

# **MEASURING THE PERFORMANCE OF SPATIAL INTERACTION MODELS IN PRACTICE**

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The candidate confirms that the work submitted is her own and that appropriate credit  
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## ABSTRACT

Spatial decision support systems are widely used throughout the retail industry. As a result of this, a competitive industry has developed in order to provide such systems. One such company is GMAP Ltd who provide information systems for companies such as WH Smith, Halifax Plc and Toyota. Due to the competitiveness of the market analysis industry it is necessary for companies such as GMAP to endeavour to continually improve their products in order to remain at the leading edge of the decision support industry. Research is required in order to discover methods of improving the performance of the spatial interaction models that GMAP have developed for their clients. This thesis looks at two case studies of modelling work undertaken by GMAP for clients, the WH Smith model and the model for Halifax Plc new mortgage sales and tries to improve the performance of these models through a variety of methods.

Each of the three components of spatial interaction models, demand, supply and interaction, are analysed in turn and attempts are made to improve the representation of these components in the case study models. The demand component is investigated using the WH Smith model and different methods of estimating demand using alternative data sources are investigated in order to discover if improvements can be made to the existing demand estimation procedure used by GMAP. The representation of the supply side is investigated by attempting to identify centre and store characteristics that influence the attractiveness of destinations and subsequently include such variables in the attractiveness calculation. Several aspects of the interaction component are investigated in order to determine if they can be improved. The measurement of impedance and the form of the impedance function together with alternative measurements of accessibility are investigated to see whether interaction patterns can be predicted more accurately.

The conclusions that arise from the investigations undertaken are presented as a series of recommendations for GMAP Ltd that can be implemented to improve the performance of their models. The implications for the specification of spatial interaction models in other contexts are also identified.

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## RETAIL SITE LOCATION ANALYSIS

### 1.1 INTRODUCTION

The service sector has increased in importance dramatically in the post industrial economy. The retail industry, as part of the service sector, has therefore undergone extensive restructuring throughout the post war era and grew to account for 10% of the UK labour force by 1992 (Bromley and Thomas, 1993) and employed in excess of 2 million workers by 1994 (Guy, 1994). This restructuring has led to an increase in the need to understand the role of place within the retail sector and hence the development of a large industry dedicated to the analysis of retail site location. The processes underlying the restructuring of the UK retailing industry are reviewed in Wrigley (1988), Bromley and Thomas (1992) and Guy (1994) and are briefly described below.

Several aspects of the change in structure of the retail sector have led to an increased need for accurate spatial analysis and planning due to the increased competition between retailing organisations. Since the abolition of Resale Price Maintenance (RPM), the balance of power has shifted from the manufacturer to the retailer with retailers being able to pass on efficiency savings to the consumer through lower prices. Thus there is a constant requirement for retailers to improve efficiency in order to further decrease prices and hence gain an advantage over their competitors.

The growth of large retail corporations is another aspect of restructuring that has occurred in the retail industry since the 1950s. By 1992, 10 of the largest 100 companies in the UK were all or mainly retail based, with Marks and Spencer and Sainsbury both appearing in the top 15 (Guy, 1995). This process has been accompanied by a decrease in the number of small independent retailers. Such expansion of chain retailers has been made possible by the removal of RPM, because large firms are more able to increase efficiency through economies of scale and are therefore able to decrease prices. A significant amount of efficiency improvement has

been achieved through the use of improved technological resources such as EPOS (Electronic Point Of Sale) systems which have allowed an increase in the efficiency of stock control and distribution as well as an improvement in the productivity of labour resulting in labour shedding. These technological advances have been easier to finance and implement for large retail firms and has therefore led to a further increase in the concentration of capital within the retail sector. This increase in the concentration of capital in the retail industry has produced further intensity of competition between large retail corporations.

## **1.2 THE NEED FOR STORE LOCATION ANALYSIS**

The changes that have occurred in the retail sector that have been illustrated above have led to increased competition for market share within the retail sector. There is therefore a need to increase efficiency in order to improve profitability. Firms also seek to increase in size in order to improve their market share and turnover. Thus retail firms are required to continually expand and improve their store network in order to increase efficiency and market share.

The spatial distribution of outlets is of extreme importance to the retailer and the location chosen for a new store by a retail firm will be vital to the success of that store and hence the firm's success. The relationship between level of sales and the location of a store means that the choice of location for a retail firm is one that should be undertaken with extreme care. Retailers are therefore becoming increasingly aware of the importance of making the correct locational decision. As Ghosh and McLafferty, state:

“Whether selling goods or services, the choice of outlet locations is perhaps the most important decision a retailer has to make”

(1987, p1)

The same authors also state that in the environment of intense competition that surrounds modern day retailing, in which all of the large retail corporations can compete with each other in terms of price, advertising and services, location can be one way in which a retail firm can distinguish itself from its competitors and gain a competitive

advantage. The smallest differences in location can have a large impact on the performance of retail firms.

Effective location strategy is therefore essential for retail firms and this led to the development over the last 40 years of increasingly sophisticated methods of retail site location analysis.

Once the need for effective store location was understood, the first methods used relied on the 'gut feeling' of retail managers in deciding where was the best location for a store. However, it was soon recognised that more sophisticated methods were required in order to increase the accuracy and effectiveness of retail site location analysis.

Bowlby, Breheny and Foot (1984) suggest four possible reasons for the increased use of systematic locational analysis as opposed to the traditional 'gut feeling' method.

Firstly, the easy sites are used first and therefore, as such sites are used up, the location decision becomes more difficult and more sophisticated methods of choosing sites are therefore required. This problem is particularly acute for grocery retailing, and the chairman of Tesco, Ian MacLaurin is quoted in Penny and Broom (1988) as stating

"There are only a finite number of superstore sites available in the UK. ...  
It is my job to make sure that we get our share of the remaining sites"  
(1988, p107)

Tesco have since set up a site location research unit in order to develop a location strategy to overcome such problems.

The second reason suggested for the increased need for improved locational decision making tools is that experience becomes a less reliable guide. The retail industry is in a constant state of flux, as competitiveness increases and new locational strategies such as locating in out of town centres are introduced, experience gained in the past is no longer as useful and therefore new techniques must be developed in order to cope with the new problems that are being experienced.

The cost of making locational mistakes is another important factor in the need for improved locational methods. The opening of new stores is inherently risky due to the cost involved. As the size of stores increases so does the risk and therefore these irreversible decisions concerning location should be undertaken in the most informed manner possible.

Retailers are also experiencing increased pressure to expand through investment in new outlets. The growing number of retail firms that have become Public Limited Companies (Plcs) has also led to pressures to increase profitability due to the obligation to pay increased dividends that is expected within the retail sector as the size and turnover of retail firms increases. The intensification of competition in retailing that has been experienced has made this expansion more difficult and therefore the correct locational decision is vital.

In response to this need for systematic location analysis tools by retailers, a lot of academic research has been undertaken.

### **1.3 THE DEVELOPMENT OF STORE LOCATION ANALYSIS**

#### ***1.3.1 Academic research***

The academic community became involved in the field of retail modelling from the 1960s. The different types of retail analysis research techniques that have been developed are reviewed in Birkin *et al.* (1995) and are briefly described here.

The first work undertaken on this topic was produced by Huff in 1964 and Applebaum in 1965. This work represented early attempts to estimate the level of sales that would be achieved at a given location. Huff (1964) predicted sales based on the probability that the population of an area would choose to use a particular destination for their purchasing needs. Applebaum (1965) used an analogue approach which involved the assessment of a location through the comparison of that location with a similar location in a different part of the country (the assumption being that sales will be roughly equivalent in areas that are similar to each other).

A further method that has been used extensively in the assessment of store locations is catchment area analysis in which the characteristics of the population of the catchment area of the store are used to predict store sales at that location. Academic work on this type of analysis was begun by Reilly in 1931 when he used an analogy to Newton's Law of Gravitation to estimate the trade areas for retail destinations. This type of store location analysis has been greatly enhanced through the development of geodemographic packages such as ACORN and MOSAIC that attempt to profile customer types. These enable catchment areas to be disaggregated according to the profiles of the customers contained within the area. The development of Geographical Information Systems (GIS) has also enabled the development of this technique.

Regression techniques have also been used extensively in the field of store revenue estimation. In this case a regression model is calibrated using data available for existing stores and subsequently used to produce predictions for new stores.

Mathematical models have been developed in order to predict retail sales at a given destination. The spatial interaction model as developed by Wilson in the 1970s is an example of one type of mathematical model that has been utilised in the context of retail location planning. Spatial interaction models attempt to quantify flows of expenditure from residential areas to shopping destinations dependent on the characteristics of those origins and destinations and the degree of spatial separation that exists between them. The use of this type of model in the retailing context will be the main focus of this thesis and spatial interaction models will be described in detail in Chapter 2.

### ***1.3.2 The role of the commercial sector***

Section 1.2 indicated the need of retailers to utilise systematic planning methods in determining their retail location strategy and Section 1.3 introduced the vast amount of academic research that has been undertaken in this field. However, Penny and Broom (1988) state that in 1980, when Tesco began its research into retail site location, there was little commercial help available. The dramatic increase in the amount of spatially referenced customer data that is available within retail companies has led to a change in this situation and meant that the academic research described above can increasingly be

used in real retail applications as a method of improving the location strategy of retail firms.

Thus the large amount of academic research undertaken in the area of retail site location has been transformed into commercial success by a large number of companies. Commercial companies such as CACI, Pinpoint and GMAP Ltd. (Geographical Modelling And Planning) have developed as a response to the need for efficient and accurate store location analysis in the retail sector and this has also led to the development of a number of 'in-house' store location research units such as can be seen for Tesco and Marks and Spencer. The locational research unit at Tesco expanded from no staff in 1981 to 25 in 1986.

Such commercial companies perform a variety of retail analysis services including the following:

- store location assessment
- catchment area analysis
- assessment of store performance
- forecasting of store turnover
- estimation of market share
- geodemographic classification
- customer targeting
- credit scoring

However, despite the progress achieved in the development in the commercial sector of retail location analysis, Simkin states that

“While mathematical models have been created, there is a dearth of operationally predictive models capable of reproducing meaningful and usable information for a company’s management.”

(1990, p33)

Thus, continued research is necessary in order to improve the applicability of the store location models that are being produced.

One of the commercial companies, GMAP Ltd, that has attempted to undertake continued research, will now be introduced in detail.

### *1.3.3 GMAP Ltd*

GMAP was first founded in the late 1980s by Alan Wilson and Martin Clarke as a consultancy section of the School of Geography at the University of Leeds and was part of the University of Leeds Industrial Services (ULIS). The company separated from ULIS in 1991 and became GMAP Ltd although the University of Leeds remained a major shareholder. The first major clients were Toyota and WH Smith. Small, local pilot studies were undertaken for these clients which were subsequently developed into national models for prediction of sales. GMAP has experienced rapid growth throughout the 1990s reaching a maximum of 110 staff in 1998 before the company was sold to POLK, an American market analysis firm for the car market. GMAP has emerged from this take-over as a smaller company that is now independent from the University but still retains strong links to the academic community.

A key factor in the success of GMAP has been the abundance of intellectual capital at its disposal. The spatial interaction modelling theory as developed by Wilson (1967) is the basis of the decision support systems produced and this theory has subsequently been studied by Clarke (1984), Birkin (1986) and Clarke (1986) all of who are still involved with GMAP. The company has also maintained its links with the School of Geography at the University of Leeds which has been a source of trained labour. This factor, along with collaborative research undertaken with the School of Geography, such as this research project, has enabled GMAP to maintain its intellectual lead in the market analysis industry.

Current clients of GMAP include WH Smith, Halifax Plc and Asda. Therefore a wide variety of retail service types are provided for and hence the models have to be adapted in order to account for differences between the retail sectors for which sales are being predicted. It is therefore necessary for companies such as GMAP to not only have knowledge of the systems which they develop for use in the retail sector but also to have knowledge of the section of the retail sector for which the models will be used. It is also necessary for GMAP to ensure that clients get the maximum benefit from their products.

To facilitate this GMAP not only produce decision support systems for their clients but they also provide a consultancy service. This is a vital service because it enables clients to enjoy the maximum benefit of the systems provided by GMAP

#### 1.4 CONTEXT OF RESEARCH

Once working models have been developed for use in real retail applications it is still necessary for continued research to be undertaken in order to continually improve the performance of models used in the retail sector. In the late 1980s Wrigley commented on the need for fresh thinking in retailing research:

“The store location research and market analysis which has developed so rapidly in the UK commercial sector over recent years is typically conducted under intense pressures both of time and/or the need to create new business. Such pressures, and the need for a relatively ‘fail safe’ product, are not conducive to the development of new approaches to retail analysis and forecasting, and the danger is that the locational analysis methods being adopted in the industry may fossilise and, therefore, perhaps ultimately disappoint.”

(1988, p32)

This statement is still relevant now and therefore firms such as GMAP that provide a service to the retail sector must endeavour to continually undertake research in order to attempt to improve the quality of the service which they provide. To this end GMAP, in conjunction with the ESRC (Economic and Social Research Council), have provided this studentship to undertake an evaluation of the performance of a selection of the spatial interaction models produced by GMAP. This investigation will be undertaken independent of the model builders and the client in order to see if the performance of such models can be improved through the use of different model forms, new model variables or alternative datasets. Two case studies of models developed by GMAP for clients will be used. The clients are WH Smith and Halifax Plc and the models attempt to predict sales of books, music, stationery, cards, video and newspapers and financial service sales respectively.

## 1.5 AIMS OF RESEARCH

The main objective of this research project is to evaluate and attempt to improve the performance of spatial interaction models produced by GMAP for clients in the retail sector. There are three main components of the spatial interaction models utilised by GMAP. These are demand, supply and interaction. The demand component of the model represents the attempts to quantify the amount of expenditure that is apparent in residential zones. The supply component is concerned with the definition of the ability of retail destinations to attract expenditure from residence zones. The third component, interaction, is the factor in the model that represents the effect of the separation between origin residential zones and destinations in terms of time, space or cost. It is proposed to examine each of these components of the spatial interaction model in turn in order to discover if the methods currently used in the GMAP models are the most appropriate or whether alternative methods of formulating these components can be used in order to improve model performance.

Improvements to model performance could be achieved through a variety of different methods. In the case of demand estimation, it is possible that the use of alternative data sources to represent the population of origin zones could produce more accurate estimates. For the supply side, the method with which the attractiveness of retail destinations is measured could be improved through detailed analysis of store and shopping centre characteristics in relation to the performance of such stores and centres. In the case of the interaction component of the spatial interaction model it could transpire that the function representing the separation between origins and destinations taken from the Wilson derivation of spatial interaction models may not be the most appropriate function. There is also the possibility that the measurement of the separation between origin zones and destinations could be improved, or that alternative model forms or additional model variables could improve the performance of the model.

Thus, there are several possible ways to improve the performance of spatial interaction models. How the research will be undertaken is briefly described in the next section.

## 1.6 THE EVALUATION OF SPATIAL INTERACTION MODEL PERFORMANCE

Chapter 2 of the thesis presents a systematic review of developments witnessed in the area of spatial interaction prediction in order to explain how the sophisticated models used today by GMAP were developed over the years. The early academic work undertaken by researchers such as Reilly (1931) and Converse (1946) that attempted to define the trade areas of centres and was based on an analogy with Newton's Law of Gravitation is reviewed. Early models that attempted to quantify sales at a given location such as those proposed by Huff (1962, 1964) and Lakshmanan and Hansen (1965) are introduced. Wilson's family of spatial interaction models and their theoretical derivation through entropy maximising techniques are described, followed by descriptions of several alternative methods that have been developed for modelling spatial interaction between zones. Such models include the intervening opportunities model developed by Stouffer (1940, 1960), the competing destinations model proposed by Fotheringham (1981, 1983a, 1983b, 1984, 1985, 1986), the multinomial logit model, log-linear models, Alonso's theory of movement, Poisson regression models and the two stage interaction model described by Liaw and Bartels (1982). Finally, the integration of spatial interaction models as undertaken by Yano (1991, 1993) is reviewed.

The possible sources of error that are apparent in spatial interaction models will be outlined in Chapter 3 along with possible solutions to the problems encountered. This process is necessary in order to identify the possible methods of improving the performance of spatial interaction models in practice.

The two case studies that are to be analysed, the spatial interaction models for WH Smith and Halifax Plc as they are currently formulated for use by GMAP are described in Chapter 4.

The analysis of the demand component of the WH Smith spatial interaction model is the subject of Chapter 5. The current method utilised by GMAP for demand estimation is described and subsequently two alternative methods of demand estimation for goods

sold by WH Smith are proposed and tested. These alternatives utilise different data sources as the basis from which to estimate demand.

Chapter 6 is concerned with the analysis of the supply component of the WH Smith spatial interaction model. The method of formulating the attractiveness of destinations (both shopping centres and individual stores) as used by GMAP at the present time are described. Subsequently, alternative methods of calculating destination attractiveness are formulated and tested in order to see if they will improve the performance of the spatial interaction model for WH Smith.

The interaction component of the spatial interaction model is investigated in Chapter 7 with reference to the Halifax Plc model as developed by GMAP. As with the demand and supply components of the model, the current GMAP method of accounting for the separation between origins and destinations is first explained. This is followed by an investigation into alternative methods of measuring the separation between zones. Alternative methods of including the level of impedance between origins and destinations is also studied, along with the possibility that additional variables added to the spatial interaction model (such as the competing destinations variable, which is a destination attribute that reflects the accessibility of destinations and is described in Chapter 2) and alternative model formulations for including the impedance term (such as Stouffer's intervening opportunities model, also described in Chapter 2).

Once all three components of the spatial interaction model have been studied, they will be combined for both the WH Smith model and the Halifax model in order to see if overall model performance can be improved. Any model improvements concerning the inclusion of the interaction component for the Halifax model will be combined with the conclusions arisen to for the demand and supply components for the WH Smith model. This will be undertaken for both case studies. The results of this analysis will be presented in Chapter 8. The overall improvements achieved for both case studies will therefore be able to be stated. It will also be possible to investigate which model alterations undertaken contribute the most to improvement in model performance in each case.

Chapter 9 comprises conclusions arising from both case studies. It will be necessary to state whether model improvements discovered are applicable to both model types or whether different factors will affect the performance of the model in each case study because this will have implications for future model development undertaken by GMAP. This final chapter also contains an executive summary with recommendations, both specific and general, which will be supplied as a separate document for GMAP's managers to read. The general implications for the specification of spatial interaction models will also be stated.

## **1.7 SUMMARY**

It can be seen that retailers require increasingly sophisticated methods of strategic store location planning in order to increase their competitiveness and market share. A significant amount of academic research has been undertaken in this field, but there is the need to link such research to real retail applications in the commercial sector. It is hoped that this thesis will be a valuable contribution to research into the use of spatial interaction models in retailing through its analysis of case studies of retail sectors in which models have been used for strategic location planning.

## A REVIEW OF SPATIAL INTERACTION MODELS

### 2.1 INTRODUCTION

This thesis investigates possible ways of improving spatial interaction models. In order to do this it is first necessary to review the work that has already been undertaken in the field of predicting spatial interaction to see how the method can be improved.

This review first considers how spatial interaction models have developed over time up to the late 1960s. Subsequently the work of Wilson during the late 1960s and 1970s, especially his development of the family of spatial interaction models will be reviewed (Wilson 1967, 1970, 1971, 1974, 1981). The alternative derivations of the gravity model will also be looked at, including the entropy maximisation method, random utility theory, and the information minimisation approach to the formulation of spatial interaction models. Alternative methods of modelling spatial interaction including the intervening opportunities approach, competing destinations, log-linear modelling, the multinomial logit model, the multi-stage migration model and the Poisson regression model will then be introduced. Finally the work that has been undertaken by Yano (1991; 1993), who has attempted to integrate the different forms of spatial interaction models will be studied.

### 2.2 SPATIAL INTERACTION MODELS

Spatial interaction models are formulated to predict flows between zones. The flows could consist of goods, information, money or persons. Locations are separated in space and different locations specialise in separate functions, therefore there is interaction between locations. Spatial interaction models attempt to quantify this interaction.

Spatial interaction models take the form of an equation which is made up of the independent variables believed to influence the level of spatial interaction between

zones, which is the dependent variable to be estimated. An example of a spatial interaction model for estimating flows to shopping centres is as follows:

$$T_{ij} = A_i O_i W_j f(c_{ij}) \quad (2.1)$$

where

- $T_{ij}$  = the flow between an origin  $i$  and a destination  $j$
- $A_i$  = a balancing factor
- $O_i$  = demand produced by origin  $i$
- $W_j$  = attractiveness of centre  $j$
- $f(c_{ij})$  = a function of the cost of interaction between  $i$  and  $j$

Spatial interaction models such as this are based on three hypotheses.

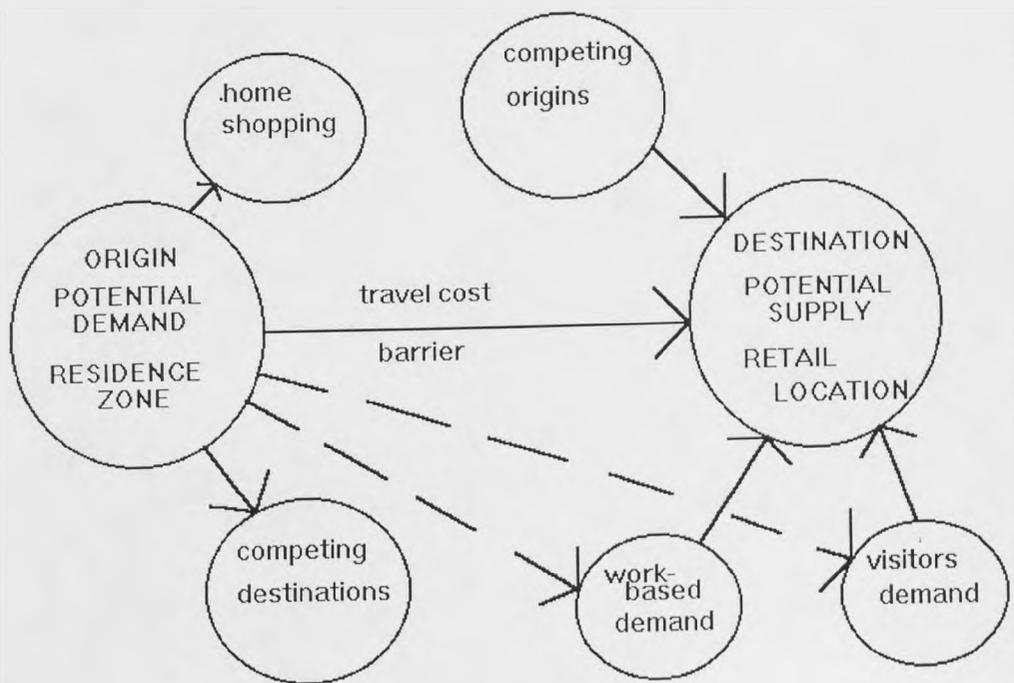
- (1) Level of interaction is proportional to the variable representing trip producing capacity of origin  $i$  (the level of demand,  $O_i$ , in the above example).
- (2) Level of interaction is proportional to the attractiveness of the destination ( $W_j$  in the above example).
- (3) Level of interaction is inversely proportional to the distance between the origin and the destination ( $c_{ij}$  in the above example). This third hypothesis introduces the concept of the cost of interaction decay which is crucial to spatial interaction modelling.

### ***2.2.1 The components of the spatial interaction system***

It can be seen from Figure 2.1 below that the interaction system for a retail model is comprised of several elements. The origin represents the source of expenditure in the system which is usually taken as the place of residence of the population. The destination is the supply point and the location of the retail outlet which can be a destination for the expenditure from the origin. There is also the possibility of expenditure from the origin not being undertaken through spatial interaction *e.g.* home shopping through mail order, the internet, cable or telephone. Between the origin and the destination is the travel cost which represents the cost to the consumer of

overcoming the spatial separation of the origin and the destination *i.e.* the cost of travelling to the destination from the origin. The system is not made up of a single origin and a single destination, there are also competing origins and destinations. Competing origins are alternative origins that compete with the origin to use the retail facilities at the destination. Competing destinations are alternative destinations in the system which could attract expenditure from the origin. Within the system there is also external demand which does not originate at the place of residence of the consumer. Work based demand is demand that occurs from the workplace and not from the place of residence. Demand from visitors outside the area also occurs through tourism.

Figure 2.1: The spatial system involved in retail spatial interaction



Before looking at the spatial interaction model more closely I will review the development of spatial interaction models and describe their progression to the form of model used today.

## 2.3 THE DEVELOPMENT OF SPATIAL INTERACTION MODELS UP TO 1967

### 2.3.1 Newtonian gravity models

The first spatial interaction models to be developed were called gravity models. This is because of their analogy with the Newtonian concept of gravity developed in physics. Newton's Law of Gravitation states that gravitational force ( $F$ ) between two masses ( $M_1$ ,  $M_2$ ) is given by

$$F_{12} = G \frac{M_1 M_2}{d_{12}^2} \quad (2.2)$$

where  $d_{12}$  is the distance between the two masses and  $G$  is the gravitational constant. The use of Newton's law to develop a model of geographical flows is usually attributed to Stewart (1941) cited in (Thomas & Huggett, 1980). An equivalent geographical formulation of the Law of Gravitation would be

$$T_{ij} = k \frac{O_i D_j}{c_{ij}^2} \quad (2.3)$$

for some constant  $k$ .  $O_i$  and  $D_j$  act as the mass elements in this case and the force is interpreted as the size of flows between two zones. In this basic gravity model the parameter on the distance function is given as 2, *i.e.* distance is squared. However this is an arbitrary figure as is the formulation of the distance function as a power function and a general form of the geographical gravity model can be given by

$$T_{ij} = k O_i D_j f(c_{ij}) \quad (2.4)$$

This social physics analogy draws on the hypothesis that the interaction between two masses will vary proportionately with the product of the two masses, and inversely with the distance between the two masses.

### 2.3.2 Reilly's Law of Retail Gravitation

Even before the work of Stewart there was evidence of geographers applying gravitational concepts to the problem of analysis of retail trade areas. For example,

Reilly (1931) developed his Law of Retail Gravitation to delimit the trading areas of cities. Reilly used the variables of population size and distance between centres in order to estimate the relative pulling power of two cities on an intermediate area. Reilly developed the following formulation

$$\frac{B_a}{B_b} = \frac{P_a}{P_b} \left( \frac{D_b}{D_a} \right)^2 \quad (2.5)$$

where

$B_a$  = proportion of retail business from an intermediate town attracted by city  
 $a$

$B_b$  = proportion of retail business from an intermediate town attracted by city  
 $b$

$P_a$  = population of city  $a$

$P_b$  = population of city  $b$

$D_a$  = distance from intermediate town to  $a$

$D_b$  = distance from intermediate town to  $b$

Converse (1946) developed Reilly's Law of Retail Gravitation to make it possible to calculate the breaking point between two cities. This is the point at which the influence of each city is equal. Such points could be used to estimate the trading area of cities. Break point from city  $b$  is defined as

$$D_b = \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}} \quad (2.6)$$

However, such methods as this made no attempt to calculate the actual amount of interaction that took place between different locations. Also the idea of a definite breaking point is unrealistic as in reality the trading influence of a city will decline gradually with distance. Break points also do not allow for overlapping trading areas.

### 2.3.3 Huff's probabilistic model

Many researchers worked with gravity models in the 1950s and 1960s, but Wilson (1974) has stated that no real progress was made until the work of Huff (1962, 1964). Huff (1964) developed an alternative model to that proposed by Reilly which he believed overcame the problems of Reilly's model. In Huff's model the principal focus became the consumer, and the model attempts to describe the process undertaken by individuals when choosing from alternative centres. The model calculates the probability that a consumer from origin  $i$  will travel to centre  $j$  to satisfy their needs. This is undertaken by dividing the attractiveness of centre  $j$  by the sum of the attractiveness of all centres. The model can be stated formally as in Huff (1964) in the following way

$$P_{ij} = \frac{s_j / t_{ij}^\alpha}{\sum_j s_j / t_{ij}^\alpha} \quad (2.7)$$

where

$P_{ij}$  = probability of consumer going from residence zone  $i$  to centre  $j$

$s_j$  = size of centre  $j$

$t_{ij}$  = travel time between the two zones

$\alpha$  = a parameter which is to be estimated empirically to reflect the effect of travel time on various kinds of shopping trips

The expected number of consumers shopping at centre  $j$  is subsequently calculated as

$$E_{ij} = P_{ij} C_i \quad (2.8)$$

where

$E_{ij}$  = expected number of consumers from residence zone  $i$  going to centre  $j$

$C_i$  = number of consumers in residence zone  $i$ .

This model is an improvement on that of Reilly because it attempts to quantify the actual level of interaction that takes place between two zones and also it allows trading areas to be graduated according to probability contours.

### 2.3.4 The model of Lakshmanan and Hansen

Lakshmanan and Hansen (1965) also used a gravity model in their study of Metrotown Centres in Baltimore. They stated that the level of retail provision would be dependent on the number of consumers and their purchasing power. The model they developed indicates that sales are directly related to size of centre and to its relative location to consumers and the purchasing power of those consumers. Sales would also be affected by the location of the centre in relation to other competing centres. The model used in the study took the following form

$$S_{ij} = O_i \frac{f_j / d_{ij}^\alpha}{\sum_{k \neq i}^n f_k / d_{ik}^\alpha} \quad (2.9)$$

where

$S_{ij}$  = consumer expenditure of population of residence zone  $i$  at centre  $j$

$O_i$  = consumer expenditure in residence zone  $i$

$f_j$  = size of centre  $j$

$f_k$  = size of other centre  $k$

$d_{ij}$  = distance between residence zone  $i$  and centre  $j$

$d_{ik}$  = distance between residence zone  $i$  and other centre  $k$

$\alpha$  = an exponent applied to the distance variable

This model indicates that there is no definite boundary to a trade area, but that there is a differing degree of interaction between all zones.

It can therefore be seen that these writers developed the concept of gravity modelling and indicated their possible use in real retail applications.

Much of the subsequent work on spatial interaction models has been undertaken by Wilson, who developed a family of spatial interaction models in an alternative derivation that gives them a firmer theoretical basis.

## 2.4 WILSON'S FAMILY OF SPATIAL INTERACTION MODELS

Wilson (1967) put forward a spatial distribution model that estimates flows ( $T_{ij}$ ) as a function of the variables  $O_i$ ,  $D_j$  and  $c_{ij}$ . Where  $T_{ij}$  represents flows between origin zone  $i$  and destination  $j$ ,  $O_i$  is a measure of demand for interaction from origin zone  $i$ ,  $D_j$  is the attractiveness of destination  $j$  in producing interaction and  $c_{ij}$  is the cost of interaction between origin  $i$  and destination  $j$ . The simplest form of such a model is the Newtonian gravity model described above in equation (2.4).

$$T_{ij} = kO_i D_j f(c_{ij}) \quad (2.10)$$

However, Wilson (1967) points out that this equation has a deficiency in that if a particular  $O_i$  and  $D_j$  are both doubled, then the number of trips between the zones would quadruple using the above equation, when it would be expected that the flow would also double. Therefore constraints need to be added to the equation in order to rectify this deficiency. The number of constraints that can be added to any particular model will depend on the amount of knowledge that can be built in to the model.

For example, if the total flow originating in the origin zone  $i$ ,  $O_i$ , is known then the constraint of

$$\sum_j T_{ij} = O_i \quad (2.11)$$

can be added to the model. This constraint requires that predictions of the trips,  $T_{ij}$ , sum to the known number of trips originating in zone  $i$ .

If the total flows ending at destination  $j$  are known,  $D_j$ , then the following constraint can be added to the gravity model.

$$\sum_i T_{ij} = D_j \quad (2.12)$$

This constraint means that the predicted number of trips,  $T_{ij}$ , must sum to the known number of trips ending in zone  $j$ .

Different situations will lead to different amounts of knowledge being built into models. For example, in a model of retail activity, the demand (expenditure) from an origin can be estimated and this knowledge can be incorporated in to the model, and in a residential location model the number of workers in each destination zone is known and this can be added as a constraint term in the model. In different situations different constraints can be used in the model.

Wilson (1971) has developed a family of spatial interaction models that are differentiated by the constraints that are placed on each member of the family. Wilson identifies four different cases based on which of  $O_i$  and  $D_j$  are known.

- (1) Both  $O_i$  and  $D_j$  are unknown - the unconstrained case.
- (2)  $O_i$  known, but  $D_j$  unknown - the production constrained case.
- (3)  $D_j$  known, but  $O_i$  unknown - the attraction constrained case.
- (4) Both  $O_i$  and  $D_j$  known - the production-attraction constrained case.

These different cases can be used to build alternative spatial interaction models based on the amount of knowledge held on the system being modelled.

When building such models it has been noted by Wilson (1971) that, if either the  $O_i$  or  $D_j$  is not known, then they must be replaced in the gravity model by an attractiveness term. Wilson uses the terms  $W_i^1$  to replace  $O_i$ , and  $W_j^2$  to replace  $D_j$ . In his 1971 paper Wilson also states that in the formulation of the family of spatial interaction models the

constant of proportionality,  $k$ , must be replaced by sets of balancing factors that are appropriate to the constraints assumed in each case.

The family of spatial interaction models developed by Wilson (1971) is now described.

#### 2.4.1 *The unconstrained spatial interaction model*

In this case both  $O_i$  and  $D_j$  are unknown and therefore neither of the constraints hold. Therefore  $O_i$  and  $D_j$  are both replaced by the attractiveness factors  $W_i^1$  and  $W_j^2$ .  $k$  is not replaced because neither of the constraints hold and therefore no origin or destination balancing factors are required. The unconstrained gravity model takes the form

$$T_{ij} = kW_i^1W_j^2f(c_{ij}) \quad (2.13)$$

However, when this type of model is used, the following overall constraint is often applied

$$\sum_i \sum_j T_{ij} = T \quad (2.14)$$

This constraint ensures that the predicted number of flows is equal to the known flow total,  $T$ .

The constant of proportionality  $k$ , is derived by substituting the right hand side of equation (2.13) for  $T_{ij}$  in equation (2.14) which gives

$$\sum_i \sum_j kW_i^1W_j^2f(c_{ij}) = T \quad (2.15)$$

rearranging the terms we obtain

$$k = \frac{T}{\sum_i \sum_j W_i^1W_j^2f(c_{ij})} \quad (2.16)$$

### 2.4.2 The production constrained spatial interaction model

In this case  $O_i$  is known and  $D_j$  is not. Therefore the constraint shown in equation (2.11) holds and  $D_j$  is replaced by  $W_j^2$ .  $k$  has to be replaced by a balancing factor that is dependent on  $i$  to ensure that the constraint is satisfied. This balancing factor is  $A_i$ . The production constrained spatial interaction model is

$$T_{ij} = A_i O_i W_j^2 f(c_{ij}) \quad (2.17)$$

This model is subject to the constraint

$$\sum_j T_{ij} = O_i \quad (2.18)$$

Substitution for  $T_{ij}$  in equation (2.18) gives

$$\sum_j A_i O_i W_j^2 f(c_{ij}) = O_i \quad (2.19)$$

The  $O_i$  terms in this equation cancel out and the  $A_i$  term can be brought outside of the summation. This leads to

$$A_i \sum_j W_j^2 f(c_{ij}) = 1 \quad (2.20)$$

Therefore, the balancing factor  $A_i$  is

$$A_i = \frac{1}{\sum_j W_j^2 f(c_{ij})} \quad (2.21)$$

### 2.4.3 The attraction constrained spatial interaction model

$O_i$  is unknown and  $D_j$  is known in this case. Therefore the constraint represented in equation (2.12) holds, and  $O_i$  is replaced by the attractiveness factor  $W_i^1$ .  $k$  is replaced by the balancing factor  $B_j$  to ensure that the constraint holds. The model is presented as

$$T_{ij} = B_j W_i^1 D_j f(c_{ij}) \quad (2.22)$$

This equation is subject to the constraint

$$\sum_i T_{ij} = D_j \quad (2.23)$$

Substitution for  $T_{ij}$  in equation (2.23) gives

$$\sum_i B_j W_i^1 D_j f(c_{ij}) = D_j \quad (2.24)$$

The  $D_j$  terms cancel out and the  $B_j$  term can be brought outside the summation to give

$$B_j \sum_i W_i^1 f(c_{ij}) = 1 \quad (2.25)$$

Therefore

$$B_j = \frac{1}{\sum_i W_i^1 f(c_{ij})} \quad (2.26)$$

### 2.4.4 The production-attraction constrained spatial interaction model

In this case both  $O_i$  and  $D_j$  are known and therefore neither are replaced by attractiveness factors.  $k$  is replaced by both  $A_i$  and  $B_j$  which are calculated to ensure that both constraints are satisfied simultaneously. This gives the following model

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \quad (2.27)$$

Substituting for  $T_{ij}$  in equations (2.11) and (2.12) gives  $A_i$  and  $B_j$ .

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})} \quad (2.28)$$

$$B_j = \frac{1}{\sum_i A_i O_i f(c_{ij})} \quad (2.29)$$

It can be seen that the value of  $A_i$  is dependent on  $B_j$  and the value of  $B_j$  is dependent on  $A_i$ . Therefore the above equations have to be solved iteratively.

Thus it can be seen that various spatial interaction models can be formulated, depending on the amount of information that is known.

#### ***2.4.5 Applications of the family of spatial interaction models***

Each of the family of spatial interaction models are used in different situations. For instance it can be seen that the production constrained case is applicable to the shopping model. Estimates of consumer demand can be made and therefore  $O_i$  is regarded as known. However,  $D_j$ , the size of flows ending at destinations is usually not known and therefore an attractiveness factor such as centre size is used instead.  $T_{ij}$  represents the flow of consumer expenditure from residents in zone  $i$  at centre  $j$ . A balancing factor such as  $A_i$  must be included in the equation to ensure that constraint equation (2.11) is met. Thomas and Huggett (1980) state that  $A_i$  can be expressed as the ratio between known expenditure per zone  $O_i$ , and the sum of the predicted expenditures leaving  $i$  for each  $j$  *i.e.*

$$A_i = \frac{O_i}{\sum_j O_i W_j^2 f(c_{ij})} \quad (2.30)$$

$O_i$  cancels out to give

$$A_i = \frac{1}{\sum_j W_j^2 f(c_{ij})} \quad (2.31)$$

as derived earlier.

The attraction constrained model is often used to model residential location. In this case the destinations of trips  $D_j$ , the number of workers in workplace zone  $j$ , are known and the model allocates workers to origins. The simple residential location model allocates individuals who work in workplace zone  $j$  to residences in residence zone  $i$ .  $T_{ij}$  represents the number of workers living in residence zone  $i$  who work in workplace  $j$  and the model must satisfy constraint equation (2.12). This constraint states that the number of people arriving to work in a zone must be equal to the number of jobs in that zone. The balancing factor  $B_j$  ensures that this constraint is met. Thomas and Huggett (1980) state that for any  $j$ ,  $B_j$  is calculated as the ratio between  $D_j$  (the known number of jobs), and the sum of the predicted flows arriving at  $j$  from  $i$ , *i.e.*

$$B_j = \frac{D_j}{\sum_i D_j W_i^1 f(c_{ij})} \quad (2.32)$$

$D_j$  in this equation cancels out to give the expression

$$B_j = \frac{1}{\sum_j W_i^1 f(c_{ij})} \quad (2.33)$$

as derived earlier.

For the production-attraction constrained case of the spatial interaction model both trip end totals are known, therefore this model predicts the size of individual flows between each origin  $i$  and each destination  $j$ . This type of model is usually used to model journey to work patterns in cases where both the number of workers residing in each zone and

the number of jobs in each zone are known and therefore both constraints must be satisfied.

## **2.5 EARLY THEORETICAL CRITICISMS OF GRAVITY MODELS**

Theoretical criticisms of gravity models have arisen and are stated as:

"objections all arise because the basic gravity propositions of attraction, production and distance decay are derived from trends in flow data and not from a theory of individual trip making behaviour"

(Thomas and Huggett, 1980, p152)

Huff (1962) has also criticised gravity models for being unable to account accurately for the behaviour of individuals. This is because flows are modelled at an aggregate level and not an individual level.

Foot (1981) has also stated that gravity models have been criticised for their lack of theoretical basis, due to being based purely on an analogy with Newton's Law of Gravitation. Wilson (1967, 1970) attempted to rectify this situation and justify the use of spatial interaction models by producing an alternative derivation of the gravity model based on statistical mechanics that would give the model a firmer theoretical foundation. The method used by Wilson (1967, 1970) is that of entropy maximisation, which is described in Section 2.6. However, this method of deriving models has since been criticised for not being based on the behaviour of individual trip makers.

## **2.6 ENTROPY MAXIMISATION**

### ***2.6.1 Introduction***

Wilson has used the physical law of entropy maximisation which describes the movement of gas particles. Wilson draws an analogy between the movement of such gas particles and the movement of people in space.

The entropy maximisation method looks at individuals and estimates the probability of them undertaking a particular trip. Wilson (1970), using the production-attraction

constrained spatial interaction model, has argued that if the known constraints on the flow matrix can be expressed in equation form, then the entropy of a probability distribution associated with the flows can be maximised and a maximum probability of the flow matrix can be obtained.

It has already been seen that the production-attraction constrained model is subject to the constraints:

$$\sum_j T_{ij} = O_i \quad (2.34)$$

$$\sum_i T_{ij} = D_j \quad (2.35)$$

Wilson also states that for entropy maximisation an additional constraint is also required. This constraint is the cost constraint and means that the total amount spent on travel cannot exceed the total amount of money available for travel. This constraint is

$$\sum_i \sum_j T_{ij} c_{ij} = c \quad (2.36)$$

where  $c_{ij}$  is the cost of travel between origin  $i$  and destination  $j$ , and  $c$  is the total cost of travel.

In reality there are many distributions of trips that will satisfy these constraints. The entropy maximisation method determines which of these distributions is most probable.

### ***2.6.2 Levels of resolution***

For entropy maximisation it is necessary to define three levels of resolution at which the state of the system can be described. These are the macro state, the meso state, and the micro state. The micro level is the most detailed state, in which individuals are assigned to particular trip categories. The meso state is the intermediate level which describes the total interaction between zones. The macro level imposes the restrictions of the above three constraints.

As pointed out by Wilson, it can be seen that there are many micro states which can give rise to a certain meso state, and many meso states that give rise to the given macro state. It is now necessary to make the assumption that all micro states are equally probable. Subsequently, it can be seen that the most probable meso state is the one that has the greatest number of micro states associated with it. Therefore, it is possible to calculate the most probable distribution of trips by finding the set of  $T_{ij}$ s that has the greatest number of individual trips (micro states). Entropy maximisation allows the prediction of the meso state without having to explain the behaviour of individuals at the micro level.

### 2.6.3 Creation of the model

The number of micro states *i.e.* the number of different ways of assigning individuals to trips and giving rise to the desired trip matrix  $\{T_{ij}\}$  is defined by Wilson (1970) as  $W(\{T_{ij}\})$ , where:

$$W(\{T_{ij}\}) = \frac{T!}{\prod_{ij} T_{ij}!} \quad (2.37)$$

Wilson (1970) has shown, that if the logarithm of  $W(\{T_{ij}\})$  is maximised subject to the constraints outlined above, then the resulting trip matrix  $\{T_{ij}\}$  is given by:

$$T_{ij} = A_i B_j O_i D_j \exp^{-\beta c_{ij}} \quad (2.38)$$

where:

$$A_i = \frac{1}{\sum_j B_j D_j \exp^{-\beta c_{ij}}} \quad (2.39)$$

and

$$B_j = \frac{1}{\sum_i A_i O_i \exp^{-\beta c_{ij}}} \quad (2.40)$$

Where  $\beta$  is the Lagrangian multiplier associated with the cost constraint equation and the balancing factors  $A_i$  and  $B_j$  ensure that the other two constraints are met.

It can be seen that the above equation is equivalent to the doubly constrained, production-attraction gravity model except that the distance function has been replaced with a negative exponential distance function.

Wilson (1970) argues that the other members of the family of spatial interaction models can be derived in the same way by dropping constraint equation (2.35) for the production constrained spatial interaction model, and constraint equation (2.34) for the attraction constrained spatial interaction model. However, the results are different from before in that in the case of the production constrained model there is no  $D_j$  term, although one can be introduced.

#### ***2.6.4 Conclusion on entropy maximisation***

Through entropy maximisation Wilson has derived the gravity spatial interaction model in a way that gives the model a more theoretical basis. The entropy maximisation derivation of spatial interaction models also overcomes one of the other criticisms of the gravity model. As was seen above gravity models have been criticised for not explaining why an individual undertakes a particular trip. The entropy maximisation model avoids this problem because it explains the macro state of the system without having to explain the behaviour of individuals. However, Thomas and Huggett (1980) argue that the entropy maximisation model has a behavioural meaning:

"When we construct entropy maximisation models it is assumed that we will never bother to find out which route each of the individuals actually assigned themselves. Given this assumption, the entropy maximisation criterion has a behavioural meaning because we select the solution that maximises the individual's freedom to choose between available trips. For this reason the entropy maximisation solution is said to be the most likely trip matrix."

(Thomas and Huggett, 1980, p156)

It was therefore argued that entropy maximising models overcame some of the problems associated with gravity models and gave gravity spatial interaction models a sounder theoretical basis. However, the theory is not based on the behaviour of individuals but on an analogy to gas particles, and therefore the theory is no longer seen as providing a behavioural basis for the spatial interaction model.

## 2.7 ALTERNATIVE DERIVATIONS OF THE SPATIAL INTERACTION MODEL

### 2.7.1 *Random utility theory*

Wilson and Bennett (1985) and Anas (1982) describe another possible method of deriving the spatial interaction model. Random utility theory can be used to develop models that are based on a probabilistic choice process and this method attempts to replicate the behaviour of individuals directly using utility theory. Random utility theory assumes that individuals will maximise their utility by making choices between alternatives. However, there are problems with this method concerned with measuring utility and ensuring that utility provides an adequate representation of individual behaviour.

One way in which it is possible to ensure that behaviour is more accurately represented is through the addition of a random term which is assumed to have a particular distribution. Total utility is assumed to be able to be separated additively into systematic utility and random utility. The systematic utility is that part of total utility that is dependent on measurable characteristics. One interpretation of the random utility aspect is that it represents the variations between households with the same characteristics that make different choices. The basic assumption of random utility theory is that the random and systematic parts of total utility are separable. Thus, an additive function for total utility for individual  $j$  making choice  $k$ ,  $u_{jk}$ , can be derived

$$u_{jk} = u_{jk}^*(z^{jk}) + \varepsilon_{jk} \quad (2.41)$$

where

$u_{jk}^*$  = the average value of  $u_{jk}$  and is a function of characteristics of individual  $j$  and choice  $k$  which are given by  $z^{jk}$

$\epsilon_{jk}$  = random utility

The probability that an individual  $j$  will make choice  $k$  is then calculated and is a function of the differences between the utility of that alternative and the utility of all other alternatives. If the common utility and random utility terms are non-linearly related, the choice probabilities of individual  $j$  making choice  $k$ ,  $P_{jk}$  can be given as

$$P_{jk} = f_{jk} \left[ u_{jk}^* (z^{lm}), all(l, m) \right] \quad (2.42)$$

where  $l$  and  $m$  represent all other alternatives and  $f_{jk}$  is the choice probability function of individual  $j$  making choice  $k$ .

The type of model that is generated using random utility theory will depend on the probability distribution which is assumed for  $\epsilon_{jk}$ . If the random variable is assumed to take a Weibull distribution (*i.e.* the probability that a random variable takes a certain value is the same as the probability that  $u$  takes a corresponding value) then the multinomial logit model, which will be outlined later, is derived. This derivation can be found in Anas (1982) or Domencich and McFadden (1975) but will not be formally expressed here.

### 2.7.2 Information minimisation

Pooler (1994) describes the information minimisation approach to deriving spatial interaction models. This methods includes prior information ( $S_{ij}$ ) when producing the model. Such previously obtained information usually takes the form of observed trips. The basic idea is to minimise

$$\sum_i \sum_j T_{ij} \ln(T_{ij}/S_{ij}) \quad (2.43)$$

subject to the same constraints as used in the entropy maximisation formulation of the spatial interaction model.

Pooler (1994) states that this process produces spatial interaction models of the following form.

$$T_{ij} = S_{ij} A_i O_i B_j D_j \exp(-\beta c_{ij}) \quad (2.44)$$

where

$$A_i = \frac{1}{\sum_j S_{ij} B_j D_j \exp(-\beta c_{ij})} \quad (2.45)$$

and

$$B_j = \frac{1}{\sum_i S_{ij} A_i O_i \exp(-\beta c_{ij})} \quad (2.46)$$

It is also possible to derive the singly constrained members of the family of spatial interaction models using information minimisation. This is through minimising

$$\sum_i \sum_j T_{ij} \ln(T_{ij}/V_i W_j) \quad (2.47)$$

where  $V_i$  and  $W_j$  are measures of the attraction of origins and destinations.

Pooler (1995) argues that the strength of this approach to modelling is that the use of prior information on trip patterns in the model can lead to improved performance of models.

However, it can be seen that there is a problem with this approach to the derivation of spatial interaction models in that if there are new residential zones or new destinations,

prior information is not available on trips to these zones. Zero cannot be input as the  $S_{ij}$  value in the information minimisation equation and therefore this method cannot be used when there are new zones.

## 2.8 ALTERNATIVE SPATIAL INTERACTION MODELS

(Several alternative methods of modelling spatial interactions have been proposed. These will be introduced in the following sections.)

### 2.8.1 *The intervening opportunities model*

The intervening opportunities model was first proposed by Stouffer in 1940. Stouffer proposed that the propensity to travel was not necessarily dependent upon the distance between two zones but was more likely to be directly proportional to opportunities perceived at a destination  $j$  and inversely proportional to the number of intervening opportunities between  $i$  and  $j$ . Figure 2.2 below illustrates the idea proposed by Stouffer in the context of the retail model.

Figure 2.2: Stouffer's theory of intervening opportunities

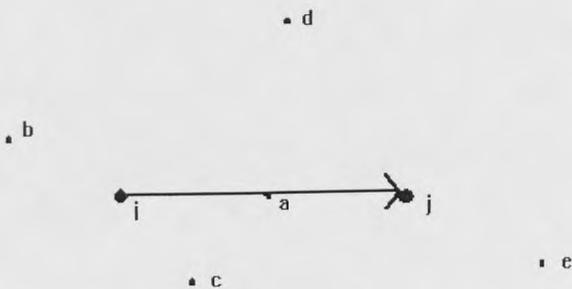


Figure 2.2 concerns the prediction of the flow from origin  $i$  to centre  $j$ . Within the spatial system shown, centres  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  compete with  $j$  for flows from  $i$ , although the competition from centres  $a$ ,  $b$  and  $c$  is stronger because they are closer to  $i$  than  $j$  is. Stouffer argues that the distance between origins and destinations is not the only factor effecting impedance between the two zones. He argues that centre  $a$  provides more competition for the flow between  $i$  and  $j$  because it intervenes between the two places

and consumers may choose to travel the shorter distance to  $a$  if the good is available in that centre rather than pass the centre and continue to  $j$ .

Stouffer (1960) also introduced the notion of competing migrants which states that migrants from different origins compete for access to the destination. However, this is unlikely to be important in a model of retail activity because there is no rationing of supply of retail facilities.

Gonclaves and Ulyseia-Neto (1993) have developed a spatial interaction model which incorporates both the distance function and an intervening opportunities function. This model is based on the argument that gravity models are flawed because they take no account of intervening opportunities, and that the intervening opportunities model suffers from the lack of a distance variable. Therefore this type of model adds an extra perspective to the interaction process based on the argument that destination choice is not solely determined by distance and could therefore more accurately reproduce actual decision making behaviour. A production constrained version of this hybrid model is stated as follows

$$T_{ij} = A_i O_i W_j \exp(-\lambda X_{ij} - \beta c_{ij}) \quad (2.48)$$

where  $X_{ij}$  is equal to the number of opportunities between  $i$  and  $j$ , according to distance.  $\lambda$  is a parameter on the intervening opportunities term that determines the importance of the term within the model.

However, it can be argued that the standard Wilson spatial interaction models do incorporate competing origins and destinations. This can be seen by considering the balancing factors  $A_i$  and  $B_j$ . The basic spatial interaction model is as follows:

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \quad (2.49)$$

where

$$A_i = \frac{1}{\sum_j B_j D_j f(c_{ij})} \quad (2.50)$$

so that

$$A_i D_i = \frac{D_j}{\sum_j B_j D_j f(c_{ij})} \quad (2.51)$$

and

$$B_j = \frac{1}{\sum_i A_i O_i f(c_{ij})} \quad (2.52)$$

so that

$$B_j O_i = \frac{O_i}{\sum_i A_i O_i f(c_{ij})} \quad (2.53)$$

The  $A_i$  term represents competition between destinations, in that this accessibility term shows how competitive each destination is in relation to other destinations, defined by the accessibility. The  $B_j$  term represents competing origins, *i.e.* the term shows how competitive each origin is in relation to all origins. The difference between these terms and those proposed by Stouffer is that the accessibility terms in the Wilson model are general and do not have a directional component as is apparent in Stouffer's definition of intervening opportunities.

### 2.8.2 *The competing destinations model*

Another model that has been developed, that accounts for the accessibility of destinations to each other is the competing destinations model which has been derived from a theory of destination choice formulated by Fotheringham. Much has been

written concerning an individual's destination choice, for example, Thill (1992) provides an overview of destination choice theories. Fotheringham (1982, 1985), Fik and Mulligan (1990), Lo (1991) and Horowitz (1991) also provide useful insights into destination choice theory. (The debate concerning destination choice is related to how individuals determine their choice set.)

(Fotheringham (1983a, 1984) has stated that traditional gravity models are misspecifications of reality because they assume that distance is the only factor affecting distance decay parameters and therefore choice sets.) He argues that the relationship that can be observed between spatial structure in a system and values of the distance decay parameters (this will be discussed fully in Section 3.5.4) means that gravity models are misspecified due to parameter bias. It is therefore necessary to attempt to try and decrease the spatial variations in parameter estimates and this has been attempted by Fotheringham through the development of an alternative theory of destination choice.

(Fotheringham (1983a) has argued that gravity models take no account of the location of destinations in relation to other destinations and therefore fail to take into account the competition or agglomeration effects that arise due to the grouping of destinations. In Fotheringham's theory of competing destinations, hierarchical destination choice is the basis for the model and he uses this theory as a means of accounting for the influence of spatial structure on decision making.) This theory of hierarchical decision making is developed from propositions from cognitive theory such as the idea of information processing.

(Cognitive theory is concerned with an individual's ability to process information) when considering alternative spatial choices (and has shown that individuals do not consider all alternatives due to their limited information. Thill (1992) has also stated that it is difficult to assume that individuals will consider all alternative destinations when undertaking a destination choice, as is assumed in the gravity model. It is therefore necessary to decrease the choice set and this is undertaken through some method of cognitive information processing.) Through this information processing, alternatives are eliminated or grouped in order to simplify the decision making procedure. (Fotheringham argues that a hierarchical decision making process is the most likely method of decreasing the choice set.) He proposes that the choice of destination is a two

stage process whereas interaction in the gravity model is represented as a one stage process. The two stages are, firstly an individual makes a decision on which group of destinations to travel to and subsequently decides which individual destination to choose. If people do not look at all destinations but restrict their choice to a certain set of destinations then the decision is a hierarchical procedure *i.e.* individuals first choose a cluster of destinations and then choose a specific destination. If this is the case then individuals do not consider all destinations equally as is assumed in the gravity model. Within the gravity model the assumption is made that destination choice is not hierarchical and therefore choice of destination will not be influenced by the spatial structure of alternative destinations. Therefore the gravity model takes no account of any advantage or disadvantage the individual may perceive from destinations being located in clusters which means that competition and agglomeration forces are ignored. The competing destinations model allows for modelling of spatial interaction to be undertaken allowing for an alternative theory of destination choice. Fotheringham argues for the inclusion of an additional accessibility variable,  $Z_j$  in the spatial interaction model that represents the accessibility of centre  $j$  with respect to other centres.) A potential accessibility variable is equal to

$$Z_j = \sum_{k=1, k \neq i, k \neq j}^n \frac{W_k}{C_{jk}^\sigma} \quad (2.54)$$

as defined in Fotheringham (1985), where  $k$  represents all other possible destinations,  $W_k$  is the attractiveness of alternative destinations and  $\sigma$  is a measure of the importance of distance in determining the perception of accessibility. Therefore an origin specific, production constrained, competing destinations model would be

$$T_{ij} = A_i O_i W_j Z_j^\delta c_{ij}^{\beta_i} \quad (2.55)$$

where the distance decay parameters are origin specific ( $\beta_i$ ). The  $\delta$  parameter in the accessibility term allows for the effects of competition or agglomeration to be incorporated into the model. When  $\delta > 0$  agglomeration forces are present *i.e.* the more accessible to other destinations a destination is the higher will be the interaction between the origin zone and the destination. If  $\delta < 0$  then competition forces are present

and as the accessibility of a destination to other destinations increases, interaction to that destination will decrease.

Comparing this equation to the Wilson production constrained spatial interaction model it can be seen that Fotheringham has included an additional accessibility variable and has also used origin specific distance decay parameters, indicating that residents of different zones will perceive distance in different ways.

Fotheringham (1984) used a matrix of airline passenger flows in 1970 between the 25 largest cities in the USA in order to test the competing destinations model against the gravity spatial interaction model. He found that for 19 out of the 25 origins the addition of  $Z_j$ , the destination accessibility variable produced an increase in the adjusted  $r^2$  statistic and that for 14 out of 25 regions the estimated origin specific accessibility parameter  $\delta_i$  was significantly different from 0 at the 95% confidence level.

Fotheringham has therefore argued that the inclusion of the competing destinations accessibility variable is a valuable extension to the gravity model enabling such models to predict interaction patterns more satisfactorily.

