PREDICTION OF INTERIOR DAYLIGHT
UNDER CLEAR SKY CONDITIONS

Khalid Asker Alshaibani

A Thesis submitted for the degree of
Doctor of Philosophy

School of Architectural Studies

November 1996
ACKNOWLEDGEMENTS

It is with great appreciation that I thank my supervisor, Prof. Peter R. Tregenza, for his encouragement and guidance during the process of this thesis.

Thanks are due to Dr. Steve Sharples for his comments and advice. I would like also to thank several persons in the School of Architectural Studies at the University of Sheffield who provided help and assistance during the progress of this thesis. I would like to thank Mrs. Hazel Hall, Prof. Tregenza's Secretary, Melvyn Broady, Chief Technician, Jules Alexandrou, Peter Williams, and Lois Burt & Elaine Ashton from the School Library.

My thanks are also due to Dr. Ibrahim Al-Naimi, Dr. Mohammed Y. Numan, Dr. Abdulaziz Alssati and Mr. Javid from King Faisal University in Saudi Arabia.

Finally I would like to thank Ms. Lois Burt for reviewing the English of this study.
Most available techniques for predicting internal daylight illuminance do not take into account reflected sunlight, nor the fact that under clear sky conditions the direction of the illuminance is usually upwards, not downwards from the sky.

The general goal of this study is to investigate the issue of predicting the internal illuminance from natural light in clear sky conditions. This includes the possibility of proposing a method based on the concept of the average daylight factor for use in sunny climates.

This thesis is divided into eight chapters. After introducing the problem in Chapter One, Chapter Two is a literature review of problems associated with utilising natural light in sunny regions.

Chapter Three is the statement of the problem and how it will be solved. Existing equations for finding the average daylight factor have one thing in common: they assume that the incident light on the window comes directly from an overcast sky or by external reflection from it. If any of these equations are to be used under clear sky conditions, or a new method is to be developed based on the same concepts, the sensitivity of average internal illuminance to the direction of external light needs to be tested. A study of this is described in Chapter Four.

Chapter Five tests, by numerical simulation, the performance of existing average daylight factor methods under clear sky conditions. It is concluded that they are not appropriate for sunny regions. The tests, and the conclusions from Chapter Four, do, however, suggest a new approach. This has two bases. The first is that it has been shown to be possible to relate incident light on the window plane to horizontal sky illuminance, and this sets a minimum condition for window design. In practice this can be used in conjunction with a limiting maximum window size based on heat gain and other environmental issues. The second basis is a new formula for relating average internal illuminance to external window plane illuminance. The overall result is a formula for predicting internal illuminance as a ratio of external horizontal sky illuminance.

In Chapter Seven this approach is tested. Two methods are used: field measurements under real sky conditions, and comparison with detailed calculations made with SUPERLITE. The proposed method is found to be reasonably accurate.
### TABLE OF CONTENTS

Acknowledgements
Summary
Table of Contents

#### CHAPTER ONE: INTRODUCTION
1.1 General
1.2 Natural light under sunny skies
1.3 The thesis and its objectives
1.4 Preview of the thesis

#### CHAPTER TWO: LITERATURE REVIEW
2.1 Introduction
2.2 Natural light in sunny regions
   2.2.1 Direct sunlight
   2.2.2 Skylight
   2.2.3 Reflected light
   2.2.4 Summary
2.3 Natural light prediction techniques
   2.3.1 The daylight factor method
      2.3.1.1 The sky component
      2.3.1.2 The internally reflected component
      2.3.1.3 The external reflected component
      2.3.1.4 The orientation factor
      2.3.1.5 Summary
   2.3.2 Modifications to the daylight factor method
   2.3.3 The average daylight factor method
   2.3.4 The lumen method
   2.3.5 Other methods
      2.3.5.1 Summary
   2.4 Discussions and conclusions
      2.4.1 Solar and sky illuminance
      2.4.2 Prediction techniques
7.2.1 Location and climate .......................................................... 141
7.2.2 Measurement of the natural light factor ............................... 144
7.2.3 Average natural light factor estimation .............................. 149
7.2.4 Measurements for the vertical to the horizontal ratio ......... 150
7.2.5 Results and analysis ...................................................... 152
7.3 Validation based on SUPERLITE analysis.................................... 156
  7.3.1 Comparison of equation 6.5 against SUPERLITE ............... 156
     7.3.1.1 Results and analysis ........................................ 157
  7.3.2 Validation of the vertical to horizontal ratio against
    SUPERLITE ..................................................................... 172
7.4 Conclusion ....................................................................... 175

CHAPTER EIGHT: CONCLUSION
8.1 General .............................................................................. 177
8.2 Main results and conclusions ..................................................... 179
8.3 Further research .................................................................... 181

APPENDIX A ........................................................................ 184
APPENDIX B ........................................................................ 197
APPENDIX C ........................................................................ 208
APPENDIX D ........................................................................ 210
REFERENCES ....................................................................... 213
CHAPTER ONE

INTRODUCTION
CHAPTER ONE: INTRODUCTION

1.1 General

Daylight has been utilised as a design element in buildings throughout history. One of the main reasons for using daylight in recent years is its benefit in reducing energy consumption through its use as a main or secondary illumination source in order to replace the use of electrical lighting.

The use of such lighting in office buildings usually accounts for 40 - 70% of total energy consumption. This percentage includes energy consumed in providing illumination, and that consumed in removing heat generated by such illumination (1, 2). In several studies concerning the use of daylight in office buildings, potential energy savings have been demonstrated. It has been shown that daylight can be used as an important tool for passive energy conservation (3-6).

However, daylight is not only an energy strategy,

"If energy conservation were the only criterion for success of a commercial building, a windowless underground building would be the ultimate energy-conscious design".*

Reasons for utilising daylight may vary from one designer to another. One main reason for using daylight is its physiological and psychological impact on people. According to some researchers (7), the most important benefit from the application of daylighting in a commercial building is not energy conservation, but increased occupant satisfaction. Collins (8), in a study of the psychological reaction to

windows reported in 1976, indicates the basic need for a window as a medium of contact with the outside for human beings. Collins stated that,

"while light is essential for vision, there is some possibility that its biological benefits may extend beyond this to the maintenance of health and well being. In addition to physiological benefits, natural illumination in the form of both daylight and sunshine adds desirable psychological qualities to an interior environment "*.

Other psychological and physiological benefits resulting from the admission of daylight are:

- the feeling of spaciousness which daylight adds to the room (8);
- the feeling of a living environment inside the space, because of the continuous change in daylight quantity and quality due to the movement of the sun and changes in the weather, which can not be achieved with artificial lighting;
- the sense of orientation within the surroundings, and connection with external weather conditions.

These valuable effects make daylight an aesthetic tool for the architect and a qualitative asset for the users of the building (9).

When designing for natural light under overcast sky conditions the designer can use simple, well established techniques, such as the daylight factor method and the average daylight factor method, to calculate the internal illuminance. However, in the case of sunny skies the issue is different.

---

1.2 Natural light under sunny skies

The design of buildings, in general, is influenced by several factors. Cultural, climatic, economical, social and technological factors are among the main ones. Although natural light could be a main determinant of the design of some buildings (9), it is not the main determinant of the shape and location of fenestration in such areas.

In hot arid regions, due to the high intensity of solar radiation, the design of buildings will be very much influenced by solar gain. The utilisation of natural light in such areas should be integrated with a range of thermal considerations that affect the geometry of a building. Problems associated with admitting natural light through windows in such regions are the effects of heat gain, glare and fading and deterioration of materials (10). Solving these problems requires careful consideration of climatic conditions and an understanding of the surrounding luminous environment.

The subject of planning and design in hot regions has been addressed by several researchers and recommendations have been proposed (11-14). These recommendations, mainly related to protection from the effects of solar radiation, have a major influence on designing for natural lighting. Proposed methods of protection from direct solar radiation may be classified into four categories:

1. Protection at the urban level

Buildings in such regions tend to be closely grouped together in order to shade each other and to provide shade for the nearby spaces. The compact layout of buildings helps to minimise the exposed area of surfaces to direct sun radiation (figure 1.1).
Figure 1.1 A compact layout of buildings gives some protection from direct solar radiation.


2. Reduction of window area

The size of the window is one of the key decisions in building design because of its effect on the interior visual, thermal and acoustic environment. Heat gain usually occurs more through glass than through a conventional wall of the same area. To minimise solar heat gain, windows in hot climates tend to be small in area (figure 1.2).
Figure 1.2 Another form of protection from direct sun radiation and heat gain is the use of small windows.


3. The use of shading devices

The selection and design of effective shading devices to satisfy thermal requirements is covered in detail by Victor Olgyay (15) and Olgyay and Olgyay (16). Devices can take several forms; horizontal, vertical and their combination. Another form of shading is the use of wooden screening devices which are popular in hot climates (figure 1.3). Such screening is a protection against direct solar radiation and reduces the problem of glare.
Figure 1.3 Wooden screening helps to reduce the problems of heat gain and glare.

Source: 
- a; El-Wakil Buildings in the Middle East, MIMAR, 1981. pp. 55.
- b; Taylor, B. B. University, Qatar. MIMAR 16, 1985. pp. 25.
4. **Choice of orientation**

In hot regions, orientation is a major issue from a thermal comfort standpoint. Movement of the sun across the sky will affect the quantity and angle of incidence of solar radiation on a vertical surface. Hence, different wall orientations will require different treatments of shading devices.

In general, the north and south facades receive less radiation gain if the appropriate shading device is used. Therefore, these are considered to be the best orientations from a thermal point of view in hot arid regions (17).

Openings on the west and east facades are not recommended due to the high heat gain and the difficulties in protection from direct solar radiation. In addition, it is recommended that these facades are to be as small in area as possible (17).

All the above forms of protection will affect the design and size of openings and the way of utilising natural light in sunny regions. The use of shading devices, screens, small windows, and the compact form of layout of buildings will minimise the exposure of windows to sky and will not allow for direct sunlight inside the rooms. These factors, in addition to the low level of brightness of the clear blue sky in a hot climate (18), indicate that the optimum source of natural light is reflected sunlight. This is reflected from the ground, opposite facades, shading devices and from the screens in front of the windows. Because these tend to have high reflectance, often in order to minimise internal heat gain, the intensity of the reflected light is also often high.

However, most available techniques for predicting internal daylight illuminance are not appropriate. They do not take into account reflected sunlight, nor the fact that the direction of the illuminance is usually upwards, not downwards from the sky.
Some of the available techniques for predicting the internal illuminance due to natural light consider the reflected sunlight from obstruction and nearby surface to be insignificant (as in the daylight factor method). Other techniques do not allow for the effect of the obstruction and assume that the window is unobstructed (as in the lumen Method).

Various techniques for expanding the Daylight Factor Method to include clear skies have been proposed (19-24). These techniques are either incomplete, too complex, or involved long periods of a designer's time (25). In addition, the role of the reflected component (reflected light from surfaces receiving direct sunlight) is not considered a major factor.

There are several computer programs such as 'SUPERLITE' which can be used to predict the internal illuminance from natural light under different sky conditions. However, these programs have been designed for use by researchers or designers working on large projects where a detailed analysis is required (26).

1.3 The thesis and its objectives

The general goal of this study is to investigate the issue of predicting the internal illuminance from natural light in clear sky conditions.

The objectives are as follows:

1. To evaluate the available calculation techniques in terms of their applicability for sunny climates.

2. To propose a method based on the concept of the average daylight factor to be used in sunny climates.
It is recognised that the utilisation of natural light must be integrated with other environmental factors. The aim of this research is to develop a method for analysing daylight which can be used in conjunction with basic thermal, airflow and acoustic analysis at the early design stages of a building.

1.4 Preview of the thesis

This thesis is divided into eight chapters. After introducing the problem in Chapter One, Chapter Two is a literature review of problems associated with utilising natural light in sunny regions. This includes a description of the solar and sky illuminance for sunny conditions and a review of the available prediction techniques.

The conclusion from the literature review is that the average daylight factor is the most simple method available for testing natural light conditions in a room, but that such a simple system is not available for clear sky conditions.

Chapter Three is the statement of the problem and how it will be solved. Existing equations for finding the average daylight factor have one thing in common: they assume that the incident light on the window comes directly, or by external reflection, from an overcast sky. If any of these equations are to be used under clear sky conditions, or a new method is to be developed based on the same concepts, the sensitivity of average internal illuminance to the direction of external light needs to be tested. A study of this is described in Chapter Four. It is found that the mean working plane illuminance does vary with the direction of light on the window but, when light from below the horizontal is separated from light from above, this variation is insignificant if direct sunlight is excluded.

Chapter Five tests, by numerical simulation, the performance of existing average daylight factor methods under clear sky conditions. It is concluded that they are not
appropriate for sunny regions. The tests, and the conclusions from Chapter Four, do, however, suggest a new approach. This has two bases. The first is that it is shown to be possible to relate incident light on the window plane to horizontal sky illuminance, and this sets a minimum condition for window design. In practice, this can be used in conjunction with a limiting maximum window size based on heat gain and other environmental issues. The second basis is a new formula for relating average internal illuminance to external window plane illuminance. The overall result is a formula for predicting internal illuminance as a ratio of external horizontal sky illuminance.

In Chapter Seven this approach is tested. Two methods are used; field measurements under real sky conditions, and comparison with detailed calculations made with SUPERLITE. The proposed method is found to be reasonably accurate.
CHAPTER TWO

LITERATURE REVIEW
CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

The aim of this chapter is to review existing methods for predicting interior daylight under clear sky conditions. This is to highlight problems associated with utilising natural light for hot regions, and to highlight the need for a new technique for such utilisation. This will include a description of the solar and sky illuminance for sunny conditions and a review of the available prediction techniques.

2.2 Natural light in sunny regions

Natural light reaching the interior of a room usually takes three forms: direct sunlight, direct skylight and reflected sunlight or skylight from nearby surfaces or from the ground. In order to utilise natural light, and in order to achieve the objective of good interior natural lighting, knowledge of the quantity and quality of the sources of natural light is necessary. The following discussion will involve an investigation into factors affecting the natural light under clear skies.

2.2.1 Direct sunlight

The amount of sunlight reaching a surface is affected by the geometrical relationship between the sun and the surface. With the exception of the two periods just after sunrise and just before sunset, direct normal illuminance values do not vary greatly during the hours of a particular day at a given location (18).
Direct sunlight in sunny climates is usually high. As an example, and based on equations 2.1 and 2.2 (27) which estimate the horizontal sun and sky illuminance as a function of solar altitude ($\gamma_s$), figure (2.1) shows that illuminance received on an unobstructed horizontal surface directly from the sun can reach up to seven times as high as illuminance from the sky under clear sky conditions.

Horizontal sun illuminance = $\sin \gamma_s \left[ 127.5 \exp \left( -\frac{0.21}{\sin \gamma_s} \right) \right]$ klx  \hspace{1cm} (2.1)

Horizontal sky illuminance = $0.8 + 15.5 \sqrt{\sin \gamma_s}$ klx \hspace{1cm} (2.2)

![Figure 2.1. Illuminance received on the horizontal from sun and sky.](image)

* There are several equations for estimating sun and sky illuminance. Different equations will give different results. However, the purpose here is to give only a general, rather than a detailed, picture.
This fact illustrates the importance of direct sunlight as a major source of natural light in sunny areas.

The direct illuminance reaching the earth is a function of several variables. It is influenced by the location of the sun, the ellipticity of the earth's orbit, atmospheric conditions (cloud cover, haze, dust etc.) and the length of the sun path through the atmosphere.

2.2.2 Skylight

Cloud cover in tropical arid climates is low. It usually covers less than 30% of the sky dome (28). Under such a sky, sunlight is intercepted by the molecules of the earth's atmosphere causing the wavelengths of light in the blue portion of the spectrum to be scattered, which produces the characteristic blue colour (skylight) of the clear sky (29). The luminance of such a sky is low and may be insufficiently bright to act as a major source of interior illumination (18).

The clear blue sky produces a far from uniform distribution of brightness. The brightest region of the sky is the zone around the sun. The darkest is the region which is at 90° from the sun in the plane of the solar azimuth (30). However, in general, the clear sky has a horizon brighter than the zenith (31, 32), except at high solar elevation.

The luminance distribution of the clear sky changes almost constantly due to the movement of the sun. However, the average luminance of the sky remains fairly constant throughout the day (18).
Figure 2.2 Clear Blue Sky Distribution model showing anisotropic brightness distribution due to the sun's position, with reference to azimuth and altitude


2.2.3 Reflected light

Sun is the main source of light. However, it is not necessarily the case that light reaching room surfaces through a window is directly from it. Sunlight and skylight will be modified and changed in term of quantity and direction before reaching a room surface. In hot arid climates, and when there is a choice between skylight and reflected light, skylight is not the most effective source of natural light. As was mentioned earlier, the luminance of the sky is low, and direct sunlight is not welcome due to problems of heat gain. Reflected sunlight seems to be the optimum option in such regions.
Light sources in this case (sunny regions) are usually the reflecting surfaces of buildings adjacent to the window wall. The sky luminance, compared with the luminance of adjacent surfaces of building and low-lying roofs receiving direct sunlight, is low (except around the sun). The difference may reach a factor of five (18). The luminance of the clear sky may be 3500-5000 cd/m² (33), when the luminance of the sunlit surfaces exceeds 25000 cd/m² (34).

It has been shown, for example, in a study by Hopkinson and Petherbridge reported in 1953 on the use of reflected sunlight (35), that under clear sky conditions, the use of such light alone can provide the required illumination level for the whole of the working day.

The amount of light received on any surface is influenced by location of the sun, the geometrical relationship with its surroundings, reflectance of external surfaces and global illuminance.

2.2.4 Summary

Window design is influenced by several factors, particularly, in arid climates, protection from direct solar radiation. Such protection (by the use of shading techniques or by reducing the window area) reduces light from the sky and excludes direct sunlight. Hence, a major source of natural lighting in hot dry regions is the reflected sunlight from surrounding surfaces, i.e., ground and external surfaces. Therefore, any method for predicting the internal illuminance in such regions should take into consideration the importance of the reflected sunlight.
2.3 Natural light prediction techniques

Daylight prediction techniques are methods which predict the daylight level at a given point in a room. Such techniques are used for designing openings and for analysing the natural light performance of an opening. Prediction of interior illuminance levels has been addressed by several researchers over the past years (29, 36) and several methods have been proposed. These methods can be grouped into two categories (37); methods that predict internal illuminance as a ratio of external illuminance (relative illuminance); methods that predict internal illuminance in quantity (absolute illuminance).

A review of methods proposed for use under clear sky conditions now follows.

2.3.1 The daylight factor method

The daylight factor method has been adopted by the C.I.E. (International Commission on Illumination) and is, therefore, internationally used in over 100 countries (36, 37).

By definition, the daylight factor (DF) is the ratio of the daylight illuminance on a given plane at a point (Ei), due to the light received directly or indirectly from the sky, to the simultaneous exterior illuminance on a horizontal plane (Edh), from the whole of an unobstructed sky of assumed or known luminance distribution. Direct sunlight is excluded from both values of illuminance (38).

The daylight factor is expressed as a percentage as follows:

\[
DF = \left( \frac{E_i}{E_{dh}} \right) \times 100
\]  

(2.1)
This method is usually based on the assumption that the sky is overcast, where a change of orientation would have no effect on the internal illuminance.

The basic principle of this method is that light at a given point is the sum of light from three sources: light coming directly from sky; light reflected to the station point from external obstruction; and light received through inter-reflection between the room's surfaces.

$$DF = SC + ERC + IRC$$  \hspace{1cm} (2.2)

where:

- $SC$ = the sky component
- $ERC$ = the externally reflected component
- $IRC$ = the internally reflected component

### 2.3.1.1 The sky component

The sky component ($SC$) is the ratio of the light received at a station point in a room directly from the sky to the simultaneous unobstructed horizontal illuminance received from the sky. The basic equation for determining illuminance ($Ep$) reaching a reference plane from the sky, seen from any opening of an area from $\theta_1$ to $\theta_2$ and from $\phi_1$ to $\phi_2$ respectively, is:

$$Ep = \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \tau(\theta,\phi) L(\theta,\phi) \sin \theta \cos \theta \, d\theta \, d\phi$$  \hspace{1cm} (2.3)

where:
L is the luminance of a patch from the sky

τ is the directional transmittance of light by the window system

The horizontal illuminance from a complete sky of uniform luminance is;

\[ E = L \pi \]  

(2.4)

In this case the Sky Component is known as the Sky Factor, with an unglazed opening, and,

\[ SF = \left( \frac{1}{\pi} \right) \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} L(\theta, \phi) \sin \theta \cos \theta \, d\theta \, d\phi \]  

(2.5)

It can be seen that the Sky Component is a geometrical quantity which varies with sky luminance distribution.

2.3.1.2 The internally reflected component

The internally reflected component (IRC) is the ratio of light received at a station point in a room through inter-reflection (between the room surfaces) to the simultaneous external unobstructed illuminance from the sky.

The most frequently used method for calculating this component is the BRE split-flux formula, where the light entering the room is considered in two parts: a) light
received directly from the sky and from obstructions above the horizon and b) light received directly from the ground below the horizon (39, 40).

Figure 2.5 The split-flux principle applied to the calculation of the internally reflected component of the daylight factor.


\[ IRC = \frac{0.8 \ W}{A(1 - R)} \left( C_1 R_{fw} + C_2 R_{cw} \right) \] (2.6)

where:

- \( W \) = area of window of 0.8 diffuse transmittance
- \( A \) = total area of room surfaces
- \( R \) = average reflectance of room surfaces
R_{fw} = \text{average reflectance of the lower parts of room surfaces}

R_{cw} = \text{average reflectance of the upper parts of room surfaces}

C_1 = \text{window factor due to the light incidence on the window from above the horizontal}

C_2 = \text{window factor due to the light incidence on the window from below the horizontal}

Tregenza, in a study reported in 1989 (41), proposed a modification of the split-flux formula to include the effect of large vertical obstructions (such as projecting wings of a building and overhanging canopies). In the original split-flux formula, C_1 is based on the assumption that the obstruction is infinite and has its outlines horizontal and parallel to the plane of the window wall. The modification proposed here was aimed at providing a more accurate assessment of light reflected from an obstruction. C_1 was calculated as a function of the exact geometry of the obstruction.

2.3.1.3 The externally reflected component

The externally reflected component (ERC) is the ratio of light received at a station point in a room directly from external surfaces through reflection to the simultaneous external unobstructed illuminance from the sky.

The ERC is calculated in the same way as the SC, by treating the obstruction as a patch of the sky. For a uniform sky the equivalent sky component should be multiplied by 0.1, based on the assumption that the obstruction is illuminated by half of the sky dome, and its reflectance is 0.2 (29). For an overcast sky the equivalent
multiplied by 0.2. This is because its luminance near the horizon is approximately half the average of the luminance of the sky (42).

Figure 2.4  The three components of daylight factor.

2.3.1.4 The orientation factor

The daylight factor concept is based on the worst case condition (the completely overcast sky). It does not change with orientation. According to Littlefair (43, 44), a room modelled with the CIE overcast sky luminance will make any solution based on providing the required daylight factor either achieve that factor or provide a higher internal daylight illuminance. However, when maximum integration between natural and artificial light is required, such a concept might not be appropriate (45). Studies at the Building Research Establishment (BRE) have shown that orientation is a key factor in the prediction of energy savings from natural and artificial light integration (44).

Hunt (46) in a 1979 study about the use of artificial lighting in relation to daylight levels and occupancy, proposed the use of an orientation factor, which is a factor which can be multiplied by the daylight factor to allow for the effect of orientation. Hunt based this proposed factor on daylight availability data collected in England (43). In 1990 Littlefair (44) proposed total orientation factor and a diffuse orientation factor. The diffuse orientation factor has been recommended by the British Standards Institute (42).

2.3.1.5 Summary

Under clear sky conditions, Hopkinson, Petherbridge and Longmore (18) stated that "the concept of the Daylight Factor as a ratio of the internal illumination to the illumination from the whole sky, with the effects of sunlight excluded, ceases to have any useful function. The effects of sunlight clearly have to be included".*

However, in this study it was necessary to look at such a method because of its advantages and because of the important modifications which have been suggested to make it usable under clear sky conditions.

The daylight factor method has two main advantages. Firstly, it expresses the amount of light in a room as a ratio of the light outside, so such a method does not require a detailed estimation of the external illuminance. Secondly, it is based on the worst possible condition (CIE overcast sky), whereby any extra light will be welcomed. If these two advantages (concepts) could be applied to clear sky conditions, the complexity of predicting the internal illuminance under such sky conditions could be reduced.

2.3.2 Modifications to the daylight factor method

Modification of the daylight factor method has been proposed by some researchers to allow for the effect of a clear sky and direct sunlight.

