Essays on Individual Decision Making under Risk and Uncertainty

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Abstract

This thesis aims to empirically test the validity of economic theories of the individual decision making under risk and uncertainty with a laboratory experiment. The first chapter outlines this thesis. The second chapter experimentally tests Manski's theory of satisficing (2017). He proposes solutions to two key questions: *when* should the decision-maker (DM) satisfice?; and *how* should the DM satisfice? The results show that some of Manski's proposition (those relating to the "*how*") appear to be empirically valid while others (those relating to the "*when*") are less so. The third chapter extends the findings from the previous chapter, mainly relating to "how to satisfice". I propose an alternative story with a different assumption of the subjects' preference functional and of the payoff distribution. The results suggest that my alternative story appears to better-explain the subjects' behaviour than that of Manski's story. The fourth chapter explores the individual behaviour towards randomisation of the choice. I use the elicitation method that provides an additional option between two alternatives, namely "I am not sure what to choose" as an alternative of two standard options: "I choose A" or "I choose B". It gives a consequence where the subjects' payoff is determined by a randomisation of two alternatives through the flipping a coin. I propose four stories to account for the choice of this option. The results show that the most of the subjects either have strictly convex preferences with random risk attitude or simply cannot distinguish the two alternatives. The fifth chapter empirically tests Nicolosi's model (2018). He derives the optimal strategy for the fund manager under a specific payment contract and the investment environment. I compare his model with other strategies. The results show that Nicolosi's model receives strong empirical support to explain the subjects' behaviour.

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Finally I personally present this thesis to my little family (Avita, Masha and Jay). They all are my life support. We have made it through this little step of our life. I wish we are always blessed throughout life.

Declaration

I, Yudistira Hendra Permana, declare that this thesis entitled "Essays on Individual Decision Making under Risk and Uncertainty" is a presentation of either my own or my joint work. Section 2 (pp. 5-30) is a joint work with my supervisor, Prof. John Hey, and my colleague, Nuttaporn Rochanahastin. The version presented in this thesis is the same version which has been published in *Theory and Decision* (doi: 10.1007/s11238-017-9600-5). Some slight amendments of notation and numbering are made in order to be consistent with the rest of the thesis.

The experimental interface in the Section 2 was programmed by Paolo Crosetto, while calculations and simulations are our own work. The experimental interface in the Section 4 and 5 was programmed by Alfa Ryano, while calculations and simulations are my own work.

I also confirm that:

- This work has not previously been presented for an award at this, or any other, University.
- Where I have consulted the published work of other, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made reference to that.

1 Preface

The hunt for the economic theories of the individual decision making under risk and uncertainty is a never-ending story. The vast-growing literature of this interest is unsurprising since most of our daily life has to deal with both environments, whether or not we can attach probability in some events. The traditional economic theory assumes that the decision maker (DM) is fully rational in ways that he or she always achieve optimality; to some extent, it leads to the theory of deterministic choice. This means that the DM fully maximises his or her cognitive ability, without any logical and arithmetical errors, to compare the choices in order to choose accordingly. Moreover, it has brought economics to become a study that provides a prediction of the individual choice. However, a large body of empirical evidence suggests that individuals often behave in ways that contradict the predictions of the theory. Allais Paradox perhaps is the famous example of such violation to the rational assumptions of the traditional economic theory. Instead of making decisions that could be predicted by the theory, people most of the time behave inconsistently to the prediction of the theory. Since then, economic theory often incorporates psychological and philosophical perspectives into the general assumptions, i.e. the concept of *bounded rationality* by Herbert Simon (1947). Simon maintained that the DM is bounded by “cognitive limits”, therefore in some situations, he or she does not optimise his or her choice but rather seeking something satisfactory.

The concept of bounded rationality gives economics a new perspective. It leads into explorations of the alternative to understand the individual behaviour under risk and uncertainty. One particular interest is the incorporation of the stochastic choice. In this case, decision-making is naturally noisy because of mistakes or errors that the DM makes when implementing his or her decisions. Let me give a sensible example of this. A subject takes a part in an experiment to weigh between two objects. In the first attempt, he or she is presented two objects without being told their weights: object 1 weighs 500 grams and object 2 weighs 300 grams. He or she may easily say object 1 is heavier than that of object 2. In the second attempt, he or she, again, is presented two objects without being told their weights: object 1 weighs 500 grams and object 2 weighs 505 grams. What is the likelihood for him or her to say that object 2 is higher than object 1? And what if we repeat this procedure with different weight of the objects? This example may be provoking, however, would this produce consistent answers from the subject with precise comparison?

This naturally drives us into the main issue: how we incorporate the noise into theories of deterministic choice. Some of the early works can be found in Davidson and Marschak (1957) and Debreu (1958). It then further has been explored by Hey and Orme (1994), Harless and Camerer (1994), and Loomes and Sugden (1995). The key issue of what it now refers to as *stochastic choice* is the appropriate stochastic specification given the individual true preference functional, hence one most-likely uses the econometric approach to make it operational.

So there are two important elements here: the identification of the true preference functional; and the identification of the noise. Although the development of descriptive theories under risk and uncertainty keeps going on, we perhaps should also discuss the specification of the stochastic structure.

The scope of this thesis is to provide the empirical testing of economic theories of the individual decision making under risk and uncertainty. The thesis consists of five sections, three of them involving experiments and one uses the data from another section to extend the analysis of that section.

Section 1 is preface. Section 2 tests the empirical validity of the theory of satisficing of Manski (2017) with a lab experiment. Satisficing is defined when the DM is satisfied with achieving some objective, rather than in obtaining the best outcome. Manski's model proposes solutions to two key questions: *when* should the DM satisfice; and *how* should the DM satisfice. Rather ironically, the DM should employ a satisficing strategy if it is optimal to satisfice. Manski envisages the DM being in an ambiguous situation and assumes that the *Minimax Regret* criterion is the objective function of the DM. We test his model experimentally, and implement ambiguity in the laboratory by using Stecher *et al*'s (2011) method to generate ambiguous numbers. The results show that some of Manski's proposition (those relating to the "how to satisfice") appear to be empirically valid while others (those relating to the "when to satisfice") are less so.

Building on the findings from Section 2, I explore further the subjects' decisions relating to "how to satisfice" in Section 3. In this section, I try to find a better explanation for the behaviour of the subjects in the experiment reported in Section 2 than that of Manski. This alternative story assumes that the DM is an Expected Utility maximiser, rather than the Minimax Regret agent, and that he or she perceives the payoffs as having a uniform risky distribution, rather than an ambiguous distribution; I call this the EU story. Given these alternative assumptions, one can derive the DM's optimal strategy; clearly, this depends upon the DM's risk aversion. I fit this EU story to the data from the experiment of Section 2 with two different stochastic specifications (of errors in the subjects' choices) beta and normal. I also fit the data using Manski's optimal strategy under both stochastic specifications. The results show that the EU story appears to be a better explanation of the data than that of Manski's story. Interestingly, the subjects appear to be risk-loving under the EU story.

In Section 4, I explore the individual behaviour towards randomisation of the choice. I use the elicitation method that provides an additional option between two alternatives. I label this option as "I am not sure what to choose" as an alternative of two standard options: "I choose option A" or "I choose option B". The implication for making this choice is that if the subjects choose this option, then their payoff is determined by a randomisation of A and B through the flipping a coin. I ask the question as to why subjects might choose this option, that is, "why do subjects prefer randomisation?", and explore four distinct stories to explain the subjects' behaviour. The *first story* is that the DM has strictly convex preferences (which vary randomly from problem to problem) and actually prefers a mixture of A and B. The *second* is that the DM prefers a mixture of A and B only if it gives the highest utility, but he may tremble in expressing his preference. The *third* is that the DM cannot distinguish between A and B unless their difference exceeds some threshold. The *fourth* is that the DM actually prefers to delegate the choice (to the coin), shifting the 'responsibility' to the coin, though the DM may tremble in expressing his preference. I compare the goodness-of-fit between the stories to see which story better explains the data. The results show that the first and third stories have the most empirical support.

In Section 5, I examines the empirical validity of Nicolosi's model (2018) which investigates the optimal strategy for a hedge fund manager under a specific payment contract. The contract specifies that the manager's payment consists of a fixed payment and a variable payment, which is based on the over-performance with respect to a pre-specified benchmark. The model assumes that the manager is an Expected Utility agent who maximises his expected utility by buying and selling the asset at appropriate moments. Nicolosi derives the optimal strategy for the manager. To find this, Nicolosi assumes a Black-Scholes setting where the manager can invest either in an asset or in a money account. The asset price follows geometric Brownian motion and the money account has a constant interest rate. I experimentally test Nicolosi's model. The results show that Nicolosi's model receives strong empirical support to explain the subjects' choice, though not most of the subjects follow Nicolosi's model. Despite this, the subjects somehow follow the intuitive prediction of Nicolosi's model where the subjects respond to the difference between their managed portfolio and the benchmark to determine their portfolio allocation.

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2 When and How to Satisfice: An Experimental Investigation

Abstract — This section is about satisficing behaviour. Rather tautologically, this is when decision-makers are satisfied with achieving some objective, rather than in obtaining the best outcome. The term was coined by Simon (1955), and has stimulated many discussions and theories. Prominent amongst these theories are models of incomplete preferences, models of behaviour under ambiguity, theories of rational inattention, and search theories. Most of these, however, seem to lack an answer to at least one of two key questions: *when* should the decision-maker (DM) satisfice; and *how* should the DM satisfice. In a sense, search models answer the latter question (in that the theory tells the DM when to stop searching), but not the former; moreover, usually the question as to whether any search at all is justified is left to a footnote. A recent paper by Manski (2017) fills the gaps in the literature and answers the questions: when and how to satisfice? He achieves this by setting the decision problem in an ambiguous situation (so that probabilities do not exist, and many preference functionals can therefore not be applied) and by using the MiniMax Regret criterion as the preference functional. The results are simple and intuitive. This section reports on an experimental test of his theory. The results show that some of his propositions (those relating to the 'how') appear to be empirically valid while others (those relating to the 'when') are less so.

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2.1 Introduction

This section is about satisficing behaviour. Way back in 1955 Herbert Simon made a call for a new kind of economics stating that:

> "the task is to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environment in which such organisms exist." (p. 99)

There is a fundamental conflict here provoked by the use of the word 'rational', and economists' obsession with it. The problem is that the expression 'rational behaviour' covers virtually all forms of behaviour, as long as it is motivated by some 'rational' objective function, and the decision-maker has all relevant information available to him or to her, and the decision-maker (henceforth, DM) can perform all the necessary calculations costlessly. If calculations are costly, then we are led into the infinite regression problem, first pointed out by Conlisk (1996), and rational behaviour, as defined by economists, cannot exist. We are, therefore, constrained to operate with rational models, defined as above. The way forward, within the economics paradigm, is therefore to weaken our ideas of what we mean by rational behaviour. This is the way that economics has been moving. Prominent amongst these latter weaker theories are theories of incomplete preferences (Ok *et al.* 2012; Nau 2006; Mandler 2005; Dubra *et al.* 2004); theories of behaviour under ambiguity (Etner *et al.* 2012; Gajdos *et al.* 2008; Ghirardato *et al.* 2004; Hayashi and Wada 2010; Klibanoff *et al.* 2005; Schmeidler 1989; Siniscalchi 2009); theories of rational inattention (Sims 2003; Manzini and Mariotti 2014; Matejka and McKay 2015; Caplin and Dean 2015); and search theories (Masatlioglu and Nakajima 2013; McCall 1970; Morgan and Manning 1985; Stigler 1961). A useful survey of satisficing choice procedures can be found in Papi (2012).

Almost definitionally, *models of incomplete preferences* have to be concerned with satisficing: if the DM does not know his or her preferences, it is clearly impossible to find the best action. These models effectively impose satisficing as the only possible strategy. The problem here is that complete predictions of behaviour must also be impossible. Prediction is possible in *models of behaviour under ambiguity*. But here again satisficing behaviour 'must' occur, if only because not all the relevant information is available to the DM. Unless the DM's information is objectively correct, there is presumably always some action that is better than the one chosen by the DM. But here the DM does not choose to satisfice; nor does he or she choose how to satisfice. *Models of rational inattention* also capture the idea of 'satisficing' behaviour — in that choice is made from a subset of the set of possible actions — those which capture the attention of the DM, that is, those which are in the consideration set of the DM. However, these theories are silent on the reasons for the formation of a consideration set, and, in some of them, on how the consideration set is formed.

We examine a new theory — that of Manski (2017) — which might be classified as an extended search model. *Search models* seem to be closest to the scenario in which Manski's paper is set. Standard search models assume that the DM is searching for the highest number in some distribution and that there is a cost of obtaining a drawing from that distribution. Because of this cost, the DM does not keep on searching until he or she finds the highest number: generally he or she should keep on searching until a 'sufficiently' high number is found. This could be termed the DM's *aspiration level*. One interpretation of Manski's paper is that he generalises the story: in addition to being able to search for numbers greater than some (or several) aspiration level(s), the DM can pay a higher search cost and be able to find the highest number, and also the DM can choose not to indulge in any search and simply receive a lower number. Manski not only considers choice between these three strategies, but also the choice of the aspiration level(s). This is the 'how' of Manski's theory: he explains how many times satisficing should be implemented, how aspiration levels should be formed and how they should be changed in the light of the information received.[1]

We experimentally test this new theory. Some of the other models that we have discussed have also been tested experimentally; for incomplete preferences we refer the reader to Cettolin and Riedl (2019), Costa-Gomes *et al.* (2014) and Danan and Ziegelmeyer (2006); for behaviour under ambiguity to Abdellaoui *et al.* (2011), Ahn *et al.* (2010), Halevy (2007), Hey and Pace (2014) and Hey *et al.* (2010); for rational inattention to Chetty *et al.* (2009), De Los Santos *et al.* (2012); and for search theories to Caplin *et al.* (2011), De Los Santos *et al.* (2012), Hayashi and Wada (2010) and Reutskaja *et al.* (2011). Our experimental test has some similarities in common with some of these and some differences. In some ways our test is closest to that of Hayashi and Wada (2010), though they test minimax, α-maximin and the (linear) contraction model (Gajdos *et al.* 2008). We test Manski's model and have a different way of generating imprecise information/ambiguity.

In the next section we describe the Manski model, while in Section 2.3 we discuss the experimental design. Our results are in Section 2.4, and Section 2.5 concludes.

[1] There are echoes of this in Selten (1998), though he notes on page 201 that "In this respect, the role of aspiration levels in [Selten's] model is different from that in the satisficing processes described by Simon, where it is assumed that it can be immediately seen whether an alternative satisfies the aspiration level or not. The situation of the decision maker in [Selten's] model is different."

2.2 Manski's model of satisficing

In the model, the DM has to choose some *action*. The DM knows that there is a set of actions, each member of the set implying some payoff. The payoffs of these actions are bounded between a lower bound, L, and an upper bound, U, which are known to the DM. Hence, without costly deliberation, the DM faces a problem under ambiguity as he or she does not have sufficient knowledge to determine the optimal decision — that of choosing the action which yields the highest payoff. However, the DM can learn more about the payoff values subject to different costs, which in turn, yield different benefits. There are three available deliberation strategies: 'No Deliberation', 'Satisficing', and 'Optimising'. 'No Deliberation' incurs no cost and yields only the value of the payoff of an arbitrarily chosen action. 'Optimising' has a positive cost (K) and reveals the maximum payoff value. 'Satisficing' has a positive cost (k) and provides information whether there are actions that are at least as large as some specified aspiration level.

Crucial to the model is that the assumed objective of the DM is the minimisation of maximum regret (MMR). One reason for this is that there is no known probability distribution of the payoffs, so, for example Expected Utility theory and its various generalisations cannot be applied.[2] Additionally, and crucially for our experiment, the solution is an *ex ante* solution, saying what the DM *should* plan to do as viewed from the beginning of the problem. As Manski writes "I study *ex ante* MiniMax-Regret (MMR) decision making with commitment". So the DM is perceived of as choosing a strategy at the beginning of the problem, and then implementing it. This implies a *resolute* decision-maker. If the DM is not resolute the solution may not be applicable.

The paper applies the *ex ante* MiniMax-Regret rule to this environment and derives a set of simple, yet intuitive, decision criteria for both the static and the dynamic choice situation. Simon (1955) also suggested that there can be a *sequence* of deliberations/satisficing where the DM adjusts his or her aspiration level in the light of information discovered. Hence, the dynamic choice situation is of particular interest. Manski's theory (in his Proposition 2) is that:

1. The optimal (maximum number of rounds of deliberation (M^*) if the DM uses a satisficing strategy is given by:

$$M^* = int\left[\frac{\log\left(\frac{U-L}{k}\right)}{\log(2)}\right]$$

2. If the DM uses a satisficing strategy, the DM sets the aspiration level t_m in the m'th

[2] Manski notes that "The maximin criterion gives the uninteresting result that the person should always choose the null option when deliberation is costly."

round of satisficing as follows:

$$t_m = \frac{L_m + U_m}{2}$$

Here t_m denotes the aspiration level in round m and L_m and U_m are the lower and upper bounds on the payoffs given what the DM has observed up to round m.

3. (a) Optimisation is an MMR decision if

$$K \leq U - L \quad and \quad K \geq kM^* + \tfrac{U-L}{2^{M^*}}$$

(b) Satisficing with M^* and t_m $(m = 1, \ldots, M^*)$ is an MMR decision if

$$k \leq \tfrac{U-L}{2} \quad and \quad K \geq kM^* + \tfrac{U-L}{2^{M^*}}$$

(c) No Deliberation is an MMR decision if

$$k \geq \tfrac{U-L}{2} \quad and \quad K \geq U - L$$

The intuition of the theory is simple. Deliberation costs play a central role. 'Optimising' or 'Satisficing' will be the decision if their respective associated cost (K, k) is low enough. If both costs are sufficiently large then 'No Deliberation' will be preferred. If 'Satisficing' is chosen, the aspiration level is midway between the relevant lower bound and the relevant upper bound, while the number of deliberation rounds is decreasing in its associated cost. This theory is different from the existing search literature in that it provides the concept of satisficing search that follows more closely Simon's perception of adaptive aspiration levels than standard search models. It clearly states *when* the DM should satisfice. It also provides a solution to the choice of aspiration levels.

Before we move on to the experiment, let us briefly translate the above theory into a description of behaviour. The DM starts with knowing that there is a set of payoffs (the number of them is unknown) lying between some lower bound L and some upper bound U. The DM is told the values of k and K. The first thing that the DM needs to do is to design a *strategy*. This depends on the values of k and K. If these are sufficiently large (see 3c above), the DM decides not to incur these costs and chooses 'No Deliberation'. The DM is then told and given the payoff of the first action in the choice set, and that is the end of the story.

If K is sufficiently small (see 3a above) the DM decides to incur this cost and 'Optimise' and hence learn the highest payoff. He or she gets paid the highest payoff minus K, and that is the end of the story.

The interesting case is 3b, where k is sufficiently small and K is sufficiently large. The DM then decides to satisfice with (a maximum[3] of) M^* rounds (as given by 1 above)[4] of satisficing. In each of these M^* rounds, the DM sets an aspiration level, pays k and is told at the end of the round whether or not there are payoffs greater than or equal to the stated aspiration level. More precisely, the DM is told whether there are 0, 1 or more than 1 payoffs greater than or equal to the stated aspiration level. The DM then updates his or her views about the lower and upper bounds on the payoffs in the light of the information received. This updating procedure is simple:

- If there are ***no*** payoffs greater than aspiration level t_m then $L_{m+1} = L_m$ and $U_{m+1} = t_m$
- If there ***are*** payoffs greater than aspiration level t_m then $L_{m+1} = t_m$ and $U_{m+1} = U_m$,

where L_m and U_m are the lower and upper bounds after m rounds of satisficing.

When at most M^* rounds have been completed, the DM gets paid the payoff of the first action in the range between his or her current lower bound and the current upper bound minus kM (the costs of deliberation), where M is the actual number of rounds of satisficing implemented ($M \leq M^*$).

This section reports on an experiment to test the theory. We test whether subjects choose between 'No Deliberation', 'Satisficing' and 'Optimising' correctly (as in (3) above). We also test, when subjects choose to satisfice, whether they choose the correct number of rounds of satisficing (as in (1) above), and whether aspiration levels are chosen correctly (as in (2) above).

2.3 Experimental design

The actual experimental design differs in certain respects from the design of the theory. First, we told subjects that if they implemented 'No Deliberation' they would be paid the *lowest* payoff in the choice set, rather than the payoff of the first-ordered element of the choice set. Second, we only told subjects, when they chose to satisfice with an aspiration level t, whether there were or were not payoffs greater than or equal to t, and not whether there were 0, 1 or more than 1. Moreover, if after satisficing for m rounds, and discovering that there were payoffs in a set $[L_m, U_m]$, if they chose 'No (further) Deliberation' at that point they would get a payoff equal to the *lowest* payoff in the set $[L_m, U_m]$ minus mk. These differences do not change the predictions of the theory in that an MMR decision-

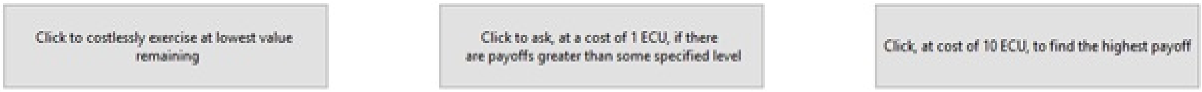
[3]Depending on what the DM learns. He or she may not implement all M^* rounds.

[4]After these M^* rounds, the DM should choose 'No Deliberation'. Subjects were informed about that.

maker will always assume that the first element is the lowest element. Additionally, the *ex ante* choice of M^* remains the same.

Let us give an example (which was included in the Instructions to the subjects). To make this example clear, we need to introduce some notation: the variable *lvgeal* is defined as the *lowest payoff greater than or equal to the highest aspiration level for which there are payoffs greater than or equal to the aspiration level.*

On the screen (see the screenshot below) there were three buttons



The one on the left corresponds to 'No Deliberation', the one in the middle to 'Satisfice' and the one on the right to 'Optimise'. In this example $k = 1$ and $K = 10$.

Suppose—**though the DM does not know this and our subjects were not told this**—that the payoffs are 55 18 75 19 9

If the DM clicks on the left-hand button straight away the income would be 9 (the lowest payoff).

If the DM clicks on the right-hand button straight away the income would be 65 (the highest payoff, 75, minus K).

If the DM clicks on the middle button and specifies an aspiration level of 40, he or she would be told that there *are* payoffs greater than this, but would not be told how many nor what they are. The software would, however, note that the lowest payoff greater than or equal to 40 is 55. This would be the *lvgeal* defined above. If the DM clicked on the left-hand button at this stage his or her income would be 54 (*lvgeal* minus k). After this first round of satisficing the DM's L_1 and U_1 are 40 and 100, respectively.

If the DM now clicks on the middle button again and now specifies an aspiration level of 70, he or she would be told that there *are* payoffs greater than this, but would not be told how many nor what they are. The software would, however, note that the lowest payoff greater than or equal to 70 is 75. This would become the *lvgeal*. If the DM clicks on the left-hand button at this stage the income for this problem would be 73 (*lvgeal* minus $2k$). After this second round of satisficing the DM's L_2 and U_2 are 70 and 100, respectively.

If the DM now clicks on the middle button a third time, and now specifies an aspiration level of 80, he or she would be told that there are *no* payoffs greater than this. The software would, however, keep the *lvgeal*, 75, in memory. If the DM clicks on the left-hand button at this stage the income for this problem would be 72 (*lvgeal* minus $3k$). After this third round of satisficing the DM's L_3 and U_3 are 70 and 80, respectively.

Subjects could keep on clicking on the middle button as often as they wanted, but they were told that the cost would be deducted from the payoff each time.

Note that in this particular case, it is better to click on the middle button twice (with aspiration levels of 40 and 70) and then on the left-hand button, rather than to click on either the left-hand button or the right-hand button straight away, and better than to click on the middle button one or three times (with aspiration levels of 40, 70 and 80) and then on the left-hand button. *But this is not always the case.*

In the experiment, 48 subjects were sequentially presented with 100 problems on the computer screen, all of the same type. They were given written Instructions and then shown a PowerPoint presentation of the instruction before going on to the main experiment. Subjects were informed of the lower (L) and upper (U) bounds on the payoffs in each problem; these were fixed at 1 and 100, respectively. They were also told the two types of cost: the cost of finding out whether there are any payoffs greater or equal to some specified aspiration level (k) and the cost of finding the highest payoff (K). The number of payoffs (N) was fixed at 5, though subjects were not given this information.[5] We used the procedure in Stecher *et al.* (2011) to generate the ambiguous distributed payoffs. This procedure creates complete ambiguity for subjects as they have no way to put any probabilities on the payoffs. To make this clear to the subjects we inserted Figures 2.5.1 and 2.5.2, which can be found in the Appendix B.2, in the Instructions. Each of them contains 49 distributions, each of 10,000 replications. In the Figure 2.5.1 in the Appendix B.3, the drawings were from a uniform distribution over the entire range, while in the Figure 2.5.2, the drawings were from an ambiguous distribution as derived using the Stecher *et al.* (2011) method. It will be seen that all the distributions in Figure 2.5.1 are approximately uniform, while those in Figure 2.5.2 are all completely different. We told the subjects that "this means that one cannot attach probabilities to each of the numbers coming up. Probabilities are undefined."

We ran two different treatments, Treatment 1 and Treatment 2. In each of these subjects were presented with 100 problems. In Treatment 1, we had four different values for k and K (with N, L and U were fixed across the 100 problems); and we gave the subjects these 4 problems in 4 blocks of 25, with the order of the blocks randomised across subjects. In Treatment 2, we had 100 different values for k and K in each of the 100 problems and presented the problems in a randomised order (again with N, L and U fixed across the 100 problems). Figure 2.3.1 illustrates. Figure 2.3.2 shows the predictions of the theory.

All 48 subjects completed the experiment which was conducted in the EXEC Lab at the University of York. Subjects' ages ranged from 18 to 44 years. Educational backgrounds

[5]This is not relevant to the theory.

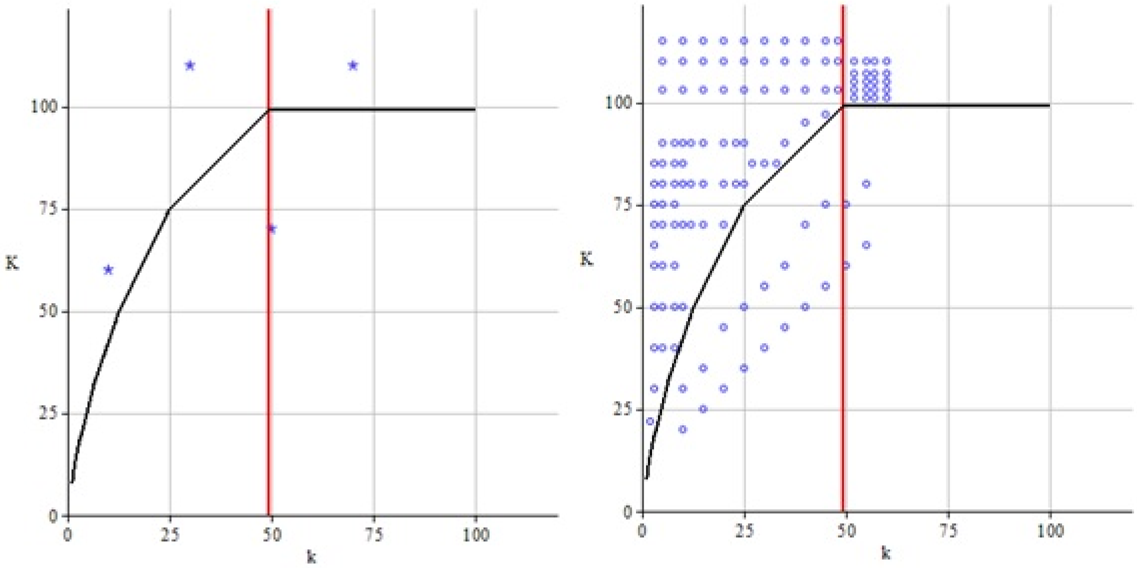


Figure 2.3.1: Sets of k and K for Treatment 1 (*left*) and Treatment 2 (*right*) plotted in the parameter space

were: high school graduate or equivalent (9 subjects); college credit (8); bachelor degree (19); master degree (11); and professional degree (1). 46 subjects reported themselves as a student (8 subjects in a bachelor degree, 9 subjects in a master degree and 11 subjects in doctoral degree); one subject was a member of staff at the University of York; one subject did not report his or her current degree/position. Subjects' ethnicities were mainly White (26 subjects) while 18 were Asian/Pacific Islander, 3 were Black or African American and 1 other. There were only 5 subjects who had any work experience related to finance or economics, but most of them (34 subjects) had previously participated in an economics experiment.

To be a fair test of the theory, we need to give incentives to the subjects to act in accordance with it. We should repeat the fact that the theory is an *ex ante* theory: it tells DMs what to do as viewed from the beginning of a problem; it assumes commitment. Clearly, given the nature of the experiment, we cannot observe what the subjects planned *ex ante*, nor can we check whether they implemented their plan. All we can observe is what they did, so we are testing the theory in its entirety — meaning the validity of *all* its assumptions.[6] *Ex ante* the objective of the theory is to minimise the maximum regret. *Ex ante* Regret is the difference between the maximum possible income and their actual income. The maximum possible value of the former is exogenous — it depends upon the

[6] An alternative design would be to ask subjects to state a plan and then *we* implement it. But 'stating a plan' is not straightforward — not only would subjects have to state whether they want to have 'No Deliberation', 'Optimise' or 'Satisfice', they would also have to specify their rules for choosing their aspiration levels. Asking subjects to do this would be immeasurably more difficult than asking them to play out the problems. We expand on this in our conclusions.

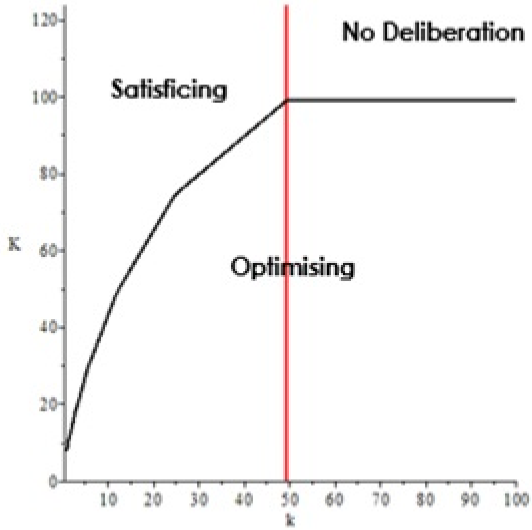


Figure 2.3.2: Partition of the parameter space into areas corresponding to the theoretical predictions

problem which in our case is always 100 *ex ante*. So minimising the *ex ante* maximum regret is achieved by maximising their income. So we paid them their (average[7]) income.

The subjects' payment from the experiment was their average income from all 100 problems plus the show-up fee of £2.50. Average income was expressed in Experimental Currency Units (ECU). Each ECU was worth 33¹/₃*p*; that is 3 ECU was equivalent to £1. They filled in a brief questionnaire after completing all problems on the computer screen, were paid, signed a receipt and were free to go. The average payment was £13.05. This experiment was run using purpose-written software written (mainly by Paolo Crosetto) in *Python* 2.7.

2.4 Results and analyses

The purpose of the experiment was to test Proposition 2 of Manski (2017) as stated in Section 1.3. First, we compare the actual and theoretical decisions for all subjects and in each treatment. Second, we compare the actual and theoretical predictions for income and regret. Third, we analyse the number of rounds of satisficing by comparing the theoretical and actual number for all subjects and both treatments. Finally, we analyse the subjects' actual aspiration levels and compare them with those of the theory.

[7] If subjects are maximising their income on each problem, they are maximising their average income, and *vice versa*, as problems are independent.

Table 2.1: Actual vs theoretical decisions for all the subjects

	Subjects' choices			
	No deliberation	Satisfice	Optimise	Totals
Manski's theory				
No deliberation	*717* *(85.36%)*	98 (11.67%)	25 (2.98%)	840 (17.5%)
Satisfice	1,079 (34.58%)	*1,895* *(60.74%)*	146 (4.68%)	3,120 (65%)
Optimise	598 (71.19%)	161 (19.17%)	*81* *(9.64%)*	840 (17.5%)
Totals	2,394 (49.88%)	2,154 (44.88%)	252 (5.25%)	

2.4.1 When to satisfice

Our experiment gives us 4,800 decisions (between 'No Deliberation', 'Satisficing' and 'Optimising') across 48 subjects and 100 problems. Table 2.1 gives a comparison of the actual and the theoretical decisions; here the main diagonal indicates where subjects followed the theoretical prediction. From this table it can be seen that 2,693 out of the 4,800 decisions (56.10%) are in agreement with the theoretical predictions. The number of theoretical predictions for each strategy can be found at the end of each row while the total number of subjects' decisions can be found at the bottom of each column. Subjects appear to choose 'No Deliberation' significantly more than the theoretical prediction (49.88% compared with 17.50%). Comparing Treatment 1 with Treatment 2 shows that Treatment 2 is closer to the Manski optimal than Treatment 1; 1,476 out of 2,400 actual decisions (61.50%) match with the theoretical in Treatment 2 compared to 1,217 out of 2,400 actual decisions (50.71%) in Treatment 1.[8]

In Table 2.2 we compare the actual and theoretical average income and average regret. Obviously, it must be the case that actual regret is higher than the theoretical regret (as subjects were not always following the theory). Subjects also have a higher average income. This suggests that subjects may have been working with a different objective function,[9] or making some assumption about the distribution of the payoffs that was not true.[10] Comparing the two treatments, we see that subjects in Treatment 2 have relatively better results in terms of the average income (33.40 ECU to 30.10 ECU) and regret (95.20 ECU to 121.10 ECU) than in Treatment 1. This is interesting, as the idea of Treatment

[8]Tables reporting for Treatment 1 and Treatment 2 can be found in the Appendix B.1.
[9]For example, maximising Expected Utility.
[10]For example, assuming that the distribution was uniform.

