Adaptive Beamforming for Distributed Relay Networks

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and

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Abstract

Tremendous research work has been put into the realm of distributed relay networks, for its distinct advantages in exploiting spatial diversity, reducing the deployment cost and mitigating the effect of fading in wireless transmission without the multi-antenna requirement on the relay nodes. In typical relay networks, data transmission between a source and a destination is assisted by relay nodes with various relaying protocols.

In this thesis, we investigate how to adaptively select the relay weights to meet specific interference suppressing requirements of the network. The thesis makes original contributions by proposing a filter-and-forward (FF) relay scheme in cognitive radio networks and an iterative algorithm based transceiver beamforming scheme for multi-pair relay networks. In the firstly proposed scheme, the relay nodes are adapted to deal with the inter-symbol-interference (ISI) that is introduced in the frequency-selective channel environment and the leakage interference introduced to the primary user. Our proposed scheme uses FF relay beamforming at the relay nodes to combat the frequency selective channel, and our scheme also aims to maximize the received SINR at the secondary destination, while suppressing the interference introduced to the primary user (PU). This scheme is further extended to accommodate a relay nodes output power constraint. Under certain criteria, the extended scheme can be transformed into two sub-schemes with lower computational complexity, where their closed-form solutions are derived. The probability that we can perform these transformations is also tested, which reveals under what circumstances our second scheme can be solved more easily.

Then, we propose an iterative transceiver beamforming scheme for the multi-pair distributed relay networks. In our scheme, we consider multi-antenna users in one user group communicating with their partners in the other user group via distributed single-antenna relay nodes. We employ transceiver beamformers at the user nodes, and through our proposed iterative algorithm the relay nodes and user nodes can be coordinatively adapted to suppress the inter-pair-interference (IPI) while maximize the desired signal power. We also divide the rather difficult
transceiver beamforming problem into three sub-problems, each of which can be solved with sub-optimal solutions. The transmit beamforming vectors, distributed relay coefficients and the receive beamforming vectors are obtained by iteratively solving these three sub-problems, each having a closed-form solution. The tasks of maximizing desired signal power, and reducing inter-pair interference (IPI) and noise are thus allocated to different iteration steps. By this arrangement, the transmit and receiver beamformers of each user are responsible for improving its own performance and the distributed relay nodes can be employed with simple amplify-and-forward (AF) protocols and only forward the received signal with proper scalar.

This iterative relay beamforming scheme is further extended by distributing the computation tasks among each user and relay node, through which high computational efficiency can be ensured while extra overhead of bandwidth is need for sharing beamforming vector updates during the iteration steps. Furthermore, with respect to the channel uncertainty, two more relay strategies are proposed considering two different requirements from the communication network: sum relay output power and individual relay output power.

At last, the application of the iterative relay beamforming method in cognitive radio networks is studied, where multiple pairs of users are considered as secondary users (SUs), and the designed transmit beamforming vector, relay beamforming vector and receive beamforming vector together guarantee that the inner interference of their transmissions is well suppressed while the interference introduced by them to the PU is restricted under a predefined threshold.
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<th>Description</th>
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<tbody>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
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<td>DF</td>
<td>Decode-and-Forward</td>
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<td>FF</td>
<td>Filter-and-Forward</td>
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<td>CSI</td>
<td>Channel State Information</td>
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<td>ISI</td>
<td>Inter-Symbol-Interference</td>
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<td>IPI</td>
<td>Inter-Pair-Interference</td>
</tr>
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<td>SOCP</td>
<td>Second-Order Cone Programming</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference-plus-Noise Ratio</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>EP</td>
<td>Eigenvector Problem</td>
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<td>ZF</td>
<td>Zero Forcing</td>
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<td>UE</td>
<td>User Equipments</td>
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<td>SU</td>
<td>Secondary Users</td>
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Chapter 1

Introduction

Within the realm of both academia and industrial wireless communications, the pursuit for better throughput, reliability and capacity has always been the main task of relevant research. The widely studied multiple-antenna based multiple-input-multiple-output (MIMO) scheme is one of the successful techniques to achieve those goals, and it has been intensively adopted in standards such as WLAN [1], WiMAX [2], LTE and LTE-Advanced [3]. However, in most practical applications, the wireless terminal devices are employed with single antenna, due to the limitation of physical size, battery life and processing power.

In order to exploit the merits of MIMO techniques in networks with single-antenna devices, the concept of distributed relay network has been proposed, and it has demonstrated its distinct advantages in exploiting the spatial diversity, reducing the deployment cost and mitigating the effect of fading in wireless communications through user-cooperation [4–10]. In such networks, communication resources are shared by different users to assist each other in transmitting the information stream by means of relaying messages from the source to
destination through multiple independent paths, which forms a virtual MIMO communication scheme. In a typical relay network, data transmission between a source and a destination is assisted by relay nodes with various relaying protocols, among which two most studied strategies are the amplify-and-forward (AF) scheme [11–16] and the decode-and-forward (DF) scheme [17–22]. In the AF scheme, relay nodes aim at compensating the power loss of the received signal by amplifying it with a proper scale and phase-shift, while in the DF scheme, the relay nodes are required to decode and re-encode the received signals before retransmitting them.

Furthermore, various strategies have also been proposed with different network constructions and different tasks assigned to relay nodes. One particular research area is the multipair relay network, where multiple peer-to-peer user pairs communicate with each other simultaneously, which significantly increases the overall throughput and efficiency of the relay network [23–27]. For such relay networks, the inter-pair interference (IPI) caused by simultaneous signal transmissions is a crucial issue. In [25, 26, 28], the authors used different zero-forcing (ZF) based methods to eliminate the IPI among users, while block diagonalization (BD) was employed on a central relay node with multiple antennas to reduce interferences experienced by each user pair in [29, 30]. In [31], the beamforming vectors for the multi-antenna user pairs were jointly decided to null out the IPI and maximize the effective channel gain between user pairs. In [23], an ad hoc network with multi-pair communications was studied with the one-way strategy. In [24] and [25], the authors investigated the
multipair two-way relay networks, where the bi-directional transmissions are supported by a multi-antenna central relay node. And in [26], a central relay node equipped with a very large array of antennas was considered, which can substantially reduce the interference with simple signal processing techniques.

For multipair relay networks with distributed single-antenna relays, [32–37], the desired signals at the destinations suffer from a higher level of interference, due to the assumption that the distributed relays do not share their received signals and thus cannot cooperate as effectively as the former network to suppress the interference accumulated in the source-relay transmission stage. In [32], the authors demonstrated that such network with destination quality-of-service constraints will lead to a non-convex relay beamforming problem, that can be turned into a semi-definite programming (SDP) optimization, and can be solved using interior point methods. And in [33] the authors proposed to approximate the same relay beamforming problem by a convex second-order cone programming (SOCP) problem with drastic reduction in computational complexity. On the basis of it, [37] developed a distributed optimization method that achieves faster convergence rates, using the accelerated distributed augmented lagrangians (ADAL) algorithm [38]. Another method was proposed in [35], where the authors used ZF to cancel the inter-user interference with the assumption that the global channel state information (CSI) is known at every relay node. In [27] the author proposed a different beamforming method on the assumption of a large number of relay nodes, where the signal-to-interference-plus-noise ratio (SINR) is maximized instead of having a predefined lower bound.
1.1 Original Contributions

Although much research has been performed in the field of distributed relay networks, some specific areas remain unstudied, and in this thesis, we study the FF distributed relay beamforming problem in cognitive radio networks and proposed the iterative transceiver beamformer designs for multi-pair two-way distributed relay networks.

1.1.1 Filter-and-Forward Distributed Relay Beamforming for Cognitive Radio Network

Among the relay protocols used on the distributed relay nodes, the AF scheme is of more interest due to simplicity of the algorithm and its implementation. As an extension in [39], Chen et al proposed a new FF relay scheme for frequency selective channel. In this scheme, finite impulse response (FIR) filters are employed at the relay nodes to combat inter symbol interferences (ISI) in frequency-selective channels and the results has shown that the FF scheme can provide significant performance enhancement for communication between single user pair. However, when additional suppression of interference is demanded aside of ISI, for example, in cognitive radio networks, the original FF relay scheme should be amended.

Motivated by that, we proposed the FF relay beamforming scheme in cognitive radio networks in Chapter 3, aiming at maximizing the received SINR at the secondary destination, while suppressing the interference introduced to
the primary user (PU). This scheme is further extended by considering a relay nodes output power constraint. It is proved that under certain criteria, the extended scheme can be transformed into two sub-schemes with lower computational complexity, where their closed-form solutions are derived. And the probability that we can perform these transformations is tested, which reveals that with certain network settings our second scheme can be solved more easily.

1.1.2 Iterative Transceiver Beamformer Design for Multi-Pair Two-Way Relay Networks

For another aspect, in all the aforementioned multi-pair distributed relay network designs, user nodes are assumed to have single antenna implementation. However, in next generation wireless communication systems like LTE/LTE-Advanced/5G [40], multi-antenna user equipments (UEs) are accepted as elementary system setup, and with the development of multi-antenna devices and coordinated multi-point joint-transmission techniques [41–43], where multiple UEs collaborate and jointly steer the transmit signal, investigating the problem of how the communication of multi-antenna devices and/or virtual multi-antenna devices can benefit in a distributed relay network becomes more and more practical and important.