Bryan and Clear (47) proposed some modifications to the daylight factor method for use under clear sky conditions. The authors proposed an equation for calculating the sky component (SC) for the CIE clear sky luminance distribution based on solving the integral.

\[
SC = \frac{1.018 \, T(0)}{N_{sc}} \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \left( \frac{\sin^2 \theta \cos \theta \sin^3 \phi}{x^{5/2}} \right) \\
\left( 1 + \left( \frac{\sin^2 \theta + \cos^2 \phi \left( \cos^2 \theta - \sin^2 \phi \right)}{x} \right)^{3/2} \right) \\
\left( 1 - \exp \left( \frac{-0.32 \, x^{1/2}}{\cos \theta \cos \phi} \right) \right) \\
\left( 0.91 + 10 \exp \left( -3 \, \zeta \right) + 0.45 \cos^2 \zeta \right) \, d\theta \, d\phi
\]  

(2.7)
where:

\[ \zeta = \text{angle of the sun from L (}\theta) \]

\[ \text{Nsc} = \text{Normalization factor (given)} \]

\[ x = \cos^2 \theta \cos^2 \phi + \sin^2 \theta \]

In addition, the internally reflected component equation (the split-flux method) was modified to include the effect of direct sunlight. The modifications were related to window factors \( C_1 \) and \( C_2 \) (factors due to the light incidence on the window from above and below the horizontal).

In the modified formula, \( C_1 \) and \( C_2 \) were calculated in a different way to allow for direct sun light. The values of \( C_1 \) were given in tables, and \( C_2 \) was given as follows:

\[ C_2 = \frac{(E_{\text{sun}} + E_{\text{sky}}) R_g G_c}{E_{\text{sky}}} \quad (2.8) \]

where:

\( E_{\text{sun}} \) = illumination from the sun

\( E_{\text{sky}} \) = illumination from the sky

\( R_g \) = reflectance of ground

\( G_c \) = ground configuration factor (0.5 for horizontal surface)

Muneer and Angus in a study reported in 1994 (32), proposed a method to find what they called the Daylight Illuminance Factor, which is the sky component in the
daylight factor method, where the effect of the changing sky luminance distribution is considered.

\[
\text{Daylight Illuminance Factor} = \frac{L_z \left( \frac{I_1}{1 + b} + \frac{b}{1 + b} \frac{I_2}{1 + b} \right)}{\text{Illuminance under the sky hemisphere}} \quad (2.9)
\]

where:

\[
I_1 = \int_{0}^{H/D} \int_{0}^{W/D} \frac{Y}{(1 + x^2 + y^2)^2} \, dx \, dy
\]

\[
I_2 = \int_{0}^{W/D} \int_{0}^{H/D} \frac{Y}{(1 + x^2 + y^2)^{5/2}} \, dx \, dy
\]

\(b\) is the sky luminance distribution index and is equal to 0.0 when the window is shaded, and equal to -0.69 when the window is exposed to the sun.

\(L_z\) is an estimated zenith luminance

\(H, W\) and \(D\) are illustrated in figure 2.6
Shukuya and Kimura, 1983 (23), proposed a new concept for the daylight factor in a study to calculate the work plane illuminance from daylight, including the effect of direct sunlight through windows with horizontal or vertical louvers. They divided the daylight factor into three new components: direct sunlight factor (the work plane illuminance from sunlight divided by direct sunlight on a horizontal surface); skylight factor (the work plane illuminance from skylight divided by skylight on a horizontal surface) and reflected daylight factor (the work plane illuminance from daylight reflected from the ground divided by luminance of ground due to sunlight and skylight). In this method the luminous distribution of the sky and the ground are assumed to be uniform. With horizontal and vertical louvers, the direct sunlight factor was expressed as a function of surface reflectance, solar position, and the geometry of the louvers. The sky light factor and reflected daylight factor for the windows with horizontal or vertical louvers were expressed as a function of surface reflectance and geometry of the louvers. The general idea of the concept seems to be a good modification. However, the theoretical estimation for these factors, which is
the key issue, is more complicated than the daylight factor method. Also, reflected light from nearby buildings is not included.

2.3.2.1 Summary

All the methods above neglect the component reflected from obstructions. Such a component is a very important element in hot dry climates and should be included. Bryan’s modification to the daylight factor method to enable it to be used for clear sky conditions is considered (28) to be the most important work in this topic in the last decade. However, it is not easy to use either this or the other techniques proposed. It is clear that further research is necessary.

2.3.3 The average daylight factor method

The daylight factor can be calculated for a given opening where its shape and location are known, so it is a method which checks the performance of a given opening. Such a method requires a trial and error approach to reach the required design. Based on this, the average daylight factor, which is less dependent on window shape and location and can be used at the early design stages, was proposed (48, 49). By definition, the average daylight factor is the spatial average of all daylight factors on a working plane under an overcast sky (49).

The first widely accepted average daylight factor formula for a side lit room (40) was that proposed by Longmore in 1975 (50).
\[
\overline{DF} = \tau W \left( \frac{C}{A_{fw}} + \frac{CR_{fw} + 5R_{cw}}{A (1-R)} \right) \% \tag{2.10}
\]

Where:

\(\overline{DF}\) = average daylight factor on the working plane level

\(W\) = glazing area

\(\tau\) = the directional transmittance of light by the window system (in the original equation \(\tau = 0.85\), however, later on 0.80 is used instead)

\(C\) = a function of the daylight flux incidence on the window plane from above the horizontal

\(A_{fw}\) = area of floor and lower parts of the walls below the mid-height of the window (not including the window wall)

\(R_{fw}\) = average reflectance of the \(A_{fw}\)

\(R_{cw}\) = average reflectance of the ceiling and upper walls above the mid-height of the window (not including the window wall)

\(A\) = area of all surfaces in the room

\(R\) = average reflectance of all surfaces in room

Another formula was proposed by Lynes in 1979 (51):

\[
\overline{DF} = \frac{W \tau \theta}{2A (1-R)} \% \tag{2.11}
\]

Where:

\(\overline{DF}\) = average daylight factor of all room surfaces

\(W\) = glazing area
\[ \tau = \text{diffuse light transmittance of the glazing} \]

\[ \theta = \text{vertical angle subtended at the centre of the window by the visible sky} \]

\[ A = \text{total area of room surfaces} \]

\[ R = \text{average reflectance of all room surfaces} \]

Later Lynes' formula was modified by Crisp and Littlefair in 1984 (48) to give the average daylight factor on the working plane:

\[ \bar{DF} = \frac{\tau \ W \ \theta}{A \ (1-R^2)} \]  

(2.12)

where:

\[ \bar{DF} = \text{average daylight factor on the working plane} \]

\[ \tau = \text{diffuse light transmittance of the glazing} \]

\[ W = \text{area of window} \]

\[ \theta = \text{vertical angle subtended at the centre of the window by the visible sky} \]

\[ A = \text{total area of room surfaces} \]

\[ R = \text{average reflectance of room surfaces} \]

Equation (2.12), has been adopted by the British Standards Institute (42, 52).
Figure 2.7  Angle of visible sky used in calculating the average daylight factor


The average daylight factor method is simple to use. However, it is based on the overcast condition where the effect of sunlight is not included. Such a method has not been tested under clear sky conditions. Therefore, an investigation in this direction is required in order to find out whether or not it could be used under such conditions, and to discover whether a modification of such a method might be possible.
2.3.4 The lumen method

The lumen method is recommended by the Illuminating Engineering Society of North America. In this method, natural light received inside a room is a function of light incidence on the window plane.

This method allows for the estimation of absolute illuminance values at three stations inside a room. The first point is located at 5 feet from the window wall, the second is in the centre of the room and the third is 5 feet from the back wall of the room. All these points are on a centre line perpendicular to the window wall and at 0.75 meters above the floor level.

The illuminance at any of the three station points can be found as follows:

\[ E_p = E_s \cdot A_g \cdot T_g \cdot C_s \cdot K_s + E_g \cdot A_g \cdot T_g \cdot C_g \cdot K_g \]  \hspace{1cm} (2.13)

where:

- \( E_p \) = absolute illuminance value at the station point
- \( E_s \) = exterior illuminance from the sun and the sky incident on the window
- \( A_g \) = area of glazing
- \( T_g \) = transmission of glazing
- \( E_g \) = exterior illuminance reflected from the ground to the window
- \( C_s, C_g \) = coefficients representing the relationship between the light reaching the window and the room length, room width and the reflectance of the interior walls of the room
$K_s, K_g =$ coefficients representing the relationship between the light reaching the window and the ceiling height, room width and the reflectance of the interior walls of the room.

These coefficients (C & K) are the main principles on which this method is based and were obtained through physical scale model tests.

Figure 2.8 Illustration for the station points in a room based on the lumen method.
2.3.5 Other methods

Other methods have been proposed for use under clear sky conditions. One of the earliest methods was proposed by Plant, Longmore and Hopkinson in an experimental study reported in 1967 (53), exploring interior illumination due to skylight under tropical conditions. In this study the sun was located at an altitude of 90° on a vertical line above the window. The study was carried out under an artificial sky with luminance distribution based on the CIE overcast sky. Based on fixed room geometry, one type of shading (an egg-crate at a 45° angle to the horizontal and to the vertical) and an obstruction of 20° from the window sill, a nomogram was produced to calculate the illumination from reflected sunlight and skylight for positions near to the window, at the centre of the room and at the back of the room. The proposed nomogram was not validated under real sky conditions.

Tambal in 1978 (54) developed other nomograms in another study in a tropical region (Sudan), and based on measurements under real clear sky conditions with the use of a scale model. In this study, the variables investigated were: exterior and interior surface reflectance, solar azimuth, solar altitude, room depth, sill height and opposite facade height. The proposed nomograms are for finding the interior illuminance levels at certain points on the centre line of a room. These nomograms are divided into two sets. The first set is for use with a window shaded with horizontal louvers and the second set is for use with a window shaded with an overhang. Each of these sets contains three nomograms for finding the illuminance level at points nearest to the window, at the centre of the room and at the back of the room. Each nomogram is accompanied by a table for correction factors for ground reflectance, louver reflectance, solar altitude angles and room depth, making a total of sixty correction factors.
Although the proposed nomograms have not been validated with real buildings, they can still be considered the best available manual choice for predicting the internal illuminance without knowing the external illuminance. In general, there are some limitations to these nomograms. These are: the nomograms consider the effect of the angle of obstruction to be insignificant; only two types and one angle (45°) of shading devices are considered; the effect of sun azimuth is limited (the sun was in a line perpendicular to the window wall, either in front of the window or at the back of the window); the window tested was a window wall and there is no consideration of the effect of other geometry and locations; and, finally, the nomograms are based on the specific climatic conditions which were present over the five day period during which this study was carried out.

Another solution for estimating the internal illuminance under clear sky conditions was proposed by Tregenza in 1995 (55). In a study to find the mean illuminance in rooms facing sunlit streets, Tregenza proposed a method for estimating the mean illuminance on the working plane and on other room surfaces. This method uses either computed or measured solar normal illuminance and the diffuse horizontal illuminance. The general approach of the method is based on estimating the three components of illuminance on the external face of the window. These are; direct illuminance from sun and sky; reflected illuminance from the ground; and reflected illuminance from above the horizon. Due to the fact that different window systems transmit light differently inside the room, Tregenza estimated the fractions of each incident component of light, that is incident on the different surfaces of the room for six window systems and presented them in six tables based on the window system used. By the use of the proposed fractions, the direct illuminance incident on the ceiling, on the walls above the working plane level, and on the working plane level is estimated. Then, the final average illuminance on these surfaces will be equal to the
estimated direct illuminance plus the inter-reflected light which is calculated by the use of Sumpner's equation for inter-reflected light.

This method can be applied by manual calculations or by the use of a simple computer spreadsheet. Although it has not been validated against real measurements, the theoretical part of it seems to be acceptable. In addition, this method appears to be the only method which considers externally reflected light as an important source of internal illuminance. The proposed method is not difficult to use. However, it is not simple for architects who might wish to avoid long calculations. One important benefit of this method is that it allows for control over the level of accuracy. Such accuracy will depend on the initial illuminance data used.

2.3.5.1 Summary

All the methods above can be used to find the internal illuminance under clear sky conditions. Tambal's nomograms and Tregenza's method are probably more accurate, due to the wide range of variables involved. However, they have not been validated under real sky conditions.
2.4 Discussions and conclusions

This chapter reviewed the use of natural light under clear sky conditions, highlighting problems associated with utilising natural light for regions where these conditions are prevalent.

2.4.1 Solar and sky illuminance

Determining the optimum source from which illuminance inside a room will be provided is a major factor in the design of the building. Under overcast sky conditions the source is assumed to be the sky. So, generally speaking, the maximum amount of daylight under an overcast sky can be achieved by having the opening exposed as much as possible to the sky. This is not the case in sunny climates. Maximum exposure to the sky does not necessarily mean a maximum amount of natural light.

If we start from the basic concept that at any point exposed to a lighting source, the illuminance level at that point is a function of the intensity of the source and the geometrical relationship between the source and the point, it could be concluded that the optimum source of natural light is the source having the highest luminance. This indicates that the use of natural light in hot dry climates should be based on utilising sunlight reflected from adjacent buildings and ground.

Another important factor relating to such regions is that the external illuminance received on the window plane is a function of several variables. These include: location of the sun, geometrical relationship with surroundings, reflectance of external surfaces, and sky condition. Such issues make it difficult to estimate easily the external illuminance levels. Different windows will usually receive different amounts of illuminance. A detailed estimation of such sources for each window will add complexity to the issue. Therefore, there is a need to simplify the estimation of
the illuminance incident on the window plane in order to reduce the complexity of such a subject.

2.4.2 Prediction techniques

The Daylight Factor method is based on an overcast sky with the exclusion of sunlight. Such a method is not suitable for predicting the internal illuminance under clear sky conditions, because the effect of sunlight must be taken into consideration.

Bryan and Clear's (47) modification to the daylight factor method for use with clear sky conditions is considered to be an important work. However, it is not easy to use their technique. The same could also be said about other modified methods (Muneer and Angus (32) and Shukuya and Kimura (23)). The fact that they are not easy to use is considered a main drawback for any method intended to be used manually. Moreover, all these methods neglect sunlight reflected from obstructions.

The lumen method can predict the illuminance inside a room with an acceptable accuracy. However, the accuracy of the method depends on the accuracy of illuminance data used. One of the major drawbacks of this method is that it requires the knowledge of external illuminance on the vertical plane, which is information not available for many places in the world. Another drawback is that the effect of external obstruction is not taken into consideration. In addition, this method is based on several assumptions in terms of room size, reflectance and window geometry (the window is assumed to extend from one side of the room to the other and extend in height to the ceiling).
From all the methods reviewed, there are two which are considered to be a better choice than the others for clear sky conditions. These are Tambal's nomograms and the Tregenza method.

Tambal's nomograms are a good tool due to the wide range of variables tested. Although the proposed nomograms have not been validated with real buildings, they can be considered the best available manual choice for predicting the internal illuminance without knowing the external illuminance. In general, there are some limitations to these nomograms which should be taken into consideration when using this method. These include; limitation of the effect of angle of obstruction which the nomograms consider to be insignificant; limitation on the types and angles of shading devices; limitation on the effect of sun azimuth (the sun was in a line perpendicular to the window wall, either in front of the window or at the back of the window); limitation on the geometry of the window (no consideration of the effect of other geometry and location); and, finally, limitation based on the fact that the nomograms were developed under certain climatic conditions. Such limitations indicate the need for further development of these nomograms.

Tregenza's method for estimating the mean illuminance on the working plane and on other room surfaces is another good tool for predicting internal illuminance, although it has not been validated against real measurements. This method can be considered as the only one which regards externally reflected light as an important source of internal illuminance. This fact makes its use with obstructed windows more appropriate than any other method. The method is not difficult to use. However, it is not simple for architects who might wish to avoid long calculations. One main drawback of this method, when used manually, is the need to repeat the estimation of external illuminance for each window having a different orientation and a different obstruction angle.
From the review above, it is believed that there is need for further research in the field of predicting internal illuminance due to natural light under clear sky conditions. Although there is a well established method which has the potential to predict the internal illuminance under clear sky conditions accurately (Tregenza's method), this method cannot be considered user-friendly.

Predicting internal illuminance under clear sky conditions is not a simple task due to the many variables involved. Therefore, a major aim of this study is to investigate ways of simplifying such a process. The average daylight factor method is the most simple method available. Recalling a statement which was quoted on the use of daylight in buildings,

"the average daylight factor proposed by Lynes is a useful design aid which is also simple to use. Since the daylight distribution and intensity inside the building will vary with time, whichever sky distribution is used as a basis for design, then it would seem that great precision in this is unnecessary.......... I have strong reservations over the provision of detailed and complex daylight prediction system. What is required, in my opinion, is a relatively simple system that architects must be encouraged to use and to combine this with the provision of daylight controls to adjust the daylight conditions when necessary. Electronic control of the electric lighting should then integrate the electric lighting with the daylighting for an energy-efficient design." (David Loe, in the discussion in reference (56)).

A new method is required for clear sky conditions. There are two constraints.

1. The method should be simple, like the use of an average daylight factor.

2. It should predict the minimum likely illuminance on the working plane.

These are the aims of the following chapters.
CHAPTER THREE

THE METHODOLOGY
3.1 The methodology

In the previous chapter it was concluded that an approach similar to the average daylight factor method was needed for practical design for daylighting under clear sky conditions. The key requirements are that any formula should be simple and that it should take reflected sunlight into account. The existing average daylight factor methods might be suitable, but appear to have drawbacks: the luminance distribution of the clear sky changes with solar position, unlike the CIE overcast sky, and the ratio of upward to downward light incident on the window is not constant.

The methodology of the research was as follows:

1. Investigation into the relationship between the angle of incidence of light on the window and the mean illuminance on the working plane. This was done in three stages:

   a. Radiosity method calculation, comparing the illuminance on different individual patches in a room with the resulting mean working plane illuminance after inter-reflection. This led to a new concept, termed 'grid-element coefficients'.

   b. Theoretical analysis of patch illuminance in relation to window plane daylight illuminance.

   c. Scale model testing of the results.
2. Numerical comparison of existing average daylight factor methods with mean working plane illuminances derived from point-by-point calculations under clear sky conditions. The relationships between external horizontal illuminance and window plane illuminance were calculated with an original computer program. Internal point illuminances were computed with SUPERLITE.

3. Proposal of a relation between the illuminance on the window wall and the horizontal diffuse illuminance.

4. Development of a modified formula for mean relative internal illuminance under clear sky conditions. This was termed the 'average natural light factor'.

5. Testing both the relation between the vertical and horizontal illuminances and the new formula:
   a. By measurements in real buildings in Dammam, Saudi Arabia.
   b. By measurements in scale models under real skies.
   c. By comparison with detailed calculations made using SUPERLITE.

In general, the approach has been to use numerical simulation methods for the main experimental stages and use measurements in scale models and real buildings for validation. This approach was taken for the following reasons:

1. Numerical methods allow systematic testing of the affects of different parameters within a controlled environment.
2. They are significantly faster, allowing a large number of different cases to be studied.

3. Major errors associated with physical scale models are avoided. These errors can result from inaccuracy in measuring the exact area receiving direct light, inaccuracy in estimating reflectance of surfaces, inaccuracy in the levelling of the photocells and inaccuracy of the photocells’ response to light.
CHAPTER FOUR

THE RELATIONSHIP BETWEEN ANGLE OF INCIDENCE OF LIGHT AND MEAN ILLUMINANCE ON THE WORKING PLANE LEVEL
4.1 Introduction

The aim of this chapter is to examine the relationship between the angle of incidence of light transmitted from a vertical window and the mean illuminance on the working plane. The chapter begins with a literature review, then describes a numerical investigation into the relationship between room patch illuminance and working plane illuminance, and links this with window plane illuminance. It then describes a scale model test.

The general results of the chapter can be summarised as follows:

1. To estimate the average illuminance on the working plane under the clear sky conditions adopted, the sky can be treated as a uniform source.

2. Ground reflected light can be a significant component of the working plane illuminance.

3. Acceptable accuracy is achieved when window illuminance is taken to have two components (i.e., light incident above the working plane level and light incident below the working plane level).

4. When window illuminance from sky and ground are equal, approximately 75% of working plane illuminance is due to skylight.
4.2 Theoretical analysis

Under clear sky conditions, light can reach the internal space of a room from different directions (figure 4.1). It can come directly from the sky or be reflected from the ground or from any external surface. The window transmittance system can direct the light entering the room to any selected surface.

![Figure 4.1](image)

**Figure 4.1** Under clear sky conditions light can reach the room surfaces from any direction in front of the window. For example, it might come directly from the sky (a), reflected from the ground (b) or reflected from an obstruction (c).

In order to deal with the light transmitted to the internal surfaces as a quantity without consideration of the direction from which it arrives or the direction to which it is transmitted, the variation of the mean working plane illuminance due to variation in such a direction should be insignificant.

This necessitates a primary investigation into the relationship between the angle of incidence of light on the internal room surfaces and the average illuminance on the working plane.

To examine such a relationship, a fixed window area is assumed. Then, the light transmitted inside the room can be assumed to reach a certain location on the room surfaces. For such condition, the mean illuminance on the working plane is calculated. To test another angle of incidence of light, the same illuminance on the
window pane is assumed to reach a different location on the room surfaces, and then, the mean illuminance on the working plane is calculated. The task here is to find the average internal illuminance on the working plane for each location receiving the light as a single case. Such an investigation will show the effect of the change in direction of the incident illuminance on the average internal illuminance.

For a detailed investigation, all possible angles of incidence of light on the room surfaces should be investigated.

Different angles of incidence of light mean different locations of light received on the room surfaces. Therefore, the investigation could involve the different locations where light falls on the room surfaces rather than angles of incidence of light.

In order to investigate the relationship between the location of the light received and the mean illuminance on the working plane level, the following procedure was used:

1. The illuminance on the external plane of the window and the transmitted light were assumed.

2. For such light, different locations were assumed to receive such illuminance one at a time.

3. For the light received at each location, the average internal illuminance was estimated.
4.3 Estimating the mean illuminance on the working plane level

The mean illuminance on the working plane due to an external lighting source is the sum of two components; the direct component and the inter-reflected component. The direct component is light transmitted directly from the external source to the working plane level. The inter-reflected component is light reaching the working plane level from the visible portions of the room surfaces after the inter-reflection process.

4.3.1 Estimating the direct component

Estimating the direct component reaching the internal surface of a room is a straightforward process. Figure 4.2 illustrate such an estimation.

\[ E = \frac{E_W A_W T}{A_C} \]

- \( E_W \) = illuminance on window surface
- \( A_W \) = area of window
- \( A_A \) = area of location A, \( A_B \) = area of location B
- \( A_C \) = area of location C
- \( T \) = transmittance of window

Figure 4.2 An illustration of how to estimate the direct Illuminance (E).
4.3.2 Estimating the inter-reflected component

The inter-reflected component is not easy to find. There follows a review of techniques for estimating such an inter-reflected component.

4.3.2.1 Review of methods for estimating inter-reflected light

One of the earliest methods is the BRE Split-Flux method (39). This method, which has been explained in Chapter Two, is based on the theory of the integrating sphere, where its reflectance is equal to the mean internal reflectance of all surfaces (18). Therefore, its accuracy is influenced by how close the room is to a spherical shape and how small the window is (the window will transmit inter-reflected light to the outside). For example, when the depth of the room is equal to three or more times of its height, the BRE method can predict the internal reflected illuminance towards the back of the room as twice or more the actual amount (57).

Gillete and Kusuda recommended another method as being more accurate than the BRE (58, 59). This method is based on Dresler's formula for inter-reflected light (as in the BRE method) (60). Initial light is estimated by averaging the initial illuminance on each of the six surfaces of the room, making the illuminance uniformly distributed over each surface. This initial illuminance is then used to find the inter-reflected light based on the theory of the integrating sphere. The reflected illuminance on each surface is calculated as follows:

\[
E_{ref} = \frac{E_1 \rho_1 A_1 + E_2 \rho_2 A_2 + \cdots + E_n \rho_n A_n}{A_T (1 - \rho)} \tag{4.1}
\]

Where:

\[E_{ref} = \text{inter-reflected light}\]
Another more accurate approach is based on dividing room surfaces into small finite elements (61) (figure 4.2). Initial illuminance on each element is computed and considered to be uniform. This approach to measuring inter-reflected light is known as the radiosity method (62) which is based on calculating light transfer between each element and the other visible elements.

Figure 4.2 Dividing the room into small surface areas will increase the accuracy in finding the inter-reflected light.
The total illuminance on element i, after each reflection, is equal to the direct illuminance incident on this element plus light reflected from all other visible elements. This total illuminance can be expressed by a linear equation. The reflected light from the other elements is a function of the luminance of each visible element and the geometric relationship between each element and element i.