1 (where each problem was repeated 25 times) was to give subjects a chance to learn; we had expected performance to be better there. Perhaps they learnt about the 'distribution' of payoffs and therefore departed from the theory?

Table 2.2: Actual average vs theoretical average for income and regret

	Average income and regret	
	Theoretical	Actual
All subjects		
Income	24.30	31.80
Regret	65.70	108.20
Treatment 1		
Income	21.60	30.10
Regret	72.70	121.10
Treatment 2		
Income	270	33.40
Regret	58.70	95.20

2.4.2 How to satisfice

Table 2.3 compares the theoretical (maximum[11]) and the actual number of rounds of satisficing (obviously restricted to the cases where they actually satisficed). There are 452 problems out of 3,120 problems (14.49%), where the subjects should satisfice, and where they choose the same number of rounds of deliberation as the theoretical prediction. The difference between treatments is small: 16.67 and 11.89% matches of theoretical and actual number of rounds of deliberation, for treatments 1 and 2 respectively. Generally they choose fewer rounds of satisficing than the theory predicts.[12]

[11] Note that if subjects were following the theory with our design, the actual number of rounds of satisficing would be equal to the M^*, while in the theory the actual number of rounds of satisficing could be less than M^* (because they would stop satisficing if they discovered the highest payoff).

[12] This is not a consequence of our experimental design which encourages subjects to choose the maximum number of rounds of satisficing. Indeed with the theory we might observe numbers below the theoretical maximum.

Table 2.3: Actual vs theoretical number rounds of satisficing

M	Actual number round of satisficing											
	0	1	2	3	4	5	6	7	8	9	11	Totals
Manski's theory												
0	*1,448*	200	19	8	1	2	1	0	0	0	1	1,680
1	852	*312*	46	8	4	0	0	1	0	1	0	1,224
2	132	163	*34*	7	0	0	0	0	0	0	0	336
3	190	532	248	*69*	13	4	0	0	0	0	0	1,056
4	19	89	85	38	*27*	4	1	0	1	0	0	264
5	18	71	67	44	26	*10*	1	2	0	1	0	240
Totals	2,659	1,367	499	174	71	20	*3*	3	1	2	1	4,800

Figure 2.4.1 shows a plot of actual vs theoretical aspiration levels for all subjects (and separately for those in Treatments 1 and 2) where the subjects chose to satisfice.[13] We calculate the theoretical aspiration level based on the relevant lower and upper bounds at the time of choosing satisficing. The 45° line shows what subjects should do if they select their aspiration level according to the theory. The figure shows that subjects' aspiration levels increase with the theoretical levels, although the mean equality test shows a rejection of equal means between the actual and theoretical aspiration levels when subjects do satisficing (t-test $= 15.19$, $p = 0.000$) for all the subjects. Doing this analysis for each treatment separately shows the same result.

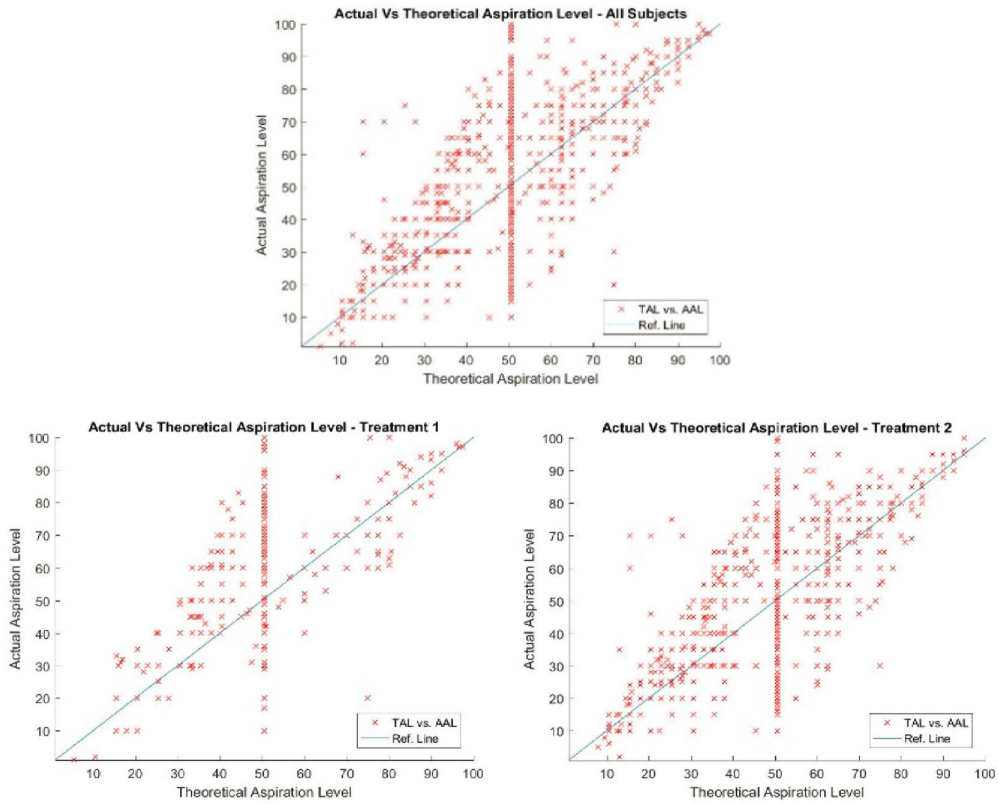


Figure 2.4.1: Actual vs theoretical aspiration levels

We now investigate more closely whether subjects set their aspiration level as the theory predicts: equal to the mid-point between the relevant upper and lower bounds. We report below regressions of the actual aspiration level against the optimal level. If the theory holds, the intercept should be zero and the slope should be equal to 1. We omit observations where the aspiration level was above the upper bound (see footnote 8), and accordingly, carry out truncated regressions. Before we proceed to the regressions, we note that the correlations between the actual and theoretical aspiration level 0.544 over

[13]We exclude the few outliers when the subjects put their aspiration level above 100. There were 39 (1.2%) out of 3,347 cases where this happened.

all subjects, 0.513 for Treatment 1 and 0.569 for Treatment 2.

Table 2.4 shows that the coefficient on the theoretical aspiration level is not significantly different from 1 in Model 1. However, in Model 1 we have included a constant term which should not be there; unfortunately, it is significantly different from 0, which should not be. If we remove the constant term to get Model 2, we find that the slope coefficient is almost significantly different from 1. So Table 2.4 tells us that the subjects are almost but not quite following the Manski's rule.

Table 2.4: Regressions of the actual aspiration level on the theoretical aspiration level for all subjects

	Model 1	Model 2
Theoretical aspiration level	0.994 (0.0208)	1.144 (0.0071)
Constant	7.662* (1.035)	
Observations	3,308	3,308
Wald Chi^2	2,273.52	25,592,94

* Significance at 1% against the null that the true is 1.0 or 0.0 as appropriate.

Table 2.5: Regressions of the actual aspiration level on the lower and upper bounds for all subjects

	Model 1	Model 2
Lower bound	0.439* (0.0156)	0.441* (0.0158)
Upper bound	0.546* (0.0153)	0.583* (0.0042)
Constant	3.489* (1.315)	
Observations	3,308	3,308
Wald Chi^2	2,457.69	32,335.35
Likelihood ratio	710.82	2,113.65

* Significance at 1% against the null that the true is 0.5 or 0.0 as appropriate.

We break down the analysis of Table 2.4 by treatments. The results are similar for Model 1 in both treatments. In Model 2, we find that the slope coefficient is significantly different from 1 in both treatments.

We now delve deeper and try to understand how the actual aspiration levels are determined, and in particular, how they are related to the upper and lower bounds. We present below regressions of the subjects' aspiration level as a function of these bounds. If following the theory the relationship should be $AL_{im} = 0.5L_{im} + 0.5U_{im}$ (where AL_{im} is subject

i's aspiration level in round m of satisficing and L_{im} and U_{im} are the relevant lower and upper bounds). As before, we have excluded outliers (aspiration levels greater than the upper bound) from the regression and performed truncated regressions.

Table 2.5, over all the subjects, shows that the estimated parameters on the bounds are significantly different from the theoretical value of 0.5 and that the subjects put more weight on the upper bound and less on the lower bound when they select their aspiration levels.

If we break down the analysis of Table 2.5 by treatments, we see some differences between them. In Treatment 1 the estimated parameters are significantly different from the theoretical 0.5 (with more weight put on the upper bound than the lower), while in Treatment 2 they are much closer (and indeed only significantly different from 0.5 for one estimated parameter). So in Treatment 2, the subjects are closer to the theory in this respect than in Treatment 1. This confirms an earlier result. Possibly it was a consequence of the fact that in Treatment 2 each problem was an entirely new one, while in Treatment 1 (where 4 problems were given in blocks of 25) subjects were 'learning' about the distribution of payoffs[14] and thus departing from the theory: as the subjects were working through the 25 problems they felt that they were getting some information about the 'distribution'.

2.5 Conclusions

The overall conclusion must be that subjects were not following the part of the theory regarding the 'when' question: the choice between 'No Deliberation', 'Satisficing' and 'Optimising', possibly as a consequence of our experimental design.[15] However, the choice of the number of rounds of satisficing is closer to the theory. The first of these is a particularly difficult task and the second slightly less difficult, and, therefore, these results may not be surprising. In addition, subjects may have experienced difficulties in understanding what was meant by an ambiguous distribution. However, when it comes to the choice of the aspiration levels, subjects are generally close to (though sometimes statistically significant from) the optimal choice of $(L+U)/2$. This latter task is easier and more intuitive. So it seems that the 'when' part of the theory is not empirically validated, while part of the 'how' part receives more empirical support.

One serious problem with our experimental test (which we have already mentioned) is that the theory is an *ex ante* theory, and one with commitment (so the DM is resolute),

[14]This raises an interesting theoretical point: if we observe 25 repetitions of an ambiguous process, can we learn about it?

[15]Though we should re-iterate that, even though our design differs from that of the theory, the theoretical predictions should be the same.

while our experimental test involves observing what subjects actually do. A full *ex ante* test is difficult as we would have to ask subjects to specify, not only their choice of deliberation strategy, but also their choice of conditional aspiration levels. Perhaps we could go part-way there by getting the computer to implement some stated aspiration levels, telling subjects the computer algorithm, and asking subjects simply to choose between 'No Deliberation', 'Satisficing' and 'Optimisation'. This would be a partial test — one answering only the 'when' of the title. Other variations are possible, but all appear to be difficult.

Let us restate that the theory is an *ex ante* theory and one with commitment: the DM is committed to his or her *ex ante* plan and implements it resolutely. The theoretical predictions may be different if the DM is not resolute. Let us illustrate this with the choice of M^*. At the beginning of the problem the DM calculates M^* — which depends on L and U at the beginning. After m rounds of satisficing the DM will have updated lower and upper bounds. Suppose he or she re-calculates the relevant M^* — call this M^*_m. Will it be true that M^*_m is equal to $M^* - m$? We see no reason why that should be so — it depends upon the information that the DM has acquired. So it seems perfectly reasonable that a DM should revise his or her plan as he or she works through a problem. But then this is not the optimal way to solve the problem even if the DM is a MMR agent — backward induction should be employed. Perhaps this is what our subjects were doing?

In conclusion, we should note that there are three crucial elements to the theory: the use of the MMR preference functional, commitment and the perception of the payoffs as having an ambiguous 'distribution'. The violation of any of these would lead to a breakdown of the theory. We tried to ensure that subjects perceived the 'distribution' as being ambiguous in our experiment. We tried to incentivise the use of the MMR preference functional by our payment rule, but the subjects could well have had a different objective function.[16] Unfortunately, it seems difficult to force commitment on the subjects, and they may well have been revising their strategy as they were working through a problem. Nevertheless, subjects seem to have been following the theory in at least one key respect — the choice of their aspiration levels.

[16] For example they could have been Expected Utility maximisers operating under the (wrong) assumption that the distributions were uniform.

Appendices

Appendix B.1 — Actual vs theoretical decisions

Table 2.6: Actual vs theoretical decisions in Treatment 1

		Subjects' choice			
		No Deliberation	Satisfice	Optimise	Totals
Manski's theory	No Deliberation	524 (87.33%)	64 (10.67%)	12 (2.00%)	600
	Satisfice	522 (43.50%)	645 (53.75%)	33 (2.75%)	1,200
	Optimise	452 (75.33%)	100 (16.67%)	48 (8.00%)	600
	Totals	1,498 (62.42%)	809 (33.71%)	93 (3.88%)	*2,400*

Note: the number in parentheses indicates the percentage by row

Table 2.7: Actual vs theoretical decisions in Treatment 2

		Subjects' choice			
		No Deliberation	Satisfice	Optimise	Totals
Manski's theory	No Deliberation	193 (80.42%)	34 (14.17%)	13 (5.42%)	240
	Satisfice	557 (29.01%)	1,250 (65.10%)	113 (5.89%)	1,920
	Optimise	146 (60.83%)	61 (25.42%)	33 (13.75%)	240
	Totals	896 (37.33%)	1,345 (56.04%)	159 (6.63%)	*2,400*

Note: the number in parentheses indicates the percentage by row

Appendix B.2 — Uniform risky and ambiguous distributions

Figure 2.5.1 and Figure 2.5.2 show uniform risky and ambiguous distributions respectively. Each of them contains 49 distributions, each of 10,000 replications. Figure 2.5.1 and Figure 2.5.2 are Figure 1 and Figure 2 respectively in the Instructions shown to the subjects.

Figure 2.5.1: Uniform risky distributions

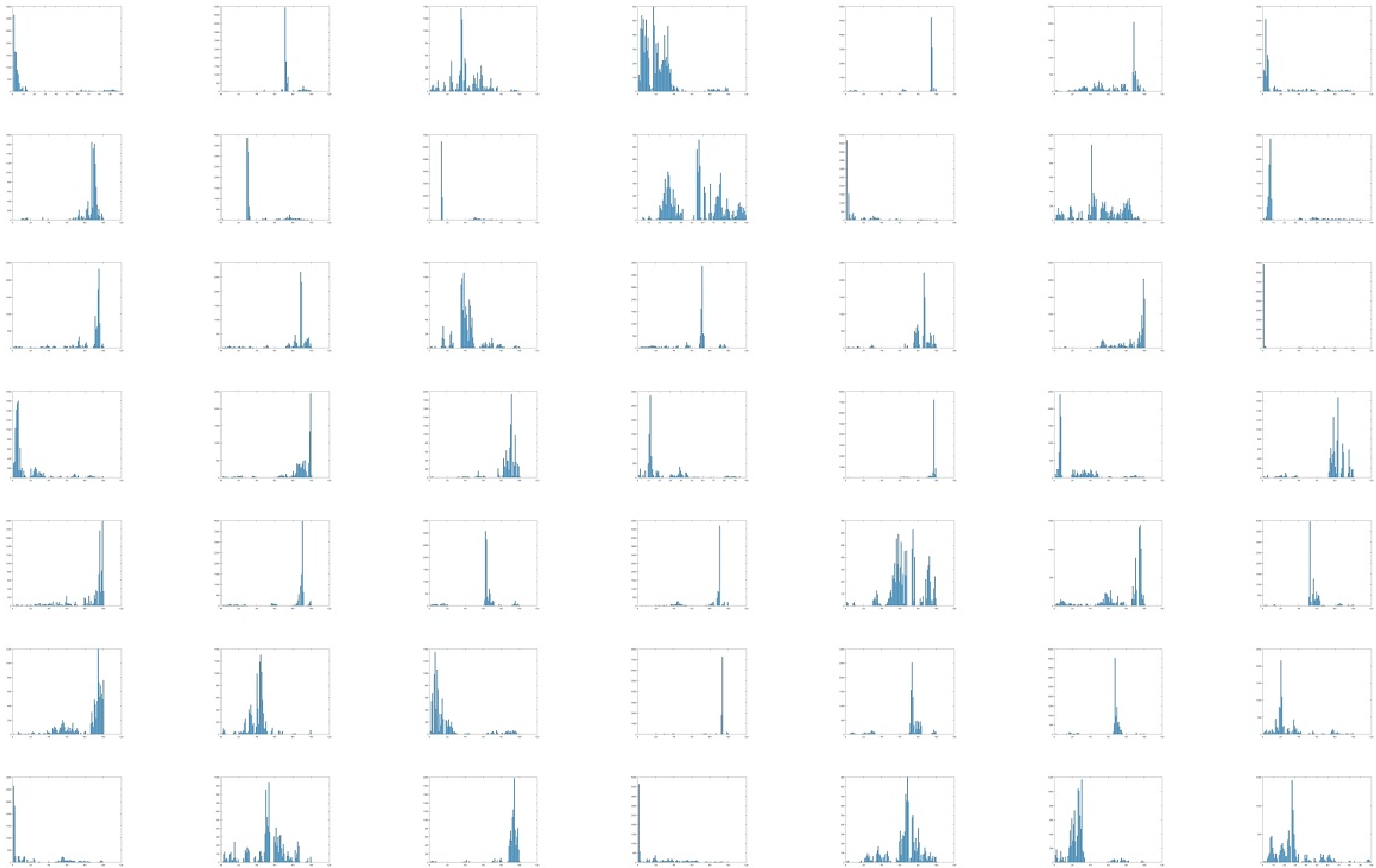


Figure 2.5.2: Ambiguous distributions

Appendix B.3 — Instructions

Instructions

Preamble

Welcome to this experiment. Thank you for coming. Please read carefully these Instructions. They are to help you to understand what you will be asked to do. You are going to earn money for your participation in the experiment and you will be paid immediately after its completion.

The Experiment

You will be presented with a series of 100 problems, all of the same type. In each problem, there are a set of integer *payoffs*, about which you initially know nothing. During any problem, you might choose to incur some costs to get information about the payoffs. At the end of any problem you will get a particular one of these payoffs. We call your *income* for any problem this payoff *minus* any costs of information that you expended in that problem. Your payment for participating in this experiment will be determined by the average income from these problems, plus a £2.50 show-up fee.

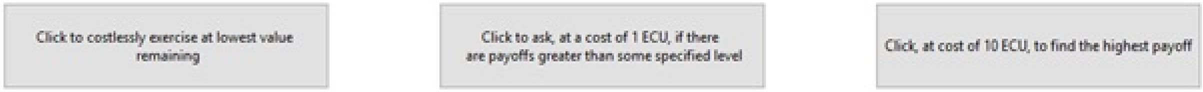
At the beginning of each problem you will not be told anything about these payoffs other than they are between 1 and 100; the payoffs can be anywhere between and including 1 and 100. In fact, they will be randomly distributed between these bounds with what is known as an *ambiguous* distribution. As such a distribution is important to the experiment; we should describe it in more detail.

Ambiguous and Uniform Risky Distributions

Examine Figures 1 and 2 at the end of these instructions. To produce each of these figures we replicated 49 times the drawing of 10,000 random numbers. For Figure 1 we generated them as uniformly distributed random numbers. You will see that the number of times that each number between 1 and 100 came up was roughly the same (around 100) on each replication; so one can conclude that the probability of any number coming up in the experiment is 1 in 100. For Figure 2, we generated them as ambiguously distributed random numbers. You will notice that, whereas in Figure 1, each of the 49 replications the distributions are approximately the same, in Figure 2, this is emphatically not the case:

the distributions vary enormously across the replications. This means that one cannot attach probabilities to each of the numbers coming up. Probabilities are undefined.

Part of the Screen



On the screen you will see some information about the payoffs and you will also see three buttons — an example is above. These relate to information that you can buy if you wish.

Information

You can choose, if you want, to buy information about the payoffs, but you do not need to.

If you do not want to buy information, then you should click on the left-hand button shown above, and then your income for that problem will simply be the *lowest* payoff in the set of payoffs.

If you do decide to buy information, there are two types you can buy — with high (denoted by K) and low (denoted by k) costs.

If you spend the high cost, K, by clicking on the right-hand button above, then the software will tell you the highest payoff in the set of payoffs, so that your income for that problem would be the highest payoff minus the high cost. In the example screen shot above, the high cost is 10 ECU.

If you want to spend the low cost, k, then you should click on the middle button above (in the screen shot above this low cost is 1 ECU), and then you will be asked to specify an *aspiration* level. The software will tell you whether there are any payoffs greater than or equal to this value. You will be told *either* that "there *are* payoffs greater than or equal to your aspiration level" *or* that "there are *no* payoffs greater than or equal to your aspiration level". If there are payoffs greater than or equal to the aspiration level, then the software will keep a record of these payoffs, and, in particular, will keep a record of the lowest one of these payoffs (greater than or equal to the aspiration level). We call this payoff the *lowest payoff greater than or equal to the highest aspiration level for which there are payoffs greater than or equal to the aspiration level.* For succinctness in what follows, we denote this by *lvgeal.* We note that the software automatically updates *lvgeal* in the sense that if you try a higher aspiration level and there are payoffs greater than or equal to this aspiration level, then *lvgeal* will become the lowest payoff greater than or equal to this new aspiration level.

You can pay this low cost as many times as you wish (though the costs will be deducted from your final payoff to determine your income for this problem) and you can change your aspiration level.

When you have decided that you have obtained enough information, simply click on the left-hand button, and your income for that problem will be *lvgeal* minus the costs you incurred in finding it. You could, of course, click on the right-hand button and your income for that problem will be the highest payoff minus all the costs you incurred up to that point, including the K.

Payment

Your payment from the experiment will be the average income from these problems plus the show-up fee of £2.50. When you have finished all 100 problems, the software will calculate your average income across all 100 problems. In the experiment all amounts are denominated in ECU (Experimental Currency Units). Each ECU is worth 33$^1/_3$*p*; that is 3 ECU is equivalent to £1. The show up fee is £2.50 and this will be added to your payment from the experiment, as described above.

Example (**Note crucially — you will NOT be told the values of the payoffs**. This example is simply to demonstrate how the software works.)

Suppose that $k = 1$ and $K = 10$. Suppose — **though you will not be told this** — that the payoffs are

55 18 75 19 9

If you clicked on the left-hand button straight away your income for this problem would be 9 (the lowest payoff).

If you clicked on the right-hand button straight away your income for this problem would be 65 (the highest payoff, 75, minus the high cost).

If you clicked on the middle button and specified an aspiration level of 40, you would be told that there *are* payoffs greater than this, but you would not be told how many nor what they are. The software would, however, note that the lowest payoff greater than or equal to 40 is 55. This would be the *lvgeal* referred to earlier. If you clicked on the left-hand button at this stage your income for this problem would be 54 (*lvgeal* minus the low cost).

If you now clicked on the middle button again and now specified an aspiration level of 70, you would be told that there *are* payoffs greater than this, but you would not be told how many nor what they are. The software would, however, note that the lowest payoff greater than or equal to 70 is 75. This would become the *lvgeal*. If you clicked on the left-hand button at this stage your income for this problem would be 73 (*lvgeal* minus the low cost twice).

If you now clicked on the middle button again and now specified an aspiration level of 80, you would be told that there are *no* payoffs greater than this. The software would, however, keep the *lvgeal*, 75, in memory. If you clicked on the left-hand button at this stage your income for this problem would be 72 (*lvgeal* minus the low cost three times).

You can keep on clicking on the middle button as often as you want, but you should note that the costs will be deducted from the payoff each time. You should also note that your income from a problem can be negative.

Note that in this particular case, it is better to click on the middle button twice (with aspiration levels of 40 and 70) and then on the left-hand button, than to click on either the left-hand button or the right-hand button straight away, and better than to click on the middle button three times (with aspiration levels of 40, 70 and 80) and then on the left-hand button. **But this is not always the case**.

What to do next

After you have read and understood these Instructions (and had any doubts clarified by asking an experimenter), please click once on your computer screen. This will activate a PowerPoint presentation of these Instructions — which goes at a predetermined speed. When it gets to the end — to a screen saying 'THANK YOU' — please hit the *escape* button — at the top-left of your keyboard. The PowerPoint presentation will disappear and you will see the first screen of the experiment proper. At this point, please call over an experimenter, and, if necessary, clarify any doubts with him or her. You will then be told how to start the experiment properly.

If you have any questions, please raise your hand and an experimenter will come to you.

John Hey   Yudistira Permana   Nuttaporn Rochanahastin

May 2016

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3 Explaining Satisficing through Risk Aversion

Abstract — This section extends the analysis of the data from the experiment of Hey *et al.* (2017), which was designed to test Proposition 2 of the theory of Manski (2017). I focus on how the subjects select the aspiration levels when they choose to satisfice, and try to find a better explanation for that story than that of Manski. I assume that the subjects are the Expected Utility (EU) (rather than MiniMax Regret) agent and that they think of the payoffs as having a uniform risky (rather than an ambiguous) distribution. I consider two special cases of the EU preferences: CRRA and CARA; and I combine these with two different stories for the stochastic specification of errors: beta and normal. To give a fair comparison in finding a better explanation of the individual behaviour, I also fit the data using Manski's optimal strategy under both stochastic specifications. I estimate using maximum log-likelihood. The estimation is done subject by subject. The results tell us that assuming that the subjects are EU agents and that they see the payoffs as uniformly distributed produces a better statistical explanation than that of Manski. That is the actual aspiration levels are statistically closer to the optimal aspiration levels assuming CRRA and CARA than those of Manski's prediction. Interestingly, the subjects in the Hey *et al.* (2017) experiment appear to be risk loving when selecting their aspiration levels.

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3.1 Introduction

Satisficing is a term introduced by Simon (1955): a decision-maker (henceforth DM) satisfices if he or she seeks an outcome greater than or equal to some 'satisfactory level'; the latter is usually referred to as the DM's *aspiration* level. However, Simon never articulated precisely how aspiration levels should be chosen. In a recent paper, Manski (2017) rectifies this deficiency by providing a theoretical analysis of when and how a DM should satisfice, and how they should choose aspiration levels.

Manski considers the DM in the following situation: he or she must choose a strategy, each strategy leading to a choice from a set of actions, each of which implies some payoff. There are three available strategies: 'No Deliberation', 'Satisficing' and 'Optimising'. *No deliberation* incurs no cost and yields the payoff of the first-ordered element of the choice set. *Satisficing* incurs a cost k and provides information as to whether there is at least one action that has a payoff greater than or equal to some specified aspiration level. *Optimising* incurs a cost K and reveals the maximum payoff in the choice set.

In determining the optimal strategy, Manski makes three key assumptions: (1) the DM's objective function is that of minimising the maximum regret (MMR) from the choice of the strategy; (2) the DM has to commit to a chosen strategy at the beginning of the problem; and (3) the payoffs have an ambiguous[1] distribution (bounded between a lower bound and an upper bound). The rationalisation for an MMR objective function stems from the assumption of ambiguity: if no probabilities exist, then the objective function of the DM cannot be that of Expected Utility, or indeed any objective function which relies on probabilities.

Hey *et al.* (2017) provide an experimental test of Proposition 2 of Manski (2017). This proposition shows that: (1) optimising is chosen if the cost K is sufficiently low; (2) satisficing with possibly a sequence of rounds is chosen if k is sufficiently low while K is sufficiently large; (3) no deliberation is chosen if both K and k are sufficiently large. Moreover, and crucially for this section, Manski's story proposes that if satisficing is chosen, the optimal aspiration level is the midpoint between the relevant lower and upper bound in every round of deliberation or satisficing. This section concentrates here on this latter prediction.

The results of Hey *et al.* (2017) experiment show that the subjects partly follow Manski's solution, in that the subjects' aspiration levels were quite close to the theoretical prediction. This current study tries to get closer on how the subjects select the aspiration level when they choose to satisfice. My alternative story is that subjects perceive (incorrectly)

[1]By which is meant that probabilities cannot be attached to the possible payoffs.

that the payoffs have a uniform risky distribution (which, in some sense, is simpler than to think of them as being ambiguously distributed);[2] as a consequence, they can and do behave as Expected Utility (EU) maximisers since they are able to attach probabilities to the payoffs. This leads to predictions that the optimal aspiration level may be above or below the midpoint between the relevant bounds depending upon the DM's attitude to risk.

This section is constructed as follows: in the next section, I describe the experiment in Hey *et al.* (2017) and the data used for this chapter; Section 3.3 discusses the preference functional; while Section 3.4 discusses the econometric specification; the results and analyses are discussed in Section 3.5, while Section 3.6 concludes.

3.2 The experiment and the data

I use the data from Hey *et al.* (2017), specifically relating to the choice of the aspiration level (obviously restricted to the cases when they chose to satisfice). The data was obtained from 48 subjects, all from the University of York. All 48 subjects were sequentially presented[3] with 100 problems all of the same type. Subjects were asked to choose between three available strategies: 'No Deliberation', 'Satisficing' and 'Optimising'. The subjects were told the lower (L) and upper (U) bounds, which were fixed at 1 and 100 respectively, in each problem. They were also told the two types of cost; the cost of finding out whether there are any payoffs greater or equal to some specified aspiration level (k) and cost of discovering the highest payoff (K). In addition, they were told[4] that the payoffs are ambiguously distributed; to generate the payoffs we used the procedure of Stecher *et al.* (2011). The number of payoffs (N) was fixed at 5 in each problem, though the subjects were not given this information.

If a subject chose satisficing, he or she was asked to specify an aspiration level. Having done this, the subject was told only whether there was at least one payoff greater than or equal to the specified aspiration level, or whether there were none. A subject was allowed to choose this strategy as many times as he or she wished. When the subject had done as much satisficing as he or she wanted, he or she would close the problem by then choosing 'No Deliberation' and then being credited with a payment equal to the smallest payoff above the highest aspiration level minus mk where m was the number of rounds of

[2] It may be simpler for one to predict what the payoffs are by thinking that they are unifromly distributed, hence it is possible for one to attach probabilities to each of the numbers coming up, instead of thinking them as having ambiguous distribution so no one can attach probabilities to each of the numbers coming up.

[3] On the computer screen; the experiment was computerised (in *Python* 2.7).

[4] How we told them this can be seen in the Instructions in Appendix B.3.

satisficing performed. At the end of the experiment, the subjects were paid their average payment over all the problems, plus a show-up fee.

There were two different treatments, each with 24 subjects. Treatment 1 had four different parameters for k and K (with N, L and U were fixed across all problems). Treatment 2 had 100 different k and K values for every 100 problems. The experiment was conducted in the EXEC Lab at the University of York.

3.3 Modelling the preference functional

Unlike Manski (2017), I assume that the DM is an EU maximiser and believes that he or she faces uniformly distributed payoffs: the DM might have not believed the experimenter (that the payoffs were ambiguously distributed with a discrete setting) and instead assumed that the payoffs were uniform; or simply assumed uniformity for simplicity.

Let me start briefly by explaining the updating process when the DM chooses to satisfice. At the beginning of the problem, the DM faces lower (L_1) and upper (U_1) bounds. The DM then is asked to choose one of the three options (*optimising*, *satisficing* or *no deliberation*). If the DM chooses to satisfice then he or she is asked to select an aspiration level t_1, and then they are told whether there *is* at least one payoff greater than or equal to t_1 or *not*. If there is at least one, then the DM can update the lower bound from L_1 to t_1, while keeping the upper bound unchanged; if there is not, then the lower bound remains unchanged, while the upper bound can be updated from U_1 to t_1. In a similar fashion, the bounds are updated with further rounds of satisficing (m); an example is given below.

Suppose that the payoffs — **though the DM does not know this information** — in this problem are: 15, 28, 55, 63 and 71. At the beginning of the problem the DM is only told that the payoffs are bounded between 1 and 100, all integers; suppose k is 5.

If, for example, the DM selects an aspiration level at 50, he or she would be told that there *is* at least one payoff greater than or equal to 50. The remaining payoffs are now 55, 63 and 71, **though the DM is not told this information**. The DM can decide to stop satisficing in this round and then receives an income of 55 (*the lowest payoff greater than or equal to the aspiration level*) minus 5 (*the cost of satisficing*). After this round of satisficing the DM's L_m and U_m are 50 and 100 respectively.

If the DM chooses to satisfice again and now specifies the aspiration level of 80, he or she will be told that there is *no* payoff greater than 80. After this round of satisficing the DM's L_m and U_m are 50 and 80 respectively. The experiment software will keep 55 as the lowest payoff greater than or equal to the aspiration level. If the DM, once again, chooses to satisfice and now specifies the aspiration level of 70, he or she will be told that there *is* at least one payoff greater than 70. After this round of satisficing the DM's L_m and U_m are 70 and 80 respectively. The experiment software notes that now the lowest payoff greater than or equal to the aspiration level is 71. If the DM stops at this round, he or she will get an income of 56 = 71 – 3k). The DM can continue to satisfice as he or she wants, but then he or she is told that the cost (mk) will be deducted from the payoff each time.

