INVESTIGATION OF THE FLEXURAL PROPERTIES OF REINFORCED CONCRETE BEAMS STRENGTHENED BY EXTERNALLY BONDED STEEL PLATES

by


Thesis submitted to the University of Sheffield for the Degree of Doctor of Philosophy in the Faculty of Engineering

September 1980
Dedicated to my parents.
SUMMARY

It is sometimes necessary to strengthen in situ concrete structures. Externally bonded plate reinforcement has been successfully applied to structures with subsequent satisfactory performance. However, little research has been reported, especially in respect to long term behaviour.

The present study investigated the flexural behaviour of normal under-reinforced concrete beams with plate reinforcement bonded to their tensile face. Furthermore, long term studies of loaded and unloaded plated beams were initiated for testing after exposure to natural weathering conditions. The parameters under investigation were adhesive and steel plate thickness, the degree of cracking in the beam prior to bonding on the plates, multiple plate layers and plate jointing techniques.

Test results showed that although the increase in ultimate load produced by the bonded plates was only 17%, the service loads were increased up to 90%. The deformations at service loads were reduced up to 65%. In general, the deformations decreased for an increase in adhesive or plate thickness, the latter having the larger effect. The maximum crack widths in the plated beams were up to 63% lower than those in the unplated beam. The ACI and CP 110 crack width prediction formulae overestimated the measured values. Within the limitations of the present test series, empirical formulae were derived for calculating rotations, crack widths, crack spacings and concrete surface strains. Tests on specimens after 18 months weathering showed no loss of flexural performance nor any visual deterioration of the adhesive or adhesive/steel interface. Further tests will be reported.

Bonded plate reinforcement can only enhance the ultimate strength of a beam to limited extent. More important are the decrease in deformations and consequently the increase in service loads, thus making it a viable technique for up-rating the load carrying capacity of existing structures.
ACKNOWLEDGEMENTS

The author wishes to thank his Supervisors, Dr. R. N. Swamy and Mr. R. Jones, for their guidance, assistance and helpful advice throughout the research and for their help during the presentation of this thesis.

Professors D. Bond and T. H. Hanna of the Department of Civil and Structural Engineering are also to be thanked for their assistance.

Thanks are due to McCall and Company (Sheffield) Ltd., the Associated Portland Cement Manufacturers Ltd., and Colebrand Ltd. for their generous supply of materials.

The author also acknowledges the assistance of the Technical and Secretarial staff of the Department, in particular Mr. R. Newman.

Finally the author wishes to thank Linda Bell and Chris Harrison for so patiently typing the text of this thesis.
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>(i)</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>(ii)</td>
</tr>
<tr>
<td>Contents</td>
<td>(iii)</td>
</tr>
<tr>
<td>List of Figures</td>
<td>(vii)</td>
</tr>
<tr>
<td>List of Tables</td>
<td>(ix)</td>
</tr>
<tr>
<td>List of Plates</td>
<td>(x)</td>
</tr>
<tr>
<td>Notation</td>
<td>(xi)</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 GENERAL INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.2 OUTLINE OF THESIS</td>
<td></td>
</tr>
<tr>
<td><strong>CHAPTER 2</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 EPOXY RESINS</td>
<td></td>
</tr>
<tr>
<td>2.1.1 General Introduction</td>
<td></td>
</tr>
<tr>
<td>2.1.2 Materials and Mixes</td>
<td></td>
</tr>
<tr>
<td>2.1.3 Mixing</td>
<td></td>
</tr>
<tr>
<td>2.1.4 Temperature</td>
<td></td>
</tr>
<tr>
<td>2.1.5 Surface Preparation</td>
<td></td>
</tr>
<tr>
<td>2.1.6 Bonding</td>
<td></td>
</tr>
<tr>
<td>2.1.7 Curing</td>
<td></td>
</tr>
<tr>
<td>2.1.8 Safety and Health Provisions</td>
<td></td>
</tr>
<tr>
<td>2.2 GLUED JOINTS IN CONCRETE</td>
<td></td>
</tr>
<tr>
<td>2.2.1 General Uses</td>
<td></td>
</tr>
<tr>
<td>2.2.2 Surface Preparation</td>
<td></td>
</tr>
<tr>
<td>2.2.3 Moisture Effects</td>
<td></td>
</tr>
<tr>
<td>2.2.4 Miscellaneous</td>
<td></td>
</tr>
<tr>
<td>2.2.5 Summary</td>
<td></td>
</tr>
<tr>
<td>2.3 STRESS DISTRIBUTION IN LAP JOINTS</td>
<td></td>
</tr>
<tr>
<td>2.4 GLUED JOINTS IN METALS</td>
<td></td>
</tr>
<tr>
<td>2.4.1 General</td>
<td></td>
</tr>
<tr>
<td>2.4.2 Surface Preparation</td>
<td></td>
</tr>
<tr>
<td>2.4.3 Moisture Effects</td>
<td></td>
</tr>
<tr>
<td>2.4.4 Miscellaneous</td>
<td></td>
</tr>
<tr>
<td>2.4.5 Summary</td>
<td></td>
</tr>
<tr>
<td>2.5 GLUED JOINTS BETWEEN STEEL AND CONCRETE</td>
<td></td>
</tr>
<tr>
<td>2.5.1 General</td>
<td></td>
</tr>
<tr>
<td>2.5.2 L'Hermite and Bresson</td>
<td></td>
</tr>
</tbody>
</table>

(iii)
<table>
<thead>
<tr>
<th>Section</th>
<th>Text</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5.3</td>
<td>Transport and Road Research Laboratories</td>
<td>22</td>
</tr>
<tr>
<td>2.5.4</td>
<td>Dundee University</td>
<td>24</td>
</tr>
<tr>
<td>2.5.5</td>
<td>Warwick University</td>
<td>25</td>
</tr>
<tr>
<td>2.5.6</td>
<td>Miscellaneous</td>
<td>25</td>
</tr>
<tr>
<td>2.5.7</td>
<td>Swiss Federal Laboratories for Testing Materials and Research</td>
<td>26</td>
</tr>
<tr>
<td>2.5.8</td>
<td>Sheffield University</td>
<td>27</td>
</tr>
<tr>
<td>2.6</td>
<td>CONCLUSIONS</td>
<td>28</td>
</tr>
</tbody>
</table>

**CHAPTER 3**

**MATERIAL PROPERTIES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Text</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>CONCRETE</td>
<td>29</td>
</tr>
<tr>
<td>3.1.1</td>
<td>Experimental Procedure</td>
<td></td>
</tr>
<tr>
<td>3.1.2</td>
<td>Results</td>
<td>30</td>
</tr>
<tr>
<td>3.1.3</td>
<td>Conclusions</td>
<td>30</td>
</tr>
<tr>
<td>3.2</td>
<td>EPOXY RESINS</td>
<td>33</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Lap Shear Tests - Steel/Steel</td>
<td></td>
</tr>
<tr>
<td>3.2.2</td>
<td>Tension Tests</td>
<td>36</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Compression Tests</td>
<td>39</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Lap Shear Tests - Steel/Concrete</td>
<td>41</td>
</tr>
<tr>
<td>3.3</td>
<td>REINFORCEMENT</td>
<td>43</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Bars</td>
<td></td>
</tr>
<tr>
<td>3.3.2</td>
<td>Plates</td>
<td>46</td>
</tr>
</tbody>
</table>

**CHAPTER 4**

**PRELIMINARY TEST SERIES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Text</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>EXPERIMENTAL PROGRAMME</td>
<td>48</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Beam Details</td>
<td></td>
</tr>
<tr>
<td>4.1.2</td>
<td>Material Properties</td>
<td>50</td>
</tr>
<tr>
<td>4.1.3</td>
<td>Preparation of Test Specimens</td>
<td>54</td>
</tr>
<tr>
<td>4.1.4</td>
<td>Testing Procedure</td>
<td>54</td>
</tr>
<tr>
<td>4.2</td>
<td>TEST RESULTS AND DISCUSSION</td>
<td>55</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Deflections and Strains</td>
<td></td>
</tr>
<tr>
<td>4.2.2</td>
<td>Modes of Failure</td>
<td>59</td>
</tr>
<tr>
<td>4.2.3</td>
<td>First Crack and Ultimate Loads</td>
<td>62</td>
</tr>
<tr>
<td>4.3</td>
<td>CONCLUSIONS</td>
<td>68</td>
</tr>
</tbody>
</table>

**CHAPTER 5**

**STRENGTH PROPERTIES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Text</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>INTRODUCTION</td>
<td>71</td>
</tr>
<tr>
<td>5.2</td>
<td>EXPERIMENTAL PROGRAMME</td>
<td>72</td>
</tr>
<tr>
<td>5.3</td>
<td>TEST PROCEDURE</td>
<td>73</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page No.</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>5.4</td>
<td>DISCUSSION OF TEST RESULTS</td>
<td>86</td>
</tr>
<tr>
<td>5.4.1</td>
<td>First Crack Loads</td>
<td></td>
</tr>
<tr>
<td>5.4.2</td>
<td>Increase of Service Loads</td>
<td>86</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Ultimate Loads</td>
<td>91</td>
</tr>
<tr>
<td>5.4.4</td>
<td>Modes of Failure</td>
<td>93</td>
</tr>
<tr>
<td>5.5</td>
<td>CONCLUSIONS</td>
<td>95</td>
</tr>
<tr>
<td>CHAPTER 6</td>
<td>DEFORMATION PROPERTIES</td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td>INTRODUCTION</td>
<td>98</td>
</tr>
<tr>
<td>6.2</td>
<td>EXPERIMENTAL PROGRAMME/PROCEDURE</td>
<td>99</td>
</tr>
<tr>
<td>6.3</td>
<td>DISCUSSION OF RESULTS</td>
<td></td>
</tr>
<tr>
<td>6.3.1</td>
<td>Introduction</td>
<td>99</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Load-Strain Characteristics</td>
<td>107</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Load-Deflection Characteristics</td>
<td>128</td>
</tr>
<tr>
<td>6.3.4</td>
<td>Moment-Rotation Characteristics</td>
<td>141</td>
</tr>
<tr>
<td>6.4</td>
<td>CONCLUSIONS</td>
<td>151</td>
</tr>
<tr>
<td>CHAPTER 7</td>
<td>CRACKING PROPERTIES</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>INTRODUCTION</td>
<td>155</td>
</tr>
<tr>
<td>7.2</td>
<td>REVIEW OF LITERATURE</td>
<td>155</td>
</tr>
<tr>
<td>7.3</td>
<td>EXPERIMENTAL PROGRAMME</td>
<td>158</td>
</tr>
<tr>
<td>7.4</td>
<td>DISCUSSION OF RESULTS</td>
<td></td>
</tr>
<tr>
<td>7.4.1</td>
<td>General - Statistical Analysis</td>
<td>159</td>
</tr>
<tr>
<td>7.4.2</td>
<td>Maximum Crack Widths</td>
<td>171</td>
</tr>
<tr>
<td>7.4.3</td>
<td>Crack Width Prediction Formulae</td>
<td>178</td>
</tr>
<tr>
<td>7.4.4</td>
<td>Relation Between Maximum and Average Crack Widths</td>
<td>182</td>
</tr>
<tr>
<td>7.4.5</td>
<td>Concrete Surface Strain</td>
<td>184</td>
</tr>
<tr>
<td>7.4.6</td>
<td>Stresses Carried by Concrete in the Tension Zone</td>
<td>191</td>
</tr>
<tr>
<td>7.5</td>
<td>CONCLUSIONS</td>
<td>199</td>
</tr>
<tr>
<td>CHAPTER 8</td>
<td>LONG TERM TEST SERIES</td>
<td></td>
</tr>
<tr>
<td>8.1</td>
<td>INTRODUCTION</td>
<td>201</td>
</tr>
<tr>
<td>8.2</td>
<td>EXPERIMENTAL PROCEDURE</td>
<td>201</td>
</tr>
<tr>
<td>8.3</td>
<td>SHRINKAGE TESTS</td>
<td>201</td>
</tr>
<tr>
<td>8.4</td>
<td>SUSTAINED LOADING/LONG TERM TESTS</td>
<td></td>
</tr>
<tr>
<td>8.4.1</td>
<td>Introduction</td>
<td>202</td>
</tr>
<tr>
<td>8.4.2</td>
<td>Discussion of Results</td>
<td>207</td>
</tr>
</tbody>
</table>
8.5 DURABILITY TESTS
8.5.1 Introduction
8.5.2 Coating Details
8.5.3 Experimental Procedure
8.5.4 Discussion of Results
8.6 CONCLUSIONS

CHAPTER 9 LIMITATIONS OF PRESENT WORK, OVERALL CONCLUSIONS,
RECOMMENDATIONS FOR FUTURE WORK
9.1 LIMITATIONS OF PRESENT WORK
9.2 OVERALL CONCLUSIONS
9.3 SUGGESTIONS FOR FUTURE WORK

APPENDIX 1 GLOSSARY OF TERMS USED IN ADHESIVES TECHNOLOGY
APPENDIX 2 THEORETICAL STRESS DISTRIBUTION IN A COMPRESSION
LAP JOINT
APPENDIX 3 FIRST CRACK AND ULTIMATE LOADS - PRELIMINARY
SERIES OF TESTS
APPENDIX 4 ULTIMATE LOAD CALCULATIONS - MAIN TEST SERIES
APPENDIX 5 DEFLECTION CALCULATIONS - MAIN TEST SERIES
APPENDIX 6 ROTATION CALCULATIONS - MAIN TEST SERIES
APPENDIX 7 INTERFACIAL STRESSES BETWEEN GLUE AND STEEL/OR
CONCRETE
APPENDIX 8 CRACK WIDTH CALCULATIONS - MAIN TEST SERIES
APPENDIX 9 STATISTICAL METHODS EMPLOYED IN ANALYSES
REFERENCES
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Fig. No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Grading of Aggregates</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>Details of Adhesive Test Specimens</td>
<td>34</td>
</tr>
<tr>
<td>3.3</td>
<td>Mean Shear Stress v. Adhesive Thickness</td>
<td>35</td>
</tr>
<tr>
<td>3.4</td>
<td>Tensile Stress-Strain Curve for Epoxy Adhesive CXL 194</td>
<td>37</td>
</tr>
<tr>
<td>3.5</td>
<td>Compressive Stress-Strain Curve for Epoxy Adhesive CXL 194</td>
<td>40</td>
</tr>
<tr>
<td>3.6</td>
<td>Details of Shear Specimen</td>
<td>42</td>
</tr>
<tr>
<td>3.7</td>
<td>Experimental and Theoretical Stress Distribution in a Bonded Steel/Concrete Lap Joint under Compression</td>
<td>44</td>
</tr>
<tr>
<td>3.8</td>
<td>Tensile Stress-Strain Curves for 6 mm and 20 mm Diameter Bars</td>
<td>45</td>
</tr>
<tr>
<td>3.9</td>
<td>Tensile Stress-Strain Curves for Steel Plates</td>
<td>47</td>
</tr>
<tr>
<td>4.1</td>
<td>Details of Test Beams : Series A</td>
<td>51</td>
</tr>
<tr>
<td>4.2</td>
<td>Details of Test Beams : Series B</td>
<td>52</td>
</tr>
<tr>
<td>4.3</td>
<td>Load-Deflection Curves : Series A</td>
<td>56</td>
</tr>
<tr>
<td>4.4</td>
<td>Load-Deflection Curves : Series A</td>
<td>56</td>
</tr>
<tr>
<td>4.5</td>
<td>Load-Deflection Curves : Series B</td>
<td>57</td>
</tr>
<tr>
<td>4.6</td>
<td>Load-Deflection Curves : Series B</td>
<td>57</td>
</tr>
<tr>
<td>4.7</td>
<td>Typical Strain Distributions</td>
<td>58</td>
</tr>
<tr>
<td>4.8</td>
<td>Load-Strain Curves : Series A</td>
<td>60</td>
</tr>
<tr>
<td>4.9</td>
<td>Load-Strain Curves : Series A</td>
<td>60</td>
</tr>
<tr>
<td>4.10</td>
<td>Load-Strain Curves : Series B</td>
<td>61</td>
</tr>
<tr>
<td>4.11</td>
<td>Load-Strain Curves : Series B</td>
<td>61</td>
</tr>
<tr>
<td>5.1</td>
<td>Loading Rig and Mechanical Instrumentation</td>
<td>75</td>
</tr>
<tr>
<td>5.2</td>
<td>Details of Reinforcement and Strain Gauge Locations</td>
<td>76</td>
</tr>
<tr>
<td>5.3</td>
<td>Variation of First Crack Load</td>
<td>88</td>
</tr>
<tr>
<td>5.4</td>
<td>Variation of Experimental Service Load</td>
<td>88</td>
</tr>
<tr>
<td>5.5</td>
<td>Variation of Ultimate Load</td>
<td>96</td>
</tr>
<tr>
<td>5.6</td>
<td>Effect of Adhesive Thickness</td>
<td>96</td>
</tr>
<tr>
<td>6.1</td>
<td>Typical Load-Strain Curves</td>
<td>108</td>
</tr>
<tr>
<td>6.2-6.11</td>
<td>Load-Strain Curves - Internal Bars, Centre Section</td>
<td>109</td>
</tr>
<tr>
<td>6.12-6.21</td>
<td>Load-Strain Curves - External Plates, Centre Section</td>
<td>116</td>
</tr>
<tr>
<td>6.22</td>
<td>Typical Longitudinal Plate Strain Distributions</td>
<td>122</td>
</tr>
<tr>
<td>6.23</td>
<td>Internal Steel Bar Strain v. Plate and Glue Thickness</td>
<td>123</td>
</tr>
<tr>
<td>6.24</td>
<td>External Steel Plate Strain v. Plate and Glue Thickness</td>
<td>124</td>
</tr>
<tr>
<td>6.25</td>
<td>Typical Strain Distributions in the Concrete</td>
<td>127</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>Title</td>
<td>Page No.</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>6.26-6.35</td>
<td>Load-Deflection Characteristics</td>
<td>133</td>
</tr>
<tr>
<td>6.36</td>
<td>Central Deflection v. Plate and Glue Thickness</td>
<td>139</td>
</tr>
<tr>
<td>6.37</td>
<td>Moment-Curvature Relationship</td>
<td>142</td>
</tr>
<tr>
<td>6.38-6.47</td>
<td>Moment-Rotation Characteristics</td>
<td>144</td>
</tr>
<tr>
<td>6.48</td>
<td>Rotation v. Plate and Glue Thickness</td>
<td>149</td>
</tr>
<tr>
<td>6.49</td>
<td>Empirical Formula for Theoretical Rotations</td>
<td>152</td>
</tr>
<tr>
<td>6.50</td>
<td>Experimental v. Theoretical Rotations</td>
<td>153</td>
</tr>
<tr>
<td>7.1-7.3</td>
<td>Mean Crack Width and Standard Deviation v. Concrete Strain at the Level of the Internal Reinforcement</td>
<td>163</td>
</tr>
<tr>
<td>7.4</td>
<td>Slope of Mean Crack Width v. Concrete Strain v. Plate and Glue Thickness</td>
<td>167</td>
</tr>
<tr>
<td>7.5</td>
<td>Linear Regression Analysis</td>
<td>168</td>
</tr>
<tr>
<td>7.6</td>
<td>Experimental v. Theoretical Values of the Slope of Mean Crack Width v. Concrete Strain</td>
<td>169</td>
</tr>
<tr>
<td>7.7</td>
<td>Ultimate Mean Crack Spacing v. Initial Crack Height</td>
<td>170</td>
</tr>
<tr>
<td>7.8</td>
<td>Theoretical v. Experimental Ultimate Crack Spacings</td>
<td>172</td>
</tr>
<tr>
<td>7.9-7.18</td>
<td>Maximum Crack Width v. Applied Load</td>
<td>173</td>
</tr>
<tr>
<td>7.19</td>
<td>Mean Crack Width v. Plate and Glue Thickness</td>
<td>179</td>
</tr>
<tr>
<td>7.20</td>
<td>Modification of CP110 Crack Width Formula</td>
<td>181</td>
</tr>
<tr>
<td>7.21</td>
<td>Modification of ACI Crack Width Formula</td>
<td>181</td>
</tr>
<tr>
<td>7.22</td>
<td>Slope of Standard Deviation v. Concrete Strain v. Slope of Mean Crack Width v. Concrete Strain</td>
<td>183</td>
</tr>
<tr>
<td>7.23</td>
<td>Theoretical v. Experimental Maximum Crack Width</td>
<td>185</td>
</tr>
<tr>
<td>7.24-7.26</td>
<td>Calculated Steel Stress v. Measured Concrete Strain</td>
<td>188</td>
</tr>
<tr>
<td>7.27</td>
<td>Modification to Beeby's Formula</td>
<td>192</td>
</tr>
<tr>
<td>7.28</td>
<td>Stress Distribution in Plated Beams at Service Load</td>
<td>194</td>
</tr>
<tr>
<td>7.29</td>
<td>Tensile Stress in Concrete v. Plate and Glue Thickness</td>
<td>197</td>
</tr>
<tr>
<td>7.30</td>
<td>Experimental v. Theoretical Values of the Tensile Stress in Concrete</td>
<td>198</td>
</tr>
<tr>
<td>8.1</td>
<td>Long Term Loading Rig</td>
<td>204</td>
</tr>
<tr>
<td>8.2</td>
<td>Variation of Strains with Time</td>
<td>205</td>
</tr>
<tr>
<td>8.3</td>
<td>Load-Strain Curves, External Steel Plate, Centre Section</td>
<td>211</td>
</tr>
<tr>
<td>8.4</td>
<td>Load-Deflection Characteristics</td>
<td>212</td>
</tr>
<tr>
<td>8.5</td>
<td>Moment-Rotation Characteristics</td>
<td>213</td>
</tr>
<tr>
<td>8.6</td>
<td>Mean Crack Width and Standard Deviation v. the Concrete Strain at the Level of Reinforcement</td>
<td>219</td>
</tr>
<tr>
<td>8.7</td>
<td>Slope of the Standard Deviation v. Concrete Strain v. Slope of Mean Crack Width v. Concrete Strain</td>
<td>220</td>
</tr>
<tr>
<td>8.8</td>
<td>Durability Test Specimens</td>
<td>223</td>
</tr>
</tbody>
</table>

(viii)
<table>
<thead>
<tr>
<th>Table No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Concrete Compressive Strength</td>
<td>32</td>
</tr>
<tr>
<td>3.2</td>
<td>Modulus of Elasticity</td>
<td>32</td>
</tr>
<tr>
<td>3.3</td>
<td>Modulus of Rupture</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Details of Plain Concrete Test Beams : Series A</td>
<td>49</td>
</tr>
<tr>
<td>4.2</td>
<td>Details of Plain Concrete Test Beams : Series B</td>
<td>49</td>
</tr>
<tr>
<td>4.3</td>
<td>Test Results for Plain Concrete Beams : Series A</td>
<td>65</td>
</tr>
<tr>
<td>4.4</td>
<td>Test Results for Plain Concrete Beams : Series B</td>
<td>66</td>
</tr>
<tr>
<td>4.5</td>
<td>Test Results for Plain Concrete Beams : Series A</td>
<td>67</td>
</tr>
<tr>
<td>4.6</td>
<td>Test Results for Plain Concrete Beams : Series B</td>
<td>67</td>
</tr>
<tr>
<td>5.1</td>
<td>Details of Test Beams</td>
<td>74</td>
</tr>
<tr>
<td>5.2</td>
<td>Strength Characteristics</td>
<td>87</td>
</tr>
<tr>
<td>5.3</td>
<td>Increase of Service Loads</td>
<td>89</td>
</tr>
<tr>
<td>5.4</td>
<td>Ultimate Loads</td>
<td>92</td>
</tr>
<tr>
<td>6.1.A</td>
<td>Deformation Characteristics at First Visible Crack Load</td>
<td>100</td>
</tr>
<tr>
<td>6.1.B</td>
<td>Deformation Characteristics at 60 kN Load</td>
<td>101</td>
</tr>
<tr>
<td>6.2.A</td>
<td>Deformation Characteristics at Design Service Load A</td>
<td>102</td>
</tr>
<tr>
<td>6.2.B</td>
<td>Deformation Characteristics at Design Service Load B</td>
<td>103</td>
</tr>
<tr>
<td>6.2.C</td>
<td>Deformation Characteristics at 130 kN Load</td>
<td>104</td>
</tr>
<tr>
<td>6.3</td>
<td>Deformation Characteristics Near Failure Load</td>
<td>105</td>
</tr>
<tr>
<td>6.4</td>
<td>Deflection Characteristics</td>
<td>140</td>
</tr>
<tr>
<td>6.5</td>
<td>Comparison of Experimental and Theoretical Rotations</td>
<td>150</td>
</tr>
<tr>
<td>7.1-7.6</td>
<td>Cracking Characteristics</td>
<td>160</td>
</tr>
<tr>
<td>7.7</td>
<td>Crack Widths at 130 kN Load</td>
<td>180</td>
</tr>
<tr>
<td>7.8</td>
<td>Ratio of Maximum to Mean Crack Width</td>
<td>180</td>
</tr>
<tr>
<td>7.9</td>
<td>Comparison of Measured and Calculated Values of Concrete Surface Strain at the Level of the Internal Reinforcement</td>
<td>186</td>
</tr>
<tr>
<td>7.10</td>
<td>Measured and Calculated Values of the Difference Between the Internal Bar Strain and the Concrete Surface Strain at the Same Level</td>
<td>186</td>
</tr>
<tr>
<td>7.11</td>
<td>Tensile Stress in the Concrete</td>
<td>195</td>
</tr>
<tr>
<td>7.12</td>
<td>Mean Values of Tensile Stress in the Concrete</td>
<td>196</td>
</tr>
<tr>
<td>7.13</td>
<td>Tensile Stress in the Concrete from Empirical Equation</td>
<td>196</td>
</tr>
<tr>
<td>8.1</td>
<td>Details of Long Term Test Beams</td>
<td>203</td>
</tr>
<tr>
<td>8.2</td>
<td>Ultimate Moments</td>
<td>208</td>
</tr>
<tr>
<td>8.3</td>
<td>Comparison of Experimental and Theoretical Deflections and Rotations</td>
<td>215</td>
</tr>
</tbody>
</table>
**Table No.** | **Title** | **Page No.**
--- | --- | ---
8.4 | Cracking Characteristics | 217
8.5 | Crack Widths at 130 KN | 218
8.6 | Durability Test Results | 224

**LIST OF PLATES**

| Plate No. | **Title** | **Page No.**
--- | --- | ---
3.1 | Glue Test Specimens | 38
4.1 | Typical Beams After Failure : Series A | 63
4.2 | Typical Beams After Failure : Series B | 64
5.1 | Loading Arrangement | 78
5.2 | Crack Patterns - PLated and Unplated Beams | 79
5.3 | Crack Patterns - Beams with 1.5 mm Glue Thickness | 80
5.4 | Crack Patterns - Beams with 3 mm Glue Thickness | 81
5.5 | Crack Patterns - Beams with 3 mm Glue Thickness | 82
5.6 | Crack Patterns - Beams with 6 mm Glue Thickness | 83
5.7 | Crack Patterns - Beams with 1.5 mm Plate Thickness | 84
5.8 | Crack Patterns - Preloaded Beams | 85
8.1 | Long Term Tests - Loaded Beams | 206
8.2 | Long Term Tests - Unloaded Beams | 206
8.3 | Long Term Test Beams After Failure | 209
8.4 | Durability Specimens After Testing | 225
8.5 | Durability Specimen Details | 226
8.6 | Crack Patterns - Age of Beams 18 Months | 227
NOTATIONS

\( a \)  
Deflection

\( \bar{a} \)  
Distance from compression face to point of measurement

\( a_{cr} \)  
Distance from point considered to the surface of the nearest reinforcing bar

\( A \)  
Area, as defined

\( b \)  
Width of section

\( b_p \)  
Width of plate

\( c_{min} \)  
Minimum cover to the main reinforcing bars

\( C \)  
Coefficient or constant, as defined

\( d \)  
Effective depth to centroid of reinforcement

\( d^1 \)  
Thickness of adhesive

\( E \)  
Youngs Modulus, as defined

\( f_{bs} \)  
Local bond stress due to ultimate load

\( f_{bsa} \)  
Anchorage bond stress due to ultimate load

\( f_{bsd} \)  
Local bond stress due to service load

\( f_{bsd} \)  
Anchorage bond stress due to service load

\( f_{cu} \)  
Concrete cube strength

\( f_{cg} \)  
Compressive strength of glue

\( f_p \)  
Stress in plate

\( f_r \)  
Stress in reinforcing bars

\( f_{tg} \)  
Tensile strength of glue

\( f_{yp} \)  
Yield stress of plate

\( f_{yr} \)  
Yield stress of reinforcing bars

\( G \)  
Modulus of rigidity

\( h \)  
Overall depth of section

\( h_1 \)  
Distance from neutral axis to tension steel

\( h_2 \)  
Distance from neutral axis to tension face

\( I \)  
Moment of inertia, as defined

\( k, K \)  
Coefficient or constant, as defined

\( I, L \)  
Span

\( M \)  
Bending moment, as defined

\( M_{cr} \)  
Cracking moment for concrete

\( P \)  
Axial force

\( r \)  
Correlation coefficient

\( S \)  
Slope, as defined

\( S_{u}, S_{rm} \)  
Ultimate mean crack spacing

\( t \)  
Thickness of adherend

\( t_b \)  
Bottom cover to the centre of longitudinal reinforcing bar

\( t_g \)  
Glue thickness
tp  Plate thickness  

\( t_s \)  Side cover to the centre of longitudinal reinforcing bar  

\( u \)  Axial deformation  

\( V \)  Shear force  

\( W, W_m \)  Mean crack width  

\( W_{cr} \)  Maximum crack width  

\( W_k \)  Characteristic crack width  

\( x \)  Neutral axis depth  

\( \bar{X} \)  Mean value  

\( z \)  Lever arm  

\( \alpha_e \)  Modular ratio  

\( \gamma \)  Angular deformation  

\( \gamma_m \)  Partial safety factor for materials  

\( \varepsilon \)  Strain, as defined  

\( \phi \)  Curvature  

\( \rho \)  Proportion of tension reinforcement  

\( \sigma \)  Standard deviation  

\( \sigma_f \)  Tensile stress in the concrete  

\( \tau \)  Shear stress in the glue
CHAPTER 1
INTRODUCTION AND OUTLINE OF THESIS

1.1 GENERAL INTRODUCTION

In practice, situations can often arise when the behaviour of a conventional reinforced or prestressed concrete beam is found to be inadequate, and replacement or strengthening then becomes necessary. Such inadequacy may be related to the ultimate strength of the member or to its behaviour under service loading. The cause may be inferior materials, design or constructional faults, external damage or deterioration. Alternatively it may arise from requirements to increase the imposed loading above the original design load. When such situations arise, it has to be considered whether it is more economical to strengthen the existing member or to replace it.

Unless additional supports can be provided, any attempt to strengthen an existing structure is generally a difficult operation because of the fact that its strength properties are essentially determined during construction. The main problem in strengthening techniques is that of ensuring adequate connection and composite action between the reinforcing element and the existing structure. Methods of strengthening are generally straightforward to carry out, but may need great care and control. However, when extensive strengthening is required, there is generally a great deal of labour, plant and disruption involved whether the structure be steel or concrete.

Strengthening of concrete structures can be carried out by several techniques. Provision of additional reinforcement along with guniting and external prestressing have been successfully used. More recently polymer impregnation techniques have been successfully used to restore a severely disintegrated reinforced concrete slab at a cost of about £35 per sq m (75). The development of glues based on synthetic resins has, on the other hand, opened up another method of structural repair in which steel plates are bonded to the structural element with epoxy glue. These glues have adequate bonding strength and it has been shown that they provide effective composite action between the steel plate and the concrete element to be strengthened. However, careful
attention is necessary at all stages of the strengthening operation to ensure adequate interaction.

The method has been used to strengthen a variety of existing structures including: bridges in France (55), Japan (51) and England (53), a concrete crane gantry in France (54) and floor slabs of a telephone exchange in Zurich (56).

Despite the advantages and future potential of the technique only a limited amount of systematic research has so far been reported. A review of the literature which has been published on this subject is given in Chapter 2. In concrete joints (4) shear strengths in the glue can reach 7 N/mm$^2$ without much difficulty, so that failure would occur by shearing of the concrete. Although great emphasis has been placed on the considerable care needed in preparing the adherend surfaces, there is a degree of latitude which should allow satisfactory bonding when prepared to the tolerances attainable on site.

1.2 OUTLINE OF THESIS

The purpose of the present work is to investigate the static flexural behaviour, in both long and short term, of reinforced concrete beams strengthened by externally bonded steel plates glued to their tension faces. The tests were carried out to investigate the effects of varying:

(a) thickness of the glue layer
(b) thickness of the reinforcing plate
(c) number of layers of plate
(d) plate lapping techniques
(e) degree of cracking prior to bonding on the plates.

Points of stress concentration were formed in some beams by cutting notches in their tension faces.

The aims of the study were:

(i) To investigate the deformation behaviour, i.e. load-deflection and moment rotation characteristics.

(ii) To determine the internal bar strains, external plate strains, and concrete strains through the elastic, inelastic and ultimate regions.

(iii) To determine the first crack load in the concrete and study crack
propagation and distribution.

(iv) To investigate interfacial stress properties.

(v) To compare theoretical analysis with experimental results.

The properties mentioned above were studied (a) in beams tested approximately 14 days after bonding on the plates and (b) in similar beams tested 18 months after plating.

(vi) To study the durability of the concrete/epoxy/steel bond.

Before studying the technique of externally bonded plate reinforcement an appraisal of the materials involved was made. Chapter 3 reports the tests carried out on the materials used in the manufacture of the beams, i.e. concrete, epoxy resin, steel plates and bars.

Chapter 4 gives details of some preliminary testing on plain, unreinforced concrete beams which was performed to gain experience in the plating technique. This testing was made up of series A and B. Altogether eighteen beams were tested; two sizes of beams 150 x 150 x 710 mm (Series A) and 100 x 150 x 1200 mm (Series B) were used. Two epoxy resin systems were used in both series. The steel plates were of the same material for both series.

In series A the effects of uniform and tapering glue thickness were studied. In series B the provision of steps in the tension face of a beam forming stress concentrations was studied, together with lapping techniques. All the beams were tested under centre point loading over a single span.

The major testing programme was performed on reinforced concrete beams 155 x 255 x 2500 mm loaded at the 1/3 points of the span, which was 2300 mm. Firstly, the long term testing programme was started.

Concrete and epoxy resins are both susceptible to time dependent deformation due to shrinkage and creep. These deformations may be critical to the serviceability and sometimes to the safety of structural elements, particularly if accompanied by a reduction in cohesive or adhesive strength of the epoxy resin.

Eight beams, all of which have identical internal reinforcement are being subjected to sustained loading. The parameters under investigation are:
(a) glue layer constant - 3 mm
(b) glue layer constant - 6 mm
(c) glue layer variable - 3 mm to 8 mm
(d) glue layer variable - 25 mm x 25 mm notches at load points
(e) single plate layer, central lap
(f) single plate layer, laps at the 1/3 points of span
(g) two layers of plate, central lap in outer layer
(h) two layers of plate, laps at 1/3 points of the span in the outer layer.

All beams have plates 1.5 mm thick, yield stress 250 N/mm², with internal reinforcement 3 x 20 mm diameter bars, 0.2% proof stress 470 N/mm². There are two unloaded beams corresponding to each loaded beam. All the beams were left outside open to the weathering effects of the elements. Eight unloaded beams were to be returned for testing after eighteen months. These results are given in Chapter 8. The remaining eight unloaded beams and the eight loaded specimens will be tested, in due course after 5 years exposure.

The short term test series of twenty four beams was performed on beams identical to those described above for the long term tests with these additional parameters under investigation:

(a) plate thicknesses 3 mm and 6 mm, glue thicknesses 1.5, 3 and 6 mm
(b) plate thickness 1.5 mm, glue thickness 1.5 mm
(c) plate and glue thickness 3 mm, central lap
(d) glue thickness 3 mm, no plate
(e) plate thickness 3 mm, glue thickness 3 mm, beam loaded to 50% theoretical ultimate load prior to plating
(f) plate thickness 1.5 mm, glue thickness 3 mm, beams loaded to 50% and 90% theoretical ultimate load prior to plating.

An unplated control beam was also tested. 6 mm diameter stirrups were provided at 75 mm centres in the shear spans of all these beams, to prevent shear failure.

The results from the tests were reported as follows:
Chapter 5 - strength characteristics.

Chapter 6 - load-strain; load-deflection and moment rotation characteristics.

Chapter 7 - cracking characteristics.

In most environments the hardened epoxy resins will undergo both heat variation and moisture changes, but unlike many other adhesives, cured epoxies are said to be very resistant to water. However, moisture will eventually penetrate most epoxies and the bond between the resin and steel or concrete may be weakened. This problem was investigated using small concrete prisms 100 x 100 x 500 mm plated on one face. The plate and glue layer were coated with different sealing agents to prevent the ingress of moisture. The specimens were then placed in a fog chamber at 100% relative humidity for 18 months prior to testing. Control beams with no sealant were also kept in the fog chamber and in dry conditions for comparison. These prisms, were tested with a single, central point load over a span of 450 mm. The results are given in Chapter 8.

Wherever possible, the results of the present investigation are compared with those found by others.

Lastly Chapter 9 concerns the limitations of the present work, the overall conclusions, design recommendations and proposals for future work.

Appendix 1 gives a glossary of terms used in adhesives technology. Appendix 2 outlines the calculation of the theoretical stress distribution in a bonded lap joint under compression. Appendix 3 gives the methods of calculation of the first crack and ultimate loads for the preliminary test beams A and B. Appendix 4 gives the calculations of ultimate load for the main series of 24 beams. Appendix 5 gives calculations of the deflections of these 24 beams by methods recommended by CP110, CEB and ACI. Appendix 6 gives calculations for rotations. Appendix 7 discusses the interfacial stresses between the glue and steel or concrete. Appendix 8 outlines the calculations for crack widths and Appendix 9 gives a brief outline of the statistical methods used in the analyses.
CHAPTER 2
REVIEW OF LITERATURE

2.1 EPOXY RESINS
(Appendix I gives a glossary of terms relating to adhesive technology.)

2.1.1 General Introduction (1-5)

The use of adhesive materials to join two or more objects together can be traced back for centuries. The Ancient Egyptians used a type of glue to stick decorations onto their coffins. The Phonecians used bitumen as a crude jointing for their boat timbers and for sealing purposes. By the Middle Ages the art of making adhesives had made little progress, glues being made from animal bones and blood. From these cases it can be seen that the materials used for bonding were all obtained from natural products and no real progress was made until the nineteenth century.

The advent of systematic chemistry, particularly in respect of physical and organic chemistry, has led to an attempt by scientists to understand the process of adhesion. This in turn has led to the development of synthetic adhesives. A change in intellectual approach, rather than any specific invention, has given adhesives a new dimension: adhesives are now engineering materials. A tremendous impetus was given to the research, by the need to form high strength bonds or joints, especially in the lightweight structures of the aircraft industry.

The first practical application of epoxy resins took place in Germany and Switzerland in the 1930's, although their basic chemistry had been known for several decades. Limited production of epoxy resins started in the late 1940's and they became available in the early 1950's on a commercial scale.

Epoxy formulations developed until there were available systems with a combination of properties which made them suitable for use as an adhesive with
concrete. Epoxy resin systems cure without release of water or other by-products of a condensation reaction, and can consequently have low autogenous shrinkage (3). Because of the high degree of cross linking between long chain epoxide molecules there is little tendency to creep under sustained loading and moisture resistance is good.

The confusingly large variety of available products has hindered the progress of the use of epoxy resins for structural uses in the construction industry. The engineer requires the resin to have a consistency such that it allows ease of application and satisfactory curing under the prevailing weather conditions found on site. The bond thus formed must show little or no loss in adhesion as a function of time, or on exposure to moisture, sustained load and temperature variations. With these properties in mind the resin manufacturers should be able to formulate satisfactory structural engineering materials.

2.1.2 Materials and Mixes

A single epoxy resin system cannot be found to suit all applications. It is for this reason that epoxy resin systems which are sold commercially are generally the products of formulators who specialise in modifying the system with flexibilisers, extenders, diluents and fillers to meet specific end-use requirements. It logically follows that it is important to follow the manufacturers recommendations for use.

Conversely, the successful use of a resin system depends on the preparation of an adequate specification which must include such requirements as; adherend material, mixing/application temperatures and techniques, curing temperatures, surface preparation techniques, thermal expansion, creep properties, abrasion and chemical resistance etc. The specification should be so worded as to avoid any misunderstanding in these provisions for anyone concerned in the design, manufacture and application of the resin system from the formulating chemist to the site labourer.
2.1.3 Mixing

The accuracy of the required proportioning of resin and hardener is very important and a tolerance of plus or minus 2% is desirable. Some compounds can tolerate a wider variation but such variations should only be allowed if the manufacturer has test data available which show the complete effect of the variation on both mechanical and chemical resistance properties of the cured compound. The most accurate method of proportioning is by the use of pre-proportioned units supplied by the manufacturer so that the contents of one component container can be emptied into the other, usually hardener into resin, and then mixed together.

A low speed electric drill with a mixing paddle may be used with the caution that paddle type mixers can introduce air which can reduce adhesive and cohesive strengths if the system cures with the air entrapped. Mixing should continue until the mixture is homogeneous. The manufacturers often facilitate this by giving the resin and hardener two distinct colours which merge to form one colour when mixing is complete.

2.1.4 Temperature

Most epoxy resin formulations available today react favourably in the temperature range 40 - 150°F, although below 60°F mixing usually becomes difficult, and above 100°F the pot life may be shortened too much. In Great Britain, temperatures are often below 40°F so it is helpful to raise the temperature of the resin system prior to mixing. This reduces the viscosity of the resin system which in turn reduces the tendency to whip air into the compound during mixing. It may also be necessary to heat the adhered surfaces. Direct flame heating of concrete surfaces is difficult to control and warm air circulation heating or radiant heaters are preferable.

2.1.5 Surface Preparation

The strength of a bonded joint depends on the degree of adhesion to the adherends as well as the cohesive strength of the resin. The aim of surface
preparation is to ensure that adhesion develops to the extent that the cohesion is the weaker link in the system.

2.1.5.1 Concrete

Concrete surfaces must be cleared to remove all substances detrimental to bond of epoxy compounds such as laitance, curing compounds, dust, dirt and other debris resulting from surface preparation operations. The simplest method of achieving this is to shotblast the surface and then remove dust and debris by jetting with compressed air. The result should be a surface abraded to the extent that large aggregate particles are exposed and free from dust and contaminants. Care should be taken to ensure that good water and oil traps are incorporated in the compressed air system to prevent contamination after shotblasting is completed.

2.1.5.2 Steel

As rolled, metals have a contaminated surface layer, which is usually so thick that the metal surface exhibits the properties of the layer and not the metal itself. There are three methods which can be used to remove these surface contaminants: solvent cleaning or degreasing; mechanical abrasion; or chemical etching.

For site applications in the construction industry it is unlikely that chemical treatments would be used on a large scale. Cleaning and degreasing using solvents is practical, but adequate time must be allowed for their evaporation before mechanical treatment, otherwise they can be forced deeper into the metal causing weakening of the adhesive bond. The only practical, consistent field method of assuring an adequate bonding surface is shotblasting. After solvent cleaning and shotblasting any dust created by the mechanical cleaning must be removed by jetting with compressed air. A cleaned metal surface is very susceptible to corrosion, particularly in a humid atmosphere,
so the work should be planned to permit the epoxy application as soon as possible after cleaning.

2.1.6 Bonding

In its broadest sense adhesive bonding includes the application of the adhesive to the adherends and holding them in position until the joint acquires some strength.

The applicator should ensure that the epoxy is applied at a rate compatible with the pot life and rate of hardening of the system. Both are affected by the temperature at which the epoxy is applied.

Intimate contact is essential and all measures should be taken to ensure complete wetting. This is often more difficult to achieve with higher viscosity systems or when fillers are present.

The application process applies a certain amount of pressure to assist the resin penetration of the adherend surface. This penetration can be increased by applying pressure to the closed joint during the curing process.

In the region of the adherend surface the adhesive hardly moves when pressure is applied, but further away the glue is squeezed out taking unwanted air bubbles with it. For this reason it is usual to apply more adhesive than is necessary in the completed joint and squeeze out the excess under pressure to remove air bubbles. This also gives an indication of how even the pressure was by inspecting the amount of resin pressed out along the joint.

2.1.7 Curing

To cure an epoxy resin system means to alter the physical properties by chemical change. This usually means polymerisation brought about by either heating or the use of a catalyst. This causes the union of adjacent molecules
of adhesive, often existing as long chains, to form a tough solid resin. The interaction of such long chain molecules is known as cross-linking.

During curing the joint must not be moved otherwise cracks can develop at the interface which could lead to loss of bond by the ingress of moisture.

As curing progresses, strains are set up due to the differential expansion of the resin and adherends, and also due to polymerisation. Initially the resin is a liquid or paste, and as polymerisation starts it becomes a gel. During this period autogenous shrinkage is not important as both liquid and gel can accommodate volume change by flowing. The process continues and results in a hard polymer. At some point its strength increases so that shrinkage cannot be accommodated without producing internal stresses. These observations have led to the development of compounds in which most shrinkage occurs during the gel stage. Fillers can be used to reduce shrinkage but do not adversely affect adhesion when used in normal amounts.

2.1.8 Safety and Health Provisions

Just as there are proper, safe practices for handling acids, portland cement, etc., there are also precautions which should be observed when handling epoxy resins and materials used with them.

Two typical health problems encountered with epoxy materials when carelessly handled are skin irritations, such as burns and rashes, and skin sensitisation, which is an allergic reaction.

Safe handling can be accomplished by working in a well ventilated area and using disposable equipment whenever possible. Disposable suits and gloves are readily available. Goggles are strongly recommended, and involuntary habits such as eyeglass adjustment should be avoided. In the case of direct skin
contact solvents other than soap and water should not be used. Most solvents merely dilute the epoxy compound, aiding their penetration into the skin.

The solvents used for precleaning and equipment cleaning require additional precautions. Many have low flash points and these should be avoided. Ketones are a fire hazard and if used good ventilation is required. Smoking and other fire initiating devices should be barred from the area of use.

Chlorinated solvents, while not presenting a fire hazard, will present a toxicological problem if smoking takes place in the area or if a fire occurs in the immediate area. Many can be toxic when inhaled.

No amount of equipment will substitute for worker education. Those involved with the use of epoxy materials should be informed of the hazards of the particular materials they must handle. The handling of epoxy materials is not dangerous as long as reasonable care is taken and personnel and equipment are kept clean.

Instances of sensitisation are rare but the possibility of burns, loss of an eye and other time losing accidents makes knowledge and observance of safe handling practices absolutely essential.

2.2 GLUED JOINTS IN CONCRETE

2.2.1 General Uses

Epoxy resins have been used in concrete repair work since the 1950's. Tremper (7) describes the use of epoxies in repairing concrete highways; Gaul and Apton (9) for repair of runways and roads; Wakeman, Stover and Blye (10) and Ciesielski (11) describe resin injection of cracked pile caps and beams; and Levy (14) reports their use in the repair of precast elements. In general (8) it was found that bond strengths were greater than concrete strength in flexure, shear and direct tension.
2.2.2 Surface Preparation

The generally accepted methods of concrete surface preparation are discussed by Batchelor (21) and Moar (22). The relative convenience of these methods will vary with location of construction site and availability of equipment. Moar found that mechanical treatment was less important for short term strength, which depended mainly on the adequate removal of dirt, grease and laitance. For long term strength the amount of mechanically exposed aggregate greatly increased the durability of the joint.

Gorgol (23) considered concrete surfaces to be adequate, "as stripped", but he was only working with compression joints.

Hallquist (24) found wire brushing to be inadequate but does not compare this with other preparations, nor does he make any positive recommendations.

Lee and Neville (6), state that porous surfaces such as concrete require no special surface treatment. This has been shown to be incorrect by many other researchers.

Guttman (12) tabulated the properties of 26 adhesives suitable for structural bonding. Skin grease deposits on the surfaces led to 75% reduction in bond strength in some cases, minute quantities of solvents used for cleaning were found to inhibit curing in most of the adhesives.

Johnson (13) suggested the surfaces should be cleaned by sand blasting and then air blown, followed by degreasing and flushing. It would seem more sensible to degrease and remove dirt before shotblasting as this operation would tend to force the contaminants deeper into the surface.

2.2.3 Moisture Effects

Lee and Neville (6) state that moisture accelerates the curing process in
agreement with the findings of Caron (29) who also found water reduced the mechanical properties.

Hallquist (24) tested epoxies after exposure to humidities ranging from 30 - 85% and found no significant effect on 7 day strength.

Ciba Geigy (25) state that certain resins, containing polyamides exhibit comparatively high water absorption, which can reduce joint strengths.

Shue Fai (26) and Batchelar (21) tested mixes in both wet and dry conditions. The ultimate strength was not affected, but the curing rate was slower in water, contradicting the opinion of Caron, Lee and Neville.

Shaw (27) describes the use of three different epoxy resins. The strength of wet joints varied from 16 - 65% of the dry joints in the short term and 24 - 75% in the long term.

Moar (22) found moisture to have no apparent affect on initial strength but to be beneficial for long term strength.

Cusens and Smith (28) tested concrete prisms, with 45° glued scarf joints, to compare the water resistance of four adhesive systems. Three resins gave only 25% of the strength of dry prisms after only 8 weeks immersion in water, whereas the fourth resin showed an increase in strength.

Some of these apparent disagreements can be accounted for by the different chemical compositions of the resins used. A full comparison of the conclusions drawn by individual researchers could only be made considering this factor and also the surface preparation techniques used. The various fillers, flexibilisers, dilutents etc. that can be added to the basic resin can all affect the behaviour of the resin system in relation to moisture.
2.2.4 Miscellaneous

O'Brien (18) stated that the lack of knowledge of the long term behaviour of bonded joints is the main factor restricting their use to gap filling compression joints.

Johnson (15) performed a series of tests on glued lap joints which showed the long term strength to be only 50% of the short term strength. He suggested that vibrations may also cause creep.

Taylor (16) investigated long term vibration of concrete scarf joints under compression and found the creep to be no greater than in control specimens.

Kreigh (17) reports tests on composite beams using epoxy mortar as a shear connector. No debonding was evident after seven million cycles. The shear stress was a maximum of 2N/mm² which is approximately only 50% of the failure stress, but is still significant as it is approximately the working stress.

Johnson (20) formulated an epoxy suitable for structural joints transmitting loads by shear and compression. Flexibility and creep were little greater than in concrete, and in order of magnitude less than in his earlier tests. No shear or tensile test results were given for the glue so its use in anything but compression joints is not clear.

2.2.5 Summary

Lee and Neville (6), and Gorgol (23) found no special surface preparation to be required, contrary to the conclusions of Batchelor (21), Moar (22), Hallquist (24), Guttman (12), Johnson (13) and Cusens and Smith (28).

Lee and Neville (6), Caron (29), Moar (22) and in one system Cusens and Smith (28) found moisture to be advantageous to curing and in some cases joint strength. Hallquist (24), Shue Fai (26) and Batchelor (21) found moisture had
no effect on joint strength whereas Shaw (27), Ciba Geigy (25) and Cusens and Smith (28) found that moisture reduced joint strengths considerably.

Johnson (15) (20) tested various resin systems to produce one with creep properties with the same order of magnitude as concrete. However, to what extent this affects its ability to resist moisture, temperature cycling etc., is not reported.

It can be seen that there is a great deal of contradictory information available and comparisons could only be made when all the facts, involving glue chemistry; surface preparation; application; curing method and test technique; are known.

2.3 STRESS DISTRIBUTION IN LAP JOINTS

External forces seldom produce a uniform field of stress, even in an homogeneous body. When a body consists of two or more materials joined together a uniform stress is even less likely. The stress concentration, (ratio of highest to mean stress), depends on many factors including the elastic moduli of adherends and adhesive, and the shape and size of the specimen. The more highly stressed areas fail first and a progressive failure of the joint follows.

The earliest theoretical analysis of lap joints due to Volkersen (30), considered the distribution of shear forces, in the adhesive layer, for the case of very stiff adherends which do not bend on loading. It was soon recognised that the loading of a lap joint gives rise to bending, tending to peel the adherends apart.