However, there are also other alternative destination choice theories. For example, Lo (1991) argues that the instability of distance decay parameters over space is due to differences in consumer preferences as well as differences in spatial structure. Fotheringham only uses spatial structure to attempt to reduce choice sets, whereas Lo (1991) argues that the processes that effect consumer perception of alternative destinations as either substitutes or compliments will have at least the same impact on destination choice as spatial structure. Golledge and Timmermans (1990) also argue that non-locational characteristics can cause variation in choice set determination.

Lo also adds that it is necessary to note that the pattern of spatial preference will be dependent on the spatial structure of the system. As each system has a unique spatial structure it is therefore difficult to produce a general preference pattern. This means that consumer preferences are hard to introduce into the model and it may therefore be necessary to accept that parameter variation is an inherent problem that can be decreased but not eliminated. Through the introduction of the accessibility variable, Fotheringham

has achieved a decrease in parameter variation by accounting for the effect of spatial structure on destination choice.

Pooler (1994) has also argued that there may be an alternative form of decision making which is not based on the relative location of facilities and that if this form of destination choice is accepted as being more representative of reality then there will be implications for Fotheringham's competing destinations model. To explain this alternative form of decision making Pooler (1994) cites the example of choice of restaurants. Pooler argues that individuals will reduce the size of the destination choice set through a form of mental simplification which involves elimination and grouping of destinations. Remaining destinations are subsequently grouped into non-spatial sets, for example, the type of restaurant. Only when the set of destinations has been reduced in this way will location of the facilities have an impact on destination choice.

Pooler (1994) states that if this form of decision making is representative it will have the following implications for the competing destinations model.

(1) The theoretical and behavioural foundations of the model will become suspect because spatial hierarchies are not being used to simplify the choice set.

(2) Even though the competing destinations model may reduce the observed spatial bias in the origin specific distance decay parameters we do not know why it does and therefore an alternative theory is required to explain why the addition of an accessibility term in a spatial interaction model decreases the spatial bias in parameter estimates.

However, Fotheringham (1986) provides evidence for the existence of spatial hierarchical decision making through the calibration of a model for migration flows in the United States. In this paper Fotheringham shows that destination choice is a spatially influenced hierarchical decision making process and therefore Fotheringham's theory of competing destinations can explain why the addition of an accessibility variable reduces the spatial bias in distance decay parameters.

### 2.8.3 A two stage interaction model

Liaw and Bartels (1982) have stated that migration models are non linear and that a two level logistic model of migration that is equivalent to the production constrained gravity model developed by Wilson (1971) can be used to model migration.

Liaw and Bartels (1982) argue that migration is the result of a two stage decision making process: the first stage being the decision to move, and the second stage the decision on where to move to.  $m_{ij}$  is the probability that a person in zone  $i$  will migrate to zone  $j$  in time period  $t$  and is calculated as follows:

$$m_{ij} = p_{ii} p_{ij} \quad (2.56)$$

Where  $p_{ii}$  is the probability of an individual leaving  $i$  in time period  $t$ , and  $p_{ij}$  is the probability that an individual leaving  $i$  will go to  $j$  in time period  $t$ .

Liaw and Bartels (1982) argue that evidence shows that the two stages of the decision making process are determined by different variables and can therefore be investigated by a two stage model.

The departure model takes the following form:

$$p_{ii} = \frac{\exp^{\alpha_0 + \alpha_1 x_{ii1} + \dots + \alpha_k x_{iik}}}{1 + \exp^{\alpha_0 + \alpha_1 x_{ii1} + \dots + \alpha_k x_{iik}}} \quad (2.57)$$

The destination choice model takes the form:

$$p_{ij} = \frac{\exp^{\beta_1 x_{ij1} + \beta_2 x_{ij2} + \dots + \beta_k x_{ijk}}}{\sum_{j=1}^D \exp^{\beta_1 x_{ij1} + \beta_2 x_{ij2} + \dots + \beta_k x_{ijk}}} \quad (2.58)$$

where

$x_{iik} \dots x_{iik}$  = observable factors controlling departure probabilities

$x_{ij1} \dots x_{ijk}$  = observable determinants of destination choice probabilities

$D$  = the number of alternative destinations.

However, it can be seen that this model has not added anything to the spatial interaction models already described because, in the example of the retail model the departure model is equivalent to the estimation of retail sales in an origin and the destination choice model represents the interaction function that divides sales between destinations.

#### 2.8.4 The multinomial logit model

Section 2.7.1 introduced random utility theory. The most simple model created through this theory is the multinomial logit model. An example of a multinomial logit model used to model destination choice and transport mode used is cited in Birkin *et al.* (1995) as

$$P(j, k: JK_t) = \frac{\exp(u_{jkt})}{\sum_{jk} \exp(u_{jkt})} \quad (2.59)$$

where

$P(j, k: JK_t)$  = the probability that an individual  $t$  will choose destination  $j$  and mode  $k$ , from  $JK_t$ , the full range of opportunities available to the individual.

$u_{jkt}$  = the utility that individual  $t$  derives from going to  $j$  by  $k$  and is a function of the variables that describe destinations, modes and the individual.

Anas (1982) has also described how this type of model has been used to estimate housing demand, and Domencich and McFadden (1975) use a multinomial logit model to estimate urban travel demand. Retailing applications have also been undertaken. For example, McCarthy (1980) used this method to determine which variables influence the attractiveness of shopping centres.

### 2.8.5 Log-linear models

Willekens (1983) developed a log-linear model of migration flows. This is a statistical approach to the formulation of spatial interaction models and focuses on the pattern of association among cross-classified variables that influence spatial interaction.

Willekens (1983) argues that log-linear modelling has great potential for the study of spatial interaction, for the following reasons.

(1) Log-linear modelling is formally equivalent to traditional models of spatial interaction.

(2) Log-linear modelling enhances the structural analysis of tables on spatial interaction and helps focus on the whole data structure as opposed to focusing on the individual elements.

(3) Log-linear modelling clarifies and simplifies the estimation of spatial interaction flows

Yano (1993) states the log-linear model in its multiplicative form as follows

$$T_{ij} = WW_i^A W_j^B W_{ij}^{AB} \quad (2.60)$$

where

$$W = \left[ \prod_{ij} T_{ij} \right]^{1/RC} = \quad (2.61)$$

$$W_i^A = \left( \frac{1}{W} \right) \left[ \prod_j T_{ij} \right]^{1/C} = \quad (2.62)$$

$$W_j^B = \left( \frac{1}{W} \right) \left[ \prod_i T_{ij} \right]^{1/R} = \quad (2.63)$$

$$W_{ij}^{AB} = \frac{T_{ij}}{(WW_i^A W_j^B)} = \quad (2.64)$$

$W$  represents the overall mean effect, this takes the value of the geometric mean of all  $T_{ij}$ s.  $W_i^A$  is the main effect of row  $i$  and represents the effect of the emissiveness of origin  $i$ . The main effect of row  $j$ , the influence of the attractiveness of destination  $j$  is given by  $W_j^B$ . The relationship between origin  $i$  and destination  $j$  is given by the interaction effect  $W_{ij}^{AB}$ . Thus it can be seen that the log-linear model is similar in structure to the entropy maximising spatial interaction model in that it consists of a proportional factor, variables that represent the influence of origins and destinations and a variable that accounts for the interaction between origin  $i$  and destination  $j$ .

### 2.8.6 Alonso's theory of movement

Alonso first proposed his general theory of movement in 1973 and it is described in Alonso (1978). The theory attempts to build a model that reproduces flows between zones in a closed system using not only characteristics of origins and destinations and the impedance between them, but also impacts on movements that are exerted by the system itself.

The movements ( $M_i$ ) that originate from origin zone  $i$  are stated to depend on the characteristics of the zone and its population

$$M_i = v_i D_i^{\alpha_i} \quad (2.65)$$

where

$v_i$  = function of the characteristics of origin  $i$  and/or its population

$D_i$  = the pull-in exerted by the system at origin  $i$

$\alpha_i$  = movement response from origin  $i$  to its relation to other zones in the system

Movements arriving at destinations ( $M_j$ ) are given by

$$M_{.j} = W_j a_j^{\beta_j} \quad (2.66)$$

where

$W_j$  = function of the characteristics of destination  $j$

$a_j$  = the push-out exerted by the system at destination  $j$

$\beta_j$  = rate of arrival at destination  $j$

The level of movements from origin  $i$  to destination  $j$ ,  $M_{ij}$ , are based on the following suppositions. The rate of movement from origin  $i$  to destination  $j$  will be

(1) proportionate to the attractiveness of destination  $j$ ,  $W_j$ .

(2) proportionate to the probability of a potential arrivals entry into destination  $j$ ,  $a_j$  with the exponent  $\beta_j$ .

(3) proportionate to any special relation that may exist between origin  $i$  and destination  $j$ , for example ease of movement. This factor is given by  $t_{ij}$ .

(4) inversely proportionate to the total opportunities or alternative attractions available to origin zone  $i$ .

This gives the following equation

$$\frac{M_{i.}}{M_{.j}} = W_j a_j^{\beta_j} t_{ij} D_i^{\alpha_i} \quad (2.67)$$

By combining equations (2.65) and (2.67) and rearranging, the following model of flows is produced

$$M_{ij} = v_i D_i^{\alpha_i} W_j a_j^{\beta_j} t_{ij} \quad (2.68)$$

Ledent (1981) has shown that this model (produced from Alonso's theory of movement) is equivalent to the family of spatial interaction models formulated by Wilson which

were described in Section 2.4. Fotheringham and Dignan (1984) also state that the generality that is apparent in the Alonso model due to the presence of the two parameters  $\alpha$  and  $\beta$  that are associated with the variables  $D_i$  and  $a_j$ , means that an infinite number of gravity models can be derived by varying the values of these parameters. They also state that the addition of parameters on the  $D_i$  and  $a_j$  variables allows various relationships between interaction and the relative location of origins and destinations to be modelled. For example, Fotheringham's competing destinations model as described in Section 2.8.2 uses the  $\beta$  parameter on the  $a_j$  variable to introduce the accessibility variable.

### 2.8.7 Poisson regression models

Flowerdew and Aitkin (1982) state that if in the interaction variable is viewed as the result of a discrete probability process then Poisson regression methods are appropriate, because, as is argued by Yano (1993), if there is a constant probability that any person will move to destination  $j$ , and if movements of people are independent, then if the population of the origin zone is large, the number of people moving from  $i$  to  $j$  will have a Poisson distribution. He states that if the flow does conform to a Poisson distribution then the probability of a flow occurring between  $i$  and  $j$ ,  $p(t_{ij})$  is given by

$$p(t_{ij}) = \frac{\exp^{-\sigma_{ij}} \sigma_{ij}^{t_{ij}}}{t_{ij}!} \quad (2.69)$$

If it assumed that the parameter  $\sigma_{ij}$  is logarithmically linked to a linear combination of the logged explanatory variables the following equation for the Poisson regression model is formulated:

$$\sigma_{ij} = \exp(\beta_0 + \beta_1 \ln U_i + \beta_2 \ln V_j + \beta_3 \ln d_{ij}) \quad (2.70)$$

where

$U_i$  = origin population

$V_j$  = destination population

$d_{ij}$  = distance between origin  $i$  and destination  $j$

$\beta_{0,\dots,3}$  = the parameters attached to the above stated variables

This formulation of the Poisson regression model is equivalent to the model derived by Flowerdew and Aitkin (1982).

Several studies have been undertaken using the Poisson regression model. Guy (1991) undertook a comparison of Poisson models with gravity models in order to determine if the Poisson method produced similar parameter estimates to those found in gravity models and to see if the Poisson models produced an acceptable level of goodness of fit. Guy concluded that the Poisson model produced consistent parameter estimates and that the method allowed complex models to be calibrated more easily.

Flowerdew and Aitkin (1982) compared the Poisson regression model with the log-normal model. The authors criticise the log-normal model and argue that the assumption that values of flows are normally distributed is invalid as this implies that flow values could be below zero. A further criticism arises out of the logarithmic transformation when some flow values are zero. The logarithm of zero cannot be calculated and so when using the log-normal model it is necessary to use a small positive number in place of zero. This can have a large impact on the results produced by the model and means that such a model may be inappropriate for analysis of flows such as migration where a considerable amount of flows could be zero. Flowerdew and Aitkin (1982) argue that the problems associated with the log-normal model arise from the assumption that flows are normally distributed and that these problems can be overcome if it is assumed that flows have a Poisson distribution and a Poisson regression model as described above is used. If it is assumed that the flow values have a Poisson distribution then, unlike the log-normal model, the dependent variable is always positive, and where the mean is a low value a value of zero is not unlikely to occur. Therefore Flowerdew and Aitkin (1982) argue that the Poisson regression model can help solve the problems associated with the log-normal model.

The results of the comparison of the log-normal and Poisson regression models undertaken by Flowerdew and Aitkin (1982) were that the Poisson model provided parameter estimates different from those provided by the log-normal model and closer to parameter estimates predicted by models based on the gravity analogy. However, the results show that for the Poisson regression model the deviance is still too great to state that the model provides an accurate description of the data. Flowerdew and Aitkin (1982) propose two reasons why this might occur when fitting a gravity model to migration data using a Poisson regression model. Firstly, they state that the populations of the origins and destinations were the only independent variables used in their analysis and that there are other variables that will be important in determining migration flows. Secondly, the Poisson method assumes that trips are independent, but this may not be true in reality because migration flows from the same household are not independent of each other. Flowerdew and Boyle (1995) have attempted to alleviate this second problem by developing a Poisson model that uses the probability of households moving as opposed to individuals.

## 2.9 INTEGRATION OF SPATIAL INTERACTION MODELS

Yano (1993) has argued that a majority of the different types of spatial interaction models *i.e.* the entropy maximising models, Poisson regression gravity models, log-linear models and multinomial logit models, are equivalent and produce the same parameter estimates when calibrated. Yano argues that all of the above spatial interaction models can be specified by Poisson regression and are differentiated by the variables included in them. He has undertaken a study of migration flows in Japan in order to compare spatial interaction models with an integrated Poisson model that includes the intervening opportunities effect, the hierarchical effect and the competing effect.

The competing effect, the intervening opportunities effect and the hierarchical effect are origin-destination pair specific effects which Yano argues can be added to spatial interaction models to improve their specification. This is because these effects are not included in the conventional spatial interaction model. The competing effect shows the relationship of other destinations to the origin-destination pair and is measured by the accessibility of  $j$  to all other destinations as perceived by residents of origin  $i$ . This is

equivalent to Fotheringham's idea of competing destinations. The intervening opportunities effect indicates the relationship of other destinations to the origin and is measured by the accessibility of  $i$  to all other alternative destinations except  $i$  and  $j$  as perceived by residents of origin  $i$ . The hierarchical effect represents the hierarchical relation of the origins or the destinations and is incorporated in the competing and intervening opportunities effects. The hierarchical competing effect measures the accessibility of the destination to all other equal or higher order destinations and the hierarchical intervening opportunities effect measures the accessibility of origin  $i$  to all other destinations that are of an equal or higher order than destination  $j$ .

Yano (1993) tested these additional variables in a study of migration in Japan using a doubly constrained spatial interaction model and concluded that the addition of the new origin-destination pair specific did lead to improvements in the goodness of fit of the model.

## 2.10 CONCLUSIONS

This review has shown how spatial interaction models have developed from the first attempts to delimit trade areas, through the first simple attempts to model spatial interaction through the formulation of a gravity model analogous with Newton's law of Gravitation, into a sophisticated method of modelling spatial interaction.

The development of alternative methods of deriving the spatial interaction model, such as the entropy maximisation, statistical averaging approach, has made spatial interaction models more robust and given them an increased theoretical meaning than when the models were based on an analogy with Newton's law of Gravitation.

It has also been seen that there have been several attempts to alter and extend the spatial interaction model as it was developed by Wilson. For instance, Fotheringham's competing destinations model with its accessibility variable and Gonclaves and Ulysea-Neto's hybrid distance and intervening opportunities model. All such developments have occurred in order to enhance the understanding of the processes involved in spatial interaction which will enable interactions to be predicted more accurately.

There are also alternative forms of spatial interaction models to the Wilson models. For example, Stouffer's intervening opportunities model, the multinomial logit model, the log-linear model, Poisson regression models, the competing destinations and the multi-stage migration model. It can be seen that these models are all attempting to model the same process *i.e.* spatial interaction. It has also been argued by Yano (1991, 1993), that all the above forms of spatial interaction models are equivalent and can be combined into a generalised linear model that can be represented by a Poisson regression model. The model developed by Yano (1993) includes both the effect of competing destinations and the intervening opportunities effect and therefore attempts to integrate different models in order to increase the predictive capacity of spatial interaction models.

The insights gained through the production of this review will be used in the next chapter in order to tease out operational problems that seem to be apparent in spatial interaction models at present and attempt to overcome some of these problems in order to improve the performance of spatial interaction models.

## SOURCES OF ERROR IN SPATIAL INTERACTION MODELS

### 3.1 INTRODUCTION

The focus of this thesis is to attempt to improve spatial interaction models developed for business applications. These models will be introduced in the next chapter. There are various possible sources of error in spatial interaction models that can be investigated in order to improve the performance of such models. This chapter identifies such possible error sources apparent in spatial interaction modelling of retail services.

Section 2.2.1 showed that there were several components to the interaction system and it will be seen throughout this chapter that each component as well as the overall system and modelling procedure are subject to errors.

### 3.2 GENERAL SOURCES OF ERROR IN SPATIAL INTERACTION MODELS

#### *3.2.1 Data problems*

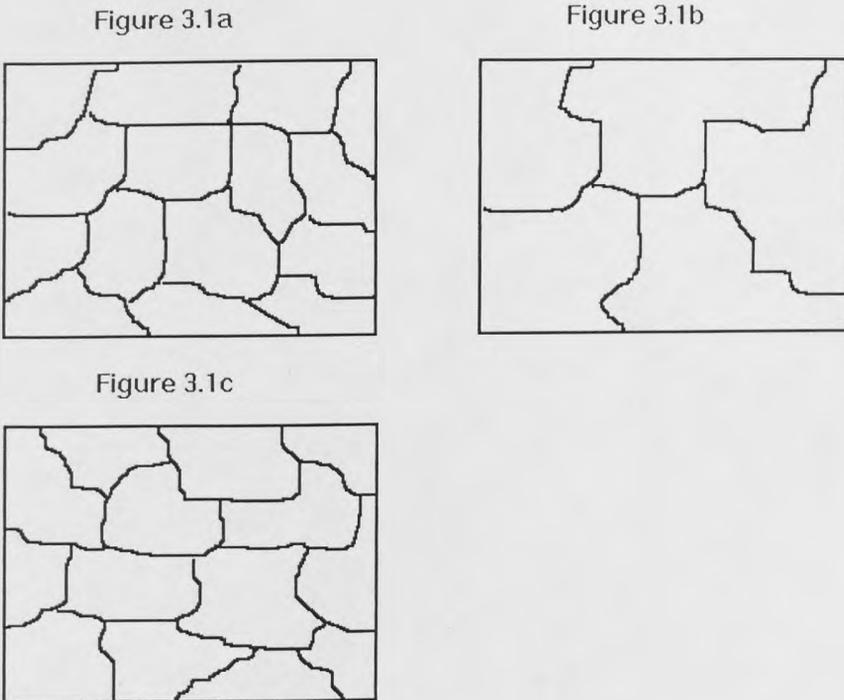
Spatial interaction models can suffer from a lack of available data, as is stated by Openshaw (1976). For example, the spatial interaction model for retailing requires information concerning shopping trips and cashflow between zones but there is no generally available source of such information. The lack of this information makes the estimation of parameters more difficult and is also indicative of another criticism of spatial interaction models as stated by Thomas and Huggett (1980): there are problems with the measurement of independent variables within the spatial interaction model. For example, there are no data concerning demand for certain products and therefore demand must be estimated. This means that the demand component of the model may not be totally accurate and this can be detrimental to model performance. Such independent variables can be estimated in different ways, and each method will produce

a different model performance. Therefore it is necessary to investigate which method is the best for estimating the independent variables.

### *3.2.2 Zoning problems*

A further problem involved in spatial interaction modelling concerns the choice of zones into which the study area is divided. Thomas and Huggett (1980) state that predictions of spatial interactions vary with the way in which the study area is divided into zones. When modelling spatial interactions, it is necessary to decide on the size and configuration of the areal units to be used in the model. This choice will affect model performance due to the existence of the modifiable areal unit problem (Openshaw, 1984; Fotheringham and Wong, 1991). The modifiable areal unit problem describes the incidence of different model results occurring for alternative spatial configurations of zones and also at different levels of spatial aggregation. The modifiable areal unit problem is split into two parts: the scale effect and the zonation effect. The scale problem is described by Batty and Sikdar (1982a) as the variation in model predictions as the level of aggregation changes. The zonation problem is defined in Boyle and Shen (1995) as occurring when the study area is divided into a number of zones in different ways. These problems are illustrated in Figure 3.1 below. Comparison of Figure 3.1a and Figure 3.1b shows the scale problem, with Figure 3.1b being an aggregation of zones from Figure 3.1a. Figure 3.1a and Figure 3.1c represent the zonation problem where Figure 3.1c shows an alternative zonation system to Figure 3.1a but on the same scale.

Figure 3.1: The modifiable areal unit problem



The scale problem can cause errors in a spatial interaction model because as the size of zones increases the modelling of intra-zonal trips becomes more inaccurate. This is because the larger the zone the more inaccurate the travel cost function will be for intra-zonal trips. The calculation of the travel cost function will also become more inaccurate for inter-zonal trips as the size of zones increases.

Thus it is necessary to choose a level of aggregation that is appropriate for the type of trips being made in the system being modelled. Therefore the size of zones appropriate to the model will depend on the type of good for which spatial interaction is being modelled. Ghosh and McLafferty (1987) state that generally, zones should be smaller for goods which have a more compact trading area. Also zones should ideally be relatively small in order for zones to be as homogeneous as possible concerning the population resident in the zone. However, the smaller the zone the lower the sample size will be of any survey data that is utilised and this could affect model performance. Therefore a compromise must be reached which will be dependent on the situation being modelled.

In the retail case there is also a problem in the choice of areal units because most of the information used to calculate the independent variables is based on the geography of the census which is incompatible with postal geography which is the basis for commerce. Therefore there is the problem of converting census geography to postal geography which can cause errors in the model.

### ***3.2.3 Level of disaggregation***

There is also the issue of what level of disaggregation to use in the model. A retail model could be disaggregated in many ways *i.e.* by person type, good type or transport mode. More disaggregate models provide a better representation of the real world, but there is an increased data requirement and it is more difficult to model the lower levels of interaction that are apparent in disaggregate models. An additional problem involved with disaggregation is mentioned by Openshaw (1974) who states that disaggregation increases the relative importance of sampling errors in any survey data used in the model due to the decrease in sample size. Therefore a decision must be made concerning the level of disaggregation to be used in the model.

## **3.3 POSSIBLE ERRORS IN DEMAND ESTIMATION**

### ***3.3.1 Data problems***

The calculation of demand for retail models is one area where the problem of estimating independent variables is apparent. The estimation of demand can cause errors in models because they are based on sample surveys, and therefore partial data which is subject to sampling error.

A major problem concerning demand estimation is deciding which method of calculating demand is the most appropriate. Alternative demand estimates need to be validated but this is problematic because of the lack of data against which estimates can be validated. Therefore survey data has to be used in validation which means that two estimates are being compared and therefore it is not clear if poor correlations are due to the demand estimates or the survey data.

### ***3.3.2 Variable definition***

There is also a problem in the estimation of demand of choosing which variables to include in the calculation. Ideally all variables believed to influence demand for the good being modelled should be included but in some cases this can prove difficult. For example, it would be expected that income would be an important factor determining levels of demand but this information is not available in the census and is therefore difficult to include in the demand model. The failure to include determining variables such as this could cause demand estimates to be sub optimal.

A further problem concerning the estimation of demand is the choice of the level at which demand should be modelled *i.e.* at the individual or household level. In some cases this decision will be clear cut, but for some goods this decision is more difficult and an incorrect choice could lead to model error.

### ***3.3.3 Zone specific error sources***

Unique characteristics of postal zones for which demand is being modelled could also affect model performance. For example, if the area is a holiday resort or a retirement area this could affect demand but may not be reflected in the demand model and will therefore cause demand estimates to be incorrect. If an area is a holiday resort then demand could be higher than expected because visitors bring their demand with them, therefore increasing demand in resort areas. Therefore demand in holiday resort areas will be dependent on the number of visitors as well as the number of residents. It should also be noted that this resort effect will follow a seasonal pattern causing demand to vary according to the time of year.

### ***3.3.4 Workplace based demand***

There is also the problem of how to incorporate demand based on the workplace. Demand is usually calculated as occurring from the zone of residence. This will cause errors in demand calculation because demand will, in some cases, be recorded at the incorrect location. This occurs because a certain amount of demand will occur from the workplace. This could have the effect of depressing demand estimates in areas such as

city centres where a lot of people work but which is usually a long distance from people's residences. The problem of accounting for work based flows has often been tackled through the addition of an extra variable to the spatial interaction model that makes city centres (where most workers are located) more attractive.

Problems also arise due to the difficulty in quantifying the level of home shopping that occurs. If home shopping is left out of the model then the amount of expenditure flowing to destinations will be overestimated.

### *3.3.5 Postal geography changes*

Changes in postal geography can also affect demand estimates made between censuses because census data for postal areas that have changed will be incorrect. Postal areas that have undergone change in recent years include Manchester, Derby and Aberdeen.

## **3.4 SUPPLY SIDE ERRORS FOR RETAIL MODELS**

### *3.4.1 Data problems*

Data are not often available on store or zonal revenues. Therefore it is necessary to calculate a proxy variable to represent the attractiveness of supply points.

Errors can also arise on the supply side due to inadequate data availability concerning actual supply points. There is no comprehensive list of all retail facilities and therefore a combination of retail directories, telephone directories and market surveys must be used to compile a list of available retail outlets. If supply points are omitted, or the location of supply points is incorrect, this will cause errors in the spatial interaction model. This is a particular problem concerning competitors of the organisation for which spatial interaction is being modelled.

### *3.4.2 Measuring attractiveness*

It was seen in Section 2.4.2 that the production constrained spatial interaction model is used to estimate revenues based partly on a proxy supply side (variable  $W_j$ ). This variable

represents the attractiveness of destinations. Many factors can be seen to affect the attractiveness of retail stores and it is therefore necessary to devise the best method of calculating a destination's attractiveness. A consumer's choice of destination will be dependent on both individual store characteristics and the characteristics of the centres in which they are located and therefore attractiveness factors have to take this in to account.)

Vickerman (1974) states that size and turnover, as used to calculate attractiveness in the early retail models of Huff (1964) and Lakshaman and Hansen (1965) do not measure the real elements of attraction because other factors also influence destination choice. However, store size or centre size are often used as the attractiveness term in spatial interaction models and therefore this could cause errors through the imperfect calculation of the attractiveness of shopping centres.)

Store choice by consumers is dependent on more than store size and ideally all store and centre characteristics that may have an influence on shopping behaviour should be included in the attractiveness calculation. A whole range of both objective and subjective measures can be used to calculate the attractiveness of destinations. Objective factors such as store size, parking facilities and relative prices are useful. Subjective measures such as the consumers' perception of brand image and customer service should also be used to measure attractiveness. Fotheringham and Trew (1993) used a household survey and a logit model to come to the conclusion that chain image is at least as important as store size and competition in determining destination choice.)

McCarthy (1980) undertook a multinomial logit analysis which indicated that generalised shopping centre attributes as derived from attitudinal information are significant) in determining destination choice behaviour. (The generalised attributes found to be significant included generalised shopping area attractiveness and generalised shopping area mobility.) Oppewal *et al.* (1997) undertook a study using conjoint analysis and multinomial logit models in order to determine centre characteristic factors important in influencing centre choice for food retailing and for clothes and shoes. They discovered that although the size of the retail destination was the most important factor determining centre choice, other variables such as centre location (convenience) and physical appearance and layout of centres were also influential.) The available selection

of competing stores and the mix of types of store in the centre were also significant determinants of a centre's attractiveness to consumers.

It can therefore be seen that simply using store size to measure destination attractiveness may cause errors in the model because this attractiveness measure does not reflect all of the factors that influence destination choice in the real world. However the inclusion of all the variables that affect destination choice is difficult due to the inadequacy of data available concerning supply of the products of both the organisation on which the model is based and on the competitors of the organisation which must also be included in the estimation of supply. It is especially difficult to estimate the subjective measures of attractiveness as this would involve the use of consumer survey information. Therefore it is necessary to decide which outlet characteristics to include in the attractiveness measure. Unique site attractiveness of particular shopping facilities are also problematic because they cannot be included in the model. This omission will therefore be detrimental to model performance.

### 3.4.3 *The alpha parameter*

A further issue concerning the attractiveness factor is whether to include a parameter on the attractiveness term and therefore raise the attractiveness function to some power *i.e.*  $W_j^\alpha$ . The  $\alpha$  parameter is often included to take account of economies of scale in destination choice. If the  $\alpha$  parameter is given a value of greater than one then this reflects consumers perceiving the benefits of larger centres, *i.e.* as the size of a centre increases, interaction to that centre increases disproportionately due to benefits such as better parking facilities and opportunities for comparison shopping.

## 3.5 ERRORS IN THE CALCULATION OF THE IMPEDANCE FUNCTION

The interaction impedance variable links demand and supply in the spatial interaction model. The interaction variable is a function of the accessibility of the destination zones in the system to the origin zones and takes the form of a function of the impedance between the origin and the destination. This measure of impedance represents whatever factor affects the level of interaction between origins and destinations due to their separation in space. There are several alternative operational measures of impedance,

which are listed below, that can be used to represent the separation of origins and destinations in the spatial interaction model.

- 1) Straight line (Euclidean) distance
- 2) Distance along the transport network
- 3) Travel time
- 4) Cost, both in terms of time and money

### ***3.5.1 Problems associated with measuring impedance***

Sources of errors are apparent in the calculation of impedance for use in the spatial interaction model because, as was seen above, the separation between origins and destinations can be measured in a variety of ways. The easiest method of measuring impedance between zones is to use the straight line distance between the zones. This is simple to calculate but will be a source of error because straight lines do not represent actual travel routes because consumers cannot travel from home to shop in a straight line and have to use road networks and public transport networks. It is this consumer perception of distance that will determine the level of interaction between zones. Therefore it is necessary to develop a measure of impedance that better reflects distance as it is experienced in the real world in order to decrease the level of error apparent in the spatial interaction model. Wilson (1971, 1974) has argued for the use of a composite travel cost function containing several elements of travel cost such as money cost, travel time and convenience. A full representation of travel cost would also involve information concerning other barriers to travel such as information, psychological, social and historical barriers. However, Wilson has also pointed out that there is a problem in the estimation of such a function due to the difficulty of estimating the weight each factor takes in the decision making process. Therefore travel time is usually used in the distance deterrence function in retail models.

### ***3.5.2 Allocation of zone centroids***

The point between which impedance between origins and destinations is measured will also influence model performance. This is because the actual centre of demand may be in a different location to the geographic centroid of the origin zone and therefore

measures of impedance calculated for the travel cost function will be inaccurate. The centroid of a zone is a measure of the central tendency in a spatial distribution and can be calculated in several ways. The simplest calculation of the centroid of a zone is to find the centre of gravity based on the area of the zone which is undertaken by finding the mean co-ordinates of the zone. This is calculated as follows, where  $AC$  is the area centroid and  $X_k$  and  $Y_k$  are the co-ordinates of subzones  $k$  in postal district  $i$ .

$$AC_i = \frac{\sum_k X_k}{n}, \frac{\sum_k Y_k}{n} \quad (3.1)$$

An alternative calculation of the centroid is the population weighted mean centre. In this case points in the zone are given weights based on the population at that point and these weights influence the location of the population weighted centroid. The population weighted centroid is calculated in the following way, where  $PWC$  is the population weighted centroid,  $X$  and  $Y$  are the co-ordinates of the centroids of subzones  $k$  in postal district  $i$ , and  $W$  represents the weight assigned to each point.

$$PWC_i = \frac{\sum_k W_k X_k}{\sum_k W_k}, \frac{\sum_k W_k Y_k}{\sum_k W_k} \quad (3.2)$$

An additional alternative method of defining the centroid of a zone is to locate the centroid at the location of the largest concentration of population.

The definition of the centroid of the zone is an important issue because it affects the calculation of the distance between zones. If the centroid is calculated inaccurately then the distance matrix used in the calculation of interaction will be incorrect and cause the predicted levels of interaction to be wrong.

Boyle & Flowerdew (1997) argue that the use of the population weighted centroid introduces bias into impedance calculations because it is not representative of the separation between zones. This is related to the modifiable areal unit problem in that the effect of the use of zone centroids to calculate distances will be dependent on the size of zones. The problem of impedance calculation is more acute for larger zones and

in such cases, on average, the actual distances travelled for migration purposes will be less than is calculated using population weighted centroids. It is also stated that the spatial structure of the zones in the region for which spatial interaction is being modelled will affect the level of bias that is introduced into the model through the use of population weighted centroids. This is because for zones with long shared boundaries more short trip migrants will cross the boundary between zones. Boyle & Flowerdew (1997) argue for the use of a migration weighted impedance calculation similar to that described by Webber (1980). In this case the zones for which migration is being modelled are split into smaller zones and the flows are estimated between these smaller units. The migration weighted distance between origin  $i$  and destination  $j$  is the average of the distance between each smaller zone in  $i$  to each smaller zone in  $j$ , weighted by the number of migrants in each flow, and is given as

$$MWD_{ij} = \frac{\left( \sum_{a \in i} \sum_{b \in j} M_{ab} d_{ab} \right)}{M_{ij}} \quad (3.3)$$

where

- $a$  = a zone within origin  $i$
- $b$  = a zone within destination  $j$
- $M_{ab}$  = estimated migration between  $a$  and  $b$
- $d_{ab}$  = distance between  $a$  and  $b$
- $M_{ij}$  = migration between origin  $i$  and destination  $j$

The process of estimating migration weighted distances between  $i$  and  $j$  is an iterative process because the distances calculated in equation (3.3) are used to produce a new estimate of  $M_{ij}$  which is subsequently used to calculate a new value of the migration weighted distance. This process continues until the predictions of  $M_{ij}$  become stable.

However, in the case of retailing this problem may not be as serious as has been encountered for migration models. This is because in retail models the destination is a point location and not a zone and therefore there will be no bias on the destination side of the distance calculation. For origins, in the retail model the size of origins tends to be

significantly smaller, such as the use of postal sectors or postal districts and therefore the bias introduced by the use of population weighted centroids will be reduced.

### 3.5.3 *Formulating the distance deterrence term*

Once the level of impedance between origins and destinations has been estimated there is also the issue of the form of the distance deterrence term. In early gravity models the Newtonian distance function  $d_{ij}^2$  was used, but there is no theoretical justification for expecting interaction to vary directly with the square of the distance between zones. This leads to the distance function being raised to some other power, *i.e.*  $d_{ij}^\beta$ . The value of this power is determined from empirical data through the calibration process which will be described in Section 3.6.

Alternative forms of the distance function have also been suggested, for example Wilson's entropy maximisation models produce a negative exponential distance function.

Foot (1981, p77) argues that the negative exponential distance function reproduces trip patterns better than an inverse power function. He states that the negative exponential function is more appropriate in a Western developed society where residents are relatively mobile because this function shows a less sharp distance decay. It has also been argued by Wilson (1970), that the negative exponential function is better on theoretical grounds due to its appearance in the entropy maximising derivation of the model. However, it is possible to derive other distance functions through entropy maximisation depending on how it is believed costs are related to distance. McCarthy (1980), in a study of the influence of generalised attributes on trip making behaviour, found the negative exponential function was a significant variable in explaining destination choice. However, Wilson (1971) has pointed out that for long trips people are likely to perceive travel cost in a logarithmic way. For instance, for longer trips an increase in travel cost due to increased trip length is perceived as less of an increase in travel cost than it would be on a short trip and this is reflected in the power distance function as opposed to the negative exponential distance function. Wilson (1971) has therefore argued that for spatial systems that consist of mainly short trips, the negative exponential function will fit well, but that for systems containing a mixture of trip

lengths but with a large number of long trip lengths, the power function may be more appropriate.

Taylor (1975) identified five possible alternative distance functions for use in spatial interaction models. These will be discussed in Chapter 7.

It is therefore important to investigate which impedance functions are appropriate for different goods and the most appropriate function should be found by comparison to empirical data. However, in the case of retailing there is a lack of empirical flow data and this could lead to an inappropriate distance function being included in the model.

#### *3.5.4 The distance decay parameter*

A major issue in spatial interaction modelling is the derivation of the correct value for the distance decay parameter  $\beta$ . The value of  $\beta$  will vary depending on the type of good for which spatial interaction is being modelled. A high value of  $\beta$  indicates a low average trip cost and therefore means that distances travelled to purchase that good will be low. If the value of  $\beta$  is low, the average trip cost will be higher indicating that longer trips will be undertaken to purchase goods. The value of  $\beta$  is determined through the process of calibration, and as there is a shortage of interaction data for retailing, this could be a potential source of error in retailing spatial interaction models. Calibration involves attempting to minimise the difference between observed interactions and predicted levels of interaction. Calibration is an iterative procedure and involves successive model runs with different values of  $\beta$  until the goodness of fit function is optimised and the best parameter estimates are achieved. Calibration will be discussed in more detail in Section 3.6.

There is also the possibility of using zone specific distance decay parameters. Such parameters can either be origin or destination specific. Stillwell (1978) has stated that a single distance decay parameter for a system will hide spatial variation in propensity to travel. Therefore origin or destination specific parameters are more likely to allow trips to be modelled more accurately.

Fotheringham (1981) also uses origin specific beta parameters to identify a relationship between spatial structure and estimates of origin specific distance decay parameters. It is argued that the observed relationship between accessibility of origins and distance decay parameters (less accessible origins have more negative distance decay parameters and more accessible origins have less negative parameter estimates) is evidence of a relationship between spatial structure and parameter estimates. Fotheringham cites earlier studies such as Chisholm and Sullivan (1973) and Gould (1975) to provide evidence for the relationship between the accessibility of origins and the value of distance decay parameters. Chisholm and Sullivan undertook a study of flows of British freight and found that the variable  $A_i$  in the spatial interaction model accounted for 48% of the variation in  $\beta_i$ . Gould stated that the square root of the distance from an origin to a point of minimum aggregate travel explained 64% of the variation in  $\beta_i$ . Fotheringham also uses distance decay parameter estimates from a spatial interaction model of airline passengers travelling between cities in the United States to show that there is a wide range of different parameter values between origins (from 0.5 to -2.6). It is argued that this indicates a large variation in interaction behaviour in the United States. The variation in parameter values is not restricted to airline passengers in the United States, Chisholm and Sullivan (1973) found variation in distance decay parameter estimates between origins from -1.3 to -4.8, Gould (1975) from -0.2 to -1.0 and Stillwell (1978) from -0.8 to -2.9. It is therefore clear that different origins produce different interaction behaviour and therefore origin specific distance decay parameters are required in order to account for such differences in the spatial interaction model.

### ***3.5.5 Alternative accessibility functions***

It is possible that alternative accessibility functions to the purely distance based function proposed in the Wilson family of spatial interaction models could be more appropriate and could therefore reproduce trip patterns more accurately in some situations. It has already been seen in Chapter 2, (Sections 2.8.1 and 2.8.2), that Fotheringham's competing destinations model and the intervening opportunities model both propose alternative accessibility functions. The competing destinations model adds an additional accessibility variable to the production constrained spatial interaction model, whereas Stouffer's formulation of the intervening opportunities model replaces the distance accessibility function with an alternative based on intervening opportunities. Gonclaves

and Ulyssea-Neto (1993) combined the ideas of distance based accessibility functions with an intervening opportunities variable to produce a hybrid model. Therefore it is necessary to determine which method of including accessibility in the model is the most appropriate. This could vary for different good types and different trip types.

Pooler (1992) has also discussed the roles of spatial uncertainty and spatial dominance on spatial competition between destinations. Pooler argues that destinations compete for potential flows and that destinations have different levels of spatial dominance dependent on their relative locations and attractiveness. This idea is similar to that proposed by Fotheringham concerning competing destinations. The level of spatial dominance,  $\rho_{ij}$ , is given by

$$\rho_{ij} = \frac{W_j f(c_{ij})}{\sum_m W_j f(c_{im})} \quad (3.4)$$

where  $m$  is equal to alternative destinations. Pooler states that this variety in spatial dominance leads to differing levels of spatial uncertainty at origins. People who are in relatively close proximity to a number of alternative destinations, all of which exert a strong spatial dominance, will suffer from higher spatial uncertainty than people in origins that are influenced by less destinations. Spatial uncertainty,  $H_i$  is stated as

$$H_i = \sum_j \rho_{ij} \ln \rho_{ij} \quad (3.5)$$

It is argued that for those origins with a high level of spatial uncertainty it is more difficult to predict model interactions, and therefore levels of spatial uncertainty within the system being modelled will influence model error levels.

A further problem associated with the relative spatial location of alternative destinations is the possibility for multi-purpose and multi-stop trips and how these should be included in models of retail flows. Dellaert *et al.* (1998) observe that due to changes in society, people now have less time to spend on activities such as shopping and therefore there is an increase in the incidence of multi-purpose and multi-stop trips. They therefore argue that a standard production constrained spatial interaction model ignores

spatial agglomeration effects that could be apparent for centres that are in close proximity to each other. However, the competing destinations model with a parameter value on the accessibility variable of greater than one, does account for agglomeration effects within spatial systems and could therefore help to alleviate this source of model error in predicting interaction.

### 3.6 CALIBRATION OF SPATIAL INTERACTION MODELS

The process of calibration involves the comparison of predicted flows to a set of observed flows in order to discover the value of model parameters which provide the best fit between predicted and observed flows.

#### 3.6.1 Problems associated with calibration

##### 3.6.1.1 Choice of calibration method

There are several alternative methods of calibrating spatial interaction models. However, Baxter (1982) undertook a comparison of several alternative parameter estimation methods including maximum likelihood estimation using both the Poisson assumption and the multinomial assumption, entropy maximisation, weighted least squares regression and logit regression analysis. He concluded that

“if the spatial interaction model, including its stochastic properties, is correctly specified the choice among the available methods of estimation is not critical.”

(Baxter, 1982, p271)

Therefore, the calibration procedure used by GMAP will be used to calibrate the models produced in this thesis. The method used is maximum likelihood, which involves the alteration of parameter estimates in order to equate observed average drive time with predicted average drive time. Thus, the objective of this process is to minimise

$$\frac{\sum_j T_{ij}^{(pred)} c_{ij}}{\sum_j T_{ij}^{(pred)}} = \frac{\sum_j T_{ij}^{(obs)} c_{ij}}{\sum_j T_{ij}^{(obs)}} \quad (3.6)$$

### 3.6.1.2 Data problems

Problems are apparent in the fitting of retail models due to lack of data against which to calibrate. In the example of the retail model there is usually no comprehensive flow data available and therefore survey data must be used which will introduce error due to bias and sampling error. A further problem involved with calibration is the choice of calibration statistic. This is because parameter values obtained through calibration are only optimal in relation to that particular calibration statistic. Thus, care must be taken to choose the most appropriate goodness of fit statistic for the purpose of calibration. Alternative goodness of fit statistics will be discussed in the next section.

## 3.7 EVALUATION OF THE GOODNESS OF FIT OF SPATIAL INTERACTION MODELS

### 3.7.1 Alternative goodness of fit statistics

There are a multitude of possible goodness of fit statistics that can be used to evaluate the performance of spatial interaction models. Several possible alternative statistics will be described along with an evaluation of their usefulness for evaluating performance.

(1) The sum of absolute deviations of the model predictions from the observed values. This method compares the flows between zones for observed and predicted interactions.  $f$  is equal to the error, where  $T_{ij}$  is the observed interaction and  $\hat{T}_{ij}$  is the predicted level of interaction.

$$f = \sum_i \sum_j |T_{ij} - \hat{T}_{ij}| \quad (3.7)$$

(2) The sum of squared deviations between model predictions and observed values. This measure is calculated as (1) but using the square of the difference between observed and predicted interactions, and is given by:

$$f = \sum_i \sum_j (T_{ij} - \hat{T}_{ij})^2 \quad (3.8)$$

(3) Correlation coefficient. This is a measure of the covariance between two variables *i.e.* observed and predicted interaction and is given by  $r^2$ .

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}} \quad (3.9)$$

(4) Mean trip cost. This statistic involves the comparison of observed and predicted mean interaction cost.

$$\bar{c}_{ij} = \frac{\sum_{ij} c_{ij}}{n} \quad (3.10)$$

(5) Spearman's rank correlation coefficient. This measure involves the comparison of two variables (observed and predicted trips) that are both ranked in order to test if the rankings are the same. Spearman's rank correlation coefficient is given by  $r_s$  and is calculated as follows, where  $N$  is equal to the number of trip pairs and  $d_k$  is the difference between the ranks of the observed and predicted trips.

$$r_s = \frac{6 \sum d_k^2}{N^3 - N} \quad (3.11)$$

where

$$d_k = r(T_{ij}) - r(\hat{T}_{ij}) \quad (3.12)$$

in which

$$r(T_{ij}) = \text{the rank of trips from } i \text{ to } j$$

(6) Wilcoxon matched pairs signed ranks test. This statistic compares the magnitudes of a pair of interaction variables (observed and predicted trips). The measure is calculated by finding the signed difference between observed and predicted interaction. These differences are then ranked in order of magnitude irrespective of sign and assigned the sign associated with the difference, zero differences are removed at this stage.  $t$  takes the value of the sum of positive ranks or the sum of negative ranks, whichever is smaller. Siegal (1956) states the Wilcoxon matched pairs statistic can be calculated as follows, where  $\mu_T$  represents the mean of the distribution of ranks and  $N$  is the number of non-zero differences.

$$f = T - \mu_T \quad (3.13)$$

$$\mu_T = \frac{N(N+1)}{4} \quad (3.14)$$

The Spearman's rank statistic and the Wilcoxon matched pairs measure can be used in conjunction with one another in order to test that both the ranks and the magnitudes of predictions are similar to those for observed trips.

(7) The chi-square statistic is stated as

$$\chi^2 = \sum_i \sum_j \frac{(T_{ij} - \hat{T}_{ij})^2}{\hat{T}_{ij}} \quad (3.15)$$

and has a lower limit of zero when total observed interaction is equal to total predicted interaction.

(8) Knudsen and Fotheringham (1986) describe four types of goodness of fit statistic based on information, the first of which is the information gain statistic which is defined as

$$I(P:Q) = \sum_i \sum_j p_{ij} \ln \left( \frac{p_{ij}}{q_{ij}} \right) \quad (3.16)$$

where

$P$  = a posterior discrete probability distribution

$Q$  = a prior discrete probability distribution

and

$$p_{ij} = \frac{T_{ij}}{\sum_i \sum_j T_{ij}} \quad (3.17)$$

$$q_{ij} = \frac{\hat{T}_{ij}}{\sum_i \sum_j \hat{T}_{ij}} \quad (3.18)$$

The relationship between the statistic and the minimum discrimination information statistic (MDI) can be used to test the significance of the information gain statistic.

$$MDI = 2. N. I(P: Q) \quad (3.19)$$

where

$$N = \sum_i \sum_j T_{ij} \quad (3.20)$$

(8) The phi statistic is another form of information based statistic and is also defined in Knudsen and Fotheringham (1986).

$$\Phi = \sum_i \sum_j p_{ij} \left| \ln \left( \frac{p_{ij}}{q_{ij}} \right) \right| \quad (3.21)$$

(9) A further information statistic is the psi statistic which is defined as

$$\Psi = \sum_i \sum_j p_{ij} \ln \left( \frac{p_{ij}}{s_{ij}} \right) + \sum_i \sum_j q_{ij} \ln \left( \frac{q_{ij}}{s_{ij}} \right) \quad (3.22)$$

where

$$s_{ij} = (p_{ij} + q_{ij})/2$$

It is necessary to take absolute values of over and under predictions because the psi statistic is sensitive to the distribution of errors.

(10) A final information based statistic is also discussed in Knudsen and Fotheringham (1986). This is the absolute entropy difference (AED) and is given by the absolute value of the difference in entropies of the observed and predicted probabilities.

$$AED = |H_P - H_Q| \quad (3.23)$$

where

$H$  = Shannon's entropy measure, therefore

$$H_P = -\sum_i \sum_j p_{ij} \ln p_{ij} \quad (3.24)$$

and

$$H_Q = -\sum_i \sum_j q_{ij} \ln q_{ij} \quad (3.25)$$

Therefore the lower limit of AED is zero when  $H_P = H_Q$ .

### 3.7.2 *Choosing the most appropriate goodness of fit statistic*

With all of these goodness of fit statistics being available it is necessary to choose the most appropriate statistic because each will give an alternative measure of performance. Batty and Mackie (1972) state that when the spatial interaction model contains a negative exponential distance function, the mean trip cost function is the most appropriate goodness of fit statistic. Diplock (1996) has undertaken a study of goodness of fit statistics for use in the evaluation of spatial interaction models. Spearman's rank and Wilcoxon matched pairs were considered inappropriate due to their lack of sensitivity to model errors. A cumulative sum of squares measurement of model error was discounted because the unstable nature of the measure meant that significant differences in observed values could produce near optimum error values. A measure based on the squared difference in drive times between observed and predicted (*i.e.*

similar to the mean trip cost function described above) was found to ignore trip volumes and was relatively insensitive to changes in parameter values. This study by Diplock (1996) concluded that of those studied the sum of squared deviations was the best goodness of fit statistic because it behaves consistently over a range of parameter values and is relatively simple to implement.

In their comparison of goodness of fit statistics, Knudsen and Fotheringham (1986) also conclude that general distance statistics, of which sum of squares of error is an example, are the most useful for comparing the performance of spatial interaction models because of all the statistics they tested, this type has the most linear relationship between error levels and the value of the statistic. Chi-square, information gain,  $\Psi$ ,  $r^2$  and AED were seen to have non-linear relationships between error levels and value of the statistic.

For the purpose of this research the mean trip cost will be used for calibration, as described in Section 3.6.1.1. The sum of squares of errors, Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient will be used as a method of comparing model performance after models have been calibrated.

### 3.8 CONCLUSIONS

Thus it can be seen that there are a multitude of possible error sources apparent in the modelling of spatial interaction for retail activities. The three main components of the spatial interaction model, demand, supply and interaction were all seen to have significant problems associated with their inclusion in the model. There are also general modelling problems such as choosing the correct method of calibration and the most appropriate goodness of fit statistic to use in the evaluation of the model.

Several questions have been raised by the contents of this discussion of error sources in spatial interaction modelling which it will be necessary to address. For example, which distance deterrence function is the most appropriate? Should distance decay parameters be origin or destination specific? Are alternative accessibility functions such as described for competing destinations and intervening opportunities better representations of consumer behaviour?

There are several possible methods of improving spatial interaction models, through the alleviation of the error sources described in this chapter. Possible ways of improving spatial interaction models will be investigated throughout the remainder of this thesis with respect to two spatial interaction models. These models have been developed by GMAP for WH Smith and Halifax Plc and will be described in the next chapter.

## A DESCRIPTION OF THE WH SMITH AND HALIFAX SPATIAL INTERACTION MODELS

### 4.1 INTRODUCTION

Each of the three components of spatial interaction models (demand, supply and interaction) described in Chapter 2 will be analysed in the context of two case studies, WH Smith stores and new mortgage sales for Halifax Plc branches. GMAP have developed working models of retail sales for these two organisations and the aim of the subsequent chapters is to explore and improve these models by considering each of the three components in turn. The WH Smith case will be used to investigate the demand and the supply sections of the spatial interaction model. The Halifax new mortgage sales case will be used to investigate the interaction component of the spatial interaction model because excellent data on origin-destination flows are available and these allow possible innovations in the representation of interaction to be evaluated against real data.

This chapter is concerned with the introduction and description of the two case studies and the associated GMAP models and the identification of problems that require solving in each case.

### 4.2 THE GMAP MODEL OF WH SMITH SALES

#### 4.2.1 *WH Smith Plc*

WH Smith is a large public limited corporation that has recently undergone extensive restructuring and rationalisation (Financial Times, 1996). Therefore it is vital that this corporation has an accurate model to aid planning of their store network. At present

WH Smith has approximately 400 stores nation-wide. These stores are typically located in city centre shopping areas.

The WH Smith Group currently runs six high street retail operations. These are WH Smith Retail, Waterstones, Virgin, Our Price, Paperchase and Playhouse. Each of these store types sell different types of goods. In the GMAP models of WH Smith sales, the good type is denoted by  $g$ ,

$g=1, \dots, 6$ , where

$g=1$ =newspapers,  $g=2$ =books,  $g=3$ =music,  $g=4$ =stationery,  $g=5$ =cards,

$g=6$ =video

WH Smith Retail sell all the above goods, Waterstones sell books, Virgin and Our Price provide music and videos, Paperchase sell cards and Playhouse is a specialist video vendor. The model developed by GMAP produces sales estimates for all good types in all store types for the WH Smith Group.

#### ***4.2.2 The spatial scale of the WH Smith Model***

The model that has been developed by GMAP for WH Smith is a national model, reflecting the geographical spread of the outlets of the retail group. The geographical scale used in the model for assessing demand for the goods that are sold by WH Smith, the origin zones in the spatial interaction model, are postal districts. Postal geography is used as opposed to administrative geography because this is the norm in the commercial sector and allows links to be made with market survey data which is usually based on postal geography, and also customer data within the companies themselves will be geographically referenced by postal address.

Address based postal geography data can be aggregated into a nested hierarchy of postal zones: postcodes (*e.g.* LS2 9JT), postal sectors (*e.g.* LS2 9), postal districts (*e.g.* LS2) and postal areas (*e.g.* LS). For the WH Smith model postal districts have been chosen as opposed to postal areas or postal sectors for the following reasons.

Postal areas contain large numbers of people and therefore using this level of spatial aggregation would not capture variations in the population that occur at the local level. The use of postal sectors as the level of geography would give a higher level of homogeneity within areas but survey data for postal sectors would have very small sample sizes. The origin zones (postal districts) are represented by  $i$  in the GMAP model and  $i$  can have any value between 1 and 2593.

#### 4.2.3 The WH Smith Model

The specification of the spatial interaction model developed by GMAP is described in Codling (1995a). The WH Smith model attempts to predict retail sales of the six good types by predicting flows between origins and destinations. The origins in this case are postal districts and the destinations are shopping centres. The money flow that the model attempts to predict is given as

$$S_{ij}^{glt} \tag{4.1}$$

It can be seen that the money flow is disaggregated by several factors:

$i$  = residential zone

$j$  = retail centre

$g$  = product type

$l$  = competitor type, where a competitor type represents a retail chain in competition with WH Smith

$t$  = trip type

where

$i = 1, \dots, 2593$  (number of postal districts)

$j = 1, \dots, 3414$  (number of retail centres)

$g = 1, \dots, 6$  (number of good types)

$l = 1, \dots, 29$  (number of competitor types)

$t = 1, \dots, 3$  (number of trip types) where,

$t = 1 =$  residential,  $t = 2 =$  work based,  $t = 3 =$  tourist

The model takes the form of a production constrained spatial interaction model which is structured in the following way

$$S_{ij}^{gt} = A_i^{gt} O_i^{gt} (W_j^g F_j^t)^\alpha \exp(-\beta_i^g c_{ij}) \quad (4.2)$$

where

$A_i^{gt}$  = a balancing factor

$O_i^{gt}$  = demand for good  $g$  at origin  $i$ , for trip type  $t$

$W_j^g$  = attractiveness of centre  $j$  for good  $g$

$F_j^t$  = shopping centre adjustment factor designed to capture unobserved attributes of centre  $j$ , for trip type  $t$

$\alpha$  = attractiveness parameter

$c_{ij}$  = a measure of the separation between origin  $i$  and destination  $j$

$\beta_i^g$  = an origin and good specific distance decay parameter

Each of these factors will now be discussed in more detail.

$A_i^{gt}$  is a balancing factor in the production constrained spatial interaction model to ensure that predicted sales add up exactly to origin zone demand

$$O_i^{gt} = \sum_{jl} S_{ij}^{gt} \quad (4.3)$$

and is derived by substituting equation (4.2) for  $S_{ij}^{gt}$  in equation (4.3) above.

$$O_i^{gt} = \sum_{jl} A_i^{gt} O_i^{gt} (W_j^g F_j^t)^\alpha \exp(-\beta_i^g c_{ij}) \quad (4.4)$$

Rearranging equation (4.4) we obtain an expression for  $A_i^{gt}$

$$\begin{aligned}
A_i^{gt} &= \frac{O_i^{gt}}{\sum_{jl} O_i^{gt} (W_j^g F_j^t)^\alpha \exp(-\beta_i^g c_{ij})} \\
&= \frac{1}{\sum_{jl} (W_j^g F_j^t)^\alpha \exp(-\beta_i^g c_{ij})} \tag{4.5}
\end{aligned}$$

$O_i^{gt}$  represents demand over a twelve month period for good  $g$  in residential zone  $i$ , for trip type  $t$ . At present this is calculated for the WH Smith model using census data and the Family Expenditure Survey. This process will be described in more detail in Chapter 5.

$W_j^g$ ,  $F_j^t$  and  $\alpha$  represent the supply side in the model.  $W_j^g$  is a sum of the individual attractiveness measures for competitors in the centre. The competitor attractiveness measure is calculated differently for WH Smith Group stores and other competitor types and is made up of a combination of a measure of brand attractiveness and an individual store attractiveness factor. The brand attractiveness variable is included to account for the fact that people choose which retail brand to use by comparing their attractiveness. The measure of brand attractiveness,  $R^{gl}$  is supplied by the WH Smith Location Planning Department.

For the WH Smith Group, individual store attractiveness is calculated from observed data consisting the EFT (store shelf space) of the store and store revenue. Therefore WH Smith Group store attractiveness is calculated as follows

$$\theta_j^{gln} = a_j^{gln} R^{gl} E_j^{gln} \tag{4.6}$$

where

$E_j^{gln}$  = EFT of store  $n$  of competitor type  $l$  in centre  $j$ , for product  $g$

$R^{gl}$  = brand attractiveness of competitor  $l$  for product  $g$

$a_j^{gln}$  = variable used to explain remaining disparity between observed and predicted revenue after calibration, for store  $n$  of competitor type  $l$  for product  $g$ .

