DEVELOPMENT OF NEW METHODOLOGIES FOR THE WEIGHT ESTIMATION OF AIRCRAFT STRUCTURES

by

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others. This copy has been supplied on the understanding that it is copyright material and that no quotation from the thesis may be published without proper acknowledgement.
Declaration

Some parts of the work presented in this thesis have been published in the following articles. In each case, details of the contributions by myself and other authors are detailed, as well as which chapters the contents of the articles feature in:


My contributions: The work in this paper is all my own.

Contributions from other authors: Prof. Toropov and Dr. Querin provided supervision, feedback and general guidance. Lucy Agyepong contributed with information and data pertaining to system design within the fixed trailing edge.

Chapters based on this work: Part of the theory about ANFIS appears in the definition of the methodologies used in Chapter 3. The case study and results are part of Chapter 4.


My contributions: The work in this paper is all my own.

Contributions from other authors: Prof. Toropov and Dr. Querin provided supervision, feedback and general guidance.

Chapters based on this work: Part of the theory about NEFPROX appears in the definition of the methodologies used in Chapter 3. The case study and results are part of Chapter 6.

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Chapters based on this work: Part of the theory about NEFPROX appears in the definition of the methodologies used in Chapter 3. The case study and results are part of Chapter 6.
ABSTRACT

The problem of weight estimation in the aerospace industry has been acquiring considerably greater importance in recent years, due to the numerous challenges frequently encountered in the preliminary phases of the design of a new aircraft. This is the stage where it is possible to make design changes without incurring into excessive cost penalties. On the other hand, the knowledge of the design, of the relationships existing between the different variables and their subsequent impact on the final weight of the structure is very limited. As a result, the designer is unable to understand the true effect that individual design decisions will produce on the weight of the structure. In addition to this, new aircraft concepts end up being too conservative, due to the high dependency of current weight estimation methods to historical data and off-the-shelf design solutions.

This thesis aims at providing an alternative framework for the weight estimation of aircraft structures at preliminary design stages. By conducting a thorough assessment of current state-of-the-art approaches and tools used in the field, fuzzy logic is presented as an appropriate foundation on which to build an innovative approach to the problem. Different adaptive fuzzy approaches have been used in the development of a methodology which is able to combine an analytical base to the structural design of selected trailing edge components, with substantial knowledge acquisition capabilities for the computation of robust and reliable weight estimates. The final framework allows considerable flexibility in the level of detail of the estimate consistent with the granularity of the input data used. This, combined with an extensive uncertainty analysis through the use of Interval Type-2 fuzzy logic, will provide the designer with the capabilities to understand the impact of error propagation within the model and increase the confidence in the final estimate.
"If we worked on the assumption that what is accepted is true, then there would be little hope for advance"

-Orville Wright-

To both my grandmas, the kindest souls I know.
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7.8 Uncertainty characteristics for the type-2 fuzzy partitions of input and output variables within the aileron weight module.
**NOMENCLATURE**

Most of the symbols used in this thesis have different meanings in different chapters while others are only relevant to short sections of text. Below are listed those symbols which have a general meaning, however specific definitions will depend on the context.

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>A, a</td>
<td>Constants of proportionality</td>
</tr>
<tr>
<td>$A_{BOT}$</td>
<td>Cross sectional area for rib bottom beam section</td>
</tr>
<tr>
<td>$A_{TOP}$</td>
<td>Cross sectional area for rib top beam section</td>
</tr>
<tr>
<td>$A_{VERT}$</td>
<td>Cross sectional area for rib vertical beam section</td>
</tr>
<tr>
<td>$A_{VERTb}$</td>
<td>Cross sectional area for rib back vertical beam section</td>
</tr>
<tr>
<td>$A_{VERTf}$</td>
<td>Cross sectional area for rib front vertical beam section</td>
</tr>
<tr>
<td>$B, b$</td>
<td>Constants of proportionality</td>
</tr>
<tr>
<td>$C$</td>
<td>Engine performance parameter</td>
</tr>
<tr>
<td>$C_{gf}$</td>
<td>Correlation factor</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>$F_{hyd}$</td>
<td>Load from hydraulic system attachment</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Hinge load</td>
</tr>
<tr>
<td>$F_z$</td>
<td>Axial load</td>
</tr>
<tr>
<td>$h$</td>
<td>Spar height</td>
</tr>
<tr>
<td>$I_{BOT}$</td>
<td>Second moment of area for rib bottom beam section</td>
</tr>
<tr>
<td>$I_{TOP}$</td>
<td>Second moment of area for rib top beam section</td>
</tr>
<tr>
<td>$I_{VERT}$</td>
<td>Second moment of area for rib vertical beam section</td>
</tr>
<tr>
<td>$K_{ca}$</td>
<td>Correlation factor for extra fuel burnt in climb and acceleration</td>
</tr>
<tr>
<td>$K_{rsv}$</td>
<td>Constant for reserve fuel</td>
</tr>
<tr>
<td>$K_w, K_{wev}$</td>
<td>Coefficient relating aircraft variable weight to its gross weight</td>
</tr>
<tr>
<td>$L$</td>
<td>Hinge line datum</td>
</tr>
<tr>
<td>$L_d$</td>
<td>Aircraft lift to drag ratio</td>
</tr>
<tr>
<td>$m$</td>
<td>Mean for primary membership function</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>Maximum resultant bending moment</td>
</tr>
<tr>
<td>$n$</td>
<td>Propeller cruise efficiency</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of components</td>
</tr>
<tr>
<td>$n_{hyd}$</td>
<td>Number of attachment points for hydraulic systems</td>
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$O^k_i$  Membership function associated with input $i$ on network layer $k$

$P_r$  Potential of numerical data point

$P_r$  Strut load

$R$  Aircraft design range

$r_a$  Cluster radius

$r_a$  Radius of cluster with reduced potential

$r_{type}$  Fuzzy rule

$S$  Type of support rib

$SFC$  Estimated standard error

$SFC$  Aircraft average cruise specific fuel consumption

$TSFC$  Aircraft average cruise thrust specific fuel consumption

$w$  Fuzzy rule firing strength

$\bar{w}$  Normalised firing strength

$W$  Weight

$W_c$  Aircraft constant weight component

$W_{fuel}$  Fuel weight

$W_g$  Aircraft gross weight

$W_o$  Aircraft fixed weight

$W_{pl}$  Payload weight

$W_v$  Aircraft variable weight component

$x_i$  Input variable

$y_i$  Output variable

Greek letters

$\mu$  Shape of membership function

$\omega_{aero}$  Aerodynamic load

$\omega_{fuel}$  Fuel load

$\Phi$  Fuzzy basis function

$\sigma$  Standard deviation

$\sigma_{th}$  Applied thermal stress

$\sigma_{ult}$  Ultimate tensile stress
\[ \theta \] Variable real value
\[ \hat{\theta} \] Variable predicted value

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AAW</td>
<td>Active Aeroelastic Wing</td>
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<tr>
<td>ANFIS</td>
<td>Adaptive Network-based Fuzzy Inference System</td>
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<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<tr>
<td>ATM</td>
<td>Advanced Technology Multiplier</td>
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<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
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<tr>
<td>CONSIZE</td>
<td>CONfiguration SIZing Program</td>
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<tr>
<td>DoE</td>
<td>Design of Experiments</td>
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<tr>
<td>FE</td>
<td>Finite Element</td>
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<td>FEA</td>
<td>Finite Element Analysis</td>
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<tr>
<td>FEM</td>
<td>Finite Element Model</td>
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<tr>
<td>FEMMAS</td>
<td>Finite Element Method for Mass Estimation</td>
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<tr>
<td>FIS</td>
<td>Fuzzy Inference System</td>
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<tr>
<td>FLOPS</td>
<td>FLight OPtimisation System</td>
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<tr>
<td>FLS</td>
<td>Fuzzy Logic System</td>
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<tr>
<td>FTE</td>
<td>Fixed Trailing Edge</td>
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<tr>
<td>FOU</td>
<td>Footprint of Uncertainty</td>
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<tr>
<td>FS</td>
<td>Fuzzy Set</td>
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<td>GF</td>
<td>Growth Factor</td>
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<td>IFTE</td>
<td>Inboard Fixed Trailing Edge</td>
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<td>IMPACT</td>
<td>Innovative Mass Properties Analysis CATIA Tool</td>
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<tr>
<td>IT2</td>
<td>Interval Type-2 fuzzy system</td>
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<tr>
<td>MANFIS</td>
<td>Multiple Adaptive Network-based Fuzzy Inference System</td>
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<tr>
<td>MAPE</td>
<td>Mean Absolute Percentage Error</td>
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<tr>
<td>MF</td>
<td>Membership Function</td>
</tr>
<tr>
<td>MFTE</td>
<td>Midboard Fixed Trailing Edge</td>
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</tbody>
</table>
MPE  Mean Percentage Error
MTOW  Maximum Take-Off Weight
NEFPROX  Neuro Fuzzy Approximator
NOMF  Non-Optimal Mass Factor
OFTE  Outboard Fixed Trailing Edge
OWE  Operational Wight Empty
PDF  Probability Density Function
RBF  Radial Basis Function
RMSE  Root Mean Square Error
SR  Specimen Rib
SSSP  Space Shuttle Synthesis Program
SVD  Singular-Value Decomposition
TOS  Theoretical Optimal Structure
TSK  Takagi-Sugeno-Kang fuzzy system
WAATS  Weight Analysis of Advanced Transportation Systems
WER  Weight Estimating Relationships
WISE  Weight Integrated Sizing Evaluation
YEIS  Year of Entry Into Service
Chapter 1

Introduction

Contents

1.1 The challenges for weight estimation in the aircraft industry 2
1.2 Scope and objectives of this research 6
1.3 Thesis layout 7
1.1 The challenges for weight estimation in the aircraft industry

The success of a new aircraft program is directly proportional to the ability of the new design to satisfy the operational needs and requirements set by the customers. In addition to being reliable and technically robust, the aircraft needs to be able to justify its selling price and operating costs by providing the performance levels stipulated with the customer. The preliminary stage of the design of a new aircraft, in particular, is the most critical point for the attainment of the required commercial competitiveness. It is at this time that the design team determines whether the agreed operational capabilities are technically feasible and defines the best combination between performance and cost within the limits of available technology and constraints.

Weight control, namely the process by which the lightest possible airplane is derived within the constraints of the design criteria (Niu, 1988), is an essential module of the design process of any aerospace vehicle. In turn, the fundamental task in a weight control program is weight estimation. Accurate estimations of aircraft weight are vital in the early stages of an aircraft design process. They concretely drive all the major choices in configuration and layout as well as being the main foundation of performance predictions. An overestimate of Maximum Take-Off Weight (MTOW) will result in the aircraft being not competitive enough on the market. Conversely, if the weight of the aircraft after production is higher than expected, the company will incur financial penalties related to both time spent for the post-production weight shedding task and to its inability to meet contractual guarantees (Sparaco, 2003). The recent example of Airbus losing the FedEx contract for its A380 Freighter due to the uncertainties in its final weight and performance levels is just the last in a long list of economic losses related to weight issues. The weight weaknesses in the A380 program development have also led Airbus to lose up to 160 orders between Virgin Atlantic, Thai Airways and Emirates (Mecham and Wall, 2006).

Weight estimation has acquired considerably greater relevance in the aerospace industry from the moment it emerged as an individual analysis field in the 1930s (Bechdolt et al., 1996). Recent aerospace periodicals are filled with examples of manufactur-
Chapter 1

 ers struggling with aircraft structures being overweight. The new Boeing Dreamliner needs a 2 percent weight reduction in order to meet its target performance, which will only be achieved from the seventh aircraft produced onwards (Schoefield, 2006). The direct consequence of this is a reduction in achievable range between 10 and 15 percent, which can translate into a diminished range capability of up to 12800 kilometres below the initially advertised values (Ostrower, 2010). The A380 was 5.5 tons overweight at its launch, with up to 5 percent exceeding weight across the whole family (Sparaco, 2003; Wallace, 2011). The initial promise to the customers of a 555-seat 15 percent to 20 percent cheaper to operate than the Boeing 747, with 35 percent more passengers and 10 percent greater range has been hard to achieve. In terms of profit and performance, each ton over the original weight prediction for the Dreamliner compares to 12 less passengers for a total of up to 55 less people. The program itself was delayed 2 years in order to solve the weight issue (Wallace, 2006). Lockheed Martins Joint Strike Fighter (JSF) was 1400 pounds over its target take off weight by the first critical design review in 2003 (Selinger, 2003).

Figure 1.1: Aircraft empty weight breakdown.

These numbers could be mistaken with being irrelevant penalties on the large
scale. This could not be further from the truth. The weight of aircraft structures in particular, however, has a snowball effect on a different number of performance parameters, from maximum operative ceiling and endurance to maximum payload capacity. Aircraft structures, in fact, account for about 50 percent of the total empty weight (Figure 1.1), thus it is the area where inaccuracies in the estimation of weight mostly influence the efficiency of the design.

Overall, the key point for an aircraft manufacturer is that an increase in MTOW will ultimately mean that the vehicle will not be able to carry a specific payload from point A to point B (Sparaco, 2003). Figure 1.2 shows how weight can affect the range for propeller driven aircraft. An aircraft which is, at production stage, 1.5 times heavier than expected will incur in up to 20 percent reduction in available range. This value could double in the case of commercial jets (Bechdolt et al., 1996). Conversely, a 50 percent reduction in weight could result in up to 40 percent increased maximum range attainable by the design.

![Figure 1.2: Influence of weight on range for propeller driven aircraft (Bechdolt et al., 1996).](image)

The effort towards more effective and precise weight estimation methodologies has also been spurred in recent years by an increasing demand for designs which are simultaneously cost effective as well as more environmentally friendly. The aerospace
industry has, therefore, redirected its focus towards new configurations, weight saving materials and alternative production methods in order to satisfy the market demand (Jankowski, 1990; Udin and Anderson, 1992). As a result, traditional approaches to weight prediction have become obsolete as well as very limited in their reliability and accuracy. The majority of these methodologies, in fact, rely on past experiences: they base weight and performance predictions for new designs on databases which are representatives of conventional configurations and technology rather than new trends.

In order to improve weight estimation capabilities, empirical techniques have, therefore, been substituted by or combined with more accurate analytical and semi-analytical formulations. The incorporation of load analysis within statistical techniques has been seen as a way to encompass in greater detail the nature of aerospace structures and reduce the error in the prediction of their weight. Initially these methods used to be stand-alone processes, aimed at generating final weight breakdowns for the only purpose of performance estimation. This has changed considerably in the past few years, when the analytical equations for weight derivation have started appearing as integral parts of structural analysis (Droegkamp, 1992; Sensburg et al., 1994; Sensmeier et al., 2006) and solid modelling packages (Flamand, 2001; Zaidel, 1992).

The current trend is converging towards a more concurrent approach to the design process as a whole. Efforts and research are aimed at concretely integrating the different analysis, from aerodynamics to structures as well as system implementation and weight control, in a coherent multidisciplinary framework able to evolve and progress in parallel with the design sequence (Bos, 1998; Kroo et al., 1994). Weight estimation has, therefore, acquired increased importance not only as the linking ring between the various discipline areas but also as the main focus for the development of optimisation techniques. An accurate and rigorous weight prediction is, as a consequence, the starting point for an optimal design. Clear identification and traceability of the sources of weight inefficiencies can focus the efforts on their elimination or substitution, resulting in a more efficient feature/component design, a concrete reduction in the overall assembly weight and consequent performance enhancement.

The concurrent view on the design process combined with the increasing market demand for shorter delivery time on highly cost and performance efficient designs,
has thus pushed the boundary on weight estimation frameworks. The focus is on the production of fast and reliable methodologies, which are able to converge towards an economical and technical optimum at the early stages of the design to avoid costly changes later in the process. The need for the application of this kind of tools early in the design results in the requirement for a procedure which could work with a minimum amount of input data for the production of an optimum initial solution. In addition to this, flexibility must be a key characteristic of any newly developed weight prediction method, to allow for a proportional increase the level of detail in output in parallel with the design process itself.

A few attempts at this have been made over the years. However, the results produced are still not satisfactory enough in terms of accuracy and versatility. Moreover, the process of weight estimation, although extremely important in the aerospace design cycle, does not seem to have raised as much interest in the engineering community as other disciplines have. The progress made in the development of new techniques and approaches to the problem seem to have come to a halt.

1.2 Scope and objectives of this research

The aim of this research is to develop a new methodology for the weight estimation of aircraft structures, which is able to fulfil the current requirements and demands of the aerospace engineering field.

The proposed methodology will be centred on adaptive fuzzy logic techniques and tools. The use of fuzzy logic principles will be explored in relation to the extraction of knowledge and design rules from the design domain of the structures being analysed. Fuzzy principles will be used in conjunction with a modular model structure to enhance the applicability of the approach to different structures and facilitate its integration in the design process. The approach will be assessed in terms of its ability to provide accurate and reliable weight estimates at the conceptual and preliminary design stages. In addition to this, the quality of the final knowledge base produced by the model will also be investigated in parallel with an evaluation of the capabilities of the framework to perform a robust uncertainty analysis on the design domain.
The research presented in this thesis was conducted in order to fulfil the following objectives:

1. Identification of major structural weight estimation techniques, their development and formulation, with focus on benefits and flaws related to the individual procedures;

2. Identification of innovative approaches and techniques to be used as foundation for the development of an alternative weight estimation methodology;

3. Implementation of reliable structural analysis and parameterisation approaches in the final weight estimation model;

4. Investigation and implementation of robust uncertainty analysis across the proposed framework;

5. Application of the methodology to aircraft structural examples for performance assessment and validation;

6. Development of suitable structure/framework for the weight prediction methodology which enables to satisfy different levels of granularity in the analysis.

1.3 Thesis layout

This thesis presents the development and analysis of a weight estimation methodology for aircraft structures. The body of this thesis explores the development process for the design of the approach, based on an initial assessment of current methodologies and tools used within the field of weight estimation. The discovery of fuzzy logic as a potential aid to the process of weight estimation follows as a direct consequence of the analysis of the pitfalls of traditional methods for weight analysis. This thesis highlights the evolution of the framework across different design requirements and through the use of various fuzzy logic techniques. Some of the issues explored relate to the knowledge extraction capabilities of the selected tools, to the formulation of a rulebase for the formalisation of the weight estimation process, to the transparency of the model and its ability to trace weight inefficiencies within the design and to the analysis and propagation of uncertainty within the framework.
The analysis of the issues and concepts related to the design of the approach has been structured across 9 chapters, in a way that closely follows the development process carried out for the design of the methodology itself (Figure 8.3).

Chapter 2 provides a critical review of the current techniques and approaches used in the weight estimation of aircraft structures. The analysis has been conducted by exploring the traditional categorisation of weight estimation methodologies as empirical, semi-analytical and analytical. Within these categories, both theoretical methodologies and practical tools used in industry have been assessed in relation to their benefits and pitfalls. The focus was also directed at examining new techniques and efforts within the engineering community, aiming at creating a more multidisciplinary view of the derivation of structural weight for new design concepts.

Chapter 3 presents an introduction to fuzzy logic as a way to overcome the limitation of traditional and current weight estimation methods. A background of the general principles of fuzzy logic is presented and put into the context of the different tools used within the research.

Chapter 4 presents an initial application of a fuzzy logic based weight estimation method. The approach introduced is based on the application of Adaptive Network-based Fuzzy Inference System (ANFIS) for a feature-based weight analysis of spoiler attachment ribs. Although only approaching the problem from a general geometrical and load definition of the structure, the chapter highlights the initial benefits obtained by the use of a fuzzy approach for this type of problem.

Chapter 5 expands on the concepts from Chapter 4 by extending the ANFIS method to a more physics-based weight analysis. The approach is further developed to be able to mirror closely the actual design process of aircraft structures, by combining weight estimation with the analytical sizing of the structural component. The process is validated across two case studies, spoiler and aileron attachment ribs. The performance of ANFIS is assessed in terms of knowledge acquisition capabilities, transparency of the knowledge based derived and through the overall interpretability of the method and its results.

Chapter 6 introduces the Neuro-Fuzzy Function Approximator (NEFPROX)
Figure 1.3: Thesis layout.
as an additional fuzzy tool to be used for weight estimation. The chapter focuses on the comparisons between ANFIS and NEFPROX in terms of the quality of both approximation and rulebase produced in the weight estimation environment.

Chapter 7 expands on the model structure presented in Chapter 5 and 6 by adopting Interval Type-2 (IT2) fuzzy logic. The use of this tool allows the integration of uncertainty analysis and propagation within the network. The principles of IT2 fuzzy systems are presented in relation with the creation of a more comprehensive and robust analysis. Through the use of case studies, the benefits of using IT2 are highlighted and critically assessed with respect to both ANFIS and NEFPROX.

Chapter 8 presents the concept of granularity in the field of weight estimation. The advantages of three fuzzy tools introduced in the previous chapters are set into the context of a flexible and versatile framework for the weight estimation of aircraft structures at preliminary design stages.

Chapter 9 provides a conclusion to the thesis, highlighting the achievements and main contributions of this research, while critically assessing the benefits of conducting weight estimation using fuzzy logic techniques.
Chapter 2

Weight Estimation for Aircraft Structures: Theory and Practice

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2.1 Introduction

The aim of this section is to provide a clear overview of existing weight prediction methodologies and formulations for aerospace structures. The different approaches to the derivation of weight formulations are defined, highlighting assumptions as well as the main steps for their development. Examples of some of the techniques within the literature are also presented, with emphasis on the relative flaws as well as the benefits of the individual solutions. An outline of the development and evolution of structural weight estimation techniques is also drawn, with reference to frameworks for weight analysis currently being used in the aerospace industry.

2.2 Classification of current approaches

Weight estimation has acquired increasing importance within the aircraft design community since the moment it became recognised as an individual analysis field in the 1930s. The weight of air vehicles has always been a point of concern and aircraft designers have been "in a continuous struggle with the laws of weight" since the beginning of aviation (Bechdolt et al., 1996). From the designers' viewpoint, however, the struggle has always been twofold. On one hand, the major challenge is to find practical and effective design solutions for an overall weight reduction and consequent performance enhancement (Pollard, 1928). At the same time, this cannot be done without reliable weight estimates which are built on methodologies and approaches that are able to embody the principal features and characteristics of the proposed design (Barlow, 1999).

The task of estimating the weight of a new aircraft concept is not an activity that is carried out only once during the design process and whose results are stored somewhere until one of the disciplines domain feels the need for them for one of its analyses. On the contrary, weight estimation spans the whole of the design cycle and continually evolves in parallel with the maturity of the project itself (Figure 2.1). It starts with a weight assessment at configuration level during the conceptual design stage and the level of granularity mirrored by the estimate increases to system and
component level between preliminary and detail design phases, as more information becomes available. The challenge is to be able to deliver representative and reliable weight estimates for each individual milestone in the design, using the information available at the time. At early stages, the knowledge of the design itself is very limited and the concept undergoes a series of re-evaluations that have to be individually weight reviewed in order to assess their viability. The analysis then shifts to the estimation of weight at subsystem level and all the way down to individual component weights towards the detail design phase proportionally to the amount of data available and the knowledge of the design. For these reasons, the development of weight estimation methodologies and tools is tailored to address the specific requirements related to the design phase in which they will be applied (Raymer, 2006).

![Figure 2.1: Schematic representation of the aircraft design process highlighting the relationship between weight estimation methodologies and individual stages in the process (Komarov and Weisshaar, 2002).](image)

At the beginning of the design process, configuration level weight assessments tend to be conducted using empirical formulations, which are statistically drawn from databases comprising of data related to existing design examples. Normally this category of formulae relate crucial overall weights (i.e. gross take-off weight, landing weight,
fuel weight, etc.) to specific parameters relative to top level design requirements, ranging from payload to range and operational factors (Torenbeek, 1985; Corke, 2003; Saelman, 1975). They can also be formulated towards the end of the conceptual stages and extended to estimate the weight of major subsystems (i.e. wing, fuselage, landing gear, etc.) (Roskam, 2003; Svoboda, 1999). Although these formulations are normally easy to use and do not require a high computational effort, the credibility of the estimates that they are able to provide is limited since they tend to be representative of designs which are similar, technologically and performance wise, to those in the database of reference.

Most aircraft manufacturers tend to prefer semi-analytical approaches in order to compensate for these pitfalls. These formulations combine a structural weight picture, which is analytically derived, with statistically drawn factors to account for specific items and features related to aspects such as manufacturing and installation. This approach extends the applicability of these types of methods to designs which differ from the ones in the reference database, from either a feature-based point of view (Saelman, 1964; Niu, 1988), or due to the presence of additional components (Udin and Anderson, 1992) or to specific materials and manufacturing processes used (St. John, 1969). Although these formulations are able to provide a more comprehensive picture of what makes up the weight of the structure, they tend to become increasingly complex in proportion to the level of detail required, with the risk of presenting erroneous trends and interactions between the numerous parameters involved.

Analytical methods are generally preferred in the design environment due to a higher rigorousness in their derivation. This ensures not only that the final results fully represent and embody the physics behind the design, but also allows a greater traceability of the sources of weight inefficiencies in the design at hand. These methods, however, are not suitable for conceptual and preliminary design stages. The majority of analytically derived weight estimation methodologies require a number of detailed information that normally is not available in the early phases of the design and end up being computationally expensive if coupled with structural analysis (Droegkamp, 1992; Zaidel, 1992) and CAD modelling software (Flamand, 2001). If applied based on erroneous initial inputs, analytical weight methods not only will produce a result
that is as realistic and accurate as one obtained via a simpler empirical model, but also they will not be able to provide any means of assessing the viability of the results themselves.

It is important to consider, however, that the boundaries between the different phases of the aircraft design process are not clearly distinguishable and often they tend to overlap quite considerably. For this reason, it is crucial to be able to identify the scope of individual weight estimation methodologies, their applicability within the specific design context and the tolerance and robustness of the results they can provide according to the quality of the information at hand at the time of the analysis.

2.3 Empirical weight estimation

Empirical methodologies represent one of the earliest approaches to the weight estimation of aircraft structures and have been the most commonly adopted formulations at preliminary aircraft design stages in particular. These relationships are statistically drawn from individual databases providing information on the component or assembly weight being considered. The source of the data mainly relates to aircraft which are already operative, with similar characteristics or configuration as well as to experimental data acquired from scaled models developed for particular studies.

2.3.1 Derivation of empirical formulations

The general formulation of weights based on empirical data tends to assume the form of the power law (Equation 2.1)

\[ W_i = A_i \phi^{B_i} \]  

(2.1)

where:

- \( W_i \) represents the component weight to be analysed;
- \( \phi \) is the dependent variable on which to base the analysis;
\( A_i \) and \( B_i \) are the constants of proportionality determined via the chosen statistical method.

The general approach for the determination of a component weight can be broken down into four individual steps (Torenbeek, 1985):

1. Definition of the component weight \( Y \) as a sum of individual contributions \( X_i \) (Equation 2.2)

\[
Y = \Sigma X_i
\]  \hspace{1cm} (2.2)

where:
- \( X_i \) represents the single items making up the component weight;
- \( Y \) represents the component being analysed.

2. Choice of relevant parameters for each contribution;

3. Definition of functional variation of \( Y \) with respect to \( X_i \). This choice will depend on factors such as range of data available as well as variation among the data itself. In the case of a limited size database, a relationship involving the linear variation of the component weight will usually be sufficient to accurately represent the trend amongst the data (Equation 2.3).

\[
Y = a + bX
\]  \hspace{1cm} (2.3)

On the other hand, in the case of a larger data set where the value of the component weight changes considerably, power law (Equation 2.4) or logarithmic log fittings (Equation 2.5) are preferred.

\[
Y = kX^n
\]  \hspace{1cm} (2.4)

\[
\log(Y) = \log(k) + n \log(X)
\]  \hspace{1cm} (2.5)

4. Estimation of standard error between actual and estimated weight (Equation 2.6)

\[
S = \sqrt{\frac{1}{N-1} \left| \Sigma m_i^2 - (\Sigma m_i)^2 \right|}
\]  \hspace{1cm} (2.6)

where:
Chapter 2

$N$ represents the number of components under study;

$m_i$ the ratio of actual to estimated weight for the chosen sample.

The literature provides numerous examples of statistically drawn weight estimating relationships (WERs). Regression analysis has been used extensively in the derivation of WERs, ranging from overall aircraft weight (Anderson, 1972) to subsystem (Svoboda, 1999) and component level weight breakdown for items such as high-lift devices (Macci, 1995). However, one of the earlier pitfalls of using this type of derivation is the creation of misleading and erroneous trends due to correlations between independent parameters which were either overlooked or difficult to detect (Bechdolt et al., 1996). Staton (1969) contributed to overcome this by adopting constrained regression to the derivation. In this case, the best curve fit is determined within a set of limits specified by the user over the whole set of statistically determined values. More recently, Rocha et al. (2006) compared the results of several model building techniques, ranging from polynomial interpolation, all the way to radial basis function (RBF) and Gaussian interpolation to evaluate the benefits of the methods in the wing weight data fitting problem. The results of his study proved that models built using principal component regression (PCR) with multiquadratic RBF interpolation were not only more accurate than the other example, but also able to depict more representative weight trends by an a priori selection of the optimum combination of input variables for the type of model building technique to be used in the analysis (Rocha, 2008).

2.3.2 Level of granularity of empirical weight estimation methods

Empirical WERs mainly differ among themselves in terms of the initial assumptions adopted for their derivation and the focus of the analysis. They can range from formulations designed to estimate the weight of the aircraft as a whole depending on the type of load carrying material used (Caldell, 1969; Arjomandi and Liseytsev, 2000) or to the weight of individual structural assemblies (Macci, 1995; Udin and Anderson, 1992; Corke, 2003). The nature of these methods makes them better suited to either overall aircraft level (Mack, 1999) or to main subassembly level (i.e. wing group, tail group..) weight analysis (Kyser, 1977). Relationships and general trends are drawn between some defining geometrical parameters by looking at historical examples from
similar aircraft, mainly as a way of establishing at a preliminary stage of the design how the variation of the size or location of a particular subgroup can affect the overall aircraft performance. On the other hand purely analytical methods, which are based on the determination and analysis of load cases on particular components, are better suited to weight estimation of single components (i.e. ribs, spars..). Single elements are looked at according to the function they need to accomplish and loading to be sustained and sized accordingly, limiting the application of these methodologies at later stages of the design process where more details are known.

The simplest form of empirical weight relationship method is the *Fixed Fraction Method*. Very suited for the early conceptual design stage, it allows to compute the weight of individual components and structural assemblies as a fraction of either the vehicles empty weight or its maximum take-off weight (Gersh and York, 1979). The most recent example of the development of this method is provided by Scott and Nguyen (1996). The analysis is based on the consideration that the gross weight of the aircraft can be considered as the summation of its corresponding fuel weight, payload weight and Operational Weight Empty (OWE) (Equation 2.7).

\[
W_g = OWE + W_{pl} + W_{fuel} \tag{2.7}
\]

where:

- \(W_g\) is the gross weight of the aircraft;
- \(W_{pl}\) is the payload weight;
- \(W_{fuel}\) is the fuel weight required for the completion of the mission.

In addition to this, the aircraft OWE can be regarded as the summation of two different weight components. The first one is a constant weight component, which does not vary during the sizing process but only depends on the number of passengers to be carried and the vehicles year of entry into service in order to account for possible technological advancement. The second term identifies a variable mass component changing proportionally to gross weight in terms of a coefficient identifying the ratio of variable weights to the aircrafts design gross weight (Equation 2.8).
where:

\[ OW_E = W_c + K_w W_g \]  \hspace{1cm} (2.8)

\( W_c \) is the constant weight component of the aircraft;

\( K_w \) is the statistical coefficient relating variable weight to the gross weight of the aircraft.

\textbf{Figure 2.2:} Graph showing the variation in weight due to passenger capacity for commercial transports (Scott and Nguyen, 1996).

Weight analysis at this level of granularity identifies structures and systems as functional groups. Systems such as avionics, instruments and electrical equipment as well as fuselage structure and furnishing are incorporated in the constant weight component since they only depend on the passenger capacity and the specific level of technological advancement applied to the vehicle (Figure 2.2). The remaining load carrying structures as well as systems such as propulsion, flight controls and landing gear are included in the varying weight component (Table 2.1).
TABLE 2.1: Identification of functional weight groups for the Fixed Fraction Method (Scott and Nguyen, 1996).

<table>
<thead>
<tr>
<th>CONSTANT WEIGHT ($W_{cj}$)</th>
<th>VARIABLE WEIGHT ($W_{vj}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body/Fuselage</td>
<td>Wing</td>
</tr>
<tr>
<td>Auxiliary Power Plant</td>
<td>Rotor</td>
</tr>
<tr>
<td>Instruments</td>
<td>Tail Group</td>
</tr>
<tr>
<td>Electrical</td>
<td>Alighting Gear</td>
</tr>
<tr>
<td>Avionics</td>
<td>Engine/Nacelle</td>
</tr>
<tr>
<td>Armament</td>
<td>Air Induction</td>
</tr>
<tr>
<td>Furnishing/Equipment</td>
<td>Propulsion</td>
</tr>
<tr>
<td>Air Conditioning</td>
<td>Flight Controls</td>
</tr>
<tr>
<td>Load and Handling</td>
<td>Hydraulic/Pneumatic</td>
</tr>
<tr>
<td>Fixed Useful Load</td>
<td>Anti-Icing</td>
</tr>
</tbody>
</table>

$W_c = \Sigma W_{c_j}$  

$K_w = \frac{\Sigma W_{v_j}}{W_g}$

Rather than being used for a detailed weight analysis, methods like this provide a quick estimation of the efficiency of a particular design. In this case it is useful to adopt the ratio of the Operational Weight Empty to Maximum Take-Off Weight (OWE/MTOW) to compare the performance of a particular configuration to that of aircraft employed for similar purposes. The method assumes conventional commercial transports to be characterized by 54 percent of variable weight components and the remaining 46 percent related to fixed weight structures and systems. In addition to this, the use of the Fixed Fraction Method allows for an easier definition of the design space being explored, for a ready identification of the parameters with the highest degree of influence on the desired performance characteristics of the design as well as highlighting the possible consequences of changing any of these parameters in the configuration. This can be readily seen in the incorporation of the Breguets equation for range in the computation of the aircraft gross weight (Equation 2.9).

$$W_g = \frac{W_c + W_{pl}}{(1 + K_{wev}) \times 10^{\frac{W}{2}} - (K_{wev} + K_{rsv})}$$ (2.9)
where:

\[ K_{w_{ev}} \] correlates variable and gross weight in the same way as \( K_w \) in the method of Scott and Nguyen (1996);

\( K_{r_{sv}} \) is a factor used to include the effects of reserve fuel on gross weight;

\( R \) indicates the design range of the aircraft.