To our best knowledge, the beamforming problem in such a network has not been investigated yet. On the one hand, to take advantage of the multi-antenna implementations of the user nodes, we propose to utilize transceiver beamforming on the user nodes, and thus the quality-of-service (evaluated by
SINR) of each user node will be jointly determined by three beamforming vectors: transmit beamforming vector, relay coefficients and receive beamforming vectors. The overall beamforming problem becomes more difficult than the single-antenna-user case. On the other hand, unlike in the single-antenna-user network, where the relay nodes are in charge of almost all the signal processing tasks, in our considered network, since each user nodes are equipped with transceiver beamformers, the main part of the signal processing tasks can be shifted to the user side, and the relay nodes can be relieved of their usual dominant role in suppressing relevant interferences and noises. Therefore, more idle devices in a certain area can meet the simple signal processing requirement of our scheme and be employed as a relay node to help the communication of the user pairs.

We also propose to divide the overall beamforming problem into three sub-problems based on the different roles of the three beamforming vectors in their contribution to the received SINR, each of which has a closed-form solution and decides one of the three beamforming vectors, and through an iterative algorithm, a sub-optimal solution can be obtained. To achieve a satisfactory performance while relieving relay nodes of the usual computation task, two iteration-based transceiver beamforming schemes are proposed to coordinate the operation of the users from the two user groups, where the beamforming vectors are determined at the user nodes through an iterative process.
1.1.3 A Distributed Iterative Transceiver Beamforming Algorithm for Multi-pair Two-Way Relay Networks

The transceiver beamforming design for multi-pair two-way relay networks is further studied where we study the potential of totally distributing the computation tasks among each user and relay node, through which high computational efficiency can be ensured; however certain degradation of SINR performance will occur at the user nodes. And we investigate the situation when the channel state information is continuously changing and we considered different rate of the change in simulations.

Although the preliminary idea of uniformly amplify-and-forward (AF) protocol was easy to implement, it has limited contribution to the enhancement of signal performances. Therefore, we further propose a different relay strategy with consideration of sum relay power constraint that can significantly improve the SINR performance of each user.

1.1.4 Robust Iterative Transceiver Beamforming For Multi-pair Two-Way Distributed Relay Networks

CSI is one of the very essential factors that can significantly affect the performance of transmission in distributed relay networks. When it is not available at the relay nodes, distributed space-time coding and distributed space-time block coding can be used to obtain proper cooperative diversity gain [11, 44–47]. However, with available CSI estimated by the user nodes and/or the relay nodes,
distributed relay beamforming can provide much better performance.

However, CSI estimation errors can potentially lead to significant performance degradation, and such errors can hardly be avoided in distributed relay networks, due to inaccurate channel estimation, mobility of relays, and quantization errors. Much work has been done in proposing robust designs in distributed relay networks [48–53, 53–56]. In [54], the robust distributed relay beamforming problem was investigated for single-pair one-way relay network, and a robust relay scheme for multi-user single-destination one-way relay network was proposed in [51] with the decode-and-forward protocol. In [55], a worst-case based distributed beamforming scheme was developed for a single communication pair with norm-bounded CSI errors. The filter and forward relay beamforming scheme was studied with spherical CSI uncertainties in [56], while in [52] ellipsoidal CSI uncertainties were considered for a multi-pair one-way communication network.

On the basis of preliminary study of the iterative transceiver beamforming for multi-pair two-way distributed relay networks in Chapter 4, we propose and compare two different relay strategies, with consideration of sum relay power constraint and individual relay power constraint, respectively. Moreover, based on the structure, we also investigate the robustness of our proposed methods at the presence of CSI errors and propose the worst-case based beamforming strategies for transmit beamformers and the relay nodes, and simulation results demonstrate that both of the proposed methods are extremely robust against CSI errors.
1.1.5 Iterative Transceiver Beamforming of Distributed Relay Network in Cognitive Radio Networks

Multi-pair communications are then considered in cognitive radio networks, where our iterative transceiver beamforming scheme is investigated. Two schemes with different relay beamforming strategies are proposed where one aims at maximizing SINR at each user node while keeping leakage interference power at the primary destination under a predefined threshold and the other one adds a total relay output power constraint to the previously studied problem. It is proved that under certain criteria specified in Chapter 3, the second scheme can be transformed into two sub-schemes with lower computational complexity. The probability of performing those two transformations are studied and relevant network settings are investigated.

1.2 Thesis Outline

This thesis is organised as follows. In Chapter 2, the background of relay networks beamforming is presented. In Chapter 3, the cognitive radio network is introduced together with the distributed relay beamforming techniques and FF strategy. Our two proposed schemes are presented, where one of them is solved by either transforming it to a standard second-order-cone-programming (SOCP) problem or to a sub-scheme with a closed-form solution. The condition of performing such transformation is discussed.

In Chapter 4, the multi-pair communication network is introduced and we
look into the transceiver beamforming problem with multiple multi-antenna user nodes and single-antenna relay nodes, where transceiver beamformers are used on the user nodes and uniformly AF protocol is applied on the relay nodes.

Then, the iterative transceiver beamforming schemes with totally distributing the iteration steps on each user node and each relay node is studied in Chapter 5, where a different phase-rotate relay protocol is proposed with much better performance. Following that, in Chapter 6, we consider the situation when CSI errors are at presence and two robust iterative transceiver beamforming schemes are proposed with considering total relay output power constraint and individual relay output power constraint, respectively.

In Chapter 7, cognitive network with multi-pair communication between SUs is introduced and we investigated the iterative transceiver beamforming scenario in such a network. Two schemes are proposed, and one of them can also be solved by transforming it into a sub-schemes with a closed-form solution. The condition and relevant settings of performing such transformation is investigated.

Notations: $[\cdot]^T$, $[\cdot]^H$ and $[\cdot]^*$ stand for transpose, Hermitian transpose and conjugate, respectively. $||\cdot||$ denotes the Frobenius norm of a vector and $|\cdot|$ the absolute value of a scalar. $\mathbb{E}[\cdot]$ represents the expectation operator and $\text{Var}[\cdot]$ the variance operator. $\mathbf{I}_N$ is the $N \times N$ identity matrix.
Chapter 2

Background and Related Work

This chapter presents the preliminaries, background and progress of distributed relay beamforming techniques, as well as our motivations of our investigations in this thesis.

2.1 MIMO and Beamforming Techniques

As has been mentioned preliminarily, the idea of distributed relay beamforming is proposed to exploit the advantage of MIMO system and they are closely related. The concept of MIMO systems is firstly introduced for communication systems to achieve multiplexing and spatial diversity where multiple antennas are deployed at the source node and destination node [57–59]. By providing multiple antennas at both sides of a radio link, a typical MIMO transmission system is formed. Spatial multiplexing can be performed by the additional antennas and the system throughput can be enhanced by transmitting multiple replicas of a signal simultaneously. More specifically, in MIMO systems, the
source node transmits several replicas of a signal through several different and independent channels, and thus additional message is transmitted through this redundancy to significantly reduce the fading effect on the signal quality. And by applying certain processing steps at the destination node, the transmitted signal can be recovered with fading and distortion being massively reduced. On the one hand, even if the CSI is unknown, space-time coding techniques can be used to achieve the full diversity promised by the transmit and receive antennas [60, 61]. On the other hand, if the CSI information is known, beamforming techniques can be adopted. It is a very efficient way to reveal the whole picture of the communication system, and it is also capable to introduce additional array gain over the space-time coding technique [46, 62, 63], and the knowledge of the channel state information can also be used to optimally allocate the transmitting power and/or the power of relay nodes, which can lead to a better control of the communication system.

2.2 Distributed Relay Beamforming

As mentioned in Chap. 1, wireless devices are usually unable to be employed with multiple antennas to form a MIMO communication environment due to their limitation of physical sizes and power. Distributed relay beamforming schemes are the remedies for this problem and they have attracted the attention of the research community for a relatively long time. In such schemes, several relay nodes with single antenna cooperatively forward the message from
the source node to the destination node, and thus they form a virtual MIMO system with multiple independent transmitting channels. The distributed relay beamforming problems are usually similar to relay beamforming problems mathematically, although they are quite different in implementation aspects, and for the distributed relay beamforming problems, whether to allow information share between the relay nodes should be a problem to consider.

In the literature, different models and scenarios have been proposed for distributed relay beamforming. Defined by relay protocols, two most studied strategies are amplify-and-forward (AF) relay scheme and decode-and-forward (DF) relay scheme. In AF scheme, relay nodes simply amplify the received signal with a proper scale and phase-shift [7, 21, 64]. While in DF scheme, the relay nodes are required to decode and re-encode the received signals before re-transmitting them [21]. Due to the simplicity of implementation and algorithm, AF scheme is of more interest.