Goland and Reissner (31) formulated a theory, taking these stresses into account, considering two distinct cases. Firstly, a thin stiff layer of adhesive is assumed to bond flexible adherends, and secondly vice-versa.
Cornell (32) varied this theory by assuming the adherends behave like simple beams and the adhesive is represented by shear and tension springs.

Wooley and Carver (33), and Amijima, Fujii and Yashida (34) carried out finite element analyses to investigate stress concentrations in lap joints. Many other theories have been suggested and are critically compared by Mylonas (35).

Bresson (54) gives a theory for stress distributions in bonded steel/concrete joints in both tension and compression.

2.4 GLUED JOINTS IN METALS

2.4.1 General

Gilibert, Delmas and Collot (36, 37) studied the fabrication of test specimens and apparatus to ensure good reproducibility of tests for comparing different glue systems. The fact that there was little scatter in their results should facilitate research into long term properties.

Allen (38) states that a primary loss of bond strength is due to the failure of molecules to achieve full cross linking due to poor batching and mixing.

2.4.2 Surface Preparation

Cagle (39) states that the service environment and life expectancy play the most important role in the selection of surface preparation techniques. This would seem to assume a greater understanding of the bonding process than is generally accepted.

Smith (40) and Olsen (41) both criticise mechanical abrasion prior to solvent cleaning as contaminants can be driven deeper into the metal.
Shields (42) recommends sharp jagged grits as round shots produce a peened surface consisting of many loose pieces of metal bent over each other resulting in a weak surface layer.

Jennings (43), and Delmas and Collot (36) (37) showed optimum adhesion is achieved by shotblasting. However, different resins, temperatures and humidities would most likely give different optimum grit size for the same metal adherend. It should also be remembered that the fluid properties of a resin change with time during curing. The rate of displacement of air from the surface pockets is essentially controlled by viscosity, and so the application of resin to a rough surface can trap air, causing stress concentrations. Thus a rough surface does not necessarily mean a high specific contact area. If heavily filled resins are used, displacement of air may be incomplete due to the increased viscosity of the resin system.

Ramel (44) found that rusting had little effect on bonding for various resins. Some could withstand 40 - 50% rusting. De Lollis (45), on the other hand, stated that corrosion was an important factor in loss of bond, soon after surface preparation.

Shields (42) found that in addition to their cleaning action, chemicals modify the surface physically and chemically.

Ciba Geigy (25) point out that chemicals can lead to inferior bond strengths if not used in the correct strength, or for the right duration. Flushing after treatment to remove all traces of chemical is also very important. Chemical treatment is unlikely to be used in the construction industry, however.

2.4.3 Moisture Effects

Buck and Hockney (46) immersed lap shear specimens for up to 1000 hours in
water and water vapour. It was found that at 20°C the joint strength was not affected. At 45°C there was a 25% reduction.

Kinloch and Gledhill (47) exposed lap joints in distilled water at 20, 40, 60 and 90°C and control 20°C; 56% relative humidity. They considered that immersion in water reduced the strength considerably, particularly at high temperature.

2.4.4 Miscellaneous

Tests (48) have been carried out on joints which had been held together at pressures ranging from 2 - 400 lb/in² during curing. There was no effect on joint strength, except when the joints were placed in natural weathering conditions when the low pressure bonded joints were adversely affected. In another series the rate of loading was varied from 0.35 to 26.5 tons/minute. In the limited number of tests no significant change in strength was observed. However, McNicholas (49) performed tests which showed that the high loading rates produced premature failure.

Delmas and Collot (37) varied the joint thickness from 0.05 mm to 1.5 mm and the optimum was found to be 0.5 mm. As the thickness increased the failure was due to a combination of adhesive and cohesive failure, i.e. a fracture plane which passed partly along the interface between adherend and adhesive, and partly through the body of the glue. In thinner joints failure was completely adhesive. It was concluded from this that the joint became brittle when thicker. Theoretically, the mean stress in a joint should increase with joint thickness, but practically thicker joints are more likely to have air bubbles, flaws and internal stresses. In filled resins the shrinkage and temperature stresses are much lower so that a filled thick joint could possibly be stronger than a thinner unfilled one.
Cusens and Smith (28) have performed comparative tests on four resin systems in steel/steel lap joints under static and cyclic loading. The effects of curing at elevated temperature and of temperature cycling were included. It was found that shear strengths increased with the roughness of the blasted surface, and slightly with increase in curing temperature. Temperature cycling between -7°C and +35°C had little effect on joint strength. There was satisfactory fatigue performance, with all specimens sustaining a stress range of 4.5 N/mm² for 10⁷ cycles. It was found that slight contamination with dust or water was not harmful.

The practical application of all these findings to predict breaking loads is very difficult. It is easy to control the surface preparation and application/curing techniques in small test specimens, but more difficult in actual site applications.

2.4.5 Summary

There is general agreement that the best surface preparation is shot-blasting. However, different resins, temperatures and humidities give different optimum grit sizes for the same metal adherend. Various glue thicknesses are recommended but these again would depend on the type of glue etc. for optimum strength.

Smith (40), Olsen (41), Shields (42) and Ciba Geigy (25), all agree that cleaning solvents and chemical treatments can lead to loss of bond if not removed completely from the surface prior to bonding. The use of chemicals prior to mechanical blasting is not recommended as this would lead to the solvents being driven deeper into the metal surface.

Buck and Hockney (46), and Kinloch and Gledhill (67) found water immersion to produce a loss in joint strength, which increased with temperature.
McNicholas (49) and others (48) give contradictory results on the effect of varying the loading rate.

Again, it is almost impossible to compare the different author's findings without knowing the resin chemistry; the exact surface preparation, loading techniques etc.

2.5 GLUED JOINTS BETWEEN STEEL AND CONCRETE

2.5.1 General

The application of epoxy resins to civil engineering structures may be classified under two main headings:

(a) resin used as a filler
(b) applications which depend on the shear strength of adhesive.

Tabor (50) gives examples of type (a) uses in his general review of the uses of epoxies in civil engineering.

Type (b) applications still remain rare and are largely confined to repair and/or strengthening of bridges. In Japan (51), by 1975, some 240 bridges had been strengthened, against increased vehicle loading, by the addition of steel plates glued to their upper and lower surfaces. In South Africa 11 bridges have been similarly strengthened and in Britain the technique has been used at Quinton (52) and Swanley (53). In France (54) (55) the technique has been used to strengthen a travelling crane and a motorway bridge, and in Switzerland (56) telephone exchange floors. Franke (57) describes the use of reinforcing bars bonded with epoxy to strengthen a spherical prefabricated concrete tank.

2.5.2 L'Hermite and Bresson (54, 55, 58, 59)

From L'Hermite's first experiments he concluded that the metal surface must be freed from contaminants and surface oxidation, and to prevent oxides reforming the joint must be bonded quickly, or alternatively a primer applied.
Beams with tension reinforcing plates all failed by concrete crushing or shearing, not by debonding. They behaved as normal reinforced concrete beams, with regard to failure load, however, the onset of cracking was delayed and crack propagation reduced indicating a close combined action between the steel plate and concrete beam.

Beams with plate reinforcement on their sides, to increase shear capacity, also behaved well and both types of beam, under cyclic loading, sustained half their ultimate loads for one million cycles without failure.

It is stated that after one year long term deflections increased by 20% but no information is given on stress level, glue type, surface preparation, etc.

In tests on plated slabs it was found indispensible to secure the edges of the plates with bonded angles to prevent premature and sudden failure.

2.5.3 Transport and Road Research Laboratories (60) (61) (62)

Flexural Tests have been reported on six reinforced concrete beams. The specimens used were in fact I shaped columns with a corbel towards one end which had a considerable local stiffening effect. However, useful information was obtained from observations of the beams away from the corbel. The columns, 4.9 m long, were positioned horizontally so that they could be regarded as under reinforced concrete beams.

The tests were used to study the effects of changes in type of adhesive, plate thickness, a joint in the external reinforcement and load cycling. In all cases full composite action was developed between the steel, resin and concrete. Failure occurred by horizontal shear in the concrete adjacent to the glue layer. The main structural benefits were:

(a) For a given crack width, the applied load for the plated beam was nearly double that for an unplated beam.
(b) The post cracking stiffness was increased by 35 - 105%.
(c) There was an increase in failure load ranging from 12 - 24%.

A seventh beam has since been tested which was cracked under flexural loading before the plate was bonded to it. The results from this indicate that immediately prior to failure the strain in the plate was considerably less than in a similar beam that was not precracked.

Long term exposure tests are being carried out on small concrete prisms, 500 x 100 x 100 mm, reinforced with epoxy-bonded steel plates, in marine, industrial and high rainfall sites for periods of 1, 2, 5 and 10 years. Half the prisms are subject to sustained loading during their exposure period.

The results to date from 1 and 2 year tests, have shown that the failure loads were slightly lower for high rainfall and marine sites. On the whole, the beams under sustained load during exposure were stronger.

All the beams from the exposure sites showed varying amounts of steel corrosion, which had been in contact with the adhesive, and become partially debonded. The control beams kept in the laboratory showed no signs of corrosion.

If such corrosion occurred on full scale bridge structures failure may occur after only a few years. Methods of preventing corrosion, and surface priming techniques are being investigated and checked with additional exposure tests.

Beams of 3.5 m length are at present being used to investigate:
(a) Four different resin systems.
(b) Four different glue line thicknesses.
(c) Different concrete strengths.
(d) Jointing, end bolting and multiple plate layers.
2.5.4 Dundee University (28) (44) (63) (70)

Development work at the Wolfson Bridge Research Unit, Dundee, is investigating the practical feasibility of two main forms of decking for medium and long span bridges. Both forms consist of concrete slabs reinforced with epoxy bonded steel plates. In each case the concrete is cast directly on to the steel plate which has been coated with epoxy resin. Thus the plate serves both as formwork and reinforcement.

Solomon (63) used this method of bonding steel plates on to hardened concrete. He found a lack of ductility in the mode of failure which was probably due to the plate thickness used.

For these purposes a resin which retains its strength in the presence of prolonged damp is required and which also has a rust inhibiting quality. The problem of aggregate particles penetrating the glue line during compaction would seem to be most important as this could facilitate permeation of water through the concrete and glue to cause steel corrosion and bond degradation, as well as forming points of weakness in the glue layer.

Cusens and Smith (28) report on the behaviour of concrete beams of 2.0 m and 344 mm length. Cusens' earlier tests with Solomon confirmed the feasibility of using a steel plate as external reinforcement, with the procedure of casting fresh concrete onto the plate coated with epoxy. In more recent tests (28), beams 2 m long have been used to study the ductility and the effects of adhesive type and thickness on the static flexural behaviour. It was found that a minimum glue thickness of 1 mm was required to ensure bond between concrete and steel and that ductility was good, especially for thicker glue layers.

The smaller beams, 344 mm long, were used to investigate the effect of curing temperature, cyclic loading and immersion in water. In both static and fatigue tests the type of adhesive seemed to have little significance. The
majority of beams survived 1.5 million cycles having a load range between 52 and 70% of their static failure load. Water immersion of beam specimens sealed on all surfaces except the top causes serious loss of flexural strength (> 33%) with some adhesives, mainly due to corrosion on the plate surface indicating ingress of moisture at the interface.

Solomon and Gopalani (70) report tests on beams similarly formed by pouring wet concrete onto the fresh adhesive applied to the steel sheet, which acts as formwork to the beam soffit. The tests were part of a feasibility study for a new type of concrete floor for buildings. All the beams failed in flexure and showed good ductility. No mention, however, is made of durability tests. All the beams mentioned in this section had the reinforcing plates along their entire length. The loading rig supports would therefore hold the plate ends onto the beam under testing conditions.

2.5.5 Warwick University

This work concerns the strengthening of a concrete bridge by epoxy bonding plates, to the top surface, above supports. The beams used were subject to a combination of longitudinal tension, bending and vertical shear. Most beams failed by a crack forming at the end of the plate and spreading towards the central section at the level of the internal reinforcement. The results show that the stress concentration at the end of the plate produces a shear/bond failure and thus prevents the concrete member from achieving its full flexural strength. This effect is probably greater in these specimens, which resisted longitudinal tension as well as bending and shear, than it would be in a beam, where the deeper compression zone would provide additional stiffness.

2.5.6 Miscellaneous

Lerchenthal (65) carried out tests on model slabs 300 x 300 x 30 mm reinforced with 0.25 mm sheets. Simultaneously, tests were carried out on slabs reinforced with strips, in both directions, cut from the sheets. For the same
quantity of steel the slab with a complete sheet had almost twice the capacity, showing the exploitation of the biaxial strength of the sheet. Three methods of bonding were used; bonding onto cured concrete; pouring fresh concrete onto sheets with resin applied; and fresh concrete poured onto a sheet with a grip layer of grit and sand glued to it. No significant difference was found, and all failures were by rupture of sheet or concrete, not by debonding. The sheet reduced the depth and spacing of cracks, relative to slabs reinforced conventionally with the same area of reinforcement. Because thin sheets were used no problem of the edges lifting was encountered as found by Bresson, who had to hold the edges down with bonded angle plates.

Cirodde (66) describes beam tests with steel and aluminium plates bonded to concrete. The resin used showed a large amount of creep.

Fleming and King (67) plated beams 150 x 150 x 1680 mm with no internal reinforcement. Failure occurred in the concrete along a plane parallel to the adhesive layer.

Kaifasz (68) describes tests carried out on rectangular and T section concrete beams having externally bonded bars and plates. Except in the case of reinforcing bars simply glued to the underside of the beam, where debonding occurred, satisfactory results were obtained with good agreement with theoretical predictions.

2.5.7 E.M.P.A. Swiss Federal Laboratories for Testing Materials and Research (56) (69)

Tests were carried out at E.M.P.A. to investigate the bonding properties of a steel/epoxy/concrete joint. The tests studied ways of anchoring the plate ends and the effects of long term fatigue loading. The technique of plated concrete as applied to floor slabs in a Zurich telephone exchange is reported.
The first short term static tests indicated that special attention should be given to anchor the plate ends. A second series of tests on plated Tee beams, subjected to both static and dynamic loading showed an 85% increase of static deflection after two million load cycles between 0.8 and 1.2 times the working load. No details are given of surface preparation, glue thickness and formulation.

The efficiency of the floor slab strengthening was checked by field measurements. Deflections were measured before, during and after strengthening, for different loadings. It was concluded that the strengthening had provided an increase in bending stiffness thereby reducing deflections and crack widths.

At present there is an extensive series of long and short term testing in progress, investigating different glue types, steel quality and thickness and lapping techniques. So far no report has been published on these.

2.5.8 Sheffield University

Bouderbalah (71) used reinforced concrete beams 100 x 150 x 1200 mm to investigate the adhesive type, glue thickness, plate lapping and precracking prior to plating. Beams were tested in flexural and shear modes and it was found that the addition of the steel plate increased the ultimate flexural capacity and the serviceability range, but had no effect on the shear capacity. The variation of glue thickness from 1.6 mm to 8 mm had no significant effect. The application of bonded plates to a precracked beam, and lapping of bonded plates were shown to be successful.

Reinforced concrete beams 100 x 150 x 2400 mm were used by Ang (72) to investigate the effects of plate thickness. Five beams were under-reinforced and three over reinforced, before plating. For the under-reinforced beams the plates increased bending stiffness and flexural capacity and reduced crack widths. The beams with 1.6 mm, 3 mm and 5 mm thick plates failed in a flexural mode by
yielding of the steel followed by local crushing of concrete. The beam with 10 mm thick plate failed due to plate separation at its end.

One over-reinforced beam was plated on its tension face and the other on the compression face. Both gave an increase in capacity of approximately 22%. The mode of failure was by debonding at the plate ends followed by shearing of the concrete.

The failure loads of all the plated beams could be satisfactorily predicted by CP 110 methods.

A limited series of tests (73) was performed to study the effect of cyclic loading during glue curing. The results showed no adverse effects on flexural capacity when compared with control beams which were not loaded during the curing period.

2.6 CONCLUSIONS

It is apparent that many factors affecting the behaviour of bonded joints are still not fully understood, and consequently not fully controlled. As is evident from the published literature there is some degree of disagreement and lack of detail in the information given on the following:

(a) glue composition and thickness
(b) plate thickness, lapping techniques, end anchorage
(c) behaviour of precracked beams
(d) long term behaviour under sustained load
(e) durability of epoxy resin bonded joints exposed to moisture.
CHAPTER 3

MATERIAL PROPERTIES

Practical experience and test results such as were outlined in Chapter 2 illustrate that the properties of epoxy adhesives are retained under varying conditions. The possibility of reinforcing concrete structures simply and effectively using epoxies could be of particular interest to the civil engineer. Nevertheless, efficient use of the adhesives will depend on the engineer having confidence in their properties and knowing their limitations.

Similarly, the properties of steel and concrete, involved in such strengthening operations, are of considerable importance. The resistance to cracking of concrete depends on its tensile strength and the control of cracking is important in maintaining the continuity of a structure; in many cases in the prevention of corrosion of reinforcement and in this application to minimise the degradation of the concrete/epoxy/steel bond. Modulus of elasticity is of importance in controlling the deflection of members, and in addition to the tensile and compressive strengths of the steel and concrete respectively, in the analysis of structural members.

3.1 CONCRETE

3.1.1 Experimental Procedure

A trial mix was performed to assess the workability and strength properties of the concrete, which was consistent with that used in precast prestressed beams used in bridge construction. Bearing in mind that in practice strengthening is often carried out after several years in service, the concrete would have matured and acquired substantial increase in strength.

The proportions of the concrete constituents were approximately 1:1.05:2.45, with a water/cement ratio of 0.4 and plasticised by the use of Febflow. The quantities for 1m$^3$ are given below.

- sand: 480 kg
- 19 mm aggregate: 1124 kg
- cement: 450 kg
Rapid hardening cement, Ferrocrete, was used. The coarse aggregate was 19 mm maximum, uncrushed gravel and the fine aggregate was natural river sand. The gradings for these are shown in Fig. 3.1.

The mixing of concrete was carried out in a non tilting pan type mixer with 0.127 m³ capacity. The materials were dry mixed for two minutes, and after the addition of water, for a further two minutes. A poker vibrator was used for compaction and the specimens were cast in steel moulds as follows,

(a) 26 - 100 mm cubes for compressive strength
(b) 12 - 500 x 100 x 100 mm prisms for modulus of rupture
(c) 4 - 300 x 100 x 100 mm prisms for modulus of elasticity.

The moulds were stripped after 24 hours and the specimens were then placed in a mist room at 21°C and 100% relative humidity, until required for testing.

Compressive strength, modulus of rupture and Young's modulus tests were carried out in accordance with the recommendations of British Standards.

3.1.2 Results

Table 3.1 shows the compressive strength results. The mean strength at 28 days was 69.5 N/mm².

Table 3.2 shows the modulus of elasticity results. The mean Young's Modulus at 28 days was 36.0 kN/mm² with a Poisson's ratio of 0.16.

Table 3.3 shows the modulus of rupture results. The mean value at 28 days was 5.59 N/mm².

3.1.3 Conclusions

The results for mean compressive strength modulus of rupture and Young's Modulus were consistent and their standard deviations fell within acceptable limits.

3.2 EPOXY RESINS

In the preliminary test series, two types of adhesive were used.

Type A  CIBA GEIGY  XD 808
Type B  COLEBRAND  CXL 194
### TABLE 3.1 CONCRETE COMPRESSIVE STRENGTH

<table>
<thead>
<tr>
<th></th>
<th>3 DAYS</th>
<th></th>
<th>7 DAYS</th>
<th></th>
<th>14 DAYS</th>
<th></th>
<th>28 DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/mm²</td>
<td>mean</td>
<td>N/mm²</td>
<td>mean</td>
<td>N/mm²</td>
<td>mean</td>
<td>N/mm²</td>
<td>mean</td>
</tr>
<tr>
<td>47.6</td>
<td>60.3</td>
<td>48.0</td>
<td>64.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.0</td>
<td>62.4</td>
<td>70.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48.4</td>
<td>61.4</td>
<td></td>
<td>64.4</td>
<td></td>
<td>70.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.7</td>
<td>62.4</td>
<td>60.6</td>
<td>64.8</td>
<td>64.7</td>
<td>69.6</td>
<td>69.5</td>
<td></td>
</tr>
<tr>
<td>43.5</td>
<td>52.3</td>
<td></td>
<td>65.2</td>
<td></td>
<td>68.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47.0</td>
<td>55.0</td>
<td></td>
<td>56.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47.6</td>
<td>63.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.2</td>
<td>64.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3.2 MODULUS OF ELASTICITY

<table>
<thead>
<tr>
<th>28 DAYS</th>
<th>kN/mm²</th>
<th>mean</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.2</td>
<td>36.0</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>35.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3.3 MODULUS OF RUPTURE

<table>
<thead>
<tr>
<th>28 DAYS</th>
<th>N/mm²</th>
<th>N/mm²</th>
<th>N/mm²</th>
<th>mean</th>
<th>coeff. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.71</td>
<td>5.52</td>
<td>5.64</td>
<td></td>
<td>5.59</td>
<td>4 %</td>
</tr>
<tr>
<td>5.21</td>
<td>5.82</td>
<td>5.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.33</td>
<td>5.47</td>
<td>5.65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.00</td>
<td>5.31</td>
<td>5.91</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the main series of long and short term tests only glue type A was used.

3.2.1 Lap Shear Tests – Steel/Steel in Tension

A qualitative series of lap-shear tests was carried out to find the optimum glue thickness to be used when lapping two layers of plate, as judged by the average shear strength of the various glue thicknesses used.

3.2.1.1 Experimental Procedure

Double lap specimens were prepared, as shown in Fig. 3.2 cut from the same sheets of steel used as external reinforcement in the preliminary series of tests. The steel was shotblasted prior to bonding using steel grit with a mean particle size of 340 microns at a pressure of 0.55 – 0.75 N/mm². The steel pieces were then bonded together within one hour after shotblasting.

The glue was mixed using a low speed drill, operating at 280 rpm fitted with a paddle, for at least two minutes, but no more than three minutes. Bonding took place under controlled conditions at 15°C and 56% relative humidity. The central lapping plates were slightly offset to induce failure in the shorter side. The specimens were allowed to cure for ten days before testing. Four thicknesses of glue were used for each glue type, and two specimens for each thickness.

3.2.1.2 Results

Fig. 3.3 shows the results of the lap shear tests. The value of average shear stress was found for each glue and plotted against glue thickness. These tests showed an almost linear reduction in strength over the range of thicknesses used, 0.5 mm – 3.5 mm. This series was very limited in scope, and the results should be treated qualitatively. The control was sufficient to enable a choice of 0.5 mm joint thickness between lapping plates.

3.2.1.3 Conclusions

The alignment of all the pieces in the lap joint is very important so as not to induce peeling stresses in the glue when load is applied. The coefficient of variation for any batch varied from 1 to 24% and this would suggest that some specimens had better alignment than others, rather than any
FIGURE 3.2 DETAILS OF ADHESIVE TEST SPECIMENS
DOUBLE LAP SHEAR TESTS

FIGURE 3.3 MEAN SHEAR STRESS × ADHESIVE THICKNESS
fault in the resin itself.

The general conclusions seemed to be in agreement with the findings of others (37) who tested lap shear joints with glue thicknesses ranging from 0.05 mm - 1.5 mm and found 0.5 mm as the optimum. However, the fact that 0.5 mm is the optimum in both cases seems to be coincidental as the resin formulation, grit size for blasting and chemical treatments, were different.

3.2.2 Tension Tests

Tensile tests were performed for adhesive CXL 194 only.

3.2.2.1 Experimental Procedure

Three types of specimen were used as shown in Fig. 3.2. Type (A) was cast in steel moulds of the required shape; types (B) and (C) were made from prisms, which were cast in steel moulds, and subsequently milled and cut to shape after curing. A total of 6 type (A) specimens; 24 type (B) and 12 type (C), were made. Histograms were drawn for each type as shown in Fig. 3.4. Three type (C) specimens from each casting were fitted with 2 mm gauge length electrical resistance strain gauges, set at right angles to obtain Poisson's ratio. To determine the stress-strain curve, demec points were also fitted for strain readings over a gauge length of 50 mm. The test results are shown in Fig. 3.4. Type (C) specimens are shown in Plate 3.1.

3.2.2.2 Results

(a) Tensile strength ($f_{tg}$)

Type (A) specimens gave a mean value of 13.2 N/mm² and a standard deviation of 0.92 N/mm², coefficient of variation of 7%.

Type (B) gave corresponding values of 14.7 N/mm², 0.77 N/mm² and 5%.

Type (C) gave corresponding values of 16.6 N/mm², 0.79 N/mm² and 5%.

(b) Modulus of elasticity ($E_{tg}$)

Young's Modulus varied considerably with the stress range as shown in Fig. 3.4. Between zero and 2500 microstrain $E_{tg}$ was 2060 N/mm² and between 2500 and 7000 microstrain it was 1650 N/mm²; these values are the average of three specimens.

(c) Poisson's ratio ($\nu_{tg}$)
FIGURE 3.4 TENSILE STRESS STRAIN CURVE FOR EPOXY ADHESIVE CXL 194
a - compressive Young's modulus

b - compressive strength

c - tensile strength and Young's modulus

d - compressive lap shear
The average value from three specimens over the range zero to 7000 microstrain was 0.33.

3.2.2.3 Conclusions

For calculation purposes the values of the tensile properties of the epoxy adhesive system CXL 194 were taken as:

\[
\begin{align*}
E_{tg} &= 2000 \text{ N/mm}^2 \\
v_{tg} &= 0.33 \\
f_{tg} &= 16 \text{ N/mm}^2 
\end{align*}
\]

The coefficients of variation were of the order expected when testing polymers and the different types of specimen gave mean values varying by 12%. The value chosen from type (C) specimens seems realistic because there was considerable deformation of type (A) specimens within the testing machine’s jaws, and in type (B) specimen fracture sometimes occurred at the jaws.

3.2.3 Compression Tests

Compression tests were carried out for glue type CXL 194 only.

3.2.3.1 Experimental Procedure

The type of test pieces used is shown in Fig. 3.2. The prisms were fitted with 2 mm gauge length electrical resistance strain gauges for determining Poisson's ratio, and demec points on a gauge length of 50 mm for determining the stress strain curve. A histogram was drawn for the compressive strength results, from tests on 50 mm cubes, as shown in Fig. 3.5, together with the stress strain curve from the prisms. Plate 3.1 shows the compressive specimens after failure.

3.2.3.2 Results

(a) Compressive strength (\(f_{cg}\))

The mean compressive strength from eighteen cubes was 40 N/mm\(^2\) with a standard deviation of 2.27 N/mm\(^2\), coefficient of variation 6%.

(b) Modulus of Elasticity (\(E_{cg}\))

The mean value from three specimens was 3050 N/mm\(^2\) between zero and 2500 microstrain and 2200 N/mm\(^2\) between 2500 and 7000 microstrain.

(c) Poisson's ratio (\(v_{cg}\))
Compressive stress - N/mm²

- 36
- 37
- 38
- 39
- 40
- 41
- 42
- 43
- 44
- 45

Room temperature
Age 28 days

Elastic Modulus 2.200 kN/mm²

Figure 3.5 Compressive Stress Strain Curve for Epoxy Adhesive CXL 194
The mean value from three specimens was 0.36 over the range zero to 7000 microstrain.

3.2.3.3 Conclusions

For calculation purposes the compressive strength properties found from the tests were taken as:

\[
\begin{align*}
E_{cg} &= 3050 \text{ N/mm}^2 \\
\nu_{cg} &= 0.36 \\
f_{cg} &= 40 \text{ N/mm}^2
\end{align*}
\]

3.2.4 Lap Shear Tests - Steel/Concrete in Compression

Shear tests were carried out for glue type CXL 194 only.

3.2.4.1 Experimental Procedure

Three shear specimens were made up from two 100 mm concrete cubes and two 3 mm thick steel plates, 80 mm x 180 mm, with a 3 mm thick glue layer as shown in Fig. 3.6. Great care had to be taken to ensure the two ends which would be loaded were parallel to each other and at right angles to the axis of the plates. The surface preparation of the steel was as described in 3.2.1.1 and the concrete surface was abraded with a disc grinder, then hand sanded and blown with nitrogen to remove all loose particles and dust. The bonding took place within two hours of surface preparation. After twenty-eight days the specimens were tested in a compression machine at a loading rate of 4 kN/minute, up to failure. Readings of plate strains were taken at five stages during loading, at the locations shown on Fig. 3.6. The mode of failure was by shearing off one of the concrete faces very close to the adhesive layer as shown in Plate 3.1.

3.2.4.2 Results

The mean shear stress at failure was calculated by dividing the failure load by the total area of glue being sheared. \((80 \times 80 \text{ mm} \times 4 = 2.56 \times 10^4 \text{ mm}^2)\). The average value from the three test specimens was 2.87 N/mm², with a coefficient of variation of 6%.

The theoretical stress distribution is derived in Appendix 2 and shown in Fig. 3.7 for both the axial stress in the plate and the shear stress in the
SIDE ELEVATIONS OF TEST SPECIMEN

FIGURE 3.6 DETAILS OF SHEAR SPECIMEN
The theoretical distribution of shear stress gives a stress intensity factor (maximum stress to mean stress) of 1.82 for the concrete, glue and plate properties used. Assuming this value and applying it to the mean experimental shear stress gives a value of 5.2 N/mm² for the maximum shear stress in the glue at failure.

The experimental and theoretical values of axial stress in the plate show good agreement as shown in Fig. 3.7. The experimental values were obtained from electrical resistance strain gauges of 2 mm gauge length.

3.2.4.3 Conclusion

The theoretical distribution of axial stress proposed by Bresson (54) was confirmed. The assessed value of maximum shear stress in the glue, 5.2 N/mm² seems reasonable as the tensile shear strength of the concrete is approximately 5 N/mm².

3.3 REINFORCEMENT

Prior to manufacturing the preliminary beams, the stress-strain behaviour, of the steel reinforcing bars and plates, was investigated.

3.3.1 Bars

3.3.1.1 Experimental Procedure

Two specimens from each bar diameter, 6 mm and 20 mm, were used to determine the Young's Modulus, yield strength and ultimate tensile strength. The tensile tests on the 6 mm standard round bar specimens were carried out as recommended in BS 18: Part 2, 1970. To eliminate any initial lack of straightness, the specimens were first loaded to about 25% of the nominal yield stress, as specified by the manufacturer, and then released. The initial readings were then taken. The strains were measured using an extensometer of 50 mm gauge length. The 20 mm diameter high yield steel bars were tested in the same manner, but without any initial "straightening". The results for both bar diameters are shown in Fig. 3.8.

3.3.1.2 Results

The high tensile steel, unlike the mild steel, had no definite yield point. The elastic modulus for both steels was 200 kN/mm². The yield stress...
FIGURE 3.7 EXPERIMENTAL AND THEORETICAL STRESS DISTRIBUTIONS IN A BONDED STEEL/CONCRETE LAP JOINT UNDER COMPRESSIVE SHEAR LOADING
20mm high yield steel
Fracture stress 507 N/mm²
0.2% Proof stress 470 N/mm²
Elastic Modulus 200 kN/m²

x 6mm mild steel
Fracture stress 336 N/mm²
Elastic Modulus 200 kN/m²

Figure 3.8 Tensile Stress Strain Curves for 6mm & 20mm Bars
of the 6 mm bar was 320 N/mm² and for the 20 mm bar the proof stress was 470 N/mm².

3.3.1.3 Conclusions

The samples of steel which were tested behaved as expected and satisfied the requirements of British Standards.

3.3.2 Plates

3.3.2.1 Experimental Procedure

The external reinforcement was in the form of mild steel plates of three different thicknesses i.e. 1.5 mm, 3 mm and 6 mm. Two specimens from each thickness of steel plate were used to determine their Young's Modulus, yield strength and ultimate tensile strength. The tests were carried out as recommended in BS 18: Part 3, 1970. The strains were measured by demountable mechanical extensometer of 50 mm gauge length. The results are shown in Fig. 3.9.

3.3.2.2 Results

The elastic modulus was 200 kN/mm², and the yield stresses for 1.5 mm, 3 mm and 6 mm thicknesses were 236 N/mm², 258 N/mm² and 248 N/mm² respectively. The respective fracture stresses were 310 N/mm², 316 N/mm² and 308 N/mm².

3.3.2.3 Conclusions

The samples of mild steel plate behaved as expected and were satisfactory for use as the external reinforcement to the plated beams.
FIGURE 3.9 TENSILE STRESS STRAIN CURVES FOR STEEL PLATES

- 0.2% Proof stress 275 N/mm²
- Elastic Modulus 200 kN/mm²

- 1.5 mm thick
- 3 mm thick
- 6 mm thick

0.2% Plastic strain
CHAPTER 4

PRELIMINARY SERIES OF TESTS

INTRODUCTION

Since so many factors affecting the behaviour of bonded joints are still not fully understood, and consequently not fully controlled, it was decided that a preliminary series of tests would be performed for two main reasons:

(a) to gain some knowledge of the properties of the epoxy resin systems used

(b) to investigate methods of surface preparation and resin application in strengthening simple unreinforced concrete beams.

It was felt that this would ensure that the main series of large test beams would have less variation in results and that wastage of materials would be reduced to a minimum.

The preliminary testing was made up of two series - A and B.

4.1 EXPERIMENTAL PROGRAMME

4.1.1 Beam Details

In series A and B plated and unplated plain concrete beams with no tension bar reinforcement were tested. Altogether eighteen beams were tested; two sizes of beams, 150 x 150 x 710 mm (Series A) and 100 x 150 x 1200 mm (Series B) were used. Two epoxy resin systems were used in both series A and series B. The steel plates used for strengthening were the same material for both series.

4.1.1.1 Series A

The beams designated A1, 3, 5, 7 (Table 4.1) were bonded with type A resin and were tested on a loading rig which had no roller support. The friction force at the supports would therefore cause a relative increase in the applied load to produce a particular bending moment.

The beams designated A2, 4, 6, 8 were bonded with type B resin and tested with a roller support.

In this series the effects of uniform and tapering glue thickness were studied. Eight beams were tested. Beams A1 and A2 had no internal reinforcement at all. The other beams had shear reinforcement at the supports, to avoid shear...
**TABLE 4.1 DETAILS OF PLAIN CONCRETE TEST BEAMS: SERIES A**

<table>
<thead>
<tr>
<th>BEAM MK. NO.</th>
<th>CONCRETE AGE AT TESTING days</th>
<th>CONCRETE</th>
<th>ADHESIVE</th>
<th>THICKNESS* mm</th>
<th>PROFILE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>86</td>
<td>A</td>
<td></td>
<td>4.0</td>
<td>Uniform glue thickness</td>
</tr>
<tr>
<td>A2</td>
<td>114</td>
<td>B</td>
<td>4.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>85</td>
<td>A</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>114</td>
<td>B</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>88</td>
<td>A</td>
<td>3.8</td>
<td></td>
<td>Tapering glue thickness</td>
</tr>
<tr>
<td>A6</td>
<td>114</td>
<td>B</td>
<td>4.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A7</td>
<td>84</td>
<td>-</td>
<td></td>
<td></td>
<td>Unplated beam</td>
</tr>
<tr>
<td>A8</td>
<td>114</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 4.2 DETAILS OF PLAIN CONCRETE TEST BEAMS: SERIES B**

<table>
<thead>
<tr>
<th>BEAM MK. NO.</th>
<th>CONCRETE AGE AT TESTING days</th>
<th>CONCRETE</th>
<th>ADHESIVE</th>
<th>THICKNESS* mm</th>
<th>PROFILE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>83</td>
<td>A</td>
<td>3.5</td>
<td></td>
<td>Uniform glue thickness</td>
</tr>
<tr>
<td>B2</td>
<td>108</td>
<td>B</td>
<td>3.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>83</td>
<td>A</td>
<td>5.0</td>
<td></td>
<td>Uniform glue, lapped plates</td>
</tr>
<tr>
<td>B4</td>
<td>108</td>
<td>B</td>
<td>3.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td>77</td>
<td>-</td>
<td></td>
<td></td>
<td>Unplated beam</td>
</tr>
<tr>
<td>B6</td>
<td>106</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B7</td>
<td>77</td>
<td>-</td>
<td></td>
<td></td>
<td>Unplated, notched beam</td>
</tr>
<tr>
<td>B8</td>
<td>106</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B9</td>
<td>85</td>
<td>A</td>
<td>3.5</td>
<td></td>
<td>Plated, notched beam</td>
</tr>
<tr>
<td>B10</td>
<td>100</td>
<td>B</td>
<td>3.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* *average thickness at midspan.*
failure outside the plated length. Beams A1, A2, A3 and A4 had uniform glue thickness, whereas beams A5 and A6 had tapering glue thickness as shown in Fig. 4.1. Beams A7 and A8 were unplated control beams.

In beams A1 to A5, the steel plates were 100 x 500 x 1 mm thick, and were stopped short of the supports by about 50 mm. The beams were tested under centre point loading, over an effective span of 610 mm.

4.1.1.2 Series B

Series B consisted of the beams in which the effects of uniform glue thickness, lapping of plates and the provision of steps on the soffit of the beam were studied. The steps were cast into the beam's tension face as shown in Fig. 4.2 and detailed in Table 4.2.

All the beams in this series had shear reinforcement at the supports and the steel plates were 75 x 1000 x 1 mm thick, again stopped short of the supports. Beams B1 and B2 had uniform glue thickness whilst B3 and B4 had lapped plates. For these latter beams, two 500 mm long plates were used, butting each other, with a cover plate 75 x 400 x 1 mm thick. The cover plate was bonded with a glue thickness of approximately 0.5 mm. Beams B9 and B10 had tapered, stepped glue lines. This was intended to simulate the soffit of a bridge deck consisting of precast, prestressed box beams spanning longitudinally. Beams B5 and B8 were unplated control beams. Beams B7 and B8 were stepped as beams B9 and B10.

As in series A the beams with even numbered marks, i.e. B2, 4, 6, 8, 10 were bonded with type B resin and tested with a roller support. Beams with odd numbered marks, i.e. B1, 3, 5, 7, 9 were bonded with type A resin and were tested without a roller support. All the beams were tested under central point loading over an effective span of 1100 mm.

4.1.2 Material Properties

4.1.2.1 Concrete

The concrete chosen for the beams was designed to be consistent with that used in bridge construction using precast prestressed beams as detailed in Section 3.1.

The concrete properties for the two series of tests were as follows. For
BEAMS A1 and A2

BEAMS A3 and A4

BEAMS A5 and A6

6mm bars at 100 mm c/c.

steel plate

tapering glue thickness 3 - 6 mm.

All steel plates 500 x 100 x 1 mm.

FIGURE 4.1 DETAILS OF TEST BEAMS: SERIES A
BEAMS B1 and B2  UNIFORM GLUE THICKNESS

Nominal glue thickness 3mm
Plate 1000 x 75 x 1 mm thick

BEAMS B9 and B10  NOTCHED BEAM

variable glue thickness 3 - 6 mm

BEAMS B3 and B4  LAPPED PLATES

Nominal glue thickness 3 mm

FIGURE 4.2 DETAILS OF TEST BEAMS : SERIES B
beams of series A, the cube strength at 28 days varied from 56-60 N/mm² with a mean value of 58.3 N/mm². The cube strength at the age of testing of the beams varied from 64 to 66 N/mm² with a mean value of 65.5 N/mm². The modulus of rupture of the beams at testing varied between 4.37 and 4.94 N/mm² with a mean value of 4.65 N/mm².

For the beams of series B, the 28 day cube strength varied from 67.2 to 75.2 N/mm² with a mean value of 70.6 N/mm². The cube strength when the beams were tested varied from 71.9 to 82.7 N/mm² with a mean value of 76.7 N/mm². The modulus of rupture at this age varied from 5.1 to 5.35 N/mm² with a mean value of 5.22 N/mm².

The two concretes had an average elastic modulus of 36 kN/mm² at 3 months, a Poisson's ratio of 0.16 and an average shrinkage of 280 x 10⁻⁶ m/m.

4.1.2.2 Steel

The 6 mm diameter steel reinforcing bars used for stirrups and hanger bars at top and bottom in the two series were as detailed in Section 3.3.1 with a yield stress of 320 N/mm².

Tensile tests were carried out on standard tensile specimens cut from the steel plates to determine the tensile strength. The average yield stress and elastic modulus were 125 N/mm² and 200 kN/mm² respectively. The ultimate tensile strength of the plates was 132 N/mm². This plate was not the same as used for the main test series, as detailed in Section 3.3.2.

4.1.2.3 Adhesives

Two epoxy adhesives, both filled systems, were used in series A and B. Glue A was a 2 part liquid system manufactured by Ciba Geigy Ltd. and designated XD 808. Glue B was also a 2 part system but with a paste consistency, manufactured by Colebrand Ltd. and designated CXL 194.

To determine the shear strength of the glue, and hence the best thickness to use when lapping plates, double lap shear tests on specimens cut from the same sheet as the tensile specimens were carried out, as detailed in Section 3.2.1. Based on these results, 0.5 mm thickness was chosen for plate lapping. The other glue properties were detailed in Section 3.2.
4.1.3 Preparation of Test Specimens

All the test beams were made in four castings. The beams and control specimens were stripped after 24 hours and then cured in a fog room at 21°C, 100% relative humidity, until required for plating or testing. After removal from the fog room, the beams were kept in a warm dry atmosphere for 24 hours prior to surface preparation.

The surface preparation of the beams consisted of the following operations. The beams were abraded, on the tension face, with a disc grinder to remove laitance and expose the aggregates. They were then wire brushed to remove all loose particles. Finally they were sanded, by hand, with 100 grit emery cloth. All remaining dust and debris were removed by blowing with nitrogen.

The steel plates were gritblasted, as for the lap shear tests, under pressure to a uniform grade, as judged visually. Mixing and bonding operations were performed as described in Section 3.2.1.1. The glue was applied to both the steel plate and the prepared surface of the beam. Small pieces of hardened epoxy resin were used as spacers to control the glue thickness. The plate was then applied and held in position by weights.

Using this technique, any entrapped air would generally be restricted to the body of the glue layer, and not to the glue/concrete or glue/steel interface, where its effect on bond is greatest. The weights were generally left for four days after which the beams were left for at least another ten days to allow the glue to cure.

4.1.4 Testing Procedure

All the beams were tested under a single central point load. The beams were loaded in stages. At each stage central deflection, concrete strain distribution, strain in the steel plate and the state of cracking were noted. The concrete and steel strains were measured at mid-span using a demountable mechanical extensometer on a 100 mm gauge length. The concrete strains were measured at the compression face and over the depth of the beam. The beams were tested at ages varying between 77 and 114 days (at ages from 14 to 30 days after
The concrete cube strength and the flexural strength at the age of testing were given in Section 4.1.2.1.

4.2 TEST RESULTS AND DISCUSSION

The load-deflection curves are shown in Figs. 4.3 to 4.6 and 4.7 shows two typical strain distributions across the concrete section. Figs. 4.8 to 4.11 show the load-strain curves. Plates 4.1 and 4.2 show typical beams after failure and Tables 4.3 to 4.6 show the test results. The properties are discussed below under the relevant sections.

4.2.1 Deflections and Strains

The load-deflection curves are shown in Figs. 4.3 and 4.4 for beams of series A and in Figs. 4.5 and 4.6 for beams of series B. These results show that glued reinforcement has four distinct effects:

1. It increases the range of elastic behaviour.
2. It increases the stiffness of the beam.
3. It increases the ultimate flexural capacity of the beam, and
4. It makes the beams more ductile.

Considering beams of series A (Figs. 4.3 and 4.4), it is seen that beam A4 with shear reinforcement at supports showed better performance than the comparable beam A2 without shear reinforcement. Beam A6 with a tapering glue thickness of 3 to 6 mm, on the other hand, showed initially higher stiffness than beam A2 with uniform glue thickness, but with cracking at higher loads this stiffness became less than that of beam A2.

In beams of series B, the presence of notches showed no adverse effects on the unplated beams; with plated beams, the notched beam B10 showed marginally better behaviour than the beam B2 with uniform glue thickness. Beam A4 with lapped plates also showed marginally better performance than beams B2 and B10 (Fig. 4.6).

The measured strain distribution over the depth of the beams is shown in Fig. 4.7 for two typical beams A2 and B5. These diagrams show that the strain distribution remained approximately linear in the compression zone throughout the loading range whereas in the tensile zone the strain distribution, approximately
**FIGURE 4.3** LOAD-DEFLECTION CURVES: SERIES A

<table>
<thead>
<tr>
<th>BEAM</th>
<th>GLUE</th>
<th>REINFORCEMENT</th>
<th>FAILURE LOAD kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Uniform thickness</td>
<td>None</td>
<td>49.9</td>
</tr>
<tr>
<td>A3</td>
<td>= =</td>
<td>Shear at supports</td>
<td>46.6</td>
</tr>
<tr>
<td>A5</td>
<td>Variable 3-6 mm</td>
<td>= =</td>
<td>48.9</td>
</tr>
<tr>
<td>A7</td>
<td>None</td>
<td>= =</td>
<td>17.7</td>
</tr>
</tbody>
</table>

**FIGURE 4.4** LOAD-DEFLECTION CURVES: SERIES A.

<table>
<thead>
<tr>
<th>BEAM</th>
<th>GLUE</th>
<th>REINFORCEMENT</th>
<th>FAILURE LOAD kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>Uniform thickness</td>
<td>None</td>
<td>42.3</td>
</tr>
<tr>
<td>A4</td>
<td>= =</td>
<td>Shear at supports</td>
<td>45.0</td>
</tr>
<tr>
<td>A6</td>
<td>Variable 3-6 mm</td>
<td>= =</td>
<td>39.0</td>
</tr>
<tr>
<td>A8</td>
<td>None</td>
<td>= =</td>
<td>18.5</td>
</tr>
</tbody>
</table>
FIGURE 4.5 LOAD-DEFLECTION CURVES : SERIES B

<table>
<thead>
<tr>
<th>BEAM</th>
<th>GLUE</th>
<th>REINFORCEMENT</th>
<th>FAILURE LOAD KN</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Uniform</td>
<td>Shear at supports</td>
<td>20.0</td>
</tr>
<tr>
<td>B3</td>
<td>=</td>
<td>=</td>
<td>24.2</td>
</tr>
<tr>
<td>B5</td>
<td>None</td>
<td>=</td>
<td>8.0</td>
</tr>
<tr>
<td>B7</td>
<td>(Notched)</td>
<td>=</td>
<td>7.0</td>
</tr>
<tr>
<td>B9</td>
<td>Uniform</td>
<td>=</td>
<td>15.5</td>
</tr>
</tbody>
</table>

FIGURE 4.6 LOAD-DEFLECTION CURVES : SERIES B

<table>
<thead>
<tr>
<th>BEAM</th>
<th>GLUE</th>
<th>REINFORCEMENT</th>
<th>FAILURE LOAD KN</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>Uniform</td>
<td>Shear at supports</td>
<td>15.0</td>
</tr>
<tr>
<td>B4</td>
<td>=</td>
<td>(Lapped plates)</td>
<td>16.0</td>
</tr>
<tr>
<td>B6</td>
<td>None</td>
<td>Shear at supports</td>
<td>7.9</td>
</tr>
<tr>
<td>B8</td>
<td>(Notched)</td>
<td>=</td>
<td>7.9</td>
</tr>
<tr>
<td>B10</td>
<td>Notched</td>
<td>=</td>
<td>15.5</td>
</tr>
</tbody>
</table>
FIGURE 4.7 TYPICAL STRAIN DISTRIBUTIONS
linear at low loads, became increasingly non-linear at higher loads due to cracking. Within the limits of experimentation, Fig. 4.7 confirms that the assumption that plane sections remain plane is valid for plated beams as well. As expected, the strain distributions showed a movement of the neutral axis towards the compression face as the loading approached failure.

The variation of strain in the steel plate at mid-span is shown in Figs. 4.8 and 4.9 for series A and Figs. 4.10 and 4.11 for series B. Bearing in mind that the steel plates used for the series A and B are of low yield strength (125 N/mm²), Figs. 4.8 - 4.11 show that the plates were well past the yielding stage, at failure, in all the beams tested in the two series. The steel strains in beams A2, A4 and A6 showed trends similar to the deflection curves shown in Fig. 4.4 and confirm the behaviour discussed earlier. In the series B, the beam B10 with notches on the tension side showed better behaviour than beams B2 and B4 (cf. Fig. 4.6).

4.2.2 Modes of Failure

The first crack in all the beams occurred in the concrete in the tension zone above the glue and the plate. With increase in load, additional cracks formed but only one major crack extended into the compression zone. Because of the lack of internal tensile reinforcement, no extensive tensile concrete cracking was observed. Generally two or three cracks formed in the shorter beams of series A, whereas in the longer beams of series B, only one or two cracks were in evidence; in all cases only one major crack developed leading to a simple tensile failure.

All the beams, except beam A3, failed by tensile yielding of the steel sheet followed by vertical (or nearly vertical) crack propagation towards the central loading point (Plates 4.1 and 4.2). Failure in all the beams was initiated by vertical cracking of the glue layer at a position coincident with the largest concrete crack. Yielding then commenced and local debonding of the plate occurred in the vicinity of the failure plane and the concrete crack propagated up to the load point leading to failure.

In the shorter beams the failure plane sometimes developed along a shear
$E_s = 190 \text{ kN/m}^2$

$F_y = 125 \text{ N/mm}^2$

$e_y = 650 \text{ microstrain}$

**Figure 4.8** Load-strain curves: Series A

**Figure 4.9** Load-strain curves: Series A
crack (due to short shear span), whereas in the longer beams, the cracks were generally vertical. Extensive concrete crushing never occurred, and although the strains, recorded at about 80-95% of the ultimate load, were only about $600-800 \times 10^{-6}$ m/m, severe strain concentration and signs of crushing occurred locally above the failure crack.

Beam A3 showed extensive debonding of the plate (Plate 4.1), and this was found to be due to the presence of large air voids in the glue line, which had reduced the bonded cross-section by about 50%. However, the beam failed at 94% of the ultimate load for a similar correctly bonded beam.

4.2.3 First Crack and Ultimate Loads

Tables 4.3 and 4.4 show the theoretical and experimental first crack and ultimate loads of all the beams tested in series A and B respectively.

The experimental first crack loads relate to the visually observed cracks both in concrete and in the glue. The latter is necessarily more approximate as the observation is confined to a thin strip equal to the glue thickness. The theoretical first crack load in concrete is based on an uncracked transformed (composite) section with an $E$ value for the glue of 6 kN/mm$^2$. The theoretical first crack load in the glue is based on a cracked transformed (composite) section, assuming a tensile strength of 60 N/mm$^2$ in the glue. These values were as supplied by the manufacturer. After the preliminary test series was complete further tests were performed on the glue as described in Chapter 3. Tables 4.5 and 4.6 show the corresponding values to Tables 4.3 and 4.4, but using the values of $E = 2$ kN/mm$^2$ and tensile strength of 16 N/mm$^2$ as determined experimentally. Sample calculations for first crack and failure loads are given in Appendix 3.

The results show that the theoretically predicted first crack loads in the concrete consistently underestimate the experimental loads. It is likely that the latter is slightly overestimated as they are based on visual examination, but even then the composite effect on delaying the formation of the first crack in the concrete is clear. The theoretical first crack loads in the glue show good agreement with experimental values when assuming 60 N/mm$^2$ for the glue's
uniform glue (A) thickness, no shear reinforcement.

uniform glue (A) thickness, shear reinforcement at supports.

tapering glue (A) thickness, shear reinforcement at supports.

PLATE 4.1 TYPICAL BEAMS AFTER FAILURE: SERIES A
uniform glue (A) thickness,

uniform glue (A) thickness, lapped plates.

glue (A) thickness varies, notched beam.