In general the radiosity method can be divided into three main steps (63). These are:

1. Estimating the initial illuminance on each element.

2. Estimating the geometric relationships that determine the transport of light between each element and all other visible elements, known as the form factor.

3. Solving the resulting system of simultaneous equations in order to find the total illuminance on each surface.

### 4.3.2.2 Conclusion

Based on the review presented in the above review (4.3.2.1), the radiosity approach is the method selected for use in this study. This is due to its accuracy (64) and to the flexibility of choosing different locations of received illuminance on the room surfaces.

In the following the mean illuminance on the working plane will be calculated by the use of the radiosity method.

### 4.3.3 The use of the radiosity method for estimating the mean illuminance on the working plane

After estimating the direct illuminance reaching the room (as described in 4.3.1), the radiosity method requires knowledge of the form factor between the different surfaces involved and then solving the inter-reflection equations.
4.3.3.1 Form factor

The accuracy in finding the final illuminance within a room depends on the evaluation of accurate form factors between the surfaces. It is, therefore, imperative that the estimation of such factors be within acceptable levels of accuracy.

The form factor between two surfaces is defined as the fraction of energy leaving one that arrives at the other (65). It is purely a geometric relationship between the two surfaces, which represents the ratio of the luminance of an emitting surface and the illuminance received by the other. Hence the form factor can be stated as:

\[
F_{d1-d2} = \frac{\text{flux reaching } d2 \text{ from } d1}{\text{Total flux leaving } d1}
\]

\[
= L_1 \frac{d1 \cos \theta_1}{d1 L_1 \pi} \cos \theta_2
\]

\[
\text{Total flux leaving } d1 = d1 L \pi
\]

Then:

\[
F_{d1-d2} = \frac{L_1 \frac{d1 \cos \theta_1}{d1 L_1 \pi}}{\cos \theta_1 \frac{d2 \cos \theta_2}{\pi S^2}}
\]
This formula represents the flux exchange between a differential area to another which is known as the configuration factor. The final exchange factor between two surfaces requires this formula to be integrated over the two surfaces (figure 4.4):

\[
F_{1-2} = \frac{1}{A_2} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2 \, d_1 \, d_2}{\pi \, S^2}
\]

(4.5)

where:

\( A_1 \) and \( A_2 \) are the areas of surface 1 and 2, \( \theta_1 \) and \( \theta_2 \) are the angles between the normal to surface differential elements \( dA_1 \) and \( dA_2 \) and the vector between those elements. \( S \) is the length of that vector.

Figure 4.4 Illustration of symbols used for finding the form factor.
4.3.3.1.1 Estimating the form factor

Solving the double integral is a time consuming process. Such a process can be simplified by averaging the inner integral over one or more sample points on surface 2 instead of over the whole surface. This simplification is acceptable when the squared distance is much greater than the area of surface 2 (62). However, in the case of a room where surfaces could be close to each other, especially near the edges, such a method will affect the accuracy of the form factors.

Another method used to solve the inner integral is the use of what is known as the Unit-Sphere Method introduced by Nusselt (65). This method is based on constructing an imaginary hemisphere of a unit radius over the element d₁ (figure 4.5), then projecting A₂ onto the unit hemisphere. This projection, which is equal to the solid angle subtended by that area, will give the configuration factor \( F_{d₁, 2} \) if projected down onto the base of the hemisphere (\( \cos θ₁ \, dω₁ \)) and the resultant area is divided by the area of the unit circle (\( π \)).

Figure 4.5 The Concept of Unit-Sphere Method.
Based on the unit hemisphere concept, Cohen & Greenberg (62) developed the hemicube method. In this method, rather than projecting onto the hemisphere, projection is done onto the upper half of a cube (figure 4.6). The hemicube surfaces are divided into small cells. For each cell a delta form factor is computed. Finding $F_{d_{1-2}}$ is done by summing the delta factors that are within the projection of $A_2$ on the face of the hemicube.

![Figure 4.6 The hemicube](image)

Another method is to transfer the area integrals into line integrals based on Stokes' theorem (65-68), where the form factor could be obtained as:

$$F_{1-2} = \frac{1}{A_2} \int_{C_1} \int_{C_2} \left( \ln S \, dx_2 \, dx_1 + \ln S \, dy_2 \, dy_1 + \ln S \, dz_2 \, dz_1 \right)$$  \hspace{1cm} (4.6)
This integration will result in a faster method, due to the difference between being integrated over two areas, involving four variables, and the integration over the two surface boundaries (65).

4.3.3.1.2 Calculated form factors

Due to the frequent use of these form factors, especially in heat transfer related subjects, several researchers have calculated form factors between different surfaces and proposed graphs and formulae to obtain such factors. A form factor for identical, parallel, opposite rectangles and a form factor for two perpendicular rectangles are examples of calculated form factors.

The proposed graphs and formulae for obtaining the form factors can be found in most of the radiation and heat transfer texts (65, 69-71) and some of the lighting texts (67, 72). In addition, some catalogues are available which contain most of the form factors used (73, 74).

4.3.3.1.3 The technique used for finding the form factor

Due to the nature of the problem being dealt with in this study, i.e., to find light inter- reflected within a room enclosure, it is most convenient to divide the surfaces of the room into typical shapes (rectangular, figure 4.7) and use the proper available formulae (which have already been proposed by other researchers) to find the form factors between any finite areas on a room surface.
For the purpose of this study there are two formulae which form the basis of form factor calculations for the surfaces of the room. These are:

1- Form factor for identical, parallel, opposite rectangles, figure 4.8, (72):

\[
F_{1-2} = \frac{\log_e A + 2 (B + C)}{\pi \times x \times y} \quad (4.7)
\]

where:

\[
A = \frac{(1 + X^2) (1 + Y^2)}{1 + X^2 + Y^2}
\]

\[
B = X \sqrt{1 + Y^2} \arctan \frac{x}{\sqrt{1 + Y^2}}
\]

\[
C = Y \sqrt{1 + X^2} \arctan \frac{x}{\sqrt{1 + X^2}} - X \arctan X - Y \arctan Y
\]
Figure 4.8 Illustration of symbols used for the form factor for identical, parallel, opposite rectangles.

2- Form factor for two perpendicular rectangles, figure 4.9, (72):

\[
F_{1-2} = \frac{1}{\pi W} \left( W \arctan \frac{1}{W} + H \arctan \frac{1}{H} - A \arctan \frac{1}{A} \right.
\]
\[
+ \log_e \left\{ \frac{[B][C]W^2[D]H^2}{4} \right\} \right) \quad (4.8)
\]

\[
A = \sqrt{H^2 + W^2}
\]

\[
B = \frac{(1+W^2)(1+H^2)}{1 + W^2 + H^2}
\]

\[
C = \frac{W^2 (1 + W^2 + H^2)}{(1+W^2)(W^2 + H^2)}
\]

\[
D = \frac{H^2 (1 + W^2 + H^2)}{(1+H^2)(W^2 + H^2)}
\]
When the above form factors are known (equations 4.7 and 4.8), factors for rectangular shape surfaces with other geometric relationships can be calculated by the use of what is known as form factor algebra.

For the cases under investigation in this study, three cases of form factor algebra can cover all the possible form factor estimations between any two rectangles within the room enclosure:

**Case 1:**

Form factor between two rectangles, where their extensions are perpendicular, figure 4.10, (69):

\[
F_{1-2} = \frac{A_{(1,3)} F_{(1,3)} - (2,4) + A_3 F_{3-4} - A_3 F_{3-2} + A_{(1,3)} F_{(1,3)} - 4}{A_1}
\]

(4.9)

Figure 4.10 Case 1
Case 2:

Form factor between rectangles located in perpendicular planes, figure 4.11 (Hamilton and Morgan (1952) as cited in reference (69)).

\[
A_1 F_{1-3'} = \frac{1}{2} \left[ K_{(1,2,3,4,5,6)}^2 - K_{(2,3,4,5,6)}^2 - K_{(1,2,5,6)}^2 + K_{(4,5,6)}^2 \right.

- K_{(4,5,6)} - (1',2',3',4',5',6') - K_{(1,2,3,4,5,6)} - (4',5',6')

+ K_{(1,2,5,6)} - (5',6') + K_{(2,3,4,5)} - (4',5') + K_{(5,6)} - (1',2',5',6')

+ K_{(4,5)}^2 - K_{5} - (2',5') + K_{5}^2 \right]

(4.10)

where:

\[ K_{m-n} = A_m F_{m-n} \]

\[ K_{m}^2 = A_m F_{m-m'} \]

Figure 4.11 Case 2
Case 3:

Form factor between rectangles located in parallel planes, figure 4.12 (Hamilton and Morgan (1952) as cited in (69)).

\[
A_1 F_{1.9} = \frac{1}{4} \left[ K_{(1,2,3,4,5,6,7,8,9)}^2 - K_{(1,2,5,6,7,8)}^2 - K_{(I,2,3,4,5,6,7,8,9)}^2 + K_{(I,2,5,6)}^2 + K_{(2,3,4,5)}^2 + K_{(4,5,8,9)}^2
\right.

- \left. K_{(1,2,3,4,5,6,7,8,9)}^2 + K_{(1,2,5,6)}^2 + K_{(2,3,4,5)}^2 + K_{(4,5,8,9)}^2
\right. - K_{(4,5)}^2 - K_{(3,8)}^2 - K_{(5,6)}^2 - K_{(4,5,6,7,8,9)}^2

\left. + K_{(5,6,7,8,9)}^2 + K_{(4,5,6,8)}^2 + K_{(2,5,8,9)}^2 - K_{(2,5)}^2 + K_5^2 \right] \quad \text{(4. 11)}

Figure 4.12 Case 3

By the use of the above formulae the form factor between any two rectangles within room surfaces can be calculated.
4.3.3.1.4 A program for estimating the form factor

Based on the above, a computer program was written in Quick Basic to calculate the form factor between any two rectangular elements on the room surfaces. The program is listed in Appendix (A).

4.3.3.1.5 Investigation into the accuracy of the program

It was necessary to validate the accuracy of the program output before using it in the study. Although the formulae used are considered to be accurate, errors may result within the structure of the program itself.

The output of the program was checked by means of the following techniques:

1. Form factors were obtained by solving the double area integrals which is considered to be an accurate simulation of energy exchange between surfaces. Such estimations were calculated for some surfaces where the distance (squared) between them was much larger than the area of the each surface. This is because the integral equation works well with such a condition (62, 75).

2. For adjacent rectangles the output of the program was compared against an accurate result, usually used as a standard test for form factor between equal-sided perpendicular squares whose form factor is equal to 0.200043 (62).

3- A useful property of form factor is that for a closed environment, the summation of form factors between one element and the rest of the elements within the enclosure is equal to one;

\[ \sum_{k=1}^{n} F_{i,k} = 1 \]  

(4.12)
Based on the use of the above techniques, the output of the program was tested. It was found that the results of the validation gave confidence in the output of the program. As an example, and by the use of the summation property, an average estimated value of 0.99999 was found for different selected cases.

4.3.3.2 Solving inter-reflection equations

One of the techniques is to simulate the inter-reflection process in the manner in which it occurs (61). Light on each element, at the first inter-reflection \( E^1 \), will equal the initial direct light plus the light reflected from all other elements. After calculating the first reflection for each element, these values, in turn, are used to simulate the second reflection, and so on, until the required accuracy is achieved.

The following formulae show the inter-reflection process for element \( i \) (61):

\[
E_{i1} = E_{di} + F_{i-1} \rho_1 E_{i0} + F_{i-2} \rho_2 E_{20} + \ldots + F_{i-n} \rho_n E_{n0} \\
E_{i2} = E_{di} + F_{i-1} \rho_1 E_{11} + F_{i-2} \rho_2 E_{21} + \ldots + F_{i-n} \rho_n E_{n1} \\
E_{i3} = \ldots \ldots \\
E_{ij} = E_{di} + F_{i-1} \rho_1 E_{ij-1} + F_{i-2} \rho_2 E_{2j-1} + \ldots + F_{i-n} \rho_n E_{nj-1} \quad (4.13)
\]

where:

\[ E_{di} = \text{initial illuminance on surface } i \]

\[ \rho_n = \text{reflectance of surface } n \]

\[ F_{n-i} = \text{form factor between element } n \text{ and element } i \]
Another approach is to generate simultaneous equations representing the final illuminance on each surface, which equals the sum of the initial illuminance and final light reflected from all other visible elements. The final illuminance on all elements can be written as follows (76):

\[
E_i = E_{d1} + F_{2-1} \rho_2 E_2 + F_{3-1} \rho_3 E_3 + \cdots + F_{n-1} \rho_n E_n
\]
\[
E_2 = E_{d2} + F_{1-2} \rho_1 E_1 + F_{3-2} \rho_3 E_3 + \cdots + F_{n-2} \rho_n E_n
\]
\[
E_3 = E_{d3} + F_{1-2} \rho_1 E_1 + F_{2-3} \rho_2 E_2 + \cdots + F_{n-3} \rho_n E_n
\]
\[\vdots\]
\[
E_n = E_n + F_{1-n} \rho_1 E_1 + F_{2-n} \rho_2 E_2 + \cdots + F_{(n-1)-n} \rho_{n-1} E_{n-1} \quad (4.14)
\]

An array of simultaneous equations gives the final illuminance of all elements. These equations can be written in a matrix notation as:

\[
D = (F) (E) \quad (4.15)
\]

where:

\[
D = \text{the matrix representing direct illuminance on each element}
\]
\[
F = \text{the matrix representing the form factors}
\]
\[
E = \text{final illuminance on each surface}
\]

The final illuminance can be found by inverting matrix \(F\) and multiplying it by matrix \(D\).

\[
E = (D) (F^{-1}) \quad (4.16)
\]
However, for a large number of elements, this technique becomes inefficient due to the fact that matrix inversion computation time increases exponentially with the number of surfaces (77). These simultaneous equations can be solved by using the Gauss-Seidel Iterative method (78). This method is easier to program and requires less time to converge (76). Once the final illuminance on each surface element is computed, it can be used in the second iteration as a starting value to produce new values for the final illuminance for use in the third iteration and so on, until the required accuracy is achieved. The illuminance on element i after mth iteration can be written as:

\[ E_i^{(m)} = \sum_{k=1}^{n} F_{i-j} \rho_j E_j^{(m-1)} \]  

(4.17)

Based on the fact that closer starting values to the final illuminance values will result in an improvement of the convergence of this iteration scheme, Littlefair (76) suggests that the best initial estimate could be direct light plus light resulting from internally reflected light, based on Dresler’s formula:

\[ I E_i = E_{di} + \frac{E_1 \rho_1 A_1 + E_2 \rho_2 A_2 + \cdots + E_n \rho_n A_n}{A (1 - \rho)} \]  

(4.18)

where:

\[ I E_i \] = Initial estimate of illuminance on element i

\[ A \] = Total area of room surfaces

\[ \rho \] = Average reflectance of room surfaces
Therefore the final illuminance on element $i$ after $m$th iteration can be written as:

$$E_{i}^{(m)} = IE_i + \sum_{k=1}^{n} F_{i-j} \rho_j E_{j}^{(m-1)}$$  \hspace{1cm} (4.19)

In the above equation, all surfaces are assumed to be Lambertian diffuse reflectors. Thus the angle of light reaching the element has no effect on the direction of the reflected component.

4.3.4 The mean illuminance on the working plane

After calculating the final illuminance on each element of the room surfaces, the final illuminance on the working plane will be equal to the light reaching from the visible portions of the surfaces to the working plane plus the direct illuminance on the working plane (figure 4.13);
\[ E_j = E_{dj} + E_i F_{i,j} \rho_j + \ldots + E_n F_{n,j} \rho_n \] (4.20)

where:

- \( E_j \) = final illuminance on element \( j \) on the working plane
- \( E_{dj} \) = direct illuminance on element \( j \) on the working plane
- \( E_i \) = final illuminance on element \( i \) on the room surfaces
- \( F_{i,j} \) = form factor between element \( i \) and element \( j \)
- \( \rho_i \) = average reflectance of element \( i \)

The mean illuminance on the working plane will equal:

\[ E_w = \left( \sum_{j=1}^{n} E_j A_j \right) / A_w \] (4.21)

where:

- \( E_w \) = mean illuminance on the working plane level.
- \( E_j \) = illuminance on element \( j \) on the working plane level.
- \( A_j \) = area of element \( j \).
- \( A_w \) = area of working plane level.
- \( n \) = number of elements on the working plane level
4.3.5 Conclusion

Based on the data presented in this section (4.3), the method for estimating the mean illuminance on the working plane level in this study can be summarised as follows:

1. Calculating the final illuminance on each element on the room surfaces as a result of a given illuminance incident on a given location by the use of the radiosity method, equation 4.19.

2. Calculating the final illuminance on each element on the working plane level from all elements above the working plane level, equation 4.20.

3. Averaging the final illuminance on all elements of the working plane level, equation 4.21.
4.4 An investigation into the relationship between the location of the received illuminance and the mean illuminance on the working plane

For the purpose of investigating the relationship between the location of the received illuminance and the mean illuminance on the working plane, this relationship was expressed by a coefficient which has been called "Grid-Element coefficient, GE", where;

\[
GE_i = \frac{\text{mean working plane illuminance due to initial illuminance of } i}{\text{initial illuminance of patch } i} \tag{4.20}
\]

A comparison between these coefficients for different locations on the room surfaces will provide the profile for such a relationship.

For this purpose, the geometry of two rooms was selected to perform the required investigation as follows:

1. Two rooms were tested. The first was 5.1 meters in width, 5.1 meters in depth and 3.4 meters in height. The second room was 5.1 meters in width, 8.5 meters in depth and 3.4 meters in height.

2. The reflectance of surfaces was assumed to be 0.25 for the floor, 0.50 for the walls and 0.70 for the ceiling.

3. The room surfaces were divided into small, square grid-elements, 0.85 meters by 0.85 meters (as in figure 4.13). Due to the fact that the height of the working plane level was assumed to be 0.85 meters and the grid-element size has to be squared for form factor estimation, the dimensions of the grid-element were influenced by the height of the working plane level.
4. During each test the GE coefficient was calculated from a given illuminance on a chosen grid element of the room surfaces.

4.4.1 Results and analysis

The Grid-Element coefficient was calculated for all elements of the two rooms tested. Figure 4.14 shows the results for the square room (5.1 by 5.1). Each grid represents a grid element on the room surface. The numbers shown are the ratios of the mean illuminance on the working plane level as a result of a light incident on a selected grid element to the illuminance on that grid element (the GE coefficient).
Figure 4.14 GE coefficients for a room of 5.1m by 5.1m in plan and 3.4m in height.

These coefficients, (figure 4.14), are based on light received on the working plane level through inter-reflection only. When direct light is incident on the working plane
level, then the coefficients for grid elements below the working plane level will be higher.

The grid-element coefficient in this case (called GE') will be equal to:

$$GE + \frac{\text{direct flux on the patch i}}{\text{illuminance on the patch i}}$$

Flux received by any patch below the working plane level from a window above the working plane level will always be equal to the flux incident on this working plane level (figure 4.15).

![Diagram showing flux incident on a patch below the working plane level]

Figure 4.15 Flux incident on any patch below the working plane level is equal to the flux received by that working plane level.

Figures 4.16 and 4.17 give the GE and GE' coefficients for the two rooms tested, where the effect of direct light reaching the working plane level is included.

The figures show very clearly the importance of the location of grid element in generating the highest amount of mean illuminance on the working plane level. It is clear that when light is incident directly in the middle of the floor surface, the highest amount of mean working plane illuminance can be achieved. On the other hand, the lowest mean illuminance on the working plane level will result when light is incident on the upper corners of the room.
Figure 4.16 An illustration for GE and GE' coefficients for a room of 5.1 x 5.1 x 3.4.
Figure 4.16 An illustration of GE and GE' coefficients for a room of 5.1 x 8.5 x 3.4.
These coefficients can be used to find the average illuminance on the working plane level. This is done by multiplying the GE coefficient for a grid element by the illuminance on such an element.

In order to illustrate the effect of a change in the angle of incidence of light, two examples were used. The first was to show the effect of a change in the altitude of the direction of light, figure 4.18, and the second was to show the effect of a change in the azimuth of the direction of light for the same room, figure 4.19. Both rooms have the same window area (0.85 by 0.85), the same vertical illuminance and the same geometry (5.1 x 5.1 x 3.4).

![Section Diagram](image)

Figure 4.18 An illustration of a room where the effect of a change in the angle of altitude of the incident light needs to be investigated.

![Plan Diagram](image)

Figure 4.19 An illustration of a room where the effect of a change in the angle of azimuth of the incident light needs to be investigated.
At each angle of incidence there is a need first to determine which surface element receives the light, and the illuminance on this surface element. The grid element which receives light and its area can be found as illustrated in figure 4.20, where either of three situations might exist when light is incident on a room surface.

![Diagram](image)

**Figure 4.20** An illustration of how to determine the grid element which receives light and its area.

Then, the mean illuminance on the working plane for each angle of incidence can be calculated as follows:
\[ E_w = \sum_{i=1}^{n} \frac{G_{E_i} E_v \tau}{\text{area of window}} \times \frac{\text{area of patch in element } i}{\text{area of patch in element } i \times \text{total area of element } i} \tag{4.22} \]

where:

- \( E_w \) = the mean illuminance on the working plane
- \( i \) = number of grid elements receiving the illuminance at the investigated angle of incidence
- \( E_v \) = the illuminance on the window wall
- \( \tau \) = transmittance of window system

An example of how to estimate the mean illuminance on the working plane level when the angle of incidence is 30° is illustrated in figure 4.21 (window transmittance = 1).

<table>
<thead>
<tr>
<th>GEi</th>
<th>illuminance x area of window</th>
<th>area of patch in element</th>
<th>element area</th>
<th>Ei</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0300</td>
<td>100x.85x.85/\tan 30 = 57.735</td>
<td>(2.94 - 2.55) x 0.85 = .454</td>
<td>.85 x .85</td>
<td>0.795</td>
</tr>
<tr>
<td>.0301</td>
<td>1.0</td>
<td>1.0</td>
<td>1.738</td>
<td></td>
</tr>
<tr>
<td>.0301</td>
<td>(1.70 - 1.47) x 0.85 / .85    = .271</td>
<td>.85 x .85</td>
<td>0.485</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td>3.018</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.21 An illustration of how to find the illuminance on the working plane level.
Based on the above approach, figures 4.22 and 4.23 were produced to illustrate the effect of the angle of incidence of the two examples.

![Figure 4.22](image1.png)

Figure 4.22 The effect of a change on the altitude of the source of light.

![Figure 4.23](image2.png)

Figure 4.23 The effect of a change on the azimuth of the source of light.
Figure 4.22 shows that there is a significant difference between the effect of light incident below the working plane level and light incident above the working plane level. Both figures show that there is a significant difference due to the change of angle of incidence when light is incident above the working plane level, while there is no such effect when light is incident below the working plane level.

Figure 4.24 illustrates a comparison between all GE and GE' coefficients of the square room (figure 4.16). This figure indicates that there is a big difference when light is received on patches below the working plane level. Therefore, a distinction should be made between light received below the working plane level and light received above the working plane level.

![Graph showing comparison of GE and GE' coefficients](image)

---

**Figure 4.24** A comparison between all GE and GE' coefficients of the square room (5.1 x 5.1).
From figure 4.24, and for light received on patches above the working plane level, the maximum difference between the GE coefficients is found to be 57%. For light incident on patches below the working plane level, the maximum difference between the GE' coefficients can reach 3.6%. The same findings could apply to figure 4.17. Therefore, when light is incident on patches below the working plane level the direction of incident light can be assumed to be insignificant. By contrast, when light is incident on patches above the working plane level, the direction of incident light is a significant issue.

These findings are for cases involving only one direction of light: light from above the working plane level (from the sky) or light from below the working plane level (from the ground). However, this is not the case under real circumstances. In reality, light will come from all angles around the window. So the effect of the angle of incidence of light will be much less, depending on how much light reaches the window plane from each direction.