2.2.1 One-way and Two-way Relay Beamforming Schemes

Defined by the data exchange scenario, there are two most widely used relay beamforming schemes: one-way and two-way. Both are perform in time-division duplexing protocol. For the one-way distributed relay beamforming, it has the very basic topology, and performs in two time slots. In the first time slot, the source node broadcasts the signal to the relay nodes. After performing certain process to the received signal (scaling, phase-shift, and/or decoding), the relay nodes transmit the processed signal to the destination node in the second
time slot. One-way relay beamforming schemes using AF and DF relay protocol were studied and compared in [64], where the problem was addressed to optimize the received SNR subjected to a total relay nodes output power constraint. In [8], the problem was studied with individual power constraint for each relay node. Also in this article, the relay beamforming schemes with and without direct link between source node and destination node were considered. When there is a direct link, in the first time slot, the source will not only broadcast the signal to relay nodes, but also broadcast it directly to the destination node. In [7], the problem with individual relay power constraint was considered as solving a convex SOCP problem, and it shows significant enhancement in efficiency.

For the two-way distributed relay beamforming, it is an extension of the one-way scheme as well as the two-way relay beamforming. In order to avoid interference, the traditional strategy of arranging two-way information exchange is as depicted in Fig.2.1. In [65], a practical design named analog network coding (ANC) was proposed to reduce the time slots required from 4 to 2. The basic idea of the approach is not to avoid the interference by scheduling information exchange at different time slots, but to strategically encourage senders to interfere (thus the source node and destination node can transmit signal simultaneously to the relay nodes), and cancel the interference when the signal is received. By applying the ANC, [66] studied the two time-slot two-way multi-antenna relay beamforming schemes and the communication scheme used in this article is provided in Fig. 2.1 as well. The two time-slot relay scheme is
widely used in studying two-way relay beamforming problems [67–69] since it can result not only significant throughput benefits but also ability to mitigate interference. Based on the minimization of the total transmit power subject to SNR constraint at two transceivers, [68] concluded that this distributed beamforming problem for two-way networks can be equivalent to another distributed beamforming problem which minimizes the total transmit power for a one-way relay network. With the similar consideration, the two-way relay beamforming problem with filter-and-forward (FF) relay protocol was studied in [70].

2.2.2 Filter-and-forward Distributed Relay Beamforming

When the communication channels are assumed to be frequency-flat, the protocols mostly used at relay nodes are AF or DF. However in practical scenarios, these channels are sometimes frequency selective, and in such case, inter symbol interferences (ISI) will be introduced at the relay nodes and receiver side. To combat the ISI, Chen et al proposed a new FF relay protocol as a solution. In this scheme, finite impulse response (FIR) filters are employed at the relay node. The simulation results of [39] and [70] have shown that the FF approach achieves significant performance improvements over the AF relay beamforming scheme when the communication channels are frequency selective. Furthermore, the optimal decision delay selecting problem at the destination node was addressed in [71]. And in [71], the FF relay beamforming problem was described directly in frequency domain, and a distortionless constraint is set to control the frequency response of the overall channel in this paper. Compared
to [72], where a problem of similar constraints and optimization goals was studied and solved in time domain, it has shown that although the original problems are similar, derivations to the solutions are quite different. More specifically, solving the FF beamforming problem of maximizing received SINR with total relay output power constraint in time domain will end up with solving a generalized eigenvector problem (GEP), and solving the similar problem in fre-
frequency domain will lead to setting a distortionless constraint as a constraint and then the problem can be solved. In [39], the transmission frequency selective channels are modeled as FIR filters, whose coefficients are given by zero-mean complex Gaussian random variables with an exponential power delay profile [73]. Moreover, the convolution of signal and channel coefficients has to be described as extended matrix multiplication in such schemes, which is more complicated compared to the frequency-flat schemes. In [72], the author presented a more straightforward and concise signal model for the FF distributed relay beamforming network.

### 2.2.3 Distributed Relay Beamforming in the Context of Cognitive Radio Networks

More recently, distributed relay beamforming has also been introduced in the context of cognitive radio (CR) to improved its performance. In cognitive radio, which was well introduced in [74], a set of secondary users (SUs) can utilize spare spectrum resources (also known as spectrum slots) for their own communication, unless the communication condition of primary users (PUs) is significantly affected. In particular, in a CR network, when an SU intends to start a communication with a secondary destination and the channel condition is poor, just like distributed relay networks a set of SUs can work as relays (referred to as CR relays) and help forward the information [75].

In such a network, when frequency selective channels are also considered, interference introduced by the secondary communication at the PU destination
must be suppressed together with ISI. Using FF beamforming seems to be a solution to this problem; however, no such work had been done in literature, and thus it motivated us to construct a new scheme specifically for its implementation in the context of cognitive radio network in Chapter 2.

2.2.4 Multi-Pair Relay Network Beamforming

In order to increase the throughput and efficiency of a distributed relay network, multi-pair distributed relay networks are proposed to support multiple pairs of users in one network [36, 76–78]. Among these literature, in [76] the SINRs at receiver nodes were ensured above predefined thresholds while the total relay transmit power is suppressed by a predefined power constraint. In this article, the original problem was solved as a semidefinite programming (SDP) problem which was convex and mathematically easy to handle but had high computational complexity. While in [36, 77], a similar problem was considered using SOCP approximation, and an iterative method was proposed in these articles where in each iteration step, an approximated SOCP problem was solved and thus making sure the performance can be consecutively improved. Meanwhile, a worst-user-SINR maximizing scheme was considered in [78] where a relay power constraint was set and the formulation was solved as an SDP problem.

The aforementioned work studied the optimization of relay output power without considering the power of all the transmitter nodes. Therefore, in [79], joint optimization of the source power allocation and relay beamforming weights in distributed multiuser relay networks was considered, in which the beamform-
ing design problem was formulated as ensuring a predefined SINR requirement for each user nodes while minimizing the sum transmit power of all the source nodes and relays.

Although a multi-pair relay networks have the advantage of increasing throughput and efficiency, they also face great challenge of suppressing the additional inter-pair interference (IPI) introduced by the additional users [5, 6, 8–10, 23, 26, 80–83]. Moreover, in all the proposed multi-pair relay beamforming schemes we can find in literature, user nodes are considered to be with single-antenna, and the main signal processing procedures and beamforming weights determination processes are performed at the relay nodes, and this will take significant resources from the relay nodes, such as time, computational capacity and processing power. This motivated us to consider the situation when the user nodes have multiple antennas and the main signal processing tasks can be shifted from relay nodes to user nodes, and thus the relay nodes can be assigned with very simple signal processing task. Therefore, in such network, many idle users in a certain area can meet the simple signal processing requirement and be utilized as relay nodes to help the communication of the user pairs. Accordingly, we investigate the beamforming problem of such networks and propose an iterative algorithm based transceiver beamforming scheme for the multi-pair distributed relay network in Chapter 4.
2.3 Channel State Information and Optimization Criteria

In distributed relay networks, channel state information is a very important information which can be used to help the relay nodes and/or source nodes with their weights decisions to improve the capacity, quality of service and power allocation of the networks. As mentioned before, space-time coding strategy is a very traditional way, based on MIMO system, to exploit the diversity gain when the channel state information (CSI) is not known by the transmitter [60, 61]. By applying some extension, a counterpart of space-time coding is adopted in the wireless relay networks [84]. However in such scheme, although the CSI is not required by the relay nodes, the received node must have the full channel information of both the channels from the transmitter to relays and the channels from relays to the receiver. In order to deal with this issue, a so called distributed differential space-time coding is proposed in [85]. In this scheme, channel information is required at neither relays nor the receiver. It is also revealed by the same author in another paper [8], that if the CSI is available distributed network beamforming can obtain better performance. Also, research of this paper reveals that the optimal weight of a relay node depends on the quality of all other channels in addition to the relay’s own channels. In another word, the derivation of the optimal relay weights requires the instantaneous global full channel state information, or otherwise the obtained weights are not the real optimal one. In fact, most of the aforementioned relay beamforming schemes face the same problem. In practical relay networks, this needs a lot of feedback.
from the receiver. A more practical assumption is that the CSI known to the relay nodes or the receiver is only partial channel information. There are two mostly used types of strategies to describe the partial channel information: second order channel statistics and quantized instantaneous CSI. The former type is studied in [86] and the latter one is studied in [87].

Furthermore, when there are estimation errors of the CSI, performance will suffer degradation. To combat this issue, robust distributed beamforming techniques are developed, among which there are two widely used types of robust designs: the worst-case optimization approach and the stochastic approach. In worst-case optimization approaches [88, 89], the channel error is considered to have a predefined uncertainty region and the design is to optimize the worst performance of the system. In stochastic approaches [90], the robustness is obtained from a probabilistic feature based on the second-order statistics of the CSI. In [88], the worst-case based robust distributed beamforming problem is studied with the objective to maximize the received SINR subject to individual relay power constraint. Then the worst-case based robust problem is extended to FF relay case in [89], with the objective to minimize the total relay power while SINR was ensured above a predefined threshold and in [90], a robust two-way FF relay beamforming design under stochastic channel uncertainties was considered.

In another aspect, in the relay beamforming schemes above, SNR criteria [7, 8, 72, 91] are mostly used in single-user-pair relay networks with flat-fading channel, while for relay networks with frequency-selective channel and mul-
user relay networks, SINR criteria [39, 70, 71] are more preferred. Furthermore, there are also schemes using mean square error (MSE) as an optimization measure [92–94]. The MMSE-based distributed relay beamforming schemes were comprehensively discussed in [93], where two different schemes were designed: 1) MMSE estimation of the signal from a source node; and 2) MMSE pre-equalization to the destination node. As demonstrated in this article, the MMSE-based beamforming problem can be solved with only local CSI of a relay node. The performance was comparable to the SNR maximization beamforming approaches and even better when long-term power constraint (LTPC) is imposed. Furthermore, a reference signal based relay beamforming scheme was proposed in [94]. In this scheme, with associated adaptive algorithms, each relay node can work out its own optimal weights based on a globally known reference signal, its own CSI and received signal fed back from the receiver.