PLATE 4 2 TYPICAL BEAMS AFTER FAILURE : SERIES B
<table>
<thead>
<tr>
<th>BEAM MK. Na</th>
<th>CONCRETE FIRST CRACK MOMENT</th>
<th>GLUE FIRST CRACK MOMENT</th>
<th>FAILURE MOMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>THEORY</td>
<td>TEST</td>
<td>RATIO</td>
</tr>
<tr>
<td></td>
<td>kNm</td>
<td>kNm</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>3.06</td>
<td>4.45</td>
<td>1.45</td>
</tr>
<tr>
<td>A2</td>
<td>3.06</td>
<td>4.81</td>
<td>1.57</td>
</tr>
<tr>
<td>A3</td>
<td>3.06</td>
<td>4.00</td>
<td>1.29</td>
</tr>
<tr>
<td>A4</td>
<td>3.06</td>
<td>5.00</td>
<td>1.63</td>
</tr>
<tr>
<td>A5</td>
<td>3.06</td>
<td>4.90</td>
<td>1.58</td>
</tr>
<tr>
<td>A6</td>
<td>3.06</td>
<td>4.27</td>
<td>1.37</td>
</tr>
<tr>
<td>A7</td>
<td>-</td>
<td>2.02</td>
<td>-</td>
</tr>
<tr>
<td>A8</td>
<td>-</td>
<td>2.82</td>
<td>-</td>
</tr>
</tbody>
</table>

a - based on uncracked transformed section, $E_{\text{glue}} = 6000 \, \text{N/mm}^2$

b - based on cracked transformed section, tensile strength of glue = 60 \, \text{N/mm}^2

c - based on yield stress of plate, plus tensile strength of glue.
d - based on ultimate stress of plate, plus tensile strength of glue.
e - based on ultimate stress of plate, no tensile strength of glue.
### TABLE 4-4  TEST RESULTS FOR PLAIN CONCRETE BEAMS : SERIES B

<table>
<thead>
<tr>
<th>BEAM MK No.</th>
<th>CONCRETE FIRST CRACK MOMENTS</th>
<th>GLUE FIRST CRACK MOMENTS</th>
<th>FAILURE MOMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>THEORY a</td>
<td>TEST</td>
<td>RATIO/TEST/ THEORY</td>
</tr>
<tr>
<td></td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
</tr>
<tr>
<td>B1</td>
<td>2.20</td>
<td>3.58</td>
<td>1.63</td>
</tr>
<tr>
<td>B2</td>
<td>2.20</td>
<td>3.03</td>
<td>1.38</td>
</tr>
<tr>
<td>B3</td>
<td>2.20</td>
<td>4.97</td>
<td>2.26</td>
</tr>
<tr>
<td>B4</td>
<td>2.20</td>
<td>3.30</td>
<td>1.50</td>
</tr>
<tr>
<td>B5</td>
<td>-</td>
<td>1.84</td>
<td>-</td>
</tr>
<tr>
<td>B6</td>
<td>-</td>
<td>2.16</td>
<td>-</td>
</tr>
<tr>
<td>B7</td>
<td>-</td>
<td>1.61</td>
<td>-</td>
</tr>
<tr>
<td>B8</td>
<td>-</td>
<td>2.10</td>
<td>-</td>
</tr>
<tr>
<td>B9</td>
<td>2.20</td>
<td>4.30</td>
<td>1.96</td>
</tr>
<tr>
<td>B10</td>
<td>2.20</td>
<td>3.30</td>
<td>1.50</td>
</tr>
</tbody>
</table>

- a - based on uncracked transformed section, \( E_{\text{glue}} = 6000 \text{N/mm}^2 \)
- b - based on cracked transformed section, tensile strength of glue\(^*\) = 60 \text{N/mm}^2
- c - based on yield stress of plate, plus tensile strength of glue.
- d - based on ultimate stress of plate, plus tensile strength of glue.
- e - based on ultimate stress of plate, no tensile strength in glue.
### TABLE 4.5 TEST RESULTS FOR PLAIN CONCRETE BEAMS: SERIES A

<table>
<thead>
<tr>
<th>BEAM MK. No.</th>
<th>GLUE CRACK MOMENTS</th>
<th>FIRST</th>
<th>FAILURE MOMENTS</th>
<th>RATIO</th>
<th>THEORY / TEST</th>
<th>THEORY</th>
<th>THEORY</th>
<th>TEST</th>
<th>RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>THEORY a</td>
<td>TEST</td>
<td>RATIO</td>
<td>THEORY c</td>
<td>THEORY</td>
<td>THEORY d</td>
<td>THEORY e</td>
<td>TEST</td>
<td>THEORY</td>
</tr>
<tr>
<td></td>
<td>kNm</td>
<td>kNm</td>
<td>c</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
</tr>
<tr>
<td>A1</td>
<td>1.40</td>
<td>5.15</td>
<td>3.68</td>
<td>2.91</td>
<td>3.02</td>
<td>2.05</td>
<td>5.71</td>
<td>1.96</td>
<td>1.89</td>
</tr>
<tr>
<td>A2</td>
<td>1.40</td>
<td>5.32</td>
<td>3.80</td>
<td>3.04</td>
<td>3.15</td>
<td>2.05</td>
<td>6.43</td>
<td>2.12</td>
<td>2.04</td>
</tr>
<tr>
<td>A3</td>
<td>1.40</td>
<td>4.24</td>
<td>3.02</td>
<td>2.78</td>
<td>2.89</td>
<td>2.04</td>
<td>5.34</td>
<td>1.92</td>
<td>1.85</td>
</tr>
<tr>
<td>A4</td>
<td>1.40</td>
<td>5.32</td>
<td>3.80</td>
<td>3.17</td>
<td>3.28</td>
<td>2.07</td>
<td>6.87</td>
<td>2.17</td>
<td>2.09</td>
</tr>
<tr>
<td>A5</td>
<td>1.40</td>
<td>4.81</td>
<td>3.44</td>
<td>2.85</td>
<td>2.97</td>
<td>2.05</td>
<td>5.60</td>
<td>1.96</td>
<td>1.89</td>
</tr>
<tr>
<td>A6</td>
<td>1.40</td>
<td>4.56</td>
<td>3.26</td>
<td>2.91</td>
<td>3.02</td>
<td>2.05</td>
<td>5.95</td>
<td>2.04</td>
<td>1.97</td>
</tr>
<tr>
<td>A7 &amp; 8</td>
<td>as on table 4.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 4.6 TEST RESULTS FOR PLAIN CONCRETE BEAMS: SERIES B

<table>
<thead>
<tr>
<th>BEAM MK. No.</th>
<th>GLUE CRACK MOMENTS</th>
<th>FIRST</th>
<th>FAILURE MOMENTS</th>
<th>RATIO</th>
<th>THEORY / TEST</th>
<th>THEORY</th>
<th>THEORY</th>
<th>TEST</th>
<th>RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>THEORY a</td>
<td>TEST</td>
<td>RATIO</td>
<td>THEORY c</td>
<td>THEORY</td>
<td>THEORY d</td>
<td>THEORY e</td>
<td>TEST</td>
<td>THEORY</td>
</tr>
<tr>
<td></td>
<td>kNm</td>
<td>kNm</td>
<td>c</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
<td>kNm</td>
</tr>
<tr>
<td>B1</td>
<td>1.10</td>
<td>4.18</td>
<td>3.80</td>
<td>2.09</td>
<td>2.17</td>
<td>1.53</td>
<td>4.74</td>
<td>2.27</td>
<td>2.18</td>
</tr>
<tr>
<td>B2</td>
<td>1.10</td>
<td>3.30</td>
<td>3.00</td>
<td>1.99</td>
<td>2.07</td>
<td>1.53</td>
<td>4.13</td>
<td>2.08</td>
<td>2.00</td>
</tr>
<tr>
<td>B3</td>
<td>1.10</td>
<td>4.18</td>
<td>3.80</td>
<td>2.38</td>
<td>2.46</td>
<td>1.54</td>
<td>5.73</td>
<td>2.41</td>
<td>2.33</td>
</tr>
<tr>
<td>B4</td>
<td>1.10</td>
<td>3.30</td>
<td>3.00</td>
<td>1.99</td>
<td>2.07</td>
<td>1.52</td>
<td>4.57</td>
<td>2.30</td>
<td>2.21</td>
</tr>
<tr>
<td>B5 - B8</td>
<td>as on table 4.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B9</td>
<td>1.10</td>
<td>3.56</td>
<td>3.24</td>
<td>2.09</td>
<td>2.17</td>
<td>1.54</td>
<td>4.70</td>
<td>2.25</td>
<td>2.17</td>
</tr>
<tr>
<td>B10</td>
<td>1.10</td>
<td>3.98</td>
<td>3.61</td>
<td>1.99</td>
<td>2.07</td>
<td>1.52</td>
<td>4.26</td>
<td>2.14</td>
<td>2.06</td>
</tr>
</tbody>
</table>

b - based on cracked transformed section, E glue 2000 N/mm², tensile strength glue 16 N/mm².
c - based on yield stress of plate, plus tensile strength of glue (16 N/mm²).
d - based on ultimate stress of plate.
e - no tensile strength in glue.
tensile strength but when assuming 16 N/mm² the theoretical loads are below the first crack load in the concrete which is not possible!

The theoretical ultimate failure loads shown in Tables 4.3 and 4.4 are based on a rectangular stress block for the concrete in compression with a stress value of 0.6 $f_{cu}$. Three stress distributions in the tension zone are considered as detailed in Appendix 3. The results suggest that the computations based on the ultimate strength of the plate and including the force in the glue give results closest to experimental values. It appears that although the plates did not fracture, considerable straining had taken place at crack positions leading to large interface shear strains and strain hardening of the steel. This, together with the spread of yield of the plate at each side of a crack, leads to local debonding.

The theoretical first crack and failure loads given in Tables 4.5 and 4.6 show poor agreement with experimental values. It would appear that although the glue tests gave a tensile strength of 16 N/mm², the actual tensile properties are different when the glue is acting compositely with the steel plate and concrete beam. The experimental failure moments were all almost twice the theoretical values assuming 16 N/mm² for the glue tensile strength (Tables 4.5 and 4.6).

4.3 CONCLUSIONS

From the results reported in this chapter, the following conclusions can be drawn. It is emphasised that these conclusions are limited to the variables studied here.

The use of external reinforcement in the form of steel plates glued to the tension face of plain concrete beams has the following effects:

(a) it increases the range of elastic behaviour

(b) for a given load, it reduces the tensile strains in the concrete, due to the composite action of the concrete, glue and steel plate, compared to those in unplated beams

(c) it delays the appearance of the first visual cracks with a resulting increase in service loads
(d) it increases flexural stiffness throughout loading and thus reduces deflections at corresponding loads

(e) it enhances the ultimate flexural capacity

(f) it increases the ductility at failure.

For a constant plate area, the stiffness of the beams increased as glue thickness increased. Lapping plates increased stiffness compared to a corresponding beam with a continuous plate, probably due to the increase in lever arm. Strain measurements showed that plane sections remained plane throughout loading, above the neutral axis. Below the neutral axis it was linear with some beams and irregular with others. This is because once the beam has cracked, the strain gauge reading is not the true strain, but is an average strain which depends on the position of the cracks.

For the variables studied in this series it was found that in plain concrete beams, with reinforcing plates of low yield strength, the glue makes a significant contribution to the ultimate strength of the composite section. The values of first crack load in the glue and the ultimate load, based on experimentally determined glue properties, were far below those observed in the tests. This suggests that the glue exhibits properties, when acting compositely with the steel and concrete, different from those found by testing samples of glue in unrestrained tests. However, further tests would be needed to confirm this.

The results indicate that the glue cannot be cracked at failure because if it were then the theoretical failure loads would be approximately half of the values found by experiment. There was, however, some evidence of surface cracking in the glue approaching failure.

It was interesting to find that even when large voids were present in the glue line, up to 50% of the width at the critical section in beam A3, the beam was able to sustain 94% of the load achieved by a similar beam with no voids in the glue.

Experience with the preparation of the steel and concrete surfaces gained from these tests emphasises the need for care at all stages. With plates only
1.5 mm thick it was essential to grit blast both sides of the plates to prevent warping. Applying the glue to both concrete and plate minimises voids at the interfaces and confines them to the body of the glue where they are less critical.
CHAPTER 5

STRENGTH PROPERTIES

5.1 INTRODUCTION

The performance of a structure is determined by the behaviour of its component members which in turn depend on the properties of the materials and the methods adopted for their design.

Plain concrete has low tensile strength and little resistance to crack propagation. Flaws or microcracks develop in the material during manufacture, even before any external load is applied, due to inherent volumetric and microstructural changes. The poor tensile strength is due to the enlargement and propagation of the internal flaws which lead to brittle fracture on loading.

In order to use concrete in a load-bearing element, it is necessary to impart tensile resistance properties to it. The use of reinforcing bars provides tensile strength in a structural member but does not increase the tensile strength of the concrete itself. For this reason it has become practice, since the establishment of reinforced concrete design techniques, to ignore the tensile strength of the concrete when estimating the flexural strength of a member.

The present trend towards using high strength materials, refined design techniques and slender members produces structures in which the serviceability conditions may be more critical than strength considerations. Codes of practice recommend that the width of surface cracks, and magnitude of deflections at service loads should not exceed certain limits, which are based on criteria such as corrosion, aesthetics or damage to non-structural elements.

The preliminary test series showed the effects of external reinforcement on plain concrete beams. It should therefore follow, that the addition of a bonded steel plate to the tensile face of a reinforced concrete member should result in:

(a) producing higher cracking loads
(b) a more even distribution of cracking
(c) a reduced crack propagation
(d) reduced deformations, i.e. rotation, deflection, strains etc. throughout the loading, up to failure.
This part of the investigation compares the experimental first crack, service and ultimate loads with those calculated by accepted methods. The increase in service load of the plated beams, above that of the unplated beam, is studied, using as a basis the different criteria of deflections, rotations, crack width and steel bar strains.

5.2 EXPERIMENTAL PROGRAMME

Previous work (54-73) has shown external bonding of steel plates onto reinforced concrete beams to be effective in controlling deformations and increasing post cracking stiffness of flexural members.

Twenty four beams were tested in this series. The main variables were glue thickness; plate thickness and lapping techniques. Details of the beams are given in Table 5.1 and the properties of the materials used were described in Chapter 4.

Three glue thicknesses were used, beams 203 to 206 had a 1.5 mm thick adhesive layer; beams 207 to 215 had a 3 mm thick adhesive layer and beams 216 to 219 had a 6 mm thick layer. For each thickness of glue three thicknesses of plate were used; beams 203, 207, 211, 212 and 216 had 1.5 mm thick plate; beams 204, 208, 215 and 217 had 3 mm thick plate; and beams 205, 209, 210, 218 and 219 had 6 mm thick plate.

Beams 206, 213 and 214 had two layers of 1.5 mm thick plate for comparison with beams with a single layer of 3 mm plate.

Beams 211, 212, 213, 214 and 215 had laps in their plates for comparison with beams having continuous layers of plate.

Beams 222 to 224 were preloaded and cracked before their plates were bonded on.

Beam 220 had an adhesive layer of variable thickness, 3 mm to 8 mm, along its length.

Beam 221 had 'V' notches cut in its tension face, at the loading points, to produce points of stress concentration.

Beam 202 had an adhesive layer 3 mm thick, but without a plate, to investigate cracking of the glue.
Beam 201 was unreinforced externally, having neither glue nor plate and was used for comparison.

The beams were all identical in size: 155 mm wide, 255 mm deep and 2.5 m long. All beams were tested under four point bending on a span of 2.3 m.

Stirrups, 6 mm diameter at 75 mm centres, were provided in the shear spans to prevent shear failure. The beam details and instrumentation are shown in Figs. 5.1 and 5.2.

5.3 TEST PROCEDURE

The beams were manufactured from the same materials and in the same manner as in the preliminary test series described in Chapter 4, with three control cubes for compressive strength and three 100 x 100 x 500 mm prisms for modulus of rupture testing, with each beam.

The beams and control specimens were stripped after 24 hours and cured in uncontrolled laboratory conditions.

After approximately fourteen days the beams were prepared for bonding as described in Chapter 4. The plates were degreased with trichlorethylene 24 hours prior to shotblasting. Because of the large size of the plates they had to be taken to a commercial shotblasting company where they were abraded with steel grit of 340 micron mean size, and then returned to the laboratory immediately for bonding to prevent surface corrosion and contamination. The mixing and bonding were carried out as described in Chapter 4. A minimum of 14 days was required between plating and testing.

Electrical resistance strain gauges of 7 mm gauge length were glued to the longitudinal bar reinforcement at three locations as shown in Fig. 5.2, one day prior to casting. Electrical resistance strain gauges of 6 mm gauge length were glued to the external reinforcing plates, after being bonded onto the beam, at locations as shown in Fig. 5.2.

The day prior to testing the beam was whitewashed to facilitate crack viewing. Demec discs were used to locate a mechanical extensometer on a base of 200 mm along the side of the beam at the centre section. The concrete surface was roughened and cleaned with acetone prior to gluing on the discs with 'Plastic
### TABLE 5.4: DETAILS OF TEST BEAMS

| BEAM NUMBER | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Glue thickness mm. | -   | 3   | 1.5 | 1.5 | 1.5 | 3   | 3   | 3   | 3   | 3   | 3   | 3   | 3   | 3   | 3   | 3   | 6   | 6   | 6   | 6   | 2-8 | 3   | 3   | 3   | 3   |
| Plate thickness mm. | -   | -   | 1.5 | 3   | 6   | 1.5 | 1.5 | 3   | 6   | 6   | 1.5 | 1.5 | 3   | 6   | 6   | 1.5 | 1.5 | 1.5 | 3   | 6   | 6   | 1.5 | 1.5 | 1.5 | 3   |
| Number of plate layers | -   | -   | 1   | 1   | 1   | 1   | 2   | 2   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   |
| Position of plate laps(3) | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   |
| % load before plating | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   |
| Cube strength N/mm² | 63.9 | 70.9 | 63.4 | 66.4 | 70.2 | 68.6 | 70.0 | 68.0 | 72.0 | 70.3 | 72.0 | 63.9 | 67.2 | 69.2 | 71.5 | 68.4 | 68.6 | 71.5 | 73.0 | 68.2 | 65.9 | 65.2 | 72.0 | 71.1 |
| Modulus of rupture N/mm² | 5.47 | 5.61 | 5.48 | 5.50 | 5.52 | 5.56 | 5.50 | 5.51 | 5.51 | 5.45 | 5.48 | 5.51 | 5.60 | 5.52 | 5.50 | 5.60 | 5.52 | 5.50 | 5.48 | 5.49 | 5.46 | 5.65 | 5.60 |
| Area of bars(4) mm² | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 | 943 |
| Area of plate(5) mm² | -   | -   | 187 | 375 | 750 | 375 | 187 | 375 | 750 | 187 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 | 375 |

(1) V Notches cut in the tension face of the beam to form stress concentrations above the load points.
(2) The second layer of plate was stopped 380 mm from the ends of the first layer.
(3) c - denotes central plate lap, lp - denotes laps above the load points.
(4) TORBAR tensile characteristic strength 425 N/mm²
(5) Plate = 250 N/mm²
FIGURE 5-1  LOADING RIG AND MECHANICAL INSTRUMENTATION
(a) REINFORCEMENT DETAILS

SIDES VIEW
OF BEAM

(b) DEMEC STUD LOCATIONS

PLAN VIEW ON TENSION FACE

(c) ELECTRICAL RESISTANCE STRAIN GAUGE LOCATIONS

FIGURE 5.2 DETAILS OF REINFORCEMENT AND STRAIN GAUGE LOCATIONS

Legend
- D demec stud
× SG electrical resistance gauge-bars
- SG plate

beams 202, 206, 210 & 219, location of strain gauges
strain gauges location, all other beams

glued steel plate external reinforcement

155

767

766

1100

2300

255

6mm. dia. at 75 mm c.c
12 no. each end

12.5
Padding'. In the first beam demec discs were stuck to both sides of the beam at midspan, but as the results showed the readings on each face to be consistent, in subsequent beams the discs were glued on one side only.

For measuring rotations, ball bearings resting on steel nuts were glued, 150 mm apart at the support and loading points. An inclinometer with one second divisions was used.

Deflections were measured at midspan and support with dial gauges having 0.01 mm divisions. On the plated beams the relative vertical movement of the plate to the beam was also measured. The location of dial gauges is shown in Fig. 5.1.

The beams were tested with their tension face uppermost to facilitate crack observation and measurements. Load was applied upwards at positions 383 mm on either side of midspan by means of a stiffened, 150 mm deep, wide flange spreader beam resting on a 50 ton capacity hand operated hydraulic jack at its mid point. The load was measured by a linear differential voltmeter and a calibrated load cell placed between the jack and the spreader beam. The ends of the beam were positioned between pairs of 35 mm diameter Macalloy bars up to crossheads that were secured at the top. One end was a fixed support and the other moving on rollers longitudinally. Both supports allowed rotation. The test rig is shown in Plate 5.1.

Load was applied gradually until the first crack appeared, which was detected visually using a magnifying glass. Load was then applied in increments up to failure. Each increment was approximately 12½% of the loading range. At each load stage the deflection; rotations; steel reinforcing bar and plate strains; concrete strains; crack width, spacing and height were measured and recorded.

The beams were loaded to failure in order to observe the mode of rupture of the plate/glue/concrete composite system. After failure the cracks were outlined by thick black marking pen and the beams were then photographed as shown in Plates 5.2 to 5.8.
a - test beam
b - spreader beam
c - loading jack
d - hand pump
e - tie rods
f - crosshead
g - strain gauge apparatus
h - digital voltmeter
UNPLATED BEAM - no glue or plate.

3mm glue thickness, no plate.

3mm glue thickness, 1.5mm plate thickness.

3mm glue thickness, 1.5mm plate thickness, centre plate lap.

3mm glue thickness, 1.5mm plate thickness, lapped plates above the load points.

PLATE 5.2 CRACK PATTERNS - PLATED AND UNPLATED BEAMS
15mm plate thickness.

3mm plate thickness.

6mm plate thickness.

2 layers of 1.5mm plate.

PLATE 5.3 CRACK PATTERNS - BEAMS WITH 1.5mm GLUE THICKNESS.
1.5 mm plate thickness.

3 mm plate thickness.

6 mm plate thickness.

6 mm plate thickness.

PLATE 5.4 CRACK PATTERNS - BEAMS WITH 3 mm GLUE THICKNESS.
3mm plate thickness.

2 layers of 1.5mm plate, centre plate lap.

2 layers of 1.5mm plate, lapped plates above the load points.
1.5 mm plate thickness

3 mm plate thickness.

6 mm plate thickness.

PLATE 5.6 CRACK PATTERNS - BEAMS WITH 6 mm GLUE THICKNESS.
1.5 mm glue thickness

3 mm glue thickness

6 mm glue thickness

2mm-8mm variable glue thickness along the beam.

3mm glue thickness with notches cut in the beam, to form stress concentrations above the load points.

PLATE 57 CRACK PATTERNS - 1.5mm PLATE THICKNESS.
3mm glue thickness, 1.5mm plate thickness

3mm glue thickness, 1.5mm plate thickness, beam loaded to 50% ultimate load before bonding the plate on.

3mm glue thickness, 1.5mm plate thickness, beam loaded to 90% ultimate load before bonding the plate on.

3mm glue thickness, 3mm plate thickness

3mm glue thickness, 3mm plate thickness, beam loaded to 50% ultimate load before bonding the plate on.

PLATE 5 8 CRACK PATTERNS - PRELOADED BEAMS.
5.4 DISCUSSION OF RESULTS

5.4.1 First Crack Loads

The first crack loads for the concrete, at the tension face of the beam, were calculated based on the uncracked, transformed moment of inertia and using the value of modulus of rupture of the concrete from the test prisms. The experimental values, given in Table 5.2, were observed with a magnifying glass. For the plated beams the experimental load was, on average, 1.38 times the theoretical value. The same ratio for the unplated beam was 1.12. The experimental values being obtained visually are not accurate and the load would be recorded after cracking had initiated. However, the restraining effect of the glue and plate on the increase of crack width is apparent from the two ratios given above. Fig. 5.3 shows the variation of first crack load with the plate and glue thickness. It can be seen that there was little effect on the cracking load for the range of glue and plate thicknesses used. However it is clear that the cracking load is well above the value for the unplated beam. The CP110 service load was calculated from the CP110 failure load, with $\gamma_m = 1.15$ for steel and 1.5 for concrete. The ratio of experimental first crack load to CP110 service load was 0.35 for the unplated beam. The mean values for beams with 1.5 mm, 3 mm and 6 mm thick plates were 0.45, 0.37 and 0.32 respectively. (Table 5.2)

5.4.2 Increase of Service Loads

The deflection, rotation, maximum crack width and strain in the reinforcing bars at the centre section were found for the unplated beam at its CP110 service load; 100 kN. This being defined as the ultimate load, as found by CP110 recommendations, divided by 1.6. The properties described above were as follows:

(a) deflection 4.6 mm

(b) rotation $113 \times 10^{-4}$ radians

(c) maximum crack width 0.09 mm

(d) steel bar strain $1020 \times 10^{-6}$ m/m.

For each plated beam the corresponding loads which produced the same respective deflection, rotation, crack width and strain were found from the experimental results. All these values are given in Table 5.3.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>EXPERIMENT FIRST CRACK LOAD kN</th>
<th>CP 110 SERVICE LOAD kN</th>
<th>(1) L 35 (2)</th>
<th>EXPERIMENT SERVICE LOAD kN</th>
<th>EXPERIMENT ULTIMATE LOAD kN</th>
<th>(3) (2)</th>
<th>(4) (2)</th>
<th>GLUE LINE THICKNESS mm</th>
<th>PLATE THICKNESS mm</th>
<th>THREADS</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>35.0</td>
<td>100</td>
<td>0.35</td>
<td>145</td>
<td>232</td>
<td>1.45</td>
<td>2.32</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>202</td>
<td>39.9</td>
<td>102</td>
<td>0.39</td>
<td>153</td>
<td>245</td>
<td>1.50</td>
<td>2.40</td>
<td>3.0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>203</td>
<td>50.1</td>
<td>116</td>
<td>0.43</td>
<td>169</td>
<td>270</td>
<td>1.46</td>
<td>2.33</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>204</td>
<td>50.1</td>
<td>134</td>
<td>0.37</td>
<td>169</td>
<td>270</td>
<td>1.26</td>
<td>2.01</td>
<td>1.5</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>205</td>
<td>54.8</td>
<td>166</td>
<td>0.33</td>
<td>133</td>
<td>213</td>
<td>0.80</td>
<td>1.28</td>
<td>1.5</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>206</td>
<td>55.0</td>
<td>135</td>
<td>0.41</td>
<td>138</td>
<td>220</td>
<td>1.02</td>
<td>1.63</td>
<td>1.5</td>
<td>2x15</td>
<td></td>
</tr>
<tr>
<td>207</td>
<td>55.0</td>
<td>119</td>
<td>0.46</td>
<td>153</td>
<td>262</td>
<td>1.38</td>
<td>2.20</td>
<td>3.0</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>208</td>
<td>49.0</td>
<td>135</td>
<td>0.36</td>
<td>155</td>
<td>264</td>
<td>1.22</td>
<td>1.96</td>
<td>3.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>209</td>
<td>52.4</td>
<td>167</td>
<td>0.31</td>
<td>138</td>
<td>220</td>
<td>0.83</td>
<td>1.31</td>
<td>3.0</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>50.1</td>
<td>166</td>
<td>0.30</td>
<td>134</td>
<td>215</td>
<td>0.81</td>
<td>1.30</td>
<td>3.0</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>211</td>
<td>54.8</td>
<td>119</td>
<td>0.46</td>
<td>158</td>
<td>253</td>
<td>1.33</td>
<td>2.13</td>
<td>3.0</td>
<td>15L</td>
<td></td>
</tr>
<tr>
<td>212</td>
<td>47.0</td>
<td>117</td>
<td>0.40</td>
<td>155</td>
<td>248</td>
<td>1.32</td>
<td>2.12</td>
<td>3.0</td>
<td>15L</td>
<td></td>
</tr>
<tr>
<td>213</td>
<td>50.1</td>
<td>135</td>
<td>0.37</td>
<td>158</td>
<td>253</td>
<td>1.17</td>
<td>1.87</td>
<td>3.0</td>
<td>2x15</td>
<td></td>
</tr>
<tr>
<td>214</td>
<td>50.1</td>
<td>135</td>
<td>0.37</td>
<td>158</td>
<td>253</td>
<td>1.17</td>
<td>1.87</td>
<td>3.0</td>
<td>2x15</td>
<td></td>
</tr>
<tr>
<td>215</td>
<td>49.8</td>
<td>134</td>
<td>0.37</td>
<td>156</td>
<td>250</td>
<td>1.16</td>
<td>1.87</td>
<td>3.0</td>
<td>30L</td>
<td></td>
</tr>
<tr>
<td>216</td>
<td>54.0</td>
<td>118</td>
<td>0.46</td>
<td>164</td>
<td>262</td>
<td>1.39</td>
<td>2.22</td>
<td>6.0</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>217</td>
<td>48.0</td>
<td>135</td>
<td>0.36</td>
<td>161</td>
<td>257</td>
<td>1.19</td>
<td>1.90</td>
<td>6.0</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>218</td>
<td>51.4</td>
<td>168</td>
<td>0.31</td>
<td>121</td>
<td>194</td>
<td>0.72</td>
<td>1.15</td>
<td>6.0</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>219</td>
<td>55.0</td>
<td>168</td>
<td>0.33</td>
<td>138</td>
<td>220</td>
<td>0.82</td>
<td>1.31</td>
<td>6.0</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>54.8</td>
<td>118</td>
<td>0.46</td>
<td>164</td>
<td>263</td>
<td>1.39</td>
<td>2.23</td>
<td>3-8</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>221</td>
<td>54.8</td>
<td>117</td>
<td>0.47</td>
<td>161</td>
<td>268</td>
<td>1.38</td>
<td>2.21</td>
<td>3.0</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>222</td>
<td>51.1</td>
<td>117</td>
<td>0.44</td>
<td>168</td>
<td>268</td>
<td>1.44</td>
<td>2.29</td>
<td>3.0</td>
<td>1.5P</td>
<td></td>
</tr>
<tr>
<td>223</td>
<td>53.0</td>
<td>118</td>
<td>0.45</td>
<td>165</td>
<td>264</td>
<td>1.40</td>
<td>2.24</td>
<td>3.0</td>
<td>1.5P</td>
<td></td>
</tr>
<tr>
<td>224</td>
<td>47.2</td>
<td>136</td>
<td>0.35</td>
<td>156</td>
<td>250</td>
<td>1.15</td>
<td>1.83</td>
<td>3.0</td>
<td>30P</td>
<td></td>
</tr>
</tbody>
</table>

L - lapped plates  P - precracked beams.
FIGURE 5.3 VARIATION OF FIRST CRACK LOAD

FIGURE 5.4 VARIATION OF EXPERIMENTAL SERVICE LOAD
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>(1) LOAD 1</th>
<th>(2) LOAD 2</th>
<th>(3) LOAD 3</th>
<th>(4) LOAD 4</th>
<th>(1) 100</th>
<th>(2) 100</th>
<th>(3) 100</th>
<th>(4) 100</th>
<th>GLUE mm. PLATE mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>202</td>
<td>101</td>
<td>102</td>
<td>106</td>
<td>102</td>
<td>1.01</td>
<td>1.02</td>
<td>1.06</td>
<td>1.02</td>
<td>3.00</td>
</tr>
<tr>
<td>203</td>
<td>104</td>
<td>106</td>
<td>144</td>
<td>120</td>
<td>1.04</td>
<td>1.06</td>
<td>1.44</td>
<td>1.20</td>
<td>1.50</td>
</tr>
<tr>
<td>204</td>
<td>110</td>
<td>124</td>
<td>153</td>
<td>151</td>
<td>1.10</td>
<td>1.24</td>
<td>1.53</td>
<td>1.51</td>
<td>1.50</td>
</tr>
<tr>
<td>205</td>
<td>131</td>
<td>138</td>
<td>170</td>
<td>171</td>
<td>1.31</td>
<td>1.38</td>
<td>1.70</td>
<td>1.71</td>
<td>1.50</td>
</tr>
<tr>
<td>206</td>
<td>113</td>
<td>123</td>
<td>158</td>
<td>155</td>
<td>1.13</td>
<td>1.23</td>
<td>1.58</td>
<td>1.55</td>
<td>1.50</td>
</tr>
<tr>
<td>207</td>
<td>111</td>
<td>106</td>
<td>153</td>
<td>116</td>
<td>1.11</td>
<td>1.06</td>
<td>1.53</td>
<td>1.16</td>
<td>3.00</td>
</tr>
<tr>
<td>208</td>
<td>126</td>
<td>134</td>
<td>160</td>
<td>150</td>
<td>1.26</td>
<td>1.34</td>
<td>1.60</td>
<td>1.50</td>
<td>3.00</td>
</tr>
<tr>
<td>209</td>
<td>130</td>
<td>144</td>
<td>184</td>
<td>210</td>
<td>1.30</td>
<td>1.44</td>
<td>1.84</td>
<td>2.10</td>
<td>3.00</td>
</tr>
<tr>
<td>210</td>
<td>133</td>
<td>146</td>
<td>199</td>
<td>170</td>
<td>1.33</td>
<td>1.46</td>
<td>1.99</td>
<td>1.70</td>
<td>3.00</td>
</tr>
<tr>
<td>211</td>
<td>111</td>
<td>117</td>
<td>145</td>
<td>130</td>
<td>1.11</td>
<td>1.17</td>
<td>1.45</td>
<td>1.30</td>
<td>3.00</td>
</tr>
<tr>
<td>212</td>
<td>111</td>
<td>114</td>
<td>137</td>
<td>130</td>
<td>1.11</td>
<td>1.14</td>
<td>1.37</td>
<td>1.30</td>
<td>3.00</td>
</tr>
<tr>
<td>213</td>
<td>110</td>
<td>130</td>
<td>138</td>
<td>154</td>
<td>1.10</td>
<td>1.30</td>
<td>1.38</td>
<td>1.54</td>
<td>3.00</td>
</tr>
<tr>
<td>214</td>
<td>119</td>
<td>137</td>
<td>142</td>
<td>153</td>
<td>1.19</td>
<td>1.37</td>
<td>1.42</td>
<td>1.53</td>
<td>3.00</td>
</tr>
<tr>
<td>215</td>
<td>120</td>
<td>133</td>
<td>158</td>
<td>168</td>
<td>1.20</td>
<td>1.33</td>
<td>1.58</td>
<td>1.68</td>
<td>3.00</td>
</tr>
<tr>
<td>216</td>
<td>111</td>
<td>113</td>
<td>145</td>
<td>143</td>
<td>1.11</td>
<td>1.13</td>
<td>1.45</td>
<td>1.43</td>
<td>6.00</td>
</tr>
<tr>
<td>217</td>
<td>123</td>
<td>133</td>
<td>145</td>
<td>165</td>
<td>1.23</td>
<td>1.33</td>
<td>1.45</td>
<td>1.65</td>
<td>6.00</td>
</tr>
<tr>
<td>218</td>
<td>134</td>
<td>156</td>
<td>-</td>
<td>190</td>
<td>1.34</td>
<td>1.56</td>
<td>-</td>
<td>1.90</td>
<td>6.00</td>
</tr>
<tr>
<td>219</td>
<td>136</td>
<td>160</td>
<td>180</td>
<td>200</td>
<td>1.36</td>
<td>1.60</td>
<td>1.80</td>
<td>2.00</td>
<td>6.00</td>
</tr>
<tr>
<td>220</td>
<td>104</td>
<td>106</td>
<td>146</td>
<td>130</td>
<td>1.04</td>
<td>1.06</td>
<td>1.46</td>
<td>1.30</td>
<td>3.00</td>
</tr>
<tr>
<td>221</td>
<td>111</td>
<td>111</td>
<td>148</td>
<td>128</td>
<td>1.11</td>
<td>1.11</td>
<td>1.48</td>
<td>1.28</td>
<td>3.00</td>
</tr>
<tr>
<td>222</td>
<td>100</td>
<td>120</td>
<td>123</td>
<td>143</td>
<td>1.60</td>
<td>1.20</td>
<td>1.23</td>
<td>1.43</td>
<td>3.00</td>
</tr>
<tr>
<td>223</td>
<td>85</td>
<td>108</td>
<td>123</td>
<td>140</td>
<td>0.85</td>
<td>1.08</td>
<td>1.23</td>
<td>1.40</td>
<td>3.00</td>
</tr>
<tr>
<td>224</td>
<td>120</td>
<td>133</td>
<td>160</td>
<td>172</td>
<td>1.20</td>
<td>1.33</td>
<td>1.60</td>
<td>1.72</td>
<td>3.00</td>
</tr>
</tbody>
</table>

L- lapped plates, P- precracked beams. *-CP110 service load = unplated beam.
LOAD 1- load in plated beam corresponding to 4.6 mm deflection (unplated beam at 100 kN)
LOAD 2- = = = = 113x10°rad. rotation ( = = = = )
LOAD 3- = = = 0.09 mm crack width ( = = = = )
LOAD 4- = = = = 1020 microstrain ( = = = = )
5.4.2.1 Deflection

The plated beams were able to sustain higher loads than the unplated beam before reaching a deflection of 4.6 mm. The increase varied as follows: 7 to 11% for beams strengthened with 1.5 mm thick plate; 10 to 23% for beams strengthened with 3 mm thick plate and 30 to 37% for beams strengthened with 6 mm thick plate.

5.4.2.2 Rotation

The plated beams again were able to sustain higher loads before reaching a rotation of $113 \times 10^{-4}$ radians. The corresponding beams as for deflections gave the following increases: 10 to 17% for 1.5 mm thick plates; 25 to 36% for 3 mm thick plates and 38 to 42% for 6 mm thick plates.

5.4.2.3 Maximum Crack Width

When considering crack width the increases for the corresponding beams were as follows: 37 to 50% for 1.5 mm thick plates; 38 to 60% for 3 mm thick plates and 70 to 99% for 6 mm thick plates.

5.4.2.4 Steel Bar Strain

When considering the steel bar strains the corresponding increases were 16 to 43% for 1.5 mm thick plates; 50 to 68% for 3 mm thick plates and 70 to 110% for 6 mm thick plates.

In general it can be seen that the addition of the external reinforcing plates had the effect of increasing the limit states for serviceability.

Table 5.2 shows the ratio of experimental service load, (experimental ultimate load + 1.6), to CP110 service load. All these ratios are greater than 1.0, except for the beams with 6 mm thick plates, which did not reach their flexural capacity due to their mode of failure. The ratios for the beams with 1.5 mm and 3 mm thick plates varied from 1.02 to 1.46, with a mean value of 1.28. This value would be expected to be greater than 1.0 as CP110 includes material safety factors.

Fig. 5.4 shows the variation of experimental service load with glue and plate thickness. The beams with 1.5 and 3 mm thick plates show an increase in service load whereas those with 6 mm plate thickness show a decrease.
5.4.3 Ultimate Loads

Three methods were used for calculating the theoretical ultimate loads of the test beams.

(a) Ultimate Limit State to CP110.
(b) Strain Compatibility - glue cracked.
(c) Strain Compatibility - glue uncracked.

The general assumptions and examples of calculations are given in Appendix 4.

The increase in the ultimate flexural moment capacity over that of an unplated beam varied from 8 to 17% for beams with 1.5 mm and 3 mm thick plates. The beams with 6 mm thick reinforcing plates showed a decrease in ultimate moment capacity compared with the unplated beam varying from 5 to 16%. Plates 5.2 to 5.8 show the beams after failure.

5.4.3.1 Beams with 1.5 mm Plate Thickness

From Table 5.4 it can be seen that there was good agreement between the theoretical and experimental failure moments. The mean ratio of experimental to theoretical moments for the three methods of calculation, i.e. CP110, strain compatibility with no tensile contribution from the glue and strain compatibility including the tensile force in the glue, were 1.09, 1.06 and 1.05 respectively, with a coefficient of variation of 3% in each case. This ratio varies only marginally when the tensile force in the glue is included. In the preliminary test series the variation was very large due to the fact that the plate used had a low yield strength and was only 1 mm thick. The glue, therefore, was providing a large proportion of the tensile force in the beam, as there was no internal reinforcement. As shown in Appendix 4, the experimental stress strain curve for the glue was used when calculating the moments by strain compatibility. More testing needs to be done to determine the actual stress condition in the glue in the composite condition. The test on beam 202, with a 3 mm thick adhesive layer and no plate, showed that the glue was not cracked at failure. Beam 221, with V notches cut in the tension face to form stress concentrations had no significant differences from similar beams without these notches.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>CP 110 ULTIMATE MOMENT (kNm)</th>
<th>STRAIN Compatibility ULTIMATE MOMENT (kNm) (NO GLUE)</th>
<th>STRAIN Compatibility ULTIMATE MOMENT (kNm) (+ GLUE)</th>
<th>EXPERTMENT ULTIMATE MOMENT (kNm)</th>
<th>(4)</th>
<th>(4)</th>
<th>(4)</th>
<th>FAILURE MODE</th>
<th>GLUE THICKNESS (mm)</th>
<th>PLATE THICKNESS (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>81.2</td>
<td>83.7</td>
<td>83.7</td>
<td>88.9</td>
<td>1.09</td>
<td>1.06</td>
<td>1.06</td>
<td>FLEXURE</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>202</td>
<td>82.8</td>
<td>85.9</td>
<td>87.0</td>
<td>93.9</td>
<td>1.13</td>
<td>1.09</td>
<td>1.08</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>0</td>
</tr>
<tr>
<td>203</td>
<td>90.1</td>
<td>92.4</td>
<td>92.8</td>
<td>103.5</td>
<td>1.15</td>
<td>1.12</td>
<td>1.12</td>
<td>FLEXURE</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>204</td>
<td>100.1</td>
<td>102.5</td>
<td>102.9</td>
<td>103.5</td>
<td>1.03</td>
<td>1.01</td>
<td>1.01</td>
<td>FLEXURE/ SHERABOND</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>205</td>
<td>118.7</td>
<td>120.3</td>
<td>120.6</td>
<td>81.7</td>
<td>0.69</td>
<td>0.68</td>
<td>0.68</td>
<td>SHEAR BOND</td>
<td>1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>206</td>
<td>100.9</td>
<td>104.3</td>
<td>104.9</td>
<td>104.3</td>
<td>1.03</td>
<td>1.00</td>
<td>0.99</td>
<td>FLEXURE/ SHERABOND</td>
<td>1.5</td>
<td>24.5L</td>
</tr>
<tr>
<td>207</td>
<td>92.2</td>
<td>95.5</td>
<td>96.6</td>
<td>100.6</td>
<td>1.09</td>
<td>1.05</td>
<td>1.04</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>208</td>
<td>100.9</td>
<td>103.6</td>
<td>104.2</td>
<td>101.2</td>
<td>1.00</td>
<td>0.98</td>
<td>0.97</td>
<td>FLEXURE/ SHERABOND</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>209</td>
<td>119.9</td>
<td>122.1</td>
<td>122.4</td>
<td>84.3</td>
<td>0.70</td>
<td>0.69</td>
<td>0.69</td>
<td>SHEAR BOND</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>210</td>
<td>118.9</td>
<td>120.8</td>
<td>121.2</td>
<td>82.4</td>
<td>0.69</td>
<td>0.68</td>
<td>0.68</td>
<td>SHEAR BOND</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>211</td>
<td>92.7</td>
<td>96.3</td>
<td>97.2</td>
<td>97.3</td>
<td>1.05</td>
<td>1.01</td>
<td>1.00</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>15.0L</td>
</tr>
<tr>
<td>212</td>
<td>90.4</td>
<td>92.9</td>
<td>93.5</td>
<td>95.8</td>
<td>1.06</td>
<td>1.03</td>
<td>1.02</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>15.0L</td>
</tr>
<tr>
<td>213</td>
<td>101.0</td>
<td>103.4</td>
<td>104.0</td>
<td>96.9</td>
<td>0.96</td>
<td>0.94</td>
<td>0.93</td>
<td>FLEXURE/ SHERABOND</td>
<td>3.0</td>
<td>24.5L</td>
</tr>
<tr>
<td>214</td>
<td>101.3</td>
<td>104.2</td>
<td>104.9</td>
<td>97.0</td>
<td>0.96</td>
<td>0.93</td>
<td>0.92</td>
<td>FLEXURE/ SHERABOND</td>
<td>3.0</td>
<td>24.5L</td>
</tr>
<tr>
<td>215</td>
<td>102.0</td>
<td>105.2</td>
<td>106.0</td>
<td>95.8</td>
<td>0.94</td>
<td>0.91</td>
<td>0.90</td>
<td>FLEXURE/ SHERABOND</td>
<td>3.0</td>
<td>30.0L</td>
</tr>
<tr>
<td>216</td>
<td>91.9</td>
<td>95.1</td>
<td>96.7</td>
<td>100.6</td>
<td>1.09</td>
<td>1.06</td>
<td>1.04</td>
<td>FLEXURE</td>
<td>6.0</td>
<td>1.5</td>
</tr>
<tr>
<td>217</td>
<td>101.3</td>
<td>104.2</td>
<td>105.5</td>
<td>98.6</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
<td>FLEXURE/ SHERABOND</td>
<td>6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>218</td>
<td>120.2</td>
<td>122.3</td>
<td>122.7</td>
<td>74.5</td>
<td>0.62</td>
<td>0.61</td>
<td>0.60</td>
<td>SHEAR BOND</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>219</td>
<td>120.8</td>
<td>123.1</td>
<td>124.3</td>
<td>84.3</td>
<td>0.70</td>
<td>0.68</td>
<td>0.68</td>
<td>SHEAR BOND</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>220</td>
<td>91.7</td>
<td>95.0</td>
<td>96.3</td>
<td>100.8</td>
<td>1.10</td>
<td>1.06</td>
<td>1.05</td>
<td>FLEXURE</td>
<td>2.8</td>
<td>1.5</td>
</tr>
<tr>
<td>221</td>
<td>91.3</td>
<td>93.9</td>
<td>94.6</td>
<td>98.9</td>
<td>1.08</td>
<td>1.05</td>
<td>1.04</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>222</td>
<td>91.4</td>
<td>93.9</td>
<td>96.3</td>
<td>102.7</td>
<td>1.12</td>
<td>1.09</td>
<td>1.07</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>1.5P</td>
</tr>
<tr>
<td>223</td>
<td>92.8</td>
<td>96.3</td>
<td>96.9</td>
<td>101.2</td>
<td>1.09</td>
<td>1.05</td>
<td>1.04</td>
<td>FLEXURE</td>
<td>3.0</td>
<td>1.5P</td>
</tr>
<tr>
<td>224</td>
<td>102.0</td>
<td>105.1</td>
<td>105.8</td>
<td>95.8</td>
<td>0.94</td>
<td>0.91</td>
<td>0.91</td>
<td>FLEXURE/ SHERABOND</td>
<td>3.0</td>
<td>30.0P</td>
</tr>
</tbody>
</table>

| MEAN VALUES | 1.05 | 1.01 | 1.00 ( * not included) |
5.4.3.2 Beams with Two Layers of 1.5 mm Thick Plate or One Layer of 3 mm Thick Plate

From Table 5.4 it can be seen that there was good agreement between the theoretical and experimental failure moments. The mean ratio's, as before, were 0.98, 0.95 and 0.95 respectively, with a coefficient of variation of 4% in each case. There was no difference between the mean ratios of failure moments, as given above, for the beams with one layer of 3 mm plate or two layers of 1.5 mm plate.

5.4.3.3 Beams with 6 mm Thick Plate

From Table 5.4 the mean ratios of experimental to theoretical failure moments, as before, were 0.68, 0.67 and 0.67 with a coefficient of variation of 5% in each case. It is apparent, therefore, that it is not possible to increase the ultimate capacity of a beam beyond a certain point by simply adding thicker and thicker plates. Nevertheless, it should be noted that the thicker plates greatly enhance the deformational properties at service loads. In general the mean ratio found from CP110 calculations were higher than those from strain compatibility. This is due to the fact that the CP110 calculation uses the proof stress of the steel and takes no account of strain hardening beyond this point.

The ratios of experimental ultimate load to CP110 service load are given in Table 5.2. Except for the beams with 6 mm thick plates, the values ranged from 1.83 to 2.33 with a mean value of 2.08. This compares with 1.60 as recommended by CP110. This difference is largely due to the material safety factors included in the CP110 design method.

5.4.4 Modes of Failure

5.4.4.1 Beams with 1.5 mm Thick Plates

All beams with 1.5 mm thick reinforcing plates failed in a flexural mode by combined yielding of the tensile bar and plate reinforcement followed by crushing of the concrete in the compression zone. As the tensile reinforcement started to yield, the width of one or two cracks at or near the critical section, increased at a faster rate. With increase in load the adjacent cracks grew wider indicating a spread of yield in the steel bars and plate along the beam. The
increase in crack width was accompanied by a slow propagation of crack height towards the compression face. Finally the concrete crushed at a point between the loads. None of these beams showed any signs of the plate debonding from the glue and/or concrete, except after failure of the compression zone. The debonding then occurred in the constant moment region and was only apparent after unloading. This debonding was probably due to the large post-failure deflections, which would have caused very high strains in the plates.

5.4.4.2 Beams with Two Layers of 1.5 mm Plate or One Layer of 3 mm Thick Plate

These beams failed in a mode which was a combination of flexure and shear/bond. Flexure, however, must have been predominant as they achieve between 94 and 103% of their theoretical CP110 ultimate moment capacity. There was no sign of debonding between the glue and concrete or plate. The strain readings in the steel bars and plates indicated that yielding had occurred prior to failure.

A shear crack initiated at the end of the plate and propagated diagonally towards the concrete compression zone widening the crack at its root and eventually causing partial separation of the plate. No debonding occurred but the concrete cover to the internal bars was ripped away.

It was thought that this shear/bond type of failure could be prevented by reducing the area of plate at the ends. The method used to accomplish this was to use two layers of 1.5 mm thick plate, the first layer being glued along the full length as usual but the second layer was stopped halfway between the loading and support points. This beam, 206, therefore had 3 mm plate thickness across the constant bending moment region and 1.5 mm plate thickness at the ends. Its behaviour was compared with beams 213 and 214 which had two full length layers of plate 1.5 mm thick. Beam 206 reached 103% of its theoretical CP110 ultimate capacity compared with only 96% for beams 213 and 214, as given in Table 5.4. The mode of failure was still by shear/bond, however, the load-deflection and moment-rotation characteristics of beams 206 and 213 were almost identical. Further tests need to be done to investigate this technique.

5.4.4.3 Beams with 6 mm Plate Thickness

All these beams failed by shearing off the concrete along the level of the
internal reinforcement, effectively ripping off the concrete cover. There was no
sign of debonding of the glue and concrete or plate prior to failure, nor was
there any sign of cracking in the glue. The strain readings in the steel bars
and plate indicated that yielding had not occurred prior to failure.

Fig. 5.5 shows the variation of the failure load with the plate and glue
thickness, and their modes of failure. It is apparent that the beams strengthened
with 6 mm plates were not satisfactory from the ultimate load point of view.
However, their behaviour when considering deflections, rotations and cracking
characteristics at service loads are discussed in Chapter 6. Fig. 5.5 is replotted
in Fig. 5.6 with ordinates changed to experimental/theoretical failure load.
Values obtained by Cusens and Smith (28) are included for comparison.

5.5 CONCLUSIONS

Based on the results presented in this section the following conclusions
can be made.

1. The use of external reinforcement delays the appearance of the first
visual cracks with a resulting increase in service loads.

2. No debonding of the glue from the concrete or steel occurred prior to
failure, thus full composite action was achieved throughout loading.

3. No cracking was observed in the glue prior to failure, including the
beam which had a layer of adhesive without a plate. Further tests need to be
performed to determine the stress conditions in the glue layer when acting
compositely with steel and concrete.

4. The maximum increase in the ultimate strength of an unplated beam by
the addition of externally bonded steel plates was found to be only 17%.

5. The ultimate flexural capacity of the concrete beams with external
plate reinforcement could be satisfactorily predicted using all three methods
detailed in Appendix 4.

6. More research is needed to investigate methods of preventing shear/
bond failures. This could possibly be achieved by:

(a) variation of plate end geometry, either by reducing the width
or thickness
**Figure 5.5** Variation of Ultimate Load

**Figure 5.6** Effect of Adhesive Thickness

- Cusens and Smith (plate thickness 3.4 mm)
(b) variation of the glue's elastic modulus at the plate ends. A lower elastic modulus near the plate ends would reduce the shear stresses and should therefore delay the onset of the propagation of large shear cracks.

(c) provision of anchor bolts or straps at the plate ends.
CHAPTER 6
DEFORMATION PROPERTIES

6.1 INTRODUCTION

In the past, the allowable stresses for both steel and concrete were low and as a result reinforced concrete members were not severely cracked under service load conditions. Therefore, small deflections and rotations resulted due to their high stiffness.