\( C \) allows to account for engine performance and depends on the type of engine used in the design (Equation 2.10, 2.11):

\[
C = 326 \times \left( \frac{L}{D} \right) \left( \frac{n}{SFC} \right) \quad \text{for propeller driven aircraft} \tag{2.10}
\]

\[
C = \left( \frac{L}{D} \right) \left( \frac{V_{cr}}{TSFC} \right) \quad \text{for jet powered aircraft} \tag{2.11}
\]

where:

\( \frac{L}{D} \) is the lift to drag ratio of the design;

\( n \) is the cruise efficiency for the propeller;

\( SFC \) is the average cruise specific fuel consumption for propeller driven aircraft;

\( TSFC \) is the average cruise thrust specific fuel consumption for jet powered aircraft.

This is a particular adaptation of the relationship between range and gross weight as presented by Bechdolt et al. (1996). Each empirical method will have a similar formulation incorporating other parameters according to the focus of the particular study. Equation (2.8), for instance, identifies payload and constant weight component as the main driving parameters for range and, consequently, for fuel reserve. Scott and Nguyen (1996) provide an equivalent type of relationship (Equation 2.12).

\[
W_g = \frac{W_c + W_{pl}}{K_{ca} \times (1 + K_{w_{ev}}) \times 10^{\frac{C}{10}} - (K_{w_{ev}} + K_{r_{sv}})} \tag{2.12}
\]
where:

\[ K_{ca} \] is a correlation factor allowing for extra fuel burnt during climb and acceleration.

The main layout of the formulation mirrors (Equation 2.9), however in this case it has been rearranged to include the effect of climbing performance on take-off weight through the correction factor.

Of a similar degree of simplicity is the risk analysis carried out for this type of methodologies. One of the first examples of the quantification of the "risk" of using a particular WER was conducted by Ballhaus (1947) who adopted probability theory to enhance the applicability of empirical WERs at aircraft subsystem level. In particular, the study focuses on examining first the effects of the individual geometrical or operational parameters chosen by the designer on the subsystem weight and, subsequently, their combined impact. Once the WERs are derived, probability theory is then applied to compute the probable error of estimate that can be expected from the formulation based on the analysis of the given statistical data. Although still basic in both the structure of the WERs and the application of the theory of probability for the solution of the problem, Ballhaus (1947) showed the first real attempt to assist the designer in judging the validity and applicability of their weight estimates.

Scott and Nguyen (1996) prefer the Growth Factor (GF) approach as a first attempt to risk analysis for empirical weight estimation at preliminary design stages. Through the computation of the GF, it is possible to estimate the relationship between increments in empty weight and desired level of performance of the design. In particular, this parameter was proven successful in determining the degree of impact of different weight variations in less than one-tenth of the time required by other statistical methods and with greater accuracy (Equation 2.13).

\[
GF = \left( \frac{C_{gf}}{K_{ca} \times (1 + K_{rs}) \times 10^{2a}} \right) - (K_w + K_{rs})
\]

(2.13)

where:
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Cgf is an additional correlation factor.

The factors relating gross weight to reserve fuel, climb performance and variable weight components are still considered. The equation itself, however, would over estimate the growth factor: for this reason the correction factor Cgf, ranging from a minimum of 0.7 up to 0.85 for long range transport.

The idea of using the growth factor as a way of evaluating overall weight penalties in the general gross weight of the design by changes in individual components, however, was first presented by Saelman (1973). By identifying the relationship between fixed weight and gross weight as a mathematical relationship (Figure 2.3), the growth factor itself can also be formalised further as the rate of change of aircraft gross weight to aircraft fixed weight (Equation 2.14).

\[
\beta = \frac{dW_g}{dW_o} = \frac{\Delta W_g}{\Delta W_o}
\]  

(2.14)

where:

\( W_o \) is the fixed weight of the aircraft.

![Graphical interpretation of the growth factor.](image)

The initial weight estimation method provided by Scott and Nguyen (1996) was applied by the same authors to a database of 17 aircraft and resulted in an absolute
average error of less than 3 percent. The method, however, strongly relies on database predictions and similarity approach. The purpose of the similarity approach is that of normalising the database in terms of critical design parameters, such as configuration layout and type of propulsion system, so as to try and minimise the error in the estimation. Additional considerations are also included in the analysis. The effect of technological advancement, for example, is also included by multiplying the constant weight component by the Advanced Technology Multiplier (ATM) (Equation 2.15).

\[
ATM = 0.9985^{(YEIS-1975)} \tag{2.15}
\]

where:

\(YEIS\) is the year of entry into service for the aircraft.

Although still applied only at early design stages and limited to first approximation studies of subsystem level, empirical WERs have proven useful in the weight estimation of spacecraft structures (Hassman, 1975) and hypersonic vehicles (Plank et al., 1970), especially when embedded in multidisciplinary analysis software such as the Weight Analysis of Advanced Transportation Systems (WAATS) program (Cook, 1981)

### 2.3.3 Benefits and limitations

One of the main benefits of weight estimation methods based on empirical formulations is their ability to produce reasonably accurate results with minimum effort and time constraints (Carreyette, 1950). It is, in fact, easy to produce simple weight equations for particular trade studies, incorporating in them the parameter in which to focus the analysis. This kind of methodology will allow a rapid evaluation of a number of configurations, structural solutions and material choices without great detailed knowledge of the design itself, making this method very suitable for weight evaluation in early design stages (Jankowski, 1990). From here it is also possible to derive trend curves to define the best correlation of two or more design parameters for the attainment of a particular degree of performance, which proves to be very useful espe-
especially when designing different design combinations for a family of aircraft (Scott and Nguyen, 1996).

![Diagram](image)

**Figure 2.4:** Commercial transport MTOW per seat trend (Scott and Nguyen, 1996).

Figure 2.4 is an example of the kind of definition of the design space obtained by using empirical methods. In particular, the trend curves in this case relate range to the general configuration of the aircraft in terms of the ratio of maximum take-off weight to passenger capacity. It is easy to understand the benefits of adopting this kind of relationship and visual description of the design space: this can be considered not only as a good starting point for the design process but also a sanity check for the more detailed weight predictions produced in the later design stages. The ease of development of empirical WERs and the limited computational effort needed for the analysis makes this type of formulations also particularly suited to early trade studies for cost and development models (Beltramo et al., 1977).

These benefits, however, are also the main limitations of this kind of methods.
From the various examples in the literature (Howe, 2000; Torenbeek, 1985) it can been seen how the simplicity of empirical WERs can only restrict their use to early and basic trade studies and not for more advanced stages in the design process. In the majority of cases, the simplicity of these formulations and the basis on average values for main variables involved in the analysis makes it hard to ensure their validity (Macci, 1995).

Moreover, this kind of weight prediction proves to be only valid when analysing designs which are mostly similar to those included in the reference database, limiting their usefulness in predicting initial performance for unusual designs or concepts involving the use of new materials of technologies. As pointed out by Scott (1992) when examining seven different wing weight estimating relationships, the variation in the value of the exponents for the same parameter can mainly be attributed to variation in the reference database. The increasingly spreading use of composites for load carrying structures in aircraft is a typical example. The use of weight fractions based on nearly all metal designs will result in highly erroneous weight estimations for new generations of aircraft which are characterised by an always higher percentage of the structure manufactured from lighter composite materials, making this approach unable to include effects of innovations in the weight prediction. It is, therefore, vital to not only build up the reference data set on similar configurations, but also to focus on the level of technological advancement to ensure that the results will be truly representative of the final design. The quality of the results produced will also be significantly dependent on the nature of the databases used. The higher the number of detailed weight estimates from other designs as well as their degree of similarity to the configuration being examined will determine the accuracy of the prediction.

To improve the degree of accuracy and reliability of empirical weight estimating relationship it is important to:

1. Adopt mathematical formulations precisely representing the degree of influence of the individual geometric parameters on the overall weight trend;
2. Combine all the necessary parameters affecting the final weight, even those contributing to it in a minor way (Scott, 1992).

Limiting the formulation to an exponential form would not allow the repre-
sentation of more complicated weight trends, such as bucket-shape ones determining the relationship between fuselage weight and finess ratio (Scott and Novelli, 1989).

If this is combined with the inclusion of secondary parameters in the analysis (i.e. wing-fuselage joint weight) the error between real and estimated weight could be considerably reduced. It is, however, important that the size of the reference database is larger than the number of parameters used in the weight estimating relationship to avoid misleading results.

2.4 Semi-analytical weight estimation

Different alternatives of semi-analytical weight prediction methods are available, mainly differing among themselves in the kind of initial assumptions on which the derivation is based. These methods are usually individually derived by the aircraft manufacturers and are based on the detailed knowledge acquired on a specific kind of component family or aircraft category, resulting in a large number of individual formulas for the estimation of the same structural component. They also tend to aim at sizing components via equations derived on the assumption of one critical design condition.

Derivation of wing group weight is the one that has acquired the major interest in the literature. Changes in the overall design of the vehicle during its evaluation often require considerable resizing of the wing. Even though only accounting for 10 percent of the structural weight of the aircraft, any design changes to the wing will have a considerable impact on the overall performance of the aircraft. Efforts have been, therefore, aimed at producing a wing weight estimation model able to yield highly accurate results with minimum time effort.

Hopton-Jones (1955) provides one of the earliest examples of rigorous structural wing weight build-up methodology by this approach. Figure 2.5 highlights the structure of the approach. The wing structural material is distinguished into basic box structure and secondary structure. The elements in the secondary structure and most of those related to the interspar weight are estimated via empirically derived WERs. However, the main bending and shear material groups depend on the loading the structure needs.
to counteract. In particular, the analysis focuses on the effect of airloads, as well as distributed and concentrated inertia, with the addition of effects from landing gear loads.

**Figure 2.5:** Diagram of semi-analytical wing weight build-up methodology (Hopton-Jones, 1955).

One of the most recently developed semi-analytical tools for wing weight estimations can be attributed to Macci (1995), and its derivation very much mirrors layout and methodology as the above example. The theoretical derivation is based on the computation of the amount of material needed in the structural wing box in order to satisfy bending and torsional stiffness under prime loading conditions of axial compression, shear and bending. In this case, aeroelastic effects are also considered in order to prevent torsional instability and flutter.

The overall mass of the wing structure is, therefore, assumed to be made up of the mass of the structural box derived analytically, as well as contributions from rib structure, control surfaces and miscellaneous elements computed via empirical methods. In particular, the structural mass of the wing box can be seen as being made up of bending material (skin cover) and shear material (shear web) both inside and outside
the fuselage. The derivation of these formulae is based on the consideration of the worst loading case, in other words the case in which the ultimate loading factor assumes its maximum value within the flight envelope, in either deep maneuvers or under gust conditions. Moreover, the derivation includes load relief effects due to inertia forces in the structure, by means of a theoretical inertia relief factor incorporating relief due to wing structure, fuel and engine attachments. Effects on allowable stress due to individual material properties are also included in the calculations by means of different formulations for design stress depending on whether the structure is fabricated from metal or laminate composites.

Slingerland et al. (2007) adopt a similar approach for the derivation of fuselage weight. By analysing the fuselage as both barrel sections and individual panels, the methodology allows for analysis of load variation in both longitudinal and circumferential directions and, consequently, a more representative load and thickness distribution. The overall structural weight is then derived by combining the analytically derived panel weights with empirical WERs for additional components. A growth factor approach is then used to evaluate weight savings achievable by using different materials both for the overall design as well as for individual fuselage sections.

In the majority of semi-analytical weight estimation approaches, the weight of secondary structures, ranging from leading and trailing edge fixed and movable components to landing gear and engine attachments, is computed empirically (Carreyette, 1950; York, 1980). The mass of the components, in this case, is not driven by stress and loading issues, but merely on a combination of geometrical parameters as well as statistical correction factors incorporated by constant exponents and multiplication factors. Another example is the weight of attachment of engine and undercarriage which is mainly driven by the total number of landing gear in the design and the number of the landing gear units attached to the main wing structure, whilst the additional components are estimated through factors accounting for miscellaneous features such as cutouts and minimum gauge design.
2.4.1 Applicability of semi-analytical weight estimation formulations

Semi-empirical weight estimation approaches, as seen in the previous section, are much more representative of the parameters affecting components and assembly weight compared to simpler empirical methodologies, thus creating a broader base for structural optimisation (Gallman et al., 1997; Huang et al., 1996). They allow a much more in depth functional level analysis by considering the different loadings the structure needs to be able to sustain as well as including weight effects induced by the application of new technology (York, 1980).

Even though the number of semi-empirical equations tends to vary largely according to the component/subassembly considered, ranging from as little as three (Macci, 1995) to hundreds (Roland, 1969), they are very suited to be incorporated in semi-automated weight estimation routines. Typical examples are programs such as the Weight Analysis of Advanced Transportation Systems (WAATS), developed as part of the Space Shuttle Synthesis Program (SSSP) (Glatt, 1974) and the Weight Integrated Sizing Evaluation (WISE) tool (Gersh and York, 1979). These types of automated frameworks for preliminary design differ amongst themselves in terms of the kind of equations used and the degree of accuracy provided in the analysis. Nonetheless, they are all based on a similar aim: a simple architecture which is at the same time flexible and highly responsive, as well as able to work with minimal inputs, but while outputting as much information on the design as possible (Glatt, 1974; St.John, 1969).

WISE, in particular, has been structured in two separate modules. WISE-One was designed for an initial rapid evaluation of the concept using empirical methods, while WISE-Two aimed at optimising the results from the first unit in terms of cost and weight using more detailed semi-analytical formulations (Gersh and York, 1979).

This kind of approach to the weight prediction problem solves some of the issues related to the more basic empirical solution. Macci (1995), however, has highlighted some of the limitations of the method, which are typical drawbacks of more general semi-analytical weight estimation procedures. The underestimation incurred in the results has to be related to the lack of information on additional penalties such as
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sealant, paint or storage tanks not accounted for in the empirical equations. The approach suggested by Hammitt (1956), although still computing weight penalties in a semi-empirical way due to the excessive time needed for computation, provides an extensive list of weight penalties incurred by both wing and fuselage structures. They range from additional weight incurred by substituting the wing edges with control surfaces, to bulkheads, joints and supports. However, this kind of approach still limits the application of the methodology to aircraft fitting the characteristics of the reference database.

This drawback is always going to be present due to the empirical contributions within these methodologies and can only be limited by increasing the size and quality of the reference database as much as possible. Moreover, care should be taken in the choice of parameters to be used. Most of the empirical correction factors used in this kind of methods are extrapolated from statistical trends. It is important to consider data points that do not fall on the regression line, since they might indicate the need of a different statistical correlation to be adopted.

2.5 Analytical methods

Purely analytical weight estimation methodologies tend to appear in later design stages, where a more detailed knowledge of the design has been acquired and the weight and balancing process itself is aimed at a specific design intent which cannot be related to any existing database. These methods are mainly designed to analyse particular structural arrangements (Kelm et al., 1995) and are structured around point sizing criteria, allowing to reach the single component weight level, not covered by the two previously described methodologies (Ritter, 1960).

Analytically derived weight statements are usually drawn around three main considerations (Bechdolt et al., 1996):

1. **Design intent**: the analysis of the component starts from the initial sketch which is then translated into weight analysis by concurrent consideration of the necessary approximations to be applied to the model. Care needs to be taken in ensuring
that the approximations applied are valid and able to include manufacturing and installation issues.

2. Sizing criteria: specified in order to satisfy strength and stiffness requirements according to the loading to which the component will be subjected, in parallel with material properties and constraints.

3. Production design: issues concerning the manufacturability of the component are included in the analysis in the form of physical constraints and calibration factors.

Analytical weight prediction methods, although often very different among themselves, are all based on theoretical formulations aimed at defining the optimum weight, in other words the minimum possible attainable weight (Shanley, 1960). The real weight of a structure, however, is a combination of the theoretical optimum and a non-optimum contributions due to inefficiencies in the design, ranging from joints to cutouts, which can add up to 80 percent above the ideal structural weight. Analytical procedures will result in formulations relating size, material properties and applied loads (Staton, 1974; Simpson, 1973).

Due to the degree of detail included in the derivation, analytical methodologies tend to work in a bottom to top way. The analysis is carried out on an individual component basis, by:

1. Simplification of the load carrying elements according to theoretical assumptions;
2. Identification of the loads driving the design of the component and their localisation;
3. Integration of the loads and derivation of load distribution for the individual component;
4. Evaluation of minimum weight to satisfy the loading conditions.

Once these steps have been carried out, the size and weight contributions from different components are available and it is possible to clearly define the effect of the component-level design on the assembly/subsystem weight (Marczi and Smrcek, 2004).

When defining the idea of optimum design, Shanley identifies in the ultimate strength the most important loading condition for the determination of the overall
weight of any load carrying structure. This work also highlights the importance of developing a method of integration in a simple manner without sacrificing the opportunity to consider the effects of different factors on the final weight (Shanley 1960). Constant allowable stress and integration methods over inaccurate load distributions are also identified as the major flaws of the weight estimation methodologies adopted until then.

In the derivation of sheet-stringer-rib type wing structure (Figure 2.6), in addition to the identification and analysis of the individual effects of the main loads acting on the different structural components, the combined effects of torsion, shear and bending are computed by means of the interaction-curve method (Shanley, 1960). Crushing and pressure loads on ribs are also accounted for. In terms of overall geometrical layout, the resulting wing weight underlines the relative effect of geometric parameters such as span, mean aerodynamic chord and wing area, as well as semi-span depth ratio and taper. The formulation also allows the analysis of both take off and landing conditions as well as providing essential information for trade studies, such as the impact of an increase in take-off weight on wing structural weight as compared to that resulting from an increase in wing span.
The derivation process follows a similar pattern in the case of the weight estimation of fuselage structures. Both general instability type failure, with buckling of stringers, skin and frames, as well as a panel-type failure, with buckling of skin-stiffener panel only in between the frames, are incorporated in the analysis due to equal probability of occurrence under loading conditions. It is assumed, however, that all loads from the wing are transmitted to a single point in the fuselage along its centerline. This assumption results in higher bending moments than those the structure will be subjected in reality due to width of fuselage and wingbox being neglected (Shanley, 1960).

To these optimum weights, weight inefficiencies are included as non-optimum factors. In the case of joints, the non-optimum factor is calculated by considering the length over which the inefficiency is present in comparison to the length of the structure affected by it. The result from the application of this analytically derived inefficiency factor is a doubler effect, underlining the increase of volume over the optimum value that the assembly of individual components would result in (Equation 2.16).

$$k_{j2} = \frac{L_D}{L_n} k_D$$  \hspace{1cm} (2.16)

where:

- $k_{j2}$ represents the increase in volume of the structure due to the presence of doublers
- $L_D$ is the equivalent length of the doubler material
- $L_n$ is the length of the joint
- $k_D$ represents the ratio of doubler cross sectional area to its ideal cross section.

Compared to previous methods, additional sources of inefficiencies are also examined, such as tapered sheets, the use of standard gauges, reinforcements due to cutouts and fixtures as well as their combined effects. Weight of high-lift devices, however, is kept semi-empirical on the assumption that the portion of volume occupied by the structure and the loadings they are subjected to are very low.
Numerous methods have been developed after these analytical formulations, but the majority of them used Shanley's assumptions and derivation methods as a basis for analysis. Razani (1965) proved the existence of a relationship between the convergence of a fully stressed design and its associated minimum weight (Singh and Yadav, 1993). Crawford and Burns (1963) expanded the concept of minimum weight analysis proposed by Shanley to a variety of structural arrangements and loading combinations for stiffened cylinders. This allowed for an extensive analysis of the efficiency of different design solutions and stiffening arrangements as well as the definition of a comprehensive set of design information to be readily applicable within the design process of fuselage structures.

In addition to the analysis of relationships for optimum weight design, the main effort behind analytical formulations for structural weight estimation developed after Shanley's example aimed at widening the applicability of the weight estimation methodology. In particular, the interest was focused on including the effect of parameters and variables which had been thus far overlooked as well as generalising the derivation so that it could be applicable to more unusual loading conditions and structural arrangements (Regis et al., 2004; Schmidt et al., n.d.). The approach proposed by Lewis and St.John (1975) tried to simultaneously simplify the problem and improve the accuracy of the results by accounting for test results on allowable stress as the main basis of the derivation. By using normalised stress and fatigue index techniques, the method manages to theoretically include all material-temperature combinations in the analysis and was easily combined with cost estimation methods for trade studies on the relative benefits of the use of different materials (Figure 2.7).

An alternative solution for wing and fuselage weight of hypersonic vehicles was provided by Ardema, and later generalised for transport aircraft (Ardema et al., 1996a; Ardema, 1972; Ardema et al., 1996b; Ardema, 1988). The method combines classical plate theory and beam theory on simplified models in order to be able to rapidly assess the benefits of different configurations but in a more accurate way than empirical models and with a less detailed knowledge of the structure compared to the finite element method. The approach examines the structure under three separate load cases: the weights of the load bearing structures resulting from the analysis were then compared
to total weights computed through PDCYL, the main subroutine for weight prediction developed at NASA as part of the weight module within their AirCraft SYNThesis program (Ardema, 1996). The accuracy of the results was measured by means of a correlation coefficient and improved for the preliminary design stage by linear regression equations relating the theoretically derived weight with those computed via PDCYL.

Eustace (April 1998) tried to concretely integrate structural weight estimation within the design process by designing a more flexible analytical framework. In addition to providing a strong base by including true loads, materials and geometries in the analysis, this method has the flexibility of considering different combinations of materials, designs and configurations as well as allowing for a preliminary optimisation of size and layout of the structures. The program works around a series of EXCEL spreadsheets linked by macros to provide a stable iterative loop. The process starts with an initial design definition, first estimate of design weight and flight loading provided by the user which are iterated on both a component and assembly level until

![Histogram showing the results of the study conducted on the effects of different material choices on the weight of the F-18 wing (Lewis and St. John, 1975).](image-url)

**Figure 2.7:** Histogram showing the results of the study conducted on the effects of different material choices on the weight of the F-18 wing (Lewis and St. John, 1975).
convergence. More than providing alternative analytical equations for the structural weight derivation, the method is based on an alternative framework for the analysis. The user is forced to address the structural layout from the start as well as their various tradeoffs. Moreover, compared to other weight estimation procedures, it includes the weight effects of different locations of sub-systems and equipment as well as their related structural implications.

A limited amount of initial data needs to be provided by the user, ranging from geometric parameters, main loading at specified stations (i.e. ribs, frames..) material properties, details on attachment masses/high lift devices as well as an initial estimate on the number of secondary structures. The outputs following the different iterations include load distribution graphs, spar and rib geometry distribution pattern and buckling ratio distribution (Figure 2.8, 2.9).

**Figure 2.8:** Spar geometry distribution (Eustace, April 1998).
2.6 Alternative solutions

In recent years, the development of new state of the art technology has pushed the engineering industry to find alternative ways of solving the weight problem in the design of aerospace vehicles. The focus has been to substantially improve the accuracy of the predictions whilst limiting the computational time mainly by increasing the degree of automation of the process. Moreover, considerable effort has been put into combining weight estimation with structural optimisation with the aim of improving the efficiency of the vehicle as much as possible by means of more significant weight reductions.

2.6.1 Solid modelling and Finite Element Analysis for weight estimation

Finite Element Analysis (FEA) has been evolving over the years. From being only relegated to the static analysis of structures, it has become an essential part of the design process. The literature proposes numerous methods of formally integrating FEA in the early stages of the design not only as a tool for structural analysis, but
also to aid the weight estimation process.

Chiesa et al. (1999) outline the framework for successful integration of FEA in the conceptual design and mission analysis of launch vehicles. The process starts with a concept definition conducted with statistical weight formulations and basic estimation of performance parameters to provide the starting points for the FEM based design. The results of the process are then iterated until convergence of mass and performance values. The core of the methodology lays on the automatic generation of the FEM model from the parametric CAD model derived from initial concept study.

Komarov and Weisshaar (2002) suggest the incorporation FEA in the design environment in two separate stages: a first simplistic model (FEM-1) for the definition of the design space, constraints and loads, and a higher fidelity one (FEM-2) which includes further details as well as additional considerations such as manufacturing constraints and product requirements. On the basis of the creation of FEM-2 a first Theoretical Optimal Structure (TOS) is produced by formal optimisation techniques and which is able to provide initial rough estimates on load path, thickness distribution as well as preliminary weight. The final weight prediction is a result of the more advanced structural analysis supplied by FEM-2 and validated by both TOS and FEM-1 and translated into real manufacturable weight by means of empirically derived conversion factors which allow to both determine the final design efficiency of the structural arrangement (load carrying factor) and convert the ideal optimised FEM weight into an "as-manufactured" structural weight (construction factor) (Komarov and Weisshaar, 1998).

Although proving to be an excellent tool for the improvement of the structural design process, FEA as a weight estimation technique has its own downsides. The issue of "weight conversion" is of primary importance when adopting FEA for weight estimation purposes. The main consideration when applying it to weight derivation is the awareness that the FEM does not represent the actual weight: the model, in fact, is built around the concept of stiffness rather than mass, therefore making conversion between the two compulsory when analysing the model. One of the main challenges, therefore, has been the integration of the factoring process in the FE routine. This was already outlined by Murphy (1987) in one of the first application of FEA to the weight estimation process.
estimation process and is still a challenge. Hutton and Richmond (1979) provided a first attempt to the solution of the problem by applying FEA to the F-15A wing structure and iteratively compare the results to manufactured weights for the optimisation and convergence of individual subfactors (Figure 2.10).

![Diagram](image)

Figure 2.10: Flow chart illustrating the development and validation of mass factors for FEM conversion (Hutton and Richmond, 1979).

The result is the complete integration in the FEA of a wide range of subfactors. This addition aims at allowing the modelling process to embody and represent unmodelled weight (i.e. joints, fasteners...) and as well as adding the capability of converting FE stiffnesses to real masses. In addition to this, the inclusion of subfactors enables the modelling process to translate more realistically approximations related to material properties and overall model calibration (Figure 2.11). The general categories, however, are further specified according to the specific component being analysed, its features and the types of elements used to model the component in the finite element environment.

A similar approach to the problem of conversion of Finite Element weight to real structural weight has been presented by Droegkamp (1992). The various element groups defining the FEM include the mass of unmodelled structures. Accounting for unmodelled elements (i.e. fastening, joints...) during the weight estimation process is vital since they can account for up to 80 percent of the as-built structure depending on the type of material used (Figure 2.12) and ignoring their effect would result in highly erroneous results.

The conversion is then carried out by means of reduction algorithms combined
with mass factors applied at the component, sub-assembly and final assembly level.

The aim of these tools is to:

1. Match neutral axis and bending moment between real and ideal strength critical structures;
2. Account for differences in structural properties for stiffness critical structures;
3. Account for weight of unmodelled structures.

Finite Element Analysis has also proved to be extremely beneficial in terms of a more disciplined weight control and systematic weight management methodology Zaidel (1992). It is easier to clearly identify inefficient areas in the target weight distribution and visualise possible solutions by including FEA in the design routine.

It is also possible, by combining FEA with CAD, to provide a more efficient solution to the problems related to the accounting phase of the weight estimation procedure. One of the traditional pitfalls incurred in the production of detailed structural weight statements is the clear definition of subassemblies and their individual components. Parts tend to be neglected or overlooked in the final weight statement as well as accounted for more than once due to lacking of a clear definition of the elements making up subassemblies. The Innovative Mass Properties Analysis CATIA Tool (IMPACT)
is a great example of the benefits of accurate weight accounting procedure in terms of time savings and accuracy of results (Flaman, 2001). The program efficiently links CAD modelled structures and Finite Element Analysis outputs with a well structured weight accounting database. The assembly tree easily developed during the creation of the model is transferred to the database by an integrated coding system able to produce extensive weight reports as well as record geographical locations of individual components and subassembly. The risk of under/over counting parts is, therefore, nonexistent thus allowing a concrete reduction of the overall error in the weight prediction process.

Of a similar nature is the framework adopted by the Vehicle Analysis Branch at NASA as part of the CONfiguration SIZing Program (CONSIZE) (Martinovic and Cerro, 2002). By coupling solid modelling tools (I-DEAS) with a Finite Element routine combining a sizing module (HyperSizer) with applied loads and individual locations (EXCEL, JAVA), greater consistency was ensured for the weight estimation process. A series of automated loops link the different units until convergence, concretely speeding

Figure 2.12: Theoretical weight as a percentage of actual as built weight (Bechdolt et al., 1996).
up the process of computing preliminary structural weight estimates (Figure 2.13).

The Finite Element Method for Mass Estimation (FEMMAS) was developed by Airbus to address the need to combine the ability to rapidly evaluate a number of different structural arrangements with finite element models that are able to truly represent in detail the defining features of the different arrangements (Wenzel, 2007). The approach for a more efficient creation of FEM in this case lays on the component-based architecture behind FEMMAS. This allows the creation of an independent library made up of a number of parametrised models for individual component definition that
can be reused for both different configuration arrangements as well as in other software environments, allowing data exchange between different disciplines more rapid and smoother.

The use of Finite Element Analysis still presents some strong limitations. It is very easy to include in the model elements which are not related to the real structure but which are necessary for the accurate design of the computational model itself. It is, therefore, necessary to identify those elements and make sure that they are not included in the conversion/accounting phase in order to allow the production of an accurate weight statement.

Of primary importance is also the accurate placement of the loads on the model. The nature of the Finite Element representation makes it necessary to apply continuous loads on an individual node basis. The choice of an excessively small number of nodes can, therefore, result in an excessively large portion of the load being carried by a discrete location. This will, therefore, result in elements being oversized in order to counteract the applied load and, consequently, in the analysis providing misleading outcomes (Hutton and Richmond, 1979). As a consequence, even though the model has been designed to allow an analysis as close to reality as possible, this so called pillow effect could produce very erroneous results.

Ledermann et al. (2006) propose the use of dynamic CAD objects to successfully link the model to the finite element structure during preliminary design. The parametric-associative methods used for CAD model definition allow for rapid changes in the design configuration by describing the interdependencies among the different elements of the design.

The accuracy of the weight prediction produced through Finite Element Analysis and CAD is directly dependent on both model maturity and consistency of application of weight factors. This kind of analysis, although providing good quality results at a later stage of model definition (approximately 3 percent for the aft fuselage structure (Zaidel, 1992)), is not highly responsive conceptual and preliminary design applications in terms of analysis flow time. The relationship between degree of model detail, structural layout definition and computational time with associated accuracy of results make it more suitable as a validation tool rather than a primary weight evaluation
methodology.

2.6.2 Functional level weight estimation methodologies

The preferred approach to combine structural analysis and weight estimation especially at a conceptual level stage can be related to the use of integrated computational analysis frameworks (i.e. FLOPS (McCullers, 1984), ASCYNT (Mason and Arledge, 1993)). The benefits of adopting this type of tools is linked to the multi-disciplinary nature of the analysis that they allow to conduct, their straightforward architecture and the structure of the codes behind it which allows easy integration of different analysis subroutines. Moreover, the flexibility of the codes enable to define an increased level of detail in the analysis in parallel with the design stage considered (Garrison, 1973). In order to make weight estimation an integral part of these tools it is, therefore, necessary to design an overall approach which:

1. Can easily accommodate different analysis levels;
2. Is rapid and cost effective;
3. Can be implemented in different analysis frameworks for a more concurrent design development.

A design-oriented stochastic approach to weight estimation has been proposed as a solution to this problem (Sexstone, 1998). The basis of the structural analysis is an extension of the Equivalent LAminate Plate Solution (ELAPS) code combined with stochastic weight analysis of the preliminary weights provided by the code itself in order to considerably reduce the uncertainty intervals at both a component and assembly level. Although similar to Finite Element Analysis in the necessary conversion between ideal and real mass as well as in the definition of non-optimum weight factoring, the definition of the structure itself follows a functional build-up methodology. The decomposition is conducted at different levels of detail according to the degree of uncertainty required by the study, with the identification of the component with the greatest impact on the range of accuracy of the results. Successive minimization of confidence intervals is, therefore, possible as the design process proceeds.
This methodology has identified the Non-Optimal Mass Factors (NOMFs) as the main source of uncertainty in the derivation of structural weight. The main condition for a successful weight estimation is, therefore, the definition of the principal sources of uncertainty in the weight prediction at a component level with the aim of producing a configuration design with minimal sensitivity to it. Compared to traditional approaches, the result of this methodology is a probabilistic weight distribution at the end of each stage of the iterative process. As the design progresses, the upper and lower boundary of the various probability distributions used to define the system get closer together thus reducing the risk of discrepancies between estimated weight and as-built weight until desired convergence (Figure 2.14).

The stochastic approach used by ELAPS is of a very basic level, where only three NOMFs sets are required: the smallest possible NOMFs, the largest and the most likely to occur which are used for a curve fitting process. However, more advanced and reliable methods could be implemented, namely Design of Experiments (DoE) and Monte Carlo Simulations (Fisherman, 1996). They would allow the random production of sub-sets of NOFMs inclusive of those representative of each component from which the as-built weight would be computed. Combined with a Pareto analysis, the rejection process of NOFMs to which the configuration would be insensitive could be significantly improved in terms of computational time (Sexstone, 1998).
Airbus has recently adopted the evolutionary feature-based weight prediction approach in order to solve some of the problems related to conventional weight estimation techniques. It can be considered as functional-level approach, even if very dissimilar to the method provided by Sextone (Baker and Smith, 2003). The evolutionary characteristics of the methodology can be linked to the way the method itself works: the weight and sizing of a component evolves gradually through the process from a combination of its detailed geometry and feature definition, themselves derived from the identification of their relative driving parameters. Rather than providing one single way of dealing with the weight estimation procedure, however, this method structures
itself around both parametric and analytical prediction methods (Figure 2.15). The choice of most suited sizing approach depends on user preferences as well as the stage of the design process in which the method is applied, allowing the procedure to evolve with its progression.