In order to estimate the effect of angle of incidence for light reaching parts of the room above the working plane level, where the effect of such angle might be significant, the following formula can be used:

\[
\text{maximum possible error} = \frac{\text{GE}_1 - \text{GE}_2}{\text{GE}_1} \times \frac{L_b}{L_b \text{GE}_1 + L_a \text{GE}'_2} \quad (4.23)
\]

where:

- GE1 = the maximum GE coefficient for locations above the working plane level
- GE2 = the minimum GE coefficient for locations above the working plane level
GE'2 = the minimum GE' coefficient for locations below the working plane level

Lb = quantity of light reaching the window from below the working plane level

La = quantity of light reaching the window from above the working plane level

Therefore, if light reaches the window from just two directions, that is, light from above and below the working plane level, and they are equal, the maximum possible error due to the effect of the angle of incidence of light reaching parts of the room surfaces above the working plane level (for the 5.1 x 5.1 room) can be estimated as follows:

\[
\frac{0.0139 - 0.0059}{0.0139} \times \frac{1 \times 0.0139}{1 \times 0.0139 + 1 \times 0.0293}
\]

\[
= 0.19
\]

When light incidence from below the working plane level is equal to three times as much as light incidence from above it, the maximum possible error will be:

\[
\frac{0.0139 - 0.0059}{0.0139} \times \frac{3 \times 0.0139}{3 \times 0.0139 + 1 \times 0.0293}
\]

\[
= 0.34
\]

These results indicate that if light reaching the window plane is dealt with as two components, that is, light incident from above the working plane level and light incident from below the working plane level, then the effect of the luminance
distribution of sky or other emitting sources, such as ground and opposing facade, is insignificant. Significant error might exist when light incidence from below the working plane level is very large compared to light incidence from above the working plane level.

When light incident on the window plane needs to be dealt with as one component, as in the BRE Average Daylight Factor method, the issue will be different. This is due to the large effect of the angle of incidence on the average internal illuminance between the two components (light from above and light from below). The ratio of the average of all GE coefficients (for locations above the working plane level) to the average of all GE' coefficients (for locations below the working plane level) is about 1:3. Therefore, any method proposed should take such an issue into consideration.
4.5 Validation for the theoretical analysis

It was necessary to validate the results of the theoretical analysis and a scale model was used for that purpose.

Theoretically, and if the effect of glass transmittance is not included, illumination effects in a physical model are scaleless. Light in the model behaves as it does in a full scale building (79). Such techniques have been used in daylighting studies and have shown a reasonably acceptable correlation when compared with natural light tested under real sky conditions (80, 81). Under the test conditions in this study, where there is no account of real sky conditions, a stronger correlation should exist.

4.5.1 The model instrumentation, components and characteristics

Based on suggestions for scale model studies (50), (82), (83), the model was designed on a scale of 1: 20 to simulate a typical room. The room was 5 meters in width, 3 meters in height and 5 meters in depth. On one of its sides the room had a window which could be extended along the width of the room to give different window areas.

The reflectance of the different surfaces were 0.62 for the ceiling, interior wall reflectance 0.48 and floor 0.25 (determined by a luminance meter).

4.5.2 Procedures of validation

Experiments were conducted in the lighting laboratory at the University of Sheffield. The position of the light source was simulated by a fixed lamp on the ceiling of the laboratory, nearly three meters away from the window plane in order to give nearly parallel rays of light (plate 4.1). The model was placed on a rotative surface which
can move vertically and horizontally, in order to achieve the required angle of incidence.

During each test, the angle of incidence, the illuminance on the centre of the window plane, and the average internal illuminance on the working plane level were recorded.

Four different categories were simulated to find the final illuminance on the working plane level. These were light incidence on ceiling only, light incidence on floor only, light incidence on the side walls where light does not fall on the working plane and, finally, light falling on the side walls where light does fall on the working plane.

The average internal illuminance was estimated as follows:

1. In each case, the vertical illuminance on the window plane was measured and then, by use of the same photocell, internal illuminance was measured. It was decided to use the same photocell in order to reduce the error which results from the different responses of different photocells (83). The photocell used was new and already calibrated by the manufacturer.

2. Based on previous studies (48, 83, 84), measurements of the internal illuminance were taken at 25 station points inside the room. Each point covered one square meter in order to represent the whole working plane of the room (figure 4.26).

3. In areas receiving direct light on the working plane, the area of the direct light was estimated and then multiplied by the average illuminance of this area. The internal average illuminance was then taken by averaging the whole area of the working plane.
Plate 4.1 A picture illustrating the scale model set up.
4.5.3 Comparison with the theoretical analysis

The theoretical analysis described earlier was used to estimate the mean illuminance on the working plane for the same examples as had been tested on the scale model. The results of the comparison between the two results are illustrated in table 4.1.

<table>
<thead>
<tr>
<th>Azimuth difference between the normal of the window and the light source</th>
<th>Altitude of light source</th>
<th>Ratio of measurements to mathematical estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
<td>1.09</td>
</tr>
<tr>
<td>0</td>
<td>-30</td>
<td>0.98</td>
</tr>
<tr>
<td>45</td>
<td>-12</td>
<td>1.12</td>
</tr>
<tr>
<td>0</td>
<td>-27</td>
<td>0.89</td>
</tr>
<tr>
<td>0</td>
<td>27</td>
<td>0.98</td>
</tr>
<tr>
<td>0</td>
<td>45</td>
<td>1.04</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>0.98</td>
</tr>
<tr>
<td>0</td>
<td>54</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 4.1 The ratio of the measured to the calculated mean working plane illuminance.
The results show a good symmetry between the scale model results and the theoretical analysis results. Although the results indicate a possible error of 12 percent, it is the belief that most of the errors exist in the scale model measurements. This is because determining the exact area of the direct light received on the room surfaces is difficult due to the non-clear boundaries of lit areas. It is believed that this causes such discrepancies. However, the findings in general give confidence in the theoretical analysis developed for this study.

4.6 Conclusion

In this chapter an investigation was carried out into the effect of the angle of incidence of light on the ratio of the average internal illuminance to the vertical illuminance on the window plane. This investigation was done by mathematical simulation analysis which was then validated by experiments on a scale model.

The main conclusions from this investigation are:

1. The effect of the non-uniform luminance of a clear sky is insignificant, in general, in relation to the average internal illuminance. The sky can be treated as a uniform light source.

2. The angle of incidence of light reflected from the ground and reaching the ceiling and upper parts of the walls might have a significant effect if such light is much larger in amount than the light incident from above the working plane level.

3. If the total amount of light used to find the average internal illuminance need to be treated as just one component (as in the BRE Average Daylight Factor), then consideration should be given to the fact that the contribution of light
incident from above the working plane level to the total internal average internal illuminance is about 75 percent. That is, when the light incidence from above the working plane level is equal to the light incidence from below the working plane level.
CHAPTER FIVE

APPLICABILITY OF THE EXISTING AVERAGE DAYLIGHT FACTOR FORMULAE UNDER CLEAR SKY CONDITIONS
CHAPTER FIVE: APPLICABILITY OF THE EXISTING AVERAGE DAYLIGHT FACTOR FORMULAE UNDER CLEAR SKY CONDITIONS

5.1 Introduction

Results from Chapter 4 indicate the possibility of dealing with the amount of illuminance transmitted to the interior of a room from above the working plane level regardless of the direction from which it arrives, in general. However, when there is much more incident light from below the working plane level than there is from above it, significant error might exist.

The aim of this chapter is to investigate the relationship between the light incident from above and below the working plane level. Based on this investigation, the possibility will be addressed of using either of the two equations available for estimating the average daylight factor at the working plane level: the Longmore equation (50) which uses the split flux approach and the BRE equation (48) which utilises the total flux approach.

Such an investigation requires knowing the two components of light, which are light incidence on the external surface of the window plane from above the working plane level and from below it.

The methodology used in this chapter is based on the following:

1. A computer program was written in BASIC to estimate the amount of light incident on the external surface of the window plane as a function of several variables.
2. A comparative analysis was carried out of light incident from below the working plane level and light incident from above it.

3. The average daylight factor for several configurations (variation of sun location, room depth, room height, etc.) was calculated, based on the two existing equations (Longmore and BRE).

4. A comparative study was carried out of the average daylight factor obtained by these two equations and by the SUPERLITE lighting program.

The general findings of this chapter can be summarised as follows:

1. A method based on the split flux approach is not, because of its complexity, the best choice for finding the average internal illuminance under clear sky conditions.

2. The BRE equation always overestimates the average internal illuminance under clear sky conditions.

3. There is a need for a new equation to be used for clear sky conditions in order to find the average internal illuminance.

4. One of the problems which needs to be solved is the variability of external illuminance on the window wall under clear sky conditions.
5.2 Light incident on the window under clear sky conditions

For the purpose of the investigations of this chapter, illuminance on the vertical window plane needs to be estimated first. This illuminance will be estimated in two components, the sky component and the ground component. The sky component will include the direct skylight and the reflected sunlight from obstruction coming from above the working plane level. The ground component will include reflected skylight and reflected sunlight coming from below the working plane level (Figure 5.1).

A program was written in BASIC to calculate, for given condition, the following:

1. The skylight received on the window plane.
2. The reflected sunlight and skylight received on the window plane.
3. The skylight on the horizontal.

The program is listed in Appendix B and an outline of it is shown in figure 5.2.
INPUT DATA
- Solar altitude and azimuth
- Orientation of window wall
- Street width and height of walls
- Reflectance of surfaces
- Window height
- Shading device depth
- Illuminance turbidity factor

COMPUTATION
- Estimate direct and diffuse light incident on all surfaces

COMPUTATION (external surfaces)
- Compute the form factor for the different surfaces
- Compute the final illuminance on each surface after the inter-reflection process

COMPUTATION
- Compute the final light received on the window plane through reflection from other surfaces and directly from the sky

OUTPUT
- Illuminance received from above the working plane level
- Illuminance received from below the working plane level
- Illuminance on the horizontal

Figure 5.2 Layout of the structure of the computer program.
5.2.1 Theoretical background to the Program

Where the window is obstructed by a nearby surface and shaded by an appropriate shading device (figure 5.3), light reaching the centre of the window plane is from the opposing facades, the ground and the sky. In order to find the illuminance received on the window plane, illuminance on each surface (ground, facade, obstruction and shading device) needs to be estimated first.

![Figure 5.3](image)

5.2.1.1 Direct illuminance on the surfaces

A. Skylight

Light received from the sky, at any station point, is a function of the sky luminance and the geometrical relationship between the station point and the sky to which it is exposed. In order to find the total sky illuminance on a station point we need first to determine the part of the sky which emits light to that point. If an obstruction exists in front of the window, the angle of that obstruction is not constant along its side.
Finding the sky line requires a detailed process. However, in order to simplify the process, and due to the fact that we are dealing with a very long obstruction, it was assumed that the angle of obstruction for the centre point will represent the skyline as a constant for the whole area seen from the sky (e.g. as a courtyard).

![Diagram](image1.png)

**Figure 5.4** Due to the complexity of determining the sky line, the angle of obstruction is used instead.

The angle at which the sky is exposed to the window is then divided vertically into 9 equal bands. Each band is then divided horizontally into 18 equal zones for the vertical plane and 36 for the horizontal plane. Such a process has been used by Eldiasty for estimating the illuminance from the sky (85, 86) and its accuracy is acceptable.

![Diagram](image2.png)

**Figure 5.5.** The division into 9 bands of the angular size of the sky seen from the window.
The illuminance received on a vertical plane from the sky will be equal to:

$$\sum_{i=1}^{9} \sum_{j=1}^{18} L_{ij} S \cos(a)$$  \hspace{1cm} (5.1)

where:

$L_{ij}$ = luminance of the patch $ij$ in the sky

$S$ = solid angle

$a$ = angle of incidence of the sky point (between normal to window and line to sky point)

For an unobstructed horizontal plane the illuminance will be equal to:

$$\sum_{i=1}^{9} \sum_{j=1}^{36} L_{ij} (S) \sin(\gamma)$$  \hspace{1cm} (5.2)

where:

$\gamma$ = altitude of sky point

This procedure is carried out for three points representing the facade, the ground and the obstruction. These points are in the centre of each surface. Although the luminance distributions of these surfaces are not uniform, it was decided to use one centre point for each surface due to the low brightness of the sky and due to the importance of the reflected sunlight.

**B. Sunlight**

Estimation of direct sunlight is based on the geometrical relationship between the surface and the sun. If the surfaces are partly exposed to the sun, the total direct
illuminance on the surface is equal to the area-weighted average of skylight and sunlight illuminance.

5.2.1.2 Inter-reflection between surfaces

The final illuminance \( (E) \) of a surface exposed to other surfaces is the sum of direct illuminance \( (E_d) \) and the illuminance from inter-reflection. Such inter-reflection is, theoretically, easy to compute if the initial illuminance is known for all the surfaces (facade, ground & obstruction, figure 5.6).

\[
E_1 = E_d + F_{2,1} \rho_2 E_2 + F_{3,1} \rho_3 \\
E_2 = E_d + F_{1,2} \rho_1 E_1 + F_{3,2} \rho_3 E_3 \\
E_3 = E_d + F_{1,3} \rho_1 E_1 + F_{2,3} \rho_2 E_2 \tag{5.3}
\]

These are a set of simultaneous equations representing the final illuminance on each surface, which is equal to the sum of the initial illuminance \( (E_d) \) and the final light reflected from all other visible elements, where \( \rho_i \) is the reflectance element \( i \) \((i = 1, 2, 3)\).
to 3) and $F_{n,i}$ is the form factor between element $n$ and element $i$. Solving these simultaneous equations is done by using the Gauss-Seidel Iterative Method (78).

5.2.1.3 Final illuminance on the window

After calculating the final illuminance on each surface, the final illuminance on the window will be equal to:

$$E_{dw} + F_{2.1} \rho_2 E_2 + F_{3.1} \rho_3 E_3$$

(5.4)

where $E_{dw}$ is the diffuse skylight received on the window plane. If a shading device is used, the diffuse skylight will be based on the angle at which the sky is exposed to the window, which is angle (A) in figure 5.7.

$E_2$ and $E_3$ are the final illuminances on surfaces 2 and 3.

![Figure 5.7](image)

Figure 5.7 When an overhang is used, the angle of sky will get smaller.
5.2.2 Basics of daylight estimation

There follows an introduction to the equations used in the simulation program. These equations, mostly, are cited in "Daylight Algorithms" prepared by Tregenza and Sharples (72), where these algorithms have been selected from several alternatives and checked numerically within computer programs.

5.2.2.1 Direct solar illuminance

The direct illuminance reaching the earth is a function of several variables. It is influenced by location of the sun, the ellipticity of the earth's orbit, atmospheric conditions (cloud cover, haze, dust etc.) and thickness of the atmosphere. These factors are determined as follows:

5.2.2.1.1 Extraterrestrial solar illuminance

The extraterrestrial solar illuminance (Evo) is the solar illuminance incident on the atmosphere of the earth. It varies according to the distance between the sun and the earth due to the ellipticity of the earth's orbit during the year (72).

\[
E_{\text{vo}} = \overline{E_{\text{vo}}} \left[ 1 + 0.034 \cos \left( \frac{2 \pi (J - 2)}{365} \right) \right] \text{klx (5.5)}
\]

where:

\[
\overline{E_{\text{vo}}} = \text{extraterrestrial illuminance at mean distance between earth and sun}
\]

\[
= 127.5 \text{ klx}
\]

\[
J = \text{day number in the year (Julian day)}
\]
5.2.2.1.2 Illuminance turbidity factor

When passing through the atmosphere, direct illuminance is scattered and absorbed several times. The combined effect of such scattering and absorption is known as atmospheric turbidity, $T_{II}$ (87).

5.2.2.1.3 Relative optical air mass

The relative optical air mass ($m$) describes the length of the sun's path through the atmosphere in comparison with the sun's path when the sun is overhead (72).

\[
m = \frac{P}{P_0} \frac{1}{\sin \gamma_s + 0.50572 \left( \frac{180 \gamma_s}{\pi} + 6.07995 \right)^{-1.6364}} \tag{5.6}
\]

\[
\frac{P}{P_0} = 1.0 - \frac{h}{10000} \tag{5.7}
\]

where:

- $\frac{P}{P_0}$ = the ratio of mean atmospheric pressure at the site to that at sea level.

- $h$ = station height < 4000 meters

- $\gamma_s$ = solar altitude

5.2.2.1.4 Mean extinction coefficient of light

The mean extinction coefficient of light ($a_{VR}$) is the optical thickness of clean dry air (77).
\[ \alpha_{VR} = \frac{0.1}{1 + 0.0045 \ m} \]  

The direct illuminance on a horizontal surface is expressed as:

\[ E_{VSH} = E_{VSN} \sin \gamma_s \text{ klx} \]  

where:

\[ E_{VSN} = \text{direct normal illuminance} \]

\[ = E_{VO} \exp (-\alpha_{VR} \ m T_n) \text{ klx} \]  

The direct illuminance on a vertical surface is expressed as:

\[ E_{VSV} = E_{VSN} \cos (a) \]  

where:

\[ a = \text{angle of incidence} \]

\[ a = \arccos (\cos \gamma_s \cos (as - ae)) \]

\[ as = \text{solar azimuth} \]

\[ ae = \text{surface azimuth} \]
5.2.2.2 Clear sky luminance

Several equations have been proposed to find the relative luminance of a clear sky (88) (89, 90). These equations are based on measurements taken in temperate regions. Narasimhan and Saxena (91) proposed an equation based on measurements in a tropical region. The CIE (89) has adopted the following equations proposed by Kittler (90):

\[
\frac{L_{vcl}(\gamma, \zeta)}{L_{vclz}} = \frac{f(\zeta)}{f(\frac{\pi}{2} - \gamma_s)} \cdot \frac{\phi(\gamma)}{\phi(\frac{\pi}{2})}
\]

(5.12)

where:

\( \zeta = \) angle between the sun and the sky point

\( \gamma = \) altitude of sky point

\( \gamma_s = \) solar altitude

\( \zeta = \arccos (\sin \gamma \sin \gamma_s + \cos \gamma \cos \gamma_s \cos(a - a_s)) \)

\[f(\zeta) = 0.91 + 10 \exp(-3 \zeta) + 0.45 \cos^2 \zeta \]

\[f(\frac{\pi}{2} - \gamma_s) = 0.91 + 10 \exp\left(-3 \left(\frac{\pi}{2} - \gamma_s\right)\right) + 0.45 \cos^2\left(\frac{\pi}{2} - \gamma_s\right) \]

\[\phi(\gamma) = 1 - \exp\left(\frac{-0.32}{\sin \gamma}\right) \]

\[\phi\left(\frac{\pi}{2}\right) = 0.27385 \]
The above equations are for a clean atmosphere (illuminance turbidity factor = 2.45), as proposed by Kittler. For an industrial atmosphere (illuminance turbidity factor = 5.5), the following equations, proposed by Gusev, are used instead of $f(\zeta)$ and $f(\pi/2 - \gamma)$ (92).

\[
f'(\zeta) = 0.856 + 16 \exp(-3 \zeta) + 0.3 \cos^2 \zeta
\]

\[
f'(\pi/2 - \gamma) = 0.856 + 16 \exp \{-3 (\pi/2 - \gamma)\} + 0.3 \cos^2 (\pi/2 - \gamma)
\]

Since the illuminance turbidity factor is a variable quantity, and for the purpose of this study, the CIE equation modified by Kittler (93) will be used where:

\[
f(\zeta) = 1 + N \{ \exp(-3 \zeta) - 0.009 \} + M \cos^2 \zeta
\]

\[
N = 4.3 \ T_{il}^{1.9} \exp(-0.35 \ T_{il})
\]

\[
M = 0.71 / \sqrt{T_{il}}
\]
In order to find the illuminance from the sky, the luminance at the zenith needs to be obtained first. The following equations were used for this purpose (72):

\[ L_{\text{vclz}} = E_0 \cdot g(\tau_v, m) \cdot f(\frac{\pi}{2} - \gamma_s) \]  \hspace{1cm} (5.13)

\[ g(\tau_v, m) = \frac{(2.5 - 1.4 \cdot \tau_v)}{19.6 (1 - m)} \cdot \frac{(\tau_v^m - \tau_v)}{19.6 (1 - m)} \], \hspace{1cm} \tau_v \geq 0.75

\[ = \frac{(1.6 - 0.4 \cdot \tau_v + 0.75 - \tau_v)}{22.07 (1 - m)} \cdot \frac{\tau_v^m - \tau_v}{22.07 (1 - m)} \], \hspace{1cm} \tau_v < 0.75

where:

\[ \tau_v = \exp(-\alpha_{VR} \cdot T_{il}) \]

\[ \tau_v = \text{the atmospheric transmittance.} \]
5.3 The relationship between the sky and ground components

Based on the program explained above and illustrated in Appendix (B), figures 5.9 and 5.10 were produced. The aim was to show the relationship between the reflected light from below the working plane level received on the window plane and the light received on the same window plane from above the working plane level. The figures indicate that, in general, more light is reflected from the ground than is received from the sky, as much as 2.4 times, in the conditions tested (illuminance turbidity = 2.45, ground reflectance = 0.25). Other conditions will produce different results. However, the issue here is to present a general idea about the sources of internal illuminance under clear sky conditions.

These results show that the illuminance received from below the working plane level is usually less than 2.5 times the amount of light incident from above the working plane level. According to the results of Chapter Four (equation 4.23), the maximum possible error due to the effect of the angle of incidence of light for such a condition might reach up to 31%. However, under real conditions light will reach different locations on the ceiling and upper parts of the walls as a diffuse source, making the possibility of having such an error reduces. Therefore, it could be assumed that the effect of angle of incidence of light under clear sky conditions is insignificant.
Figure 5.9 The relationship between the reflected light from the ground and light from the sky received on the window plane under clear sky conditions (ground reflectance = 0.25 and illuminance turbidity = 2.45).

Figure 5.10 The relationship between the reflected light from the ground and light from the sky received on the window plane under clear sky conditions (ground reflectance = 0.25, illuminance turbidity = 2.45 and the angle of obstruction = 140°).
5.4 Applicability of the existing average daylight factor formulae under clear sky conditions

The following section compares illuminances calculated on the basis of two average daylight factor equations with detailed calculation using SUPERLITE, under clear sky conditions and reflected sunlight.

The Longmore equation, when it is to be used under clear sky conditions, will be written as:

\[
\bar{D}F = \tau W \left( \frac{C_1}{A_{fw}} + \frac{C_1 \ R_{fw} + C_2 \ R_{cw}}{A \ (1-\rho)} \right) \%
\]  

(5.14)

where:

\( \bar{D}F \) = average daylight factor on the working plane level

\( W \) = glazing area

\( \tau \) = the directional transmittance of light by the window system

\( C_1 \) = a function of the light incidence on the window from above the horizontal

\( C_2 \) = a function of the light incidence on the window from below the horizontal

\( A_{fw} \) = area of floor and lower parts of the walls below the mid-height of the window (not including the window wall)

\( R_{fw} \) = average reflectance of the \( A_{fw} \)

\( R_{cw} \) = average reflectance of the ceiling and upper walls above the mid-height of the window (not including the window wall)
\[ A = \text{area of all surfaces in the room} \]

\[ \rho = \text{average reflectance of all surfaces in the room} \]

The BRE equation, when it is to be used under clear sky conditions will be written as:

\[
\overline{DF} = \frac{2 C \tau W}{A (1 - \rho^2)} \tag{5.15}
\]

Where:

\[ C = \text{a function of the total light incidence on the window} \]

\[ W = \text{glazing area} \]

\[ \tau = \text{diffuse light transmittance of the glazing} \]

\[ A = \text{total area of room surfaces} \]

\[ \rho = \text{average reflectance of all room surfaces} \]

C, C1 and C2 in equations 5.24 and 5.25 were obtained for clear sky conditions using the program listed in Appendix (B).

Because the average daylight factor, by definition, excludes the effect of sunlight, the average daylight factor under a clear sky will be called, in this study, the average natural light factor \( \overline{NF} \), where the reflected sunlight will be part of the internal illuminance of the room.
5.4.1 Estimating the average natural light factor by SUPERLITE

The lighting simulation program SUPERLITE (26) was selected because it is a well
documented program that can perform detailed analysis of natural light in buildings;

“SUPERLITE 2.0, a powerful lighting analysis program designed to
accurately predict interior illuminance in complex building spaces due to
daylight and electric lighting systems. SUPERLITE enables a user to model
interior daylight levels for any sun and sky conditions in spaces having
windows, skylights or other standard fenestration systems ....... The program
calculates lighting levels on all interior surfaces, as well as on planes that can
be arbitrarily positioned to represent work surfaces or other locations of
interest to the user. SUPERLITE 2.0 is intended to be used by researchers
and lighting designers, who require detailed analysis of the illuminance
distribution in architecturally complex spaces.” *

SUPERLITE has been tested and used in previous studies (80, 81, 94-96). Although
these studies had shown some notable differences from real measurements, major
errors are the result of the difference between the luminance of the real sky and the
luminance of the CIE sky. Hence, in the case of this study, where a comparison is
needed between the same sky such differences will not exist.

* SUPERLITE 2.0 User’s Manual. Lawrence Berkeley Laboratory (1994),
pp. 1-1.
5.4.1.1  Daylight estimation by SUPERLITE

The calculation approach used in SUPERLITE is a very detailed one which accounts for a variety of sky conditions. For the purpose of this study, estimating daylight under clear sky conditions is addressed.