The present trend towards the use of high strength steels and concrete, and the development of more accurate and sophisticated ultimate strength design procedures have made it possible to use slender members. However, there has been no corresponding increase in the elastic moduli of the construction materials. Although correctly maintaining an adequate factor of safety against collapse, this gives rise to a greater possibility of local damage due to increased deformations.

Limit design theories for statically indeterminate structures require a knowledge of the deformational capacity of hinging regions in members. The major advantage of including inelastic behaviour in a design method is that moment redistribution can be used. This means transferring some calculated moment at one position to another position in a member. If the calculated moment at a support is reduced then this means that the resistance moment at that section will be incapable of resisting the total moment it can get. So at this position the member will become plastic and yield. The amount of redistribution depends on the deformational capacity of the structure. Several theories of limit design have been proposed which are based either on a knowledge of the moment-curvature relationship, such as that proposed by Sawyer (87), or on a knowledge of the moment-rotation relationship, such as that proposed by Baker (88).

It is obvious that excessive deflection can cause structural problems and can also give rise to public concern. The prediction and control of deformation is of great importance but it is complicated by the many factors which influence
the behaviour of a member such as: the magnitude and distribution of loading; span and support conditions, materials and section properties; and the extent of cracking. Additional variables controlling long term deflections which are of primary importance are the effects of creep and shrinkage.

The object of this part of the investigation was to assess the effects of varying glue and plate thickness, lapping techniques, multiple plate layers and degree of precracking on the load-strain, load-deflection and moment-rotation characteristics.

The load-strain behaviour was observed for both the internal steel reinforcing bars and the externally bonded steel plates. In addition the concrete surface strain distribution was measured.

The use of accepted methods for predicting the deflections and rotation of normally reinforced concrete beams were assessed for the plated beams.

6.2 EXPERIMENTAL PROGRAMME/PROCEDURE

The twenty four beams described in Section 5.2 were used in this part of the investigation. The instrumentation, test apparatus and procedure are described in Section 5.3. The strains in the steel bars and plate were measured by electrical resistance strain gauges at various locations, and the concrete strains were measured by a demountable extensometer of 200 mm gauge length at the centre section only. The central deflection was measured by dial gauge and the rotations by inclinometer. The load was applied incrementally, the readings described above being taken at each stage.

6.3 DISCUSSION OF RESULTS

6.3.1 Introduction

Tables 6.1 to 6.3 summarise the main results obtained from the tests. The deflections, rotations and strains are taken directly from the measurements. The neutral axis depth is obtained from the strain distribution in the concrete. The curvature is then found by dividing the measured concrete compressive strain by the neutral axis depth. The experimental flexural rigidity was calculated by dividing the applied bending moment by the curvature.
| BEAM NUMBER | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Deflection  | mm. | 1.0 | 1.2 | 1.7 | 1.65| 1.2 | 1.4 | 1.8 | 1.4 | 1.3 | 1.2 | 1.65| 1.35| 1.45| 1.35| 1.35| 1.6 | 1.0 | 0.9 | 1.2 | 1.7 | 1.7 | 2.0 | 2.6 | 1.65|
| Span/deflection | 2300| 1916| 1394| 1916| 1642| 1278| 1645| 1769| 1916| 1393| 1704| 1586| 1704| 1704| 1437| 2300| 2556| 1917| 1353| 1353| 1150| 885| 1394|
| Total rotation- radians \(10^3\) | 3.5 | 375 | 4.5 | 375 | 3.0 | 3.5 | 5.25| 3.5 | 3.2 | 3.0 | 4.25| 4.0 | 3.75| 3.5 | 3.5 | 4.25| 3.25| 2.95| 2.5 | 4.75| 4.75| 4.5 | 5.25| 3.25|
| Bar strain \(10^6\) | 300 | 260 | 350 | 220 | 210 | 220 | 460 | 230 | 160 | 150 | 360 | 330 | 200 | 200 | 220 | 280 | 160 | 100 | 120 | 330 | 330 | 250 | 280 | 170|
| Compressive concrete strain \(10^6\) | 290 | 280 | 300 | 280 | 280 | 270 | 390 | 270 | 300 | 300 | 350 | 350 | 350 | 340 | 300 | 260 | 360 | 310 | 280 | 350 | 400 | 300 | 390 | 290|
| Neutral axis depth mm. | 110 | 110 | 110 | 120 | 125 | 125 | 115 | 120 | 120 | 125 | 120 | 115 | 135 | 120 | 125 | 125 | 125 | 125 | 130 | 100 | 110 | 100 | 100 | 125|
| Curvature- radians/m \(10^3\) | 2.63| 2.55| 2.72| 2.72| 2.48| 2.16| 2.39| 2.25| 2.50| 2.81| 2.87| 2.51| 2.50| 2.08| 2.88| 2.69| 2.24| 2.30| 3.50| 3.64| 2.73| 3.90| 2.32|
| Stiffness EI kNm\(^2\) | 5080| 6013| 7046| 8226| 9375| 8888| 6220| 8358| 8042| 8000| 7245| 6271| 7644| 7679| 9188| 7190| 6850| 8527| 9174| 6024| 5792| 6931| 5205| 7801|
| Calculated EI\(_u^'\) kNm\(^2\) | 935  | 9150| 9673| 10174| 14160| 10174| 9688| 10220| 11232| 11232| 9695| 9695| 10220| 10220| 10033| 9695| 10300| 11628| 9695| 8615| 9688| 9688| 10220|
| EI/EI\(_u^'\) | 0.56| 0.66| 0.73| 0.81| 0.84| 0.87| 0.84| 0.80| 0.82| 0.70| 0.73| 0.64| 0.75| 0.70| 0.89| 0.73| 0.66| 0.73| 0.79| 0.62| 0.60| 0.71| 0.54| 0.76|
| Experimental first crack load. kN. | 35.0 | 50.0 | 50.0 | 55.0 | 55.0 | 55.0 | 49.0 | 52.5 | 50.0 | 55.0 | 47.0 | 50.0 | 50.0 | 49.8 | 54.0 | 48.0 | 51.4 | 55.0 | 55.0 | 55.0 | 55.0 | 51.1 | 53.0 | 47.2|
| Experimental first crack moment. kNm | 13.4 | 153 | 19.2 | 19.2 | 21.0 | 19.2 | 21.0 | 18.8 | 20.1 | 19.2 | 19.2 | 19.1 | 20.7 | 18.4 | 19.7 | 21.0 | 21.0 | 21.0 | 19.6 | 20.3 | 18.1 | 13.1 | 13.1 |
| Experimental Theoretical | 1.12| 125 | 1.48| 137 | 1.31| 137 | 1.60| 1.33| 1.21| 1.15| 1.58| 1.40| 1.37| 1.36| 1.31| 1.58| 1.30| 1.15| 1.23| 1.60| 1.60| 1.60 | 1.60 | 1.60|

\(I_u' =\) uncracked, transformed moment of inertia.
| BEAM NUMBER | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Deflection  mm. | 2.0 | 2.0 | 1.92 | 1.90 | 1.88 | 1.80 | 1.65 | 1.55 | 1.60 | 1.80 | 1.80 | 1.65 | 1.70 | 1.40 | 1.20 | 1.20 | 1.80 | 2.00 | 2.50 | 2.80 | 2.00 |
| Span/deflection | 1150 | 1150 | 1211 | 1211 | 1438 | 1278 | 1394 | 1484 | 1438 | 1278 | 1278 | 1394 | 1394 | 1353 | 1643 | 1917 | 1917 | 1278 | 1150 | 920 | 871 | 1150 |
| Total rotation – radians 10^-3 | 6.50 | 6.50 | 6.00 | 4.50 | 3.75 | 4.25 | 5.50 | 4.50 | 3.75 | 5.50 | 5.00 | 5.25 | 4.25 | 4.60 | 4.25 | 5.20 | 4.50 | 3.80 | 3.25 | 5.50 | 5.50 | 5.25 | 6.00 | 4.25 |
| Compression concrete strain 10^-6 | 455 | 440 | 350 | 360 | 300 | 330 | 410 | 340 | 350 | 350 | 430 | 490 | 310 | 320 | 430 | 390 | 300 | 330 | 420 | 400 | 450 | 350 | 440 |
| Neutral axis depth mm | 100 | 108 | 108 | 118 | 123 | 123 | 112 | 118 | 118 | 123 | 118 | 114 | 130 | 118 | 123 | 123 | 114 | 123 | 128 | 99 | 108 | 108 |
| Curvature – radians/m 10^-3 | 4.55 | 4.07 | 3.24 | 3.05 | 2.44 | 2.68 | 3.66 | 2.88 | 2.97 | 2.85 | 3.31 | 3.77 | 3.77 | 2.63 | 2.60 | 3.50 | 3.42 | 2.44 | 2.58 | 4.24 | 4.17 | 3.24 | 4.40 |
| Stiffness EI kNm^2 | 5055 | 5651 | 7099 | 7541 | 9426 | 8582 | 6289 | 7986 | 7744 | 8070 | 6949 | 6101 | 6101 | 8745 | 8846 | 8571 | 6725 | 9426 | 8915 | 5425 | 5516 | 7099 | 5227 | 7443 |
| Calculated EI' kNm^2 | 9135 | 9150 | 9673 | 10174 | 1160 | 10174 | 9688 | 10220 | 11232 | 1232 | 9695 | 9695 | 10220 | 10220 | 10303 | 9695 | 10303 | 11628 | 11628 | 9695 | 9615 | 9688 | 9688 | 10220 |
| EI / EI' | 0.55 | 0.62 | 0.73 | 0.74 | 0.84 | 0.84 | 0.68 | 0.78 | 0.69 | 0.72 | 0.63 | 0.60 | 0.86 | 0.86 | 0.68 | 0.68 | 0.81 | 0.77 | 0.56 | 0.64 | 0.73 | 0.54 | 0.73 |

IU = uncracked, transformed moment of inertia.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>201</th>
<th>202</th>
<th>203</th>
<th>204</th>
<th>205</th>
<th>206</th>
<th>207</th>
<th>208</th>
<th>209</th>
<th>210</th>
<th>211</th>
<th>212</th>
<th>213</th>
<th>214</th>
<th>215</th>
<th>216</th>
<th>217</th>
<th>218</th>
<th>219</th>
<th>220</th>
<th>221</th>
<th>222</th>
<th>223</th>
<th>224</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service load (A) kN</td>
<td>100</td>
<td>102</td>
<td>116</td>
<td>134</td>
<td>166</td>
<td>135</td>
<td>119</td>
<td>135</td>
<td>167</td>
<td>166</td>
<td>119</td>
<td>117</td>
<td>135</td>
<td>135</td>
<td>136</td>
<td>118</td>
<td>135</td>
<td>168</td>
<td>168</td>
<td>118</td>
<td>117</td>
<td>135</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>Deflection mm</td>
<td>4.6</td>
<td>4.6</td>
<td>5.7</td>
<td>6.4</td>
<td>6.7</td>
<td>6.1</td>
<td>5.2</td>
<td>5.2</td>
<td>6.9</td>
<td>6.6</td>
<td>5.0</td>
<td>5.0</td>
<td>5.7</td>
<td>5.0</td>
<td>5.0</td>
<td>4.8</td>
<td>5.1</td>
<td>6.5</td>
<td>6.4</td>
<td>4.6</td>
<td>5.8</td>
<td>5.4</td>
<td>6.3</td>
<td>6.0</td>
</tr>
<tr>
<td>Span /deflection</td>
<td>500</td>
<td>500</td>
<td>404</td>
<td>359</td>
<td>343</td>
<td>377</td>
<td>442</td>
<td>442</td>
<td>333</td>
<td>348</td>
<td>460</td>
<td>460</td>
<td>404</td>
<td>460</td>
<td>460</td>
<td>479</td>
<td>442</td>
<td>354</td>
<td>359</td>
<td>500</td>
<td>397</td>
<td>426</td>
<td>365</td>
<td>460</td>
</tr>
<tr>
<td>Total rotation—radians $10^{-3}$</td>
<td>1.3</td>
<td>1.5</td>
<td>1.2</td>
<td>1.2</td>
<td>1.5</td>
<td>1.2</td>
<td>1.3</td>
<td>1.2</td>
<td>1.4</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
<td>1.2</td>
<td>1.3</td>
<td>1.2</td>
<td>12.3</td>
<td>12.2</td>
<td>11.0</td>
<td>12.5</td>
<td>11.7</td>
</tr>
<tr>
<td>Bar strain $10^{-6}$</td>
<td>1020</td>
<td>1000</td>
<td>930</td>
<td>850</td>
<td>960</td>
<td>820</td>
<td>1000</td>
<td>860</td>
<td>700</td>
<td>940</td>
<td>900</td>
<td>880</td>
<td>860</td>
<td>860</td>
<td>700</td>
<td>750</td>
<td>700</td>
<td>750</td>
<td>700</td>
<td>800</td>
<td>700</td>
<td>870</td>
<td>860</td>
<td>750</td>
</tr>
<tr>
<td>Compressive concrete strain $10^{-6}$</td>
<td>1050</td>
<td>780</td>
<td>800</td>
<td>840</td>
<td>960</td>
<td>950</td>
<td>950</td>
<td>790</td>
<td>1010</td>
<td>1000</td>
<td>1000</td>
<td>875</td>
<td>1030</td>
<td>920</td>
<td>1080</td>
<td>975</td>
<td>975</td>
<td>995</td>
<td>960</td>
<td>900</td>
<td>910</td>
<td>755</td>
<td>880</td>
<td>890</td>
</tr>
<tr>
<td>Neutral axis depth mm</td>
<td>96</td>
<td>95</td>
<td>100</td>
<td>109</td>
<td>114</td>
<td>114</td>
<td>106</td>
<td>109</td>
<td>109</td>
<td>114</td>
<td>112</td>
<td>108</td>
<td>113</td>
<td>109</td>
<td>118</td>
<td>118</td>
<td>105</td>
<td>119</td>
<td>119</td>
<td>97</td>
<td>104</td>
<td>105</td>
<td>100</td>
<td>109</td>
</tr>
<tr>
<td>Curvature—radians/m $10^{-3}$</td>
<td>10.8</td>
<td>8.2</td>
<td>8.0</td>
<td>7.7</td>
<td>8.4</td>
<td>8.3</td>
<td>9.0</td>
<td>7.2</td>
<td>9.3</td>
<td>8.8</td>
<td>8.9</td>
<td>8.1</td>
<td>9.1</td>
<td>8.4</td>
<td>9.2</td>
<td>8.3</td>
<td>9.3</td>
<td>8.4</td>
<td>8.1</td>
<td>9.3</td>
<td>8.7</td>
<td>7.3</td>
<td>8.8</td>
<td>8.2</td>
</tr>
<tr>
<td>Stiffness EI kNm$^2$</td>
<td>3537</td>
<td>4780</td>
<td>5575</td>
<td>6662</td>
<td>7571</td>
<td>6212</td>
<td>5066</td>
<td>7167</td>
<td>6892</td>
<td>7250</td>
<td>5146</td>
<td>5519</td>
<td>5670</td>
<td>6187</td>
<td>7021</td>
<td>5490</td>
<td>5570</td>
<td>7690</td>
<td>7960</td>
<td>4882</td>
<td>5172</td>
<td>6160</td>
<td>5205</td>
<td>6340</td>
</tr>
<tr>
<td>Calculated $E_{cr}'$ kNm$^2$</td>
<td>4522</td>
<td>4557</td>
<td>5519</td>
<td>6384</td>
<td>7992</td>
<td>6384</td>
<td>5568</td>
<td>6475</td>
<td>8124</td>
<td>8124</td>
<td>5665</td>
<td>5665</td>
<td>6475</td>
<td>6475</td>
<td>6599</td>
<td>5665</td>
<td>6599</td>
<td>8300</td>
<td>8300</td>
<td>5665</td>
<td>5900</td>
<td>9568</td>
<td>5568</td>
<td>6475</td>
</tr>
<tr>
<td>$EI / E_{cr}'$</td>
<td>1.04</td>
<td>0.95</td>
<td>0.97</td>
<td>0.91</td>
<td>1.11</td>
<td>0.85</td>
<td>0.89</td>
<td>0.91</td>
<td>0.97</td>
<td>0.88</td>
<td>0.95</td>
<td>1.06</td>
<td>0.97</td>
<td>0.84</td>
<td>0.93</td>
<td>0.96</td>
<td>0.86</td>
<td>0.88</td>
<td>1.11</td>
<td>0.93</td>
<td>0.98</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$E_{cr}' = $ cracked, transformed moment of inertia.

(1) Design service load (A) = calculated from CP 110 including material safety factors:

\[
1.6 \times \text{service load (A)} \times 1.4 \times \text{dead load} = 0.87 \left( \frac{1 - \frac{I}{E_{cr}'} }{f_{cu}'} \right) bd^4
\]

---

**TABLE 6.2A** DEFORMATION CHARACTERISTICS AT DESIGN SERVICE LOAD (A)
## Table 6.2B: Deformation Characteristics at Design Service Load (B)\(^{(1)}\)

<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>201</th>
<th>202</th>
<th>203</th>
<th>204</th>
<th>205</th>
<th>206</th>
<th>207</th>
<th>208</th>
<th>209</th>
<th>210</th>
<th>211</th>
<th>212</th>
<th>213</th>
<th>214</th>
<th>215</th>
<th>216</th>
<th>217</th>
<th>218</th>
<th>219</th>
<th>220</th>
<th>221</th>
<th>222</th>
<th>223</th>
<th>224</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service load (B) kN</td>
<td>137</td>
<td>144</td>
<td>153</td>
<td>168</td>
<td>197</td>
<td>170</td>
<td>157</td>
<td>170</td>
<td>170</td>
<td>198</td>
<td>159</td>
<td>153</td>
<td>170</td>
<td>171</td>
<td>172</td>
<td>157</td>
<td>171</td>
<td>201</td>
<td>202</td>
<td>157</td>
<td>154</td>
<td>154</td>
<td>159</td>
<td>173</td>
</tr>
<tr>
<td>Deflection mm</td>
<td>8.0</td>
<td>8.4</td>
<td>8.8</td>
<td>9.0</td>
<td>8.8</td>
<td>8.5</td>
<td>8.2</td>
<td>9.5</td>
<td>9.5</td>
<td>8.0</td>
<td>7.6</td>
<td>8.4</td>
<td>7.8</td>
<td>7.4</td>
<td>7.8</td>
<td>7.4</td>
<td>7.8</td>
<td>7.5</td>
<td>9.0</td>
<td>7.4</td>
<td>8.5</td>
<td>8.0</td>
<td>9.2</td>
<td>7.2</td>
</tr>
<tr>
<td>Span/deflection</td>
<td>288</td>
<td>280</td>
<td>261</td>
<td>256</td>
<td>261</td>
<td>271</td>
<td>280</td>
<td>303</td>
<td>242</td>
<td>261</td>
<td>288</td>
<td>303</td>
<td>274</td>
<td>295</td>
<td>311</td>
<td>295</td>
<td>307</td>
<td>256</td>
<td>311</td>
<td>271</td>
<td>288</td>
<td>250</td>
<td>319</td>
<td></td>
</tr>
<tr>
<td>Total rotation—radians 10(^{-3})</td>
<td>17.1</td>
<td>18.0</td>
<td>18.5</td>
<td>16.5</td>
<td>19.5</td>
<td>17.3</td>
<td>18.8</td>
<td>16.3</td>
<td>20.0</td>
<td>19.0</td>
<td>17.0</td>
<td>17.0</td>
<td>17.0</td>
<td>16.0</td>
<td>16.5</td>
<td>17.5</td>
<td>15.8</td>
<td>15.8</td>
<td>18.1</td>
<td>17.6</td>
<td>15.0</td>
<td>17.5</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>Bar strain 10(^{-6})</td>
<td>1200</td>
<td>1480</td>
<td>1530</td>
<td>1200</td>
<td>1330</td>
<td>1180</td>
<td>1470</td>
<td>1170</td>
<td>930</td>
<td>1330</td>
<td>1400</td>
<td>1250</td>
<td>1200</td>
<td>1330</td>
<td>1100</td>
<td>1020</td>
<td>1000</td>
<td>1280</td>
<td>1250</td>
<td>1080</td>
<td>1180</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive concrete strain 10(^{-5})</td>
<td>1350</td>
<td>1110</td>
<td>1070</td>
<td>1150</td>
<td>1260</td>
<td>1200</td>
<td>1400</td>
<td>1050</td>
<td>1200</td>
<td>1280</td>
<td>1260</td>
<td>1300</td>
<td>1500</td>
<td>1140</td>
<td>1350</td>
<td>1320</td>
<td>1300</td>
<td>1170</td>
<td>1400</td>
<td>1300</td>
<td>1050</td>
<td>1270</td>
<td>1180</td>
<td></td>
</tr>
<tr>
<td>Neutral axis depth mm</td>
<td>80</td>
<td>83</td>
<td>93</td>
<td>109</td>
<td>115</td>
<td>112</td>
<td>95</td>
<td>106</td>
<td>109</td>
<td>114</td>
<td>100</td>
<td>100</td>
<td>110</td>
<td>106</td>
<td>116</td>
<td>110</td>
<td>103</td>
<td>119</td>
<td>95</td>
<td>94</td>
<td>98</td>
<td>98</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>Curvature—radians/m 10(^{-3})</td>
<td>16.9</td>
<td>13.4</td>
<td>11.5</td>
<td>10.6</td>
<td>11.0</td>
<td>10.7</td>
<td>14.7</td>
<td>9.9</td>
<td>11.0</td>
<td>11.2</td>
<td>12.6</td>
<td>13.0</td>
<td>13.6</td>
<td>10.8</td>
<td>11.6</td>
<td>11.9</td>
<td>12.6</td>
<td>9.8</td>
<td>14.7</td>
<td>13.8</td>
<td>10.7</td>
<td>109</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td>Stiffness EI kNm(^{2})</td>
<td>3112</td>
<td>4112</td>
<td>5096</td>
<td>6075</td>
<td>6882</td>
<td>6090</td>
<td>6088</td>
<td>6576</td>
<td>6982</td>
<td>6777</td>
<td>4722</td>
<td>4523</td>
<td>4786</td>
<td>6050</td>
<td>6722</td>
<td>5050</td>
<td>5190</td>
<td>7898</td>
<td>4088</td>
<td>4283</td>
<td>5523</td>
<td>4700</td>
<td>6100</td>
<td></td>
</tr>
<tr>
<td>Calculated EI'(_{cr}) kNm(^{2})</td>
<td>4522</td>
<td>4567</td>
<td>5519</td>
<td>6384</td>
<td>7992</td>
<td>6384</td>
<td>5566</td>
<td>6475</td>
<td>8124</td>
<td>8124</td>
<td>5665</td>
<td>5665</td>
<td>6475</td>
<td>6599</td>
<td>5665</td>
<td>6559</td>
<td>8300</td>
<td>8300</td>
<td>5665</td>
<td>5900</td>
<td>5568</td>
<td>5568</td>
<td>6475</td>
<td></td>
</tr>
<tr>
<td>EI'/EI(_{cr})</td>
<td>0.68</td>
<td>0.90</td>
<td>0.92</td>
<td>0.95</td>
<td>0.86</td>
<td>0.95</td>
<td>0.73</td>
<td>1.02</td>
<td>0.86</td>
<td>0.83</td>
<td>0.84</td>
<td>0.80</td>
<td>0.74</td>
<td>0.93</td>
<td>1.01</td>
<td>0.89</td>
<td>1.79</td>
<td>0.95</td>
<td>0.72</td>
<td>0.73</td>
<td>0.99</td>
<td>0.84</td>
<td>0.94</td>
<td></td>
</tr>
</tbody>
</table>

\(_{cr}\) = cracked, transformed moment of inertia.

(1) Design service load (B)—calculated from strain compatibility without material safety factors.

\[1.6 \times \text{[service load]} \times 1.4 \times \text{[dead load]} = M_u\]

(2) Beam 218 failed before reaching this load.
### TABLE 6.2 C DEFORMATION CHARACTERISTICS AT 130 kN LOAD

<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>201</th>
<th>202</th>
<th>203</th>
<th>204</th>
<th>205</th>
<th>206</th>
<th>207</th>
<th>208</th>
<th>209</th>
<th>210</th>
<th>211</th>
<th>212</th>
<th>213</th>
<th>214</th>
<th>215</th>
<th>216</th>
<th>217</th>
<th>218</th>
<th>219</th>
<th>220</th>
<th>221</th>
<th>222</th>
<th>223</th>
<th>224</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>mm</td>
<td>7.3</td>
<td>6.9</td>
<td>6.6</td>
<td>6.0</td>
<td>4.6</td>
<td>5.7</td>
<td>6.0</td>
<td>5.0</td>
<td>4.5</td>
<td>4.3</td>
<td>5.8</td>
<td>5.8</td>
<td>5.4</td>
<td>4.8</td>
<td>4.7</td>
<td>5.7</td>
<td>4.8</td>
<td>4.2</td>
<td>4.2</td>
<td>5.4</td>
<td>6.6</td>
<td>6.4</td>
<td>7.0</td>
</tr>
<tr>
<td>Span/deflection</td>
<td></td>
<td>315</td>
<td>333</td>
<td>348</td>
<td>383</td>
<td>500</td>
<td>404</td>
<td>383</td>
<td>460</td>
<td>511</td>
<td>535</td>
<td>397</td>
<td>397</td>
<td>426</td>
<td>479</td>
<td>489</td>
<td>404</td>
<td>479</td>
<td>548</td>
<td>548</td>
<td>426</td>
<td>348</td>
<td>360</td>
<td>329</td>
</tr>
<tr>
<td>Total rotation—radians $10^{-3}$</td>
<td></td>
<td>16.0</td>
<td>15.7</td>
<td>15.0</td>
<td>11.6</td>
<td>10.2</td>
<td>11.6</td>
<td>14.5</td>
<td>11.0</td>
<td>9.5</td>
<td>13.0</td>
<td>13.8</td>
<td>11.3</td>
<td>11.1</td>
<td>11.1</td>
<td>11.0</td>
<td>13.3</td>
<td>11.0</td>
<td>8.8</td>
<td>8.2</td>
<td>14.3</td>
<td>14.3</td>
<td>12.7</td>
<td>15.0</td>
</tr>
<tr>
<td>Bar strain $10^{-6}$</td>
<td></td>
<td>1350</td>
<td>1320</td>
<td>1100</td>
<td>770</td>
<td>530</td>
<td>740</td>
<td>1100</td>
<td>740</td>
<td>500</td>
<td>550</td>
<td>1000</td>
<td>1020</td>
<td>750</td>
<td>760</td>
<td>750</td>
<td>900</td>
<td>670</td>
<td>500</td>
<td>460</td>
<td>1000</td>
<td>1000</td>
<td>830</td>
<td>950</td>
</tr>
<tr>
<td>Compressive concrete strain $10^{-6}$</td>
<td></td>
<td>1260</td>
<td>950</td>
<td>900</td>
<td>800</td>
<td>760</td>
<td>950</td>
<td>1000</td>
<td>750</td>
<td>770</td>
<td>800</td>
<td>1100</td>
<td>920</td>
<td>950</td>
<td>900</td>
<td>750</td>
<td>1070</td>
<td>920</td>
<td>750</td>
<td>720</td>
<td>1000</td>
<td>1000</td>
<td>860</td>
<td>1000</td>
</tr>
<tr>
<td>Neutral axis depth mm</td>
<td></td>
<td>88</td>
<td>85</td>
<td>95</td>
<td>110</td>
<td>115</td>
<td>115</td>
<td>100</td>
<td>110</td>
<td>115</td>
<td>105</td>
<td>102</td>
<td>115</td>
<td>110</td>
<td>120</td>
<td>115</td>
<td>106</td>
<td>120</td>
<td>120</td>
<td>95</td>
<td>98</td>
<td>100</td>
<td>100</td>
<td>110</td>
</tr>
<tr>
<td>Curvature—radians/m $10^{-3}$</td>
<td></td>
<td>14.3</td>
<td>11.1</td>
<td>9.5</td>
<td>7.3</td>
<td>6.6</td>
<td>8.3</td>
<td>10.0</td>
<td>6.8</td>
<td>7.0</td>
<td>10.5</td>
<td>9.0</td>
<td>8.3</td>
<td>8.2</td>
<td>6.3</td>
<td>9.4</td>
<td>8.7</td>
<td>6.3</td>
<td>6.0</td>
<td>10.5</td>
<td>10.2</td>
<td>8.6</td>
<td>10.0</td>
<td>7.7</td>
</tr>
<tr>
<td>Stiffness EI $\text{kNm}^2$</td>
<td></td>
<td>3496</td>
<td>4489</td>
<td>5262</td>
<td>6855</td>
<td>7474</td>
<td>6033</td>
<td>4980</td>
<td>7318</td>
<td>7119</td>
<td>7160</td>
<td>4690</td>
<td>5537</td>
<td>6033</td>
<td>6993</td>
<td>7840</td>
<td>5373</td>
<td>5740</td>
<td>7840</td>
<td>8300</td>
<td>4737</td>
<td>4885</td>
<td>5294</td>
<td>4983</td>
</tr>
<tr>
<td>Calculated $EI'_c$ $\text{kNm}^2$</td>
<td></td>
<td>4522</td>
<td>4557</td>
<td>5519</td>
<td>6384</td>
<td>7992</td>
<td>5384</td>
<td>5568</td>
<td>6475</td>
<td>8124</td>
<td>8124</td>
<td>5665</td>
<td>5665</td>
<td>6475</td>
<td>6475</td>
<td>6599</td>
<td>5665</td>
<td>6599</td>
<td>8300</td>
<td>8300</td>
<td>5665</td>
<td>5600</td>
<td>5568</td>
<td>6475</td>
</tr>
<tr>
<td>$\frac{EI}{EI'_c}$</td>
<td></td>
<td>0.74</td>
<td>0.99</td>
<td>0.95</td>
<td>1.07</td>
<td>0.94</td>
<td>0.96</td>
<td>0.79</td>
<td>1.13</td>
<td>0.88</td>
<td>0.88</td>
<td>0.84</td>
<td>0.84</td>
<td>0.93</td>
<td>0.94</td>
<td>1.19</td>
<td>0.95</td>
<td>0.87</td>
<td>0.94</td>
<td>1.00</td>
<td>0.84</td>
<td>0.78</td>
<td>1.04</td>
<td>0.89</td>
</tr>
</tbody>
</table>

$\frac{1}{r_c}$ = cracked, transformed moment of inertia.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>201(1)</th>
<th>202(1)</th>
<th>203</th>
<th>204</th>
<th>205</th>
<th>206</th>
<th>207</th>
<th>208</th>
<th>209</th>
<th>210</th>
<th>211</th>
<th>212(1)</th>
<th>213</th>
<th>214</th>
<th>215</th>
<th>216</th>
<th>217</th>
<th>218</th>
<th>219</th>
<th>220</th>
<th>221</th>
<th>222</th>
<th>223</th>
<th>224</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>20.0</td>
<td>17.6</td>
<td>21.6</td>
<td>16.8</td>
<td>8.4</td>
<td>15.8</td>
<td>20.8</td>
<td>15.8</td>
<td>11.6</td>
<td>8.0</td>
<td>18.4</td>
<td>14.8</td>
<td>17.3</td>
<td>15.3</td>
<td>15.1</td>
<td>19.0</td>
<td>15.0</td>
<td>8.2</td>
<td>8.0</td>
<td>18.4</td>
<td>19.8</td>
<td>18.0</td>
<td>18.5</td>
<td>14.2</td>
</tr>
<tr>
<td>Span/deflection</td>
<td>115</td>
<td>130</td>
<td>106</td>
<td>137</td>
<td>274</td>
<td>145</td>
<td>111</td>
<td>146</td>
<td>198</td>
<td>280</td>
<td>125</td>
<td>155</td>
<td>133</td>
<td>150</td>
<td>152</td>
<td>121</td>
<td>153</td>
<td>280</td>
<td>288</td>
<td>125</td>
<td>116</td>
<td>128</td>
<td>124</td>
<td>162</td>
</tr>
<tr>
<td>Total rotation-radians $10^{-3}$</td>
<td>36.2</td>
<td>35.5</td>
<td>44.3</td>
<td>38.0</td>
<td>18.8</td>
<td>36.0</td>
<td>42.5</td>
<td>36.0</td>
<td>25.5</td>
<td>17.5</td>
<td>38.0</td>
<td>30.2</td>
<td>35.5</td>
<td>36.0</td>
<td>33.7</td>
<td>43.0</td>
<td>35.2</td>
<td>17.5</td>
<td>14.5</td>
<td>40.0</td>
<td>41.0</td>
<td>35.5</td>
<td>36.0</td>
<td>31.5</td>
</tr>
<tr>
<td>Bar strain $10^{-6}$</td>
<td>4400</td>
<td>3100</td>
<td>3560</td>
<td>2500</td>
<td>1250</td>
<td>2400</td>
<td>3600</td>
<td>2430</td>
<td>1065</td>
<td>1250</td>
<td>3800</td>
<td>2720</td>
<td>2600</td>
<td>2520</td>
<td>2330</td>
<td>2950</td>
<td>2200</td>
<td>960</td>
<td>850</td>
<td>3250</td>
<td>3400</td>
<td>2700</td>
<td>2150</td>
<td>2350</td>
</tr>
<tr>
<td>Compressive concrete strain $10^{-6}$</td>
<td>3350</td>
<td>2400</td>
<td>2300</td>
<td>2050</td>
<td>1200</td>
<td>1810</td>
<td>2500</td>
<td>1980</td>
<td>1400</td>
<td>1200</td>
<td>2450</td>
<td>2150</td>
<td>2520</td>
<td>1860</td>
<td>1970</td>
<td>2600</td>
<td>2370</td>
<td>1200</td>
<td>1150</td>
<td>2500</td>
<td>2350</td>
<td>2210</td>
<td>2200</td>
<td>1920</td>
</tr>
<tr>
<td>Neutral axis depth mm</td>
<td>62</td>
<td>75</td>
<td>85</td>
<td>106</td>
<td>113</td>
<td>106</td>
<td>80</td>
<td>100</td>
<td>105</td>
<td>110</td>
<td>95</td>
<td>95</td>
<td>105</td>
<td>100</td>
<td>110</td>
<td>100</td>
<td>97</td>
<td>118</td>
<td>116</td>
<td>90</td>
<td>85</td>
<td>90</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Curvature-radians/m $10^{-3}$</td>
<td>54.0</td>
<td>32.0</td>
<td>27.0</td>
<td>19.3</td>
<td>10.6</td>
<td>17.1</td>
<td>31.3</td>
<td>19.8</td>
<td>13.3</td>
<td>10.9</td>
<td>25.8</td>
<td>22.6</td>
<td>24.0</td>
<td>18.6</td>
<td>17.9</td>
<td>26.0</td>
<td>24.4</td>
<td>10.2</td>
<td>9.9</td>
<td>27.8</td>
<td>27.6</td>
<td>24.6</td>
<td>22.0</td>
<td>192</td>
</tr>
<tr>
<td>Stiffness EI kNm $^2$</td>
<td>1562</td>
<td>2635</td>
<td>3549</td>
<td>4965</td>
<td>6871</td>
<td>5604</td>
<td>3282</td>
<td>4840</td>
<td>6340</td>
<td>6681</td>
<td>3717</td>
<td>4233</td>
<td>4008</td>
<td>5210</td>
<td>5354</td>
<td>3689</td>
<td>3927</td>
<td>7140</td>
<td>7349</td>
<td>3447</td>
<td>3630</td>
<td>3895</td>
<td>4356</td>
<td>4991</td>
</tr>
<tr>
<td>Calculated EI' $^T$ kNm $^2$</td>
<td>4521</td>
<td>4557</td>
<td>5519</td>
<td>6384</td>
<td>7992</td>
<td>6384</td>
<td>5568</td>
<td>6475</td>
<td>8124</td>
<td>8124</td>
<td>5665</td>
<td>5665</td>
<td>6475</td>
<td>6475</td>
<td>6599</td>
<td>5665</td>
<td>6599</td>
<td>8300</td>
<td>8300</td>
<td>5665</td>
<td>5900</td>
<td>5568</td>
<td>5568</td>
<td>6475</td>
</tr>
<tr>
<td>$EI/EI' T$</td>
<td>0.36</td>
<td>0.58</td>
<td>0.64</td>
<td>0.78</td>
<td>0.84</td>
<td>0.88</td>
<td>0.59</td>
<td>0.75</td>
<td>0.78</td>
<td>0.82</td>
<td>0.66</td>
<td>0.75</td>
<td>0.62</td>
<td>0.80</td>
<td>0.81</td>
<td>0.65</td>
<td>0.60</td>
<td>0.86</td>
<td>0.89</td>
<td>0.61</td>
<td>0.62</td>
<td>0.62</td>
<td>0.69</td>
<td>0.77</td>
</tr>
<tr>
<td>Experimental failure moment kNm</td>
<td>889</td>
<td>939</td>
<td>1035</td>
<td>1035</td>
<td>81.7</td>
<td>1043</td>
<td>1006</td>
<td>101.2</td>
<td>843</td>
<td>82.4</td>
<td>97.3</td>
<td>95.8</td>
<td>96.9</td>
<td>97.0</td>
<td>95.8</td>
<td>1006</td>
<td>98.6</td>
<td>74.5</td>
<td>84.3</td>
<td>1008</td>
<td>98.9</td>
<td>1027</td>
<td>1012</td>
<td>95.8</td>
</tr>
<tr>
<td>Theoretical failure moment $^{(3)}$ kNm</td>
<td>83.7</td>
<td>87.0</td>
<td>92.8</td>
<td>1029</td>
<td>120.6</td>
<td>104.9</td>
<td>96.6</td>
<td>104.2</td>
<td>122.4</td>
<td>121.2</td>
<td>97.2</td>
<td>93.5</td>
<td>104.0</td>
<td>104.9</td>
<td>1060</td>
<td>967</td>
<td>1055</td>
<td>122.7</td>
<td>124.3</td>
<td>96.3</td>
<td>94.6</td>
<td>963</td>
<td>969</td>
<td>1056</td>
</tr>
<tr>
<td>Experimental moment</td>
<td>1.06</td>
<td>1.08</td>
<td>1.12</td>
<td>1.01</td>
<td>0.68</td>
<td>0.99</td>
<td>1.04</td>
<td>0.97</td>
<td>0.69</td>
<td>0.68</td>
<td>1.00</td>
<td>1.02</td>
<td>0.93</td>
<td>0.92</td>
<td>0.90</td>
<td>1.04</td>
<td>0.93</td>
<td>0.60</td>
<td>0.68</td>
<td>1.05</td>
<td>1.04</td>
<td>1.07</td>
<td>1.04</td>
<td>0.91</td>
</tr>
<tr>
<td>Theoretical moment</td>
<td>1.06</td>
<td>1.08</td>
<td>1.12</td>
<td>1.01</td>
<td>0.68</td>
<td>0.99</td>
<td>1.04</td>
<td>0.97</td>
<td>0.69</td>
<td>0.68</td>
<td>1.00</td>
<td>1.02</td>
<td>0.93</td>
<td>0.92</td>
<td>0.90</td>
<td>1.04</td>
<td>0.93</td>
<td>0.60</td>
<td>0.68</td>
<td>1.05</td>
<td>1.04</td>
<td>1.07</td>
<td>1.04</td>
<td>0.91</td>
</tr>
</tbody>
</table>

(1) Readings at $220$ kN load
(2) Readings at $190$ kN load
(3) Calculated by strain compatibility, including the tensile force in the glue.
Tables 6.1A and 6.1B show the deformation characteristics at first visible crack load and at 60 kN, which is just above the first crack loads. The latter being used for comparisons.

Tables 6.2A, 6.2B and 6.2C show the deformation characteristics at design service loads. Table 6.2A is for design service load A, which was calculated by CP 110 methods including the material safety factors - 1.5 for concrete and 1.15 for steel. Table 6.2B is for design service load B, which was calculated by taking the ultimate load found by strain compatibility methods and dividing by 1.6. Design service load C was taken as 130 kN for comparative purposes, as given in Table 6.2C.

Table 6.3 shows the deformation characteristics near ultimate load.

The flexural rigidities found from the experimental readings were compared with the theoretical values both before and after cracking (Tables 6.1 to 6.3).

At 60 kN load, the flexural rigidities of the plated beams were between 3% and 86% higher than the unplated beam. When compared with the calculated rigidity for the uncracked sections the unplated beam gave a ratio of experimental to theoretical of 0.56 while the plated beams gave values ranging from 0.56 to 0.86, except the precracked beam 223 which gave 0.54. For beams with 1.5 mm thick plates the mean value was 0.66, for 3 mm thick plates it was 0.76 and for 6 mm plates 0.77. Thus it can be seen at this load, just above the first crack load, the plates have reduced the extent of cracking and therefore increased the stiffness of the beams.

At 130 kN the flexural rigidities of the plated beams were between 40 and 140% higher than the unplated beam. However, these figures represent the behaviour only in the constant moment region and a better representation of the flexural rigidity of the beams as a whole is found through the deflections and rotations. The ratio of experimental to theoretical flexural rigidity for the unplated beam was 0.74. For beams reinforced with 1.5 mm thick plates the mean ratio was 0.86; for beams with 3 mm plates it was 1.0 and for 6 mm plates 0.93. At service loads, therefore, the plates are effectively increasing the
At 190 kN load, the flexural rigidities of the plated beams were between 100 and 300% higher than the unplated beam. However, again a better representation of the total beam behaviour is obtained from the rotation and deflection characteristic as reported below.

6.3.2 Load-Strain Characteristics

6.3.2.1 Literature Review

It has been shown by many authors (94, 96, 97) that the tensile strain on the concrete surface between cracks is a very small amount that may be neglected. Since the crack width at the surface is given by the extension of the bars between two cracks, it can be concluded that the strain in the reinforcing bar, and therefore the strain in the concrete surface across cracks, at the same level are identical. The demec gauge had a gauge length of 200 mm so that it crossed at least one crack each time it was read. The load-strain curves given in Fig. 6.1 confirmed that the bar and concrete surface strains are almost identical.

6.3.2.2 Reinforcing Bar Strains

Fig. 6.2 shows that for a glue thickness of 1.5 mm, the bar strains are decreased as the plate thickness increases. Beam 204, with 3 mm thick plate, behaved identically to Beam 206, with two layers of 1.5 mm plate up to service load, and very closely thereafter.

Figs. 6.3 and 6.4 show similar behaviour for beams with 3 mm and 6 mm thick glue layers respectively. Beams 209 and 210 both had 6 mm thick plates and 3 mm thick glue layer, but Beam 210 was considerably more flexible above service loads, in fact its behaviour was almost identical to Beam 205 (Fig. 6.9) which had a 1.5 mm thick glue layer. It is thought, therefore, that some of the spacers placed in the glue line to control its thickness must have been squeezed out with the excess glue.

Fig. 6.4 shows that a beam with a glue layer only was slightly stiffer than the unplated beam. Beams 211 and 212, which had lapped plates, behaved almost identically with Beam 207 which had a continuous layer of plate. Up to
FIGURE 6.4  LOAD-STRAIN CURVES  INTERNAL STEEL BARS - CENTRE SECTION

FIGURE 6.5  LOAD-STRAIN CURVES  INTERNAL STEEL BARS - CENTRE SECTION
service load the beams with lapped plates were slightly stiffer. This would be expected as the lever arm is slightly increased.

Fig. 6.5 compares Beam 208 with an unlapped 3 mm thick plate, to Beam 215 which had a 3 mm thick lapping plate at the centre section. Beams 213 and 214 had two layers of 1.5 mm plate, the upper layers were lapped. These four beams behaved in a similar manner; the beams having lapped plates being slightly stiffer and the beam with a single 3 mm lapped plate showed lower strains than the beams with two layers of 1.5 mm plate. This could be explained by the fact that there must be a certain amount of movement between the two layers of 1.5 mm plate.

Figs. 6.7, 6.8 and 6.9 show the behaviour of beams with constant plate thicknesses of 1.5 mm, 3 mm and 6 mm respectively, when the glue thickness is varied. There is a general reduction in bar strains for an increase in glue thickness, although the effect was not as large as the reduction found when increasing the plate thickness for constant glue thickness. The behaviour of the beams with variable and notched glue layers, 220 and 221, was between that of the beams with 3 mm and 6 mm glue thickness.

Figs. 6.10 and 6.11 show that for the beams which were precracked, before bonding on the plates, the reinforcing bar strains were reduced in comparison to those found in a similar beam which was not precracked. This behaviour seems somewhat anomalous. It could be expected that the precracked beams would be relatively less stiff than the beams that were not cracked prior to bonding on their plates. However, this behaviour could perhaps be explained by assuming that the glue has penetrated into the existing cracks at the tensile face and is resisting their propagation. This should result in an increased number of cracks with a reduced mean height and spacing. If the cracking results for beams 207 (no precracking) and 223 (90% ultimate load before bending) are compared the above hypothesis is not confirmed.

Alternatively the reduced bar strains could be explained by assuming that bond failure had occurred adjacent to cracks when loading to 90% ultimate before bonding. The bar extension, after bonding on the plate, would then give
FIGURE 6-6 LOAD-STRAIN CURVES  INTERNAL STEEL BARS - CENTRE SECTION

- 216  6mm glue, 15 mm plate.
- 217  = 3mm plate.
- 218  = 6mm plate.
- 219  =

design service load (c) 130 kN

- 203  15mm glue, 15mm plate
- 207  3mm glue, =
- 216  6mm glue, =
- 220  2.8mm glue, =
- 221  3mm glue, notched, =

comparison fist crack load 60 kN

FIGURE 6-7 LOAD-STRAIN CURVES  INTERNAL STEEL BARS - CENTRE SECTION
Figure 6.10 Load-Strain Curves Internal Steel Bars - Centre Section

- Design service load (c) 1130 kN
- Comparison 1st crack load 60 kN

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>207</td>
<td>3 mm glue, 15 mm plate</td>
</tr>
<tr>
<td>222</td>
<td>3 mm glue, preloaded to 50% ultimate</td>
</tr>
<tr>
<td>223</td>
<td>3 mm glue, 15 mm plate, preloaded to 90% ultimate</td>
</tr>
<tr>
<td>208</td>
<td>3 mm glue, 3 mm plate</td>
</tr>
<tr>
<td>224</td>
<td>3 mm glue, preloaded to 50% ultimate</td>
</tr>
</tbody>
</table>

Figure 6.11 Load-Strain Curves Internal Steel Bars - Centre Section

- Design service load (c) 1130 kN
- Comparison 1st crack load 60 kN
smaller strain. However, the same behaviour is also apparent in the beams preloaded to only 50% ultimate load and it is unlikely that bond failure would have occurred at this stage. This area of research must be studied further.

As shown in Table 6.1B at 60 kN load the internal bar strains were reduced to 64-80%, of the values for the unplated beam for beams strengthened with 1.5 mm thick plates; 47-58% for beams strengthened with 3 mm thick plates; and 27-33% for beams with 6 mm plates. At 130 kN load these same strains were reduced to 70-81% (1.5 mm); 50-58% (3 mm); and 34-41% (6 mm). Similarly at 190 kN load the strains were reduced to 46-62% (1.5 mm); 38-46% (3 mm); and 24-43% (6 mm). This information is given in Fig. 6.23. If, instead of adding external reinforcement, the bar area had been increased by the same area as the plates, the calculated stresses in the bars would be reduced to 84%, 72% and 55% of the original beam stresses, for 1.5 mm, 3 mm and 6 mm plates respectively. The decrease in strains obtained with the plated reinforcement is greater than would have been found for the same increase in bar area. This would be expected to some extent due to the increased lever arm of the plates. However, it shows that there is good composite action between the glue, plate and concrete.

The bar strains for the beams strengthened with 1.5 mm thick plates varied from 2520 to 3600 microstrain at the load stage prior to failure. This indicates that the bars were yielding as the elastic limit of the steel is approximately 2000 microstrain, see Fig. 3.8.

The beams strengthened with 3 mm thick plates had bar strains varying from 2200 to 2600 at the same load stage, again indicating yielding of the bars.

The beams with 6 mm thick plates had bar strains varying from 850 to 1250 microstrain indicating that the steel was well below its elastic limit.

6.3.2.3 External Plate Reinforcement Strains

Figs. 6.12 to 6.21 show similar behaviour for the plate strains as for the internal bar strains. The behaviour of the precracked beams was again not as expected.
Design service load (c) 130 kN

- 207: 3mm glue, 1.5 mm plate.
- 222: + preloaded to 50% ultimate.
- 223: 1.5 mm plate, preloaded to 90% ultimate.

Comparison 1st crack load 60 kN

---

**Figure 6.20 Load-Strain Curves**

**Figure 6.21 Load-Strain Curves**
Fig. 6.22 shows the longitudinal distribution of plate strains. For a constant glue thickness, increasing the plate thickness reduces the general level of straining. Increasing the glue thickness for a constant plate thickness has a similar but much reduced effect. It can be seen that the strain increases along the beam, as the bending moment, until it reaches the load points. The strain varies only slightly in the constant moment region. The strain gradient is largest at the end of the plate, but the 6 mm gauge length was not suitable for obtaining an accurate measurement of this gradient. A more accurate measurement of the rapidly changing stress fields around the end of the plate, at joints and around cracks, must be developed.

The measured values of strain can only be an approximation to the local strains as they are based on average values over a finite gauge length. Therefore, it follows that the smaller the gauge length the more accurate a representation of the strain distribution is obtained. It should be noted that the strain in the plate at any particular distance from the end of the plate is not constant across the width. To obtain the stress contours in such areas, or around joints and cracks in the concrete, many strain gauges of small gauge length would be needed in a grid pattern over the entire area. Such an array of gauges would best be monitored by a data logger. Alternatively it may be possible to use photoelastic techniques. It would also be useful to have strain gauges embedded within the glue layer to obtain similar distributions of adhesive stresses.

The variations of external plate strains with plate and glue thickness are shown diagrammatically in Fig. 6.24. The effect of plate thickness can be seen to be greater than that of glue thickness.

At the load stage prior to failure, the strain gauges at the centre section of the beams with 1.5 mm thick plate reinforcement indicated that the strains in the plates varied from 3000 to 3900 microstrain. The yield strain of the 1.5 mm thick plate is 1300 microstrain, as shown in Fig. 3.9. This indicates that extensive yielding has occurred prior to failure. Although this was not visually evident while the beam was under load, it became obvious when
Figure 6.22: Typical Longitudinal Strain Distributions

- **203**: 1.5mm glue, 15mm plate.
- **204**: = 3mm plate.
- **205**: = 6mm plate.
- **206**: = 2x15mm plate.
- **207**: 3mm glue, 1.5mm plate.
- **208**: = 3mm plate.
- **209**: = 6mm plate.
- **210**: = 6mm plate.
- **216**: 6mm glue, 1.5mm plate.
- **217**: = 3mm plate.
- **218**: = 6mm plate.
- **219**: = 6mm plate.

End of beam  End of plate

Distance along beam (mm)
FIGURE 6.23 INTERNAL STEEL BAR STRAIN vs PLATE & GLUE THICKNESS

- 15 mm. plate.
- 3 mm. plate.
- 6 mm. plate.
Figure 6.24: External steel plate strain vs. plate & glue thickness.

- X: 1.5 mm plate.
- ▲: 3 mm plate.
- ●: 6 mm plate.

Loads:
- 190 kN
- 130 kN
- 60 kN
releasing the load after failure. At this stage the plastically deformed plate warped away from the concrete beam.

At the load stage prior to failure for the beams with 3 mm thick plates, the plate strains varied from 2300 to 2900 microstrain, again indicating yielding of the plates.

For the beams with 6 mm thick plates the strains varied from 1200 to 1300 microstrain. The yield strain, Fig. 3.9, was 1500 microstrain. This indicates that the plates are approaching their yield point at failure.

In Appendix 7 some assessments of interfacial stresses are made. The shear stress and the anchorage bond stress are calculated from the measured plate strains. Local bond stresses are also given. It should be emphasised that these evaluations of interfacial stresses should be treated qualitatively. The tests were not designed to study such properties and the values found are not limiting nor ultimate stresses.

6.3.2.4 Concrete Strains

At 60 kN load, the compressive concrete strain in the unplated beam was 455 microstrain. For the plated beams the mean values were as follows: for beams with 1.5 mm thick plates - 410 microstrain; for beams with 3 mm thick plates - 356 microstrain and for beams with 6 mm thick plates - 226 microstrain.