Due to the important role of channel state information in distributed relay networks, we investigate our proposed multi-pair distributed relay schemes with different channel state stationarity level in Chapter 5, and we also consider the channel state information uncertainty and propose a worst-case based robust scheme for multi-pair distributed relay networks in Chapter 6.
Chapter 3

Filter-and-forward Distributed Relay Beamforming for Cognitive Radio Network

As preliminarily noted, distributed relay beamforming can exploit spatial diversity of distributed network nodes without requiring multiple antennas at each relay node. Among the relay protocols applied at the relay nodes, the FF relay beamforming method is proposed by Chen et al in [39] to specifically combat the frequency selective environments, where the transmitter-to-relay and relay-to-destination channels are assumed to be frequency selective. In such a case, there is a significant amount of inter-symbol-interference (ISI) which will lead to significant performance degradation when flat fading scenarios are used. To combat the ISI, some communication systems employ orthogonal frequency-division multiplexing (OFDM) [95–98]. Although OFDM is gaining much popularity, there are still many applications where multi-carrier transmission tech-
niques are not applicable (like GSM/EDGE mobile communication systems) or not preferred because of the disadvantages of OFDM such as a high peak-to-average power ratio. In such applications, using the FF relay beamforming method can significantly alleviate the IPI.

Meanwhile, distributed relay beamforming has been introduced in the context of cognitive radio to improve its performance in recent years [99–103]. In cognitive radio [74, 104–111], a set of secondary users (SUs) can utilize unused spectrum resource (also known as spectrum slots) for their communication, when the communication condition of PU allows. In particular, in a CR network, when a SU intends to communicate with a secondary destination and the channel condition is poor, a set of SUs can work as relays (referred to as CR relays throughout the thesis) and help forward the information [75]. In such a network, interference introduced by the secondary communication at the PU destination must be suppressed. By assuming the interference from each CR relay nodes to the primary receiver is synchronized, [75] proposed a distributed relay beamforming method called zero forcing beamforming (ZFBF), which can maximize the signal-to-noise ratio (SNR) at the secondary destination and completely eliminate the interference to the PU. In [112], a so-called leakage beamforming (LBF) method is proposed that maximizes the signal power at the secondary destination while suppressing the asynchronous leakage interference at the primary receiver.

In the work of [75] and [112], the communication channels are assumed to be flat-fading and the CR relay nodes are working in the DF mode. However,
in practical scenarios, since these channels are more likely to be frequency selective, the FF protocol could be employed at CR relay nodes instead. In this chapter, we study the FF relay distributed beamforming problem for CR systems with frequency selective channels. In the frequency-selective-channel scheme, due to employment of the FF protocol instead of the DF protocol, there would be noise introduced by CR relay nodes. Thus, the ZFBF method can not be used and the interference cannot be eliminated completely. In this situation, our proposed method shows its capability of combating the frequency selective channel distortion and suppressing the leakage interference power to the PU. Furthermore, in our second proposed scheme, we study the situation when the CR relay output power constraint is applied, which can be transformed to a SOCP problem [113] by applying bisection search procedure, and solved using the interior point method [114]. At last, we present the conditions under which the second scheme can be simplified to either of two sub-schemes, with closed-form solution provided.

In this chapter, we first present the overall system model in Section 3.1. Then in Section 3.2, we develop the formulation of maximizing SINR at the secondary destination node while suppressing the leakage interference at the PU, and then we further extend the scheme by adding the relay output power constraint. Section 3.3 presents the numerical simulation results. Finally, conclusions are given in Section 3.4.
3.1 System Model

3.1.1 General System Model

We consider a time-slotted dual-hop distributed CR relay network with frequency-selective channels, with one single-antenna SU source node, one single-antenna secondary destination node and R single-antenna CR relay nodes. As the same in [39], we assume that the direct link between source and destination nodes does not exist and that the transmission is divided into two time-slotted stages. In the first stage, the SU source broadcasts a signal to all CR relay nodes, and in the second stage, each CR relay node filters the received signal and then re-transmit it to the secondary destination. Similar to most of the models in aforementioned literature, the instantaneous CSI of all transmission channels is perfectly known by the secondary receiver node.

Fig. 3.1: The FF relay network signal model
As shown in Fig. 3.1, the SU-source-to-relay channel (fore-channel), relay-to-secondary-destination channels (back-channel) and relay-to-PU channels (interference channels) are represented by FIR filters with impulse responses $f_i$, $g_i$ and $g_i^{(PU)}$, respectively, where

$$f_i = [f_i(0), \ldots, f_i(L_f - 1)]^T$$

$$g_i = [g_i(0), \ldots, g_i(L_g - 1)]^T$$

$$g_i^{(PU)} = [g_i^{(PU)}(0), \ldots, g_i^{(PU)}(L_g^{(PU)} - 1)]^T$$

(3.1)

for the $i$th relay node, with $(\cdot)^T$ denoting transpose, $L_f$, $L_g$ and $L_g^{(PU)}$ are the corresponding FIR filter length. Hence, the signal received at the relay nodes can be modeled as an $R \times 1$ vector $r(n) = [r_1(n), \ldots, r_R(n)]^T$, with $r_i(n)$ given by

$$r_i(n) = \sum_{l=0}^{L_f-1} s(n - l)f_i(l) + n_i(n)$$

(3.2)

where, $s(n)$ is the information-bearing sequence of symbols transmitted by the SU source node with power of $P_s = \mathbb{E}[|s(n)|^2]$, $\mathbb{E}[\cdot]$ is the expectation operation, $\ast$ denotes the convolution sum, and $n_i(n)$ is the additive white Gaussian noise (AWGN), with power of $\sigma_n^2 = \mathbb{E}[|n_i(n)|^2]$.

Then, the received signal $r_i(n)$ passes through the $i$th CR relay filter, with an impulse response $h_i = [h_i(0), \ldots, h_i(L_h - 1)]^T$ and thus generates the transmitted signal $t_i(n)$ from the $i$th relay nodes. Note that the channel impulse responses are assumed to be independent quasi-static, which means that $h_i$ re-
mains unchanged over a frame time.

$$t_i(n) = \sum_{l=0}^{L_h-1} r_i(n - l) h_i(l)$$  \hspace{1cm} (3.3)

Thus, the signal received by the secondary destination node is given by

$$y(n) = \sum_{i=1}^{R} \sum_{l=0}^{L_q-1} t_i(n - l) g_i(l) + v(n)$$

$$= \sum_{i=1}^{R} \sum_{l_e=0}^{L_{eqv}-1} s(n - l_e) h_{eqv}(l_e) + n_{pro}(n) + v(n)$$  \hspace{1cm} (3.4)

where $v(n)$ is the AWGN with power $\sigma_v^2 = \mathbb{E}[|v_i(n)|^2]$, $h_{eqv}(l_e)$ is the $l_e$th vector of $h_{eqv}$ and $h_{eqv} = \sum_{i=1}^{R} f_i \ast h_i \ast g_i$ is the overall equivalent channel impulse response from the SU source to the secondary destination node with the length being $L_{eqv}$. $n_{pro}(n)$ denotes the propagation noise, which are the noise parts in $y(n)$ that are related to $n_i(n)$.

Let $\delta_i^{(PU)}(n)$ denotes the part of leakage signal from the $i$th CR relay nodes, and the leakage signal introduced by CR relays at the primary receiver can be expressed as

$$y^{(PU)}(n) = \sum_{i=1}^{R} \delta_i^{(PU)}(n) = \sum_{i=1}^{R} \sum_{l=0}^{L_q-1} t_i(n - l) g_i^{(PU)}(l)$$  \hspace{1cm} (3.5)

### 3.1.2 System Model in Matrix Form

For convenience of subsequent derivations, we next rewrite the signal model in Section 3.1.1 to a matrix form as in [72]. Firstly, we consider the model between the SU source and the secondary destination.
To begin with, the convolution sum of fore-channel and back-channel related to the $i$th CR relay node can be expressed as follows

$$c_i = f_i * g_i = \tilde{F}_i \cdot g_i = [c_{i,1}, \ldots, c_{i,L_c}]$$  \hspace{1cm} (3.6)

where $L_c=(L_f+L_g-1)$, and $\tilde{F}_i$ is a column-circulant matrix of size $L_c \times L_g$

$$\tilde{F}_i = [F_i(0), \ldots, F_i(L_g - 1)]$$  \hspace{1cm} (3.7)

$$F_i(l) = \begin{bmatrix} l \text{ columns} & (L_g-l-1) \text{ columns} \end{bmatrix}^T$$  \hspace{1cm} (3.8)