In SUPERLITE, the illuminance on a station point is the sum of direct illumination from sun and sky and reflected illuminance from surfaces. Therefore, the luminance of each surface must be found first.

For the external surfaces, the luminance distribution of each surface is assumed to be uniform with a differentiation between shaded and non shaded areas. Luminance on every external surface is equal to its reflectance multiplied by the sum of direct illumination from sun and sky and reflected illuminance from surfaces. A light flux balance on each surface yields (97):

\[ \sum_{j=1}^{Nwp} \left( \frac{\delta_{ij}}{\rho_{jp}} - F_{i-j} \right) L_{jp} = L_z F_{i-sky} + \frac{f_{jp}}{\pi} E_s \cos \beta_i, \quad i = 1, Nwp \]  (5.16)

where:

- \( Nwp \) = number of opaque walls in enclosure \( p \)
- \( L_{jp} \) = unknown luminance of surface \( j \)
- \( \rho_{jp} \) = reflectance of surface \( j \)
- \( F_{i-j} \) = light exchange factor from surface "i" to "j"
- \( F_{i-sky} \) = light exchange factor from surface "i" to sky
- \( L_z \) = sky luminance at zenith
- \( E_s \) = direct normal illuminance from the sun
\( f_{ip} \) = fraction of surface \( A_{ip} \) receiving direct sunshine

\( \beta_{si} \) = angle between surface normal to enclosure surface \( i \) and vector toward the sun

\( \delta_{ij} \) = kronecker delta (i.e. =1 if \( i = j \), otherwise = 0)

This equation forms a system of \( N_{wp} \) equations with \( N_{wp} \) unknown luminance, \( L_{jp} \). This set of simultaneous equations is solved by matrix inversion to find the luminance of each external surface.

For the internal surfaces, each surface is divided into a finite number of grid elements, whose size can be determined by the user. For each grid element the direct (sunlight and skylight) and reflected light is estimated. Then, the resultant light flux balance for each grid element is expressed as (97):

\[
L_{ik} = \rho_i \sum_{j=1}^{N} \sum_{l=1}^{N_j} F_{ikjl} L_{jl} + \rho_i L_{ck} + L_{ii}, \quad i = 1, N; \quad k = 1, N_i
\]  

(5.17)

where:

\( L_{ik} \) = luminance of node \( k \) on surface \( i \)

\( N \) = total number of surfaces

\( N_i \) = total number of surfaces

\( \rho_i \) = reflectance of nodes in surface \( i \)

\( F_{ikjl} \) = light exchange factor between node \( k \) on surface \( i \) to node \( l \) on surface \( j \)
Leik = the directly transmitted luminance penetrating through a clear or sheer-curtained window (as seen by node k)

Ldi = the contribution of diffusing or sheer-curtained windows to their own luminance

5.4.1.1.1 Daylight factor by SUPERLITE

SUPERLITE is capable of calculating the illuminance at any given point within an internal space. Finding the average daylight factor through SUPERLITE was done by first selecting an imaginary working surface and dividing it into 30 to 40 grid elements as measurement stations (48, 83, 84), depending on the depth of the room.

Final illuminance on each element (Eik) is calculated as (97):

\[
E_{ik} = \pi \sum_{j=1}^{N} \sum_{l=1}^{N_j} F_{ikjl} L_{jl} + \pi L_{cik} \quad (5.18)
\]

Then the average daylight factor;

\[
= \left[ \frac{\sum_{ik=1}^{N} E_{ik}}{N} \right] / \text{Eh} \quad (5.19)
\]

where:

Eh is the illuminance from a clear sky on the horizontal. Direct sunlight is excluded from both values, Eik and Eh.
5.4.1.1.2 Direct sun light

The direct illuminance on a horizontal surface (Evsh) is estimated by SUPERLITE as:

\[ Evsh = Evo \exp (-avR \cdot m \cdot T) \cdot \sin \gamma_s \]  \hspace{1cm} (5.20)

where:

- \( \gamma_s \) = solar altitude
- \( Evo \) = extraterrestrial solar illuminance
- \( m \) = relative optical air mass
- \( avR \) = mean extinction coefficient of light
- \( T \) = Linke's turbidity factor.

where:

\[ Evo = 128.82 + 4.248 \cos J + 0.0825 \cos 2J - 0.00043 \cos 3J \]
\[ + 0.169 \sin J + 0.000914 \sin 2J + 0.01728 \sin 3J \]  \hspace{1cm} (5.21)

\[ J = (2\pi / 366) \text{ (day of the year)} \]

\[ m = \frac{1 - 0.1 \cdot h}{\cos \gamma_s + 0.15 (-\gamma_s + 93.89)^{-1.255}} \]  \hspace{1cm} (5.22)

\( h \) = building altitude in km

\( avR = 0.1512 - 0.0262 T \)  \hspace{0.5cm} \text{for} \hspace{0.5cm} \beta = 0.05
\( avR = 0.1656 - 0.0215 T \)  \hspace{0.5cm} \text{for} \hspace{0.5cm} \beta = 0.10
\( avR = 0.2021 - 0.0193 T \)  \hspace{0.5cm} \text{for} \hspace{0.5cm} \beta = 0.20 \]  \hspace{1cm} (5.23)
Linke's turbidity factor, is given by (87):

\[ T = \left( \frac{\gamma s + 85.0}{39.5 \exp(-w) + 47.4} \right) + 0.1 + (16.0 + 0.22 \, w) \beta \]  

(5.24)

where:

\[ \beta = \text{Angstrom turbidity coefficient.} \]

\[ w = \text{Thickness of condensable water.} \]

5.4.1.1.3 Clear sky luminance

SUPERLITE uses the CIE clear sky formula (89) described in section 5.2.2.2 (equation 5.12, illuminance turbidity = 2.45)

Zenith luminance, is given by (98):

\[ L_z = (1.376 \, T - 1.81) \tan \gamma s + 0.38 \]  

(5.25)

5.4.1.1.4 Estimating the light incidence on the window plane

The SUPERLITE simulation program does not calculate the illuminance on the window plane. However, the program described in Appendix (B) can estimate the amount of light reaching the window as a ratio of the horizontal sky illuminance. By using the sky illuminance and direct sunlight on the horizontal, which are found by SUPERLITE, as input data in the program, the total light reaching the window plane could be easily estimated for the data obtained by SUPERLITE.
Therefore, estimating the average natural light factor in this case will be based on calculating the mean illuminance on the working plane level by SUPERLITE and by calculating the illuminance on the window plane by the use of the program listed in Appendix (B).

If a shading device is used, reflected light from the shading device to the window is estimated as follows:

1. After estimating the final illuminance reaching the ground, the luminance of the ground will be multiplied by the appropriate configuration factor between the ground and the shading device.

![Diagram of light reflection](image)

Figure 5.11 When a shading device is used, an exact estimation of light received from the sky and light reflected from shading is required.

From a diffuse surface the illuminance at point \( p \) is:

\[
E_p = L_i \cdot c_{f_{i\cdot j}} \tag{5.26}
\]

where:
Li = luminance of the surface i.

c_{f_i-j} = configuration factor.

Figure 5.12

c_{f_i-j} (for this case (72))

\[ c_{f_i-j} = \frac{1}{2\pi} \left( \frac{x}{\sqrt{1 + x^2}} \tan^{-1} \frac{y}{\sqrt{1 + x^2}} + \frac{y}{\sqrt{1 + y^2}} \tan^{-1} \frac{x}{\sqrt{1 + y^2}} \right) \]  \hspace{1cm} (5.27)

2. Light received from the shading device on the window plane is obtained by multiplying the luminance of the shading device by the appropriate form factor between the shading device and the window plane.

5.4.2 The average natural light factor obtained by the existing equations and SUPERLITE

A comparison of the average natural light factor obtained by the BRE and the Longmore average daylight factor equations and SUPERLITE was made for several configurations. These include variation of sun location, room depth, room height, window area, average reflectance of internal surfaces and orientation of window wall. These configurations are listed in Appendix (C).
5.4.3 Results and analysis

Data obtained for the comparison of the calculated average natural light factor is illustrated in table 5.1 and figure 5.12.

### Table 5.1 Comparison of the two equations for the average natural light factor on the working plane with values calculated by SUPERLITE.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>SUPERLITE</th>
<th>BRE</th>
<th>Longmore</th>
<th>BRE SUPERLITE</th>
<th>Longmore SUPERLITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.8</td>
<td>13.16</td>
<td>9.28</td>
<td>1.22</td>
<td>.86</td>
</tr>
<tr>
<td>2</td>
<td>11.75</td>
<td>15.13</td>
<td>9.9</td>
<td>1.29</td>
<td>.84</td>
</tr>
<tr>
<td>3</td>
<td>12.76</td>
<td>16.95</td>
<td>10.66</td>
<td>1.33</td>
<td>.84</td>
</tr>
<tr>
<td>4</td>
<td>12.46</td>
<td>16.21</td>
<td>10.52</td>
<td>1.3</td>
<td>.84</td>
</tr>
<tr>
<td>5</td>
<td>12.82</td>
<td>16.17</td>
<td>11.08</td>
<td>1.26</td>
<td>.86</td>
</tr>
<tr>
<td>6</td>
<td>14.47</td>
<td>18.41</td>
<td>12.41</td>
<td>1.27</td>
<td>.86</td>
</tr>
<tr>
<td>7</td>
<td>13.35</td>
<td>17.52</td>
<td>11.29</td>
<td>1.31</td>
<td>.85</td>
</tr>
<tr>
<td>8</td>
<td>12.88</td>
<td>17.07</td>
<td>10.79</td>
<td>1.33</td>
<td>.84</td>
</tr>
<tr>
<td>9</td>
<td>11.35</td>
<td>15.47</td>
<td>9.91</td>
<td>1.36</td>
<td>.87</td>
</tr>
<tr>
<td>10</td>
<td>10.35</td>
<td>14.23</td>
<td>9.32</td>
<td>1.37</td>
<td>.90</td>
</tr>
<tr>
<td>11</td>
<td>8.79</td>
<td>12.04</td>
<td>7.63</td>
<td>1.37</td>
<td>.87</td>
</tr>
<tr>
<td>12</td>
<td>7.14</td>
<td>9.86</td>
<td>6.2</td>
<td>1.38</td>
<td>.87</td>
</tr>
<tr>
<td>13</td>
<td>11.92</td>
<td>16.07</td>
<td>10.96</td>
<td>1.35</td>
<td>.92</td>
</tr>
<tr>
<td>14</td>
<td>15.17</td>
<td>19.0</td>
<td>13.38</td>
<td>1.25</td>
<td>.88</td>
</tr>
<tr>
<td>15</td>
<td>17.49</td>
<td>21.14</td>
<td>15.28</td>
<td>1.21</td>
<td>.87</td>
</tr>
<tr>
<td>16</td>
<td>18.44</td>
<td>21.74</td>
<td>16.67</td>
<td>1.18</td>
<td>.90</td>
</tr>
<tr>
<td>17</td>
<td>9.52</td>
<td>12.81</td>
<td>8.13</td>
<td>1.35</td>
<td>.85</td>
</tr>
<tr>
<td>18</td>
<td>16.05</td>
<td>22.38</td>
<td>13.92</td>
<td>1.39</td>
<td>.87</td>
</tr>
<tr>
<td>19</td>
<td>10.63</td>
<td>13.64</td>
<td>8.57</td>
<td>1.28</td>
<td>.81</td>
</tr>
<tr>
<td>20</td>
<td>8.15</td>
<td>10.29</td>
<td>6.47</td>
<td>1.26</td>
<td>.79</td>
</tr>
<tr>
<td>21</td>
<td>14.05</td>
<td>18.81</td>
<td>11.8</td>
<td>1.34</td>
<td>.84</td>
</tr>
<tr>
<td>22</td>
<td>11.32</td>
<td>15.51</td>
<td>10.22</td>
<td>1.37</td>
<td>.90</td>
</tr>
<tr>
<td>23</td>
<td>11.69</td>
<td>16.48</td>
<td>9.82</td>
<td>1.41</td>
<td>.84</td>
</tr>
<tr>
<td>24</td>
<td>13.87</td>
<td>17.48</td>
<td>11.58</td>
<td>1.26</td>
<td>.83</td>
</tr>
<tr>
<td>25</td>
<td>5.49</td>
<td>6.90</td>
<td>4.34</td>
<td>1.26</td>
<td>.79</td>
</tr>
<tr>
<td>26</td>
<td>5.34</td>
<td>6.90</td>
<td>4.34</td>
<td>1.29</td>
<td>.81</td>
</tr>
<tr>
<td>27</td>
<td>5.15</td>
<td>6.90</td>
<td>4.34</td>
<td>1.34</td>
<td>.84</td>
</tr>
</tbody>
</table>
The results indicate that the BRE formula overestimates the average natural light factor while the Longmore formula underestimates it. The BRE formula, used to find the average daylight factor under an overcast sky, was developed and checked against a fixed relation between light coming from the sky and light coming from the ground (48). This relation is based on a non-obstructed window, that is, a ratio of 39:5, the ratio of light received from the sky to light received from the ground. Large variations were found when external obstructions were placed near to the window (99).
Such variations are believed to be due to the different relationship which exists between light coming from the sky and light coming from the ground (different from the ratio of 39 : 5). Light from the sky will be influenced by the angle of obstruction, while light coming from the ground will not be affected, according to the formula. The greatest error will occur when the angle of obstruction is $80^\circ$, because in this situation the ratio of light coming from the sky will equal light coming from the ground. The error will increase as the angle of obstruction increases.

The Longmore equation produces better results than the BRE although in every case the values were slightly lower than those calculated with SUPERLITE. The split-flux approach depends, though, on knowing the parameters $C_1$ and $C_2$. Using existing methods, this is a lengthy calculation with several variables – relative solar position, illuminance turbidity, external reflectances, angles of obstruction. If it is to be simplified, the effect of window illuminance variability must be studied further, especially its relationship with horizontal external illuminance.

5.5 Conclusion

In this chapter, an investigation was carried out on the applicability of use of the available average daylight factor equations under clear sky conditions. It was decided that a method based on the split flux method is not the best choice for finding the average internal illuminance. This is due to the fact that the estimation of the light incidence on the window from above and below the horizontal is influenced by external variables. These variables include the illuminance turbidity factor, reflectance of external surfaces, shape and size of external surfaces and the geometrical relationship between these surfaces. A method based on the split flux
will require either a detailed estimation of incident light or a simplification where error will result. The BRE formula does not require the relation between the light from below and above the horizontal to be available. It requires only that total illuminance should be known. However, it always overestimates the average internal illuminance. This is because the BRE average daylight factor equation is based on the assumption that the contribution of light reaching the room from below the horizontal is not a major source of internal illuminance.

Based on the above, two items need to be addressed. These are:

1. It seems that there is a need for a new equation to be used for clear sky conditions in order to find the average internal illuminance. Such an equation should take into consideration the importance of reflected sunlight.

2. There is a need to solve the problem of the variability of external illuminance on the window wall.

In the following chapter these two items will be discussed.
CHAPTER SIX

THE PROPOSED SOLUTION
6.1 Introduction

In the previous chapter two problems were presented. First, neither the Longmore equation nor the BRE equation is appropriate for use under clear sky conditions. Longmore's equation requires a knowledge of light incidence on the window plane (knowledge of the upper and lower components), something which is not constant under real sky conditions. The BRE equation was developed for conditions where the sky component is a major contributor to the internal illuminance of a room.

Second, under clear skies the external illuminance on the window surface is a function of several variables, in particular the sun's position in the sky.

The aim of this chapter is to investigate a solution to these two problems. This will involve the following:

1. An investigation into the relationship between the average internal illuminance and the horizontal and vertical external illuminances.


3. An investigation into the relationship between the illuminance incident on the window plane and the horizontal sky illuminance.

Findings from this chapter show that it is possible to propose a solution for predicting the average internal illuminance under clear sky conditions, based on the following:
1. A new formula for predicting the average internal illuminance as a ratio of the external illuminance on the external surface of the window plane.

2. A minimum condition in which the received illuminance on the window plane is equal to the horizontal diffuse illuminance.

6.2 Comparison of vertical and horizontal external illuminances with the mean internal illuminance

A numerical experiment was undertaken to compare the relative variation of the mean internal illuminance with respect to the vertical illuminance on the window plane and the external horizontal illuminance under varies sky conditions.

The room illustrated in figure 6.1 was used as a model and then illuminances calculated under different conditions of solar position, ground reflectance and sky conditions. The room was 5 meters in depth, 5 meters in width and 2.5 meters in height. Reflectance of ceiling was 0.7, floor was 0.25 and walls was 0.5. The window was 1.2 meters in height by 5.0 meters in width with a transmittance of 0.85.
In the first case, the investigated illuminances were calculated as a function of solar position (table 6.1). The ground reflectance was 0.25 and Linke’s turbidity factor was 3.9 (2.5 illuminance turbidity factor - clear sky). A horizontal shading device of a projection of 1.25 meters from window wall was used.

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the sun and the normal of the window wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>180</td>
</tr>
<tr>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>180</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.1 An illustration for sun positions examined in the first case.
In the second case, the investigated illuminances were calculated as a function of solar position with an existing obstruction at a distance of 4.0 meters from the window wall. The azimuthal difference between the sun and the normal of the window wall was 180°. The reflectance of ground was 0.25 and the reflectance of the obstructions was 0.5. Linke’s turbidity was 3.9.

In the third case the investigated illuminances were calculated as a function of ground reflectance. The ground reflectance was 0.15, 0.20, 0.25 and 0.30. Solar altitude was 60° with 180° azimuth difference between the sun and the normal of window wall. Linke’s turbidity factor was 3.9.

In the fourth case the investigated illuminances were calculated as a function of Linke’s turbidity factor. This factor was 3.1, 3.9, 4.4 and 5.4. Solar altitude was 60° with 180° azimuth difference between the sun and the normal of window wall.

In the above four cases, the horizontal sky illuminance and the mean illuminance were calculated by the use of the lighting simulation program SUPERLITE. The vertical illuminance was calculated by the use of SUPERLITE output data and the program listed in Appendix C, as described in 5.4.

6.2.1 Results of the comparison

Figure 6.2 shows a comparison of vertical and horizontal external illuminances with the mean internal illuminance as a function of solar altitude. In this case, the ratio of the average internal illuminance to the vertical illuminance is more stable than the ratio of the average internal illuminance to the horizontal sky illuminance. This is due to the effect of reflected sunlight from the ground and the luminance of the sky facing
the window, which both vary with sun position, while the horizontal sky illuminance will not vary in the same ratio. As an example, a room facing east will not have the same average internal illuminance throughout the day. In the morning the room will face a brighter sky and a brighter ground, while in the afternoon it will face a less bright sky and ground. The horizontal sky illuminance will be nearly the same in the morning and afternoon, while the illuminance on the window plane will differ.

Figure 6.2 A comparison of vertical and horizontal external illuminances with the mean internal illuminance as a function of solar altitude.

The above findings, could also be found from the second case where the window is facing an obstruction. Due the effect of reflected sunlight from the obstruction, the illuminance reaching the window plane and the room surfaces will increase while the horizontal illuminance will not be effected by such reflected light.
In the third case the illuminance reaching both the window and the room surfaces will vary with the reflectance of the ground, while the horizontal illuminance will stay the same. Therefore the internal illuminance will be a function of the illuminance incident on the window plane and not the sky illuminance on the horizontal.

Figure 6.3 A comparison of vertical and horizontal external illuminances with the mean internal illuminance as a function of solar altitude with an obstruction facing the window.

Figure 6.4 A comparison of vertical and horizontal external illuminances with the mean internal illuminance, as a function of the reflectance of ground.
In the fourth case the effect of sky conditions was investigated. Linke's turbidity factor varied from 3.1 to 5.4. Again, in this case the ratio of change in the illuminance on the window plane and the horizontal sky illuminance is not the same. When the sky becomes clearer, the diffuse (sky) illuminance will decrease while the sunlight will increase. Hence the reflected sunlight received on the window plane and the room surfaces will increase and at the same time the horizontal sky illuminance is decreasing.

Figure 6.5 A comparison of vertical and horizontal external illuminances with the mean internal illuminance, as a function of Linke's turbidity factor.
6.2.2 Conclusions

The above analysis indicate that the ratio of the average internal illuminance to the external vertical illuminance is more stable than if we relate the average internal illuminance to the horizontal sky illuminance. This fact could be generalised under clear sky conditions for typical rooms. This is due to the fact that under such conditions, the total flux entering the room is related to the vertical illuminance rather than to the horizontal.

6.3 A simplified formula for estimating the mean illuminance on the working plane level

The proposed formula will be based on finding the two components of the internal illuminance, the direct and inter-reflected light, separated as in the Longmore approach. However, the total flux will be used, as in the BRE approach. The aim is to develop a simple equation which gives a useful prediction method for working plane illuminance under clear skies.

A. The direct illuminance on the working plane level

Light entering a room will reach all the surfaces of the room except the window wall. The average illuminance of these surfaces can be written as:

\[ E_s = \frac{W \tau E_w}{A_1} \]  

(6.1)

where:

\[ E_s = \text{average illuminance on the total area of room surfaces, excluding the window wall (lux)} \]
W = glazing area

Eₜₜ = total light incidence on the external surface of the window plane

τ = diffuse light transmittance of the glazing

A₁ = total area of room surfaces (excluding the window wall)

The mean internal illuminance on the working plane is assumed to be equal to the mean illuminance on the total area of room surfaces (excluding the window wall). Therefore:

\[ E_{wp} \approx \frac{W \tau E_t}{A_1} \]  \hspace{1cm} (6.2)

where:

\[ E_{wp} = \text{average direct illuminance on the working plane level (lux)} \]

B. Inter-reflected light on the working plane level

Estimation of the inter-reflected light is based on the theory of the integrating sphere (100), where the total flux (F) entering the sphere is uniformly distributed over the sphere area \( (F/4 \pi r^2) \). This flux is then reflected over the entire sphere \( (F \rho /4 \pi r^2) \). In turn, another reflection occurs over the whole sphere surface \( (F \rho^2 /4 \pi r^2) \) and so on. The total illuminance at any point on the sphere surface is:

\[ \frac{F}{4 \pi r^2} + \frac{F \rho}{4 \pi r^2} + \frac{F \rho^2}{4 \pi r^2} + \frac{F \rho^3}{4 \pi r^2} + \ldots \ldots \]

\[ = \frac{F}{4 \pi r^2} (1 + \rho + \rho^2 + \rho^3 + \ldots) \]

\[ = \frac{F}{4 \pi r^2} (1/(1-\rho)) \]
Assuming that the distribution of light in the room is nearly the same as the integrating sphere, the inter reflected light for the room is:

\[ F \rho / (A (1 - \rho)) \]  

(6.3)

where:

\[ A = \text{total area of room surfaces} \]

\[ \rho = \text{area-weighted average reflectance of all room surfaces} \]

Then, assuming that the mean reflectance of the entire room is close to the mean reflectance of all the surfaces except the window wall, the total mean working plane illuminance is:

\[ E_W \ W \ \tau \left( \frac{1}{A} + \frac{\rho}{A (1 - \rho)} \right) \text{ lux} \]  

(6.4)

Taking the external window plane illuminance to be 100%, the mean natural light factor will be defined to be:

\[ \overline{NF} = W \ \tau \left( \frac{1}{A} + \frac{\rho}{A (1 - \rho)} \right) \times 100 \]  

(6.5)

where:

\[ \overline{NF} = \text{the average natural light factor, \%} \]

\[ W = \text{glazing area} \]

\[ \tau = \text{diffuse light transmittance of the glazing} \]
A₁ = total area of room surfaces except the window wall

A = total area of room surfaces

ρ = area-weighted average reflectance of all room surfaces

6.4 Comparison between vertical window plane illuminance and external horizontal illuminance

It was shown in 6.2 that the vertical illuminance is a better predictor of internal illuminance than the horizontal sky illuminance. However, the external horizontal illuminance is a much easier quantity to derive. This section therefore examines the conditions under which the vertical illuminance may be taken as a fixed ratio of the horizontal illuminance where no obstruction exist.

The program listed in Appendix "B" and explained in 5.2, was used to estimate the total illuminance on a window plane as a ratio of the horizontal sky illuminance. The window illuminance includes reflected sunlight but excludes direct sunlight. Table 6.1 gives the results.
<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the normal of window wall and sun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.70</td>
</tr>
<tr>
<td>45</td>
<td>1.57</td>
</tr>
<tr>
<td>60</td>
<td>1.44</td>
</tr>
<tr>
<td>75</td>
<td>1.32</td>
</tr>
<tr>
<td>90</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 6.1 The ratio of the total illuminance received on the window plane (including reflected sunlight) to the horizontal sky illuminance (illuminance turbidity = 2.45, ground reflectance = 0.25).

Another table of illuminance data was produced, this time for an illuminance turbidity of 5.0 (table 6.2).