At the load stage prior to failure the values were: unplated beam 3350 microstrain; 1.5 mm plate - 2407 microstrain; 3 mm plate - 2080 microstrain; 6 mm plate - 1230 microstrain.

The determination of the ultimate strains involves having the gauge length of the demec extensometer straddling the section at which the beam would fail. Furthermore, the strains recorded are those measured prior to, and not at failure.

Values of 1200 to 5000 microstrain for the ultimate limiting strain of normal strength concrete have been reported (98). For beams with 1.5 mm thick plates and some beams with 3 mm thick plates it would appear that the full strain capacity had been achieved, as was exhibited by crushing of the concrete at failure. The beams with 6 mm plate did not reach their full strain capacity.
Typical concrete strain distributions are given in Fig. 6.25. Before the beams cracked the difference between the strains at the outer tension and compression faces was a small amount arising from the fact that the neutral axis of the composite section is slightly removed from the half depth. This indicates that the modulus of elasticity of the concrete is of the same order in both tension and compression. Above the first crack load, the strain distributions were approximately linear above the neutral axis but non-linear below it. This is because once the concrete has cracked the demec reading is not the true strain, but is an average 'strain' which depends on the positions of cracks. In all the beams the neutral axis position moved closer to the compression face prior to failure, as the steel started to yield. The rise of neutral axis position was greater in the beams with 1.5 mm plate than for beams with 3 mm and 6 mm plates. This showed that the beams were changing from being well under reinforced (1.5 mm plate) to an almost balanced section (6 mm plate).

The mean ratios of neutral axis to effective depths, at the load stage prior to failure were: 0.31 for the unplated beam; 0.40 for beams with 1.5 mm plate; 0.44 for beams with 3 mm plates; and 0.47 for beams with 6 mm plates.

In normally reinforced concrete beams some limitation is placed on the percentage of tension steel in a beam, during the design process. The purpose of this is to ensure that the steel will reach its yield stress before the concrete fails in compression, and thus avoid brittle failure. As tension steel is added to a beam the ratio x/d increases. It has been found by experiment that the steel does not yield when x/d is approximately 0.6, therefore for design it is generally limited to 0.5. For the plated beams in the present series the transition from purely flexural to a more brittle shear/bond failure appears to come between the beams with 1.5 mm and 3 mm plates. Their values of x/d being 0.4 and 0.44 respectively. It would seem, therefore, that a limiting value of x/d of 0.4 would be better for plated beams.
(i) first crack load
(ii) service load
(iii) near failure load
The beams tested by Ang (72) showed a transition from flexural to shear/bond also. This occurred between beams with 3 mm and 5 mm plate, their values of x/d being 0.39 and 0.47 respectively.

However, it is thought that the type of shear bond failure found in the beams with thicker plates could be alleviated, as discussed in Chapter 5 under ultimate loads. By modifying the plate ends such failures could be changed back to a flexural mode. Further testing is needed to verify this, and then the limiting value of x/d suggested by the present series may be increased.

6.3.3 Load Deflection Characteristics

6.3.3.1 Review of Literature

The actual deflection behaviour in beams is probabilistic in nature and requires statistical methods for a rational analysis. Even with the most sophisticated methods of analysis using experimentally determined material properties, the range of variation between measured and computed results is surprisingly high. Studies (82) have shown that the coefficient of variation for such deflections is of the order of 15 to 20%. In addition this difficulty is compounded when applied to actual structures, since the only property of the concrete known to the designer is the specified characteristic strength.

Because of the variability of deflection, it would appear to be not only feasible but also essential that relatively simple procedures be used so that engineers will not place undue reliance on predicted results.

When subjected to a load, a section of a beam undergoes compression on one side and tension on the other, which gives rise to a local curvature at that section. This curvature \( \phi \) may be expressed by

\[
\phi = \frac{\varepsilon_c + \varepsilon_t}{h}
\]

where \( \varepsilon_c \) is the maximum compressive strain at the section,
\( \varepsilon_t \) is the maximum tensile strain at the section, and
\( h \) is the depth of the section.
For a beam consisting of an elastic material, the local curvature $\phi$ is given by
\[ \phi = \frac{M}{EI} \]
where $M$ is the applied moment at the section,
$E$ is the elastic modulus of the beam material, and
$I$ is the second moment of area of the section.

The rotation or deflection of the beam may be found by computing the single or double integral of the local curvatures respectively. In general the deflection, $a$, is given by
\[ a = k \phi x^2 \]
where $k$ is a constant depending on the variation of curvature along the beam and $x$ is the span of the beam.

The most important methods of predicting short term deflections of reinforced concrete members, proposed over the last 20 years are given below:

(a) 1960 - Yu and Winter (76) presented two simple methods for the calculation of instantaneous deflections under service load.

Method A. The deflections are calculated using elastic methods. The cracked, transformed moments of inertia at the midspan is used as a constant value throughout the length of the beam for simple spans.

Method B. To account for the contribution of the concrete between tension cracks to the rigidity of the beam the deflections from method A were multiplied by a correction factor $F$
\[ F = 1 - \frac{M_1}{M} \]
where $M_1 = 0.1 f_{cu}^{2/3} h(h - x)$ and $M$ = service moment. The derivation of the correction factor followed an elastic theory approach with the factor 0.1 having been determined empirically. Comparison with test data for 90 beams indicated that method B gave somewhat better results.
(b) **1961 - Comité European du Beton (78) (CEB)** gave a simplified method for determining deflections under short term loads, in which the value of instantaneous deflection was considered equal to the sum of the deflection of the uncracked section under the moment at which cracking is produced, and the deflection of the cracked section under a moment equal to the working moment less the moment at which cracking is produced. This latter moment being calculated from the tensile strength of the concrete in bending. This can be expressed by:

\[
a = k \left( M_{cr} + \frac{4}{E_C I_u} \frac{(M - M_{cr})}{3 E_s A_s d^2 (1 - 2q)(1 - 2q/3)} \right)
\]

where \(M_{cr}\) is the moment at which cracking is produced,
\(M\) is the moment under consideration,
\(E_C, E_s\) are the elastic moduli of concrete and steel,
\(I_u\) is the uncracked transformed moment of inertia,
\(A_s\) is the area of reinforcement,
\(d\) is the effective depth, and
\[q = \frac{A_s}{bd} \frac{f_y}{f_{cu}}\]

This is a sound and logical method which can allow for the application of a load lower than the working load. In practice, however, there is some uncertainty in assessing the tensile strength of the concrete as this can be reduced considerably by shrinkage.

(c) **1963 - D. E. Branson (77)** gave a form of expression for the effective moment of inertia of a beam in which the effect of bending moment, section properties, concrete strength and extent of cracking is included. The expression satisfies the limiting conditions \(I_{eff} = I_u\) when the moment at that section \(M = M_{cr}\) the cracking moment, and \(I_{eff} = \lim I'_{cr}\) when \(M\) is very large in relation to \(M_{cr}\):

\[
I_{eff} = I_u - \left( I_u - I'_{cr} \right) \left[ 1 - \left( \frac{M_{cr}}{M} \right)^{n} \right]
\]
where \( n \) is an unknown power. A precedent for a power function relation relative to the distribution of cracking effects was established by Murashev (83). Branson used the results from some 58 laboratory beams, (simple and two span rectangular beams and simply supported T beams), to empirically determine the value for \( n \). For the effective moment of inertia at an individual section \( n = 4 \) was found to give good agreement. It was further determined that the same type of equation but with \( n = 3 \) could be used for an average \( I_{\text{eff}} \) for simply supported beams with uniform loading. The equation given above was rewritten as:

\[
I_{\text{eff}} = \left( \frac{M_{\text{cr}}}{M} \right)^3 I_u + \left[ 1 - \left( \frac{M_{\text{cr}}}{M} \right)^3 \right] I'_{\text{cr}}
\]

The deflection, \( a \), is then given by

\[
a = \frac{k M L^2}{E c \ I_{\text{eff}}}
\]

where \( k \) is a coefficient which depends on the type of loading and support conditions.

(d) 1968 - Beeby (84) proposed an idealised form of the moment curvature relationship where \( \phi \), the curvature was given by:

\[
\phi = \frac{M_{\text{cr}}}{E c \ I_u} + \frac{M - M_{\text{cr}}}{X}
\]

Firstly, it was assumed that \( X = E c \ I'_{\text{cr}} \) but this overestimated the values when compared with the results from 133 test beams. Alternative values for \( X \) were given as:

\[
X = E c (0.825 I'_{\text{cr}})
\]

and

\[
X = (0.57 E c) I'_{\text{cr}}
\]

(e) 1970 - CEB (85) for beams of constant section, loaded in simple bending and subjected to symmetrical loading the midspan deflection is given by:

\[
a = k z^2 \left( \frac{M_{\text{cr}}}{E c \ I_u} + \frac{4}{3} \frac{M - M_{\text{cr}}}{E s A z (d - x)} \right)
\]
where $M_{cr}$, $M$, $E_c$, $E_s$, $I_u$ have the same meaning as before.

- $z$ = lever arm,
- $d$ = effective depth, and
- $x$ = neutral axis depth.

This is similar to their earlier recommendations (1961) with the part relating to post cracking stiffness being modified.

(f) **1972 - Stevens (80)** describes two methods for determining deflections from curvatures determined using the maximum tensile and compressive strain in the steel and concrete respectively. In the first method it was assumed that the concrete in the tension zone has no contribution to the rigidity of the beam. In the second method the concrete resists tension between the cracks with as much as 75% of its flexural strength. The second method was found to give better results.

(g) **1972 - CP 110 (86)** suggests an approach which involves finding the curvatures of sections initially, and then calculating the deflection by numerical integration along the beam. The code does, however, suggest a simplified approach.

6.3.3.2 Load-Deflection Curves

Figs. 6.26 to 6.35 show the load-deflection curves for the test beams. Fig. 6.26 shows that for a constant 1.5 mm glue thickness the deflections are reduced with increasing plate thickness. The beam with two layers of 1.5 mm plate gave almost identical results to the beams with a single layer of 3 mm plate. Figs. 6.27 and 6.30 show similar behaviour for beams with 3 mm and 6 mm plates respectively.

Fig. 6.28 shows that the beam with a glue layer, but no plate, is slightly stiffer than the unplated beam. The beams with lapped plates showed the same behaviour as for strains, i.e. slightly reduced deflections compared with those of the beam with a continuous plate layer. Fig. 6.29 also compares a beam with an unlapped 3 mm thick plate to another beam having 3 mm plate and a central lap joint. Two other beams are shown which had two layers.
FIGURE 6.30 LOAD-DEFLECTION CHARACTERISTICS

FAILURE LOADS

- 216 6mm glue, 1.5mm plate.
- 217 = 3mm plate.
- 218 = 6mm plate.
- 219 =

design service load (c) 130 kN

comparison 1st crack load 60 kN

FIGURE 6.31 LOAD-DEFLECTION CHARACTERISTICS

FAILURE LOADS

- 203 1.5mm glue/5mm plate.
- 207 3mm glue, =
- 216 6mm glue, =
- 220 2-8mm glue, =
- 221 3mm glue-notched =

design service load (c) 130 kN

comparison 1st crack load 60 kN

CENTRAL DEFLECTION mm
of 1.5 mm plate, the outer layer being lapped. The behaviour of all four beams fell within ±6% of the beam with the unlapped 3 mm thick plate.

Fig. 6.31 compares beams with a 1.5 mm plate thickness for varying glue thickness. The deflections are reduced as the glue thickness increases, but not so much as when the plate thickness is increased for constant glue thickness. The beams with variable and notched glue lines follow very closely to similar beams with constant glue thickness. Figs. 6.32 and 6.33 show similar behaviour for beams with 3 mm and 6 mm thick plates. The stiffening produced by increasing the glue thickness is reduced as the plate thickness increases.

Figs. 6.34 and 6.35 indicate that the beams loaded prior to bonding on the plates have deflections greater than their control beams up to working load, but less than the control beams above this. The control beams were not precracked. This behaviour, as for load-strains needs further examination.

From Table 6.18 it can be seen that at 60 kN load the deflections, except for the precracked beams, were decreased to 85 to 100% (1.5 mm plate), 70 to 95% (3 mm plate) and 60 to 94% (6 mm plate) of the values of the unplated beam. At 130 kN the deflections were similarly reduced to 78 to 96% (1.5 mm plate), 68 to 87% (3 mm plate), and 61 to 67% (6 mm plate) of the unplated beam's deflection. At 190 kN the deflections were reduced to 69 to 83% (1.5 mm plate), 59 to 69% (3 mm plate) and 51 to 54% (6 mm plate). These results are shown diagrammatically in Fig. 6.36.

6.3.3.3 Theoretical Predictions of Deflections

Table 6.4 shows experimental deflections at 220 kN, 130 kN and 60 kN loads. A measure of the ductility of the beams was found by comparing the deflections at 220 kN and 130 kN loads for each of the test beams. There is a small reduction in ductility in the plated beams, however, only one value was obtained for an unplated beam. The reductions were as follows: 10% for beams with 1.5 mm thick plates; 13% (3 mm plates) and 11% (6 mm plates). It should be noted that two beams, 210 and 218 failed before reaching 220 kN load and their values were extrapolated.
FIGURE 6.36 CENTRAL DEFLECTION v. PLATE & GLUE THICKNESS

- 1.5 mm plate.
- 3 mm plate.
- 6 mm plate.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>GLUE THICKNESS</th>
<th>PLATE FIRST CRACK</th>
<th>130 kN</th>
<th>220 kN</th>
<th>EXPERIMENTAL DEFLECTIONS - mm.</th>
<th>THEORETICAL DEFLECTIONS - (130 kN)</th>
<th>EXPERIMENTAL THEORETICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>EXP.</td>
<td>THEORY</td>
<td>EXP.</td>
</tr>
<tr>
<td>201</td>
<td>-</td>
<td>-</td>
<td>1-00</td>
<td>7-30</td>
<td>20-40</td>
<td>6-12</td>
<td>6-10</td>
</tr>
<tr>
<td>202</td>
<td>3</td>
<td>-</td>
<td>1-20</td>
<td>6-90</td>
<td>17-70</td>
<td>6-12</td>
<td>6-10</td>
</tr>
<tr>
<td>203</td>
<td>1-5</td>
<td>1-5</td>
<td>1-70</td>
<td>6-60</td>
<td>16-20</td>
<td>5-00</td>
<td>5-00</td>
</tr>
<tr>
<td>204</td>
<td>1-5</td>
<td>3</td>
<td>1-65</td>
<td>6-00</td>
<td>13-80</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>205</td>
<td>1-5</td>
<td>6</td>
<td>1-20</td>
<td>4-60</td>
<td>11-00</td>
<td>3-50</td>
<td>3-43</td>
</tr>
<tr>
<td>206</td>
<td>1-5</td>
<td>2x1-5</td>
<td>1-40</td>
<td>5-70</td>
<td>13-00</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>207</td>
<td>3</td>
<td>1-5</td>
<td>1-80</td>
<td>6-00</td>
<td>16-00</td>
<td>5-00</td>
<td>5-00</td>
</tr>
<tr>
<td>208</td>
<td>3</td>
<td>3</td>
<td>1-40</td>
<td>5-00</td>
<td>12-60</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>209</td>
<td>3</td>
<td>6</td>
<td>1-30</td>
<td>4-50</td>
<td>11-50</td>
<td>3-50</td>
<td>3-43</td>
</tr>
<tr>
<td>210</td>
<td>3</td>
<td>6</td>
<td>1-20</td>
<td>4-30</td>
<td>11-00</td>
<td>3-50</td>
<td>3-43</td>
</tr>
<tr>
<td>211</td>
<td>3</td>
<td>1-5L</td>
<td>1-65</td>
<td>5-60</td>
<td>14-00</td>
<td>5-00</td>
<td>4-89</td>
</tr>
<tr>
<td>212</td>
<td>3</td>
<td>1-5L</td>
<td>1-35</td>
<td>5-60</td>
<td>14-80</td>
<td>5-00</td>
<td>4-89</td>
</tr>
<tr>
<td>213</td>
<td>3</td>
<td>2x4.5L</td>
<td>1-45</td>
<td>5-40</td>
<td>13-00</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>214</td>
<td>3</td>
<td>2x4.5L</td>
<td>1-35</td>
<td>4-80</td>
<td>12-00</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>215</td>
<td>3</td>
<td>3L</td>
<td>1-35</td>
<td>4-70</td>
<td>11-60</td>
<td>4-20</td>
<td>4-30</td>
</tr>
<tr>
<td>216</td>
<td>6</td>
<td>1-5</td>
<td>1-60</td>
<td>5-70</td>
<td>14-00</td>
<td>5-00</td>
<td>4-89</td>
</tr>
<tr>
<td>217</td>
<td>6</td>
<td>3</td>
<td>1-00</td>
<td>4-80</td>
<td>11-60</td>
<td>4-20</td>
<td>4-22</td>
</tr>
<tr>
<td>218</td>
<td>6</td>
<td>6</td>
<td>0-90</td>
<td>4-20</td>
<td>10-80</td>
<td>3-50</td>
<td>3-43</td>
</tr>
<tr>
<td>219</td>
<td>6</td>
<td>6</td>
<td>1-20</td>
<td>4-20</td>
<td>10-40</td>
<td>3-50</td>
<td>3-43</td>
</tr>
<tr>
<td>220</td>
<td>2</td>
<td>8</td>
<td>1-70</td>
<td>5-40</td>
<td>13-80</td>
<td>5-00</td>
<td>4-89</td>
</tr>
<tr>
<td>221</td>
<td>3</td>
<td>1-5</td>
<td>1-70</td>
<td>6-60</td>
<td>16-00</td>
<td>5-00</td>
<td>4-89</td>
</tr>
<tr>
<td>222</td>
<td>3</td>
<td>1-5P</td>
<td>2-00</td>
<td>6-40</td>
<td>13-50</td>
<td>5-00</td>
<td>5-00</td>
</tr>
<tr>
<td>223</td>
<td>3</td>
<td>1-5P</td>
<td>2-60</td>
<td>7-00</td>
<td>14-60</td>
<td>5-00</td>
<td>5-00</td>
</tr>
<tr>
<td>224</td>
<td>3</td>
<td>3P</td>
<td>1-65</td>
<td>4-80</td>
<td>11-00</td>
<td>4-20</td>
<td>4-30</td>
</tr>
</tbody>
</table>

Mean ratios: 1-23 1-23 1-05

L - lapped plates.
P - precracked.
The deflections of all the beams, as calculated by the three methods described in Appendix 5, are given in Table 6.4 for comparison with experimental values at 130 kN load. The mean ratio of measured to predicted deflection for all the beams were as follows: 1.23 (CP110); 1.23 (A.C.I.); 1.05 (CEB).

Within the present tests the CEB method for predicting deflections gives the best results. The test results by Ang (72) were studied and the three methods of calculation were found to give mean ratios of measured to theoretical deflections of 1.15 (CP110), 1.21 (A.C.I.) and 1.01 (CEB), confirming the latter as the most appropriate method.

6.3.4 Moment-Rotation Characteristics

6.3.4.1 Literature Review

Figure 6.37(a) shows the moment-curvature diagram of a typical reinforced concrete beam. The total rotation can be found by integrating the local curvatures along the beam. The shape of the curve is a reflection of the behaviour of the materials making up the cross section. The curve can be divided into three parts corresponding to the different stages of behaviour. The first part, OX, is characterised by a linear relationship between the moment and curvature, since the section is uncracked and both concrete and steel behave elastically. The second part, XY, is characterised by a changing slope of the curve. In this region the area of concrete in compression is decreasing as the cracks spread towards the compression face. The concrete compression fibres and the steel are approaching their inelastic strain range. At point Y, either the steel or concrete begins to behave inelastically. The third part, YZ, is characterised by a rapidly changing slope of the curve. Between Y and Z the section reaches its ultimate capacity, the concrete under compression starts cracking and the tension steel reaches its strain hardening stage, at least in under-reinforced beams. The curvatures at these stages are shown in Fig. 6.37(b). Distribution 1 is typical when the beam is in the uncracked state and the characteristics of the beam along its length are the
FIGURE 6.37  MOMENT CURVATURE RELATIONSHIP

(a) MOMENT CURVATURE RELATIONSHIP

(b) MOMENT AND CURVATURE DISTRIBUTION
same, thus the curvature distribution follows the bending moment distribution. As a result of the spread of cracks along the tension side, the characteristics of the cross-section vary along the beam. This corresponds to Distribution 2 and point Y. The distribution is non-linear, especially near the loading point. Distribution 3 corresponds to point Z on the moment curvature curve. The curvature increases very rapidly over the region near the critical section, while it remains nearly linear over the rest of the beam. The total rotation over the length of the beam at any particular stage of loading can be obtained either by integrating the curvature along the length of the beam or by measuring the support rotations and adding them together.

6.3.4.2 Moment Rotation Curves

Figs. 6.38 and 6.47 show the moment-rotation curves of the twenty four test beams. The same general behaviour, discussed under load-strain and load-deflection, was observed for the moment-rotation curves.

In Tables 6.1 and 6.3 the experimental rotations are given. At 60 kN load, which is slightly above their first crack load, the total rotations of the plated beams were decreased to between 50 and 92% of the value for the unplated beam. For beams strengthened with 1.5 mm plate the rotations were reduced to 80-92% of the unplated beams value. Similarly for 3 mm and 6 mm plates the reductions were to 65-70% and 50-58% respectively. At 130 kN load the rotations were reduced in a similar manner to 81-94% (1.5 mm plate); 69-73% (3 mm plate) and 51-69% (6 mm plate). At 190 kN the rotations were reduced to 83-94% (1.5 mm plate); 69-75% (3 mm plate) and 51-70% (6 mm plate). These results are shown diagramatically in Fig. 6.48.

6.3.4.3 Theoretical Prediction of Rotations

Table 6.5 shows the comparison between the measured rotations and those calculated in Appendix 6. It can be seen that as the load increases, the difference between experiment and theory increases. The rotations are greatly under estimated above service load. The calculated values assuming the tensile strength of concrete was 1 N/mm² gave slightly better values than for 3 N/mm².
FIGURE 6.38  MOMENT - ROTATION CHARACTERISTICS

FIGURE 6.39  MOMENT - ROTATION CHARACTERISTICS

design service load (c) 130 kN

comparison #1 crack load - 60 kN

- 203 1.5 mm glue, 15 mm plate.
- 204 3 mm plate.
- 205 6 mm plate.
- 206 2x15 mm plates.

- 207 3 mm glue, 15 mm plate.
- 208 3 mm plate.
- 209 6 mm plate.
- 210 = = = =

0 10 20 30 40 50 60 70 80 90 100 110 120
APPLIED MOMENT kN

50 100 200 300 400
TOTAL ROTATION radians 10^-4
**FIGURE 6.40**  MOMENT-ROTATION CHARACTERISTICS

- **DESIGN SERVICE LOAD (C) 130 kN**
- **Comparison**
  - Crack load = 60 kN

Legend:
- □ 216 6 mm glue, 1.5 mm plate
- ○ 217 = 3 mm plate
- △ 218 = 6 mm plate
- □ 219 = *
- ○ 201 no glue, no plate
- △ 202 3 mm glue, no plate
- □ 207 = 1.5 mm plate
- ▽ 211 = centre lap
- × 212 = 1/3 point laps

**FIGURE 6.41**  MOMENT-ROTATION CHARACTERISTICS
FIGURE 6.48  ROTATION CHARACTERISTICS

ADHESIVE THICKNESS mm.

-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
-190
-200
-210
-220
-230
-240
-250

1.0 2.0 3.0 4.0 5.0 6.0

x 1.5 mm plate
△ 3 mm plate
○ 6 mm plate

190 kN
130 kN
60 kN
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>PLATE THICKNESS (mm)</th>
<th>GLUE THICKNESS (mm)</th>
<th>EXPERIMENTAL ROTATIONS (radians 10^-4)</th>
<th>ROTATION FROM THEORY IN APP. 6 (radians 10^-4)</th>
<th>ROTATION FROM EMPIRICAL FORMULA (radians 10^-4)</th>
<th>% DIFFERENCE BETWEEN MEASURED &amp; EMPIRICAL FORMULA</th>
</tr>
</thead>
<tbody>
<tr>
<td>203</td>
<td>1.5</td>
<td>1.5</td>
<td>60 150 260 443</td>
<td>60 134 195 253</td>
<td>61 152 258 426</td>
<td>+2  +1  -1  -4</td>
</tr>
<tr>
<td>207</td>
<td>1.5</td>
<td>3.0</td>
<td>55 145 250 430</td>
<td>61 135 195 247</td>
<td>61 150 255 413</td>
<td>+11 +3  +2  -4</td>
</tr>
<tr>
<td>216</td>
<td>1.5</td>
<td>6.0</td>
<td>52 133 235 430</td>
<td>56 131 199 263</td>
<td>56 146 253 418</td>
<td>+8  +9  +8  -3</td>
</tr>
<tr>
<td>204</td>
<td>3.0</td>
<td>1.5</td>
<td>45 116 200 380</td>
<td>50 116 172 224</td>
<td>50 128 215 341</td>
<td>+11 +12 +8 -10</td>
</tr>
<tr>
<td>208</td>
<td>3.0</td>
<td>3.0</td>
<td>45 110 195 360</td>
<td>50 115 169 212</td>
<td>50 121 209 330</td>
<td>+11 +10 +7 -12</td>
</tr>
<tr>
<td>217</td>
<td>3.0</td>
<td>6.0</td>
<td>45 110 185 352</td>
<td>50 115 168 224</td>
<td>50 122 209 323</td>
<td>+11 +11 +11 -9</td>
</tr>
<tr>
<td>205</td>
<td>6.0</td>
<td>1.5</td>
<td>37 102 145 188</td>
<td>42 93 115 137</td>
<td>42 100 128 160</td>
<td>+8  -2  -12 -10</td>
</tr>
<tr>
<td>209</td>
<td>6.0</td>
<td>3.0</td>
<td>37 110 140 185</td>
<td>41 92 133 153</td>
<td>41 98 148 179</td>
<td>+8  -11 +6  -3</td>
</tr>
<tr>
<td>218</td>
<td>6.0</td>
<td>6.0</td>
<td>38 88 120 175</td>
<td>41 91 112 134</td>
<td>41 98 125 155</td>
<td>+8  +11 +4  -11</td>
</tr>
</tbody>
</table>

**Upper value** $f_{ct} = 1 \text{N/mm}^2$  $f_{cr} = 3 \text{N/mm}^2$

**Lower value** $f_{ct} = 1 \text{N/mm}^2$  $f_{cr} = 3 \text{N/mm}^2$
To produce a more accurate prediction of rotational behaviour, some allowance must be made for the change in Young's Modulus which occurs as the strain in the compressive concrete increases. In order to check the validity of the present test results the 'correction' to $E_c$ should include a factor which can be applied to other test beams. The compressive strain in the concrete at any particular load is dependant upon the degree of loading in relation to its theoretical capacity. Then we have

$$E_{\text{corrected}} = E_c - k \left( \frac{W}{W_{\text{ultimate}}} \right)^c$$

where $k$ and $c$ are constants,

$W$ = load stage under consideration,

$W_{\text{ultimate}}$ = theoretical ultimate load.

Then $\log (E_c - E_{\text{corrected}}) = \log k + c \log \left( \frac{W}{W_{\text{ultimate}}} \right)$.

The experimental rotations are used to find the $E_{\text{corrected}}$ required at each load stage, using 1 N/mm² for the tensile strength of concrete. The value of $E_c - E_{\text{corrected}}$ is then found and the log values were plotted against $\log \left( \frac{W}{W_{\text{ultimate}}} \right)$ in Fig. 6.49. The values of $k$ and $c$ were found by plotting the best fit line by linear regression.

The resulting formula is: $E_{\text{corrected}} = E_c - 1.37.10^4 \left( \frac{W}{W_{\text{ultimate}}} \right)^{1.88}$.

Table 6.5 gives the results from this formula and also the percentage difference between these values and experiment. In Fig. 6.50 the calculated values are plotted against experimental values and for the range of glue and plate thicknesses used, all points fell within ±12% of the experimental = theoretical line.

6.4 CONCLUSIONS

Based on the results presented in this Chapter the following conclusions can be drawn.

1. As the thickness of the reinforcing plate was increased, for a constant glue line thickness, there was a corresponding reduction in: plate, bar and concrete strains, central deflection and total rotation.
FIGURE 6.49

EMPIRICAL FORMULA FOR THEORETICAL ROTATIONS

\[
\log_e \left( \frac{E_c - E_{\text{corrected}}}{W_{\text{Ultimate}}} \right) = 9.53 - 1.68 \log_e \left( \frac{W}{W_{\text{Ultimate}}} \right)
\]
FIGURE 6.50 EXPERIMENTAL V. THEORETICAL ROTATIONS
2. As the thickness of the glue line was increased, for a constant plate thickness, there was a reduction in these same properties but to a lesser degree.

3. Beams with multiple layers of plate behaved almost identically to beams with a single plate of the same total thickness.

4. Beams with lapped plates were slightly stiffer than beams with continuous plates of the same thickness. This was probably due to the slightly increased lever arm of the lapping plate at the critical section.

5. The beams with notched and variable glue line thickness behaved almost identically to the corresponding beam with a constant glue thickness.

6. The preloaded beams had smaller strains, deflections and rotations, in general, than corresponding beams which had not been cracked prior to bonding on the plates. This behaviour does not seem logical and further testing should be performed to investigate these findings more fully.

7. The deflections at service loads were slightly underestimated using CP110 and A.C.I. recommended calculations. The method given by CEB gave the best results having a mean ratio of experimental to theoretical values of 1.05.

8. The ductility of the plated beams was approximately 12% less than the unplated beam. However, as there was only one result for an unplated beam this figure can only be approximate.

9. The rotations could be predicted to within ±12% of the experiment using the following formula for the Young's Modulus of the concrete.

\[ E_{\text{corrected}} = E_C - 1.37.10 \left( \frac{W_{\text{ultimate}}}{W} \right)^{1.88} \]
CHAPTER 7
CRACKING PROPERTIES

7.1 INTRODUCTION

The tensile strength of concrete is of the order of one tenth of its compressive strength, and the tensile strain at which it cracks is of the order of 100 microstrain. This is only a fraction of the ultimate strain of the steel which it surrounds. Obviously, therefore, the formation of cracks, even in well designed reinforced concrete structures, is unavoidable. Since cracking is one of the criteria which a design has to satisfy in the limit state design of reinforced concrete structures, it is necessary that the cracks should be kept as small as possible for two main reasons. Firstly, wide cracks are aesthetically unpleasant, and can cause public concern. Secondly, corrosive elements can penetrate to the main steel which can lead to weakening of the structure. In addition, in plated structures, such penetration could cause degradation of the adhesive bonding mechanism. In the past, flexural cracks cause little concern since the relatively low permissible steel stresses used in design ensured that cracks would not be large. Recently there has been a considerable increase in permissible stresses in steel, and since crack widths are proportional to steel stress the crack widths are increased. It has therefore become necessary to know, with greater certainty the factors which govern cracking and to be able to predict and control crack widths and spacings.

7.2 REVIEW OF LITERATURE

Gergely and Lutz (89) analysed, statistically, information from six experimental investigations. The major conclusions which were drawn, regarding factors affecting crack widths were:

1. The steel stress is the most important variable.
2. The cover thickness is an important variable.
3. The bar diameter is not a major variable.
4. The size of the side crack width is reduced by the proximity of the compression zone in flexural members.
5. The bottom crack width increases with concrete strain gradient across the section in flexural members.
Illston and Stevens (90) investigated surface and internal cracking in reinforced concrete beams through a resin injection technique. At working loads, the spacings of flexural cracks were found to be a function of concrete cover and the surface crack width was a function of spacing and steel stresses.

The Cement and Concrete Association conducted an extensive investigation by Base et al (91). Their report gave factors which influenced the width and distribution of cracks in zones of uniform bending moment in reinforced concrete beams. The most important factors affecting crack widths were:

1. The magnitude of the steel stress.
2. The cover thickness.
3. No evidence was found that there was any effect on the crack width when varying the type of reinforcement, and the percentage within the range 0.85 to 2.29%.
4. No evidence was produced that variation of concrete strength or curing conditions had a significant effect on cracking.

The British Code of Practice CP110, (86) recommends that the surface crack width shall not exceed certain limits depending on the environment, exposure and service requirements of a member. When exposed to aggressive environments, the assessed surface crack widths at points nearest to the main reinforcement should not exceed 0.004 times the nominal cover. The formula in Appendix A of the code was proposed by Beeby (92).

\[
\varepsilon_m = \varepsilon_1 - \frac{1.2.2.b.h.(\bar{a} - x).10^{-3}}{A_s.f_y.(h - x)}
\]

The formula for determining the design crack width, \( W_{cr} \), is then given by:

\[
W_{cr} = \frac{3 \cdot a_{cr} \cdot \varepsilon_m}{1 + \frac{2(a_{cr} - \varepsilon_{min})}{h-x}}
\]

All symbols as defined earlier.
The American Concrete Institute Code (89) recommends that the maximum crack width at the level of the reinforcement is given by:

\[ W_{\text{max}} = \frac{0.0913 \sqrt{t_s A}}{1 + t_s/h_1} (fs - 5) \times 10^{-3} \] (IMPERIAL UNITS)

and that the maximum crack width at the tension face is given by:

\[ W_{\text{max}} = 0.0913 \sqrt{t_b A} \times R(fs - 5) \times 10^{-3} \] (IMPERIAL UNITS)

where:
- \( t_s \) = side cover to centre of longitudinal reinforcing bar, (inches)
- \( t_b \) = bottom cover to centre of longitudinal reinforcing bar, (inches)
- \( fs \) = steel stress based on an elastic cracked section, (kips/in²)
- \( R = h_2/h_1 \)
- \( h_1 \) = distance from neutral axis to the tension steel (inches)
- \( h_2 \) = distance from neutral axis to the tension face, (inches)
- \( A = \frac{2b(h - d)}{\text{number of bars}} \)
- \( b \) = width of beam, (inches)
- \( h \) = overall depth of beam, (inches)
- \( d \) = effective depth, (inches)

It is also stated that the cross section at the maximum bending moment should be proportioned so that \( fs \times \sqrt{t_b A} \) does not exceed 170 kips/inch for interior exposure and 145 kips/inch for exterior exposure. These values correspond to crack widths of 0.4 and 0.3 mm, respectively.

The Comité Euro-International du Beton (85) recommend that the characteristic value of the crack width, \( W_{k} \), at the reinforcement level should not exceed 1.7 times the mean crack width, \( W_{m} \).

\[ W_{m} = S_{\text{rm}} \times \varepsilon_{\text{sm}} \]

where:
- \( S_{\text{rm}} \) = the final mean crack spacing at the reinforcement level
- \( \varepsilon_{\text{sm}} \) = the mean elongation of the reinforcement allowing for the contribution of the concrete in tension.

and:
\[ S_{\text{rm}} = 2c + \frac{k\Phi}{\rho} \]
where,
\[ c = \text{cover to reinforcement} \]
\[ \phi = \text{diameter of largest bars} \]
\[ \rho = \frac{A_s}{2b(h-d)}, \quad A_s = \text{area of steel bars} \]

\( k \) is a coefficient which depends on the type of steel and the mode of loading.

also:
\[ \varepsilon_{sm} = \frac{\sigma_s}{E_s} \left[ 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2 \right] \]

where,
\[ \sigma_s = \text{steel stress based on an elastic cracked section} \]
\[ \sigma_{sr} = \text{steel stress calculated on the assumption that the concrete in tension reaches its maximum tensile strength} \]

\( \beta \) is a coefficient which depends on bond characteristics; 0.7 for high bond bars and zero for smooth bars.

7.3 **EXPERIMENTAL PROGRAMME**

Previous research (89-93) has revealed that certain variables have a strong influence on cracking. The most important of these being

1. The stress in the steel
2. The cover to reinforcement
3. The proximity of the concrete compression zone.

The addition of an externally bonded steel plate should effectively satisfy all three of these requirements. As has been shown in the section on strains; for the plated beams the internal bar strains are reduced considerably, and also the neutral axis is brought closer to the level of reinforcement.

The test beams 201 to 224, already described, were used to investigate the cracking behaviour. The side and bottom cover to the reinforcement were kept constant and the effect on crack width at the level of reinforcement would then be related to the glue and plate parameters.

At each load stage the widths, heights and spacings of all the cracks in the constant moment region were noted. The crack widths and spacings were read at the level of the internal reinforcement.
7.4 DISCUSSION OF RESULTS

7.4.1 General-statistical analysis

The assessment of crack widths was made on a statistical basis. The widths of all the cracks that appeared within the constant moment region were read at the level of reinforcement, and the values of crack width which have a 1% chance of being exceeded were determined. Crack widths determined in this manner are subject to less experimental error than the measured values of maximum crack width, since the former are determined from measurements on the entire population of cracks. Normal distribution of crack widths is a generally accepted phenomenon, and has been proved by Base et al (91) in their investigations.

Instead of plotting the width of a single crack, the mean width of all cracks in the constant moment region, at the reinforcement level, was plotted against the average strain in the concrete at the same level, the latter being determined from Demec gauge readings on a 200 mm gauge length at the centre section. As was stated earlier, the strain in the reinforcing bar and the surface concrete strain are approximately equal. Figure 6.1 confirms this. The Demec readings were used, instead of electrical strain gauge readings, as they gave average strain rather than local strains. At each load stage the following values were measured and are given in Tables 7.1 - 7.6.

(i) maximum crack width
(ii) mean crack width, height, spacing and standard deviations
(iii) number of cracks in the constant moment region.

Figs. 7.1 - 7.3 show the graphs of mean crack width versus the concrete strain at the reinforcement levels. A roughly linear relationship can be obtained. Although the stress in the reinforcement is not used directly it may be calculated by reading off the strain in the concrete surface at the reinforcement level and multiplying by the elastic modulus of steel. For stresses within the linear range of the steel behaviour, the graphs indicated that the mean crack width was proportional to the stress in the reinforcement, as confirmed by many other authors. (89-93).
### TABLE 7-3: CRACKING CHARACTERISTICS

<table>
<thead>
<tr>
<th>No.</th>
<th>Unconfined</th>
<th>width</th>
<th>Standard Deviation</th>
<th>Coeff. of Variation</th>
<th>Mean Crack Spacing</th>
<th>Standard Deviation</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3</td>
<td>200</td>
<td>117</td>
<td>0.29</td>
<td>25</td>
<td>62</td>
<td>47</td>
<td>7</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>350</td>
<td>180</td>
<td>0.60</td>
<td>33</td>
<td>101</td>
<td>32</td>
<td>21</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>580</td>
<td>280</td>
<td>1.20</td>
<td>43</td>
<td>79</td>
<td>23</td>
<td>30</td>
</tr>
<tr>
<td>150</td>
<td>11</td>
<td>830</td>
<td>390</td>
<td>2.20</td>
<td>56</td>
<td>60</td>
<td>21</td>
<td>35</td>
</tr>
<tr>
<td>140</td>
<td>14</td>
<td>1030</td>
<td>610</td>
<td>4.60</td>
<td>56</td>
<td>50</td>
<td>20</td>
<td>31</td>
</tr>
<tr>
<td>250</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) beam 209 \( W_{\text{max}} / W_{\text{mean}} = 1.63 \)

(b) beam 210 \( W_{\text{max}} / W_{\text{mean}} = 1.66 \)

(c) beam 211 \( W_{\text{max}} / W_{\text{mean}} = 2.20 \)

(d) beam 212 \( W_{\text{max}} / W_{\text{mean}} = 2.0 \)

### TABLE 7-4: CRACKING CHARACTERISTICS

<table>
<thead>
<tr>
<th>No.</th>
<th>Unconfined</th>
<th>width</th>
<th>Standard Deviation</th>
<th>Coeff. of Variation</th>
<th>Mean Crack Spacing</th>
<th>Standard Deviation</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>8</td>
<td>250</td>
<td>2.50</td>
<td>0.75</td>
<td>30</td>
<td>4.0</td>
<td>101</td>
<td>16</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>450</td>
<td>3.80</td>
<td>1.70</td>
<td>45</td>
<td>7.0</td>
<td>87</td>
<td>22</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>780</td>
<td>3.80</td>
<td>1.70</td>
<td>57</td>
<td>8.0</td>
<td>80</td>
<td>23</td>
</tr>
<tr>
<td>150</td>
<td>11</td>
<td>1200</td>
<td>5.00</td>
<td>2.83</td>
<td>54</td>
<td>10.0</td>
<td>65</td>
<td>28</td>
</tr>
<tr>
<td>150</td>
<td>12</td>
<td>1930</td>
<td>10.53</td>
<td>3.24</td>
<td>31</td>
<td>20.0</td>
<td>65</td>
<td>28</td>
</tr>
<tr>
<td>250</td>
<td>15</td>
<td>2560</td>
<td>14.26</td>
<td>5.80</td>
<td>57</td>
<td>29.0</td>
<td>59</td>
<td>30</td>
</tr>
</tbody>
</table>

(a) beam 213 \( W_{\text{max}} / W_{\text{mean}} = 2.0 \)

(b) beam 214 \( W_{\text{max}} / W_{\text{mean}} = 1.62 \)

(c) beam 215 \( W_{\text{max}} / W_{\text{mean}} = 2.20 \)

(d) beam 216 \( W_{\text{max}} / W_{\text{mean}} = 1.70 \)
### Table 7-5: Cracking Characteristics

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>400</td>
<td>1.80</td>
<td>0.45</td>
<td>138</td>
<td>22</td>
</tr>
<tr>
<td>100</td>
<td>700</td>
<td>1.60</td>
<td>48</td>
<td>93</td>
<td>37</td>
</tr>
<tr>
<td>130</td>
<td>670</td>
<td>1.80</td>
<td>45</td>
<td>82</td>
<td>22</td>
</tr>
<tr>
<td>160</td>
<td>1060</td>
<td>1.60</td>
<td>44</td>
<td>62</td>
<td>21</td>
</tr>
<tr>
<td>220</td>
<td>1800</td>
<td>1.40</td>
<td>43</td>
<td>53</td>
<td>20</td>
</tr>
<tr>
<td>250</td>
<td>2200</td>
<td>1.40</td>
<td>45</td>
<td>53</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) beam 217: $W_{\text{max}} / W_{\text{mean}} = 1.75$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>1.33</td>
<td>0.41</td>
<td>144</td>
<td>36</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>2.56</td>
<td>43</td>
<td>90</td>
<td>40</td>
</tr>
<tr>
<td>130</td>
<td>11</td>
<td>3.78</td>
<td>57</td>
<td>71</td>
<td>27</td>
</tr>
<tr>
<td>160</td>
<td>12</td>
<td>4.71</td>
<td>56</td>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>220</td>
<td>15</td>
<td>7.37</td>
<td>55</td>
<td>12</td>
<td>111</td>
</tr>
<tr>
<td>250</td>
<td>15</td>
<td>10.6</td>
<td>67</td>
<td>51</td>
<td>12</td>
</tr>
</tbody>
</table>

(b) beam 218: $W_{\text{max}} / W_{\text{mean}} = 1.94$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>1.80</td>
<td>23</td>
<td>138</td>
<td>42</td>
</tr>
<tr>
<td>100</td>
<td>7</td>
<td>1.29</td>
<td>31</td>
<td>90</td>
<td>40</td>
</tr>
<tr>
<td>130</td>
<td>12</td>
<td>1.58</td>
<td>35</td>
<td>88</td>
<td>29</td>
</tr>
<tr>
<td>160</td>
<td>14</td>
<td>1.68</td>
<td>34</td>
<td>68</td>
<td>19</td>
</tr>
<tr>
<td>220</td>
<td>14</td>
<td>3.59</td>
<td>48</td>
<td>100</td>
<td>57</td>
</tr>
<tr>
<td>250</td>
<td>14</td>
<td>4.65</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(c) beam 219: $W_{\text{max}} / W_{\text{mean}} = 1.55$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>2.25</td>
<td>0.36</td>
<td>107</td>
<td>24</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>2.05</td>
<td>33</td>
<td>80</td>
<td>23</td>
</tr>
<tr>
<td>130</td>
<td>12</td>
<td>2.25</td>
<td>38</td>
<td>60</td>
<td>27</td>
</tr>
<tr>
<td>160</td>
<td>13</td>
<td>2.54</td>
<td>47</td>
<td>58</td>
<td>27</td>
</tr>
<tr>
<td>220</td>
<td>16</td>
<td>4.16</td>
<td>58</td>
<td>47</td>
<td>18</td>
</tr>
<tr>
<td>250</td>
<td>17</td>
<td>3.14</td>
<td>78</td>
<td>55</td>
<td>45</td>
</tr>
</tbody>
</table>

(d) beam 220: $W_{\text{max}} / W_{\text{mean}} = 1.71$

---

### Table 7-6: Cracking Characteristics

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>5</td>
<td>1.60</td>
<td>0.94</td>
<td>38</td>
<td>35</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>1.90</td>
<td>1.67</td>
<td>53</td>
<td>60</td>
</tr>
<tr>
<td>130</td>
<td>13</td>
<td>1.38</td>
<td>1.10</td>
<td>28</td>
<td>60</td>
</tr>
<tr>
<td>160</td>
<td>14</td>
<td>1.55</td>
<td>1.68</td>
<td>32</td>
<td>11.0</td>
</tr>
<tr>
<td>220</td>
<td>17</td>
<td>2.71</td>
<td>3.85</td>
<td>17.0</td>
<td>45</td>
</tr>
<tr>
<td>250</td>
<td>17</td>
<td>4.65</td>
<td>8.47</td>
<td>27.0</td>
<td>45</td>
</tr>
</tbody>
</table>

(a) beam 221: $W_{\text{max}} / W_{\text{mean}} = 1.71$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>9</td>
<td>1.83</td>
<td>0.80</td>
<td>44</td>
<td>35</td>
</tr>
<tr>
<td>100</td>
<td>12</td>
<td>3.13</td>
<td>1.38</td>
<td>42</td>
<td>55</td>
</tr>
<tr>
<td>130</td>
<td>13</td>
<td>5.56</td>
<td>2.10</td>
<td>38</td>
<td>10.0</td>
</tr>
<tr>
<td>160</td>
<td>13</td>
<td>6.68</td>
<td>2.36</td>
<td>35</td>
<td>12.0</td>
</tr>
<tr>
<td>220</td>
<td>15</td>
<td>9.51</td>
<td>3.31</td>
<td>35</td>
<td>14.0</td>
</tr>
<tr>
<td>250</td>
<td>15</td>
<td>15.72</td>
<td>4.91</td>
<td>31</td>
<td>24.0</td>
</tr>
</tbody>
</table>

(b) beam 222: $W_{\text{max}} / W_{\text{mean}} = 1.72$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>1.80</td>
<td>0.98</td>
<td>54</td>
<td>3.0</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>4.80</td>
<td>2.10</td>
<td>60</td>
<td>6.0</td>
</tr>
<tr>
<td>130</td>
<td>11</td>
<td>9.30</td>
<td>2.57</td>
<td>48</td>
<td>10.0</td>
</tr>
<tr>
<td>160</td>
<td>14</td>
<td>12.00</td>
<td>3.30</td>
<td>51</td>
<td>12.0</td>
</tr>
<tr>
<td>220</td>
<td>14</td>
<td>17.00</td>
<td>3.30</td>
<td>36</td>
<td>15.0</td>
</tr>
<tr>
<td>250</td>
<td>14</td>
<td>22.00</td>
<td>4.30</td>
<td>30</td>
<td>20.0</td>
</tr>
</tbody>
</table>

(c) beam 223: $W_{\text{max}} / W_{\text{mean}} = 1.73$

<table>
<thead>
<tr>
<th>Width</th>
<th>Mean Crack Width</th>
<th>Coefficient of Variation</th>
<th>Mean Crack Spacing</th>
<th>Mean Crack Height</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>10</td>
<td>2.70</td>
<td>0.44</td>
<td>30</td>
<td>7.9</td>
</tr>
<tr>
<td>100</td>
<td>12</td>
<td>5.90</td>
<td>1.16</td>
<td>4.6</td>
<td>4.5</td>
</tr>
<tr>
<td>130</td>
<td>12</td>
<td>7.00</td>
<td>1.43</td>
<td>3.9</td>
<td>7.0</td>
</tr>
<tr>
<td>160</td>
<td>12</td>
<td>9.00</td>
<td>1.78</td>
<td>3.2</td>
<td>9.0</td>
</tr>
<tr>
<td>220</td>
<td>13</td>
<td>14.80</td>
<td>3.40</td>
<td>42</td>
<td>14.0</td>
</tr>
<tr>
<td>250</td>
<td>13</td>
<td>18.50</td>
<td>5.10</td>
<td>48</td>
<td>19.0</td>
</tr>
</tbody>
</table>

(d) beam 224: $W_{\text{max}} / W_{\text{mean}} = 1.72$
FIGURE 7.1  MEAN CRACK WIDTH & STANDARD DEVIATION  V.  CONCRETE
STRAIN AT THE LEVEL OF INTERNAL REINFORCEMENT
FIGURE 7.2 MEAN CRACK WIDTH & STANDARD DEVIATION V. CONCRETE STRAIN AT THE LEVEL OF INTERNAL REINFORCEMENT
FIGURE 7.3 MEAN CRACK WIDTH & STANDARD DEVIATION V. CONCRETE STRAIN AT THE LEVEL OF INTERNAL REINFORCEMENT
The method of analysis of the results was based upon the slopes of the graphs of mean crack width and standard deviation against the average concrete surface strain, at the level of reinforcement. The slopes in all cases were found from computed "best fit" lines based on a linear regression analysis. (See also Appendix 9). The values, as shown on Figs. 7.1 to 7.3, were then plotted against the thickness of reinforcing plate for each glue line thickness as shown in Fig. 7.4.

It can be seen from Fig. 7.4 that the results are relatively scattered. However, this is not unexpected as it has been shown that cracking is a random phenomenon (89,93), and a scatter of ±50% could be expected. For each glue thickness a best fit line was plotted resulting in an experimental expression for the relationship between the slope of the mean crack width v. concrete strain, S, and the plate thickness, t_p. To combine the three equations shown on Fig. 7.4 it is assumed that there is a linear variation of S with change of glue thickness, t_g. The combined equation is then in the form:

$$S = (K_1 + K_2 t_g) t_p + K_3 t_g + K_4$$

To find the values of K_1 to K_4, the slopes and intercepts of each of the three relationships given on Fig. 7.4 are plotted against glue thickness in Fig. 7.5, and the best fit lines were then drawn. The resulting relationship for all three glue and plate thicknesses is:

$$S = [(47.2 - 1.55 t_g) t_p + 34.3 t_g + 210]. 10^{-1}$$

The slopes found from this expression, over the range of experimental glue and plate thicknesses, were plotted against the experimental values as shown in Fig. 7.6. All values fell within +13% and -5%.

The results found by Ang (72) were also plotted for comparison. These values are consistently greater than those found in the present investigation. More beam test results need to be added to refine the proposed formula.

In Fig. 7.7 the mean ultimate crack spacings, S_u, were plotted against the mean initial crack heights, for all the beams except those which had been precracked and beams 210 and 218, with 6 mm thick reinforcing plates, which failed
Best fit lines from linear regression:

\[ S = (46.7t_p + 250) \times 10^{-6} \quad (t_g = 1.5 \text{mm}) \]

\[ S = (39.8t_p + 330) \times 10^{-6} \quad (t_g = 3 \text{mm}) \]

\[ S = (38.8t_p + 410) \times 10^{-6} \quad (t_g = 6 \text{mm}) \]
LINEAR REGRESSION ANALYSIS FOR COEFFICIENTS K1 AND K2

\[ Y = (47.2 - 1.55t_g) \times 10^{-1} \]

FIGURE 7.5 LINEAR REGRESSION ANALYSIS FOR COEFFICIENTS K3 AND K4

\[ Y = (34.3t_g + 210) \times 10^{-1} \]
FIGURE 7.6 EXPERIMENTAL V. THEORETICAL VALUES OF THE SLOPE OF MEAN CRACK WIDTH V. CONCRETE STRAIN
\[ S_u = 0.32h + 38 \quad t_p = 1.5\text{mm} \]

\[ S_u = 0.26h + 37 \quad t_p = 3\text{mm} \]

\[ S_u = 0.23h + 36 \quad t_p = 6\text{mm} \]

**Figure 7.7 Ultimate Mean Crack Spacing vs. Mean Crack Height at 60 kN**
before their ultimate spacings had been reached. The initial crack height is that at 60 KN, just above the first crack load. Best fit lines were drawn for each plate thickness and the following relationship was found when combining the three plate thicknesses:

\[ S_u = (0.02 \, t_p + 0.2)h_{60} + 0.43 \, t_p + 35.5 \]

The values found from this formula were plotted against the results from experiments in Fig. 7.8. All values fell within ±5% of the experimental - theoretical line. However, the above formula does not contain a term relating to concrete cover, which is a major factor in crack formation. Further tests need to be done with different covers to produce a formula for crack spacings which could be applied to any plated beam.