For competitors, observed data are not available and therefore an alternative method of calculating store attractiveness has to be found. For competitors located in centres that contain WH Smith Group stores the EFT is known from the WH Smith Group competitor survey which is undertaken by store managers. For centres for which this information is not available, EFTs are estimated based on observed data from other centres. For these competitors store attractiveness is given by

$$\theta_j^{gln} = R^{gl} E_j^{gln} \quad (4.7)$$

Centre attractiveness for each competitor is therefore

$$W_j^{gl} = \sum_{n \in j} \theta_j^{gln} \quad (4.8)$$

and total centre attractiveness is given as

$$W_j^g = \sum_l W_j^{gl} \quad (4.9)$$

The second component of the supply side,  $F_j^t$  is a centre and trip type specific variable called a City Centre Factor (CCF) that is added to account for any remaining differences between observed and predicted centre revenues. CCFs are included separately for the three different trip types, residential, work based and tourist.

The attractiveness parameter  $\alpha$  is as described in Section 3.4.3, and takes into account whether consumers see the advantages of larger centres.

A further process is undertaken concerning the attractiveness of centres. Market segmentation accounts for variations in brand attractiveness dependent on the profile of the catchment population of a centre. Different types of people find different brands more attractive, for example, a study undertaken by GMAP (Codling, 1995b) found that customers of social class AB who have a degree are more likely to purchase books at Waterstones than at WH Smith. The general centre specific market segmentation brand attractiveness,  $M_j^{gl}$  is calculated as follows

$$M_j^{gl} = \sum_{m=1}^N \left( R_m^{gl} \frac{P_{mj}}{P_{ml}} \right) \quad (4.10)$$

where

$R_m^{gl}$  = attractiveness of brand  $l$ , selling good  $g$  as perceived by population segment  $m$

$P_{mj}$  = proportion of the catchment population in segment  $m$

$P_{ml}$  = proportion of national population in segment  $m$

This market segmentation procedure is used in the model to adjust individual store attractiveness,  $\theta_j^{gln}$ , when sharing out the revenue of the centre between different competitors.

In equation (4.2), the functional expression  $\exp(-\beta_i^g c_{ij})$  denotes the interaction component of the spatial interaction model. This is a negative exponential distance decay function as described in Section 3.5.3. Within this function  $c_{ij}$  is a measure of separation between origins and destinations. For the WH Smith model the measure of distance used is drive time.  $\beta_i^g$  is a product and origin specific distance decay parameter which is a measure of the level of distance deterrence as perceived by residents of zone  $i$  when purchasing product  $g$ .

For the purposes of release of the spatial interaction model to WH Smith, GMAP undertakes a process of calibration that provides the values of the parameters  $a_j^{gln}$  (the store attractiveness parameter),  $F_j^f$  (the CCF) and  $\beta_i^g$  (the distance deterrence parameter) that will optimise model predictions and hence model performance. In this way, the model provided by GMAP produces revenue estimates that are nearly equivalent to observed values. This is undertaken in order for the model to be used predictively for strategic planning by WH Smith. The model parameters are calibrated to ensure that store revenues are predicted correctly. If this is achieved, it is hoped that new store scenarios considered by WH Smith will be predicted accurately and this will enable informed store location decisions to be made. However, this means that the values of the parameters used in the model have little relation to actual store or centre characteristics and are therefore difficult to interpret. Thus, the focus of the modelling procedure undertaken by GMAP is to produce a predictive model that produces accurate model predictions. One of my aims during this thesis is to produce an explanatory model that uses parameter values that are based on the characteristics of the centres and stores in the model in order to discover if such observed characteristics can be used to explain differences in performance between destinations.

#### *4.2.4 Current performance of the WH Smith Model*

Goodness of fit statistics will be reported for both the GMAP model in its present predictive state (this will be called the GMAP Full Model), and for the GMAP model with the parameters that are used to equate observed and predicted revenues removed, which will be called the GMAP Base Model. My model results will be compared to both these models but for analysis undertaken on producing new parameter values, the GMAP Base Model will be used. The results are reported for centres in the Yorkshire TV region that contain WH Smith Group stores. There are 29 such centres whose locations are shown in Figure 4.1. Postal district boundaries are also shown.

Figure 4.1: Centres in the Yorkshire TV region containing WH Smith group stores

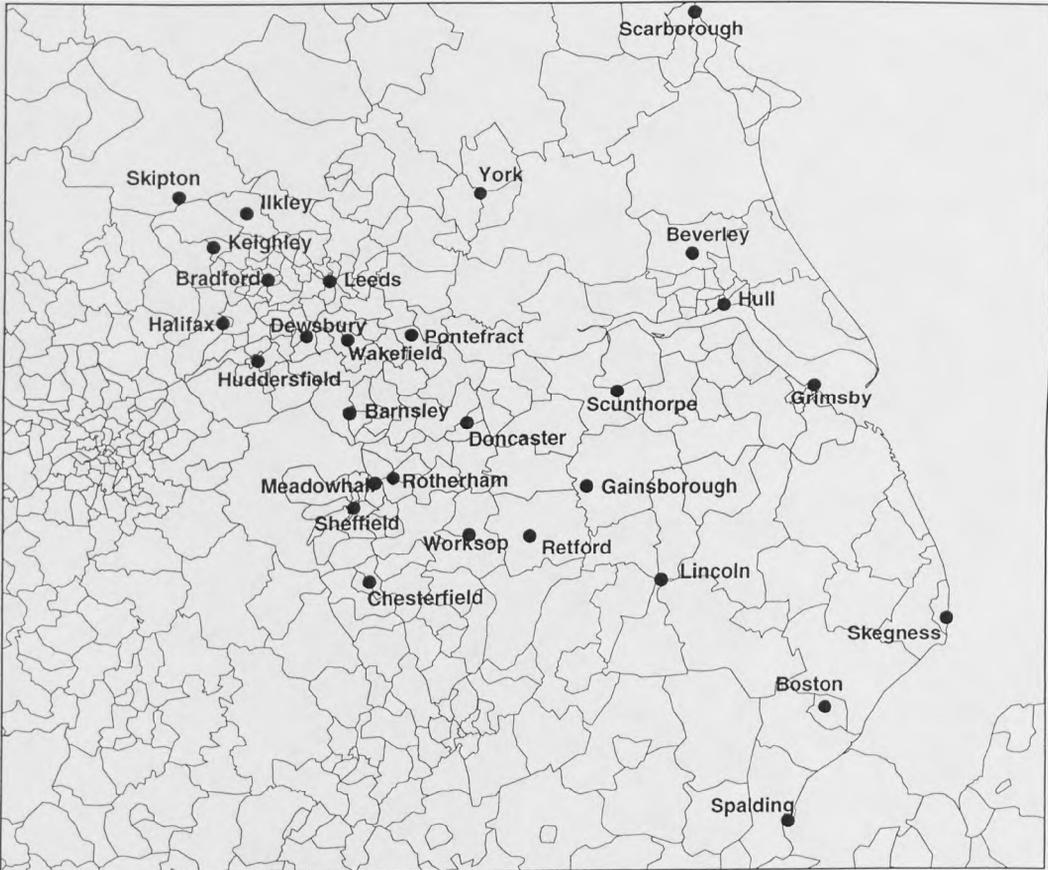


Table 4.1: Goodness of fit statistics for the GMAP Full Model and the GMAP Base Model, for all goods, for centres in the Yorkshire TV region containing WH Smith Group stores

Model	PADT (minutes)	SSE (£1000s)	$r^2$	$r_s$
GMAP Full Model	8.52	8559324	0.98	0.98
GMAP Base Model	8.47	60299628	0.92	0.96

PADT represents the predicted average drive time for customers travelling to WH Smith group stores. The SSE is the sum of squares of errors between observed and predicted centre revenues for WH Smith Group stores in the Yorkshire TV region. The  $r^2$  represents the square of Pearson's product moment correlation, which was described in Section 3.7.1 and the variables used in its calculation are observed and predicted centre revenues. The same variables are used to calculate  $r_s$ , the Spearman's rank correlation coefficient, which is also described in Section 3.7.1. These goodness

of fit statistics will be used throughout the thesis in order to compare the model experiments undertaken.

It can be observed from Table 4.1 that the calibration of parameters in order to optimise predictions that is undertaken by GMAP has a significant effect on model performance with the GMAP Full Model performing better than the GMAP Base Model and having a SSE value 86% lower than the GMAP Base Model.

Figures 4.2 and 4.3 are scatterplots of observed and predicted centre revenue totals for WH Smith Group stores for the GMAP Full Model and the GMAP Base Model respectively. These figures are used as an indicator of which centres are being the most over or under predicted and comparison of the two figures will indicate the effect the parameter values used in the GMAP Full Model will have on model outliers.

Figure 4.2: Scatterplot of observed and predicted centre revenues for the GMAP Full Model

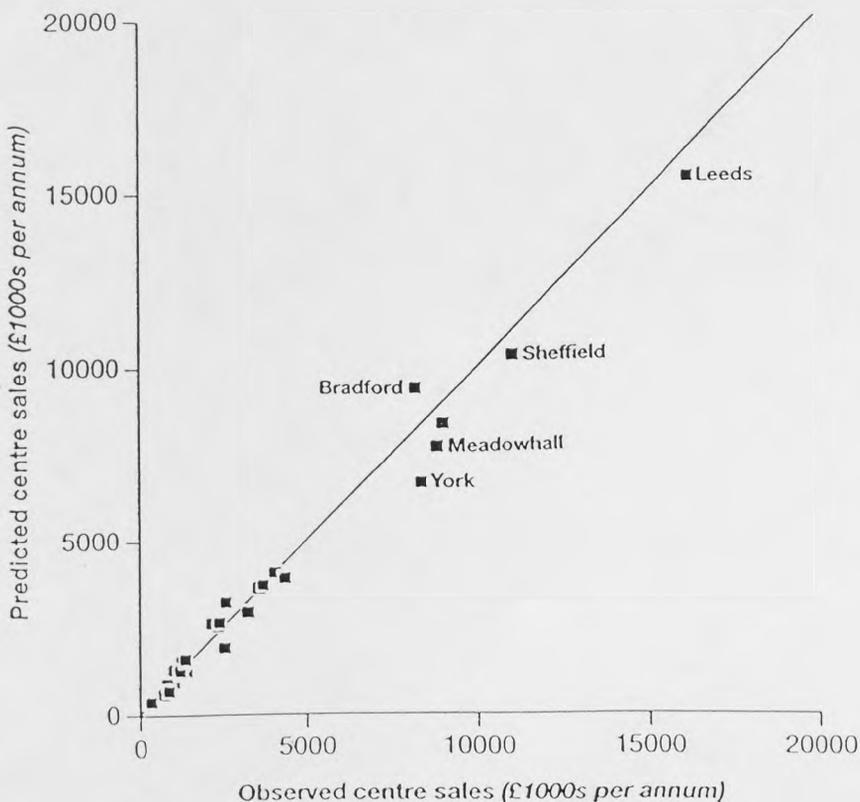


Figure 4.3: Scatterplot of observed and predicted centre revenues for the GMAP Base Model

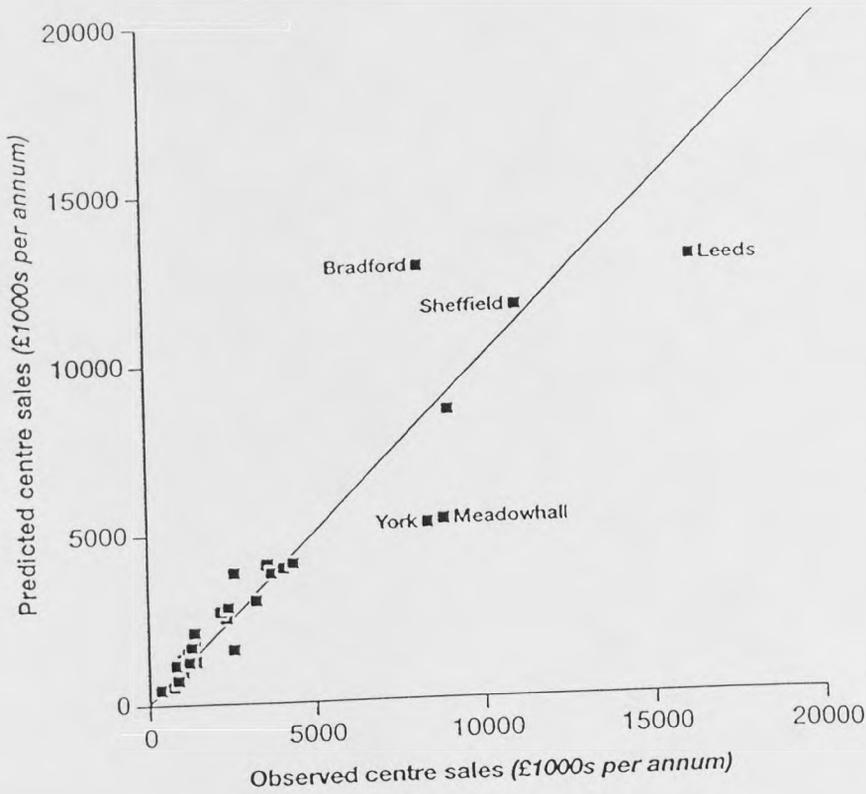


Figure 4.2 indicates that the calibration techniques undertaken to equate observed and predicted revenues for the GMAP Full Model creates no significantly outlying centres. However, with such parameters removed, the GMAP Base Model has several outliers. Centres such as Bradford and Sheffield are being over predicted, whereas Meadowhall, York and Leeds are being under predicted by the model.

The aim of this research is to improve model predictions for the outliers shown in Figure 4.3 by using alternative model inputs and formulations, by substituting variables based on real characteristics of stores and centres for internally calibrated adjustment factors.

### 4.3 THE GMAP MODEL FOR NEW MORTGAGE SALES BY HALIFAX PLC

#### 4.3.1 *The Halifax Plc*

The Halifax Building Society became a bank in 1997 and is now known as Halifax Plc. The Halifax is the largest mortgage provider in the UK and also provides savings and current accounts. The Halifax has 1138 branches across the country, GMAP have produced a national spatial interaction model which aims to replicate customer behaviour for current, savings and mortgage accounts. However I will concentrate on analysing the prediction of mortgage sales.

The origin zones used in the Halifax model are postal sectors, of which there are 8850 in Great Britain.

#### 4.3.2 *The Halifax Model*

The specification of the GMAP Model for Halifax is contained in Urquhart (1997). The spatial interaction model for the Halifax is a production constrained model. The variable to be estimated in this case is the number of new mortgages purchased by residents of origins (residential zones) in destinations (financial centres) this variable is given as

$$S_{ij}^{gl} \tag{4.11}$$

where

$i$  = residence zone

$j$  = financial service centre

$g$  = product type

$l$  = competitor type

where

$i = 1, \dots, 8850$  (number of postal sectors)

$j = 1, \dots, 3478$  (number of financial centres)

$g = 1, \dots, 3$  (number of good types)

$l = 1, \dots, 63$  (number of competitor types)

Financial service centres are derived by GMAP and are defined as clusters of one or more financial service outlets. Competitor types are defined as financial service companies in competition with Halifax Plc. It can be seen that, unlike the WH Smith model, the Halifax model is not disaggregated by trip type.

The structure of the model produced by GMAP for the Halifax is as follows

$$S_{ij}^{gl} = A_i^g O_i^g (W_j^{gl} F_j)^{\alpha_J} \exp(-\beta_i^g c_{ij}) \quad (4.12)$$

where

$A_i^g$  = a balancing factor

$O_i^g$  = demand for good  $g$  in origin  $i$

$W_j^{gl}$  = attractiveness of centre  $j$ , for competitor type  $l$ , for product  $g$

$F_j$  = shopping centre adjustment factor designed to capture unobserved attributes of centre  $j$

$\alpha_J$  = parameter on the attractiveness factor, disaggregated by region  $J$

$c_{ij}$  = a measure of separation between origin  $i$  and destination  $j$

$\beta_i^g$  = a distance decay parameter disaggregated by origin,  $i$  and good type,  $g$

The balancing factor  $A_i^g$  is applied to ensure that the following constraint is met.

$$O_i^g = \sum_{jl} S_{ij}^{gl} \quad (4.13)$$

This factor is derived by substituting equation (4.12) into equation (4.13), which gives

$$O_i^g = \sum_{jl} A_i^g O_i^g (W_j^{gl} F_j)^{\alpha_J} \exp(-\beta_i^g c_{ij}) \quad (4.14)$$

Rearranging equation (4.14) gives the following expression for  $A_i^g$

$$A_i^g = \frac{O_i^g}{\sum_{jl} O_i^g (W_j^{gl} F_j)^{\alpha_J} \exp(-\beta_i^g c_{ij})}$$

$$A_i^g = \frac{1}{\sum_{jl} (W_j^{gl} F_j)^{\alpha_J} \exp(-\beta_i^g c_{ij})} \quad (4.15)$$

The variable  $O_i^g$  represents the demand component of the Halifax spatial interaction model. This factor is calculated for mortgages using the National Mortgage Database which is created by the market analysis company CACI using pooled observed data from banks and building societies concerning the number of new mortgages per annum for postal sectors and is therefore relatively accurate.

$W_j^{gl}$ ,  $F_j$  and the  $\alpha_J$  parameter represent the supply component in the model.  $W_j^{gl}$  is made up of a combination of competitor attractiveness and the number of stores of each competitor type in a centre. Competitor attractiveness is calculated using the Financial Research Survey (FRS) and GMAP's competitor database. The FRS has been purchased by Halifax and provided to GMAP for use in model building and is a market research survey concerning the propensity of different types of people to purchase financial services from alternative sources. The calculation undertaken is to estimate average sales per branch for the main competitor types. This competitor attractiveness variable is denoted by,  $R^{gl}$ . This calculation is undertaken separately for each region to account for spatial differences in brand attractiveness. Therefore,  $R_j^{gl}$  is equal to the brand attractiveness of competitor type  $l$ , for good  $g$ , in region  $J$

For smaller competitors for which no information is available, the competitor attractiveness is set to an average value. Centre attractiveness is then calculated as

$$W_j^{gl} = \sum_l \chi_j^l R_j^{gl} \quad (4.16)$$

where

$\chi_j^l$  = the number of branches of competitor type  $l$  in centre  $j$

Market segmentation is not undertaken for the Halifax model because it is unlikely that choice of mortgage provider will vary by person type.

Drive times between origins and destinations ( $c_{ij}$ ) and the origin and product specific distance decay parameter  $\beta_i^g$  are combined to form a negative exponential function which represents the interaction component in the Halifax spatial interaction model.

As for the WH Smith model GMAP produces a predictive model that uses calibrated parameter values to ensure predicted flows equal observed flows. The values of the parameters  $F_j$  (the City Centre Factor) and  $\beta_i^k$  (the distance decay parameter) are calibrated to optimise model predictions but are not based on observed characteristics of centres or origins. Again I will aim to produce an explanatory model that uses observed characteristics to produce interpretable variable values.

### ***4.3.3 Current performance of the Halifax spatial interaction model***

Goodness of fit statistics will be provided for the current formulation of the GMAP spatial interaction model for Halifax new mortgage sales (the GMAP Full Model) and the GMAP model with the  $F_j$  (CCF) variables removed (GMAP Base Model). The results are reported for financial service centres in the Yorkshire TV Region containing Halifax branches. There are 68 such centres. Figure 4.4 shows the locations of a majority of these centres and Figure 4.5 shows the locations of centres in and around Leeds and Bradford. Postal sector boundaries are also shown.

Figure 4.4: Centres in the Yorkshire TV region containing Halifax Plc branches

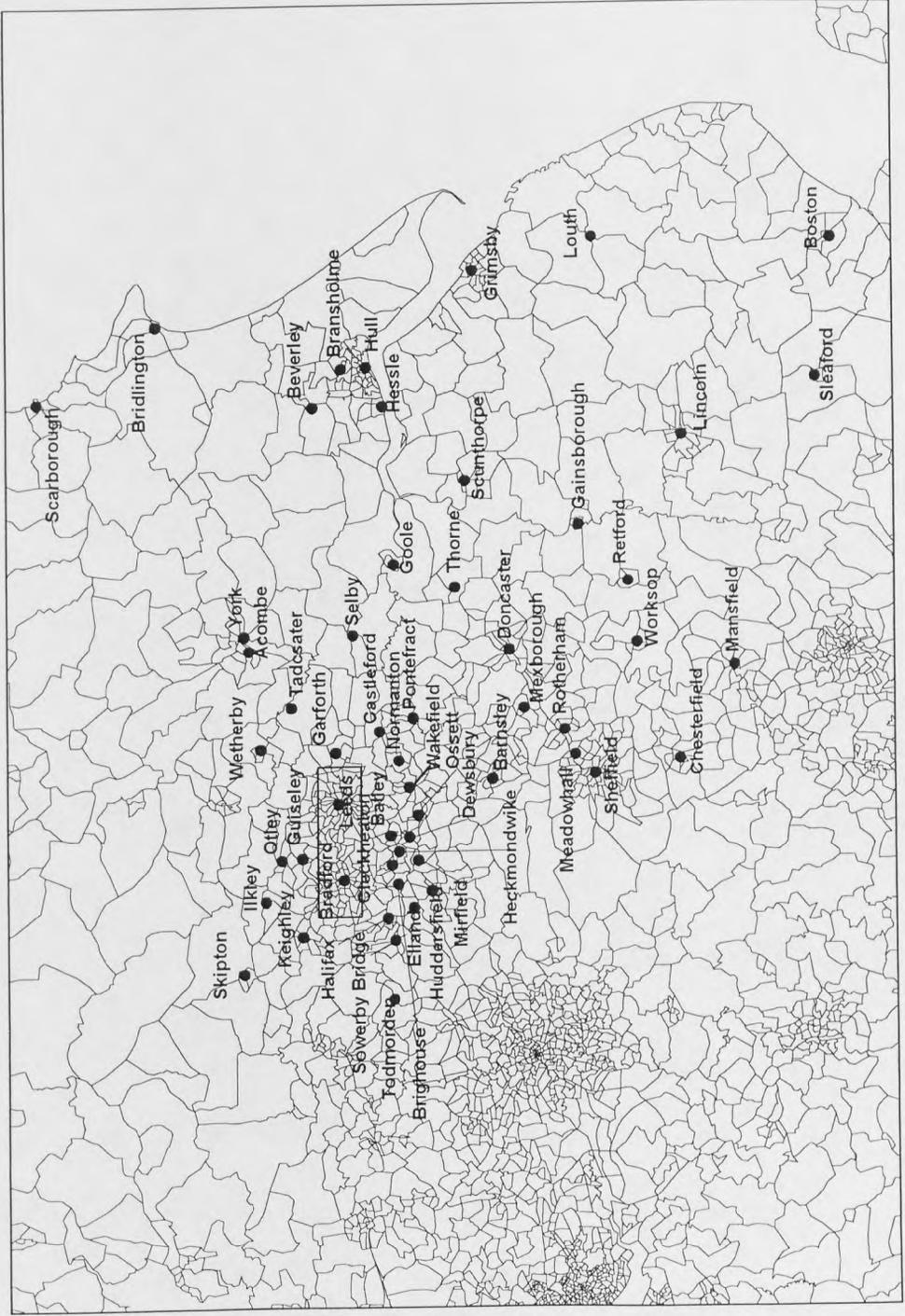
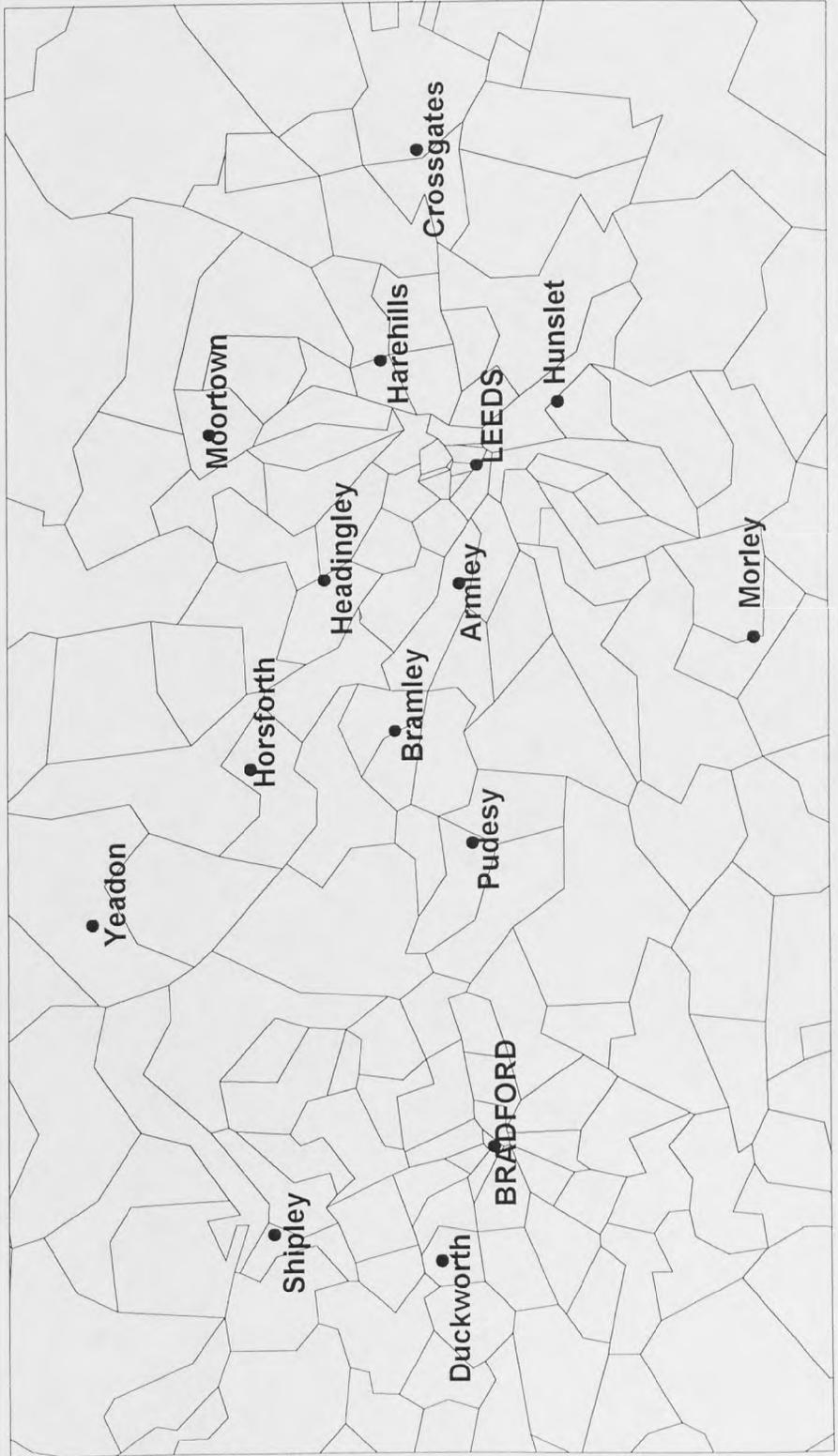


Figure 4.5: Centres in the Leeds and Bradford area containing Halifax Plc branches



The same goodness of fit statistics are calculated as were calculated for WH Smith only in this case the variables being compared are the observed and predicted interactions between origins and destinations and not observed and predicted centre totals.

Table 4.2: Goodness of fit statistics for the GMAP Full Model and the GMAP Base Model for financial centres in the Yorkshire TV region

Model	PADT (minutes)	SSE (£1000s)	$r^2$	$r_s$
GMAP Full Model	8.90	18584	0.66	0.49
GMAP Base Model	8.82	19655	0.65	0.48

The observed average drive time for mortgage sales for the Halifax is 9.32 minutes.

Figures 4.6 and 4.7 below show scatter plots of observed and predicted centre totals for the GMAP Full Model and the GMAP Base Model. This will enable the identification of outlying centres that are being predicted the most incorrectly.

Figure 4.6: Scatterplot of observed and predicted centre totals for the GMAP Full Model

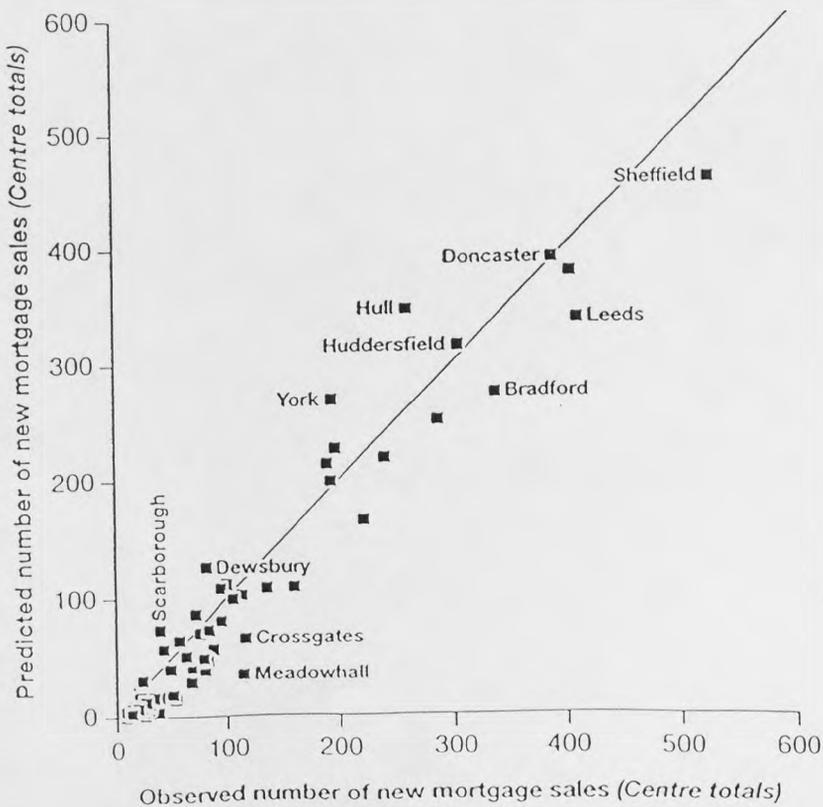


Figure 4.7: Scatterplot of observed and predicted centre totals for the GMAP Base Model

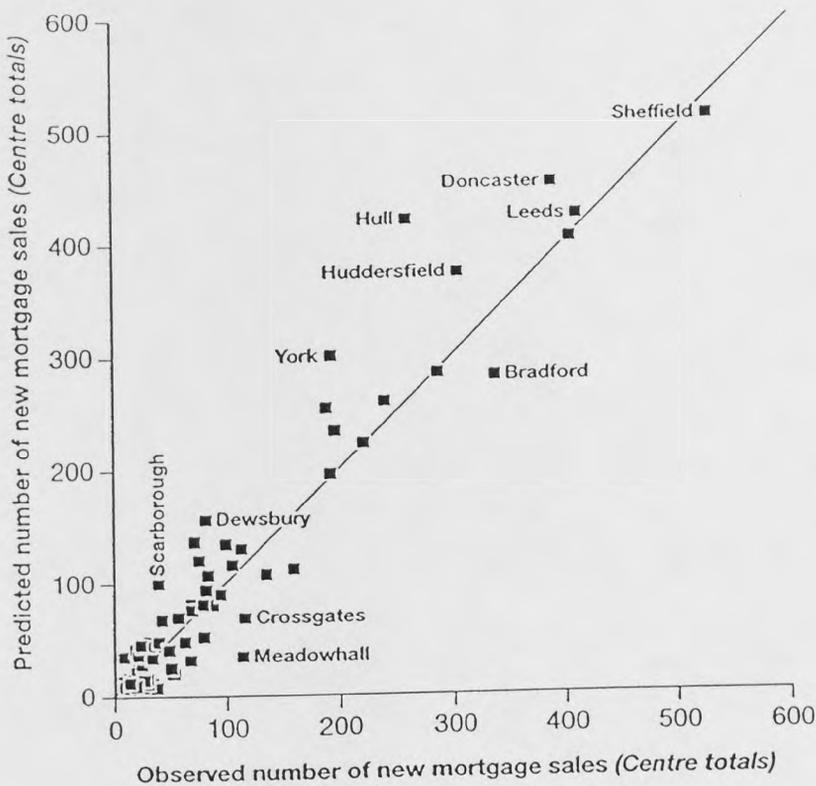


Figure 4.6 shows that for the GMAP Full Model there are no largely outlying centres although there is overall tendency for under prediction within the model. Comparison of this figure with Figure 4.2 also indicates that the WH Smith GMAP Full Model produces better model results than the Halifax GMAP Full Model. Figure 4.7 shows that with the removal of the calibration parameters the tendency towards under prediction is reversed and replaced with a tendency towards over prediction. Several significant outliers can also be identified. Doncaster, Hull, Huddersfield and York are being over predicted, whereas Meadowhall and Bradford are being under predicted.

#### 4.4 CONCLUSIONS

GMAP has developed two predictive models that when calibrated to equate observed and predicted revenues produce good model performance levels. However, when the

adjustment factors are removed the models perform less well. In this thesis I will consider each of the three components of spatial interaction modelling in turn, and attempt to improve the performance of the GMAP Base Models in several ways. Alternative data sources will be used to try and improve factors such as demand estimates. Alternative formulations of certain functions in the model, such as the interaction function will be tested to see if they produce improved model performance levels. Alternative formulations of the adjustment factors will also be undertaken to produce an explanatory rather than a predictive model that uses factor values that are based on actual centre and store characteristics as opposed to being given an arbitrary value that brings predicted revenue closer to observed revenue.

The next chapter is concerned with an investigation of possible improvements to the demand component for the WH Smith spatial interaction model.

## ESTIMATING DEMAND FOR THE WH SMITH SPATIAL INTERACTION MODEL

### 5.1 INTRODUCTION

This chapter is concerned with the origin term,  $O_i^g$ , and therefore the estimation of the demand for good  $g$  that is apparent in each origin zone  $i$ . An estimation procedure is necessary because there is no comprehensive census of expenditure on goods by zone of residence. Therefore demand by residence zone must be modelled. There are a variety of ways in which demand can be calculated. These will be considered in turn and compared in an attempt to identify the most appropriate method of demand estimation. This process will be undertaken for demand at WH Smith stores.

### 5.2 DEMAND ESTIMATION

#### *5.2.1 Disaggregation of the demand term*

The estimation of the origin term,  $O_i^g$ , involves the further disaggregation of this term. This is because demand for a certain good will vary by person type, and therefore disaggregation of the demand term will lead to an increase in accuracy. How the term is disaggregated will depend on what factors it is believed will affect the demand for a certain good. The different factors that are used to disaggregate the population will be reflected in the method used for demand estimation. For example, the first demand model to be tested is disaggregated by the age and social class of the population of the origin zone, whereas in the geodemographic model described in Section 5.5, the population will be disaggregated by the geodemographic type of the neighbourhood in which they live.

### *5.2.2 Alternative demand models*

Three possible demand estimation models are formulated and then tested using data from the National Market Survey. The demand models are the GMAP Demand Model, the Geodemographic Demand Model and the Income Demand Model.

#### *5.2.2.1 The GMAP Demand Model*

The first model is the current GMAP formulation which is based on the use of Census and Family Expenditure Survey data to disaggregate the population of the origin zone according to age and social class in order to produce estimates of household expenditure for each of the main good types sold by WH Smith. This method involves the adjustment of average household expenditure on items bought at WH Smith according to the age and class composition of each postal district. Demand is also adjusted according to the region in which the origin is located. This is undertaken through the use of data on the personal characteristics of the population in the Census, combined with information from the Family Expenditure Survey concerning expenditure estimates for households broken down by the characteristics of those households. This information is used to create class, age and regional weights for each postal district.

#### *5.2.2.2 The Geodemographic Demand Model*

A second demand model uses geodemographics which involves the classification of the population of the origin zones into geodemographic types using the GB Profiler system, developed by Openshaw and Blake (1995). Geodemographic profiles of postal districts have been produced by aggregating from postcode level using the GB Profiler system. This information is subsequently combined with an estimate of average expenditure on books by each geodemographic type. This average expenditure estimate is produced by using market survey information on the number of books purchased by each geodemographic type which is employed in conjunction with a calculation of the average price of a book as estimated from the 1994/95 Family Expenditure Survey. In this way a demand estimate can be made for each postal district based on the disaggregation of each postal district by the population of each geodemographic type resident in that postal district.

### 5.2.2.3 The Income Demand Model

The income model is the third model of demand, which combines expenditure estimates by income band from the Family Expenditure Survey with data on the number of households in each income band in each origin zone.

## 5.3 THE GMAP DEMAND MODEL

### 5.3.1 Introduction

The aim in this section is to recreate and verify the existing method of demand estimation undertaken at GMAP for the WH Smith model. This process of replication is undertaken in order to reveal the structure and assumptions present in the current demand model used by GMAP in the WH Smith spatial interaction model. Demand is calculated through the application of social class, age and regional weights for each postal district to the average expenditure. Demand is calculated for households.

Let  $\bar{s}_i^g$  represent the average household demand for good  $g$  in postal sector  $i$ . This is calculated in the following way

$$\bar{s}_i^g = \bar{s}^g \times A_i^g \times C_i^g \times R^r \quad (5.1)$$

where

$A_i^g$  = the age weight for postal district  $i$  for good  $g$

$C_i^g$  = the social class weight for postal district  $i$  for good  $g$

$R^r$  = the regional weight for region  $r$

Each of the above weights are dimensionless ratios which are used to multiplicatively modify the national average household demand for good  $g$ ,  $\bar{s}^g$ . The total demand for good  $g$  in postal district  $i$  is estimated by multiplying the average household demand in the postal district by the number of households in the postal district in the following way

$$O_i^s = \bar{s}_i^s H_i \quad (5.2)$$

where  $H_i$  is equal to the number of households in postal district  $i$ .

### 5.3.2 Data requirements

Data from the 1991 Census of Population Small Area Statistics are required in order to disaggregate the population of the origin zones according to their age and social class composition. These data are required for all postal districts in England, Scotland and Wales. The number of households resident in each postal district is also taken from the Census. These data are available for academic use on the MIDAS system at Manchester Computing, having been purchased by the ESRC and the JISC under the 1991 Census of Population Programme (Rees 1995a, 1995b).

The 1994/5 Family Expenditure Survey (CSO, 1995a) is utilised as a source of information on the average expenditure on relevant goods, disaggregated by household type. The Family Expenditure Survey is a continuous national survey with a sample of approximately 10,000 households per annum, that is collected by the Office of National Statistics.

### 5.3.3 Model assumptions

- (1) All households of the same type conform to the mean expenditure for that type of household as identified from the Family Expenditure Survey.
- (2) Age, social class, and region of residence are the only factors that affect demand for WH Smith goods.
- (3) The values of variables obtained for postal districts will apply equally to the whole of that postal district, *i.e.* there will be no recognition of local variations.

### 5.3.4 Formal exposition of the GMAP Demand Model

The first step in demand estimation is to identify  $P_i^a$ , the population (individuals) in each postal district that are in age group  $a$ , where  $a$  takes four values:

$$a=1=<30, a=2=30-49, a=3=50-64, \text{ and } a=4=65+.$$

$H_i^c$ , the number of households in social class  $c$  in postal district  $i$ , must also be identified. Four social classes are identified:

$$c=1=AB, c=2=C1, c=3=C2, \text{ and } c=4=DE.$$

This information is combined with expenditure estimates from the Family Expenditure Survey that are broken down by age and class to provide weights for each postal district. These weights are then applied to the average expenditure per household for each good type. Regional weights are also applied to average expenditure, the differences in expenditure between regions are calculated from the Family Expenditure Survey.

The average expenditures,  $\bar{s}^{ag}$  (average spending on good  $g$  by people in age group  $a$ ), and  $\bar{s}^{cg}$  (average spending on good  $g$  by households in social class  $c$ ) are calculated from the Family Expenditure Survey, and in the case of cards and stationery the national market size is also required in order to calculate the average expenditure for each type of good. This is because in the Family Expenditure Survey, cards and stationery are classed as one expenditure item and therefore the market sizes are required in order to calculate the proportion spent on each type of good.

#### 5.3.4.1 The age weight, $A_i^g$

The age weight for each postal district is calculated in the following way. Firstly, spending ratios for each age group are calculated by dividing the average expenditure for each age group by the average expenditure for all age groups. This is undertaken for each good. Let  $R^{ag}$  represent the spending ratio for each age group  $a$  on each good  $g$ , which is calculated in the following way

$$R^{ag} = \frac{\bar{s}^{ag}}{\bar{s}^g} \quad (5.3)$$

In order to calculate an age weighting for each postal district for each age group the spending ratio was multiplied by the number of people in each age group resident in that postal district. These figures were then summed together and divided by the total population of the postal district to give an expenditure weighting for each postal district based on the age composition of that postal district. Let  $A_i^g$  be equal to the age weight for each postal sector  $i$ , for good  $g$ , where  $A_i^g$  is calculated thus

$$A_i^g = \frac{\sum^a (R^{ag} \times P_i^a)}{P_i} \quad (5.4)$$

#### 5.3.4.2 The social class weight, $C_i^g$

The social class weights for each postal district are calculated in a similar way. Spending ratios for each class are calculated by dividing the average expenditure of each class group by the average expenditure across all classes. Let  $R^{cg}$  equal the spending ratio for each social class on good  $g$

$$R^{cg} = \frac{\bar{s}^{cg}}{\bar{s}^g} \quad (5.5)$$

For each postal district the spending ratio for each class is multiplied by the number of households in that class. These figures are subsequently summed together and divided by the total number of households in each postal district to provide a class based expenditure weighting for each postal district. Let  $C_i^g$  represent the class weight for postal sector  $i$  for good  $g$ , which is computed thus

$$C_i^g = \frac{\sum^c (R^{cg} \times H_i^c)}{H_i} \quad (5.6)$$

### 5.3.4.3 The regional weight, $R^r$

It was also necessary to adjust expenditure according to which region each postal district was located in. Expenditure variation by standard region is available in the Family Expenditure Survey, and it is therefore possible to calculate regional weights.

However, some of the regional differences in expenditure evident in the Family Expenditure Survey will have already been accounted for in the class weights because different regions have different class structures and this will affect the level of expenditure in the region. Therefore it is necessary to calculate alternative regional weights using the regional expenditure estimates from the Family Expenditure Survey and information on the class structure of each region.  $R^r$  represents the regional weight for region  $r$

$r=1, \dots, 10$  where:

$r=1$ =North,  $r=2$ =Yorkshire and Humberside,  $r=3$ =East Midlands

$r=4$ =East Anglia,  $r=5$ =South East,  $r=6$ =South West,  $r=7$ =West Midlands

$r=8$ =North West,  $r=9$ =Wales,  $r=10$ =Scotland

The first step in the calculation of the regional weight is to calculate an index (shown by the symbol  $I$ ) for the UK expenditure on the goods sold at WH Smith, based on the class structure of the UK. This is undertaken using the average expenditure for the UK for all the goods sold at WH Smith, which is given by  $\bar{x}$ . The value of  $\bar{x}$  calculated from the Family Expenditure Survey is £7.14 per fortnight.

In order to calculate  $I$  it is also necessary to know how many households of each social class there are in the UK. This information is contained in Table 5.1, below.

Table 5.1: The social class structure of the UK

Index	Social Class	Households ( $H^c$ ) *	Percentage of households ( $Y^c$ )
c=1	AB	3,545,685	16
c=2	C1	3,135,725	14
c=3	C2	3,791,054	17
c=4	DE	2,362,458	11
	other	9,193,698	42
	TOTAL	22,028,620	100

\* Source: CSO (1995b)

As can be seen from Table 5.1, a large proportion of households are not classified as belonging to a particular social class. The figure for other social class is so large that they cannot be accounted for as solely armed forces and retired households. Therefore the households classified as anything other than AB, C1, C2, DE in the Census are excluded from the analysis.

The total market size for WH Smith goods,  $M$ , can be computed as follows:

$$M = \bar{s} \times H \quad (5.7)$$

For the UK,  $M = £7.14 \times 22,028,620 = £157,284,350$ .

It is then necessary to calculate the percentage of the total market that is contributed by each social class. This is found by multiplying the average expenditure of the social class (calculated from the Family Expenditure Survey) by the number of households in the class. Let  $\bar{s}^c$  represent the average expenditure of social class  $c$  on all WH Smith goods. The values of  $\bar{s}^c$  are shown in Table 5.2 below.

Table 5.2: The average expenditure of each social class on all WH Smith goods

Index	Social Class	Average Expenditure ( $\bar{s}^c$ ) *
c=1	AB	£11.88
c=2	C1	£6.86
c=3	C2	£7.86
c=4	DE	£6.28

\* Source: CSO (1995a)

Let  $T^c$  represent the percentage contribution to total sales from social class  $c$

$$T^c = \left( \frac{(\bar{s}^c \times H^c)}{M} \right) \times 100 \quad (5.8)$$

The values for  $T^c$  for the UK are shown in Table 5.3.

Table 5.3: The percentage of sales of WH Smith goods purchased by each social class in the UK

Index	Social Class	Percentage Of Sales ( $T^c$ )
c=1	AB	27
c=2	C1	14
c=3	C2	19
c=4	DE	9
	other	31
	total	100

Weights are then calculated for each class in the following way, where  $C^c$  represents the weight for social class  $c$

$$C^c = \frac{T^c}{Y^c} \quad (5.9)$$

Table 5.4 contains the class weights as calculated for the UK.

Table 5.4: The social class weights for the UK

Index	Social Class	Class Weight $C^c$
c=1	AB	1.67
c=2	C1	0.96
c=3	C2	1.10
c=4	DE	0.88

The index ( $I$ ) for the UK, is subsequently calculated as follows:

$$I = \frac{\sum (C^c \times H^c)}{H} \quad (5.10)$$

For the UK,  $I = 0.69$ . It would usually be expected that the value for the UK would be 1, but because 31% of households were excluded, the value is 0.69.

Then, for each region a regional index,  $I^r$ , is calculated as follows:

$$I^r = \frac{\sum (C^c \times H^{cr})}{H^r} \quad (5.11)$$

The process of the calculation of the regional index will be illustrated using the example of the South East region. It is therefore necessary to know the class structure of the South East region. This information is shown in Table 5.5.

Table 5.5: The social class structure of the South East

Index	Social Class	Households ( $H^{cr}$ ) *
c=1	AB	1,374,289
c=2	C1	1,173,952
c=3	C2	1,068,082
c=4	DE	635,271
	other	4,068,502
	total	6,945,944

\* Source: CSO (1995b)

Substituting these values into equation (5.11) gives a value of 0.74 for  $I^r$  for the South East.

This regional index is then compared to the UK index in order to identify the difference in the class structure of the region and the effect that this will have on regional expenditure.  $B^r$  represents the ratio of the regional index to the index for the UK, and is calculated as follows

$$B^r = \frac{I^r}{I} \quad (5.12)$$

For  $r=5$ , the South East

$$B^r = \frac{0.74}{0.69} = 1.07$$

This figure of 1.07 indicates that for the South East, the class structure of the region causes average expenditure on WH Smith goods to be 7% higher than average expenditure for the UK as a whole.

It is also necessary to utilise  $E^r$ , the ratio of the average expenditure in the region to the average expenditure across the UK.  $E^r$  is calculated as follows:

$$E^r = \frac{\bar{s}^r}{\bar{s}} \quad (5.13)$$

For the South East:

$$E^5 = \frac{7.83}{7.14} = 1.10$$

Therefore for the South East, average expenditure is 10% higher than for the UK as a whole.

$E^r$  indicates how much the expenditure varies between the region and the UK,  $B^r$  indicates how much of the difference in expenditure between the region and the UK is due to the class structure of the region. Therefore by using these two figures it is possible to identify how much regional expenditure should be altered, taking into account the fact that some regional difference in expenditure has already been included in the social class weight. Let  $R^r$  equal the regional weight after accounting for the class structure of region  $r$ .

$$R^r = 1 - (B^r - \bar{s}^r) \quad (5.14)$$

Thus for the South East

$$R^r = 1 - (1.07 - 1.10) = 1.03$$

This calculation indicates that the regional weight for the South East, taking into account the class structure of the region is 1.03.

#### 5.3.4.4 Demand estimation

Once all the weights have been calculated for each postal sector, each weight is applied to the overall average expenditure estimate, as in equation (5.1). This is undertaken for each individual postal district in order to derive expenditure estimates that account for the age and class composition of each postal district, and of the region in which the postal district is located.

#### 5.3.5 Individual demand estimates

An alternative method of demand estimation is to calculate demand in the same way as described above but for individuals as opposed to households. This involves the calculation of a social class weight based on individuals as opposed to households. Data on social class by respondent are available from the census as are data on age of the population. Therefore  $P_i^c$  is calculated, as opposed to  $H_i^c$  in the previous method.

These data are subsequently used to calculate age and class weights for each postal district in the same way as for the estimation of household demand. The same regional weights were used as were used in the household demand estimates.

### *5.3.6 Validation of GMAP Demand Model estimates*

After demand estimates for each postal district have been calculated it is necessary to validate those estimates. The National Market Survey produced by ICM Research (1995) and purchased by WH Smith for use by GMAP has been used for this purpose.

#### *5.3.6.1 Data description*

The National Market Survey contains information from approximately 17,000 respondents and is concerned with the frequency with which respondents buy certain goods. The data for the National Market Survey has been collected between May 1994 and April 1995 and is spatially referenced to the postcode of the respondent's residence. Responses were obtained through telephone interviews.

#### *5.3.6.2 Method of validation*

The information used for validation concerns whether the respondent purchased each type of good over the four week survey period. This information was aggregated for each postal district and the number of responses per postal district was calculated as was the number of positive responses in each postal district (*i.e.* how many respondents responded yes for each specific good type within the broad good types described above). A ratio of positive responses to total responses for a four week period was calculated as follows

$$\text{NMS Ratio} = \text{number of positive responses/number of responses} \quad (5.15)$$

This ratio could have a value of greater than one because within each broad good type there are several different specific good types *i.e.* different kinds of books. If respondents stated that they had bought two different types of books this was counted as two positive responses. Validation involved the comparison of the ratio for each postal

district to the average household demand estimate for that postal district in order to see if there was any correlation between the two variables.

Table 5.6 below contains a sample of the postal districts for which correlations were performed. The sample contains all Leeds postal districts for which National Market Survey data is available. Correlation coefficients were calculated between the National Market Survey ratio of each postal district and the average household demand for each postal district.

Table 5.6: A sample of postal districts showing the values used in the calculation of correlation coefficients

Postal District	Number of Positive Responses	Number of Responses	NMS ratio of Responses	Average Demand per Household (£s per month)
LS8	24	21	1.14	5.01
LS9	6	5	1.20	3.96
LS12	0	1	0.00	4.85
LS14	105	111	0.95	4.33
LS15	37	33	1.12	5.59
LS25	123	126	0.98	6.03
LS26	21	20	1.05	5.36

### 5.3.6.3 Results of validation

This analysis was undertaken for model average demand estimates and the ratio calculated from the National Market Survey. There was a problem in using this validation dataset in that some postal districts had small sample sizes. To decrease the error caused by this, correlations were undertaken at two levels, for postal districts with ten or more responses and postal districts with fifty or more responses. The results are shown in Table 5.7.

Table 5.7: Correlations between GMAP Demand Model average demand estimates and National Market Survey ratios, for postal districts in the National Market Survey

Good Type	Response Level	$r$	$r^2$	$r_s$
books	10	0.24	0.06	0.25
	50	0.29	0.09	0.31
cards	10	0.16	0.02	0.16
	50	0.14	0.02	0.18
music	10	0.27	0.07	0.27
	50	0.23	0.05	0.21
newspapers	10	-0.01	0.00	0.00
	50	0.04	0.00	0.07
stationery	10	0.15	0.02	0.16
	50	0.18	0.03	0.19
video	10	0.05	0.00	-0.01
	50	0.01	0.00	0.00

As can be seen from Table 5.7, the analysis produced different results for different good types. The correlation coefficients between average household demand and National Market Survey ratio were significant at the 95% level at both levels of analysis for books and music, whereas for cards and stationery the coefficient was only significant for the sample of postal districts with over ten responses. For newspapers and videos the coefficient was not significant at the 95% level for either the sample with more than ten responses or the sample with over fifty responses, and in fact for postal districts with over ten responses for newspapers the  $r$  value indicated a negative relationship between average demand and ratio. However, even for those good types that had significant  $r$  value, the value of  $r$  is still low, below 0.3 in all cases, which indicates that there is not a high degree of correlation between the two variables. The  $r_s$  value follows broadly the same pattern as  $r$ .

Further analysis was undertaken to discover if there were differences in the correlation coefficients between GMAP Demand Model average household demand estimates and National Market Survey ratios for different regions. The results are set out in Table 5.8.

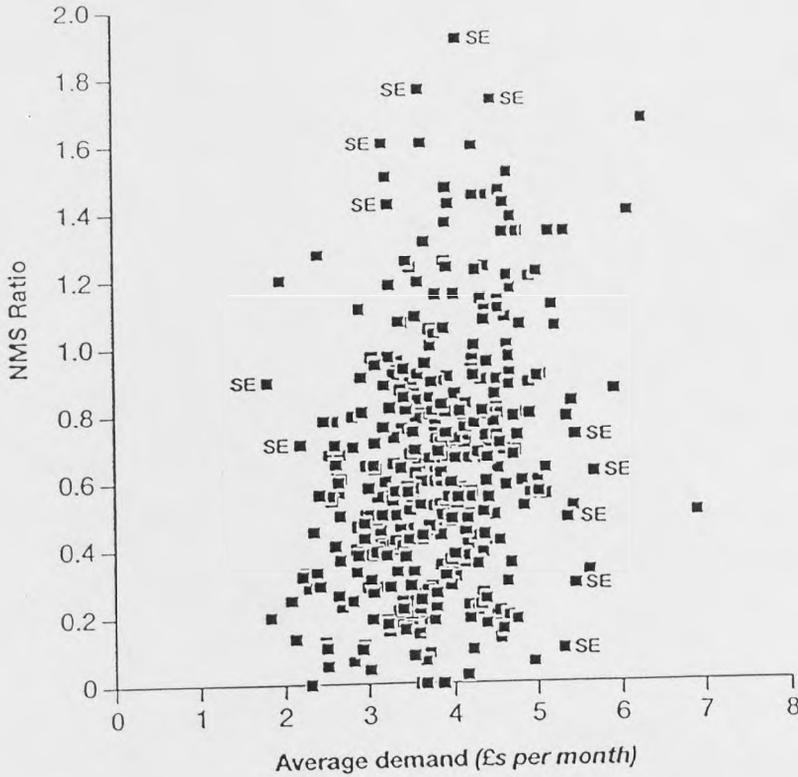
Table 5.8: Correlations between GMAP Demand Model demand estimates and National Market Survey ratios, disaggregated by standard region

Region	$r$	$r^2$
East Anglia	-0.06	0.00
East Midlands	0.40	0.16
North	0.07	0.01
North West	0.16	0.03
Scotland	-0.14	0.02
South East	0.11	0.01
South West	0.65	0.42
Wales	0.44	0.20
West Midlands	0.38	0.15
Yorkshire and Humberside	-0.14	0.02

These results indicate that there are substantial variations in the  $r$  values between regions. The East Midlands, South West, Wales and the West Midlands all have correlation coefficients that are significant at or above the 95% level. The North, North West and South East all have low values of  $r$  that are not significant, and East Anglia and Yorkshire and Humberside actually have negative values of  $r$ . Therefore it is possible that low values of the correlation coefficient for different goods could be due to regional bias in the model or validation dataset.

Scatterplots, as in Figure 5.1 below, also indicate that errors could be caused by regional bias, because they reveal that many outliers are from the same regions. For example all the points labelled SE in Figure 5.1 are in the South East, and it can be seen that such postal districts predominate in the outliers. Outliers are also caused by postal districts that have been subject to changes in postal geography *e.g.* Derby. However, even when these factors are considered there is still wide variation between the two data sets.

Figure 5.1: Scatterplot of GMAP Demand Model average household demand for books and National Market Survey ratio for books, for postal districts with ten or more responses



The National Market Survey data also allows validation to be undertaken for books using data collected concerning the number of books purchased by respondents. It was thought that this information was more representative of demand for books than simply if the respondent purchased books. Correlations were undertaken for average demand estimates and the average number of books bought in each geographical zone. This analysis was undertaken at two levels of aggregation, postal districts and postal areas. Also correlations for standard regions were undertaken using total estimated demand in each region and total regional demand calculated from the FES.

Table 5.9: Correlations between GMAP Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal districts

	$r$	$r^2$	$r_s$
postal districts with $\geq 10$ responses	0.27	0.07	0.27
postal districts with $\geq 50$ responses	0.29	0.08	0.29

Comparison of Table 5.9 with Table 5.7 indicates that the use of this alternative information for validation provides higher correlation coefficients for both postal districts with over 10 responses and postal districts with over 50 responses. For Table 5.9 the correlation coefficients are significant at the 99.99% level, which is higher than for the coefficients shown in Table 5.7. This could be due to the fact that the information used in this analysis is more representative of demand for books. However, it is not possible to use this information in validation for all the goods concerned because data on the number of goods bought is only available for books.

Correlations were also undertaken at coarser geographical levels because this will decrease the errors due to small sample sizes in the National Market Survey.

Table 5.10: Correlations between GMAP Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal areas

	$r$	$r^2$	$r_s$
all postal areas	0.31	0.09	0.29
postal areas with $\geq 50$ responses	0.41	0.17	0.33
postal areas with $\geq 100$ responses	0.41	0.17	0.36

Table 5.11: Correlations between GMAP Demand Model total regional demand estimates for books and the regional demand calculated from the Family Expenditure Survey, for standard regions

	$r$	$r^2$	$r_s$
regions	0.98	0.97	0.90

Both the Pearson's product moment correlation coefficient and the Spearman's rank correlation coefficient indicate that the GMAP Demand Model performs better at higher levels of geographical aggregation. This is shown by the increase in both the  $r^2$  value and the  $r_s$  value as the level of aggregation increases. This could be due to the paucity of market survey data at the postal district level. Therefore, it could be that it is not the model that is improving in performance, but that variation in the survey data is decreasing leading to better goodness of fit statistics.