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the normal of window wall and sun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.66</td>
</tr>
<tr>
<td>45</td>
<td>1.43</td>
</tr>
<tr>
<td>60</td>
<td>1.14</td>
</tr>
<tr>
<td>75</td>
<td>1.09</td>
</tr>
<tr>
<td>90</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 6.2 The ratio of the total illuminance received on the window plane (including reflected sunlight) to the horizontal sky illuminance (illuminance turbidity = 5.0, ground reflectance = 0.25).
Figure 6.9 provides a histogram for tables 6.1 and 6.2.

![Histogram](image)

The ratio of illuminance on the vertical to illuminance on the horizontal.

Figure 6.9 A histogram for the data in tables 6.1 and 6.2.

The calculations show that:

1. At a first approximation, the vertical illuminance on the window plane is equal to the horizontal sky illuminance.

2. The greatest difference between the two quantities occurs when the sun is directly in front of the window at a low altitude. Assuming that the vertical illuminance is equal to the horizontal diffuse illuminance would, in the worst cases, give an overestimate of window illuminance by 39% with high turbidities and an underestimate by 70% at low solar altitudes.

This may be a very useful result, giving a simple estimation method for window illuminances under clear skies, but its generality needs to be tested under various conditions of obstruction.
6.5 Conclusions

1. Equation 6.5 may give a simple method for prediction mean working plane illuminance in relation to the vertical window illuminance.

2. Under some conditions, the window illuminance is approximately equal to the diffuse horizontal illuminance.

These combine to give a technique for predicting window size in climates with clear skies. Except at high turbidities an assumption based on conclusion 2 would give an underestimate of window performance, hence in design the technique could be used to determine as a 'worst case' condition, a minimum window size. The maximum window size would be based on thermal criteria.

The accuracy of these assumptions will be examined in the following chapter.
CHAPTER SEVEN

VALIDATION
7.1 Introduction

In the previous chapter, a solution was proposed for predicting average internal illuminance under clear sky conditions, where direct sunlight is excluded. This solution is divided into two parts.

The first part assumes that the illuminance on the external plane of a vertical window is equal to the horizontal diffuse illuminance; direct sunlight is not considered in either quantity, but the window illuminance includes reflected sunlight. Such a hypothesis reduces the complexity of estimating the illuminance incidence on the window under clear sky conditions. Designing for natural light, in this case, will be based on what, in most situations, is the worst condition, that the illuminance on the vertical plane equals the horizontal diffuse illuminance. If more light is needed, the maximum window size will be determined by the heat gain issue, which is a major factor influencing the design of openings in hot arid regions.

The second part estimates the average internal illuminance on the working plane level as a percentage of the illuminance on the external surface of the vertical window plane (which has been called the average natural light factor - $\overline{NF}$) by use of the following equation:

$$\overline{NF} = W \tau \left( \frac{1}{A_1} + \frac{\rho}{A \left( 1 - \rho \right)} \right) \times 100$$
where:

\[
\frac{\bar{NF}}{100} = \frac{\text{mean illuminance on the working plane level}}{\text{simultaneous illuminance on the vertical external plane of the window}}
\]

\(W\) = glazing area

\(\tau\) = diffuse light transmittance of the glazing

\(A_1\) = total area of room surfaces except window wall

\(A\) = total area of room surfaces

\(\rho\) = area-weighted average reflectance of all room surfaces

The aim of this chapter is to validate the proposed solution. The validation will involve three different methods:

The first is a validation of the proposed average natural light factor against measurements taken in real rooms under real sky conditions.

The second is a validation of the assumed vertical to horizontal ratio against measurements taken in a scale model under real sky conditions.

The third is a numerical comparison of the proposed solution against results obtained by the use of the SUPERLITE simulation program, since this takes account of a wide range of conditions.
7.2 Validation based on real measurements

The real measurement study was carried out in the Dammam area of Saudi Arabia. The cloud cover in this area is very low, which represents a typical clear sky condition (< 2.0 oktas (101)).

7.2.1 Location and climate

The Dammam Metropolitan Area is located on the eastern coast of Saudi Arabia, north of the tropic of Cancer, at a latitude of 26° 30' North, and a longitude of 50° 09' East. The climate of the area is fairly complex.

The eastern coast, as a whole, is classified as being in the Hot-Dry climate zone (13). However, because the Dammam Area is located on the shoreline of the Gulf, the relative humidity is high, and the area may be classified as a Hot-Dry Maritime Desert (13).

The Area is characterised by two climatic periods, an extremely hot season, which lasts from May to October, and a relatively cool season, from November to April (102). During the hot season, the temperature reaches a mean maximum of 46.7° C in July and August. In the cool season the mean minimum is around 6.3° C and 6.2° C in December and January (103).

The mean maximum relative humidity exceeds 91% for the whole year. The mean minimum is around 20% for the whole year, except for the months of November (26.9%) and December (28.4%). Solar radiation is intense in the area. The monthly maximum radiation values on the horizontal surface range from 764 W/m² in December to over 1100 W/m² in May, June and July (103). The wind velocity is higher during the hot season, reaching 9 m/s. These winds blow predominantly
from the north to the north-west and pass through the desert, usually making them dusty and sandy. However, occasionally there are cool breezes from the south-east (102, 104). Annual rainfall is very low, around 71 mm annually (13).

Figure 7.1 Map of Saudi Arabia, showing the location of the Dammam Area.
Figure 7.2 Summary of the climatic data for the Dammam Area
The high humidity in the Dammam area results in high turbidity values. These are higher than the standard ones assumed for clear sky conditions.

<table>
<thead>
<tr>
<th>Month</th>
<th>Linke's turbidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>4.77</td>
</tr>
<tr>
<td>Feb.</td>
<td>5.82</td>
</tr>
<tr>
<td>March</td>
<td>6.13</td>
</tr>
<tr>
<td>April</td>
<td>6.82</td>
</tr>
<tr>
<td>May.</td>
<td>7.51</td>
</tr>
<tr>
<td>June</td>
<td>7.11</td>
</tr>
<tr>
<td>July</td>
<td>7.42</td>
</tr>
<tr>
<td>Aug.</td>
<td>6.21</td>
</tr>
<tr>
<td>Sep.</td>
<td>6.30</td>
</tr>
<tr>
<td>Oct.</td>
<td>6.18</td>
</tr>
<tr>
<td>Nov.</td>
<td>5.53</td>
</tr>
<tr>
<td>Dec.</td>
<td>4.48</td>
</tr>
</tbody>
</table>

Table 7.1 Linke's turbidity factor for Dhahran city (105).

7.2.2 Measurement of the natural light factor

Three rooms were selected for measuring the average natural light factor. The first two rooms were located in a building on the campus of King Faisal University. Figure 3 shows the plan of this building.
The two rooms are identical in geometry and window area, but differ in orientation and window transmittance. Each room is 4.77 meters by 4.77 meters in plan and 3.14 meters in height. The net area of the glazing for each window is 1.66 m².

Reflectance of each surface was estimated by measuring both the light incidence on the surface and the reflected light from the same surface. The ratio of reflected light to incident light is the reflectance of the surface (29, 37). Reflectance of the surfaces are 0.35 for floors and 0.77 for ceilings and walls. The average reflectance of each room was estimated to be 0.61.

Transmittance of glazing was estimated by measuring the light incidence on the window plane and then measuring the light transmitted directly from the window system (106). The south room has horizontal louvers for solar radiation protection (plate 7.1). The transmittance for this window was estimated to be 0.18. The north window has an estimated transmittance of 0.28.
Plate 7.1 The facade of the south room.

Plate 7.2 The facade of the north room.

146
Plate 7.3 The external view from the window of the south room.

Plate 7.4 The external view from the window of the north room.
Both rooms have an almost non-obstructed view of the sky. Therefore, it was beneficial to select another room with a different angle of obstruction. Such a room was selected from a building in the Dammam Area and is illustrated in figure 7.4.

![Figure 7.4](image)

Figure 7.4 Plan showing the third room facing a very close obstruction.

The room is 3.87 meters by 4.15 meters in plan and 2.79 meters in height. The net glazing area is 1.24 m². The transmittance of the window was estimated to be 0.26. Surface reflectance are 0.31 for the floor, 0.43 for the walls and 0.63 for the ceiling. The average reflectance of the room surfaces was estimated to be 0.45.
7.2.3 Average natural light factor estimation

Estimating the average natural light factor was based on estimating the average internal illuminance inside the room and then dividing it by the vertical illuminance incidence on the external surface of the window system. Measuring the average internal illuminance was based on dividing each room into equal grids (48). Figure 7.5 illustrates the suggested grid system for the north room. Under a clear sky and when the sun is high, the external illuminance is generally stable. Therefore, finding

Plate 7.5 Illustration of the proximity of the obstruction in the case of the third room.
the average natural light factor was based on measuring the external vertical illuminance on the window, then measuring the light on a working plane level of 0.85 cm for the first row of grids and repeating this procedure for each row of grids. Sun location was calculated by means of the equations listed in Appendix “D”.

Figure 7.5 Plan of the north room showing position of photocells

7.2.4 Measurements for the vertical to the horizontal ratio

The second study concerned the validation of the proposed relationship between the illuminance on the window plane and the horizontal diffuse illuminance. A scale model was built to simulate a window wall facing a movable obstruction, where different angles of obstruction could be tested. Measurements were taken simultaneously of the illuminance on the vertical plane, which included direct and reflected sky light and reflected sunlight, and of the sky horizontal illuminance. When measuring the diffuse illuminance, direct sunlight was excluded manually. Several readings were taken for each case. The orientation of the surfaces was estimated by the use of a compass and sun location was calculated by means of the equations listed in Appendix “D”.

150
Plate 7.6 & 7.7 Illustration of how the model was used in measuring the vertical and the horizontal illuminance.
7.2.5 Results and analysis

Data for the measurements of the average natural light factor (\(\overline{NF}\)) for the three rooms are illustrated in table 7.2.

<table>
<thead>
<tr>
<th>Room no.</th>
<th>Orientation</th>
<th>Measured NF</th>
<th>Estimated NF</th>
<th>Solar Altitude</th>
<th>Solar Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>North</td>
<td>1.07</td>
<td>72</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.04</td>
<td>65</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06</td>
<td>67</td>
<td>235</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.05</td>
<td>57</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.04</td>
<td>73</td>
<td>199</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>South</td>
<td>1.02</td>
<td>62</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.98</td>
<td>71</td>
<td>213</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.02</td>
<td>72</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06</td>
<td>73</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.12</td>
<td>52</td>
<td>251</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>24° south of the east</td>
<td>1.69</td>
<td>44</td>
<td>264</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.76</td>
<td>42</td>
<td>265</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.20</td>
<td>72</td>
<td>229</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.33</td>
<td>67</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.84</td>
<td>76</td>
<td>204</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.41</td>
<td>59</td>
<td>252</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2 Ratio of measured to estimated average natural light factors for the three rooms.

From the table above, and without considering the results of the SUPERLITE simulation program, it seems that the proposed \(\overline{NF}\) equation can be used to find the average natural light factor under a clear sky for cases with similar conditions to those of rooms one and two. In the case of room 3, the formula does not work well, due, mainly, to the reflected sunlight from the obstruction which is very close to the window (figure 7.6). The issue of the angle of obstruction requires more investigation.
Table 7.3 illustrates the ratio of the illuminance on the vertical plane, which includes direct and reflected sky light and reflected sunlight, to the diffuse horizontal illuminance.
<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference*</th>
<th>Angle of obstruction</th>
<th>Vertical/Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>115</td>
<td>29</td>
<td>0.93</td>
</tr>
<tr>
<td>64</td>
<td>115</td>
<td>15</td>
<td>1.01</td>
</tr>
<tr>
<td>64</td>
<td>115</td>
<td>no obstruction</td>
<td>1.01</td>
</tr>
<tr>
<td>64</td>
<td>135</td>
<td>29</td>
<td>1.08</td>
</tr>
<tr>
<td>64</td>
<td>135</td>
<td>15</td>
<td>0.96</td>
</tr>
<tr>
<td>64</td>
<td>135</td>
<td>no obstruction</td>
<td>0.80</td>
</tr>
<tr>
<td>64</td>
<td>180</td>
<td>29</td>
<td>0.99</td>
</tr>
<tr>
<td>64</td>
<td>180</td>
<td>15</td>
<td>0.89</td>
</tr>
<tr>
<td>64</td>
<td>180</td>
<td>no obstruction</td>
<td>0.76</td>
</tr>
<tr>
<td>29</td>
<td>110</td>
<td>15</td>
<td>1.07</td>
</tr>
<tr>
<td>29</td>
<td>110</td>
<td>no obstruction</td>
<td>0.97</td>
</tr>
<tr>
<td>29</td>
<td>150</td>
<td>no obstruction</td>
<td>0.93</td>
</tr>
<tr>
<td>20</td>
<td>180</td>
<td>29</td>
<td>0.89</td>
</tr>
<tr>
<td>20</td>
<td>180</td>
<td>15</td>
<td>0.82</td>
</tr>
<tr>
<td>20</td>
<td>180</td>
<td>no obstruction</td>
<td>0.81</td>
</tr>
<tr>
<td>20</td>
<td>145</td>
<td>no obstruction</td>
<td>0.77</td>
</tr>
<tr>
<td>20</td>
<td>145</td>
<td>15</td>
<td>0.80</td>
</tr>
<tr>
<td>49</td>
<td>98</td>
<td>29</td>
<td>0.84</td>
</tr>
<tr>
<td>49</td>
<td>98</td>
<td>15</td>
<td>0.98</td>
</tr>
<tr>
<td>49</td>
<td>98</td>
<td>no obstruction</td>
<td>0.96</td>
</tr>
<tr>
<td>47</td>
<td>112</td>
<td>29</td>
<td>0.90</td>
</tr>
<tr>
<td>47</td>
<td>112</td>
<td>15</td>
<td>0.91</td>
</tr>
<tr>
<td>47</td>
<td>112</td>
<td>no obstruction</td>
<td>0.97</td>
</tr>
<tr>
<td>46</td>
<td>180</td>
<td>29</td>
<td>1.23</td>
</tr>
<tr>
<td>46</td>
<td>180</td>
<td>15</td>
<td>0.97</td>
</tr>
<tr>
<td>46</td>
<td>180</td>
<td>no obstruction</td>
<td>0.66</td>
</tr>
<tr>
<td>45</td>
<td>110</td>
<td>29</td>
<td>1.02</td>
</tr>
<tr>
<td>45</td>
<td>135</td>
<td>29</td>
<td>0.98</td>
</tr>
<tr>
<td>45</td>
<td>135</td>
<td>15</td>
<td>0.89</td>
</tr>
<tr>
<td>45</td>
<td>135</td>
<td>no obstruction</td>
<td>0.74</td>
</tr>
<tr>
<td>44</td>
<td>80</td>
<td>29</td>
<td>0.75</td>
</tr>
<tr>
<td>44</td>
<td>80</td>
<td>15</td>
<td>0.98</td>
</tr>
<tr>
<td>44</td>
<td>80</td>
<td>no obstruction</td>
<td>1.07</td>
</tr>
<tr>
<td>44</td>
<td>60</td>
<td>29</td>
<td>0.70</td>
</tr>
<tr>
<td>44</td>
<td>60</td>
<td>15</td>
<td>0.98</td>
</tr>
<tr>
<td>44</td>
<td>60</td>
<td>no obstruction</td>
<td>1.14</td>
</tr>
<tr>
<td>67</td>
<td>75</td>
<td>29</td>
<td>0.98</td>
</tr>
<tr>
<td>67</td>
<td>0</td>
<td>29</td>
<td>0.96</td>
</tr>
<tr>
<td>70</td>
<td>85</td>
<td>29</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 7.3 Data for the horizontal diffuse illuminance and vertical illuminance readings.
* Azimuth difference is between the sun and the normal of the window wall. Horizontal shading devices have been used when the azimuth difference is less than 90.
The results above show that the vertical illuminance is usually less than the horizontal illuminance. The same conclusion was drawn from the mathematical simulation used to generate the same relationship for a highly turbid sky. Again these results show that it is worth considering the illuminance on the window plane as approximately equal in quantity to the horizontal diffuse illuminance.

Figure 7.7 A histogram for the data illustrated in table 7.3.
7.3 Validation based on SUPERLITE analysis

There follows a comparative analysis between the average natural light factor estimated by the proposed methodology and the average natural light factor estimated by the use of the lighting simulation program SUPERLITE. This analysis will involve both the proposed average natural light factor (equation 6.5) and the relationship between the vertical illuminance on the window plane and the horizontal diffuse illuminance.

7.3.1 Comparison of equation 6.5 against SUPERLITE

Several tests were carried out, covering the most important factors that affect the internal illuminance inside a room under clear sky conditions:

1. The geometrical relationship between the sun and the window wall.
2. Geometry of the room.
3. Reflectance of the surfaces.
5. Shading devices.
6. Window area and location.

Based on the arguments presented above, the average natural light factor ($\bar{NF}$) was estimated for different cases by both equation 6.5 and SUPERLITE. For all of these cases, a general base room, which is illustrated in figure 7.8, was used. However, changes occur depending on the status of each case. The reflectance of the
surfaces of the base room are 0.7 for the ceiling, 0.5 for the walls and 0.25 for the floor. Linke's turbidity factor was 3.9.

Figure 7.8 The geometry of the base room used for finding the average natural light factor by the use of SUPERLITE and the proposed average natural light factor equation.

7.3.1.1 Results and analysis

A. The effect of solar altitude

Below is an investigation into the effect of solar altitude on the accuracy of the $\frac{\bar{N}}{F}$ equation. The base room illustrated in figure 7.8 was used here. However, due to the need for protection from direct solar radiation, shading was employed (figure 7.9).

Figure 7.9 A horizontal shading device is used when there is a need for protection from direct sun radiation.
The average natural light factors were calculated by both the proposed equation and SUPERLITE. Figure 7.10 illustrates the ratio of both quantities as a function of solar altitude (the sun is perpendicular to the window wall).

![Graph showing the ratio of natural light factors obtained by equation 6.5 to those obtained by SUPERLITE as a function of solar altitude.](image)

Figure 7.10 The ratio of the $N_{F}$ obtained by equation 6.5 to the $N_{F}$ obtained by SUPERLITE as a function of solar altitude.

Results of the test reported above show that with the change of solar altitude, equation 6.5 usually gives an average natural light factor of within 10% above or below the average natural light factor obtained by SUPERLITE.

If an obstruction exists in front of the window wall, the performance of the investigated ratio changes. With an obstruction at an angle of 32° (figure 7.11), and solar altitude of 90°, an error of up to 14% can occur (figure 7.12). However, when the angle is smaller, e.g. 22.6°, the error is within the range of 10% (figure 7.13).

![Illustration of the location of angle of obstruction.](image)

Figure 7.11 Illustration of the location of angle of obstruction.
Figure 7.12 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of solar altitude and angle of obstruction of 32°.

Figure 7.13 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of solar altitude and angle of obstruction of 22.6°.
B. The effect of solar azimuth

For the cases tested (figures 7.14 - 16), the effect of the azimuth difference between the sun and the vertical of the window wall can be considered insignificant. Equation 6.5 usually produces an average natural light factor to within 4% accuracy of the average natural light factor obtained by SUPERLITE. This is for both an unobstructed window (figure 7.14), or for a window with obstruction at an angle of less than $23^\circ$. Obstruction closer to the window wall will result in a larger error (figure 7.16). However, generally speaking, if the angle of obstruction is less than $32^\circ$, the average natural light factor can be considered to be represented with acceptable accuracy. The solar altitude for the cases above was $60^\circ$.

![Figure 7.14 The ratio of the NF obtained by equation 6.5 to the NF obtained by SUPERLITE as a function of solar azimuth (unobstructed window).](image)
Figure 7.15 The ratio of the $\frac{N_F}{N_F}$ obtained by equation 6.5 to the $\frac{N_F}{N_F}$ obtained by SUPERLITE as a function of solar altitude and angle of obstruction of 22.6°.

Figure 7.16 The ratio of the $\frac{N_F}{N_F}$ obtained by equation 6.5 to the $\frac{N_F}{N_F}$ obtained by SUPERLITE as a function of solar altitude and angle of obstruction of 32°.
C. Geometry of the room

The purpose of equation 6.5 is for use with the geometry of a typical room, i.e., rectangular spaces. Figures 7.17 and 7.18 set the boundaries of the geometry of spaces within which the proposed equation for finding the average natural light factor can be used. Figure 7.17 illustrates that the equation can be used when the ratio of height to depth is less than 0.88. The depth of a room is not very critical. It seems that the proposed equation can work well with rooms as deep as four times their height (figure 7.18).

![Figure 7.17](image-url)
D. Reflectance of the surfaces

Reflectance of ceiling and walls are more important than that of the floor. This is because the ceiling and walls are more directly related to the illuminance on the working plane level than is the floor. Equation 6.5 seems to work well with normal room reflectance (figures 7.19 – 21). When ceiling reflectance becomes low (less than 0.55) an error of more than 10% will occur (figure 7.22).

Figure 7.18 The ratio of the $\bar{N}\bar{F}$ obtained by equation 6.5 to the $\bar{N}\bar{F}$ obtained by SUPERLITE as a function of the depth of the room (room height = 3.0 meters).

Figure 7.19 The ratio of the $\bar{N}\bar{F}$ obtained by equation 6.5 to the $\bar{N}\bar{F}$ obtained by SUPERLITE as a function of average reflectance.
Figure 7.20 The ratio of the $\frac{\bar{NF}}{F}$ obtained by equation 6.5 to the $\frac{\bar{NF}}{F}$ obtained by SUPERLITE as a function of wall reflectance.

Figure 7.21 The ratio of the $\frac{\bar{NF}}{F}$ obtained by equation 6.5 to the $\frac{\bar{NF}}{F}$ obtained by SUPERLITE as a function of floor reflectance.
E. Reflectance of external surfaces

Under clear sky conditions the reflectance of obstruction and ground are important due to their being an important source of internal illuminance. The importance of the reflectance of the obstruction increases when the obstruction is closer to the window wall. When the reflectance of an obstruction drops below 0.35, the error will be more than 10% (figure 7.23).
However, when the angle of obstruction becomes larger, the reflectance of the obstruction becomes less important (figure 7.24).

Light reflected from the ground is an important factor. When such a factor becomes small, then the equation will produce a result with an error of more than 10%. The equation seems to work well with normal ground reflectance (figure 7.25).

---

**Figure 7.24** The ratio of the $\frac{NF}{F\text{ by } \text{SUPERLITE}}$ obtained by equation 6.5 to the $\frac{NF}{F\text{ by } \text{SUPERLITE}}$ obtained by SUPERLITE as a function of the reflectance of the obstruction (angle of obstruction of 22.6°).

**Figure 7.25** The ratio of the $\frac{NF}{F\text{ by } \text{SUPERLITE}}$ obtained by equation 6.5 to the $\frac{NF}{F\text{ by } \text{SUPERLITE}}$ obtained by SUPERLITE as a function of ground's reflectance.
F. Angle of obstruction

Normal angles of obstruction do not have much significance on the ratio investigated. Usually, the result can be considered acceptable (angle of obstruction is illustrated in figure 7.11). However, when the angle of obstruction increases, a larger error will result. It can reach up to 35% (figure 7.26). It seems that angles of obstruction of less than 40° can work well with the proposed equation.

![Figure 7.26 The ratio of the $N_F$ obtained by equation 6.5 to the $N_F$ obtained by SUPERLITE as a function of angle of obstruction.](image)

If the effect of illuminance turbidity is taken into consideration, the results above agree with the results of the real measurements study, where an error of up to 76% was found for very close obstruction.
G. Shading devices

The effect of shading devices should be taken into consideration. This is due to the need for protection from direct sun radiation. Although they minimise direct skylight, they reflect some light inside the room.

![Figure 7.27 Angle of horizontal shading device.](image)

In the first case (Figure 7.28), the azimuth difference between the sun and the normal of the window wall was $0^\circ$ and solar altitude was at $75^\circ$.

![Figure 7.28 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of shading length. (Azimuth difference between the vertical of the window and the sun = $0^\circ$.)](image)
In the second case (Figure 7.29), the azimuth difference between the sun and the normal of the window wall was 180°. Solar altitude was at 75°.

![Graph](image)

Angle of shading device

Figure 7.29 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of shading length. (Azimuth difference between the vertical of the window and the sun = 180°.)

In the third case (Figure 7.30), the effect of the reflectance of the shading itself was investigated.

![Graph](image)

Figure 7.30 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of shading reflectance. (Azimuth difference between the vertical of the window and the sun = 180°, and angle of shading = 40°.)
The figures illustrate that equation 6.5 can work to within an acceptable accuracy with normal horizontal shading devices.