Plates 5.2 to 5.8 show the number of cracks that reach almost to the neutral axis is less than at the reinforcement level. Broms (93) pointed out that in a reinforced concrete member primary cracks formed first which nearly extend to the neutral axis; with a further increase in load secondary cracks of shorter length are formed between these. Base et al (91) explained this in a different way. In the zone just below the neutral axis there would be no cracking since the tensile strain in the concrete is very small. In the zone near the tension face the crack spacing would be a certain value, while in between there would be a transition from this certain spacing to an infinite spacing.

7.4.2 Maximum Crack Widths

Figs. 7.9 to 7.18 show the relationship between the maximum crack width, in the constant bending moment region, and the applied load. The unplated beam had a crack width of 0.12 mm at 130 KIN load, approximately the service load condition. For beams with 1.5 mm and 3 mm thick reinforcing plates this width was reduced by 33 to 50%. For beams with 6 mm thick plates the reduction was 58 to 63%. The beams with 1.5 mm lapped plates had between 1.0 and 1.33 times the crack width of similar beams with a continuous layer of plate. Similarly, the beams with two layers of 1.5 mm plate, or one layer of 3 mm plate with lapped joints had between 1.0 and 1.33 times the crack width of similar beams with a continuous layer of 3 mm thick plate. The precracked beams with 1.5 mm thick
FIGURE 7-8 THEORETICAL V. EXPERIMENTAL ULTIMATE CRACK SPACINGS
plates had maximum crack widths 83-100% of the unplated beam while the precracked beam with 3 mm plate had 58% of the crack width of the unplated beam. In general, the maximum crack width of the precracked beams was greater than for similar beams which had not been precracked.

The variation of mean crack width with glue and plate thickness is shown diagrammatically in Fig. 7.19. In general, there is an increase in crack width for a decrease in plate thickness and for an increase in glue thickness, the latter having the lesser effect.

7.4.3 Crack Width Prediction Formulae

The crack widths were calculated, as outlined in Appendix 8, for all beams with a single layer of plate which had not been precracked. The experimental and theoretical crack widths are shown in Table 7.7. The CP 110 and ACI methods greatly overestimated the crack widths for the plated beams. This is further evidence that the bonded plates reduce crack widths considerably. It is apparent that good composite action is being achieved and that the presence of the bonded plate at the concrete tensile surface is having a restraining effect on the increase of crack width.

Modifications were made to the two methods by factoring the equations to produce the best agreement with experimental results. This produces equations in which the crack width is dependent on the plate thickness, in addition to the other variables applicable to normally reinforced concrete beams.

In Fig. 7.20, the coefficient, \( C_1 \), by which \( a_{cr} e_m \) must be multiplied for each test beam, in order to produce the experimental crack width, was plotted against the plate thickness. The best fit line produces the modified CP 110 formula:

\[
W_{cr} = (2.0 + 0.1 t_p) a_{cr} e_m
\]

As shown in Table 7.7 this formula, which is derived from experimental results, gave ratios of theoretical to experimental crack widths varying from 0.90 to 1.09.

The ACI formula was treated in a similar manner. The coefficient \( C_2 \), required to produce the experimental crack width is plotted against plate thickness.
FIGURE 7.19  MEAN CRACK WIDTH v PLATE & GLUE THICKNESS
TABLE 7.7 CRACK WIDTHS AT 130 kN LOAD

<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>EXPERIMENT WIDTH mm</th>
<th>CP 110 THEORY mm $10^{-2}$</th>
<th>A.C.I. THEORY mm $10^{-2}$</th>
<th>CP 110 CORRECTED mm $10^{-2}$</th>
<th>A.C.I. CORRECTED mm $10^{-2}$</th>
<th>$\frac{2}{T}$</th>
<th>$\frac{3}{T}$</th>
<th>GLUE THICKNESS mm</th>
<th>PLATE THICKNESS mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>12.0</td>
<td>10.0</td>
<td>10.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>203</td>
<td>6.5</td>
<td>8.8</td>
<td>9.3</td>
<td>6.3</td>
<td>6.4</td>
<td>0.97</td>
<td>0.98</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>204</td>
<td>6.0</td>
<td>7.4</td>
<td>7.6</td>
<td>5.7</td>
<td>5.9</td>
<td>0.95</td>
<td>0.98</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>205</td>
<td>5.0</td>
<td>5.6</td>
<td>5.4</td>
<td>4.9</td>
<td>5.0</td>
<td>0.98</td>
<td>1.00</td>
<td>1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>207</td>
<td>6.0</td>
<td>8.8</td>
<td>9.3</td>
<td>6.3</td>
<td>6.4</td>
<td>1.05</td>
<td>1.07</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>208</td>
<td>6.0</td>
<td>7.4</td>
<td>7.6</td>
<td>5.7</td>
<td>5.9</td>
<td>0.95</td>
<td>0.98</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>209</td>
<td>4.5</td>
<td>5.6</td>
<td>5.4</td>
<td>4.9</td>
<td>5.0</td>
<td>1.09</td>
<td>1.11</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>210</td>
<td>5.0</td>
<td>5.6</td>
<td>5.4</td>
<td>4.9</td>
<td>5.0</td>
<td>0.98</td>
<td>1.00</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>216</td>
<td>7.0</td>
<td>8.8</td>
<td>9.3</td>
<td>6.3</td>
<td>6.4</td>
<td>0.90</td>
<td>0.91</td>
<td>6.0</td>
<td>1.5</td>
</tr>
<tr>
<td>217</td>
<td>6.0</td>
<td>7.4</td>
<td>7.6</td>
<td>5.7</td>
<td>5.9</td>
<td>0.95</td>
<td>0.98</td>
<td>6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>218</td>
<td>5.0</td>
<td>5.6</td>
<td>5.4</td>
<td>4.9</td>
<td>5.0</td>
<td>0.98</td>
<td>1.00</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>219</td>
<td>5.0</td>
<td>5.6</td>
<td>5.4</td>
<td>4.9</td>
<td>5.0</td>
<td>0.98</td>
<td>1.00</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>220</td>
<td>6.0</td>
<td>8.8</td>
<td>9.3</td>
<td>6.3</td>
<td>6.4</td>
<td>1.05</td>
<td>1.07</td>
<td>30 - 8.0</td>
<td>1.5</td>
</tr>
<tr>
<td>221</td>
<td>6.0</td>
<td>8.8</td>
<td>9.3</td>
<td>6.3</td>
<td>6.4</td>
<td>1.05</td>
<td>1.07</td>
<td>3.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

TABLE 7.8 RATIO OF MAXIMUM TO MEAN CRACK WIDTH

<table>
<thead>
<tr>
<th>RATIO</th>
<th>Hognestad. (94)</th>
<th>Broms. (93)</th>
<th>Base et al. (91)</th>
<th>Borges. (95)</th>
<th>Illston. (90)</th>
<th>Clark (96)</th>
<th>Present investigations</th>
</tr>
</thead>
<tbody>
<tr>
<td>RANGE</td>
<td>1.03 – 210</td>
<td>1.20 – 240</td>
<td>–</td>
<td>1.72 or 1.65</td>
<td>2.00</td>
<td>1.64</td>
<td>1.78 or 2.25*</td>
</tr>
</tbody>
</table>

* PROBABILISTIC VALUES
FIGURE 7.20 MODIFICATION OF CP 110 CRACK WIDTH FORMULA

best fit line \( C_1 = 2.0 + 0.1t_p \)

FIGURE 7.21 MODIFICATION OF ACI CRACK WIDTH FORMULA

best fit line \( C_2 = 0.056 + 0.0046t_p \)
as shown in Fig. 7.23. The best fit line produces the modified ACI formula:

\[ W_{\text{max}} = \frac{(0.056 + 0.0046 t_p)^{3/3} s_A (f_s - 5)}{1 + ts/h_1} \times 10^{-3} \]

As shown in Table 7.7 this formula, derived from experimental results, gives ratios of theoretical to experimental crack widths varying from 0.91 to 1.11.

### 7.4.4 Relationship between Maximum and Average Crack Width

Different values of the ratio of maximum to average crack width have been suggested by many authors, as shown in Table 7.8. In Fig. 7.22 the slopes of the mean crack width against concrete strain are plotted against the slopes of their standard deviations against concrete strain (Figs. 7.1 to 7.3). The best fit line, forced through the origin is given by:

\[ \frac{\sigma}{\varepsilon} = 0.5 \frac{W}{\varepsilon} \]

If a one percent chance of a certain crack width being exceeded is chosen, the maximum crack width is given by:

\[ W_{\text{max}} = W_{\text{mean}} + 2.5 \times (\text{Standard Deviation}) \]  
(Appendix 9)

It should be mentioned here that the average coefficient of variation \( \frac{\sigma}{W_{\text{mean}}} \), calculated at the reinforcement level at each load stage from Tables 7.1 to 7.6 (108 results) was 0.46 which is in good agreement with the value from Fig. 7.22. These values are a little higher than those found by Base et al (91) 42% and by Borges (95) 40%. It is thought that the random nature of cracking and the effects of experimental error in the techniques of crack measurement may account for this. The resulting relationship between maximum and average crack widths is given by:

\[ W_{\text{max}} = W_{\text{mean}} + 2.5 \times (0.5 \times W_{\text{mean}}) \]

or \[ W_{\text{max}} = 2.25 \times W_{\text{mean}} \]

The average ratio of maximum crack width to average crack width found in the tests was 1.78, ranging from 1.5 to 2.2. These are from 108 sets of readings as given in Tables 7.1 to 7.6. Both the range and the mean are in good agreement with the values found by others as shown in Table 7.8.
FIGURE 7-22  SLOPE OF STANDARD DEVIATION V. CONCRETE STRAIN
V.
SLOPE OF MEAN CRACK WIDTH V. CONCRETE STRAIN

\[ \frac{\sigma}{\varepsilon} = 0.50 \frac{W}{\varepsilon} \]
From Figs. 7.4 and 7.5 the following relationship was found:

\[ W_{\text{mean}} = [(47.2 - 1.55 t_g) t_p + 34.3 t_g + 210] \cdot 10^{-1} \]

Using the probabilistic value of \( W_{\text{max}} = 2.25 \cdot W_{\text{mean}} \) the maximum crack width which has only a 1% chance of being exceeded can be computed from:

\[ W_{\text{max}} = 2.25 \cdot \varepsilon_m \cdot [(47.2 - 1.55 t_g) t_p + 34.3 t_g + 210] \cdot 10^{-1} \]

Using the calculated values of \( \varepsilon_m \) (CP 110 assumptions, Appendix 8) for the plated beams, the values of \( W_{\text{max}} \) were calculated from the above formula. In Fig. 7.23 these are plotted against the measured maximum crack widths. In no case did the experimental value exceed the predicted value, which in theory has only a 1% chance of being exceeded.

It should be emphasised that this formula is only valid within the limitation of the present test series and further tests would be required to check its validity for other plated beams.

7.4.5 Concrete Surface Strain

The concrete in the tension zone of a flexural member contributes to its stiffness. Therefore, the concrete strain is less than the value calculated on the basis of zero tensile strength in the concrete. Various expressions have been proposed, by many authors, to account for the reduction of the calculated value of strain, by allowing for the tension stiffening effect of the tensile concrete. Two are given below:

\[ \varepsilon_m = \varepsilon_1 - 1.2 \cdot b_t \cdot h \cdot (a^1 - x) \cdot 10^{-3} \]

\[ \frac{A_s}{(h - x) f_y} \]

\[ \varepsilon_m = \varepsilon_1 - 4.5 \cdot b \cdot (h - x) \cdot 10^{-6} \]

\[ \frac{A_s}{Beeby (92)} \]

\[ \varepsilon_m \] - concrete surface strain

\[ \varepsilon_1 \] - value calculated on basis of zero tensile strength in concrete.

When calculating these values of \( \varepsilon_m \) the combined centroid position for bars and plate must be used. The values of steel area are for both steel and plate.

Table 7.9 shows the average strain, at the level of intonal reinforcement for the beams, from three sources:

(a) experimental values

-184-
FIGURE 7.23  THEORETICAL V. EXPERIMENTAL MAXIMUM CRACK WIDTH

Theoretical Crack Width

$W_{\text{max}} = \frac{\sigma}{E}\left[1 - \left(\frac{r}{15}\right)^{2}\right]\left(1 - \frac{1}{2\gamma}\right)$

Experimental Maximum Crack Width (mm)

-185-
## TABLE 7.9
COMPARISON OF MEASURED AND CALCULATED VALUES OF CONCRETE SURFACE STRAIN AT THE LEVEL OF THE INTERNAL REINFORCING BARS.

<table>
<thead>
<tr>
<th>BEAM NUMBERS</th>
<th>203, 207, 211, 212, 216, 220, 221</th>
<th>204, 206, 213, 214, 215, 217</th>
<th>205, 209, 210, 218, 219</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLATE THICKNESS</td>
<td>1.5</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>MEASURED STRAINS $10^6$</td>
<td>1170</td>
<td>841</td>
<td>624</td>
</tr>
<tr>
<td>MEAN</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RANGE</td>
<td>960 - 1260</td>
<td>670 - 1000</td>
<td>580 - 780</td>
</tr>
<tr>
<td>VALUES FROM CP110 EQUATION $10^6$</td>
<td>1130</td>
<td>94.6</td>
<td>725</td>
</tr>
<tr>
<td>VALUES FROM BEEBY EQUATION $10^6$</td>
<td>1130</td>
<td>960</td>
<td>751</td>
</tr>
<tr>
<td>RATIO EXPERIMENTAL</td>
<td>1.04</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>THEORETICAL BEEBY</td>
<td>1.04</td>
<td>0.87</td>
<td>0.83</td>
</tr>
</tbody>
</table>

## TABLE 7.10
MEASURED AND CALCULATED VALUES OF THE DIFFERENCE BETWEEN THE INTERNAL BAR STRAIN AND THE CONCRETE SURFACE STRAIN AT THE SAME LEVEL.

<table>
<thead>
<tr>
<th>BEAM NUMBERS</th>
<th>203, 207, 211, 212, 216, 220, 221</th>
<th>204, 206, 213, 214, 215, 217</th>
<th>205, 209, 210, 218, 219</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLATE THICKNESS</td>
<td>1.5</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>MEAN MEASURED DIFFERENCE $10^6$</td>
<td>96</td>
<td>134</td>
<td>148</td>
</tr>
<tr>
<td>MEAN DIFFERENCE FROM FIGURES $10^6$</td>
<td>85</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>CP110 $\frac{1.2 bh (d-x) 10^{-3}}{A_s} f_y (h-x)$</td>
<td>86</td>
<td>80</td>
<td>69</td>
</tr>
<tr>
<td>BEEBY $\frac{4.5 b (h-x) 10^{-6}}{A_s}$</td>
<td>80</td>
<td>65</td>
<td>43</td>
</tr>
<tr>
<td>PRESENT INVESTIGATION $\frac{(23+222 t_p) b (h-x) 10^{-6}}{A_s}$</td>
<td>100</td>
<td>129</td>
<td>149</td>
</tr>
</tbody>
</table>
For plate thicknesses of 1.5 mm, 3 mm and 6 mm the number of beams used to find the mean experimental readings were 7, 6 and 5 respectively. These values and the range for each plate thickness are given in Table 7.9. The ratio of experimental to theoretical values was 1.04 (1.5 mm plate) for both CP 110 and Beeby's equation. For 3 mm and 6 mm plate thicknesses the agreement between experiment and theory was not so good. In order to improve the prediction equations Figs. 7.2, 7.25 and 7.26 were plotted. These show the calculated steel stress, on the basis of zero tensile strength in the concrete, plotted against the measured value of concrete strain at the reinforcement level.

As shown in Appendix 8, at service load beams with 1.5 mm plate have a steel stress of 230 N/mm². Similarly, beams with 3 mm and 6 mm plates, at service load, have steel stresses of 195 and 150 N/mm² respectively. Calculations for steel stresses were also made at 60 kN and 190 kN loads. These values are then plotted against the experimental value of concrete strain, for each beam, at the level of the internal reinforcement.

Best fit lines were plotted by linear regression and all the experimental points fell within ±15% for beams strengthened with 1.5 mm thick plate, +28 and -30% for beams with 3 mm thick plate and +35 and -19% for beams with 6 mm thick plates.

Since crack widths are usually checked at working load conditions the difference between the actual measured strain and the strain in the steel at the same stress, calculated from the steel's elastic stress/strain relationship was found. A comparison similar to that in Table 7.9 is shown in Table 7.10. The differences from Figs. 7.24 to 7.26 are compared with the values from measured readings and the two theoretical methods given below.

(a) \[ \frac{1.2 \Delta h (a - x) \cdot 10^{-3}}{A_s (h - x) f_y} \]
(b) \[ 4.5 \frac{b (h - x) \cdot 10^{-6}}{A_s} \]
FIGURE 7.24  CALCULATED STEEL STRESS V MEASURED CONCRETE STRAIN

PLATE THICKNESS 1.5 mm

Design service load (c) 130 kN

+15%  -15%

DIFFERENCE 85 microstrain

Measured concrete strain at the level of internal bar reinforcement

Best fit line
Stress-strain relationship for the steel bars
Limits within which all points lie
PLATE THICKNESS 6mm

MEASURED CONCRETE STRAIN AT THE LEVEL OF INTERNAL BAR REINFORCEMENT

CALCULATED STEEL STRESS V. MEASURED CONCRETE STRAIN
The measured values are the difference between the mean steel strain, from Table 6.2 (c) and the mean concrete strain at the reinforcement level from Tables 7.1 to 7.6. The values for 1.5 mm plate gave good agreement but 3 mm and 6 mm plates gave progressively worse agreement.

The modified formula: \( \epsilon_m = \epsilon_1 - (2.22 \cdot t_p + 2.30) \cdot \frac{b(h - x)}{A_s} \cdot 10^{-6} \) was found to give good agreement as shown in Table 7.10.

It should be noted that in calculations for \( \epsilon_m \), the centroid of both plate and bars was found and then the neutral axis position. The value of \( h \) is taken as the depth of the beam i.e. 255 mm. In Fig. 7.27 the method of derivation of the above formula is shown. The coefficient, \( C_3 \), required to multiply Beeby's expression \( \frac{b(h - x)}{A_s} \cdot 10^{-6} \) to produce the measured value was plotted against plate thickness. The best fit line was then plotted by linear regression.

7.4.6 Stresses Carried by Concrete in the Tension Zone

It is generally accepted that concrete in a tension or flexural member adds considerably to its stiffness through the tension zone. Yu and Winter (76) were the first researchers who took into account the contribution of the concrete in tension to the stiffness of a member. However, they did not give any numerical value for the effective tensile stress of the concrete to be used for this contribution. In 1972 Beeby (97) found that stresses in the concrete, for members subjected to pure tension and reinforced with deformed bars, remain approximately constant at about 1.0 N/mm². The results for specimens reinforced with plain bars were considerably lower.

A series of sustained loading tests on reinforced concrete beams was described by Stevens (80), in which the major factors affecting the development of deflection were varied. He found that there was no consistent difference between beams reinforced with round or deformed bars; and that varying the cover from 25 mm to 50 mm gave no consistent difference either. An expression was proposed for the average tensile force, \( T \), in the concrete after cracking.

\[ T = \frac{3}{16} b \cdot h \cdot f_{rm} \cdot f_{rm} = \text{strength in bending.} \]

CP 110 recommends that the curvature of any section may be calculated by assuming that the stress distribution in the concrete is triangular, having a value of
FIGURE 7.27 MODIFICATION TO BEEBY'S FORMULA

best fit line $C_3 = 2.22t_p + 2.3$
zero at the neutral axis and 1.0 N/mm² at the reinforcement level.

Fig. 7.28 shows the stress distribution in the plated beams at service load. The concrete is assumed to be elastic up to a compression of 1000 micro-strain.

Taking moments of the tension steel about the centroid of the compression block: \( M_{TS} = f_s \cdot A_s \cdot (d - \frac{x}{3}) \), but, as shown in Fig. 7.28, the steel is composed of bars and plates and \( d \) is the distance from the compression face to their combined centroid.

Hence \( M_{TS} = (f_b \cdot A_b + f_p \cdot A_p) \cdot (d - \frac{x}{3}) \)

The difference between the applied moment \( M_a \) and \( M_{TS} \) is therefore the contribution of the concrete in the tension zone, \( M_c \).

But \( M_c = \sigma_f \cdot \frac{h - x}{3} \cdot b \cdot h \).

or \( \sigma_f = \frac{3M_c}{b \cdot h} \cdot (h - x) \)

where \( \sigma_f \) = the tensile stress in the concrete at the tension face.

The calculated values are given in Table 7.11. Since the beams were loaded incrementally, three values for each beam in the elastic range of both steel and concrete are given. These values did not show any significant change with increasing load and the average values were found for each beam. In Table 7.11, the strains given for the bars and plates are obtained from Figs. 6.2 to 6.21 and the neutral axis positions are from demec readings at 100 KN, plus the values given in Tables 6.1 (b) (60 kN) and 6.2 (c) (130 kN). The average values of tensile stress for each glue and plate thickness are shown in Table 7.12. The mean tensile stress in the concrete was then plotted against the plate thickness for each glue thickness, as shown in Fig. 7.29. Best fit lines were plotted and the combined equation, allowing for a linear change in tensile concrete stress with glue thickness was given by:

\[ \sigma_f = (0.81 - 0.058 \cdot t_g) \cdot t_p + 0.59 \cdot t_g + 0.36. \]

The values found from this expression are given in Table 7.13 for comparison with Table 7.12.
FORCES

\[ F_c = b \left( \frac{x}{2} \right) \text{ compressive force in the concrete above the neutral axis.} \]

\[ T_c = q \cdot b \left( h - \frac{x}{2} \right) \text{ tensile force in the concrete below the neutral axis.} \]

\[ T_s = (f_p A_p + f_r A_r) \text{ tensile force in the steel plate and bars.} \]

SYMBOLS

\( d \) combined centroid of steel bars and plate.

subscript \( p \) denotes plate

\( r \) bars

\( x \) neutral axis depth found from the measured strain distribution in the concrete.

\( h \) overall beam depth.

FIGURE 7.28 STRESS DISTRIBUTION IN PLATED BEAMS AT SERVICE LOAD.
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>60kN</th>
<th>100kN</th>
<th>130kN</th>
<th>60kN</th>
<th>100kN</th>
<th>130kN</th>
<th>60kN</th>
<th>100kN</th>
<th>130kN</th>
<th>60kN</th>
<th>100kN</th>
<th>130kN</th>
<th>60kN</th>
<th>100kN</th>
<th>130kN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>550</td>
<td>0</td>
<td>1030</td>
<td>0</td>
<td>1350</td>
<td>0</td>
<td>220</td>
<td>100</td>
<td>96</td>
<td>88</td>
<td>943</td>
<td>0</td>
<td>194</td>
<td>365</td>
<td>485</td>
</tr>
<tr>
<td>202</td>
<td>550</td>
<td>0</td>
<td>960</td>
<td>0</td>
<td>1320</td>
<td>0</td>
<td>220</td>
<td>108</td>
<td>96</td>
<td>85</td>
<td>943</td>
<td>0</td>
<td>191</td>
<td>340</td>
<td>447</td>
</tr>
<tr>
<td>203</td>
<td>440</td>
<td>500</td>
<td>780</td>
<td>900</td>
<td>1100</td>
<td>1280</td>
<td>226</td>
<td>108</td>
<td>102</td>
<td>95</td>
<td>943</td>
<td>187</td>
<td>193</td>
<td>347</td>
<td>490</td>
</tr>
<tr>
<td>204</td>
<td>260</td>
<td>400</td>
<td>500</td>
<td>830</td>
<td>770</td>
<td>1200</td>
<td>231</td>
<td>118</td>
<td>114</td>
<td>110</td>
<td>943</td>
<td>375</td>
<td>151</td>
<td>302</td>
<td>457</td>
</tr>
<tr>
<td>205</td>
<td>180</td>
<td>185</td>
<td>500</td>
<td>450</td>
<td>530</td>
<td>650</td>
<td>238</td>
<td>123</td>
<td>119</td>
<td>115</td>
<td>122</td>
<td>321</td>
<td>394</td>
<td>62</td>
<td>3.5</td>
</tr>
<tr>
<td>206</td>
<td>260</td>
<td>185</td>
<td>400</td>
<td>450</td>
<td>740</td>
<td>700</td>
<td>231</td>
<td>123</td>
<td>119</td>
<td>115</td>
<td>943</td>
<td>375</td>
<td>120</td>
<td>283</td>
<td>370</td>
</tr>
<tr>
<td>207</td>
<td>440</td>
<td>500</td>
<td>800</td>
<td>900</td>
<td>1100</td>
<td>1280</td>
<td>226</td>
<td>112</td>
<td>106</td>
<td>100</td>
<td>943</td>
<td>187</td>
<td>192</td>
<td>352</td>
<td>490</td>
</tr>
<tr>
<td>208</td>
<td>320</td>
<td>370</td>
<td>560</td>
<td>750</td>
<td>740</td>
<td>1080</td>
<td>231</td>
<td>118</td>
<td>114</td>
<td>110</td>
<td>943</td>
<td>375</td>
<td>168</td>
<td>312</td>
<td>429</td>
</tr>
<tr>
<td>209</td>
<td>180</td>
<td>220</td>
<td>330</td>
<td>480</td>
<td>500</td>
<td>700</td>
<td>238</td>
<td>118</td>
<td>114</td>
<td>110</td>
<td>943</td>
<td>750</td>
<td>133</td>
<td>259</td>
<td>401</td>
</tr>
<tr>
<td>210</td>
<td>180</td>
<td>200</td>
<td>370</td>
<td>480</td>
<td>490</td>
<td>680</td>
<td>238</td>
<td>123</td>
<td>119</td>
<td>115</td>
<td>943</td>
<td>750</td>
<td>126</td>
<td>281</td>
<td>388</td>
</tr>
<tr>
<td>211</td>
<td>430</td>
<td>465</td>
<td>750</td>
<td>850</td>
<td>1000</td>
<td>1250</td>
<td>226</td>
<td>118</td>
<td>111</td>
<td>105</td>
<td>943</td>
<td>187</td>
<td>184</td>
<td>327</td>
<td>450</td>
</tr>
<tr>
<td>212</td>
<td>430</td>
<td>600</td>
<td>750</td>
<td>1050</td>
<td>1020</td>
<td>1550</td>
<td>226</td>
<td>114</td>
<td>108</td>
<td>102</td>
<td>943</td>
<td>187</td>
<td>195</td>
<td>343</td>
<td>481</td>
</tr>
<tr>
<td>213</td>
<td>280</td>
<td>300</td>
<td>520</td>
<td>600</td>
<td>750</td>
<td>930</td>
<td>231</td>
<td>130</td>
<td>120</td>
<td>115</td>
<td>943</td>
<td>375</td>
<td>141</td>
<td>266</td>
<td>407</td>
</tr>
<tr>
<td>214</td>
<td>280</td>
<td>250</td>
<td>520</td>
<td>530</td>
<td>760</td>
<td>800</td>
<td>231</td>
<td>118</td>
<td>114</td>
<td>110</td>
<td>943</td>
<td>375</td>
<td>137</td>
<td>266</td>
<td>395</td>
</tr>
<tr>
<td>215</td>
<td>230</td>
<td>250</td>
<td>460</td>
<td>550</td>
<td>700</td>
<td>900</td>
<td>231</td>
<td>123</td>
<td>121</td>
<td>120</td>
<td>943</td>
<td>375</td>
<td>136</td>
<td>244</td>
<td>381</td>
</tr>
<tr>
<td>216</td>
<td>350</td>
<td>350</td>
<td>600</td>
<td>700</td>
<td>950</td>
<td>1100</td>
<td>226</td>
<td>123</td>
<td>119</td>
<td>115</td>
<td>943</td>
<td>187</td>
<td>146</td>
<td>260</td>
<td>413</td>
</tr>
<tr>
<td>217</td>
<td>280</td>
<td>350</td>
<td>470</td>
<td>650</td>
<td>670</td>
<td>1000</td>
<td>231</td>
<td>114</td>
<td>110</td>
<td>106</td>
<td>943</td>
<td>375</td>
<td>153</td>
<td>267</td>
<td>394</td>
</tr>
<tr>
<td>218</td>
<td>150</td>
<td>230</td>
<td>330</td>
<td>460</td>
<td>500</td>
<td>660</td>
<td>238</td>
<td>123</td>
<td>121</td>
<td>120</td>
<td>943</td>
<td>750</td>
<td>124</td>
<td>259</td>
<td>383</td>
</tr>
<tr>
<td>219</td>
<td>150</td>
<td>265</td>
<td>300</td>
<td>520</td>
<td>460</td>
<td>750</td>
<td>238</td>
<td>128</td>
<td>124</td>
<td>120</td>
<td>943</td>
<td>750</td>
<td>133</td>
<td>265</td>
<td>394</td>
</tr>
<tr>
<td>220</td>
<td>380</td>
<td>400</td>
<td>700</td>
<td>780</td>
<td>1000</td>
<td>1180</td>
<td>226</td>
<td>99</td>
<td>97</td>
<td>95</td>
<td>943</td>
<td>187</td>
<td>167</td>
<td>312</td>
<td>452</td>
</tr>
<tr>
<td>221</td>
<td>380</td>
<td>400</td>
<td>700</td>
<td>780</td>
<td>1000</td>
<td>1250</td>
<td>226</td>
<td>108</td>
<td>103</td>
<td>98</td>
<td>943</td>
<td>187</td>
<td>165</td>
<td>309</td>
<td>455</td>
</tr>
<tr>
<td>222</td>
<td>350</td>
<td>350</td>
<td>600</td>
<td>660</td>
<td>830</td>
<td>950</td>
<td>226</td>
<td>108</td>
<td>104</td>
<td>100</td>
<td>943</td>
<td>187</td>
<td>150</td>
<td>264</td>
<td>370</td>
</tr>
<tr>
<td>223</td>
<td>350</td>
<td>270</td>
<td>650</td>
<td>560</td>
<td>950</td>
<td>750</td>
<td>226</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>943</td>
<td>187</td>
<td>14.7</td>
<td>27.7</td>
<td>400</td>
</tr>
<tr>
<td>224</td>
<td>230</td>
<td>270</td>
<td>430</td>
<td>500</td>
<td>650</td>
<td>750</td>
<td>231</td>
<td>123</td>
<td>116</td>
<td>110</td>
<td>943</td>
<td>375</td>
<td>12.1</td>
<td>22.8</td>
<td>34.8</td>
</tr>
</tbody>
</table>

**TABLE 7.11**  
**TENSILE STRESS IN THE CONCRETE**
**TABLE 7.12** MEAN VALUES OF TENSILE STRESS IN THE CONCRETE FROM TABLE 7.11

<table>
<thead>
<tr>
<th>Glue plate thickness</th>
<th>1.5</th>
<th>3.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.4</td>
<td>3.7</td>
<td>5.1</td>
</tr>
<tr>
<td>3.0</td>
<td>2.4</td>
<td>5.7</td>
<td>5.9</td>
</tr>
<tr>
<td>6.0</td>
<td>4.2</td>
<td>5.5</td>
<td>6.4</td>
</tr>
</tbody>
</table>

**TABLE 7.13** TENSILE STRESS IN THE CONCRETE FROM THE DERIVED EQUATION $\sigma_f = (0.81 - 0.058 t_p + 0.59 t_g + 0.36$

<table>
<thead>
<tr>
<th>Glue plate thickness</th>
<th>1.5</th>
<th>3.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>2.3</td>
<td>3.4</td>
<td>5.6</td>
</tr>
<tr>
<td>3.0</td>
<td>3.1</td>
<td>4.4</td>
<td>6.0</td>
</tr>
<tr>
<td>6.0</td>
<td>4.6</td>
<td>5.3</td>
<td>6.7</td>
</tr>
</tbody>
</table>
FIGURE 7.29 TENSILE STRESS IN THE CONCRETE VS. PLATE THICKNESS FOR EACH GLUE THICKNESS

TG = glue thickness

- 1.5 mm
- 3 mm
- 6 mm

Best fit lines:
- \( \sigma_f = 0.46 t_p + 3.75 \)
- \( \sigma_f = 0.68 t_p + 2.30 \)
- \( \sigma_f = 0.77 t_p + 0.70 \)
Figure 7.30 Experimental v. Theoretical Values of the Tensile Stress in the Concrete
In Fig. 7.30 the calculated values of $\sigma_f$ from this formula were plotted against the experimentally determined values. All points, except one, lie within ±25%.

7.5 CONCLUSIONS

Based on the tests carried out in this investigation the following conclusions are drawn:

1. At a given steel stress and at a certain distance from a reinforcing bar, crack widths in concrete beams with externally bonded steel plates were less than those which would result in ordinary reinforced concrete beams. This allows a higher internal bar stress for a particular crack width to be developed, therefore increasing the limit state of cracking.

2. The mean crack width was found to be proportional to the steel stress and the following relationship was derived between the slope of the mean crack width v. concrete strain and the plate and glue thicknesses.

$$S = \left[ (47.2 - 1.55 \tan \theta) t_p + 34.3 \tan \theta + 210 \right] \times 10^{-6}$$

3. The ultimate crack spacings were related to the plate thickness and the crack height just after first crack. The mean ultimate crack spacing was given by:

$$S_u = (0.02 t_p + 0.2) h + 0.43 t_p + 35.5$$

4. The application of crack width prediction formulae recommended by Codes of Practice, for normally reinforced concrete beams, highly overestimated the crack widths for the plated beams. This shows that the presence of the bonded steel plates effectively reduces the crack widths, both by physically restraining the increase in crack width and by reducing the strain in the internal bars at any particular load, relative to an unplated beam. The CP 110 and ACI formulae were modified to satisfactorily agree with the experimental results. The resulting equations were:

$$W_{cr} = (0.1 t_p + 2.0) a_{cr} \epsilon_m$$  \hspace{1cm} (CP 110)

and

$$W_{max} = \frac{(0.056 + 0.0046 t_p)^3 \sqrt{ts} \cdot A (f_s - 5) \times 10^{-3}}{1 + ts/h_1}$$  \hspace{1cm} (ACI)
5. The relationship between maximum and average crack widths was derived statistically for the plated beams, as follows:

\[ W_{\text{max}} = 2.25 \cdot W_{\text{mean}} \]

This compared with the experimental mean value of:

\[ W_{\text{max}} = 1.78 \cdot W_{\text{mean}} \quad \text{(range 1.5 to 2.2)} \]

Both the range and mean values were in agreement with those found by other authors for normally reinforced concrete beams.

6. The application of accepted formulae for predicting concrete surface strain was found to overestimate the strains for beams strengthened with 3 mm and 6 mm thick plates. A modified formula was derived from the experimental results:

\[ \varepsilon_m = \varepsilon_1 - (2.22 \cdot t_p + 2.3) \cdot \frac{b(h - x)}{A_s} \cdot 10^{-6} \]

7. The contribution of the concrete in the tension zone to the stiffness of plated beams was more than that found in the case of normally reinforced concrete beams. A formula was derived for the tensile stress in the concrete:

\[ \sigma_f = (0.81 - 0.058 \cdot t_g) \cdot t_p + 0.59 \cdot t_g + 0.36 \]

8. The number of cracks at the 'service load' of 130 kN is between two thirds and three quarters of those fully developed at failure. This means that the average crack spacing at service load is greater than at failure. The number of cracks at failure was generally more in the case of plated beams than for the un-plated beam.

9. It should be emphasised that the formulae derived in this section are applicable only within the limitations of the present tests. Other variables such as concrete cover and cube strength, beam size and a wider range of plate and glue thicknesses should be investigated in order to prove the validity and refine the proposed formulae.
CHAPTER 8
LONG TERM TESTING

8.1 INTRODUCTION

Concrete and epoxy resins are subject to time dependent deformations due to creep and shrinkage. Reinforced concrete elements composed of these materials are, therefore, subject to long term deformations under sustained load. These deformations may be critical to the serviceability, and sometimes to the safety of a structure. If such deformations were accompanied by a loss of cohesive and/or adhesive strength of the resin bond there could be serious results to the safety of the structure.

Highway bridges are designed for lives of 120 years, and it is necessary to have stable behaviour of the construction materials over this period. The performance of epoxy resins needs checking because experience in the aerospace industry has indicated that under unfavourable conditions the strength of bonded metal joints may be reduced drastically over periods as short as six years. Calder (62) reports some loss of bond and corrosion of the bonding surface over two years between metal/concrete bonds.

In this chapter the long term performance and durability of loaded and unloaded externally plated reinforced concrete beams, under external weathering conditions are reported.

8.2 EXPERIMENTAL PROCEDURE

The same concrete, glue and steel types which were used in the short term testing were adopted for the long term test series. The material properties were reported in Chapter 3 and the manufacture of beams was described in Chapter 5. Prisms, 100 x 100 x 500 mm, were cast for shrinkage and durability specimens. All specimens were demoulded after 24 hours and kept in the laboratory under uncontrolled conditions until they were either plated, tested or transported to their test sites.

8.3 SHRINKAGE TESTS

Three prisms were used as shrinkage specimens, these were fitted with demec points, on a 200 mm gauge length on opposite faces. Strains were read
within 24 hours after demoulding and then at increasing intervals with time. At 18 months the shrinkage was 320 microstrain.

8.4 SUSTAINED LOADING/LONG TERM TESTS

8.4.1 Introduction

The details of the twenty four test beams are given in Fig. 5.2 and Table 8.1. Eight beams were subjected to sustained loading, with two unloaded beams corresponding to each loaded beam. The loaded beams were placed in rigs, as shown in Fig. 8.1, and Plate 8.1, in such a way that the maximum stress in the concrete under sustained load was approximately equal to one third of the twenty eight day cube strength. The control cubes and prisms were kept under the same conditions as the beams throughout testing. Each loading frame was designed to take two beams. The assembly and loading was performed carefully to ensure that the correct load was applied. This was checked both on a pressure gauge attached to the pump which operated the loading jack, and by checking the extensions of the Macalloy tie bars by means of a demountable mechanical extensometer of 300 mm base. All measurements of strain on the beams tension, side and compression faces were taken before and immediately after loading and thereafter at increasing intervals with time, using a demec of 200 mm gauge length. The strains in the steel plate were also measured at the centre section. Fig. 8.2 shows a typical graph of the change of maximum observed concrete compressive and steel tensile strains with time. The change in strain distribution across the beam depth is also shown. The loaded and unloaded beams were left exposed to the elements at a sewage treatment works (Plates 8.1 and 8.2).

After 18 months exposure eight of the unloaded beams were brought back for testing in the laboratory. The test rig, loading procedure, instrumentation etc. were identical to that described for the short term tests in Chapter 5, with the exception that there were no strain gauges on the internal bar reinforcement. After testing three of the eight beams it was found that there was no loss of bond due to their 18 months exposure. It was decided, therefore, to leave the remaining five beams for a longer period before testing.
| BEAM NUMBER | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 123 | 124 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| GLUE THICKNESS mm | 3   | 6   | 2-8 | (1) | 3   | 3   | 3   | 3   | 3   | 6   | 2-8 | (1) | 3   | 3   | 3   | 3   | 3   | 6   | 2-8 | (1) | 3   | 3   | 3   | 3   |
| PLATE THICKNESS mm | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| NUMBER OF LAYERS OF PLATE | 1   | 1   | 1   | 1   | 1   | 1   | 2   | 2   | 1   | 1   | 1   | 1   | 2   | 2   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 2   | 2   |
| POSITION OF LAP JOINTS | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   |
| CUBE STRENGTH AT 28 DAYS N/mm² | 66.4 | 63.2 | 68.5 | 65.5 | 76.8 | 76.4 | 65.0 | 77.0 | 70.5 | 69.2 | 78.5 | 73.3 | 73.0 | 74.5 | 72.2 | 80.2 | 60.4 | 71.2 | 81.2 | 80.6 | 69.2 | 79.1 | 79.1 | 81.1 |
| CUBE STRENGTH AT PLATING N/mm² | 80.0 | 76.6 | 78.3 | 78.3 | 60.3 | 81.7 | 78.3 | 80.6 | 77.8 | 83.0 | 86.0 | 78.4 | 78.4 | 82.2 | 86.4 | 85.3 | 83.2 | 82.2 | 86.8 | 83.4 | 77.7 | 79.4 | 81.5 | 86.9 |
| AGE AT TESTING (3) months | 18  | 18  | 30  | 30  | 30  | 30  | 30  | 30  | 18  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  |
| CUBE STRENGTH AT TESTING N/mm² | 66.3 | 89.0 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 | 86.6 |
| MODULUS OF N/mm² | 5.85 | 5.92 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 | 5.73 |
| L - LOADED | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | UL | L | L | L | L |
| UL - UNLOADED | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L | L |

(1) glue thickness varies along the length of the beam.
(2) V notches cut in the tension face of the beam to form stress concentrations above the load points.
(3) all beams to be tested at 30 & 60 months by others.
All stiffeners from 12mm thick plate.
All roller bearings 20mm diameter unless shown otherwise.

FIGURE 8-1
LONG TERM LOADING RIG
BEAM No. 120

- concrete in compression
- steel plate in tension

STRESS STRAIN CURVES FOR STEEL AND CONCRETE

CONCRETE STRAIN DISTRIBUTION

FIGURE 8.2 VARIATION OF STRAIN WITH TIME
PLATE 8.1  LONG TERM TESTS : LOADED BEAMS

PLATE 8.2  LONG TERM TESTS : UNLOADED BEAMS
8.4.2 Discussion of Results

8.4.2.1 Strength Characteristics

The ultimate loads of the three beams tested at 18 months were calculated as detailed in Appendix 4 and are given in Table 8.2 together with the experimental values and the corresponding loads for similar beams tested at 28 days. For the CP110 method of calculation, the mean ratio of experimental to theoretical ultimate moment was 1.05, as compared to 1.05 at 28 days. The same ratios for the calculations by strain compatibility were 1.03 (1.01 at 28 days) assuming the glue to be cracked, and 1.02 (1.00 at 28 days) assuming the glue is not cracked.

After testing three of the beams, which had been exposed for 18 months after plating, it was thought that there was nothing to be gained, at this stage, by testing the remaining five of the set of eight beams. The beams were resisting greater moments at failure than the beams tested at 28 days as would be expected due to the ageing of the concrete, assuming that there was no degradation of the bond between the glue and plate or concrete. The plates were stripped from the beams after failure as shown in Plate 8.3. Although there was evidence of a small amount of air pockets within the glue line (less than 5% of the area) and areas of insufficiently mixed resin hardener (less than 5% of the area), there was no sign of corrosion of the steel plate, except along its edges where some of the protective paint had been chipped off during transportation. The concrete beam was cut through with a circular saw to produce a small element of the beam (Plate 8.3).

The amount of calcium hydroxide available in a hardened cement paste depends on the amount and composition of the calcium silicate phases in the cement and their degree of hydration. In time the alkalies react with the acidic constituents in the atmosphere, particularly carbon dioxide and sulphur dioxide, so that the alkalinity of the concrete is progressively reduced. Corrosion of the steel bar reinforcement occurs when moisture and oxygen gain access into the concrete and also there must be a value of pH less than 11, in other words the environment must be acidic. The diffusion of acidic vapours into the concrete converts the free lime to calcium carbonate thus reducing the pH, and consequently
<table>
<thead>
<tr>
<th>PLATE THICKNESS</th>
<th>GLUE THICKNESS</th>
<th>FAILURE MODE</th>
<th>EXPERIMENTAL STRAIN COMPARISON</th>
<th>ULTIMATE MOMENT (NO GLUE)</th>
<th>ULTIMATE MOMENT (+ GLUE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0 1.5</td>
<td>FLEXURE</td>
<td>1.09</td>
<td>95.6</td>
<td>100.6</td>
</tr>
<tr>
<td></td>
<td>6.0 1.5</td>
<td>FLEXURE</td>
<td>1.06</td>
<td>96.7</td>
<td>100.6</td>
</tr>
<tr>
<td></td>
<td>3.0 1.5</td>
<td>FLEXURE / SHEAR BOND</td>
<td>0.96</td>
<td>104.9</td>
<td>97.0</td>
</tr>
<tr>
<td></td>
<td>3.0 1.5</td>
<td>FLEXURE / SHEAR BOND</td>
<td>0.96</td>
<td>104.9</td>
<td>97.0</td>
</tr>
<tr>
<td></td>
<td>6.0 1.5</td>
<td>FLEXURE</td>
<td>1.03</td>
<td>99.6</td>
<td>101.5</td>
</tr>
<tr>
<td></td>
<td>3.0 1.5</td>
<td>FLEXURE</td>
<td>1.11</td>
<td>111.9</td>
<td>111.6</td>
</tr>
<tr>
<td></td>
<td>6.0 1.5</td>
<td>FLEXURE</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BEAM NUMBERS</th>
<th>AGE AT TESTING</th>
<th>28 DAYS</th>
<th>18 MONTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>207</td>
<td>216</td>
<td>214</td>
<td>210</td>
</tr>
<tr>
<td>101</td>
<td>102</td>
<td>103</td>
<td>108</td>
</tr>
<tr>
<td>95.5</td>
<td>96.4</td>
<td>97.1</td>
<td>107.8</td>
</tr>
</tbody>
</table>

TABLE 8.2
Section cut through a beam to test for carbonation of cement.

Plate peeled off the beam.

a. air pockets
b. glue not fully hardened due to insufficient mixing.
the protective value of the concrete. Carbonation also tends to increase the shrinkage of the concrete and thus promotes the development of cracks. This in turn increases the penetration of moisture and chemicals which further assists corrosion. If the carbonation front reaches the steel bars then corrosion will start. Since the corrosion products occupy a greater volume than the original steel the concrete cover cracks and spalls off.

The piece of beam was treated with phenolphthalein indicator on both ends to test for carbonation. The change in colour (colourless to pink) of the phenolphthalein in the pH range between 8.2 and 9.8 indicated clearly the boundary of complete carbonation. The depth of carbonation was measured at several points and the average depth was estimated to be 2 mm at the top surface and 3 mm on the side faces. On the bottom concrete face which had the epoxy resin bonded to it there was no carbonation whatever.

It is clear from these results that the increase in ultimate strength provided by the bonded steel plate has not been adversely affected by the weathering over a period of eighteen months. The presence of the glue and plate has prevented carbonation proceeding at the bottom surface of the beam.

However, this test period is very short in comparison with the design life of a structure and the long term behaviour of plated beams must be studied for much longer periods.

It is interesting to note, nevertheless, that the plating technique not only strengthens the beams satisfactorily but also reduces the crack widths and carbonation at the beam soffit. The possibility of corrosion of the interval reinforcement should therefore be reduced.

8.4.2.2 Deformation Characteristics

Figs. 8.3, 8.4 and 8.5 show the comparison between the load-strain, load-deflection and moment rotation characteristics, respectively, of the beams tested at 28 days and 18 months. There was an increase in stiffness for both increase in glue or plate thickness and, as at 28 days, the effect of plate thickness was greater than that of glue thickness.

The load-strain curves of the beams tested at 18 months generally indicated
Figure 8.3: Load strain curves - external steel plate - centre section.

- 101(207) 3mm glue, 1.5 mm plate.
- 102(216) 6mm glue.
- 108(214) 3mm glue, 2x1.5 mm plates.

Age of beam:
- 18 months.
- 28 days.
\(290 \text{ kN}\)  102  \(286 \text{ kN}\)  108  101  207  216

FAILURE LOADS

\(\sqrt{3}\) mm glue, 1.5 mm plate.

\(\sqrt{2}\) 6 mm glue, =

\(\sqrt{1}\) 3 mm glue, 2x15 mm plates.

AGE OF BEAM 18 months.

CENTRAL DEFLECTION mm

APPLIED LOAD kN

5 10 15 20 25 30

FIGURE 8-4 LOAD-DEFLECTION CHARACTERISTICS
FIGURE 8.5  MOMENT - ROTATION CHARACTERISTICS

- 101 (207) 3mm glue, 1.5mm plate.
- 102 (216) 6mm glue,
- 108 (214) 3mm glue, 2x1.5mm plates.

AGE OF BEAM 18 months.
28 days.
an increase of rigidity when compared with the behaviour at 28 days. The plate
strains of beam 108 were greater than the comparable beam (214) at 28 days,
throughout its loading. This could indicate that beam 214 had a certain amount
of slipping between its plate layers. Beam 102 showed slightly higher plate
strains up to service load, but above this the strains were lower than the
corresponding beam tested at 28 days.

The load-deflection curves of the beams tested at 18 months all indicated
an increased stiffness when compared with the behaviour at 28 days.

The values of ductility and theoretical deflections at 130 kN are given
in Table 8.3 together with the experimental deflections. The ductility
of the three beams tested at 18 months was slightly lower than the corresponding
beams at 28 days, and the theoretical predictions gave good agreement with
experiment. The calculation of deflections were as described in Appendix 5,
with the Youngs Modulus of concrete = 38.9 kN/mm² (from experimental tests
at 18 months). The mean ratios of experimental to theoretical deflections
were 1.12 (CP110), 1.12 (ACI) and 0.95 (CEB), compared with 1.23 (CP110),
1.23 (ACI) and 1.05 (CEB) for all the beams tested at 28 days.

The experimental rotations were compared with theoretical values as
calculated in Appendix 6. Again the Youngs Modulus of the concrete was
38.9 kN/mm², at 18 months. Table 8.3 shows both the experimental and
theoretical rotations. As in the case of the 28 day tests, the difference
between theory and experiment increases as the load increases. The empirical
formula derived in Chapter 6 was also used to predict the rotations. These
values, and the percentage difference between them and the measured rotations
are also given in Table 8.3. For the limited series of three beams tested at
18 months the maximum difference between the predictions of the empirical
formula and the measured values was ±11%, compared with ±12% at 28 days.

The moment-rotation curves closely reflect the load-deflection
behaviour, all the beams being stiffer than at 28 days.
TABLE 8.3 COMPARISON OF EXPERIMENTAL AND THEORETICAL DEFLECTIONS AND ROTATIONS

<table>
<thead>
<tr>
<th>AGE AT TESTING</th>
<th>18 MONTHS</th>
<th>28 DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEAM NUMBER</td>
<td>101</td>
<td>102</td>
</tr>
<tr>
<td>GLUE THICKNESS mm</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>PLATE THICKNESS mm</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>FIRST CRACK LOAD</td>
<td>180</td>
<td>160</td>
</tr>
<tr>
<td>130 KN LOAD</td>
<td>140</td>
<td>120</td>
</tr>
<tr>
<td>220 KN LOAD</td>
<td>160</td>
<td>140</td>
</tr>
<tr>
<td>CP 110</td>
<td>4.90</td>
<td>4.90</td>
</tr>
<tr>
<td>ACI</td>
<td>4.90</td>
<td>4.76</td>
</tr>
<tr>
<td>CEB</td>
<td>5.70</td>
<td>5.70</td>
</tr>
<tr>
<td>CP 110</td>
<td>1.12</td>
<td>1.10</td>
</tr>
<tr>
<td>ACI</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>CEB</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>DUCTILITY DEFN. 2.20 kN DEFN. 130 kN</td>
<td>2.54</td>
<td>2.31</td>
</tr>
<tr>
<td>EXPERIMENTAL ROTATIONS</td>
<td>60 KN LOAD</td>
<td>55</td>
</tr>
<tr>
<td>130</td>
<td>145</td>
<td>125</td>
</tr>
<tr>
<td>190</td>
<td>245</td>
<td>210</td>
</tr>
<tr>
<td>250</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td>THEORETICAL ROTATIONS</td>
<td>60</td>
<td>56 (50)</td>
</tr>
<tr>
<td>130</td>
<td>125 (118)</td>
<td>121 (113)</td>
</tr>
<tr>
<td>190</td>
<td>181 (175)</td>
<td>183 (175)</td>
</tr>
<tr>
<td>250</td>
<td>235 (220)</td>
<td>242 (236)</td>
</tr>
<tr>
<td>FORMULA</td>
<td>60</td>
<td>56</td>
</tr>
<tr>
<td>130</td>
<td>130</td>
<td>125</td>
</tr>
<tr>
<td>190</td>
<td>217</td>
<td>215</td>
</tr>
<tr>
<td>250</td>
<td>355</td>
<td>383</td>
</tr>
<tr>
<td>FORMULA</td>
<td>60</td>
<td>+2</td>
</tr>
<tr>
<td>130</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>190</td>
<td>-11</td>
<td>+2</td>
</tr>
<tr>
<td>220</td>
<td>-9</td>
<td>+1</td>
</tr>
</tbody>
</table>

Figures in brackets assuming tensile stress in concrete = 3 N/mm². Otherwise 1 N/mm².
8.4.2.3 Cracking Characteristics

The crack analysis was performed as for the beams tested at 28 days. Table 8.4 shows the test results and compared them with the 28 day tests. Plate 8.6 shows the beams after failure.