Then the equivalent channel factor expression can be rewritten as

$$h_{eqv} = \sum_{i=1}^{R} c_i * h_i = \sum_{i=1}^{R} \tilde{C}_i \cdot h_i = \Psi \mathbf{w}$$  \hspace{1cm} (3.9)

where $\Psi=[\tilde{C}_1, \ldots, \tilde{C}_R]$, $\mathbf{w} = [h_1^T, \ldots, h_R^T]^T$, and $\tilde{C}_i$ is also a column-circulant matrix, with the size of $L_c \times L_h$ ($L_c=L_f+L_g+L_h-2$), defined by

$$\tilde{C}_i=[C_i(0), \ldots, C_i(L_h)]$$  \hspace{1cm} (3.10)

$$C_i(l) = \begin{bmatrix} l \text{ columns} & (L_h-l-1) \text{ columns} \end{bmatrix}^T$$  \hspace{1cm} (3.11)

The propagation noise $n_{pro}(n)$ can also be expressed in a matrix form

$$n_{pro}(n) = \sum_{i=1}^{R} h_i^H \bar{G}_i^T \mathbf{n}_i(n)$$  \hspace{1cm} (3.12)
where \((\cdot)^H\) denotes the Hermitian transpose and \(\bar{G}_i\) is a column-circulant matrix with a similar form as \(\bar{F}_i\), given by

\[
\bar{G}_i = [G_i(0), \cdots, G_i(L_h - 1)]
\]

(3.13)

\[
G_i(l) = \begin{bmatrix}
0 \cdots 0 \\
\vdots \\
(\text{l columns}) \\
\vdots \\
0 \cdots 0 \\
(\text{L}_h - \text{l} - 1 \text{ columns})
\end{bmatrix}^T
\]

\[l = 0, \cdots, L_h - 1\]

(3.14)

\(n_i(n)\) in (3.12) is the relay noise vector with \(n_i(n) = [n_i(n), n_i(n-1), \cdots, n_i(n-L_g - L_h + 2)]^T\).

From (3.9) and (3.12), we can rewrite the signal model (3.4) into

\[
y(n) = w^H \bar{\psi} s(n) + \sum_{i=1}^{R} h_i^H \bar{G}_i^T n_i(n) + v(n)
\]

(3.15)

where \(s(n) = [s(n), s(n-1), \cdots, s(n-L_e + 1)]^T\)

Let \(\bar{\psi}\) and \(\bar{\Psi}\) denote the first row and the remaining part of \(\Psi\) and define \(\bar{s}(n) = [s(n-1), s(n-2), \cdots, s(n-L_g - L_h + 2)]\). Then,

\[
y(n) = \underbrace{w^H \bar{\psi} s(n)}_{\text{Desired signal}} + \underbrace{w^H \bar{\Psi} \bar{s}(n)}_{\text{ISI}}
\]

\[+ \sum_{i=1}^{R} h_i^H \bar{G}_i^T n_i(n) + v(n)
\]

(3.16)

Thus, the expression for the desired signal, ISI and noise components at the secondary destination are obtained. Now for the model between the CR relay
nodes and the primary receiver, with similar derivation as above, (3.5) can be rewritten in matrix form as

\[ y^{(PU)}(n) = \sum_{i=1}^{R} \delta_i^{(PU)}(n) \]

\[ = \sum_{i=1}^{R} h_i^H \bar{C}_i^{(PU)^T} s_i(n) + h_i^H \bar{G}_i^{(PU)^T} n_i(n) \]  \hspace{1cm} (3.17)

where \( \bar{C}_i^{(PU)} \) and \( \bar{G}_i^{(PU)} \) are two column-circulant matrices with the similar structure as \( \bar{C}_i \) and \( \bar{G}_i \), respectively. They can be obtained by equations (3.6), (3.10), (3.11), (3.13) and (3.14), with \( g_i \) replaced by \( g_i^{(PU)} \) and \( L_g \) replaced by \( L_g^{(PU)} \).

### 3.2 Problem Formulation

In the following, two FF beamforming schemes are proposed. In both schemes, global CSI is assumed known by the secondary user nodes, and the derivations of relay weights are performed on either of the secondary user nodes and then the relay nodes are informed with their weights by a backhaul link. In our first scheme, the aim is to maximize the SINR at the secondary destination node, while minimizing the leakage interference signal power at the PU. In the second one, a CR relay output power constraint is added into the firstly considered problem.
3.2.1 Maximization of SINR at the Secondary Destination

We first derive the expressions for the power of desired signal, ISI and propagation noise at the secondary destination from (3.16).

\[ E\{|y(n)|^2\} = w^H Q_s w + w^H Q_i w + w^H Q_n w + \sigma_v^2 \]  

(3.18)

where,

\[ Q_s = P_s \cdot \bar{\psi}^T \bar{\psi}^* \]

\[ Q_i = P_s \cdot \bar{\Psi}^T \bar{\Psi}^* \]

\[ Q_n = \sigma_n^2 \cdot \text{blkdiag}\{\bar{G}^T_1 \bar{G}_1^*, \ldots, \bar{G}^T_R \bar{G}_R^*\} \]  

(3.19)

Accordingly, power of the desired signal part, inter-symbol-interference and propagation noise can be expressed by,

\[ P_{\text{desire}} = w^H Q_s w \]

\[ P_{\text{ISI}} = w^H Q_i w \]

\[ P_{\text{Npro}} = w^H Q_n w \]  

(3.20)

And the leakage signal power at the PU can be derived from (3.17),

\[ P_{\text{leak}} = E\{|y^{(PU)}(n)|^2\} = w^H Q_{\text{leak}} w \]  

(3.21)
where $Q_{\text{leak}}$ is given by

\[
Q_{\text{leak}} = Q_{\text{leak,s}} + Q_{\text{leak,n}}
\]

\[
Q_{\text{leak,s}} = \begin{bmatrix}
\bar{C}_1^{(PU)T} & \bar{C}_1^{(PU)*} & \cdots & \bar{C}_1^{(PU)T} & \bar{C}_R^{(PU)*} \\
\bar{C}_2^{(PU)T} & \bar{C}_1^{(PU)*} & \cdots & \bar{C}_2^{(PU)T} & \bar{C}_R^{(PU)*} \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
\bar{C}_R^{(PU)T} & \bar{C}_1^{(PU)*} & \cdots & \bar{C}_R^{(PU)T} & \bar{C}_R^{(PU)*}
\end{bmatrix}
\]

\[
Q_{\text{leak,n}} = \sigma_n^2 \cdot \text{blkdiag}\{ \bar{G}_1^{(PU)T}, \cdots, \bar{G}_R^{(PU)T} \}
\]

Using the above results, we have the following problem formulation

\[
\max_w SINR = \frac{w^H Q_s w}{w^H Q_i w + w^H Q_n w + \sigma_v^2} \\
s.t. \quad w^H Q_{\text{leak}} w \leq P_N
\]

(3.23)

$P_N$ is the pre-defined threshold for the leakage signal power at the primary receiver. It can be easily proved that the constraint is always active (satisfied with equality) to obtain a maximum SINR, i.e., $w^H Q_{\text{leak}} w = P_N$. By further defining $Q_{in} = Q_i + Q_n$ and $\hat{w} = P_N^{-1/2} \cdot Q_{\text{leak}}^{1/2} w$, problem (3.23) can be changed to

\[
\max_w SINR = \frac{\hat{w}^H \hat{Q}_s \hat{w}}{\hat{w}^H (\hat{Q}_{in} + \frac{\sigma_v^2}{P_N} \cdot I) \hat{w}} \\
s.t. \quad ||\hat{w}||^2 = 1
\]

(3.24)
where
\[ \hat{Q}_s = Q_{\text{leak}}^{-1/2} Q_s Q_{\text{leak}}^{-1/2} \]  

(3.25)

This is a standard generalized eigenvalue problem (GEP) that can be transformed into an eigenvector problem (EP) [115]. Defining \( \hat{Q} = (\hat{Q}_{\text{in}} + \sigma_v^2 P_N^{-1} \mathbf{I}) \), the solution of this EP is given by the principle eigenvector of \( \hat{Q}^{-1} \hat{Q}_s \) [6], i.e.,

\[ \hat{w}_{\text{opt}} = \rho\{\hat{Q}^{-1} \hat{Q}_s\} = \rho\{P_S \cdot \hat{Q}^{-1} Q_{\text{leak}}^{-1/2} \psi \psi^T \} \]  

(3.26)

where \( \rho\{\cdot\} \) denotes the principle eigenvector of a matrix. As in [116], the closed-form solution to such a problem in (3.26) can be obtained by

\[ \hat{w}_{\text{opt}} = \hat{Q}^{-1} \hat{\alpha} / \sqrt{\hat{\alpha}^H \hat{Q}^{-2} \hat{\alpha}} \]

\[ w_{\text{opt}} = P_1^{1/2} \cdot Q_{\text{leak}}^{-1/2} \hat{w}_{\text{opt}} \]  

(3.27)

where \( \hat{\alpha} = P_S^{1/2} \cdot Q_{\text{leak}}^{-1/2} \psi \mathbf{T} \), and the optimal SINR is

\[ \text{SINR}_{\text{max}} = P_N \cdot \hat{\alpha}^H \hat{Q}^{-1} \hat{\alpha} \]  

(3.28)