I now turn to the determination of the optimal aspiration level. Suppose after m rounds of satisficing, the lower and upper bounds are L_m and U_m. If the DM sets an aspiration level t_m at this point, there are two possible outcomes: (1) there is *no* payoff greater than or equal to t_m; (2) there *is*. Case (1) has probability $\frac{t_m-L_m}{U_m-L_m}$ (that the payoff exists between t_m and L_m) and I assume that the DM expects to earn L_m, and the utility would be $L_m - mk$; Case (2) has probability $\frac{U_m-t_m}{U_m-L_m}$ (that the payoff exists between U_m and t_m) and I assume that the DM expects to earn t_m, and the utility would be $t_m - mk$. Therefore the expected utility from choosing an aspiration level t in round m of satisficing is:

$$EU\left(t_m\right) = u\left(L_m - mk\right)\left\{\frac{t_m - L_m}{U_m - L_m}\right\} + u\left(t_m - mk\right)\left\{\frac{U_m - t_m}{U_m - L_m}\right\} \tag{3.3.1}$$

The optimum value of t_m is given by the first-order condition of Equation 3.3.1 with respect to t_m:

$$u\left(L_m - mk\right) - u\left(t_m - mk\right) + u'\left(t_m - mk\right)\left(U_m - t_m\right) = 0 \tag{3.3.2}$$

Equation 3.3.1 and 3.3.2 show that both the expected utility of the aspiration level ($EU(t_m)$) and the optimal aspiration level (t_m^*) in round m of satisficing depend on the past information (the relevant bounds L_m and U_m). The EU has a parameter risk aversion (r). I use the Constant Relative Risk Aversion (CRRA) and Constant Absolute Risk Aversion (CARA) utility functions to calculate the optimal choice of the aspiration level in accordance with the EU specification above (Equation 3.3.1). Details of the functional forms are given in Appendix C.1. In the particular case of risk neutrality, the solution to Equation 3.3.2 is $t_m^* = \frac{U_m+L_m}{2}$ — the same as Manski's solution which is the midway between the relevant lower and upper bounds. However, the solution is different if the DM is not risk-neutral. One cannot find the optimal aspiration level analytically except for the case of risk neutrality. This is done numerically. It is clear that the t_m^* must lie between L_m and U_m.

3.4 The econometric specification

I estimate the parameters of the functional forms (EU story with either CRRA or CARA) using maximum likelihood. The parameters of each estimate are estimated subject by subject, as subjects are different, as has been shown in the countless experiment. Maximum likelihood estimation requires a specification of the stochastic nature of the data. Let me start with a normalisation of the optimal and actual aspiration levels. The optimal aspiration level must lie between L_m and U_m. So I normalise it as follows, which means that normalised optimal aspiration level, $NOAL$, must be between 0 and 1:

$$NOAL_m = \frac{OAL_m - L_m}{U_m - L_m} \tag{3.4.1}$$

I do the same for the actual aspiration level

$$NAAL_m = \frac{AAL_m - L_m}{U_m - L_m} \tag{3.4.2}$$

Unlike $NOAL_m$, the $NAAL_m$ can be outside of the relevant bounds $[0, 1]$ if the AAL_m is outside $[L_m, U_m]$ — which is irrational behaviour on the part of the subject, but possible.

Now I need to talk about noise or error. Clearly, subjects make mistakes when choosing their aspiration levels, and I must specify some stochastic story to account for this. I assume this error is independent in every round of satisficing. To specify the noise in subjects' choice I must consider two possible cases: (1) when a subject *always* sets the aspiration level between the lower and upper bounds; and (2) when a subject *sometimes* sets the aspiration level outside the lower and upper bounds.

The first case is simple — since the normalised actual aspiration level (like the normalised optimal aspiration level) is between 0 and 1. Let x^* denotes the optimal aspiration level and x denotes the actual aspiration level. The obvious stochastic specification is a *beta* distribution for x. Further, if we posit that x has a beta distribution parameters α and β, and we put $\alpha = x^*(s-1)$ and $\beta = (1-x^*)(s-1)$, then it follows that the mean and the variance of x are x^* and $\frac{x^*(1-x^*)}{s}$ respectively; these parameters must satisfy the condition of $\alpha > 0$ and $\beta > 0$. Here the parameter s indicates the *precision* of the subject — the higher is the s, the less noisy is the subject. This specification implies that the DM does not make a biased decision if $x = x^*$ and that the magnitude of the noise decreases towards the bounds.

The second case is more problematic: clearly, this beta story cannot explain the aspiration levels outside the relevant bounds. So what I do here is to simply assume a *tremble* (ω).

The tremble parameter appears here to capture the behaviour when the DM selects the aspiration level outside the relevant bounds. This can possibly happen if the subjects simply make a *mistake* when selecting the aspiration level. By this, I mean that the DM chooses an aspiration level in every round of satisficing between L_m and U_m with probability $(1-\omega)$, otherwise with probability ω.

Before I turn to the specification of the log-likelihood function, I introduce some further notation which I will need in the estimation (it takes into account the fact that subjects were asked to report their aspiration levels as integers).

$$NAAL_m^+ = \frac{AAL_m + 0.05 - L_m}{U_m - L_m} \tag{3.4.3}$$

$$NAAL_m^- = \frac{AAL_m - 0.05 - L_m}{U_m - L_m} \tag{3.4.4}$$

Given these notations, the choice of aspiration level t, after normalisation, could result from an optimal value between $t-0.05$ and $t+0.05$. Thus the log-likelihood function finds the probability that the actual aspiration levels lie within the interval ($t-0.05$ and $t+0.05$) for any given level of risk aversion r. These notations are important here to capture the possibility of unobserved behaviour. The subjects might have thought of a non-integer number as the aspiration level (i.e. 49.5) but they could not do that. With the notation as in the Equations 3.4.3 and 3.4.4, we can now specify the log-likelihood function. Under 'beta with tremble' story, the contribution to the log-likelihood of an observation $NAAL_m$ is (recall that subjects had to state their aspiration levels as an integer):

$$\begin{gathered}(1-\omega)\log\left[\Psi\left(NAAL_m^+,\alpha,\beta\right)-\Psi\left(NAAL_m^-,\alpha,\beta\right)\right];0\leq NAAL_m\leq 1\\ \omega;0>NAAL_m\,||\,NAAL_m>1\end{gathered} \tag{3.4.5}$$

where $\Psi(x,\alpha,\beta)$ is the cumulative distribution function (*cdf*) of a beta distribution with parameters α and β. Here $\alpha=(s-1)\,NOAL_m$ and $\beta=(1-NOAL_m)(s-1)$.

However, one does not need to follow this 'beta with tremble' story. The stated (actual) aspiration level could be *normally* distributed. This solves the problem of rationalising actual aspiration levels outside the lower and upper bounds. I adopt this story and call it the 'normal' story; I assume that the $NAAL$ is normally distributed with mean $NOAL$ (so that there is no bias) and standard deviation $1/s$ — so that s again indicates the precision. With this 'normal' story the contribution to the log-likelihood of an observation $NAAL_m$ is:

$$\log\left[\Phi\left(NAAL_m^+, NOAL_m, \frac{1}{s}\right) - \Phi\left(NAAL_m^-, NOAL_m, \frac{1}{s}\right)\right] \tag{3.4.6}$$

where $\Phi(x, \mu, \sigma)$ is the *cdf* of a normal distribution with mean μ and standard deviation σ; as stated previously, $\sigma = 1/s$ in this case.

I also fit Manski's rule for the choice of the aspiration levels using both the 'beta with tremble' and the 'normal' stories. This enables me to compare his story with mine. Notice that there is no preference parameter in Manski's model. However, I can estimate s and ω for the 'beta with tremble' story and s for the 'normal' story and find the goodness-of-fit. In summary, I report the estimation of the EU CARA, EU CRRA and Manski's models each combined with the 'beta with tremble' and 'normal' error stories. Let me refer to these six combinations as 'specifications'.

3.5 Results and analyses

The main purpose of this study is to find a better explanation of individuals' behaviour in selecting the aspiration level when they choose to satisfice than that in Proposition 2 of Manski (2017). First, I compare the actual and optimal aspiration levels, given the estimated risk aversion, for five[5] of the six specifications. The optimal aspiration levels are estimated by fitting the data except for Manski's model, where they are given, irrespective of the subject's risk aversion. Second, I report the estimates of individual risk preferences, r, for four of the six specifications (CRRA and CARA each with the 'beta with tremble' and 'normal' stories). Third, I report the estimates of the precision s in all six specifications. Last, I report the individual log-likelihoods and the corrected log-likelihoods in each specification; I use the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQC) to correct for differing degrees of freedom.[6]

Rather trivially, the maximum likelihood of the tremble parameter ω under the beta story is always equal to the proportion of rounds of satisficing in which the aspiration level was outside the relevant bounds. This is because the estimate of ω takes the observations that are not involved in the log-likelihood function — the non-optimal actual aspiration levels. Figure 3.5.1 shows the comparison of optimal and actual aspiration levels for each specification.

[5]There is no risk aversion parameter to estimate in Manski's model and thus only one set of optimal aspiration levels to compare with the actual aspiration levels.

[6]Four of the specifications have one more parameter than the two Manski's specifications.

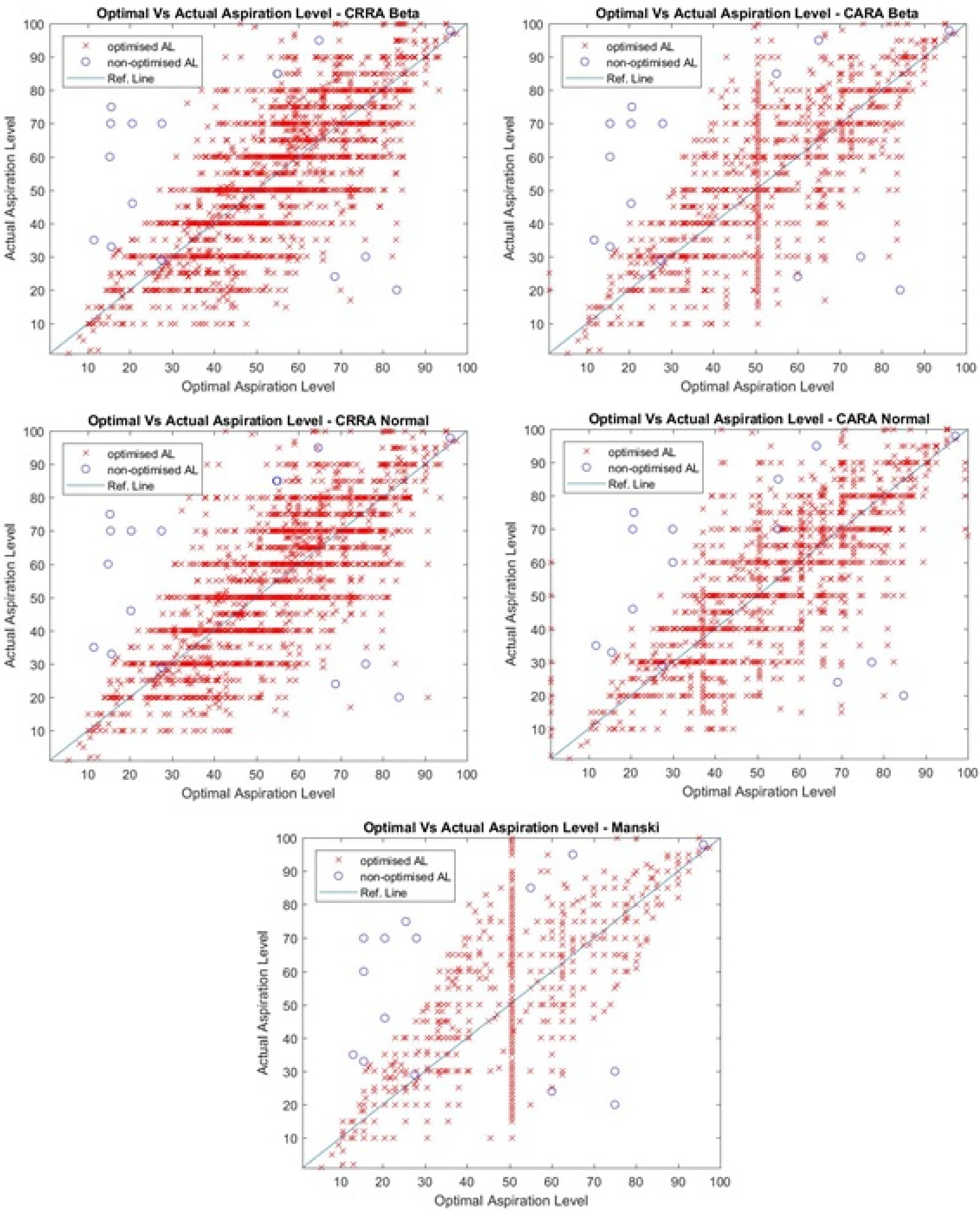

Figure 3.5.1: Optimal vs actual aspiration levels

First I focus on a comparison of the optimal and actual aspiration levels for each specification as shown in Figure 3.5.1. There are 3,347 observations when the subjects choose to satisfice. I do not exclude any observations in this figure — thus including the trembles.[7] Plots within the relevant bounds in Figure 3.5.1 are shown with a *red cross*; those outside

[7]The estimated values of the tremble parameter for each subject are reported in Figure 3.4.2.

are shown with a *blue circle*. The forty-five-degree line shows what subjects should do if they select the aspiration level equal to its optimal.

I use a Pearson correlation and the mean equality test to see which specification provides a better story. Table 3.1 shows that there is a significant relationship between the optimal and actual aspiration levels in all specifications. In addition, the coefficient indicates a positive correlation in all specifications. However, the mean equality test shows that the mean of optimal and actual aspiration levels in Manski specification and EU CARA within the beta story are not equal while other specifications are equal. According to this, the EU story may well produce a better explanation than that of Manski.

Table 3.1: Pearson correlation coefficient and the mean equality test of each estimate

Estimation	**Pearson correlation**	**Mean eq. test**
EU CRRA Beta	0.561 (0.0000)	1.021 (0.3072)
EU CARA Beta	0.484 (0.0000)	4.517 (0.0000)
EU CRRA Normal	0.564 (0.0000)	1.768 (0.077)
EU CARA Normal	0.523 (0.0000)	0.829 (0.407)
Manski	0.436 (0.0000)	11.251 (0.0000)

The number in parentheses indicates p-value.

Turning into the EU stories, it is clear that different subjects have different risk aversion. Figure 3.5.2 shows the estimated risk aversion coefficients under the non-Manski specifications.

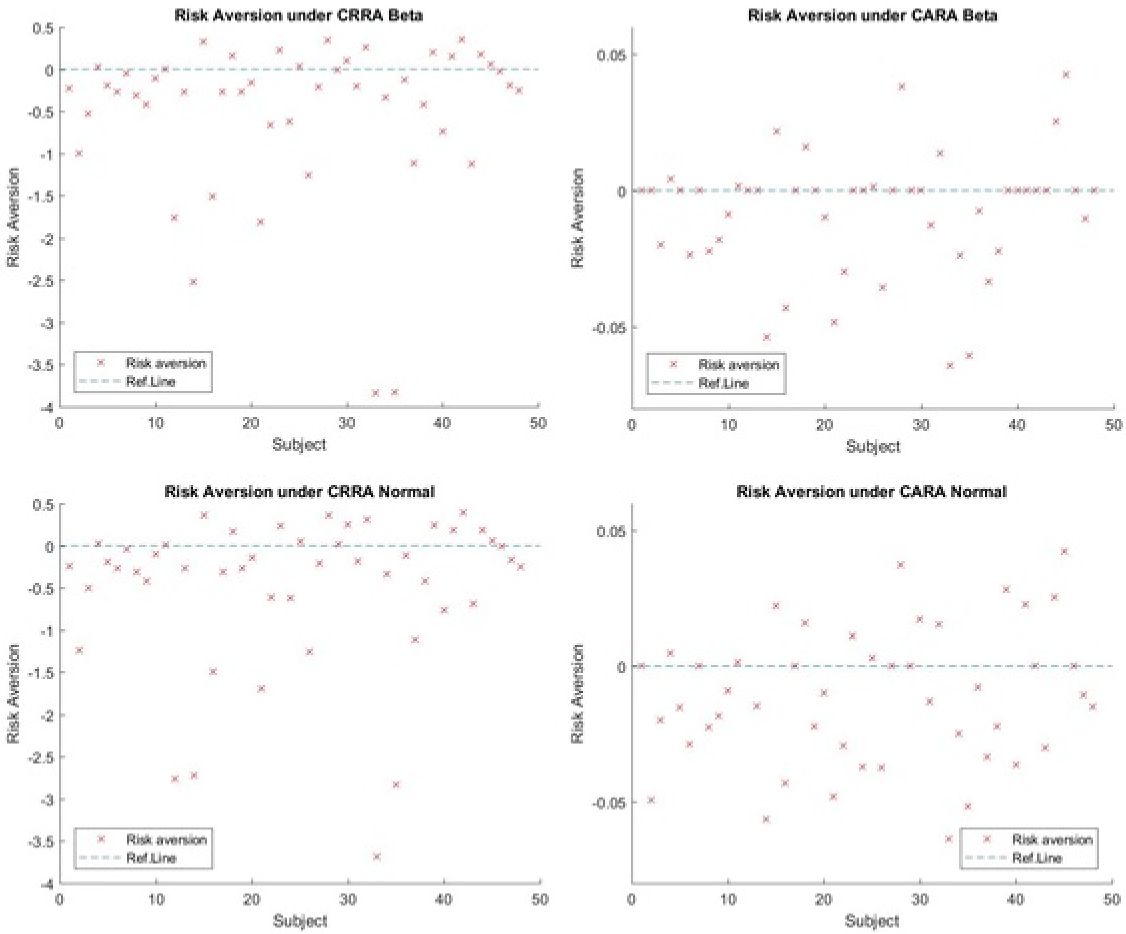

Figure 3.5.2: Individual estimated risk aversion

Figure 3.5.2 shows the estimates of individual risk aversion for each specification; except Manski's model. The dashed-blue line is horizontal at 0, indicating risk neutrality. Four estimates suggest a similar result: the majority of the subjects are risk-lover. The details can be seen in Table 3.2. The results are similar in each treatment; but Treatment 1 has more risk-loving subjects than that in Treatment 2.

Table 3.2: Number of individual estimated risk aversion

		EU CRRA Beta	**EU CARA Beta**	**EU CRRA Normal**	**EU CARA Normal**
Entire subjects	Risk loving	34	19	33	29
	Risk neutral	0	20	0	5
	Risk averse	14	9	15	14
Treatment 1	Risk loving	19	10	19	16
	Risk neutral	0	10	0	3
	Risk averse	5	4	5	5
Treatment 2	Risk loving	15	9	14	13
	Risk neutral	0	10	0	2
	Risk averse	9	5	10	9

Each specification admits there is noise in fitting the actual data. Following the stochastic story in this study, I report precision (s) from all estimates. They are shown in Figure 3.5.3. The higher is the precision (s), the less is the noise in the subjects' choice.

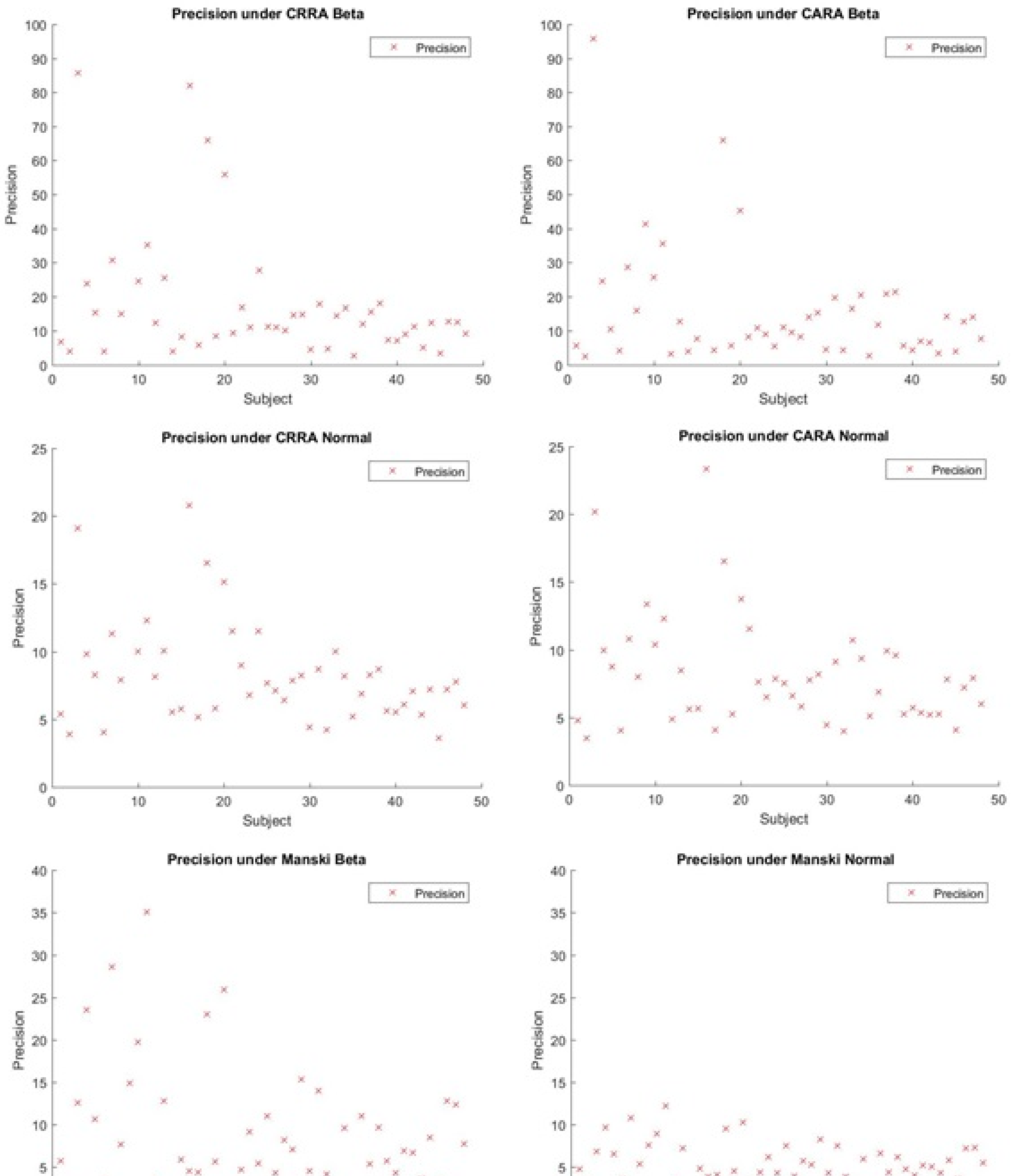

Figure 3.5.3: Individual estimated precision

The beta story captures the mistake in selecting the aspiration level. I report the tremble (ω) from the beta story; the ω will be identical in CRRA and CARA estimates. It is shown in Figure 3.5.4. There are 11 subjects who have made a mistake in selecting the aspiration level. The higher is the tremble (ω), the more is the mistake in the subjects' choice.

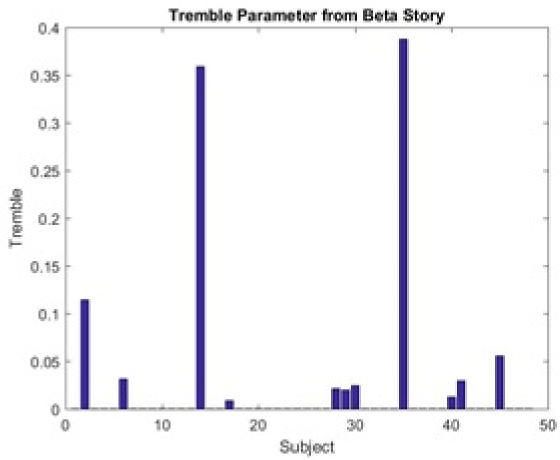

Figure 3.5.4: Individual estimated tremble

Now I report the log-likelihood in all specifications for all subjects. These are measures of the goodness-of-fit (the details in table form are given in Appendices C.2 – C.5) of the various specifications. However, I have already noted that different specifications have different degrees of freedom: the non-Manski ones have three parameters, while the Manski ones have just two parameters. In order to correct for differing degrees of freedom, I calculate the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and the Hannan-Quinn Information Criterion (HQC). [8] I compare them and have a conclusion accordingly for each subject.

The measures of the goodness-of-fit show that the EU story used in this study is better than that of Manski in explaining the subjects' behaviour when decided to satisfice. The details of the judgment can be seen in Appendix C.6. Of all subjects, 43 subjects are better explained with the EU story; 21 subjects are best-fitted using the beta story and 22 subjects are best-fitted using the normal story. Whether the subjects are best-fitted using either beta or normal story, the CRRA specification clearly appears as the better utility function. One advantage of the EU story is that it explains why many subjects selected the aspiration level close to the upper bound. Under the assumption that subjects are EU-maximisers, many subjects appear to be risk-loving.

[8]The AIC is given by $2v$- $2\log(LL)$, the BIC is given by $v\log(n) - 2\log(LL)$ and the HQC is given by $2v\log(\log(n)) - 2\log(LL)$; where v is the number of estimated parameters, n is the number of observations and LL is the maximised likelihood. By this, BIC and HQC employ a penalty function for additional coefficients while AIC does not—BIC penalises more than does HQC. The BIC and HQC penalise the number of estimated parameters using the number of observations so they are considered to be a consistent model selection for a large sample size. However, AIC remains simple and effective in practice, and very general methodology for the analysis of empirical data (Anderson and Burnham 2002). I report all three model selection criteria in this section and make a judge of which specification is best according to all criteria.

3.6 Conclusion

Manski's story is that subjects (correctly) perceived the payoff distribution as ambiguous and therefore adopted a MiniMax Regret objective function. I have investigated a different story in which subjects, finding it difficult to understand what ambiguity means, adopted a simpler perception — namely that the payoff distribution was uniform. Given this perception, my story then assumes that the objective function of the subjects was that of Expected Utility, which needs probabilities. Overall results show that my story better explains the satisficing behaviour in the part of the subjects of Hey *et al.* (2017) experiment than that of Manski's story. Of course, other stories are possible. One could assume some other perceptions, for example, subjects may have perceived the distribution of the payoffs as being that of a beta distribution. Also, instead of assuming an Expected Utility objective function of the subject, one could assume some non-Expected Utility functional — i.e. Loomes and Sugden 1983 (regret theory); Chorus 2010 (random regret minimisation); Gonzalez-Valdes and Ortuzar 2018 (stochastic satisficing in discrete choice). Alternatively, one could go further to the literature of objective function within ambiguity framework — for example, Etner *et al.* (2012). However, all these extensions would involve extra parameters, and therefore might not be better fitting than my story. But that, as I have shown, is better than that of Manski.

Appendices

Appendix C.1 — CRRA and CARA utility functions

This is the applications of CRRA and CARA in the EU specification as in Equation 3.3.1:

$$CRRA: \begin{matrix} x \geq 0 \begin{cases} u(x) = \frac{x^{1-r}}{1-r}; r \neq 1 \\ u(x) = \log(x); r = 1 \end{cases} \\ x < 0 \begin{cases} u(x) = \frac{-(-x)^{1-r}}{1+r}; r \neq 1 \\ u(x) = \log(x); r = 1 \end{cases} \end{matrix}$$

$$CARA: \begin{matrix} u(x) = \frac{1-\exp^{-rx}}{1-\exp^{-rX}}; r \neq 0 \\ u(x) = \frac{x}{X}; r = 0 \end{matrix}$$

where $u(x)$ is the utility of getting x (the DM's income), X is the maximum possible income $(U_m - mk)$ and r is the risk aversion parameter. In both CRRA and CARA above, the r takes any values between $-\infty$ and ∞, with a positive r indicating a risk averse, a negative r indicating a risk loving and $r = 0$ indicating a risk neutral. Note that the parameters L_m, U_m, k and r are exogenous to the optimal aspiration level in the EU specification. Substituting the EU function as in Equation 3.3.1 into CRRA and CARA forms results in:

$$CRRA: EU(t_m) = \frac{(L_m - mk)^{1-r}}{1-r}\left(\frac{t_m - L_m}{U_m - L_m}\right) + \frac{(t_m - mk)^{1-r}}{1-r}\left(\frac{U_m - t_m}{U_m - L_m}\right)$$

$$CARA: EU(t_m) = \frac{1-\exp^{-r(L_m-mk)}}{1-\exp^{-r(U_m-mk)}}\left(\frac{t_m - L_m}{U_m - L_m}\right) + \frac{1-\exp^{-r(t_m-mk)}}{1-\exp^{-r(U_m-mk)}}\left(\frac{U_m - t_m}{U_m - L_m}\right)$$

Note that the CRRA is conditional on $r \neq 1$ and the CARA is conditional on $r \neq 0$. They are also conditional on the value of x — either $(L_m - mk)$ or $(t_m - mk)$. Given these functional forms, the optimal aspiration level is calculated by taking the first derivative with respect to t_m. The optimal aspiration level from both CRRA and CARA may be different with the one from Manski's optimal aspiration level which is always the midpoint of L_m and U_m, depending on the degree of the risk aversion.

Appendix C.2 — Log-likelihood of each class of estimate by subject

Subject	Log-Likelihood					
	EU CRRA Beta	EU CARA Beta	EU CRRA Normal	EU CARA Normal	Manski Beta	Manski Normal
1	-137.0818	-140.4459	-137.361	-141.8459	-140.4459	-141.8459
2	-117.2147	-126.183	-127.3944	-131.1898	-126.183	-133.1838
3	-82.5899	-81.029	-82.7833	-81.2951	-109.3809	-110.3939
4	-160.4996	-159.6992	-160.3705	-159.7086	-160.7526	-160.9417
5	-142.9282	-150.4336	-141.9707	-139.9121	-150.4336	-150.4549
6	-142.7936	-144.6828	-147.6134	-147.6569	-146.7577	-152.42
7	-170.9142	-172.6853	-169.9894	-172.2301	-172.6853	-172.2301
8	-195.4035	-193.6349	-196.8826	-196.2193	-214.4524	-217.4165
9	-69.5953	-122.7041	-79.1499	-122.2347	-142.826	-143.7574
10	-119.2678	-118.5873	-119.2943	-118.1065	-123.088	-122.8984
11	-186.0771	-185.8757	-184.3621	-184.2638	-186.2113	-184.5798
12	-356.7305	-425.6602	-360.9915	-410.1529	-425.6602	-451.8336
13	-191.7149	-211.3424	-192.6701	-202.4645	-211.3424	-211.3852
14	-100.4139	-98.9567	-105.5605	-105.1277	-110.3323	-122.4522
15	-68.2335	-67.8882	-68.2823	-68.488	-70.4649	-71.2063
16	-156.9663	-149.3406	-157.8336	-151.4357	-241.5274	-250.1756
17	-456.2044	-476.2547	-459.1101	-485.3185	-476.2547	-485.3185
18	-32.1148	-32.1148	-32.107	-32.107	-37.5147	-37.6116
19	-360.9719	-380.1267	-365.4331	-374.6104	-380.1267	-387.1399
20	-85.7851	-88.5424	-85.7547	-88.2763	-95.9878	-95.8618
21	-47.5193	-48.105	-42.6578	-42.6039	-55.5046	-57.4771
22	-128.2903	-136.9631	-130.262	-135.8589	-153.9751	-155.7088
23	-291.0074	-298.2927	-290.666	-293.7398	-298.2927	-298.191
24	-197.5677	-247.1025	-197.7136	-219.6665	-247.1025	-254.0015
25	-324.1581	-324.7408	-313.7025	-315.0042	-324.8619	-315.6486
26	-381.7141	-410.1681	-386.0406	-393.085	-432.9103	-444.9089
27	-297.0586	-305.6377	-298.3949	-305.4319	-305.6377	-307.1325
28	-330.4155	-332.1696	-333.0814	-334.338	-367.7292	-370.5831
29	-669.5666	-664.7455	-656.1102	-656.238	-664.7455	-656.238
30	-518.3362	-519.8223	-521.2735	-520.476	-519.8223	-525.7093
31	-278.0399	-274.2338	-276.7565	-273.1816	-287.9179	-288.0319

32	-388.7663	-392.2955	-399.3369	-404.6544	-395.9684	-408.672
33	-255.0719	-249.2691	-263.2727	-258.394	-325.3427	-353.1646
34	-344.9115	-345.7437	-347.6672	-335.1391	-373.4001	-378.2545
35	-109.6705	-110.2196	-130.9613	-131.432	-123.5889	-150.1689
36	-360.557	-360.9986	-362.323	-362.0929	-364.8646	-366.1326
37	-267.923	-255.7363	-270.3283	-257.5517	-309.4383	-316.0274
38	-329.245	-319.8938	-329.0403	-320.2271	-359.6061	-361.6542
39	-298.8213	-308.576	-299.4934	-304.0832	-308.576	-311.315
40	-642.6974	-684.4305	-649.2134	-644.5289	-684.4305	-702.1708
41	-274.5945	-284.9113	-276.6887	-285.5894	-284.9113	-287.4787
42	-285.6887	-307.1449	-285.6726	-308.2354	-307.1449	-310.9337
43	-461.9904	-483.2967	-450.639	-452.759	-483.2967	-476.5856
44	-355.0234	-348.5793	-356.2455	-348.903	-374.2465	-375.7026
45	-64.3419	-63.0725	-67.7099	-65.6977	-65.2678	-67.8672
46	-435.1923	-435.3229	-434.625	-434.6867	-435.3229	-434.6867
47	-321.2582	-315.4041	-312.3625	-310.4275	-321.9624	-317.9618
48	-394.2274	-403.9283	-397.1945	-397.65	-403.9283	-406.7582

Appendix C.3 — AIC of each class of estimate by subject

Subject	Akaike Information Criterion					
	EU CRRA Beta	EU CARA Beta	EU CRRA Normal	EU CARA Normal	Manski Beta	Manski Normal
1	280.16	286.89	278.72	287.69	284.89	285.69
2	240.43	258.37	258.79	266.38	256.37	268.37
3	171.18	168.06	169.57	166.59	222.76	222.79
4	327.00	325.40	324.74	323.42	325.51	323.88
5	291.86	306.87	287.94	283.82	304.87	302.91
6	291.59	295.37	299.23	299.31	297.52	306.84
7	347.83	351.37	343.98	348.46	349.37	346.46
8	396.81	393.27	397.77	396.44	432.90	436.83
9	145.19	251.41	162.30	248.47	289.65	289.51
10	244.54	243.17	242.59	240.21	250.18	247.80
11	378.15	377.75	372.72	372.53	376.42	371.16
12	719.46	857.32	725.98	824.31	855.32	905.67
13	389.43	428.68	389.34	408.93	426.68	424.77
14	206.83	203.91	215.12	214.26	224.66	246.90
15	142.47	141.78	140.56	140.98	144.93	144.41
16	319.93	304.68	319.67	306.87	487.05	502.35
17	918.41	958.51	922.22	974.64	956.51	972.64
18	70.23	70.23	68.21	68.21	79.03	77.22
19	727.94	766.25	734.87	753.22	764.25	776.28
20	177.57	183.08	175.51	180.55	195.98	193.72
21	101.04	102.21	89.32	89.21	115.01	116.95
22	262.58	279.93	264.52	275.72	311.95	313.42
23	588.01	602.59	585.33	591.48	600.59	598.38
24	401.14	500.20	399.43	443.33	498.20	510.00
25	654.32	655.48	631.40	634.01	653.72	633.30
26	769.43	826.34	776.08	790.17	869.82	891.82
27	600.12	617.28	600.79	614.86	615.28	616.26
28	666.83	670.34	670.16	672.68	739.46	743.17