The key innovation provided by this approach is the identification of the driving parameters at the individual component level, including:

1. Component loading;
2. Geographical positioning in the assembly;
3. Component family;
4. Specialist function.

These parameters, however, are individually ranked in order to determine their relative influence on the design of the component itself and, consequently, on its weight. Traditionally, the analysis in the design and weight estimation process is based exclusively on component loading to determine its size and features. However, this could be very limiting not only in the eye of component development, but especially when approaching the accounting stage of weight estimation. The simultaneous consideration of all these different factors allows for a more systematic and rigorous procedure. As a result, it is possible to obtain a clear identification of both the single parts of assembly (component level weight accounting) and the individual features characterizing the layout of the component (volume based weight accounting) by linking them to a specific function. Moreover, this framework provides not only the benefit of including manufacturing and assembly considerations very early in the design process, but also allows for a continuous questioning and challenge of the design itself. The review of each individual feature will make it easy to identify redundancies in the component, thus concretely integrating weight reduction efforts in the overall analysis framework.
2.7 Weight optimisation and management in the design process

With the aircraft industry currently increasing the pressure for shorter development times for designs that ensure better performance and increased reliability, the demands are pushing for the application of improved weight and costs management frameworks as early as preliminary design phases. The focus has shifted from accuracy to confidence levels in the weight estimates. Design teams are more interested in knowing how likely the weight of the proposed design is to change according to possible modifications that might occur in later stages rather than having a fixed single weight with no knowledge of the likelihood of matching it at the end of the design process (Monroe et al., 1998).

![Funnel of weight and cost](image)

**Figure 2.16:** Funnel of weight and cost (Dahm, 2007).

Computer aided weight and cost management tools have so far successfully aided the designers in actively managing different design scenarios and configurations. The risk and opportunities driven approach proposed by Dahm (2007) in the object oriented
aircraft weight management software SMART ACT allows the designer to work within the weight bandwidths for individual design solutions at different development stages. In addition to this, the design team is also able to compare the selected arrangement with alternative weight variants. This enables the designer to identify and account for any possible snowball effect within the configuration studied from the beginning of the design development and ensure a fast and reliable convergence of the cost and weight bandwidths to a desirable target (Figure 2.16).

Mauersberger et al. (2007) prefer a stochastic approach to the use of more simplistic and less reliable WERs to solve the problem of weight estimation in early project phases. In particular, the use of a probabilistic approach within the weight management environment allows for a consistent and more robust way of handling uncertainties within the mass properties life cycle. Mavris and DeLaurentis (2000) adopt a stochastic approach for a life-cycle process management from the point of view of exploring the feasibility of different design concepts with respect to their affordability. In this case the focus is not the lowest cost or weight, but rather a product that achieves the right balance between effectiveness and the costs and potential risks associated with its development.

The degree of complexity associated with aerospace systems has recently moved the focus towards the optimization of the overall design process (Sobieszczanski-Sobieski and Haftka, 1997). Great effort has been directed towards the integration of several disciplines in the preliminary stages of the design, aiming at achieving a more well-rounded optimum design rather than excellence in a single discipline (Bonardi, 1990; Meledy, 1974; Tong and Naylor, 2009; Carrera et al., 2003).

In order to more deeply understand the impact of external influences on the design and conduct a more comprehensive analysis, Frank (1997) added decisions and constraints from both customers and manufacturers to the design optimisation approach. In particular, the definition of a market-based weight metric in the analysis allows the identification of weight changes that specific departure from the original specifications can have on the design and the their consequent impact on the final revenue.

In terms of wing design and weight optimisation, a great effort has been directed
towards the inclusion of aeroelastic effects in the analysis and the sensitivity studies aimed at understanding the relative impact of the individual disciplines on the final design weight (Malone and Mason, 1995; Sensburg et al., 1994; Robinson and Heal, 1959). Barthelemy et al. (1994) provides one of the first examples of multidisciplinary weight analysis applied to supersonic wing models with the inclusion of aeroelastic considerations. The approach, based on the coupled effect of aerodynamics and structures, provides a basis for trade studies on the effects of material selection on the wing minimum weight.

Zink et al. (1999) were able to evaluate the variation in wing structural weight with the inclusion of aeroelastic effects through multidisciplinary design optimisation and response surface methods. The results of the study provided a comprehensive set of weight relationships derived via a parametrically-defined FEM combined with an aerodynamic wing model able to compare the weight impact of conventional control as compared to Active Aeroelastic Wing (AAW) technology with respect to minimum weight design parameters. FEM based optimisation was also used in conjunction with statistical weight equations by Huang et al. (1994). The study aims at both verifying the applicability of the selected WERs to the weight analysis of optimally designed high speed wing structures but most importantly at defining a variable-fidelity optimisation and weight estimation methodology able to provide a compromise between computational expenses and model accuracy.

A more multidisciplinary approach for aircraft design synthesis has been proposed by DeLaurentis et al. (1996) who structured a complex aircraft design framework on a combination of Design of Experiment (DoE) and response surface methods which can guide the overall design process from the very early phases. The framework combines mission requirements with an assessment of aerodynamic, structural and propulsion technologies as well as market demands and economic constraints to direct the optimisation process from both a weight and an overall design efficiency perspective. The results is a design that is not only the optimum configuration choice based on weight, performance and structural arrangement, but it is also an economically viable solution. From a deterministic perspective, a similar approach has been presented by Hwang et al. (2005) who propose a strategy for a completely automated and more ef-
Klemt and Oltmann (2007) suggest building a multidisciplinary design environment on the entire definition of the design concept by parametric-associative models where individual parts are automatically generated and parametrically defined so that any change at component level can be immediately reflected in a rearrangement of the structural definition of the overall configuration. By linking this type of model with analysis routines within the different design disciplines, the design definition can be continually updated to respond to the individual discipline requirements whilst maintaining consistency in the data exchange process.

More recently, topology optimisation has become more and more relevant within the weight estimation community as a way of more efficiently shedding extra pounds. In particular, the focus is on trying to merge topology optimisation activities with the weight reduction techniques early in the design development. At the aircraft wing level, Sensmeier et al. (2006) suggest combining a parametric definition of the configuration with moderate-level fidelity FEA which, through the use of specifically designed algorithms for model definition and analysis, enable the designer to evaluate a greater number of possible topologies with a much reduced computational effort. Having identified manufacturing feasibility as one of the key requirements in the early concept definition, Thomas (2005) adopts topology optimisation to be able to address the issue of system-structure integration for an optimised structural design from early project phases. By including specific constraints related to individual manufacturing methods within the optimisation process, he combines minimum weight design strategies with optimum topologies and a global optimum layout for a solution that can be concretely manufactured. The solution is a structure that, through particular manufacturing techniques, can accommodate the integration of specific features or systems in an optimal way with minimum weight penalties (Figure 2.17).
2.8 Summary

Weight estimation has been fundamental to the design of aerospace structures from the beginning of flight, although it only started to receive the attention of the engineering community when it was first recognised as and individual analysis field in the 1930s. Since then, the development of weight estimation methodologies has been taking primary importance within the aircraft design process.

Weight estimation methods, although still classifiable according to traditional groupings, have undergone drastic changes with the introduction of new technologies
and algorithms. The rise of FEA and CAD techniques within the design process has greatly influenced the redesign of what weight estimation truly is and the focus has moved from dry mathematical relationships to multidisciplinary analysis and frameworks to allow an effective management of structural weight from the very early stages of the design all the way to production and delivery to the customer.

There is still, however, a lack of tools and techniques to address the sizing and weight estimation of especially secondary structures at preliminary design stages. The methodologies already established in the design community are able to tackle quite successfully primary structures whose design is primarily driven by load considerations which can be closely embodied by analytical approaches and computational tools. The majority of frameworks for the weight estimation of aircraft structures still use empirical relationships to "guesstimate" the weight of components whose primary function is not that of sustaining loads. Important issues, such as individual features of different structural layouts, structural and system integration as well as manufacturing constraints, although thoroughly considered within the design process, do not seem to be successfully included in weight estimation techniques and relationships. There is also no established way of including the uncertainty related to these factors within the analysis as early as concept definition and propagate its effect not only from component to overall configuration level but also all the way through the design process.

Although they have become an integral part of the multidisciplinary design environment, weight estimation methodologies in general do not seem to be currently designed with the aim of contributing to the knowledge base of the overall design process. Weight estimation naturally links all the various design disciplines together and current approaches end up retaining important knowledge and information on how the different fields impact on each other and how, in turn, they influence the final design.
Chapter 3

An Alternative Approach to Weight Estimation

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3.1 Introduction

This section introduces the theory behind the computational tools used within this thesis. From the analysis of the issues permeating the weight estimation task at the preliminary stages of aircraft design, fuzzy logic theory is identified as a suitable aid to the problem. In particular, it is emphasised how fuzzy logic can be applied to help in acquiring more in depth knowledge about the system at hand as well as with dealing with the uncertainties within the problem itself.

Fuzzy logic has been extensively applied in this research through the use of different Fuzzy Inference Systems (FIS). Details of the theory and structure of Adaptive Network-based Fuzzy Inference Systems (ANFIS), Multiple Adaptive Network-based Fuzzy Inference Systems (MANFIS) and Neuro-Fuzzy Approximator (NEFPROX) are presented in this chapter, highlighting the differences amongst them and how their individual characteristics are valuable in handling the specific requirements encountered in the weight estimation process.

Type-2 fuzzy logic theory is then introduced to complete the picture, as a way to combine the knowledge mining properties of traditional type-1 fuzzy logic with uncertainty management and quantification, for a more comprehensive and exhaustive weight analysis.

3.2 A new perspective on weight estimation

The design of a new aircraft is characterised by a complex iterative nature. At the beginning of any design process the design parameters are only approximated quantities, which are identified within vague and imprecise ranges of possible values and are coupled with a large number of safety factors as an attempt to account for variability in the estimate. From the very start, these parameters are modified, restructured and redefined until the design is as close to the desired target as possible.

The weight estimations resulting from the analysis of these variables will, therefore, always be fuzzy in their nature: they will be always associated with an inherent degree of imprecision due to having been derived from loosely approximated parame-
The degree of fuzziness associated with design variables and, consequently, with the estimated weight, will increase proportionally to the degree of innovation that the organisation wants to embed in the final product. At the same time, the system of design iterations will become longer and more complicated. The further the design is away from conventional configurations and design solutions, the greater the number of loops to be undertaken resulting in longer completion times. Moreover, the lack of a basis for comparison and reasonability checks makes it harder to get it right the first time around, resulting in costly changes further along the design process (Mauersberger et al., 2007).

**Figure 3.1:** Graph showing the relationship between risk and cost for changes in the design process (Neff, 2001).

Figure 3.1 underlines the dichotomy between cost and knowledge during the
design process. In the early stages of the design of a new vehicle, only limited knowledge of the required design parameters is available. However, it is right at the conceptual and basic development stages that the most crucial decisions are being made. The cost of making decisions and modifying them rises exponentially with program development. It is vital to concretely consider the risks and uncertainties associated with each piece of information and subsequent design decision in order to speed up the loops and avoid costly changes later in the process.

For these reasons, when designing a model for weight estimation, it is important to couple a substantial understanding of the system under analysis with the ability to identify the unknowns in the problem and account for them within the estimate itself.

The weight estimation task can, therefore, be thought as comprising of two fundamental phases. To begin with, it is vital to obtain extensive knowledge about the system under consideration. This will range from information regarding its different parts and features, details of the processes needed for its manufacture and assembly as well as how the combination of these factors ends up influencing the final design itself. This will allow complete traceability of the set of design decisions taken as well as their combined impact on the design, thus not only improving the overall accuracy and credibility of the weight estimate but also resulting in added confidence during the decision making process.

Complete knowledge of the system, however, will never be possible. The number of variables and the iterative nature of the design process itself permeate the weight estimation task with a high degree of uncertainty, which grows exponentially with the application of new technologies within the design concept. There is the need, therefore, to combine a comprehensive knowledge acquisition with the ability to identify the uncertainties in the problem and account for them in the estimate itself (Figure 3.2).
3.3 Fuzzy logic for knowledge acquisition and management

In the design of a weight estimation model, the focus is on the use of tools and techniques that are able to provide robust and reliable approximations when adopted in noisy and uncertain environments. The literature provides examples of successful applications of soft computing techniques for modelling problems which are characterised by missing and imprecise information which are comparable to that present in the preliminary stages of the design of a new aircraft (Fonseca et al., 2001; Chawdhry and Pant, 1997).

Among these, fuzzy logic appears to be extremely suited for the task, due to its capability to translate the interdependencies between the different variables within the problem into a series of rules which can then be included as an integral part of a dynamic knowledge base to be used during both the design and weight estimation of aircraft structures.
3.3.1 Fuzzy logic vs. classical logic

Fuzzy logic was first introduced by Prof. Lofti Zadeh in 1965 as a mathematical framework designed to deal with uncertainty (Zadeh, 1965). The initial inspiration behind the development of fuzzy set theory was based on the notion that the information that a mathematical model is able to provide rapidly declines as the system under analysis becomes more complex, thus considerably affecting the capability of the engineer to take the most appropriate decisions. Fuzzy logic was, for this reason, introduced as a tool to enable to formalise and analyse ill-defined problems.

The theory behind fuzzy logic parallels that of classical logic. Both environments are built around the notion of sets as collections of elements which share a specific characteristic or property. Within classical logic, the sets are defined in such a way that members and non-members of a specific sets are unambiguously defined. An element, therefore, either belongs or does not belong to a set, and the transition between membership and non-membership to the set is crisp (Ross et al., 2002). A membership value of "1" will identify a member of the set, whilst a value of "0" will be associated to an element that does not belong to such set.

\[ \begin{array}{c}
\text{(a)} \\
\end{array} \]

\[ \begin{array}{c}
\text{(b)} \\
\end{array} \]

**FIGURE 3.3:** Diagram showing the difference between crisp sets (a) and fuzzy sets (b) according to membership function definition.

In everyday situations, however, such sharp classification is often impossible. The perception and description of the real world is often done through concepts which can be vague and imprecise, and through statements which can be true or false only
to some degree. Elements will belong to a specific fuzzy set according to various degrees of membership which indicate the extent to which the element itself is associable to the concept represented by the fuzzy set. As a result, membership to a fuzzy set can be defined via a characteristic membership function (MF) which maps the individual elements to a specific value between 0 and 1 according to its specific degree of membership to that set (Figure 3.3).

3.3.2 Reasoning with fuzzy logic

Within the fuzzy logic environment, the description and approximation of a system is obtained by mapping an input space to an output space through a set of rules of the form:

$$\text{IF premise (antecedent), THEN conclusion (consequent).} \quad (3.1)$$

![Figure 3.4: Schematic representation of the definition of the design space though membership functions and fuzzy rules.](image)
Chapter 3

The IF-THEN rule base is used to represent the condition that if a specific fact is known, then it is possible to deduce a conclusion. In the case of a mathematical system, the process of fuzzy inference can be expressed as:

\[ \text{IF } x \text{ is } A, \text{THEN } y \text{ is } B. \]  

(3.2)

where \( x \) and \( y \) are the variables of interest, and \( A \) and \( B \) relate to individual fuzzy sets within the universe of discourse of the problem.

Each rule defines a distinct fuzzy patch in the design space of interest, depending on the shape and properties of the different membership functions used (Figure 3.4).

\[ \text{y} \vphantom{\text{x}} \]  

\[ \text{x} \vphantom{\text{y}} \]

**FIG URE 3.5**: Schematic representation of the evolution of fuzzy rules in the design process and its impact on the accuracy of system approximation.

Such representation of the system under study can greatly aid the visualisation of the effects of several different combinations of design variables on the final solution. In terms of building a weight estimation model, this type of approach could also enable the accuracy of the approximation to grow in parallel with the design process itself. At the very early stages of concept definition the fuzzy sets will be large and able to approximate the system loosely. With an increased definition of the design, the rule patches will get smaller, leading to improved and more representative estimates (Figure 3.5) (Kosko, 1994).

The application of fuzzy reasoning principles and techniques for system modelling is achieved through the use of fuzzy inference systems (FIS). FIS are computational frameworks based on the principles of fuzzy reasoning and fuzzy set theory and are structured around five functional blocks (Figure 3.6):
1. The rule base which holds the necessary IF-THEN rules;

2. The database which manages the information about the membership functions of the relevant fuzzy sets used within the rules;

3. The decision-making unit which performs the inference on the rules;

4. The fuzzification unit which converts the variables of interest into fuzzy quantities;

5. The defuzzification unit which translates the fuzzy outputs into crisp quantities at the end of the process (Sivanandam et al., 2006).

![Fuzzy inference system diagram](image)

**FIGURE 3.6: Fuzzy inference system.**

Two types of FIS in particular have been successfully applied to a variety of engineering problems and they differentiate themselves in both the nature of their outputs and the way they are derived (Figure 3.7). Mamdani fuzzy inference systems were firstly introduced by Mamdani and Assilian (1975) as a tool for the design of automatic controllers. In this type of FIS, each rule consequent will be represented by a fuzzy set. Once all the rule consequents have been evaluated, they are combined together to get a output distribution, which can be defuzzified or maintained as a fuzzy quantity according to the specific needs of the study. In the Takagi-Sugeno-Kang (TSK) FIS, on the other hand, the consequents of the individual rules are formulated as crisp polynomial functions, which relate the input variables to the desired output within the fuzzy region specified by the individual rules (Jang and Sun, 1997).

As shown by figure 3.7, the principal difference between the two fuzzy inference systems lays in the nature of the consequent of the fuzzy rules and, as a consequence, in the methods of defuzzification employed by the FIS. This strongly influences both
the quality of the estimation provided by the system, the final FIS structure as well as the overall interpretability of the resultant network and of the rulebase it derives.

In terms of approximation qualities, the complexity of TSK FIS depends on the nature of the function being analysed: the higher the number of extrema, the larger the number of fuzzy sets needed. In addition to this, TSK FIS are characterised by a higher number of adjustable parameters especially within the rule consequents, as opposed to the Mamdani type. For this reason, in the case of larger scale problems, the resultant system structure for TSK FIS could potentially become too complex and unmanageable due to the "curse of dimensionality" (Guney and Sarikaya, 2008).

The literature, however, identifies TSK FIS not only as being able to achieve higher accuracy in approximations environments, but also better suited at being coupled with algorithms for automated learning due to the more explicit functional relationship between inputs and outputs (Jassbi et al., 2006; Jang and Sun, 1997).

Mamdani FIS appear more largely in industrial applications, mainly due to their ability to provide high accuracy through a relatively simple network structure. The attractiveness of Mamdani over TSK FIS lays on the more intuitive nature of its

**Figure 3.7:** Schematic representation of the differences in output derivation between Mamdani and TSK fuzzy inference systems.
rulebase. Since both input and output are fully described as fuzzy sets, the system becomes highly more interpretable from a visual perspective as well as more intuitive in its design, which eases the process of converting the designer's knowledge into fuzzy rules.

3.4 Neuro-fuzzy systems

Fuzzy systems have been a source of growing interest across the engineering community in recent years (Chawdhry and Pant, 1997; Dhingra et al., 1990; Ghorbani and Ghasemi, 2009; Topçu and Sarıdemir, 2008). The attractiveness of the fuzzy approach for engineering problems lays in:

1. The capability of fuzzy systems to incorporate the uncertainties within the problem in the analysis in a way which can be easily interpreted and modified by the user;
2. The flexibility of expanding and enhancing the analysis by adding expert knowledge to the framework;
3. The robustness of fuzzy systems to noisy environments.

The design of a conventional fuzzy system requires the users to convert their knowledge of the problem into the fuzzy rules required for its complete definition. In the case of problems such as that of weight estimation, however, the designer does not have the complete knowledge of the system a priori; on the contrary, the aim of the analysis is to gather as much information about the system as possible in order to be able to make informed and efficient design decisions. It is, therefore, vital to have a system which is capable of deriving its own set of fuzzy rules, which in turn can be used in the description and approximation of the system.

This can be achieved through neuro-fuzzy systems, mathematical frameworks which integrate Artificial Neural Network (ANN) theory for parameter derivation and optimisation with fuzzy logic (Vieira et al., 2004). These are multi-layered, feed forward networks which are trained by a set of algorithms to learn relationships between the variables defining the problems from a set of given data. The learning process consists in
the modification of the structure of the FIS itself on the basis of input-output patterns in order to allow the network to match the system response and provide an improved numerical approximation of the problem under analysis.

The fusion of FIS with ANNs has been a source of great interest for the solution of real life problems, due to the ability of the hybrid system to combine the expression of knowledge through linguistic rules with adaptive learning (Guney, 2006; Lotfi, 2001; Dinh and Afzulpurkar, 2007). The main reason for the interest of the research community in these modelling tools lays in their ability to combine the low-level learning and minimal computational effort required by neural networks with the higher-level transparent linguistic system description which is distinctive of fuzzy logic. The most widely used types of neuro-fuzzy systems belong to the fused category: the learning algorithms from ANNs drive the computation of the parameters within the FIS structure via an ANN-based network (Abraham, 2001).

3.4.1 Adaptive Network-based Fuzzy Inference Systems (ANFIS)

Adaptive Network-based Fuzzy Inference Systems (ANFIS) represent the most successful and widely used type of neuro-fuzzy architecture for the optimisation of TSK fuzzy inference systems. Firstly developed by Jang (1993), ANFIS was designed as a way of deriving "an input-output mapping based on both human knowledge (in the form of fuzzy if-then rules) and stipulated input-output data pairs". For a given data set, an ANFIS network can be created and subsequently optimised by adaptive learning.

Adaptive techniques are aimed at changing selected parameters within the FIS in order to better reflect the relationships existing between the different variables in the problem. This is achieved by linking the FIS to a multilayered feed forward network made up of nodes and directional links. Each node performs a particular function based on both the incoming signals related to the input variables and the specific parameters pertaining to the node itself. The network is made up of adaptive nodes, traditionally represented by squares and whose parameters are updated during network training, and fixed nodes, which define the necessary operations to be carried out on the adaptive parameters. The parameters associated with the adaptive nodes can be updated using back propagation and hybrid learning techniques in order to match a given training
Figure 3.8 represents the case of a network with two inputs, \(x\) and \(y\), and an output \(z\), and which is described by two fuzzy rules of the form:

- Rule 1: if \(x\) is \(A_1\), and \(y\) is \(B_1\), then \(z_1 = p_1 x + q_1 x + r_1\)
- Rule 2: if \(x\) is \(A_2\), and \(y\) is \(B_2\), then \(z_2 = p_2 x + q_2 x + r_2\).

The adaptive (square) nodes occur in layers 1 and 4 and the fixed (circular) nodes in layers 2, 3 and 5. In layer 1, the adaptive node yields a nodal output given by:

\[
O_1^i = \mu_{A_i}(x) \quad (3.3)
\]

where \(O_1^i\) is the membership function which determines the degree to which a given input \((x)\) belongs to a defined fuzzy set and \(\mu_{A_i}\) is associated with the shape of the membership function being used. For instance, in the case of a bell shaped membership function with maximum and minimum values of 1 and 0 respectively, \(\mu_{A_i}\) will be represented by:

\[
\mu_{A_i}(x) = \frac{1}{1 + \left[\left(\frac{x - c_i}{a_i}\right)^2\right]}^{b_i} \quad (3.4)
\]
where \( a_i, b_i, c_i \) is the set of adaptable parameters associated with this layer.

The process of inference of the fuzzy rules in the problem occurs in layer 2, where the system picks the specific rules to apply based on the value of their firing strength \( w_i \) which is calculated by multiplying together the signals coming into the node from layer 1.

\[
w_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad i = 1, 2
\]  

(3.5)

The normalised firing strength \( \tilde{w}_i \) which is the specific weight of the rule based on the structure of the entire network, is then calculated in layer 3, according to the individual firing strengths present within the network, as,

\[
\tilde{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2
\]  

(3.6)

Partial node outputs are calculated in layer 4 as,

\[
O_i^4 = \tilde{w}_i z_i = \tilde{w}_i (p_i x + q_i y + r_i)
\]  

where \( (p_i, q_i, r_i) \) is the adaptable parameter set associated with each square node in this layer. The overall output \( O_i^5 \) is then computed in layer 5 as a summation of all the incoming signals from the individual nodes within layer 4,

\[
O_i^5 = \text{overall output} = \sum_i \tilde{w}_i z_i = \frac{\sum_i w_i z_i}{\sum_i w_i}
\]  

(3.8)

When the data is fed through the network for the first time, however, the final output may not match the training data set accurately. In such cases, the adaptable parameters sets associated with layers 1 and 4 can be changed to improve the quality of the approximation via a hybrid learning technique combining gradient based and least squares methods (Jang and Sun, 1997; Gallo et al., 1999). Each step (epoch) of the hybrid learning cycle comprises two phases: a forward pass and a backward pass. In the forward pass, the input data and functional signals are sent forward and used in the calculation of the node output. The parameter set associated with the calculated output node is then evaluated using least squares method. The functional signal is then carried forward throughout the network until the error measure is calculated.
The derivative of the error measure with respect to the parameters in each output node (error rates) is then calculated and propagated from the output end towards the input end (back propagation) and the parameters set updated accordingly using gradient based optimization methods. The parameters can either be updated after the complete training data set has been examined by the system (batch or offline learning), or they can be sequentially modified after each input-output pair has been presented.

In its traditional layout, however, ANFIS can only provide an analysis framework for single output problems. For this reason, the principles behind ANFIS have also been extended for the development of Multiple Adaptive Neuro-Fuzzy Inference System (MANFIS) (Cheng et al., 2002). MANFIS represent a generalisation of ANFIS for handling the modelling of systems with multiple outputs and responses. In this case, the network can be visualized as a combination of a number of individual ANFIS structures simulating a single response (Figure 3.9). In the case of MANFIS, however, the mapping between individual inputs and the desired multiple outputs can be obtained by the minimisation of the error measure obtained by summing the squared errors of the $m$ ANFIS used in the network structure. This, in turn, can be approached as the learning of $m$ individual ANFIS (Dhingra et al., 1990).

![Figure 3.9: Schematic representation of a MANFIS network.](image-url)
3.4.2 Neuro-Fuzzy Approximator (NEFPROX)

The Neuro-Fuzzy Function Approximator (NEFPROX) is a neuro-fuzzy architecture designed to derive fuzzy systems of the Mamdani type via back propagation and reinforcement learning (Nauck, 1997). The network is structured in 3 layers: the first denoting the input variables \((x_1, \ldots, x_n)\), the second the fuzzy rules \((R_1, \ldots, R_k)\) and the last the output variables \((y_1, \ldots, y_m)\) (Figure 3.10). One of the characteristics of this network structure, as opposed to other fused neuro-fuzzy systems like ANFIS, is the sharing of the weights across different rules. This ensures that each fuzzy set and associated linguistic value are uniquely defined and that all the fuzzy weights related to them evolve in the same way during the learning process to guarantee consistency.

![Figure 3.10: Structure of the NEFPROX network.](image)

The system evolves through supervised learning in a heuristic manner. The error between the system output and the expected value is computed and used to modify the membership functions of the consequent part of the rule to a higher or lower value, then it is propagated back through the network. At this stage the individual error of
each rule node is computed and used to modify the membership functions relative to the antecedent part of the fuzzy rule. The new output is then computed in a similar manner and the training proceeds until the desired level of convergence is achieved.

The difference in learning methodology between NEFPROX and ANFIS lays in the nature of the rulebase itself. The overall output of a TSK FIS is represented by a linear combination of the consequent parameters of the rules used and therefore the error rates are differentiable functions. On the other hand, the rules within the Mamdani FIS in NEFPROX are fuzzy in both their premise and consequent side and can only be optimised using heuristic approaches.

**3.5 Designing under uncertainty**

In the conceptual stage of the design of a new vehicle the engineer is faced with the possibility of highly influencing the final product though the decision making process. However, at this point, only limited information and details of the system itself are available. The first step to try and adapt to this kind of uncertain environment is the clear identification of the possible sources of uncertainties that the program will be affected by.

![Uncertainties in Design](image)

**Figure 3.11: Uncertainties in the design process.**

Hahn and Shapiro (1994) defined that the complete knowledge of a system is usually prevented by different types of uncertainties related to unavailable information, erroneous information as well as misinterpretation of available information (Figure
3.11). In the preliminary stages of a design process, the whole range of parameters affecting the design is unavailable. The engineer has, therefore, to deal with incomplete set of data (i.e. skin thicknesses not defined, exact location of cutouts, etc.) and it is forced to make assumptions. The result of this is an inherent degree of inaccuracy in the output of the analysis. Erroneous information comprises of both lack of confidence in the data as well as inconsistency in data itself. Misinterpretation of information is the one that occurs the most in the weight engineering environment. The lack of a standard and recognised weight accounting system is usually one of the main sources of misinterpretation. Parts of a subassembly or individual features can be considered more than once or even ignored if mistakenly attributed to a nearby subassembly.

Probability theory so far has been the preferred method for the quantification of uncertainty within engineering design (?). More recently, however, the engineering community has highlighted that complex systems are characterised by more multifaceted and varied types of uncertainty which traditional probability analysis is not fully capable of handling. In addition to this, probabilistic frameworks are usually based on strong assumptions for the complete characterisation of the required uncertain parameters. A typical example is the definition of probability density functions (PDFs) derived without any sufficient supporting evidence (Bae et al., 2004). As a consequence, the quality of the results from this type of analysis will only be a reflection of the quality of the assumptions used.

Traditional fuzzy logic represent as a suitable alternative to probability analysis as a framework for weight estimation. It is able to:

1. Comprehensively deal with the vagueness and imprecision which permeates this type of analysis;
2. Provide a model which is sufficiently flexible to be adapted according to the level of information available at specific stages of the design;
3. Capture knowledge about the system under study in terms of both the various interdependencies between the different variables and their impact on the final design solution.

FIS are, however, subject to inherent uncertainties:
1. Uncertainties related to the definition of antecedents and consequents especially if extracted from a group of experts,
2. Noisy measurement in the activation of membership functions,
3. Noisy training data.

### 3.5.1 Interval type-2 fuzzy sets

Zadeh (1975) was the first to address the issue of uncertainty quantification within fuzzy logic by providing a generalisation of conventional (type-1) fuzzy set theory by introducing the notion of *type-2 fuzzy sets*. A general type-2 fuzzy, as opposed to type-1, is characterised by membership functions that are themselves fuzzy. In particular, within the research presented in this thesis, only *interval type-2 fuzzy sets* (IT2 FS) will be considered, due to their greater computational efficiency as opposed to general type-2 fuzzy sets.

The reasoning behind the transition from classical logic to fuzzy logic has its foundation in the inability of determining the membership of an element to a specific set in a crisp and unambiguous way. Membership functions of a type-1 fuzzy system can be defined either through expert knowledge or adaptive procedures. In situations such as the weight estimation of an aerospace structural system, the data driving the FIS adaptive learning is, itself, fuzzy and subjected to variability which is difficult and computationally expensive to quantify at the preliminary phases of the design process. The membership functions derived during the network optimisation will, therefore, be characterised by a degree of uncertainty.

Figure 3.12 highlights the differences between type-1 and type-2 fuzzy sets. It is possible to visualise type-2 fuzzy sets as a "blurring" of type-1 membership functions, which is obtained by shifting the points to the left or to the right of the original MF. As a result, at specific values of x, the type-2 membership functions will be characterised by an interval of possible values. The type-2 fuzzy set is bounded by 2 type-1 membership functions, an upper \( \bar{\mu} \) and a lower \( \hat{\mu} \) MF.

The shape of the region bounded by the two MFS, called *footprint of uncer-
\[ \mu_{A_i} = \exp\left[-\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right] \quad m \in [m_1, m_2] \]  

(3.9)

The second is represented by the case where the set is identified by a Gaussian primary membership function characterised by a fixed mean \( m \) but uncertain standard deviation with values \([\sigma_1, \sigma_2]\) (3.10).

\[ \mu_{A_i} = \exp\left[-\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right] \quad \sigma \in [\sigma_1, \sigma_2] \]  

(3.10)

It is easy to understand why the different choices of FOU are an excellent way of assessing quantitatively and visually the uncertainty within the system under study.
A larger FOU will highlight a more uncertain environment, and vice versa.

3.5.2 Type-2 fuzzy systems and their structure

The general principles behind type-2 fuzzy inference systems do not change greatly from type-1. What differs between the two types of FIS is the nature of the membership functions used to describe the problem and, as a consequence, the operations that are based on them. The structure of type-2 fuzzy inference systems, for this reason, is not dissimilar to that of a type-1 (Figure 3.14). The fuzzifier still embodies the function of mapping crisp inputs into fuzzy sets, in this case $\text{IT}_2$. The formulation of the rulebase still follows the traditional IF-THEN structure, with the only difference that some or all the sets associated with rule antecedents and consequents
are IT2. Consequently, the inference engine in this case will combine the necessary rules together to provide a mapping from type-2 inputs to type-2 outputs. The main difference lays in the computation of the output. In type-1 FIS, the defuzzifier block enables the translation of the fuzzy output into a crisp quantity (i.e. type-1 to type-0 transformation). This functional unit has been replaced in type-2 FIS by an output processing block. Since the output of the inference engine is a type-2 set, an "extended version" of type-1 defuzzification is necessary to go from type-2 to type-1 fuzzy set (Zadeh, 1975). This is achieved through the type reducer. The resultant type-1 set can then be defuzzified into a crisp output.

Type-2 fuzzy systems can be interpreted as a blurred type-1 FIS due to the effect of uncertainties within the problem (Karnik et al., 1999). For this reason, it is possible to interpret the type-1 fuzzy set obtained from the type reduction operations in the FIS as a measurement of the uncertainty of the system. By assessing measures of spread within the resultant type-reduced fuzzy sets, it is therefore possible to understand the variability in the outputs of the systems and trace them back to the uncertainties within the initial quantities inputted to the FIS. This allows a complete and exhaustive visualisation of the sources of uncertainties and risk within the problem itself and their impact on the final solution.

3.6 Summary

This chapter has introduced the computational tools used within this research. The analysis of the task of structural weight estimation at the preliminary stages of the design of a new aircraft, as presented in Chapter 2, highlighted significant points to consider when designing a new approach to the problem. In particular, it is important to keep in mind that the basis for an accurate and reliable weight model is the efficient coupling of a substantial knowledge of the design of the component itself with a framework that is capable of accounting for and propagating the uncertainties permeating the problem at hand throughout the computational modelling.

Fuzzy logic appears to be able to combine:
1. Effective knowledge acquisition attributes;
2. Intuitive visualisation of knowledge of the design space of interest and causality among the variables;
3. Extensive and robust uncertainty management and propagation capabilities.