The mean crack width and standard deviation were plotted against the surface concrete strain at the level of the internal reinforcement. The slopes were computed by linear regression as shown in Fig. 8.6. If the empirical formula derived from the 28 day tests, for the slope of the mean crack width against concrete strain, is used to compute values for the 18 month old test beams the values are very close to the experimental values.

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Experimental Value</th>
<th>Formula Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>102</td>
<td>52</td>
<td>47</td>
</tr>
<tr>
<td>108</td>
<td>44</td>
<td>44</td>
</tr>
</tbody>
</table>

The empirical formula derived from the 28 day tests, for finding the ultimate crack spacing was then used to compute values for the 18 month old beams.

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Experimental Spacing (mm)</th>
<th>Formula Spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>53</td>
<td>55</td>
</tr>
<tr>
<td>102</td>
<td>61</td>
<td>55</td>
</tr>
<tr>
<td>108</td>
<td>52</td>
<td>50</td>
</tr>
</tbody>
</table>

The agreement between the formula's prediction and experiment is good.

In Fig. 8.7 the slope of the mean crack width against concrete strain is plotted against the slope of the standard deviation against concrete strain. The best fit lines for the beams tested at both 28 days and 18 months are given. The limited accuracy obtained from only 3 tests at 18 months gives close agreement with the 28 day tests.

The modified CP110 and ACI crack width prediction formulae, derived from the tests at 28 days, were used to calculate the maximum crack widths for the three beams tested at 18 months, as shown in Table 8.5. The average ratio of theory to experimental values was 0.85 (ACI) or 0.84 (CP110), as compared with 0.91 at 28 days for the three comparable beams.
<table>
<thead>
<tr>
<th>LOAD (kn)</th>
<th>NUMBER OF CRACKS</th>
<th>MEAN CONCRETE STRAIN</th>
<th>MEAN CRACK WIDTH</th>
<th>STANDARD DEVIATION</th>
<th>COEFFICIENT OF VARIATION</th>
<th>MAXIMUM CRACK WIDTH</th>
<th>MEAN CRACK SPACING</th>
<th>STANDARD DEVIATION</th>
<th>MEAN CRACK HEIGHT</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>467</td>
<td>2.42</td>
<td>0.93</td>
<td>3.8</td>
<td>92</td>
<td>62</td>
<td>54</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>405</td>
<td>2.28</td>
<td>0.76</td>
<td>3.3</td>
<td>94</td>
<td>21</td>
<td>80</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>820</td>
<td>2.72</td>
<td>0.87</td>
<td>4.2</td>
<td>79</td>
<td>32</td>
<td>82</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>11</td>
<td>840</td>
<td>3.00</td>
<td>1.40</td>
<td>4.7</td>
<td>90</td>
<td>22</td>
<td>105</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>14</td>
<td>1200</td>
<td>3.95</td>
<td>0.83</td>
<td>2.8</td>
<td>68</td>
<td>25</td>
<td>92</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>15</td>
<td>1565</td>
<td>4.39</td>
<td>2.33</td>
<td>5.3</td>
<td>54</td>
<td>25</td>
<td>94</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>16</td>
<td>2800</td>
<td>7.53</td>
<td>3.22</td>
<td>12.2</td>
<td>53</td>
<td>25</td>
<td>97</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>3700</td>
<td>14.18</td>
<td>8.17</td>
<td>30.0</td>
<td>48</td>
<td>21</td>
<td>108</td>
<td>54</td>
<td></td>
</tr>
</tbody>
</table>

Upper figures - 28 day test.
Lower figures - 18 month test.

(a) beams 207 \( W_{\text{max}} / W_{\text{mean}} = 1.98 \)

(b) beams 216 \( W_{\text{max}} / W_{\text{mean}} = 1.70 \)

(c) beams 214 \( W_{\text{max}} / W_{\text{mean}} = 1.82 \)
<table>
<thead>
<tr>
<th>AGE AT TESTING</th>
<th>BEAM NUMBER</th>
<th>EXPERIMENT WIDTH $\times 10^{-2}$ mm</th>
<th>CP 110 THEORY $\times 10^{-2}$ mm</th>
<th>ACI THEORY $\times 10^{-2}$ mm</th>
<th>CP 110 CORRECTED $\times 10^{-2}$ mm</th>
<th>ACI CORRECTED $\times 10^{-2}$ mm</th>
<th>$\frac{2}{1}$</th>
<th>$\frac{3}{1}$</th>
<th>GLUE THICKNESS mm</th>
<th>PLATE THICKNESS mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 DAYS</td>
<td>207</td>
<td>6.0</td>
<td>9.3</td>
<td>9.0</td>
<td>6.4</td>
<td>6.4</td>
<td>1.07</td>
<td>1.07</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>216</td>
<td>7.0</td>
<td>9.3</td>
<td>9.0</td>
<td>6.4</td>
<td>6.4</td>
<td>0.91</td>
<td>0.91</td>
<td>6.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>214</td>
<td>8.0</td>
<td>7.9</td>
<td>7.7</td>
<td>5.8</td>
<td>5.9</td>
<td>0.73</td>
<td>0.74</td>
<td>3.0</td>
<td>2 x 1.5</td>
</tr>
<tr>
<td>18 MONTHS</td>
<td>101</td>
<td>7.0</td>
<td>9.3</td>
<td>9.0</td>
<td>6.4</td>
<td>6.4</td>
<td>0.91</td>
<td>0.91</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>102</td>
<td>8.0</td>
<td>9.3</td>
<td>9.0</td>
<td>6.4</td>
<td>6.4</td>
<td>0.80</td>
<td>0.80</td>
<td>6.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>7.0</td>
<td>7.9</td>
<td>7.7</td>
<td>5.8</td>
<td>5.9</td>
<td>0.82</td>
<td>0.84</td>
<td>3.0</td>
<td>2 x 1.5</td>
</tr>
</tbody>
</table>
FIGURE 8.6 MEAN CRACK WIDTH AND STANDARD DEVIATION V. THE CONCRETE STRAIN AT THE LEVEL OF INTERNAL REINFORCEMENT
Figure 8.7 W/ε MEAN CRACK WIDTH V. CONCRETE STRAIN AT REINFORCEMENT LEVEL
The formulae derived at 28 days to predict the concrete surface strain and the tensile stress in the concrete could not be checked in the long term beams as there were no strain gauges on the reinforcing bars. Therefore the difference between the internal bar strain and the concrete surface strain was not measured.

8.5 DURABILITY TESTS

8.5.1 Introduction

This part of the test programme was designed to investigate the effects of various sealing agents on the durability of the concrete/epoxy/steel joints. Obviously, to be effective, the sealing agent must show stable behaviour under moist conditions in the long term and have good bonding to concrete, epoxy resin and steel. The following products were used:

8.5.2 Coating Details

The coatings used were easily applied using a spatula or paintbrush, as indicated.

8.5.2.1 Polyurethane Rubber (Brush)

This was a two component liquid whose shear resistance is good up to temperatures of 70°C. The product used was FLEXANE 30 manufactured by DEVCON Ltd.

8.5.2.2 Silicone Rubber (Spatula)

Generally these have the same type of resistance as the polyurethanes, but are one component systems, which cure in air at room temperature to produce a resilient rubber with heat resistance up to 260°C. The product used was SILITE 100 manufactured by DEVCON Ltd.

8.5.2.3 Acrylonitrile Phenolic (Brush)

Again these show the same sort of resistance as the polyurethanes and have heat resistance to temperatures of 150°C. They are one component systems which dry in air at room temperature. The product used was K7066 manufactured by SWIFT Ltd.

8.5.2.4 Paint (Brush)

Two coats of primer and two coats of finish coat were applied. The paint used was MANDERLAC manufactured by MANDERS Ltd.
8.5.2.5 Control Specimens

Four prisms were made with steel plates having no protective coating. Two were stored with the other test specimens described above and the other two were stored in a controlled atmosphere at 17°C, 56% relative humidity. Four unplated prisms were also cast and kept in the mist room at 20°C, 100% relative humidity.

8.5.3 Experimental procedure

The test specimens were concrete prisms 100 x 100 x 500 mm as shown in Fig. 8.8. Four prisms were cast for each of the four types of coating and the control specimens as detailed above. In all, therefore, twentyfour prisms plus three control cubes were cast. All specimens were stripped after 24 hours and placed in a mist room at 22°C, 100% relative humidity for 7 days before removal for surface preparation and plating. Ciba Giegy XD808 was used to bond on the plates, as used in the preliminary test series.

After plating, the beams were left to cure in uncontrolled laboratory conditions for seven days before application of the coatings and for a further seven days afterwards. The beams were then replaced in the mist room except two control beams with no coating which were kept at 17°C, 56% relative humidity. The beams were left in the mist room for 10 months or 20 months before testing. The loading arrangement is shown in Fig. 8.8. The beams were tested under central point loading over a span of 450 mm. The loading rate was 4 kN/minute. The first crack load in the concrete, and the ultimate load were noted. After failure the plates were stripped off the beams to investigate corrosion of the plate, and plate/glue interface.

8.5.4 Discussion of result

The results are given in Table 8.6. After 10 months in the mist room the plated control beam, with no protective coating, had a failure load of 31.6 kN. The average failure loads (2 specimens each) for the coated prisms were 34.3 kN (FLEXANE); 32.5 (SILITE); 35.3 (SWIFT) and 34.1 (PAINT). The uncoated beam which was kept at 17°C, 56% relative humidity had a failure load of 34 kN. Therefore after 10 months in the mist room there was no significant difference
DETAILS OF DURABILITY SPECIMENS

All dimensions in mm.

LOADING ARRANGEMENT

FIGURE 8-8 DURABILITY TEST SPECIMENS
<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>AGE AT TESTING MONTHS</th>
<th>TYPE OF COATING</th>
<th>CP 110 THEORY FIRST CRACK LOAD kN</th>
<th>CP 110 THEORY FAILURE LOAD kN</th>
<th>EXPERIMENT FIRST CRACK LOAD kN</th>
<th>EXPERIMENT FAILURE LOAD kN</th>
<th>EXP. FAIL. LOAD/EXP. THEOR.</th>
<th>EXP. FAIL. THEOR.</th>
</tr>
</thead>
<tbody>
<tr>
<td>301</td>
<td>10</td>
<td>NONE</td>
<td>12.6</td>
<td>32.8</td>
<td>18.0</td>
<td>31.6</td>
<td>1.50</td>
<td>0.96</td>
</tr>
<tr>
<td>302 (1)</td>
<td>10</td>
<td>NONE</td>
<td>12.6</td>
<td>32.8</td>
<td>16.0</td>
<td>34.0</td>
<td>1.27</td>
<td>1.04</td>
</tr>
<tr>
<td>303</td>
<td>20</td>
<td>NONE</td>
<td>13.0</td>
<td>34.0</td>
<td>20.0</td>
<td>32.7</td>
<td>1.54</td>
<td>0.96</td>
</tr>
<tr>
<td>304 (1)</td>
<td>20</td>
<td>NONE</td>
<td>13.0</td>
<td>34.0</td>
<td>18.0</td>
<td>36.1</td>
<td>1.38</td>
<td>1.06</td>
</tr>
<tr>
<td>305</td>
<td>10</td>
<td>FLEXANE</td>
<td>12.6</td>
<td>32.8</td>
<td>16.0</td>
<td>32.3</td>
<td>1.27</td>
<td>0.98</td>
</tr>
<tr>
<td>306</td>
<td>10</td>
<td>FLEXANE</td>
<td>12.6</td>
<td>32.8</td>
<td>17.0</td>
<td>36.2</td>
<td>1.35</td>
<td>1.10</td>
</tr>
<tr>
<td>307</td>
<td>20</td>
<td>FLEXANE</td>
<td>13.0</td>
<td>34.0</td>
<td>16.0</td>
<td>37.2</td>
<td>1.23</td>
<td>1.09</td>
</tr>
<tr>
<td>308</td>
<td>20</td>
<td>FLEXANE</td>
<td>13.0</td>
<td>34.0</td>
<td>16.0</td>
<td>38.7</td>
<td>1.23</td>
<td>1.14</td>
</tr>
<tr>
<td>309</td>
<td>10</td>
<td>SILITE</td>
<td>12.6</td>
<td>32.8</td>
<td>18.0</td>
<td>31.4</td>
<td>1.43</td>
<td>0.96</td>
</tr>
<tr>
<td>310</td>
<td>10</td>
<td>SILITE</td>
<td>12.6</td>
<td>32.8</td>
<td>20.0</td>
<td>33.6</td>
<td>1.59</td>
<td>1.02</td>
</tr>
<tr>
<td>311</td>
<td>20</td>
<td>SILITE</td>
<td>13.0</td>
<td>34.0</td>
<td>18.0</td>
<td>36.8</td>
<td>1.38</td>
<td>1.08</td>
</tr>
<tr>
<td>312</td>
<td>20</td>
<td>SILITE</td>
<td>13.0</td>
<td>34.0</td>
<td>20.0</td>
<td>38.3</td>
<td>1.54</td>
<td>1.13</td>
</tr>
<tr>
<td>313</td>
<td>10</td>
<td>SWIFT</td>
<td>12.6</td>
<td>32.8</td>
<td>18.0</td>
<td>36.2</td>
<td>1.35</td>
<td>1.10</td>
</tr>
<tr>
<td>314</td>
<td>10</td>
<td>SWIFT</td>
<td>12.6</td>
<td>32.8</td>
<td>17.0</td>
<td>34.4</td>
<td>1.46</td>
<td>1.05</td>
</tr>
<tr>
<td>315</td>
<td>20</td>
<td>SWIFT</td>
<td>13.0</td>
<td>34.0</td>
<td>19.0</td>
<td>35.4</td>
<td>1.46</td>
<td>1.04</td>
</tr>
<tr>
<td>316 (1)</td>
<td>20</td>
<td>SWIFT</td>
<td>13.0</td>
<td>34.0</td>
<td>16.0</td>
<td>37.6</td>
<td>1.23</td>
<td>1.11</td>
</tr>
<tr>
<td>317</td>
<td>10</td>
<td>PAINT</td>
<td>12.6</td>
<td>32.8</td>
<td>18.0</td>
<td>34.4</td>
<td>1.43</td>
<td>1.05</td>
</tr>
<tr>
<td>318</td>
<td>10</td>
<td>PAINT</td>
<td>12.6</td>
<td>32.8</td>
<td>17.0</td>
<td>33.8</td>
<td>1.35</td>
<td>1.03</td>
</tr>
<tr>
<td>319</td>
<td>20</td>
<td>PAINT</td>
<td>13.0</td>
<td>34.0</td>
<td>17.0</td>
<td>36.0</td>
<td>1.31</td>
<td>1.06</td>
</tr>
<tr>
<td>320</td>
<td>20</td>
<td>PAINT</td>
<td>13.0</td>
<td>34.0</td>
<td>17.0</td>
<td>36.0</td>
<td>1.31</td>
<td>1.06</td>
</tr>
<tr>
<td>321 (2)</td>
<td>10</td>
<td>NONE</td>
<td>9.2</td>
<td>9.2</td>
<td>9.5</td>
<td>9.5</td>
<td>-</td>
<td>1.03</td>
</tr>
<tr>
<td>322 (2)</td>
<td>10</td>
<td>NONE</td>
<td>9.2</td>
<td>9.2</td>
<td>9.2</td>
<td>9.2</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td>323 (2)</td>
<td>20</td>
<td>NONE</td>
<td>9.5</td>
<td>9.5</td>
<td>9.7</td>
<td>9.7</td>
<td>-</td>
<td>1.02</td>
</tr>
<tr>
<td>324 (2)</td>
<td>20</td>
<td>NONE</td>
<td>9.5</td>
<td>9.5</td>
<td>10.0</td>
<td>10.0</td>
<td>-</td>
<td>1.05</td>
</tr>
</tbody>
</table>

(1) Uncoated, plated beams kept at 17°C, 56% RH.
(2) Uncoated, unplated beams.
All beams other than those marked (1) were kept at 20°C, 100% R.H.
a - no coating - control specimen kept at 16°C, 56% Relative humidity
b - no coating - kept at 22°C, 100% Relative humidity
c - SWIFT K 7066 Acrylonitrile phenolic
d - FLEXANE 30 Polyurethane
e - MANDERLAC Paint
f - SILITE 100 Silicone

PLATE 84 DURABILITY SPECIMENS AFTER TESTING
DURABILITY SPECIMENS BEFORE TESTING

a-f see Plate 8.4

REINFORCING PLATES AFTER TESTING

PLATE 8.5 DURABILITY SPECIMEN DETAILS
BEAM 101  
3 mm glue thickness, 1.5 mm plate thickness

BEAM 102  
6 mm glue thickness, 1.5 mm plate thickness

BEAM 108  
3 mm glue thickness, 2 layers of 1.5 mm plate, lapped plates above the load points.

PLATE 8-6  
CRACK PATTERNS - AGE OF BEAMS - 18 months.
between the protection given by the different types of coating. The control beam, without a coating, kept with the coated beams in the mist room had a slightly lower failure load than the uncoated beam which was kept under dry conditions.

After 20 months in the mist room, the control beam with no coating had a failure load of 32.7 kN. The similar beam kept at 17°C, 56% relative humidity had a failure load of 36.1 kN. There would therefore appear to be some loss in strength due to prolonged exposure to moisture when no coating is applied. The average failure loads (2 specimens each) for the coated prisms were 38.0 (FLEXANE): 37.6 (SILITE); 36.5 (SWIFT); and 37.1 (PAINT). These values all exceed the control specimen kept at 56% relative humidity showing the effectiveness of the coatings.

The failure loads were calculated by CP110 methods with material safety factors equal to unity as described in the previous Chapters. The tensile stresses in the glue and steel were taken as 16 N/mm² and 275 N/mm² respectively. The cube strengths of concrete at 10 months and 20 months were 75 and 90 N/mm² respectively. The average ratio of experimental to theoretical values was 1.05 at 10 months and 1.08 at 20 months.

The first crack obtained visually were all higher than the predicted loads. As explained previously, this would be expected. The composite behaviour of the steel/epoxy/concrete system was good, and in no case did failure occur by debonding. Plates 8.4 and 8.5 show the beams after failure. In no case was there any visual deterioration of the epoxy or steel plate, even in the uncoated specimens, except of course on the exposed face of the latter. The slightly lower failure loads of the uncoated, exposed prisms could be explained, not due to any loss of bond through ingress of moisture to the adhesive, but rather because of the excessive rusting and resulting loss in thickness of the steel plate. This was found to be up to 0.3 mm in places, with a mean value of approximately 0.2 mm.
8.6 CONCLUSIONS

Based on the results presented in this Chapter the following conclusions can be made.

1. The ultimate loads sustained by the beams tested after 18 months exposure were compatible with what would be expected assuming that there was no degradation of the epoxy bond and that the concrete was ageing normally. The ultimate loads could be predicted accurately by strain compatibility assuming the glue was not cracked.

2. Virtually no corrosion of the steel plates had occurred in the test beams after 18 months exposure. The only signs of rusting occurred at the edge of the plate where the paint had been chipped off during transportation. Carbonation of the cement paste was limited to 2-3 mm depth and had not occurred at all on the tension face where the concrete was protected by the epoxy bonded plate.

3. Even though there was evidence of inadequate mixing of resin and hardener, and inclusion of air pockets in the glue line (up to 10% of the area), the beams ultimate strength and deformation properties were not adversely affected.

4. The load-strain, load-deflection and moment rotation properties closely followed those for similar beams tested at 28 days. In general the older beams showed slightly stiffer behaviour.

5. The deflections calculated by the methods outlined in Appendix 5 were as accurate as those calculated at 28 days when compared with experimental values. The CEB method gave the best results.

6. The empirical formula derived for the beams tested at 28 days predicted the rotations of the beams tested at 18 months within ±11%.

7. For the limited number of tests at 18 months (3 beams only) the cracking characteristics were found to be in close agreement with the 28 day test results.

8. The durability tests showed the effectiveness of the coatings in protecting the epoxy/steel system from the penetration of moisture. However, even in the prisms which had no protective coating there was no visual evidence of corrosion of the epoxy/steel interface after 20 months. The coatings themselves showed no visual deterioration whatever.
CHAPTER 9
LIMITATIONS OF PRESENT WORK, OVERALL CONCLUSIONS AND
RECOMMENDATIONS FOR FUTURE WORK

9.1 LIMITATIONS OF PRESENT WORK

The test data presented here are considered to be the first set of results systematically covering a range of glue and plate thicknesses. Limitations on the number of beams and the duration of testing has restricted this range. Nevertheless, it is hoped that this work will add to our knowledge of the flexural behaviour of reinforced concrete beams strengthened with externally bonded steel plates. The limitations within which the main series of flexural testing were conducted are:

1. The concrete strength of all the beams varied between 60-80 N/mm².
2. Only one type of epoxy adhesive, CXL 194, was used.
3. Only flexural tests were performed. The shear span was kept constant.
4. The amount of internal bar reinforcement was kept constant. All the beams were under reinforced prior to plating. The behaviour of actual bridge members may be different as prestressed beams often behave in an over reinforced manner.
5. Due to the duration of testing only a limited amount of replication of testing was performed.
6. The range of adhesive and plate thickness was 1.5 to 6 mm.
7. All beams had the same dimensions.
8. The long term tests were performed after a relatively short period of only 18 months.
9. The concrete cover to the internal bars was not varied.

9.2 OVERALL CONCLUSIONS

Although the conclusions derived from each chapter are summarised at the end of that chapter, the general conclusions which can be extracted from the test data presented in this thesis may be summarised as follows, (these conclusions are limited by the test conditions and procedures as outlined above):
1. The maximum increase in ultimate flexural capacity on addition of bonded
plate reinforcement was only 17%. The ultimate loads could be accurately predicted by CP 110 or strain compatibility methods when the mode of failure was flexural. The failure mode changed from purely flexural, for beams with 1.5 mm plate, to a shear/bond type failure, for beams with 6 mm plate thickness.

2. The service loads for the plated beams, assessed as the loads at which corresponding deformations in an unplated beam at its service load were attained, were up to 90% higher depending on which criterion was chosen from: deflections, rotations, steel bar strains or maximum crack widths.

3. The service load deflections in the plated beams were up to 40% less than in the unplated specimen. The measured deflections could be predicted satisfactorily by accepted methods. The CEB recommendations were found to give the best correlation.

4. The rotations were up to 48% less in the plated beams than in the unplated beam at service loads. The measured rotations were used to produce an empirical formula for the Young's Modulus of the concrete at any stage of loading. Using this, the rotations could be predicted within ±12%.

5. The crack widths were reduced in the plated beams by up to 63%. The maximum crack widths at working loads were overestimated by the crack width prediction formulae recommended by both CP 110 and ACI. These formulae were modified to produce empirical formulae satisfying the measured values.

6. The flexural behaviour of the beams had not been adversely affected by exposure to natural weathering over a period of 18 months. No deterioration of the adhesive or adhesive/adhered interface was found over this period. Carbonation had been eliminated on the beam soffit by the presence of the bonded plate reinforcement.

7. Inspection of the plates removed from the long term specimens showed signs of inadequate mixing of the resin/hardener system and inclusions of air. In one beam these areas covered up to 10% of the bonded surface. This beam showed no loss of strength during testing.

8. The durability specimens showed that there are various sealing agents available for coating the epoxy/plate element, which effectively prevent the ingress of moisture.
9. The preliminary test series of unreinforced concrete prisms with externally bonded plates seemed to indicate that the tensile strength of the epoxy resin when incorporated in the steel/epoxy/concrete composite system is considerably higher than the unrestrained tensile strength of the epoxy.

9.3 SUGGESTIONS FOR FUTURE WORK

The author endeavoured, within the limited time available to assess the effects of the following:

(a) glue and plate thickness, multiple layers of plate, plate jointing
(b) degree of precracking of the beams
(c) long term exposure.

The range of glue and plate thickness was from 1.5 mm to 6 mm. The thinner layers of glue behaved as well as the thicker layers from a bonding point of view although the latter did provide slightly more stiffness. It is thought that in practice the glue layer would be applied in the thinnest layer which would produce a durable high strength bond. Cusens and Smith (28) tested plated beams with glue thicknesses ranging from 0.25 to 1.5 mm and found the minimum acceptable thickness to be 1 mm. However, they were using a different technique of pouring wet concrete onto an uncured epoxy layer applied to the reinforcing plate. The range of glue thicknesses employed when bonding plates onto hardened concrete should, therefore, be extended below 1.5 mm.

The degree of precracking was investigated to a very limited extent. The tests showed that even with beams loaded up to 90% of their failure load prior to bonding on the plate, satisfactory performance was achieved with the added reinforcement. However, some of the results seemed anomalous, as the deformations of the precracked beams were generally found to be less than in the beams which were not cracked prior to plating. This aspect of behaviour needs further examination.

The mode of failure of the plated beams changed, as the plate thickness increased, from purely flexural to a shear/bond type. The latter should be avoided as there is less warning and it is more brittle. It is thought that this type of failure could be avoided by modifying the ends of the plate. The
plate thickness or width near the ends of the beams could be reduced, or some means of anchoring the plate ends may be effective. In one beam of the present investigation, the thickness of the plate at the ends was reduced and this produced a slightly higher failure load. This is an area which requires further study.

The beams in the present series were all tested with an identical shear span. The effect of varying this should be investigated.

Some assessments of interfacial stresses were made from the test data of the present investigation. These were not limiting values. Tests should be designed to assess the limiting shear and bond stresses in plated beams.

The behaviour of the epoxy resin, especially its tensile contribution in the calculation of the ultimate strength of a beam, should be studied further as it is thought that its tensile strength in the composite system could be considerably higher than the unrestrained value.

Long term test beams have been set up. The tests performed after 18 months showed no loss of performance or deterioration of the epoxy resin. However, 18 months is a very short period in comparison to the life of a structure. Tests must be carried out on beams after increasing lengths of exposure to the elements.
The rapid growth in the use of adhesives has led to an extensive technical vocabulary. The following pages present a selection from various sources.

**ADHERE**
fasten together two surfaces by adhesion.

**ADHEREND**
a body which is attached to another body by an adhesive.

**ADHESION**
the state of being held together by means of an interlayer of adhesives between adherend interfaces; the attachment of two surfaces by interfacial forces consisting of molecular forces, chemical bonding forces, interlocking action, or combinations of these.

**ADHESIVE**
a material that binds other materials together by surface attachment.

**BATCH**
a production quantity derived from a manufacturing process or a mixture of these resulting from the same process conditions.

**BOND**
the union of two materials by adhesives.

**CATALYST**
a chemical substance which accelerates adhesive curing when added in small amounts to the larger quantities of the reactants: material which promotes cross linking in a polymer or accelerates adhesive drying.

**COHESION**
internal adhesion; the ability to resist rupture within the bulk material.
CORROSION the chemical reaction between the adhesive or contamination and the adherend surfaces, due to reactive components in the adhesive leading to deterioration of the bond strength.

CREEP the dimensional change with time of a material under sustained load.

CROSS LINKING the union of adjacent molecules of uncured adhesives (often existing as long polymer chains) by catalytic or curing agents.

CURE to alter the physical properties of an adhesive by chemical change, e.g. polymerisation, brought about by the agency of heat, pressure, or catalysts.

DECREASE to remove oil and grease from adherend surfaces.

DELAMINATION the separation of layers due to adhesive failure.

DURABILITY the resistance to reduction in joint strength shown by adhesives to moisture, heat, chemicals and biodeterioration etc.

EXTENDER a non reactive liquid substance added to epoxy compounds to extend pot life, increase flexibility and lower the cost.

FAILURE, ADHESIVE joint failure such that the separation occurs at the surface of the adherend, e.g. the failure in adhesion of a pressure sensitive tape when peeled from an adherend.

FAILURE, COHESIVE joint failure within the adhesive.
FATIGUE  a condition of stress from repeated flexing or impact force 
on the adhesive-adherend interface.

FILLER  an adhesive additive intended to improve their strength 
and performance.

FLEXIBILISER  a substance which will react with epoxy compounds to impart 
flexibility.

GEL a semi-solid system consisting of a network of solid 
aggregates in which liquid is held.

HARDENER a catalytic or cross linking material used to promote 
setting of adhesives.

INTERFACE the contact area between adherend and adhesive surfaces.

JOINT the junction of two adherends which are held together by 
an adhesive layer.

PASTE a high viscosity adhesive composition.

PENETRATION the passage of an adhesive into an adherend.

PH a measure of acidity or alkalinity.

PHASE a homogenous and physically distinct part of a system 
separated from other parts by definite bounding surfaces.

PLASTICISER a material added to an adhesive to increase its flexibility.

POLYMER a compound formed by the reaction of identical simple 
molecules containing active functional groups to produce a 
high molecular weight material.
POROSITY  the ability of an adherend surface to absorb an adhesive.

POT-LIFE  the effective working time for an adhesive after preparation.

PRETREATMENT  those treatments, mechanical, chemical or physical which are applied to adherends to promote adhesive properties.

PRIMER  an adherend surface coating applied before the adhesive to improve bond performance.

REACTIVE DILUTENT  a low viscosity liquid dilutent for solvent free thermosetting resins of high viscosity. The dilutent undergoes chemical reaction with the adhesive whilst curing proceeds.

RELATIVE HUMIDITY  the ratio of the weight of water in a given volume of air to the weight required to saturate it at the same temperature.

RESIN  the general term for natural and synthetic polymers which are amorphous and have no fixed melting point.

RETARDER  an additive which slows the rate of a chemical reaction.

RHEOLOGY  the study of deformation and flow behaviour of materials under stress.

SET  the conversion of an adhesive into a permanently cured state by chemical or physical means.

SHRINKAGE  the volume reduction occurring during adhesive curing.

SLIPPAGE  the movements of adherends relative to each other during bonding.

STORAGE LIFE  the time period for which an adhesive remains usable when stored under specified temperature conditions.
STRENGTH, CLEAVAGE: The tensile load expressed as force per unit width of bond required to cause cleavage separation of a test specimen of unit length.

STRENGTH, FATIGUE: The maximum load that a joint will sustain when subjected to repeated stress application under specified conditions, i.e. range of stress, mean value, frequency of application.

STRENGTH, IMPACT: Ability of material to resist shock.

STRENGTH, PEEL: The resistance of an adhesive joint to peel stress, the force per unit width of bond at failure.

STRENGTH, SHEAR: The resistance of an adhesive joint to shearing stress. The force per unit area sheared at failure.

STRENGTH, TENSILE: The resistance of an adhesive joint to tensile stress; the force per unit area under tension at failure.

SUBSTRATE: The material surface to which an adhesive material is applied for bonding or coating or other purposes.

SURFACE PREPARATION: The physical and chemical methods employed to prepare an adherend surface for bonding.

THERMO PLASTIC: Susceptible to repeated softening by heating and hardening by cooling.

THERMO SET: A material which does not soften on heating, as a result of being formed from an irreversible chemical reaction initiated by a catalyst.

VIScosity: A measure of resistance to flow of a liquid.
THEORETICAL STRESS DISTRIBUTION IN A BONDED LAP JOINT UNDER COMPRESSION

Notation

\( u \) \quad \text{axial deformation of adherends}
\( \gamma \) \quad \text{angular deformation}
\( \tau \) \quad \text{shear stress in the adhesive}
\( E \) \quad \text{modulus of elasticity of adherends}
\( G \) \quad \text{modulus of rigidity of adhesive} \left[ \frac{E_G}{1(2+\nu_G)} \right]
\( t_1 \) \quad \text{semi-thickness of concrete}
\( t_2 \) \quad \text{thickness of steel plate}
\( d' \) \quad \text{thickness of adhesive}
\( c \) \quad c = \frac{G}{d'}
\( P \) \quad \text{applied load}

\[ \text{concrete cube.} \]

\[ \begin{align*}
\Phi &= \frac{c}{P} \\
\Phi &= \frac{c}{P} + \frac{dP}{dt}
\end{align*} \]

\[ \Phi = \frac{u_1}{d} \quad \text{and} \quad \Phi = \frac{u_2}{d} \]

\[ \text{steel plate} \]

\[ \text{glue} \]
\[ P_1 + P_2 = P \]

\[ \gamma = \frac{u_2 - u_1}{d} = \frac{\tau}{G} \]

or \[ \tau = c (u_2 - u_1) \]

\[ \begin{align*} 
\frac{dt}{dx} &= c \left( \frac{du_2}{dx} - \frac{du_1}{dx} \right) \\
\frac{du_1}{dx} &= \frac{P_1}{E_1 t_1} \quad \frac{du_2}{dx} = \frac{P_2}{E_2 t_2} 
\end{align*} \]

\[ \begin{align*} 
\frac{dt}{dx} &= c \left( \frac{P_2}{E_2 t_2} - \frac{P_1}{E_1 t_1} \right) \quad \text{but } P_1 = (P - P_2) \\
\frac{dt}{dx} &= c \left( \frac{P_2}{E_2 t_2} + \frac{P_2}{E_1 t_1} \right) - \frac{cP}{E_1 t_1} \\
\frac{d^2t}{dx^2} &= c \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right) \frac{dP_2}{dx} 
\end{align*} \]

putting \( w^2 = c \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right) \) and \( \tau = \frac{dP_2}{dx} \)

\[ \frac{d^2t}{dx^2} - w^2 \tau = 0 \]

The general solution of this differential equation is:

\[ = A \sinh wx + B \cosh wx \]

when \( x = 0 \quad P_2 = 0 \)

\[ \frac{dt}{dx} = A w = -\frac{cP}{E_1 t_1} \quad A = -\frac{c}{w} \frac{P}{E_1 t_1} \]

when \( x = 1 \quad P_2 = P \)

\[ \frac{dt}{dx} = cP \left( \frac{1}{E_1 t_1} - \frac{1}{E_2 t_2} \right) - \frac{cP}{E_1 t_1} = -\frac{c}{w} \frac{Pw}{E_1 t_1} \cosh wL + Bw \sinh wL \]

\[ \therefore \quad B = \frac{c}{w} \frac{P}{\sinh wL} \left( \frac{1}{E_2 t_2} + \frac{\cosh wL}{E_1 t_1} \right) \]

-240-
\[ \tau(x) = \frac{cP}{w \sinh w} \left( \frac{\cosh w(1-x)}{E_1 t} + \frac{\cosh wx}{E_2 t^2} \right) \]  

(1)

But \( \tau = \frac{dP_2}{dx} = c(u_2 - u_1) \)

\[ \frac{d^2P_2}{dx^2} = c\left( \frac{du_2}{dx} - \frac{du_1}{dx} \right) \]

so

\[ \frac{d^2P_2}{dx^2} = cP_2 \left( \frac{1}{E_1 t} + \frac{1}{E_2 t^2} \right) - \frac{Pc}{E_1 t} \]

then

\[ \frac{d^2P_2}{dx^2} = w^2P_2 - \frac{Pc}{E_1 t} \]

The general solution of this differential equation is:

\[ P_2 = A \cosh wx + B \sinh wx + C \]

Then

\[ \frac{d^2P_2}{dx^2} = w^2 \left( A \cosh wx - B \sinh wx \right) \]

\[ w^2(A \cosh wx + B \sinh wx) = w^2(A \cosh wx - B \sinh wx) + w^2C - \frac{Pc}{E_1 t} \]

\[ \therefore C = \frac{Pc}{w^2 E_1 t} \]

when \( x = 0 \)

\[ P_2 = 0 \]

\[ 0 = A + \frac{Pc}{w^2 E_1 t} \]

\[ \therefore A = -\frac{Pc}{w^2 E_1 t} \]

when \( x = 1 \)

\[ P = \cosh w + B \sinh w + \frac{Pc}{w^2 E_1 t} \]

\[ \therefore B = \left( \frac{Pc}{w^2 E_1 t} \cosh w - \frac{Pc}{w^2 E_1 t} \right) \frac{1}{\sinh w} + \frac{P}{\sinh w} \]

\[ \therefore P_2 = \frac{Pc}{w^2 E_1 t} \left[ 1 - \cosh wx + \frac{(\cosh w - 1) \sinh wx}{\sinh w} \right] + \frac{P \sinh wx}{\sinh w} \]

\[ P_2 \]  

(2)

These theoretical distributions are plotted in Fig. 3.7.
APPENDIX 3
PRELIMINARY TEST SERIES

LOADING SYSTEM - Series A, Odd numbered beams.

\[ F = \mu W/2 \]

Bending moment at X:

\[ \frac{W \cdot 0.61}{4} - \frac{W \cdot 0.15 \cdot \mu}{2} \]

Assuming \( \mu = 0.5 \) for concrete on steel:

\[ M_X = 0.152 W - 0.0375 W = 0.1145 W. \]

LOADING SYSTEM - Series A, Even numbered beams.

\[ M = \frac{W \cdot 0.61}{4} = 0.152 W \]
Calculation of First Crack Load in Concrete - Beam A1

Modulus of elasticity of steel plate = 200 kN/mm² \( \alpha_s = 5.56 \)

" " " concrete = 36 kN/mm²

" " " glue = 6 kN/mm² \( \alpha_g = 0.167 \)

Location of Neutral Axis:

\[
\bar{x}(152.152 + 16.4 + 556.1) = (152.152^2 + 16.4.154 + 1.556.156.5)
\]

\[
\bar{x} = 78.09 \text{ mm}
\]

Taking moments about the Neutral Axis:

\[
I = \frac{152.78.09^3 + 152.78.09(78.09)^2 + 152(73.91)^2}{12} + 16.4.75.91^2 + 555.78.41^2 \quad \text{(Concrete)}
\]

\[
I_{u}^1 = 4.84 \times 10^7 \text{ mm}^4.
\]

\[
M = \frac{\sigma I}{y}
\]

Modulus of rupture \( \sigma = 4.37 \text{ N/mm}^2 \)

\( y = 73.91 \text{ mm} \)

\[
\therefore \text{ First Crack Moment} = \frac{4.84 \times 10^7 \times 4.37}{73.91} = 3.06 \text{ kNm.}
\]
CALCULATION FOR FIRST CRACK LOAD IN THE GLUE - BEAM A1

Location of Neutral Axis:

\[ \bar{x} (152 \bar{x} + 16.4 + 556.1) = \frac{(152 \bar{x}^2 + 16.4 \cdot 154 + 556.1 \cdot 156.5)}{2} \]

\[ 152 \bar{x}^2 + 619 \bar{x} = 76 \bar{x}^2 + 96714 = \]

\[ \bar{x}^2 + 8.14 \bar{x} - 1273 = 0. \]

Hence \( \bar{x} = 31.8 \text{ mm} \).

Moment of Inertia, taking moments about the Neutral Axis:

\[ I = \frac{152 \cdot 31.8^3 + 152 \cdot 31.8}{12} \frac{31.8^2}{2} (\text{Concrete}) \]

\[ + 16.4 \cdot 122.2^2 (\text{Glue, neglecting inertia about its own axis}) \]

\[ + 555.1 \cdot 124.7^2 (\text{Steel, neglecting inertia about its own axis}) \]

\[ I_{CR} = 1.12 \cdot 10^7 \text{ mm}^4. \]

\[ M = \frac{\sigma I}{y} \]

Tensile strength of glue = 60 N/mm²

\[ y = 120.2 \text{ mm} \]

First Crack Moment = 5.59 kNm.
CALCULATIONS FOR ULTIMATE LOAD. TABLES 4.3 - 4.6.

(c) Based on yield stress of plate, plus tensile strength of glue.

Beam A1 (glue thickness 4 mm).

Force in concrete in compression (CP110)
0.6. fcu. b. \( \bar{x} = 0.6 \times 64.6 \times 152 \times \bar{x} \).
FC = 5891 \( \bar{x} \). N.
FG = 60.4.100 = 24000 N.
FS = 125.1.100 = 12500 N.

\( f_{cu} = 64.6 \text{ N/mm}^2 \)
\( f_{g} = 60 \text{ N/mm}^2 \)
\( f_{y} = 125 \text{ N/mm}^2 \).

Hence \( \bar{x} = 6.20 \text{ mm} \).

Then taking moments about the centroid of the compressive force:
\[
M = 24000 \ (154 - 3.1) + 12500 \ (156.5 - 3.1) = 5.539 \ \text{kNm}.
\]

(d) Based on ultimate stress of plate, plus tensile strength of glue.

As for (c) except \( F_S = 13200 \text{ N} \).

Hence \( \bar{x} = 6.31 \text{ mm} \).

\[
M = 24000 \ (154 - 3.155) + 13.2 \ (156.5 - 3.155) = 5.644 \ \text{kNm}.
\]

(e) Based on ultimate stress of plate, plus no tensile strength in glue.

\( \bar{x} = 2.24 \text{ mm} \).

\[
M = 13.2 \ (156.5 - 1.12) = 2.051 \ \text{kNm}.
\]
APPENDIX 4

CALCULATION OF ULTIMATE LOADS: BEAMS 201 - 224

Three methods are used for calculating the theoretical ultimate capacities of the test beams.

(a) Ultimate Limit State to CP110

The following assumptions are made:

1. The strain distribution in the concrete in compression is derived from the assumption that plane sections before loading remain plane up to failure.

2. The resistance of concrete in tension is ignored.

3. The relationships between stress and strain in the reinforcing bar, plate and glue are as shown in Figs. 3.8, 3.9 and 3.4 respectively.

4. The distribution of stress in the concrete at failure may be assumed to be equal to a uniform stress of 0.6 $f_{cu}$ over the entire compression zone. The maximum strain at the compression face is taken as 0.35% and the centroid of the stress block is at half the depth of the compression zone.

5. For the purpose of calculating the ultimate capacity of the test beams the material safety factors are equal to 1.0 and the stresses in the bar and plate are their respective 0.2% proof stresses, as taken from Figs. 3.8 and 3.9.
Tensile force = Ast. fp where fp = 0.2% proof stress

Compressive force = 0.6 fcu x b where b = width of beam

Hence \( x = \frac{\text{Ast. fp}}{0.6 \cdot b \cdot fcu} \)

Also \( z = d - \frac{x}{2} = d(1 - 0.83 \cdot \text{Ast. fp}) \)

Therefore the ultimate moment capacity \( M_u \) is equal to:

\[
M_u = \frac{\text{Ast. fp} \cdot d(1 - 0.83 \cdot \text{Ast. fp})}{b \cdot d \cdot fcu}
\]

But also \( \frac{\text{Ast.}}{bd} = \rho \)

\[
M_u = \frac{fp \cdot \rho \cdot bd^2 (1 - 0.83 \cdot \rho \cdot fp)}{fcu}
\]

---

**EXAMPLE BEAM 207**

Plate thickness 1.5 mm
Glue thickness 3 mm
fcu 70.2 N/mm²

<table>
<thead>
<tr>
<th>Plate</th>
<th>Bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>fp = 275 N/mm²</td>
<td>470 N/mm²</td>
</tr>
<tr>
<td>( \rho = 0.0047 )</td>
<td>0.0277</td>
</tr>
<tr>
<td>d = 258.75 mm</td>
<td>220 mm</td>
</tr>
</tbody>
</table>
Then the ultimate moment is given by

\[ Mu = 275.0 \times 0.0047 \times 155.258 \times 75^2 \left(1 - 0.83(0.0047 \times 275 + 0.0277 \times 470) \right) \]

\[ + 470.0 \times 0.0277 \times 155.220^2 \left(1 - 0.83(0.0047 \times 275 + 0.0277 \times 470) \right) \]

\[ \frac{70.2}{70.2} \]

Plate Bars

\[ = (11.14 + 81.10) \times 10^6 \text{ Nmm} \]

\[ = 92.24 \text{ kNm} \]

(b) **Strain Compatibility**

The basic principle of strain compatibility is that for a given section, including the reinforcement, a neutral axis depth can be found so that the total compression force equals the total tension force; and hence the ultimate moment.

The following assumptions are made:

1. The strain distribution in the concrete in compression is derived from the assumption that plane sections before loading remain plane up to failure.
2. The resistance of concrete in tension is ignored.
3. The relationship between the stress and strain in the reinforcements are as shown in Figs. 3.8 and 3.9, found by experimental tests.
4. The relationship between the stress and strain in the concrete is the rectangular stress block with a maximum concrete compression strain of 0.0035, and a compressive stress of 0.60 \( f_{cu} \).
5. The relationship between the stress and strain in the glue is as shown in Fig. 3.4 found by experimental tests.
6. All material safety factors, \( Y_m \), are equal to unity.
(i) First it is assumed that no tensile contribution is made by the glue

\[
\text{Guess } f_{\text{plate}} = 250 \text{ N/mm}^2 \quad f_{\text{bars}} = 425 \text{ N/mm}^2
\]

Tensile force = Compressive force

\[
250 \times 187.5 + 425 \times 943 = 0.6 f_{\text{cu}} \times 155 x
\]

\[
x = 75.74 \text{ mm}
\]

By strain compatibility:

\[
E_{\text{bars}} = \frac{220 - 75.74 \times 0.0035}{75.74} = 0.0067
\]

\[
E_{\text{plate}} = \frac{258.75 - 75.74 \times 0.0035}{75.74} = 0.0085
\]

From the experimental stress strain curves for the steel bar and plate, as shown in Figs. 3.8 and 3.9, the stresses corresponding to these strains are

\[
f_{\text{bars}} = 490 \text{ N/mm}^2
\]

\[
f_{\text{plate}} = 325 \text{ N/mm}^2
\]

The initial guess, therefore, was incorrect.

Try \( f_{\text{plate}} = 300 \) and \( f_{\text{bars}} = 490 \)

Then

\[
300 \times 187.5 + 490 \times 943 = 0.6 f_{\text{cu}} \times 155 x
\]

\[
x = 79.39
\]
Hence \( E_{\text{bars}} = \frac{220 - 79.39}{79.39} \cdot 0.0035 = 0.0062 \)

\( E_{\text{plate}} = \frac{258.75 - 79.39}{79.39} \cdot 0.0035 = 0.0079 \)

From the experimental stress strain graphs:

\( f_{\text{bars}} = 490 \text{ N/mm}^2 \)  
\( f_{\text{plate}} = 297 \text{ N/mm}^2 \)

\( f_{\text{rebar}} = 490 \text{ N/mm}^2 \left( \frac{258.75 - 79.39}{2} \right) + 943.490 \left( \frac{220 - 79.39}{2} \right) \)

\( \mu = \frac{95.5}{2} \text{ kNm} \)

(ii) Assuming the glue remains uncracked up to failure the force it imparts is included in the tensile component

Assume \( f_{\text{glue}} = 15 \text{ N/mm}^2 \) \( f_{\text{plate}} = 300 \text{ N/mm}^2 \) \( f_{\text{rebar}} = 490 \text{ N/mm}^2 \)

\( 300 \cdot 187.5 + 490 \cdot 943 + 15 \cdot 375 = 0.6 f_{\text{cu}} \cdot 155 x. \)

\( x = 80.25 \text{ mm} \)

\( E_{\text{bars}} = \frac{220 - 80.25}{80.25} \cdot 0.0035 = 0.0061 \)

\( E_{\text{plate}} = \frac{258.5 - 80.25}{80.25} \cdot 0.0035 = 0.0078 \)

\( E_{\text{glue}} = \frac{256.5 - 80.25}{80.25} \cdot 0.0035 = 0.0077 \)

from experimental stress/strain graphs:-

\( f_{\text{rebar}} = 490 \text{ N/mm}^2 \)

\( f_{\text{glue}} = 14 \text{ N/mm}^2 \)

\( f_{\text{plate}} = 297 \text{ N/mm}^2 \)

Then \( \mu = 187.5 \cdot 300 \left( \frac{258.75 - 80.25}{2} \right) + 943.490 \left( \frac{220 - 80.25}{2} \right) + 15.375 \left( \frac{256.5 - 80.25}{2} \right) \)

\( = 96.6 \text{ kNm} \)

The calculated ultimate loads are shown in Table 5.4 together with the experimental failure loads.
(c) Strain Compatibility

An alternative, and more refined method of calculation than in (b) is to use the stress distribution in the concrete as suggested by Hognestad, as shown below. ($k_1$, $k_2$, $\epsilon_c$ from Fig. 4.4-1, Reinforced and Prestressed Concrete - Kong and Evans.)

Beam 207

\[ \epsilon_b = \epsilon_c \left( \frac{220 - x}{x} \right) \quad \text{and} \quad \epsilon_p = \epsilon_c \left( \frac{258.75 - x}{x} \right) \]

\[ k_1 \cdot f_{cu} \cdot b \cdot x = A_p \cdot f_p + A_b \cdot f_b \]

Assume $x = 94$ mm

we also have: $f_{cu} = 70.2$ N/mm$^2$

$f = 295$ N/mm$^2$

$k_1 = 0.48$

$k_2 = 0.41$

Then $0.48 \cdot 70.2 \cdot 155.94 = 295.187.5 + 943.94 \cdot f_b$

\[ f_b = 462 \text{ N/mm}^2 \]

With $\epsilon_c = 0.0028$, $\epsilon_b = 0.0028 \left( \frac{220 - 94}{94} \right) = 0.00375$, $\epsilon_p = 0.00491$

From the experimental stress/strain curve, Fig. 3.8, the steel stress in the bars corresponding to this strain is 464 N/mm$^2$, and for the plate the stress is found from Fig. 3.9, 295 N/mm$^2$. \text{.} \text{.} \text{.} assumptions were OK. The ultimate moment is then given by:

\[ M_u = 464.943 \cdot (220 - 0.41.94) + 295.187.5 \cdot (258.75 - 0.41.94) \]

\[ = 91.6 \text{KNm} \]

-251-
Similar calculations for beams 208 (3 mm plate) and 209 (6 mm plate) give values of $100.6 \, \text{N/mm}^2$ and $120.1 \, \text{N/mm}^2$. These differ from method (b) (i) by 2 to 4\% only.
APPENDIX 5

CALCULATION OF DEFLECTIONS

(a) CP 110 Recommendations

This is dealt with in Appendix A of the code. In clause A.1 it states that in general it will be sufficiently accurate to assess the moments and forces at serviceability limit states by using an elastic analysis. When the deflections of reinforced concrete members are calculated there are a number of factors which are difficult to allow for but which have a considerable effect on the reliability of the result:

(a) support conditions
(b) precise loading conditions, especially long term
(c) extent of cracking.

The approach used is to assess the curvatures of sections under the appropriate moments and then calculate the deflections from the curvatures. The recommended procedure involves calculating the curvatures at successive sections along the beam and using numerical integration to compute the deflection. For calculating the curvatures clause A.2.2 gives a procedure which employs an appropriate set of assumptions depending on whether the section is cracked or uncracked. The test beams were all cracked at service load. The assumptions for this are straightforward and are illustrated diagrammatically in Fig. A.5.1.

(i) Strains are calculated on the assumption that plane sections before loading remain plane after loading.

(ii) The reinforcement is assumed to be elastic and its modulus of elasticity is taken as 200 kN/mm².

(iii) The concrete in compression is assumed to be elastic. Under short term loading, the modulus of elasticity $E_c$ is used.

(iv) Stresses in the concrete in tension may be calculated on the assumption that the stress distribution is triangular having a value of zero at the neutral axis and a value of 1 N/mm² at the centroid of the tension steel.