29	1345.10	1335.50	1316.20	1316.50	1333.50	1314.50
30	1042.70	1045.60	1046.50	1045.00	1043.60	1053.40
31	562.08	554.47	557.51	550.36	579.84	578.06
32	783.53	790.59	802.67	813.31	795.94	819.34
33	516.14	504.54	530.55	520.79	654.69	708.33
34	695.82	697.49	699.33	674.28	750.80	758.51
35	225.34	226.44	265.92	266.86	251.18	302.34
36	727.11	728.00	728.65	728.19	733.73	734.27
37	541.85	517.47	544.66	519.10	622.88	634.05
38	664.49	645.79	662.08	644.45	723.21	725.31
39	603.64	623.15	602.99	612.17	621.15	624.63
40	1291.40	1374.90	1302.40	1293.10	1372.90	1406.30
41	555.19	575.82	557.38	575.18	573.82	576.96
42	577.38	620.29	575.35	620.47	618.29	623.87
43	929.98	972.59	905.28	909.52	970.59	955.17
44	716.05	703.16	716.49	701.81	752.49	753.41
45	134.68	132.14	139.42	135.40	134.54	137.73
46	876.38	876.65	873.25	873.37	874.65	871.37
47	648.52	636.81	628.73	624.86	647.92	637.92
48	794.45	813.86	798.39	799.30	811.86	815.52

Appendix C.4 — BIC of each class of estimate by subject

Subject	Bayesian Information Criterion					
	EU CRRA Beta	**EU CARA Beta**	**EU CRRA Normal**	**EU CARA Normal**	**Manski Beta**	**Manski Normal**
1	285.00	291.72	281.94	290.91	288.11	287.30
2	245.10	263.03	261.90	269.49	259.48	269.92
3	175.07	171.95	172.16	169.18	225.35	224.08
4	332.49	330.88	328.40	327.07	329.16	325.71
5	296.69	311.70	291.16	287.05	308.09	304.52
6	295.98	299.76	302.16	302.25	300.45	308.31
7	353.44	356.98	347.72	352.20	353.11	348.33
8	402.66	399.12	401.67	400.34	436.81	438.78
9	150.10	256.32	165.57	251.74	292.93	291.15
10	249.03	247.66	245.58	243.21	253.17	249.29
11	384.07	383.66	376.66	376.47	380.36	373.13
12	727.15	865.01	731.11	829.43	860.45	908.23
13	395.51	434.76	393.39	412.98	430.74	426.80
14	211.82	208.90	218.45	217.58	227.99	248.57
15	144.78	144.09	142.11	142.52	146.48	145.19
16	325.95	310.70	323.68	310.89	491.07	504.36
17	926.67	966.77	927.73	980.14	962.02	975.39
18	71.14	71.14	68.82	68.82	79.64	77.53
19	735.41	773.72	739.84	758.20	769.23	778.77
20	181.34	186.86	178.03	183.07	198.49	194.98
21	102.49	103.66	90.29	90.18	115.98	117.44
22	267.25	284.59	267.63	278.83	315.06	314.97
23	594.97	609.54	589.97	596.11	605.22	600.70
24	407.26	506.33	403.51	447.42	502.29	512.05
25	661.42	662.59	636.14	638.75	658.46	635.67
26	777.24	834.15	781.29	795.38	875.03	894.42
27	607.07	624.23	605.42	619.50	619.91	618.58
28	674.49	678.00	675.27	677.78	744.57	745.72

29	1355.10	1345.40	1322.80	1323.10	1340.10	1317.80
30	1051.10	1054.10	1052.20	1050.60	1049.30	1056.20
31	569.11	561.50	562.20	555.05	584.52	580.41
32	791.10	798.16	807.72	818.35	800.98	821.87
33	522.97	511.37	535.10	525.34	659.24	710.61
34	703.52	705.18	704.46	679.41	755.93	761.07
35	231.02	232.11	269.71	270.65	254.96	304.23
36	734.74	735.63	733.73	733.27	738.82	736.81
37	548.68	524.30	549.21	523.66	627.43	636.33
38	672.09	653.39	667.15	649.52	728.28	727.84
39	610.43	629.94	607.51	616.69	625.68	626.89
40	1300.60	1384.10	1308.60	1299.20	1379.00	1409.40
41	561.85	582.48	561.82	579.62	578.26	579.18
42	584.29	627.20	579.95	625.08	622.90	626.17
43	938.22	980.83	910.77	915.01	976.08	957.92
44	723.58	710.69	721.51	706.83	757.51	755.92
45	137.35	134.82	141.20	137.18	136.32	138.62
46	884.72	884.98	878.81	878.93	880.20	874.15
47	656.02	644.31	633.72	629.85	652.92	640.42
48	802.27	821.67	803.60	804.51	817.07	818.12

Appendix C.5 — HQC of each class of estimate by subject

Subject	Hannan-Quinn Information Criterion					
	EU CRRA Beta	EU CARA Beta	EU CRRA Normal	EU CARA Normal	Manski Beta	Manski Normal
1	281.87	288.60	279.86	288.83	286.03	286.26
2	242.04	259.98	259.86	267.45	257.44	268.90
3	172.34	169.21	170.34	167.36	223.53	223.17
4	329.05	327.45	326.11	324.79	326.88	324.57
5	293.56	308.57	289.08	284.96	306.00	303.48
6	293.04	296.82	300.20	300.29	298.49	307.33
7	349.95	353.49	345.39	349.87	350.78	347.17
8	399.05	395.51	399.26	397.93	434.40	437.58
9	146.94	253.16	163.47	249.63	290.82	290.10
10	246.05	244.69	243.60	241.22	251.18	248.30
11	380.43	380.02	374.24	374.04	377.94	371.92
12	722.57	860.43	728.06	826.38	857.39	906.70
13	391.79	431.04	390.91	410.50	428.26	425.56
14	208.62	205.70	216.31	215.45	225.86	247.50
15	142.59	141.90	140.64	141.06	145.01	144.45
16	322.26	307.01	321.22	308.42	488.61	503.13
17	921.76	961.86	924.46	976.87	958.75	973.75
18	69.23	69.23	67.55	67.55	78.37	76.89
19	730.95	769.26	736.87	755.23	766.26	777.28
20	178.66	184.17	176.23	181.28	196.70	194.09
21	100.50	101.67	88.96	88.85	114.65	116.77
22	264.19	281.54	265.60	276.79	313.02	313.95
23	590.79	605.36	587.18	593.33	602.44	599.31
24	403.52	502.59	401.02	444.92	499.79	510.80
25	657.16	658.33	633.30	635.91	655.62	634.25
26	772.59	829.50	778.19	792.28	871.93	892.87
27	602.89	620.05	602.64	616.71	617.13	617.19
28	669.93	673.44	672.23	674.74	741.52	744.20

29	1349.20	1339.50	1318.90	1319.20	1336.20	1315.80
30	1046.10	1049.10	1048.80	1047.20	1045.90	1054.60
31	564.89	557.28	559.39	552.24	581.71	579.00
32	786.59	793.64	804.71	815.34	797.97	820.36
33	518.86	507.26	532.36	522.60	656.50	709.24
34	698.93	700.60	701.41	676.35	752.87	759.55
35	227.49	228.59	267.36	268.30	252.61	303.06
36	730.20	731.08	730.70	730.24	735.78	735.29
37	544.57	520.19	546.47	520.92	624.69	634.96
38	667.56	648.86	664.13	646.50	725.26	726.33
39	606.34	625.85	604.79	613.97	622.95	625.53
40	1295.10	1378.60	1304.90	1295.60	1375.40	1407.60
41	557.83	578.46	559.14	576.94	575.58	577.84
42	580.13	623.05	577.18	622.31	620.13	624.79
43	933.32	975.94	907.51	911.75	972.82	956.29
44	719.09	706.20	718.52	703.83	754.52	754.42
45	135.05	132.51	139.67	135.64	134.78	137.86
46	879.77	880.03	875.51	875.63	876.90	872.50
47	651.54	639.83	630.74	626.87	649.94	638.93
48	797.62	817.02	800.50	801.41	813.97	816.57

Appendix C.6 — Model selection based on the corrected log-likelihood (by majority)

Subject	Model Selection			Decision
	AIC	BIC	HQC	
1	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
2	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
3	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
4	EU CARA Normal	Manski Normal	Manski Normal	Manski Normal
5	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
6	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
7	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
8	EU CARA Beta	EU CARA Beta	EU CARA Beta	EU CARA Beta
9	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
10	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
11	Manski Normal	Manski Normal	Manski Normal	Manski Normal
12	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
13	EU CRRA Beta	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
14	EU CARA Beta	EU CARA Beta	EU CARA Beta	EU CARA Beta
15	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
16	EU CARA Beta	EU CARA Beta	EU CARA Beta	EU CARA Beta
17	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
18	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
19	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
20	EU CRRA Nomal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
21	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
22	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
23	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
24	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
25	EU CRRA Normal	Manski Normal	EU CRRA Normal	EU CRRA Normal
26	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
27	EU CRRA Beta	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
28	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
29	Manski Normal	Manski Normal	Manski Normal	Manski Normal

30	EU CRRA Beta	Manski Beta	Manski Beta	Manski Beta
31	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
32	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
33	EU CARA Beta	EU CARA Beta	EU CARA Beta	EU CARA Beta
34	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
35	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta
36	EU CRRA Beta	EU CARA Normal	EU CRRA Beta	EU CRRA Beta
37	EU CARA Beta	EU CARA Normal	EU CARA Beta	EU CARA Beta
38	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
39	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
40	EU CRRA Beta	EU CARA Normal	EU CRRA Beta	EU CRRA Beta
41	EU CRRA Beta	EU CRRA Normal	EU CRRA Beta	EU CRRA Beta
42	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
43	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal	EU CRRA Normal
44	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
45	EU CARA Beta	EU CARA Beta	EU CARA Beta	EU CARA Beta
46	Manski Normal	Manski Normal	Manski Normal	Manski Normal
47	EU CARA Normal	EU CARA Normal	EU CARA Normal	EU CARA Normal
48	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta	EU CRRA Beta

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4 Why Do People Prefer Randomisation? An Experimental Investigation

Abstract — Increasingly, experimental economists, when eliciting risk preferences using a set of pairwise choice problems (between two risky lotteries A and B), have given subjects a third choice (in addition to 'I prefer A' and 'I prefer B'), namely that of saying, for example, 'I am not sure about my preference' or 'I am not sure what to choose'. The implications for subjects of choosing this third option (which we call the 'middle column') vary across experiments depending upon the incentive structure. Some experiments provide no direct financial implications: what is 'played out' at the end of the experiment is not influenced by subjects choosing this middle column. In other experiments, if the middle column has been checked, then the payoff is determined by a randomisation of A and B. I report on an experiment, which adopts this latter incentive mechanism, and ask the question as to why people might choose this option, that is "why do they prefer randomisation?" I explore four distinct stories and compare their goodness-of-fit in explaining the data. My results show that the two of the four have the most empirical support. I include a discussion of whether my results have anything to say about preference imprecision.

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4.1 Introduction

Increasingly, experimental economists, when eliciting risk preferences using a set of pairwise choice problems (between two risky lotteries A and B), have given subjects a third choice (in addition to 'I prefer A' and 'I prefer B'), namely that of saying, for example, 'I am not sure about my preference' or 'I am not sure what to choose'. We call this choice 'choosing the middle column'. Some experimental economists use this design to investigate the difficulty in making a straight choice (either Option A and Option B), while some use it to explore the possibility of a preference for randomisation.

The implications for subjects of choosing this middle column vary across experiments — it depends on the incentive mechanism. In some experiments, for example Cubitt *et al.* (2015), there are no financial implications: what is 'played out' at the end of the experiment is not influenced by subjects choosing this middle column. Cubitt *et al.* use this procedure to associate the subjects' decision choosing the middle column with preference imprecision. In other experiments, for example Cettolin and Riedl (2019), if the middle column has been checked, then the payoff is determined by a randomisation of Option A and Option B (a mixture of A and B). Recent literature adopts this procedure to allow the investigation of a preference for randomisation (Dwenger *et al.* 2018) and stochastic choice (Agranov and Ortoleva 2017).

I report on an experiment which adopts this latter incentive mechanism, and ask the question as to why people might choose this option, that is "why do they prefer randomisation?" I explore four distinct stories and compare their goodness-of-fit in explaining the data: the random-convex preference story, the tremble story, the threshold story and the delegation story. The first story is that the decision-maker (DM) has convex indifference curves within the Marschak-Machina Triangle (MMT) and actually prefers a mixture of A and B. To make it operational, this story is embedded in the Random Preferences Model (Loomes and Sugden 1995; Loomes *et al.* 2002) in which the risk aversion parameter varies randomly from problem to problem. My second story is that the DM prefers a mixture of A and B only if it gives the highest utility; however, the DM simply makes a mistake in expressing the preferences. By this, the DM is assumed to be able to calculate the subjective utility of an alternative but that does not guarantee him or her choosing the optimal choice. This stochastic specification follows the tremble specification as in Harless and Camerer (1994), and Moffatt and Peters (2001). My third story is that the DM cannot distinguish between A and B unless their difference exceeds some threshold. This story follows the same logic as in Khrisnan (1977). Here the DM prefers an alternative if he or she subjectively perceives the utility of an alternative exceeding another one by at least some threshold (a minimum perceivable difference). Otherwise, the DM perceives that he or she is indifferent between the two alternatives. So the choice of a mixture A and B depends on the magnitude of the threshold — the higher is the threshold, the more likely is the choice of a mixture of A and B. I also add in a tremble. Lastly, the delegation story follows Vickers (1985), and Armstrong and Vickers (2010). The DM will delegate decisions if it gives the highest utility. Hence, I assume that the DM gets an additional utility when stating their preference with "*I am not sure what to choose*" — delegating the decision to the coin toss. Again, I add in a tremble.

I note that my incentive mechanism is different from that in Cubitt *et al.* The latter was concerned with preference imprecision; this section is concerned with preference for randomisation. These are different things, but this chapter may have something to tell us about preference imprecision — this depends upon how subjects view the choice problem and the incentive mechanism. We shall have more to say on this in Section 4.4.2 and 4.5.

This section is organised as follows: the next section discusses the experimental design; Section 4.3 describes the four stories in detail; Section 4.4 presents the empirical results and analyses; Section 4.5 discusses and concludes.

4.2 Experimental design

I used 72 'response tables'. In each of these, subjects were presented with a number (which varied from table to table) of pairwise-choice problems (we refer this to problem) between a certainty and a (two- or three-outcome) lottery. in every response table, the lottery remained unchanged, while the certain amount varied from the highest amount in the lottery, in steps of 25 pence to the lowest amount in the lottery. This determines the number of rows or problems in each response table.

These tables were similar to those used in Cubitt *et al.* (giving subjects choices which cpanned most possible preferences), though I duplicated the tables (in order to have a sufficient number of problems for estimation). There are seven lottery sequences: payoff scale (Seq.1), mean preserving spread (Seq.2), risky common consequence (Seq.3), safe common consequence (Seq.4), safe common ratio (Seq.5), risky common ratio (Seq.6) and betweenness (Seq.7). Subjects were given three alternative answers to state their preference on a particular problem in each response table: 1) *I choose Option A*; 2) *I am not sure what to choose*; or 3) *I choose Option B*. Figure 4.2.1 illustrates a response table used in the experiment. In the Instructions, these were called 'Preference Sheets'.

You are asked to state your preference between certain money proposed in the first column (**Option A**) and a lottery (Option B). There are 3 answer options to represent your preference: (i) **I choose Option A**; (ii) **I am not sure what to choose**; (iii) **I choose Option B**. You should click **CONFIRM** once you have finished completing this Preference Sheet, otherwise click **CLEAR** to modify your answer. The **CONFIRM** button will appear after 10 seconds. Please notice that you cannot go back to the previous Preference Sheet.
Option A: You will receive a proposed amount of money for sure.
Option B: You will have a chance of 0.65 to win £30.00 and a chance of 0.35 to win £15.00.

No.	**Proposed certain money**	**I choose Option A**	**I am not sure what to choose**	**I choose Option B**
1	For £30.00			
2	For £29.75			
3	For £29.50			
⋮	⋮	⋮	⋮	⋮
59	For £15.50			
60	For £15.25			
61	For £15.00			

Figure 4.2.1: A response table in the experiment

As Figure 4.2.1 shows, a short explanation of what subjects have to do is given at the top of the table. There is a description of Option A and Option B. Option A is always a sure amount of money whereas Option B is a fixed lottery in any one table. There were five columns in each table: the first column was the problem number in the particular response table; the second column was the certain amount of money in Option A; the third to the fifth columns are the three answer boxes.

The response table is implemented as follows. Subjects had to choose one answer in each row of the table. Unlike Cubitt *et al.* (who forced subjects not to switch between columns as they moved down the table), I did not restrict the subjects in any way. There were two buttons to confirm and to modify the answer. The confirm button became active after ten seconds; before that it was inactive. However, there was no maximum time to complete the tasks, so subjects were free to think as long as they wished. Subjects were given the instructions (paper and on-screen) and two practice tables prior to the main tasks. The experimental software was written (mainly by Alfa Ryano) in *Python* 2.7.

Monetary incentives were provided to reveal the subjects' true preferences. Subjects were told in the instructions that one of the problems from one of the tables would be the basis of their payment (additionally they were given a show-up fee of £2.50). The subject's response in a randomly chosen problem would be played out for real. First, the subject drew a disk from a closed bag containing the numbered disks from 1 to 72 — this identified a particular response table. Then, the subject drew another disk from a different closed bag to choose the problem in the selected table to play out — the number of disks depending upon the number of rows or problems in the randomly-chosen table. For the selected problem, the following rules were used to determine the subjects' payment: 1) if a subject chose Option A, then he or she would get the sure amount of money; 2) if a subject chose Option B, then he or she would play the lottery in that particular response table; 3) if a subject stated that he or she was unsure, then he or she would flip a coin to determine which option to play — then either rule (1) or rule (2) would be applied for the chosen option from the coin toss. This payment mechanism implied that the choice of the middle column was a choice for randomisation. Note that this is a different incentive mechanism than that used by Cubitt *et al.*[1] as we are specifically interested in a preference for randomisation.

The experiment was conducted in the EXEC Lab at the University of York. Invitation messages were sent through *hroot* (Hamburg registration and organization online tool) to all registered subjects in the system. There were 77 subjects who participated in the main experiment; this was preceded by a pilot experiment; I do not report its results here. They were all members of the University of York: 73 subjects were students and 4 subjects were staff members. The gender composition was: 32 subjects were male and 45 subjects were female. Subjects read the instructions together and were free to ask anything about the experiment before starting the experiment. After subjects had completed all tasks, they were paid as previously explained. Then they were free to leave. The average payment to the subjects was £11.72 and the average duration of the experiment (including reading the instructions) was a little below one and a half hours. Communication was prohibited during the experiment.

4.3 Modelling the choice

I bring four stories to try to explain the subjects' decisions: *the random-convex preference story* (henceforth the RCP story)*, the tremble story, the threshold story* and *the delegation story.* These four stories have different ways of interpreting a statement of choosing the middle column; hereafter we use A, B and M to refer Option A, Option B and the mixture of A and B (the middle column) respectively. To model the stories in this paper, I use either the Expected Utility (EU) or the Rank-Dependent Expected Utility (RDEU)

[1] In that, there was no incentive for choosing the middle column: subjects were additionally asked to indicate the row on which their preference changed from A to B, and their payment depended on the position of this row relative to the randomly-chosen row.

functional to specify the DM's preference function; so that the DM chooses option with the highest expected (rank-dependent) utility. This is crucial as I put important assumptions on the DM's preference within the MMT; in which I describe later in this section. EU has a risk-aversion parameter (r), while RDEU has two parameters (risk aversion, r, and probability-weighting-function parameter, g). I use the Constant Absolute Risk Aversion (CARA) and the Constant Relative Risk Aversion (CRRA) to specify the DM's utility function in both EU and RDEU. In addition, I use the Power function[2] to specify the probability weighting function in RDEU to rationalise the strictly convex preference, which is necessary in the RCP story.[3]

All stories share the common assumption that the subjects answer each pairwise-choice problems independently.[4] So crucially, all stories can rationalise the DM's decision to switch between columns as he or she moves down the table. We should notice that A first-order stochastically dominates B in the first problem in each table, and *vice versa* in the last problem in each table. Hence, these two problems are dominance problems, since, using either the EU or the RDEU, they should be chosen with certainty. I also assume that the DM perceives M as a single lottery through the use of the reduction of compound lotteries (ROCL). This may raise an issue as I use RDEU in some of my stories (the RCP story and the tremble story).[5] For example, Harrison *et al.* (2015) find the violation of ROCL as they assume RDEU preference and implement random-lottery incentive mechanism. This violation occurs since their subjects attach additional value to the compound lottery hence evaluate it differently problem to problem. However, in order to keep my stories as simple as possible, I assume ROCL.

The first story is that the DM has strictly convex indifference curves within the MMT — this can rationalise the choice of M. **The second** is that the DM simply makes a mistake though he or she is fully able to determine the best choice. **The third** is that the DM cannot distinguish between A and B unless their utility difference exceeds some threshold; if not, the DM chooses M;[6] I specify the threshold in two ways, a random and a fixed threshold. **The fourth** is that the DM actually prefers to delegate the choice (to the coin), shifting the 'responsibility' to the coin; here the DM receives an extra utility from delegating the choice. The details will be explained later. In each story there is

[2] $f(p) = p^g$

[3] Other specifications, i.e. the Quiggin and the Prelec functions, cannot rationalise a strictly convex preference within MMT as they produce an S-shape form. Detailed specifications can be found in the Appendix D.2.

[4] I have to assume this since I allowed subjects to switch between columns as they moved down the table: in fact, in 253 out of 5,544 tables (4.56%) from 27 subjects saw such switches.

[5] Other evaluations are possible to used, for example the compound independence (Segal 1990), in the case of RDEU preference.

[6] This idea is different than that of noisy preferences in which the DM is assumed to calculate precisely if the utility difference between two alternatives exceeds some threshold.

inevitably some randomness.

Table 4.1: A short explanation of each story

Story	Preference within MMT	Source(s) of stochasticity	Why the DM states that he or she prefers randomising
The random-convex preference	Strictly convex	The DM picks risk parameters randomly	If it gives the highest utility.
The tremble	It can be convex, concave or linear	Mistake in expressing the preference function.	If it gives the highest utility or if the DM trembles
The threshold	Strictly linear	The threshold to make a precise calculation on the preference function; and mistake in expressing the preference function when the DM is able to calculate precisely.	**Fixed threshold**: randomly chosen if the utility difference of A and B is less than some threshold. **Random threshold**: strictly chosen if the utility difference of A and B is less than some threshold.
The delegation	Strictly linear	Mistake in expressing either the DM's or other's preference function.	If delegating the choice gives the highest utility.

I apply RDEU to the RCP and the tremble stories as it allows the indifference curves (IC) in the MMT to be non-linear in probability, hence these stories can explain why randomising might be preferable. The DM prefers the mixture of A and B if it gives the highest utility. This may occur if the indifference curves are strictly convex within the MMT (Starmer 2000).[7] Figure 4.3.1 illustrates concave preferences (left panel) and convex preferences (right panel) within the MMT.[8] In the left panel, A and B are preferred to M, while in the right panel, M is preferred to both A and B.

I apply EU to the threshold story and to the delegation story; with EU the DM's indifference curves are linear within the MMT. For the threshold story, in particular, one can assume other preference functions to calculate the utility, but I assume that the DM is an EU agent to have the simplest version of this story; I explain these in each story's specification. Building on these four stories, I have sixteen variants depending upon the stochastic specification and the utility function. I fit the various stories using maximum likelihood.

[7] Of course one can assume other preference functionals that allow for non-linear preference within the MMT to explain why the mixture of A and B might be preferable.

[8] I specify the MMT accordingly with p_1 is the probability of the highest outcome and p_3 is the probability of the lowest outcome.

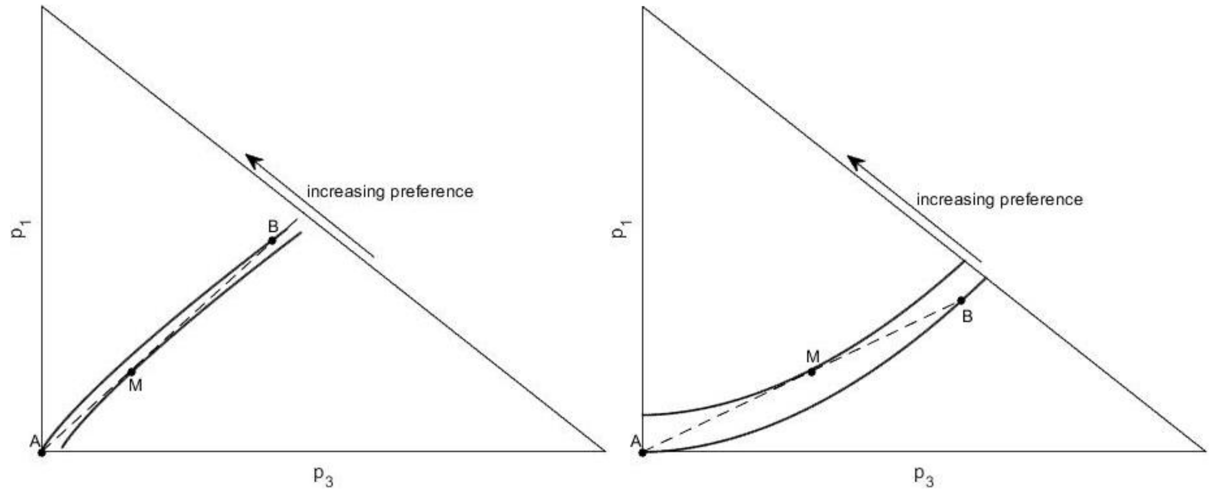


Figure 4.3.1: Indifference curves within the MMT

4.3.1 The random-convex preference story

In this story, I assume that the DM's preference function is that of RDEU and the DM has strictly convex indifference curves within the MMT. This convex preference implies the possibility that the DM prefers M. Given this, I assume that the DM always chooses the option with the highest expected rank-dependent utility. I also assume the Random Preference Model (Loomes and Sugden 1995) in which the DM's preferences vary randomly from problem to problem. By this, I mean that the DM's risk aversion parameter varies randomly from problem to problem.

Let me explain how I implement this story. Let $V(A)$ be the RDEU value of A, $V(B)$ be the RDEU value of B and $V(M)$ be the RDEU value of M. To proceed to a decision, the DM makes the following comparisons: i) $V(A, M) = V(A) - V(M)$ to compare A and M; ii) $V(B, M) = V(B) - V(M)$ to compare B and M. Therefore, the DM's preferences on each comparison are given by:

$$A\left(\substack{\succsim\\ \precsim}\right)M \Leftrightarrow V(A, M)\left(\gtreqless\right)0 \quad and \quad B\left(\substack{\succsim\\ \precsim}\right)M \Leftrightarrow V(B, M)\left(\gtreqless\right)0 \tag{4.3.1}$$

I specify the r (risk-aversion parameter) in the RDEU to be random across problems, while the g (probability-weighting parameter) is fixed; that is why I call this the random-convex preference story. So there will be an r^* in every comparison indicating that the DM is indifferent between two options for any given fixed g; $V(A, M) = 0$ and $V(B, M) = 0$. This setup is to simplify the estimation.[9] I arbitrarily assume that the r has a normal

[9]However, it is possible to allow both r and g random, and find a combination of r^* and g^* on each $V(A, M)$ and $V(B, M)$ — when the DM is indifferent between A and M, and between B and M respectively. Thus, to make it operational, it needs a joint distribution to define the simultaneous

distribution with parameters (μ and σ) — mean and standard deviation.

Each comparison, $V(A, M)$ and $V(B, M)$, defines a function between r and g. Since I assume random r and fixed g, this implies a unique r for each comparison. I define r_1^* and r_2^* as follows: $r_1^* \Leftrightarrow V(B, M) = 0$ and $r_2^* \Leftrightarrow V(A, M) = 0$. These must satisfy $r_1^* \leq r_2^*$ since the DM must be less risk-averse to be indifferent between B and M than when he or she is indifferent between A and M. The implication is that the DM chooses A if $r \geq r_2^*$; that the DM chooses B if $r \leq r_1^*$; and that the DM chooses M if $r_1^* < r < r_2^*$. As the preferences are convex, so that $0 < g < 1$, there exists the solution for r_1^* and r_2^*. However, I exclude the dominance problems, for which r_1^* and r_2^* do not exist. It follows that this story has fewer observations than the other stories by excluding the first and the last row in each response table since they are dominance problem.[10]

Using these two 'boundary' risk attitudes (r_1^* and r_2^*), I can now specify the log-likelihood function. Let $y \in \{1, 2, 3\}$ be the DM's decision in any problem; taking the value 1, 2 and 3 if the DM chooses A, M and B respectively. The contribution to the log-likelihood of the observation y in any problem is:

$$\frac{(y-3)(y-2)\log(1-\Phi_2)}{2} + (3-y)(y-1)\log(\Phi_2 - \Phi_1) + \frac{(y-2)(y-1)\log\Phi_1}{2} \quad (4.3.2)$$

where Φ_2 is the cumulative distribution function (*cdf*) of a normal distribution with parameters μ and σ given an observation r_2^*, and Φ_1 is the *cdf* of a normal distribution with parameters μ and σ given an observation r_1^*.[11] However I will report $s = 1/\sigma$, the precision. I implement this story with two variants — being the two utility functions CARA and CRRA.

4.3.2 The tremble story

As with the RCP story, I use RDEU to specify the DM's preference function, and assume that he or she always prefers the option that yields the highest expected rank-dependent utility. I assume that r and g are fixed across the problems. For this story, I assume that the DM is able to make a correct calculation but he or she sometimes trembles when expressing his or her preference. Hence I involve a tremble parameter, which I denote by ω, in this story to capture the DM's mistake.

relationship of r^* and g^*.

[10] I could include these problems within the RCP story by involving a tremble in its specification. But I want to keep this story as simple as possible.

[11] Strictly, Φ_2 is the probability that a variable with the given distribution takes value less than r_2.

I specify this story in two ways: ***the tremble 1*** and ***the tremble 2***. The former specification assumes that the tremble is the same in all possible non-optimal decisions. The tremble parameter for this specification takes a value $0 \leq \omega \leq 0.5$. The probability distribution of all decisions within this sub-story therefore is:

Table 4.2: Probability distribution of the tremble 1 story

		Optimal decisions (y^*)		
		A	M	B
Actual decisions (y)	A	$1-2\omega$	ω	ω
	M	ω	$1-2\omega$	ω
	B	ω	ω	$1-2\omega$

Following Table 4.2, the contribution to the log-likelihood of the observation y conditional to y^* is:

$$\begin{gathered} P\left(y = y^* \mid r, g, \omega\right) = \log\left(1-2\omega\right) \\ P\left(y \neq y^* \mid r, g, \omega\right) = \log\left(\omega\right) \end{gathered} \tag{4.3.3}$$

The *tremble 2* specification assumes that the error can be different across the non-optimal decisions. The probability distribution of all decisions within this sub-story therefore is:

Table 4.3: Probability distribution of the tremble 2 story

		Optimal decisions (y^*)		
		A	M	B
Actual decisions (y)	A	$1-\omega_1-\omega_2$	ω_1	ω_2
	M	ω_1	$1-\omega_1-\omega_2$	ω_1
	B	ω_2	ω_1	$1-\omega_1-\omega_2$

It is not necessarily the case that $\omega_1 > \omega_2$ in the tremble 2 specification. The tremble parameters in this particular specification take values $0 \leq \omega_1, \omega_2 \leq 0.5$. I assume that the tremble is shared equally if the optimal decision is choosing M, otherwise it is not necessary. As with the tremble 1 specification, this specification has two variants depending upon the utility function (CARA or CRRA) in the RDEU. Following the table above, the contribution to the log-likelihood of the observation y conditional to y^* is:

$$\begin{gathered} y^* \neq M \begin{cases} P\left(y = y^* \mid r, g, \omega_1, \omega_2\right) = \log\left(1-\omega_1-\omega_2\right) \\ P\left(y \neq y^* \mid r, g, \omega_2\right) = \log\left(\omega_{|y^*-y|}\right) \end{cases} \\ y^* = M \begin{cases} P\left(y = y^* \mid r, g, \omega_1, \omega_2\right) = \log\left(1-2\omega_1\right) \\ P\left(y = y^* \mid r, g, \omega_1\right) = \log\left(\omega_1\right) \end{cases} \end{gathered} \tag{4.3.4}$$

This story has four variants from the implementation of the tremble specification and the utility function in the EU.

4.3.3 The threshold story

Unlike the two previous stories, I assume that the DM has a limitation in making a precise calculation of a straight option (either A or B). The implication is that the DM can clearly distinguish between A and B only if the difference in their evaluation exceeds some threshold, otherwise the DM's optimal decision is to choose M. Here is a simple example: someone is shown two bars, they are similar in length with a difference of 0.5 mm. Those bars are seen from a distance of 1 meter. It is highly likely that someone would say that those bars are exactly identical. So he or she thinks that those bars give the same level of utility.