H. Window area and location

Variables of ratio of window area to wall area were tested. A window to wall ratio of 0.19 to 0.64 seems to give acceptable results (figure 7.31). Figure 7.33 indicates that the location of the window is insignificant.

![Graph](image)

*Figure 7.31* The ratio of the $\overline{IF}$ obtained by equation 6.5 to the $\overline{IF}$ obtained by SUPERLITE as a function of window area. (The window wall area = 12 m²)

![Locations](image)

*Figure 7.32* The different locations of the tested window.
Figure 7.33 The ratio of the $\overline{NF}$ obtained by equation 6.5 to the $\overline{NF}$ obtained by SUPERLITE as a function of window location.
7.3.2 Validation of the vertical to horizontal ratio against SUPERLITE

SUPERLITE lighting program was used to investigate the relationship between the illuminance on the vertical window plane and the unobstructed horizontal illuminance, where direct sunlight is excluded from both quantities.

For this purpose, the vertical to horizontal ratio was estimated for three different cases. In the first case there is no obstruction in front of the window (table 7.4), in the second case the window faces an obstruction with an angle of 32° (table 7.5), and in the third case the angle of obstruction is equal to 22.6° (table 7.6).

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the normal of window wall and sun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.82</td>
</tr>
<tr>
<td>45</td>
<td>1.78</td>
</tr>
<tr>
<td>60</td>
<td>1.70</td>
</tr>
<tr>
<td>75</td>
<td>1.44</td>
</tr>
<tr>
<td>90</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Table 7.4 The ratio of vertical illuminance to unobstructed horizontal illuminance (non-obstructed window). Linke's turbidity factor = 3.9.
Azimuth difference between the normal of window wall and sun

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the normal of window wall and sun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.04</td>
</tr>
<tr>
<td>45</td>
<td>1.15</td>
</tr>
<tr>
<td>60</td>
<td>1.21</td>
</tr>
<tr>
<td>75</td>
<td>1.09</td>
</tr>
<tr>
<td>90</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 7.5  The ratio of vertical illuminance to unobstructed horizontal illuminance.  (Angle of obstruction = 32°.) Linke's turbidity factor = 3.9.

Azimuth difference between the normal of window wall and sun

<table>
<thead>
<tr>
<th>Solar altitude</th>
<th>Azimuth difference between the normal of window wall and sun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1.26</td>
</tr>
<tr>
<td>45</td>
<td>1.33</td>
</tr>
<tr>
<td>60</td>
<td>1.35</td>
</tr>
<tr>
<td>75</td>
<td>1.20</td>
</tr>
<tr>
<td>90</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Table 7.6  The ratio of vertical illuminance to the unobstructed horizontal illuminance.  (Angle of obstruction = 22.6°.) Linke's turbidity factor = 3.9.
The three tables show that the illuminance on the window plane is generally larger than the illuminance on the horizontal. The histogram, illustrated in figure 7.34, indicates the possibility of setting the illuminance on the window plane as being equal to the horizontal illuminance, representing, in this case, a minimum possible condition.

![Histogram for the vertical to horizontal ratio](image)

**Figure 7.34** Histogram for the vertical to horizontal ratio for the data of tables 7.4, 7.5 and 7.7.
7.4 Conclusion

Equation 6.5 has been tested against real measurements and against the simulation program SUPERLITE. The variables tested cover most possible conditions in sunny climates. These include sun location, room geometry, surfaces reflectance (internal and external), angle of obstruction, shading devices, window area and window location. Based on the results of these tests, boundaries can be set around the proposed equation to ensure that it will estimate the average natural light factor to within an error of 10%, obtained by both the real measurements and SUPERLITE. One clear finding is that the equation should not be used for a window which has a very close obstruction. However, in general, the results give confidence in the equation proposed.

The assumption that the vertical illuminance is equal to the horizontal has been tested by both real measurements and SUPERLITE. The results indicate that this assumption is permissible. Although considerable variation between the illuminance on the vertical window plane and the diffuse illuminance might exist sometimes, due to the continuously changing nature of natural light, advantages here will far outweigh the disadvantages. It will be possible to deal with the average internal light as a percentage of the horizontal sky illuminance, as in the average daylight factor method.
CHAPTER EIGHT

CONCLUSION
CHAPTER EIGHT: CONCLUSION

8.1 General

The general aim of this study was to investigate the issue of predicting the internal illuminance from natural light under clear sky conditions. This included evaluation of the calculation techniques available, in terms of their applicability to sunny regions.

One of the objectives was to propose a method for predicting the internal illuminance in sunny regions, using the concept of the average daylight factor as a basis. However, it was uncertain as to how such a method was to be developed. The research carried out in the previous chapters can be categorised as exploratory, in which the issue of using natural light in sunny climates was tackled (107). It consists of five parts.

The first part (Chapter Two) reviewed the use of natural light under clear sky conditions. This included a description of the solar and sky illuminance for sunny conditions and a review of the prediction techniques available. This literature review demonstrated the need for a prediction method as simple as the average daylight factor method.

However, before such a method can be developed, the complexity of dealing with the continuous changeable luminance distribution of the sky and the surrounding surfaces (ground, obstruction, shading device, etc.) must be eliminated. Therefore, the second part investigated the possibility of dealing with the illuminance on the window wall rather than with the luminance distribution of light sources.
The relationship between the angle of incidence of light and the mean illuminance on the working plane level was investigated by mathematical simulation analysis. A computer program was written in BASIC to estimate the average internal illuminance on the working plane level as a function of a given illuminance incident on the window plane. The variable in this analysis was the angle of incidence of light. Results from the analysis were validated against results which had been obtained from measurements from a scale model.

It was found that the mean working plane illuminance does vary with the direction of light on the window. However, this variation can be assumed to be insignificant when light from below and above the horizontal are separated and direct sunlight is excluded. This means that it is possible to deal with the illuminance on the external window plane in order to estimate the mean illuminance on the working plane level.

The third part deals with testing, by numerical simulation, the performance of existing average daylight factor equations under clear sky conditions.

It was concluded that they are not appropriate for sunny regions. One important finding from this research is that it was shown that splitting the light incident on the window is not recommended under clear sky conditions. This is because the illuminance from above and below the working plane level is a function of several variables. Utilising the BRE split flux approach with a fixed ratio of upward and downward components, for every case, will not be considered as a good prediction tool for the mean illuminance on the working plane level under clear sky conditions. Utilising such an approach with variable components will complicate it.

The fourth part deals with proposing the new approach. This has two bases. The first is that it has been shown to be possible to relate incident light on the window plane to horizontal sky illuminance, and this sets a minimum condition for window design. In practice, this can be used in conjunction with a limiting maximum window size, based on heat gain and other environmental issues. The second basis
is the introduction of a new formula for relating average internal illuminance to external window plane illuminance (average natural light factor). The overall result is a formula for predicting internal illuminance as a ratio of external horizontal sky illuminance.

The fifth part is the validation of the proposed technique. It was carried out as follows:

1. The average natural light factor was estimated by taking real measurements in real buildings. This was carried out under real clear sky conditions in the Dammam area of Saudi Arabia.

2. Real measurements were taken of the light incident on the window plane and the horizontal of a scale model under real clear sky conditions in the Dammam area of Saudi Arabia.

3. A lighting simulation program was used to compare the proposed solution with a detailed calculation.

8.2 Main results and conclusions

The main results and conclusions from this study are:

1. A new approach for finding the mean illuminance on the working plane level as a result of the luminance of room surfaces was proposed. This concept was based on what has been called the Grid Element coefficient (GE). That is, the ratio of the average internal illuminance on the working plane level to the illuminance of a grid element on the room surfaces. This coefficient is helpful not only in daylight studies but in illuminating engineering studies in general. Such an approach is very helpful in reducing the computation time. Once these coefficients are known for a given
room, the average illuminance on the working plane can be calculated with less computation time than if the calculation is repeated every time the illuminance, or grid area changes.

The radiosity method itself can benefit from such a concept. The GE coefficient can be estimated for one grid element to another grid element, based on the form factors and the effects of inter-reflection between all elements. Therefore, once the later coefficients are known there is no need to use the radiosity method every time the amount of incident light changes. Calculating the illuminance on all grid elements therefore will be much faster.

2. The illuminance on the external plane of a vertical window can be assumed to be approximately equal to the horizontal diffuse illuminance, where direct sunlight is not considered in either quantity. Such an assumption reduces the complexity of estimating the illuminance incident on the window under clear sky conditions. Designing for natural light, in this case, will be based on the assumed condition. If more light is needed, the maximum window size will be determined by the heat gain issue, which is a major factor influencing the design of openings in such areas.

3. The study proposed a new prediction equation for finding the average internal illuminance on the working plane level as a percentage of the illuminance on the external surface of the vertical window plane (which has been called the Average Natural light Factor - NF).

The proposed method (the average natural light factor and the assumption that the external illuminance on the window plane is equal to the horizontal diffuse illuminance) if compared with the methods which already exist in the field of natural
light estimations does provide some advantages. Most of the external illuminance data used in the Lumen method is based on lighting calculation where large errors exist (37). Even with the most sophisticated prediction tools, large errors have been found when compared to real measurements (81, 94). What the average natural light factor method offers is an elimination of errors resulting from lighting estimation between the external and the internal. It always deals with what exists outside the window surface. In addition, the assumption that the vertical illuminance is equal to the horizontal diffuse illuminance allows us to set an easily predictable minimum condition for clear sky conditions, although considerable variation between the illuminance on the vertical window plane and the diffuse illuminance might exist sometimes. However, due to the continuously changing nature of natural light, advantages here will far outweigh the disadvantages. It will be possible to deal with the average internal light as a percentage of the horizontal sky illuminance, as in the average daylight factor method.

It is believed that the methodology proposed (the average natural light factor and the vertical to horizontal ratio) can help in solving the problem of the complexity of predicting internal illuminance under clear sky conditions. Therefore, designers may be encouraged to use it.

8.3 Further research

Although this study does propose some techniques for internal illuminance prediction which have been validated, more analysis and development are needed.

In order for such a method to be successful, standards or guidelines should be proposed concerning these factors. For example, average natural light factors should
give some indication as to the internal natural light performance, such as how bright or dim the space is.

More research is needed to estimate the horizontal diffuse illuminance in sunny regions and to compare it to the vertical illuminance on the window plane.

Also, more development on the concept of the Grid Element coefficient (GE) is a possibility. Investigation is necessary into the speed of such a method as compared with others, such as the full radiosity method.
APPENDIX A

This program estimates the form factor between any two rectangular finite areas within a room enclosure.

Khalid Alshaibani
University of Sheffield, School of Architectural studies 1995

OPEN "FormFactor" FOR OUTPUT AS 1

' Input data
widthr = 5 'Width of room (window wall)
depth = 5 'depth of room and height of room
height = 3 'height of room
element = 1 'the size of surface which the form factor is needed for

w = widthr / element
h = height / element
dc = depth / element
k = 2 * (w * h) + 2 * (dc * h) + 2 * (dc * w) 'k is number of elements

DIM f(k,k)
DIM x(k)
DIM y(k)
DIM z(k)
DIM R(k,k)

GOSUB co-ordinates 'to set the co-ordinates of the centre of each element

FOR i = 1 TO k
Start! = TIMER
FOR j = 1 TO k
    IF i = j THEN 10 'same element
    IF f(i,j) > 0 THEN 10
    GOSUB ANGELS:
    LINE1 = d1*(x(i)-x(j)) + d2*(y(i)-y(j)) + d3*(z(i)-z(j))
    IF LINE1 = 0 THEN 'elements are in the same plane
        f(i,j) = 0
    ELSE
        GOSUB Selector
        f(i,j) = f
        f(j,i) = f
    END IF
10 total = total + f(i,j)
NEXT j
Finish! = TIMER
total = 0
PRINT i, Finish!-Start!
NEXT i
CLOSE
STOP
CASE1:
' sub program to find the form factor between two surfaces within this case (as illustrated in 4.3.3.1.3)
******************************************************************
• bi width of element 2
' b2 the distance between 2 and 1
' b3 the distance between 1 and 2
' b4 width of element 1
' c length of element 1 and element 2

a (1) = c * b4
a (2) = c * b1
a (3) = c * b3
a (6) = a (1) + a (3) ' area of elements 1 and 3 , which is considered to be
' a new elements no. (6)
a = b1 + b2
b = b3 + b4
c = element
GOSUB formfactor1: ' to find F(6,5)
F65 = f
a = b2
b = b3
c = element
GOSUB formfactor1: ' to find F(3,4)
F34 = f
a = b1 + b2
b = b3
c = element
GOSUB formfactor1: ' to find F(3,5)
F35 = f
a = b2
b = b3 + b4
c = element
GOSUB formfactor1: ' to find F(6,4)
F64 = f
f = (a (6) * F65 + a (3) * F34 - a (3) * F35 - a (6) * F64 ) / a (1)
b2 = 0
b3 = 0
b4 = 0
c = 0
RETURN
CASE2:
'sub program to find the form factor between two surfaces within this case (as illustrated in 4.3.3.1.3)

IF d > 0 THEN
    a = c1 + c2 + c3
    b = b1 + b2
    c = d
    GOSUB formfactor2
ELSE
    a = b3 + b4
    b = b1 + b2
    c = c1 + c2 + c3
    GOSUB formfactor1
END IF

k1 = (b1 + b2) * (c1 + c2 + c3) * f
IF d > 0 THEN
    b = b1 + b2
    a = c2 + c3
    c = d
    GOSUB formfactor2
ELSE
    b = b1 + b2
    a = b3 + b4
    c = c2 + c3
    GOSUB formfactor1
END IF

k2 = (b1 + b2) * (c2 + c3) * f
IF d > 0 THEN
    b = b1 + b2
    a = c2 + c3
    c = d
    GOSUB formfactor2
ELSE
    b = b1 + b2
    a = b3 + b4
    c = c1 + c2
    GOSUB formfactor1
END IF

k3 = (b1 + B2) * (c1 + C2) * f
IF d > 0 THEN
    a = b2
    b = c1 + c2 + c3
    c = d
    GOSUB formfactor2
ELSE
    a = b3
    b = b2
    c = c1 + c2 + c3
    GOSUB formfactor1
END IF

k4 = (b2) * (c1 + c2 + c3) * f
a = b3 + b4
b = b2
IF d > 0 THEN
    c = d
GOSUB formfactor2
ELSE
   c = c1 + c2 + c3
   GOSUB formfactor1
END IF
k5 = (b2) * (c1 + c2 + c3) * f
a = b3
b = b1 + b2
c = d
IF d > 0 THEN
   GOSUB formfactor2
ELSE
   c = c1 + c2 + c3
   GOSUB formfactor1
END IF
k6 = (b1 + b2) * (c1 + c2 + c3) * f
a = b3
b = b1 + b2
c = d
IF d > 0 THEN
   GOSUB formfactor2
ELSE
   c = c1 + c2
   GOSUB formfactor1
END IF
k7 = (b1 + b2) * (c1 + c2) * f
a = b3
b = b1 + b2
c = d
IF d > 0 THEN
   GOSUB formfactor2
ELSE
   c = c2 + c3
   GOSUB formfactor1
END IF
k8 = (b1 + b2) * (c2 + C3) * f
a = b3 + b4
b = b2
c = d
IF d > 0 THEN
   GOSUB formfactor2
ELSE
   c = c1 + c2
   GOSUB formfactor1
END IF
k9 = (b2) * (c1 + C2) * f
a = b3 + b4
b = b2
c = d
IF d > 0 THEN
   GOSUB formfactor2
ELSE
   c = c2 + c3
   GOSUB formfactor1
END IF
k10 = (b2) * (c2 + c3) * f
a = b3
b = b2
c = d
IF d > 0 THEN
  GOSUB formfactor2
ELSE
  c = c2 + c3
  GOSUB formfactor1:
END IF
k14 = (b2) * (c2 + c3) * f
a = b3 + b4
b = b2
c = d
IF d > 0 THEN
  GOSUB formfactor2
ELSE
  c = c2
  GOSUB formfactor1
END IF
k15 = (b2) * (c2) * f
a = b3
b = b2
c = d
IF d > 0 THEN
  GOSUB formfactor2
ELSE
  c = c2
  GOSUB formfactor1
END IF
k16 = (b2) * (c2) * f
k1 = k1 - k2 - k3 + k4 - k5 - k6
k2 = k7 + k8 + k9 + k10 + k11
k3 = - k12 - k13 - k14 - k15 + k16
f = (((1 / 2) * (k1 + k2 + k3)) / (c1 * b1))

Cl = 0
C2 = 0
C3 = 0
b1 = 0
b2 = 0
b3 = 0
b4 = 0
d = 0
RETURN
CASE3:
' sub program to find the form factor between two surfaces within this case (as illustrated in 4.3.3.1.3)

\[ a = c_1 + c_2 + c_3 \]
\[ b = b_1 + b_2 + b_3 \]
\[ c = d \]
GOSUB formfactor2
\[ k_1 = (b_1 + b_2 + b_3) * (c_1 + c_2 + c_3) * f \]
\[ a = c_1 + c_2 \]
\[ b = b_1 + b_2 + b_3 \]
\[ c = d \]
GOSUB formfactor2
\[ k_2 = (b_1 + b_2 + b_3) * (c_1 + c_2) * f \]
\[ a = c_3 + c_2 \]
\[ b = b_1 + b_2 + b_3 \]
\[ c = d \]
GOSUB formfactor2
\[ k_3 = (b_1 + b_2 + b_3) * (c_3 + C_2) * f \]
\[ a = c_1 + c_2 + C_3 \]
\[ b = b_2 + b_3 \]
\[ c = d \]
GOSUB formfactor2
\[ k_4 = (b_2 + b_3) * (c_1 + C_2 + C_3) * f \]
\[ a = c_1 + c_2 \]
\[ b = b_3 + b_2 \]
\[ c = d \]
GOSUB formfactor2
\[ k_5 = (b_3 + b_2) * (c_1 + C_2) * f \]
\[ a = c_2 + c_3 \]
\[ b = b_2 + b_3 \]
\[ c = d \]
GOSUB formfactor2
\[ k_6 = (b_2 + b_3) * (c_2 + C_3) * f \]
\[ a = c_2 + C_3 \]
\[ b = b_2 + b_1 \]
\[ c = d \]
GOSUB formfactor2
\[ k_7 = (b_1 + b_2) * (c_2 + C_3) * f \]
\[ a = c_2 + c_3 \]
\[ b = b_2 \]
\[ c = d \]
GOSUB formfactor2
\[ k_8 = (b_2) * (c_2 + C_3) * f \]
k9 = (b1 + b2) * (c2) * f
a = c1 + c2
b = b2
c = d
GOSUB formfactor2
k10 = (b2) * (c1 + c2) * f

a = c1 + C2 + C3
b = b1 + b2
c = d
GOSUB formfactor2
k11 = (b1 + b2) * (c1 + C2 + C3) * f

a = c1 + c2
b = b1 + b2
c = d
GOSUB formfactor2
k12 = (b1 + b2) * (c1 + C2) * f

a = c1 + C2 + C3
b = b2
c = d
GOSUB formfactor2
k13 = (b2) * (c1 + C2 + C3) * f

a = c2
b = b1 + b2 + b3
c = d
GOSUB formfactor2
k14 = (b1 + b2 + b3) * (c2) * f

a = c2
b = b3 + b2
c = d
GOSUB formfactor2
k15 = (b3 + b2) * (c2) * f

a = c2
b = b2
c = d
GOSUB formfactor2
k16 = (b2) * (c2) * f
k1 = k1 - k2 - k3 - k4 + k5 + k6
k2 = k7 - k8 - k9 - k10 - k11
k3 = k12 + k13 + k14 - k15 + k16
f = (((1/4) * (k1 + k2 + k3)) / (c1 * b3)
c1 = 0
c2 = 0
c3 = 0
b1 = 0
b2 = 0
b3 = 0
d = 0
RETURN
formfactor1:
'Subroutine to calculate the form factors for two perpendicular rectangles

\begin{align*}
&\text{IF } a = 0 \text{ THEN 100} \\
&\text{IF } b = 0 \text{ THEN 100} \\
&\text{IF } c = 0 \text{ THEN 100} \\
&h = a / c \\
&w = b / c \\
&aa = (h^2 + w^2)^{0.5} \\
&BB = (((1 + w^2) * (1 + h^2)) / (1 + w^2 + h^2) \\
&CC = ((w^2 * (1 + w^2 + h^2)) / ((1 + w^2) * (w^2 + h^2)) \\
&dd = (h^2 * (1 + h^2 + w^2)) / ((1 + h^2) * (h^2 + w^2)) \\
f = (1 / (3.141592654 * w)) * (w * ATN (1 / w) + h * ATN (1 / h) \\
- aa * ATN (1 / aa) + (LOG (BB * CC*(w^2) * dd*(h^2))) / 4) \\
\text{GOTO 200} \\
100 \ f = 0 \\
200 \\
a = 0 \\
b = 0 \\
c = 0 \\
\text{RETURN}
\end{align*}

formfactor2:
'form factor for two identical parallel rectangles

\begin{align*}
&\text{IF } a = 0 \text{ THEN 100} \\
&\text{IF } b = 0 \text{ THEN 100} \\
&\text{IF } c = 0 \text{ THEN 100} \\
&x = a / c \\
&y = b / c \\
&aa = ((1 + x^2) * (1 + y^2)) / (1 + x^2 + y^2) \\
&bb = x * ((1 + y^2)^{0.5}) * ATN (x / ((1 + y^2)^{0.5})) \\
&cc = y * ((1 + x^2)^{0.5}) * ATN (y / ((1 + x^2)^{0.5})) \\
&\quad - x * ATN (x) - y * ATN (y) \\
f = (LOG (aa) + 2 * (bb + cc)) / (3.141592654 * x * y) \\
\text{GOTO 200} \\
100 \ f = 0 \\
200 \\
a = 0 \\
b = 0 \\
c = 0 \\
\text{RETURN}
\end{align*}
co-ordinates:
'sub program to determine the co-ordinates of all sub elements in the room surfaces.