### 5.3.6.4 Validation of individual demand estimates

The individual demand estimates were subsequently correlated with the market survey data from the National Market Survey. The results of these correlations are shown in Table 5.12.

Table 5.12: Correlations between GMAP Demand Model individual average demand estimates and National Market Survey ratios for WH Smith goods

Good Type	Response Level	$r$	$r^2$	$r_s$
books	10	-0.05	0.00	0.00
	50	-0.03	0.00	0.10
cards	10	0.05	0.00	0.05
	50	0.05	0.00	0.07
music	10	-0.01	0.00	0.06
	50	-0.04	0.00	-0.01
newspapers	10	-0.05	0.00	-0.11
	50	-0.05	0.00	-0.10
stationery	10	-0.05	0.00	-0.05
	50	-0.01	0.00	0.05
video	10	-0.04	0.00	-0.01
	50	-0.12	0.01	-0.12

It can be seen from Table 5.12 that the results obtained are not significant, with none of the correlation coefficients being significant at the 95% level. Many of the  $r$  and  $r_s$  values indicate negative correlations between average demand and ratio. Those instances that showed a positive correlation only had small values of  $r$ . The results of correlations for individual demand estimates were worse than for household demand estimates in all cases except for videos in postal districts with fifty or more responses.

### *5.3.6.5 Problems with the validation of demand estimates*

The fact that both demand estimation methods investigated so far produced such low correlation coefficients could indicate that the validation method is flawed. It can be seen that there are several problems concerning the validation process.

- (1) The survey does not contain data on how much respondents actually spent on goods and is therefore not fully representative of the demand for goods.
- (2) The validation procedure involves the comparison of two estimates that are both based on assumptions. Neither data set represents the actual demand situation. This means that it is difficult to detect which data set is producing the errors apparent in the demand model.
- (3) The National Market Survey is a general purpose survey and is therefore not specifically geared towards WH Smith. Validation would be facilitated if there were specific data available such as point of sale data, but these are expensive and difficult to obtain.
- (4) Within the National Market Survey, postal districts do not have an equal probability of being selected for analysis, because only postal districts with over a certain number of responses were chosen there is the possibility of a bias towards the selection of larger postal districts.
- (5) There is not equal coverage of respondents across the UK.
- (6) The National Market Survey has a relatively small sample size.

It can be seen from the analysis undertaken that the current GMAP Demand Model does not perform well when compared to market survey data. Therefore alternative models for demand estimation need to be produced and tested.

## 5.4 AN ANALYSIS OF VARIABLES THOUGHT TO INFLUENCE DEMAND

One reason why the GMAP Demand Model could be performing badly is that it does not contain the correct variables that determine purchasing behaviour. There could be other variables that are more important in determining demand levels and if these variables are used to estimate demand it could lead to an improvement in the performance of the demand model.

One method of determining which variables will be important in determining demand is logit analysis. This is a type of generalised linear model that is described in Appendix A. The use of this method for the investigation of variables to be used in demand estimation will be described using the example of demand for books. The logit modelling for this analysis was undertaken using the GLIM 4 statistical package on the MIDAS machine at the Manchester Computing Centre.

In this example, the dependent variable is whether a respondent purchased books. The data used were drawn from the 1994/95 Family Expenditure Survey. The independent variables chosen for analysis were as follows

- (1) social class
- (2) age of head of household
- (3) region
- (4) socio-economic group
- (5) disposable income
- (6) education age
- (7) household composition
- (8) household members

Social class, age of head of household and region were included because they are the variables currently used by GMAP for the calculation of demand for the WH Smith model. Socio-economic group was used in order to test if this was a more accurate method of classifying the population than social class. Disposable income, household composition, number of household members and education age were included because it is thought that these variables could influence demand for books.

Table 5.13: Deviance results from logit models for book buying behaviour in the Family Expenditure Survey 1994/95

Variable	Model Deviance	Degrees of Freedom	Null Deviance - Model Deviance
Null Model	7913	6978	
Social Class	7679	6974	234
Age of Head of Household	7771	6975	142
Region	7868	6967	45
Socio-economic Group	7646	6972	267
Disposable Income	7489	6974	424
Education Age	7762	6975	151
Household Members	7749	6974	164
Household Composition	7677	6974	236
GMAP Demand Model	7602	6959	311
Optimal Model	7318	6956	595

It can be seen from Table 5.13 that of the single variable models, disposable income decreases the deviance by the largest amount and is therefore the most significant discriminatory variable concerning the purchase of books. The second most significant variable is socio-economic group which can be seen to decrease variance by more than social class. However, this is also associated with a larger decrease in the degrees of freedom. Therefore there is probably not a significant difference between the use of social class and socio-economic group as a method of classifying the population. Household composition is the next most important explanatory variable, followed by number of household members and education age. Analysis of the parameters for the variables age of head of household and region, which are shown in Appendix B, shows that these are not significant variables at the 95% level.

Thus it can be seen that two of the variables used in the GMAP Demand Model, age of head of household and standard region, do not cause large decreases in deviance and are not significant at the 95% level. Other variables, such as disposable income cause larger decreases in deviance indicating that they are more important determinants of consumer purchasing habits and could therefore produce improved models for the estimation of demand.

The multiple variable models shown in the last two rows of Table 5.13 also indicate that the variables used in the GMAP Demand Model, when combined do not cause as large a decrease in deviance as the disposable income variable alone (the three variables used in the GMAP Demand Model only cause a decrease in deviance of 311). The optimal model consists of the variables disposable income, socio-economic group, education age, household composition and number of household members. This combination of variables produces the largest overall decrease in deviance although this is associated with a large decrease in degrees of freedom. However, this result is still indicative of the possibility that demand models made up of several alternative variables could be more appropriate. One method of introducing a range of variables into the demand estimation process is through the use of geodemographics as will be discussed in the next section.

## **5.5 THE GEODEMOGRAPHIC DEMAND MODEL**

### ***5.5.1 Introduction***

One method of introducing a wide range of census variables into the demand estimation model, including those in Table 5.13 that are available in the Census (all except disposable income) is through the use of geodemographics.

Geodemographics are descriptions of areas based on the characteristics of their populations. Such descriptions are created using a variety of data concerning the socio-economic and demographic characteristics of the populations of the areas. The most important data source for the production of geodemographic classifications is small area census data. Every postal code is assigned to a geodemographic group.

Brown (1991) states that geodemographic classifications are a means of distinguishing between variations in the behaviour of consumers. Birkin (1995) also illustrates an example of how geodemographics, in conjunction with the Target Group Index can be used to identify the types of people that frequent holiday camps.

Thus, geodemographics can be used alongside market survey data (such as the National Market Survey) in order to discover the propensity of people of different geodemographic types to purchase certain goods.

Demand by postal district is calculated by finding the number of people in each geodemographic group in each postal district and subsequently combining this information with an estimate of the amount a person from each group will spend on the good in question. For this analysis, demand estimates are only produced for books for this is the only good for which there is data available concerning the number of items bought by respondents in the National Market Survey.

### *5.5.2 Data requirements*

The GB Profiler system developed by Openshaw and Blake (1995) was used to identify which geodemographic group each postal code is assigned to. GB Profiler is a small area classification system based on 85 variables from the 1991 census and uses a neural net approach to classify areas. Each classification produced by GB Profiler has a label attached to it which is based on the characteristics of the area. Each label has three parts. Firstly, there is a single word description. Each classification is one of five types: struggling, aspiring, established, climbing or prosperous. The second part of the label concerns the description of the residents of the area, typically including information concerning age, ethnicity, family status and occupation. The third section of the label is a description of the physical amenities of the areas, for example, the typical tenure that is apparent in the area. These descriptions can be found in Appendix C.

The National Market Survey described above is used to obtain estimates of how many books are purchased on average by each geodemographic type. This is achieved by discovering which geodemographic type each respondent of the survey corresponds to and extracting from the survey the number of books purchased by each respondent. This information can then be aggregated for each geodemographic group and divided by the number of respondents of each geodemographic type to obtain an estimate of the average number of books purchased by each geodemographic type.

The 1994/95 Family Expenditure Survey (Central Statistical Office, 1995a) in machine readable form, available from the ESRC Data Archive at the Manchester Computing Centre is also used in order to produce an estimate of the average price of a book. The average price of a book is calculated by extracting from the survey the number of books bought by respondents and the total amount spent on books. Division of the total expenditure on books by the number of books bought produces an average book price.

The data required for the Geodemographic Demand Model are reproduced in Appendix C. Appendix C is comprised of a table containing geodemographic information concerning the respondents in the National Market Survey. The table includes an aggregation of the number of respondents that are in each geodemographic type and the number of books purchased by respondents of each type. Therefore it is possible to calculate an average number of books bought by people in each geodemographic group. This average, combined with the average book price provides an average spend on books for each type. The geodemographic descriptions are also available for each type, in Appendix C. The table in Appendix C is ranked, with the geodemographic types which have the highest average spend on books at the top of the table.

### *5.5.3 Statistical justification of the use of geodemographics*

In order to discover if geodemographic classifications are of use in demand estimation it is necessary to identify if there is a significant grouping of book buyers in certain geodemographic types. This was undertaken by producing a Lorenz curve and a Gini coefficient for book buyers within the National Market Survey.

#### *5.5.3.1 Lorenz curves*

A Lorenz curve indicates if there is any difference in the distribution of a variable between two populations. In this case a comparison is being made between the distribution of geodemographic groups between the whole survey population and the respondents who bought books.

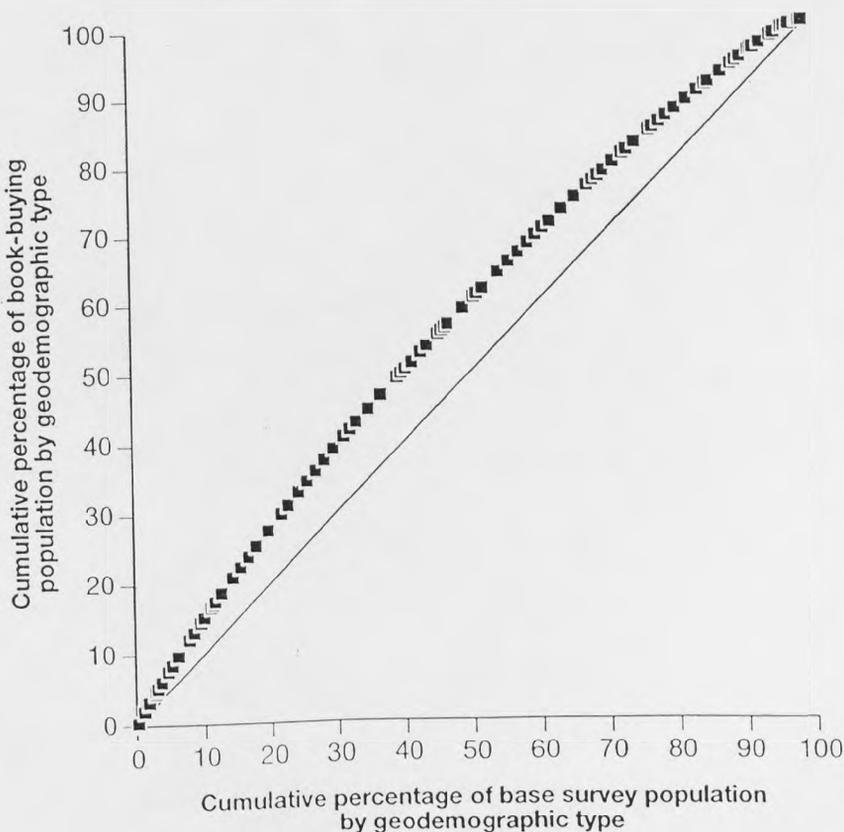
The first step in the production of a Lorenz curve is to calculate a penetration index in order to compare the base survey population with the book buying population for each

geodemographic type. This penetration index is calculated by dividing the frequency of the geodemographic type for the book buying population by the frequency of the geodemographic type for the survey as a whole. This figure is then multiplied by one hundred. This process is undertaken for each geodemographic type.

The next stage is to rank the geodemographic types by the penetration index in descending order. Once the types have been ranked, the cumulative population percentages are calculated for the book buyers and for the whole survey population. These cumulative percentages are then plotted against each other to produce a Lorenz curve. If the curve that has been plotted varies from the 45 degree line then this is indicative of a difference in the distribution of the variable between the two populations.

The level of difference between the distributions is measured by the Gini coefficient (Gini, 1915-6). This index is measured by comparing the difference between the area under the Lorenz curve and the area below the 45 degree line. Figure 5.2 shows the Lorenz curve as calculated for the comparison of book buyers to the whole survey population.

Figure 5.2: The Lorenz curve for the geodemographic types of book buyers compared to the whole survey population



Analysis of the Lorenz curve shown in Figure 5.2 for the National Market Survey data indicates that there is a difference in the distribution of geodemographic types between book buyers and the survey population. This shows that people from some geodemographic types are more likely to buy books than respondents from other geodemographic types. The Gini coefficient for this case is 1.20. This figure of 1.20 is different from 1.0 and therefore indicates that there is some grouping of book buyers within certain geodemographic types. Therefore geodemographics can be used to disaggregate the population in order to calculate demand for books.

By studying Appendix C and analysing the rankings of the penetration index it is possible to identify which geodemographic types are more or less likely to purchase books. The patterns that are revealed are consistent with what would be expected in that the geodemographic types that are more likely to buy books are generally prospering or aspiring groups that contain well off educated populations. The types that were indicated to not be as likely to buy books were mostly struggling groups, which contain a relatively poor population who are less educated.

#### *5.5.4 Assumptions on which the Geodemographic Demand Model is based*

- (1) People in each geodemographic group are homogeneous and will therefore have the same spending habits.
- (2) The geodemographic classification given to an area is representative of all residents of that area.

#### *5.5.5 Formal exposition of the Geodemographic Demand Model*

The population of geodemographic type  $t$  resident in postal district  $i$  is defined as  $P_i^t$ , and is the sum of the number of people in each postcode who are in each geodemographic type in the postal district. Therefore

$$P_i^t = \sum_{u \in t} P_i^u \quad (5.16)$$

where  $P_i^u$  is the number of people in each postcode  $u$  in each postal district  $i$ . The populations are subsequently summed for postcode units in each geodemographic type.

$O_i^g$ , the demand for good  $g$  in postal district  $i$ , is estimated as follows

$$O_i^g = \sum_t (\bar{s}^{tg} P_i^t) \quad (5.17)$$

where  $\bar{s}^{tg}$  is the level of spending on good  $g$  by people of geodemographic type  $t$  which is calculated thus

$$\bar{s}^{tg} = \bar{p}^g \bar{n}^{tg} \quad (5.18)$$

where

$\bar{p}^g$  = the average price of a good  $g$

$\bar{n}^{tg}$  = the average number of good  $g$  bought by people in geodemographic group  $t$

### 5.5.6 Validation of results obtained from the Geodemographic Demand Model

The demand estimates from the Geodemographic Demand Model were validated using the National Market Survey. Two types of correlation were undertaken, using different information from the National Market Survey. Correlations were calculated between average demand estimates for books and the National Market Survey ratios for books, and demand estimates and the average number of books bought.

Table 5.14: Correlations between Geodemographic Demand Model average demand estimates for books and National Market Survey ratios, for postal districts

	r	r <sup>2</sup>	r <sub>s</sub>
postal districts with ≥ 10 responses	0.26	0.07	0.22
postal districts with ≥ 50 responses	0.26	0.07	0.24

Table 5.14 indicates that the correlation coefficient  $r$ , is significant at the 95% level at both levels of analysis. The  $r^2$  value is the same at both levels of analysis, although  $r_s$  is slightly higher for postal districts with more than fifty responses. Comparison of Table 5.14 and Table 5.7 shows that for postal districts with ten or more responses, the Geodemographic Demand Model performs better, and has a higher  $r^2$  value and a higher significance level than the GMAP Demand Model, but a lower value of  $r_s$ . However, for postal districts with fifty or more responses the Geodemographic Demand Model produces a lower  $r^2$  and  $r_s$ , but the same significance level of  $r$ .

Table 5.15: Correlations between Geodemographic Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal districts

	$r$	$r^2$	$r_s$
postal districts with $\geq 10$ responses	0.34	0.11	0.28
postal districts with $\geq 50$ responses	0.23	0.05	0.23

The information shown in Table 5.15, indicates that the correlation coefficient is higher for postal districts with ten or more responses than for postal districts with fifty or more responses, although  $r$  is significant at the 95% level at both levels. Comparison with the GMAP Demand Model correlations between average demand and average number of books bought, which are shown in Table 5.9, shows that again the Geodemographic Demand Model outperforms the GMAP Demand Model for postal districts with ten or more responses but not for postal districts with fifty or more responses.

Comparison of scatterplots of National Market Survey ratio against average demand for the GMAP Demand Model and the Geodemographic Demand Model, as shown in Figure 5.3 and Figure 5.4 below show that the patterns are similar and that several of the outliers are the same in the two plots. This could indicate that it is the survey data that is causing these outliers to occur and therefore the survey data could be the cause of the low correlation coefficients being calculated for the demand models.

Figure 5.3: Scatterplot of average demand for books calculated from the Geodemographic Demand Model and National Market Survey ratio for books, for postal districts with ten or more responses

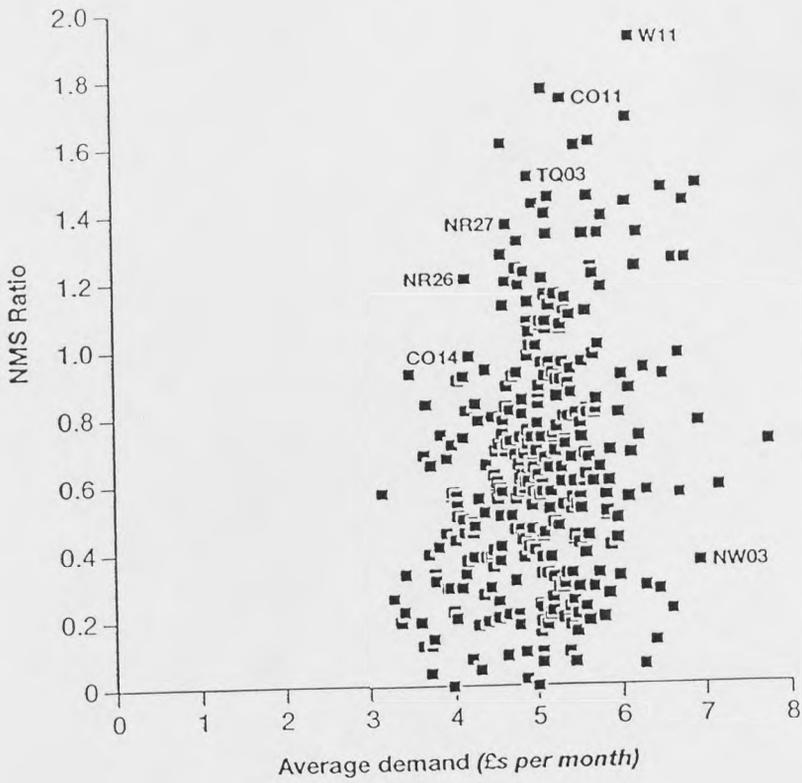
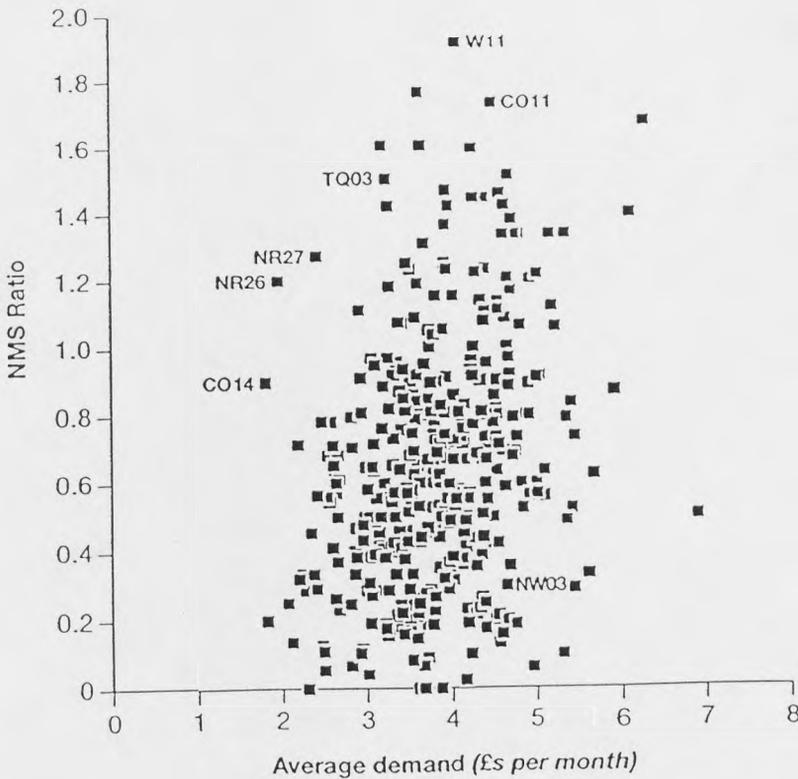


Figure 5.4: Scatterplot of average demand for books calculated by the GMAP Demand Model and National Market Survey ratio for books, for postal districts with ten or more responses



Correlations were also undertaken at higher levels of geographical resolution. The results are as follows:

Table 5.16: Correlations between Geodemographic Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal areas

	$r$	$r^2$	$r_s$
all postal areas	0.10	0.01	0.29
postal areas with $\geq 50$ responses	0.14	0.02	0.33
postal areas with $\geq 100$ responses	0.14	0.02	0.34

Unlike for the GMAP Demand Model, the Geodemographic Demand Models average demand estimates do not improve at this geographical level.

Table 5.17: Correlations between Geodemographic Demand Model total regional demand estimates for books and the regional demand calculated from the Family Expenditure Survey, for standard regions

	$r$	$r^2$	$r_s$
regions	0.98	0.96	0.90

Comparison of Tables 5.16 and 5.17 with Tables 5.10 and 5.11 indicate that at these higher levels of geographical resolution the GMAP Demand Model produces better average demand estimates than the Geodemographic Demand Model.

## 5.6 THE INCOME DEMAND MODEL

### 5.6.1 Introduction

It was seen in Section 5.4 that disposable income was the most important variable in determining book buying behaviour. Therefore, a further method of estimating demand is to break down the population of postal districts according to which income band each household belongs to. Once the population has been broken down into income bands, this information can be combined with estimates of demand by income bands in order to calculate average expenditure estimates for postal districts.

### 5.6.2 Data requirements

Data concerning the income of individuals and households is not available in the census and therefore an alternative data source must be found. The income data used in this model is taken from a National Demographics and Lifestyles (NDL) database which has been purchased by GMAP. This database is compiled from customer questionnaires obtained from guarantee forms completed on the purchase of durable consumer goods. NDL have collected over 10 million questionnaires through this process and hold information on approximately half of the individuals and households in the country (Birkin 1995). The NDL database is used to provide the number of households in each postal district that are in each income band. The income band (where income relates to disposable income per annum) is denoted by  $y$ .

$y=1, \dots, 12$ , where:

$y=1=<£5000$ ,  $y=2=£5000-<£7500$ ,  $y=3=£7500-<£10000$ ,  $y=4=£10000-<£12500$ ,  $y=5=£12500-<£15000$ ,  $y=6=£15000-<£17500$ ,  $y=7=£17500-<£20000$ ,  $y=8=£20000-<£22500$ ,  $y=9=£22500-<£25000$ ,  $y=10=£25000-<£30000$ ,  $y=11=£30000-<£35000$ ,  $y=12=£35000+$

Data is also required concerning the average expenditure of households in each income band. This information is extracted from the 1994/95 Family Expenditure Survey at MIDAS.

### 5.6.3 Assumptions on which the Income Demand Model is based

(1) Income is the only variable to affect the demand for books.

(2) All households in each income group will conform to the mean expenditure for that income group as calculated from the Family Expenditure Survey.

### 5.6.4 Formal exposition of the Income Demand Model

The number of households in income band  $y$  in postal district  $i$  is defined as  $H_i^y$ .  $O_i^g$  the demand for good  $g$  in postal district  $i$  is calculated by multiplying average demand estimates by income range,  $\bar{s}_i^{gy}$  with the number of households in each income range in the postal district.

$$O_i^g = \sum_y (\bar{s}_i^{gy} \times H_i^y) \quad (5.19)$$

Therefore average demand per household for good  $g$  in postal district  $i$  is calculated as follows

$$\bar{s}_i^g = \frac{\sum_y (\bar{s}_i^{gy} \times H_i^y)}{H_i} \quad (5.20)$$

### 5.6.5 Validation of results obtained from the Income Demand Model

The demand estimates were validated using the National Market Survey. Validations were undertaken for books in order to allow comparisons to the other demand model estimates to be made. Correlations have been undertaken between average household demand for books as estimated by the Income Demand Model and average number of books bought taken from the National Market Survey, and also between average demand estimates and National Market Survey ratios. Correlations have been calculated at three levels of analysis, postal districts, postal areas and standard regions. Correlations at regional level are between estimated total regional demand and regional demand calculated from the Family Expenditure Survey.

Table 5.18: Correlations between Income Demand Model average demand estimates and National Market Survey ratios, for postal districts

	$r$	$r^2$	$r_s$
postal districts with $\geq 10$ responses	0.22	0.05	0.21
postal districts with $\geq 50$ responses	0.19	0.04	0.21

The value of  $r$  is significant at both levels of analysis for the Income Demand Model. Comparison to Tables 5.7 and 5.14 show that there is not a large amount of difference in performance of the Income Demand Model to the other models, but that the goodness of fit statistics are slightly lower.

Comparison of Figure 5.5 below, with Figures 5.3 and 5.4 indicates that for the Income Demand Model there is less variation in the demand estimates and that the estimates tend to be slightly higher than those for the GMAP Demand Model and the Geodemographic Demand Model. However, the labelled postal districts are still outliers which is further evidence of problems in using the National Market Survey in the validation process.

Figure 5.5: Scatterplot of average demand for books calculated by the Income Demand Model and National Market Survey ratio for books, for postal districts with ten or more responses

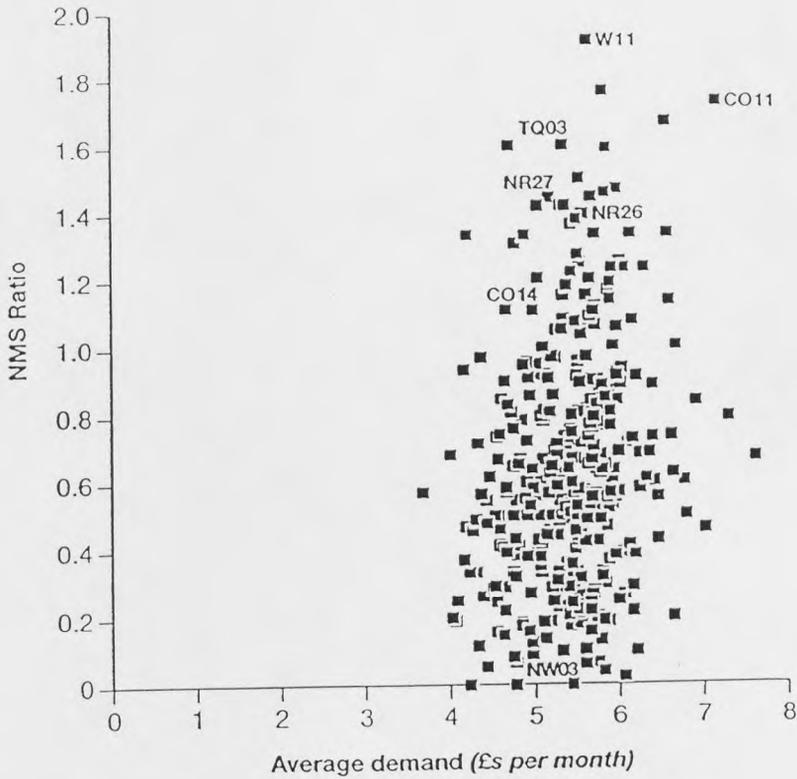


Table 5.19: Correlations between Income Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal districts

	$r$	$r^2$	$r_s$
postal districts with $\geq 10$ responses	0.22	0.05	0.22
postal districts with $\geq 50$ responses	0.20	0.04	0.24

The  $r^2$  and  $r_s$  both improve slightly through the use of average number of books bought as the validation variable, but comparison with Tables 5.9 and 5.15 show that the Income Demand Model still does not perform as well as the GMAP Demand Model or the Geodemographic Demand Model for postal districts.

Table 5.20: Correlations between Income Demand Model average demand estimates for books and the average number of books bought (taken from the National Market Survey), for postal areas

	$r$	$r^2$	$r_s$
all postal areas	0.38	0.15	0.35
postal areas with $\geq 50$ responses	0.41	0.17	0.39
postal areas with $\geq 100$ responses	0.45	0.20	0.45

The performance of the Income Demand Model has improved significantly with the move to postal area level. Comparison with Tables 5.10 and 5.16 also indicate that the Income Demand Model now produces goodness of fit statistics that are higher than for both the GMAP Demand Model and the Geodemographic Demand Model at this geographical scale.

Table 5.21: Correlations between Income Demand Model total regional demand estimates for books and regional demand calculated from the Family Expenditure Survey, for standard regions

	$r$	$r^2$	$r_s$
regions	0.98	0.97	0.90

At the regional level, as was seen with the other models, the Income Demand Model performs well, with values of  $r^2$  and  $r_s$  that are the same as for the GMAP Demand Model and marginally higher than for the Geodemographic Demand Model.

## 5.7 WORKPLACE BASED AND TOURIST DEMAND

As was seen in Section 4.2.3 the WH Smith model is disaggregated into three trip types, residential, work based and tourist flows. The demand estimation procedures described above produce estimates of total demand for each postal district. To create separate residential, work based and tourist demands, this total demand estimate is divided. Firstly, before this is undertaken the demand estimate is decreased by 12% to account for book purchases that are undertaken through mail order.

GMAP currently estimate that 40% of the demand of a postal district will be work based as opposed to residence based. Separate tourism demand is only used in a few centres

that are being predicted incorrectly such as some Central London centres and at places where a lot of travel occurs such as airports and the Channel Tunnel terminal. There are no such places in the Yorkshire TV region, therefore tourism demand is set to zero for the postal districts in this region.

Thus, of the total postal district demand estimates produced for the Yorkshire TV region, 60% is allocated to residential demand, 40% to work based demand and 0% for tourist demand.

Analysis concerning workplace based interactions and tourism interactions will not be undertaken here but will be looked at in Chapter 6.

## 5.8 TESTING THE DEMAND ESTIMATES IN THE WH SMITH MODEL

In order to test which of the demand estimates produced the best model results, the new estimates from the Income Demand Model and the Geodemographic Demand Model were used in the GMAP Full Model as described in Chapter 4. This was undertaken for books. The results are shown in Table 5.22.

Table 5.22: Goodness of fit statistics for the GMAP Full Model using the alternative demand models, for books

Demand Model	SSE	$r^2$	$r_s$
GMAP	3941151	0.97	0.88
Geodemographic	4968828	0.96	0.87
Income	3421808	0.96	0.89

The  $r^2$  and  $r_s$  statistics do not show much variation between the performance of the model using each different demand type. However, the SSE indicates that of the three alternative demand models the Income Demand Model produces the best model performance with a decrease in the SSE of 13% from the GMAP Demand Model. The Geodemographic Demand Model causes SSE to increase by 26% from the GMAP Model.

## 5.9 CONCLUSIONS

The analysis undertaken for testing the alternative demand estimates indicates that at the postal district level there is not a significant difference between the performance of the three demand models, although the Geodemographic Demand Model did perform slightly better. At the postal area level, the Income Demand Model produced performance levels that were significantly higher than for the GMAP Demand Model and the Geodemographic Demand Model. For correlations undertaken at the geographical scale of standard regions, all three models performed well, with the Geodemographic Demand Model performing only slightly less well than the other two demand models. Therefore, from this analysis it is difficult to distinguish between the alternative demand models and make a decision on which model is the most appropriate for use in the WH Smith spatial interaction model.

Another reason why it is difficult to base judgements on which is the best demand model to use on the analysis undertaken in Sections 5.3, 5.5 and 5.6 is the validation procedure used. It can be seen that at the lower levels of geographical resolution (postal districts and postal areas) the correlations produced are low. This could be due to the National Market Survey not being an ideal data source for use in the validation of demand estimates. Analysis of the data contained in the National Market Survey shows that there is uneven coverage of respondents across the country which could cause bias in the validation procedure. There are also very small sample sizes for some postal districts. Comparison of the scatterplots in Figures 5.3, 5.4 and 5.5 show that several of the outliers are the same in all models, this would indicate that it is the survey data that is causing these postal districts to be outliers and therefore the survey data is contributing to the low values for the correlation coefficients achieved in the validation procedure. The fact that good correlations are achieved at regional level, where the demand estimates are correlated against demand estimates calculated from the Family Expenditure Survey could also be indicative of the National Market Survey contributing to the low correlation coefficients produced in this analysis. Therefore, in order to undertake an improved validation of the demand estimation models it would be necessary to use an alternative source of data.

Due to the difficulties encountered in using the off-line demand model experiments to choose which model to use, the results obtained from using the alternative demand estimates in the WH Smith spatial interaction model, and the logit analysis will be used.

The logit analysis undertaken in Section 5.4 indicated that two of the variables used in the GMAP Demand Model were not significant at the 95% level in determining book buying behaviour, but the variable disposable income caused a large decrease in the deviance of the logit model. This could indicate that the Income Demand Model will be the most appropriate to use for estimating demand for the WH Smith model. The integration of the new demand estimates into the GMAP Full Model also indicated that the Income Demand Model produces improved goodness of fit for the spatial interaction model when compared to both the Geodemographic Demand Model and the currently used GMAP Demand Model.

The second component of the spatial interaction model is the supply side, this will now be considered in Chapter 6.

## AN INVESTIGATION OF THE SUPPLY SIDE IN THE WH SMITH SPATIAL INTERACTION MODEL

### 6.1 INTRODUCTION

As was seen in Section 2.2.1, the second component in spatial interaction modelling is the supply side. The supply component of the WH Smith spatial interaction model is denoted by  $W_j^\alpha$  and is a measure of the attractiveness of the shopping destination  $j$ .

During this chapter I will describe how the supply side is handled at present in the WH Smith spatial interaction model and then proceed to investigate possible methods of enhancing the supply side attractiveness measure in order to improve the performance of the model.

### 6.2 THE CURRENT METHOD OF CALCULATING ATTRACTIVENESS

At present, the attractiveness of a retail outlet is based upon the size of the outlet (which is given by the effective footage of the facility) and the size of the centre in which the outlet is located. Therefore the supply component of the spatial interaction model is made up of two main parts, store attractiveness and centre attractiveness. The effective footage (EFT) of a store is a measure derived by WH Smith and one EFT is approximately three feet high and one foot wide (Codling, 1995a). Factors called City Centre Factors (CCFs) are also included in the centre attractiveness factor as are individual store attractiveness factors. At present these factors are heuristic and are used to make centres more attractive if sales to centres or stores are being underpredicted by the model and vice versa for destinations that are being over predicted. Three types of CCFs are used, one each for residential, work and tourist attractiveness. The residential CCF is based on the size of the centre, the work CCF on the number of workers in the centre, and the tourism CCF is applied to centres at which a lot of travel occurs, such as Victoria and Waterloo in London. These CCFs are not empirically based and are not

imposed systematically on all centres, just those whose attractiveness needs adjusting in order to make the model an accurate predictive tool. It can therefore be seen that the main empirical basis of supply side attractiveness used at present is the size of the store and the centre in which it is located. However, as was seen in Section 3.4.2, a substantial amount of literature has been produced concerning other factors believed to be important in determining the attractiveness of retail stores and centres. Therefore it is possible that by implementing some alternative attractiveness measures, the explanatory power of the model could be improved.

To make the model an explanatory as opposed to a predictive tool the CCFs need to be given a more empirical and systematic basis. At present CCFs are calibrated by trial and error to find a value that makes the model fit the observed data in order to improve predictions made by the model. In order to make the model explanatory it is necessary to discover why centres are being over and under predicted rather than simply accepting that they are being predicting incorrectly and subsequently adjusting their attractiveness.

### **6.3 METHOD OF ANALYSIS**

In order to investigate the supply side I will use the Yorkshire TV region. This region has been chosen because local knowledge of the centres will aid the analysis. It was decided to restrict the analysis to one region because this will enable a more detailed analysis of the supply side to be undertaken. Within the Yorkshire TV region there are 29 centres that contain WH Smith Group stores, these centres are shown in Figure 4.1.

In order to investigate the supply side for WH Smith I will use the spatial interaction model developed by GMAP for WH Smith as described in Chapter 4. In order to build up an alternative view of the supply side I will take out the CCFs and individual store attractiveness factors used at present in order to produce the GMAP Base Model introduced in Section 4.2.3 and this will be used as the basis for analysis of the supply side. I will leave the basic EFT based size attractiveness as it is because size will be an important factor in the determination of attractiveness. From this basic model I will try several alternative additions to attractiveness in order to see if model performance can be improved on a more empirical basis than the method used at present for the creation of the CCFs and individual store factors.

Table 6.1: Goodness of fit statistics for the GMAP Base Model, for centres in the Yorkshire TV region containing WH Smith Group stores

Good Type	SSE	$r^2$	$r_s$
All Goods	60299628	0.92	0.96
Newspapers	922499	0.83	0.87
Books	13397108	0.92	0.86
Stationery	767143	0.73	0.87
Music	5307744	0.94	0.95
Cards	86690	0.87	0.82
Video	1475182	0.89	0.93

Table 6.2: Goodness of fit statistics for the GMAP Full Model, for centres in the Yorkshire TV region containing WH Smith Group stores

Good Type	SSE	$r^2$	$r_s$
All Goods	8559324	0.98	0.96
Newspapers	84204	0.97	0.87
Books	3941151	0.97	0.86
Stationery	82946	0.96	0.87
Music	1042299	0.97	0.95
Cards	10917	0.97	0.82
Video	247220	0.97	0.93

It can be seen from Tables 6.1 and 6.2 that the current GMAP formulation of the supply side does produce good model performance figures and is significantly better than the GMAP Base Model (in which attractiveness is based simply on store and centre EFT). However, this is due to the fact that the CCFs and individual store factors are used to improve model performance, but these factors do not have any theoretical or empirical justification at present. Therefore the aim of this chapter is to improve the performance of the model from the GMAP Base Model in a more empirical manner. It may not be possible to reach model performance levels equal to those achieved using heuristic CCFs and individual store attractiveness factors, but if the model is to have any explanatory meaning then the attractiveness factor should be based on empirical analysis.

### *6.3.1 Possible supply side improvements*

There are several possible ways of improving the supply side measure in the WH Smith spatial interaction model.

#### *6.3.1.1 Centre revenue prediction*

The first step in the analysis of the attractiveness of supply points is to ensure that the amount of revenue being predicted to centres is correct and therefore ensure that centre attractiveness is being devised correctly. If it is found that large centres are being consistently under or over predicted, then it is possible to rectify this through an alteration of the  $\alpha$  parameter. The alpha parameter is added to the  $W_j$  variable to take account of consumers' perceptions of the advantages of larger centres. Centre attractiveness and therefore revenue predictions are also effected by the CCFs, which will be analysed.

#### *6.3.1.2 Individual site attractiveness*

Attractiveness factors associated with single stores could also affect the performance of stores. Several factors could influence the performance of an individual store and these will be investigated to see if their inclusion will improve model performance.

#### *6.3.1.3 Market segmentation*

Once centre revenue is being predicted accurately then there is the question of how that revenue is to be split between different store brands. Should revenue be split according to national market share or are there local variations that should be taken into account?

## 6.4 CENTRE REVENUE PREDICTION

### 6.4.1 Introduction

It is necessary to investigate if the revenues of centres are being predicted accurately in order to discover if centre attractiveness is correct and therefore identify if there is more to centre attractiveness than simply the combined EFTs of the stores contained within the centre. To do this, the performance of WH Smith Group stores are investigated in relation to the size of the centre in which they are located. If it is found that stores are over performing (*i.e.* the model is under predicting flows to these stores) in large centres this could indicate that large centres have an added attractiveness above the combined EFTs of their component stores.

One way of investigating if performance is related to centre size is to undertake correlations between the centre performance factor which is shown in equation (6.1) below and certain centre size proxies.

$$\text{centre performance} = (\text{observed centre revenue}/\text{predicted revenue}) * 100 \quad (6.1)$$

The centre size proxies used were

- Centre EFT
- Centre weighted EFT (weighted according to brand attractiveness)
- Catchment population of centre
- Centre revenue
- Number of stores in the centre
- Centre type

The centre type variable was produced by WH Smith, and classifies centres as follows

- 1) Metropolitan area
- 2) Outer metropolitan area

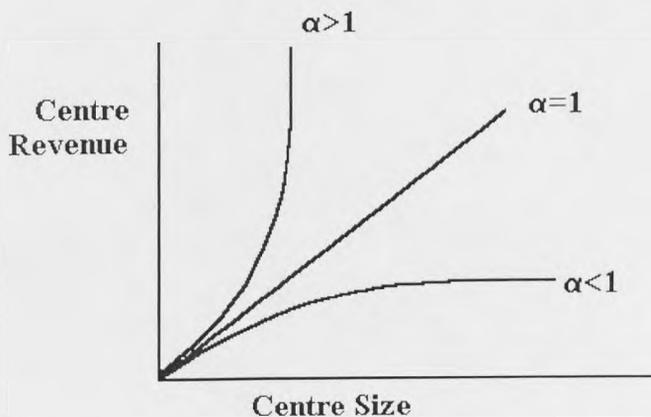
- 3) Stand alone centre
- 4) More rural centres
- 5) Villages
- 6) Airports, stations

If it is found that larger centres are systematically over performing then it is necessary to make larger centres more attractive in the model. This is undertaken through the alteration of the alpha value.

#### 6.4.1.1 The alpha parameter

The alpha parameter is added to the attractiveness term in the following way,  $W_j^\alpha$ . This parameter represents consumers' perception of economies of scale in the retailing trade. If consumers find large centres proportionately more attractive then the alpha value is greater than one. If consumers find small centres proportionately more attractive then alpha is less than one. Alpha is set equal to one if consumers are neutral to the effect of centre size on attractiveness. The relationship between centre size and centre revenue for different alpha values are shown in Figure 6.1 below.

Figure 6.1: The alpha parameter



Therefore, if it is found that larger centres are consistently overperforming then this can be rectified through alterations in the value of the alpha parameter. At present the value of alpha in the WH Smith spatial interaction model is set to 1.3 for all goods in all centres.

### 6.4.2 Good-specific alpha parameter

Both Pearson's product moment correlations and Spearman's rank correlations will be useful in indicating the relationship between store performance and the centre size proxies. For the Pearson's correlation  $r$  is indicated rather than  $r^2$  in order to display the direction of the relationship.

Table 6.3: Correlations between centre performance and centre size proxies, for all goods, for centres in the Yorkshire TV region containing WH Smith Group stores

Centre Size Proxy	$r$	$r_s$
Centre Revenue	0.01	0.05
Catchment Population	0.07	0.07
Centre EFT	0.22	0.21
Centre Weighted EFT	0.21	0.21
Centre Type	-0.10	0.05
Number Of Stores	0.09	0.15

It can be seen from Table 6.3 that there is no consistent relationship between centre size proxies and centre performance. However this table shows results for all good types, but there is no reason to believe that consumers will behave in the same way for each good type that is sold by WH Smith Group stores. Each product sold by WH Smith is very different and will therefore instigate different consumer behaviour. Therefore it is necessary to undertake the correlations for each good type to see if the relationship between performance and centre size varies by good type. If it is found that there are individual relationships for certain good types then it may be necessary to disaggregate the alpha parameter according to good type in order to account for differing consumer behaviour for different good types.

Table 6.4: Correlations between centre performance and centre size proxies, for individual good types, for centres in the Yorkshire TV region containing WH Smith Group stores

Centre Size Proxy	Papers	Books	Stationery	Music	Cards	Video
Centre Revenue	-0.23	0.24	0.01	-0.09	-0.28	-0.12
Catchment Population	-0.23	0.26	0.00	-0.08	-0.28	-0.09
Centre EFT	-0.12	0.45	0.14	0.09	-0.09	0.11
Centre WEFT	-0.13	0.43	0.12	0.05	-0.11	0.10
Centre Type	0.16	-0.23	-0.18	-0.11	0.03	0.12
Number Of Stores	-0.09	0.43	0.12	-0.04	-0.01	-0.14

Table 6.5: Spearman's rank correlations between centre performance and centre size proxies, for centres in the Yorkshire TV region containing WH Smith Group stores

Centre Size Proxy	Papers	Books	Stationery	Music	Cards	Video
Centre Revenue	-0.26	0.43	0.10	0.11	-0.33	-0.24
Catchment Population	-0.17	0.41	0.11	-0.09	-0.34	-0.20
Centre EFT	-0.05	0.67	0.18	0.11	-0.13	-0.04
Centre WEFT	-0.13	0.67	0.19	0.09	-0.19	-0.21
Centre Type	0.34	-0.23	-0.06	0.10	0.15	0.19
Number Of Stores	-0.08	0.38	0.21	0.03	-0.07	-0.06

From Table 6.4 and Table 6.5 it can be seen that most of the correlation figures are low, with the exception of books which show a fairly strong positive relationship between centre size and centre performance in the GMAP Base Model. This would indicate that consumers see more of an advantage of larger centres concerning book buying than is currently accounted for in the model and therefore an alternative alpha value may be required. It can also be seen that the relationship between centre performance and centre size proxies varies for different good types. Therefore an analysis was undertaken using good-specific alpha values to see if the performance of the GMAP Base Model could be improved in this way.

The process of investigating alternative alpha values was undertaken iteratively. For each good at each alpha value the predicted revenue of the 29 centres was compared against the observed centre revenue. If the goodness of fit statistics improved then the

alpha value was either increased or decreased more. This process was continued until the goodness of fit statistics indicated that no improvement in model performance is achieved through the alteration of the alpha value. The goodness of fit statistic used was sum of squares of error (SSE) between observed and predicted centre revenues.

Table 6.6: Goodness of fit statistics for the GMAP Base Model with good-specific alpha values for centres in the Yorkshire TV region containing WH Smith Group stores

Good	Optimal Alpha	SSE	$r^2$	$r_s$
All Goods		59382412	0.86	0.96
Newspapers	1.20	889721	0.68	0.87
Books	1.40	12605170	0.85	0.87
Stationery	1.25	766463	0.54	0.86
Music	1.10	4836109	0.87	0.95
Cards	1.20	83151	0.75	0.82
Video	1.25	1471973	0.79	0.93

It can be seen from Table 6.6 that it was found that 1.3 was not the optimal value of alpha for any good type. The change in alpha values for each good type has led to a slight improvement in goodness of fit statistics in each case. The new model performance figures for the GMAP Base Model including the good-specific alpha values are shown in Table 6.6 in the all goods row. Through comparison of Table 6.1 and Table 6.6 it can be seen that  $r^2$  has increased slightly and the SSE has also decreased by a small amount, but only by 1.5%. Therefore, although model performance has been improved by the inclusion of good-specific alpha values it is only by a small amount. Thus good specific alpha values will not be used in the following analysis due to their only having a small effect on model performance.

### 6.4.3 Centre attractiveness

Another way of improving centre revenue predictions is to investigate the way centre attractiveness is formulated. At present centre attractiveness is based on; the size of centre which is included in the model through the EFT of the centre, the attractiveness of stores in the centre and the three types of CCFs. These factors are not systematically empirically formulated and therefore need to be reconsidered in order to increase the

explanatory power of the model. As was seen in Section 3.4.2 there are also many other factors thought to influence the attractiveness of centres that should therefore be included in the model.

For this analysis the three types of CCFs will be removed from the GMAP Full Model to produce the NoCCF Model. The other factors in the attractiveness calculation such as individual store attractiveness will be retained in the model. The goodness of fit statistics for the NoCCF Model are presented in Table 6.7 below.

Table 6.7: Goodness of fit statistics for the NoCCF Model for centres in the Yorkshire TV Region containing WH Smith Group stores

Good	SSE	$r^2$	$r_s$
All Goods	43628664	0.89	0.97
Newspapers	463470	0.86	0.94
Books	7989307	0.92	0.91
Stationery	284348	0.83	0.96
Music	4587059	0.89	0.99
Cards	51245	0.86	0.90
Video	1341480	0.82	0.96

It can be seen through comparison of Table 6.1 and Table 6.6 that the removal of the CCFs causes a significant decrease in model performance. Therefore it is necessary to investigate alternative centre attractiveness factors to the CCFs in order to increase the performance of the NoCCF Model.

The aim of the centre attractiveness analysis is to try and discover factors that will aid the explanation of why some centres are over or under performing. The centre performance is derived from summing the observed and predicted revenues for all WH Smith Group stores, for all goods within each centre. Subsequently, a centre performance factor, as in equation (6.1) is calculated. The centre performance levels for the 29 Yorkshire centres for the NoCCF Model, ranked with the highest performing centres at the top, are shown in Table 6.8 below.

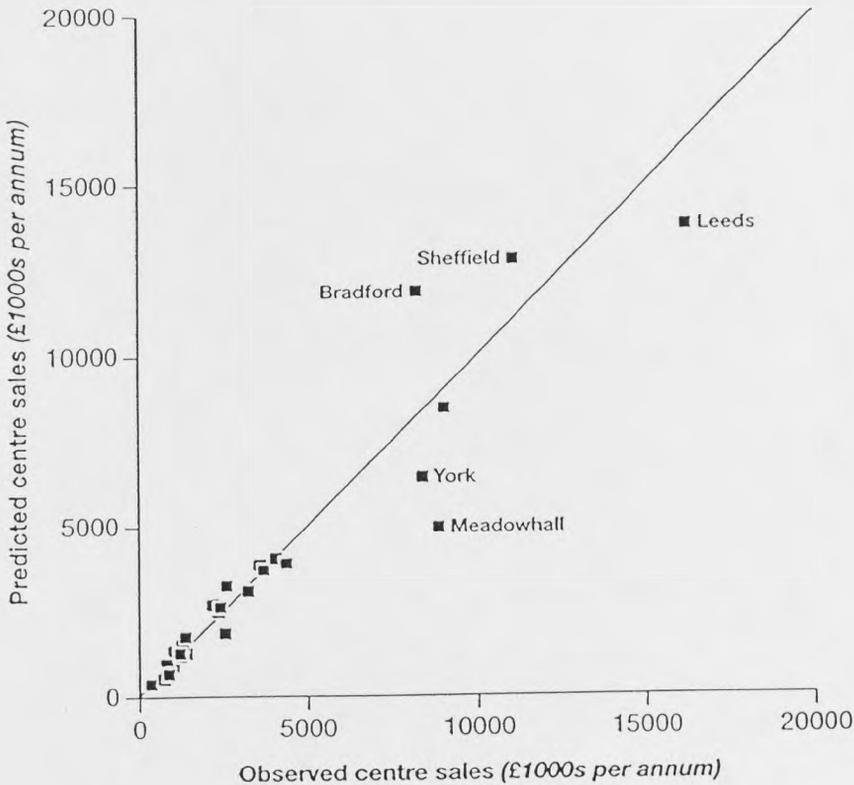
Table 6.8: Centre performance levels for centres in the Yorkshire TV region containing WH Smith Group stores, for the NoCCF Model

Centre	Observed Revenue	Predicted Revenue	Centre Performance
Meadowhall	8822	4980	177.2
Scarborough	2530	1886	134.1
Ilkley	733	557	131.5
York	8360	6460	129.4
Skipton	863	692	124.6
Leeds	16215	13764	117.8
Lincoln	4351	3939	110.5
Boston	1387	1284	108.0
Beverley	1003	935	107.3
Hull	8997	8469	106.2
Wakefield	3223	3123	103.2
Huddersfield	4054	4087	99.2
Dewsbury	1229	1244	98.8
Grimsby	3685	3737	98.6
Spalding	1198	1293	92.6
Halifax	2318	2509	92.4
Doncaster	3581	3894	92.0
Scunthorpe	2377	2645	89.9
Rotherham	1188	1350	88.0
Sheffield	11028	12800	86.2
Gainsborough	338	408	82.8
Retford	818	993	82.4
Skegness	1263	1543	81.8
Keighley	2160	2707	79.8
Barnsley	2566	3279	78.2
Pontefract	1341	1744	76.9
Chesterfield	1040	1360	76.4
Worksop	1364	1786	76.3
Bradford	8191	11871	69.0

Source: WH Smith Sales Dept. Units: £1000s per annum. 1995

The scatter plot shown in Figure 6.2 below also represents the differences in performance between centres for the NoCCF Model, with centres below the 45 degree line over performing (being under predicted in the model) and those above the line under performing (being over predicted in the model).

Figure 6.2: Scatterplot between observed and predicted centre revenues for centres in the Yorkshire TV region Containing WH Smith Group stores, for the NoCCF Model



It can be seen from Table 6.8 and Figure 6.2 that there is a significant variation in performance levels between centres with some centres over performing and others under performing. Table 6.8 indicates that the attractiveness of centres is not simply dependent on their size, with small centres such as Ilkley and Skipton having high performance figures and large centres such as Bradford and Sheffield underperforming by a large amount.

There could be several reasons for such variations in performance. Meadowhall receives centre revenue nearly double that expected for its size. This centre is a special case in that it is a large, out of town, regional shopping centre that attracts people from

long distances in a way that city centres do not. Also, from considering the EFT of WH Smith goods in Meadowhall, it can be seen that it is not a particularly large centre in terms of these goods. However, the WH Smith Group stores still do exceptionally well in this centre. This could be because a large amount of people go to this centre in order to take advantage of the large amount of comparison shopping that is available for goods such as clothing and shoes. Therefore WH Smith Group stores may not be the primary destination of the customer in Meadowhall but such stores benefit from the attractiveness of the centre in terms of other store types. Therefore there are other factors at work besides the size of the WH Smith Group stores in the centre. This factor makes revenue for centres such as Meadowhall difficult to predict accurately because different factors are at work in such a centre than are apparent in city centres.

Other centres are also over predicted but this could be due to different reasons. For instance it can be seen that the centre of Scarborough is under predicted by 35%. This could be due to the large amount of tourism that occurs in the town. Tourism could also be a factor in the under prediction of centre revenues for York, Ilkley and Skipton.

The centre that is being most significantly over predicted is Bradford. Bradford contains lots of independent shops, many of which are low class bargain shops, and this could explain why the city doesn't attract as much revenue as it should given its size. The under performance of centres such as Worksop, Pontefract and Barnsley could also be explained through this factor.

It is necessary to try and formulate a centre attractiveness function that will explain these differences in performance. Such a function could be made up of several factors. The factors decided upon to investigate in relation to centre attractiveness are discussed in the next section.

#### *6.4.3.1 Possible factors influencing centre attractiveness*

A variety of factors thought to influence shopping behaviour will be analysed with reference to the centre attractiveness of the centres in the Yorkshire TV region that contain WH Smith Group stores.

- 1) Provision of parking facilities and general transport factors will be analysed to see if centres with better access are more attractive.
- 2) The number of key stores such as department stores in centres will be tested to see if this factor influences performance and therefore attractiveness of centres.
- 3) The number of banks and building societies will also be investigated because of the possibility of multiple purpose trips.
- 4) The proportion of stores in the centre that are retail multiples. This factor is based on the hypothesis that multiples will be more attractive to consumers and therefore centres with a large percentage of retail multiples will be more attractive.
- 5) The percentage of stores in a centre that are in undercover centres or in pedestrianised areas will also be calculated in order to see if this factor is important in determining centre attractiveness.
- 6) General attractiveness of the shopping centre including factors such as general centre appearance and cleanliness will be tested as an attractiveness measure. The level of facilities available in the centre as well as the number of alternative attractions will also be included in this measure.
- 7) A tourism factor will also be investigated. This will be based on the number of hotels in the centre. This factor will be used to replace the tourism CCF in the spatial interaction model.
- 8) A work based factor based on the number of workers in the centre will also be analysed. This factor will be used to replace the work CCF in the model.

#### *6.4.3.2 Data sources*

The data required for the centre attractiveness analysis will come from a variety of sources. Information concerning the types of shops in centres and parking facilities can be extracted from GOAD city centre plans. These maps provide a detailed breakdown

of which shops are located in centres. The plans also provide information concerning car parks, including the number of spaces in each car park. These maps can be used to find out how many key stores, banks and parking spaces are present in centres, and also provide the information required to calculate the proportion of retail multiples in each centre. The GOAD city centre plans will also be used to calculate the percentage of a centre's stores that are undercover or in a pedestrianised area.

Fieldwork involving visiting the centres in question was also undertaken. This was in order to update the GOAD plans because many that are available are out of date. Fieldwork also enabled subjective judgements on the quality of the shopping centre to be made more accurately, for the formulation of the field survey attractiveness factor.

For the replacement of the tourism and work CCFs census data will be utilised. The Special Workplace Statistics will be used to collect data concerning the characteristics of workers in centres. Information from the 1991 Census Small Area Statistics concerning the number of hotels in a centre will be used to construct a tourism factor.

#### *6.4.3.3 Objective attractiveness factors*

The objective factors concerning attractiveness are those that can be stated specifically, such as number of parking spaces, percentage of retail multiples, percentage of shops undercover or pedestrianised, number of banks and building societies and the number of department stores. The data collected through fieldwork and from GOAD city centre plans for the 29 chosen centres concerning the objective attractiveness factors are shown in Table 6.9 below. The centres are listed in order of performance, with the best performing centres at the top.

#### *6.4.3.4 Subjective attractiveness factors*

Subjective factors are based on personal opinions and value judgements. In the case of shopping centre attractiveness, factors such as the general desirability of the shopping centre will be formulated subjectively based on judgements made during fieldwork. Centres were marked on a scale of 1 to 5 (with 5 being the highest score) on five factors. These factors were the number of restaurants in the centre, level of facilities, other

attractions in the centre, general centre attractiveness and cleanliness. These five scores were subsequently added together to give the field survey attractiveness factor with a maximum score of 25. The values calculated for this factor are also shown in Table 6.9 below.