For this story, I use EU to specify the DM's preference function. The DM prefers A if $EU(A) - EU(B) > \varphi$ and prefers B if $EU(B) - EU(A) > \varphi$, where φ is the threshold of the EU difference. Additionally, I will have to involve a tremble (ω) to capture the DM's mistake in expressing the EU when the DM is fully able to calculate the EU precisely; the DM cannot express the EU preference if the EU difference is less than the threshold.

I specify this story in two ways according to the DM's threshold: ***a random*** and ***a fixed threshold***. The *random threshold* specification assumes that the DM has a different calculation ability across problems because he or she may understand each problem differently — that the φ may be different across problems. Here I assume that the DM cannot distinguish A and B if he or she prefers M. So the choice of either A or B implies that the DM can clearly distinguish A and B. The *fixed threshold* specification assumes that the DM has fixed calculation ability across problems — that the φ is fixed across problems. I arbitrarily assume that the DM can choose any option when he or she cannot distinguish A and B with a probability of 1/3.

I involve tremble in both specifications to capture the DM's mistake in expressing his or her preference. However, since the two specifications have key differences, they will have different probability distribution of the DM's decision. For the random threshold specification, the probability distribution of the DM's decisions where he or she does not choose M is:

Table 4.4: Probability distribution of the random threshold story

		Optimal decisions (y^*)	
		A	B
Actual decisions (y)	A	$1-\omega$	ω
	B	ω	$1-\omega$

The tremble parameter takes value $0 \leq \omega \leq 1$ in this specification. To proceed to the estimation I have to assume the distribution of φ as it is assumed to be random across all problems. I use the exponential and log-normal distribution of φ. Both distributions take into account the non-negative nature of φ. The exponential distribution has a parameter of λ (the inverse of the mean), whereas the log-normal distribution has two parameters: $\log(\mu)$ and $\log(\sigma)$. Instead I will report the mean of φ ($\Lambda = 1/\lambda$) for the exponential distribution, and the mean (μ) and the precision ($s = 1/\sigma$) of φ to make it easy to read — the higher is the precision, the less is the noise. Following the decision matrix above, the contribution to the log-likelihood of the observation y conditional to y^* is:

$$
\begin{aligned}
&P(y = M \mid r, \delta) = \log(1 - \Theta(\varphi)) \\
y \neq M &\begin{cases} P(y = y^* \mid r, \omega, \delta) = \log((1-\omega)\Theta(\varphi)) \\ P(y \neq y^* \mid r, \omega) = \log(\omega) \end{cases}
\end{aligned}
\tag{4.3.5}
$$

where Θ is *cdf* of the φ following either a log-normal or an exponential distribution, and δ is a set of parameter(s) of either a log-normal or an exponential distribution. This random threshold specification has four variants from the implementation of the threshold distribution and the utility function in the EU. So I have: i) the log-normal threshold combined with CRRA and CARA, and ii) the exponential threshold combined with CRRA and CARA.

Move on to the fixed threshold specification, the probability distribution of all decisions within this sub-story therefore is:

Table 4.5: Probability distribution of the fixed threshold story

		Actual decisions (y)		
		A	M	B
What the DM reveals (y^*)	$EU(A) - EU(B) > \varphi$	$1-2\omega$	ω	ω
	$EU(A) - EU(B) \leq \varphi$ and $EU(B) - EU(A) \leq \varphi$	1/3	1/3	1/3
	$EU(B) - EU(A) > \varphi$	ω	ω	$1-2\omega$

As in the decision matrix in Table 4.5, each option shares an equal probability to be optimal decision when the DM cannot distinguish A and B. I assume that ω is the same in all possible non-optimal decisions when the DM can distinguish A and B. This is to keep this story as simple as possible.[12] The tremble parameter takes value $0 \leq \omega \leq 0.5$. The contribution to the log-likelihood of the observation y conditional to y^* is:

$$\begin{aligned} &|EU(A) - EU(B)| \leq \varphi \Leftrightarrow P(y = A, M, B \mid r) = \log(1/3) \\ &|EU(A) - EU(B)| > \varphi \begin{cases} P(y = y^* \mid r, \omega) = \log(1 - 2\omega) \\ P(y \neq y^* \mid r, \omega) = \log(\omega) \end{cases} \end{aligned} \tag{4.3.6}$$

This fixed threshold story has two variants depending on the specification of the utility function: the fixed threshold with CRRA and the fixed threshold with CARA.

4.3.4 The delegation story

As in the threshold story, I assume that the DM is the EU agent so he or she always prefers the option that yields the highest expected utility. In addition, I assume that the DM receives an extra utility if he or she chooses M — delegating the decision to the coin toss. Therefore the expected utility of M is defined as: $EU(M) = 0.5[EU(A)] + 0.5[EU(B)] + a$ where $EU(M)$ is the expected utility of M, and a is an extra utility. This setup differentiates this story from the tremble story.

Also, I involve a tremble to capture the DM's mistake in expressing his or her preference. I specify the tremble in two ways: *tremble 1* and *tremble 2* as in the tremble story. Since there is always the best of all decisions in this story, the probability distribution of the DM's decision in both tremble 1 and tremble 2 specifications is identical to that one in the tremble story; likewise the contribution to the log-likelihood. This story has four variants arising from variations of the tremble specification and of the utility function in the EU.

4.4 Results and analyses

I start with some simple descriptive statistics. Then I proceed to a more formal analysis.

[12] Of course one can assume that the tremble can be different across non-optimal decisions when the DM can distinguish A and B.

4.4.1 Descriptive statistics

I have already noted that I allowed subjects to switch between columns as they moved down a table. We observed 253 (4.56%) such switches in 5,544 tables from 27 subjects. This means that the entries in the middle column may not have been continuous. There are 114 (2.06%) of 5,544 tables from 10 subjects see a non-continuous range in the choice of the middle column. However, in either case, I can measure the percentage of middle column responses in each table. This I call $PROPMID$. Note that this is not the same as $INTSIZE$ as used in Cubitt *et al.*, though it is closely related to it.

First I report the subjects' behaviour when they chose M. There are 14,761 cases (6.22%) out of 237,314 decisions in which subjects choose M — with 47 subjects choose M at least once. Figure 4.4.1 shows the histogram of $PROPMID$ in all problems. It is clear that some subejcts hardly ever choose the middle column, while a few choose it rather often.

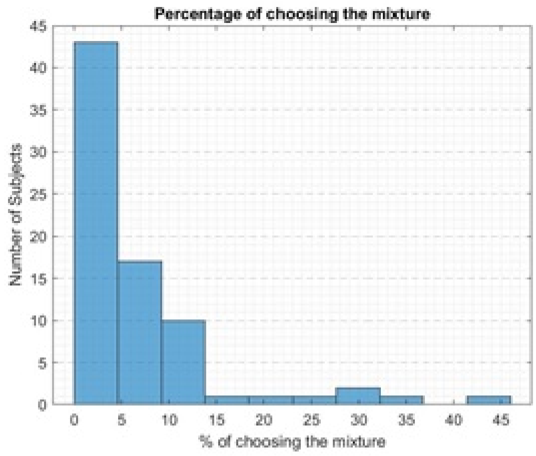


Figure 4.4.1: The percentage of choosing M

Now we break down $PROPMID$ by the lottery sequence — there are seven basic lottery sequences following the design in Cubitt *et al.* The percentage of choosing M is slightly different between the seven sequences, and there is a slight tendency for $PROPMID$ to be lower when the problems are repeated: in the first half (problems 1 to 36), $PROPMID$ averages 6.95% compared to 5.49% in the second half (problem 37 to 72).[13]

Note that, not only does $PROPMID$ decreases when the lottery sequences are repeated, but also the subjects show different pattern across lotteries within sequence. We can use this to see whether there is any connection between our subjects' behaviour and those of Cubitt *et al.* Let us focus attention on Sequences 3 to 6 where Cubitt *et al.* find that

[13]Sequence 5 has six lotteries with lottery 2 is slightly different kind of lottery 1 in this sequence. Lottery 1 is a certain lottery while lottery 2 is a close-to-certainty lottery.

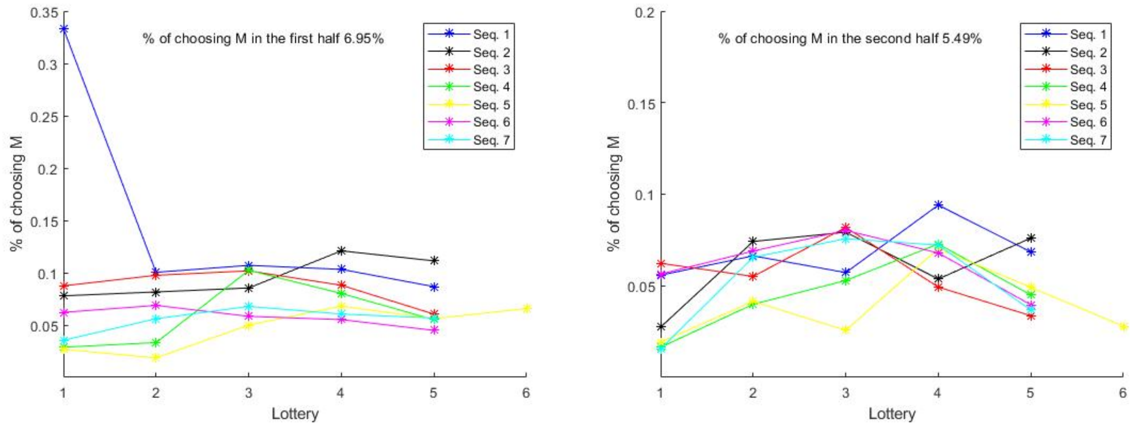


Figure 4.4.2: The percentage of subjects choosing M by sequence in each half of the problem set

the size of the imprecision decreases as the lotteries approach certainty. The left panel in Figure 4.4.2, for problems 1 to 36, shows that only Sequences 3 and 6 have an apparent decrease in $PROPMID$ as the lotteries approach to certainty; however, Sequences 4 and 5 do not have a systematic pattern across lotteries. Different results are shown in the right panel, for problems 37 to 72, where there is an apparent decrease of $PROPMID$ as the lotteries approach to certainty in Sequence 3 to 6. These findings are different from those in Cubitt *et al.*, since subjects do not show a strong systematic pattern of in Sequences 3 to 6. The differences almost certainly arise because of the different incentive mechanism: a point that is reinforced by our regression results below.

4.4.2 Formal analyses

Now I report formal results investigating my stories. There are sixteen variants of the four stories to try to explain why subjects choose M. I estimate subject by subject using maximum likelihood. Below is a summary of the stories. Following that are the results.

I separate the formal analysis into two parts. The first part is the main analysis of this paper where it tries to find the best story to account for the subjects' behaviour. I do this subject by subject. For this, I run a *horse-race* between the four stories and their some descriptive statistics, before proceeding to some more formal analyses. Second, I perform a regression analysis that serves as a complement to the first part and tries to identify the connection between preference for randomisation and preference imprecision.

Table 4.6: Summary of the stories

Story	**Preference functional**	**Random variable**	**Number of variants**	**Estimated parameters**
The RCP	RDEU	The risk parameter.	2	μ and s of r, and g within all variants.
The tremble	RDEU	n.a. in both tremble specifications.	4	r, g, ω for the tremble 1 specifications. r, g, ω for the tremble 2 specifications.
The threshold	EU	The threshold in the random threshold specifications.	4	r, Λ, ω for the exponential threshold specifications within all variants. r, μ, s, ω for the log-normal threshold specifications within all variants.
		n.a. in the fixed threshold specifications.	2	μ, φ, ω in all variants.
The delegation	EU	n.a. in all variants.	4	r, a, ω for the delegation with tremble 1 specifications. r, a, ω_1, ω_2 for the delegation with tremble 2 specifications.

To find the best fitting variant and story, I compare the individual average corrected log-likelihood in each model. Note that the RCP story has fewer observations than other stories due to exclusion of the dominance problems in this story.[14] So this compares the contribution of the corrected log-likelihood from each problem. I use the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQC).[15]

[14] This leaves the RCP story to have 2,694 observations for each subject compared to 3,082 observations for each subject in other stories.

[15] AIC is given by $2k - 2LL$; BIC is given by $\ln(n)\,k - 2LL$; HQC is given by $-2LL + 2\ln(\ln(n))\,k$;

Based on the variant comparisons, the RCP with CARA and the fixed threshold with CARA receive the most empirical support. These two variants best explain the choice of the majority of the subjects. Within the story comparison, it follows that the RCP story and the threshold story receive the most empirical support.[16] The results show that 38 subjects are best explained by the threshold story; 33 subjects are best explained with the RCP story.

Now I turn to the subjects' risk aversion. All variants used involve risk aversion, and it is obvious that different subjects have different attitudes to risk.[17] All variants show that most of the subjects are risk averse. The details of each variant can be seen in Appendix D.6. I report the tremble (ω) from variants within those stories in Appendix D.7 following the tremble parameter used in the tremble, the threshold and the delegation stories to capture the mistake in expressing the subjects' preference. I also report the extra utility parameter within the delegation story in Appendix D.8.

4.4.3 Regression analysis of the choice on the mixture of A and B

This section's main purpose is to see whether there is a connection between preference for randomisation and preference imprecision. I follow the regression model as in Cubitt *et al.* to explore what determines the choice on M. Since we saw switching amongst the subjects, I collect the percentage ($PROPMID$) of the choice of M in each response table — this differs with Cubitt *et al.* who use the range of the choice of M as measured in a monetary sum ($INTSIZE$). I use the lottery characteristics and the subjects' experience as the determinant of $PROPMID$ for each regression. Outcomes in the particular lottery are constructed through the MMT: each lottery has three outcomes with their corresponding probabilities. x_1 is the highest outcome with probability p_1, x_2 is the middle outcome with probability p_2, and x_3 is the lowest outcome with probability p_3. For any lottery with two outcomes, I interpret it as having x_3 zero with p_3 zero. I also involve the ratio of the middle to the highest outcome ($RATIO\, x_2 x_1$), the expected value of each lottery (EV), and the range between the highest and the lowest outcome stated in the lottery ($RANGE$) as the lottery characteristics in the regression. To capture the subjects' experience, I involve a number of response table that the subjects had completed ($ORDER$) and a dummy variable to indicate the repeated response table ($REPEAT$). I report the significant variables only from stepwise regression in Equation 4.4.1 below:[18]

where k is the number of parameters, LL is the maximised log-likelihood and n is the number of observations.

[16] The overall comparisons to find the best variant and the best story are in Appendix D.3 and D.4 respectively.

[17] The detail results can be seen in Appendix D.5

[18] Standard errors are in parentheses; * and ** denote significance at the 5% and 1% levels respectively.

$$PROPMID = \underset{(1.099)^{**}}{5.317} + \underset{(0.078)^{**}}{0.284x_1} - \underset{(0.586)^{*}}{1.456p_1} + \underset{(0.104)^{**}}{0.354RATIO\,x_1x_2} - \underset{(0.009)^{**}}{0.051ORDER} \quad (4.4.1)$$

The regression model shows reasonable results as all coefficients are jointly not equal from zero.[19]

The regression results in Equation 4.4.1 show that both x_1 and $RATIO\,x_1x_2$ have a positive effect to $PROPMID$; the higher are x_1 and $RATIO\,x_1x_2$, the higher is $PROPMID$. Meanwhile, both p_1 and $ORDER$ have a negative effect to $PROPMID$; the higher are p_1 and $ORDER$, the lower is $PROPMID$. One interesting finding here is that of experience ($ORDER$) is negatively significant to $PROPMID$. This implies that randomising behaviour is a temporary phenomenon; there is a tendency of the choice of M to decrease as subjects continued to the next tables. This confirms the descriptive analysis in the previous sub-section where the subjects are found to have different behaviour of choosing M when the problems were repeated.

Below I reproduce the same regression from Cubitt *et al.* (the only difference being the definition[20] of the dependent variable)[21]:

$$INTSIZE = \underset{(0.315)}{0.294} + \underset{(0.280)^{**}}{2.211p_1} - \underset{(0.377)^{*}}{0.841p_3} + \underset{(0.013)^{**}}{0.206RANGE} - \underset{(0.016)^{**}}{0.049EV} \quad (4.4.2)$$

Comparing these equations (4.4.1 and 4.4.2), it is seen that, not only does significance change, but also the magnitude of the coefficients. The conclusion seems to be that preference for randomisation and preference imprecision are two different things. The incentive mechanism would appear to be the key reason for these different results.

4.5 Discussion and conclusion

The motivation for this paper is to explore possible stories to help understand subjects' behaviour when they are given an additional option when stating their preference between two options, namely, "I am not sure what to choose", and when, if they chose this option, their payment would depend upon the tossing of a coin. It means that this choice

[19] Stepwise deletion ($p \leq 0.2$) and stepwise addition ($p \geq 0.2$) produce identical results. I also report simple and truncated regressions. The detail of all regression results can be seen in Appendix D.9.

[20] My $PROPMID$ is the *percentage* of response using M; Cubitt *et al*'s $INTSIZE$ is the *interval size* of the choice of M as measured in a monetary sum.

[21] I reproduce the detail of regression results from Cubit *et al.* in Appendix D.9

has a direct financial implication through randomisation, and leads to the discussion of preference for randomisation.

The main contribution of this section is to try to understand the nature of preference for randomisation. I propose four stories of why someone may randomise the choice. This complements previous studies that rarely provide a formal model to account for this behaviour. My analysis shows, that of the four stories, the random-convex preference story and the threshold story receive the most empirical support in explaining the subjects' behaviour. Further research, of course, is necessary to disentangle these two since they are principally different for why the DM may randomise the choice.

The four stories in this section consider that preference for randomisation is a deliberate choice. Cettolin and Riedl (2019) investigated if the subjects prefer to randomise the choice between risky and uncertain options, and between risky and sure options. They found that randomisation is a deliberate decision and is consistent across problems since their subjects' behaviour does not change with the magnitude of the incentives. An investigation by Dwenger *et al.* (2018) shows a similar pattern; they conducted experiments where the subjects' choice is implemented (in the payout rule) with a certain known probability. In one treatment, subjects were allowed to make a choice twice, the idea being that subjects with a strict preference will have the same choice in both attempts. However, the results show that a significant proportion of the subjects have choice reversals, indicating deliberate randomisation. A different approach has been done by Dominiak and Schnedler (2011) who tried to challenge the classical prediction in which uncertainty-averse individual is supposed to prefer randomisation. The experiment elicited both randomisation and uncertainty attitudes, and identified their connection. Their findings show that randomisation and uncertainty attitude are not negatively associated; it is either randomising-averse subjects are uncertainty-averse or otherwise.

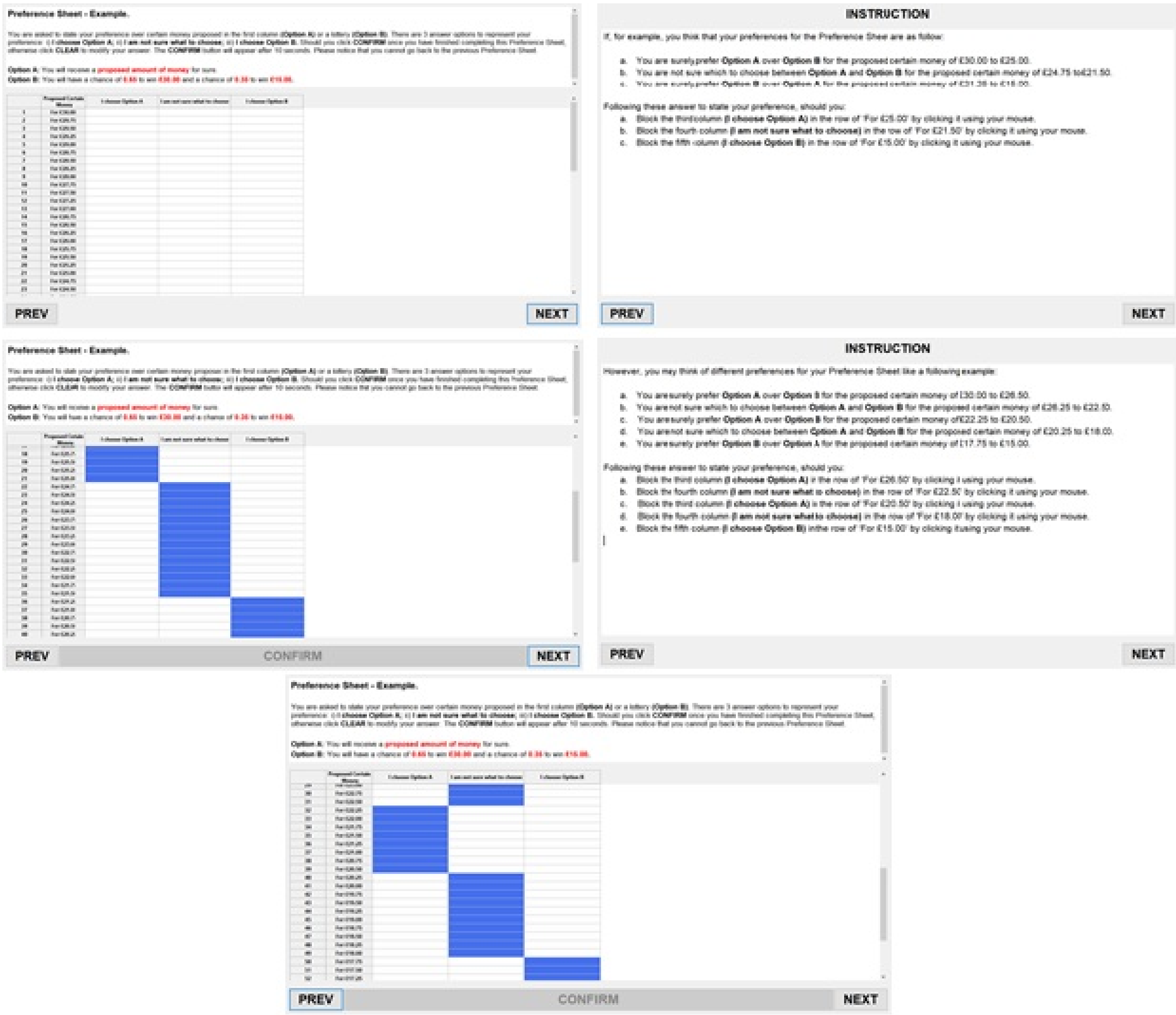
An interesting exploration has been made by Agranov and Ortoleva (2017) that is related to one concern of my stories: the source of the stochastic process. They ran an experiment to find the relationship between preference for randomisation and the stochastic choice. In their experiment, the subjects faced repeated problems in two treatments: far repetition and in-a-row repetition. Both treatments differed in how the binary-problems were presented. The far repetition treatment repeated the same problems far apart and the subjects were not told about that, whereas the in-a-row treatment repeated the same problems in a row and the subjects were told about that. Moreover, subjects were allowed to randomise the choice in both treatments at a fixed cost. They found that subjects who randomise the choice are significantly more likely to report inconsistent choice in both treatments. This indicates that the desire to randomise plays an important role in driving stochastic choice.

One recent popular topic that may have a connection with preference for randomisation is preference imprecision. This has been the main subject explored in Cubitt *et al.* (2015) and in Butler and Loomes (2007, 2011); these papers give an interpretation of the choice of the middle column despite there being no financial implication of subjects choosing it. I try to find a connection between these two topics, though they have different incentive mechanisms, by the descriptive statistics and constructing a regression model following Cubitt *et al.* Analyses on the descriptive statistics show that subjects in both studies have a different behaviour in choosing the middle column. This is clarified in my regression estimation that shows the different results than that of Cubitt *et al.* since no explanatory variables have the same magnitude and significance. This suggests that there is no association between preference for randomisation and preference imprecision as defined by Cubitt *et al.*

However, it still might be possible to link these phenomena by having a different interpretation of what we mean by preference imprecision. As Loomes and Pogrebna (2014) recommend, eliciting preferences should take care of the context where it is elicited, and that it is necessary to develop a model engaging the inherently stochastic nature of human decision-making; though they avoid using some deterministic theory combined with the error term. This paper tries to address this issue by identifying the possible source of stochasticity given the elicitation procedure, that is, the two stories that receive most empirical support in this section involve an imprecision element: in the threshold story, the imprecision takes occurs because of the DM's (in)ability to calculate utility; surely this has an implication for the DM's preference? In the random-convex preference story, the DM does not have a single preference since his or her risk attitude changes across problems. Perhaps we should pay more attention to defining what we mean by preference imprecision.

Appendices

Appendix D.1 — On the screen example to complete the tasks in the particular response table



Appendix D.2 — Specification of the EU and the RDEU

The general form of the EU is $EU\,(.) = \sum_i^I p_i u_i$ and of the RDEU is $RDEU\,(.) = \sum_i^I P_i u_i$; where p_i is the set of true probabilities, (u_i) is a set of the utility indices and (P_i) is the set of weighted probabilities. For the RDEU specification, I assume that the DM ranks the outcomes from the highest to the lowest. So I can define P_i as: $P_i = \sum_1^i w\,(p_i) - w\,(p_{i-1})$ — where $w\,(.)$ is the probability weighting function. I use the Power weighting function for $w\,(.)$ which can formally be written as: $w\,(p) = p^g; g > 0$ — where g is the parameter of $w(.)$.[22] Given this, $P_1 = w(p_1)$ and RDEU will reduce to EU if $w(p_1) = p_1$ everywhere. The $w(p_1)$ is monotonically increasing in the area of $[0, 1]$ with $w(0) = 0$ and $w(1) = 1$.

To complete the specification of the EU and RDEU, I use CARA and CRRA to specify the utility function. The general form of CARA and CRRA, and its application in this paper, are:

$$CARA : u(x) = \begin{cases} \frac{1-\exp(-rx)}{1-\exp(-rX)}; r \neq 0 \\ \frac{x}{X}; r = 0 \end{cases}$$

$$CRRA : u(x) = \begin{cases} (x+e)^{1-r}; r \neq 1 \\ \log(x+e); r = 1 \end{cases}; e > 0$$

where x is the outcome received by the DM in a choice problem, X is the highest outcome for all choice problems and r is the parameter of risk aversion. I normalise CARA so the utility index will always be $0 \leq u \leq 1$. For CRRA, I need to add e because CRRA does not fully accommodate the case when $x = 0$ and $r < 0$, otherwise the function is undefined.

[22]There are several forms to specify the probability weighting function, such as Quiggin and Prelec weighting function. However, a power function is used due to a technical reason.

Appendix D.3 — Count for best-fitting variant according to the average corrected log-likelihood[23]

Variant	AIC	BIC	HQC
RCP CARA	29	29	29
RCP CRRA	4	4	4
CT CARA	0	0	0
CT CRRA	0	0	0
NCT CARA	1	1	1
NCT CRRA	5	5	5
RTE CARA	2	2	2
RTE CRRA	1	1	1
RTL CARA	0	0	0
RTL CRRA	1	0	0
FT CARA	23	24	24
FT CRRA	11	11	11
DCT CARA	0	0	0
DCT CRRA	0	0	0
DNCT CARA	0	0	0
DNCT CRRA	0	0	0

Appendix D.4 — Count for story selection according to the average corrected log-likelihood

Story	AIC	BIC	HQC
The RCP	33	33	33
The Tremble	6	6	6
The Threshold	38	38	38
The Delegation	0	0	0

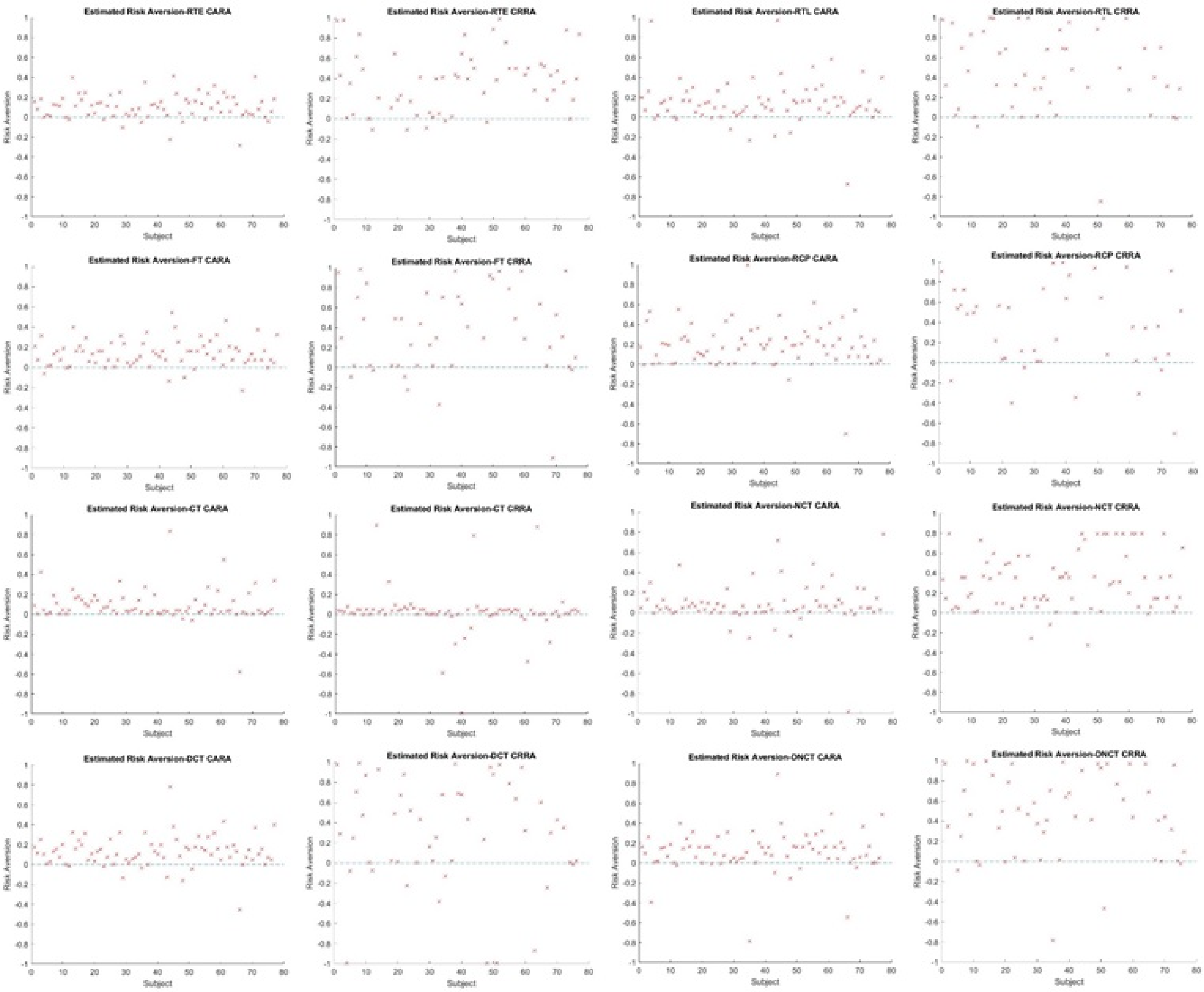
[23]Each variant is abbreviated as follows: RCP = the random-convex preference; CT = the tremble 1; NCT = the tremble 2; RTE = the random threshold with an exponential distribution of the threshold; RTL = the random threshold with a log-normal distribution of the threshold; DCT = the delegation with the tremble 1 specification; DNCT = the delegation with the tremble 2 specification; FT = the fixed threshold.

Appendix D.5 — Count for risk aversion from each variant[24]

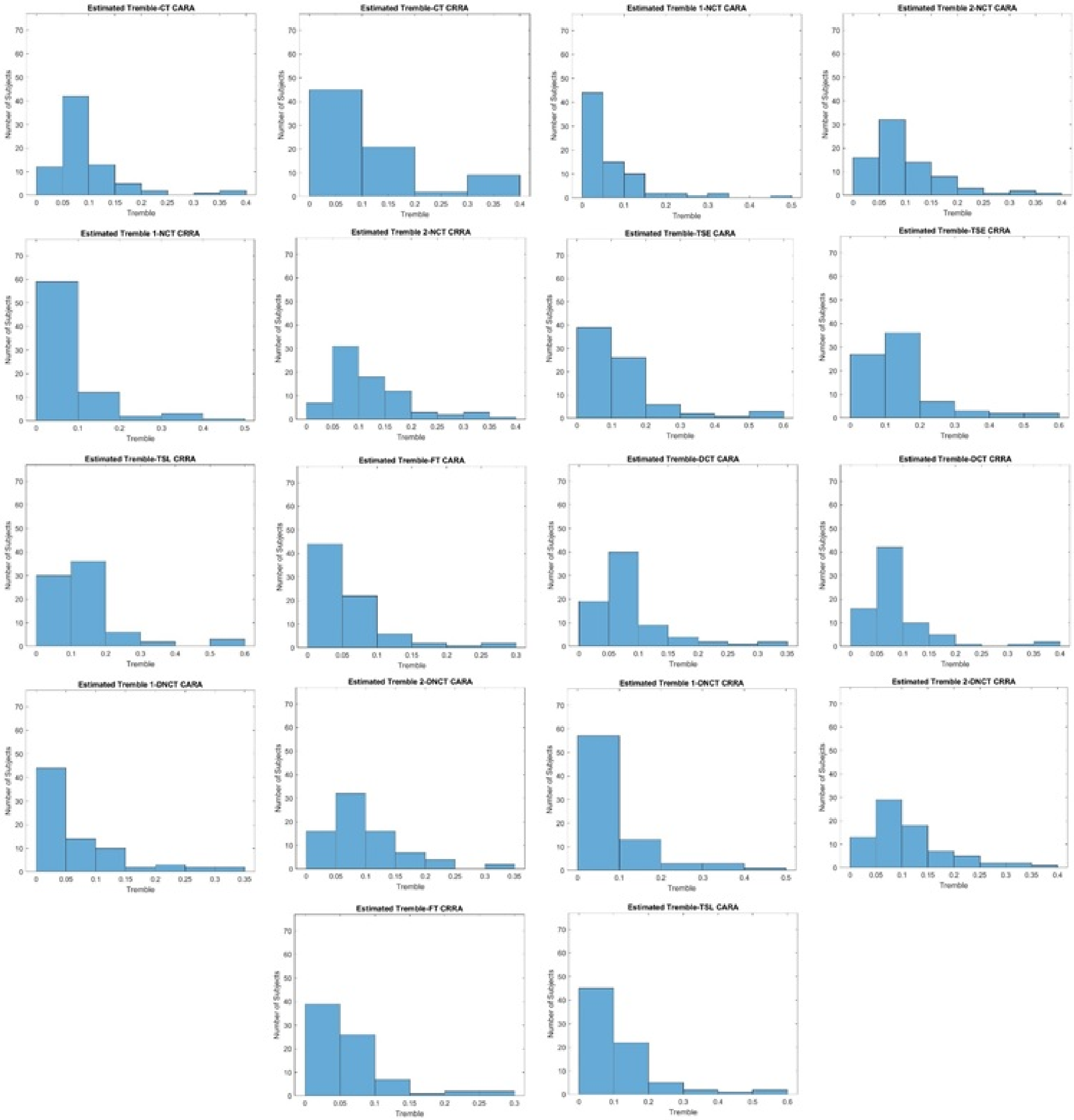
Variant	**AIC**	**BIC**	**HQC**
RCP CARA	70	0	7
RCP CRRA	68	0	9
CT CARA	73	0	4
CT CRRA	60	0	17
NCT CARA	63	0	14
NCT CRRA	72	0	5
RTE CARA	67	1	9
RTE CRRA	71	1	5
RTL CARA	66	1	10
RTL CRRA	68	0	9
FT CARA	69	0	8
FT CRRA	64	0	13
DCT CARA	69	0	8
DCT CRRA	63	0	14
DNCT CARA	65	2	10
DNCT CRRA	64	1	12

[24]The reported count within the RCP story indicates that most of the subjects have their mean of the r at the risk aversion state.