In particular, a number of framework within the fuzzy logic environments have been introduced and their specific attributes will be analysed in relation to the design of an optimal weight estimation model in the later chapters of this thesis.

Neuro-fuzzy systems have been introduced as a suitable way of combining the ability to extract knowledge from data with a fuzzy rule-based structure and visualisation. Within this category, Adaptive Network-based Fuzzy Inference Systems (ANFIS) and Neuro-Fuzzy Approximator (NEFPROX) were selected. Numerous examples within the literature have showed how both FIS structures are able to combine great modelling accuracy with simple networks and minimal computational effort. The following chapters will explore how both approaches compare in terms of:

1. The accuracy within the estimation;
2. The interpretability of both the resultant network structure and rules extracted from the data;
3. The complexity of the final network and rulebase;
4. The flexibility to incorporate different requirements in parallel with the evolution of the design process itself.

In terms of uncertainty management, the concepts have been extended to Type-2 fuzzy logic. This tool has been identified as a suitable means of combining the knowledge mining properties of traditional type-1 fuzzy logic with uncertainty management and propagation within the computational model, for the design of a more comprehensive robust weight estimation framework.
Chapter 4

ANFIS for Weight Estimation Problems

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4.1 Introduction

This chapter introduces the application of neuro-fuzzy systems for the weight estimation of aircraft structures. In particular, this section will assess how the performance of an ANFIS-based framework compares against the design requirements and expectations relative to the initial definition and preliminary design of a structural component.

The model, in this first instance, can only be formulated on the basis of space requirements from a preliminary assessment of the location where the structure itself will be placed and of its overall function. The variables of interests, therefore, will relate to the location of the component within the major subassembly, to a preliminary geometrical definition of the structure itself and of its predominant features as well as to an initial characterisation of its surroundings and to the different loading applied to it.

In addition to the requirements established within the design process, the focus of the design of the weight model will cover issues such as structure parameterisation, variable selection and model optimisation with the aid of a specific structural example.

4.2 Weight estimation for aircraft secondary structures

While major structural assemblies, such as wing or fuselage, justify the use of computationally expensive modelling tools to aid the weight estimation process due to their size and function, the methods used for assessing the weight of secondary structures appear to be mostly empirically based even at later stages of the design process. On one hand, the design of this type of structures is driven by a high number of variables related to both the individual structure itself as well as the surrounding elements. This makes the development of analytical methods that are able to represent the numerous functions covered by secondary structures a very challenging task. On the other hand, the weight of these structures is minimal compared to that of primary structural elements. As a result, it is currently infeasible to apply computationally expensive analytical tools such as FEA for the weight estimation of secondary structures,
from both a cost-to-weight point of view as well as due to their inability to fully capture
the weight implications of major issues such as system installation. There is, however,
a lack of empirical or semi-analytical approaches able to provide reliable results and
incorporate the effects of additional factors such as manufacturing and installation
within the estimates themselves.

The fixed trailing edge (FTE) is the section of the wing extending aft of the
rear spar and acts as support for ailerons, spoilers, shroud box and shroud panels. It is
mainly made up of ribs which are designed to transmit the aerodynamic loads acting
on the movable surfaces and panels to the rear spar. The wing FTE can be split into
three sections:

1. Inboard Fixed Trailing Edge (IFTE), which houses landing gear attachments and
   false rear spar assembly;
2. Midboard Fixed Trailing Edge (MFTE), which comprises spoiler and flap track
   attachments;
3. Outboard Fixed Trailing Edge (OFTE), which includes aileron supports and outer
   falsework.

4.2.1 Case study: spoiler attachment ribs

Spoiler attachment ribs are part of the wing fixed trailing edge and their main
purpose is to provide fixed support for the spoilers (Figure 4.1). For the purpose of
this study only spoiler attachment ribs in the MFTE have been considered. All spoiler
hinge ribs are shaped as an A structure and their main function is that of ensuring fixed
support for the spoilers. In addition to this, they also allow the aerodynamic contour
of the wing to be preserved during flight and provide the necessary space allocation,
attachment points and support for the systems running through the wing trailing edge.
Their individual functions depend on their location along the span of the spoiler itself
and the type of loading that they need to sustain. This allows the classification of the
ribs according to 4 different categories:

1. Actuator hinge ribs: they provide a restraint for the spoiler in the hinge line
direction, and in both perpendicular directions to the hinge line. They carry loads acting on the spoiler and distribute them to the upper and lower wing skins and into the rear spar.

2. Common ribs: they provide a common attachment point to the adjacent movable surfaces.

3. Failsafe ribs: they appear in along the span of critical spoilers to prevent the detachment of the spoiler in case of failure of any of the actuator hinge ribs.

4. Intermediate ribs: they provide attachment points and support for top and bottom secondary structural panels as well as system routing. They also allow aerodynamic and system loads to be transferred into the rear spar as well as upper and lower skins.

![Figure 4.1: A general midboard fixed trailing edge assembly (a), highlighting spoiler attachment ribs and their nomenclature (b).](image-url)
In order to make the weight model representative of the real structure, it is important to be able to embody the actual design of the component/assembly being evaluated. In the case of spoiler hinge ribs, the design is driven by both loading consideration as well as the need to maintain the aerodynamic integrity of the wing.

A typical spoiler attachment rib needs to sustain the following loads:

1. Aerodynamic loads \((\omega_{\text{aero}})\), which are applied to the upper section of the rib through its direct attachment to the fixed upper skin panel.

2. Hinge loads \((F_r)\) resulting from the axial hinge force components from the spoiler and acting on the spoiler hinge line.

3. Strut loads \((P_r)\), which are the effect of aerodynamic loads acting on the fixed lower skin panel and transmitted to the bottom section of the rib via a strut.

4. Fuel loads \((\omega_{\text{fuel}})\) acting on the vertical section of the rib, which can be found in those ribs that are positioned where an external integral spar stiffener would have been.

5. System attachment loads resulting from the routing of system runs across the trailing edge and fixed on individual rib locations.

6. Applied thermal stresses \((\sigma_{\text{th}})\) arising from the differences in thermal expansion at composite to metal interfaces. For the purpose of this study a constant 20MPa was applied on metallic sections connected to composite components.

Figure 4.2 shows a schematic representation of the positive loads acting on a spoiler hinge rib. System installation considerations have been taken into account in the analysis. This was achieved by including within the input variable set the total axial load resulting from system attachment \((F_{\text{hyd}})\) on individual ribs as well as the number of hydraulic system attachment points on the rib structure \((n_{\text{hyd}})\). For the purpose of this study, only hydraulic installation has been taken into account due to the greater proportion of its loading on the rib structure compared to that resulting from electrical installation and other miscellaneous systems.
Figure 4.2: Typical spoiler attachment rib (a) and its schematic representation (b), highlighting its main three sections, the positive forces applied on them and global geometrical parameters.

4.3 Model development

4.3.1 Subtractive clustering for fuzzy model extraction

Subtractive clustering was adopted for model initialisation in order to derive an optimal and concise model structure and ensure a rapid convergence during network training. The method was initially proposed by Chiu (1994) as a way to identify natural groupings of data within the original input-output data pairs and formulate from these an initial fuzzy model to further optimise. By identifying cluster centres within the data set, it is possible to determine the initial rules needed to describe the system by associating each cluster with the presence of a rule. In addition to this, the techniques also helps establish initial values for the premise parameters for the individual rules.

The determination of cluster centres and initial estimation of rule parameters can be formalised as follows. Consider a set of data points $x_1, x_2, ..., x_n$ in an $M$-dimensional space which have all been normalised in each dimension. Initially it is reasonable to assume that each single data point is a potential cluster centre. It is possible to measure the individual potential of each data point $x_i$ to be a cluster centre as:

$$P_i = \sum_{j=1}^{n} \exp(-\alpha \|x_i - x_j\|^2)$$  (4.1)
where:

$$\alpha = \frac{4}{r_a^2}$$  \hspace{1cm} (4.2)

$r_a$ is a positive constant indicating the cluster radius selected for the problem and $\|x_i - x_j\|$ indicates the Euclidean distance between the points considered. This formulation for the potential of a data point depends on its distance from the other points in the set, with $r_a$ indicating the radius of the neighbourhood under analysis. In other words, the impact of neighbouring data points diminishes exponentially with the square of the distance between the points.

Once the potential of all data points has been calculated, the point characterised by the highest value of $P_i$ will be singled out as the first cluster centre. $x_i^*$ and $P_i^*$ will subsequently be used to identify its location and potential respectively. The potential of the remaining data points will then be computed again as:

$$P_i = P_i - P_i^* \exp^{-\beta \|x_i - x_i^*\|^2}$$  \hspace{1cm} (4.3)

where:

$$\beta = \frac{4}{r_b^2}$$  \hspace{1cm} (4.4)

Equation 4.3 highlights how the new potential of the single data points decreases proportionally with its distance from the cluster centre. The value of $r_b$ will define the radius of the neighbourhood affected by a reduction in potential and normally it is set at a value higher than $r_a$ to avoid closely spaced clusters.

A new cluster centre will then be identified once the new potential of all remaining points has been computed. The process continues by further reducing the potential of the data points with respect to their distance from the second cluster centre. In general terms, once the $k$th cluster has been located, the potential of each point can be adjusted according to:

$$P_i = P_i - P_k^* \exp^{-\beta \|x_i - x_k^*\|^2}$$  \hspace{1cm} (4.5)

where $x_k^*$ identifies the location of the $k$th cluster centre and $P_k^*$ the value of its poten-
tial. The process continues until the condition $P_k^* < 0.15P_1^*$ is met.

Each cluster will identify a specific input-output behaviour present in the system to be modelled. As a consequence, each cluster centre can be adopted as a starting point for the formulation of a fuzzy rule for the description of the system behaviour.

### 4.3.2 Variable selection

One of the major issues regarding the acquisition of knowledge of a system in the weight estimation environment is the large number of associated variables. Fuzzy models have the capability of dealing with multiple combinations of input variables. However, with such capabilities come associated problems, including overcomplicated models, which are computationally expensive. It is crucial, therefore, from the modelling perspective, to be able to reduce the number of parameters to an optimum, by eliminating variables that have little or no impact on the performance of the model itself. This not only makes the model much simpler, but also improves its usability and reliability.

One of the most efficient ways of selecting input variables and rapidly simplifying the ANFIS network structure present in the literature is the method of variable removal introduced by Chiu (Chiu, 1996). The process is initiated with the development of an initial fuzzy model containing all possible input variables through subtractive clustering. This method determines the number of rules and the associated rule parameters, which can in turn be tuned or optimized using ANFIS to minimize the root mean square error (RMSE) of the output with respect to the checking data, computed as:

$$RMSE = \sqrt{\frac{\sum_{j=1}^{n} (\hat{\theta}_i - \theta_i)^2}{n}}$$  (4.6)

where $\theta_i$ and $\hat{\theta}_i$ indicate respectively the real and predicted values for the variable of interest for $n$ number of data points.

The importance of each input variable is then determined by the systematic elimination of variables and their associated rules. This allows the effect on the per-
formance of the full model to be analysed and rapidly determine the optimal variable set for the modelling of the problem, as a compromise between model complexity and accuracy in the estimation. This process is deployed in five main steps (Figure 4.3):

1. Evaluation of model performance on checking data according to RMSE analysis, based on the model built with all input variables;
2. Evaluation of model performance with systematic variable removal from the original model;
3. Identification of the most efficient partial set of input variable for model definition;
4. Subsequent variable elimination from best performing model from step 3 and re-iteration of steps 2-3;
5. Selection of the best performing variable set based on the minimum RMSE calculated across the various models.

![Figure 4.3: Method of systematic variable selection proposed by Chiu (Chiu, 1996).](image)

A final fuzzy model can then be generated using subtractive clustering in conjunction with ANFIS based only on the best performing set of variables as inputs to the system.
4.3.3 Parameterisation and data pre-processing

One of the principal aims of the model was to identify the effect of individual design parameters on the component structural weight. In order to achieve this, the initial parameterisation of the problem for ANFIS modelling was developed by considering three main parameter classifications:

1. Global variables;
2. Local variables;
3. Loads.

Spar height \((h)\) at the individual rib location and hinge line datum \((L)\) were chosen as global geometric definition of the fixed trailing edge (Figure 4.2). These variables would be readily available from the onset of the design as soon as the team has agreed on a wing geometrical definition. Moreover, these quantities will be able to link the rib to a specific spanwise location and an unambiguous rib type by considering geometry and location of the individual spoilers. Second moments of areas have been selected as variables to locally define the different rib sections: \(I_{\text{TOP}}\), \(I_{\text{BOTTOM}}\) and \(I_{\text{VERT}}\) represent sectional properties for top, bottom and vertical section respectively. This has been preferred to the geometrical definition of single flanges through individual variables such as thicknesses and length, in an attempt to both reduce the number of variables to a minimum and allow the design to be more generic.

The different loads acting simultaneously on the ribs have all been included as variables. Their values are the maximum that the structure would be designed for, including retracted and extended spoiler setting as well as intact and failed conditions where applicable.

Input data pre-processing is crucial for the attainment of a well performing adaptive fuzzy model, both in terms of its accuracy and convergence rate. Data normalisation, in particular, is of primary importance. In the case of complex problems, the variables required for a reliable estimate might be numerous and represented by very different scales. Numerical models based on adaptive network structures tend to ascribe higher importance to those variables characterised by higher values (Sola and
Sevilla, 1997). In addition to this, when adopting subtractive clustering for the initial model extraction, the lack of normalisation in the pre-processing of the training data strongly biases the location of the cluster centres towards the high valued regions of the design space. The final model, therefore, will be unrepresentative since it will neglect the impact that the lower valued quantities have on the final output.

Variable normalisation is of primary importance to ensure that all variables of interest get equal weighting during the training process. Moreover, by adopting normalised and compressed scales, the search space is reduced in all directions thus significantly condensing the distance to be covered by the backpropagation algorithm. This also aids the gradient descent algorithm which is used in parallel with backpropagation during network training. In this case, large values tend to slow down considerably the algorithm, due to the gradient of the activation function for the individual rules approaching zero (Dawson and Wilby, 2001).

For these reasons, prior to the development and application of the model, all variables in the input-output data pairs were normalised according to the formula:

$$Z = \frac{X - \mu}{\sigma}$$

(4.7)

where $X$ is the variable of interest characterised by the mean $\mu$ and standard deviation $\sigma$ while $Z$ represents the variable rescaled to a Gaussian distribution with a mean of zero and unit standard deviation. This process sets the variables in a non-dimensional form and ensures that all the inputs receive equal weighting during the network training.

Two weight models were created. Model A evaluates the rib weight without considering the impact of system loads, whilst model B includes variables linked to loads due to hydraulic system installation. As a consequence, model A was initialised with 10 input variables while model B with 12 (Table 4.1). Both ANFIS models were designed and optimised using Matlab and the ANFIS toolbox (MATLAB, 2008; Jang, 1993).

One vital part of developing a fuzzy model that is representative of the relationships between the variables of interest, is the selection of appropriate data sets for
TABLE 4.1: Input variables used for the definition of the geometrical fuzzy models for the weight estimation of spoiler attachment ribs.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GLOBAL</strong></td>
<td><strong>GLOBAL</strong></td>
</tr>
<tr>
<td>(L)</td>
<td>(L)</td>
</tr>
<tr>
<td>(h)</td>
<td>(h)</td>
</tr>
<tr>
<td><strong>LOCAL</strong></td>
<td><strong>LOCAL</strong></td>
</tr>
<tr>
<td>(I_{TOP})</td>
<td>(I_{TOP})</td>
</tr>
<tr>
<td>(I_{BOT})</td>
<td>(I_{BOT})</td>
</tr>
<tr>
<td>(I_{VERT})</td>
<td>(I_{VERT})</td>
</tr>
<tr>
<td><strong>LOADING</strong></td>
<td><strong>LOADING</strong></td>
</tr>
<tr>
<td>(\omega_{aero})</td>
<td>(\omega_{aero})</td>
</tr>
<tr>
<td>(\omega_{fuel})</td>
<td>(\omega_{fuel})</td>
</tr>
<tr>
<td>(\sigma_{th})</td>
<td>(\sigma_{th})</td>
</tr>
<tr>
<td>(F_r)</td>
<td>(F_r)</td>
</tr>
<tr>
<td>(P_r)</td>
<td>(P_r)</td>
</tr>
<tr>
<td>(F_{hyd})</td>
<td>(F_{hyd})</td>
</tr>
<tr>
<td>(n_{hyd})</td>
<td>(n_{hyd})</td>
</tr>
</tbody>
</table>

network training and testing. Studies have shown that the way that the reference data is selected and split into these categories has a considerable effect on the accuracy of the estimations provided by the adaptive model (Tokar and Johnson, 1999; Shahin et al., 2004). When splitting the data, care should be taken in ensuring that both training and testing data sets include the descriptive trends characterising the full database. This allows the model to be optimised in terms of both extrapolation and generalisation capabilities. As a consequence, the statistical properties of the produced data set need to be analogous between both training and testing sets as well as with the full database of reference, so as to ensure that the model is representative of the same population.

Shahin et al. (2004) highlight how, in the case of an adaptive network structure, the performance of the model is noticeably improved by adopting statistically similar data sets during the process of model derivation. In this work, a similar data splitting process is used. The available data is manually separated into training and testing sets, following a 70-30 split between the two categories. The data is selected by ensuring that elements from each individual spoiler attachment rib groups are included in both sets and that the training comprises also of individual examples with features or characteristics which are unusual within the overall database (e.g. additional parts,
attachments, minimum/maximum values of variables, etc.). Once these are produced, t- and F- tests are carried out to assess that their statistical similarity. The t-test examines the null hypothesis of no difference in the means of two data sets and the F-test examines the null hypothesis of no difference in the standard deviation of the two sets (Shahin et al., 2004). A level of significance of 0.05 is chosen to as a threshold for the tests, which highlights a 95 percent confidence level of statistical consistence between the two derived sets.

The reference database was built on 36 examples of spoiler attachment ribs, related to two different aircraft models. The first design considered (Aircraft 1) is representative of a long-range civil transport. Its wing is of a traditional layout, with composite wing panels and metallic spars. In the case of Aircraft 2, both wing covers and spars are of composite design. The reference database was split into 25 examples for model training and 11 for testing of the optimised model structure.

4.4 Results

4.4.1 Sensitivity and model selection

The method of variable selection was applied for the optimisation of the two fuzzy models, with checking RMSE error on the testing database as the selection criterion. Subtractive clustering was preferred for the derivation of the initial model structure. A cluster radius of $r_a = 0.4$ and accept and reject ratio of 0.5 and 0.15 were used for both model A and B, since it allowed for a good compromise between accuracy of solution and overall model complexity.

An initial checking RMSE of 0.144 on the initialized model A was achieved, which was reduced to 0.050 at 8 variables (Figure 4.4). The graphs shows the variable removal process for model A. Each point indicates the normalised checking error associated with the removal of a specific variable at each stage of the analysis, as annotated. The point represents the model at the last stage of the process after the elimination of variable $L$, leaving $h$ as the only input. The optimum model was attained by removing both thermal effects and strut loads from the initial input variable set, thus defining them.
as the least influential parameters. This is reasonable if related to the design process of the component, which is primarily driven by spoiler loads. This is also confirmed by the results of the model optimisation process, where hinge load is the last loading variable eliminated, thus making it the primary one necessary for model formulation.

From the point of view of the geometrical definition of the ribs, the most significant parameter for the evaluation of the weight is the spar height, being the last parameter left after the removal of the hinge line location.

Model B, on the other hand, showed a better initial performance, with a checking RMSE of 0.077 on the full set of 12 inputs (Figure 4.5). This was reduced to an optimum value of 0.056 with 9 inputs. In this case, the optimum model was obtained by subsequent removal of three variables, namely thermal stress, and second moment of area for bottom and vertical section. The small relative importance of these parameters is understandable. In a similar way to model A, thermal loading is not a design driver for the component. In addition to this, the vertical section only appears in a limited number of examples and its properties are relatively minor compared to the other two sections. The results of the optimisation process also suggest the smaller influence of the bottom section of the rib on the final design weight, mainly due to the fact that

Figure 4.4: Effect of variable removal on the accuracy of model A for spoiler attachment ribs.
the load sustained by this part of the structure is comparatively lower than that acting on the top section. The most significant parameter was found to be the hinge load, as it had the greatest impact on model accuracy.

Although the best performance occurs with 8 input variables for model A and 9 for model B, it can be seen from the results that 7 and 8 variables for A and B respectively are still capable of achieving a relatively accurate approximation. A compromise can therefore be made between accuracy of results and model simplicity based on the information at hand at the time of the analysis. In the case of the selection of a simpler model, this process allows the quantification of the error resulting from the choice of a smaller variable set, therefore enabling the designer to compensate for this in weight estimation process.

### 4.4.2 Model performance

Figure 4.6 shows the individual results from model A and B on hinge ribs from the two representative transport aircraft. The addition of system integration considerations in the model, although slightly increasing its complexity, has improved its
generalization capabilities. It also reduces the average error in the estimation from 10.4 percent (Model A) to 8.5 percent (Model B). As shown, both models have been able to approximate the ribs closely, with the exception of Aircraft 2 hinge rib spoiler 1 inboard, hinge ribs spoiler 3 inboard and outboard. This can be attributed to the simplistic way thermal effects have been accounted for. A constant thermal stress of 20MPa was applied to all the ribs without considering the proportion of their areas interfacing with a composite component.

For example, in the case of Aircraft 1 only the skin panels are composite while the rear spar is of metallic design. This results in the top surface of the rib top section being in full contact with a composite skin panel and the bottom section being attached to it only via a small fraction of its lower area. In the case of the Aircraft 2 on the other hand, both skins as well as the rear spar are composite which results in the addition of a vertical component where spar stiffening is required. As a consequence, a higher fraction of the rib is subjected to thermal effects. This, however, has not been fully accounted for in the fuzzy model, which may be the cause of the higher discrepancies in the estimated results. Had this been represented more accurately rather than with a
constant value, the system would have recognized its impact on the final design weight, yielding improved performance in both models.

Both models have managed to accurately capture existing relationships between the different variables and the final output. The addition of the hydraulic system installation parameters, however, has impacted the degree with which the chosen variables affect the rib weight. Figure 4.7 shows the combined effect of rib height and length on the component weight. In both cases, the direct proportionality between the input variables and the output has been identified, however the proportion to which they impact the output has diminished in model B. In terms of weight prediction, the results show that, for the same applied loading, model A attributes a maximum of 20 percent additional weight to the structure, a proportion which relates to the impact of hydraulic system loads on the final component weight.

The hinge load is the loading parameter which has the greatest effect on the rib weight. Model B is able to represent this more closely, as shown in Figure 4.8. Both the rib height and the hinge loading, contribute to the increase in the final output. The representation of the true impact of the loading, however, is more closely embodied in model B than model A, where the proportion of the rib weight associated to its height is much higher.

Model B also captured a more representative picture of the role that the different types of loading play on the structure. Aerodynamic and hinge load influence the

![Figure 4.7: Effect of height and length on the weight of spoiler attachment ribs for model A (a) and model B (b).](image-url)
majority of the structure. For a rib with spar height equal to the hinge line datum, an increase of both loading will result in the increase of the structural weight of the component with a greater weight impact attributed to hinge loading (Figure 4.9). Model A, however, erroneously applies an additional 8 percent of structural weight on the rib from this types of loading, which in model B is related to systems being attached to the structure itself.

**FIGURE 4.8**: Effect of height and hinge load on the weight of spoiler attachment ribs for model A (a) and model B (b).

**FIGURE 4.9**: Effect of aerodynamic and hinge loads on the weight of spoiler attachment ribs for model A (a) and model B (b).

Overall, model B provides a better representation of the multidisciplinary nature of the problem. The addition of system installation parameters allows a more complete understanding of the sources of weight inefficiencies. During the design process, the design of the structures tends to be conducted separately from that of the system architecture and it assumes an overall greater importance. From figure 4.10, however,
it is possible to note how hinge load and the load resulting from hydraulic installation impact rib weight. The impact of system loading on the rib structural weight, although not as considerable as that resulting from hinge loading conditions, is still noticeable and neglecting it would result in an incomplete and unrepresentative estimation of the component weight.

![Figure 4.10: Effect of hinge and hydraulic system attachment loads on the weight of spoiler attachment ribs.](image)

### 4.5 Summary

This chapter has introduced ANFIS as a computational tool for the weight estimation of aircraft structures at preliminary design stages. In particular, the focus has been primarily on the design of a structural weight model based purely on specific geometrical variables, location parameters and initial loadings applied on the structure.

Specific techniques for ANFIS model design and optimisation have been highlighted. Subtractive clustering was chosen for the initial fuzzy model extraction based on data clusters present in the available data set of reference. An iterative variable selection process was also used in parallel with this in order to evaluate the combination of input variables providing greatest accuracy in the estimation of the chosen component weight.

The literature highlights a lack of representative and reliable weight estimation
methodologies for secondary structures. Major structural assemblies, such as wing or fuselage, justify the use of computationally expensive modelling tools to aid the weight estimation process due to their size and function. The methods used for assessing the weight of secondary structures, however, appear to be mostly empirically based even at later stages of the design process. The high number of variables involved in the weight estimation problem, combined with the numerous interactions between structural and systems components within secondary structural assemblies makes them ideal candidates for the application of neuro-fuzzy modelling. In particular, spoiler attachment ribs were selected as case study for this chapter.

In order to make the weight model representative of the design of the component, ANFIS was structured on the basis of 3 input variable categories. Global variables were used to relate the individual spoiler attachment ribs to their location across the wing fixed trailing edge. Local variables allowed to characterise the three individual sections of the structure by identifying their second moments of area. Lastly, loading variables allowed for a full definition of the applied loads on the individual rib structures. In particular, within the loading categories, parameters related to system attachments on the ribs as well as the loading resulting from them were considered. In addition to this, installation issues such as additional thermal stresses at metallic to composite interfaces were also included where applicable.

Two separate models were derived in order to assess the impact of adding variables related to systems installation issues on the final estimation accuracy of the model. Adding system installation in the analysis reduces the average error by approximately 2 percent, which highlights the importance of a more multidisciplinary weight analysis as early as preliminary design stages. The study also emphasises that it is possible to conduct a more comprehensive weight analysis that includes system attachments and other installation considerations even with the specific extent and quality of information available at early project phases. In terms of the representation of the relationships between the different variables of interest, the addition of system installation parameters allowed a more exhaustive representation of the sources of weight inefficiencies as well as more representative trends.

ANFIS proved to be a successful modelling tool when applied to weight estima-
tion problems, both in terms of accuracy and its ability to handle complex nonlinear relationship between variables. The following chapters will explore more in-depth the use of ANFIS in more complex problems where weight estimation is conducted in parallel with the design of the structure within the neuro-fuzzy environment. In particular, its performance will be evaluated and compared to that of NEFPROX with focus on the comparison between both accuracy and interpretability of the models derived. In addition to this, the focus will be on the analysis of the rulebase developed with the two neuro-fuzzy tools and its impact on the weight estimation problem.
Chapter 5

Integration of Network-based Weight Estimation in the Design Process

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5.1 Introduction

One of the crucial challenges for any weight estimation framework to be used at preliminary design stages is the ability to provide a weight solution that is straightforward to obtain and requires minimal computational effort. At the same time, the weight model should still be able to capture the physics behind the design of the component at hand even with the limited information and data available at this phase in the design.

It has been demonstrated in Chapter 4 that fuzzy logic could be adopted as a basis for a weight model through the use of an ANFIS structure designed around a preliminary geometrical and functional definition of the structure to be analysed. This chapter aims at expanding the concepts presented in Chapter 4, by providing practical examples of how to construct an ANFIS-based model that can be successfully integrated in the preliminary design process of aircraft secondary structures.

This chapter will highlight the definition of a model structure for the derivation of both preliminary sizing as well as weight estimates for structural components. The design of the computational model will be based on the information and knowledge of the design which would be normally available at the early stages of its development. The model will be designed around a combination of ANFIS and MANFIS network structures.

The case studies used for model validation within the chapter will highlight the potential benefits of adopting such framework. In particular, results will focus on the ability of the methodology to identify the major trends and relationships between the number of design variables involved, the reliability and quality of the estimates provided as well as on the specific features of the rulebase derived through the framework.

The framework will be analysed from the point of view of its ability to combine an analytical methodology for component design with a fast and computational inexpensive tool, which is able to improve the quality of the estimates very early in the design of the component itself. Parameterisation and model optimisation issues are emphasised in order to enhance model performance. The benefits of implementing fuzzy logic techniques in the process will also be highlighted and placed into the over-
all context of obtaining a more comprehensive knowledge acquisition phase within the weight estimation process.

5.2 Structural design with fuzzy logic

The weight estimation of an aerospace structural component during the preliminary design stages is a very complex process, at the basis of which there is a flow of information and data from the different departments taking part in the analysis. The route to a representative weight assessment of the component under analysis can be delayed by bottlenecks within the sharing of information across the different disciplines until the right information gets to the mass estimation department. In addition to this, once all the necessary data is gathered, more often than not the mass properties engineers have little or no insight in the reliability and accuracy of it. As a consequence, there is very limited control on the final weight estimate provided at the end of the cycle.

For this reason, the weight estimation teams are always in search of modelling tools which can replicate the necessary design steps to get the right information needed for the weight analysis but with minimal computational effort and time. These are not only seen as a way to optimise the process itself in the case of delays within the exchange of data across the different departments, but also as a way of carrying out inexpensive safety checks on the high fidelity models built with the full set of data.

These tools and frameworks should be able to provide robust weight estimates with minimal computational effort whilst capturing the physics behind the design of the component at hand even with the limited information and data available at this phase in the design. In order to do this, it is vital that the methodology at the basis of them replicates the design process for the structure being analysed, even if just on a smaller scale.

The preliminary design of a structural component can be condensed and viewed as an iterative 3-tiered flow of information (Figure 5.1). The process starts with the definition of the necessary design requirements, namely the function of the component, its location within the main assembly and relative spatial constraints, materials and
processes to be used for its manufacturing, as well the various loads that it needs to be able to sustain. All this information is then used for the sizing process and comprehensive design definition. The overall component function and its relative location within the assembly initially define the configuration and individual features of the structure. The loads are then translated into minimum sizing requirements (e.g. sectional properties). Finally, space constraints define the global geometry, while manufacturing issues drive local geometry and specific features. Once these parameters are derived, they are used to formulate a weight estimate which is evaluated and reviewed with respect to some of the input parameters. If the estimate is considered erroneous, too conservative or too ambitious, the process is reiterated by modifying the design definition at any of the intermediate stages.

![Figure 5.1: Schematic representation of the flow of information used within the weight estimation process.](image)

5.2.1 Model framework

When applied to the weight estimation of the spoiler attachment ribs in Chapter 4, ANFIS modelling proved to be a viable option, providing accurate results as well as realistic trends between the variables of interest. The model approached the problem from the point of view of combining loading information with details of the geometry, function and location of the selected structures in order to derive weight estimates. However, the results highlighted the need to provide more knowledge and insight into the sizing methodology for the component to be able to fully support the design process.
For this reason, the neuro-fuzzy approach was extended to supply initial sizing information as well as weight estimates for the selected structural example based on the type and nature of the information available in preliminary design stages.

Figure 5.2 shows an illustrative example of the architecture of a generic neuro-fuzzy framework for the weight estimation of aircraft structures, derived following the main design process steps at the preliminary stages of structural definition of the component. The majority of the initial requirements can be translated into input variables for the sizing part of the neuro-fuzzy system developed to derive sectional properties and local geometry parameters for the structure. These results will indicate the minimum sizing parameters for the structures. The remaining initial spatial requirements can then be combined with the outputs of the first neuro-fuzzy unit as well as with additional parameters relative to the component local geometry, configuration and features that the designers can agree upon once minimum sizing has been derived. These will represent the inputs for the secondary module of the neuro-fuzzy system which will be used to compute the weight estimate.

![Figure 5.2: Illustrative framework architecture for neuro-fuzzy sizing and weight estimation of structural components.](image)

According to the results obtained, the process can be reiterated and the relevant variables modified according to varying design requirements or optimised in view of weight reduction efforts. The results themselves will highlight trends and relationship
between the different variables, causality of design changes and sensitivities of the final weight of the component to different design solutions, which will enable the designer in the optimisation process.

5.3 ANFIS-based structural sizing and weight analysis: spoiler attachment ribs

In order to assess the applicability of the method, the sample framework illustrated in the previous section was implemented on two separate structural examples. The first test was carried out on spoiler attachment rib structures.

Figure 5.3 shows the model structure for the sizing and weight estimation of spoiler attachment ribs. The general architecture has been adapted to work within an ANFIS and MANFIS network structures as well as to fit the specific inputs and design requirements for the component. In line with the general illustrative framework, the aims of the computational model were to provide an accurate weight estimate for the component at hand as well as to act in parallel to the design process in order to fulfil the needs of the design team during both preliminary design and weight estimation tasks. For this reasons, the model was structured in 3 parts:

1. An interactive Microsoft Excel based loading module, was developed and used to evaluate resultant bending moments, shear and axial forces in the structure from applied loads

2. A multiple output MANFIS-based sizing module to evaluate sizing parameters for the structure by combining the results of the loading module with relevant material properties

3. An ANFIS-based weight module evaluating the structural weight of the component by combining local sizing parameters generated by the sizing module with global sizing variables related to the specific location and function of the spoiler attachment rib within the fixed trailing edge.
5.3.1 Loading and structural sizing

From the schematic representation of the ANFIS-based framework for spoiler attachment ribs, it is easy to understand the process of translating the general framework to a practical, real life structural problem. In the case of spoiler attachment ribs, the overall process was based on the idealisation of the structure as a combination of three individual beams, the design of which is driven by their different functions and loading scenarios (Figure 5.4).

The requirements posed by the design process for the component are mainly covered in the sizing module. Based on the individual loads acting on the spoiler...
attachment rib, an Excel/Visual Basic system solved the individual beam components for maximum bending moment \( M_{\text{max}} \) axial forces \( (F_x) \) according to beam theory principles. Within the loading module, issues such as manufacturing and installation are included, such as additional thermal stresses occurring at the composite to metallic interfaces as well as loading resulting from the addition of system installation within the structure itself. Maximum resultant axial and bending loads are then used as input for the MANFIS based sizing module in combination with the Youngs modulus \((E)\) and ultimate tensile stress of the material \((\sigma_{\text{ult}})\).