### 3.2.2 Maximization of SINR at the Secondary Destination With CR Relay Output Power Constraint

Now let us consider the beamforming problem with CR relay output power constraint. As noted before, \( r(n) = [r_1(n), \ldots, r_R(n)]^T \) denotes the received signal vector at relay nodes. The transmitted signal vector from each relay node to destination node is thus given by

\[ t_i(n) = r_i(n) * h_i(n) = s(n) * f_i(n) * h_i(n) + n_i(n) * h_i(n) \]  

(3.29)
Furthermore, we rewrite (3.29) in matrix form

\[ t_i(n) = (\tilde{F}_i h_i)^T \hat{s}(n) + h_i^T \hat{n}_i(n) \]  

(3.30)

\( \hat{s}(n) \) is the received signal vector at relay node, \( \hat{s}(n) = [s(n), s(n-1), \cdots, s(n-L_f - L_h + 2)]^T \), and \( \hat{n}_i(n) = [n_i(n), n_i(n-1), \cdots, n_i(n-L_h + 1)]^T \) represents the relay noise vector. \( \tilde{F}_i \) is an column-circulant matrix with a similar form as \( \bar{F}_i \)

\[ \tilde{F}_i = [\tilde{F}_i(0), \cdots, \tilde{F}_i(L_h - 1)] \]  

(3.31)

\[ \tilde{F}_i(l) = [0, \cdots, 0, f_i, 0, \cdots, 0] \]  

(3.32)

Now the output power at relay node is given by

\[ P_0 = \sum_{i=1}^{R} \{E|t_i(n)|^2\} \]

\[ = \sum_{i=1}^{R} h_i^T (P_s \cdot \hat{F}_i \hat{F}_i^* + \sigma^2 \cdot I_{L_h}) h_i^* = w^H D w \]  

(3.33)

where

\[ D = P_s \cdot \text{blkdiag}\{F_1^T F_1^*, \cdots, F_R^T F_R^*\} + \sigma^2_{n} \cdot I_{RL_h} \]  

(3.34)

Accordingly, the problem is formulated as

\[ \max_{w} \quad \text{SINR} \]

s.t. \[ w^H Q_{\text{leak}} w \leq P_N \]

\[ w^H D w \leq P_0 \]  

(3.35)
By introducing an auxiliary variable $\mu < 0$ [113], (3.35) can be rewritten as

$$\max_{w, \mu} \mu$$

s.t. $\frac{w^HQ_sw}{w^HQ_iw + w^HQ_nw + \sigma_v^2} \geq \mu^2$

$$w^HQ_{\text{leak}}w \leq P_N$$

$$w^HDw \leq P_0$$

(3.36)

which can be changed into a standard SOCP, as in [39]

$$\max_{w, \mu} \mu$$

s.t. $\sqrt{P_s}\tilde{w}^H\tilde{h} \geq \mu||\tilde{U}\tilde{w}||$

$||\tilde{V}_Q\tilde{w}|| \leq P_N$

$||\tilde{V}_D\tilde{w}|| \leq P_0$

$$\tilde{w}_{\text{first}} = 1$$

(3.37)

where $\tilde{w} = [1, w^T]^T$, $\tilde{V}_D = [0_{RL_h \times 1}, V_D]$, $\tilde{V}_Q = [0_{RL_h \times 1}, V_Q]$, $\tilde{h} = [0, h^T]^T$, and $\tilde{w}_{\text{first}}$ denotes the first element of $\tilde{w}$, with

$$\tilde{Q} = \begin{bmatrix} \sigma_v^2 & 0_{1 \times RL_w} \\ 0_{RL_w \times 1} & Q_i + Q_n \end{bmatrix} = \tilde{U}^H\tilde{U}$$

$$D = \tilde{V}_D^H\tilde{V}_D$$

$$Q_{\text{leak}} = \tilde{V}_Q^H\tilde{V}_Q$$

(3.38)

Note that $U$, $V_D$ and $V_Q$ are the Cholesky factorization product of matrix $\tilde{Q}$, $D$ and $Q_{\text{leak}}$, respectively.
The SOCP (3.37) can be solved by firstly reduce it to a SOCP feasibility problem by assigning a value of $\mu$ using bisection search procedure [117] and then solved using the interior point method [113] or some other interior-point-based methods, for example, the SeDuMi package [118] which produces a feasibility certificate if the problem is feasible.

Now consider the following sub-schemes which are both related to the previously presented problem formulation:

$$\max_\mathbf{w} \quad \text{SINR}$$
$$s.t. \quad \mathbf{w}^H \mathbf{Q}_{\text{leak}} \mathbf{w} \leq P_N$$  \hspace{1cm} (3.39)

and

$$\max_\mathbf{w} \quad \text{SINR}$$
$$s.t. \quad \mathbf{w}^H \mathbf{Dw} \leq P_0$$  \hspace{1cm} (3.40)

Both can be solved using the same approach as in Section 3.2.1. Let us denote the solution to problem (3.39) and (3.40) as $\mathbf{w}_{\text{opt1}}$ and $\mathbf{w}_{\text{opt2}}$, respectively. Under specific conditions, problem (3.35) can be transformed into either of the above sub-schemes.

**Condition 1:** If $\mathbf{w}_{\text{opt1}}^H \mathbf{Dw}_{\text{opt1}} \leq P_0$, $\mathbf{w}_{\text{opt2}}^H \mathbf{Q}_{\text{leak}} \mathbf{w}_{\text{opt2}} > P_N$, (3.35) is transformed to sub-scheme (3.39), and the solution is $\mathbf{w}_{\text{opt1}}$.

**Condition 2:** If $\mathbf{w}_{\text{opt1}}^H \mathbf{Dw}_{\text{opt1}} > P_0$, $\mathbf{w}_{\text{opt2}}^H \mathbf{Q}_{\text{leak}} \mathbf{w}_{\text{opt2}} \leq P_N$, (3.35) is transformed to sub-scheme (3.40), and the solution is $\mathbf{w}_{\text{opt2}}$.

**Condition 3:** $\mathbf{w}_{\text{opt1}}^H \mathbf{Dw}_{\text{opt1}} \leq P_0$, $\mathbf{w}_{\text{opt2}}^H \mathbf{Q}_{\text{leak}} \mathbf{w}_{\text{opt2}} \leq P_N$ can only be satisfied when $\mathbf{w}_{\text{opt1}}^H \mathbf{Dw}_{\text{opt1}} = P_0$, and $\mathbf{w}_{\text{opt2}}^H \mathbf{Q}_{\text{leak}} \mathbf{w}_{\text{opt2}} = P_N$. And in this case,
\( w_{opt1} \) and \( w_{opt2} \) are identical.

**Condition 4:** If \( w_{opt1}^H D w_{opt1} > P_0 \), \( w_{opt2}^H Q_{\text{leak}} w_{opt2} > P_N \), (3.35) cannot be transformed into either of the sub-schemes, and it remains being solved as SOCP.

### 3.3 Simulation Results

In our simulations, we consider the FF beamforming relay cognitive network with the number of CR relay nodes being \( R = 5 \). The transmission channels between source to relay nodes and relay to destination nodes are quasi-static frequency selective Rayleigh fading channels, with channel impulse response coefficients being zero-mean complex Gaussian random variables with exponential power delay profile [73] \( p(n) = (1/\sigma_t) \cdot \sum_{l=0}^{L_k-1} e^{-n/\sigma_t} \delta(n - l) \), where \( \delta(\cdot) \) represents the Dirac delta function, and \( \sigma_t \) represents the delay spread factor (here \( \sigma_t = 2 \) is used). \( L_k \in \{L_f, L_g, L_g^{(PU)}\} \), represents the length of fore-channel, back-channel and interference channel, and in our case, \( L_f = L_g = L_g^{(PU)} = 5 \) is assumed, unless otherwise specified. For the channels between the CR relay nodes and the primary receiver, log-distance path loss model is also considered with a path loss exponent value of 3. The average distance between the CR relay nodes and the primary receiver is assumed to be 10 times the average distance between the SU source and the CR relay nodes. Thus an additional 30 dB path loss is considered for the leakage interference signal at the primary receiver. It is assumed that the noise power at CR nodes and secondary destination nodes
are identical, $\sigma_v^2 = \sigma_n^2 = 1$, and the transmitted signal power at the SU source node is 10 dB higher than the noise.

![Figure 3.2: SINR versus the leakage threshold $P_N$ and relay filter length $L_h$.](image)

In Fig. 3.2, we present the output SINR performance of our proposed approach of (3.23), versus the threshold of leakage interference power on the primary receiver, for different relay filter length. It can be seen that when the length of relay filter increases, the SINR is improved. And SINR also improves as the threshold $P_N$ is set higher, meaning the primary receiver will suffer higher leakage interference from our secondary communication. However, it can also be seen from our result that the SINR turns to be stable before the leakage interference power becomes significantly high.
Fig. 3.3: Probability of Condition 1, $L_h=4$ (second scheme)

Fig. 3.4: Probability of Condition 2, $L_h=4$ (second scheme)
Figs. 3.3-3.5 depict the probability of beamforming weights satisfying each decision condition of scheme 3.35, versus the ratio of the relay power threshold $P_0$ to the leakage signal power threshold $P_N$, when different $P_N$ is chosen. Note that, since **Condition 3** is proved to be only valid when it is a special case of **Condition 1** and **Condition 2**, it is not considered here. It can be seen from Fig. 3.3 and Fig. 3.5, that the probability of satisfying **Condition 1** increases when there is a higher ratio of $P_0$ to $P_N$, and the situation for **Condition 2** is just the opposite. For **Condition 4**, Fig. 3.5 indicates that its probability first increases from a very low level and then decreases, as the ratio increases. Also we can see that the probability of **Condition 2** is not much affected by the choice of $P_N$. However, when $P_N$ increases the probability of **Condition 1** increases, and
for Condition 4 the probability peak decreases rapidly. This indicates that a larger value of $P_N$ is good for reducing the computational complexity of problem (3.35). However, this will lead to high leakage interference power at the primary receiver. Above all, $P_0$ and $P_N$ will jointly determine the probability of each condition, and when the probability peak in Fig. 3.5 is avoided, problem (3.35) is more likely to be transformed into simpler sub-schemes.