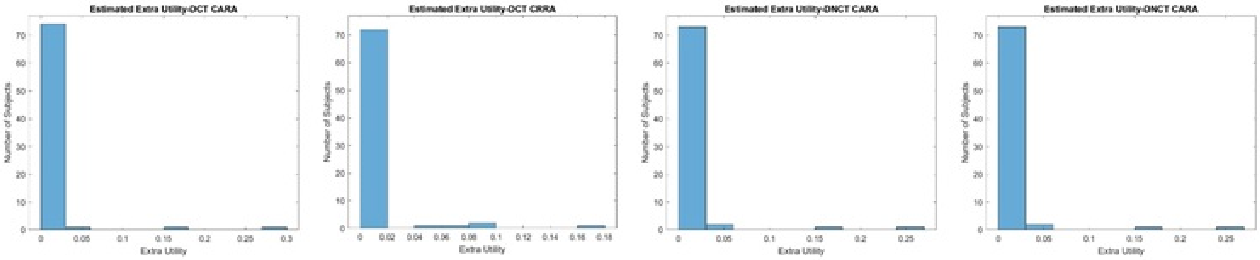
Appendix D.6 — Risk aversion of each variant



Appendix D.7 — Tremble parameter of the related variant



Appendix D.8 — Extra utility parameter of the related variant



Appendix D.9 — Reported regression model results for $PROPMID$ and $INTSIZE$[25]

Dependent Variable	$PROPMID$		$INTSIZE$	
Procedure	**Simple Linear**	**Stepwise**	**Simple Linear**	**Stepwise**
$Constant$	5.998^{**} (1.255)	5.317^{**} (1.099)	-0.285 (1.329)	0.294 (0.315)
x_1	0.185 (0.122)	0.185^{**} (0.078)	0.058 (0.079)	
p_1	-2.455^{**} (0.898)	-1.456^{*} (0.586)	2.816^{*} (1.132)	2.211^{**} (0.280)
p_3	-0.766 (1.111)		-2.262 (2.386)	-0.841^{*} (0.377)
$RATIO\, x_2 x_2$	0.312^{**} (1.112)	0.354^{*} (0.104)	0.870 (1.650)	
EV	0.108 (0.093)		-0.122 (0.286)	-0.049^{**} (0.016)
$RANGE$	-0.016 (0.108)		0.203^{**} (0.019)	0.206^{**} (0.013)
$REPEAT$	0.311 (1.119)		0.081 (0.186)	
$ORDER$	-0.055 (0.029)	-0.051^{*} (0.009)	-0.014 (0.018)	
Observations	5,544	5,544	1,830	1,830
$Adj - R^2/R^2$	0.012	0.011	0.162	0.161
Subjects	77	77	79	79
Prob. > F	0.000	0.000	0.000	0.000
RMSE	14.206	14.206	N.A	N.A
Mean of $\hat{y}$	6.319	6.319	N.A	N.A
Min. $\hat{y}$	3.469	3.749	N.A	N.A
Max. $\hat{y}$	10.424	10.267	N.A	N.A

[25] Standard errors are in parenthesis; * and ** denote significance at the 5% and 1% respectively. Stepwise addition (p-values ≥ 0.2) and stepwise removal (p-values ≤ 0.2) produce identical results.

Appendix D.10 — Instructions

centre for experimental economics

Instructions

Preamble

Welcome to this experiment and thank you for coming. Please read these Instructions carefully. They will help you to understand what the experiment is all about and what you are being asked to do during the experiment. This experiment gives you the opportunity to earn money which will be paid to you in cash after you have completed the experiment. However there is no participation fee in this experiment; what you earn in the experiment is what you will be paid. So you must take this experiment seriously. The payment is described below and it will be added to a show-up fee of £2.50 that you will be paid independently of your answers.

The Experiment

This is an experiment involving pairwise choices. The pairwise choices will be presented in a series of preference sheets. There are 72 preference sheets for you to complete, all of the same type. Each preference sheet has a number of pairwise choices that we will call it as row. Each pairwise choice will ask you to state your preference between a certain amount of money (**Option A**) and a lottery (**Option B**). On any one preference sheet, the certain amount (**Option A**) changes, while the lottery (**Option B**) remains the same. Each lottery involves either two or three possible outcomes. All outcomes are positive amounts, so you are guaranteed not to lose money. You should look at the figure below for an example of a preference sheet. In each of these, the unchanging lottery (**Option B**) is described at the top of the sheet. In each preference sheet, there are several rows, in each of which there is a pairwise choice — the number of rows depending on the range between the lowest and highest outcomes in the lottery (**Option B**) with a decrement of 25 pence. We refer to the number of row as the number of the pairwise choice. There are 5 columns in each preference sheet: the first column is the row number; the second column is the certain amount of money in **Option A**; the third to the fifth columns are three answer boxes. You are given 3 answer options to state your preference over **Option A** or **Option** B: i) I choose **Option A**; ii) I am not sure what to choose; iii) I choose

Option B. What answer option "ii" means will become clearer when we describe below how you will be paid.

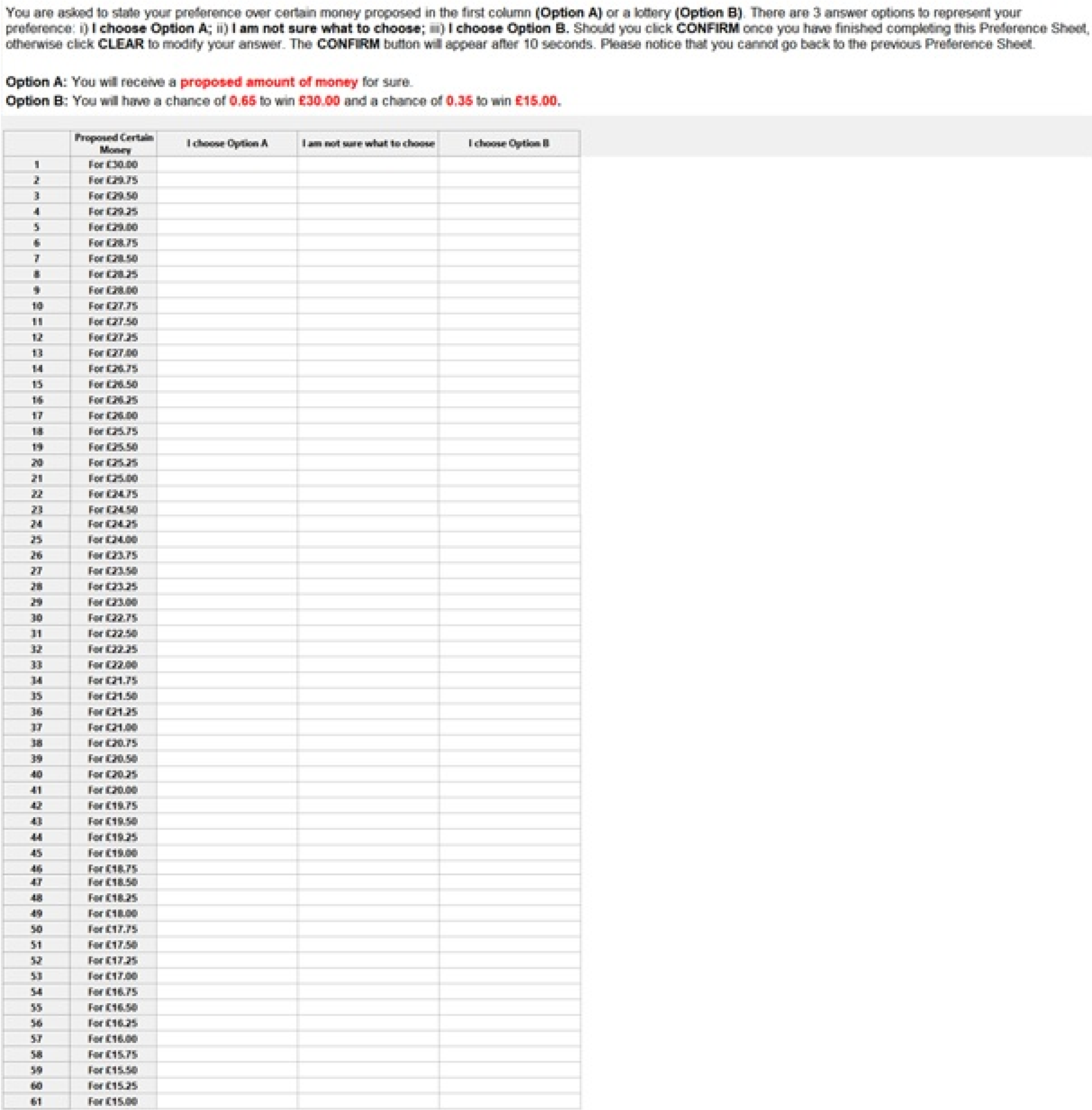
To state your preference in the sheet, you have to block one answer box in the particular row. To save your time, if you want, you can block an answer box at the last row that you think it represents your preference in the particular row and its previous row(s). The computer will automatically block the previous row(s) for you. There are 2 buttons to confirm and to modify your answer. If you think your answers represent your preference, you should click on **CONFIRM** and you will then go on to the next preference sheet. The **CONFIRM** button will be active after 10 seconds if you have answered to all pairwise choices in a preference sheet. Otherwise it remains inactive. If you wish to modify your answer, you should click on the **CLEAR** button. It will clear all of your answers and you will have to fill in the answer boxes again from the start.

Please notice that you can **only** choose one answer column in a particular row.

<u>Example</u>

Suppose that you are offered a lottery (this will be the Option B) that gives you a 0.65 chance of winning £30 and a 0.35 chance of winning £15. It means that you can earn either £30 or £15 if you play out the lottery, each depends on its chance to win (0.65 and 0.35 respectively). This lottery will determine the certain amounts of money (this will be the Option A) and the number of pairwise choices offered to you in this preference sheet. In this case, the preference sheet will have 61 rows with the highest certain amount of money £30 and the lowest certain amount of money £15. So you have 61 pairwise choices (rows) between 61 different certain amounts of money (as Option A) and an unchanged lottery as Option B.

On the screen, you will see the problem instructions, Option A and Option B, the preference sheet, and the control buttons. An example of the preference sheet on your screen based on the problem above is:



You are asked to state your preference over certain money proposed in the first column **(Option A)** or a lottery **(Option B)**. There are 3 answer options to represent your preference: i) **I choose Option A;** ii) **I am not sure what to choose;** iii) **I choose Option B.** Should you click **CONFIRM** once you have finished completing this Preference Sheet, otherwise click **CLEAR** to modify your answer. The **CONFIRM** button will appear after 10 seconds. Please notice that you cannot go back to the previous Preference Sheet.

Option A: You will receive a **proposed amount of money** for sure.
Option B: You will have a chance of **0.65** to win **£30.00** and a chance of **0.35** to win **£15.00.**

	Proposed Certain Money	I choose Option A	I am not sure what to choose	I choose Option B
1	For £30.00			
2	For £29.75			
3	For £29.50			
4	For £29.25			
5	For £29.00			
6	For £28.75			
7	For £28.50			
8	For £28.25			
9	For £28.00			
10	For £27.75			
11	For £27.50			
12	For £27.25			
13	For £27.00			
14	For £26.75			
15	For £26.50			
16	For £26.25			
17	For £26.00			
18	For £25.75			
19	For £25.50			
20	For £25.25			
21	For £25.00			
22	For £24.75			
23	For £24.50			
24	For £24.25			
25	For £24.00			
26	For £23.75			
27	For £23.50			
28	For £23.25			
29	For £23.00			
30	For £22.75			
31	For £22.50			
32	For £22.25			
33	For £22.00			
34	For £21.75			
35	For £21.50			
36	For £21.25			
37	For £21.00			
38	For £20.75			
39	For £20.50			
40	For £20.25			
41	For £20.00			
42	For £19.75			
43	For £19.50			
44	For £19.25			
45	For £19.00			
46	For £18.75			
47	For £18.50			
48	For £18.25			
49	For £18.00			
50	For £17.75			
51	For £17.50			
52	For £17.25			
53	For £17.00			
54	For £16.75			
55	For £16.50			
56	For £16.25			
57	For £16.00			
58	For £15.75			
59	For £15.50			
60	For £15.25			
61	For £15.00			

The picture above is an example of a particular sheet. Please notice that it has 61 rows and you must drag down the slider in the right to see all 61 rows. The Option A always changes 61 times from £30 to £15 with a decrement of 25 pence whereas the Option B remains fixed when you are asked to state your preference in this particular preference sheet. The chance of winning each lottery outcome (£30 or £15) in the example above is 0.65 and 0.35 respectively. You have to click one of the three answer boxes to state your preference between Option A and Option B. So one answer for one row. There are 2 control buttons as shown below to confirm and to modify your answers. The left one is the **CONFIRM** button with a timer on it — this button is to confirm your answers. It remains inactive for 10 seconds and if you have not completed all pairwise choices. The right one is the **CLEAR** button — this button is to modify your answers.

After 10 seconds and if you have answered to all pairwise choices, the **CONFIRM** button will be active. You have to click this button to confirm your answers and to go to the next problem. If you want to modify your answer, you should simply click the **CLEAR** button and it will clear your answers in the current sheet.

Payment

Your payment in this experiment will be determined by your responses during the experiment plus a show-up fee of £2.50. There is no participation fee in this experiment. What you have done is what you will be paid with. So make sure that you answer every task seriously. The row in a preference sheet will be a basis of your payment. To determine the number of the preference sheet, you will be presented a closed bag containing the numbered disks from 1 to 72, each indicating the number of a preference sheet and is in integer number. Then you draw a disk by yourself. After that, once again, you will be presented a closed bag containing numbered disks depending on how many rows there are in the chosen preference sheet. You pick a disk from the bag to determine which row in the chosen preference sheet is to be played for real. After you have got the row as your basis of your payment, these following rules are used to determine your payment:

- If, on the chosen row, your answer is "**I choose Option A**" then you will get paid the certain amount of money.
- If, on the chosen row, your answer is "**I choose Option B**" then you will play out the lottery for real, being paid one of the outcomes in the lottery.
- If, on the chosen row, your answer is "**I am not sure what to choose**" then you will be asked to toss a fair coin. If the coin lands head you will get paid the amount of money specified in **Option A**. If the coin lands tail you will play out the lottery for real, being paid one of the outcomes in the lottery.

How is the lottery played out

You will play out the lottery if your payment basis is the lottery. The lottery gives you an opportunity to win either £X or £Y or £Z, depending on the lottery setting, each with its own chance. We will use disks in the closed bag that is numbered from 1 to 100 to represent the chance of winning some money from the lottery. The disks are in integer number. There are two basic lottery design: a 2-outcome lottery and a 3-outcome lottery. Here is the example of a 2-outcome lottery. Suppose you get a lottery that gives you a 0.35 chance of winning £20 and a 0.65 chance of winning £10. Then you are presented a closed bag with 100 disks in it that are numbered from 1 to 100 and you draw a disk by yourself. If you draw a disk numbered from 1 to 35 then you will get paid £20. If you draw a disk numbered from 36 to 100 then you will get paid £10. Here is the example of 3-outcome lottery. Suppose you get a lottery that gives you a 0.25 chance of winning £20, a 0.50 chance of winning £15 and a 0.25 chance of winning £10. Then you are presented a closed bag with 100 disks in it that are numbered from 1 to 100 and you draw a disk by yourself. If you draw a disk numbered from 1 to 25 then you will get paid £20. If you draw a disk numbered from 26 to 75 then you will get paid £15. If you draw a disk numbered from 76 to 100 then you will get paid £10.

How long will the experiment last?

It is important for you to consider the problems carefully because your answers determine how much you can earn from this experiment. Hence we impose a minimum time of 10 seconds for you to respond on each preference sheet. This means that you can go on to the next preference sheet after 10 seconds providing that you have answered all pairwise choices in that sheet. There is no maximum time to complete each preference sheet but you still have to complete all pairwise choices to go on to the next preference sheet. Please notice that you cannot go back to the previous preference sheets. We estimate that the experiment will take at least 45 minutes of your time. You can take longer and it is clearly up to your interests to be as careful as you can when you are answering the questions.

Questionnaire

Subject Number:

Please provide us the following information about you.

Q. Sex: Male/Female (Cycle the right one)

Q. Age: What is your age?

Q. Ethnicity origin: Please specify your ethnicity. (Cycle the right one)

• White • Hispanic or Latino • Black or African American • Native American or American Indian • Asian / Pacific Islander • Other

Q. Education: What is the highest degree or level of school you have completed? If currently enrolled, highest degree received. (Cycle the right one)

• No schooling completed • Nursery school to 8th grade • Some high school, no diploma • High school graduate, diploma or the equivalent (for example: GED) • Some college credit, no degree • Trade/technical/vocational training • Associate degree • Bachelor's degree • Master's degree • Professional degree • Doctorate degree

Please answer also the following questions

Q. Are you currently a student? If so, in which level you are currently enrolled?

Q. What are you studying?

Q. Do you have any work experience in Economics? If so, for how long did/do you work in this field and which was/is your job title/titles?

Q. Have you participated in economics experiments in the past?

Q. Did you feel impatience during the experiment? (Cycle the right one)

1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Did you feel stress during the experiment? (Cycle the right one)

1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Which is your risk aversion level? From 1 to 5 the risk aversion level is increasing.

(Cycle the right one)

1 2 3 4 5

Q. What did you like in the experiment?

Q. What you did not like in the experiment?

Q. Any suggestions for improvement?

Thank you for your participation!

Yudistira Permana

October 2017

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5 On the optimal strategy for the hedge fund manager: An experimental investigation

Abstract – This section examines the empirical validity of Nicolosi's model (2018) which investigates the optimal strategy for a hedge fund manager under a specific payment contract. The contract specifies that the manager's payment consists of a fixed payment and a variable payment, which is based on the over-performance with respect to a pre-specified benchmark. The model assumes that the manager is an Expected Utility agent who maximises his or her expected utility by buying and selling the asset at appropriate moments. Nicolosi derives the optimal strategy for the manager. To find this, Nicolosi assumes a Black-Scholes setting where the manager can invest either in an asset or in a money account. The asset price follows geometric Brownian motion and the money account has a constant interest rate. I experimentally test Nicolosi's model. To meet the aim of this paper, I compare the empirical support of Nicolosi's model with other possible strategies. The results show that Nicolosi's model receives strong empirical support for explaining the subjects' behaviour, though not all of the subjects follow Nicolosi's model. Having said this, it seems that the subjects somehow follow the intuitive prediction of Nicolosi's model in which the decision-maker responds to the difference between the managed portfolio and the benchmark to determine the portfolio allocation.

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5.1 Introduction

The hedge fund industry has grown enormously in the last few decades. It may be best defined as the private investment vehicle deploying a wide range of investment strategies in order to achieve a high rate of return, though there are alternative definitions for it (Hildebrand 2005). It has a wide variety of investments such as stock, bonds, real estate and other commodities. The hedge fund manager is then responsible to manage the investor's funds under a specific contract. The contract initially specifies the investor's target (usually referred to as the benchmark), the investment period and the payment scheme. The payment typically is based on the manager's performance with respect to a pre-specified benchmark; though the benchmark may be arbitrarily set by the investor. The better is the manager's performance with respect to the benchmark, the higher is the manager's payment.

Clearly, the payment scheme determines the manager's behaviour, given his or her risk attitude, in managing the investor's funds (Palomino and Prat 2003). The investor employs this payment scheme to meet his or her benchmark, and the manager maximises his or her expected utility by buying and selling the asset at appropriate moments given the payment scheme. So once the contract is agreed, the manager chooses his or her portfolio strategy, given the risk attitude, to ensure beating the benchmark at maturity, in order to maximise the manager's utility.

Much literature has explored the optimal portfolio strategy for the hedge fund manager in order to maximise his or her expected utility under a specific contract. Notable amongst these recently are Browne (1999), Carpenter (2000), Gabih *et al.* (2006), Hodder and Jackwerth (2007), Panageas and Westerfield (2009), Guasoni and Obloj (2016) which investigate the optimal portfolio choice for the manager in continuous-time with respective to a selected benchmark by the investor; this literature being motivated by the work of Merton (1969, 1971).[1] One clear conclusion from this literature is that the benchmark level determines the manager's behaviour, given his or her risk attitude (measured by the level of risk aversion). In particular, this literature investigates how the manager's risk attitude affects his or her allocation decision: that is, how much to allocate in the risky asset. Generally, the literature shows that the manager is highly likely to hold more of the asset (that is, take on more risk) when the portfolio value is below the benchmark, in order to increase his or her chance of ending up with a higher payment. Contrariwise, the manager should reduce his or her portfolio volatility (by holding less of the asset) when

[1] This benchmark can be either fixed or variable. The fixed benchmark usually is the expected return from the investment funds whereas the variable benchmark usually is the portfolio value at maturity following the investor's portfolio allocation choice.

the performance is relatively above the benchmark.

This section examines the empirical validity, with a laboratory experiment, of a recent theory — that of Nicolosi (2018). This theory investigates the dynamic optimal strategy for the hedge fund manager under a performance-based payment. In his model, Nicolosi specifies two types of payment: a *fixed payment* and a *variable payment*, where the variable payment is based on the over-performance at maturity with respect to the benchmark. So the manager surely earns the fixed payment and will earn the variable payment depending on what he or she achieves. The benchmark is a linear combination of the investment invested in the risky and riskless assets, and the over-performance is achieved if the manager makes a higher portfolio value than that of the benchmark at maturity. Nicolosi imposes two important rules of the game: 1) the manager allocates the fund between an asset (risky) and a money account (riskless), and 2) the manager's performance is assessed by the value of the portfolio at maturity. He also assumes that the asset price follows geometric Brownian motion while the money account provides a constant interest rate.

Nicolosi derives the optimal portfolio strategy for the manager to maximise his or her expected utility subject to the given investment funds. The optimal strategy is dynamic portfolio choice decisions that maximizes the manager's expected utility at maturity. The intuition behind this solution is similar to the existing literature in which the optimal strategy manages the manager's risk-taking behaviour, given his or her level of risk aversion, in order maximise his or her expected utility at maturity. One crucial implication of Nicolosi's story is that, during the trading period, the manager should not hold a high allocation in the asset when his or her portfolio is above the benchmark. *Mutatis mutandis*, the manager should allocate his or her portfolio to the asset when his or her portfolio value is lower than that of the benchmark. Following the optimal strategy, thus, helps the manager to end up earning both fixed and variable payments as his or her portfolio value is higher than that of the benchmark at maturity — hence receiving the maximum utility.

The aim of this section is to investigate how close is actual behaviour to the optimal strategy of Nicolosi's model given the estimated risk aversion. Actual behaviour is then compared with other strategies to check the empirical validity of Nicolosi's model. I estimate the individual risk aversion — elicited from the actual choice — which best explains behaviour and use it to compute the optimal strategy and the portfolio value at maturity. In the next section, I describe Nicolosi's model. Section 5.3 describes the experimental design, Section 5.4 describes the econometric specification, Section 5.5 presents the results and analysis, and Section 5.6 discusses and concludes.

5.2 Nicolosi's model of the fund manager

Nicolosi explores the optimal strategy for the hedge fund manager who wants to maximise his or her expected utility subject to the investment funds. The hedge fund manager receives investment funds (W_0) from the investor and takes responsibility to invest in the financial market. There are two types of the financial market where the manager can invest, the risky asset market and the money market. The risky asset market trades an asset whose price (S) fluctuates over time t. The money market is riskless and gives a constant return (r). What the manager does then is to set portfolio allocation to be invested in the asset (θ) and in the money market ($1-\theta$).

The investor asks if the manager can achieve, at least, the benchmark (Y) from the investment funds over an investment period T.[2] This benchmark is the basis of the manager's performance measure and it is used to determine his or her payment; this payment will be explained later. The investor arbitrarily sets his or her benchmark as the value at maturity of a portfolio with a constant proportion (β) invested in the asset and a constant proportion ($1-\beta$) in the money market. The investor then sees what would happen to his or her benchmark value at maturity (Y_T) following this scheme.

The manager agrees on a contract, determined by the investor, which sets the investment period and the payment scheme for the manager. The investor pays the manager depending on what the manager achieves at maturity (W_T). The payment (Π) consists of two terms, a fixed and a variable payment. The fixed payment is a percentage (K) of the initial investment funds and the variable payment is a share (α) on the over-performance $(W_T - Y_T)^+$ relative to the benchmark. It is assumed that there is no penalty for the manager if he under-performs relative to the benchmark.[3] It follows that the manager will always earn non-negative payment irrespective of his or her performance. However the better is the performance compared to the benchmark, the higher is the payment for the manager.

Nicolosi assumes a Black-Scholes setting[4] with the asset price following standard geometric

[2] One may also refer this to as the "investment planning horizon".

[3] Despite this assumption, there may be various implementation to be taken for the case of under-performance considering that the investor pays a relatively high amount payment for the manager. For example, a percentage deduction to the fixed payment depending on the magnitude of the under-performance.

[4] Black-Scholes setting has following assumptions: a) there are two types of market, the asset market (risky) and the money market (risk-free), b) asset pays no dividend and there is no transaction cost in the market, c) asset price reflects all information in the asset market, d) asset price is exogenous to all agents, e) it is possible to borrow and lend cash at riskless rate as well as doing short-selling, f) the asset price change is random with known parameters, and g) it is possible to buy and to sell asset at any time. This assumption is important in the model in order to draw stochasticity of the asset price.

Brownian motion. We can write this as: $dS_t = S_t\,(\mu dt + \sigma dZ_t)$; where S_t is the asset price at time $t \in [0\ T)$, μ and σ are trend and volatility of the asset price respectively, and Z is a standard Brownian motion which follows $N\,(0, 1)$. The asset price follows a geometric Brownian motion, hence it is defined as:

$$S_t = S_0 \exp^{\left(\mu - 0.5\sigma^2\right)t + \sigma V_t} \tag{5.2.1}$$

where $V_t = Z_t d_t^{0.5}$ is the increment of a Wiener process[5] and S_0 is the initial asset price. Both the manager and the investor are aware of this process and its parameters.

Given the contract, the investor will pay the manager with a linear combination of the *fixed* and the *variable* payment which can be written as: $\Pi = K + \alpha\,(W_T - Y_T)^+$. The first term is the fixed payment (K) and the second term is the variable payment where α is a proportion of the positive underlying managed portfolio minus the benchmark at maturity $(W_T - Y_T)^+$. So the higher is the $(W_T - Y_T)^+$, the higher is the manager's payment.

The manager is assumed to be an Expected Utility (EU) agent who maximises his or her expected utility from the payment in managing the investment fund. The model assumes that the utility function of the manager is that of constant relative risk aversion (CRRA) with a parameter risk aversion γ.[6] In addition, it assumes that the manager is strictly risk-averse, so that $\gamma > 0$. Therefore the manager's problem is written as follows:

$$\max_{W_T} E\left[u\left(\alpha\,(W_T - Y_T)^+ + K\right)\right]\ st\ E\left[\frac{\xi_T}{\xi_0} W_T\right] = W_0 \tag{5.2.2}$$

where ξ is the state price density[7] which is defined as $\xi_T = \exp^{-\left(r + 0.5X^2\right)T - XZ_T}$ and $\xi_0 = 1$ – where $X = \frac{(\mu - r)}{\sigma}$ and $X > 0$. Although Equation 5.2.2 is a static problem, it is maximised through optimising the dynamic problem throughout the investment period by setting the optimal allocation θ^* subject to W_t.[8] Crucial to this approach is to define the optimal portfolio at maturity (W_T^*). Carpenter (2000) proposes the solution of this

[5] Wiener process (V_t) has the following properties: a) it is continuous, b) its change process is independent of the previous values, c) its increment process follows $N\,(0, d_t)$, d) $V_0 = 0$.

[6] See Appendix E.1 for the specification of CRRA utility function.

[7] State price density contains important information on the behaviour and expectations of the market (Hardle and Hlavka 2009). It follows a log-normal distribution in the Black-Scholes setting.

[8] Equation 5.2.2 is the implication of the martingale approach used in the model which decomposes a dynamic optimisation problem $\max_{\theta} E\left[u\left(\alpha\,(W_T - Y_T)^+ + K\right)\right]$ *s.t.* W_t into a static optimisation problem as in Equation 5.2.2. This determines the optimal condition at maturity. Next step is to find the portfolio strategy that leads to the optimal condition at maturity. This approach was notably developed by Pliska (1986), Karatzas *et al.* (1987), Cox and Huang (1989) among others.

problem in which W_T^* depends on Y_T, since the manager would never want $W_T \in (0, Y_T]$, and the realisation of ξ_T. It is given by:

$$W_T^* = \left\{ \left[I\left(\frac{\lambda^* \xi_T}{\alpha}\right) - K \right] \frac{1}{\alpha} + Y_T \right\} I_{\{\xi_T \leq \xi^*\}} \tag{5.2.3}$$

where $I(X) = (u')^{-1}(x)$ is the inverse function of the marginal utility and $I_{\{.\}}$ is the indicator function over $\{.\}$ and $\hat{\xi}$ is the threshold state price density. As a part of the solution, there exists a unique Lagrange multiplier $\lambda^* > 0$ to ensure that $E\left[\frac{\xi_T}{\xi_0} W_T\right] = W_0$ is satisfied for any $W_T^* > Y_T$.

Proposition 1 of Nicolosi's model proposes the optimal portfolio strategy throughout the investment period (W_t^*) that leads to the optimal portfolio at maturity (W_T^*). Given the manager's risk aversion γ, the optimal portfolio strategy W_t^* for any $\beta \leq \beta_m$ — where $\beta_m = \frac{X}{\sigma}$ — is:

$$W_t^* = \frac{1}{\xi_t} E_t \left[\xi_t W_t^* \right] \tag{5.2.4}$$

where $E_t[.]$ is the expectation of the optimality conditional to the information at time t which is ξ_t. Since ξ follows Markovian process, for which the future probability is determined by its most recent value, we can rewrite ξ_T as:

$$\xi_T = \xi_t \exp^{-\left(r + 0.5X^2\right)(T-t) - X(Z_T - Z_t)} \tag{5.2.5}$$

The corresponding optimal strategy to achieve W_t^* as in Equation 5.2.4 given the manager's risk aversion γ is:

$$\begin{aligned} \theta_t^* = \theta^M + \frac{\beta_m}{W_t^*} \Bigg(& -\frac{1}{\gamma} C_2(t) N\left(d_2(t, \xi_t)\right) + \left(\frac{\beta}{\beta_m} - \frac{1}{\gamma}\right) C_3(t) \xi_t^{-\frac{\beta}{\beta_m}} \Phi\left(d_3(t, \xi_t)\right) \\ & + \frac{C_1(t) \xi_t^{-\frac{1}{\gamma}} \exp^{-0.5 d_1(t, \xi_t)^2}}{X\sqrt{2\pi(T-t)}} + \frac{C_2(t) \exp^{-0.5 d_2(t, \xi_t)^2}}{X\sqrt{2\pi(T-t)}} + \frac{C_3(t) \xi_t^{-\frac{\beta}{\beta_m}} \exp^{-0.5 d_3(t, \xi_t)^2}}{X\sqrt{2\pi(T-t)}} \Bigg) \end{aligned} \tag{5.2.6}$$

where $\theta^M = \frac{\beta_m}{\gamma}$ is the Merton's strategy (1971) in the dynamic optimisation problem without compensation scheme and $\Phi(.)$ is the cumulative distribution function (*cdf*) of the normal distribution. What C_1, C_2, C_3, d_1, d_2 and d_3 mean are defined in Appendix E.2. All parameters in Nicolosi's model are pre-determined except the risk aversion γ; both λ^*

and $\hat{\xi}$ are solved from the solution to the final optimal portfolio (W_t^*) as in Equation 5.2.3. Therefore this section reports on an experiment to see how close the subjects' choices are, of the θ_t, to those optimal choices as in the theory and elicit the risk aversion γ from the subjects' choice.