The initial weight estimation study described in Chapter 4 showed the importance of describing the structure of spoiler attachment ribs through local sectional properties, global parameters and loading information when approaching the problem of structural weight estimation with ANFIS. Within the new 3-layered framework, the same information is kept throughout the process but fed into the system at different stages.

Loading variables appear at the beginning of the process and they are used to derive the local sectional properties for the 3-beam structure, namely sectional moments of inertia \((I_{\text{TOP}}, I_{\text{BOT}} \text{ and } I_{\text{VERT}})\) and cross sectional areas \((A_{\text{TOP}}, A_{\text{BOT}} \text{ and } A_{\text{VERT}})\). The cross sectional areas for the 3 beams are then combined in the weight module with global geometry, spatial requirements and specific feature definition em-
bodied by the individual variables of spar height \((h)\), location of hinge line datum \((L)\) and rib type \((r_{\text{type}})\) for a full definition of the structure.

Overall, this type of framework encompasses the main analytical steps of the structural design and weight estimation of the component by combining an analytical loading solution with a neuro-fuzzy derivation of sizing and weight.

### 5.3.2 Sizing module performance

The database of reference provided a total of 77 beam components for spoiler attachment rib structures, of which 59 were used for training the MANFIS network at the basis of the sizing module and 18 for its performance assessment. During the training process, subtracting clustering was applied for initial rule derivation to avoid combinatorial explosion of rules and to ensure quick and consistent convergence of the model optimisation process. A radius of \(r_a = 0.4\) and accept and reject ratios of 0.5 and 0.15 respectively were chosen as suitable cluster parameters for the chosen model.

In addition to RMSE, the performance of the ANFIS models was assessed using *Mean Percentage Error (MPE)* and *Mean Absolute Percentage Error (MAPE)*, which are defined as:

\[
MPE = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{\theta}_i - \theta_i}{\theta_i}
\]

\[
MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\hat{\theta}_i - \theta_i}{\theta_i} \right|
\]

where \(\theta_i\) and \(\hat{\theta}_i\) indicate respectively the real and predicted values for the variable of interest for \(n\) number of data points. The model managed to provide satisfactory results in terms of the accuracy in the computation of sizing and weight variables (Table 5.1). In particular, the model shows greatest modelling accuracy in the estimation of second moment of area \(I\) across the different beam types, with lowest values of RMSE and MAPE.

A more in depth analysis of the performance of the sizing module reveals a
Table 5.1: Performance assessment of ANFIS framework applied to the sizing and weight estimation of spoiler attachment ribs.

<table>
<thead>
<tr>
<th></th>
<th>MANFIS</th>
<th>1 MANFIS</th>
<th>W ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Training</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>Testing</td>
<td>18</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.131</td>
<td>0.117</td>
<td>0.140</td>
</tr>
<tr>
<td>MPE</td>
<td>-7.07</td>
<td>-3.31</td>
<td>1.97</td>
</tr>
<tr>
<td>MAPE</td>
<td>11.26</td>
<td>8.87</td>
<td>10.25</td>
</tr>
</tbody>
</table>

A tendency from MANFIS to under estimate the required cross sectional areas and second moments of inertia for the given condition, which can be rectified by applying correction factor on the final results (Figure 5.5). The reason behind this could be attributed to the fact that analytically, a load and sizing analysis normally leads to minimum sizing requirements for the structure, whilst the values used for the design of the model represent the structure "as built". The application of a correction factor within the structural assessment could potentially aid the designer in understanding the relative effect of additional fabrication and installation issues on the final design of the component which cannot be readily incorporated in the ANFIS model.

Figure 5.5: Performance of the MANFIS sizing module for spoiler attachment ribs on testing database for cross sectional area A and second moment of area I.
Overall, the model has accurately represented the physical relationships between the input variables and the structural sizing parameters chosen. As expected, the trends are highly nonlinear, in particular within the effect of the axial forces on the output variables. The rate of change of cross sectional area with respect to bending is inconspicuous if compared to the impact that a change in axial force will have on it (Figure 5.6(a)). Conversely, the second moment of area of the beam increases steadily with bending but its value will only fluctuate minimally with respect to changes in axial forces (Figure 5.6(b)).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig5_6}
\caption{The figure shows the results from MANFIS with regards to the variation in beam cross sectional area (a) and second moment of area (b) due to applied bending and axial loads on spoiler attachment ribs.}
\end{figure}

\subsection*{5.3.3 Weight module performance}

The method of variable selection proposed by Chiu (Chiu, 1996) was used to better understand the relative importance of the different input variables on the performance of the ANFIS weight model, with respect to the resultant RMSE on the testing database and to optimise the final model itself. As shown in Figure 5.7, the greatest accuracy is attained with the full variable set and the removal of any of the input parameters generates a substantial deterioration in the accuracy of the estimation. The results highlight how the global geometrical definition of the rib through $h$ and $L$ as well as its function, determined by the rib classification parameter $r_{\text{type}}$, are essential variables for the design of a reliable weight model for this structural component, as demonstrated also by the previous numerical example of Chapter 4.
Weight data for a total of 79 examples of spoiler support ribs was collected, of which 59 sample ribs were used for network training and 20 for testing following the method of statistical similarity described in Chapter 4 (Shahin et al., 2004). The network shows high generalisation capabilities on the testing dataset as well as satisfactory overall performance (Table 5.1). The results show that the ANFIS network acquired higher generalisation capabilities in the case of intermediate ribs compared to hinge ribs (Figure 5.8). This underlines the need to improve the definition of the spoiler support rib structure by adding additional parameters in the analysis, in particular with respect to specific features related to manufacturing and fabrication.

The results show that global rib geometry and the location of the structure within the trailing edge have a higher impact on the final structural weight of intermediate ribs as opposed to hinge ribs (Figure 5.9). The design of intermediate ribs is mainly driven by spatial requirements, whereby the main aim of the structure is that of maintaining the aerodynamic profile of the fixed trailing edge, as opposed to loading considerations.

In terms of local geometry, the model was capable of identifying the main con-
Chapter 5

25

• Hinge Ribs
• Intermediate Ribs

10

5

0

-5

-10

-15

-20

Specimen Rib ID

Figure 5.8: Performance of the ANFIS weight module for spoiler attachment ribs on testing database.

tributions to the structural weight in the cross sectional areas of the vertical and top sections of the structure. In the case of hinge ribs, the vertical component is of prominent influence due to the size required to replace vertical spar stiffeners and sustain fuel loads (Figure 5.10(a)). Vertical and top components have a similar effect on the weight of intermediate ribs but altogether show a lower combined effect on the output compared to that on hinge ribs (Figure 5.10(a)). This can be attributed to the minimal variation in beam size, which characterizes this type of component.

The results also help in understanding the different contributions that the choice of a specific design for the rib has on the final weight of the structure. The effect of top and bottom section sizes is approximately 3 times higher on a hinge rib as opposed to an intermediate rib of the same global geometry. This also confirms how both the design and final weight of hinge ribs is highly dependent on loading and, consequently, size of its individual sections, whilst global geometry is the major factor affecting the weight of intermediate ones.
FIGURE 5.9: Variation in spoiler attachment rib weight with respect to hinge line datum ($L$) and spar height ($h$) for hinge ribs (a) and intermediate ribs (b).

FIGURE 5.10: Effect of local geometry on rib structural weight for spoiler hinge ribs (a) and intermediate ribs (b).

5.4 ANFIS-based structural sizing and weight analysis: aileron attachment ribs

Aileron attachment ribs were chosen as an additional case study to explore the validity of the methodology. As with spoiler attachment ribs, these are secondary structures which are part of the fixed trailing edge and for which weight estimation activities are still being carried out with basic empirical methods. They are located in the outboard part of the fixed trailing edge and their purpose is that of sustaining the loads transmitted from the aileron, as well as maintaining the aerodynamic integrity of the outboard fixed trailing edge whilst providing space allocation and attachment points for system routing.
Figure 5.11: Schematic representation of Design A for the aileron attachment rib with the relevant loads acting on it (a) and its idealisation into a 4-beam structure with the relevant loads and boundary conditions for sizing within the ANFIS framework (b).

Figure 5.12: Schematic representation of Design B for the aileron attachment rib with the relevant loads acting on it (a) and its idealisation into a 2-beam structure with the relevant loads and boundary conditions for sizing within the ANFIS framework (b).

Normally, only hinge ribs are present within the outboard fixed trailing edge (OFTE). Their design is driven by the need to sustain the following loads:

1. Aerodynamic loads ($\omega_{aero}$), which are applied to the upper section of the rib through its direct attachment to the fixed upper skin panel.

2. Hinge loads ($F_r$) resulting from the axial hinge force components from the aileron and acting on the spoiler hinge line.

3. Strut loads ($P_r$), which are the effect of aerodynamic loads acting on the fixed lower skin panel and transmitted to the bottom section of the rib via a strut.

4. Fuel loads ($\omega_{fuel}$) acting on the vertical section of the rib, which can be found in those ribs that are positioned where an external integral spar stiffener would have been.

5. System attachment loads resulting from the routing of system runs across the
trailing edge and fixed on individual rib locations.

6. Applied thermal stresses ($\sigma_{th}$) arising from the differences in thermal expansion at composite to metal interfaces. For the purpose of this study a constant 20MPa was applied on metallic sections connected to composite components.

These loading requirements have generated 3 different designs within commercial and military aircraft, all of which made up by I cross-sectional beam structures. Figure 5.11 highlights the structure of Design A, which is representative of aileron attachment ribs for larger aircraft. This design is characterised by the presence of two vertical beam sections, of which the back provides additional stability to the spar in the absence of a vertical stiffener and the front one helps the structure carry a larger hinge load. Design B (Figure 5.12) is not dissimilar from the general layout of spoiler attachment ribs, with a two-beam configuration. In the case of Design C (Figure 5.13), the same requirements are satisfied by a single tapered beam structure occupying the majority of the available cross sectional trailing edge space. In this case, there is no need to provide additional space allocation for system installation due to re-routing and attachment of system lines onto the the rear spar instead than onto the rib itself. All three designs were used for model training and validation by idealising the structure as individual beams, in a similar way as for spoiler attachment ribs.

The model structure follows the initial general illustrative framework and is similar to that used for sizing and weight estimation of spoiler attachment ribs. In order to increase the flexibility of the model to allow the analysis of the different design variants being included in this case study, the model has been modified to account for
the additional beam elements within the structure (Figure 5.14). By modifying the model for the concurrent analysis the three different aileron support ribs, it will be possible to understand the weight penalties resulting from adopting alternative design solutions as early as conceptual design stages.

![Figure 5.14: Schematic representation of the three-level sizing and weight estimation framework for aileron attachment ribs, highlighting the variables and structure of MANFIS-based sizing module and ANFIS-based weight estimation module.](image-url)
5.4.1 Sizing module performance

The database of reference provided sizing information for a total of 63 beam components of aileron attachment rib structures, of which 44 were used for training the MANFIS network at the basis of the sizing module and 19 for the assessment of model performance. During the training process, subtracting clustering was applied for initial rule derivation to avoid combinatorial explosion of rules and to ensure quick and consistent convergence of the model optimisation process. A radius of $r_a = 0.4$ and accept and reject ratios of 0.5 and 0.15 respectively were chosen as suitable parameters for cluster definition.

**Table 5.2:** Performance assessment of ANFIS framework applied to the sizing and weight estimation of aileron attachment ribs.

<table>
<thead>
<tr>
<th></th>
<th>MANFIS</th>
<th>MANFIS</th>
<th>ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Training</td>
<td>44</td>
<td>44</td>
<td>46</td>
</tr>
<tr>
<td>Testing</td>
<td>19</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.121</td>
<td>0.125</td>
<td>0.132</td>
</tr>
<tr>
<td>MPE</td>
<td>-1.72</td>
<td>0.37</td>
<td>-3.65</td>
</tr>
<tr>
<td>MAPE</td>
<td>11.65</td>
<td>10.48</td>
<td>6.49</td>
</tr>
</tbody>
</table>

The model managed to provide satisfactory results in terms of the accuracy in the computation of both sizing and weight variables (Table 5.2). As for spoiler attachment ribs, the sizing module performed better when estimating second moment of area for the individual beams rather than for the computation of their cross sectional areas. In this case, however, the difference in accuracy is minimal from the point of view of both MAE and MAPE and with MANFIS showing slightly lower RMSE in the estimation of cross sectional area.

MANFIS continues to show a general tendency to under estimate the cross sectional areas for the beam sections. This can be linked to the presence of constraints and minimum gauges driving the geometry of the manufactured sections in parallel with loading requirements. In the case of second moment of areas, on the other hand, the results balance out, showing no clear trend within the estimation capabilities of the network for these type of designs.
In addition to an overall satisfactory performance in the estimation of sizing parameters, the model has been able to analyse and represent clear and accurate relationships between the different variables of interest (Figure 5.16). As for spoiler attachment ribs, the trends produced are representative of the physics behind the design of the component. Bending is shown to have only a minimal impact on the cross sectional area of the beam elements whilst it drives the choice of second moment of area. Conversely, the lower values of axial force appear to produce a considerable linear increase in cross-sectional area, compared to higher ones. The profile of the relationship, in this case, suggests the presence of additional factors, on top of loading considerations, which contribute to the choice of beam cross sectional area for aileron attachment ribs.

5.4.2 Weight module performance

Weight data for a total of 66 examples of spoiler support ribs was collected and split into training and testing datasets, according to the method of statistical similarity
Figure 5.16: The figure shows the results from MANFIS with regards to the variation in the beam cross sectional area (a) and second moment of area (b) due to applied bending and axial loads on aileron support ribs.

formalised by Shahin et al. (2004). As a result, 46 sample ribs were used for network training and 20 for testing.

Due to the presence of additional beam sections in Design A, the weight module for the analysis of aileron support ribs was initialised with seven input variables. The method of variable selection (Chiu, 1996) was also used in this case to both optimise the model with respect to input variables and to understand the impact of the individual parameters on the performance of the ANFIS weight model.

As for spoiler attachment ribs, the full variable set provides the best estimation accuracy with an RMSE of 0.132. A quick deterioration in performance occurs with the removal of the cross sectional areas of the two vertical sections $A_{VERT}$ and $A_{VERT_b}$. Global geometrical parameters still have the greatest impact on the quality of the approximation, with location of hinge line datum $L$ being the last variable removed. The cross sectional area of the top beam $A_{TOP}$, however, has a much greater impact on the overall model accuracy for aileron attachment ribs as opposed to spoiler ones: its removal from the analysis could result in an error in the estimation up to 7 times higher than the one obtained with the full set of input variables in the worse case scenario (Figure 5.17).

Overall, the weight module for aileron attachment ribs shows greater generalisation capabilities compared to that for spoiler attachment structures (Figure 5.18). Both RMSE and MAPE have improved, with MAPE reduced by over 4 percent. In this case, however, the model has a much greater tendency to underestimate the weight.
of the structure on average by nearly 4 percent. Compared to spoiler attachment ribs, in fact, the three designs analysed here appeared to have a higher number of specific features which have not been explicitly accounted for within the model itself, such as a higher number of stiffening elements in the larger beam structures or the presence of holes for system routing within Design C.

The results across the three different designs agree when it comes to the weight impact of global geometry on the structure. Overall, an increase in the height of the structure $h$ and, therefore, its location further inboard along the spar, influence the final weight the most, as opposed to the position of the hinge line datum $L$. In particular, the highest impact from this parameter occurs in the case of Design C. Since this type of attachment rib is designed on a single beam structure, an increase of height will cause a higher volumetric expansion. As a consequence, the weight penalty resulting from the location of the rib in the wing spanwise direction is double that which would be incurred by Design B and is up to 7 times higher compared to the 4-beam solution of Design A (Figure 5.19).
For both Design A and B, the geometry of the top beam section appears to have the highest impact on the final structural weight, validating the results of the model optimisation process. In particular, $A_{BOT}$ shows to be inducing the same weight increase in both cases (Figure 5.20). The proportion of weight dependent on $A_{TOP}$ is approximately 1.5 times higher in the case of Design B, due to the lack of the additional vertical beam sections sharing the load and, therefore, the extra weight of the structure. Figure 5.21 shows how the added structural weight is mainly driven by the back vertical beam section, which is responsible to both stiffening the spar and carrying the fuel load from the wing box, whilst only approximately one fifth of the overall weight of the rib is dependent on the inclusion of the front vertical beam section.

In the case of Design C, where only a single beam structure is present, the cross sectional area $A_{TOP}$ contributes to approximately a third of the rib weight when compared to its global hinge line datum location $L$ (Figure 5.22). The dip in weight occurring at the maximum values of the two geometrical parameters can be attributed to the maximum influence of spar height $h$ on the weight of the structure. This is
particularly relevant in the case of the larger aileron attachment ribs, which are located on the inboard side of the OFTE where spar height is at its maximum, as previously highlighted in Figure 5.19(c).

5.5 The TSK fuzzy knowledge base: structure, interpretability and versatility

In addition to the numerical estimation of sizing and weight for spoiler and aileron attachment rib structures, both modules are accompanied by a full rulebase describing the overall behaviour of the system.

In the case of ANFIS and MANFIS modelling, the set of rules derived by the model have the form "IF – THEN" with the antecedent side of the rule defined by the membership functions for each individual input applicable for the selected rule. Since the fuzzy model derived through ANFIS is of the Takagi-Sugeno type (Jang, 1993),
the consequent part, or the output side of the rule, is represented by a single value or singleton.

Figure 5.23 shows the rulebase for the sizing module for spoiler attachment ribs. As shown, the optimised system consists of 4 inputs and 2 outputs each. For each rule, each input is defined by a relevant membership function within the variable design space which is derived during the network training process. For the purpose of the figure, inputs have been set at random values. Once the input values have been selected and fuzzified according to the relevant membership functions, the rules are weighted and the individual output from the rules is computed. According to the weight assigned to each rule, the individual outputs are aggregated into a one single fuzzy output which is then defuzzified into a crisp quantity.
In this case, the system derived through the optimisation process is simple, due to the small number of rules necessary for its full description. However, its interpretability is limited because of the nature of the outputs. By looking at the rulebase, the designer could easily understand where the chosen design solution is located within the design space, in terms of input variables. In addition to this, for each rule it will be possible to intuitively understand the initial effect that the variability in the design parameters could have on the rule weighing, by looking at the applicable membership functions for the individual variables. By using the TSK fuzzy systems, however, the representation of the outputs as singletons prevents the immediate visual understanding of the degree of uncertainty within the outputs themselves and how this changes according to individual design decisions.

In the case of the rulebase behind the weight module, the structure itself is more complex (Figure 5.24). The higher number of input parameters has led to an exponential increase in the number of rules necessary to describe the system. This is typical of TSK FIS derived through ANFIS. The network behind the weight estimation module is characterised by 20 rules linking the 6 input variables to the single output. The higher number of rules contributes to the reduction in the interpretability of the system, especially in the analysis the consequentiality between the individual inputs and the final output.

Figure 5.25 and figure 5.26 highlight the rulebase for both sizing and weight
modules for aileron attachment ribs. The rule structure behind the sizing module appears to be not very dissimilar from that of spoiler attachment ribs, with 4 inputs, 2 outputs and 6 rules. The rulebase is very compact due to the few rules describing the system, however its interpretability is still limited due to the crispness of the outputs. ANFIS, however, was able to derive a somewhat simpler rulebase for the weight module. Despite having 7 inputs, as opposed to the 6 inputs of the weight module for spoiler attachment ribs, only 18 rules are sufficient for a full definition of the problem.

5.6 Summary

This chapter presented the definition of a model structure based on the ANFIS and MANFIS network structures, for the derivation of both preliminary sizing and weight estimates for aircraft structural components. The model was designed with the aim of closely embodying the general structural sizing and weight estimation process within the preliminary stages of the design of new components.

The initial framework was applied on two separate case studies for testing and validation: spoiler and aileron attachment ribs. The general design process for these two types of structures translated into a 3-stage model, consisting of an analytical Microsoft Excel module for derivation of loading scenarios, a MANFIS-based sizing module and an ANFIS-based weight estimation module.

The flow of input-output data between the different parts of the model replicates the data and information transfer occurring during the preliminary design process of the structural components and is able to translate the physics behind the design itself in the actual model structure and analysis. This ensures that the weight engineer has a way of deriving the required structural and sizing data in the case of delays within the data transfer between the departments involved in the design or if selected information is missing or unreliable. In addition to this, the tool represents a fast and computationally inexpensive way to obtain reliable and traceable estimates which can act as safety checks for higher fidelity models.

The network-based modules of the framework were trained and optimised using databases with real structural test cases. The final models succeeded in deriving
accurate estimates for sizing and weight for both structure types. In particular, the estimates for both cases never exceeded an overall average error of 12 percent during the validation process. The major source of error can be attributed to the absence of specific feature analysis within the estimation process. The neuro-fuzzy model derived the minimum sizing needed to sustain the applied loading, but in essence this estimate was compared to "as built" structural examples. In depth feature analysis is beyond the scope of weight estimation activities within the preliminary design stages also due to the lack of information typical of this phase. However, the general underestimation of the model could be compensated either by statistically derived adjustment factors specifically design to incorporate the effects of typical manufacturing and assembly related features in the estimate itself.

In addition to accurate approximations, the model was able to successfully derive reliable trends and to identify the principal causalities between the numerous variables of interest. By idealising both the spoiler and aileron support ribs as aggregations of beam structures, the results were also able to highlight the possible weight penalties resulting from the selection of a particular design solution instead of another or from the possible integration of system routing within the structural assembly itself.

The rulebase derived through the ANFIS-based network optimisation process, however, presents the typical restrictions of a TSK fuzzy system. Although ANFIS allows for fast computation and a compact rule and network structure, the derivation of a TSK FIS instead than a Mamdani type one results in a rulebase that fully explores the design space in terms of variability of input parameters but lacks interpretability of the causality of this on the outputs. These, in fact, are only represented by singletons rather than through membership functions, which prevents both a visual and intuitive definition of the full design space and a comprehensive understanding of how the uncertainty in the inputs translates into the definition of the outputs.

The next chapter will explore the use of Mamdani systems derived through the Neuro-Fuzzy Approximator (NEFPROX) in order improve the modelling capabilities of the framework from the point of view of the extraction of a more intuitive and comprehensive knowledge base able to aid the designer during the sizing and weight estimation process.
FIGURE 5.23: Rulebase for the estimation of sectional properties for spoiler attachment ribs.
Figure 5.24: Rulebase for the estimation of structural weight of spoiler attachment ribs.
Figure 5.25: Rulebase for the estimation of sectional properties for aileron attachment ribs.
Figure 5.26: Rulebase for the estimation of structural weight of aileron attachment ribs.
Chapter 6

Fuzzy Systems and Their Interpretability for Design Applications

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Chapter 6

6.1 Introduction

Accuracy and reliability are only two of the desirable qualities of a successful weight estimation methodology. At preliminary design stages in particular, the focus of the weight engineer is to obtain both a meaningful solution as well as a way of understanding how possible future design changes will influence the final weight of the structure.

The rule base structure characterising neuro-fuzzy systems is very attractive since it allows the designer to derive a set of mathematical guidelines which are able to illustrate how the chosen structural and design parameter interact with each other and, in turn, impact the weight of the structure itself. The definition of the design variables by means of membership functions also aids the designer in deriving a visual and more intuitive definition of the design space. The fuzzy rulebase which is derived via neuro-fuzzy systems can act as a visual map of the design space itself to be used as a guide during the decision making process. The rules highlight how different combinations of input variables impact the structural weight of the component. In addition to this, the representation of the variables by means of membership functions helps bring focus to the effects of the variability within the inputs themselves on the final design solution.

Chapter 5 demonstrated how neuro-fuzzy systems derived though ANFIS and MANFIS can successfully support both the sizing and weight estimation processes for structural components at preliminary design stages. The results derived using adaptive nero-fuzzy techniques proved to be accurate as well as able to extract a comprehensive knowledge base for the structural examples being analysed. The nature of the TSK fuzzy systems derived though ANFIS, however, results in the outputs being computed as singletons rather than through membership functions, thus limiting the interpretability of the knowledge base from the point of view of output characterisation and definition.

This chapter aims at overcoming this pitfall by deriving a Mamdani-type neuro-fuzzy systems for sizing and weight estimation using NEFPROX. The use of spoiler and aileron attachment ribs as case studies for model validation within the chapter will highlight the potential benefits of adopting Mamdani-type FIS for the derivation
of a more efficient and explicit rulebase for sizing and weight estimation. In particular, the results will focus on the analysis of the system from the point of view of accuracy, network complexity, fuzzy variable definition and quality of the structure as well as according to the rationale behind the rulebase derived within the analysis.

### 6.2 Selecting a fuzzy system for design applications: accuracy vs. interpretability

The choice of neuro-fuzzy systems for structural sizing and weight estimation at preliminary design stages was made based on:

1. The ability of the system to learn from given examples;
2. The capability of translating the acquired knowledge of the system under study into a set of rules to be used within the design process;
3. The possibility of combining the results with knowledge from experts;
4. The ability to provide reliable and accurate results even in the presence of noise;
5. The capability of incorporating the uncertainties within the problem in the analysis in a way which can be easily interpreted and modified by the user.

In certain cases, however, the adaptive learning algorithms used to extract the optimal FIS structure from data, such as those used by neuro-fuzzy systems, can lead to the derivation of fuzzy system that lack model interpretability (Zhou and Gan, 2008). In the development of models built using adaptive learning, accuracy and the need to preserve the interpretability of the final solution tend to be two conflicting objectives. This is normally the basis of the dichotomy between Mamdani and Takagi-Sugeno-Kang FIS. The former allows a complete fuzzy visualisation of both input and output spaces, with a more legible rulebase and an increase insight into the complex system. This attempt to move from a black box setting to a more grey box environment, usually comes at the price of lower accuracy and additional computational effort. In contrast, TSK FIS combine rapid computation and improved modelling accuracy at the expense of the readability and transparency of the solution. It is, therefore, important to evaluate the benefits of using Mamdani or TSK FIS for a specific modelling problem and asses
the relative benefits of preferring added interpretability to accuracy, or vice versa.

6.2.1 Interpretability of fuzzy systems

It is relatively easy to assess and quantify the accuracy of a FIS, by looking at error measures obtained by applying the model derived from training data on a testing set, which comprises of data points relative to examples previously "unseen" by the FIS itself. In terms of interpretability, however, research shows no agreement in the definition of an appropriate measure for its quantification, resulting in assessments which predominantly follow a more qualitative approach Castellano et al. (2003); Alonso et al. (2009); Alonso and Magdalena (2010).

Zhou and Gan (2008) suggest that, when trying to evaluate the interpretability of a fuzzy system, the user will have to consider two different aspects:

1. The architecture of the rulebase;
2. The expression of the fuzzy sets within it.

Low-level interpretability is specifically connected to the design of the membership functions at fuzzy set level and their consequent ability of unequivocally represent a particular fuzzy partition. Conversely, high-level interpretability is concerned with the overall FIS structure and the transparency of its rulebase. Both of these levels of analysis are characterised by specific criteria and conditions which enable a more coherent evaluation of the FIS itself.

When it comes to membership function analysis, it is vital to appraise the model by considering:

1. Distinguishability: the domain of interest of input and output variables should be represented by clearly defined and distinct fuzzy partitions.
2. Number of membership functions: the amount of fuzzy partitions used to defined each variable should stay within 7±2, which identify the limits of human information processing capability (Miller, 1956).
3. **Completeness**: the entire domain of each individual variable should be covered by MFs.

Full interpretability, however, can only be achieved if the rulebase as a whole is "readable". High-level interpretability is based on:

1. **Completeness**: at least one rule should be activated for each instance, thus ensuring that the entire design space is examined and considered.

2. **Readability of rules**: as for membership functions, the elements in the premise part of each rule should align with the $7\pm 2$ rule.

3. **Consistency**: rules should not be conflicting (i.e. for similar combination of inputs, the rules should produce similar outputs).

4. **Transparency**: this is related to the inner structure of the fuzzy system itself. For instance, Mamdani FIS are technically transparent by nature, due to the way their outputs are represented, as opposed to TSK.

To be able to select the most appropriate type of fuzzy system for design and weight estimation applications, NEFPROX (Nauck and Kruse, 1999) will be used to derive Mamdani-type FIS equivalent to the TSK ones presented in previous chapters. They will be compared to be able to gauge the best trade-off between accuracy and interpretability of rulebase.

### 6.3 NEFPROX-based structural sizing and weight estimation: spoiler attachment ribs

The illustrative framework for structural sizing and weight estimation was translated into a NEFPROX-based model in a similar way as for the ANFIS testcase (Figure 6.1). For the structural sizing and weight estimation of spoiler attachment ribs, the model framework follows an equivalent 3-layered architecture:

1. An interactive Microsoft Excel based **loading module**, used to evaluate resultant bending moments, shear and axial forces in the structure from applied loads;
2. A multiple output NEFPROX-based **sizing module** to evaluate sizing parameters for the structure by combining the results of the loading module with relevant material properties;

3. A NEFPROX-based **weight module** evaluating the structural weight of the component by combining local sizing parameters computed by the sizing module with global sizing variables related to the specific location and function of the spoiler attachment rib within the fixed trailing edge.

![Schematic representation of the three-level sizing and weight estimation framework for the spoiler attachment ribs, highlighting the variables and structure of NEFPROX-based sizing and weight estimation modules.](image)

**Figure 6.1:** Schematic representation of the three-level sizing and weight estimation framework for the spoiler attachment ribs, highlighting the variables and structure of NEFPROX-based sizing and weight estimation modules.

From the point of view of the architecture, there is no substantial difference between the ANFIS and NEFPROX framework, as they were both designed in order to closely mirror the information flow and processing structure within the preliminary design stage of a structural component. In the case of the NEFPROX architecture, however, there is a much comprehensive preservation of information between the different stages of the process. The reason for this relates to the derivation of Mamdani FIS
through NEFPROX rather than TSK ones. In this case, the output of the sizing module will be fuzzy in nature and the output domain will be characterised by membership functions rather than individual singletons. The translation of the cross-sectional area from outputs of the MANFIS sizing module to inputs of the ANFIS weight estimation module means converting a singleton into a fuzzy membership function.

This can lead to an inevitable loss or alteration of the qualities and properties of the variable under study. Firstly, the process of aggregating the individual rule outputs in a TSK system is achieved via the process of weighted average of the partial individual rules. Although computationally faster than the various defuzzification methods used in Mamdani systems, averaging across a range of singletons inevitably leads to results which are highly approximated and whose accuracy is strongly dependent on the quality of the input data. In the case of weight estimation at the preliminary design stages, the information and data available for the analysis is not only limited but also permeated with noise. The reliability of the results will, in turn, be greatly affected as well as hard to quantify.

In addition to this, singletons are characterised by a reduction in the representation capabilities of the fuzzy region analysed. Singletons represent one dimensional fuzzy sets, whose membership function is unity at a particular point and zero everywhere else in the universe of discourse. On the other hand, a traditional Gaussian membership function is able to incorporate, both visually and mathematically, information about noise and spread of the variable under consideration due to its two dimensional profile. The consequence of this lack of dimensionality within the fuzzy singleton is the loss of detail in the representation of the variable as well as a reduction in the visual and intuitive interpretability of the model.

In the specific case of the NEFPROX-based model derived for sizing and weight estimation of spoiler attachment ribs, the additional benefit of using Mamdani FIS as opposed to TSK ones derived via ANFIS, is the ability to easily translate the cross sectional areas computed from the sizing module into inputs for the weight estimation module without having to empirically modify the relative fuzzy sets from singleton to gaussian definition.
6.3.1 Sizing module performance

Training and testing of the NEFPROX sizing network was conducted using the same database of reference as for the ANFIS model, with a total of 77 beam components for spoiler attachment rib structures, 59 of which were selected for training and 18 for its performance assessment. In order to be able to compare the performance of the NEFPROX-derived model with ANFIS, subtracting clustering was applied for initial rule derivation during the training process. The same parameters of cluster radius of $r_a = 0.4$ and accept and reject ratios of 0.5 and 0.15 respectively were chosen.

![Performance of the NEFPROX sizing module for spoiler attachment ribs on testing database for A and I.](image)

NEFPROX shows satisfactory generalisation capabilities when applied to the beam sizing problem. An analysis of the model performance highlights an average absolute error of prediction of 9.6 percent and 7.7 percent in the estimation of beam cross section area and second moment of area respectively (Figure 6.2). In a similar way as shown by the testing procedure of the MANFIS sizing module, the results underline a general tendency of the network to underestimate the outputs by approximately 6 percent for both cases (Table 6.1). Overall, the NEFPROX model shows improved performance.
generalisation capabilities as opposed to MANFIS. The model RMSE has reduced by approximately 17 percent for both cross sectional area and second moment of area which is also mirrored by the lower values of MAPE. On the other hand, the MAE shows a 2 percent larger underestimation of the second moment of area by NEFPROX compared to MANFIS, whilst the average underestimation level for the cross-sectional area is of approximately the same magnitude between the two fuzzy models.

**TABLE 6.1: Performance assessment of the NEFPROX framework applied to the sizing and weight estimation of spoiler attachment ribs.**

<table>
<thead>
<tr>
<th></th>
<th>A NEFPROX</th>
<th>I NEFPROX</th>
<th>W NEFPROX</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Training</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>Testing</td>
<td>18</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
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<td>0.096</td>
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</tr>
<tr>
<td>MPE</td>
<td>-7.68</td>
<td>-5.13</td>
<td>-1.93</td>
</tr>
<tr>
<td>MAPE</td>
<td>9.54</td>
<td>7.69</td>
<td>8.80</td>
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</table>

In terms of the impact of the individual input variables on the final outputs, the trends shown by both models are highly nonlinear, but are able to highlight the different dependencies between the variables of interest. In particular, the relationships derived through NEFPROX are strongly validated by those previously obtained with ANFIS. Cross-sectional area, as expected, is highly dependent on the value of axial force (Figure 6.3(a)). More specifically, the rate of change of cross sectional area with respect to axial force is comparable to that computed by the previous model. Similarly to ANFIS, NEFPROX was able to capture the higher influence of bending loads on the second moment of area compared to axial loads (Figure 6.3(b)). In addition to this, NEFPROX provided a more realistic approximation at higher values of applied loads, with a steady increase of sectional properties in the design space. On the other hand, the results from the ANFIS model highlight a noticeable dip in second moment of area at higher values of applied axial force.