### 3.4 Summary

In this chapter, the problem of distributed relay beamforming in cognitive network with frequency-selective channel has been studied. A beamforming method was proposed to combat the leakage signal interference and frequency-selective channel distortion at the same time in such a network. Then, we further extended the method to restrict the output relay power in our second scheme. We also provided the conditions under which the extended scheme can be transformed into two simpler sub-schemes, and the probability of beamforming weights satisfying such conditions is demonstrated by our numerical simulation results. The results also reveal that by carefully designing each parameter, computational complexity of the given FF relay scheme of (3.35) can be modified and the corresponding simulation results can be used to assist this work.
Chapter 4

Iterative Transceiver Beamformer Design for Multi-Pair Two-Way Distributed Relay Networks

Multipair relay networks are one of the specific research area of relay networks. Due to its advantages in coverage extension, mitigating the effect of fading and enhancement of network throughput, distributed relay assisted networks have attracted much attention in the past decade [5, 6, 8–10, 23, 26, 80–83]. In such networks, distributed relay nodes create a virtual multiple-input multiple-output (MIMO) environment, where beamforming techniques can be applied to regulate the performance of the network.

For a multipair two-way relay network, the main bottleneck is the inter-pair interference (IPI) caused by simultaneous signal transmission of multiple user pairs. In [25, 26, 28, 35], beamforming methods base on zero forcing (ZF) were proposed for IPI cancellation. Meanwhile, [30] studied the scheme of block-
diagonalization (BD), which is employed at one central relay node with multiple antennas. In [31], a coordinated eigen-beamforming scheme was proposed where multi-antenna user node and multi-antenna relay node are assumed, and the beamforming weights at user nodes and relay node are jointly determined to maximize the effective channel gain between user pairs. A similar scheme was studied in [119], where the user pairs are also equipped with multiple antennas, and the signal space alignment (SSA) method is used for transceiver beamforming to reduce the effective number of interference, with an enhanced ZF method for relay beamforming. The work in [35] studied the distributed single-antenna relay networks with multipair two-way communication, where a relatively complicated ZF method was applied to eliminate the IPI completely and guided the relay weights setting. In [27], a similar network was considered, and the implementation of the relay nodes was simplified. However, both methods require a very large relay number.

In all the aforementioned multi-pair distributed relay network designs, user nodes are assumed to have single antenna implementation, and the main signal processing procedures and beamforming weights determination processes are performed at the relay nodes, and this will take significant resources from the relay nodes, such as time, computational capacity and processing power. Moreover, in next generation wireless communication systems like LTE/LTE-Advanced/5G [40], multi-antenna user equipments (UEs) are accepted as elementary system setup, and with the development of multi-antenna devices and coordinated multi-point joint-transmission techniques [41–43] where multiple
UEs collaborate and jointly steer the transmit signal, investigating the problem of how the communication of multi-antenna devices and/or virtual multi-antenna devices can benefit in a distributed relay network becomes more and more practical and important.

To our best knowledge, the beamforming problem in such a network has not been investigated yet. In our considered network, the user pairs are assumed to have multiple antennas, some or the main parts of the signal processing tasks could be shifted to the user nodes. If the resources requirement for the relay is reduced, more devices can potentially be utilized as distributed relay nodes, and help forward signals for user pairs with their spare resources.

Motivated by this, in this chapter and the chapters that follow, we focus on a multipair two-way distributed relay network with multi-antenna users from one user group simultaneously transmitting signals to their user partners in the other user group via distributed relay nodes working in the simple amplify-and-forward (AF) mode, and two iteration-based transceiver beamforming schemes are proposed for coordination of the user pairs, where the beamforming vectors are decided at the user side, instead of the relay nodes. Furthermore, supported by simulation results, we propose a possible way to reduce iteration steps without noticeable performance sacrifice in this scenario.

In this chapter, the transceiver beamforming design problem for multipair two-way distributed relay networks is first introduced in Section 4.1, where each multi-antenna user in one user group communicate with its partner in the other user group via distributed single-antenna relay nodes. The proposed iterative
ZF scheme and iterative SINR optimization scheme are presented in Section 4.2. Simulation results and relevant discussions are provided in Section 4.3 and conclusions are drawn in Section 4.4.

### 4.1 System Model

We consider a time-slotted dual-hop multipair two-way distributed relay network consisting of $K$ multi-antenna communication pairs (each is equipped with $N$ antennas) which are divided into two groups ($X_a$, $X_b$) as shown in Fig. 4.1. We assume that the distance between the two groups are long enough compared to their transmission power that the direct link does not exist, and the transmission between user pairs is assisted by $M$ single-antenna distributed relay nodes between them.

![Fig. 4.1: Model for the distributed relay network.](image)

Two transmission phases are considered. In the multiple-access phase, the
users transmit information stream to the relay nodes simultaneously with transmit beamforming. Then in the broadcast phase, the distributed relay nodes use low-complexity AF protocols to broadcast the signals back to the user nodes.

The transmission channels are assumed to be Rayleigh fading, reciprocal and quasi-stationary, so that the channel gains remain unchanged during the two time slot phases.

In the first time slot, the transmitted signal from user \( X_{a,i} \) and \( X_{b,i} \) \((i = 1, \ldots, K)\) to the relay nodes are

\[
x_{a,i} = a_i x_{a,i}, \quad x_{b,i} = b_i x_{b,i}, \quad i \in \{1, \cdots, K\},
\]

(4.1)

where \( x_{a,i} \) and \( x_{b,i} \) are the data symbol. \( a_i, b_i \in \mathbb{C}^{N \times 1} \) are the transmit beamforming vectors, which satisfy the total transmit power constraint \( ||a_i||^2 \leq P_s \) and \( ||b_i||^2 \leq P_s \), with \( P_s \) being the upper bound. Then the signals received at the relay can be represented by an \( M \times 1 \) vector \( r \), given by

\[
r = \sum_{i=1}^{K} F_i a_i x_{a,i} + \sum_{i=1}^{K} G_i b_i x_{b,i} + n_R,
\]

(4.2)

where \( F_i, G_i \in \mathbb{C}^{M \times N} \) are the channel matrix from user \( X_{a,i} \) and \( X_{b,i} \) to the relay nodes, respectively. \( n_R \in \mathbb{C}^{M \times 1} \) denotes the complex Gaussian noise vector of relay nodes with the distribution \( \mathcal{CN}(0, \sigma_r^2 I) \). Then, each relay node amplifies the received signal to generate the transmit signal \( r_T \) as

\[
r_T = Wr,
\]

(4.3)

where \( W \in \mathbb{C}^{M \times M} \) is diagonal, and \( r_T \) is subject to a total power constraint \( P_R \).
In the second time slot, the relay nodes broadcast the scaled versions of the received signals to all users. Let $y_{a,i}$ and $y_{b,i}$ represent the signal received at the user node $X_{a,i}$ and $X_{b,i}$, respectively. Due to the reciprocal channel assumption, we have

$$
y_{a,i} = F_i^T W G_i b_{i,a,i} + F_i^T W F_i a_{i,a,i} + F_i^T W n_{R} + n_{a,i} + F_i^T W \sum_{j \neq i}^K (F_j a_j x_{a,j} + G_j b_j x_{b,j}), \tag{4.4}
$$

where $n_{a,i}, n_{b,i} \in \mathbb{C}^{N \times 1}$ denote the complex Gaussian noise vectors of user node $X_{a,i}$ and $X_{b,i}$, respectively, with the distribution $\mathcal{C}\mathcal{N}(0, \sigma_u^2 I)$. Here the expressions for the desired signal, self interference (SI), IPI and noise are obtained. Since each user knows its own transmitted signal, the SI signal can be removed from $y_{a,i}$ and $y_{b,i}$ through some standard adaptive filtering techniques and for simplicity, we will omit them in the following derivation. The estimated desired symbol after cancelling SI and applying receive beamforming can be expressed as

$$
\bar{y}_{a,i} = c_i^H y_{a,i}, \quad \bar{y}_{b,i} = d_i^H y_{b,i}, \tag{4.6}
$$
where \( c_i, d_i \in \mathbb{C}^{N \times 1} \) denote the beamforming vectors, and they are assumed to be unit vectors in our work (\( ||c_i||^2 = 1, ||d_i||^2 = 1 \)).

### 4.2 Problem Formulation

In the following, two transceiver beamforming designs will be proposed for the multipair two-way distributed relay beamforming network. In the first design, an iterative zero-forcing-based scheme is proposed aiming at eliminating the IPI, where an iterative algorithm is used to achieve coordination of beamforming vectors of the two user groups. In the second one, it is focused on an iterative transceiver beamforming scheme by maximizing the SINR at each user node.