#### *6.4.3.5 Variable selection*

In order to test which of the above attractiveness factors were important in determining centre performance a logit analysis was carried out. A detailed description of the features of logit analysis is provided in Appendix A. For this logit analysis the categorical variable is whether a centre is over or under performing. Correlations were also undertaken in order to analyse the relationship between the attractiveness factors and the performance variable. The results of the logit analysis are as shown in Table 6.10.

Table 6.9: The observed attractiveness factors for centres in the Yorkshire TV region containing WH Smith Group stores

Centre	Parking Spaces	% Multis	% Covered/ Pedestrian	Banks/ Building Societies	Dept Stores	Field Survey Attractive - ness
Meadowhall	12000	77	100	6	7	25
Scarborough	1806	44	60	18	6	22
Ilkley	764	35	10	11	5	21
York	643	50	30	27	12	24
Skipton	840	33	15	10	5	21
Leeds	4680	57	60	40	15	22
Lincoln	2224	51	48	31	8	21
Boston	2090	32	21	15	7	17
Beverley	590	54	56	15	2	19
Hull	3019	56	59	41	13	19
Wakefield	2343	50	57	20	5	17
Huddersfield	2669	41	28	25	7	14
Dewsbury	1890	32	18	10	4	9
Grimsby	1953	47	60	19	6	14
Spalding	668	30	22	13	6	12
Halifax	1504	33	42	19	5	13
Doncaster	2530	34	25	27	11	16
Scunthorpe	2059	28	34	14	8	11
Rotherham	1578	39	30	16	3	10
Sheffield	4097	40	37	44	11	11
Gainsborough	385	25	31	8	4	11
Retford	642	30	45	10	2	15
Skegness	227	30	16	8	5	13
Keighley	1670	44	46	15	6	10
Barnsley	2320	48	34	23	6	10
Pontefract	1150	38	33	11	5	8
Chesterfield	1741	36	27	20	7	11
Worksop	1066	26	34	12	4	8
Bradford	1820	28	21	27	9	10

Table 6.10: Results of the logit analysis to test the importance of different attractiveness factors in determining centre performance

Attractiveness Factor	Deviance	Degrees of Freedom	Null Deviance - Model Deviance
Null model	38.50	28	
Parking spaces	31.65	25	6.85
% multiples	21.27	25	17.23
% covered	27.81	26	10.69
Banks/building societies	37.28	25	1.21
Dept. stores	37.28	26	1.21
Field Survey Attractiveness	20.72	25	17.77

Table 6.11: Results of the correlations between attractiveness factors and performance for centres in the Yorkshire TV region containing WH Smith Group stores

Attractiveness Factor	r	r <sup>2</sup>
Parking spaces	0.67	0.45
% multiples	0.67	0.45
% covered	0.54	0.29
Banks/Building Societies	-0.11	0.01
Dept. stores	0.16	0.03
Field Survey Attractiveness	0.83	0.69

Tables 6.10 and 6.11 show that the field survey attractiveness factor is the most important determinant of variations in centre performance, causing the largest decrease in deviance for the logit model and having the highest value of  $r^2$ . This is followed by the percentage multiples factor, the number of parking spaces and the percentage of stores that are undercover or in a pedestrianised area. The other variables examined were found not to be significant at the 95% level.

Therefore the centre attractiveness factors to be used to replace the current residential CCFs will comprise of data concerning the percentage of retail multiples in the centre, number of parking spaces, percentage of stores undercover or pedestrianised and the field survey attractiveness factor. It is now necessary to decide on the form of the attractiveness factor to be used.

#### 6.4.3.6 Formulation of the new attractiveness factor

The method chosen to include the new data concerning the attractiveness of centres was to find a level of factor that would make the revenue prediction for Meadowhall (the best performing centre) correct and subsequently factor the attractiveness measures for other centres in relation to their scores on the attractiveness variables.

A decision must also be made concerning how the attractiveness factor should be applied within the model. The new data on centre attractiveness could be incorporated in two ways, either through the alpha value or multiplicatively on the value of the size attractiveness variable, shown here as  $\sigma_j$ , in the same way that the CCFs are currently applied. Ideally an  $\alpha_j$  location-specific alpha should be used because this reflects that added attractiveness will be related to centre size. However, it was found that an  $\alpha$  value could not be found that would make Meadowhall attractive enough to produce a correct revenue prediction. Therefore it was decided to apply the new attractiveness factor multiplicatively to the  $\sigma_j$  value in the following way

$$W_j = \sigma_j \times \gamma_j \quad (6.2)$$

where  $W_j$  denotes total attractiveness and  $\gamma_j$  is the centre attractiveness factor.

The value of the attractiveness factor,  $\gamma_j$  that will make Meadowhall's revenue prediction correct is 2.0. Therefore this was set as the maximum value of the attractiveness measure  $\gamma_j$ .  $\gamma_j$  for other centres is calculated by factoring other centres according to their values on variables such as percentage of multiples compared to the value for Meadowhall. The scaled value of each factor for each centre is given by  $\eta_j^f$ , where  $f$  is the factor being considered,

$f = 1, \dots, 4$  where

$f = 1 =$  percentage multiples factor,  $f = 2 =$  parking factor,  $f = 3 =$  percentage covered or pedestrianised factor and  $f = 4 =$  field survey attractiveness factor

$\eta_j^f$  is formulated in the following way

$$\eta_j^f = \frac{v_j^f}{v_{j=M.hall}^f} \times 2.0 \quad (6.3)$$

where  $v_j^f$  is equal to the value of factor  $f$  for centre  $j$ .

However, as has been seen by the correlation coefficients in Table 6.11 each of these factors are not of equal significance in determining centre attractiveness, therefore weights have to be devised. These weights were calculated using the correlation coefficients shown above in the following way. The weight for each factor is given by  $\chi^f$  and is formulated as follows

$$\chi^f = \frac{r^f}{\sum_f r^f} \quad (6.4)$$

where  $r$  is the correlation coefficient.

The field survey attractiveness factor has the highest correlation coefficient of 0.83 and this relates to a weight of 0.3 in the attractiveness factor. The percentage of retail multiples and the number of parking spaces each have the same correlation coefficient of 0.67 and therefore receive equal weighting in the attractiveness factor of 0.25. The percentage of stores undercover or in a pedestrian area has the lowest correlation coefficient of the four factors at 0.54 and has a weighting of 0.2.

The proportion of the centre attractiveness factor  $\gamma_j$  taken up by each factor is the weight of the factor ( $\chi^f$ ) multiplied by the scaled value of that factor ( $\eta_j^f$ ). Therefore the value of the centre attractiveness variable for each centre is given as

$$\gamma_j = \sum_f \chi^f \times \eta_j^f \quad (6.5)$$

The values calculated for  $\gamma_j$  for the 29 centres in the Yorkshire TV region are shown in Table 6.12 below.

Table 6.12: Values of the  $\gamma_j$  centre attractiveness factor for centres in the Yorkshire TV region containing WH Smith Group stores

Centre	Scaled Multiples factor	Scaled Parking factor	Scaled Covered factor	Scaled Field Survey Attractiveness factor	Weighted Multiples factor	Weighted Parking factor	Weighted covered factor	Weighted Field Survey attractiveness factor	$\gamma_j$
Meadowhall	2.00	2.00	2.00	2.00	0.50	0.50	0.40	0.60	2.00
Scarborough	1.14	0.30	1.20	1.76	0.28	0.08	0.24	0.53	1.13
Ilkley	0.90	0.13	0.20	1.68	0.23	0.03	0.04	0.50	0.80
York	1.31	0.11	0.60	1.92	0.33	0.03	0.12	0.58	1.05
Skipton	0.86	0.14	0.30	1.68	0.22	0.04	0.06	0.50	0.81
Leeds	1.47	0.78	1.20	1.76	0.37	0.20	0.24	0.53	1.33
Lincoln	1.34	0.37	0.96	1.68	0.33	0.09	0.19	0.50	1.12
Boston	0.83	0.35	0.42	1.36	0.21	0.09	0.08	0.41	0.79
Beverley	1.39	0.10	1.12	1.52	0.35	0.02	0.22	0.46	1.05
Hull	1.45	0.50	1.18	1.52	0.36	0.13	0.24	0.46	1.18
Wakefield	1.30	0.39	1.14	1.36	0.33	0.10	0.23	0.41	1.06
Huddersfield	1.08	0.44	0.56	1.12	0.27	0.11	0.11	0.34	0.83
Dewsbury	0.84	0.32	0.36	0.72	0.21	0.08	0.07	0.22	0.58
Grimsby	1.21	0.32	1.20	1.12	0.30	0.08	0.24	0.34	0.96
Spalding	0.79	0.11	0.44	0.96	0.20	0.03	0.09	0.29	0.60
Halifax	0.87	0.25	0.84	1.04	0.22	0.06	0.17	0.31	0.76
Doncaster	0.88	0.42	0.50	1.28	0.22	0.11	0.10	0.38	0.81
Scunthorpe	0.73	0.34	0.68	0.88	0.18	0.09	0.14	0.26	0.67
Rotherham	1.01	0.26	0.60	0.80	0.25	0.07	0.12	0.24	0.68
Sheffield	1.09	0.68	0.74	0.88	0.27	0.17	0.15	0.26	0.85
Gainsborough	0.65	0.06	0.62	0.88	0.16	0.02	0.12	0.26	0.57
Retford	0.79	0.11	0.90	1.20	0.20	0.03	0.18	0.36	0.76
Skegness	0.77	0.04	0.32	1.04	0.19	0.01	0.06	0.31	0.58
Keighley	1.15	0.28	0.92	0.80	0.29	0.07	0.18	0.24	0.78
Barnsley	1.26	0.39	0.68	0.80	0.31	0.10	0.14	0.24	0.79
Pontefract	0.98	0.19	0.66	0.64	0.24	0.05	0.13	0.19	0.62
Chesterfield	0.95	0.29	0.54	0.88	0.24	0.07	0.11	0.26	0.68
Worksop	0.69	0.18	0.68	0.64	0.17	0.04	0.14	0.19	0.54
Bradford	0.72	0.30	0.42	0.80	0.18	0.08	0.08	0.24	0.58

#### 6.4.3.7 Model results using the new residential CCFs

The model including the new residential CCF values will be called the NewresCCF Model. An indication of how well the new centre attractiveness factor  $\gamma_j$  will represent the centre performance situation can be found by producing a correlation coefficient between  $\gamma_j$  and centre performance. The result of that correlation is an  $r^2$  value of 0.72. This indicates that the new centre attractiveness variable will be useful in improving the inclusion of centre attractiveness in the spatial interaction model.

The new revenue predictions and performance indicators for the 29 centres for the NewresCCF Model are shown in Table 6.13.

Table 6.13: New centre revenue predictions and centre performance for centres in the Yorkshire TV region containing WH Smith Group stores, for the NewresCCF Model

Centre	Observed Revenue	Predicted Revenue	Centre Performance	% Improvement in Performance
Meadowhall	8822	8940	98.7	75.9
Scarborough	2530	1915	132.1	2.0
Ilkley	733	515	142.4	-10.9
York	8360	6793	123.1	6.3
Skipton	863	668	129.2	-4.6
Leeds	16215	15447	105.0	12.8
Lincoln	4351	4078	106.7	3.8
Boston	1387	1252	110.8	-2.8
Beverley	1003	917	109.3	-2.0
Hull	8997	8853	101.6	4.6
Wakefield	3223	3218	100.2	3.0
Huddersfield	4054	3939	102.9	-2.1
Dewsbury	1229	851	144.4	-43.2
Grimsby	3685	3704	99.5	0.9
Spalding	1198	1063	112.6	-5.2
Halifax	2318	2444	94.8	2.4
Doncaster	3581	3611	99.2	7.2
Scunthorpe	2377	2452	97.0	7.1
Rotherham	1188	914	130.0	-18.0
Sheffield	11028	10982	100.4	13.4
Gainsborough	338	328	103.1	14.1
Retford	818	933	87.7	5.3
Skegness	1263	1374	91.9	10.1
Keighley	2160	2604	82.9	3.1
Barnsley	2566	2796	91.8	13.6
Pontefract	1341	1258	106.6	16.5
Chesterfield	1040	1174	88.6	12.2
Worksop	1364	1301	104.8	18.9
Bradford	8191	9770	83.8	14.8

Table 6.14: Goodness of fit statistics for the NewresCCF Model, for centres in the Yorkshire TV region containing WH Smith Group stores

Good Type	SSE	$r^2$	$r_s$
All Goods	6715813	0.98	0.97
Newspapers	132308	0.98	0.97
Books	3330732	0.97	0.91
Stationery	70643	0.96	0.98
Music	1049086	0.97	0.99
Cards	11849	0.97	0.93
Video	380756	0.96	0.95

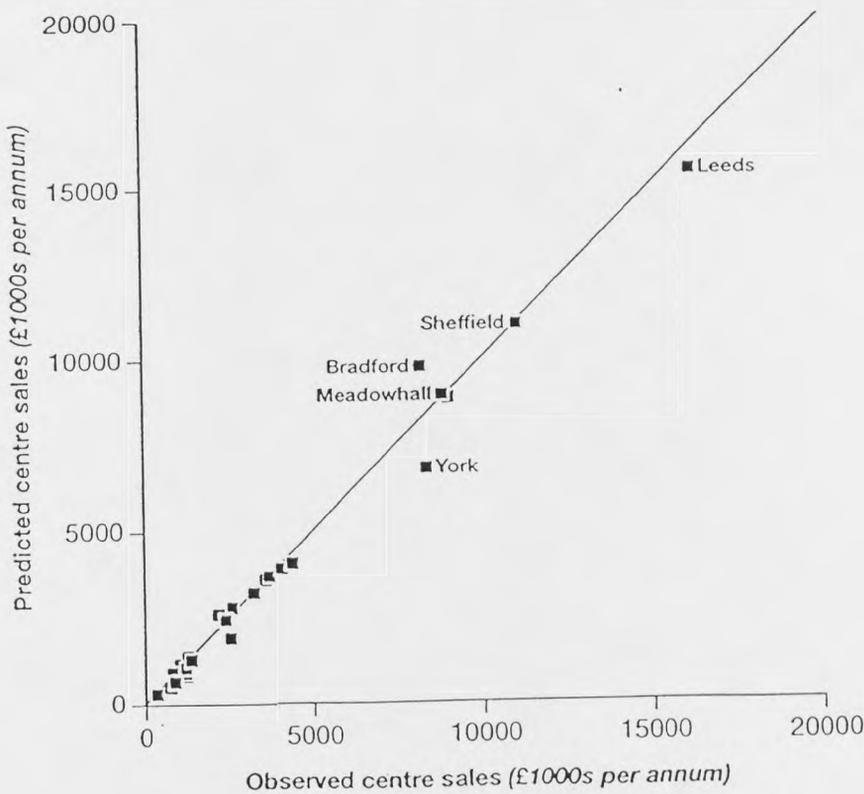
Through comparison of Table 6.14 and Table 6.7 it can be seen that the NewresCCF Model has significantly improved performance from the NoCCF Model. For all goods the SSE has decreased by 85% from the NoCCF Model and the  $r^2$  value has also increased significantly from 0.89 to 0.98. It can also be observed that model performance improved for all individual good types although the improvements in performance are not uniform across all good types. The SSE improved by 77% for cards, 75% for stationery, 77% for music, 72% for video, 58% for books and 72% for newspapers. The  $r^2$  value increased for all good types.

Comparison of Table 6.14 with Table 6.2 also shows that the NewresCCF Model produces goodness of fit statistics that are improved over the GMAP Full Model. The SSE for the NewresCCF Model is 22% lower than for the GMAP Full Model. The value of  $r^2$  is the same for both models.

Figure 6.3 also indicates the improvement of the centre revenue predictions caused by the inclusion of the new residential CCF. By comparing Figure 6.3 with Figure 6.2 it can be seen that there is less variation away from the 45 degree line in Figure 6.3, the scatterplot showing revenue predictions for the NewresCCF Model. Table 6.13 also indicates that model performance improved for 21 of the 29 centres and that there is only one centre whose performance decreases significantly, Dewsbury. The WH Smith Group performs significantly better than would be expected given the attractiveness values for Dewsbury. One possible explanation for this is that the shopping centre in which the WH Smith store is located in Dewsbury is a new development and is by far

the most attractive part of the centre. Several of the centres that were being excessively incorrectly predicted such as Meadowhall and Bradford are now having their sales predicted much more accurately. Therefore this is an indication of the improved explanatory and predictive power of the model subsequent to the addition of the  $\gamma_j$  variable, the new residential CCF.

Figure 6.3: Scatterplot between observed and predicted centre revenue for the NewresCCF Model, for centres in the Yorkshire TV region containing WH Smith Group stores



#### 6.4.3.8 Removal of the field survey attractiveness variable

Goodness of fit statistics were also calculated for the NewresCCF Model with the CCF formulated without the field survey attractiveness factor. This was undertaken because there is a danger of bias in this factor due to its subjectivity. Also, if the new formulation of the CCFs was to be undertaken by GMAP for use in the model, the exclusion of the field survey attractiveness factor would help to reduce the costs of CCF calculation because if up to date GOAD maps were available it would not be necessary to visit each centre.

Table 6.15: Goodness of fit statistics for the NewresCCF Model excluding the field survey attractiveness factor, for centres in the Yorkshire TV region containing WH Smith Group stores

	SSE	$r^2$	$r_s$
All Goods	7598129	0.98	0.98

The SSE has increased by 13% subsequent to the exclusion of the field survey attractiveness variable, but the goodness of fit statistics are still better than for the NoCCF Model and the GMAP Full Model. The implications of this with reference to GMAP are discussed in Chapter 9.

#### 6.4.3.9 The tourism and work CCFs

The variable used for the formulation of a tourism factor was the number of hotels in the centre. This data was extracted from the 1991 Census and is shown below in Table 6.16.

Also shown in Table 6.16 are data concerning the number of workers in each centre. This will be used to investigate if the number of workers in a centre will have an influence on that centre's performance. Information for the production of work based CCFs was extracted from the Census Special Workplace Statistics (SWS). SWS Set B was the dataset utilised, this set of Census tables provides information on the number and characteristics of workers at their workplace and therefore allows work based centre profiles to be produced for each centre.

Work based data and the number of hotels are available at ward level, therefore it was necessary to find out which ward each centre was located in. This process could be problematic because wards will not necessarily coincide with centres. Some larger centres such as Leeds will be covered by more than one ward whereas some wards will cover more than one centre. Thus, this process could introduce errors into the calculation of work based and tourism CCFs.

Table 6.16: The number of hotels and workers in centres in the Yorkshire TV region containing WH Smith Group stores

Centre	Number of Hotels	Number of Workers
Barnsley	40	1311
Beverley	48	666
Boston	0	322
Bradford	9	844
Chesterfield	0	230
Dewsbury	18	1395
Doncaster	73	950
Gainsborough	0	156
Grimsby	15	238
Halifax	0	324
Huddersfield	0	420
Hull	14	3231
Ilkley	9	550
Keighley	16	911
Leeds	129	11637
Lincoln	12	781
Meadowhall	0	546
Pontefract	2	1325
Retford	0	195
Rotherham	21	1744
Scarborough	14	218
Scunthorpe	34	613
Sheffield	13	4359
Skegness	25	127
Skipton	0	64
Spalding	6	229
Wakefield	0	876
Worksop	0	535
York	76	1798

Source: 1991 Census SAS/SWS. Crown Copyright.

ESRC/JISC purchase supplied by CDU on MIDAS.

In order to get an indication of whether the number of hotels or the number of workers will have an impact on model performance, correlations between number of hotels and workers in a centre and centre residuals were undertaken for the NewresCCF Model to see if these factors could further improve the performance of the model. The results are shown in Table 6.17.

Table 6.17: Correlations between numbers of hotels and workers and model residuals for centres in the Yorkshire TV region containing WH Smith Group stores

	$r$	$r^2$	$r_s$
Number of Hotels	0.07	0.00	0.09
Number of Workers	0.04	0.00	0.09

It can be seen that there is not a significant relationship between number of hotels or number of workers and model residuals for centres. Therefore these factors will not be included in the model.

Consideration of Table 6.16 shows that the number of hotels in a centre may not be necessarily indicative of the level of tourism in a centre. Large cities have a relatively large number of hotels but this will be determined not only by tourism but by business activities. The number of hotels in a centre will also not capture day visitors which could have an influence on sales in centres such as Ilkley and Skipton, which are being under predicted but have low values for the number of hotels. Thus, in order to include an accurate tourism variable in the spatial interaction model an alternative measure of the level of tourism would have to be found, but this has not been undertaken in this study due to time constraints.

## 6.5 INDIVIDUAL STORE ATTRACTIVENESS

### 6.5.1 *Current GMAP individual store attractiveness factors*

At present in the GMAP spatial interaction model, product specific, individual store attractiveness factors are used to adjust store revenue for those stores that are being

predicted incorrectly. The store attractiveness factor for a particular store is one of four measures.

- 1) The ratio of observed average sales per EFT for each store and product against the average for that store type and product.
- 2) The ratio of EFT per square footage for that store type and product against the average for that store type and product.
- 3) Average performance by store type and product of stores where observed sales per EFT is above average is applied to those stores with above average sales per EFT for that store type and product. Average performance by store type and product of stores where observed sales per EFT are below average is applied to those stores with below average sales per EFT for that store type and product.
- 4) An average of ratios 1 and 3.

The factor that is chosen for a particular store is the one that produces the best model fit. It can be seen that these individual store attractiveness factors are reactionary and are applied after model calibration to enhance the predictive power of the spatial interaction model. There is no attempt to explain why individual stores perform as they do. Therefore I will undertake an investigation to see if there are any individual store characteristics that can help explain differences between the performance of stores.

### *6.5.2 New store attractiveness factors*

Four factors will be investigated with reference to individual store performance. These are: whether a store is in an undercover or pedestrianised area, the size of the frontage of the store, the centrality of the store within the centre and the proximity of the store to Marks and Spencer. The information to build these factors was observed during fieldwork or, in the case of distance from Marks and Spencer it was taken from the GOAD city centre plans.

Table 6.18 below indicates the performance of the NewresCCF Model with the GMAP store attractiveness factors removed. This will be called the Nostoreattr Model. In this case, because the focus of the analysis is the store and not the centre, the goodness of fit statistics are calculated using observed and predicted store revenues as opposed to centre revenues.

Table 6.18: Goodness of fit statistics for the Nostoreattr Model for WH Smith Group stores in the Yorkshire TV region

Good	SSE	$r^2$	$r_s$
All Goods	17171180	0.79	0.87
Newspapers	463452	0.63	0.85
Books	6272389	0.80	0.89
Stationery	450006	0.74	0.92
Music	2636077	0.89	0.95
Cards	55791	0.79	0.85
Video	848400	0.76	0.89

An attempt will now be made to investigate if observed individual store characteristics can be used to improve the performance of the Nostorattr Model. A logit analysis (as described in Appendix A) is used to determine if the individual store characteristics will be significant in affecting the performance of the model. The dependent categorical variable for this analysis is whether centre sales are being over or under predicted. The results of this analysis are shown in Table 6.19 below.

Table 6.19: Results of the logit analysis to test the importance of individual store characteristics in explaining differences in store performance in the Nostorattr Model, for all goods

	Deviance	Degrees of Freedom	Null Model Deviance - Model Deviance
Null Model	86.56	62	
Covered	85.97	61	0.59
Front Size	83.01	60	3.55
Centrality	86.32	61	0.24
Distance to Marks & Spencer	85.11	58	1.45

Table 6.19 shows that each individual store attractiveness factor only caused a small decrease in deviance for the logit model and none of the variables were significant at the 95% level.

A regression was also undertaken between store performance and distance to Marks and Spencer, this was only possible for this variable because it is the only continuous variable in the analysis. The  $r$  value for the correlation was -0.05 which is not significant. Therefore it seems that for all goods the individual store characteristics are not significant variables in determining the difference in performance between stores. However, Table 6.18 indicated that there were differences in the performance of the Nostorattr Model between different good types. Thus a logit analysis using the four store characteristic variables was undertaken for each good type, the results of which are shown in Table 6.20.

Table 6.20 shows that the addition of individual store characteristic variables for each good type did not lead to large decreases in model deviance and the variables were not significant at the 95% level. Thus, it can be seen that the characteristics studied in relation to individual store performance for WH Smith Group stores will not add to the explanatory power of the spatial interaction model.

Table 6.20: Results of the logit analysis to test the importance of individual store characteristics in explaining differences in store performance in the Nostorattr Model, for individual good types

Store Characteristics	Deviance	Degrees of Freedom	Null Deviance - Model Deviance
NEWSPAPERS			
Null Model	45.72	62	
Covered	42.5	61	3.22
Front Size	44.04	60	1.68
Centrality	45.70	61	0.02
Distance to Marks & Spencer	40.90	58	4.82
BOOKS			
Null Model	55.35	62	
Covered	54.77	61	0.58
Front Size	51.58	60	3.77
Centrality	55.35	61	0.00
Distance to Marks & Spencer	51.37	58	3.99
STATIONERY			
Null Model	43.26	62	
Covered	43.23	61	0.03
Front Size	37.99	60	3.27
Centrality	43.01	61	0.25
Distance to Marks & Spencer	40.90	58	2.36
MUSIC			
Null Model	68.59	62	
Covered	67.31	61	1.28
Front Size	65.71	60	2.88
Centrality	68.51	61	0.08
Distance to Marks & Spencer	63.50	58	4.09
CARDS			
Null Model	44.99	62	
Covered	44.82	61	0.17
Front Size	44.33	60	0.66
Centrality	44.95	61	0.04
Distance to Marks & Spencer	42.09	58	2.90
VIDEO			
Null Model	72.09	62	
Covered	71.99	61	0.10
Front Size	65.29	60	3.80
Centrality	71.97	61	0.12
Distance to Marks & Spencer	72.09	58	0.00

## 6.6 MARKET SEGMENTATION

### *6.6.1 Market segmentation through the use of census variables*

Market segmentation within the WH Smith model takes into account differences in purchasing behaviour dependent on certain population characteristics. This process is concerned with which stores are chosen by different sections of the population. Some research on this subject has already been undertaken by GMAP, the results of which are shown in Codling (1995b). This study investigated whether the profiles of shoppers at different store types was related to the following combinations of population characteristics, social class and age, social class and terminal education age, and age and terminal education age. The data used for the analysis was taken from the National Market Survey (NMS). The analysis was undertaken for book buyers.

It was found that such population characteristics were important in determining store choice by consumers. It was discovered that WH Smith had a buyer profile that was very similar to the general profile of book buyers in the NMS, whereas Waterstones attracted book buyers who were of a higher class and with a higher terminal education age. Blackwells is also attractive to book buyers of a higher class and also attracted a large number of students.

The conclusion of the market segmentation study undertaken by Codling (1995b) was that social class and terminal education age was the best combination of variables for determining store choice by book buyers.

It can be seen that the inclusion of market segmentation within the WH Smith Model has a significant impact on model performance. The model containing new residential CCFs but no individual store attractiveness factors or market segmentation will be called the Noseg Model. The goodness of fit statistics for the Noseg Model are shown in Table 6.21.

Table 6.21: Goodness of fit statistics for the Noseg Model for centres in the Yorkshire TV region containing WH Smith Group stores

Good	SSE	$r^2$	$r_s$
All Goods	22760766	0.95	0.97
Newspapers	599919	0.80	0.93
Books	8777082	0.91	0.88
Stationery	424918	0.74	0.92
Music	2185268	0.95	0.95
Cards	42901	0.88	0.88
Video	1145577	0.88	0.93

By comparison of Table 6.21 with Table 6.18 it can be seen that for all goods the SSE has increased by 35% on the removal of market segmentation, although the values of  $r^2$  and  $r_s$  have improved. For individual good types it can be observed that for stationery, music and cards, taking out the market segmentation has actually led to a decrease in the SSE, but an increase in  $r^2$  and  $r_s$ .

### 6.6.2 Market segmentation through the use of geodemographics

Further research that could be undertaken regarding market segmentation would be to investigate if there is a better way of segmenting the population. One method of doing this is through the use of geodemographics. It has already been seen in Section 5.5.3 that geodemographics were important in characterising the book buying population of the NMS. An analysis was undertaken to see if geodemographics can be used to distinguish between choice of store by the book buying population of the survey.

For the demand analysis 100 geodemographic types from GB Profiler were used to disaggregate the population. For market segmentation analysis this is too many because sample sizes will be too small, therefore the geodemographic types have been condensed into 5 main types in order for sample sizes to be large enough. The five main types are as described in Section 5.5.2: struggling, climbing, established, aspiring and prosperous.

The analysis was carried out for book buyers in the NMS for four store types, WH Smith, Blackwells, Woolworths and Waterstones. The percentage of book buyers of each type that use each type of store was calculated and this is shown in Table 6.22.

Table 6.22: Percentages of each geodemographic classification that use each store type, for book buyers in the NMS

	Blackwells	WH Smith	Woolworths	Waterstones
Aspiring	86.2	10.8	2.5	0.4
Climbing	86.8	9.2	2.7	1.3
Established	89.4	7.5	2.9	0.3
Prosperous	86.8	10.1	2.8	0.3
Struggling	92.9	3.9	2.8	0.5

From Table 6.22 it can be observed that for each geodemographic classification the percentages frequenting each type of shop are very similar. Thus the five broad geodemographic classifications do not provide a means of profiling which types of book buyers use which shops. Thus, the current GMAP method of market segmentation, using social class and terminal education age, will be retained in the model.

## 6.7 BEST PERFORMING MODELS

### 6.7.1 *The Explanatory Model*

The analysis undertaken in this chapter has shown that the best explanatory model for WH Smith sales prediction consists of the new residential CCFs and the GMAP formulation of market segmentation. The other factors analysed such as tourism, work and individual store characteristics were found to be insignificant in determining sales. The goodness of fit statistics for this explanatory model are shown in Table 6.23.

Table 6.23: Goodness of fit statistics for the Explanatory Model, for centres in the Yorkshire TV region containing WH Smith Group stores

Good	SSE	$r^2$	$r_s$
All Goods	20866654	0.95	0.97
Newspapers	599919	0.80	0.93
Books	7817671	0.91	0.90
Stationery	423766	0.75	0.91
Music	212747	0.95	0.97
Cards	39859	0.88	0.86
Video	1085049	0.89	0.94

Comparison of Table 6.23 and Table 6.1 which shows goodness of fit statistics for the GMAP Base Model with no explanatory variables indicates that the SSE for all goods has been decreased by 65% subsequent to the introduction of the new residential CCFs,  $r^2$  and  $r_s$  have also improved. Improvements in goodness of fit are experienced for all good types. Comparison of Figures 6.4 and 6.5 below which show observed and predicted sales for centres for the Explanatory Model and the GMAP Base Model also indicates how the inclusion of the new explanatory variables has improved the prediction of centre sales. Centres such as Meadowhall and Bradford which were extreme outliers are now predicted significantly more accurately. Sales predictions for Leeds and York have also improved.

Figure 6.4: Scatterplot between observed and predicted centre revenues, for the GMAP Base Model, for centres in the Yorkshire TV region containing WH Smith Group stores

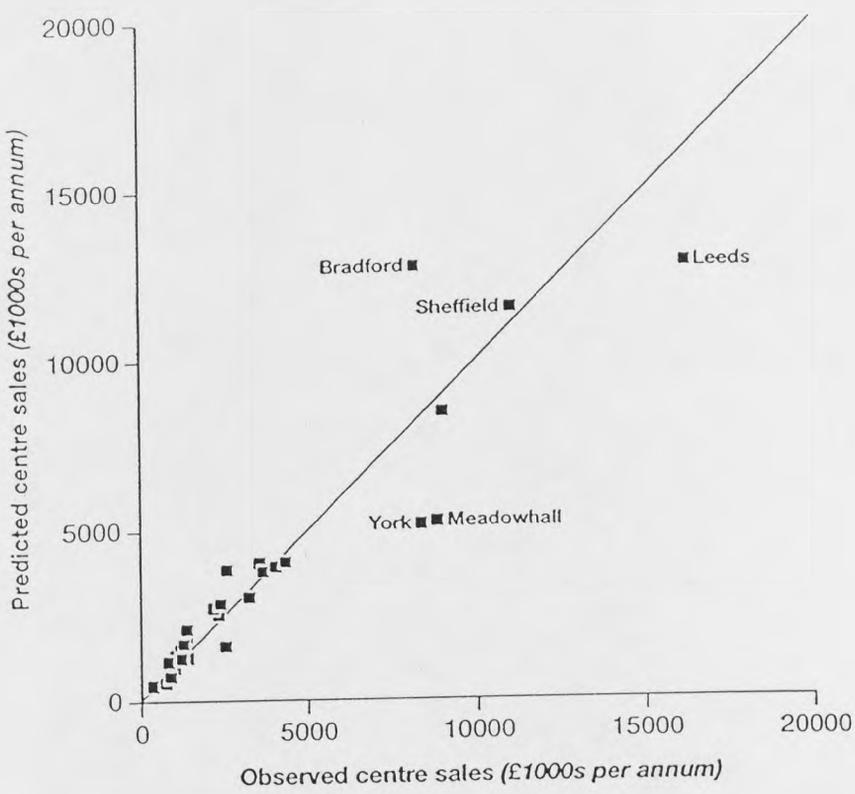
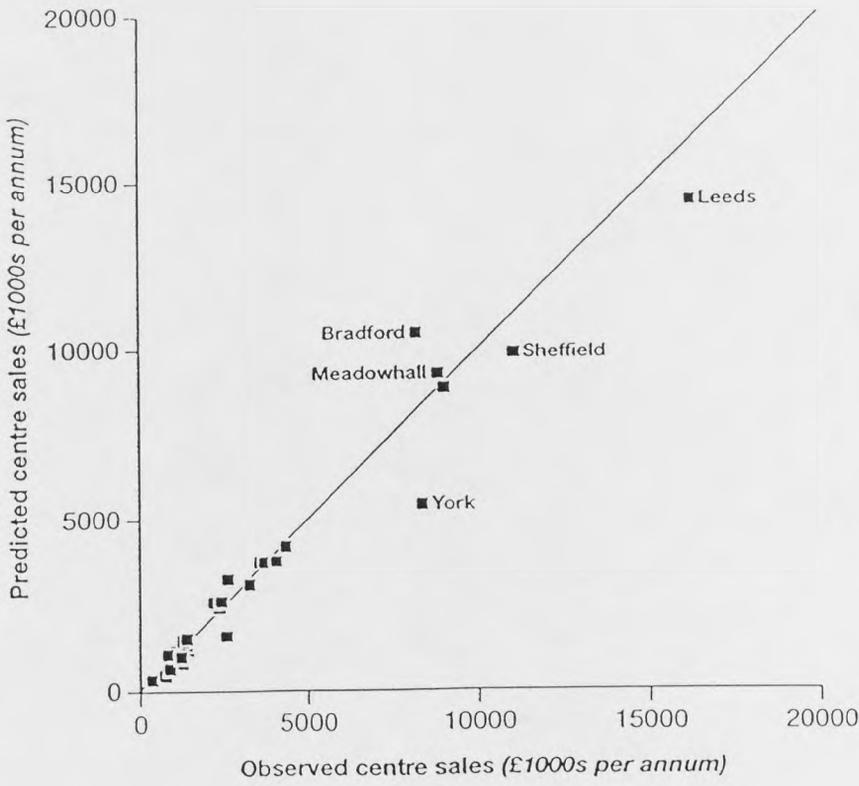


Figure 6.5: Scatterplot between observed and predicted centre revenues, for the Explanatory Model, for centres in the Yorkshire TV region containing WH Smith Group stores



### 6.7.2 The Predictive Model

Not only has the creation of the new residential CCFs based on centre characteristics improved the performance from the GMAP Base Model to produce the Explanatory Model, it has also enabled improvements to be made in the predictive capability of the model. At present the predictive model is represented by the GMAP Full Model. However, if the residential CCFs used at present are replaced by the new residential CCFs produced in Section 6.4, then a new Predictive Model is created that performs better than the GMAP Full Model.

Table 6.24: Goodness of fit statistics for the Predictive Model, for centres in the Yorkshire TV region containing WH Smith Group stores

Good	SSE	$r^2$	$r_s$
All Goods	5716508	0.99	0.97
Newspapers	97150	0.97	0.98
Books	3186598	0.97	0.91
Stationery	79742	0.95	0.98
Music	877130	0.98	0.99
Cards	11724	0.97	0.93
Video	404881	0.96	0.95

Comparison of Table 6.24 with Table 6.2 which shows the performance of the GMAP Full Model shows that for all goods the SSE has been improved by 33% by using the new residential CCFs, the values of  $r^2$  and  $r_s$  have also improved. However, improvements in goodness of fit have not been achieved for all good types, with newspapers, cards and video experiencing an increase in SSE. The scatterplots in Figures 6.6 and 6.7 indicate that there is slightly less variation from the 45 degree line for the Predictive Model than for the GMAP Full Model, and it can be seen that predictions for Meadowhall and Bradford have been improved.

Figure 6.6: Scatterplot between observed and predicted centre revenues for all goods, for the GMAP Full Model, for centres in the Yorkshire TV Region containing WH Smith Group stores

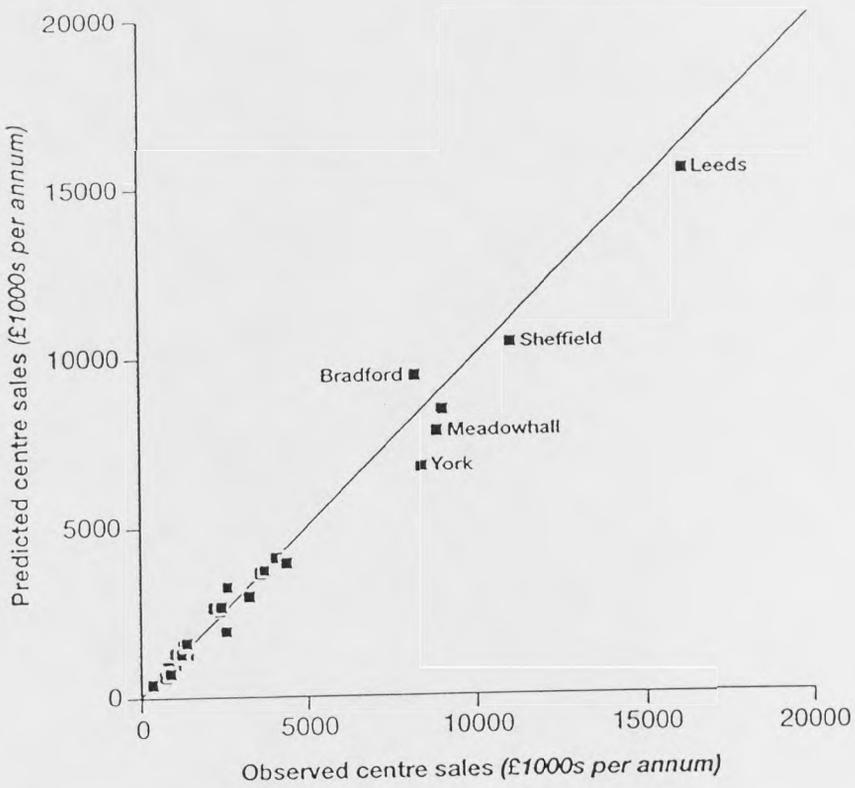
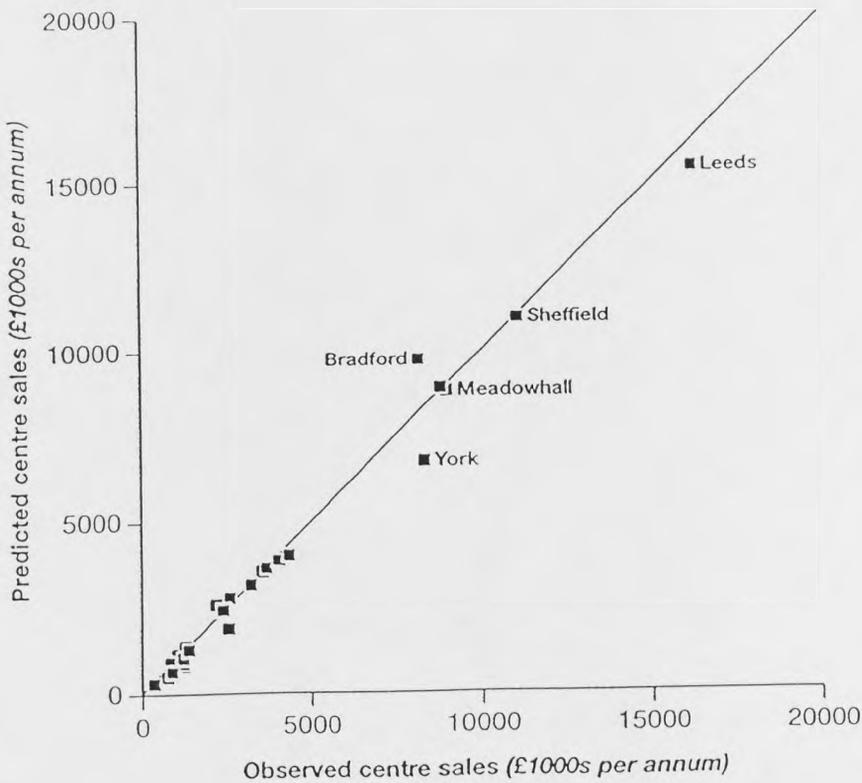


Figure 6.7: Scatterplot between observed and predicted centre revenues for all goods, for the Predictive Model, for centres in the Yorkshire TV region containing WH Smith Group stores



## 6.8 CONCLUSIONS

The attempt to produce an explanatory as opposed to predictive model for WH Smith sales, which is based on real, observable characteristics of the system has had mixed success. The new residential CCFs, which were formulated using observed centre characteristics, proved to be an important addition to the model, producing goodness of fit statistics that were better than the current formulation of the GMAP model for WH Smith.

The new residential CCFs will also have a further predictive advantage in that changes in the attractiveness situation of a centre can easily be incorporated in to the model. At present the CCFs are created through trial and error calibration procedures using the observed revenue. However, in some modelling problems the observed value is not known and therefore this method cannot be used. The new residential CCFs centre

attractiveness variable does not rely on observed centre revenue for its formulation and can therefore respond to changes in centre attractiveness in a way that the original CCFs cannot. For example, if a new centre is created such as the White Rose centre, there are no observed revenue values for this centre and so a CCF will be difficult to calculate. However, the new residential CCF value will be able to be formulated in the same way as it is for all other centres, using information on the number of multiples, number of parking spaces, amount of undercover and pedestrianised area, and a day's fieldwork to collect information for the creation of the field survey attractiveness factor.

The new method also allows for centre attractiveness to be changed due to changes that occur in existing centres such as an increase in parking spaces or an increase in the amount of the centre that is pedestrianised. In this respect, the new centre attractiveness factor is also capable of 'what if' forecasting in a way that a model containing heuristic CCFs is not. A model containing the new residential CCF variable can predict the change in revenue that will occur due to changes in centre attractiveness. For example, if a new undercover centre is built in the centre, or a new car park the  $\gamma_j$  value for the centre can be adjusted accordingly in order to take account of these changes. The CCFs as calculated by GMAP are not capable of such forecasting because they are not comprised of information concerning the actual characteristics of centres.

Thus to give the model more explanatory power, a more empirical basis and the capacity to provide 'what if' forecasts concerning centre characteristics, information such as that used for the 29 Yorkshire centres for the creation of  $\gamma_j$  should be included for all centres in the model. This would be a time consuming and expensive exercise but would give the centre attractiveness component of the WH Smith spatial interaction model a more empirical basis and would therefore aid understanding of the model. However, it has also been seen that model performance can be improved through the use of new residential CCFs at a lower cost, through the exclusion of the field survey attractiveness variable.

The replacement of the tourism and work CCFs using Special Workplace Statistics and the number of hotels in a centre was unsuccessful because it was found that these variables did not affect centre performance.

Individual store characteristics were also found to be unimportant in determining the difference between observed and predicted revenues for WH Smith Group stores. Thus the observed difference must be being caused by other factors that have not been observed.

The alternative method of segmenting the book buying population, through the use of geodemographics was also found to add no extra insights into the WH Smith model and therefore the current GMAP formulation of market segmentation, using social class and terminal education age will be retained in the model.

Thus, as was seen in Section 6.7 the best explanatory model that has been found includes new residential CCFs, no work based or tourist CCFs, no individual store attractiveness factors, and market segmentation based on social class and terminal education age. The best predictive model is comprised of new residential CCFs, the individual store attractiveness factors as calculated by GMAP, and market segmentation using social class and terminal education age.

There has therefore been some success achieved in the formulation of an explanatory model for WH Smith sales. However, there are still a significant amount of model residuals that have not been explained. It could be that these errors could be decreased by changing the formulation of the interaction component of the spatial interaction model. The interaction component will be analysed in the next chapter with reference to the Halifax spatial interaction developed by GMAP, and any conclusions arisen to will be tested for the WH Smith model in Chapter 8.

## AN INVESTIGATION OF THE INTERACTION FUNCTION FOR THE HALIFAX PLC SPATIAL INTERACTION MODEL

### 7.1 INTRODUCTION

The third component of the spatial interaction model is the interaction function. This variable provides the link between the demand and the supply components of the spatial interaction model and is represented in the model by a function of the impedance between demand points (origins) and supply points (destinations) and is usually based on a function of interaction cost

$$f(c_{ij}) \tag{7.1}$$

where  $c_{ij}$  represents some measure of the cost of travel between origin  $i$  and destination  $j$ . The interaction function is therefore the component of the model that determines the degree of interaction between residential areas (origins) and shopping centres (destinations). It is generally accepted that as the distance between origins and destinations increases, *ceteris paribus*, the interaction between the two zones will decrease. This is because given a choice of two identical destinations it is irrational for an individual to choose to travel to the furthest of those destinations because they would incur a higher cost of interaction in terms of both time and money. This process of decreasing interaction with distance is termed the distance decay effect and is included in the spatial interaction model through a parameter that is contained within the interaction function. This parameter  $\beta$  reflects the fact that people will generally prefer to travel shorter distances as opposed to longer distances. As was seen in Section 3.5.4, the value of the  $\beta$  parameter will reflect the level of distance decay that is apparent and this will depend on the good for which spatial interaction is being modelled. For a low order good such as food, people will be unwilling to travel long distances. Therefore there is a high level of distance decay and this is reflected by a high negative value for the  $\beta$  parameter. For higher order goods such as furniture, distance will have less of a

detering effect and this is reflected in the model through a lower negative value of  $\beta$ . The value of  $\beta$  can also be dependent on person type because a persons ability to travel will depend on factors such as age and social class.

The aim of this chapter is to investigate the best possible way of formulating the interaction function with reference to Halifax Plc new mortgage sales. For Halifax Plc there exist data on the flows between origins and destinations for Halifax customers. This flow data consists of the number of new mortgages sold at a destination to residents of an origin. The analysis in this chapter will involve the investigation of the best way of reproducing these flows using the spatial interaction model currently used for the Halifax and varying the interaction function within the model in order to discover which formulation of the interaction function reproduces trip flows the most accurately.

The current Halifax spatial interaction model, as described in Section 4.3.2 will be used as the base from which to test the alternative interaction functions in order to attempt to improve flow prediction. As was seen in Section 4.3.3, the goodness of fit statistics for the GMAP Full Model and the GMAP Base Model for Halifax new mortgage sales are as follows, where the GMAP Full Model represents the current formulation of the model as used by GMAP, and the GMAP Base Model has the CCF values removed.

Table 7.1: Goodness of fit statistics for the GMAP Full Model and the GMAP Base Model

Model	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18584	0.66	0.49
GMAP Base Model	8.82	19655	0.65	0.48

The interaction analysis will be based on the GMAP Full Model and will consist of three sections. Firstly, the correct method of calculating the cost of interaction between zones will be studied. The most appropriate function to be attached to the interaction cost term is the second factor to be analysed. The third section of the interaction investigation will involve analysis of whether a purely interaction cost based accessibility function is the most appropriate way of incorporating the interaction component into the spatial interaction model. Finally, a model will be produced that

combines all the best formulations from the three sections of analysis concerned with the interaction function.

## 7.2 CALCULATING TRAVEL COST

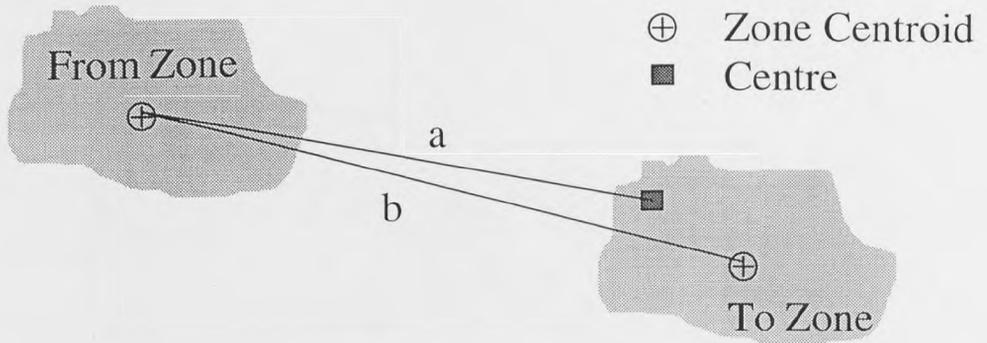
As was seen in Section 3.5.1 there are a variety of different methods of measuring the separation between origins and destinations. The interaction cost variable is usually based on some form of distance measurement, the simplest of which is the straight line distance between the two zone centroids. However, as was discussed in Chapter 3, straight line distances are not an accurate reflection of actual distance travelled because people have to travel using road and public transport networks. Therefore it is necessary to develop a measure of interaction cost that more accurately reflects distance as perceived by consumers.

The measure of interaction cost most commonly used in retail models is drive time. The use of such a measure is more appropriate than straight line distance because it is more representative of the consumers experience of interaction cost. An interaction matrix comprising of drive times between postal sector centroids is available at GMAP, and this is the method currently used to calculate interaction cost in the Halifax spatial interaction model. However, there is a problem with the data that is available in that it is for postal sector centroid to postal sector centroid, whereas the interaction cost that is required is for postal sector centroid to financial centre. At present in the Halifax spatial interaction model the problem is solved as shown in Figure 7.1 below.

The weighted drive time from zone centroid to financial centre is calculated as

$$\text{Weighted Drive time}=(a/b)*\text{Drive Time} \quad (7.2)$$

Figure 7.1: The calculation of postal sector centroid to financial centre drive times



### 7.2.1 Proposals for analysis on measuring distance

The method shown in Figure 7.1 above may not be the most appropriate way of estimating the drive time from postal sector centroids to financial centres. An alternative would be to develop a function showing the relationship between distance and drive times, this could be achieved using the centroid to centroid drive time data that is available. This function could then be applied to the distance between the postal sector centroid and the financial centre.

The use of straight line distance between residence zones and financial centres will also be analysed in order to determine the level of performance improvement that is achieved through the use of drive times. There is the possibility that the constraints built into the model are so stringent that changing the calculation of the degree of separation between origins and destinations will not have an effect on the performance of the model. This hypothesis can be tested for the Halifax model by using straight line distance as opposed to drive times for the distance calculation.

### 7.2.2 Results of analysis on alternative calculations of the distance variable

The results produced from using the straight line distance function are shown in Table 7.2 below.

Table 7.2: Goodness of fit statistics for the GMAP Full Model using straight line distance

	SSE	$r^2$	$r_s$
Straight Line Distance Calculation	18823	0.67	0.50

It can be seen from the comparison of Table 7.2 and Table 7.1 that the use of straight line distance decreases the performance of the model. Therefore the constraints apparent in the model do not mean that the distance calculation is unimportant and therefore drive times should be used in the calculation of the interaction term.

The function calculated between distance and drive time was a quadratic regression function which took the following form

$$c_{ij} = 2.7063 + 117.488(d_{ij}) - (65.303(d_{ij})^2) \quad (7.3)$$

Several alternative regression functions were calculated, the function shown in equation (7.3) was chosen because it created the highest value of  $r$  for the regression, with a value of 0.829. The results achieved using this function are shown in Table 7.3 below.

Table 7.3: Goodness of fit statistics for the GMAP Full Model using the function conversion of straight line distance

	SSE	$r^2$	$r_s$
Function Conversion Distance Calculation	18664	0.66	0.49

The results in Table 7.3 indicate that using the function to convert the straight line distance from postal sectors to financial centres into drive times increases the SSE slightly and the  $r^2$  and  $r_s$  values remain the same. Therefore the use of this function does not improve model performance and the current method of calculating postal sector centroid to financial centre drive times is retained in the model as the most appropriate method.

## 7.3 FORMULATING THE INTERACTION TERM

### 7.3.1 Introduction to alternative distance decay functions

Once travel cost has been estimated there is also the issue of the form of the distance deterrence term and the value of the distance decay parameter,  $\beta$ . Section 3.5.3 indicated that the power distance function  $c_{ij}^\beta$  was used in early gravity models but that there was no theoretical justification for the use of this function. Therefore, alternative distance functions have been suggested such as the negative exponential distance function proposed by Wilson, which is the function used in the Halifax model.

Taylor (1971, 1975) has also identified several alternative distance decay functions which could be tested in the context of the Halifax mortgage spatial interaction model. Taylor describes five distance functions as proposed by Goux (1962). These five functions are all exponential curves of the form

$$I = ke^{-bf(c)} \quad (7.4)$$

This is the general model on which the other functions are based. The first three of the remaining five functions are single log models of the form

$$f(c) = c^m \quad (7.5)$$

The alternative single log models are defined by varying the value of  $m$  in the above function. The square root exponential model is produced by setting the value of  $m$  to 0.5 and is given by

$$f(c) = e^{-bc^{0.5}} \quad (7.6)$$

If the value of  $m$  is set to one then the simple exponential model is produced. This model is of the following form

$$f(c) = e^{-bc} \quad (7.7)$$

It can be seen that this function is equivalent to the negative exponential distance function discussed above as proposed by Wilson (1970) and used in the current formulation of the Halifax model. The normal model is produced when the value of  $m$  is two, as below

$$f(c) = e^{-bc^2} \quad (7.8)$$

There are also two double log models described by Taylor, these functions take the following general form

$$f(c) = (\log c)^m \quad (7.9)$$

Again, the alternative distance functions are found by varying the value of  $m$ . If  $m$  is equal to one the pareto model is produced

$$f(c) = e^{-b \log c} \quad (7.10)$$

This model can be seen to be equivalent to the power function distance decay term described above. If  $m$  is set to two then the log normal distance function is produced. This function is given as follows

$$f(c) = e^{-b(\log c)^2} \quad (7.11)$$

The two distance functions that have been used the most in spatial interaction modelling are the negative exponential function and the power function. These two functions are compared in Section 3.5.3. The square root exponential function, the normal function and the log normal function are less well documented and have not been used widely in applications. However, they could provide a more accurate measure of distance decay in certain circumstances.

Different distance functions should be used in different circumstances and the most appropriate function should be found by comparison to empirical data.

### 7.3.2 *Distance decay parameters*

There is also the problem of deriving the correct value for the distance decay parameter  $\beta$ . The value of  $\beta$  will vary depending on the type of good for which spatial interaction is being modelled. A high value of  $\beta$  indicates a low average trip cost and therefore means that distances travelled to purchase that good will be low. If the value of  $\beta$  is low, the average trip cost will be higher indicating that longer trips will be undertaken to purchase goods. The value of  $\beta$  is determined through calibration.

The calibration procedure used for the Halifax spatial interaction model is the maximum likelihood method as described in Section 3.6.1.1, which involves equating observed and predicted drive times. Therefore it will be necessary to analyse how far people travel to purchase mortgages in order to determine the value of the distance decay parameter.

#### 7.3.2.1 *Zone specific distance decay parameters*

As was discussed in Section 3.5.4 there is also the possibility of using zone specific distance decay parameters. Such parameters allow for spatial variations in the propensity to travel to be incorporated into the model. Therefore origin or destination specific parameters may allow trips to be modelled more accurately.

Fotheringham (1981) argues for the use of origin-specific distance decay parameter values to account for what he argues is the influence of spatial structure on interaction behaviour. By spatial structure he means the configuration in space of the origins and destinations, and Fotheringham argues that this spatial structure affects interaction but can be accounted for by using origin specific distance decay parameters. In his 1981 paper, Fotheringham uses the example of 1970 airline passenger interaction in the USA, which is split in to 100 zones which are both origins and destinations. Fotheringham calibrated a spatial interaction model to produce a distance decay parameter for each origin. He argued that if there was a spatial pattern in the distribution of different distance decay values this would have been caused by the spatial structure of the zones. He found that less accessible origins had more negative distance decay parameter

estimates and that more accessible origins had less negative parameter estimates and that in these cases some estimates of parameter values were even positive. A further piece of evidence cited for the existence of a spatial structure effect was the wide range in parameter values found across the USA. Fotheringham therefore argues that the location of the origin is a large factor in the determination of the value of  $\beta$  (because  $\beta$  is misspecified).

Destination specific distance decay parameters can also be used in order to take into account the effect of the spatial structure of destinations on peoples' willingness to travel.

### *7.3.3 Proposals for analysis on alternative distance functions and distance decay parameters*

Each of the five distance decay functions proposed by Taylor will be tested with reference to Halifax Plc in order to discover which function provides the best estimate of mortgage sales.

It can be seen that the current formulation of the Halifax spatial interaction model uses an origin specific beta parameter  $\beta_i$ , on the distance function. This is justified by the argument that the different characteristics of origins will effect the distance that people are willing to travel in order to purchase a mortgage. One possibility is that the accessibility of an origin will effect the distance travelled, *i.e.* those origins that are more accessible, such as those in city centres, would warrant a higher distance decay parameter because people would be less willing to travel long distances in order to receive services. In order to test if there is a significant relationship between distance travelled and accessibility it is necessary to develop a proxy variable for accessibility. One such variable is population density because accessible zones such as those in city centres and inner city areas will have a higher population density whereas origin zones in rural areas where it is proposed people will travel further will have a lower population density.