### 6.3.2 Weight module performance

The derivation of the NEFPROX weight module was also conducted using the same database of reference as for the corresponding ANFIS model. A total of 79
examples of spoiler attachment rib structures were employed, 59 of which were selected for training and 20 for its performance assessment.

Table 6.1 summarises the results from the application of the weight estimation model on spoiler attachment ribs. It is clear that the overall performance of the model has improved greatly, with over 35 percent reduction in the testing RMSE and an overall decrease in MAPE from 10.25 to 8.8 percent in the estimation of structural weight. Similarly to ANFIS, the model appears more accurate in the analysis of hinge ribs as opposed to intermediate ones, but shows a higher tendency to provide a lower estimate across the full range of ribs (Figure 6.4). This restates the need to further improve the model at later stages with additional considerations related to features which are rib specific or which link the structural element to a particular manufacturing process. This will enhance the approximation capability of the model and allow it to better discriminate across a range of different design solutions.

The results of the variable selection process also validates the model. As shown in figure 6.5, the greatest accuracy is obtained with the full set of variables and the removal of any of the input parameters noticeably deteriorates the accuracy of the final weight estimate. In particular, the removal sequence is the same as for the ANFIS model. In line with the previous model, the results stress the importance of the definition of the rib through global geometrical variables and rib function. It is also important to note that NEFPROX provides a much lower RMSE at each stage of the removal process,
thus proving it to be a better solution in terms of accuracy, even in cases where the information relative to specific variables is missing.

The approximations from the two models appear very similar in terms of weight estimates. Both NEFPROX and ANFIS highlight how the weight penalties from global geometry on the final structural weight are shared in equal proportions between hinge line datum and spar height. In particular, in the case of spoiler hinge ribs, NEFPROX is able to derive a more realistic trend between global geometrical parameters and structural weight, with a steady direct proportionality between the variables and without displaying anomalous decrease in weights at higher values of $L$ and $h$ (Figure 6.6(a)). Results are in strong agreement in the case of intermediate ribs. The surface displayed in figure 6.6(b) closely matches that developed through ANFIS, both in terms of dependencies between the variables and overall profile.

Cross-sectional areas of top and vertical beams are identified as the main contributions to the rib structural weight in terms of local geometry, as well as their higher
impact on weight as compared to global geometrical parameters. NEFPROX, however, suggests a lower weight penalty resulting from the inclusion of the vertical beam in the design of the spoiler rib as compared to that coming from top beam in the case of hinge ribs (Figure 6.7(a)). In both cases, dip in weight is clearly identifiable at higher values of cross sectional areas for the two selected beams, suggesting that in the case of bigger spoiler ribs, the bottom section suffers a more considerable increase in size thus contributing more to the final structural weight. In terms of intermediate ribs, the influence of the local geometry of both top and vertical beams on the structural weight of the rib is highly consistent across the results of both ANFIS and NEFPROX. The surface profile in figure 6.7(b) clearly matches that computed via the TSK fuzzy system previously, with the exception of minor additional nonlinearities.

**Figure 6.5:** Effect of variable removal on the accuracy of the NEFPROX weight module for spoiler attachment ribs.
6.4 NEFPROX-based structural sizing and weight estimation: aileron attachment ribs

The same process employed in the case of the spoiler attachment ribs was used for the conversion of the general illustrative framework for structural sizing and weight estimation into a NEFPROX-based mode architecture. Figure 6.8 highlights the 3-layer model structure for the aileron problem.

As in the case of the ANFIS model structure, this framework allows for the analysis of different aileron designs. This is possible by idealising the aileron sup-

---

**Figure 6.6:** Variation in spoiler attachment rib weight with respect to the hinge line datum \( L \) and spar height \( h \) for hinge ribs (a) and intermediate ribs (b), as derived through the NEFPROX weight module.

**Figure 6.7:** Effect of local geometry on the rib structural weight for spoiler hinge ribs (a) and intermediate ribs (b), as derived through the NEFPROX weight module.
Figure 6.8: Schematic representation of the three-level sizing and weight estimation framework for the aileron attachment ribs, highlighting the variables and structure of NEFPROX-based sizing and weight estimation modules.

port rib structure as a combination of individual rib components, giving the model increased flexibility when dealing with possible changes in design configurations. The three designs analysed are the same as in the previous case study:

1. Design A identifies a 4-beam configuration, characterised by both a front and a back beam in addition to the traditional top and bottom ones;
2. Design B characterises the more common 3-beam rib structure, with a vertical support beam as a replacement for the spar stiffener;
3. Design C identifies a single tapered beam structure.
6.4.1 Sizing module performance

The same database of reference used for the development of the ANFIS framework was employed in the testing and training of the NEFPROX-based sizing model. Data for a total of 63 beam components of aileron attachment ribs was collected, of which 44 were used for training and 19 for module testing. Initial rule derivation was obtained through subtractive clustering with the same parameters as for the ANFIS sizing model to ensure consistency within the results and to allow for unbiased comparison. A cluster radius of $r_a = 0.4$ and accept and reject ratios of 0.5 and 0.15 respectively were chosen as suitable parameters for cluster definition.

The model performance has greatly improved through the use of NEFPROX. Both RMSE and MAPE have considerably decreased with the estimation of cross-sectional area and second moment of area (Table 6.2). In particular, results show a reduction of 30 percent on average in the RMSE for both variables. This highlights the higher capability of the MANFIS fuzzy inference system, compared to the TSK FIS derived through ANFIS, to both learn underlying trends within the given sample dataset as well as generalise to unseen examples. The results show no clear tendency of the model to either over or under estimate the output variables, as opposed to the ANFIS model (Figure 6.9).

<table>
<thead>
<tr>
<th></th>
<th>A NEFPROX</th>
<th>I NEFPROX</th>
<th>W NEFPROX</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Training</td>
<td>44</td>
<td>44</td>
<td>46</td>
</tr>
<tr>
<td>Testing</td>
<td>19</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.088</td>
<td>0.081</td>
<td>0.073</td>
</tr>
<tr>
<td>MPE</td>
<td>-0.07</td>
<td>-0.66</td>
<td>-2.45</td>
</tr>
<tr>
<td>MAPE</td>
<td>9.29</td>
<td>8.38</td>
<td>6.39</td>
</tr>
</tbody>
</table>

The model performance appears to have improved even in terms of the influences of the different variables on the sizing parameters of the structure. Overall, the trends derived by the model follow the same general pattern as those obtained via ANFIS. Result highlight how MANFIS FIS is able to derive more credible and realistic trends,
especially in the regions around the boundaries of the design domain or in areas where the TSK FIS displayed unlikely maxima or minima.

The relationship between beam cross sectional area and axial loads shows a steadier direct proportionality that plateaus only at the very edge of the domain of interest (Figure 6.10(a)). This is in strong contrast to the results from ANFIS: although the rate of change between the variables is the same, the maximum value of cross sectional area is reached at much lower values of $F_x$ and then maintained, indicating the inability of ANFIS to handle limit regions. Similarly, in the case of second moment of area, NEFPROX derives a more reliable trend. The model has been able to handle the nonlinearities especially in the relationship between $I$ and $F_X$ in parallel with highlighting the a steadier dependency between second moment of area and bending moment (Figure 6.10(b)).
Chapter 6

Figure 6.10: Effect of axial and bending loads on beam cross-sectional area (a) and second moment of area (b) for aileron attachment ribs.

6.4.2 Weight module performance

The derivation of the NEFPROX weight modules was also conducted using the same database of reference as for the corresponding ANFIS model, with a total of 66 examples of aileron support ribs, 46 of which were selected for training and 20 for performance assessment.

Although the model performance has not changed significantly in terms of MAPE between ANFIS and NEFPROX frameworks, the model has acquired greater generalisation capabilities with the use of a MANFIS FIS, which is confirmed by 44 percent reduction in RMSE. In addition to this, the tendency of the model to underestimate the structural weight has also improved, with MAE reducing from -3.65 to -2.45 percent. The underestimation is more prominent when the model tackles the weight of both Design A and B, whilst no clear tendency appears within the weight analysis of Design C (Figure 6.11).

The model is further validated by the results of the variable selection process. As in the case of the ANFIS model, the greatest accuracy is obtained when the analysis is conducted using the full set of variables (Figure 6.12), with noticeable decrease in effectiveness with the subsequent removal of variables. The removal pattern is the same across both ANFIS and NEFPROX, with hinge line datum $L$ and top beam local geometry having the greatest influence on the final model performance. In this case, the magnitude of RMSE at the different stages of the approximation does not
change when ANFIS and NEFPROX models are compared, as opposed to the spoiler attachment rib case.

Overall, the results corroborate the findings obtained via the ANFIS model: the weight penalties resulting from global geometrical parameters $L$ and $h$ between the two models are very similar in terms of scale and trend especially for Design B and C. In particular, the findings highlight once again how the weight penalty coming from an increase in the height of the structure and, therefore, from its location along the OFTE, is higher than that resulting from a change in hinge line location. The general pattern of dependency between the variable is clearly maintained across the three different designs, as shown by figures 6.13 (a), (b) and (c). The trends characterising Design B and Design C are in very close agreement between ANFIS and NEFPROX, in terms of nature and magnitude of the dependency between the global geometry and the structural weight, with the only difference lying in the convexity of the curves produced by NEFPROX. The main discrepancies can be found within the results of Design A between the two different models. The curve produced by NEFPROX follows the convexity of the
other two designs, but with higher gradient relating $L$ to the structural weight of the component. This strongly contrast with the lower weight impact attributed by ANFIS on the two geometrical parameters for this type of design solution and, combined with the difference in concavity between the two surfaces, highlights the lower generalisation capability of the TSK fuzzy inference system in the case of aileron attachment ribs.

NEFPROX continues to attribute the highest weigh impact to the top beam section, when it comes to local geometry (Figure 6.14). Even in this case, $A_{TOP}$ appears to impact the weight of both Design A and B by a comparable magnitude, which is also in agreement with the results from the variable removal process. The findings from ANFIS, however, highlight an inverse proportionality between the cross sectional area of the bottom beam and the structural weight of Design A ribs. This, however, is in contrast with the trend derived by NEFPROX, where a more realistic direct proportionality is shown between the variables, with a plateau at higher values of $A_{TOP}$ and $A_{BOT}$ suggesting that a lower proportion of the weight of the larger ribs within the Design A category can be attributed to the size of the two additional vertical beams (Figure 6.14(a)). The same conclusion can be reached in the case of Design B.
The profile of the trend derived by NEFPROX is very similar to that provided by ANFIS, with inverse proportionality between lower beam geometry and rib structural weight (Figure 6.14(b)). The gradient of the NEFPROX curve, however, is much lower than the ANFIS one, denoting the lower impact of the vertical rib geometry on the weight of larger ribs.

NEFPROX confirms how this added structural weight can be attributed specifically to the back vertical beam section geometry, as opposed to the front one. Figure 6.15 shows how, even in this case, the weight penalty related to the inclusion of the front vertical beam section is minimal compared to that of the back beam component. In particular, in this case, the proportion of the structural weight related to the back section is about 30 percent higher than the additional weight incurred by the inclusion of a front beam component. The Mamdani fuzzy inference system is also able to highlight the marked nonlinearities associated with the front beam section which the TSK FIS did not detect. These can be attributed to both the spread in the values
of cross sectional for this particular beam section and the specific features associated with some individual vertical beam elements present in the data sets which contribute with some of the higher weight penalties. A concrete example could be the inclusion of larger hinge attachments which are present on specific vertical beam members but not explicitly accounted for within the model.

When analysing Design C, NEFPROX produces very similar results as compared to ANFIS. Within the single beam design, even in this case there is a 3:1 ratio of influence of hinge line datum and beam cross sectional area on the overall rib structural weight respectively (Figure 6.16). In this case, however, there is no dip in the trend at higher values of $L$ and $A_{TOP}$, but rather a steady direct dependencies between
the input variables and the outputted weight. This further reiterates the inability of ANFIS-developed model to accurately portray the boundary regions of the domain of interest, but rather produce erroneous and misleading trends during the approximation process. The reason behind this can be linked with the way ANFIS represents the final output. By using a linear combination of the individual outputs rather than partitioning the output region systematically as in a Mamdani model, TSK FIS incur the risk of misinterpreting the underlying relationship between the variables of interest, especially when it comes to limit regions within the domain, as shown throughout this chapter.

6.5 Mamdani vs. Sugeno for knowledge acquisition in the design process

The results so far have shown a considerable improvement in the accuracy of the modelling approximation by opting for a Mamdani instead of a TSK fuzzy inference system, for the sizing as well as the weight estimation modules in both structural examples used. In addition to this, the Mamdani FIS derived using NEFPROX overcome the problem of producing erroneous approximations at the limits of the design domain, which were evident whilst using ANFIS.
Chapter 6

The additional benefits of adopting a Mamdani FIS structure instead of a TSK one are linked with the:

1. Definition of the fuzzy variables;
2. More efficient partitioning of the design space;
3. Increased interpretability of the FIS due to the improved visualisation of the rulebase itself.

In the case of spoiler attachment ribs, the quality of the approximation has improved with a reduction of approximately 17 percent on the RMSE for the testing data for the sizing module and with over 25 percent improvement on the RMSE for the weight estimation module (Table 6.3). As expected, the use of Mamdani systems produces a more complex network, with nearly twice as many rules as its TSK counterpart. Despite this, NEFPROX is able to describe the individual variables with a reduced number of membership functions and, consequently, a lower number of fuzzy partitions.

The Mamdani system is more complex also in the case of the FIS for the aileron support rib problem, with twice the number of rules compared to the TSK correspondives for both sizing and weight estimation. The fuzzy inference system derived by NEFPROX, however, appear highly more efficient than the ANFIS one. In the case of the weight module for the aileron support ribs in particular, the Mamdani system is able to describe each variable, both input and output, with less than half the number of membership functions, but still maintaining a substantially higher accuracy than the TSK one.

This allows for an overall more interpretable definition of the system itself. Interpretability for a fuzzy system lays on both system description and network structure (Alonso et al. (2009)), the former identifying the complexity of the individual system components (i.e. rulebase, fuzzy sets, ...) and the latter the network structure itself (i.e. network operations, number of variables, rule structure, ...). In this case, the TSK FIS developed through ANFIS are defined by simpler network structures, being described by overall fewer rules and modifiable connections. Their descriptive elements however are much less interpretable compared to those in the Mamdani FIS developed.
TABLE 6.3: Comparison of results for the architecture sizing and weight fuzzy inference systems built with ANFIS and NEFPROX for both spoiler and aileron attachment ribs.

<table>
<thead>
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<th>SPOILER</th>
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<tr>
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<td>I No. of Rules</td>
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</tr>
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<td>W No. of Rules</td>
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<td>MFs*</td>
<td>20</td>
<td>15</td>
<td>18</td>
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</table>

through NEFPROX.

Figure 6.17 shows the way in which input and output variables within both the ANFIS and the NEFPROX-derived FIS are defined by the use of membership functions. In the case of the Mamdani FIS derived through NEFPROX, all the variables appear characterised by clearly distinguishable and complementary fuzzy partitions. The additional difference is in the output W, which is not defined by fuzzy partitions in the ANFIS structure, since the outputs of a TSK FIS are identified by singletons. On the other hand, the Mamdani system allows for a visual definition of the output variables too. This ensures that the design space is fully defined, thus providing a clearer and more transparent problem characterisation.

In the case of the networks developed for the weight module of spoiler attachment ribs, ANFIS derived 6 membership functions more per variable, however the distribution and shape of those used within the NEFPROX network is more consistent and allows a more even description of the variables themselves. All the variables within the NEFPROX network are fully defined within the design space by uniformly distributed membership functions. On the other hand, input variable description within ANFIS appears more irregular, with noticeable gaps in locations where the system does not seem able to describe the input domain, as well as numerous duplicated fuzzy partitions. This discrepancy is particularly evident in the definition of the input variable.
AVERT. The fuzzy definition of the variable by the ANFIS network is ambiguous and incomplete: the figure highlights the ability of the network to describe only the leftmost part of the variable domain with repeated and highly overlapping membership functions. In contrast, the definition of the same variable provided by NEFPROX appears highly more transparent (Castellano et al., 2003), providing a complete and unequivocal description of the design space being considered by adopting clearly defined and discernible fuzzy partitions.

Figure 6.18 highlights similar trends within the definition of variables in the aileron attachment rib problem. Each input variable within the NEFPROX architecture, as well the the output one, is defined by 8 membership functions which evenly span the entire variable domain and are clearly distinguishable. Conversely, ANFIS produces over twice the amount of membership functions per input variable, the majority of which tend to show more than 90 percent overlap, causing fuzzy partitions to be ambiguous and hard to identify. This is evident especially the case of variables such as AVERTb and AVERTf. These two variables do not appear as often, due to the difference between the three designs for aileron support ribs considered in the problem. As a consequence, the fuzzy system has less instances to learn the behaviour of these variables. The performance of ANFIS appears to be strongly affected by this factor: although still using 18 membership functions to define them, they are only able to interpret the extreme regions within the full domain of the variables.

The fuzzy rather than crisp description of the output variables within the Mamdani systems developed with NEFPROX is also more advantageous from a design perspective, since it allows a more visual assessment of the impact of the individual variables on the final output, resulting in a more manageable and easy to read system (Alonso et al., 2009). By looking at the rulebase at the basis of the sizing module for spoiler attachment ribs, it is easy to appreciate how the definition of the individual rule outputs as fuzzy partitions helps visualising how, for each specific input condition, the design will have to focus on a particular region of the design space (Figure 6.19).

The additional benefit of adopting a Mamdani FIS is the possibility of understanding as well as visualise the impact of the variability and uncertainty of the design parameters on the final input. According to the value of each individual input for
the specific design under study, each rule will be fired with a different strength. For instance, in the case of the determination of cross sectional area for aileron attachment ribs, with the input given, rule number 7 has the highest influence on the final output (Figure 6.20). The weighted outputs from the individual rules are then aggregated to determine a final overall output distribution. The fuzzy definition of this final output will change in profile and spread very dynamically in parallel with variations in input values. In a real-life design scenario, this will enable the engineer to visualise very rapidly the impact of individual design decisions on the sizing and consequent weight of the structure. In addition to this, the designer will be able to get a better understanding of how the variability within the output will translate into an altogether different fuzzy partition within the output space, from the point of view of the magnitude of the spread of the resultant fuzzy set and, as a consequence, the uncertainty of its crisp counterpart.

6.6 Summary

This chapter explored the issue of interpretability within a data-driven fuzzy inference system. In particular, one of the main topics analysed was the critical evaluation of the trade-offs between modelling accuracy and interpretability in involved in the choice of a Mamdani or of a TSK FIS. Definitions of low-level and high-level interpretability were introduced with explicit criteria for a comprehensive qualitative assessment of fuzzy inference systems.

This formed the background for a comparative evaluation of Mamdani and TSK system within the sizing and weight estimation framework for aircraft structures. NEFPROX was used to derive Mamdani-type FIS for both spoiler and aileron attachment ribs, using the same reference datasets adopted for TSK FIS extraction and optimisation through ANFIS. The new fuzzy inference systems were also designed following the same illustrative general framework for design and weight estimation produced following the flow of information within the design process, making the new FIS equivalent in terms of analysis and overall structure to the previous ones and, thus, easily comparable.
The final models were firstly assessed in terms of their modelling accuracy. The Mamdani FIS developed using NEFPROX showed greater generalisation capabilities than their TSK counterparts from the point of view of both sizing and weight estimation. In particular, RMSE decreased by approximately 20 percent on average across the different sizing modules and 30 percent on the weight ones. In addition to this, the Mamdani FIS were able to produce more reliable trends for the relationships between the different variables of interest and proved to be more efficient than TSK in analysing the limit region of the design domain.

NEFPROX was also able to derive more streamlined FIS structure. Although the overall number of rules was on average twice as high as those derived through ANFIS, the system was successful in describing each single variable by a substantially lower number of fuzzy partitions without affecting the final modelling accuracy. This was also coupled with a better coverage of the universe of discourse of all input and output variables though better defined and distinguishable fuzzy partitions. For this reason, the rulebase derived within the Mamdani environment was substantially more transparent, with a clear fuzzy definition of the design space through a set of simpler and more readable rules.

The next chapter will focus on enhancing the capabilities of the Mamdani FIS for both spoiler and aileron attachment ribs by implementing the derived architectures within a Type-2 fuzzy logic environment. This will allow to combine the accuracy and interpretability of this system with a more explicit and understandable way of dealing with the uncertainties permeating the variables and their impact on the final outputs.
Figure 6.17: Comparison between the membership functions for the individual variables in the weight module for spoiler attachment ribs, as derived using ANFIS and NEFPROX.
FIGURE 6.18: Comparison between the membership functions for the individual variables in the weight module for aileron attachment ribs, as derived using ANFIS and NEFPROX.
Figure 6.19: Rulebase for the estimation of sectional properties for spoiler attachment ribs, as derived with NEFPROX.
Figure 6.20: Rulebase for the estimation of sectional properties for aileron attachment ribs, as derived with NEFPROX.
Chapter 7

Type-2 Fuzzy Logic for Uncertainty Management and Propagation in Weight Estimation

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7.1 Introduction

When designing a computational model for approximation applications, it is of vital importance to be able to correctly embed uncertainty information within the model itself. It is of even greater significance to be able to construct a model which is able to propagate the uncertainties permeating its variables all the way down to the final approximation and visualise the effect they have on it.

Although probabilistic approaches continue to dominate the field of uncertainty analysis, they can prove to be the wrong choice of modelling technique. This is particularly true of applications where a complete knowledge of the variables themselves is not sufficiently extensive to be able to build reliable probability density functions (PDFs). The lack of knowledge about the variables and the uncertainties associated with them, can induce a considerable error propagation within the model itself, producing misleading results. In addition to this, the limited understanding of the correlation between the variables of interest, combined with PDFs which are constructed under erroneous assumptions, will inevitably lead to a biased representation of the combined uncertainty across the model.

Previous chapters have proven how type-1 fuzzy logic, implemented through both TSK and Mamdani fuzzy systems, can help in achieving both approximation accuracy and modelling transparency when applied to the estimation of sizing and weight of aeronautical structural components at preliminary design stages. The additional benefit of adopting fuzzy techniques lays also in the possibility of deriving a rulebase which highlights the relative impact of the individual variables on the approximation and which can be used as a set of visual guidelines in the design process.

Although type-1 FIS are able to handle the uncertainties and noise within the data, they are unable to fully visualise and propagate them within the model. It is possible to view the output of a type-1 FLS as the mean of a PDF. When dealing with uncertainties, it is important, however, to know also the variance of the distribution. Type-2 fuzzy logic can be seen as a tool to derive a measure of dispersion about the mean and capture a more comprehensive uncertainty picture of the problem at hand.

This chapter aims at analysing the potential benefits of using type-2 FIS in the
sizing and weight estimation of aircraft structures. In particular, background theory on interval type-2 FIS and their design will be presented. This will then be put into context through the description and analysis of interval type-2 FIS for the sizing and weight estimation of spoiler and aileron attachment ribs. Their performance will be firstly evaluated by assessing their approximation capabilities and comparing it with the type-1 FIS derived in previous chapter. In addition to this, they will be analysed from the point of view of uncertainty analysis, with particular focus on the propagation of uncertainties within the model structure. The final rulebase will also be reviewed in terms of its ability to combine transparency in the representation of the causalities among the system variables and readability, with a comprehensive overall visualisation of the uncertainties within the system itself.

7.2 Type-2 fuzzy systems theory for scenario analysis

To better understand the translation from type-1 to type-2 fuzzy set, imagine adding the uncertainty information about the variables to the initial fuzzy partition, by blurring the original type-1 membership functions by transposing the points within the curve to the left or right of the membership function itself. By doing so, for each value of $x$, the membership function will no longer assume a single value. Instead, the membership of each individual point will be represented by a fuzzy interval, whose values will be themselves weighted differently through secondary membership functions. This results in type-2 fuzzy sets being defined by a three dimensional membership function.

A type-2 fuzzy set $\tilde{A}$, defined by the membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$, $u \in J_x \subseteq [0, 1]$, can be identified as:

$$
\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u)/(x, u) J_x \subseteq [0, 1]
$$

where $0 \geq \mu_{\tilde{A}}(x, u) \leq 1$ and $\int \int$ indicates the union over all the possible $x$ and $u$. $J_x \subseteq [0, 1]$ indicates the primary membership of $x$ in the fuzzy set, whilst $\mu_{\tilde{A}}(x, u)$ is used to represent a type-1 fuzzy set acting as a secondary set. In other words, a type-2
membership can be represented by any subset in \([0, 1]\) as primary membership, and each primary membership will be associated to a secondary one defining the uncertainty of the primary MF itself (Sepúlveda et al., 2006).

In order to significantly reduce the computational effort required to analyse type-2 fuzzy systems, the engineering community has been focusing on interval type-2 fuzzy sets (IT2 FS) (Melin et al., 2010; Mendoza et al., 2009; Lee et al., 2009; Ranjbar-Sabraie et al., 2011). These are special cases of type-2 fuzzy sets, occurring when all \(\mu_{\tilde{A}}(x, u) = 1\) and can be mathematically represented as:

\[
\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x, u), J_x \subseteq [0, 1] \tag{7.2}
\]

Figure 7.1 shows a representation of the membership function of an interval type-2 fuzzy set in the case of discrete \(x\) and \(u\) for \(X = 1, 2, 3, 4, 5\) and \(U = 0, 0.2, 0.4, 0.6, 0.8, 1\). The individual lines on the graph identify values of \(\mu_{\tilde{A}}(x, u)\) at
discrete \((x, u)\) locations. The grey area in the graph is the *footprint of uncertainty* (FOU) for the membership function considered.

The FOU is a region in the set of interest which represents the union of all the primary membership functions within the region of uncertainty for the set itself (Equation 7.3).

\[
FOU(\tilde{A}) = \bigcup_{x \in X} J_x
\]  

(7.3)

The FOU is of primary importance in the analysis of interval type-2 fuzzy systems. Firstly, it fully conveys the uncertainties and variability within the membership function itself, since they directly impact its shape and size. Secondly, since the secondary membership is constant for IT2 FS, the footprint of uncertainty represents the complete definition of the fuzzy set itself. As a consequence, the uniformly shaded FOU used to describe IT2 FS, highlights the uniform secondary membership characterising it.

Gaussian primary membership functions with uncertain means have been selected to be the basis of the IT2 fuzzy sets for both antecedents and consequents of the FLS. These are formalised as:

\[
\mu_l^i(x_i) = \exp \left[ -\frac{1}{2} \left( \frac{x_i - m_l^i}{\sigma_l^i} \right)^2 \right] \quad m_l^i \in [m_{l1}^i, m_{l2}^i]
\]  

(7.4)

where \(m_l^i \in [m_{l1}^i, m_{l2}^i]\) indicates the uncertain mean, with \(i = 1, \ldots, p\) (number of antecedents) and \(l = 1, \ldots, M\) (number of M rules), and \(\sigma_l^i\) is the standard deviation.

The type-2 fuzzy rules for fuzzy inference assume a similar form as their type-1 counterparts:

\[
\text{IF } x \text{ is } \tilde{A}, \text{ THEN } y \text{ is } \tilde{B}.
\]  

(7.5)

where \(x\) and \(y\) are the variables of interest, and \(\tilde{A}\) and \(\tilde{B}\) relate to individual type-2 fuzzy sets within the universe of discourse of the problem.
As mentioned in Chapter 3, the difference between traditional type-1 and type-2 fuzzy systems lays in the addition of an extra processing block in the system. The *type reducer* is included in the framework to allow the translation of consequents fuzzy sets from type-2 to type-1 for output computation, before moving onto a crisp solution.

Once type-reduction operations have been carried out, the resultant type-1 sets can be viewed as output sets of a type-1 FLS. As a consequence, the original IT2 FLS is an aggregation of of the individual type-1 systems, which are themselves embedded in it. The type-reduced set is, therefore, an aggregation of the outputs of all the embedded type-1 FLS (Mendel, 2001). For this reason, the type-reduced set can be regarded as the fuzzy representation of the output of the type-2 FLS. In turn, the membership functions of the type-reduced set can be seen as a way of understanding and defining the level of uncertainty of the embedded type-1 systems. By viewing type-2 FLS as type-1 FLS that have been blurred due to the presence of uncertainties within the system itself, the type-reduced set can be thought of indication the characteristic uncertainties within the crisp output of its respective type-2 FLS.

By analysing the shape and spread of the type-reduced set, it is then possible to understand the variability of the output due to uncertainties, as well as assess the reliability of the approximations derived by the system.

### 7.2.1 Development and optimisation of interval type-2 fuzzy logic systems

For the purpose of this study, the development of the interval type-2 fuzzy system has been designed to follow from the results of the best performing type-1 FLS from derived within previous chapters. In terms of both approximation accuracy and system interpretability, the fuzzy systems obtained using NEFPROX substantially outperformed those produced by ANFIS. For this reason, the type-2 fuzzy logic system was initialised using the fuzzy sets built and optimised through NEFPROX for both inputs and outputs.

An interval type-2 fuzzy logic system is characterised by a number of design parameters, namely mean bounds $m_{11}^1$ and $m_{12}^1$ and standard deviations $\sigma_i^1$ for each
antecedent and consequent, as well as input measurement parameters $\sigma_k$. There are different design approaches for an interval type-2 fuzzy systems, all aiming at establishing the different parameters of the membership functions for antecedent and consequents (Mendel, 2001):

1. All the design parameters for antecedents, consequents and input measurement parameters, thus establishing the shape of FOU. The data is only used to determine the rules.

2. All design parameters for antecedents and consequents are fixed as well as the shape of the membership functions, but not the input measurement parameters. The data is used to optimise input measurement parameters and fuzzy rules.

3. The shape of all antecedent, consequents and input measurement parameters is fixed, thus establishing the shape of the FOU. The data is used to optimise all the design parameters, allowing the size the individual FOU and the input measurement parameters to reflect the patterns within the specific data set.

The third approach is the most suitable for this particular study. By using the data to optimise the majority of the FLS structure, it will be possible to evaluate the uncertainties within the system itself, to gain a better understanding of uncertainties in the approximation process as well as assess the reliability of the final solution on the basis of these factors.

7.2.2 The iterative design approach

An iterative design process was set up and carried out in order to both improve the structure of the interval type-2 fuzzy system and optimise its approximation capabilities. The design and optimisation process follows four main steps:

1. Conversion of the type-1 fuzzy logic system derived through NEFPROX into an interval type-2 FLS. Within this phase, the design parameters for antecedent and consequent membership functions are initialised, whilst the rule structure as well as the number and shape of the individual fuzzy partitions and membership function are retained from the type-1 FLS.
2. Design and training of the type-2 fuzzy logic system using backpropagation. In this step, all design parameters for antecedents and consequents are tuned by a steepest descent optimisation algorithm based on training and testing data until desired approximation error threshold is met.

3. Rule reduction and fuzzy logic system structure optimisation by Singular-value decomposition (SVD) combined with QR decomposition. The combined SVD-QR process allows the identification of the most important rules within the rulebase of the FLS to overcome the problem of combinatorial rule explosion (Liang and Mendel, 2000).

4. Reiterations of steps 2 and 3 for further parameter and rulebase optimisation until approximation performance is acceptable.

7.2.3 SVD-QR routine

Combinatorial explosion of rules is a common problem encountered by fuzzy logic systems, both type-1 and type-2. In the case of previous examples with type-1 FLS within this research, subtractive clustering was used in combination with variable selection routines in order to overcome this problem (Chiu, 1994, 1996).

In the case of IT2 FLS, SVD has proven successful in the identification and extraction of the most important rules for output approximation within an initial rulebase (Mouzouris and Mendel, 1996; Yam et al., 1999). From a general perspective, SVD is an effective and widely used mathematical approach for the solution of algebraic problems, such as the determination of the rank of a matrix and the computation of numerical solutions of least squares problems.

Consider $H$ a $K \times M$ matrix, and $U$ and $V$ two $K \times K$ and $M \times M$ unitary matrices respectively. The SVD of $H$ can be computed as:

$$H = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^T$$  \hspace{1cm} (7.6)

In particular, the attractiveness of the SVD method lies in its straightforward way of identifying dominant and subdominant subspaces within a particular domain of
interest. By looking at the rulebase as a matrix $\Phi$ made up of the individual rules or fuzzy basis functions (FBFs) (Wang and Mendel, 1992), as:

$$
\Phi = \begin{pmatrix}
\phi_1(x^{(1)}) & \ldots & \phi_M(x^{(1)}) \\
\vdots & \ddots & \vdots \\
\phi_1(x^{(N)}) & \ldots & \phi_M(x^{(N)})
\end{pmatrix}
$$

(7.7)

where $\Phi_l(x)(l = 1, \ldots, M)$ represents the single fuzzy basis function with $l$ indicating the rule number. The FBF can itself be formalised as:

$$
\Phi_l(x) = \frac{\prod_{i=1}^{p} \mu_{F_i}(x_i)}{\sum_{l=1}^{M} \min_{i=1,\ldots,p} \mu_{F_i}(x_i)} \quad l=1, \ldots, M
$$

(7.8)

where $\mu_{F_i}(x_i)$ relates to the membership grade of input $x_i$ for the $l$th rule.

By considering $\Phi_l(x)$ as a span of the input domain, the SVD method allows to translate it into an equivalent orthogonal span (Mendel, 2001). This enables both the determination of the most dominant and subdominant FBFs as well as the combination of FBFs which is able to represent the system most reliably and accurately.

The SVD routine essentially orders the individual fuzzy basis functions according to their importance within the matrix, based on their numerical rank. The rules with the least impact, in other words those whose rank is below a specific user-defined threshold, are removed. This results in an optimised fuzzy logic system with only the minimum number of rules needed to fully describe the problem, without compromising its approximation accuracy.