**********
k=1 ' start counting number of elements
FOR i=1 TO h
  FOR j=1 TO w
    x(k)=0
    y(k)=element *j - element /2
    z(k)=element *i - element /2
    k=k+1
  NEXT j
NEXT i

'for side 1
FOR i=1 TO h
  FOR j=1 TO de
    y(k)=widthr
    x(k)=element *j - element /2
    z(k)=element *i - element /2
    k=k+1
  NEXT j
NEXT i

'for back wall
FOR i=1 TO h
  FOR j=1 TO w
    x(k)=depth
    y(k)=element *j - element/2
    z(k)=element *i - element/2
    k=k+1
  NEXT j
NEXT i

'for side 2
FOR i=1 TO h
  FOR j=1 TO de
    y(k)=0
    x(k)=element *j - element/2
    z(k)=element *i - element/2
    k=k+1
  NEXT j
NEXT i

'for ceiling
FOR i=1 TO de
  FOR j=1 TO w
    z(k)=height
    x(k)=element *j - element/2
    y(k)=element *i - element/2
    k=k+1
  NEXT j
NEXT i

'for floor
FOR i=1 TO de
  FOR j=1 TO w
    z(k)=0
    x(k)=element *i - element/2
    y(k)=element *j - element/2
    k=k+1
  NEXT j
NEXT i
k=k-1
RETURN

'*----------------------------------------------------------------------------------
Selector:
' sub program to decide which case the elements are from
'*----------------------------------------------------------------------------------

IF ABS (x (i) - x (j)) = width THEN 101
IF ABS (y (i) - y (j)) = depth THEN 101
IF ABS (z (i) - z (j)) = height THEN 101

' if one of the above is true then the elements are parallel to each other
' it is needed to knew which category they are, either to go sub formfactor2
' or to got to case 2 or to case 3

'if the above is not true the elements are from case I or case 2
GOTO 280 'that means case 1 or 2

101

' R (i, j)  distance between two points
IF ABS (x (i) - x (j) + y (i) - y (j) + z (i) - z (j)) = R (i, j) THEN
a = element
b = element
c = R (i, j)
GOSUB formfactor2
GOTO 300
ELSEIF (x (i) - x (j)) * (y (i) - y (j)) * (z (i) - z (j)) = 0 THEN
' in this case the elements are parallel and they are in the same axis
cl = element
c3 = element
bl = element
b4 = element
d = ABS (d1* (x (i) - x (j)) + d2 * (y (i) - y (j)) + d3 * (z (i) - z (j)))
c2 = ABS (x (i) - x (j)) + ABS (y (i) - y (j))
+ ABS (z (i) - z (j)) - d - element
GOSUB CASE2
GOTO 300
ELSE
the elements are parallel and they are not in the same axis (case3)
bl = element
b3 = element
c1 = element
c3 = element
END IF

d = distance 1
IF d = ABS (x (i) - x (j)) THEN
b2 = ABS (y (i) - y (j)) - element
c2 = ABS (z (i) - z (j)) - element
ELSEIF d = ABS (y (i) - y (j)) THEN
b2 = ABS (x (i) - x (j)) - element
c2 = ABS (z (i) - z (j)) - element

193
ELSEIF d = \text{ABS} (z(i) - z(j)) \text{ THEN}
\quad b2 = \text{ABS} (x(i) - x(j)) - \text{ element}
\quad c2 = \text{ABS} (y(i) - y(j)) - \text{ element}
\text{ END IF}

GOSUB Case3
GOTO 300

280
' in this case the elements are not parallel and in either case 1 or 2
\text{ IF } (x(i) \cdot x(j)) \cdot (y(i) \cdot y(j)) \cdot (z(i) \cdot z(j)) = 0 \text{ THEN}
' in this case the elements are not parallel and they are in the same axis
\quad b1 = \text{ element}
\quad b4 = \text{ element}
\text{ IF } x(i) - x(j) = 0 \text{ THEN}
\quad b2 = \text{ABS} (y(i) - y(j)) - \text{ element/2}
\quad b3 = \text{ABS} (z(i) - z(j)) - \text{ element/2}
\text{ ELSEIF } y(i) - y(j) = 0 \text{ THEN}
\quad b2 = \text{ABS} (x(i) - x(j)) - \text{ element/2}
\quad b3 = \text{ABS} (z(i) - z(j)) - \text{ element/2}
\text{ ELSE}
\quad b2 = \text{ABS} (x(i) - x(j)) - \text{ element/2}
\quad b3 = \text{ABS} (y(i) - y(j)) - \text{ element/2}
\text{ END IF}

GOSUB CASE1
GOTO 300

ELSE
' the elements from case 2
\quad b1 = \text{ element}
\quad b4 = \text{ element}
\quad c1 = \text{ element :}
\quad c3 = \text{ element}
\quad c2 = \text{ABS} (x(i) - x(j)) + \text{ABS} (y(i) - y(j))
\quad + \text{ABS} (z(i) - z(j)) - \text{distance1 - distance2 - element}
\quad b2 = \text{distance1 - element/2}
\quad b3 = \text{distance2 - element/2}

\text{ END IF}
GOSUB case2
300
\text{ RETURN}

**********************************************************************************************************
ANGELS:
' subroutine to find the distance between two points, and the angle
' between normal of a plane and this line
**********************************************************************************************************
' input co-ordinates of the two points x1, y1, z1
' the length of line between two selected points
' Direction of normal to the selected surface
\text{ R(i,j) = ((x(i) - x(j)) \^2 + (y(i) - y(j))\^2 + (z(i) - z(j))\^2)^.5}
\text{ cc1 = \text{ABS} (x(j) - x(i)) / R(i,j)}
\text{ cc2 = \text{ABS} (y(j) - y(i)) / R(i,j)}
\text{ cc3 = \text{ABS} (z(j) - z(i)) / R(i,j)}
\[ d_1 = 0 \]
\[ d_2 = 0 \]
\[ d_3 = 0 \]

\[ \text{IF } x(j) = 0 \ \text{THEN } d_1 = 1 \]
\[ \text{IF } x(j) = \text{depth} \ \text{THEN } d_1 = 1 \]
\[ \text{IF } y(j) = 0 \ \text{THEN } d_2 = 1 \]
\[ \text{IF } y(j) = \text{width} \ \text{THEN } d_2 = 1 \]
\[ \text{IF } z(j) = 0 \ \text{THEN } d_3 = 1 \]
\[ \text{IF } z(j) = \text{height} \ \text{THEN } d_3 = 1 \]

\[ \text{distance}_1 = |x(j) - x(i)| \cdot d_1 + |y(j) - y(i)| \cdot d_2 + |z(j) - z(i)| \cdot d_3 \]

\[ d_1 = 0 \]
\[ d_2 = 0 \]
\[ d_3 = 0 \]

\[ \text{IF } x(i) = 0 \ \text{THEN } d_1 = 1 \]
\[ \text{IF } x(i) = \text{depth} \ \text{THEN } d_1 = 1 \]
\[ \text{IF } y(i) = 0 \ \text{THEN } d_2 = 1 \]
\[ \text{IF } y(i) = \text{width} \ \text{THEN } d_2 = 1 \]
\[ \text{IF } z(i) = 0 \ \text{THEN } d_3 = 1 \]
\[ \text{IF } z(i) = \text{height} \ \text{THEN } d_3 = 1 \]

\[ \text{distance}_2 = |x(j) - x(i)| \cdot d_1 + |y(j) - y(i)| \cdot d_2 + |z(j) - z(i)| \cdot d_3 \]

RETURN
APPENDIX B

This program calculates the illuminance on a window plane as a ratio from the horizontal sky illuminance, 1995

Khalid Alshaibani
University of Sheffield, School of Architectural Studies 1995

DIM c (50)
DIM S (50)
DIM ALSKY (100)
DIM AZSKY (100)
DIM J (4)
DIM R (5)
DIM Lzone (50)
DIM angle (4)
DIM Lsum (10)
DIM FF (45)
DIM F (5)
DIM L (50)

Pi = 3.141592654#
Rad = Pi / 180
deg = 180 / Pi
'Ae = elevation azimuth
'Angle = Angle of shading device
'AO = obstruction azimuth
'AZ = Solar elevation azimuth
'AZO = Solar obstruction azimuth
'HEIGHT = height of facade in meters
'H = Station height (meters)
'GLAT = Latitude of site
'GLON = Longitude of site (negative west of Greenwich)
'GLOM = Longitude of standard meridian (negative west of Greenwich)
'J = number of the day
'Reflectance.g = reflectance of the ground
'Reflectance.w = reflectance of the external walls
PRINT " height of the facade"
INPUT height ' height of facade in meters
obst = height
PRINT " Input solar altitude in degrees"
INPUT SAZH
SAZH = SAZH * Rad
PRINT " Input solar azimuth in degrees"
INPUT SAZN
SAZN = SAZN * Rad
J = 180
PRINT "input the orientation of the window wall in degrees"
INPUT ORIENTATION
PRINT "input street width"
INPUT SWIDTH
PRINT "print 1 if there is a shading device"
INPUT device
IF device = 1 THEN
  PRINT "input window height"
  INPUT wheight :
  PRINT " input shading device length"
  INPUT shade :
  PRINT " input window length"
  INPUT WindowL :
  PRINT " Input reflectance of the shading device"
  INPUT Reflectance
  R (4) = Reflectance
ENDIF

PRINT " Input reflectance of the facade"
INPUT Reflectance
R (1) = Reflectance
PRINT " Input reflectance of obstruction"
INPUT Reflectance
R (3) = Reflectance
PRINT " Input reflectance of the ground"
INPUT Reflectance
R (2) = Reflectance
Ae = ORIENTATION
Ao = Ae + 180
IF Ao > 360 THEN
  Ao = Ao-360
ELSE
  Ao = Ao
END IF
'Ang.Obst. = Angle of obstruction
Ang.Obst. = ATN ((height /2) / SWIDTH)
GOSUB SolarNormalilluminance ' (Evsn)
GOSUB Solarilluminance
Global = sum1 + Direct
GOSUB Shadow
GOSUB Angles
GOSUB Directilluminance
GOSUB Diffuseilluminance
GOSUB final.Illuminance

c = (Lsum (1) + F (5)) / horizontal
g = f (2) * R (2) * FF (21) / horizontal
w= f (3) * R (3) * FF (31) / horizontal

STOP

'******************************************************************************
ARCSIN:
' Returns the inverse sine of an x where -1 <= X <= 1
'******************************************************************************
IF x = 1 THEN  ' to prevent division by zero
  ARCSIN = Pi / 2
ELSEIF x = -1 THEN
  ARCSIN = -Pi / 2
ELSE

198
ARCSIN = ATN(x / SQR (1 - (x^2)))
END IF
RETURN

'******************************************************************************
ARCCOS:
'Returns the inverse cosine of an x where -1 <= X <= 1
******************************************************************************
IF x = 1 THEN 'To prevent division by zero
    ARCCOS = 0
ELSEIF x = -1 THEN
    ARCCOS = Pi
ELSE
    ARCCOS = Pi / 2 - ATN(x / SQR (1 - (x^2)))
END IF
RETURN

' ***************************************************************************
'SolarNormalilluminance:
'subroutine to calculate the solar normal illuminance Evsn
*****************************************************************************
SolarNormalilluminance:

' P = the ratio of mean atmospheric pressure at the site to that at sea level.
' hh = Station height (meters)
' hh = 30
' PP = 1 - (hh / 10000) ' PP is the
PRINT "input Til";
INPUT Til

'Relative optical mass (MASS) h < 4000 meters
MASS = PP * 1 / (SIN (SAZH) + 0.50572
* ((180 * SAZH / Pi + 6.07995)^ - 1.6364))

'Mean extinction coefficient
AILR = 0.1 / (1 + 0.0045 * MASS)

' atmospheric transmittance (STRV)
STRV = EXP (- AILR * Til)

' Extraterrestrial solar illuminance - Evo (klx)
Evo = 127.5 * (1 + 0.034 * COS ((2 * Pi * (j - 2)) / 365))

' solar normal illuminance - Evsn (klx)
Evsn = Evo * EXP (- AILR * MASS * Til)
PRINT " print 0 if you are not using SUPERLITE data "
INPUT SUPERLITE
IF SUPERLITE = 0 THEN 10
PRINT "input solar Component " : INPUT Evsn
Evsn = Evsn / SIN (SAZH)
10 RETURN
Solar Illuminance:
'subroutine to calculate the solar illuminance on surfaces

'solar illuminance on a horizontal plane - silh (klx)
silh = Evsn * SIN (SAZH)
PRINT "input Solar elevation azimuth"
INPUT Az
Az = Az * Rad
PRINT "input Solar obstruction azimuth"
INPUT Azo
Azo = Azo * Rad
IF Az >= Pi/2 THEN
  silk = 0
ELSE
  'Incidence angle Ai
  x= COS (SAZH) * COS (Az)
  GOSUB ARCCOS
  Ai = ARCCOS
  'solar illuminance on an inclined plane - SILK(klx)
  silk = Evsn * (COS (Ai))
  IF silk <0 THEN silk =0
END IF

IF Azo >= Pi/2 THEN
  silko = 0
ELSE
  'Incidence angle Ai
  x= COS (SAZH) * COS (Azo)
  GOSUB ARCCOS
  Ai = ARCCOS
  'solar illuminance on an inclined plane - SILKO(klx)
  silko = Evsn * (COS (Ai))
  IF silko <0 THEN silko =0
END IF
RETURN

Shadow:
'subroutine to find shadow lengths on horizontal and vertical planes

' shv = shadow length on the vertical
' shg = shadow length on the ground
' obst = height of obstruction
' there will be shadow when COS (SAZH) * COS (AZ) >0

' case one; find shadow from either surface
' check = ABS (height / (TAN (SAZH) /COS (Az))) -SWIDTH
IF check >0 THEN
  shv = check * ABS ((TAN (SAZH) /COS (Az)))
ELSE
  shv = 0
END IF
IF obst >0 THEN
  shv = shv
ELSE
shv = 0
END IF
shg = check + SWIDTH
RETURN

ANGLES:
Subroutine to find the angles of which the centre point at each surface
is exposed to the sky
ANGLES:
Subroutine to find the angles of which the centre point at each surface
is exposed to the sky
these angles are used to find the diffuse illuminance from the sky
on the parts of the three sides of the street

angle (1) = ATN (SWIDTH / (height / 2))
angle (2) = Pi - ATN (height / (SWIDTH / 2))
angle (3) = angle (1)
IF shade > 0 THEN
    angle (1) = angle (1) - ATN (Shade// weight)
ENDIF
RETURN

DIRECTILLUMINANCE:
Subroutine to find the direct illuminance on the different planes involved
DIRECTILLUMINANCE:
Subroutine to find the direct illuminance on the different planes involved
In this subroutine there are two cases;
the first when the facade is the surface casting the shadow
the second is when the obstruction is casting the shadow

IF SAZH = Pi / 2 THEN
    PRINT " solar altitude = 90 "
    STOP
ENDIF

IF ABS (Ae-SAZN*deg) < 90 THEN 100
' Case one (the facade is the surface casting the shadow)
IF SWIDTH > shg THEN
    state = 1
    Points (1) = 0
    Points (2) = 0
    Points (3) = silh
    Points (4) = silko
    length1 = height
    length2 = shg
    length3 = SWIDTH - shg
    length4 = obst
ELSE
    state = 2
    Points (1) = 0
    Points (2) = 0
    Points (3) = 0
    Points (4) = silko

201
length1 = height  
length2 = SWIDTH  
length3 = shy  
length4 = obst - shy

END IF

GOTO 200

100
' the second is when the obstruction is casting the shadow

' Case two (the obstruction is the surface casting the shadow)

'in the following the facade is exposed
' to the sun and there is a need to check
' which parts of the ground and the facade are shaded

IF SWIDTH > shg THEN
    state = 1
    Points (1) = silk
    Points (2) = silh
    Points (3) = 0
    Points (4) = 0
    length1 = height
    length2 = SWIDTH - shg
    length3 = shg
    length4 = obst
ELSE
    state = 3
    Points (1) = silk
    Points (2) = 0
    Points (3) = 0
    Points (4) = 0
    length1 = height - shy
    length2 = shy
    length3 = SWIDTH
    length4 = obst
END IF

200

RETURN

'Diffuse illuminance:
'Subroutine to find the diffuse illuminance from the sky on the centre
' point of each surface

' the illuminance from the sky is:
' first we divide the sky seen into 10 bands, each band is
' divided into 36 zone (the vertical plane is exposed to 18 only)
' so the width of each zone = 10 degrees. The height of each band is = angle /9

FOR i = 1 TO 3
    Lzone (i) = angle (i) / 9
NEXT i
'So the are three points we need to find the diffuse illuminance at; 'the diffuse illuminance at point one: 'first we need to determine the azimuth and altitude of the first sky point

" ALSKY = sky point altitude, radians
'AZSKY= azimuth of sky point
' for the two points on the vertical planes
FOR wall = 1 TO 2
  POIN = 1
  IF wall = 2 THEN
    p = POIN + 2
  ELSE
    p = POIN
  END IF
  AZSKY (1) = Ae * Rad - Pi/2 + Pi/36
  ' (orientation of the window plane - 90 + 5)
  IF AZSKY (1) < 0 THEN
    AZSKY (1) = AZSKY (1) + 2 * Pi
  ELSE
    AZSKY (1) = AZSKY (1)
  END IF
  ALSKY (1) = Ang.Obst. + Lzone (p) / 2
  Lzone = Lzone (p)
  FOR k = 1 TO 18
    FOR i = 1 TO 9
      Incidence angle Ai
      Az = ABS (Ae * Rad - AZSKY (k))
      x = COS (ALSKY (i)) * COS (Az)
      GOSUB ARCCOS
      Ai = ARCCOS
      ALSKY = ALSKY (i)
      AZSKY = AZSKY (k)
      GOSUB SKYLUMINANCE
      S = 2 * Pi * (SIN(ALSKY (i) + Lzone (p) / 2) - SIN (ALSKY (i) - Lzone (p) / 2)) / 36
      L(i) = L * S * COS (Ai)
      Lsum (p) = Lsum (p) + L(i)
      ALSKY (i +1) = ALSKY (i) + Lzone
    NEXT i
    AZSKY (k +1) = AZSKY (k) + Pi / 18
    IF AZSKY (k +1) > 2 * Pi THEN
      AZSKY (k +1) = AZSKY (k +1) - 2 * Pi
    ELSE
      AZSKY (k +1) = AZSKY (k +1)
    END IF
  NEXT k
  Ae = Ao
NEXT wall

Ae = ORIENTATION
GOSUB Horizontal.Diff.Illum.
Lsum (2) = horizontal
PRINT horizontal; "horizontal"
RETURN
SKYLUMINANCE:

A subroutine for finding the luminance (L) of a sky point altitude (ALSKY)
and angular distance (Theta) from the sun

Lz = zenith luminance kcd/m²
ALSKY = sky point altitude, radians
AZSKY= azimuth of sky point
Theta = angular distance of sky point from sun, radians

x = SIN (ALSKY) * SIN (SAZH) + COS (ALSKY) * COS (SAZH) * COS (ABS (AZSKY-SAZN))
GOSUB ARCCOS

theta = ARCCOS
MM=.7 1 / (SQR (Til))' MM & NN will be used in finding the luminance of the sky
NN = 4.3 * (Til)¹ 1.9 * EXP (-0.35 * Til)
F44 = 0.27385
F11 = 1 +NN * (EXP (- 3 * Theta) - 0.009) + MM * (COS (theta))^2
F22 = 1 - EXP (- 0.32 / SIN (ALSKY))
F33 = 1 + NN * (EXP (- 3 * (Pi /2 - SAZH)) - 0.009)
+ MM * (COS (Pi /2 -SAZH))^2

IF STRV > = 0.75 THEN
F55 = ((2.5- 1.4 * STRV) * (STRVA MASS - STRV)) / (19.6 * (1-MASS))
ELSE
F55 = (1.6- 1.4 * STRV + (0.75 - STRV) * MASS) * (STRVA MASS - STRV) / (22.07 * (1 - MASS))
END IF
Lz = Evo * F55 * F33
L = Lz * ((F11 *F22) / (F33 *F44))
RETURN

Horizontal.Diff.Illum.:

'Subroutine to find the horizontal diffuse illuminance from the sky

'first we divide the sky into 9 bands, each band is divided into 36 zone
'The width of each zone = 10 deg. The height of each band = 10 deg.

' ALSKY = sky point altitude, radians
' AZSKY= azimuth of sky point
'first we need to determine the altitude of the first sky point for each point in the
'ground
AZSKY = 10 * Rad / 2
FOR k = 1 TO 36
  ALSKY = 10 * Rad / 2
  FOR i = 1 TO 9
    S = 2 * Pi * (SIN (ALSKY +10 * Rad / 2) - SIN (ALSKY - 10 * Rad / 2))/ 36
    GOSUB SKYLUMINANCE
    L1 = L * S * SIN (ALSKY)
    suml = suml + L1
    ALSKY = ALSKY + 10 * Rad
  NEXT i
NEXT k
AZSKY = AZSKY + 10 * Rad

NEXT k
horizontal = sum1
RETURN

'********************************************************
final.Illuminance:
'********************************************************

IF SUPERLITE = 0 THEN 50
PRINT "enter sky component": INPUT sky
Lsum (1) = (Lsum (1) / horizontal) * sky
Lsum (2) = (Lsum (2) / horizontal) *sky
Lsum (3) = (Lsum (3) / horizontal) * sky
horizontal = sky
50
IF state = 1 THEN
E (1) = Points (1) + Lsum (1)
E (2) = (length2 * Points (2) + length3 * Points (3)) / SWIDTH+Lsum (2)
E (3) = Points (4) + Lsum (3)
ELSEIF state = 2 THEN
E (1) = Points (1) + Lsum (1)
E (2) = Points (2) + Lsum (2)
E (3) = (obst * Lsum (3) + length4 * Points (4)) / obst
ELSE
E (1) = (height * Lsum (1) + length1 * Points (1)) / height
E (2) = Points (3) + Lsum (2)
E (3) = Points (4) + Lsum (3)
END IF

h = height
w = SWIDTH
GOSUB formfactor

FF (12) = F1
FF (32) = F1
FF (21) = F2
FF (23) = F2
FF (13) = F3
FF (31) = F3
FOR i = 1 TO 20
  F (1) = E (1) + F (2) * R (2) * FF (21) + F (3) * R (3) * FF (31)
  F (2) = E (2) + F (1) * R (1) * FF (12) + F (3) * R (3) * FF (32)
  F (3) = E (3) + F (1) * R (1) * FF (13) + F (2) * R (2) * FF (23)
NEXT i
IF shade > 0 THEN
GOSUB FormFactor1
FF (24) = F2
FF (45) = F1
F (4) = F (2) * R (2) * FF (24)
F(5) = F(4) * R(4) * FF(45)
END IF
RETURN

formfactor:
form factor for two perpendicular plates of infinite length - from facade to street
'h = height
'W = width
F1 = .5 * (1 + (h / w) - SQR (1 + (h / w)^2))

' from street to facade
F2 = .5 * (1 + (w / H) - SQR (1 + (w / H)^2))

' form factor for two identical parallels plates of infinite length
F3 = SQR (1 + (w / H)^2) - (w / H)
RETURN

FormFactor1:
Subroutine to calculate the form factors for two perpendicular plates of from a to b
'a = shade 'shading device length
'b = wheight 'window height
'c = WindowL 'window length
'h = a / c
'w= b / c
aa = (h^2 + w^2)^0.5
bb = ((1 + w^2) * (1 + h^2)) / (1 + w^2 + h^2)
cc = (w^2 * (1 + w^2 + h^2)) / ((1 + w^2) * (w^2 + h^2))
dd = (h^2 * (1 + h^2 + w^2)) / ((1 + h^2) * (h^2 + w^2))
F1 = (1 / (3.141592654# * w)) * (w * ATN (1 / w) + h * ATN (1 / H) - aa
   * ATN (1 / aa) + (LOG (bb * CC^w^2) * dd^h^2))/ /4)

' Routine to calculate the configuration factor for an element parallel to a rectangle,
'and lies on a line perpendicular to one corner of the rectangle
'a = SWIDTH
'b = 100
'c = wheight + 0.85 'distance between the shading and ground
'v = a / c
'y = b / c
F2 = (1 / (2 * Pi)) * ((x / SQR (1 + x^2)) * ATN (y / SQR (1 + x^2))
   + (y / SQR (1 + y^2)) * ATN (x / SQR (1 + y^2)))
F2 = 2 * F2
RETURN
APPENDIX C
## APPENDIX C

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Solar altitude</th>
<th>Azimuth difference between sun and normal of window wall</th>
<th>Geometry of the room</th>
<th>Reflectance</th>
<th>Window area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>width</td>
<td>depth</td>
<td>height</td>
<td>ceiling</td>
<td>walls</td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>90</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>120</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>150</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>3.0</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>11</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>7</td>
<td>3.0</td>
</tr>
<tr>
<td>12</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>9</td>
<td>3.0</td>
</tr>
<tr>
<td>13</td>
<td>90</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>14</td>
<td>70</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>16</td>
<td>50</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>17</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>18</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>19</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>21</td>
<td>60</td>
<td>180</td>
<td>4</td>
<td>4</td>
<td>2.5</td>
</tr>
<tr>
<td>22</td>
<td>60</td>
<td>180</td>
<td>4</td>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td>23</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>24</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>25</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>26</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>27</td>
<td>60</td>
<td>180</td>
<td>5</td>
<td>5</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table c1 An illustration for the cases tested to generate table 5.1.
APPENDIX D

The mathematical determination of the position of the sun

Solar altitude is given by:
\[ \gamma_S = \arcsin \left( \sin \varphi \sin \delta_S - \cos \varphi \cos \delta_S \cos \zeta \right) \]

where:
\[ \gamma_S = \text{Solar altitude} \]
\[ \varphi = \text{Latitude of site, radians} \]
\[ \delta_S = \text{Solar declination} \]
\[ \zeta = \text{hour angle} \]

Solar declination \( (\delta_S) \)
\[ \delta_S = 0.006918 - 0.399912 \cos \tau_d + 0.070257 \sin \tau_d \]
\[ - 0.006758 \cos d + 0.000907 \sin 2 \tau_d \]
\[ - 0.002697 \cos 3 \tau_d + 0.00148 \sin 3 \]

Day angle \( (\tau_d) \)
\[ = 2 \tau (J - 1) / 365 \]

Solar hour angle \( (\zeta) \)
\[ \zeta = (\pi / 12) \text{TST} \quad \text{radians} \]

TST = True solar time \( \quad \text{(hours)} \)

TST = LT + \( (\lambda^o - \lambda_s^o) / 15 \) + ET

LT = Local clock time
\[ \lambda^o = \text{Longitude of site (negative west of Greenwich)} \]
\[ \lambda_s^o = \text{Longitude of standard meridian (negative west of Greenwich)} \]
ET = Equation of time

\[ ET = 0.170 \sin \left[ \frac{4 \pi (J - 80)}{373} \right] - 0.129 \sin \left[ \frac{2 \pi (J - 8)}{355} \right] \text{ hours} \]

**Solar azimuth**

\[ \alpha_s = \arccos \frac{- \sin \varphi \sin \gamma_s + \sin \delta_s}{\cos \varphi \cos \gamma_s}, \quad 0 < \zeta < \pi \]

\[ \alpha_s = 2 \pi - \arccos \frac{- \sin \varphi \sin \gamma_s + \sin \delta_s}{\cos \varphi \cos \gamma_s}, \quad \pi < \zeta < 2\pi \]
REFERENCES


103. KFUPM. Data obtained personally from King Fahad University. (1992).