To obtain a relationship between the bending moment and the curvature a force diagram is drawn and the bending moments taken. The equation for the
Compressive force = Tensile force

\[ \frac{bx}{2} E_c \epsilon_c = E_s \epsilon_s A_s \]

but \( \epsilon_s = \epsilon_c \left( \frac{d-x}{x} \right) \), \( E_s / E_c = \alpha_e \) and \( A_s / bd = \rho \)

therefore \( \frac{bx}{2} E_c \epsilon_c = E_s \epsilon_c \left( \frac{d-x}{x} \right) A_s \)

or \( x^2 = 2\alpha_e \rho d (d-x) \)

then \( x^2 + 2\alpha_e \rho dx - 2\alpha_e \rho d^2 = 0 \)

solving this quadratic equation gives:

\[ x = -\alpha_e \rho d \pm d \sqrt{\alpha_e \rho (2 + \alpha_e \rho)} \]

or \( x = -\frac{\alpha_e \rho + \sqrt{\alpha_e \rho (2 + \alpha_e \rho)}}{d} \)

also \( z = d - x = d (1 - x_r) \)

and \( I_e = \frac{bx^3}{12} + bx \left( \frac{x}{2} \right)^2 A_s \alpha_e (d-x)^2 \)

or \( \frac{I_e}{bd^3} = \frac{1}{3} \left( \frac{x}{d} \right)^3 + \alpha_e \rho \left( 1 - \frac{x}{d} \right)^2 \)

FIGURE A5.1 ELASTIC ASSUMPTIONS FOR CALCULATING CURVATURES
neutral axis depth becomes rather complicated and a further assumption is made to simplify this. It is assumed that the neutral axis depth is calculated on the basis of zero stress in the concrete in the tension zone. This will slightly underestimate the neutral axis depth. Thus, the concrete in tension is ignored when calculating the neutral axis depth, but taken into account when calculating the resistance moment.

In the case of the plated beams the plate area has been added to the bar area and the effective depth is taken to their combined centroid.

**EXAMPLE** Beam 207

<table>
<thead>
<tr>
<th>Glue thickness</th>
<th>3 mm</th>
<th>(E_c = 36) kN/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate thickness</td>
<td>1.5 mm</td>
<td>(E_s = 200) kN/mm²</td>
</tr>
</tbody>
</table>

\[\alpha_e = \frac{200}{36} = 5.56\]

\[\rho = \frac{187 + 943}{155.226} = 3.23 \times 10^{-2}\]

\[A_{\text{bars}} = 943\text{mm}^2\]

\[A_{\text{plate}} = 187\text{mm}^2\]

From Fig. A.5.1 \(\frac{x}{d} = -0.179 + \sqrt{0.179(2.179)}\) Hence \(x = 100.6\) mm

\[\frac{I}{bd^3} = \left(0.445\right)^3 + 0.179 (1 - 0.445)^2\] Hence \(I = 1.51 \times 10^8\) mm⁴

Next the resistance moment is calculated allowing for the tension stiffening of the concrete. As shown in Fig. A.5.2 the moment due to the concrete in tension is given by \(\frac{fc}{3}\left\frac{b(h - x)^3}{d - x}\right\)

The deflections of the test beams were calculated at 130 KN load for comparison with experimental values. The applied moment \(\frac{WL}{6}\) corresponding to 130 KN load is 49.8 KNm.
**TYPICAL MODE OF LOADING**

---

**BENDING MOMENT DIAGRAM**

---

**TENSILE CONCRETE**

- Stress at reinforcement level: $f_{ct}$
- Stress at extreme fibre: $f_{ct} \frac{(h-x)}{(d-x)}$

- Force in tensile concrete: $F_{ct}$
  
  $$F_{ct} = f_{ct} \frac{b(h-x)}{2}$$

- Moment of tensile concrete about neutral axis: $M_{ct}$
  
  $$M_{ct} = f_{ct} \frac{b(h-x)}{3(d-x)}$$

---

**FIGURE A5.2** BENDING MOMENT DUE TO THE CONCRETE IN THE TENSION ZONE
The resistance moment \( = 49.8 - 1.155'(255' - 100.7)^3.10^{-6} \text{ KNm} \)

\[ \frac{3}{2} (226 - 100.7) \]

\( = 48.3 \text{ KNm} \)

The curvature is then found from simple bending theory. \( \frac{M}{E} \frac{I}{R} \)

Therefore \( \frac{1}{R} = \frac{48.3 \times 10^6}{36.10^5 \times 1.51 \times 10^6} = 8.9 \times 10^{-6} \) radians.

The deflection can then be found by the simplified approach recommended in clause A.2.3 from the equation \( a = k_1^2 \frac{1}{R} \)

Where \( K \) is a constant which depends on the loading and support conditions, and \( l \) is the effective span. The Code Handbook, in Table A3 gives values of \( K \) for various loadings and support conditions. In general the deflection of a beam is given by the formula \( a = \frac{K_1 W I^3}{EI} \), but also the bending moment,

\( M = K_2 W I \).

For the test beams the loadings are symmetrically placed at the \( l \) points of the span and the value of \( K_2 \) is \( \frac{1}{6} \) or \( 0.167 \).

For the value of \( K_1 \) Macaulay's method can be used as shown in Fig. A.5.3 and is equal to \( \frac{23}{1296} = 0.01775 \).

Combining the equations containing \( K_1 \) and \( K_2 \): \( a = \frac{K_1 \cdot 1^2 \cdot M}{K_2 \cdot EI} \)

or \( K = \frac{0.01775}{0.1666} = 0.1065 \) as shown in Table A3 of the code.

Then the deflection for beam 207 is given by:

\( a = 0.1065 \times 2300^2 \times 0.89 \times 10^{-5} = 5.0 \text{ mm} \).

(b) **ACI Recommendations**

From the work performed by Branson (77) the effective moment of inertia is given by \( I_e = \left( \frac{M_{cr}}{M} \right)^3 I_u + \left[ 1 - \left( \frac{M_{cr}}{M} \right)^3 \right] \cdot I_{cr} \).

The deflection is then given by \( a = \frac{23 \times W I^3}{1296 \cdot E I_e} \).

The same assumptions apply as for CP 110, i.e.:

(i) plane sections remain plane

(ii) reinforcement is elastic
General expression for bending moment at distance \( x \) from the end.

\[
M = EI \frac{d^2a}{dx^2} = \frac{-Wx}{2} \cdot \frac{W}{2} \cdot \frac{x-\frac{L}{3}}{\frac{2}{3}} \cdot \frac{W}{2} \cdot \frac{x-\frac{2L}{3}}{\frac{2}{3}}
\]

\[
J_M = EI \frac{d^2a}{dx^2} = \frac{-Wx}{4} \cdot \frac{W}{4} \cdot \frac{x-\frac{L}{3}}{\frac{3}{3}} \cdot \frac{W}{4} \cdot \frac{x-\frac{2L}{3}}{\frac{3}{3}} \cdot A
\]

\[
J_M = EI \frac{d^2a}{dx^2} = \frac{-Wx}{12} \cdot \frac{W}{12} \cdot \frac{x-\frac{L}{3}}{\frac{3}{3}} \cdot \frac{W}{12} \cdot \frac{x-\frac{2L}{3}}{\frac{3}{3}} \cdot A x \cdot B
\]

Boundary conditions:
when \( x = 0 \) & \( L_a = 0 \)
Therefore \( B = 0 \) and \(-AL = \frac{Wl^3}{12} \cdot \frac{W}{12} \cdot \frac{L}{2} \cdot \frac{L}{2} \cdot \frac{L}{3} \cdot \frac{L}{3} \)

or \( A = \frac{2WL^3}{36} \)

Then:

\[
EIa = \frac{-Wx}{12} \cdot \frac{W}{12} \cdot \frac{x-\frac{L}{3}}{3} \cdot \frac{W}{12} \cdot \frac{x-\frac{2L}{3}}{3} \cdot \frac{2WL^3}{36}
\]

The central deflection:
when \( x = \frac{L}{2} \)

\[
a = \frac{1}{EI} \left[ \frac{-Wl^3}{96} \cdot \frac{W}{12} \cdot \frac{L}{216} + \frac{2WL^3}{3} \cdot \frac{24}{3} \right]
\]

or \( a = \frac{23}{1296} \frac{WL^3}{EI} \)

---

**FIGURE A5.3 CENTRAL DEFLECTION BY MACAULAY'S METHOD**
(iii) concrete in compression is elastic.

EXAMPLE Beam 207

Mcr, the theoretical cracking moment, depends on the moment of inertia of the uncracked section (Fig. A.5.4) and the modulus of rupture which was determined by experiment - 5.56 N/mm².

\[ M_{cr} = \frac{2.694.10^8 \times 5.56}{113.83} = 13.2 \text{kNm} \]

M, the moment under consideration = 49.8 kNm (130 kN load). The value of the cracked, transformed moment of inertia = 1.54.10⁸ mm⁴ (Fig. A.5.5).

Hence the effective moment of inertia

\[ I_e = \left( \frac{13.2}{49.8} \right)^2 \times 694.10^8 + \left[ 1 - \left( \frac{13.2}{49.8} \right)^3 \right] \times 1.54.10^8 \text{ mm}^4 \]

\[ = 1.561.10^8 \text{ mm}^4 \]

Then \( a = \frac{23 \times 130.10^3 \times 2300^3}{1296 \times 36.10^3 \times 1.561.10^8} \)

\[ = 5.0 \text{ mm} \]

(c) CEB Recommendations

The deflections are calculated from considerations of whether the section is cracked or uncracked. In simply supported structures the deflections under short term loading may be calculated on the assumption that the stiffness in the cracked state = \( E_s A z (d-x) \). Thus the total deflection is split into two parts, one applying before cracking and the other after.

Before cracking the curvature, \( \frac{1}{r_1} = \frac{M_{cr}}{E_c I^1_u} \), notation as before

After cracking the curvature, \( \frac{1}{r_2} = \frac{4 (N-M_{cr})}{3 E_s A z (d-x)} \)

The deflection, a, is then given by \( a = kl^2 \left( \frac{1 + \frac{1}{r_1}}{r_2} \right) \)

EXAMPLE Beam 207

Mcr as in ACI method = 13.2 kNm

\( I^1_u \) = 2.694.10⁸ mm⁴

K as in CP 110 method = 0.1065

x = 100.6 mm

z = 192.5 mm
Neutral axis position - $\bar{x}$

$$
\bar{x} = \frac{(155.255 \cdot 260.20 \cdot 7.3 \cdot 695.15 \cdot 5 \cdot 695.15 \cdot 5.258.75)}{2}
$$

Hence $\bar{x} = 141.17$ mm.

Moment of Inertia - taking moments about the Neutral Axis:

$$
I_{concrete} = \frac{141.17^3 \cdot 155}{12} \cdot \frac{155 \cdot 141.17}{2} = 1.46 \cdot 10^8 \text{ mm}^4 \quad \text{(concrete above N.A.)}
$$

$$
I_{concrete} = \frac{113.83^3 \cdot 155}{12} \cdot \frac{155 \cdot 113.83}{2} = 0.76 \cdot 10^8 \text{ mm}^4 \quad \text{(concrete below N.A.)}
$$

$$
I_{steel} = 943 \cdot 5.56 \cdot 78.83 = 0.33 \cdot 10^8 \text{ mm}^4 \quad \text{(steel bars, neglecting inertia about their own axis)}
$$

$$
I_{plate} = 187 \cdot 5.56 \cdot 117.56 = 0.14 \cdot 10^8 \text{ mm}^4 \quad \text{(steel plate, glue negligible.)}
$$

TOTAL $I_u = 2.69 \cdot 10^8 \text{ mm}^4$

FIGURE A5.4 MOMENT OF INERTIA OF UNCRACKED TRANSFORMED SECTION
Neutral axis position - $x$

$$x(155x + 260.20 \cdot 73 \cdot 695.15) = (155x^2 + 260.20 \cdot 220 \cdot 73.256.5 \cdot 695.15 \cdot 258.75)$$

Hence $x = 101.15$ mm

Moment of inertia:

taking moments about the Neutral Axis:

$$I = \frac{155 \cdot 101.15^3 + 155.101.15 \cdot 10.15}{12} = 0.5410 \text{ mm}^4$$

(concrete)

$$+ 943.5 \cdot 5.56 \cdot 188.85 = 0.7410 \text{(steel bars, neglecting inertia about their own axis.)}$$

$$+ 187.5 \cdot 5.56 \cdot 157.60 \cdot 0.2630 \text{(steel plate)}$$

+ glue negligible

TOTAL $I'_{cr} = 1.5410 \text{ mm}^4$
\[ A_s = 943 \text{ (bars)} + 187 \text{ (plate)} = 1130 \text{ mm}^2 \]

Hence
\[
a = 0 \cdot 1065.2300 \left[ 36 \cdot 10^3 \cdot 2.10^6 + 4 \cdot \frac{(49 \cdot 8 - 13 \cdot 2) \cdot 10^5}{3 \cdot 2.10^5 \cdot 1130 \cdot 192 \cdot 5.125 \cdot 4} \right] = 5.81 \text{ mm}\
\]
APPENDIX 6

CALCULATION OF ROTATIONS

From basic bending theory \( \frac{M}{I} = \frac{E}{R} \)

where \( M \) = applied bending moment
\( I \) = moment of inertia of the section
\( E \) = Young's Modulus
\( \frac{1}{R} \) = curvature at the section under consideration

The integral of the curvatures at each section along a beam produces the rotation. Hence the integral of \( \frac{M}{EI} \) along the beam also gives the rotation. This is equivalent to the area under the bending moment diagram divided by \( EI \), assuming a constant stiffness along the length of the beam. As in the calculation of deflections two cases must be considered, one before cracking occurs and the other after. Before cracking occurs the rotation is

\[
\frac{E_c}{I_u} \int \text{Area under Bending Moment Diagram}
\]

where \( E_c \) = Young's Modulus of concrete, short term

and \( I_u \) = uncracked, transformed moment of inertia.

In the second case the bending moment, resisted by the concrete in compression and the tension steel, is reduced by the moment resisted by the concrete in tension between the cracks. This is taken as, \( M_{ct} \),

\[
M_{ct} = f_{ct} \frac{6}{3} \left( \frac{(h-x)^3}{(d-x)} \right) \text{ Fig. A5.2}
\]

The bending moments were calculated at four stages after cracking, up to failure, allowing for the tensile contribution of the concrete. The neutral axis positions found from the measured strain distributions were used and the effective depth was taken to the centroid of the steel bar and plate areas.

The second moment of area was calculated at each stage using the same neutral axis positions. The rotations were then calculated as described above for a cracked section. Two values of tensile stress in the concrete were assumed. Firstly, \( f_{cr} = 1 \text{ N/mm}^2 \) as recommended by CP110 and secondly, \( 3 \text{ N/mm}^2 \). The
latter value was not thought to be unreasonable as the modulus of rupture had an
average value of 5.6 N/mm². A value of approximately half this did not seem too
high.

EXAMPLE Beam 207

(i) Moments to be resisted by the section

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Moment (\frac{WL}{6}) (Nmm)</th>
<th>(M_{ct})</th>
<th>(f_{ct}=1) N/mm² (10⁶)</th>
<th>(f_{ct}=3) N/mm² (10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>60.2300.1000/6</td>
<td>(\frac{155}{3}(\frac{255-112}{226-112})^3)</td>
<td>21.7</td>
<td>19.0</td>
</tr>
<tr>
<td>130</td>
<td>130.2300.1000/6</td>
<td>(\frac{155}{3}(\frac{255-100}{226-100})^3)</td>
<td>48.3</td>
<td>45.3</td>
</tr>
<tr>
<td>190</td>
<td>190.2300.1000/6</td>
<td>(\frac{155}{3}(\frac{255-90}{226-90})^3)</td>
<td>71.1</td>
<td>67.7</td>
</tr>
<tr>
<td>250</td>
<td>250.2300.1000/6</td>
<td>(\frac{155}{3}(\frac{255-80}{226-80})^3)</td>
<td>93.9</td>
<td>90.1</td>
</tr>
</tbody>
</table>

(ii) Moment of Inertia

The section is assumed to be cracked to some degree at all the load
stages shown above. The neutral axis positions are as shown in the moment
calculations, and concrete below the neutral axis is assumed to have no
contribution to the moment of inertia, and \(\alpha_e = 5.56\).

<table>
<thead>
<tr>
<th>Section</th>
<th>Load (kN)</th>
<th>Moment of Inertia (mm⁴) (10⁸)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>(\frac{155.112^3}{12}) +155.112⋅(\frac{(112)^2}{2})+5243.108²+1040.146⋅75²=1.52</td>
<td>[943\times5.56]</td>
</tr>
<tr>
<td>130</td>
<td>(\frac{155.100^3}{12}) +155.100⋅(\frac{(100)^2}{2})+5243.120²+1040.158⋅75²=1.53</td>
<td>(258.75)</td>
</tr>
<tr>
<td>190</td>
<td>(\frac{155.90^3}{12}) +155.90⋅(\frac{(90)^2}{2})+5243.130²+1040.168⋅75²=1.56</td>
<td>(187\times5.56)</td>
</tr>
<tr>
<td>250</td>
<td>(\frac{155.80^3}{12}) +155.80⋅(\frac{(80)^2}{2})+5243.140²+1040.178⋅75²=1.62</td>
<td>(258.75)</td>
</tr>
</tbody>
</table>

neglecting the inertia of the steel bar and plates about their own axes.
(iii) Rotations - Area under B.M. diagram ÷ EI.

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Rotation (radians $10^4$)</th>
<th>$f_{ct} = 1$ N/mm²</th>
<th>$f_{ct} = 3$ N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>$\frac{2300.2 \cdot M \div 36000.1 \cdot 52 \cdot 10^8}{3}$</td>
<td>61</td>
<td>53</td>
</tr>
<tr>
<td>130</td>
<td>$\frac{2300.2 \cdot M \div 36000.1 \cdot 53 \cdot 10^8}{3}$</td>
<td>135</td>
<td>126</td>
</tr>
<tr>
<td>190</td>
<td>$\frac{2300.2 \cdot M \div 36000.1 \cdot 56 \cdot 10^8}{3}$</td>
<td>195</td>
<td>186</td>
</tr>
<tr>
<td>250</td>
<td>$\frac{2300.2 \cdot M \div 36000.1 \cdot 62 \cdot 10^8}{3}$</td>
<td>247</td>
<td>237</td>
</tr>
</tbody>
</table>

These rotations are given in Table 6.5. It is clear that at higher loads the predicted rotations are greatly exceeded. The main reason for this is that as cracking becomes more widespread and the concrete compressive strain increases the value assumed for $E_C$ is not realistic.
APPENDIX 7

INTERFACIAL STRESSES

In this appendix an assessment is made of the bond stresses between the adhesive and plate and the shear stresses within the adhesive. It must be emphasised that the values should be treated qualitatively and that they in no way represent limiting or ultimate stresses. The beam tests were not designed to investigate such stress conditions but rather to study the flexural behaviour.

Bond Stresses

For concrete and steel to work in a beam it is necessary that stresses be transferred between the two materials. The term 'bond' can be used to describe the means by which slip, between the steel and concrete, is prevented or at least minimised. Wherever the tensile or compressive stresses in the reinforcing element change, bond stresses must act along their surface to produce this change.

Research on normal reinforced beams has shown that the bond stresses in a beam is neither uniform nor gradually varying from point to point. Rather, it has been found that very large bond stresses exist adjacent to cracks, essentially ultimate bond stresses, even at low loads. Very much smaller bond stresses exist close by on the same bar. Thus there is a practical problem as to how to describe, measure or evaluate such a fluctuating stress condition, and large bond stresses can exist in members at relatively low loads without signs of distress. Codes of Practice use two approximate methods to measure bond stress.

(a) Local Bond Stress

These are the shear stresses at the bar surface which prevent longitudinal movement of the bar in the concrete. Local bond failures are produced by large changes in the tensile force over a short length of bar. This change in tensile force is produced by a change in bending moment and the rate of change of bending moment is the shear force.
For plated beams the local bond stress at the plate/glue or concrete/glue interface is a horizontal shear stress given by
\[
\frac{V.A.y}{I.b_p}
\]
where \( A \) = area of plate or area (plate + glue),
\( b_p \) = width of plate,
\( V \) = shear force,
\( y \) = distance from the neutral axis to the section under consideration,
\( I \) = second moment of area of the cracked transformed section.

The results are given in Table A.7.1. The mean local bond stress near failure was 1.46 N/mm². CP110 limits the local bond stress in plain bars to 2.7 N/mm² (grade 40 concrete and above). These stresses are for the glue/plate interface.

(b) Anchorage Bond Stresses

This is the average bond stress over a particular length of bar. The removal of a bar from the concrete is resisted by shearing stresses, between the concrete and steel, which are assumed to be uniform along the length of the bar.

\[
\frac{n}{4} \theta^2 \cdot f_u = \pi \cdot \theta \cdot f_{bs} \cdot l
\]

In CP110 \( f_u \) for the steel is limited to \( \frac{f_Y}{f_m} \) to produce the allowable ultimate anchorage bond stresses. Certain experimentally determined limiting values are tabulated for different steel and concrete properties in Table 22 (CP110).

For evaluating the actual average anchorage bond stress between the plate and glue, in the test beams, a similar expression is used:
**TABLE A 7.1 LOCAL BOND STRESSES IN PLATED BEAMS**

<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>PLATE THICKNESS mm</th>
<th>PLATE THICKNESS mm</th>
<th>SHEAR FORCE AT SERVICE LOAD kN</th>
<th>COMBINED CENTROID AT SERVICE LOAD mm</th>
<th>LOCAL BOND STRESS N/mm²</th>
<th>SHEAR FORCE AT ULTIMATE LOAD kN</th>
<th>COMBINED CENTROID AT ULTIMATE LOAD mm</th>
<th>LOCAL BOND STRESS N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>203</td>
<td>1.5</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>204</td>
<td>1.5</td>
<td>3.0</td>
<td>65</td>
<td>231</td>
<td>0.89</td>
<td>125</td>
<td>231</td>
<td>1.70</td>
</tr>
<tr>
<td>205</td>
<td>1.5</td>
<td>6.0</td>
<td>65</td>
<td>238</td>
<td>1.23</td>
<td>95</td>
<td>238</td>
<td>1.80</td>
</tr>
<tr>
<td>207</td>
<td>3.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>208</td>
<td>3.0</td>
<td>3.0</td>
<td>65</td>
<td>231</td>
<td>0.90</td>
<td>125</td>
<td>231</td>
<td>1.72</td>
</tr>
<tr>
<td>209</td>
<td>3.0</td>
<td>6.0</td>
<td>65</td>
<td>238</td>
<td>1.24</td>
<td>95</td>
<td>238</td>
<td>1.81</td>
</tr>
<tr>
<td>210</td>
<td>3.0</td>
<td>6.0</td>
<td>65</td>
<td>238</td>
<td>1.24</td>
<td>95</td>
<td>238</td>
<td>1.81</td>
</tr>
<tr>
<td>216</td>
<td>6.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.57</td>
<td>125</td>
<td>226</td>
<td>1.09</td>
</tr>
<tr>
<td>217</td>
<td>6.0</td>
<td>3.0</td>
<td>65</td>
<td>231</td>
<td>0.90</td>
<td>125</td>
<td>231</td>
<td>1.74</td>
</tr>
<tr>
<td>218</td>
<td>6.0</td>
<td>6.0</td>
<td>65</td>
<td>238</td>
<td>1.24</td>
<td>95</td>
<td>238</td>
<td>1.81</td>
</tr>
<tr>
<td>219</td>
<td>6.0</td>
<td>6.0</td>
<td>65</td>
<td>238</td>
<td>1.24</td>
<td>95</td>
<td>238</td>
<td>1.81</td>
</tr>
<tr>
<td>220</td>
<td>3.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>221</td>
<td>3.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>222</td>
<td>3.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>223</td>
<td>3.0</td>
<td>1.5</td>
<td>65</td>
<td>226</td>
<td>0.55</td>
<td>125</td>
<td>226</td>
<td>1.05</td>
</tr>
<tr>
<td>224</td>
<td>3.0</td>
<td>3.0</td>
<td>65</td>
<td>231</td>
<td>0.90</td>
<td>125</td>
<td>231</td>
<td>1.74</td>
</tr>
</tbody>
</table>

(1) notched beam.
(2) precracked beams.

**PLATE THICKNESS mm**

<table>
<thead>
<tr>
<th>PLATE THICKNESS mm</th>
<th>MEAN SERVICE STRESS N/mm²</th>
<th>MEAN ULTIMATE STRESS N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.55 (0.55)</td>
<td>1.05 (1.05)</td>
</tr>
<tr>
<td>3.0</td>
<td>0.90 (0.90)</td>
<td>1.72 (1.74)</td>
</tr>
<tr>
<td>6.0</td>
<td>1.24</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Figures in brackets - precracked beams.
\[ f_{bs} = f_p \frac{A_p}{Ac} \]

where \( f_p \) is the stress in the plate \( (= \varepsilon_p E_p) \)

\( A_p \) is the cross section area of the plate \( (= t_p b_p) \)

\( A_c \) is the contact area between glue and plate \( (= b_p \ell) \)

\( \varepsilon_p, E_p, t_p \) and \( b_p \) are the strain, Young's Modulus, thickness and width, respectively, of the steel plate, and \( \ell \) is the anchorage length.

Then,

\[ f_{bs} = \varepsilon_p E_p \frac{t_p b_p}{b_p \ell} \]

For the test beams:

- \( t_p \) = thickness of plate varies - 1.5 mm to 6 mm
- \( b_p \) = width of plate constant - 125 mm
- \( E_p \) = Young's Modulus 200,000 N/mm²
- \( \varepsilon_p \) = strain in the plate varies
- \( \ell \) = anchorage length is taken as the distance from the end of the plate to the loading point = 742 mm.

Substituting we have:

\[ f_{bs} = 269 \varepsilon_p t_p \]

Using the strains obtained by experiment at both working load and near ultimate load the anchorage bond stresses were calculated for beams with single layers of continuous plate. The results are given in Table A7.2.

The mean anchorage bond stress at the load stage prior to failure was 2.12 N/mm², and at service load it was 0.81 N/mm². The limiting values, at these load stages, given in CP110 for plain bars are 1.9 and 1.0 N/mm² respectively, (for concrete \( f_{cu} \) 40 N/mm² and above.)

(c) **Shear Stresses in the Glue**

Considering longitudinal forces on a short length of steel plate \( \delta \varepsilon_p \), the change in load corresponding to a change in strain of \( \delta \varepsilon_p \) is given by:

\[ \delta \varepsilon_p E_p b_p t_p \] (Symbols as before)
### Table A 7.2: Anchorage Bond Stresses in the Plated Beams

<table>
<thead>
<tr>
<th>Beam Number</th>
<th>Glue mm Thickness</th>
<th>Plate mm Thickness</th>
<th>Design Service Load kN</th>
<th>Plate Strain at Centre microstrain</th>
<th>Anchorage Bond Stress N/mm²</th>
<th>Ultimate Load kN</th>
<th>Plate Strain at Centre microstrain</th>
<th>Anchorage Bond Stress N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>203</td>
<td>1.5</td>
<td>1.5</td>
<td>130</td>
<td>1300</td>
<td>0.5</td>
<td>250</td>
<td>4900</td>
<td>2.0</td>
</tr>
<tr>
<td>204</td>
<td>1.5</td>
<td>3.0</td>
<td>130</td>
<td>1200</td>
<td>1.0</td>
<td>250</td>
<td>3700</td>
<td>2.9</td>
</tr>
<tr>
<td>205</td>
<td>1.5</td>
<td>6.0</td>
<td>130</td>
<td>650</td>
<td>1.0</td>
<td>190</td>
<td>1200</td>
<td>1.9</td>
</tr>
<tr>
<td>207</td>
<td>3.0</td>
<td>1.5</td>
<td>130</td>
<td>1300</td>
<td>0.5</td>
<td>250</td>
<td>4900</td>
<td>2.0</td>
</tr>
<tr>
<td>208</td>
<td>3.0</td>
<td>3.0</td>
<td>130</td>
<td>1100</td>
<td>0.8</td>
<td>250</td>
<td>3200</td>
<td>2.6</td>
</tr>
<tr>
<td>209</td>
<td>3.0</td>
<td>6.0</td>
<td>130</td>
<td>700</td>
<td>1.1</td>
<td>190</td>
<td>1350</td>
<td>2.2</td>
</tr>
<tr>
<td>210</td>
<td>3.0</td>
<td>6.0</td>
<td>130</td>
<td>650</td>
<td>1.0</td>
<td>190</td>
<td>1250</td>
<td>2.0</td>
</tr>
<tr>
<td>216</td>
<td>6.0</td>
<td>1.5</td>
<td>130</td>
<td>1100</td>
<td>0.4</td>
<td>250</td>
<td>4200</td>
<td>1.6</td>
</tr>
<tr>
<td>217</td>
<td>6.0</td>
<td>3.0</td>
<td>130</td>
<td>1000</td>
<td>0.8</td>
<td>250</td>
<td>2900</td>
<td>2.3</td>
</tr>
<tr>
<td>218</td>
<td>6.0</td>
<td>6.0</td>
<td>130</td>
<td>660</td>
<td>1.0</td>
<td>190</td>
<td>1200</td>
<td>1.9</td>
</tr>
<tr>
<td>219</td>
<td>6.0</td>
<td>6.0</td>
<td>130</td>
<td>750</td>
<td>1.2</td>
<td>190</td>
<td>1350</td>
<td>2.2</td>
</tr>
<tr>
<td>220</td>
<td>3.8</td>
<td>1.5</td>
<td>130</td>
<td>1200</td>
<td>0.5</td>
<td>250</td>
<td>4200</td>
<td>1.7</td>
</tr>
<tr>
<td>221</td>
<td>3.0</td>
<td>1.5</td>
<td>130</td>
<td>1300</td>
<td>0.5</td>
<td>250</td>
<td>4500</td>
<td>1.8</td>
</tr>
<tr>
<td>222</td>
<td>3.0</td>
<td>1.5</td>
<td>130</td>
<td>950</td>
<td>0.4</td>
<td>250</td>
<td>2900</td>
<td>1.2</td>
</tr>
<tr>
<td>223</td>
<td>3.0</td>
<td>1.5</td>
<td>130</td>
<td>750</td>
<td>0.3</td>
<td>250</td>
<td>2300</td>
<td>0.9</td>
</tr>
<tr>
<td>224</td>
<td>3.0</td>
<td>3.0</td>
<td>130</td>
<td>750</td>
<td>0.6</td>
<td>250</td>
<td>2400</td>
<td>1.9</td>
</tr>
</tbody>
</table>

(1) notched beam
(2) precracked beams

Figures in brackets - precracked beams.
This must be balanced by a shear force in the resin which is given by:
\[ \tau \cdot \delta \cdot b_p \]
where \( \tau \) = shear stress at the plate/glue interface.

Hence \( \tau = \frac{\delta \varepsilon_p}{E_p \cdot t_p \cdot \delta \ell} \)

In the limit \( \tau = \frac{d\varepsilon_p}{E_p \cdot t_p \cdot d\ell} \) where \( \frac{d\varepsilon_p}{d\ell} \) is the strain gradient.

This analysis has ignored the thickness of the glue and therefore assumes that the strain in the contact face of the steel is the same as the contact surface strain in the concrete.

It is evident that the glue will be subjected to high shear forces where the strain gradient is high. Such gradients occur where there is a sudden change in section, for example at the end of the plate, a joint in a plate or at a crack in the concrete. The strain gradients were measured at the plate ends for the test beams. However, the measured values of strain can only be an approximation to the local strains as they are based on average values over a finite length. Gauges of 6 mm length were used and in such a region of rapidly changing strain are not small enough to determine an accurate strain gradient. Nevertheless, the tests do give some indication of the order of magnitude of the shear stresses. The shear stresses are given in Table A7.3, for beams with single continuous layers of plate.
### TABLE A.7.3  SHEAR STRESS IN THE GLUE LAYER AT THE END OF THE PLATE.

<table>
<thead>
<tr>
<th>BEAM NUMBER</th>
<th>GLUE THICKNESS mm</th>
<th>PLATE THICKNESS mm</th>
<th>DESIGN SERVICE LOAD kN</th>
<th>STRAIN GRADIENT per mm</th>
<th>SHEAR STRESS N/mm²</th>
<th>ULTIMATE LOAD kN</th>
<th>STRAIN GRADIENT per mm</th>
<th>SHEAR STRESS N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>203</td>
<td>1.5</td>
<td>1.5</td>
<td>130</td>
<td>2.4</td>
<td>0.7</td>
<td>250</td>
<td>5.4</td>
<td>1.6</td>
</tr>
<tr>
<td>204</td>
<td>1.5</td>
<td>3.0</td>
<td>130</td>
<td>1.8</td>
<td>1.1</td>
<td>250</td>
<td>4.4</td>
<td>2.6</td>
</tr>
<tr>
<td>205</td>
<td>1.5</td>
<td>6.0</td>
<td>130</td>
<td>1.4</td>
<td>1.7</td>
<td>190</td>
<td>3.0</td>
<td>4.1</td>
</tr>
<tr>
<td>207</td>
<td>3.0</td>
<td>1.5</td>
<td>130</td>
<td>2.6</td>
<td>0.8</td>
<td>250</td>
<td>6.6</td>
<td>2.0</td>
</tr>
<tr>
<td>208</td>
<td>3.0</td>
<td>3.0</td>
<td>130</td>
<td>2.0</td>
<td>1.2</td>
<td>250</td>
<td>5.5</td>
<td>3.3</td>
</tr>
<tr>
<td>209</td>
<td>3.0</td>
<td>6.0</td>
<td>130</td>
<td>1.3</td>
<td>1.6</td>
<td>190</td>
<td>3.8</td>
<td>4.6</td>
</tr>
<tr>
<td>210</td>
<td>3.0</td>
<td>6.0</td>
<td>130</td>
<td>1.8</td>
<td>2.2</td>
<td>190</td>
<td>3.6</td>
<td>4.3</td>
</tr>
<tr>
<td>216</td>
<td>6.0</td>
<td>1.5</td>
<td>130</td>
<td>2.7</td>
<td>0.8</td>
<td>250</td>
<td>8.5</td>
<td>2.6</td>
</tr>
<tr>
<td>217</td>
<td>6.0</td>
<td>3.0</td>
<td>130</td>
<td>2.2</td>
<td>1.3</td>
<td>250</td>
<td>6.0</td>
<td>3.6</td>
</tr>
<tr>
<td>218</td>
<td>6.0</td>
<td>6.0</td>
<td>130</td>
<td>1.5</td>
<td>1.8</td>
<td>190</td>
<td>4.0</td>
<td>4.8</td>
</tr>
<tr>
<td>219</td>
<td>6.0</td>
<td>6.0</td>
<td>130</td>
<td>2.0</td>
<td>2.4</td>
<td>190</td>
<td>4.2</td>
<td>5.0</td>
</tr>
<tr>
<td>220</td>
<td>3.8</td>
<td>1.5</td>
<td>130</td>
<td>3.7</td>
<td>1.1</td>
<td>250</td>
<td>8.2</td>
<td>2.5</td>
</tr>
<tr>
<td>221</td>
<td>3.6(1)</td>
<td>1.5</td>
<td>130</td>
<td>3.7</td>
<td>1.1</td>
<td>250</td>
<td>8.2</td>
<td>2.5</td>
</tr>
<tr>
<td>222</td>
<td>3.0(2)</td>
<td>1.5</td>
<td>130</td>
<td>2.0</td>
<td>0.6</td>
<td>250</td>
<td>6.5</td>
<td>2.0</td>
</tr>
<tr>
<td>223</td>
<td>3.0(2)</td>
<td>1.5</td>
<td>130</td>
<td>2.0</td>
<td>0.6</td>
<td>250</td>
<td>6.7</td>
<td>2.0</td>
</tr>
<tr>
<td>224</td>
<td>3.0(2)</td>
<td>3.0</td>
<td>130</td>
<td>1.2</td>
<td>0.7</td>
<td>250</td>
<td>4.4</td>
<td>2.6</td>
</tr>
</tbody>
</table>

(1) notched beam.
(2) precracked beams.

### PLATE THICKNESS MEAN SERVICE SHEAR STRESS MEAN ULTIMATE SHEAR STRESS

<table>
<thead>
<tr>
<th>PLATE THICKNESS mm</th>
<th>MEAN SERVICE SHEAR STRESS N/mm²</th>
<th>MEAN ULTIMATE SHEAR STRESS N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.85 (0.60)</td>
<td>2.18 (2.00)</td>
</tr>
<tr>
<td>3.0</td>
<td>1.43 (0.70)</td>
<td>3.18 (2.60)</td>
</tr>
<tr>
<td>6.0</td>
<td>1.72</td>
<td>4.56</td>
</tr>
</tbody>
</table>

Figures in brackets - precracked beams.
APPENDIX 8

CALCULATION OF CRACK WIDTHS

The crack widths were calculated by two methods:


As with deflections an elastic analysis of the concrete section was used for calculating the stiffness. Any calculations to determine the moments and forces for deflections can also be used for cracking.

In Clause A.3.2 of the code, Equation 61 is given for determining the crack width

\[ W_{cr} = \frac{3 \cdot acr \cdot \varepsilon_m}{1 + 2 \left( \frac{acr - C_{min}}{h-x} \right)} \]

At the level of reinforcement \( acr = C_{min} \) hence the formula becomes:

\[ W_{cr} = 3 \cdot acr \cdot \varepsilon_m \]

The average strain at the level of reinforcement, \( \varepsilon_m \), is calculated from \( \varepsilon_1 \), the strain calculated ignoring the stiffening effect of the concrete in the tension zone, and then allowing for the tensile stiffening as shown by:

\[ \varepsilon_m = \varepsilon_1 - \frac{1.2 \cdot b_t \cdot h \cdot (a'-x) \cdot 10^{-3}}{A_s \cdot (h-x) \cdot f_y} \]

The code states that the formula only applies if the strain in the tension reinforcement is limited to 0.8 \( f_y/E_s \), and that when calculating strains the modulus of elasticity of the concrete should be taken as half the instantaneous value obtained from Table 1 in the code or by experiment. However, the proposed crack width formula gives a value which has a certain probability of being exceeded during the life of a structure. With repeated and sustained loadings, crack widths can increase over a period of time. To compute values for beams tested in the laboratory at an age of 28 days and loaded for only a few hours it is more reasonable to use the full elastic modulus of the concrete.

Calculations were performed using both values, i.e. \( E_c \) and 0.5 \( E_c \). The values given in brackets at the end of the calculations are for 0.5 \( E_c \) for comparison.
Beam 207  
Glue thickness  3 mm  
Plate thickness 1.5 mm  

$$ae = \frac{200,000}{36,00} = 5.56$$  

$$\rho = \frac{A_b}{bd}$$  

d, the effective depth, is taken to the combined centroid of the plate and bars.

$$d \cdot (943+187) = 220.943 + 187.158.75$$  
$$d = 226 \text{ mm}$$  

Thus $$\rho = \frac{(187 + 943)}{155.226} = 0.032$$  

$$\frac{x}{d} = \sqrt{ae \cdot \rho (2 + ae \cdot \rho)} - ae \cdot \rho$$  

$$= 0.44$$  
Hence $$x = 100.5 \text{ mm}$$  
and $$Z = d - \frac{x}{3} = 192.5 \text{ mm}$$  

The crack widths are all calculated at 130 KN load which corresponds to a moment of 49.8 KNm.

The stress in the reinforcement (at the combined centroid) is given by:

$$f_s = \frac{M}{Z \cdot A_s} = \frac{49.8 \cdot 10^6}{192.5 \cdot 1130} = 230 \text{ N/mm}^2$$  

Hence the strain, $$\varepsilon_1 = \frac{230}{200000} = 0.00115.$$  

Correcting for the tension stiffening of the concrete:

$$\varepsilon_m = 0.00115 - \frac{1 \cdot 2.155.255. \cdot (226-100.5) \cdot 10^{-3}}{(187.250+943.410)(255-100.5)}$$  

$$= 0.00107$$

The crack width is then given by:

$$W_{cr} = 3 \cdot a_{cr} \cdot \varepsilon_m$$

For all the beams the cover to the bars; $$a_{cr}$$, is 27.5 mm.
Hence \( W_{cr} = 3.27 \times 0.00107 \)
\[ = 0.088 \text{ mm (0.093)} \]

For beams with 3 mm plate the values are 0.074 mm (0.079)
and for beams with 6 mm plate the values are 0.056 mm (0.061).
The values are shown in Table 7.7.

b. American Concrete Institute.

The formula for crack width at the reinforcement level, as recommended by Gergely and Lutz (89) is given by:

\[
W_{\text{max}} = \frac{0.091 \sqrt{\tau_s \cdot A} \cdot (f_s - 5) \cdot 10^{-3}}{1 + \frac{\tau_s}{h_1}} \] (IMPERIAL UNITS)

Beam 207 Value \( E_C \) assumed.

The depth to the centroid of steel = 226 mm
Neutral axis depth = 100.5 mm

\( h_1 \), the distance from the neutral axis to the centroid of the tension steel = 226 - 100.5 = 125.5 mm.

For normally reinforced concrete beams \( A = \frac{2b(h-d)}{\text{number of bars}} \)

For the plated beams it is assumed that the concrete surrounding the plate is equal to that surrounding each bar, and that the plate is the same width as the beam.

\[
b.c = \left[ 2(h-220) - c \right] \cdot \frac{b}{3}
\]

\[
\therefore \ c = \frac{(h-220)}{2}
\]

and \( A = b \cdot \frac{h-220}{2} = 2712 \text{ mm}^2 \)

Thus for the plated beams, the thickness of plate is assumed not to affect \( A \), but does affect the positions of the combined centroid of steel and hence \( x \) and \( d \), which in turn affect \( h_1 \) and the steel stress, \( f_s \).

As for CP110 \( f_s = \frac{M}{zA_s} = 230 \text{ N/mm}^2 \)
Converting to Imperial Units

\( t_s = 37.5 \text{ mm} \) (to centre of bar) = 1.48 inches

\( A = 2712 \text{ mm} = 4.2 \text{ in} \)

\( f_s = 230 \text{ N/mm} = 33.3 \text{ Kips/in} \)

\( h_1 = 125.5 \text{ mm} = 4.94 \text{ in} \)

Then

\[
W_{\text{max}} = 0.091 \sqrt[3]{1.48 \cdot 4.2 \cdot (33.3 - 5) \cdot 10^{-3}} \frac{1.48}{1 + \frac{1.48}{4.94}}
\]

Hence \( W_{\text{max}} = 0.093 \text{ mm} \)

The results are given in Table 7.7.
APPENDIX 9

STATISTICS

a. STANDARD DEVIATION. \( \sigma \)

When calculating the standard deviation of a set of numbers

\[ \sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} \]

where \( N \) is the number of elements
\( \bar{x} \) is the mean

b. COEFFICIENT OF VARIATION

This is taken as \( \frac{\text{standard deviation}}{\text{mean}} \) \( \frac{\sigma}{\bar{x}} \)

c. LINEAR REGRESSION

In many disciplines it is desirable to express one variable in terms of another even though the variables are not necessarily analytical functions of each other. An accepted practice is to perform a least-squares regression which is designed to minimise the sum of the squares of the deviation of the actual data points from the straight line of best fit. In practice we are essentially constructing a plot of the variables, called a scatter diagram, and drawing the best straight line which uniformly divides the points. The result is a linear equation in the form \( y = mx + b \).

It can be shown that the slope and y intercept are determined as follows:

\[ m = \frac{\sum x_i y_i}{N} - \frac{\sum x_i y_i}{\sum x_i^2} \]

and \( b = \bar{y} - m\bar{x} \)

\[ \bar{x} = \frac{\sum x_i}{N} \]

\[ \bar{y} = \frac{\sum y_i}{N} \]

The degree of association between the two variables \( x \) and \( y \) is called the correlation, \( r \).
where $r = m \frac{\sigma_x}{\sigma_y}$  \hspace{1cm} (\sigma = \text{standard deviation})

In most applications it is advisable to select a degree of certainty that is desired when analysing a population of numbers. To facilitate this statisticians have constructed tables based on the areas under different portions of the normal curve. These tables are called 'Z' values and enable a prediction for the range of the mean value to a specific degree of certainty.

In the present case of cracking in the test beams, the relationship between the maximum and mean crack width is required.

$$W_{\text{max}} = W_{\text{mean}} + Z \cdot \text{standard deviation}.$$  

The experimental results from 24 test beams are used and from statistical tables this gives a factor of 2.5.

This gives:

$$W_{\text{max}} = W_{\text{mean}} + 2.5 \cdot \sigma.$$
REFERENCES

1. A.C.I. COMMITTEE 403:
Guide for use of epoxy compounds with concrete.

2. A.C.I. COMMITTEE 503:
Use of epoxy compounds with concrete.

3. A.C.I.
Epoxies with concrete.

4. E.W. BAUMAN, R.B. JACKSON and W.R. McCONNELL:
Guide for use of epoxy compounds with concrete - Discussion of paper by A.C.I. 403.

5. R. HOUWINK and G. SALOMON:
Adhesion and Adhesives.
Volume 1 (1965) and Volume 2 (1967), 2nd edition,

6. H. LEE and K. NEVILLE:
Handbook of epoxy resins.

7. B. TREMPER:
Repair of damaged concrete with epoxy resins.

8. F.P. BRUINS:
Epoxy resin technology.

9. R. GAUL and N. APTON:
Epoxy adhesives in concrete construction.
Civil Engineering (New York), Volume 29, No. 11, 1959, pp 50-52.

10. C.M. WAKEMAN, H.E. STOVER and E.N. BLYE:
Glue for concrete repair.

11. J. CIESIELSKI:
Reinforcement of reinforced concrete beams by epoxy resin injection.

12. R. GUTTMAN:
Concise guide to structural adhesives.

13. R.P. JOHNSON:
Glued joints for structural concrete.

14. M. LEVY:
The use of adhesives in the bonding and repair of precast products.

-279-
15. R. P. JOHNSON:  
Creep tests on glued joints.  

16. B. TAYLOR:  
The effect of vibration on creep of glued concrete joints.  
Research Project, Cambridge University Engineering Laboratory, 1965.

17. J. D. KREIGH:  
The use of epoxy resin in reinforced concrete - Dynamic tests.  
Engineering Research Laboratories, University of Arizona, August 1963.

18. T. O'BRIEN:  
Jointing structural precast concrete units with resin adhesives.  

19. P. W. ABELES:  
Investigation of composite prestressed concrete beams comprising precast members glued together by means of resins.  
Materials and Structures, Volume 1, No. 1, Jan.-Feb. 1968, pp 33-36

20. R. P. JOHNSON:  
The properties of an epoxy mortar and its uses for structural joints.  
Structural Engineer, Volume 48, No. 6, June 1970, pp 227-232

21. M. L. BATCHELAR:  
Epoxy resin materials as structural adhesives.  

22. J. MOAR:  
The strength, creep and durability of epoxy jointed concrete members.  
PhD thesis, University of New South Wales, Australia, 1974

23. V. GORGOL:  
Epoxy resin finish of the grandstands of Paris sports stadium.  

24. A. HALLQUIST:  
An investigation on epoxy and polyester resin mortars as a jointing material.  

25. CIBA GEIGY:  
Araldite Bonding.  
Instruction Manual A15g, August 1971.

26. I. P. SHUE FAI:  
Structural applications of epoxy resin adhesives.  
National Roads Board of New Zealand, RRU Bulletin No. 29, 1974.

27. R. SHAW:  
Epikote resins in civil engineering applications - recent developments.  

28. A. R. CUSENS and D. W. SMITH:  
A study of epoxy resin adhesive joints in shear.  
Structural Engineer, Volume 58A, No. 1, January 1980

29. C. CARON:  
Synthetic Resins. Laboratory tests applied to resins injection.  
30. R. VOLKERSEN:
Stress distribution in adhesive joints.
Luftfahrtforsch, Volume 15, No. 31, 1938.

31. M. GOLAND and E. REISSNER:
Stress distribution in adhesive joints.

32. R.W. CORNELL:
Stress distribution in adhesive joints.

33. G.R. WOOLEY and D.R. CARVER:
Stress distribution in bonded lap joints.
Aircraft, Volume 8, No. 10, 1971, pp 817.

34. S. AMIJIMA, T. FUJII and A. YASHIDA:
Two dimensional stress analysis of adhesive bonded joints.

35. D. MYLONAS:
Critical comparison of theories of stress distribution in adhesive joints.

36. Y. GILIBERT, J.P. DELMAS and C. COLLOT:
Contribution to the study of plated concrete.

37. Y. GILIBERT and C. COLLOT:
Contribution to the study of adhesion between steel and adhesive as a function of the micro-geometric properties of the roughened surface.
RILEM Bulletin, No. 48, 1975

38. K.W. ALLEN:

39. C.V. CAGLE:
Adhesives Bonding.

40. D.R. SMITH:
How to prepare the surface of metals and non metals for adhesive bonding.
Adhesives Ages, March 1967.

41. J. OLSEN:
The industrial adhesives plausibility gap.
Adhesives Age, January 1975.

42. J. SHIELDS:
Adhesives Handbook.

43. C.W. JENNINGS:
Surface Roughness and bond strengths of adhesives.

44. E.B. RAMEL:
Analytical and experimental studies of adhesive bonded beams and plates.


52. ANON: 'M5 shrinkage cracks - traffic restricted.' New Civil Engineer, No. 62, October 1973, p 21.

53. T. SOMMERARD: 'Swanley's steel plate patch up' New Civil Engineer, No. 247, June 1977, p 18.


58. R. L'Hermite:
Use of bonding techniques for reinforcing concrete and masonry structures.

59. R. L'Hermite and J. Bresson:
Concrete reinforced with glued plates.
RILEM Symposium, Synthetic Resins in Building Construction, Paris 1967,
pp 175-203.

60. C.A.K. Irwin:
The strengthening of concrete beams by bonded steel plates.

61. M.B. MacDonald
The flexural behaviour of concrete beams with bonded external reinforcement.

62. Anon:
The strengthening of concrete structures by bonded external reinforcement
Long term exposure tests.

63. S.K. Solomon, D.W. Smith and A.R. Cusens:
Flexural tests on steel-concrete-steel sandwiches. Vol.2B: 94

64. R.P. Johnson and C.J. Tait:

65. C.H. Lerchenthal:
Bonded steel reinforcement for concrete slabs.
RILEM Symposium, Synthetic Resins in Building Construction, Paris 1967,
pp 165-173.

66. R. Cirodde:
Techniques of glued assembly.
RILEM Symposium, Synthetic Resins in Building Construction, Paris 1967,
pp 47-59.

67. C.J. Fleming and G.E.M. King:
The development of structural adhesives for three original uses in
South Africa.

68. S. Kaifasz:
Concrete beams with external reinforcement bonded by gluing - preliminary
investigation.

69. M. Ladner:
Field measurement on subsequently strengthened concrete slabs.

70. S.K. Solomon and L.K. Gopalani:
Flexural tests on concrete beams externally reinforced by steel sheet.
The Indian Concrete Journal, Volume 53, No. 9, September 1979, pp 249-253.

71. A. Bouderbah:
Strengthening of existing concrete beams by the use of glued steel plates.
M. Eng. dissertation, University of Sheffield, September 1978.

73. R.N. SWAMY and R. JONES: Unpublished work.


86. CP110. The structural use of concrete.

87. H.A. Sawyer:
   Elastic-plastic design of single span beams and frames.

88. A.L.L. Baker:
   The Ultimate Load Theory - applied to the design of reinforced and pre-stressed concrete frames.

89. P. Gergely and L.A. Lutz:
   Maximum crack width in reinforced concrete flexural members,
   Proceedings of the Symposium of Causes, Mechanism and Control of Cracking in Concrete.
   American Concrete Institute, SP 20, Detroit 1968, pp 87-117

90. J.M. Illston and R.F. Stevens:
   Internal Cracking in Reinforced Concrete.

91. G.D. Base et al:
   An investigation of the crack control characteristics of various types of bars in reinforced concrete.

92. A.W. Beeby:
   Suggested recommendations to the crack prediction formula in the 1970 CEB Recommendations.

93. B.B. Brooms:
   Crack width and crack spacing in reinforced concrete members.

94. E. Hognestad:
   High strength bars as concrete reinforcement, Part 2, control of flexural cracking.

95. J. Ferry-Borges:
   Cracking and deformability of reinforced concrete beams.

96. A.P. Clark:
   Cracking in reinforced concrete flexural members.

97. A.W. Beeby:
   A study of cracking in reinforced concrete members, subjected to pure tension.
   Cement and Concrete Association Technical Report No. 42,468, June 1972, 24 pp

98. A.H. Matteck:
   The strength of singly reinforced beams in bending.
   Session B, Paper 1, Symposium on the Strength of Concrete Structures.