Overall, the general layout of the SVD-QR routine can be formalised as follows:

1. Computation of the SVD of $\Phi$.
2. Computation of the rank of $\Phi$.
3. Retain the components of the SVD of $\Phi$ associated with the numerical rank of the matrix.
4. Use the QR algorithm to order the fuzzy basis functions associated with the SVD matrix according to rank. This will lead to a new matrix $\Phi_{M'}$

$$\Phi_{M'} = \begin{pmatrix} 
\phi_1'(x^{(1)}) & \ldots & \phi_{M'}'(x^{(1)}) \\
\vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots \\
\phi_1'(x^{(N)}) & \ldots & \phi_{M'}'(x^{(N)}) 
\end{pmatrix}$$

(7.9)

where $M' < M$ highlights that the number of the new fuzzy basis functions $M'$ has decreased from the original set of $M$ FBFs, due to the removal of the lower ranked functions, and $\phi'$ indicate that the fuzzy basis functions have been ordered according to their rank. The fuzzy logic system can now be formalised as

$$y(x^{(i)}) = f_s(x^{(i)}) = \sum_{i=1}^{M'} y_i^j \phi_i'(x)$$

(7.10)

5. Normalisation of the $M'$ fuzzy basis functions using the firing strengths of only those functions which have been maintained after the ordering process. This is an extremely important step: if this step is ignored, the $M'$ will be normalised at the level of the original set of fuzzy basis functions, thus nullifying the ranking effect of the previous steps in this process (Hohensohn and Mendel, 1994).

6. Determination of the parameters for the total number of remaining fuzzy basis functions ($M' y^j$) using least-squares.

For interval type-2 fuzzy logic systems, the crisp output at the end of the defuzzification process is represented by the center of the type-reduced set. In other words, the output is a type-1 set which is determined by both its left and right-most points, $y_l$ and $y_r$ (Liang and Mendel, 2000). This set can then be further defuzzified to produce a crisp output. The type-reduced set, however, is sometimes more important than the final crisp value itself since it conveys the uncertainties which have been propagated through the FLS.
In the case of SVD-QR for IT2 FLS, the starting point will be two separate fuzzy basis functions, one for the left point and one for the right point, represented by equations 7.11 and 7.12 respectively:

\[ y_l = \frac{\sum_{i=1}^{M} f_l^i y_l^i}{\sum_{i=1}^{M} f_l^i} = \sum_{i=1}^{M} y_l^i p_l^i \]  
\[ y_r = \frac{\sum_{i=1}^{M} f_r^i y_r^i}{\sum_{i=1}^{M} f_r^i} = \sum_{i=1}^{M} y_r^i p_r^i \] (7.11) (7.12)

where \( f_l^i \) and \( f_r^i \) indicate the firing strength membership grades which contribute to \( y_l \) and \( y_r \), \( p_l^i = \frac{f_l^i}{\sum_{i=1}^{M} f_l^i} \) and \( p_r^i = \frac{f_r^i}{\sum_{i=1}^{M} f_r^i} \) are two FBFs used to simplify the expansions.

In this case, the general SVD-QR process will be applied to both \( y_l \) and \( y_r \). The results from the two processes are then combined to produce one single rule set from the union of the two individual rule sets obtained from the process.

The design process for interval type-2 fuzzy logic systems as outlined here has been applied within this research in the Matlab® environment using the open source framework for the design of IT2 FLS developed by Karnik \textit{et al.} (2011).

### 7.3 Structural sizing and weight analysis using interval type-2 fuzzy systems: spoiler attachment ribs

The same illustrative framework for structural sizing and weight estimation was translated into an interval type-2 fuzzy based model in a similar way as for the NEFPROX testcase. For the structural sizing and weight estimation of spoiler attachment ribs, the model framework follows an equivalent 3-layered architecture:

1. An interactive Microsoft Excel based \textbf{loading module}, used to evaluate resultant bending moments, shear and axial forces in the structure from applied loads

2. A multiple output IT2 fuzzy logic-based \textbf{sizing module} to evaluate sizing parameters for the structure by combining the results of the loading module with
relevant material properties

3. An IT2 fuzzy logic-based weight module evaluating the structural weight of the component by combining local sizing parameters outputted by the sizing module with global sizing variables related to the specific location and function of the spoiler attachment rib within the fixed trailing edge.

From the point of view of the architecture, there is not a substantial difference between the ANFIS, NEFPROX and IT2 FIS framework: all three of them have been designed in order to closely mirror the information flow and processing structure within the preliminary design stage of a structural component. In particular, both the IT2 FIS based sizing and weight module have been initiated with the NEFPROX-derived network structure. In other words, to reduce the computational burden involved within the type-2 FIS design, the initial Mamdani FIS structure optimised using NEFPROX has been used as a starting point for the network design and optimisation process. The number and size of antecedent and consequents as well as number and structure of the individual rules was therefore maintained the same as in the NEFPROX-derived FIS.

The choice of initialising the model with the Mamdani structure obtained by NEFPROX, rather than with the TSK one derived by ANFIS, is due to both preservation of information and readability of the final FIS. Firstly, translating singletons outputs from the sizing modules into 3 dimensional type-2 fuzzy partitions will inevitably lead to a loss of information especially from the point of view of uncertainty. FOUs cannot be preserved without having to make assumptions about design parameters, when converting singletons outputs from the sizing module in the type-2 partitions used as input of the weight estimation module. In particular, in the case of type-2 fuzzy systems where the principal aim is that of being able to readily visualise and interpret the uncertainties within the system, the description of the outputs with singleton partitions causes a reduction in the representation capabilities of the fuzzy region analysed. The lack of dimensionality within singletons equates to a loss of detail in the representation of both variable and overall system.
7.3.1 Sizing module performance

The design and optimisation of the IT2 FIS for sizing derivation of the beam components of spoiler attachment ribs followed the iterative process described in previous sections. The network was initialised using the Mamdani fuzzy inference structure derived by NEFPROX in Chapter 6, where input and output variables were defined by 7 Gaussian fuzzy partitions and the overall FIS structure was based on a system of 13 rules. With this FIS definition as a starting point, the fuzzy partitions were converted into interval type-2 with Gaussian primary membership function with uncertain mean \([m_1, m_2]\), with the following general formulation:

\[
\mu_{A_i} = \exp \left[ -\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right] \quad m \in [m_1, m_2]
\] (7.13)

The process was initialised using mean and standard deviation for the individual variables based on the full data set used for training and testing process. The individual fuzzy partitions were also initially defined using the following

\[
[m_1, m_2] = [m_\mu - 2\sigma_\mu, m_\mu + 2\sigma_\mu]
\] (7.14)

where \(m_\mu\) and \(\sigma_\mu\) are the shape parameters of the type-1 fuzzy partitions optimised by NEFPROX.

Training and testing of the IT2 FIS sizing network was conducted using the same database of reference as for both the ANFIS and NEFPROX models, with a total of 77 beam components for spoiler attachment rib structures, 59 of which were selected for training and 18 for its performance assessment.

The iterative design process combining backpropagation and SVD-QR for rule reduction was able to generate a substantially more concise and compact fuzzy system, with the rulebase reduced from 13 to 4 compared to the NEFPROX-based FIS and a much more intuitive variable definition with 4 fuzzy partitions for inputs and outputs.
TABLE 7.1: Comparison of results for the architecture of sizing and weight fuzzy inference systems of type-1 built with NEFPROX and interval type-2 for spoiler attachment ribs.

<table>
<thead>
<tr>
<th></th>
<th>NEFPROX</th>
<th>IT2 FIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPOILER</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.110</td>
<td>0.156</td>
</tr>
<tr>
<td>A</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>MFs*</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.096</td>
<td>0.123</td>
</tr>
<tr>
<td>I</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>MFs*</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.102</td>
<td>0.108</td>
</tr>
<tr>
<td>W</td>
<td>34</td>
<td>4</td>
</tr>
<tr>
<td>MFs*</td>
<td>15</td>
<td>4</td>
</tr>
</tbody>
</table>

(Table 7.1). By streamlining the FIS, however, the quality of the approximation has reduced. This is evident by examining the change in RMSE between the type-1 model derived with NEFPROX and the interval type-2 FIS. The RMSE has increased from 0.110 to 0.156 in the case of estimation of cross sectional areas and from 0.096 to 0.123 for second moments of area.

TABLE 7.2: Performance assessment of the interval type-2 fuzzy logic framework applied to the sizing and weight estimation of spoiler attachment ribs.

<table>
<thead>
<tr>
<th></th>
<th>A IT2</th>
<th>I IT2</th>
<th>W IT2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Training</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>Testing</td>
<td>18</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.156</td>
<td>0.123</td>
<td>0.108</td>
</tr>
<tr>
<td>MPE</td>
<td>-2.27</td>
<td>-0.50</td>
<td>-4.63</td>
</tr>
<tr>
<td>MAPE</td>
<td>14.39</td>
<td>12.72</td>
<td>11.12</td>
</tr>
</tbody>
</table>

Despite being characterised by lower approximation accuracy, compared to both the ANFIS the NEFPROX-derived FIS, the interval type-2 fuzzy inference system, shows satisfactory generalisation capabilities when applied to the beam sizing problem. The overall approximation performance shows a mean absolute error of prediction of approximately 14.4 percent and 12.7 percent in the estimation of beam cross section area and second moment of area respectively, which still represents a satisfactory level
of approximation for a preliminary design tool (Table 7.2). As opposed to the previously derived FIS where both sizing parameters were noticeably underestimated, the approximation obtained by the IT2 network is only approximately 2 percent lower than the expected values for cross sectional areas, whilst no clear trend in the estimation of second moment of area (Figure 7.2).

### 7.3.2 Weight module performance

The derivation of the IT2 fuzzy weight module was conducted in a similar manner to the development of the sizing module. In terms of model initialisation, the network was firstly derived from the FIS structure optimised by NEFPROX, where the 6 input variables as well as the final output were defined by 15 Gaussian fuzzy partitions and the overall FIS structure was based on a system of 34 rules. The fuzzy partitions were then converted into interval type-2 with Gaussian primary membership function with uncertain mean \([m_1, m_2]\), following the same procedure as for the sizing
module. The same reference database used for the derivation of both the ANFIS and NEFPROX FIS was adopted for the IT2 network optimisation procedure. Of the 79 examples of spoiler attachment rib structure, 59 were selected for training and 20 for the performance assessment of the fuzzy inference system.

As in the case of the sizing module, the iterative design process was able to produce a substantially simpler and more transparent network structure. The initial 34 rules used within the type-1 system derived through NEFPROX were reduced to a total of 4, with input and output variables being described by 4 fuzzy partitions each instead of 7 (Table 7.1). Nonetheless, the overall generalisation capability of the IT2 FIS appear only marginally inferior to that of the equivalent NEFPROX-derived model, although still highly satisfactory. The RMSE has increased from 0.102 in the case of the NEFPROX FIS to 0.108, in parallel with just over a 2 percent increase in the mean

![Figure 7.3: Performance of the type-2 fuzzy logic weight module for spoiler attachment ribs on testing database.](image-url)
absolute percentage error between the two models (Table 7.2). Results also highlight the noticeable tendency to provide a lower estimate of structural weight for the examples provided, however the IT2 model appear to have the same level of accuracy across both hinge and intermediate rib types, as opposed to the previous fuzzy models which demonstrated greater generalisation capabilities in the case of hinge ribs (Figure 7.3). This further confirms the importance of embedding within the fuzzy definition of the design additional considerations related to specific features, manufacturing process or rib function to allow to both improve model approximation and enhance its ability to discriminate between different design solutions.

![Figure 7.4: Effect of variable removal on the accuracy of the type-2 fuzzy logic weight module for spoiler attachment ribs.](image)

Variable selection was used also in this case in order to both assess the importance of the individual variable in terms of the quality of the model approximation and to identify the optimum combination of variables for an exhaustive problem definition. The results of the variable selection process validate those obtained from both ANFIS and NEFPROX network design. Figure 7.4 illustrates how the process continues to identify the full set of variables as the optimum in terms of model performance, with the removal of any of the input parameters causing a noticeable deterioration in the ac-
The accuracy of the final weight estimate. The variables themselves appear to show the same order of importance as in the previous fuzzy models. Global geometrical variables and rib function still have the greatest impact on model accuracy, being the last ones removed in the process. The selection process also highlights that both NEFPROX-based type-1 and IT2 FIS have comparable accuracy when it comes to RMSE performance assessment, reiterating how Mamdani fuzzy systems are still a more appropriate choice when it comes to accuracy in weight estimation applications.

7.3.3 Interval type-2 function approximation and decision boundaries

The previous results have indicated that interval type-2 fuzzy modelling does not match type-1 approximations obtained with both TSK and Mamdani fuzzy systems. This was expected since the main purpose of IT2 FLS is that of understanding the impact of uncertainty propagation within the modelling process. For this reason, a more compact and concise rulebase and fuzzy system architecture for both sizing and weight estimation modules takes priority on modelling accuracy. However, the results also indicate that a considerable reduction in the size of the rulebase is mirrored only within a marginal increase in the approximation error.

This can also be seen in the functional relationships between the different variables of interests that the system is able to produce. The trends that the system was able to learn from the dataset of reference match those produced by ANFIS and NEFPROX in terms of both profile and correlation of variables. In addition to this, the use of type-2 fuzzy theory allows to establish confidence boundaries across the different functional relationships derived. This is possible thanks to the way of formalising the variables of interest, using intervals to describe the means of the input fuzzy membership functions as well as left and right boundaries for the estimation of the output centroids. This particular approach leads to the computation of two separate curves for each variable relationship: they represent the boundaries for the approximation and delimit the confidence region for the estimation of the output, given a particular set of inputs.

The trends derived by interval type-2 fuzzy models are highly nonlinear, but the different dependencies between the variables of interest highlighted within the approx-
imations are strongly validated by the previous models. In the case of the impact of global geometry on the final structural weight, the overall trend is dominated by direct proportionality between the two inputs and the output variables. The functional relationship derived by the IT2 for spoiler hinge ribs indicates a higher proportion of the weight being linked to the location of the spoiler attachment rib along the trailing edge, identified by the spar height \( h \), compared to the outputs of the previous fuzzy models (Figure 7.5(a)). In addition to this, the IT2 FIS shows a smoother profile between global geometrical parameters and structural weight, with a steady direct proportionality between the variables and without displaying anomalous decrease in weights at higher values of \( L \) and \( h \). The confidence region, in this case, is quite narrow, indicating a low level of uncertainty in the estimation of the weight of this particular type of spoiler attachment rib.

Figure 7.5 (b) on the other hand, shows a much larger confidence region in the case of intermediate ribs, in particular for those located further outboard (i.e. low values of \( L \) and \( h \)). This can be related to the limited number of these ribs at such locations, which is due to the smaller span of the spoilers at this location, and thus the limited number of training examples within the database of reference. The system, in this case, is forced to largely interpolate between the difference numerical instances, which results in added uncertainties within the final estimate. In terms of functional relationship, the trend is also highly more non linear in the case of intermediate ribs. The profile appears to match closely that of the function derived by the NEFPROX-based FIS, with an initial steep increase in weight at low values of \( L \) and \( h \) followed by a flatter profile towards the boundaries of the domain. This indicates that, for those ribs located further outboard, global geometry is the major source of structural weight, whilst, for inboard intermediate ribs, weight penalties will be more affected by other parameters such as local geometry.

This is confirmed by figure 7.6(b). From the functional representation of the relationship, it is apparent that a larger proportion of the weight of intermediate spoiler attachment ribs can be attributed to local geometry parameters, in particular from the cross sectional area of the top beam section \( A_{TOP} \). The confidence region is still large, however, in this case, the reliability of the estimation is higher for intermediate ribs.
located further outboard as opposed to the inboard ones. As a result, in a context where the design information is scarce, the model will have focus more on global geometry parameters for those ribs located inboard, in an attempt to improve the reliability of the estimates. In terms of hinge ribs, the model still confirms a lower weight penalty coming from the inclusion of the vertical beam section compare to the top one, especially when considering the lower boundary of the confidence region (Figure 7.6(a)).
7.4 Structural sizing and weight analysis using interval type-2 fuzzy systems: aileron attachment ribs

The same illustrative framework for structural sizing and weight estimation for aileron attachment ribs was translated into an interval type-2 fuzzy based model in a similar way as for the NEFPROX testcase. For the structural sizing and weight estimation of aileron attachment ribs, the model framework follows an equivalent 3-layered architecture, comprising of a module for loading computation, a multiple output IT2 fuzzy logic-based sizing module for the computation of the cross sectional areas and second moment of area for the individual beam components and an IT2 fuzzy logic-based weight module combining information on both global and local geometry with rib function for the computation of weight estimates.

As for the spoiler attachment rib case, both the IT2 FIS based sizing and weight module have been initiated with the NEFPROX-derived network structure. The initial type-1 Mamdani FIS structure optimised using NEFPROX was converted into an interval type-2 fuzzy logic system and consequently used as a starting point for the network design and optimisation process. The number and size of antecedent and consequents as well as number and structure of the individual rules was therefore maintained the same as in the NEFPROX-derived FIS.

7.4.1 Sizing module performance

The design and optimisation of the IT2 FIS for sizing derivation of the beam components of aileron attachment ribs followed the same iterative process as for the spoiler attachment rib example. The network was initialised using the Mamdani fuzzy inference structure derived by NEFPROX in Chapter 6, where input and output variables were defined by 5 Gaussian fuzzy partitions and the overall FIS structure was built on a system of 11 rules. This fuzzy system structure was then translated into an IT2 FIS, with the conversion of individual variable definition from Gaussian type-1 into type-2 fuzzy partitions based on Gaussian primary membership functions with uncertain mean.
The same database of reference used for the development of the NEFPROX framework was employed in the testing and training of the IT2 sizing module. Data for a total of 63 beam components of aileron attachment ribs was collected, of which 44 were used for training and 19 for module testing.

Table 7.3: Comparison of results for the architecture of sizing and weight fuzzy inference systems of type-1 built with NEFPROX and interval type-2 for aileron attachment ribs.

<table>
<thead>
<tr>
<th>AILERON</th>
<th>NEFPROX</th>
<th>IT2 FIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.088</td>
<td>0.162</td>
</tr>
<tr>
<td>A</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>MFS*</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.081</td>
<td>0.188</td>
</tr>
<tr>
<td>I</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>MFS*</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.073</td>
<td>0.118</td>
</tr>
<tr>
<td>W</td>
<td>34</td>
<td>5</td>
</tr>
<tr>
<td>MFS*</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

In a similar way as for the case of spoiler attachment ribs, the combination of backpropagation for model design and training and SVD-QR for rule reduction was able to generate a substantially more concise and compact fuzzy system, with the rulebase reduced from 11 to 5 individual rules compared to the NEFPROX-based FIS and a description of both input and output variables using only 5 fuzzy partitions (Table 7.3). Even in this case, however, the simplification of the FIS structure translates into a reduction in the quality of the approximation compared to the equivalent type-1 FIS. This is apparent by analysing the RMSE of the estimation, which has deteriorated from 0.088 in the case of the NEFPROX-derived FIS to 0.162 for the estimation of beam cross sectional areas from the 0.081 to 0.188 for second moments of area.

The overall approximation performance shows a mean absolute error of prediction of approximately 16.8 percent and 13.4 percent in the estimation of beam cross section area and second moment of area respectively, which still represent suitable generalisation capabilities displayed by the model are still suitable for the type of analysis conducted at preliminary design stages (Table 7.4). As for the NEFPROX-based ex-
ample, the results show no clear tendency of the model to either over or under estimate the value of cross sectional areas; on the contrary, in this case, the IT2 FIS appears to provide an approximation which exceeds the expected value of second moment of area by approximately 2 percent on average, up to a maximum of 22 percent in certain cases (Figure 7.7).

Table 7.4: Performance assessment of the type-2 fuzzy logic framework applied to the sizing and weight estimation of aileron attachment ribs.

<table>
<thead>
<tr>
<th></th>
<th>A TYPE-2</th>
<th>I TYPE-2</th>
<th>W TYPE-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Inputs</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Training</td>
<td>44</td>
<td>44</td>
<td>46</td>
</tr>
<tr>
<td>Testing</td>
<td>19</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.162</td>
<td>0.188</td>
<td>0.118</td>
</tr>
<tr>
<td>MPE</td>
<td>-0.72</td>
<td>2.23</td>
<td>-1.08</td>
</tr>
<tr>
<td>MAPE</td>
<td>16.83</td>
<td>13.49</td>
<td>11.31</td>
</tr>
</tbody>
</table>

Figure 7.7: Performance of the type-2 fuzzy logic sizing module for aileron attachment ribs on testing database for A and I.
7.4.2 Weight module performance

The derivation of the IT2 fuzzy weight module for aileron attachment ribs was conducted in a similar manner to the development of the sizing module. The initial model structure was firstly derived from type-1 FIS optimised by NEFPROX, where the 7 input variables as well as the final output were defined by 7 Gaussian fuzzy partitions and the overall FIS structure was based on a system of 34 rules. The fuzzy partitions were then converted into interval type-2 with Gaussian primary membership function with uncertain mean \([m_1, m_2]\), following the same procedure as for the sizing module. The system was then trained and validated through the same reference database used for both ANFIS and NEFPROX FIS optimisation, with a total of 66 examples of aileron support ribs, 46 of which were selected for training and 20 for performance assessment.

As in the case of the sizing module, the iterative design process was able to produce a substantially simpler and more transparent network structure. The initial 34 rules used within the type-1 system derived through NEFPROX were reduced to a total of 5, with input and output variables being described by 5 fuzzy partitions each instead of 7 (Table 7.3). The consequence of network simplification was, even in this case, a reduction in model approximation performance. The RMSE has increased from 0.073 in the case of the NEFPROX FIS to 0.118, in parallel with an decrease in approximation performance of just over a 5 percent with respect to the mean absolute percentage error between the two models (Table 7.4).

A closer look at the results from the validation process shows that the IT2 model presents a clear tendency of underestimating the structural weight of the aileron attachment ribs belonging to the Design C category, whilst no clear approximation trend appears within the weight analysis of Designs A and B (Figure 7.8). This highlights the needs to further improve the definition of the design when it comes to structural solutions, such as the ribs belonging to Design C, which have less distinguishing features or components in the situation when different design alternatives are evaluated within the same framework. This will ensure that the model is consistent and the same level of generalisation is maintained across the individual design variations.
The model is further validated by the results of the variable selection process. As in the case of both the ANFIS and NEFPROX-derived weight models, the best performing FIS is achieved by describing the structure through the full set of variables (Figure 7.9), with noticeable decrease in effectiveness in parallel with each individual variable elimination. The removal pattern is maintained across all the fuzzy weight models, with hinge line datum $L$ and top beam local geometry having the greatest influence on the final quality of the approximation. The deterioration in performance shown by the IT2 FIS is also comparable in terms of magnitude to that experienced by the NEFPROX model at each individual stage of the removal process which helps corroborating both models.
7.4.3 Interval type-2 function approximation and decision boundaries

As for the modelling example of spoiler attachment ribs, the results so far have highlighted that the approximation provided by the interval type-2 sizing and weight models is of lower quality compared to that provided by the fuzzy systems designed by both ANFIS and NEFPROX. It is to be noted, however, that the deterioration in modelling performance is minimal if compared to the considerable consolidation of both the final network structure and rulebase. The system, is therefore, still able to provide great generalisation in the case of unseen structural examples even on the basis of a much more concise fuzzy variable representation and interpretable system of rules. This is of primary importance from the point of view of providing the designer with a ready to use and intuitive system to aid the design and decision making within the preliminary phases of concept definition.

In addition to this, the most important achievement in this case is still the derivation of reliable uncertainty information when it comes to both the definition of the variable and the computation of the final output. Overall, the results corroborate
Chapter 7

the findings obtained via both the ANFIS and the NEFPROX-derived fuzzy models. The IT2 FIS weight model was still able to represent the nonlinearities present in the relationships between the different variables of interest and the overall trends learnt from the database of reference mirror those derived in previous stages.

A more in depth analysis of the results reveals how the weight penalty coming from an increase in the height of the structure and, therefore, from its location along the OFTE, is higher than that resulting from a change in hinge line location. Once again, the general shape of the dependencies between the weight of the aileron attachment rib and its global geometrical definition is maintained across the three different designs being analysed, as shown by figures 7.10 (a), (b) and (c). It is interesting to note the different levels of confidence within the approximation for the three designs. The largest confidence region appears in Design A (Figure 7.10 (a)) highlighting a higher level of uncertainties within this estimate, as opposed to the Design C which displays more compact confidence boundaries (Figure 7.10 (c)). The justification for this lays in the way the designs are defined. Since all three types of ribs are idealised as a combination of individual beam components, the discrete uncertainties associated with each of them are then cumulated into the final assembly. For this reason, confidence within the weight estimate for Design C which is designed around a single beam structure, will be higher compared to that for a structure designed around multiple beam components.

By looking at the impact of local geometry on the overall structural weight of the rib, the top beam section is still the source of the highest weight penalties, thus cross validating both the results from the variable removal process and the previous fuzzy models (Figure 7.11). Even in this case, \( A_{\text{TOP}} \) appears to impact the weight of both Design A and B by a comparable magnitude, which is also in agreement with the trends derived by the NEFPROX model. In particular in the case of Design A, the profile of graph support the findings from the NEFORX-derived model with regards to the impact of the bottom beam geometry on the final weight of the rib. As opposed to the results from the TSK FIS built with ANFIS, which established an inverse proportionality between the cross sectional area of the bottom beam and the structural weight of Design A ribs, both type-1 and type-2 Mamdani FIS derived a more realistic relationship profile between the two variables. In addition to this, for both Design A
and B. NEFPROX and IT2 FIS highlight a plateau within the trend located at higher values of $A_{TOP}$ and $A_{BOT}$ which indicating that a noticeable proportion of the weight of larger ribs can be related to the addition of additional vertical beams. With regards to the reliability of the estimates, the model highlights substantial uncertainties in both the designs solutions in particular in the region of larger $A_{TOP}$, which suggests a lower confidence level for the weight estimate of rib structures located toward the inboard side of the trailing edge, which are characterised by considerably larger top beam sections due to the larger aerodynamic loads to be sustained.

As opposed to the results from NEFPROX, however, the additional weight penalties incurred by larger aileron attachment ribs are shared in equal proportion by both front and back vertical beam components in the case of Design A. Figure 7.12 highlights a comparable gradient in the profile of both $A_{VERT}$ and $A_{VERT_1}$ as well as similar weight contribution from both beam sections even in the case of larger rib structures.
Nonetheless, the results from IT2 and NEFPROX validate each other with regards to the profile of the relationship with the analysis of local geometry. In both cases, the Mamdani fuzzy inference systems have been able to identify substantial nonlinearities associated with the inclusion of a front beam section in the final design. This is also combined with a marked degree of uncertainties in the estimation, as highlighted by the large confidence region displayed figure 7.12 (b). The analysis of these two factors reiterates the need to improve the design definition of the vertical beam sections with the inclusions of parameters related to specific features which contribute with some of the higher weight penalties.

In the case of Design C, where the structure is designed around a single beam
component, the results are consistent with both type-1 TSK and Mamdani fuzzy models. Hinge line datum $L$ still accounts for the majority of the weight penalties within the design, although in a lower proportion than that derived by ANFIS and NEFROX compared to the contributions from $A_{TOP}$ (Figure 7.13). In particular, the surface profile derived by the IT2 FIS closely match that of the NEFROX-derived FIS. As in the case of the type-1 Mamdani model, IT2 FIS displays much more realistic dependencies between the variables, especially at higher values of $L$ and $A_{TOP}$, compared to the TSK model which tends to misinterpret correlations between variables especially in the boundary regions of the design domain. As with previous cases, the uncertainties are focused at higher values of $L$ and, in particular, at larger cross sectional areas. This, combined with previous findings, highlights the importance of both improving the computational definition of the designs located further inboard as well as taking into account the variability in weight of these specific ribs when approaching individual design decisions.

### 7.5 The uncertain rulebase

The relationships between the variables of interest which have been derived by type 1 and type 2 fuzzy logic systems appear to be comparable when it comes to both magnitude and profile. Results have also shown that, although the modelling accuracy
has suffered a minor deterioration in the translation from type 1 to type 2 systems, the structure and definition of the FLS has vastly improved. The number of governing rules has noticeably decreased for both the spoiler and aileron attachment rib examples, with up to an 85 percent reduction within the rulebase in particular within the weight estimation modules.

Similarly, the definition of the variables of interest has become more streamlined, concise and, as a result, more interpretable. Figure 7.14 highlights the definition of the loading input variables and the two outputs within the sizing module for spoiler attachment ribs. In each case, the system was able to provide both a uniform and complete definition of the design space. The entire domain of interest is fully covered by the 4 membership functions which are both clearly distinguishable and complementary to each other, without any substantial duplication of information. This clearly reiterates the overall advantage of Mamdani over TSK systems in the derivation of a noticeably more intuitive and readily applicable rulebase.

In addition to this, the fuzzy variable definition obtained via the type-2 FLS includes both a visual and quantitative definition of the uncertainties within the inputs as well as a definition of how these translate into possible variability across the desired outputs. In the case of the sizing module for spoiler attachment ribs, the footprints of uncertainty (FOUs) appear analogous in terms of shape and dimensions across both input and output variables. The areas enclosed by upper and lower membership functions for each partition is reasonably large, indicating a noticeable variability within the means of the MFs. In particular, it is evident from the size of the FOUs that the highest level of uncertainty in the inputs lies in the definition of the applied axial force, whose footprints of uncertainty are much larger, both in terms of variability of means and standard deviation. The effect of these uncertainties is propagated through the network and its effect can be visually assessed on the outputs. In the case of both cross sectional area $A$ and second moment of area $I$, the region within the domain of interest with the most variability is across that of larger beams. The FOUs describing the higher values of both variables are characterised by a much larger variability in the mean as well as a larger standard deviation, as opposed to the lower end of the scale. This represents a clear warning for the designer to account for a different level
of uncertainties across the spectrum of attachment ribs.

Figure 7.14: Type-2 fuzzy partitions used to define bending moment (a), axial force (b), cross-sectional area (c) and second moment of area (d) within the spoiler sizing module.

The large variability within the inputs and, in particular, in the applied axial force, is also noticeable when it comes to the quantitative definition of the uncertainties within the variables of interest. By looking at the difference between the means in the four partitions used to define $F_x$, Table 7.5 highlights a significant variation of up to 2 nondimensional units among the values of $m_1$ and $m_2$ used to describe the same partition. This translates into an even more substantial variability within the two outputs, reaching values of over 4.50 in specific partitions used for the definition of beam cross-sectional area and second moment of area. The larger variations in the means within the outputs are due to the cumulative uncertainties in the input parameters. Although the means suffer quite noticeably from the uncertainties permeating the problem at hand, the partitions used in the definition of the variables show an acceptable level of
spread. The standard deviation of the individual partitions oscillates around the 0.5 level, with higher values of up to 0.85 in the case of some partitions used within the definition of bending moment $M$.

Table 7.5: Uncertainty characteristics for the type-2 fuzzy partitions of input and output variables within the spoiler sizing module.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>-1.28</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>-2.35</td>
<td>-0.32</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>-1.32</td>
<td>0.68</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>1.13</td>
<td>2.66</td>
<td>0.32</td>
</tr>
<tr>
<td>$M$</td>
<td>-0.67</td>
<td>0.12</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>2.03</td>
<td>3.21</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>2.76</td>
<td>3.82</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>1.14</td>
<td>1.65</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_{ULT}$</td>
<td>-0.18</td>
<td>1.83</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>-2.20</td>
<td>-0.27</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>-0.19</td>
<td>1.83</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>-0.07</td>
<td>1.83</td>
<td>0.35</td>
</tr>
<tr>
<td>$A$</td>
<td>-0.23</td>
<td>1.81</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>-2.26</td>
<td>-0.30</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>-0.22</td>
<td>1.81</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>-0.10</td>
<td>1.81</td>
<td>0.35</td>
</tr>
<tr>
<td>$I$</td>
<td>-2.03</td>
<td>-2.63</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>-0.77</td>
<td>2.71</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>-2.73</td>
<td>1.94</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>-1.99</td>
<td>2.30</td>
<td>0.46</td>
</tr>
</tbody>
</table>

In the case of variable definition within the weight estimation module, the picture is more varied. In a similar way as for the sizing module, the variable partitioning has been much more streamlined with a considerable reduction of fuzzy partitions from 15 to 4 per variable. From a general perspective this considerably enhances the overall interpretability of the system, however in the case of hinge line datum $L$ (Figure 7.15(a)) and top beam cross sectional area $ATOP$ (Figure 7.15(c)) this has resulted in a slight loss of information. In the case of hinge line datum, in particular, it is possible to
note that the extreme regions of the domain of interest are not fully defined by the fuzzy partitions, in a similar way as higher values of cross sectional areas for top beam sections. The variable definition, nonetheless, appears uniform and comprehensive across the remaining inputs as well as for the final output variable.

### Table 7.6: Uncertainty characteristics for the type-2 fuzzy partitions of input and output variables within the spoiler weight module.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>-0.25</td>
<td>0.62</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>-0.45</td>
<td>1.25</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>0.72</td>
<td>2.35</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>-0.34</td>
<td>1.46</td>
<td>0.51</td>
</tr>
<tr>
<td>( h )</td>
<td>1.25</td>
<td>1.37</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>1.43</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>-0.26</td>
<td>-0.20</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>-1.08</td>
<td>-0.04</td>
<td>0.56</td>
</tr>
<tr>
<td>( A_{TOP} )</td>
<td>-1.60</td>
<td>-0.65</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>-1.87</td>
<td>-0.81</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.61</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>-1.62</td>
<td>0.58</td>
<td>0.32</td>
</tr>
<tr>
<td>( A_{BOT} )</td>
<td>-1.86</td>
<td>-1.01</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>-1.50</td>
<td>-0.19</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>1.28</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>-2.02</td>
<td>0.15</td>
<td>0.45</td>
</tr>
<tr>
<td>( A_{VERT} )</td>
<td>-0.95</td>
<td>0.26</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>-0.95</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>-0.98</td>
<td>1.35</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>-0.96</td>
<td>1.23</td>
<td>0.34</td>
</tr>
<tr>
<td>( r_{type} )</td>
<td>0.40</td>
<td>2.15</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.89</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>-0.08</td>
<td>1.43</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.13</td>
<td>0.68</td>
<td>0.60</td>
</tr>
<tr>
<td>( W )</td>
<td>-0.54</td>
<td>2.62</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>-2.74</td>
<td>0.56</td>
<td>0.49</td>
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<td></td>
<td>2.80</td>
<td>0.36</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>-3.52</td>
<td>-1.51</td>
<td>0.57</td>
</tr>
</tbody>
</table>

In terms of definition of uncertainty, the system identifies smaller spoiler attachment ribs as most affected. This is clearly visible especially in the fuzzy definition of the global geometrical parameters \( L \) and \( h \) (Figure 7.15 (a) and (c)) where the definition of the lower values within the domain is characterised by considerably larger FOUs and,
as a consequence, higher variability. It is important to note that these have also been identified as the focal variables across the design of the fuzzy logic systems explored in this research. This highlights the importance from a designer’s perspective to include the consideration of uncertainties and their effect within the design and weight estimation process otherwise erroneous and misleading results will be produced.