5.3 Experimental design

The actual experiment design differs in two aspects from the theoretical design: a non-consequential and a consequential difference. The non-consequential difference is that the experimental design implements a discrete approximation to the continuous time problem, due to computer system limitation. Each discrete time step has a length $dt = 0.1$ second — hence the asset price changes every 0.1 second. The consequential difference is that the subjects were allowed to allocate their portfolio in the asset market (θ) only between 0% and 100%. By this, the subjects were not allowed to short-sell so avoiding a large negative payment for the subjects. However, the theory allows $-\infty < \theta < \infty$.

There were ten problems in the real experiment, all of the same type; the number of problems was chosen arbitrarily considering the experiment duration. It was preceded by two practice problems. The subjects were given paper and on-screen instructions, and a simulation practice to generate the actual asset price with adjustable parameters (μ and σ) before going on to practice session.[9] They were informed (in non-technical terms) that the asset price followed geometric Brownian motion, and were presented with as many simulations as they liked of such motion. Each simulation lasted for one minute; subjects could see how as many simulated asset price path as they wanted. After they were clear of what being asked to do and of how the asset price is generated, they started the practice session; after that, they started the real experiment. At the beginning of every problem, subjects were told all parameters for that problem ($S_0, K, \alpha, T, t, \beta, \mu, \sigma, W_0, r$); the initial price S_0, initial wealth W_0, and interest rate r are always 25 ECU, 100 ECU, and 0 ECU respectively in every problem. They were also given six examples of the asset price chart for given parameters in every problem. Given all these information, subjects were asked to set their θ_0 before the trading period.

Subjects were shown all update information during trading (the managed portfolio value in the asset, in the money account and in total, the benchmark value, the asset price, trading time and portfolio allocation in both asset and money market). They adjusted their portfolio allocation in the stock market using a slider. In addition, short instructions and the parameters used were displayed on the trading screen. They could start trading

[9] The paper instructions can be seen in Appendix E.10

anytime they wished by clicking the "**START**" button. Each problem lasted for one minute in the practice session and three minutes in the real experiment. In addition, subjects were shown their performance (the managed portfolio value, the benchmark value and the payoff) by the end of every problem.

Monetary incentives were provided in accordance with the theory. One problem from the main experiment was randomly drawn for the subjects' payment. Subjects were asked to draw a disk themselves from a closed bag containing the numbered disks from 1 to 10 — this identifies the problem number. The conversion rate is £1:3 ECU rounded up to the nearest 5 pence. The payment then will be added to a show-up fee of £3.

The experiment was conducted in the EXEC Lab, University of York. Invitation messages were sent through *hroot* (Hamburg registration and organization online tool) to all registered subjects in the system. 73 university members participated in this experiment: 46 males and 27 females. Composition of their educational degree was: 49 subjects were bachelor, 15 subjects were master, 7 subjects were PhD, 1 subject was diploma and 1 subject did not report his or her educational degree. I targeted the subjects who were or had been enrolled in the specific study that teaches finance and/or Brownian motion (e.g. Economics, Finance, Physics, Mathematics, and Statistics). Most of them (48 subjects) had participated in at least one economic experiment prior to this experiment. The average payment to the subjects was £8.1 and the average duration of the experiment (including reading the instructions) was around one and quarter hours. Communication was prohibited during the experiment. The experimental software was written (mainly by Alfa Ryano) in *Python* 2.7.

5.4 Econometric specification

I use maximum likelihood to estimate the parameter of the model — risk aversion (γ), estimating subject by subject. Maximum likelihood requires a specification of the stochastic nature of the data to capture the noise or error in the subjects' choice (θ_t). I assume this error is independent in every period during trading (t). Since the optimal choice (θ_t^*) takes any values, I consider a normal distribution to specify the stochastic story to account for this case. I assume that the choice of θ_t is normally distributed with mean θ_t^* (so that there is no bias) and standard deviation ς; I will report $s = \frac{1}{\varsigma}$ which indicates the precision. My estimation takes into account the difference between the model and the actual the latter is bounded between 0 and 1 — as I have explained above.

Before I turn to the specification of the log-likelihood function, I introduce further notations which will be used in the estimation to create an interval around θ_t since the log-likelihood function is continuous, while the actual choices were discrete, with steps of 0.1 second:

$$\begin{aligned} \theta_t^+ &= \theta_t + 0.005 \\ \theta_t^- &= \theta_t - 0.005 \end{aligned} \tag{5.4.1}$$

Given these notations, the log-likelihood function finds the probability that θ_t lies within θ_t^+ and θ_t^- for any given γ (risk aversion). Under this specification, the contribution to the likelihood of an observation θ_t is:

$$\begin{gathered} \theta_t = 0 \Leftrightarrow \Phi\left(\theta_t^+, \theta_t^*, \tfrac{1}{s_1}\right) \\ 0 < \theta_t < 1 \Leftrightarrow \Phi\left(\theta_t^+, \theta_t^*, \tfrac{1}{s}\right) - \Phi\left(\theta_t^-, \theta_t^*, \tfrac{1}{s_1}\right) \\ \theta_t = 1 \Leftrightarrow 1 - \Phi\left(\theta_t^-, \theta_t^*, \tfrac{1}{s_1}\right) \end{gathered} \tag{5.4.2}$$

where Φ is the *cdf* of a normal distribution with parameters θ_t^* (mean) and $\frac{1}{s_1}$ (standard deviation) given an observation θ_t. For this specification, I estimate γ_1 (risk aversion) and s_1 (precision).

I also estimate using the average dataset. This is addressed to minimise the noise since the discrete time step (t) is quite fast (0.1 second). By this, I take an average of the dataset on every second, excluding the initial decision which remains as a single data — that is every 10 discrete time step (t). I denote subjects' choice as $\bar{\theta}_i$ in this specification where i is the average discrete time step. Given this specification, the contribution to the likelihood of an observation $\bar{\theta}_i$ is:

$$\begin{gathered} \bar{\theta}_i = 0 \Leftrightarrow \Phi\left(\bar{\theta}_i^+, \bar{\theta}_i^*, \tfrac{1}{s_2}\right) \\ 0 < \bar{\theta}_i < 1 \Leftrightarrow \Phi\left(\bar{\theta}_i^+, \bar{\theta}_i^*, \tfrac{1}{s_2}\right) - \Phi\left(\bar{\theta}_i^-, \bar{\theta}_i^*, \tfrac{1}{s_2}\right) \\ \bar{\theta}_i = 1 \Leftrightarrow 1 - \Phi\left(\bar{\theta}_i^-, \bar{\theta}_i^*, \tfrac{1}{s_2}\right) \end{gathered} \tag{5.4.3}$$

where $\bar{\theta}_i^+ = \bar{\theta}_i + 0.005$ and $\bar{\theta}_i^- = \bar{\theta}_i - 0.005$. Let me call these two specifications as *Nicolosi 1*, following the specification in Equation 5.4.2, and *Nicolosi 2*, following the specification as in Equation 5.4.3. As with previous specification, I estimate γ_2 (risk aversion) and s_2 (precision) in this specification.

One may think of other stochastic assumptions to underlie estimation. For example, I could use a beta distribution to specify the stochastic of the subjects' choices since they are bounded between 0 and 1. However, I start simple in this section with a normal distribution specification.

To give a proper assessment to Nicolosi's model, I fit the data additionally assuming both random and risk-neutral choices. The former (random choice) assumes that the choice of

θ_t is random following a normal distribution. The latter (risk-neutral choice) assumes that the choice of θ_t follows risk-neutral behaviour. Theoretically, the risk neutrality returns either $-Inf$ or Inf, depending on the asset price change. Here I assume that θ_t is 1 if the asset price goes up, otherwise 0 if the asset price goes down. Note crucially, neither of these alternatives involves parameter risk aversion γ.

Again, I consider a normal distribution to specify the stochasticity for both random and risk-neutral choices. I also estimate using both all observations and the average dataset. The contribution to the log-likelihood for these specifications adopts Equation 5.4.2 and 5.4.3 as appropriate. For these specifications, let me call the random choice specification as *Random 1* (for the all-observation estimation) and *Random 2* (for the average-dataset estimation), and *Risk Neutral 1* (for the all-observation estimation) and *Risk Neutral 2* (for the average-dataset estimation) for risk-neutral choices.[10]

5.5 Results and analyses

One main purpose of this section is how well Nicolosi's model explains the subjects' choice compared to other strategies (random and risk-neutral choices). The analyses for this purpose use all observations and average dataset from the real experiment. The former sees 1,801 decisions while the latter sees 181 decisions in each problem for each subject. However, the risk-neutral choice will see 1,800 and 180 decisions respectively, excluding the initial decision, since it is drawn following the realisation of the price change.

Additionally, I develop a simple strategy from a regression model using variables in Nicolosi's model. This is a simplification of the theory as in Equation 5.2.6; hereinafter referred to as *Simple 1* (for the all-observation estimation) and *Simple 2* (for the average-dataset estimation). As previously, I compare Nicolosi's model and this simple strategy given the estimated risk aversion γ from both the all-observation and average-dataset estimations.

Before going on the main analyses, I estimate the individual risk aversion and precision in *Nicolosi 1* and *Nicolosi 2*, which can be found in Appendix E.3 and E.4. The results between both estimates show that estimate in *Nicolosi 2* returns less risk-averse and higher precision on average than that of estimates in *Nicolosi 1*.[11] This can be a further point of interest, but I take this merely as a consequence of using different approach since the main purpose of this study is to test the empirical validity of Nicolosi's model.

[10] I use the *patternsearch* routine in *Matlab* to maximise the log-likelihood function in all specifications.

[11] The average estimated risk aversion in Nicolosi 1 is 2.3353 compared to 0.5778 from the result in Nicolosi 2. Meanwhile, the average estimated precision Nicolosi 1 is 1.3556 compared to 1.4763 from the result in Nicolosi 2; with θ_t is bounded between 0 and 1 whereas θ_t^* is unbounded.

The estimated risk aversion then is used to compute the optimal portfolio value individually as in Equation 5.2.6. It should be the case that following the optimal strategy will return a better portfolio than that of the benchmark given the estimated risk aversion. There are 730 portfolios at maturity in total from 10 problems across 73 subjects. Appendix E.5 shows comparisons of the optimal portfolio and the benchmark values at maturity across all problems in both *Nicolosi 1* and *2*. Results from *Nicolosi 1* show that all of the optimal portfolios (730 portfolios) are better than that of the benchmark, meanwhile results from *Nicolosi 2* show that 711 optimal portfolios (97.4% of the total) are better than that of the benchmark given the individual estimated risk aversion. This shows that following the optimal strategy of Nicolosi is highly likely to end up with both payments (fixed and variable payments). In addition, the optimal portfolios return the higher utility than that of the actual portfolios — as shown in Appendix E.6.[12] This is hardly surprising.

5.5.1 Nicolosi's model vs random and risk-neutral strategies

Now we move on to the first comparison between Nicolosi's model and the random and risk-neutral strategies. The concern is to find the best-fitting strategy as the explanation of the individual behaviour in selecting the portfolio allocation between the asset and the money account with Nicolosi's model as the subject to test. I measure the goodness-of-fit by maximising the log-likelihood function, as specified in the Equation 5.4.2 and 5.4.3, but we need to correct the maximised log-likelihood for the number of parameters in each specification — Nicolosi's model has two estimated parameters while each of random and risk-neutral choices has one estimated parameter. In particular, I simulate 100 times each to generate the dataset for both random choices (*Random 1* and *2*), then taking its average log-likelihood.

I use the Akaike Information Criterion (AIC) as the measure of the goodness-of-fit to find the best explanation for each subject. The details of the judgment can be seen in Appendix E.7 and E.8. With the all-observation estimation — between *Nicolosi 1*, *Random 1* and *Risk Neutral 1* — of all 73 subjects, 49 subjects are better explained with *Risk Neutral 1* while the other 24 subjects are better explained with *Nicolosi 1*; *Random 1* is always the worst. Nevertheless, with the average-dataset estimation, 40 subjects are better explained with *Nicolosi 2* while the other 33 subjects are better explained with *Risk Neutral 2*; *Random 2* remains the worst. This finding obviously shows that subjects did

[12]This sees 535 optimal portfolios (73.29% of the total) returns the better utility than that from the actual portfolios from results in *Nicolosi 1*; and 665 optimal portfolios (91.1% of the total) returns the better utility than that from the actual portfolios from results in *Nicolosi 2*.

not randomise their choice in allocating their portfolio — that they followed some specific strategies for this. In particular, averaging the dataset improves the goodness-of-fit of Nicolosi's model. This may be the evidence that subjects somehow follow the optimal strategy as in Nicolosi's model but having difficulties to be as precise as the theory.

So far it is obvious that subjects did not randomise their choices, and that following the optimal strategy is highly likely to end up with a better portfolio than that of the benchmark. As it also has shown, Nicolosi's model receives the most empirical support on the average level. Nevertheless, subjects might find it difficult to follow the optimal strategy of Nicolosi, which involves sophisticated dynamic programming, given his or her risk aversion — calculating and implementing as precise as the optimal strategy. Results from estimated precision show that the subjects' choices are noisy compared to the optimal strategy in both *Nicolosi 1* and *2*; with average estimated precisions are 1.3556 (or standard deviation 0.7377) and 1.4763 (or standard deviation 0.6774) respectively. However, they must respond to some variables shown on the screen to determine their choice.

5.5.2 Nicolosi's model vs the simple strategy

Building on the previous results, I try to explore the determinants of the subjects' choice in a simple way using a regression model. Following Nicolosi, the portfolio allocation in the asset (θ_t) should not be constant, as the Merton's strategy $\left(\theta^M = \frac{\beta_m}{\gamma}\right)$, when the managed portfolio value is lower than that of the benchmark during trading in order to increase the chance to beat the benchmark at maturity. In particular, θ_t tends to be low, during trading, when the portfolio value (W_t) is higher than that of the benchmark (Y_t), *vice versa*. So I involve the difference between the managed portfolio and the benchmark ($W_t - Y_t$) in the regression model; I denote this as Δ_t. In addition, I also involve the asset price (S_t) and the benchmark value (Y_t) since they were shown to the subjects in the experimental interface — I denote $\bar{\theta}, \bar{S}, \bar{Y}$ and $\bar{\Delta}$ for variables used in *Simple 2*. The regression results from *Simple 1* and *Simple 2* are as follows:[13]

$$\theta_t = \underset{(0.129)\,*}{43.649} - \underset{(0.005)\,*}{0.0145}\,S_t + \underset{(0.002)\,*}{0.115}\,Y_t - \underset{(0.002)\,*}{0.008}\,\Delta_t \tag{5.5.1}$$

$$\bar{\theta}_i = \underset{(0.401)\,*}{43.672} - \underset{(0.017)\,**}{0.042}\,\bar{S}_i + \underset{(0.007)\,*}{0.115}\,\bar{Y}_i - \underset{(0.004)\,***}{0.008}\,\bar{\Delta}_i \tag{5.5.2}$$

[13] I use a simple linear procedure in both regression models. Standard errors are in parentheses and *, **, *** denote the significance at 1%, 5%, and 10% respectively. All coefficients are jointly not equal to zero in both regression models. *Adjusted* R^2 in both models are 0.0094 and 0.0097 respectively, and the number of observations is 1,314,730 and 132,130 respectively.

I use percentage values for θ, t is time step and i is average time step. Overall results from both regression model above show that all independent variables are significant in determining the subjects' choice — the signs of the independent variables are identical. Both the asset price and the difference between the managed portfolio and the benchmark have a negative effect to the subjects' choice, meanwhile, the value of the benchmark has a positive effect to the subjects' choice. These results are sensible and intuitive. Overall, subjects tended to buy the asset when its price is low and to sell the asset when its price is high; to some extent, it is commonly known as "buy low, sell high" strategy. This strategy is possibly the most famous adage in making profits from the asset market. Moreover, subjects tended to hold the asset as they saw the benchmark value was high. Lastly, subjects were consistent with the theoretical prediction in which they were unlikely to hold the asset when their portfolio value was relatively far above the benchmark.

Building on the regression results, I then run the regression model individually using the same structure as in Equation 5.5.1 and 5.5.2. This is to give a comparison of which model to have a better explanation for each subject between Nicolosi's model and the simple strategy using their measure of the goodness-of-fit; I compare between *Nicolosi 1* and *Simple 1*, and between *Nicolosi 2* and *Simple 2*. Since the models have different specifications, hence different degree of freedom, I calculate the AIC to correct for differing degrees of freedom.[14] I compare them and have a conclusion accordingly for each subject.

Results from two estimation procedures (using all observations and the average dataset) produce a slightly different AIC conclusion. With the all-observation estimation, 37 subjects are better explained with Nicolosi's model; 36 subjects are better explained with the simple strategy. Meanwhile, with the average-dataset estimation, 36 subjects are better explained with Nicolosi's model; 37 subjects are better with the simple strategy. The details of the judgment can be seen in Appendix E.9.

5.6 Discussion and conclusion

This section examines Nicolosi's model (2018) by investigating the subjects' behaviour in order to follow the optimal strategy of Nicolosi in a controlled-lab experiment. Subjects act as if they are the hedge fund manager who takes a responsibility to manage the investor's funds. The manager agrees on a contract, determined by the investor, who pays the manager with a two-term payment: the fixed payment and the variable payment, where the variable payment is a share-based on the over-performance with respect to the specific

[14]The AIC is given by $2k - 2LL$, where k is the number of estimated parameters and LL is the log-likelihood.

benchmark.

I follow Nicolosi's design in which there are two markets for the manager to invest: the asset market and the money market. The asset price follows geometric Brownian motion and subjects were aware of all parameters used in the experiment. However, there are non-consequential and consequential differences in the experimental setup from the theoretical design. The former relates to the computer limitation to implement the continuous time in generating the asset price, hence I use the discrete approximation to the continuous time with an increment of 0.1 second. The latter restricts the maximum portfolio allocation between the asset and the money account. This is addressed to prevent the subjects from a negative payoff from short-selling since it may have an unlimited loss.

To give a proper assessment of the empirical validity of Nicolosi's model, I compare its optimal choice, given the estimated subjects' risk aversion, with two alternative strategies. First, I compare the optimal choice of Nicolosi's model with random and risk-neutral choices. The random choice generates θ (portfolio allocation in the asset market) randomly following a normal distribution while the risk-neutral choice generates θ as if the subject was a risk-neutral agent; put everything on the asset if the asset price goes up, otherwise nothing if the asset price goes down. Second, I compare the optimal choice of Nicolosi's model with the simple strategy, developed using a regression model. One obvious conclusion from the first assessment is that subjects did not randomise their choice. They followed some specific strategies to maximise their utility from the experiment. Of all subjects, 24 subjects are better explained with Nicolosi's model while the other 49 subjects are better explained with risk-neutral choice using all the observations. We get a different conclusion if we use the average dataset. Of all subjects, 40 subjects are better explained with Nicolosi's model; the other 33 subjects are better explained with the risk-neutral choice.

Building on the previous results, I then develop a regression model to provide the simple strategy in which subjects may plausibly have followed. With this simple strategy, one tends to hold the asset when its price is low and to sell the asset when its price is high. In addition, one manages its portfolio value depending on the benchmark value and the difference between the managed portfolio and the benchmark. I compare Nicolosi's model with this simple strategy individually — as with the previous analysis. The comparison sees that 37 subjects are better explained with Nicolosi's model, 36 others are better explained with the simple strategy, using all observations. If we use the average dataset, we get slightly different results: of all subjects, 36 subjects are better explained with Nicolosi's model, while 37 others are better explained with the simple strategy.

Although the optimal strategy of Nicolosi ensures a high possibility to end up with both fixed and variable payments, hence the maximum utility, subjects found it difficult to follow. As it has shown, the subjects' choices are noisy compared with the optimal strategy. One may argue that subjects could have more precise computation if they were well accommodated in the experiment since Nicolosi's optimal strategy involves sophisticated dynamic programming. For example, we could ask the subject to specify their own strategy to be implemented during trading at the beginning of each problem, and they are free to adjust their strategy at any time. Will it improve the empirical validity of the theory? I may not think so because it depends on how subjects understand the random process in the asset price, hence determining the benchmark value.

Alternatively, we could go further with other models within similar substantial framework of Nicolosi's model. Among these are Nicolosi *et al.* (2018) and Herzel and Nicolosi (2019). Both provide the optimal solution for the fund manager, similar to Nicolosi (2018), who invests in one riskless asset and several risky assets. However the former assumes that there is no fixed payment, instead the manager is compensated with implicit incentives as shown in Chevalier and Ellison (1997). By this the asset under management is multiplied if the manager performs well due to the inflow in the investor's funds, otherwise a part of the asset under management is withdrawn. Other possibility is to model the subjects' choice assuming one preference function within either risk or ambiguity frameworks as shown in He and Zhou (2011) and Ahn et al. (2014), though they do not take into account the payment scheme for the fund manager; but they are not restricted only for the risk-averse agent case. Nevertheless, Nicolosi's model receives strong empirical support in explaining the subjects' behaviour. In addition, the subjects follow the intuitive prediction of Nicolosi's model where the difference between the managed portfolio and the benchmark determines the subjects' choice.

Appendices

Appendix E.1 — Specification of CRRA utility function

Nicolosi's model assumes that the manager is utility maximiser specified with CRRA utility function. It can be written as:

$$CRRA : u(x) = \frac{x^{1-\gamma}}{1-\gamma}; \gamma > 0, \gamma \neq 1$$

The manager is assumed to be strictly risk-averse and the function is undefined when $\gamma = 1$. However I apply CRRA utility function so it is able to accommodate when $\gamma = 1$ as follows:

$$CRRA : u(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}; \gamma \neq 1 \\ log(x); \gamma = 1 \end{cases}$$

Appendix E.2 — Definitions to Equation 5.2.6

Solution in Equation 5.2.6 contains some components (C_1, C_2, C_3, d_1, d_2 and d_3) where they are defined as follows:

$C_1(t) = \frac{1}{\alpha}\left(\frac{\lambda^*}{\alpha}\right)^{-\frac{1}{\gamma}} \exp\left[\left(\frac{1}{\gamma} - 1\right)\left(r + \frac{1}{2\gamma}X^2\right)(T-t)\right]$

$C_2(t) = -\frac{K}{\alpha}\exp\left[-r(T-t)\right]$

$C_3(t) = Y_0 A_T \exp\left[\left(\frac{\beta}{\beta_m} - 1\right)\left(r + \frac{1}{2}\beta\alpha X\right)(T-t)\right]$

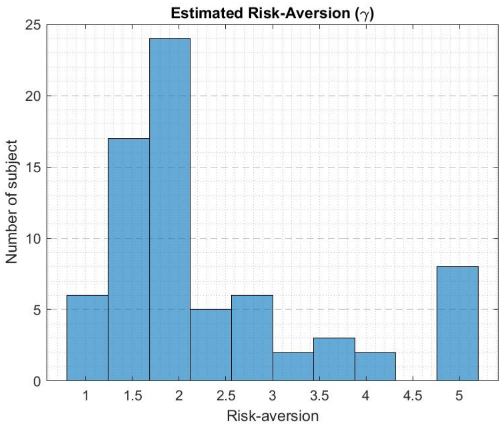
$d_1(t, \xi_t) = \frac{\ln\left(\frac{\hat{\xi}}{\xi}\right) + \left(r - \frac{1}{2}X^2\left(1 - \frac{2}{\gamma}\right)\right)(T-t)}{X\sqrt{T-t}}$

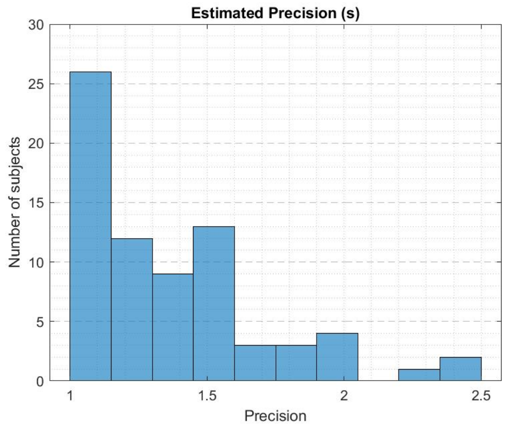
$d_2(t, \xi_t) = \frac{\ln\left(\frac{\hat{\xi}}{\xi}\right) + \left(r - \frac{1}{2}X^2\right)(T-t)}{X\sqrt{T-t}}$

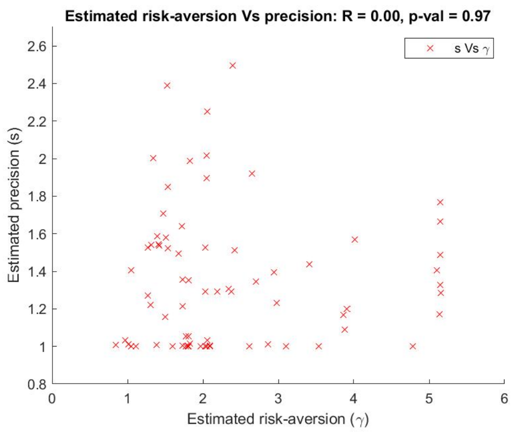
$d_3(t, \xi_t) = \frac{\ln\left(\frac{\hat{\xi}}{\xi}\right) + \left(r - \frac{1}{2}X^2\left(1 - \frac{2\beta}{\beta_m}\right)\right)(T-t)}{X\sqrt{T-t}}$

$A_T = \exp\left[\left(r + \frac{1}{2}\beta\sigma X - \frac{1}{2}\beta^2\sigma^2 - r\frac{\beta\sigma}{X}\right)T\right]$

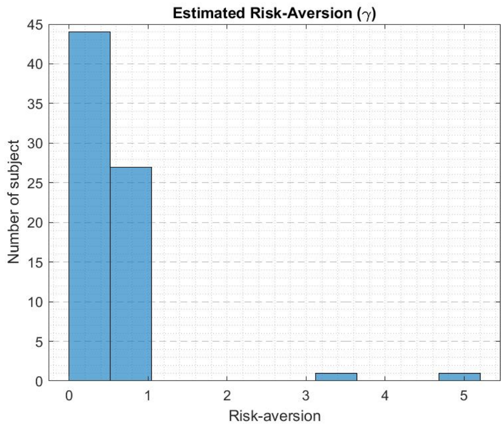
Appendix E.3 — Estimated individual risk aversion and precision in *Nicolosi 1*

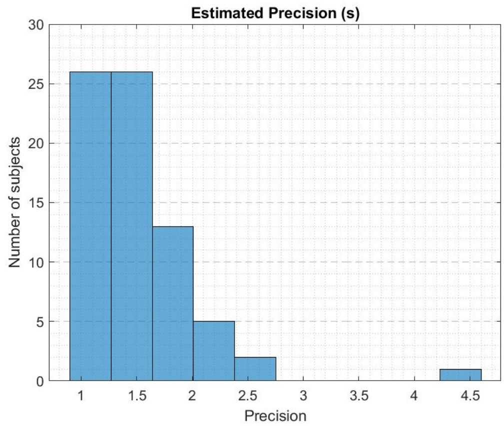


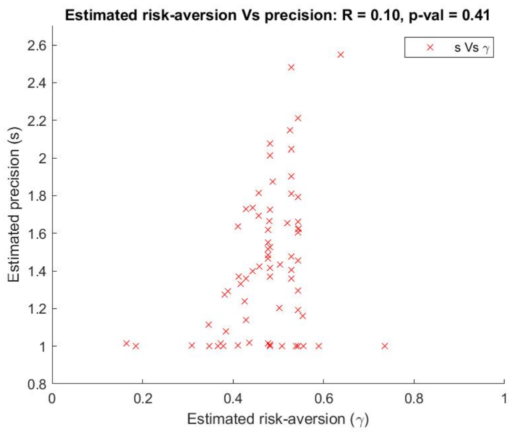




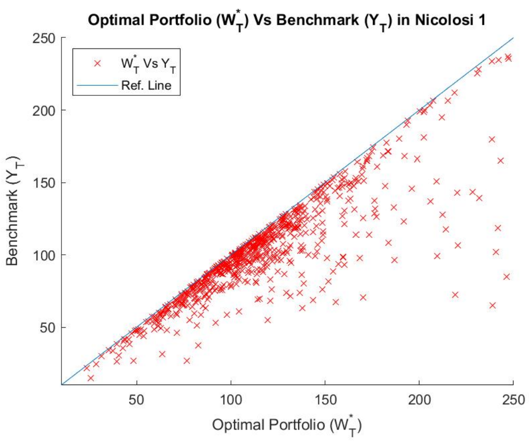
Appendix E.4 — Estimated individual risk aversion and precision in *Nicolosi 2*

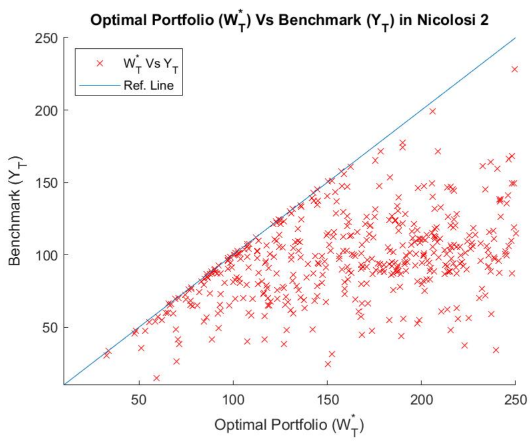




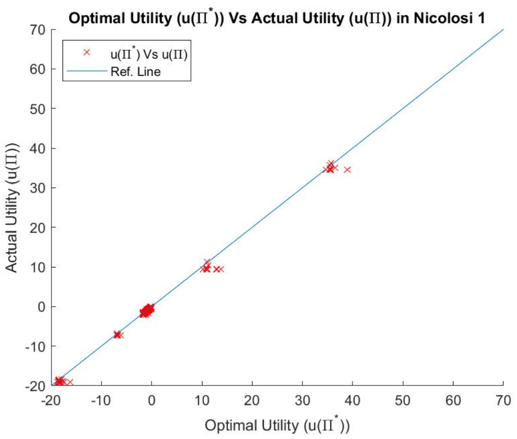


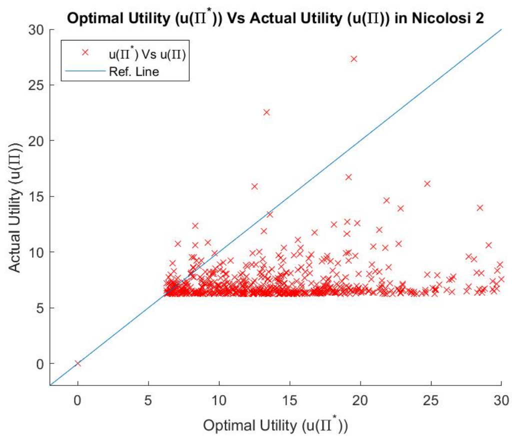
Appendix E.5 — Optimal portfolio vs the benchmark at maturity given the estimated risk aversion in *Nicolosi 1* and *Nicolosi 2*





Appendix E.6 — Optimal utility vs actual utility given the estimated risk aversion in *Nicolosi 1* and *Nicolosi 2*





Appendix E.7 — Individual AIC of Nicolosi's model vs random and risk-neutral strategies from the all-observation estimation

Sub.	Nicolosi 1	Random 1	Risk Neutral 1	Judgment
1	198,321.246	221,535.182	196,036.112	Risk Neutral 1
2	94,621.670	108,686.455	95,907.756	Nicolosi 1
3	197,009.678	217,617.004	191,808.286	Risk Neutral 1
4	192,582.322	216,253.368	189,739.869	Risk Neutral 1
5	122,903.085	138,235.154	119,387.990	Risk Neutral 1
6	189,562.792	214,315.596	189,052.356	Risk Neutral 1
7	84,860.704	99,729.582	99,462.459	Nicolosi 1
8	186,540.242	207,883.918	183,558.288	Risk Neutral 1
9	182,607.437	217,255.937	195,904.024	Nicolosi 1
10	106,135.443	120,671.830	115,009.369	Nicolosi 1
11	196,790.120	221,631.587	193,273.823	Risk Neutral 1
12	54,509.687	66,651.489	59,471.648	Nicolosi 1
13	152,116.133	169,983.115	153,503.171	Nicolosi 1
14	194,862.244	222,545.901	193,953.201	Risk Neutral 1
15	186,890.991	215,782.103	191,504.509	Nicolosi 1
16	189,016.232	207,505.756	181,871.617	Risk Neutral 1
17	148,867.582	164,916.650	147,838.409	Risk Neutral 1
18	176,018.071	151,683.890	138,768.587	Risk Neutral 1
19	163,426.340	180,998.317	161,666.209	Risk Neutral 1
20	189,294.637	207,850.753	184,832.636	Risk Neutral 1
21	160,497.587	177,756.712	155,350.027	Risk Neutral 1
22	186,078.435	208,115.105	183,769.752	Risk Neutral 1
23	170,643.478	188,881.973	170,967.209	Nicolosi 1
24	51,079.834	63,854.067	54,270.308	Nicolosi 1
25	187,165.955	209,552.850	186,916.125	Risk Neutral 1
26	126,078.497	141,947.310	116,499.274	Risk Neutral 1
27	151,085.506	168,136.912	145,798.463	Risk Neutral 1
28	175,190.721	192,642.030	169,266.254	Risk Neutral 1
29	157,320.343	174,516.666	146,924.193	Risk Neutral 1
30	190,712.679	211,741.269	186,811.810	Risk Neutral 1
31	157,683.899	173,815.791	153,422.846	Risk Neutral 1
32	188,010.084	220,763.381	195,939.363	Nicolosi 1

