} S' (I_{t+1} / I_t) \left( -\frac{I_{t+1}}{I_t^2} I_{t+1} \right) \quad (C.2.2)$$

Inserting $X_t \equiv I_t / I_{t-1}$ gives the first order condition in the main text,

$$Q_t (1 - S (X_t) - X_t S' (X_t)) + E_t \left[ \Lambda_{t, t+1} Q_{t+1} S' (X_{t+1}) X_{t+1}^2 \right] = 1 \quad (C.3.3)$$
APPENDIX C.3 DYNAMICS OF PRICE DISPERSION

Since the demand equations forming aggregate investment and government expenditures have the same functional form as aggregate consumption,

\[ I_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} I_t; G_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} G_t \]  \hspace{1cm} (C.3.1)

the equilibrium for good \( f \) is given by

\[ Y_t(f) = (1-c)A_t h_t(f) \left( \frac{K_{t-1}(f)}{Y_t^W(f)} \right)^{\frac{1-\alpha}{\alpha}} = (C_t + I_t + G_t) \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} \]  \hspace{1cm} (C.3.2)

where \( Y_t^W(f), h_t(f), K_{t-1}(f) \) are the quantities of output, hours and capital used in the wholesale sector for the production of good \( f \) in the retail sector. Using the facts that \( h_t = \int_0^1 h_t(f) df \) and the capital-labour ratio stays constant while integrating over \( f \), we have

\[ Y_t = \frac{(1-c)Y_t^W}{\Delta_t} \]  \hspace{1cm} (C.3.3)

as in the main text, where \( \Delta t = \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\zeta} df \) is the measure of price dispersion.
References