It can be seen through comparison of Figures 7.2 and 7.3, that there is a relationship between population density (Figure 7.2) and observed average drive time to get a

mortgage (Figure 7.3). The maps show that where population density is low such as in rural areas, the observed average drive time tends to be higher and when population density is high in urban areas the observed average drive time tends to be lower.

Correlations between population density of origin zones and average distance travelled from each origin also indicate that there is a relationship between origin characteristics and propensity to travel. The value of the correlation coefficient  $r$  between population density of an origin and the observed average distance travelled from that origin to purchase a mortgage is -0.4123. This figure is significant at the 99.99% level and indicates that as population density increases, the propensity of the origin's residents to travel long distances decreases.

Figure 7.2: Population densities of postal districts in the Yorkshire TV region

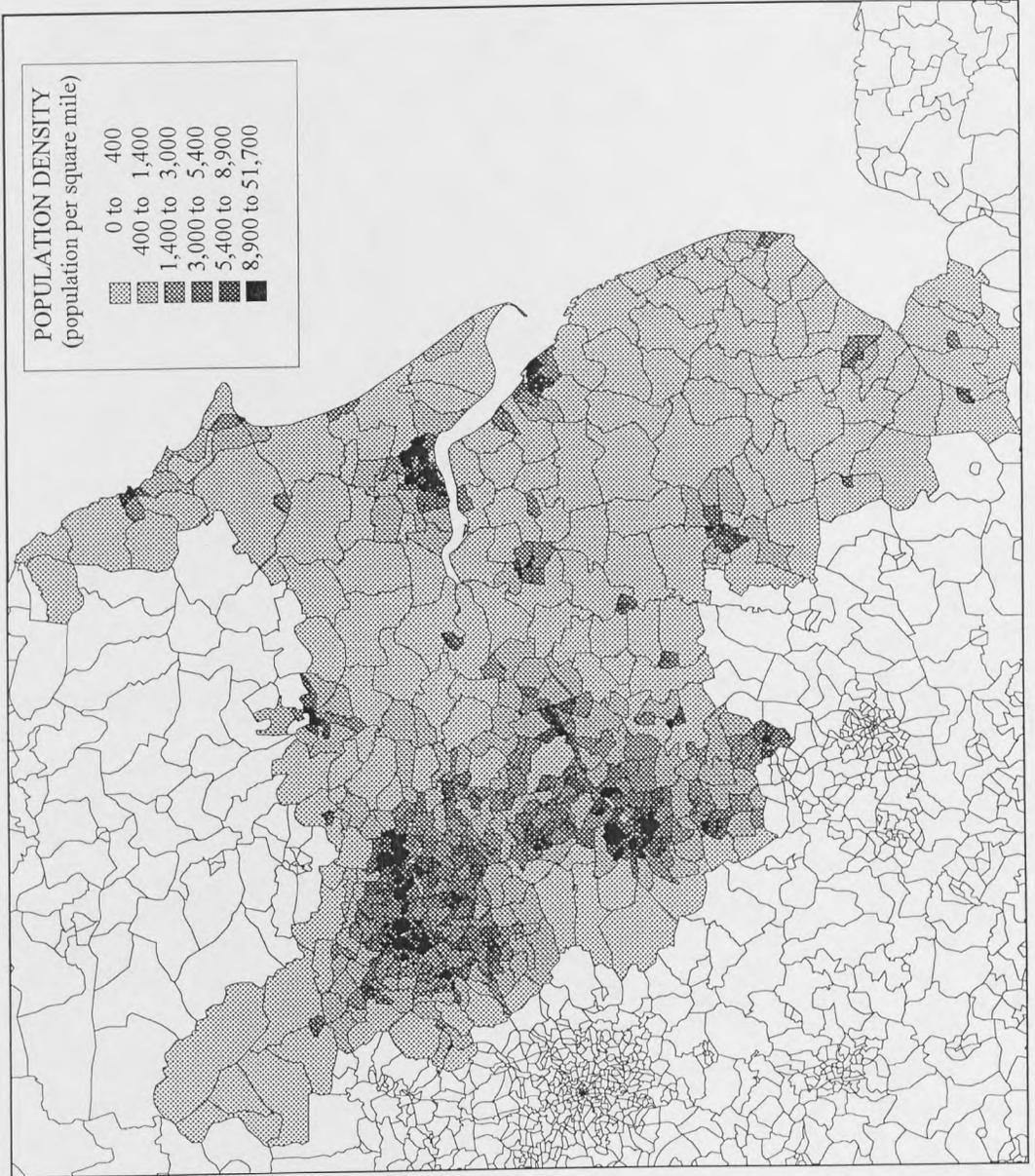
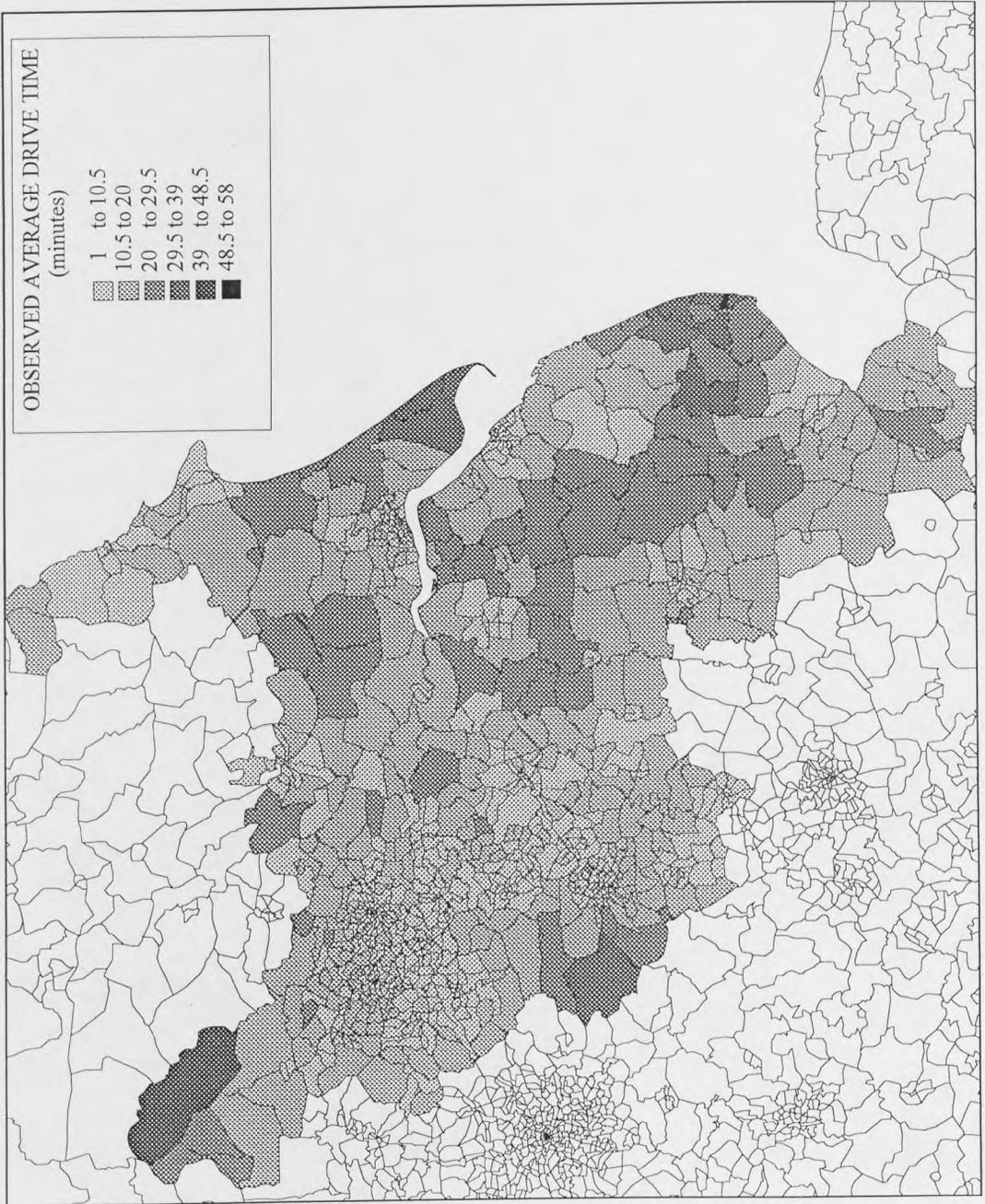


Figure 7.3: Observed average drive times for postal districts in the Yorkshire TV region



This correlation coefficient, along with Figures 7.2 and 7.3 could indicate that there is a significant enough difference between origins to warrant origin specific beta values being used in the model. However, another way of determining whether origin specific beta parameters are necessary and if they have a significant effect on model performance is to use a global beta value in the model and see how this affects model performance. This will be undertaken for the mortgage model.

Destination specific distance decay parameters will also be analysed in order to discover if destinations have more of an influence on distance decay parameters than origins.

### *7.3.4 Results of analysis on alternative distance functions and distance decay parameters*

#### *7.3.4.1 Results using global beta values*

Global beta values for each of the distance functions proposed by Taylor will be calibrated using the mean trip cost. This is an iterative process and a beta value will be considered appropriate if it produces a predicted average drive time within 0.25 minutes of the observed average drive time. The observed average drive time for mortgages for the Yorkshire TV region is 9.37 minutes.

Figure 7.4 below compares the five alternative distance functions using the global beta values as calibrated for the GMAP Full Model. The x axis shows distance and the y axis displays the value of the distance function.

Figure 7.4: A comparison of the five alternative distance functions

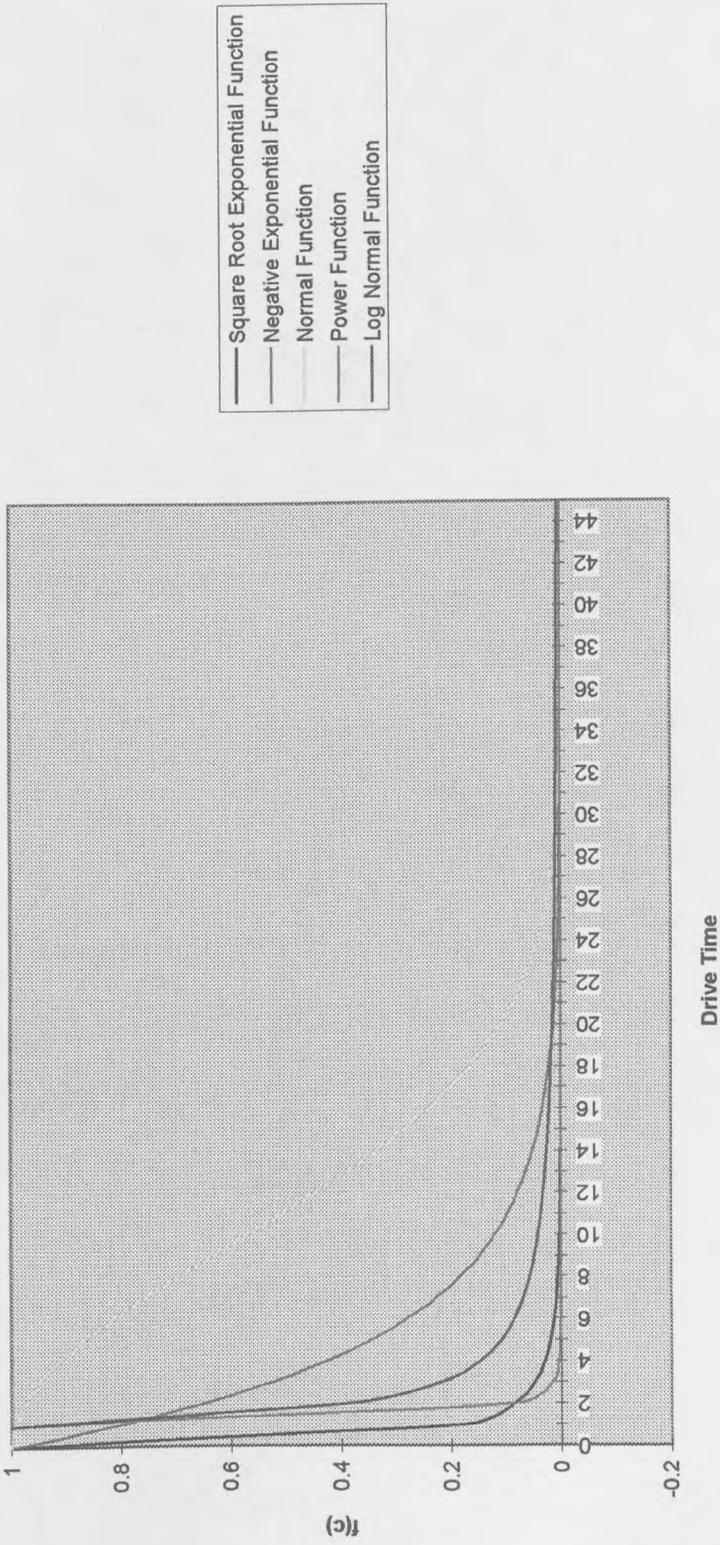
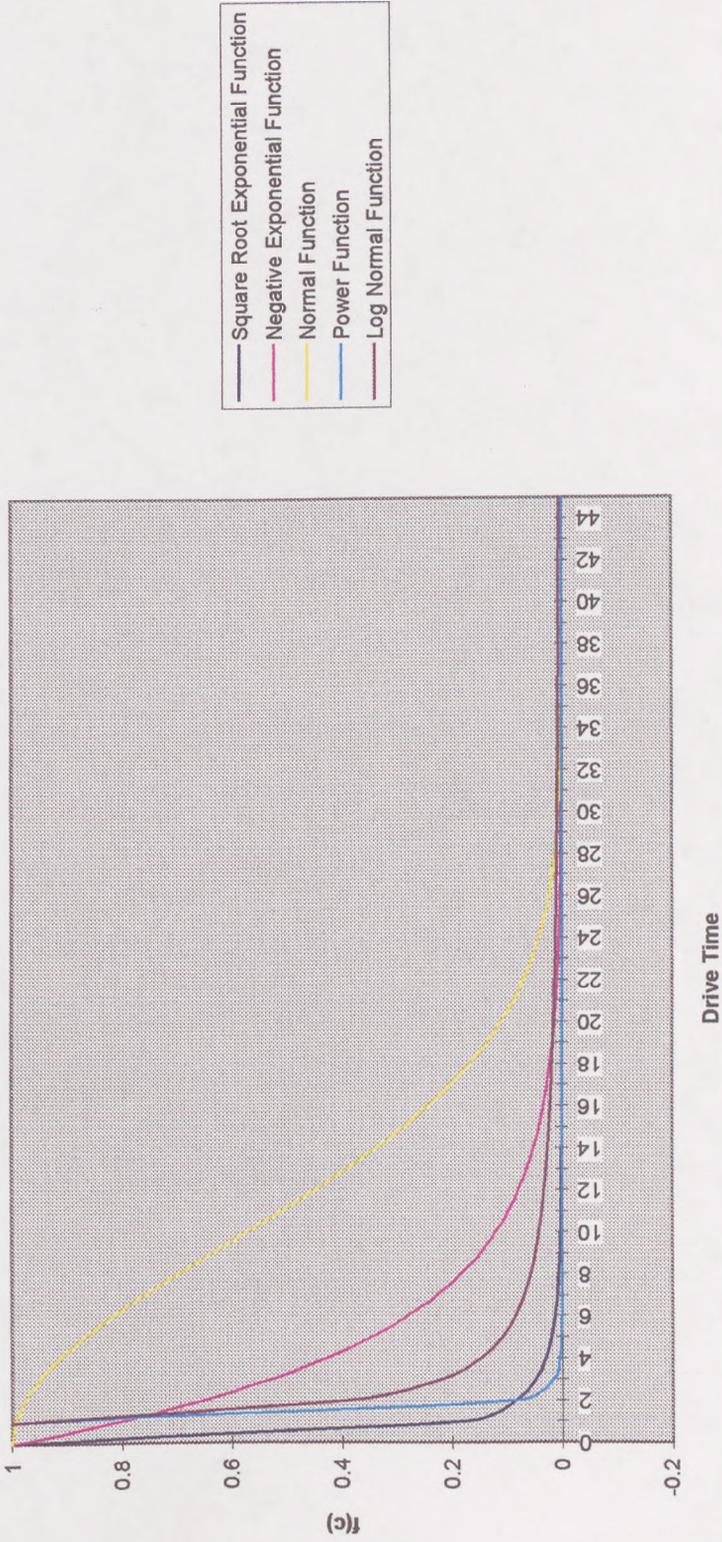


Figure 7.4: A comparison of the five alternative distance functions



It can be seen from Figure 7.4 that the normal function shows a relatively uniform level of distance decay. Of the remaining four functions the negative exponential function has the most gentle slope indicating less abrupt distance decay. The power function has the sharpest distance decay, displaying very low values of the distance function after a drive time of about 2 minutes. The square root exponential function and the log normal function both indicate a more gentle level of distance decay than the power function.

Table 7.4: Goodness of fit statistics for the five Taylor distance functions using a global beta value

Function	BETA	PADT	SSE	$r^2$	$r_s$
Square Root Exponential Function	1.7500	9.62	20302	0.63	0.46
Negative Exponential Function	0.2113	9.32	20325	0.63	0.45
Normal Function	0.0055	9.54	21587	0.61	0.41
Power Function	3.7500	9.55	21547	0.62	0.45
Log Normal Function	0.6875	9.13	20645	0.63	0.46

It can be seen from comparing Table 7.1 with Table 7.4 that for the current distance function used in the present model (the negative exponential distance function), the performance of the model using a global beta is lower than when utilising an origin specific beta, with the SSE being 9% higher than when using an origin specific beta value. Therefore there is a case for using origin specific beta values in the model.

Comparison of the performances of each of the alternative distance functions shown in Table 7.4 shows that when considering the values of  $r^2$  and  $r_s$ , there is no significant difference between the functions, although the normal function does appear to perform slightly less well than the others. When the SSE is taken into account it can also be seen that the functions do not produce significantly different model performances, although the square root exponential function actually performs better than the negative exponential function which is the function currently used in the model, although only by a small amount. Therefore these goodness of fit statistics make it difficult to see which of the distance functions is the most appropriate for modelling mortgage sales although it can be seen that the normal and power functions perform less well than the other distance functions. One method which may help in the decision of which distance

function is the most appropriate is the plotting of the flows predicted by each function against distance travelled and comparing them to the observed flows plotted against distance travelled.

Figure 7.5 shows the observed flows plotted against observed drive times. Therefore the function which looks most like this graph will be the one that best replicates the observed distance decay.

Figure 7.6 for the square root exponential function indicates that using this function, there are more flows at shorter distances than in the observed data and very few flows at long distances. However, this graph does show a similar gradient to that of Figure 7.5. The negative exponential function, shown in Figure 7.7 indicates that this function produces less short flows than the observed data, but has a similar gradient to Figure 7.5. Figure 7.8 shows the relationship between flows and drive times for the normal function; in this case the graph is dissimilar to that for the observed flows because it does not show a concave curve. The power distance function shown in Figure 7.9 shows a sharper distance decay than Figure 7.5 with very few flows at longer distances. The distance decay for the log normal function, shown in Figure 7.10, is quite similar to that for observed flows although there are fewer flows at longer distances.

Figure 7.5: Observed flows against observed drive times

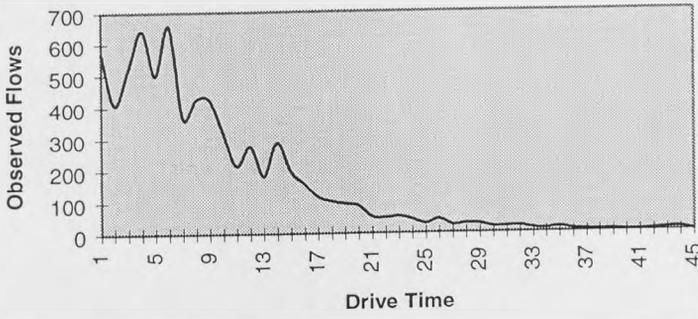


Figure 7.6: Predicted flows against drive times for the square root exponential function

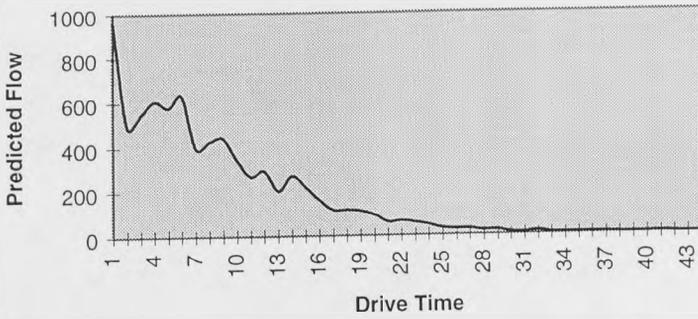


Figure 7.7: Predicted flows against drive times for the negative exponential function

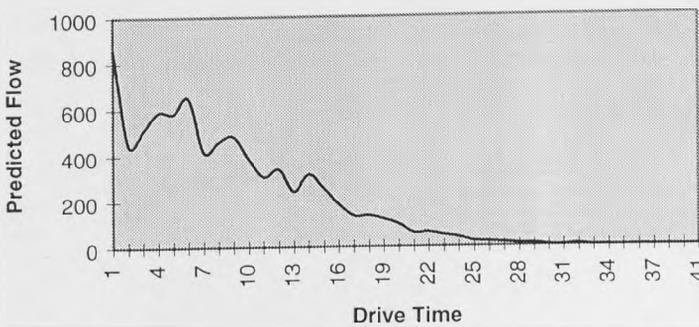


Figure 7.8: Predicted flows against drive times for the normal function

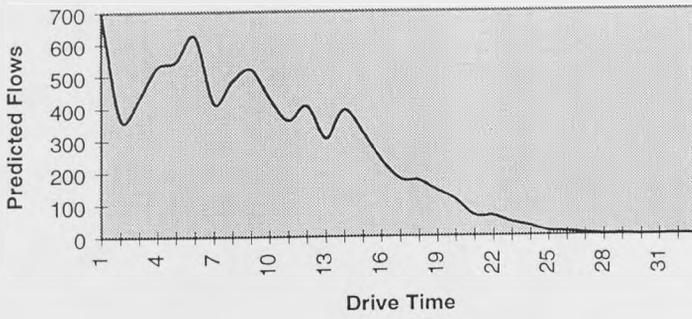


Figure 7.9: Predicted flows against drive times for the power function

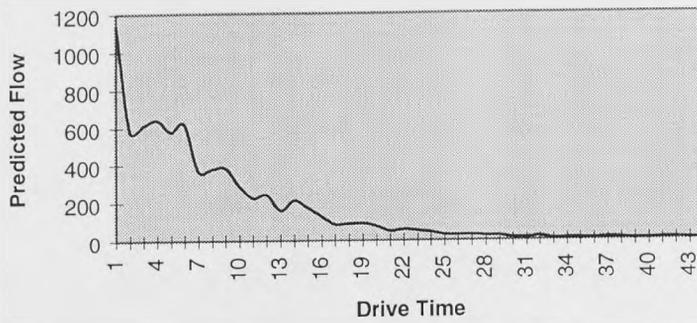
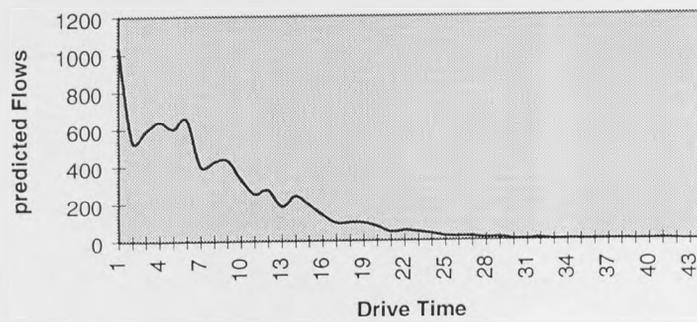


Figure 7.10: Predicted flows against drive times for the log normal function



This analysis also indicates that the square root exponential, negative exponential and the log normal functions are the best performing distance functions. However, Figures 7.5 - 7.10 do not show conclusively which form of the distance function performs the best using a global distance decay parameter. Therefore, from the results obtained the square root exponential function appears to perform the best but only by a small amount.

#### 7.3.4.2 Results using origin specific beta values

As was seen from the maps and correlation in Section 7.3.3 and the comparisons of model performance between the origin specific beta model and a global beta model, there is a case for using origin specific beta parameters in the Halifax spatial interaction model for mortgage sales.

Origin specific beta values,  $\beta_i$ , are used in the model at present and are calibrated iteratively until a value of beta for an origin is found that equates predicted average drive time for the origin with the observed average drive time for that origin. However, there is a problem in this process because there are some origins for which there are no observations but this does not mean that people will not travel from these areas to purchase mortgages. At present this problem is solved by giving origins with no observed average drive time the average beta value of the postal sectors surrounding it. This may not be the ideal solution to this problem because origins may be significantly different to those origins surrounding it and therefore produce different interaction behaviour. An alternative method of calibrating betas for origins with no observations is to relate the beta value to some origin characteristics. It has already been seen in Section 7.3.3 that there is a relationship between observed average drive times and population density. Population densities can be used to provide average drive times for those origins with no observed data. Origins with no observed drive time will be given the average of the observed average drive times for the five origins with the nearest level of population density to that origin.

The results of the origin specific beta analysis for the alternative distance functions are shown in Table 7.5 below. The results shown are after five calibration iterations. The calibration procedure was not reprogrammed to produce parameters that would optimise predicted drive times due to difficulties encountered in the alteration of the complex

FORTRAN code written by GMAP employees. The results in the table cannot be compared directly to each other because each iteration for different distance functions will have a differing level of effect depending on the values chosen for the increments of beta during calibration which are different in each case.

Table 7.5: Goodness of fit statistics using origin specific betas

Function	PADT	SSE	$r^2$	$r_s$
Square Root Exponential Function	9.34	18278	0.67	0.48
Negative Exponential Function	8.90	18584	0.66	0.49
Normal Function	9.58	19187	0.65	0.45
Power Function	9.28	20391	0.63	0.47
Log Normal Function	9.19	18718	0.66	0.47

It can be seen that the introduction of origin specific distance decay parameters has improved the performance of the model for all distance functions. In the case of the square root exponential function the goodness of fit statistics have improved compared to those of the negative exponential function which is currently used in the Halifax spatial interaction model, and the log normal function produces statistics not much worse than for the negative exponential function. The square root exponential function also produces an overall predicted average drive time of 9.34 which is closer to the observed average drive time of 9.37 than the predicted average drive time for the negative exponential distance function (8.90).

#### 7.3.4.3 Results using destination specific beta values

Destination specific betas were calibrated by attempting to equate observed and predicted average drive times for each centre. As with the origin specific betas, five iterations were undertaken. This analysis was undertaken using the negative exponential distance function.

Table 7.6: Goodness of fit statistics using destination specific betas

	PADT	SSE	$r^2$	$r_s$
Negative Exponential Function	10.95	27865	0.50	0.36

Table 7.6 shows that the use of destination specific betas values as opposed to origin specific distance decay parameters decreases the performance of the spatial interaction model for mortgage sales. This could be because people's willingness to travel different distances to alternative centres is already accounted for in the model by the centre attractiveness factor.

Thus, from the analysis undertaken so far concerning the interaction function, the best model performance is achieved using an origin specific distance decay parameter on a square root exponential distance function, where drive times are used as the method of measuring separation between origins and destinations. However, it is possible that further improvements could be made to the Halifax models performance through the use of alternative accessibility functions.

## **7.4 ALTERNATIVE ACCESSIBILITY FUNCTIONS**

It has been argued that alternative accessibility functions to the purely distance based function used in the Wilson spatial interaction models might be more appropriate and could therefore reproduce trip patterns more accurately. There are two major alternative accessibility functions which will be analysed, the intervening opportunities model and the competing destinations model, which were introduced in Section 2.8.

### ***7.4.1 The intervening opportunities model***

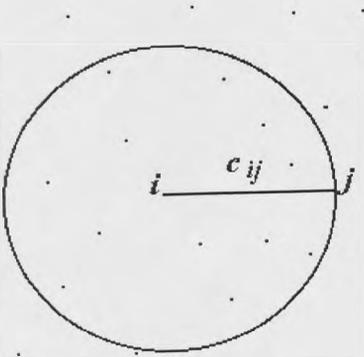
The intervening opportunities model was described in detail in Section 2.8.1.

#### ***7.4.1.1 Calculating the intervening opportunities variable***

Several alternative methods of calculating the intervening opportunities variable have been proposed, three of which will be tested in this analysis. Of these three methods, one is taken from Wilson's (1970) discussion on intervening opportunities and the second and third are derived from the work of Stouffer (1940).

The derivation of the intervening opportunities variable developed by Wilson (1970) involves ranking destinations away from origin  $i$  according to distance from  $i$ . Subsequently, all destinations that are closer to origin  $i$  than destination  $j$  is are counted as intervening opportunities and their attractiveness is included in the intervening opportunities variable. Destinations are included regardless of what direction they are located, as shown in Figure 7.11 below.

Figure 7.11: Wilson's intervening opportunities variable

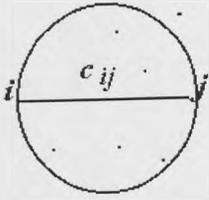


All alternative destinations that fall within the circle are counted as intervening opportunities between origin  $i$  and destination  $j$ , whereas those outside are not.

The form of the intervening opportunities variable originally proposed by Stouffer (1940) is slightly different to that of Wilson in that there is a directional component. Destinations are only seen to be intervening if they are passed on the way from origin  $i$  to destination  $j$ . This stems from Stouffer's assumption that each person considering travelling from origin zone  $i$  to destination  $j$  will consider each of the alternative destinations that they pass. Therefore destinations that are not passed on the way to destination  $j$  will not be considered as intervening even though they may be closer to origin  $i$  than destination  $j$  is.

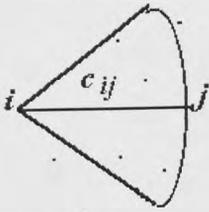
The area within which destinations are considered to be intervening in this method can vary depending on how it is believed that consumers perceive space. Two alternative versions that will be used are shown in Figures 7.12 and 7.13 below.

Figure 7.12: Stouffer's intervening opportunities variable, version A



In this case, destinations are considered to be intervening if they fall within a circle, the diameter of which is the line joining origin  $i$  to destination  $j$ .

Figure 7.13: Stouffer's intervening opportunities variable, version B



In the second version of the calculation of Stouffer's intervening opportunities, the notion of interveningness is restricted to a segment, as shown above.

It can be seen that in both of these cases the number of destinations that are considered to be intervening are fewer than in the Wilson formulation of the intervening opportunities variable.

#### 7.4.1.2 Proposals for analysis on intervening opportunities

Each of the three alternative formulations of the intervening opportunities variable are tested within the Halifax model for mortgage sales. Each variable is included in the model both as a single variable interaction term using just the intervening opportunities variable as was proposed by Stouffer, and as two variable interaction terms using both intervening opportunities and distance to determine impedance as suggested by Gonclaves and Ulysea-Neto (1993).

#### 7.4.1.3 Results using intervening opportunities

The results of the intervening opportunities analysis after five beta iterations, are given in Table 7.7 and Table 7.8 below. The model was calibrated to equate predicted average drive time to observed average drive time as before.

Table 7.7: Goodness of fit statistics for models with intervening opportunities accessibility functions

Model	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18583	0.66	0.48
Wilson	9.27	38999	0.51	0.32
Stouffer A	12.51	97640	0.07	-0.14
Stouffer B	13.71	67159	0.15	-0.18

For the single parameter intervening opportunities model for each of Stouffer's alternative formulations of the variable, it was not possible to calibrate the model to equate predicted average drive time with observed average drive time, hence the poor values of goodness of fit statistics for these models. Version A of the Stouffer intervening opportunities variable would only decrease predicted average drive time to 12.51 minutes, and version B to 13.71 minutes. One explanation for this could be that these two variables introduce less intervening opportunities than the Wilson variable and therefore people will travel further and the interaction variable cannot introduce enough impedance to decrease predicted average drive time to 9.37 minutes.

It can also be seen from Table 7.7 that the single interaction variable intervening opportunities model using the Wilson formulation does not perform well in comparison to the current formulation of the GMAP Full Model using a negative exponential distance function. All the goodness of fit statistics are worse for the intervening opportunities model indicating that this single variable formulation is not an improvement on the current formulation of the model.

Table 7.8: Goodness of fit statistics for models with intervening opportunities and drive time accessibility functions

Model	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18584	0.66	0.48
Wilson	9.33	34864	0.54	0.34
Stouffer A	9.35	75384	0.33	0.28
Stouffer B	9.38	25924	0.58	0.34

It can be seen from Table 7.8 that with the introduction of the distance variable in combination with the intervening opportunities variable it is possible to calibrate the model for all three alternative formulations of the intervening opportunities variable. However, it can also be seen that none of the alternatives produce goodness of fit statistics equal to the GMAP Full Model, which uses a negative exponential distance function. Of the three alternative intervening opportunities variables the Stouffer version B produces the best results.

#### *7.4.2 The competing destinations model*

The second alternative accessibility function is based on Fotheringham's theory of competing destinations which was described in Section 2.8.2.

##### *7.4.2.1 Proposals for analysis on competing destinations*

The inclusion of the accessibility variable as described in Section 2.8.2 could be a valuable extension to the spatial interaction model, enabling such models to predict interaction patterns more satisfactorily. This hypothesis is tested using the flow data from the Halifax spatial interaction model in order to see if the competing destinations

extension to the spatial interaction model can replicate the data better than the Wilson spatial interaction model without such a term which is the form of the model that is currently used.

The additional accessibility variable,  $Z_j$  will be included in the Halifax spatial interaction model in order to analyse the effect on predicted trip patterns. This process will also involve the discovery of the most appropriate value for the  $\delta$  parameter in the case of mortgages which will depend on whether competition or agglomeration forces are present.

#### 7.4.2.2 Calculation of the competing destinations variable

It was seen in equation (2.54) how the competing destinations accessibility variable,  $Z_j$  is calculated; however, the parameter values also have to be determined.

Fotheringham (1981) states that for  $\sigma$  the ideal calculation of this parameter would include iteration. However, an alternative used was the average  $\beta_i$  from the production constrained model. It was found that this more simple method did not significantly decrease model performance when compared to a parameter value calculated iteratively. Thus

$$\sigma = \frac{\sum \beta_i}{n} \quad (7.12)$$

where

$n$  = the number of origins

It is also necessary to determine the value of the  $\delta$  parameter for the  $Z_j$  variable. The accessibility parameter used is not origin specific in order make the calibration process quicker and simpler. The Pearson's correlation coefficient  $r$ , between centre residuals calculated for the GMAP Full Model and the accessibility variable ( $Z_j$ ) as calculated in equation (2.54) is 0.45, which is significant at the 99.99% level, where

$$\text{centre residual} = \text{observed centre sales} - \text{predicted centre sales} \quad (7.13)$$

Thus, a positive value for the centre residual indicates that the centre is over performing with observed sales in excess of predicted sales.

This positive value of the correlation between centre residuals and the accessibility variable indicates that when  $Z_j$  is high *i.e.* when a destination is accessible, then the model tends to underpredict flows to that centre. Fotheringham (1983) states that this indicates that the production constrained spatial interaction model as used for the prediction of Halifax mortgage sales is misspecified because there is a relationship between model performance and spatial structure. Therefore, according to Fotheringham (1983) it would seem that agglomeration effects are present in the system that are not being accounted for in the current formulation of the spatial interaction model for Halifax new mortgage sales. Fotheringham argues that the inclusion of a competing destinations accessibility variable such as  $Z_j^\delta$  will help to alleviate the relationship between spatial structure and model performance. Also in such a case as this a positive parameter on the accessibility variable will also incorporate the agglomeration effects that are apparent in the system by making accessible centres more attractive.

#### *7.4.2.3 Results for the competing destinations model*

Several values of  $\delta$  were tested in the competing destinations model and the optimal goodness of fit was achieved when  $\delta = 1.5$ . Therefore the accessibility variable  $Z_j^\delta$  was included in the model with  $\delta = 1.5$  to indicate agglomeration effects. The comparative goodness of fit statistics between the GMAP Full Model and the competing destinations model are shown in Table 7.9 below. The CD Base Model represents the competing destinations model with the original beta values from the GMAP Full Model, and the CD Full Model represents the competing destinations model with newly calibrated beta values.

Table 7.9: Goodness of fit statistics for the competing destinations models

MODEL	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18584	0.66	0.48
CD Base Model	8.88	18216	0.66	0.48
CD Full Model	9.10	17412	0.68	0.49

It can be seen from Table 7.9 that even without recalibrating beta to account for the new accessibility function the inclusion of a competing destinations accessibility variable improves the SSE between observed and predicted interactions. The  $r^2$  and  $r_s$  values are unchanged, but the difference between predicted average drive time and observed average drive time has increased. If model performance for each of the individual centres is considered as shown in Table 7.10, it can be seen that the CD Base Model improves the prediction for 72% of the 68 centres in the region when compared with the GMAP Full Model. The competing destinations model predicts 37% of centres within 10% of the observed value, whereas the GMAP Full Model only manages this for 12% of centres.

However, the competing destinations model, with its additional variable, will not necessarily use the same origin specific betas as the GMAP Full Model. Therefore new origin specific beta values were calibrated for the competing destinations model, to produce the CD Full Model. Five beta calibration iterations were undertaken. The goodness of fit statistics for this model are shown in Table 7.9. It can be seen that all of the goodness of fit statistics have improved from both the GMAP Full Model and the CD Base Model. The SSE between interactions has decreased by 5%,  $r^2$  and  $r_s$  have increased slightly and the difference between predicted average drive time and observed average drive time has decreased significantly.

Thus it can be seen that the CD Full Model with newly calibrated origin specific beta values does improve the performance of the model. However, the overall goodness of fit statistics hide a wide variation in the model performance between centres, as can be seen from Table 7.10.

Table 7.10: Model performance for individual centres for the GMAP Full Model and the competing destinations models

Centre	Observed Sales	% Difference Between Observed and Predicted Sales - GMAP Full Model	% Difference Between Observed and Predicted Sales - CD Base Model	% Difference Between Observed and Predicted Sales - CD Full Model
Acombe	19	49	-6	-6
Armley	25	92	13	17
Barnsley	221	25	2	1
Batley	8	43	-101	-104
Beverley	71	-23	-49	-53
Boston	81	-57	-65	-75
Bradford	337	19	6	9
Bramley	29	79	49	48
Bransholme	12	83	2	-2
Bridlington	99	-13	-12	-13
Brighouse	68	49	2	1
Castleford	87	34	11	10
Chesterfield	189	-14	-20	-15
Cleckheaton	68	41	-27	-30
Crossgates	116	42	9	7
Dewsbury	113	8	-29	-32
Doncaster	388	-1	-15	-15
Duckworth	8	86	8	10
Elland	31	50	1	0
Gainsborough	20	-13	-27	-35
Garforth	33	72	40	39
Goole	62	16	-3	-5
Grimsby	192	-4	-2	-2
Guiseley	17	62	-25	-28
Halifax	404	6	-9	-9
Harehills	52	70	20	21
Headingley	67	56	20	19
Heckmondwike	19	52	1	-1
Hessle	23	24	-44	-48
Horsforth	20	59	-50	-50
Huddersfield	304	-4	-19	-19
Hull	259	-34	-41	-39
Hunslet	37	88	42	43
Ilkley	28	70	63	56
Keighley	75	5	-20	-26
Leeds	410	18	1	3
Lincoln	196	-16	-14	-14
Louth	42	-38	-31	-52
Mansfield	105	4	-5	-6
Meadowhall	114	69	47	47
Mexborough	25	49	1	-2
Mirfield	24	59	2	1
Moortown	29	64	31	33
Morley	81	42	-26	-28
Normanton	28	58	-1	-3
Ossett	39	56	0	-2
Otley	50	62	34	33
Pontefract	83	11	-26	-26
Pudsey	79	50	-9	-7
Retford	48	15	-3	-5
Rotherham	240	9	-27	-28
Scarborough	39	-90	-94	-94
Scunthorpe	159	31	28	27
Selby	94	13	7	4
Sheffield	527	13	11	13
Shiplay	78	36	-5	-5
Skipton	22	-44	-65	-56
Sleaford	56	-16	-6	-10
Sowerby Bridge	21	58	-1	-3
Spalding	23	-36	-30	-33
Tadcaster	13	50	-11	-14
Thorne	33	49	5	-2
Todmorden	26	49	46	8
Wakefield	286	12	-7	-8
Wetherby	19	60	19	16
Worksop	135	19	12	13
Yeadon	13	73	13	15
York	193	-40	-45	-43

When individual centre predicted mortgage sales totals are considered, as shown in Table 7.10, it can be seen that the CD Full Model also out performs the GMAP Full model but not the CD Base Model. The CD Full Model improves the predictions for 71% of centres compared to the GMAP Full model, and predicts 35% of centres within 10% of the observed centre totals. Figures 7.11 and 7.12 which show scatterplots between observed and predicted centre totals for the GMAP Full Model and the CD Full Model also indicate that the CD Full Model improves centre prediction when compared to the GMAP Full Model because there is significantly less variation from the 45 degree line in Figure 7.12 than there is in Figure 7.11.

Figure 7.11: Observed and predicted numbers of new mortgage sales for centres, for the GMAP Full Model

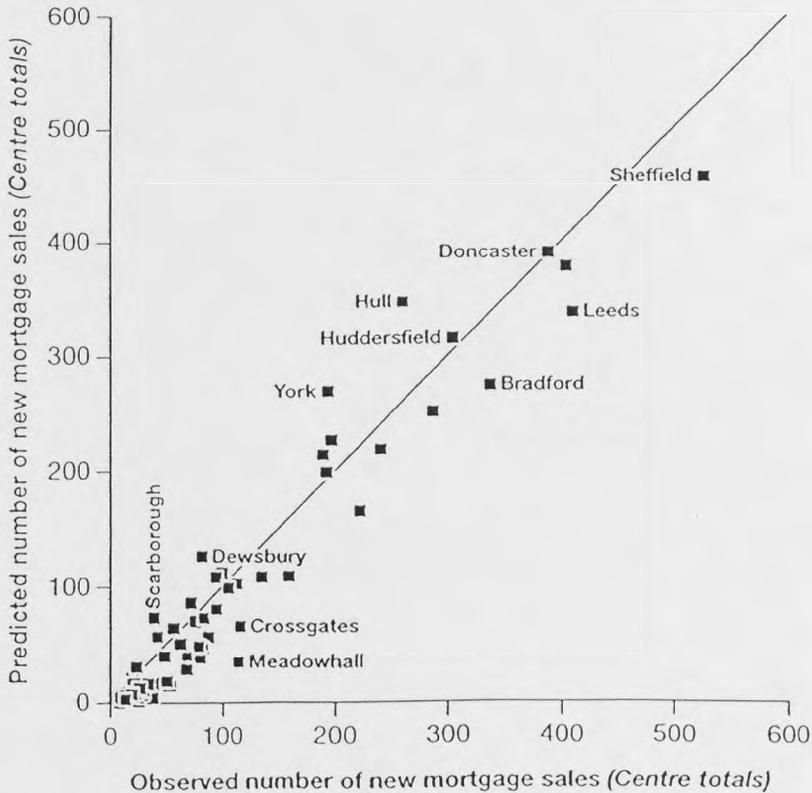
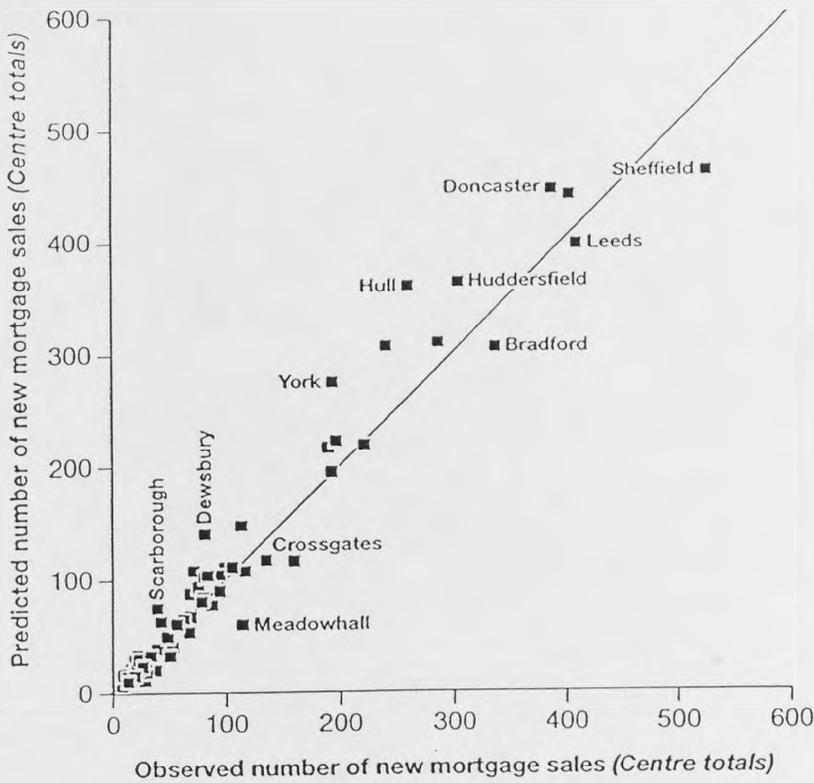


Figure 7.12: Observed and predicted numbers of new mortgage sales for centres, for the CD Full Model



Comparison of Figures 7.11 and 7.12 shows that there are several centres for which new mortgage sales predictions have been improved through the inclusion of a competing destinations accessibility variable. Predicted sales for Leeds have been increased so that predictions are only 3% lower than observed sales. It can also be seen that predictions for the centres of Crossgates and Bradford have been improved. However, the figures also show that predictions for some centres, such as Doncaster and Huddersfield have worsened.

Thus, it can be seen that the introduction of a competing destinations variable into the spatial interaction model has led to an improvement in model performance, but with a variation in performance when individual centres are considered.

## 7.5 A COMBINED ALTERNATIVE INTERACTION FUNCTION

At the end of Section 7.3 it was stated that the analysis undertaken concerning distance calculation, distance decay parameters and alternative distance functions, produced a best performing model containing origin specific beta values and a square root exponential distance function using an interaction cost measurement based on drive times. Section 7.4 indicated that there was evidence to suggest that the inclusion of an accessibility variable based on competing destinations could lead to an improvement in model performance. The aim of this Section is to calibrate a spatial interaction model for Halifax new mortgage sales using the new distance function combined with a competing destinations accessibility variable. This model will be called the CI (combined interaction) Model.

The CI Model was calibrated in the same way as above, using maximum likelihood methods to attempt to equate observed and predicted origin drive times. Five iterations on the origin specific betas were undertaken.

### 7.5.1 Results of the Combined Interaction Model

The goodness of fit statistics achieved for the CI Model after five iterations are shown in Table 7.11 below. The results for the GMAP Full Model, are also shown.

Table 7.11: Goodness of fit statistics for the CI Model

MODEL	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18584	0.66	0.48
CI Model	9.32	16880	0.69	0.48

Table 7.11 indicates that the combination of the square root exponential function and the competing destinations accessibility variable leads to an improvement in model performance. The SSE has decreased by 9% compared to the GMAP Full Model,  $r^2$  has increased and the difference between observed and predicted drive times has decreased. The goodness of fit statistics have also improved when compared to the CD Full Model.

SSE has decreased by 3% and the  $r^2$ ,  $r_s$  and difference between observed and predicted drive time have also improved.

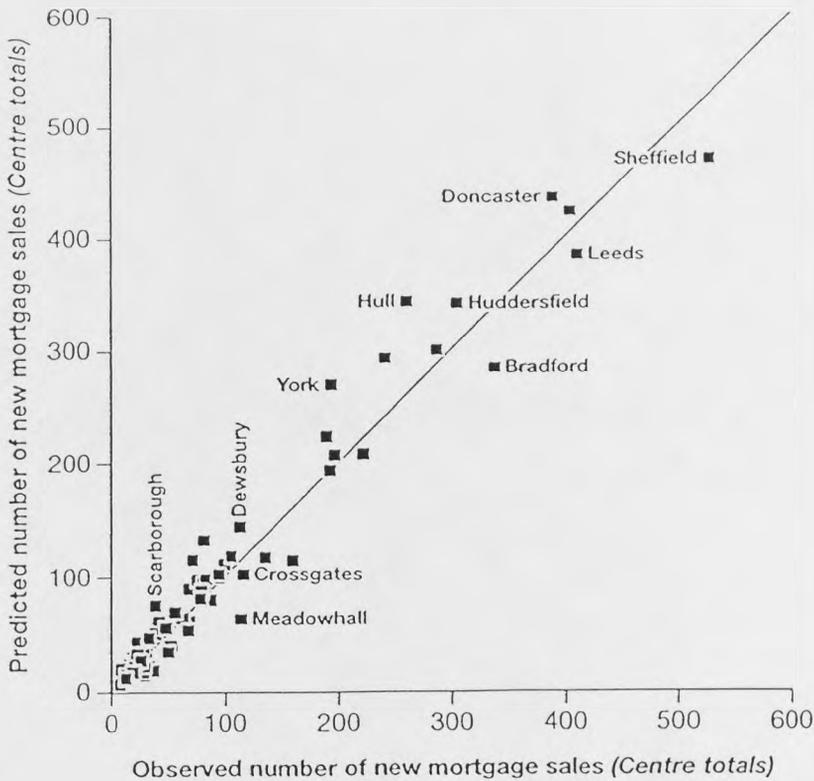
The individual centre model performance levels are shown in Table 7.12 below, again the GMAP results are shown to enable comparisons to be made.

The CI Model leads to an improvement in prediction from the GMAP Model for 74% of centres which is higher than for the CD Full Model. However, although the CI Model predicts more centres within 10% of observed sales totals (27% of centres) than the GMAP Full Model this is less than is achieved by the CD Full Model. Figure 7.13 which shows the scatterplot between observed and predicted centre totals for the CI Model indicates that centre predictions have been improved when compared to the GMAP Full Model (Figure 7.11).

Table 7.12: Individual centre model performance for the GMAP Full Model and the CI Model

Centre	Observed Sales	% Difference Between Observed and Predicted Sales GMAP Full Model	% Difference Between Observed and Predicted Sales CI Model
Acombe	19	49	-15
Armley	25	92	23
Barnsley	221	25	6
Batley	8	43	-79
Beverley	71	-23	-63
Boston	81	-57	-64
Bradford	337	19	16
Bramley	29	79	46
Bransholme	12	83	-25
Bridlington	99	-13	-13
Brighouse	68	49	9
Castleford	87	34	8
Chesterfield	189	-14	-18
Cleckheaton	68	41	-31
Crossgates	116	42	11
Dewsbury	113	8	-28
Doncaster	388	-1	-12
Duckworth	8	86	9
Elland	31	50	-8
Gainsborough	20	-13	-46
Garforth	33	72	33
Goole	62	16	-4
Grimsby	192	-4	-1
Guiseley	17	62	-41
Halifax	404	6	-5
Harehills	52	70	23
Headingley	67	56	20
Heckmondwike	19	52	-5
Hessle	23	24	-90
Horsforth	20	59	-52
Huddersfield	304	-4	-12
Hull	259	-34	-32
Hunslet	37	88	49
Ilkley	28	70	36
Keighley	75	5	-30
Leeds	410	18	6
Lincoln	196	-16	-6
Louth	42	-38	-46
Mansfield	105	4	-13
Meadowhall	114	69	44
Mexborough	25	49	-24
Mirfield	24	59	3
Moortown	29	64	20
Morley	81	42	-17
Normanton	28	58	-19
Ossett	39	56	-31
Otley	50	62	29
Pontefract	83	11	-19
Pudsey	79	50	-11
Retford	48	15	-17
Rotherham	240	9	-22
Scarborough	39	-90	-95
Scunthorpe	159	31	28
Selby	94	13	-9
Sheffield	527	13	11
Shipley	78	36	-5
Skipton	22	-44	-36
Sleaford	56	-16	-24
Sowerby Bridge	21	58	-24
Spalding	23	-36	-47
Tadcaster	13	50	-16
Thorne	33	49	-44
Todmorden	26	49	-5
Wakefield	286	12	-5
Wetherby	19	60	10
Worksop	135	19	13
Yeadon	13	73	6
York	193	-40	-39

Figure 7.13: Observed and predicted numbers of new mortgage sales for centres, for the CI Model



Comparison of Figures 7.13 and 7.12 indicates that variation from the 45 degree line has been further decreased by the introduction of the square root exponential distance function as opposed to the negative exponential function, but only slightly. All of the outlying centres such as Hull, Sheffield, Doncaster, Scarborough and York are now being predicted more accurately.

## 7.6 CONCLUSIONS

The analysis undertaken concerning alternative distance functions has shown that there is evidence to support the use of origin specific beta values in the model for mortgage sales because there is a significant relationship between origin characteristics and propensity to travel to receive a mortgage. The use of origin specific beta parameters has also been shown to improve the performance of the model significantly when compared to models using global distance decay parameters.

As concerns the most appropriate distance function of those tested, the results from both the global beta analysis and the origin specific beta analysis indicate that three functions the square root exponential function, negative exponential function and the log normal function produce the most accurate interaction patterns for the Halifax mortgage model and that of these three, the square root exponential function produces the highest level of model performance.

The analysis concerning alternative accessibility functions concluded that both the single variable intervening opportunities model and the hybrid intervening opportunities and distance model caused model performance to decrease. However, the inclusion of a competing destinations accessibility variable in the spatial interaction did lead to an improvement in model performance.

The combination of the square root exponential and the competing destinations accessibility variable and the use of origin specific betas lead to a further improvement in model performance for Halifax new mortgage sales.

It could be possible that further improvements in model performance could be achieved. For example, through undertaking further iterations of the beta calibration procedure. Experimentation with alternative values of the  $\delta$  on the competing destinations variable could also be helpful to determine the extent of the agglomeration effects apparent in the system.

Now that all three components of the spatial interaction model have been analysed individually, it is necessary to combine the recommendations for both the Halifax new mortgage sales model and the WH Smith model. This is undertaken in Chapter 8.

## **THE INTEGRATION OF DEMAND, SUPPLY AND INTERACTION ANALYSIS FOR THE HALIFAX AND WH SMITH SPATIAL INTERACTION MODELS**

### **8.1 INTRODUCTION**

This chapter brings together the three separate sections of analysis previously completed: demand, supply and interaction. This is undertaken for both the Halifax and WH Smith spatial interaction models. This analysis will enable the assessment of the overall effect on model performance of the alterations that have been made to the models in previous chapters. It is possible that the effects achieved for each section could be independent and additive or there could be interaction between the new factors introduced. Therefore the combination of new modelling practices could lead to either an increase or a decrease in model performance. This analysis will also enable a conclusion to be made concerning which of the three sections of analysis has produced the most model performance improvement. There is also the possibility that results achieved for one type of retail model will not be replicated for an alternative retail service due to differences in purchasing behaviour for alternative good types.

### **8.2 THE HALIFAX MODEL**

Demand and supply analysis have to be added to the interaction analysis undertaken for Halifax mortgage sales in Chapter 7.

#### ***8.2.1 Demand estimation***

At present demand is incorporated into the Halifax model using data purchased by Halifax from CACI. These data are a record of actual sales of new mortgages by postal sector for 1995 and information is collected from members of the National Mortgage Market Database. This is a pooled database containing data from banks and building

societies concerning the number of mortgages that they sell in each postal sector. Not all mortgage providers subscribe to this database but demand estimates are weighted to account for these non subscribers. Thus the demand estimates used in the Halifax model are based on actual sales and are therefore likely to be relatively accurate.

The alternative demand estimation procedures will not be undertaken for mortgage sales because there is no information in the Family Expenditure Survey concerning expenditure on new mortgages, only on existing ones. Also, as was seen earlier, the current demand estimates for Halifax new mortgage sales are likely to be relatively accurate.

### *8.2.2 Supply side analysis*

The centre revenue prediction analysis undertaken for WH Smith in Section 6.4 will be replicated here for the Halifax spatial interaction model for mortgage sales. In the GMAP formulation of the Halifax model there is only one type of CCF calculated, unlike the WH Smith model for which three types are calculated, for residential, work and tourist trip types. It is unlikely that tourism factors will be important in determining demand for mortgages, therefore only residential and work CCFs will be tested for the Halifax model.

#### *8.2.2.1 Residential CCFs*

The production of new residential CCFs for financial centres required additional fieldwork to be undertaken for financial centres in the Yorkshire TV region that did not appear as shopping centres in the WH Smith model. The results of the fieldwork are shown in Table 8.1 below. The field survey attractiveness factor was calculated in the same way as described in Section 6.4.3.4. The new CCFs calculated will be tested using both the GMAP Base Model (containing no CCFs) and the Halifax model using the new interaction calculation consisting of competing destinations and a square root exponential distance function (but no CCFs). This will be called the CI (combined interaction) Base Model. Table 8.2 contains the goodness of fit statistics for these models. The statistics for the GMAP Full Model and the CI Model (which contains CCFs) are also included to enable comparisons to be made.