33	171,356.837	188,166.312	163,075.761	Risk Neutral 1
34	180,645.237	207,768.161	186,878.163	Nicolosi 1
35	148,606.352	165,520.165	144,722.103	Risk Neutral 1
36	149,326.154	168,566.469	154,822.340	Nicolosi 1
37	115,501.088	131,262.478	114,856.663	Risk Neutral 1
38	164,300.775	179,758.909	163,826.794	Risk Neutral 1
39	105,318.745	119,110.311	105,075.899	Risk Neutral 1
40	193,010.105	215,980.369	191,613.115	Risk Neutral 1
41	193,079.490	215,507.103	191,125.544	Risk Neutral 1
42	102,158.038	117,273.322	102,023.025	Risk Neutral 1
43	191,534.105	217,478.581	192,924.714	Nicolosi 1
44	100,711.684	115,442.352	96,066.439	Risk Neutral 1
45	189,579.881	207,319.183	180,829.065	Risk Neutral 1
46	194,783.214	222,833.959	194,219.174	Risk Neutral 1
47	182,924.555	204,784.083	183,673.065	Nicolosi 1
48	116,701.608	132,051.666	113,366.554	Risk Neutral 1
49	124,055.332	139,689.192	119,136.480	Risk Neutral 1
50	164,785.441	182,718.537	161,453.783	Risk Neutral 1
51	161,217.997	180,715.868	163,137.731	Nicolosi 1
52	191,179.743	210,993.377	182,570.474	Risk Neutral 1
53	154,667.291	172,399.683	158,500.161	Nicolosi 1
54	180,204.121	198,634.060	176,427.731	Risk Neutral 1
55	128,189.903	143,954.123	130,924.244	Nicolosi 1
56	135,654.533	151,052.930	138,158.201	Nicolosi 1
57	170,254.851	193,062.692	175,979.028	Nicolosi 1
58	191,571.075	221,156.903	195,161.530	Nicolosi 1
59	130,256.125	146,583.512	119,740.229	Risk Neutral 1
60	190,167.843	211,890.509	190,014.183	Risk Neutral 1
61	202,235.566	223,559.600	197,466.484	Risk Neutral 1
62	172,044.525	191,046.174	170,416.059	Risk Neutral 1
63	90,625.379	104,704.284	94,595.255	Nicolosi 1
64	207,332.067	227,625.598	199,915.193	Risk Neutral 1
65	109,043.874	123,792.059	112,917.040	Nicolosi 1
66	194,080.405	222,086.179	197,334.587	Nicolosi 1
67	193,908.536	217,101.724	193,144.819	Risk Neutral 1
68	187,425.745	207,541.866	182,043.057	Risk Neutral 1

69	94,374.381	110,006.082	86,862.585	Risk Neutral 1
70	199,755.572	220,640.863	193,689.952	Risk Neutral 1
71	174,579.674	218,264.507	196,189.365	Nicolosi 1
72	127,173.519	142,300.792	126,914.538	Risk Neutral 1
73	191,689.817	210,673.852	185,153.444	Risk Neutral 1

Appendix E.8 — Individual AIC of Nicolosi's model vs random and risk-neutral strategies from the average-dataset estimation

Sub.	Nicolosi 2	Random 2	Risk Neutral 2	Judgment
1	19,645.443	22,396.753	19,707.556	Nicolosi 2
2	10,308.974	11,782.616	10,451.865	Nicolosi 2
3	19,654.644	21,920.540	19,323.457	Risk Neutral 2
4	19,110.957	21,726.471	19,102.646	Risk Neutral 2
5	12,721.576	14,567.587	12,584.114	Risk Neutral 2
6	18,881.490	21,386.711	18,992.737	Nicolosi 2
7	9,073.263	10,720.300	10,598.342	Nicolosi 2
8	18,432.750	21,020.618	18,455.694	Nicolosi 2
9	18,211.662	21,863.967	19,736.161	Nicolosi 2
10	10,910.544	12,372.939	11,842.976	Nicolosi 2
11	18,562.476	22,249.818	19,419.260	Nicolosi 2
12	7,072.808	8,472.030	7,463.668	Nicolosi 2
13	15,570.811	17,597.844	15,900.244	Nicolosi 2
14	19,276.702	22,519.796	19,518.378	Nicolosi 2
15	18,478.606	21,778.457	19,321.530	Nicolosi 2
16	18,698.295	20,953.837	18,266.338	Risk Neutral 2
17	15,867.467	17,826.628	15,956.145	Nicolosi 2
18	14,487.021	16,213.418	14,690.872	Nicolosi 2
19	16,512.099	18,530.058	16,428.442	Risk Neutral 2
20	19,139.755	21,299.027	18,857.811	Risk Neutral 2
21	16,221.765	18,209.440	15,863.659	Risk Neutral 2
22	18,577.961	20,952.206	18,524.010	Risk Neutral 2
23	17,297.021	19,396.443	17,560.452	Nicolosi 2
24	6,858.525	8,360.110	7,112.858	Nicolosi 2
25	18,743.614	21,263.016	18,818.953	Nicolosi 2
26	12,183.404	14,173.531	11,740.719	Risk Neutral 2
27	15,094.474	16,840.326	14,710.935	Risk Neutral 2
28	17,693.703	19,721.227	17,281.467	Risk Neutral 2
29	15,456.541	17,438.657	14,783.131	Risk Neutral 2
30	18,890.308	21,499.885	18,803.754	Risk Neutral 2

31	15,895.487	17,789.191	15,689.552	Risk Neutral 2
32	18,526.775	22,119.098	19,693.844	Nicolosi 2
33	17,015.108	18,982.365	16,477.604	Risk Neutral 2
34	18,015.064	20,749.880	18,891.320	Nicolosi 2
35	15,146.999	17,155.569	14,817.561	Risk Neutral 2
36	15,168.377	17,248.989	15,804.205	Nicolosi 2
37	11,896.929	13,691.741	11,961.519	Nicolosi 2
38	16,989.139	18,919.505	16,897.985	Risk Neutral 2
39	11,219.657	12,503.414	11,340.596	Nicolosi 2
40	19,240.664	21,731.483	19,315.266	Nicolosi 2
41	19,068.611	21,756.788	19,148.219	Nicolosi 2
42	10,929.215	12,372.552	10,954.997	Nicolosi 2
43	19,046.644	21,904.505	19,378.574	Nicolosi 2
44	10,393.924	12,028.812	10,090.863	Risk Neutral 2
45	18,747.958	20,850.652	18,217.787	Risk Neutral 2
46	19,584.232	22,184.205	19,521.625	Risk Neutral 2
47	18,272.897	20,726.031	18,539.222	Nicolosi 2
48	12,098.261	13,840.846	11,856.479	Risk Neutral 2
49	12,740.073	14,514.027	12,463.929	Risk Neutral 2
50	16,569.828	18,483.517	16,499.363	Risk Neutral 2
51	16,448.717	18,570.264	16,770.022	Nicolosi 2
52	18,932.483	21,183.288	18,280.755	Risk Neutral 2
53	15,785.945	17,666.946	16,236.848	Nicolosi 2
54	18,267.852	20,146.303	18,006.249	Risk Neutral 2
55	13,141.490	14,834.835	13,500.656	Nicolosi 2
56	14,167.084	15,896.999	14,434.206	Nicolosi 2
57	16,924.950	19,519.659	17,803.038	Nicolosi 2
58	19,007.689	22,125.966	19,604.083	Nicolosi 2
59	12,676.044	14,829.104	12,218.915	Risk Neutral 2
60	19,037.041	21,507.981	19,166.922	Nicolosi 2
61	20,077.660	22,429.210	19,884.740	Risk Neutral 2
62	17,388.911	19,482.250	17,368.394	Risk Neutral 2
63	10,421.870	11,906.986	10,742.597	Nicolosi 2
64	20,533.681	22,838.574	20,089.477	Risk Neutral 2
65	11,727.217	13,435.158	12,039.345	Nicolosi 2
66	19,164.185	22,282.204	19,823.653	Nicolosi 2

67	19,276.851	21,797.376	19,318.071	Nicolosi 2
68	18,828.910	20,978.008	18,379.339	Risk Neutral 2
69	10,400.811	12,102.199	9,721.980	Risk Neutral 2
70	19,798.275	22,282.207	19,464.585	Risk Neutral 2
71	16,337.286	21,896.816	19,652.329	Nicolosi 2
72	13,006.015	14,722.619	13,040.748	Nicolosi 2
73	19,224.107	21,620.434	18,838.633	Risk Neutral 2

Appendix E.9 — Individual AIC of Nicolosi's model vs the simple strategy from the all-observation and average-dataset estimations

Sub.	Nicolosi 1	Simple 1	Judgment	Nicolosi 2	Simple 2	Judgment
1	198,321.246	174,453.096	Simple 1	19,645.443	17,455.645	Simple 2
2	94,621.670	187,219.239	Nicolosi 1	10,308.974	18,728.059	Nicolosi 2
3	197,009.678	177,602.827	Simple 1	19,654.644	17,758.940	Simple 2
4	192,582.322	166,821.787	Simple 1	19,110.957	16,733.459	Simple 2
5	122,903.085	182,955.881	Nicolosi 1	12,721.576	18,304.908	Nicolosi 2
6	189,562.792	168,015.790	Simple 1	18,881.490	16,891.658	Simple 2
7	84,860.704	171,150.393	Nicolosi 1	9,073.263	17,002.175	Nicolosi 2
8	186,540.242	171,222.098	Simple 1	18,432.750	17,215.338	Simple 2
9	182,607.437	166,307.909	Simple 1	18,211.662	16,608.700	Simple 2
10	106,135.443	182,277.380	Nicolosi 1	10,910.544	18,228.427	Nicolosi 2
11	196,790.120	157,968.120	Simple 1	18,562.476	15,843.093	Simple 2
12	54,509.687	189,229.569	Nicolosi 1	7,072.808	18,861.851	Nicolosi 2
13	152,116.133	174,479.333	Nicolosi 1	15,570.811	17,457.335	Nicolosi 2
14	194,862.244	149,449.397	Simple 1	19,276.702	15,026.829	Simple 2
15	186,890.991	140,797.022	Simple 1	18,478.606	14,131.838	Simple 2
16	189,016.232	170,370.559	Simple 1	18,698.295	17,123.570	Simple 2
17	148,867.582	182,301.089	Nicolosi 1	15,867.467	18,154.031	Nicolosi 2
18	176,018.071	188,021.265	Nicolosi 1	14,487.021	18,768.873	Nicolosi 2
19	163,426.340	163,742.014	Nicolosi 1	16,512.099	16,425.107	Simple 2
20	189,294.637	182,459.465	Simple 1	19,139.755	18,084.611	Simple 2
21	160,497.587	180,933.397	Nicolosi 1	16,221.765	18,095.145	Nicolosi 2
22	186,078.435	172,963.107	Simple 1	18,577.961	17,361.759	Simple 2
23	170,643.478	183,885.162	Nicolosi 1	17,297.021	18,371.147	Nicolosi 2
24	51,079.834	189,411.817	Nicolosi 1	6,858.525	18,873.701	Nicolosi 2
25	187,165.955	176,686.211	Simple 1	18,743.614	17,630.047	Simple 2
26	126,078.497	173,352.631	Nicolosi 1	12,183.404	17,410.910	Nicolosi 2
27	151,085.506	180,397.210	Nicolosi 1	15,094.474	18,129.940	Nicolosi 2
28	175,190.721	176,939.549	Nicolosi 1	17,693.703	17,721.956	Nicolosi 2
29	157,320.343	153,376.336	Simple 1	15,456.541	15,409.923	Simple 2
30	190,712.679	170,171.255	Simple 1	18,890.308	17,041.947	Simple 2

31	157,683.899	184,829.529	Nicolosi 1	15,895.487	18,514.293	Nicolosi 2
32	188,010.084	161,722.966	Simple 1	18,526.775	16,250.788	Simple 2
33	171,356.837	166,172.225	Simple 1	17,015.108	16,676.372	Simple 2
34	180,645.237	170,523.007	Simple 1	18,015.064	17,085.953	Simple 2
35	148,606.352	181,124.271	Nicolosi 1	15,146.999	18,123.924	Nicolosi 2
36	149,326.154	173,965.530	Nicolosi 1	15,168.377	17,377.665	Nicolosi 2
37	115,501.088	185,358.890	Nicolosi 1	11,896.929	18,521.134	Nicolosi 2
38	164,300.775	190,106.051	Nicolosi 1	16,989.139	18,786.662	Nicolosi 2
39	105,318.745	188,589.613	Nicolosi 1	11,219.657	18,861.343	Nicolosi 2
40	193,010.105	179,141.643	Simple 1	19,240.664	17,959.728	Simple 2
41	193,079.490	175,035.633	Simple 1	19,068.611	17,448.843	Simple 2
42	102,158.038	187,359.316	Nicolosi 1	10,929.215	18,753.266	Nicolosi 2
43	191,534.105	168,380.417	Simple 1	19,046.644	16,883.941	Simple 2
44	100,711.684	184,714.380	Nicolosi 1	10,393.924	18,524.670	Nicolosi 2
45	189,579.881	156,042.604	Simple 1	18,747.958	15,683.010	Simple 2
46	194,783.214	154,996.682	Simple 1	19,584.232	15,558.910	Simple 2
47	182,924.555	179,923.901	Simple 1	18,272.897	18,060.958	Simple 2
48	116,701.608	188,101.443	Nicolosi 1	12,098.261	18,841.319	Nicolosi 2
49	124,055.332	179,661.868	Nicolosi 1	12,740.073	17,995.199	Nicolosi 2
50	164,785.441	182,127.024	Nicolosi 1	16,569.828	18,263.963	Nicolosi 2
51	161,217.997	180,868.857	Nicolosi 1	16,448.717	18,068.112	Nicolosi 2
52	191,179.743	150,357.056	Simple 1	18,932.483	15,120.175	Simple 2
53	154,667.291	181,851.840	Nicolosi 1	15,785.945	18,154.987	Nicolosi 2
54	180,204.121	176,885.073	Simple 1	18,267.852	17,667.756	Simple 2
55	128,189.903	183,068.355	Nicolosi 1	13,141.490	18,362.161	Nicolosi 2
56	135,654.533	187,990.605	Nicolosi 1	14,167.084	18,768.260	Nicolosi 2
57	170,247.571	177,850.007	Nicolosi 1	16,924.950	17,838.094	Nicolosi 2
58	191,571.075	157,387.741	Simple 1	19,007.689	15,819.855	Simple 2
59	130,256.125	170,843.543	Nicolosi 1	12,676.044	17,157.900	Nicolosi 2
60	190,167.843	183,068.432	Simple 1	19,037.041	18,190.008	Simple 2
61	202,235.566	174,855.835	Simple 1	20,077.660	17,534.646	Simple 2
62	172,044.525	178,631.531	Nicolosi 1	17,388.911	17,846.881	Nicolosi 2
63	90,625.379	186,551.648	Nicolosi 1	10,421.870	18,584.942	Nicolosi 2
64	207,332.067	171,978.988	Simple 1	20,533.681	17,291.396	Simple 2
65	109,043.874	190,880.268	Nicolosi 1	11,727.217	19,066.867	Nicolosi 2
66	194,080.405	164,270.808	Simple 1	19,164.185	16,518.488	Simple 2

67	193,908.536	177,807.706	Simple 1	19,276.851	17,709.987	Simple 2
68	187,425.745	171,412.721	Simple 1	18,828.910	17,112.860	Simple 2
69	94,374.381	185,129.275	Nicolosi 1	10,400.811	18,465.155	Nicolosi 2
70	199,755.572	167,452.215	Simple 1	19,798.275	16,821.980	Simple 2
71	174,579.674	117,201.218	Simple 1	16,337.286	11,732.197	Simple 2
72	127,173.519	186,266.268	Nicolosi 1	13,006.015	18,667.986	Nicolosi 2
73	191,689.817	170,526.544	Simple 1	19,224.107	17,077.428	Simple 2

Appendix E.10 — Instructions

centre for experimental economics

Instructions

Preamble

Welcome to this experiment and thank you for coming. Please read these Instructions carefully. They will help you to understand what the experiment is all about and what you are being asked to do during the experiment. This experiment gives you the opportunity to earn money which will be paid to you in cash after you have completed the experiment. However there is no participation fee in this experiment; what you earn in this experiment is what you will be paid. So you must take this experiment seriously. Your payment is described below and it will be added to a show-up fee of £3 that you will be paid independently of your answers.

The Experiment

This experiment is interested in your decision as a **fund manager** who manages the investor's funds. You are entrusted to manage his or her funds and then the investor will see if you can perform better than that of the investor's benchmark. What the investor's benchmark means will be explained later. There are two types of market that you can invest, the money market and the asset market. The money market is risk-free that gives you a constant interest rate, whereas the asset market is risky that trades assets continuously in time.

The asset price process in the asset market contains the Brownian motion. It means that the asset price changes randomly over time t. There are two parameters in the asset price process, the **Drift** (mean) and the **Scale** (sigma). Please notice that parameter **Drift** determines the **realisation** of the asset price by the maturity time, whereas parameter **Scale** determines the **fluctuation** of the asset price over all period. Those parameters remain constant in a particular problem. However the asset price is non-negative (either zero or positive) in this experiment. The asset price change therefore can be written as: $dS_t = S_t(\mu d_t + \sigma dZ_t)$; where S is the asset price, μ is the drift, σ is the scale and Z is the Brownian motion. There will be examples of the asset price process that help you to understand how it works given the parameters. Please do not ignore these examples! Moreover they are addressed to help you make your decision during the experiment.

You will be presented with a sequence of **10 problems**, all of the same type, in the main experiment, preceded by **2 practice problems**. Please do not waste the practice session! Each problem has its unique circumstances depending on its initial setup. In the beginning of each problem, you are endowed by an initial fund for you to manage in a certain time. You are told the initial asset price in the asset market and the interest rate in the money market. Given these information, you will be asked your initial allocation, of your portfolio, in each market. Then, you can adjust your allocation in both markets as a response to the current asset price in the asset market and the current benchmark portfolio.

You will receive **payoffs** depending on your performance with respect to the investor's benchmark by the end of each problem. The **benchmark** is a constant portfolio consisting of the proportions of the funds invested in the money market and in the asset market. These proportions are determined by the investor in the beginning of the problem and remain fixed throughout the problem. There are two types of the **payoff** that you will receive, a fixed payoff and an additional payoff. The fixed payoff is paid to you independently of your performance. The additional payoff is paid proportionally to the profit earned by you if the final portfolio you manage is higher than that of the final benchmark portfolio. You will be told these information in the beginning of each problem along with other information. Given this, your payoff function is:

$$\alpha\,(W_T - Y_T)^+ + K$$

where K is the fixed payoff and α is the proportion of positive margin of the final portfolio to the final benchmark $(W_T - Y_T)^+$ that you will earn—where W_T and Y_T are your portfolio and the benchmark portfolio respectively by the maturity time. All the currency used in this experiment is in ECU (Experimental Currency Unit). Therefore, you can earn more payoffs if your final portfolio is higher than that of final benchmark portfolio.

Here is to give you an example of how you will receive payoff in a particular problem:

Example: You are asked by the investor if you can optimise his or her funds in both money market and asset market. The initial fund that you can manage is 100 ECU. The investor has his or her benchmark to the portfolio allocation in both money market and asset market. He or she would allocate 0% of the initial fund to the money market, and 100% of the initial fund to the asset market. The money market return is 0%, whereas the asset market trades asset in continuously period. The investor would let his or her portfolio grows given his or her allocation to the maturity time. This will be the final benchmark portfolio and a basis to the fund manager's performance.

You will receive two types of payoffs as a part of your effort by the end of the problem: a fixed payoff and an additional payoff. The fixed payoff for you is 10 ECU. The additional payoff is 25% of the positive margin of your final portfolio minus the final benchmark portfolio. Given this information, you will start this problem by setting your initial portfolio allocation in each market. Then you can manage your portfolio by adjusting your portfolio allocation in each market for a given certain time—as an example, the total time in this problem is 3 minutes.

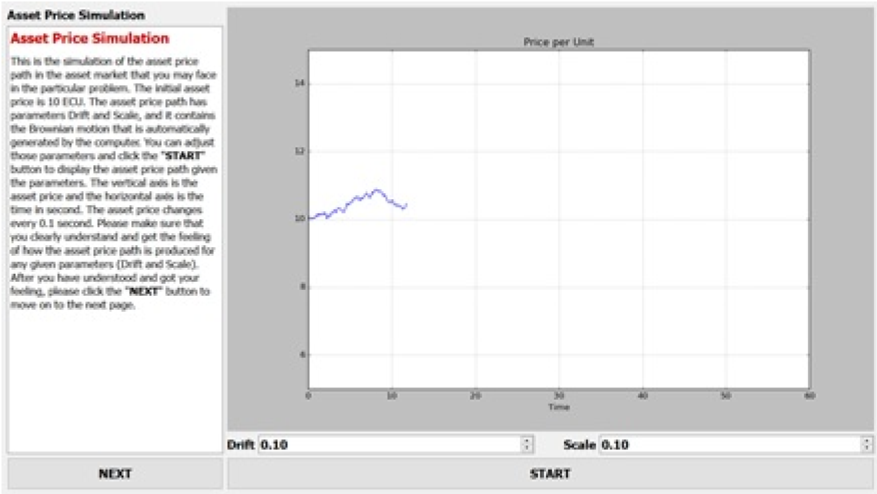
If, for example, your final portfolio is **190 ECU** and the final benchmark portfolio is **150 ECU** by the end of a problem. Your payoff is calculated as follow: $\mathbf{0.25(190 - 150) + 10 = 20}$ **ECU**. You receive both fixed payoff and additional payoff because you have managed to have a higher final portfolio than that of the final benchmark portfolio.

If, for example, your final portfolio is **190 ECU** and the final benchmark portfolio is **200 ECU** by the end of a problem. Your payoff, therefore, is **10 ECU**. You receive only a fixed payoff because you have failed to have a higher final portfolio than that of the final benchmark portfolio.

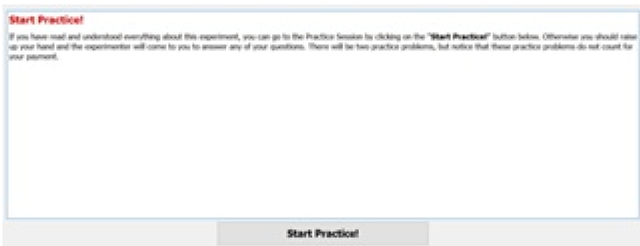
The Interface

When you arrive at the laboratory, you will find the screen displaying the EXEC logo. Do not touch the computer until all the participants have read the instructions. The screen remains inactive. When all have done so, the experimenter will let you know to go to the instructions screen. There are three instruction screens that will help you to understand the experiment.

After you think you are clear of what you are asked to do, you can practice the asset price simulation, which contains the Brownian motion, to get your feeling on how the asset price is generated. Below is the practice screen for the asset price simulation.

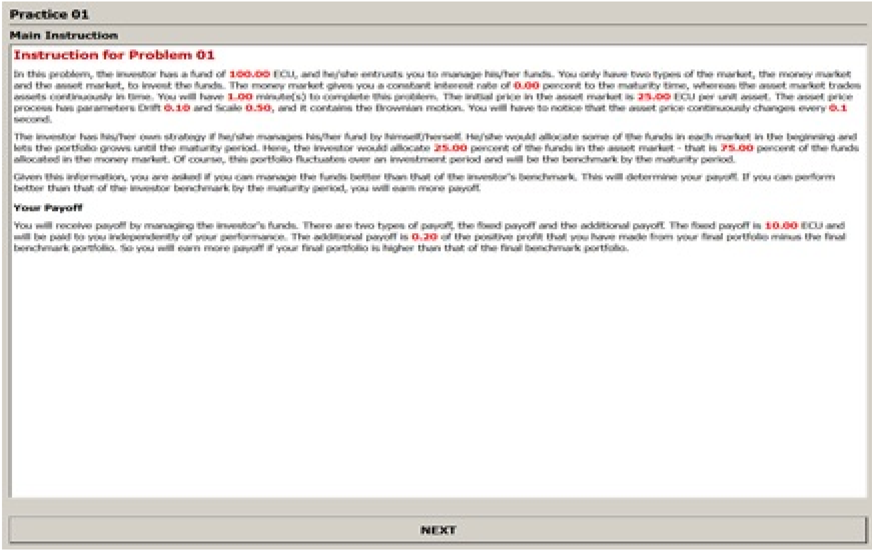


You can adjust both parameters Drift and Scale and click "**Start**" button to display the asset price path (which contains Brownian motion). There is no time limit for you in this simulation. Make sure you clearly understand and get the feeling of how the asset price path is produced for any given parameters. After you have understood and got your feeling, please click the "**NEXT**" button to move on to the next page (as shown in the figure below).

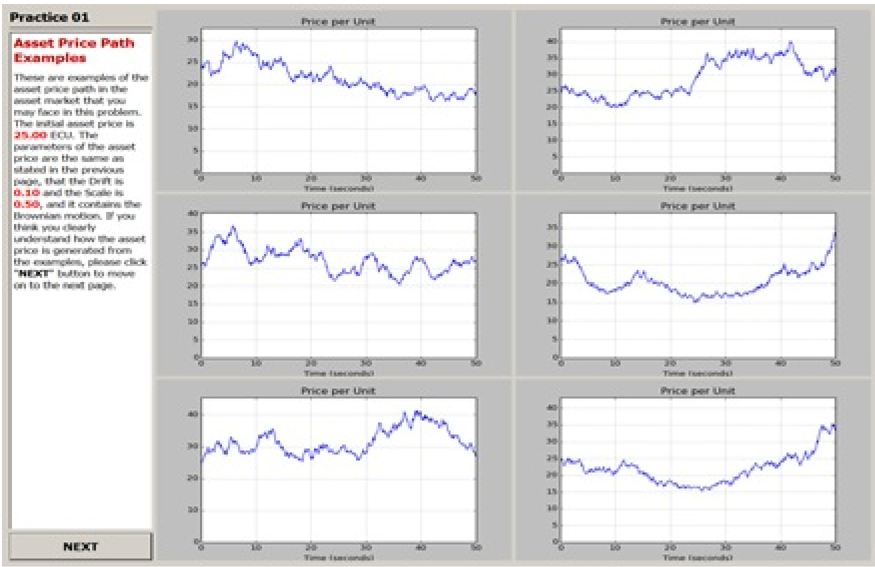


There will be two practice sessions for you before going on to the main experiment. But notice that these practice sessions do not count for your payment. You can continue to the practice session by clicking on the "**Start Practice!**" button if you think you are clear so far.

You will be told any necessary information in the particular problem. It includes the initial funds to manage, the interest rate in the money market, time maturity (in minute), the initial price per unit asset in the asset market, the investor's allocation strategy in each market, your payoffs and the asset price process parameters. Once you have understood the tasks and all information, you can continue by clicking the "**NEXT**" button (as shown in the figure below).

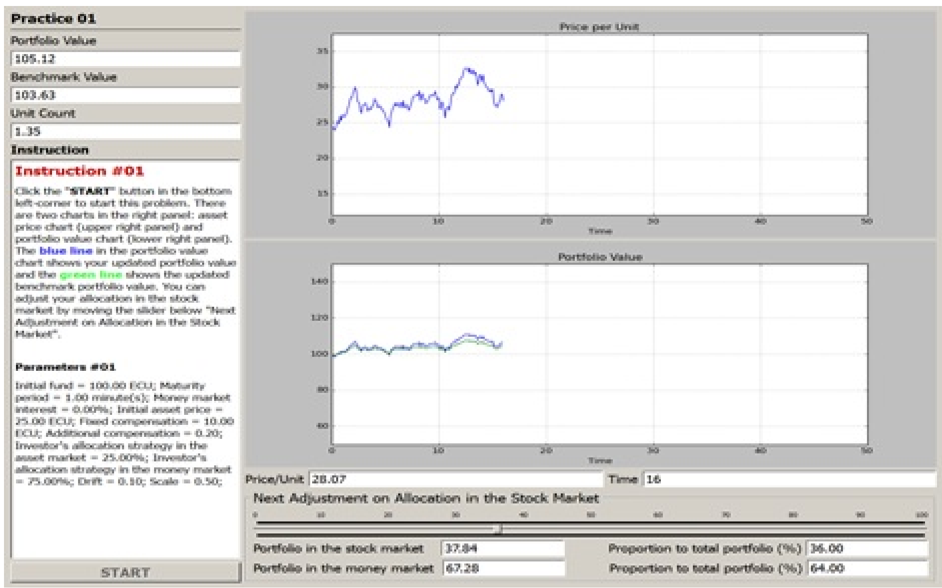


Next screen (the figure below) is examples of asset price path using the same parameters as stated in a particular problem (in the previous screen). Click "**NEXT**" button to move on to the next page if you are clear so far.

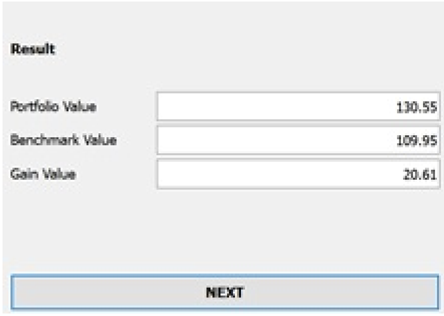


Then, in the next page, you will be asked your initial allocation of the funds to be invested in the asset market given all necessary information that you have acquired in the previous screen. You do so by making an adjustment in the "**Initial Allocation in the Asset Market**" box. Its default value is 50 percent, but you can adjust it up to two decimal places. However you can only set the initial allocation in the asset market between 0 and 100. Once you have decided your initial allocation, you can continue to have your practice by clicking the "**NEXT**" button as shown in the figure below.

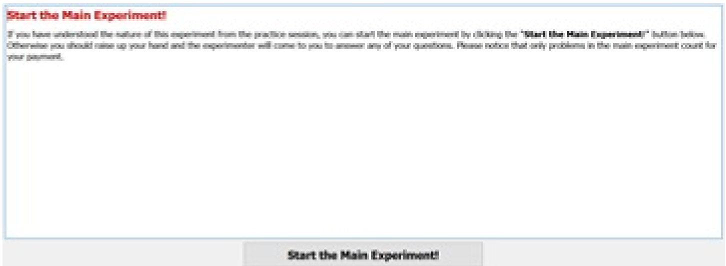
The next screen (the figure below) is the main screen of the practice session. On the left panel there are updating information in continuously period—your portfolio value, the benchmark portfolio value and the unit asset hold. There is a short instruction that tells you the parameter in the particular problem. On the right panel there are two figures that show you the asset price chart and the portfolio value chart continuously in time. The blue line in the portfolio value chart shows your updated portfolio value and the green line shows the updated benchmark portfolio value.



The "**START**" button will be inactive once you click that. You can adjust your portfolio allocation in the stock market by moving the slider below "**Next Adjustment on Allocation in the Stock Market**". Again, you can only adjust your allocation in the stock market between 0 and 100. Bottom right are the information of your portfolio in the stock market and in the money market, and their proportion to your total portfolio. Notice that the better the Portfolio Value than that of the Benchmark Value, the higher the payoff you will earn in this particular problem.



You will be told your gain value (Portfolio Value – Benchmark Value) after the problem is finished (as shown in the figure above). Please click the "**NEXT**" button to go to the next practice session and finish this session. If you have understood the experiment after finishing the practice session you can click the "**Start the Main Experiment!**" button (as shown in the figure below). Otherwise you should raise up your hand and the experimenter will come to you to answer any of your questions.



Notice that you cannot go back to the previous screen once you click the "**Start the Main Experiment!**" button. So please make sure that you clearly understand of what you are asked to do during the experiment. You will be told any necessary information and are asked to set your initial allocation in the asset market as in the practice session.

The Payment

It is important for you to take this experiment seriously because it determines your payment by the end of this experiment. Notice that you will earn cash from this experiment. Only problems in the main experiment will be basis of your payment. You will draw a disk by yourself from a closed bag containing the number disks from 1 to 10, each indicating the number of a problem. The drawn disk determine which problem to be a basis of your payment. The exchange rate of your payment is £1:3 ECU. That means if your payoff in a particular problem is 3 ECU, then you will receive £1 from this experiment. However this will be round up by 5 pence. Your payment in this experiment then will be added to a show-up fee of £3.

How Long Will the Experiment Last?

It is important for you to understand the problem carefully. You can start the experiment as you wish. However, as the timing for each problem is fixed, I estimate that the experiment will take at least 60 minutes of your time. Please notice that you cannot go back to the previous screen as you move on to the next screen, and any kind of communication is prohibited during this experiment.

Questionnaire

Subject Number:

Please provide us the following information about you.

Q. Sex: Male/Female (Cycle the right one)

Q. Age: What is your age?

Q. Ethnicity origin: Please specify your ethnicity. (Cycle the right one)

• White • Hispanic or Latino • Black or African American • Native American or American Indian • Asian / Pacific Islander • Other

Q. Education: What is the highest degree or level of school you have completed? If currently enrolled, highest degree received. (Cycle the right one)

• No schooling completed • Nursery school to 8th grade • Some high school, no diploma • High school graduate, diploma or the equivalent (for example: GED) • Some college credit, no degree • Trade/technical/vocational training • Associate degree • Bachelor's degree • Master's degree • Professional degree • Doctorate degree

Please answer also the following questions

Q. Are you currently a student? If so, in which level you are currently enrolled?

Q. What are you studying?

Q. Do you have any work experience in Economics? If so, for how long did/do you work in this field and which was/is your job title/titles?

Q. Have you participated in economics experiments in the past?

Q. Did you feel impatience during the experiment? (Cycle the right one)

1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Did you feel stress during the experiment? (Cycle the right one)

1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Which is your risk aversion level? From 1 to 5 the risk aversion level is increasing. (Cycle the right one)

1 2 3 4 5

Q. What did you like in the experiment?

Q. What you did not like in the experiment?

Q. Any suggestions for improvement?

Thank you for your participation!

Yudistira Permana

November 2018

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