As opposed to the results from the sizing module, both the visual and quantitative assessment of the weight estimation module show a lower level of uncertainty across the variables of interest. Table 7.6 highlights a less prominent variation between the means of the fuzzy partitions than those characterising the sizing module. Although $L$ and $h$ are characterised by larger FOUs at the lower values within the domain and, in turn, by a higher level of variability compared to the other variables in the problem, the difference between $m_1$ and $m_2$ across their partitions never reaches values over 1.7, as across the fuzzy definition of the other input variables in the system. The cumulative impact of the uncertainty within the mean that propagates to the final weight estimate is also lower than that experienced in the sizing module, with a maximum of 3.30. It is possible to identify an overall lesser level of uncertainty in the weight estimation process compared to the sizing one also when examining the characteristic standard deviation across the different variables. In the weight estimation module in particular, $\sigma$ never exceeds 0.65 across the inputs, with a maximum value of 0.57 in the definition of the output partitions.

As with the spoiler attachment rib case, the use of type-2 FLS as a basis for the network allowed a much more concise representation of the variables of interests within the sizing module for aileron attachment ribs. All input and output variables were successfully defined with 4 fuzzy partitions providing a full and exhaustive coverage of the design space (Figure 7.16). As highlighted in figure 7.16 (c) and (d), in the case of the output variables $A$ and $I$ in particular, the system was able to structure the partitions within the domain with a clear and distinguishable configuration. On the contrary, the definition of the applied axial force $F_x$ appears to some extent more imprecise, with partitions which considerably overlap one another.

This, combined with the overall shape of the FOUs for $F_x$, highlights a higher level of uncertainty in the definition and description of the variable itself. It is par-
particularly evident that the footprints of uncertainty for the partitions in this case are considerably large compared to those derived for the other variables in the module. The large areas covered by them, caused by the significant variation in the means, combined with the larger standard deviations is a distinct indication of the fact that this particular variable is both subjected to considerable variability and represents the major contributor to the uncertainties within the system itself.

Table 7.7: Uncertainty characteristics for the type-2 fuzzy partitions of input and output variables within the aileron sizing module.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>-0.96</td>
<td>1.89</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>-2.27</td>
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</tr>
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<td>0.67</td>
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<td>$M$</td>
<td>-4.55</td>
<td>-3.15</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>-1.28</td>
<td>0.47</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>2.57</td>
<td>3.47</td>
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</tr>
<tr>
<td></td>
<td>5.39</td>
<td>6.74</td>
<td>0.52</td>
</tr>
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<td>0.52</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>-1.74</td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>2.35</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>2.85</td>
<td>0.58</td>
</tr>
<tr>
<td>$\sigma_{ULT}$</td>
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<td>0.01</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>-1.11</td>
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<td>0.41</td>
</tr>
<tr>
<td></td>
<td>0.39</td>
<td>2.16</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>2.86</td>
<td>0.59</td>
</tr>
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As opposed to the sizing module for spoiler attachment ribs, the uncertainties within the example of aileron attachment ribs appear smaller from both a visual and quantitative perspective. Table 7.7 readily highlights overall lower values of $\sigma$ across both inputs and outputs, with a noticeably lower upper threshold of 0.69 even in the
case of lesser defined inputs. The large variability within the inputs and, in particular, in the applied axial force, is also noticeable when it comes to the quantitative definition of the uncertainties within the variables of interest. This is mirrored by a considerably lower variation within the means of the individual partitions, which are characterised by a maximum value of 3.05 across the inputs and 3.66 within the fuzzy characterisation of the outputs. This also strongly highlights a much lower level of uncertainty propagation in the aileron sizing analysis compared to the spoiler one, which can be attributed to both the type of analysis and how well it fits the structural example, as well as to the quality of data available for the analysis in the two cases.

The type-2 FLS proved to be a particularly suitable tool for the derivation of weight estimation architecture which is both simple and reliable. In the specific case of aileron attachment ribs, the network structure derived necessitated only 5 membership functions for the full definition of all input and output variables, compared to the 18 and 7 obtained via the type-1 FLS derived through ANFIS and NEFPROX respectively (Figure 7.17). This greatly contributes to the definition of a much more interpretable system, especially when combined with partitions which are, at the same time, descriptive of the overall design space and defining discernible sections within it. As opposed to the spoiler attachment rib example, however, in this case the variables illustrating a less intuitive fuzzy definition appear to be the cross sectional areas of back (AVERTb) and front (AVERTf) beam sections (Figures 7.17(e) and (f)). In line with the results from previous models, these are the variables which are both the least influential on the approximation in terms of accuracy and those with the highest level of uncertainty. This is understandable since only a limited number of the structural examples within the reference data set present these beam components in their design and, as a consequence, the system is faced with noticeably higher need to interpolate when deriving specific modelling rules for these instances.

This translates into a higher level of uncertainties especially within the definition of the front vertical beam section AVERTf. The footprints of uncertainty are much larger compared to those derived for the other variables within the systems and, in particular, the standard deviation for the partitions is particularly substantial. As previously derived within the variable selection process, however, AVERTf is the least
influential variable on the accuracy of the final approximation provided by the system. It is clearly visible in figure 7.17(h) that the large uncertainties within $A_{VERT}$ only marginally affect the final output. The fuzzy definition of the output $W$ appears particularly clear with distinguishable partitions, all of them characterised with realistic levels of variability within the means and standard deviation.

This transpires also in the quantitative definition of the variables within the weight estimation problem (Table 7.8). $A_{VERT}$ is characterised by the highest variation between $m_1$ and $m_2$ of 2.75; it is closely followed, however, by $L$ with a value of 2.42. This reiterates the need of including uncertainty analysis from the beginning of the design process. As shown in previous results, $L$ is the variable with the greatest impact on the final accuracy of the estimation, and neglecting such a variability in the parameter will results in strongly misleading and erroneous results. The results also restate how the uncertainties within the sizing problem are considerably higher than those within weight estimation. This is confirmed by both the lower variability in the means of the fuzzy partitions as well as by the smaller values of their standard deviation. Apart from the fuzzy definition of $L$ which displays the higher end of the spectrum of $\sigma$, the remaining variables are characterised by standard deviations which do not exceed 0.55, which much lower values in the definition of the output $W$.

7.6 Summary

This chapter investigated the issue of combining transparency and interpretability within a fuzzy system with a comprehensive visualisation and accounting of the uncertainties in the problem at hand. In particular, the analysis focused on the application of interval type-2 Mamdani fuzzy inference systems as an aid to include a more rigorous assessment of the uncertainties permeating both sizing and weight estimation of aircraft structural components. The chapter provided a theoretical overview and definition of the mathematical foundations of interval type-2 fuzzy systems, a critical comparison with type-1 in terms of structure, analysis and capabilities as well as how these characteristics could prove extremely beneficial when dealing with weight estimation at the preliminary design stages of aircraft structural components.
Moreover, additional tools were introduced to further optimise the performance of IT2 FIS in this type of problem. A combination of iterative design, network optimisation through an SVD-QR routine and variable selection process was adopted throughout the development of both sizing and weight estimation modules for spoiler and aileron attachment ribs, in order to both enhance their performance and increase their final interpretability.

Thanks to this process, the final FIS obtained were able to provide a good combination of modelling accuracy and transparency in the final rulebase. The accuracy of the estimation within the results is marginally lower than the approximation capabilities displayed by NEFPROX in particular. A minor increase in the final estimation error is counteracted by a much simpler network structure and rulebase. In both structural examples, sizing and weight estimation modules achieved a dramatic reduction in the overall size of the rulebase and in the number of fuzzy partitions necessary to describe the individual variables. In turn, this resulted in a much more streamlined network structure and in an overall more interpretable rulebase which could be easily integrated within the preliminary design of the structural component. In addition to this, the results were much more comprehensive in the information they were able to translate about the design of the various components. Visually, the relationships between the variables provided a cross-validation with results obtained with type-1 FIS developed using both ANFIS and NEFPROX. The trends computed by the systems also showed an additional level of insight in the understanding of the problem itself through the derivation of confidence regions and boundaries, which provide a first stage assessment of the variability in the final solution. Moreover, the final interval type-2 FIS provide means of assessing the quality of the solution and the uncertainties within it both quantitatively and qualitatively within the rulebase derived. The representation of the fuzzy partitions for the individual variables in the systems through their footprint of uncertainties represents an exceptionally intuitive way of visualising the resolution of the model in terms of variable definition, coverage of the design space and management of the uncertainties in the problem.

The next chapter will focus on the definition of a formal framework for the design and weight estimation of aircraft structures at early project phases. In particular,
the final methodology will aim at combining the various fuzzy techniques and tools described so far in order to fully exploit their capabilities, with the aim of establishing a structured approach for the application of fuzzy methods to the problem of weight estimation.
Figure 7.15(a)

Figure 7.15(b)

Figure 7.15(c)
FIGURE 7.15: Type-2 fuzzy partitions used to define input and output variables within the spoiler weight estimation module.

(a) (b) 

FIGURE 7.16: Type-2 fuzzy partitions used to define bending moment (a), axial force (b), cross sectional area (c) and second moment of area (d) within the aileron sizing module.
Chapter 7

Figure 7.17(d)

Figure 7.17(e)

Figure 7.17(f)
FIGURE 7.17: Type-2 fuzzy partitions used to define input and output variables within the aileron weight estimation module.
TABLE 7.8: Uncertainty characteristics for the type-2 fuzzy partitions of input and output variables within the aileron weight module.

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Chapter 8

Integration of Knowledge and Uncertainty in the Design Process

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8.1 Introduction

The design of a weight estimation model for structural components is usually driven by two factors: accuracy and the ability to incorporate the physics behind the design of the structure within the model itself, in a way that is both interpretable and representative of the real design process. In addition to this, it is desirable to have a model which is flexible and can adapt to different levels of quality in the input as well as output information required from the approximation.

The benefit from this is especially crucial at preliminary design stages, where the information about the design of the component is continually evolving. The quality of the input data is always refined within this phase of the design, with new information being added to the model or improved, from the point of view of uncertainty measures and variability of the data.

Previous chapters have described three different adaptive fuzzy systems, namely the Takagi-Sugeno-Kang derived through ANFIS, the Mamdani-type FIS obtained through NEFPROX and Interval Type-2 FIS, as well as their potential applications to the field of weight estimation. In particular, the sample problems presented have highlighted benefits and limitations of each of the proposed approaches, when used to produce approximations for weight estimation applications. With the ability of representing the problems through a combination of fuzzy partitions and a comprehensive rulebase, all the three methods were able to provide extremely satisfactory performance, both in terms of problem formalisation and interpretability as well as in the accuracy of the approximation. The analysis of the results, however, highlighted noticeable differences in the way the three methodologies represent the knowledge derived in the adaptive process, as well as in the details of the problem which can be extrapolated from the final model.

This chapter aims at defining an overall framework for the application of fuzzy methodologies in structural weight estimation problems. The chapter will approach the problem from two different angles. Initially, the three fuzzy methodologies explored through this thesis will be analysed in terms of their individual contributions to the derivation of both weight estimates and a comprehensive rulebase. The problem of
"level of granularity" in the definition of a weight model will also be explored in relation to the assessment of the different FIS and their performance. In parallel, this will lead to both the definition of a framework as well as general guidelines for the implementation of a fuzzy model in weight estimation problems. This will focus on combining the potential of the three methodologies together and using them to counteract their specific pitfalls, to obtain a versatile, coherent and self-sufficient method of approaching weight estimation at the preliminary phase of the design of aircraft structures.

8.2 Fuzzy systems for weight estimation applications

Fuzzy systems have, so far, proven to be very useful modelling tools for various approximation applications. In particular, this research has been trying to investigate their capabilities and potential in the field of structural weight estimation. Results, however, have brought light to some specific issues in the application of these modelling tools in this particular field.

More specifically, when designing a weight estimation approach around fuzzy methodologies, it is vital to assess the problem by focussing on the following:

1. The desired level of granularity to be achieved in the estimate;
2. The level of transparency and interpretability of the final rulebase;
3. The integration and preservation of knowledge of the design;
4. The required level of flexibility and adaptability to be achieved by the final framework.

8.2.1 Granularity and interpretability in adaptive fuzzy systems

The research presented in this thesis has highlighted the potential of adopting different fuzzy approaches to the problem of estimating the structural weight of aircraft components early in the design process. Fuzzy approaches have proven very suitable overall. In terms of approximation capabilities, the modelling accuracy shown by the
fuzzy systems used was very satisfactory, with some of the approaches even reaching estimates of just over 5 percent of the "as built" value.

The additional benefit is connected to the choice of fuzzy logic as a modelling methodology. All of the fuzzy systems employed demonstrated the capability of decomposing the design space into fuzzy patches in a way that not only contributes to the overall accuracy of the model, but which also strengthened its transparency and interpretability. This allowed the definition of individual sets of rules which were able to characterise both the problem and the design of the sample structural components with substantial depth.

The analysis of the results from the three fuzzy methodologies adopted, however, revealed significant differences between them, both from a performance perspective as well as from the point of view of their individual capabilities and strengths. By looking at the Takagi-Sugeno-Kang, the Mamdani and Interval Type-2 FIS in parallel, it is possible to highlight a noticeable difference in the way they are able to solve the problem at hand. In other words, the three fuzzy approaches differ among themselves in the "level of granularity" of the approximations they are able to produce.

Granularity, in this particular case, is associated with the both the level of detail of the approximation produced by the model as well as with the information about the design itself that the final solution is able to embody. Figure 8.1 highlights how TSK, Mamdani and IT2 FIS differ in terms of their detailed exemplification of the problem. Takagi-Sugeno-Kang fuzzy systems, although having demonstrated the ability of producing highly representative results across the case studies in this research, have also highlighted certain limitations in the quality of the rulebase derived from the data of reference. From a fuzzy system benchmark perspective, the case studies have shown that TSK systems are undeniably successful in providing great quality approximations and deriving an effective network structure for a preliminary analysis of the design space of interest. The analysis produced by this system, however, would only be of a baseline nature, since the rulebase extracted by the TSK FIS is still limited in its ability to capture representative relationship between the variables at the boundaries of the design domain. In addition to this, the transparency of the final model is very limited, from both rulebase and network perspective. This is due, one one
hand, to the derivation of input membership functions, which tend to be noticeably irregular in both shape and overall distribution across the domain of reference. In addition to this, the definition of the outputs as singletons only increases the overall lack of interpretability of the final model, in terms of both structure and rulebase.

The next level of granularity is embodied by Mamdani-type fuzzy models. In this case, results have established a higher capability of generalisation within FIS derived through NEFPROX, whose performance has proven higher in terms of both accuracy and quality of rulebase compared to TSK FIS. NEFPROX allows Mamdani systems to achieve a more streamlined network architecture, which is able to outperform TSK FIS also from the point of view of the analysis of the design domain. Variable definition is greatly enhanced, with membership functions that are able to fully define the design space in a homogeneous and transparent way. This, combined with a higher resolution in the definition of the outputs, contributes to a model which is overall more interpretable and provides a higher fidelity approximation.

The highest granularity level, however, is achieved through interval type-2 FIS.
From the point of view of accuracy, the performance is only marginally lower compared to the other two FIS types. Its overall effectiveness for weight estimation, however, is much greater. The final network and model structures produced by the fuzzy system are considerably simpler, with a much lower number of rules and network parameters. This also results in a considerably improved knowledge base, where the individual variables are fully defined by membership functions which are highly interpretable and which complement each other exhaustively, for an effective definition of the design domain. In addition to this, IT2 FIS are also able to achieve a superior level of analysis by the evaluation of the uncertainties across the variables within the problem, their combined effect and their propagation through the network, all the way down to output level uncertainty assessment.

8.2.2 The problem of preservation of information

When it comes to aircraft design, it is important to acknowledge the value of the experience of the designers and their knowledge of both the structures and their specific behaviour. Due to the multidisciplinary nature and the overall scale of the design, however, it is very common to rely on computational and modelling tools at the expense of the designer's insight. The major problems within the aircraft design process in the present day are both the ability to effectively combine the potential of the computational models with the knowledge embodied by the design team. In addition to this, there are also numerous challenges in the sequential integration of the results from the mathematical approximations and their experimental assessment back into the collective mindset and, ultimately, in the design process.

In terms of fuzzy logic, adaptive data-driven FIS are widely spread due to their ability to "discover" knowledge within the data that might have been precedently unrecognised and unaccounted for. The additional benefit of fuzzy systems is the possibility of combining the knowledge gathered through the data with that coming from experts. The subject of knowledge integration and preservation is a topic of continuos analysis from the research community (Cornelissen et al., 2003; Larichev, 2002; Pedrycz and Vukovich, 2002). The fusion of separate knowledge bases into a single, transparent and interpretable set of rules can be a labourious process subjected
by strict constraints.

The main issue lays in the fact that both sets of knowledge, expert-based and data-driven, are not sufficient on their own to achieve a comprehensive view of the problem at hand. In order to create an exhaustive fuzzy model, it is vital to define a shared input domain between the expert knowledge and the data space and analyse the compatibility of the two according to:

1. Granularity;
2. Range;
3. Interpretation of fuzzy partitions.

![Diagram showing the process of knowledge extraction and amalgamation for fuzzy systems (Guillaume and Magdalena, 2006).](image)

Figure 8.2 defines the overall process for the concurrent rule extraction and integration from both data and experts (Guillaume and Magdalena, 2006). The methodology itself can be analysed from the point of view of both fuzzy partitions and fuzzy rules. The initial part of the process focuses on the definition of a shared fuzzy parti-
tioning system between experts and data. The two types of information are different in their nature. Expert partitions are qualitative and their definition is somewhat limited to the overall number used in the definition of each individual variable considered, their range as well as linguistic definition. On the other end of the spectrum, the results of the data-driven analysis will help in expanding the quantitative definition of the individual partitions.

The process is driven by six tasks:

1. Expert definition of fuzzy partitions;
2. Derivation of fuzzy partitions from data;
3. Integration of partitions;
4. Rule elicitation from experts;
5. Derivation of rules from data;
6. Integration of the two sets of rulebase.

Only once the common domain has been established and rules are extracted from both expert and the dataset, it is possible to compare the two knowledge bases, since they are both rooted in the same common fuzzy infrastructure. The integration process itself is then conducted on the basis of the number of fuzzy partitions, their complementarity and interpretability, the coverage of the domain of interest and their overlapping (Guillaume and Magdalena, 2006). Any discrepancies between data and expert-derived rules or fuzzy partitions should be corrected by giving priority to expert knowledge, which is the most reliable. Ultimately, the conflict between the two knowledge sets will help expand and update expert knowledge in a controlled and verifiable manner.

8.3 Towards a general fuzzy logic-based framework for weight estimation

The design and weight estimation of aircraft structures is an iterative process, which involves many disciplines concurrently over large timescales. In addition to this,
the level of detail required at different stages of the process varies greatly. Each design stage on its own is a mirror image of the overall process for the component under study, but on a smaller scale. For instance, the preliminary design stage of an aircraft structural part or subassembly will involve the design of the component itself initially only from an empirical to semi-analytical perspective, with a level of detail relative only to the general layout definition of the component itself. This will then mature into a more comprehensive physics-based analysis of the structure through a series of process iterations where increasingly more detail will be added to the definition of the part producing, in turn, a higher fidelity definition and assessment of the design.

The key within this interpretation of the evolution of the design of aircraft structures is the idea of flexibility. When designing a weight estimation model for aircraft structural components, it is important to keep this image in mind and try and define a framework which can accommodate both the iterative nature of the process as well as different levels of information quality needed, within both input and output definition.

The choice of fuzzy logic as the foundation of the modelling process was taken based on the ability of FIS to be modular and modifiable, to help with the handling of a variety of modelling scenarios as well as different levels of granularity in the approximation. The fuzzy tools explored have all shown different capabilities and potential. The next step is the creation of a framework, which can fully exploit and benefit from these different resources.

8.3.1 Integration in the design process

Mass properties teams in charge of weight estimation of aircraft structure throughout the design process face four major problems:

1. Lack of data/information;
2. Delays in the knowledge sharing process across the different departments involved in the design;
3. Strict timelines for deliverables;
4. Model validation.
Most of the time mass properties teams have to produce weight estimates with missing data or lacking the full knowledge behind the design of the component. At the same time, usually they are not equipped with numerical models or tools which can derive the required data on a smaller scale. Coarser models, in fact, could help in producing weight approximations with a lower level of granularity to be used as a basis for higher fidelity analysis at later stages, or to act as validation tools when the necessary data is received. In addition to providing weight estimation capabilities, modelling frameworks for the development of weight approximations at preliminary design stages need to allow for the integration of structural design principles within their architecture.

The flowchart in figure 8.3 highlights the layout of a general framework for the weight estimation of structural components from first principles, based on the combined use of the three fuzzy modelling methodologies analysed within this thesis. The reasoning behind the integration of all three approaches within a single framework stems from the need to try and combine their specific potential and areas of excellence as the foundation of the modelling approach.

As with the examples analysed in previous chapters, the process begins with the analysis of the structure from first principles. The analysis of the requirements that the structure has to satisfy follows naturally into the initial definition of the design on a feature-base level. The problem is then structured and analysed on the basis of the leading features which uniquely define the structure at hand. Input parameters are agreed upon based on governing structural features and they will be driving the preliminary structural analysis. This can be designed around several modelling approaches, according to the relevant stage of the design the methodology will be applied in. Within this research, a combination of semi-analytical formulations and beam bending theory was adopted for the derivation of the structural properties of interest for the study. Alternatively, results from in house tools or higher fidelity models can also be used depending on the level of granularity required by the analysis.

This approach closely mimics feature-based methods typically used in the weight estimation applications (Baker and Smith, 2003), but with the additional benefits stemming from the use of the physics-based perspective on structural design, which is dis-
tinctive of analytical weight estimation methodologies. The input variables relative to structural and feature parameters as well as the outputs of the structural analysis are then used as the basis of the structural and weight modelling in the first level of fuzzy approximation. The core of this level is the ANFIS modelling framework. From this first stage approximation, the user will be able to derive an initial fuzzy network architecture which will produce a first stage approximate rulebase, based on a preliminary fuzzy definition of both input and output variables. At this point in the process, the analysis at this stage will only be of a basic level, due to the specific capabilities of the Takagi-Sugeno-Kang FIS at the basis of ANFIS. If the generalization capabilities of the model and the final approximation accuracy fall within the limits established by the user, the process can continue. Alternatively, the weight engineer can go back and re-define the initial problem definition or verify the reliability of the database of reference used for the training and testing of the fuzzy model.

The second stage is based on Mamdani FIS optimized through NEFPROX. The initial input and output variable definition optimised through ANFIS is used as a foundation of the level-2 NEFPROX-based fuzzy system optimization. Within this stage, the network would be further streamlined in order to derive an improved and more interpretable rulebase. Input and output variables will also be enhanced in their fuzzy definition, with improved membership functions and more intuitive characterisation. As for the first stage, the process can be reiterated from level 1 definition until the required performance is achieved.

The third and final level of fuzzy abstraction is represented by Interval Type-2 fuzzy systems. Even in this case, the fuzzy model definition obtained through the optimization process in level 2 is used as a basis of the analysis, as a way of reducing computational effort needed in the process. At this stage, the fuzzy input and output domains are further streamlined and stabilized, while the rulebase achieves the optimum compromise between interpretability and final modelling accuracy. In addition to this, the model is further enhanced by the introduction of an additional level of analysis, through the formalization of uncertainty estimates and their propagation from input to output levels across the fuzzy network. This information can then be fed back across the initial design definition, as well as to the previous two levels of fuzzy abstraction.
(ANFIS and NEFPROX) for further model optimization and knowledge extraction. Alternatively, the modelling outputs from the IT2 FIS can be utilized within separate structural and weight estimation models for enhanced analysis from the point of view of uncertainty and system reliability.

8.4 Summary

This chapter explored the problem of defining a general framework for weight estimation of aircraft structural components in the preliminary phases of the design process, through the use of fuzzy logic methodologies. In particular, the overall aim was that of producing both a flexible and comprehensive approach which could be adapted to different structures as well as at different levels of design granularity.

The chapter analysed the different fuzzy methodologies presented through the previous case studies in terms of the different requirements posed by the design of a framework for weight estimation. Within this context, the concept of granularity was introduced, which was formalised from the point of view of the analysis and derivation of structural weight, as well as with reference to the three fuzzy approaches analysed within this research. Takagi-Sugeno-Kang fuzzy systems developed using ANFIS, Mamdani FIS designed through NEFPROX and Interval Type-2 fuzzy logic have been critically assessed side by side in the light of the results produced by the case studies and assessed according to the general requirements of a weight estimation methodology and the various output levels expected from it.

In addition to this, the issue of knowledge completeness and integrity within the design and weight estimation of aircraft structures was examined. A general approach for the synthesis of the knowledge from experts and designers with that derived through the use of adaptive fuzzy techniques was presented and put into context. The aim of the process was that of ensuring the derivation of a framework able to provide a weight picture which benefits from both the experience of the design team as well as from the inter-variable relationships existing within the design, but which are hidden within the data.

From this analysis, it was possible to define the necessary steps and processes
for the definition of a comprehensive and stand alone weight estimation process. The approach itself builds on the different capabilities and potential offered by the three fuzzy methodologies examined. Overall, it integrates them into a multi-layered process structure which is sufficiently flexible to accommodate the changing requirements and needs of the design process, but which is still able to provide a comprehensive weight picture, with complete traceability of the different sources of weight penalties in parallel with exhaustive uncertainty analysis.
Figure 8.3: Flowchart outlining the framework for the 3-level process for the weight estimation of aircraft structures at preliminary design stages.
Chapter 9

Conclusion

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9.1 Achievements and contributions

This thesis outlines the development and analysis of a weight estimation methodology for aircraft structures suitable for the preliminary stages of the aircraft design process. In particular, the research conducted explores the potential of adopting fuzzy logic as foundation of a more comprehensive and reliable weight assessment of aircraft structural components. The material in the thesis follows the evolution of the framework through the various stages of its development and the adaptation of the different fuzzy logic techniques to the requirements of the problem.

Each chapter, with the exception of the critical review of current weight estimation methodologies and the introduction to the field and the theory of fuzzy logic, presents original contributions from the author. These include the formalisation of the problem of weight estimation for selected structural components, the identification of problematic areas within the solution, the adaptation and implementations of different adaptive fuzzy logic techniques, as well as the optimisation of the approaches to suit the specific requirements of the problems. The process culminated in the definition of an overall framework structure for the optimal implementation of the fuzzy logic approach for the weight estimation of aircraft structures.

The individual achievements of this research and its specific contributions to the field of weight estimation are presented below:

1. The field of weight estimation for the aircraft industry has never received considerable interest within the academic community. Being more of an industrial problem, the majority of the advancements and developments in the field have been restricted to the private sector. This thesis has managed to provide a comprehensive and critical overview of the state-of-the-art in field, from the traditional academic-based approaches to specific industry-led techniques, for a more comprehensive understanding of commonalities and differences between the two spheres, their relative contributions and the problems they share which have still not been resolved.

2. The identification of fuzzy logic as a suitable tool for the development of a weight
estimation approach, which is able to provide a solution to the problems within the weight estimation of structural components at the preliminary phases of the design process. In particular, adaptive fuzzy inference systems have been successfully applied in a variety of research problems characterised by similar requirements and constraints as that of weight estimation, demonstrating to be suitable and have the potential to provide an efficient solution to the drawbacks of current methodologies.

3. The creation of an approach for weight estimation with the potential for a range of applications, from a simple feature-based weight derivation to a more physics-based weight analysis of structural components.

4. The formalisation of the problem of weight estimation of aircraft structural components for implementation more robust methodology to be applied at preliminary design phases. In comparison to the majority of weight estimation methods for early project phases which are of a purely empirical nature, the approach presented in this thesis provides a weight analysis which mimics the actual design process of the components. The approach is built on an analytical component-based load derivation from first principles, which forms the basis of a fuzzy logic-based structural sizing and weight estimation of the component.

5. The creation of a weight estimation methodology for preliminary design phases which is also able to incorporate the effects of system installation as well as inputs relative to the installation of the component in the final assembly.

6. The successful application of adaptive fuzzy logic techniques to the weight estimation of real aircraft structural components. In particular, three different fuzzy tools were tested, namely Adaptive Network-Based Fuzzy Inference Systems (ANFIS), Neuro-Fuzzy Function Approximation (NEFPROX) and Interval Type-2 fuzzy systems. All three techniques were used for the sizing and weight of spoiler and aileron attachment ribs, with data related to loads and geometries of as-built structures from both categories provided by Airbus UK.
7. The derivation of a method for the creation of a usable and modifiable knowledge base for the component being analysed. The use of fuzzy logic as a basis of the methodology presented in this research, allows the creation of a set of rules governing the relationship between the different variables for both the structural sizing and weight derivation of the component. The structure of the rulebase combines a computational base with a graphical structure for ease of interpretation through the visual representation of the causalities between the different variables and their impact on the final output. This will enable the designer with concrete ways of improving the decision making process from the very early stages, by evaluating not only the impact of the individual design decisions but also their combined effect on the final structural weight.

8. The development of a method which can be used both as a stand alone weight estimation approach, by providing detailed and reliable results based on physics-based sizing derivation, as well as a validation tool for higher fidelity computational models. In addition to this, the tool represents a fast and computationally inexpensive way to obtain reliable and traceable estimates, which can act as safety checks for the results of models in later design phases.

9. The development and implementation of uncertainty analysis and propagation across the sizing and weight estimation process. This was achieved though the use of Interval Type-2 fuzzy logic within the framework. The use of footprints of uncertainty (FOUs) within the knowledge base derived by the system allows for a both quantitative and visual assessment of the effects of the uncertainties within the problem on the quality of the analysis.

10. The formalisation of an overall framework for the weight estimation of aircraft structural components, which is sufficiently flexible as well as easily adaptable to conform to different levels of granularity that may be required in the analysis at preliminary design stages.
11. A general approach for the successful synthesis of the knowledge from experts and designers with that discovered from the data through the use of adaptive fuzzy techniques was presented and integrated within the framework, to ensure the derivation of a more complete and exhaustive weight picture.

9.2 Conclusions

The research presented in this thesis has provided many significant contributions to the field of weight estimation of aircraft structures. In particular, the greatest achievement was proving the applicability of fuzzy logic theory and tools as foundation for the development of successful and reliable computational models for weight estimation. A number of valuable conclusions can be drawn from the material presented.

1. The relationships between the different variables, both within the sizing and weight analysis, are highly non-linear. The use of traditional statistical linear relationships for the weight prediction of structural components at preliminary design stages will result in erroneous and misleading estimates.

2. The results from all the different fuzzy logic approaches for both structural case studies presented in this thesis highlight the importance of the inclusion of system installation considerations as an integral part of the weight estimation process. The impact of system loading on the structural weight, although not as considerable as that resulting from other primary loading conditions, is still considerably noticeable and neglecting it would result in an incomplete and unrepresentative estimation of the component weight.

3. The results from the three fuzzy methods examined in this research, although different amongst themselves in terms of accuracy, have provided a way of cross-validating the models, by highlighting closely similar trends across the variables of interest.
4. This research also provided evidence of the superiority of Mamdani over TSK FIS when applied to the derivation of weight estimates. Mamdani-type fuzzy systems proved to derive higher quality approximations from the point of view of accuracy, interpretability, simplicity of the final network structure and rulebase.

5. Results have demonstrated the ability to achieve a reliable uncertainty analysis by using interval type-2 fuzzy logic in environments, like weight estimation at preliminary design phases, which are characterised by lack of information and a high degree of uncertainty and variability in the definition of their parameters.

6. In addition to accurate approximations, the fuzzy approaches presented were able to successful derive reliable trends and to identify of principal causalities between the numerous variables of interest. By idealising the both spoiler and aileron support ribs as aggregations of beam structures, the results were also able to highlight the possible weight penalties resulting from the selection of a particular design solution instead of another, or from the possible integration of system routing within the structural assembly itself.

9.3 Recommendations for future work

Being the first attempt at the application of fuzzy logic theory in the field of weight estimation for aircraft structures, there is plenty of scope to expand the research presented in this thesis. Firstly, there is a great potential in the methodologies analysed to provide full weight estimation capabilities for the entire wing leading and trailing edges. The 3-level sizing and weight estimation approach used for the application of ANFIS, NEFPROX and Interval Type-2 fuzzy inference systems on spoiler and aileron attachment ribs could be easily modifiable for the analysis of other fixed secondary structures, such as leading edge ribs and falsework, as well as for the movables (e.g. flaps, slats, ailerons, etc.). In addition to this, the same basic design approach could also be extended to other secondary structures, as well as to primary structural components. Ideal examples could be the wing ribs, where the fuzzy logic approach could explore
the weight inefficiencies in the structures resulting from loading considerations, system routing as well as integration of the structure within the main assembly.

Fuzzy logic could be a very powerful tool in the preliminary phases of the design of structures. The framework as well as the individual approaches presented in this thesis could be modified to provide the designer with insight on how different manufacturing processes or design philosophies can affect the final weight of the structure under consideration. The main steps that will have to be taken in approaching the problem from this angle will have to focus on the definition of a successful way to parametrise the problem, in order to link the necessary information about the to specific manufacturing, fabrication or assembly processes.

The research has also highlighted the potential of using Interval Type-2 fuzzy logic as tool to conduct more comprehensive and intuitive uncertainty analysis at the preliminary phases of the design. At this point in the design, the lack of information and data make the results of methods traditional probability theory unreliable and unrepresentative. IT2 FIS could be applied in weight estimation, as well as in other areas within the design, to improve the understanding of the impact of individual decisions on the final product.

The results have also highlighted the potential of integration fuzzy logic weight estimation in computational tools for the design and analysis of structural components at the preliminary stages of the process. From a larger scale perspective, there is scope for the integration of the proposed framework within a multidisciplinary design environment. The use of fuzzy logic could allow for an easier transfer of information between the different disciplines, as well as ensure the preservation of the information extracted from both experts and data during the design process across the various design domains.

The value of this work is emphasised by the long term research scope that it has brought to light. The hope is for the engineering community to realise the value and the importance of weight estimation, both in the aircraft sector as well as in the automotive field, in order to further the research in the area and move towards a truly multidisciplinary design.
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