Table 8.1: Observed centre characteristics of financial centres in the Yorkshire TV region containing Halifax branches

Centre	Parking Spaces	% Multiples	% Covered /Pedestrian	Banks/Building Societies	Dept./Key Stores	General Attractiveness
Acombe	50	35	42	5	2	12
Armley	50	36	0	5	4	9
Barnsley	2320	48	34	23	6	10
Batley	583	47	6	6	4	12
Beverley	590	54	56	15	2	19
Boston	2090	32	21	15	7	17
Bradford	1820	28	21	27	9	10
Bramley	127	67	100	2	3	10
Bransholme	520	63	100	2	3	11
Bridlington	650	34	22	9	4	12
Brighouse	335	33	0	9	4	9
Castleford	756	55	67	9	8	11
Chesterfield	1741	36	27	20	7	11
Cleckheaton	160	10	0	5	2	11
Crossgates	432	57	71	4	6	13
Dewsbury	1890	32	18	10	4	9
Doncaster	2530	34	25	27	11	16
Duckworth	0	0	0	1	0	5
Elland	150	0	0	4	0	6
Gainsborough	385	25	31	8	4	11
Garforth	100	36	0	4	1	9
Goole	710	39	31	8	4	10
Grimsby	1953	47	60	19	6	14
Guiseley	117	57	0	5	1	10
Halifax	1504	33	42	19	5	13
Harehills	0	19	0	3	2	6
Headingley	250	60	34	8	3	12
Heckmondwike	121	24	0	4	1	9
Hessle	205	26	41	4	2	10
Horsforth	462	14	0	8	1	10
Huddersfield	2669	41	28	25	7	14
Hull	3019	56	59	41	13	19
Hunslet	250	30	60	3	1	11
Ilkley	764	35	10	11	5	21
Keighley	1670	44	46	15	6	10
Leeds	4680	57	60	40	15	22
Lincoln	2224	51	48	31	8	21
Louth	538	32	0	10	3	11
Mansfield	1617	55	77	17	9	13
Meadowhall	12000	77	100	6	7	25
Mexborough	412	25	67	4	3	8
Mirfield	250	7	0	5	1	10
Moortown	75	50	0	6	1	11
Morley	856	33	71	10	5	11
Normanton	80	22	96	5	1	13
Ossett	206	21	83	8	2	13
Otley	353	32	16	8	4	16
Pontefract	1150	38	33	11	5	8
Pudsey	182	30	0	8	4	9
Retford	642	30	45	10	2	15
Rotherham	1578	39	30	16	3	10
Scarborough	1806	44	60	18	6	22
Seunthorpe	2059	28	34	14	8	11
Selby	536	49	35	10	4	12
Sheffield	4097	40	37	44	11	11
Shipley	552	51	36	12	5	13
Skipton	840	33	15	10	5	21
Sleaford	330	38	21	10	3	11
Sowerby Bridge	150	0	0	3	1	7
Spalding	668	30	22	13	6	12
Tadcaster	328	11	0	4	1	10
Thorne	20	21	36	6	3	9
Todmorden	210	14	82	5	3	13
Wakefield	2343	50	57	20	5	17
Wetherby	500	24	21	10	1	12
Worksop	1066	26	34	12	4	8
Yeadon	390	17	0	7	1	10
York	643	50	30	27	12	24

Table 8.2: Goodness of fit statistics for the Halifax models

Model	PADT	SSE	$r^2$	$r_s$
GMAP Full Model	8.90	18583	0.66	0.49
GMAP Base Model	8.82	19655	0.65	0.48
CI Model	9.32	16880	0.69	0.48
CI Base Model	9.21	18363	0.66	0.47

Table 8.2 indicates that when the CCFs are removed from the models the model performance does decrease for both the GMAP Full Model and the CI Model.

Tables 8.3 and 8.4 below show correlations between the centre characteristics and centre performance that were undertaken in order to see if the variables are significant in determining centre performance. Centre performance is calculated in the following way

$$\text{observed centre sales for Halifax/predicted centre sales for Halifax} * 100 \quad (8.1)$$

Therefore centres with a performance value greater than 100 are performing better than expected when compared to predicted sales whereas centres with a value less than 100 are not performing as well as predicted.

Table 8.3: Correlations between attractiveness factors and centre performance for the GMAP Base Model, for financial centres in the Yorkshire TV region containing Halifax branches

Attractiveness Factor	r	$r_s$
Parking Spaces	0.11	0.21
% Multiples	0.17	0.08
% Covered	0.17	0.11
Banks/Building Societies	-0.30	-0.25
Number of Stores	-0.26	-0.25
Key Stores	-0.20	-0.14
Field Survey Attractiveness	-0.03	-0.08

Table 8.4: Correlations between attractiveness factors and centre performance for the CI Base Model, for financial centres in the Yorkshire TV region containing Halifax branches

Attractiveness Factor	r	r <sub>s</sub>
Parking Spaces	0.17	-0.27
% Multiples	0.24	0.21
% Covered	0.01	-0.13
Banks/Building Societies	-0.16	-0.04
Number of Stores	-0.11	-0.04
Key Stores	-0.07	0.06
Field Survey Attractiveness	0.01	0.04

It can be seen that the relationship between centre characteristics and centre performance is not as pronounced as was seen for WH Smith (Table 6.11). For the GMAP Base Model none of the variables used in the creation of the WH Smith residential CCFs are correlated significantly with centre performance at the 95% level. For the CI Base Model the centre characteristics percentage multiples and number of parking spaces are significant at the 95% level although the values are considerably lower than those seen for WH Smith.

These findings indicate that consumers consider centres and their attractiveness differently for different goods. For the goods sold by WH Smith, the centre characteristics found through fieldwork were found to be important determinants of centre choice and therefore those centres that were the most attractive tended to produce better performing stores. The situation seems to be more complicated for Halifax mortgage sales. Customer behaviour and perception of centre characteristics appears to be different. Mortgages are a very different good to those sold by WH Smith which creates different customer behaviour and this could cause consumers to perceive the attractiveness of centres differently. Mortgages are a planned, one-off purchase whereas books are purchased frequently and are often impulse buys. Thus, purchasers could be choosing their destination using a different set of priorities than was seen in the case of WH Smith. The aesthetic attractiveness of the centre is not a factor in influencing destination choice and the more practical attractiveness factors such as percentage multiples and number of parking spaces are less important and this produces different interaction patterns for the different goods.

The competing destinations analysis undertaken in Chapter 7 indicated that agglomeration factors are apparent in the destination choice behaviour for mortgage sales which would indicate that a centre's location in relation to other centres providing financial services is more important in determining trip behaviour, although the negative correlation seen between centre performance and number of banks and building societies in the centre would indicate that within centres competition factors are apparent. Thus it can be seen that alternative factors are influencing the destination choice for mortgage sales than was observed for WH Smith goods.

These findings are backed up by the results produced from adding the new CCFs in to the Halifax model. As can be seen from Table 8.6 the introduction of the new CCFs produces a decrease in model performance for both the GMAP Base Model and the CI Base Model. The CCFs are calculated in the same way as described in Section 6.4.3.6 and are shown in Table 8.5. One set of CCFs was calculated using Meadowhall as the base (as was seen in Chapter 6) and another set were produced using the centre that was being predicted most accurately (Wakefield) as the base.

Table 8.5: New CCFs for the financial centres in the Yorkshire TV region containing Halifax branches

Centre	CCFs Based on Meadowhall	CCFs Based on Wakefield
Acombe	0.81	0.50
Armley	0.48	0.30
Barnsley	1.04	0.83
Batley	0.70	0.48
Beverley	1.20	0.79
Boston	0.73	0.62
Bradford	0.65	0.55
Bramley	1.67	1.04
Bransholme	1.65	1.06
Bridlington	0.67	0.47
Brighouse	0.45	0.30
Castleford	1.30	0.86
Chesterfield	0.80	0.63
Cleckheaton	0.15	0.11
Crossgates	1.35	0.86
Dewsbury	0.68	0.57
Doncaster	0.81	0.71
Duckworth	0.01	0.01
Elland	0.02	0.02
Gainsborough	0.61	0.41
Garforth	0.48	0.30
Goole	0.80	0.55
Grimsby	1.22	0.91
Guiseley	0.75	0.47
Halifax	0.87	0.66
Harehills	0.26	0.16
Headingley	1.07	0.68
Heckmondwike	0.32	0.21
Hessle	0.69	0.44
Horsforth	0.23	0.18
Huddersfield	0.93	0.79
Hull	1.40	1.11
Hunslet	0.90	0.58
Ilkley	0.61	0.44
Keighley	1.04	0.78
Leeds	1.52	1.32
Lincoln	1.20	0.92
Louth	0.46	0.33
Mansfield	1.43	1.02
Meadowhall	2.53	2.56
Mexborough	0.89	0.58
Mirfield	0.12	0.10
Moortown	0.66	0.41
Morley	1.06	0.72
Normanton	1.07	0.66
Ossett	0.97	0.61
Otley	0.59	0.39
Pontefract	0.83	0.61
Pudsey	0.40	0.26
Retford	0.81	0.55
Rotherham	0.85	0.65
Scarborough	1.19	0.88
Scunthorpe	0.77	0.64
Selby	0.96	0.63
Sheffield	1.06	1.00
Shipley	0.99	0.66
Skipton	0.63	0.46
Sleaford	0.69	0.45
Sowerby Bridge	0.02	0.03
Spalding	0.62	0.44
Tadcaster	0.18	0.14
Thorne	0.57	0.35
Todmorden	0.87	0.55
Wakefield	1.26	0.97
Wetherby	0.53	0.36
Worksop	0.68	0.51
Yeadon	0.25	0.19
York	0.96	0.64

Table 8.6: Goodness of fit statistics for models including new residential CCFs

Model	PADT	SSE	$r^2$	$r_s$
GMAP Base Model	9.72	19960	0.63	0.43
CCFs (Meadowhall)				
CI Base Model	9.53	19402	0.64	0.45
CCFs (Meadowhall)				
GMAP Base Model	9.67	19558	0.62	0.43
CCFs (Wakefield)				
CI Base Model	9.53	19040	0.63	0.46
CCFs (Wakefield)				

Table 8.6 indicates that the introduction of CCFs based on centre characteristics actually decreases the performance of both model types. This could be because there is a large variation in the values of the attractiveness factors for the different centres and this causes a large variation in the values of the CCFs produced. Some centres have values for the CCF which are very low, for example Elland, Duckworth and Sowerby Bridge. This causes the attractiveness of such centres to be decreased too much so that they hardly receive any predicted flows. In fact with the new CCF value Duckworth receives no predicted flows. This effect can be seen in Table 8.7 which contains the centre performance figures produced from the introduction of the CCFs.

Table 8.7: Centre performance for Halifax new mortgage sales for financial centres in the Yorkshire TV region containing Halifax branches

Centre	GMAP Base Model	GMAP Base Model + CCFs (Wakefield)
Acombe	100	125
Armley	262	10416
Barnsley	99	109
Batley	22	37
Beverley	52	59
Boston	52	56
Bradford	120	168
Bramley	322	152
Bransholme	73	53
Bridlington	73	101
Brighouse	83	222
Castleford	107	89
Chesterfield	74	97
Cleckheaton	88	943
Crossgates	167	154
Dewsbury	86	103
Doncaster	86	94
Duckworth	56	0
Elland	64	28181
Gainsborough	58	69
Garforth	275	1150
Goole	127	162
Grimsby	98	109
Guiseley	116	140
Halifax	100	102
Harehills	249	7536
Headingley	207	213
Heckmondwike	45	173
Hessle	92	194
Horsforth	91	735
Huddersfield	81	79
Hull	62	63
Hunslet	447	1156
Ilkley	192	172
Keighley	62	57
Leeds	97	62
Lincoln	83	94
Louth	62	111
Mansfield	90	82
Meadowhall	322	70
Mexborough	94	114
Mirfield	83	1270
Moortown	240	485
Morley	86	89
Normanton	173	184
Ossett	80	80
Otley	198	355
Pontefract	77	106
Pudsey	149	592
Retford	115	119
Rotherham	92	141
Scarborough	39	42
Scunthorpe	141	181
Selby	104	94
Sheffield	103	111
Shipley	95	90
Skipton	49	81
Sleaford	80	109
Sowerby Bridge	57	5676
Spalding	50	73
Tadcaster	151	1781
Thorne	96	122
Todmorden	175	137
Wakefield	100	85
Wetherby	184	470
Worksop	125	175
Yeadon	102	684
York	64	75

Table 8.7 shows that model performance is only improved for 18 centres, which is only 26% of centres. The centres with the low attractiveness factors and hence low CCFs have predicted flows that are extremely low which causes model residuals to increase significantly.

The correlation between number of stores in a centre and centre performance (Tables 8.3 and 8.4) was negative, indicating that smaller centres tend to perform better than larger centres. This is backed up by the centre performance figures shown above which show that smaller centres such as Hunslet and Harehills are being significantly under predicted in the model. This would help to explain why the centre attractiveness factors and therefore the CCFs are not significant in the case of mortgage sales. Consumers are (with the exception of Meadowhall) proportionately more attracted to smaller centres when only considering the  $W_j$  value (with no CCFs) whereas it is the larger centres that tend to have higher values in the attractiveness factors.

The wide variation in CCF values leads to the attractiveness factor in the model being altered too much when the CCF is included multiplicatively to the  $W_j$  attractiveness value. An analysis was undertaken using linear regression in order to investigate if the new attractiveness factors were more significant if included in the model by addition rather than multiplication because this would decrease the influence of outlying values. The dependent variable in the regression is observed flows. The correlation coefficients produced by the regression for each individual factor are shown in Table 8.8.

Table 8.8: Correlation coefficients from the regression analysis between centre attractiveness factors and model residuals

Variable	$r^2$
$W_j$	0.769
$W_j + \% \text{ Multiples}$	0.770
$W_j + \% \text{ Covered}$	0.769
$W_j + \text{ Parking Spaces}$	0.783
$W_j + \text{ General Attractiveness}$	0.769
$W_j + \% \text{ Multi+ Parking}$	0.784
All Variables	0.787

Table 8.8 shows that even used additively the variables do not have a large effect on the goodness of fit statistic. Only percentage multiples and number of parking spaces improves the fit of the regression when they are added to the  $W_j$  attractiveness calculation.

One possible method of including the significant attractiveness variables (percentage multiples and number of parking spaces) in a way that would decrease the variance in the CCF values is to use a grouped CCF value. Centres will be assigned one of three CCF values based on a comparison of the value of the attractiveness factor for that centre with the average value for that factor. If a centre's value for the factor is less than half the average the CCF will be 0.75, between a half and one and a half times the average the CCF will be 1.0, and over one and a half times the average will have a CCF of 1.25. The results of this analysis are shown below.

Table 8.9: Goodness of fit statistics for the Base Models including grouped residential CCFs based on percentage multiples

Model	PADT	SSE	$r^2$	$r_s$
GMAP Base Model	8.90	19960	0.64	0.48
CI Base Model	9.30	18489	0.66	0.48

Table 8.10: Goodness of fit statistics for the Base Models including grouped residential CCFs based on number of parking spaces

Model	PADT	SSE	$r^2$	$r_s$
GMAP Base Model	9.04	20566	0.64	0.47
CI Base Model	9.41	18604	0.67	0.47

Comparison of Table 8.2 with Tables 8.9 and 8.10 indicates that although the inclusion of the attractiveness factors through a grouped CCF value improves the predicted average drive time (observed drive time is equal to 9.37) in each case the other goodness of fit statistics are not improved.

### 8.2.2.2 Work based CCFs

Information for the production of work based CCFs was extracted from the Census Special Workplace Statistics (SWS). As in Chapter 6, SWS Set B was the dataset utilised.

Due to the fact that no data was available concerning mortgage purchasing by different person types the only variable it is possible to use in the case of mortgage sales is the total number of workers in each centre. From this information weighted work CCFs were calculated according to a centre's difference in number of workers from the average.

$$\text{CCF}(\text{work}) = \frac{v_j}{\sum_j v_j / n} \quad (8.2)$$

where  $v_j$  is the number of workers in centre  $j$  and  $n$  is the number of centres.

The number of workers in each centre and the work based CCFs are shown in Table 8.11 overleaf.

Table 8.11: The number of workers and the work based CCFs for financial centres in the Yorkshire TV region containing Halifax branches

Centre	Number of Workers	Work Based CCFs
Acombe	122	0.15
Armley	739	0.88
Barnsley	1311	1.56
Batley	1265	1.51
Beverley	666	0.79
Boston	322	0.38
Bradford	844	1.01
Bramley	502	0.60
Bransholme	409	0.49
Bridlington	158	0.19
Brighouse	568	0.68
Castleford	1055	1.26
Chesterfield	230	0.27
Cleckheaton	736	0.88
Crossgates	650	0.78
Dewsbury	1395	1.66
Doncaster	950	1.13
Duckworth	299	0.36
Elland	501	0.60
Gainsborough	156	0.19
Garforth	607	0.72
Goole	19	0.02
Grimsby	238	0.28
Guiseley	1028	1.23
Halifax	324	0.39
Harehills	331	0.39
Headingley	616	0.74
Heckmondwike	706	0.84
Hessle	233	0.28
Horsforth	879	1.05
Huddersfield	420	0.50
Hull	3231	3.86
Hunslet	1099	1.31
Ilkley	550	0.66
Keighley	911	1.09
Leeds	8850	10.56
Lincoln	781	0.93
Louth	160	0.19
Mansfield	811	0.97
Meadowhall	546	0.65
Mexborough	429	0.51
Mirfield	410	0.49
Moortown	409	0.49
Morley	2152	2.57
Normanton	453	0.54
Ossett	736	0.88
Otley	973	1.16
Pontefract	1325	1.58
Pudsey	1578	1.88
Retford	195	0.23
Rotherham	1744	2.08
Scarborough	218	0.26
Scunthorpe	613	0.73
Selby	493	0.59
Sheffield	4359	5.20
Shipley	1138	1.36
Skipton	64	0.08
Sleaford	293	0.35
Sowerby Bridge	331	0.39
Spalding	229	0.27
Tadcaster	321	0.38
Thorne	437	0.52
Todmorden	370	0.44
Wakefield	876	1.05
Wetherby	1137	1.36
Worksop	535	0.64
Yeadon	1028	1.23
York	1798	2.15

A correlation analysis was undertaken between number of workers in a centre and centre performance to indicate if this variable would be important in determining the performance of centres.

Table 8.12: Correlations between number of workers and centre performance for financial centres in the Yorkshire TV region containing Halifax branches

Model	$r$	$r_s$
GMAP Base Model	-0.05	0.15
CI Base Model	0.04	0.21

The correlations shown in Table 8.12 indicate that there is not a relationship between number of workers in a centre and centre performance. It can be seen that there is a large variation in the values of the work CCFs produced. Therefore a grouped CCF as described above for the residential CCFs in Section 8.2.2.1, will be used to include this variable in the model.

Table 8.13: Goodness of fit statistics for the Base Models including grouped work based CCFs

Model	PADT	SSE	$r^2$	$r_s$
GMAP Base Model	8.96	20032	0.64	0.46
CI Base Model	9.34	18307	0.66	0.46

Comparison with Table 8.2 shows that the PADT and SSE for the CI Base Model have improved subsequent to the addition of the grouped work based CCF, but the SSE has only decreased by 0.3% and the value of  $r_s$  has worsened. All of the goodness of fit statistics have worsened for the GMAP Base Model.

### 8.2.2.3 The alpha parameter

It can therefore be seen that centre attractiveness factors do not explain the difference between observed and predicted Halifax centre new mortgage sales. The apparent relationship between centre size and centre performance could indicate that an alteration in the model parameter  $\alpha$  on the  $W_j$  value could improve model performance. This parameter represents the perception of returns to scale by consumers. At present the alpha value in the Halifax spatial interaction model is based on standard regions and

varies around a value of approximately 1.25. For this analysis I used one alpha value for the whole of the Yorkshire TV region. The results are shown in Table 8.14.

Table 8.14: Goodness of fit statistics for new alpha values for the CI Base Model

Alpha Value	PADT	SSE	$r^2$	$r_s$
1.2	9.13	18538	0.66	0.47
1.3	9.27	18924	0.67	0.46
1.4	9.41	18472	0.66	0.48

Table 8.14 shows that the varying of the alpha value for the CI Base model produces no improvement in model performance. Therefore the region-specific alpha values currently used in the model will be retained.

### *8.2.3. The best performing model for the prediction of new mortgage sales for the Halifax*

The analysis undertaken has shown that centre attractiveness factors when included in the model through CCFs, in the same way as for WH Smith, do not have the same effect on model performance for the Halifax. This is due to the wide variation in the values seen for the attractiveness factors and hence for the CCFs. However, even when the CCFs are restricted to remove the effect of very low CCFs the model performance still does not improve. This could be caused by destination choice being influenced by different factors in the case of mortgage purchasing. Thus, residential based CCFs based on observed centre characteristics are not important in the determination of the performance of centres for the selling of mortgages.

Although the work based CCF did cause a slight improvement in the CCFs for the CI Model it was only by a small amount and it caused a decrease in model performance for the GMAP Model. Thus there is insufficient evidence for work based factors having a significant influence on centre performance.

Thus, it has not been possible to produce an explanatory model for Halifax new mortgage sales because observed centre characteristics do not influence the performance of centres.

The best predictive model for Halifax new mortgage sales is the CI Model, which uses GMAP demand estimates and CCFs but has the negative exponential distance function replaced by a square root exponential function and an agglomerative competing destinations variable added. The goodness of fit statistics for this model are shown in Table 8.2. In order to discover the extent of the model improvement that has been achieved this model should be compared to the GMAP Full Model, whose goodness of fit statistics are also shown in Table 8.2. The CI Model produces a 9% decrease in the SSE, has a better predicted average drive time and  $r^2$  value, but a slightly lower value of  $r_s$ .

Although model improvements have been made for the prediction of Halifax new mortgage sales there is still a significant level of error. However, the model cannot be expected to replicate observed flows exactly due to the number of factors that could influence the observed data. For example, if a consumer's work address has been assigned to the mortgage as opposed to their home address this will cause distortion in the observed flows. The transfer of accounts subsequent to the closure of a branch can also cause anomalies in the flow database which the model will not be able to replicate.

### **8.3 THE WH SMITH MODEL**

The demand and supply analyses undertaken in Chapters 5 and 6 for WH Smith need to be integrated and the interaction analysis from Chapter 7 has to be applied to the WH Smith model in order to see if it also applicable in this case. This analysis will be undertaken for the prediction of book sales. Only one good is chosen to speed up calibration in the interaction analysis; books are chosen because they were the basis for the demand analysis.

#### ***8.3.1 The integration of demand and supply analysis***

In Chapter 5 it was found that the income demand estimates produced a marginally improved goodness of fit for the GMAP Full Model for book sales for WH Smith than the current demand estimates used which are based on age, social class and region. Thus, these income demand estimates will be introduced into the Explanatory Model

and the Predictive Model produced in Chapter 6 in order to discover if demand estimates based on the income of the origin population will continue to produce improved model fit subsequent to the addition of the new residential CCFs that were introduced in Chapter 6.

Table 8.15: Goodness of fit statistics for the Explanatory and Predictive Income Demand Models, for centres in the Yorkshire TV region containing WH Smith Group stores, for books

Model	SSE	$r^2$	$r_s$
Explanatory Model	7817671	0.91	0.90
Explanatory Income Demand Model	8748667	0.90	0.88
Predictive Model	3186598	0.97	0.91
Predictive Income Demand Model	3402975	0.96	0.92

Table 8.15 indicates that when the income demand estimates are added to the models containing the new residential CCFs, model performance is not improved. Therefore for models containing the new CCFs the current formulation of demand estimates using age, social class and region are the most appropriate to use in the estimation of demand for books.

Thus it can be seen that the new income demand estimates and the new residential CCFs both individually improve model performance, but that when they are integrated together into the WH Smith Explanatory and Predictive Income Demand Models there is a decrease in the performance of the model for predicting book buying. It is therefore necessary to discover which of the new components contributes most to model performance improvement when used alone. It was seen in Section 5.8 that the substitution of income demand estimates for the current estimates used by GMAP caused the SSE to be decreased by 13% for the GMAP Full Model for book sales. Table 6.24 shows that the Predictive Model that includes new residential CCFs produced a SSE value of 3186598 for books, when compared to the SSE value for the GMAP Full Model for books (shown in Table 6.2) this is a decrease of 19%. Thus the new residential CCFs produce more model improvement than the use of income demand estimates. Therefore, the forthcoming analysis on the interaction function for the WH Smith model will be based on the both the Explanatory and the Predictive Models

(which include new residential CCFs) because they are the best performing models produced thus far.

### *8.3.2 Interaction analysis*

Chapter 7 concluded that the best form of the interaction function for the Halifax spatial interaction model consisted of a square root exponential distance function in combination with a competing destinations accessibility variable. These findings will now be tested using the WH Smith model in order to discover if the same interaction function is the most appropriate in this alternative retail activity.

The two factors of the new interaction function will be tested separately to begin with and then combined if it is found that they both produce improved model fit when used individually.

#### *8.3.2.1 The square root exponential function*

The calibration of the model was undertaken using maximum likelihood methods to equate observed and predicted drive times, as in the Halifax model. The observed drive times were calculated using data collected by WH Smith concerning where their customers have travelled from. This information was collected through surveys. As was experienced for the Halifax model, there were some postal districts in the Yorkshire TV region for which there were no observed data. Thus, it was necessary to determine whether there was a relationship between observed drive times and population density of origin zones in order to see if it is possible to use the method described in Section 7.3.4.2 to use population densities to fill in the gaps in the observed drive time data. The correlation coefficient between postal district population density and observed average drive times from postal districts for book buyers at WH Smith is -0.4203 which is significant at the 95% level. Thus, as was undertaken for postal sectors for the Halifax model, postal districts for which there is no observed flow data will be given the average of the observed drive times for the five origins with the nearest level of population density to that origin.

The results of using the square root exponential model in the Explanatory and Predictive Models are shown in Table 8.16. The results shown are those for which the overall predicted average drive time between origins and destinations in the Yorkshire TV region was the closest to the observed average drive time. The observed average drive time for books in the Yorkshire TV region, for WH Smith is 10.29 minutes.

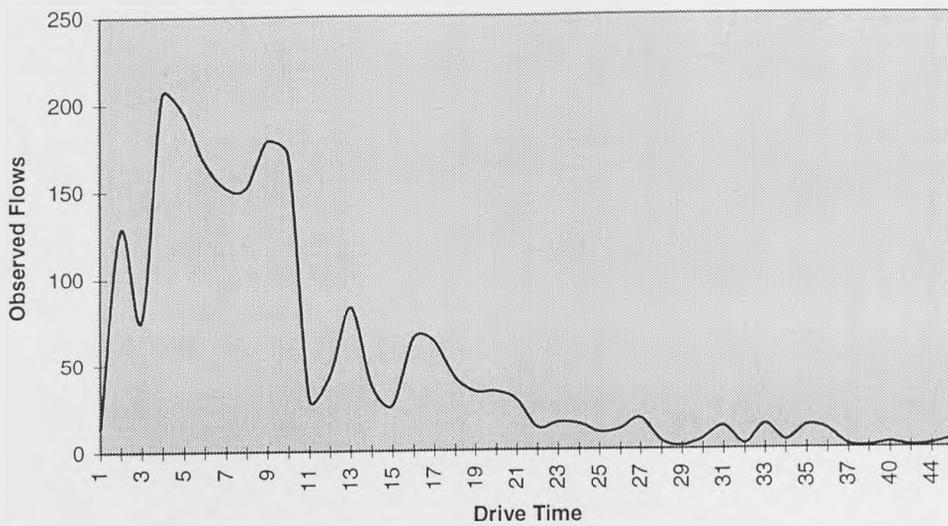
Table 8.16: Goodness of fit statistics for the Explanatory and Predictive Models, for books, using a square root exponential distance function, for centres in the Yorkshire TV region containing WH Smith Group stores

Model	PADT	SSE	$r^2$	$r_s$
Explanatory Model	10.95	7827877	0.91	0.86
Predictive Model	11.12	3342647	0.96	0.87

These statistics indicate that for the WH Smith model of book buying, the use of the square root exponential distance function does not lead to an improvement in model performance although there is not much difference between the models using different distance functions. The difference between the models using the square root exponential distance function and the model using the negative exponential distance function is more pronounced for the Predictive Model which experiences a 5% increase in SSE from the introduction of the square root exponential distance function. The  $r^2$  and  $r_s$  values are also worsened. For the Explanatory Model the SSE is less than 1% higher subsequent to the use of the square root exponential distance function. The  $r^2$  value is the same and the value of  $r_s$  has decreased. Thus, the substitution of the square root exponential distance function for the negative exponential distance function has not led to an improvement in model performance in the case of the prediction of book sales for WH Smith.

Figure 8.1 below shows observed flows against drive times for book buyers at WH Smith group stores. Comparison of this diagram with Figure 7.5 which shows observed flows against drive times for Halifax new mortgage clients shows that for book buyers at WH Smith the rate of distance decay does not appear to be as sharp. Figure 7.4 showed that the square root exponential function produces a higher rate of distance decay than the negative exponential function when calibrated for the Halifax model. This could indicate why the negative exponential distance function is more appropriate for representing distance decay for the WH Smith book buying population.

Figure 8.1: Observed flows against observed drive times for book buyers at WH Smith Group stores



Thus it is concluded that the negative exponential function is the most appropriate representation of the rate of distance decay for book buyers at WH Smith Group stores.

### 8.3.2.2 *The competing destinations accessibility variable*

The preceding section indicated that the use of the square root exponential function does not improve model performance for WH Smith book buying. Thus, the use of a competing destinations variable will be tested using the Explanatory and Predictive Models using the negative exponential distance function.

The positive relationship between the value of the competing destinations variable and model residuals for the Halifax mortgage model indicated that agglomeration effects were being experienced for new mortgage sales. For the model of WH Smith book sales there is a negative relationship between model residuals and the values of the competing destinations variable for centres in the Yorkshire TV region. This is seen for both the Explanatory Model and the Predictive Model. The correlation coefficient is  $-0.3038$  for the Explanatory Model and  $-0.3350$  for the Predictive Model. These negative values indicate that as the accessibility of a destination increases the model residuals for that centre decrease *i.e.* predicted sales become greater than observed sales in the centre, therefore centre performance, as defined in equation (6.1) tends to be lower in centres

that are more accessible. This is an indication that competition factors are being experienced between centres selling books. Thus, a negative parameter is required on the competing destinations variable in order to account for such competition effects. It can also be seen that the relationship between the competing destinations variable and centre model residuals is weaker than the relationship seen in the case of new mortgage sales, therefore a smaller value of the  $\delta$  parameter may be required.

The results for the Explanatory and Predictive Models with the addition of the competing destinations accessibility variable with a  $\delta$  parameter value of -0.5 are shown in Table 8.17. Several values of  $\delta$  were tested, but -0.5 produced the optimal model performance. These models will be called the Explanatory CD Model and the Predictive CD Model. The results shown are for the calibration iteration that produces a predicted average drive time closest to the observed average drive time of 10.29 minutes.

Table 8.17: Goodness of fit statistics for the Explanatory and Predictive CD Models, for books, for centres in the Yorkshire TV region containing WH Smith Group stores

Model	PADT	SSE	$r^2$	$r_s$
Explanatory CD Model	10.44	7265267	0.92	0.87
Predictive CD Model	10.37	3020413	0.96	0.89

The addition of the competing destinations accessibility variable leads to an improvement in model performance for both the Explanatory and the Predictive Model. For the explanatory model the SSE has decreased by 7%, the  $r^2$  value has increased, but the value of  $r_s$  has decreased. The SSE for the Predictive Model has been decreased by 5%, but the values of  $r^2$  and  $r_s$  have decreased slightly. The individual centre performance figures for the Predictive and Explanatory CD Models are shown in Tables 8.18 and 8.19 below.

Table 8.18: Centre performance for the Explanatory and Predictive CD Models, for books, for centres in the Yorkshire TV region containing WH Smith Group stores

Centre	Explanatory Model	Explanatory CD Model	% Model Improvement
Barnsley	81.4	68.1	-13.3
Beverley	87.1	76.5	-10.6
Boston	154.0	145.8	8.2
Bradford	73.1	72.5	-0.6
Dewsbury	132.4	134.1	-1.7
Doncaster	104.1	105.9	-1.0
Gainsborough	73.0	38.5	-34.4
Grimsby	87.8	84.6	-3.1
Halifax	92.4	90.2	-2.2
Huddersfield	83.6	83.3	-0.3
Hull	89.9	83.9	-6.0
Ilkley	127.4	228.0	-100.6
Keighley	84.7	84.7	-0.1
Leeds	110.0	96.9	6.9
Lincoln	145.1	147.7	-2.5
Meadowhall	142.2	134.9	7.3
Pontefract	87.3	82.8	-4.6
Retford	70.3	70.2	-0.1
Rotherham	173.4	170.2	3.2
Scarborough	167.1	168.3	-1.1
Scunthorpe	72.0	64.2	-7.8
Sheffield	168.5	161.7	6.8
Skegness	66.9	62.0	-4.9
Skipton	164.6	156.6	8.0
Spalding	73.2	74.5	1.2
Wakefield	120.3	123.3	-3.0
Worksop	91.3	71.2	-20.1
York	189.5	181.0	8.5

The scatterplots shown in Figures 8.2 and 8.3 also show the effect the introduction of the competing destinations accessibility variable has on the prediction of centre sales of books in the Explanatory Model.

Figure 8.2: Scatterplot between observed and predicted centre revenue for books, for the Explanatory Model, for centres in the Yorkshire TV region containing WH Smith Group stores

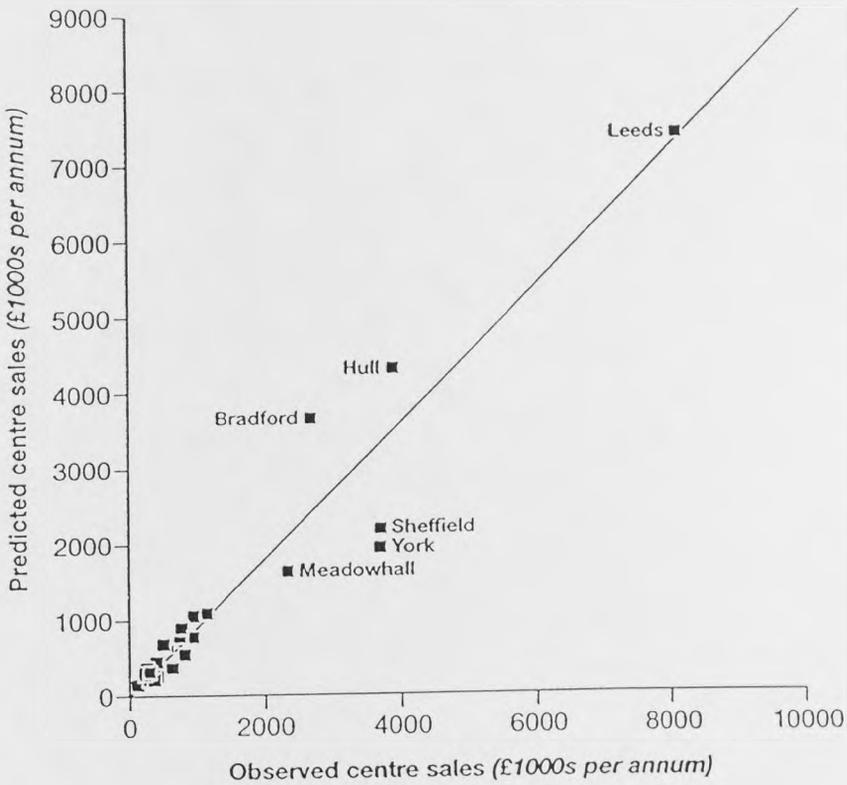
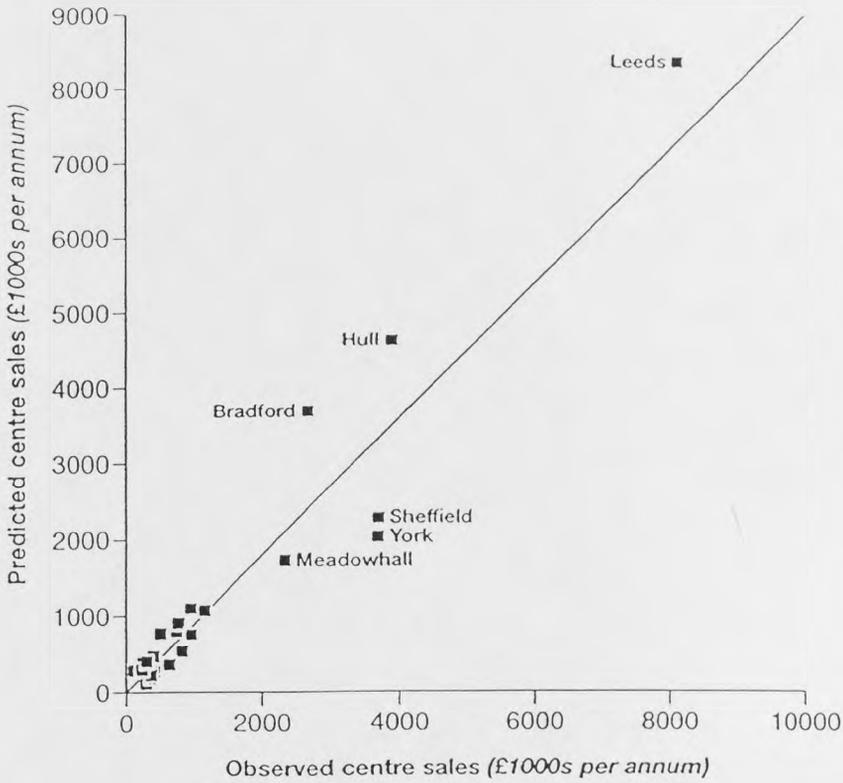


Figure 8.3: Scatterplot between observed and predicted centre revenues for books, for the Explanatory CD Model, for centres in the Yorkshire TV region containing WH Smith group stores



Although the introduction of the competing destinations variable improves overall model performance for the Explanatory Model it only improves the performance of the model for 29% of the individual centres. For most centres for which performance was decreased the reduction in model performance was small. However, some centres experienced a large decrease in model performance. For example, the centre of Ilkley experienced a 100% decrease in model performance. This occurred because this centre is being under predicted already and is relatively accessible due to its location in close proximity to Leeds, Bradford and Skipton. Therefore the value of the competing destinations variable with a parameter reflecting competition between centres leads to centre predictions being further reduced. However, for most centres, performance is not altered by a large amount.

Figures 8.2 and 8.3 also indicate that the use of a competitive competing destinations variable does not have a large effect on the performance of most centres with there

being no significant difference between the scatter plots of observed and predicted centre sales before and after the addition of the accessibility variable.

It is therefore clear that the addition of the competing destinations variable is not beneficial to all centres, but that it does lead to an overall improvement in model performance for the Explanatory Model.

Table 8.19 shows that the introduction of the competing destinations variable for the Predictive Model leads to an increase in model performance for 39% of centres. It can also be seen that more centres achieve a large improvement in model performance than was observed for the Explanatory Model. Rotherham, Dewsbury and Skipton all experience large improvements in model performance.

Table 8.19: Centre performance for the Predictive and Predictive CD Models, for books, for centres in the Yorkshire TV region containing WH Smith Group stores

Centre	Predictive Model	Predictive CD Model	% Model Improvement
Barnsley	102.3	78.7	-19.0
Beverley	106.1	96.2	2.3
Boston	154.0	137.1	16.9
Bradford	84.7	83.7	-1.0
Dewsbury	151.6	105.6	46.0
Doncaster	121.1	119.1	2.0
Gainsborough	76.8	31.1	-45.7
Grimsby	93.0	87.5	-5.5
Halifax	94.2	86.7	-7.5
Huddersfield	83.7	79.3	-4.4
Hull	98.4	93.7	-4.8
Ilkley	126.6	170.2	-43.7
Keighley	85.1	82.1	-3.0
Leeds	108.2	96.4	4.6
Lincoln	145.2	150.6	-5.4
Meadowhall	107.0	120.4	-13.4
Pontefract	96.5	76.0	-20.5
Retford	84.6	87.5	2.9
Rotherham	184.1	113.6	70.5
Scarborough	146.9	146.7	0.3
Scunthorpe	87.1	73.7	-13.4
Sheffield	132.5	130.4	2.1
Skegness	72.0	55.0	-17.0
Skipton	164.6	142.2	22.5
Spalding	81.3	73.7	-7.6
Wakefield	119.9	131.5	-11.6
Worksop	111.7	76.8	-11.5
York	148.0	142.3	5.7

Figure 8.4: Scatterplot between observed and predicted centre revenues for books, for the Predictive Model, for centres in the Yorkshire TV region containing WH Smith Group stores

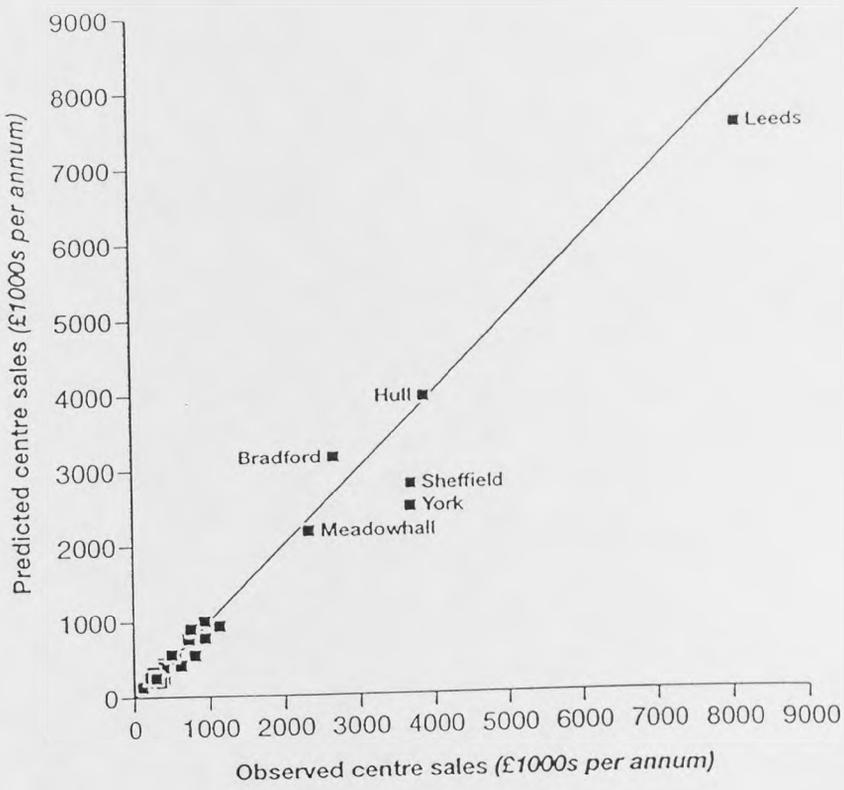
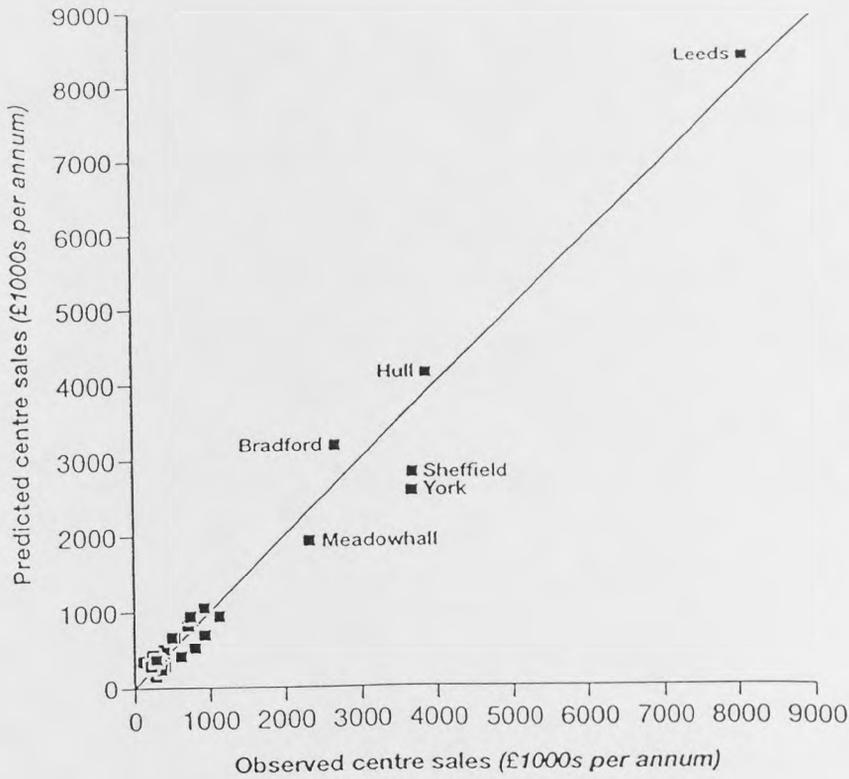


Figure 8.5: Scatterplot between observed and predicted centre revenues for books, for the Predictive CD Model, for centres in the Yorkshire TV region containing WH Smith Group stores



As was seen for the Explanatory Model, Figures 8.4 and 8.5 indicate that the introduction of the competing destinations variables in the Predictive Model does not have a large visible effect on centre performance values. However, an improvement in overall model performance has been achieved.

### 8.3.3 The best performing model for WH Smith book sales

Improvements in model performance have been achieved both for the model that attempts to predict sales based on observed destination characteristics (the Explanatory Models) and the model that uses various calibrated variables to attempt to optimise sales predictions (the Predictive Models).

When SSE are considered, the best performing models for the prediction of book sales for WH Smith Group stores are the Explanatory CD Model and the Predictive CD Model. In these models demand estimates are based on age, social class and region, as

they are currently formulated by GMAP. The supply side of the model is represented by residential CCFs that are based on observed centre characteristics, although the Predictive CD Model also includes individual store calibration variables. Such a variable is not included in the Explanatory CD Model because they are not based on observed store characteristics. The interaction component of the spatial interaction model is represented by the negative exponential distance function and a competing destinations accessibility variable.

The overall degree of model enhancement that has been achieved can be discovered by comparing the performance of the Explanatory CD Model to the GMAP Base Model and the Predictive CD Model to the GMAP Full Model. The goodness of fit statistics for the GMAP Base Model and the GMAP Full Model are shown in Table 8.20. The results shown are for the prediction of book sales only.

Table 8.20: Goodness of fit statistics for the GMAP models, for books, for centres in the Yorkshire TV region containing WH Smith Group stores

Model	PADT	SSE	$r^2$	$r_s$
GMAP Base Model	9.23	13397108	0.92	0.86
GMAP Full Model	9.34	3941151	0.97	0.86

Comparison of Table 8.20 with Table 8.17 which shows the goodness of fit statistics for the Explanatory CD Model and the Predictive CD Model shows that for the Explanatory Model the SSE has been decreased by 46% and that the values of both  $r^2$  and  $r_s$  have been improved. For the Predictive Model the SSE has been decreased by 23% and  $r^2$  and  $r_s$  have both improved. Of these improvements in model performance the new residential CCFs have been the most important influence in achieving the improved model performance, contributing 41% of the improvement for the Explanatory CD Model and 19% for the Predictive CD Model. Both the CD Models also produce predicted average drive times that are closer to the observed average drive time of 10.29 minutes than those for the GMAP Models.

## 8.4 CONCLUSIONS

This chapter has shown that improvements in model performance have been achieved for both the Halifax model of new mortgage sales and the WH Smith model of book sales. However, these model improvements have been achieved through different methods for each model and it has been found that factors found to improve the performance of one type of retail model will not necessarily reproduce the performance improvements when transferred to alternative models.

For the Halifax model it was found that the spatial distribution of centres and the way centres affected each other was the main determinant of differences in centre performance and therefore the introduction of a competing destinations variable with a positive parameter representing agglomeration effects leads to model performance improvements. This factor was also important in the WH Smith Model but to a lesser degree and with competition effects as opposed to agglomeration effects. The residential CCFs were found to be the most important factor in improving the performance in the WH Smith model, but these factors were found to be insignificant for the Halifax model of new mortgage sales.

These findings indicate that different retail activities produce different customer behaviour and therefore have to be modelled in different ways. It can be seen that destination choice for the purchase of mortgages was not influenced by the characteristics of the centres but by the location of centres in relation to other centres. The purchase of mortgages is a different trip type to that undertaken for book buying, for which centre characteristics are an important factor in the determination of centre choice. The process of purchasing a mortgage is a functional flow in which the main element of the trip is to select a mortgage, it is likely that a trip to purchase a mortgage would be a single-purpose trip and therefore the characteristics of the centre will be unimportant. However, for the good types sold at WH Smith, trips are more likely to be multi-purpose and there will be the influence of shopping as leisure which will mean that the characteristics and attractiveness of centres will have more of an influence on destination choice.

It has also been shown that there are differences in the rates of distance decay for the different good types and therefore different distance decay functions are appropriate for representing distance decay in different retail activities. It was observed that the negative exponential distance function that is usually used in retailing applications is not necessarily appropriate and in the case of Halifax new mortgage sales this function did not produce the best model estimates and the square root exponential function was found to produce improved model estimates. However, it was found that the negative exponential function was the most appropriate function in the case of WH Smith book sales which experienced less distance decay than was observed for mortgage sales.

The findings of this chapter, which has shown that improvements in a model for one retail activity are not necessarily transferable to other retail models will have implications for companies such as GMAP. These implications and recommendations that I would make to GMAP in order for them to produce improved retail models will be discussed in the next chapter.

**THESIS CONCLUSIONS  
OR  
AN EXECUTIVE SUMMARY FOR GMAP LTD**

## **9.1 INTRODUCTION**

The main aim of this thesis has been to improve the performance of two spatial interaction models that have been produced by GMAP for Halifax Plc and WH Smith. This has been achieved for both the models of new mortgage sales and book sales. The conclusions that have arisen subsequent to the analysis of the models for Halifax Plc and WH Smith will have significant implications for the methods GMAP use in the formulation of their models. It is now possible to make recommendations for the model builders at GMAP that, if acted upon, could lead to the improvement in the performance of the spatial interaction models they produce for their clients. Specific recommendations to GMAP will be shown in italics for the remainder of this chapter. The implications for spatial interaction model specification in general will also be identified.

The progress achieved through this research can be examined by considering the extent to which the error sources apparent in spatial interaction modelling, which were described in Chapter 3, have been addressed and alleviated. The implications of the analysis for demand, supply and interaction will be discussed in turn, followed by a more general discussion of the implications of the research undertaken in this thesis.

## **9.2 DEMAND**

The analysis undertaken in Chapter 5 on the estimation of demand for WH Smith showed that the current formulation of demand estimates using age, social class and region provide good approximations for the level of expenditure that is manifest in origin zones. However, the logit analysis also indicated that the income variable was

the most important single variable in the determination of levels of demand in origin residential areas and therefore income data should ideally be utilised in the estimation of demand. However, it should also be recognised that there are problems with the reliability of the income data used due to the nature of its collection and the tendency of respondents to overstate their income when asked to provide an estimate. Income data is collected by private sector organisations such as NDL through product guarantees which produces a bias towards purchasers of durable goods. There is also the problem that income data that are collected only represent a sample of the population although it is a high sample. The problem of bias has been addressed to some extent through a collaborative project between GMAP and NDL that uses microsimulation as a method of reweighting the income data in order to remove bias. This project is described in Birkin and Clarke (1995).

A further problem with the demand estimation process that must be recognised by any company wishing to produce small area expenditure estimates is that an accurate method of testing such estimates is required. Problems were experienced in testing the demand estimates for WH Smith due to the unreliability of the National Market Survey and due to the other validation problems listed in Section 5.3.6.5, which include uneven survey coverage across the UK, small sample size and lack of actual expenditure information.

**Recommendation 1:** *GMAP should encourage clients to collect accurate survey data on the location and expenditure of customers in order for demand estimates to be validated.*

This problem has been addressed to some extent by WH Smith through the introduction of a loyalty card. However, holders of such cards only represent a small proportion of the customers of WH Smith Group stores.

Such demand estimation problems are less apparent for mortgages due to the existence of the National Mortgage Database that provides small area estimates of new mortgage sales based on information provided by mortgage providers across the country.

Thus, for retailers such as WH Smith the data availability problems identified in Section 3.3.1 will persist until a comprehensive survey of expenditure is available, which seems unlikely to materialise unless major investments in data collection are made. The problem of variable definition stated in Section 3.3.2 has been addressed to some extent through the logit analysis undertaken in Chapter 5 which identified the variables that were important in the determination of levels of demand for books.

**Recommendation 2:** *GMAP should continue to investigate alternative methods of demand estimation using alternative income data sources.*

A collaborative GMAP/academic project has already been undertaken by Birkin and Clarke (1989) which used microsimulation methods to produce small area estimates of household and individual incomes. It is possible that this work could be extended and linked to expenditure in order to produce small area demand estimates.

Thus, it can be seen that with respect to demand estimation the implications for spatial interaction models are data related and do not affect model specification. The demand component of the spatial interaction model,  $O_i$  should continue to be formulated in the same way, but it is possible that model improvements could be achieved by using alternative data sources, such as income data.

### 9.3 SUPPLY

The problem described in Section 3.4 concerning the representation of the supply side in retail spatial interaction models has been addressed and to some extent solved through the calculation of new residential CCFs based on centre characteristics. The analysis undertaken concerning the production of an explanatory model for WH Smith revealed that observed centre characteristics including percentage of multiples, number of parking spaces, percentage of shops in a pedestrianised or undercover area, and general shopping centre attractiveness were important in the determination of shopping centre performance and allowed improved attractiveness calculations to be produced for centres containing WH Smith Group stores. These findings reinforce the work undertaken by authors such as McCarthy (1980) who identified the importance of generalised centre attractiveness in determining destination choice, and Oppewal *et al.*

(1997) who found that the physical appearance of a centre was an important factor in determining choice of centres.

The inclusion of these centre characteristics through a new residential City Centre Factor (CCF) led to a significant improvement in model performance for both the explanatory and the predictive models of book sales. The CCFs produced from real centre characteristics actually produced better model results in the predictive model than the CCFs values that are calibrated by GMAP to optimise model predictions. It would therefore be advantageous to produce values for CCFs for all centres in the WH Smith model to extend the work that has been undertaken here for the Yorkshire TV region. The production of CCFs for all centres would, of course, cost money. It would be necessary to purchase new data sources such as GOAD plans for each centre and fieldwork would also be required in order to collect the information that is not available on GOAD plans. There are 3411 centres in the WH Smith model and therefore this would be a significant investment. GOAD city centre plans are available at a price of £40 each, thus purchasing a plan for each centre in the WH Smith model would cost £136,440, which is a great deal of money to spend on one project although bulk purchase discounts could probably be negotiated. A further way of reducing costs would be to only produce the residential CCFs for centres that contain WH Smith Group stores because the attractiveness of these centres would have the most impact on the sales of WH Smith. This would reduce the number of centres to 456 and it would cost £18,240 to purchase GOAD Plans for each of these centres.

For the maximum model improvement to be achieved from the inclusion of the new residential CCFs, fieldwork for the calculation of the centre quality variables used to build the field survey attractiveness factor should also be undertaken. The field work for each centre took on average one hour per centre. If this process were undertaken by current WH Smith employees, it would reduce costs because no travel expenses would be incurred. However, if it is deemed that the carrying out of field work is too expensive a process, it was seen in Section 6.4.3.8, that although the omission of the field survey attractiveness factor reduces the level of model performance improvement, the percentage multiples, percentage undercover or pedestrianised and the number of parking spaces factors still cause a significant enhancement in model predictions.

**Recommendation 3:** *GMAP should investigate the financial viability of producing the new residential CCFs for all centres.*

If the GOAD plans were purchased they could also be used to check and if necessary supplement the competitor databases that are held by GMAP because they show all retail and financial service outlets that are located in a centre. Much of the information gathered through the use of GOAD plans and field work could also be sold on to other market analysis companies in order to recoup some of the costs. Thus, if it is a financially viable option, the data used for the production of residential CCFs should be collected and utilised in order to improve model performance.

The use of such CCFs will also help in the scenario modelling that is undertaken by GMAP. The effect of changes that occur in centres can be estimated through changing the values of the CCFs. For example, if a new car park is built or a new undercover centre is built, the effect on the performance of stores in that centre can be estimated.

The argument for the production of the new residential CCFs for the whole country is augmented by the possibility that the information gathered could be transferable to other models produced by GMAP. It was seen in Chapter 8 that the residential CCFs based on observed centre attractiveness characteristics did not lead to an improvement in model performance for the financial service example of mortgage sales, because of differences in destination choice behaviour. However, it is possible that other retail models produced by GMAP could benefit from the centre characteristic data. Goods that are sold by WH Smith, such as books, are likely to be impulse buys. Destination choice will therefore not be determined by the necessity of buying a book but by centre characteristics. It is likely that consumers purchasing goods such as books and music will be undertaking shopping as leisure and therefore the attractiveness of the centre will have an influence on their destination choice. Other goods for which this is likely to be the case are clothes and shoes. Therefore models to predict sales for such goods are likely to benefit from the use of the new residential CCFs.

**Recommendation 4:** *It is necessary for GMAP to identify which retail activities will benefit from the use of the CCFs. For those activities for which centre characteristics are not significant in determining destination choice, behavioural analysis must be undertaken to determine what influences the choice of destination.*

The move towards more explanatory models that build in the new residential CCFs could also have beneficial effects concerning the interpretability of the models produced by GMAP. The CCF variable in the Explanatory Model is based on observed centre characteristics as opposed to the heuristic CCF values in the Predictive Model, which are not related to centre characteristics. If this is the case then the model building process becomes more understandable to the clients who can see how the value of a variable has been calculated. This will be beneficial for GMAP because clients are more likely to trust a product that they can understand.

**Recommendation 5:** *GMAP should attempt to make their models more explanatory in order to improve their meaning and interpretability to the client for which the model is produced.*

Thus, for goods such as books and music that are likely to be impulse buys, it has been demonstrated that the spatial interaction model will benefit from the inclusion of an additional variable (the CCF) that accounts for centre characteristics other than the size of the centre. The specification of the production constrained spatial interaction model as used in retail applications should therefore become

$$T_{ij} = A_i O_i (W_j F_j)^\alpha f(c_{ij}) \quad (9.1)$$

where

$F_j$  = the new residential CCF

## 9.4 INTERACTION

For the interaction component of the spatial interaction model it was discovered that some factors were important for both types of retail activity studied but that other model alterations were not transferable between applications.

It was found that the problem of measuring impedance between origins and destinations, as described in Section 3.5.1, was already being dealt with competently. The value of the use of drive times as the distance measurement was confirmed, with the model for new mortgage sales prediction performing significantly better using a drive time measurement of distance than when straight line distance was used. However, a problem was identified concerning the possible refinement of drive time calculations at the local scale. At present in the Halifax model, drive time estimates have both their origin and destination as a postal sector centroid, but a more accurate calculation of the drive time would be between a postal sector centroid and the financial centre.

**Recommendation 6:** *GMAP should endeavour to ensure that the drive time calculation is as accurate as possible.*

Section 3.5.3 proposed the importance of investigating which distance function would be appropriate in different situations. This problem has been tackled through the analysis on interaction undertaken for Halifax new mortgage sales. In this case study it was found that the negative exponential function did perform well for the Halifax model, but that there was evidence for the use of the square root exponential function as proposed by Taylor (1975). This distance function produced improved model predictions of new mortgage sales and appeared to better replicate the pattern of observed distance decay for Halifax new mortgage sales in the Yorkshire TV region. This judgement was not replicated for the WH Smith model for which it was found that the negative exponential function produced better model predictions than the square root exponential function and was also a better representation of the observed pattern of distance decay for WH Smith book sales. Thus, it was seen that different retail activities possess different patterns of distance decay and therefore one distance function will not be appropriate for models of all types of retail activity.

**Recommendation 7:** *GMAP must endeavour to analyse observed expenditure flow patterns in an effort to reveal which distance function is the most appropriate in each case instead of simply using the negative exponential function because of its presence in the entropy maximisation derivation of the spatial interaction model.*

The issues discussed in Section 3.5.4 regarding the nature of the distance decay parameters have also been addressed and the value of employing origin specific betas has been confirmed. Chapter 7 showed that origin specific betas were necessary in order to account for differences in the propensity to travel between residents of different origins. However, experiments undertaken to test destination specific betas found that the level of distance decay was not determined by destinations.

**Recommendation 8:** *GMAP should use origin specific beta parameters in retail models, but they should consider using origin characteristics such as population density as a way of filling the gaps for origins with no observed data instead of using the values of adjacent origins.*

The possibility of using alternative accessibility functions, as described in Section 3.5.5 has also been explored. Several alternative accessibility functions were tested to identify if theories such as intervening opportunities (Stouffer, 1940) and competing destinations (Fotheringham 1981, 1983a, 1983b, 1984, 1985, 1986) were relevant when applied to real retailing applications. It is only through application to real case studies that theories such as intervening opportunities and competing destinations can be developed. It was found that Stouffer's (1940) theory of intervening opportunities and Gonclaves and Ulysea-Neto's (1992) hybrid model of intervening opportunities and distance accessibility added no explanatory or predictive power to the model. Thus, the use of the theory of intervening opportunities in the model was rejected because model performance was significantly reduced subsequent to the introduction of an intervening opportunities accessibility variable both as a replacement to and in conjunction with the existing distance function. Therefore, any intervening opportunity effects that are apparent in the system must already be captured by the distance function.

The theory of hierarchical destination choice applied through the use of a competing destinations variable did produce improved model fit by accounting for the spatial distribution of centres. The relationship between centre performance and the values of the competing destinations variable for centres indicated that the relative accessibility of centres to each other did influence the performance of centres and that agglomeration factors were apparent. This would indicate that purchasers of mortgages are more likely to travel to a centre that is located close to other centres. Mortgage buying involves a lot

of comparison between different outlets in order to get the best mortgage deal and this could lead to consumers wishing to visit multiple centres in order to undertake such comparisons, hence the agglomeration effects that are observed for Halifax new mortgage sales. The inclusion of a competing destinations accessibility variable also led to improvements in model performance for the WH Smith model of book sales. However, in this case study the relationship between centre performance and the values of the competing destinations variable indicated that competition forces were apparent between centres selling books. Comparison between centres is unlikely to be necessary for the purchase of a book and therefore centres will compete for book purchasers. However, in both cases improvements in performance were not experienced for all centres and therefore competition or agglomeration effects were seen not to be consistent across all destinations. It should also be noted that the extent of the improvement in model performance achieved through the use of a competing destinations accessibility variable varied between the two case studies, having a larger effect for Halifax mortgage sales than for WH Smith book sales. This could be because, as has already been seen, the destination choice for book buying is also determined by centre characteristics, so that the spatial distribution of centres will have less impact on destination choice. Other good types for which the competing destinations accessibility variable could be effective are pubs and restaurants. Considering the example of pubs, it is likely that customers will choose a location with several pubs and subsequently decide on the specific pub on their arrival. Thus hierarchical decision making is being experienced and therefore the competing destinations model could be appropriate.

The calculation of the competing destinations variable is simple and involves no additional data. The distance and attractiveness variables which are used to calculate the competing destinations variable are already present in the databases and therefore this model alteration could be undertaken at minimal cost, but would produce considerable benefits in the form of improved model predictions, particularly for the Halifax model.

If the competing destinations variable were to be added to further models it would be necessary to determine whether competition or agglomeration forces were being experienced. This can be undertaken by looking at the relationship between centre

performance and the value of the competing destinations variable and by considering the type of trip that is being undertaken.

**Recommendation 9:** *GMAP should include the competing destinations accessibility variable in their spatial interaction models in order to account for the spatial distribution of retail centres and the effect that this will have on interaction patterns.*

There is therefore evidence for the specification of spatial interaction models to include a competing destinations accessibility variable. The model should therefore be

$$T_{ij} = A_i O_i W_j^\alpha Z_j^\delta f(c_{ij}) \quad (9.2)$$

for good types that are not likely to be impulse buys, and

$$T_{ij} = A_i O_i (W_j F_j)^\alpha Z_j^\delta f(c_{ij}) \quad (9.3)$$

for good types such as books and music. Although in this case the  $Z_j^\delta$  competing destinations variable will not be as effective, because destination choice is also affected by  $F_j$ .

An additional implication for model specification that has been identified is that alternative formulations of  $f(c_{ij})$  to the traditional negative exponential function may be required.

## 9.5 GENERAL ISSUES

It is important that GMAP considers these possible model improvements within an integrated framework. It was seen in Chapter 8 that for WH Smith book sales prediction the income demand estimates no longer led to improvements in model performance once the new residential CCFs had been introduced.

**Recommendation 10:** *GMAP should undertake tests in order to determine whether model alterations produce model improvements that are independent and additive or whether there is interaction between new model procedures that leads to model performance being reduced when two techniques are used together.*

The analysis undertaken has shown that different modelling techniques should be used for modelling different retail activities due to the variations in consumer behaviour that are apparent in separate markets. It is therefore impossible to recommend one type of model for use in predicting retail activities. The activities to be modelled can be significantly different. Therefore, a generic model for all retail types is not appropriate. This reinforces the argument proposed by Breheny (1988) who stated that customisation of responses to retail problems was necessary due to variations in problems arising in different retailing situations.

**Recommendation 11:** *GMAP must undertake individual investigations for each retail activity in order to determine which modelling techniques should be used for each component of the spatial interaction model for each type of retail activity. A series of computer programs such as those developed for this thesis could be produced, i.e. for new residential CCFs or the competing destinations variable, that could be slotted into the basic modelling framework in order to test if such model changes would lead to performance improvement.*

The importance of data in this investigation procedure should also be recognised. The general problem of data availability and quality has been described in Section 3.2.1 and has previously been addressed in Recommendation 1. New sources of data that could be of use for processes such as demand estimation are continually being produced and there is a further data issue concerning the updating of databases. For example, variables such as disposable income that are measured for the NDL database are in a state of constant flux. Thus, the longer data are used without being updated the more inaccurate they will be.

**Recommendation 12:** *GMAP should endeavour to ensure that they have access to the most accurate and up to date data that is available in order to increase the precision of their predictions.*

The data issue is also important for the clients of GMAP. In most cases, the accuracy of the modelling procedure undertaken by GMAP is dependent on data provided by the client. For example, observed flow data is vital for the calibration procedure, but the quality of these data varies between clients. For example, for Halifax new mortgage sales there is extensive information on observed flows between branches and postal sectors. However, in the case of WH Smith, little flow information was available and the data that did exist were quite old which made the process of calibration difficult and could introduce errors into the model. The quality of data that are available for model building will be dependent on the nature of the retailing activity. For example, in the financial services' sector customer addresses are often known which produces accurate observed data, whereas retailers such as WH Smith have to rely on surveys to collect any form of flow data. However, if retailers want accurate models of their markets, they need to provide data that are of high quality.

It is also important for there to be continued collaboration between GMAP and the client. The retailer will be aware of any market changes that occur in their retail sector and it is necessary for GMAP to be informed of such changes so that they can be accounted for in the models that are produced. The retailer will also want to preserve their investment through an ongoing process of improvement of the model and this would require continued contact with GMAP.

**Recommendation 13:** *GMAP should facilitate continuing interaction between itself and the client.*

It is also important for GMAP to continue to recognise the dangers of stagnation that could arise due to competitive pressures in the market. Continued research is a time consuming and expensive process, but if the applied models are to be further improved, further research is necessary.

**Recommendation 14:** *GMAP should undertake continuing research and innovation in order to remain at the leading edge of the retail decision support market. The link between the commercial sector and academia should also be maintained and academic theories must continue to be tested in commercial applications in order to determine their validity in explaining real life interaction patterns.*

Possible avenues of future research that could be investigated by GMAP are suggested in the next section.

## 9.6 THE WAY FORWARD

Through this research, considerable progress has been made in the investigation of methods of decreasing the error in spatial interaction models. However, there is still a significant amount of error apparent in the models. Although spatial interaction models cannot be expected to exactly replicate observed flow patterns due to the inherent randomness of human behaviour, there is still potential for additional performance improvements and therefore further research is required.

This research has focused on the residential trips of the WH Smith model (although some work has been undertaken concerning work based and tourist CCFs) but it is also necessary to investigate methods of improving predictions for work and tourist based trips. At present, the workplace based demand is represented in the WH Smith model simply as a percentage of a postal district's demand and this value is uniform across all postal districts. Special Workplace Statistics (SWS) dataset A which contains the number of workers that are resident in an area could be investigated in order to determine if it could be used to improve the estimation of work based demand. SWS dataset C could be utilised to determine the amount of work based interaction that occurs between an origin and a destination. However, there would still be the problem of determining the level of sales that are undertaken from work as opposed to from home. This problem could only be solved through store surveys of customers.

Alternative data sources also need to be used in order to determine the level of tourism in areas because the number of hotels in a centre is not necessarily representative of the level of tourism in an area.

Analysis is also required concerning whether flows should be separated into the three trip types, residential, work and tourist trips for all models as they are for the WH Smith model. It is unlikely that tourism factors would be important in the determination of mortgage sales, but it is possible that consumers may travel from work to purchase a mortgage. It was seen in Chapter 8 that work based CCFs were not significant in determining destination choice, but the work variable used in the calculation of that factor was very crude and additional research using the SWS could identify a method of accounting for work based interactions in mortgage sales.

Behavioural research would also be useful in providing insights into why customer behaviour differs between alternative retail types. This could lead to the identification of additional factors that determine destination choice and interaction levels. It has been seen that the factors that affect destination choice for book sales are different from those for mortgage sales. It has not been possible to identify centre characteristics that influence level of mortgage sales. Centre agglomeration factors have been found to be important, but it is possible that behavioural analysis could identify further factors that could be used to build an explanatory model for mortgage sales, which has not been possible in this study. It is possible that the process of buying a mortgage is significantly different from that of book buying in that it warrants a different modelling procedure. For example, the increase in the use of the telephone for making financial service decisions could be affecting interaction patterns, whereas interaction levels and patterns for book buying will be affected by the increase in mail order, and the use of the internet as a means of making purchases. Thus, behavioural analysis should be undertaken in order to understand the processes that influence a consumer's decision on where to shop, in order for such interactions to be modelled more accurately. Such analysis could be undertaken using the method of stated preferences, in which respondents are asked to state their preference between a set of alternatives. This method has been used previously in behavioural research in the retailing context. For example, Moore (1989) undertook a study of consumer behaviour in grocery shopping by determining the preferred alternatives of individuals from a set of hypothetical choice scenarios. He found that the stated preference data revealed systematic variations in consumer preferences for grocery shopping and that these variations were linked to the affluence, mobility and time budgets of respondents. The segmentation of the consumer choice model according to these variables led to an increase in the accuracy of the

model for grocery retailing. Oppewal *et al* (1997) also used stated preference models to look at the effect on consumer behaviour of changing centre size and store variety, and McCarthy (1980) used the same method to determine what factors attributed to shopping choice behaviour. Both of these studies have helped in the identification of variables important in determining consumer choice, such as the finding of Oppewal *et al* that the physical appearance and layout of centres are significant in the determination of expenditure levels at a destination and McCarthy's conclusion that generalised centre attributes that are based on attitudinal information have a significant impact on destination choice. Therefore, stated preference methods could provide useful behavioural insights into consumer choice. However, such analysis is beyond the scope of this study due to a lack of available preference data.

The possibility of multi-stop and multi-purpose trips as described in Delleart *et al.* (1998) should also be investigated. The increase in multi-stop trips will have an effect on the interaction component of the spatial interaction model because consumers will not necessarily be travelling from an origin zone, but from another destination.

The findings of this research should also be tested using alternative retailing activities such as the car market and grocery retailing in order to determine the similarities and differences in the processes of interaction that are apparent in each retail sector.

Additional searches should also be undertaken in order to discover observable variables that could be used to improve and, in the case of Halifax, create explanatory models. The importance of explanatory models has already been stated and therefore it is important that every effort is made to make models more explanatory in order to improve their interpretability.

## 9.7 CONCLUSIONS

This thesis has shown that significant improvements can be made to the models currently used by GMAP and several recommendations have been made that will facilitate this process. However, there are differences in the level of performance improvement for each model. The performance of the Halifax model has been

improved by 9%, whereas the WH Smith Explanatory Model was improved by 46% and the WH Smith Predictive Model by 23%.

Systematic testing of alternative modelling techniques has shown that traditional methods are not necessarily the most appropriate and that differences in the shopping process between different retail activities must be reflected in alternative model formulations. For example, it was discovered that the new supply side variable that was developed in Chapter 6 improved the performance the WH Smith Model more than the competing destinations variable (leading to 41% of the 46% performance improvement for the Explanatory Model and 19% of the 23% for the Predictive Model). However, it did not improve the Halifax Model. The effect of the competing destinations variable on model performance also varied between the different retail models, being more important for the Halifax Model than for the WH Smith Model.

Theoretical advances in modelling approaches have been applied to real-world applications, an approach that is commonly advocated, but rarely undertaken. It has been shown that although the intervening opportunities model did not improve the model performance, the use of the competing destinations model did lead to more accurate predictions of retail flows. In conclusion, the competing destinations model is a more accurate method of predicting flows than the traditional gravity model in these retailing contexts.

However, the difficulties of producing working models in an applied context must be noted. Difficulties have been encountered concerning the transfer of theories into practical models of real life spatial systems due to the difficulty of quantifying the variables that have to be included in such models.

It has therefore been seen that, particularly in the case of the WH Smith model, important model developments and performance improvements have been achieved. However, these results emphasise the necessity of undertaking the modelling process for alternative retail activities individually because alternative processes are at work in different situations. Therefore, there is scope for a large amount of further research concerning the application of alternative modelling techniques to other retailing activities.

LOGIT MODELLING

The logit modelling for this analysis was undertaken using the GLIM 4 statistical package on the MIDAS machine at the Manchester Computing Centre. Detailed descriptions of logit modelling can be found in Aitkin *et al.* (1989), Healy (1988), Knoke and Burke (1980) and O'Brien (1992).

The logit model is a form of generalised linear model that allows the formation of a relationship similar to multiple regression, between a categorical dependent variable and several independent variables. Knoke (1980) states that the logit model is a special case of the generalised linear model. In the logit model, the dependent variable is taken to be dependent upon variation in the explanatory variables included in the model, and the criterion to be modelled is the log of the odds (the logit) of a 'success' (a value of 1 for the categorical dependent variable). The odds of a success occurring in the dependent variable is given by

$$\frac{P_i}{1 - P_i} \quad (\text{A.1})$$

where  $P_i$  is the probability of success associated with observation  $i$  and  $(1 - P_i)$  is the probability of non-success associated with observation  $i$ .

The log odds of a success are produced through the logit transformation

$$\text{logit}(P_i) = \log\left(\frac{P_i}{1 - P_i}\right) \quad (\text{A.2})$$

The general form of the logit model is given by

$$P_i = \frac{e^{f(X_i)}}{1 + e^{f(X_i)}} \quad (\text{A.3})$$

where

$$f(X_i) = \alpha + \sum_{n=1}^n \beta_n X_{in} \quad (\text{A.4})$$

$P_i$  = the probability of success for the  $i$ th observation

$X_{in}$  = is the value of the  $n$ th explanatory variable for the  $i$ th observation

$\alpha$  = parameter to be estimated

$\beta$  = parameters to be estimated

The logit transformation described above can be used to allow this model to be fitted as a generalised linear model.

The logit model produces a deviance measure  $D$ , which is represented by the following equation

$$D = -2 \sum_{i=1}^n \text{logit}(P_i) + \log(1 - P_i) \quad (\text{A.5})$$

The deviance measure allows the comparison of the explanatory power of the independent variables being investigated but cannot be used as a measure of goodness of fit on its own. Comparison of the fit of different models is possible because the difference in deviance (caused by addition of another independent variable) approximates to the Chi-square ( $\chi^2$ ) distribution.

The significance of the addition of a variable to the logit model can be tested in the following way. The decrease in deviance from the null model (a model with no additional explanatory variables) and the decrease in degrees of freedom from the addition of the variable can be analysed using the Chi-square statistic at the significance level chosen (95% in this case), where the degrees of freedom are given by the number of logits minus the number of independent parameters in the model. This method means that it is possible to find which variables are most important in determining variation in the dependent variable.

More variables can be added to the model in a stepwise fashion in order to find the optimal model. *i.e.* the model which decreases deviance the most while controlling for the associated decrease in degrees of freedom caused by adding variables. This is

undertaken by adding a variable, testing its significance and subsequently retaining or discarding the variable according to the outcome of the significance test.

The GLIM 4 statistical package produces individual parameter estimates associated with each category of the categorical independent explanatory variables. For logit models the parameter estimates are the log odds of a respondent being a success *i.e.* having a value of 1 for the categorical response variable. McCullagh & Nelder (1983) state that in the example of a logit model with two explanatory variables  $x_1$  and  $x_2$ , the model for the log odds of a positive response (*i.e.* the parameter estimate) will be as follows:

$$\log\left(\frac{P_i}{1-P_i}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \quad (\text{A.6})$$

If it is assumed that  $x_1$  and  $x_2$  are unrelated variables, then McCullagh & Nelder state that it is possible to make the following conclusions. The effect of a change in  $x_2$  is to increase the log odds by an amount given by  $\beta_2$ . Equivalently, the effect of a change in  $x_2$  is to increase the odds of a positive response multiplicatively by the factor  $\exp(\beta_2)$ . These conclusions are dependant on the value of  $x_1$  remaining fixed as  $x_2$  is allowed to vary. Thus for an explanatory categorical variable with  $i$  categories, the factor  $\beta_i$  represents the effect of category  $i$  on the response variable.

These parameters show the strength and direction of the relationship between that category and the dependent variable. Not all such parameters will be significant. Therefore it is necessary to test which parameters are significant. A parameter can be said to be significant at the 95% level, if it is approximately twice the standard error of the parameter estimate. Therefore it is possible to investigate which categories of the independent variables are most important in explaining variance in the dependent variable.

**THE UNIVARIATE LOGIT MODELS FOR BOOK BUYING**

variable	subgroup	parameter estimate	standard error of parameter estimate	deviance	degrees of freedom	null deviance-model deviance
null model		-1.076	0.027	7912.7	6978	
age				7771.0	6975	141.7
	<30	-1.181	0.077			
	30-49	0.353	0.088			
	50-64	0.288	0.096			
	65+	-0.484	0.100			
region				7867.7	6967	45.0
	Northern	-1.184	0.116			
	Y & H	-0.035	0.148			
	NW	-0.045	0.145			
	East Mids	0.105	0.156			
	West Mids	-0.016	0.151			
	E Anglia	0.143	0.184			
	Gt London	0.208	0.143			
	SE	0.395	0.130			
	SW	0.210	0.148			
	Wales	-0.108	0.174			
	Scotland	-0.084	0.150			
	N Ireland	-0.315	0.250			
social class				7679.2	6974	233.5
	AB	-0.427	0.051			
	C1	-0.653	0.106			
	C2	-0.691	0.084			
	DE	-0.892	0.101			
	other	-1.043	0.070			
seg				7645.5	6972	267.2
	1	-0.381	0.052			
	2	-0.779	0.113			
	3	-0.801	0.105			
	4	-0.778	0.080			
	5	-0.717	0.579			
	6	-1.291	0.083			
	7	-0.757	0.092			
income				7488.9	6974	423.8
	1	-2.262	0.096			
	2	1.116	0.102			
	3	1.821	0.112			
	4	2.191	0.164			
	5	2.648	0.236			
h1				7677.2	6974	235.5
	1	-1.778	0.067			
	2	0.811	0.079			
	3	0.226	0.141			
	4	1.096	0.085			
	5	1.362	0.136			
h2				7780.9	6974	131.8
	1	-1.360	0.039			
	2	0.800	0.096			
	3	0.503	0.082			
	4	0.618	0.068			
	5	-0.0264	1.118			
hh members				7749.2	6974	163.5
	1-2	-1.381	0.038			
	3-4	0.691	0.059			
	5-6	0.770	0.101			
	7-8	0.464	0.376			
	9+	-3.169	2.676			
education				7762.1	6975	150.6
	student	-1.289	0.034			
	≤ 16	0.532	0.078			
	17-18	0.869	0.076			
	19+	0.694	0.242			

**APPENDIX C**

**THE GEODEMOGRAPHIC PROFILE OF THE NATIONAL  
MARKET SURVEY**

## Description of the geodemographic type (7)

geodemo -graphic type (1)	number of responses (2)	number of books (3)	average number of books(4)	average book price (5)	average spend on books (6)	Description of the geodemographic type (7)
72	70	235	3.36	5.93	19.91	Aspiring; Better-Off Multi-ethnic Areas - Young mixed occupation singles & couples - buying & privately renting terraces & bedsits
89	38	118	3.11	5.93	18.41	Aspiring; Town & City Bedsit Areas - Young educated white collar singles & couples - privately renting bedsits
98	213	642	3.01	5.93	17.87	Aspiring; Multi-ethnic Student Areas - Well educated students & professional residents - privately renting flats and bedsits
78	153	448	2.93	5.93	17.36	Climbing; Affluent Executive Home Owning Areas - White collar families & couples - buying detached houses
18	183	534	2.92	5.93	17.30	Prospering; Affluent Rural Commuter Areas - Mature professional or farming residents - owning & privately renting detached houses
22	105	304	2.90	5.93	17.17	Struggling; Less Well-Off Council Tenants - Young unemployed blue collar families & single parents - LA rented terraces
28	170	476	2.80	5.93	16.60	Aspiring; Academic Centres & Student Areas - Young educated white collar singles & couples - privately rented terraces & bedsits
51	142	375	2.64	5.93	15.66	Prospering; Prosperous Pensioners - Mature educated professional families & elderly residents - owning large detached houses
37	296	771	2.60	5.93	15.45	Established; Comfortable Middle Agers - Mature blue collar couples without children - owning semi's
85	117	300	2.56	5.93	15.21	Prospering; Affluent Achievers - Mature educated professional couples & families - owning large detached houses
43	161	394	2.45	5.93	14.51	Climbing; Affluent Executive Home Owning Areas - Educated white collar families - buying detached & semi detached houses
66	33	80	2.42	5.93	14.38	Aspiring; Armed Services - Young families with infants & children - privately rented semi's & terraces
57	190	452	2.38	5.93	14.11	Prospering; Affluent Achievers - Mature educated professional families - owning & buying large semi-detached & detached houses
29	110	259	2.35	5.93	13.96	Aspiring; Town & City Bedsit Areas - Young educated white collar singles & couples - privately renting bedsits
47	190	445	2.34	5.93	13.89	Aspiring; Academic Centres & Student Areas - Young educated white collar singles & couples - mixed tenure terraces & bedsits
96	49	114	2.33	5.93	13.80	Established; Rural Farming Communities - Mature self-employed couples and families-privately renting or owning large houses
73	72	167	2.32	5.93	13.75	Struggling; Multi-ethnic Areas - Young Black white collar couples & students - privately rented bedsits & flats
39	260	571	2.20	5.93	13.02	Established; Comfortable Middle Agers - Mature white collar couples & families - owning and buying semi's
67	90	197	2.19	5.93	12.98	Prospering; Very Affluent Achievers - Mature educated professional families - owning & buying large detached houses
35	130	280	2.15	5.93	12.77	Climbing; Affluent Executive Home Owning Areas - Educated white collar families & couples - owning & buying terraces
48	222	467	2.10	5.93	12.47	Aspiring; Better-off Multi-ethnic Areas - Middle aged white collar families & couples - owning & buying terraces
46	289	604	2.09	5.93	12.39	Established; Comfortable Pensioners - Mature white collar couples & pensioners - owning & buying terraces
19	155	323	2.08	5.93	12.36	Aspiring; Academic Centres & Student Areas - Well educated students & professional residents - privately renting flats & bedsit
80	134	274	2.04	5.93	12.13	Aspiring; Academic Centres & Student Areas - Well educated students & professionals - privately rented bedsits & semi's
21	383	765	2.00	5.93	11.84	Aspiring; New Home Owners & Mature Communities - Blue collar families & couples - owning & buying terraces & semi's
12	241	477	1.98	5.93	11.74	Established; Comfortable Middle Agers - Mature white collar couples - owning semi's
34	91	180	1.98	5.93	11.73	Struggling; Multi-ethnic Areas - Young Black blue collar families & students - LA & privately rented flats & bedsits
90	186	362	1.95	5.93	11.54	Climbing; Affluent Executive Home Owning Areas - Educated white collar couples & families - buying detached
53	192	367	1.91	5.93	11.33	Struggling; Council Tenants - Young blue collar families & retired residents - LA rented semi's

88	260	1.88	5.93 11.18	Established; Comfortable Middle Agers - Mature blue & white collar families - buying semi's
58	202	1.87	5.93 11.07	Climbing; Well-Off Middle Class Areas - Mature white collar families - owning semi's
71	81	1.85	5.93 10.98	Struggling; Multi-ethnic Areas - Young unemployed Black families & single parents - LA rented bedsits/terraces/flats
25	278	1.84	5.93 10.90	Established; Well-Off Middle Agers - Mature white collar families - owning & buying detached & semi-detached houses
99	147	1.83	5.93 10.85	Aspiring; Multi-ethnic Student Areas - Young educated white collar singles & couples - LA rented flats
11	34	1.76	5.93 10.46	Struggling; Multi-ethnic Areas - Young Black families & single parents - severe unemployment - LA rented flats
17	97	1.76	5.93 10.45	Established; Rural Farming Communities - Mature self-employed couples - owning or privately renting large detached houses
50	32	1.75	5.93 10.38	Established; Rural Farming Communities - Mature self-employed couples & families - privately renting large detached houses
69	271	1.75	5.93 10.35	Prospering; Affluent Achievers - Mature educated professional families - buying large detached houses
76	235	1.71	5.93 10.14	Struggling; Better-off Council Tenants - Mature blue collar singles & couples - LA rented semi's & terraces
40	353	1.69	5.93 10.01	Prospering; Affluent Rural Commuter Areas - Mature professional or farming residents - owning & buying detached houses
3	330	1.63	5.93 9.65	Aspiring; Young Married Suburbia - Young less well-off blue collar families - owning & buying terraces
36	231	1.62	5.93 9.63	Established; Comfortable Pensioners - Elderly white collar residents - owning & LA renting flats
23	214	1.62	5.93 9.59	Aspiring; New Home Owners & Mature Communities - Blue & white collar families & couples - LA rented & buying terraces
86	29	1.59	5.93 9.41	Struggling; Multi-ethnic Areas - Young blue collar Pakistani singles & couples - buying & privately renting flats
41	125	1.58	5.93 9.39	Aspiring; New Home Owners - Young white collar couples - buying terraces
38	165	1.58	5.93 9.34	Struggling; Multi-ethnic Areas - Pensioners & single parents - high unemployment - Highrise LA rented flats
63	150	1.57	5.93 9.29	Aspiring; New Home Owners & Mature Communities - Blue & white collar families & couples - buying terraces
100	113	1.56	5.93 9.24	Prospering; Affluent Achievers - Mature educated professional families - buying large detached houses
68	248	1.56	5.93 9.23	Established; Well-Off Middle Agers - Mature white collar families & retired residents - owning detached & semi-detached houses
79	290	1.56	5.93 9.22	Established; Less Well-Off Middle Agers - Mature blue collar couples & pensioners - LA renting & owning terraces & semi's
45	317	1.55	5.93 9.18	Established; Comfortable Middle Agers - Mature white collar couples - owning and buying semi's
95	143	1.55	5.93 9.16	Established; Less Well-off Middle Aged - Young white collar couples & elderly residents - buying flats or privately renting bedsits
93	253	1.54	5.93 9.12	Established; Comfortable Pensioners - Mature white collar couples & pensioners - owning detached houses
70	259	1.54	5.93 9.11	Established; Comfortable Middle Agers - Mature blue collar couples & families - owning and buying detached & semi-detached houses
59	55	1.53	5.93 9.06	Established; Comfortable Pensioners - Elderly residents & some students - owning & renting flats & bedsits
49	122	1.52	5.93 9.04	Struggling; Less Well-Off Council Tenants - Young unemployed blue collar families & single parents - LA rented semi's & terraces
9	146	1.51	5.93 8.98	Prospering; Affluent Achievers - Mature educated professional families & retired residents - owning detached houses
64	162	1.46	5.93 8.64	Prospering; Prosperous Pensioners - Mature educated professional families & retired residents - owning detached houses
92	121	1.45	5.93 8.63	Struggling; Residential Homes & Retirement Areas - Retired and elderly residents - LA rented flats & privately rented bedsits
84	299	1.45	5.93 8.61	Established; Comfortable Middle Class Family Areas - Blue collar families - buying semi's
75	212	1.43	5.93 8.48	Prospering; Affluent Rural Commuter Areas - Mature couples & families in mixed occupations - owning & large detached houses

30	313	438	1.40	5.93 8.30	Established; Comfortable Middle Class Family Areas - Blue collar families - buying semi's
20	223	311	1.39	5.93 8.27	Aspiring; New Home Owners & Mature Communities - Blue & white collar families & couples - LA rented & buying terraces
77	374	511	1.37	5.93 8.10	Climbing; Well-Off Middle Class Areas - Mature white collar families & couples - owning & buying semi's
13	134	183	1.37	5.93 8.10	Struggling; Less Well-off Council Tenants - Pensioners & single parents - LA rented flats & terraces
44	81	110	1.36	5.93 8.05	Struggling; Council Tenants - Young unemployed blue collar families & single parents - LA rented flats
55	158	214	1.35	5.93 8.03	Struggling; Council Tenants - Blue collar families & single parents - LA rented terraces
54	112	148	1.32	5.93 7.84	Struggling; Less Prosperous Pensioner Areas - Retired residents - LA rented flats
87	35	46	1.31	5.93 7.79	Struggling; Council Tenants - Young white unemployed families & single parents - LA rented flats
7	270	353	1.31	5.93 7.75	Struggling; Lower Class Renters - Poor young blue collar families - privately renting terraces
91	226	295	1.31	5.93 7.74	Struggling; Council Tenants - Young blue collar families & single parents - LA rented semi's and terraces
26	275	353	1.28	5.93 7.61	Established; Less Well-off Middle Agers - Mature couples & pensioners - owning semi's
4	199	250	1.26	5.93 7.45	Established; Comfortable Pensioners - Elderly blue & white collar residents - owning & renting flats & terraces
94	66	82	1.24	5.93 7.37	Struggling; Less Well-off Council Tenants - Young unemployed blue collar families & single parents - LA rented semi's
83	243	298	1.23	5.93 7.27	Struggling; Council Tenants - Young blue collar families & pensioners - LA rented terraces
16	86	105	1.22	5.93 7.24	Struggling; Less Well-Off Council Tenants - Young unemployed families/single parents/pensioners - LA rented flats & terraces
5	213	259	1.22	5.93 7.21	Struggling; Council Tenants - Young blue collar families with infants & single parents - LA rented flats
61	15	18	1.20	5.93 7.12	Struggling; Multi-ethnic Areas - Young blue collar Indian families with children - mixed tenure terraces & bedsits
2	54	64	1.19	5.93 7.03	Struggling; Less Well-Off Council Tenants - Young blue collar families & single parents - severe unemployment - LA rented flats
24	190	215	1.13	5.93 6.71	Struggling; Less Prosperous Pensioner Areas - Retired blue collar residents - LA rented flats & semi's
74	55	62	1.13	5.93 6.68	Established; Comfortable Pensioners - Retired & elderly residents - owning detached houses
56	60	64	1.07	5.93 6.33	Aspiring; Academic Centres & Student Areas - Young educated white collar singles & couples - buying & privately renting flats & bedsits
31	212	224	1.06	5.93 6.27	Climbing; Well-Off Suburban Areas - Young white collar singles & couples - buying semi's & detached houses
33	7	7	1.00	5.93 5.93	Climbing; Well-Off Middle Class Areas - Mature white collar families & couples - owning & buying semi-detached & detached houses
6	178	176	0.99	5.93 5.86	Struggling; Fading Industrial Areas - Mature blue collar residents - owning terraces
27	61	56	0.92	5.93 5.44	Struggling; Less Prosperous Private Renters; Multi-ethnic Areas - Young Pakistani families with children - privately renting terraces
65	156	137	0.88	5.93 5.21	Struggling; Better-off Council Tenants - Mature blue collar singles & couples - LA rented terraces
82	338	287	0.85	5.93 5.04	Struggling; Council Tenants - Young blue collar families & retired residents - LA rented semi's
97	91	77	0.85	5.93 5.02	Struggling; Council Tenants - Single parents & young unemployed families - LA rented flats
60	32	27	0.84	5.93 5.00	Established; Rural Farming Communities - Mature self-employed couples - privately renting or owning large detached houses
32	159	129	0.81	5.93 4.81	Struggling; Less Prosperous Pensioner Areas - Retired blue collar residents - LA rented terraces
10	270	213	0.79	5.93 4.68	Struggling; Council Tenants - Blue collar families - LA rented terraces
62	24	16	0.67	5.93 3.95	Struggling; Multi-ethnic Areas - Young unemployed Pakistani & Bangladeshi families with children - privately renting terraces & bedsits

52	187	112	0.60	5.93 3.55	Struggling; Council Tenants - Young blue collar families & single parents - LA rented terraces
8	19	11	0.58	5.93 3.43	Aspiring; Better-Off Multi-ethnic Areas - Blue collar Indian residents - owning & buying semi's
42	86	45	0.52	5.93 3.10	Struggling; Less Prosperous Pensioner Areas - Retired blue collar residents - LA rented semi's
15	60	24	0.40	5.93 2.37	Struggling; Less Well-Off Council Tenants - Young unemployed families/single parents/pensioners - LA rented flats
1	56	21	0.38	5.93 2.22	Established; Residential Homes & Retirement Areas - Less well-off elderly residents - LA rented flats
14	1	0	0.00	5.93 0.00	Struggling; Less Well-Off Council Tenants & Students - Young blue collar families & educated white collar residents - LA rented flats
81	1	0	0.00	5.93 0.00	Struggling; Multi-ethnic Areas - Young Bangladeshi families with infants - severe unemployment - LA rented flats

- Aitkin, M., Anderson, D., Francis, B. & Hinde, J. (1989) *Statistical modelling in GLIM*. Oxford University Press, New York.
- Alonso, W. (1978) A theory of movements. In Hanson, N.M. (ed.) *Human settlement systems*. Ballinger Publishing Company, Cambridge, Massachusetts, 197-211.
- Anas, A. (1982) *Residential location markets and urban transportation*. Academic Press, New York.
- Applebaum, W. (1965) Can store location be a science? *Economic Geography*, **41**:234-237.
- Applebaum, W. (1966) Methods for determining store trade areas, marketing penetration and potential sales. *Journal of Marketing Research*, **3**:127-141.
- Applebaum, W. (1968) The analog method for estimating potential store sales. In Karblane, C. (ed) *Guide to store location research*. Addison-Wesley, Reading, Massachusetts.
- Bailey, T.C. & Munford, A.G. (1994) Modelling a large, sparse spatial interaction matrix using data relating to a subset of possible flows. *European Journal of Operational Research*, **79**:489-500.
- Batey, P. & Brown, P.J. (1995) From human ecology to customer targeting: the evolution of geodemographics. In Longley, P. & Clarke, G.P. (eds) *GIS for business and service planning*. Longman, London.
- Batty, M. (1976) *Urban modelling*. Cambridge University Press, London.
- Batty, M. & Mackie, S. (1972) The calibration of gravity, entropy, and related models of spatial interaction. *Environment and Planning A*, **4**:206-233.
- Batty, M. & Sikdar, P.K. (1982a) Spatial aggregation in gravity models: 1. An information-theoretic framework. *Environment and Planning A*, **14**:377-405.
- Batty, M. & Sikdar, P.K. (1982b) Spatial aggregation in gravity models: 2. One-dimensional population density models. *Environment and Planning A*, **14**:525-553.

- Batty, M. & Sikdar, P.K. (1982c) Spatial aggregation in gravity models: 3. Two-dimensional population density models. *Environment and Planning A*, **14**:629-658.
- Batty, M. & Sikdar, P.K. (1982d) Spatial aggregation in gravity models: 4. Generalisations and large-scale applications. *Environment and Planning A*, **14**:795-822.
- Baxter, M. (1982) Similarities in methods of estimating spatial interaction models. *Geographical Analysis*, **14**:267-272.
- Beaumont, J.R. (1988) Store location analysis: problems and progress. In Wrigley, N. (ed) *Store choice, store location and market analysis*. Routledge, London, 87-105.
- Birkin, M. (1986) *Applications of dynamical systems theory to urban modelling: new subsystems and extended comprehensive models*. Unpublished PhD thesis, School of Geography, University of Leeds.
- Birkin, M. (1995) Customer targeting, geodemographics and lifestyle approaches. In Longley, P. & Clarke, G. (eds) *GIS for business and service planning*. Geoinformation International, Cambridge, 104-149.
- Birkin, M. & Clarke, G. (1991) Spatial interaction in geography. *Geography Review*, **4**:16-24.
- Birkin, M. & Clarke, G. (1995) Using microsimulation methods to synthesise census data. In Openshaw, S. (ed) *The census user's handbook*. Geoinformation International, Cambridge, 363-387.
- Birkin, M. & Clarke, M. (1989) The generation of individual and household incomes at the small area level. *Regional Studies*, **23**:535-548.
- Birkin, M. & Foulger, F. (1992) *Sales performance and sales forecasting using spatial interaction modelling: the WH Smith approach*. Working Paper 92/21, School of Geography, University of Leeds.
- Birkin, M., Clarke, G., Clarke, M. & Wilson, A.G. (1995) *Intelligent GIS*. Geoinformation International, Cambridge.
- Blake, M. & Openshaw, S. (1995) *Selecting variables for small area classifications of 1991 Census data*. Working Paper 95/5, School of Geography, University of Leeds.

Bowlby, S., Breheny, M.J. & Foot, D. (1988) Problems and methods in store location 1: is locating a viable store becoming more difficult? *International Journal of Retail and Distribution Management*, **12**:31-33.

Boyle, P.J. & Flowerdew, R. (1997) Improving distance estimates between areal units in migration models. *Geographical Analysis*, **29**:93-107.

Boyle, P.J. & Shen, J. (1995a) *Gravity models and the modifiable areal unit problem revisited 1: identifying data, scale and zonation effects*. Unpublished research paper.

Boyle, P.J. & Shen, J. (1995b) *Gravity models and the modifiable areal unit problem revisited 2: exploring the problem*. Unpublished research paper.

Boyle, P.J. & Shen, J. (1995c) *Gravity models and the modifiable areal unit problem revisited 3: improving distance estimation and removing interaction effects between the constant and the remaining parameters*. Unpublished research paper.

Boyle, P.J. & Shen, J. (1995d) *Gravity models and the modifiable areal unit problem revisited 4: improving the model specification*. Unpublished research paper.

Breheny, M.J. (1988) Practical methods of retail location analysis: a review. In Wrigley, N. (ed) *Store choice, store location and market analysis*. Routledge, London, 39-86.

Bromley, D.F. & Thomas, C.J. (1993) *Retail change: contemporary issues*. University College London Press, London.

Brown, P.J. (1991) Geodemographics: a review of recent developments and emerging issues. In Masser, I. & Blakemore, M. (eds) *Handling geographic information: methodology and potential applications*. Longman, Harlow, 221-258.

Central Statistical Office (1995a) *Family Spending 1994-5*. HMSO, London.

Central Statistical Office (1995b) *Regional Trends*, 30. HMSO, London.

Champion, T., Fotheringham, S., Rees, P., Boyle, P. & Stillwell, J. (1998) *The determinants of migration flows in England: a review of existing data and evidence*. Department of Geography, University of Newcastle, Newcastle upon Tyne.

- Chisholm, M. & O'Sullivan, P. (1973) *Freight flows and spatial aspects of the British economy*. Cambridge University Press, Cambridge.
- Clarke, G.P. (1986) *Retail centre usage and structure: empirical and theoretical explorations*. Unpublished PhD thesis, School of Geography, University of Leeds.
- Clarke, G.P. & Clarke, M. (1995) The development and benefits of customised spatial decision support systems. In Longley, P. & Clarke, G.P. (eds) *GIS for business and service planning*. Longman, London, 227-245.
- Clarke, M. (1984) *Integrating dynamical models of urban structure and activities: an application to urban retail systems*. Unpublished PhD thesis, School of Geography, University of Leeds.
- Clarke, M. (1985) The role of attractiveness functions in the determination of equilibrium solutions to production-constrained spatial interaction models. *Environment and Planning A*, **17**:175-183.
- Codling, D. (1995a) *WH Smith model specification*. Internal document, GMAP Ltd, GMAP House, Cromer Terrace, Leeds, LS2 9JU.
- Codling, D. (1995b) *WH Smith model improvements off-line study*. Internal document, GMAP Ltd, GMAP House, Cromer Terrace, Leeds, LS2 9JU.
- Collins, A. (1989) Store location planning: its role in marketing strategy. *Environment and Planning A*, **21**:625-628.
- Company News (This week) (1996) Chief has tough debut with drop in profits likely - WH Smith. *Financial Times*, January 22nd, 19.
- Converse, P.D. (1946) *Retail Trade Areas in Illinois*. Urbana, Illinois.
- Converse, P.D. (1949) New laws of retail gravitation. *Journal of Marketing*, **14**:379-384.
- Davies, R.L. (1977) Store location and store assessment research: the integration of some new and traditional techniques. *Transactions of the Institute of British Geographers*, 141-157.

- Davies, R.L. & Rogers, D.S (1984) *Store location and store assessment research*. Wiley, Chichester.
- Dellaert, B.G.C., Artenze, T.A., Bierlaire, M., Borgers, A.W.J. & Timmermans, H.J.P (1998) Investigating consumers' tendency to combine multiple shopping purposes and destinations. *Journal of Marketing Research*, **35**:177-188.
- Dibb, S. & Simkin, L. (1991) Targeting, segments and positioning. *International Journal of Retail Distribution and Management*, **19**:4-10.
- Diplock, G. (1996) *The application of evolutionary computing techniques to spatial interaction modelling*. Unpublished PhD thesis, School of Geography, University of Leeds.
- Domencich, T.A. & McFadden, D. (1975) *Urban travel demand: a behavioural analysis*. American Elsevier Publishing Company Inc., New York.
- Fenwick, I. (1978) *Techniques in store location research: a review and applications*. Retail and Planning Associates, Corbridge.
- Fienberg, S.E. (1977) *The analysis of cross-classified categorical data*. MIT Press, Cambridge, Massachusetts.
- Fik, T.J. & Mulligan, G.F. (1990) Spatial flows and competing central places: towards a general theory of hierarchical interaction. *Environment and Planning A*, **22**:527-549.
- Flowerdew, R. & Aitkin, M. (1982) A method of fitting the gravity model based on the Poisson distribution. *Journal of Regional Science*, **22**:191-202.
- Flowerdew, R. & Amrhein, C. (1989) Poisson regression models of Canadian census division migration flows. *Papers of the Regional Science Association*, **67**:89-102.
- Flowerdew, R. & Boyle, P.J. (1995) Migration models incorporating interdependence of movers. *Environment and Planning A*, **27**:1493-1502.
- Foot, D. (1981) *Operational urban models*. Methuen, London.
- Fotheringham, A.S. (1981) Spatial structure and distance-decay parameters. *Annals of the Association of American Geographers*, **71**:425-436.

- Fotheringham, A.S. (1983a) A new set of spatial interaction models: the theory of competing destinations. *Environment and Planning A*, **15**:15-36.
- Fotheringham, A.S. (1983b) Some theoretical aspects of destination choice and their relevance to production-constrained gravity models. *Environment and Planning A*, **15**:1121-1132.
- Fotheringham, A.S. (1984) Spatial flows and spatial patterns. *Environment and Planning A*, **16**:529-543.
- Fotheringham, A.S. (1985) Spatial competition and agglomeration in urban modelling. *Environment and Planning A*, **17**:213-230.
- Fotheringham, A.S. (1986) Modelling hierarchical destination choice. *Environment and Planning A*, **18**:401-418.
- Fotheringham, A.S. (1995) Directional variation in distance decay. *Environment and Planning A*, **27**:715-729.
- Fotheringham, A.S. & Dignan, T. (1984) Further contributions to a general theory of movement. *Annals of the Association of American Geographers*, **74**:620-633.
- Fotheringham, A.S. & Trew, R. (1993) Chain image and store-choice modelling: the effects of income and race. *Environment and Planning A*, **25**:179-196.
- Fotheringham, A.S. & Wong, D.W.S. (1991) The modifiable areal unit problem in multivariate statistical analysis. *Environment and Planning A*, **23**:1025-1044.
- Getis, A. & Ord, J.K. (1992) The analysis of spatial association by use of distance statistics. *Geographical Analysis*, **24**:189-205.
- Ghosh, A. & McLafferty, S.L. (1987) *Location strategies for retail service firms*. Lexington Books, Massachusetts.
- Gini, C. (1915-6) Indici di concordanza. *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti, Series 8*, **75**:1419-1461.
- Golledge, R.G. & Stimson, R.J. (1987) *Analytical behavioural geography*. Croom Helm, London.

- Golledge, R.G. & Timmermans, H. (1990) Applications of behavioural research on spatial problems I: cognition. *Progress in Human Geography*, **14**:57-99.
- Goncalves, M.B. & Ulyssea-Neto, I. (1993) The development of a new gravity-opportunity model for trip distribution. *Environment and Planning A*, **25**:817-826.
- Goodman, L.A. & Kruskal, W.H. (1954) Measures of association for cross classifications. *American Statistical Association Journal*, **49**:732-764.
- Goodman, L.A. & Kruskal, W.H. (1959) Measures of association for cross classifications II: further discussion and references. *American Statistical Association Journal*, **54**:123-161.
- Gould, P. (1975) Acquiring spatial information. *Economic Geography*, **51**:87-99.
- Goux, J.M. (1962) Structure de l'espace et migration. In Sutter, J. (Ed.) *Human displacements*. Entretiens de Monaco en Sciences Humaines: premier session.
- Guy, C.M. (1991) Spatial interaction modelling in retail planning practice: the need for robust statistical methods. *Environment and Planning B*, **18**:191-203.
- Guy, C. M. (1994) *The retail development process: location, property and planning*. Routledge, London.
- Guy, S. (1995) *Demand estimation*. Internal document, GMAP Ltd, GMAP House, Cromer Terrace, Leeds, LS2 9JU.
- Hansen, N.M. (ed) (1978) *Human settlement systems*. Ballinger Publishing Company, Cambridge, Massachusetts.
- Haynes, K.E. & Fotheringham, A.S. (1984) *Gravity and spatial interaction models*. Sage Publications, London.
- Healy, M.J.R. (1988) *Glim: an introduction*. Clarendon Press, Oxford.
- Hopmans, A.C.M. (1986) A spatial interaction model for branch bank accounts. *European Journal of Operational Research*, **27**:242-250.
- Huff, D.L. (1961) A note on the limitations of intra-urban gravity models. *Land Economics*, **38**:64-66.

- Huff, D.L. (1963) A probabilistic analysis of shopping centre trade areas. *Land Economics*, **39**:81-90.
- Huff, D. L. (1964) Defining and estimating a trading area. *Journal of Marketing*, **28**:37-38.
- I.C.M. Research (1995) *The National Market Survey*. 56 Mortimer Street, London.
- Jones, K. & Simmons, J. (1991) *The retail environment*. Roulledge, London.
- Karblane, C. (ed) (1968) Guide to store location research. Addison-Wesley, Reading, Massachussets.
- Knoke, D. & Burke, P.J. (1980) *Log-linear models*. Sage Publications Inc, Beverley Hills.
- Knudsen, D.C. & Fotheringham, A.S. (1986) Matrix comparison, goodness-of-fit, and spatial interaction modelling. *International Regional Science Review*, **10**:127-147.
- Lakshmanan, T.R. & Hansen, W.G. (1965) A retail market potential model. *Journal of the American Institute of Planners*, **31**:134-149.
- Ledent, J. (1981) On the relationship between Alonso's theory of movement and Wilson's family of spatial interaction models. *Environment and Planning A*, **13**:217-224.
- Liaw, K.L. & Bartels, C.P.A. (1982) Estimation and interpretation of a nonlinear migration model. *Geographical Analysis*, **14**:229-245.
- Lo, L. (1991) Spatial structure and spatial interaction: a simulation approach. *Environment and Planning A*, **23**:1279-1300.
- Longley, P. & Clarke, G. (eds) (1995) *GIS for business and service planning*. Geoinformation International, Cambridge.
- Losch, A. (1954) *Economics of location*. Yale University Press, New Haven, Connecticut.

- Maguire, D.J. (1995) Implementing spatial analysis and GIS applications for business and service planning. In Longley, P. & Clarke, G.P. (eds) *GIS for business and service planning*. Geoinformation International, Cambridge.
- Malmberg, B (1996) Understanding attraction: co-operation and human intentionality as determinants of spatial interaction and corporate location. *Environment and Planning A*, **28**:651-665.
- Masser, I. & Blakemore, M. (1991) *Handling geographic information: methodology and potential applications*. Longman, Harlow.
- McCarthy, P.S. (1980) A study of the importance of generalized attributes in shopping choice behaviour. *Environment and Planning A*, **12**:1269-1286.
- McGoldrick, P.J. (1990) *Retail marketing*. McGraw-Hill, London.
- Moore, L. (1989) Modelling store choice: a segmented approach using stated preference analysis. *Transactions, Institute of British Geographers*, **14**:461-477.
- O'Brien, L. (1992) *Introducing quantitative geography: measurement, methods and generalised linear models*. Routledge, London.
- Openshaw, S. (1976) An empirical study of some spatial interaction models. *Environment and Planning A*, **8**:23-41.
- Openshaw, S. (1984) The modifiable areal unit problem. *Concepts and Techniques in Modern Geography*, **38**. Geo Books, Norwich.
- Openshaw, S. (1989) Making geodemographics more sophisticated. *Journal of the Market Research Society*, **31**:111-131.
- Openshaw, S. (1994) *Developing smart and intelligent target marketing systems*. Working Paper 94/13, School of Geography, University of Leeds.
- Openshaw, S. (1995) *The census user's handbook*. Geoinformation International, Cambridge.
- Openshaw, S., Blake, M. & Wymer, C. (1994) *Using neurocomputing methods to classify Britain's residential areas*. Working Paper 94/17, School of Geography, University of Leeds.

- Openshaw, S. & Wymer, C. (1995) Classifying and regionalizing census data. In Openshaw, S. (ed) *The census user's handbook*. Geoinformation International, Cambridge, 213-237.
- Oppewal, H., Timmermans, H.J.P. & Louviere, J.J. (1997) Modelling the effects of shopping centre size and store variety on consumer choice behaviour. *Environment and Planning A*, **29**:1073-1090.
- Pacione, M. (1974) Measures of the attraction factor: a possible alternative. *Area*, **6**:279:282.
- Penny, N.J. & Broom, D. (1988) The Tesco approach to store location. In Wrigley, N. (ed) *Store choice, store location and market analysis*. Routledge, London, 106-119.
- Plowman, J. (1996) *Halifax 96 update data specification*. Internal document, GMAP Ltd, GMAP House, Cromer Terrace, Leeds, LS2 9JU.
- Pooler, J. (1992) Spatial uncertainty and spatial dominance in interaction modelling: a theoretical perspective on spatial competition. *Environment and Planning A*, **24**:995-1008.
- Pooler, J. (1994) An extended family of spatial interaction models. *Progress in Human Geography*, **18**:17-39.
- Pooler, J. (1995) Modelling spatial interaction without distances: the use of prior spatial flows. *Geographical Systems*, **2**:309-324.
- Raper, J., Rhind, D. & Shepherd, J. (1992) *Postcodes: the new geography*. Longman, Harlow.
- Rees, P. (1995a) Research using the 1991 Census of population: development of tools for analysis. *Environment and Planning A*, **27**:349-352.
- Rees, P. (1995b) Research using the 1991 Census: findings on deprivation, unemployment, ethnicity and religion. *Environment and Planning A*, **27**:515-518.

- Rees, P. (1995c) Putting the census on the researcher's desk. In Openshaw, S. (ed) *The census user's handbook*. Geoinformation International, Cambridge, 27-82.
- Reilly, W.J. (1929) *Methods for the study of retail relationships*. Bureau of Business Research Studies in Marketing, Austin, Texas.
- Rogers, D.S. & Green, H.L. (1979) A new perspective on forecasting store sales: applying statistical models and techniques in the analog approach. *The Geographical Review*, **69**:449-458.
- Sayer, R.A. (1976) *A critique of urban modelling*. Pergamon Press, Oxford.
- Siegel, S. (1956) *Nonparametric statistics for the behavioural sciences*. McGraw-Hill Book Company, New York.
- Simkin, L.P. (1990) Evaluating a store location. *International Journal of Retail Distribution and Management*, **18**:33-38.
- Sleight, P. & Leventhal, B. (1989) Applications of geodemographics to research and marketing. *Journal of the Market Research Society*, **31**:75-101.
- Spencer, A.H. (1978) Deriving measures of attractiveness for shopping centres. *Regional Studies*, **12**:713-726.
- Stewart, J.Q. (1941) An inverse distance variation for certain social influences. *Science*, **93**:89-90.
- Stillwell, J.C.H. (1978) Interzonal migration: some historical tests of spatial interaction models. *Environment and Planning A*, **10**:1187-1200.
- Stillwell, J.C.H. & Congdon, P. (eds) (1991) *Migration models: macro and micro approaches*. Belhaven Press, London.
- Stouffer, S.A. (1940) Intervening opportunities: a theory relating mobility and distance. *American Sociological Review*, **5**:845-867.
- Stouffer, S.A. (1960) Intervening opportunities and competing migrants. *Journal of Regional Science*, **2**:1-21.

- Taylor, P.J. (1971) Distance transformation and distance decay functions. *Geographical Analysis*, **3**:221-238.
- Taylor, P.J. (1975) *Distance decay models in spatial interaction*. Catmog 2, Geo Abstracts Ltd, Norwich.
- Thill, J.C. (1992) Choice set formation for destination choice modelling. *Progress in Human Geography*, **16**:361-382.
- Thomas, R.W. & Huggett, R.J. (1980) *Modelling in geography: a mathematical approach*. Harper and Row, London.
- Timmermans, H.J.P. (1981) Multiattribute shopping models and ridge regression analysis. *Environment and Planning A*, **13**:43-56.
- Timmermans, H. & Golledge, R.G. (1990) Applications of behavioural research on spatial problems II: preference and choice. *Progress in Human Geography*, **14**:311-354.
- Urquhart, J. (1997) *Halifax 96 update model specification*. Internal document, GMAP Ltd, GMAP House, Cromer Terrace, Leeds, LS2 9JU.
- Vickerman, R.W. (1974) Accessibility, attraction, and potential: a review of some concepts and their use in determining mobility. *Environment and Planning A*, **6**:675-691.
- Walford, N. (1995) *Geographical data analysis*. John Wiley & Sons Ltd, Chichester.
- Webber, M.J. (1980) A theoretical analysis of aggregation in spatial interaction models. *Geographical Analysis*, **12**:129-141.
- Willekens, F. (1983) Log-linear modelling of spatial interaction. *Papers of the Regional Science Association*, **52**:185-205.
- Wilson, A.G. (1967) A statistical theory of spatial distribution models. *Journal of Transportation Research*, **1**:253-269.
- Wilson, A.G. (1970) *Entropy in urban and regional modelling*. Pion Ltd, London.

- Wilson, A.G. (1971) A family of spatial interaction models, and associated developments. *Environment and Planning A*, **3**:1-32.
- Wilson, A.G. (1974) *Urban and regional models in geography and planning*. John Wiley & Sons, London.
- Wilson, A.G. (1981) *Geography and the environment*. John Wiley & Sons, Chichester.
- Wilson, A.G. (1983) *A generalised and unified approach to the modelling of service-supply structures*. Working Paper 352, School of Geography, University of Leeds.
- Wilson, A.G., Coelho, J.D., Magill, S.M. & Williams, H.C.W.L. (1981) *Optimization in locational and transport analysis*. John Wiley & Sons, Chichester.
- Wilson, A.G. & Bennett, R.J. (1985) *Mathematical methods in human geography and planning*. John Wiley & Sons, Chichester.
- Wrigley, N. (1988) *Store choice, store location and market analysis*. Routledge, London.
- Yano, K. (1991) The integration of spatial interaction models using generalised linear modelling. *Geographical Review of Japan*, **64A**:367-387.
- Yano, K. (1993) Integration of spatial interaction models: towards general theory of spatial interaction. *Geographical Reports of Tokyo Metropolitan University*, **28**:33-78.
- Young, W.J. (1975) Distance decay values and shopping centre size. *Professional Geographer*, **17**:304-309.