#### 4.2.1 Iterative Zero-Forcing Design

In order to derive the expression for IPI, we first define the overall uplink channel matrix of the IPI (containing the transmit beamforming vectors) of the \( i \)th user pair as \( \tilde{\Omega}_i \in \mathbb{C}^{M \times 2K-2} \), which is given by

\[
\tilde{\Omega}_i = [\Omega_1 \cdots \Omega_{i-1} \Omega_{i+1} \cdots \Omega_K],
\]

(4.7)

where \( \Omega_i = [F_i a_i \ G_i b_i] \in \mathbb{C}^{M \times 2} \) is the uplink channel matrix of the \( i \)th pair. Then from (4.4), (4.5), (4.6) and (4.7), we can obtain the IPI signal received at the \( i \)th user pair as

\[
y_{a,i}^{IPI} = c_i^H F_i^T W \tilde{\Omega}_i \tilde{x}_i,
\]

\[
y_{b,i}^{IPI} = d_i^H G_i^T W \tilde{\Omega}_i \tilde{x}_i,
\]

(4.8)
where  \( \tilde{x}_i = [x_{a,1} \ x_{b,1} \ \cdots \ x_{a,i-1} \ x_{b,i-1} \ x_{a,i+1} \ x_{b,i+1} \ \cdots \ x_{a,K} \ x_{b,K}] \) consists of all the transmit user symbols other than those coming from the \( i \)th user pair.

According to (4.8), in order to completely eliminate the IPI, \( c_i \) and \( d_i \) should lie in the null space of \( F_i^T W \tilde{\Omega}_i \) and \( G_i^T W \tilde{\Omega}_i \), respectively. The null space exists when the condition \( N > 2K - 2 \) is satisfied. We can define the singular value decomposition (SVD) of the two matrix products as

\[
\begin{align*}
\Psi_{X_{a,i}} &= F_i^T W \tilde{\Omega}_i = [U_{X_{a,i}}^{(1)} \ U_{X_{a,i}}^{(0)}] \Sigma_{X_{a,i}} V_{X_{a,i}}^H, \\
\Psi_{X_{b,i}} &= G_i^T W \tilde{\Omega}_i = [U_{X_{b,i}}^{(1)} \ U_{X_{b,i}}^{(0)}] \Sigma_{X_{b,i}} V_{X_{b,i}}^H,
\end{align*}
\]

where \( U_{X_{a,i}}^{(1)} \) and \( U_{X_{b,i}}^{(1)} \) hold the left singular vectors of non-zero singular values of the corresponding left-hand-side matrices, while \( U_{X_{a,i}}^{(0)} \) and \( U_{X_{b,i}}^{(0)} \) hold the left singular vectors of zero singular values of \( \Psi_{X_{a,i}} \) and \( \Psi_{X_{b,i}} \), respectively.

To cancel IPI completely, for the receive beamforming vectors \( c_i \) and \( d_i \) we can choose any column vectors of \( U_{X_{a,i}}^{(0)} \) and \( U_{X_{b,i}}^{(0)} \). However, the undetermined transmit beamforming vectors \( a_i \) and \( b_i \) will affect the values of \( U_{X_{a,i}}^{(0)} \) and \( U_{X_{b,i}}^{(0)} \), and we also need to find appropriate values for \( a_i \) and \( b_i \) for a complete solution. To avoid iteration, an effective method is to apply the eigen-beamforming approach at the transmitter side. In detail, \( a_i \) and \( b_i \) are generated as the eigenvectors corresponding to the largest eigenvalues of \( F_i^H F_i \) and \( G_i^H G_i \), respectively.

However, to obtain a better performance, \( a_i \) and \( b_i \) should maximize the real equivalent channel gain, taking into consideration the effect of \( c_i \) and \( d_i \). From
(4.4), (4.5) and (4.6), we can formulate the problem as follows,

$$\max_{\mathbf{b}_i} \quad C_{X_{a,i}} = |\mathbf{c}_i^H \mathbf{F}_i^T \mathbf{W} \mathbf{G}_i \mathbf{b}_i|^2,$$

s.t. \(|\mathbf{b}_i|^2 \leq P_s,\)

$$\max_{\mathbf{a}_i} \quad C_{X_{b,i}} = |\mathbf{d}_i^H \mathbf{G}_i^T \mathbf{W} \mathbf{F}_i \mathbf{a}_i|^2,$$

s.t. \(||\mathbf{a}_i||^2 \leq P_s,\) \hspace{1cm} (4.10)

where \(C_{X_{a,i}}\) and \(C_{X_{b,i}}\) represents the overall equivalent channel gain for the desired signal received at user nodes \(X_{a,i}\) and \(X_{b,i}\), respectively. It is difficult to derive a closed-form solution for (4.9) and (4.10), and here we propose an iterative algorithm to alternately optimize the transmit and receiver beamforming vectors, and make sure no update is required during the iteration for either the relay node or the user node from the other group.

To start with, we employ the uniform AF mode at the relay node, i.e.

$$\mathbf{W} = \lambda_R \cdot \mathbf{I}_M, \hspace{1cm} (4.11)$$

where \(\mathbf{I}_M \in \mathbb{C}^{M \times M}\) is the unity matrix, and \(\lambda_R\) is a power-control scalar resulting from the total relay power constraint, which can be expressed as

$$\lambda_R = \sqrt{\frac{P_R}{\text{tr}(\mathbf{F}_i \mathbf{a}_i \mathbf{a}_i^H \mathbf{F}_i^H + \mathbf{G}_i \mathbf{b}_i \mathbf{b}_i^H \mathbf{G}_i^H + \sigma_r^2 \cdot \mathbf{I}_M)}}. \hspace{1cm} (4.12)$$

Note that the value of \(\lambda_R\) does not affect the solution of (4.9) and (4.10), we can consider it at the final step of our iteration process. First, the initial values of the receive beamforming vectors \(\mathbf{c}_i\) and \(\mathbf{d}_i\) are assigned as \([\delta_M \delta_M \cdots \delta_M] \in \mathbb{C}^{1 \times N}\), where \(\delta_M = \sqrt{M}\). Then we can calculate \(\mathbf{a}_i\) and \(\mathbf{b}_i\) at each user node.
based on (4.10), given by

\[ a_i = \lambda_{a,i} \cdot F_i^H G_i^* d_i, \quad b_i = \lambda_{b,i} \cdot G_i^H F_i^* c_i, \] (4.13)

where \( \lambda_{a,i} \) and \( \lambda_{b,i} \) are the power-control scalars resulting from the transmit power constraint, given as

\[ \lambda_{a,i} = \sqrt{\frac{P_S}{||F_i^H G_i^* d_i||^2}}, \quad \lambda_{b,i} = \sqrt{\frac{P_S}{||G_i^H F_i^* c_i||^2}}. \] (4.14)

Next, the updated values of \( c_i \) and \( d_i \) can be obtained at each user node from \( U_{X_{a,i}}^{(0)} \) and \( U_{X_{b,i}}^{(0)} \) in (4.9). The updates keep going until a preset maximum iteration number (the choice could be guided by the simulation results) is reached or convergence is achieved. When the final updates of the beamforming vectors are obtained, the power-control scalar \( \lambda_R \) is decided from (4.12). The iterative ZF method is summarized in **Iteration Algorithm Summary**.

Although the iterative ZF method can not guarantee a globally optimum solution due to the non-convexity of the problem, it still outperforms the non-iterative ZF method significantly, as will be shown in our simulations.

**4.2.2 Iterative Algorithm for SINR Optimizing**

The proposed iterative ZF method can completely eliminate the IPI signal received at each user node. However, such a beamformer may lead to undesired amplification of noise, degrading the overall performance. In this section, we propose an iterative algorithm aiming at maximizing the SINR at each user node, which has a better performance compared to the ZF based one.
Without loss of generality, we take user $X_{a,i}$ as an example. From (4.4) and (4.6), the SINR at this user node can be expressed as

$$
SINR_{a,i} = \frac{c_i^H F_i^T Q_{a,i}^{(S)} F_i^* c_i}{\sigma_u^2 + c_i^H F_i^T Q_{a,i}^{(N)} F_i^* c_i + c_i^H F_i^T Q_{a,i}^{(I)} F_i^* c_i},
$$

where,

$$Q_{a,i}^{(N)} = \lambda_R^2 \sigma_r^2 \cdot I_M,$$

$$Q_{a,i}^{(S)} = \lambda_R^2 P s \cdot G_i b_i b_i^H G_i^H,$$

$$Q_{a,i}^{(I)} = \lambda_R^2 P s \sum_{j \neq i}^K (F_j a_j a_j^H F_j^H + G_j b_j b_j^H G_j^H).$$

Apparently, if we only need to consider user node $X_{a,i}$, an ideal way to maximize the $SINR$ is to completely eliminate the IPI by $a_j$ and $b_j$ ($j = 1 \cdots K, j \neq i$), and maximize the remaining part by $c_i$ and $b_i$. However, the optimal choice of $a_j$ and $b_j$ for user node $X_{a,i}$ will unlikely result in an optimal $SINR$ for other user nodes. In fact, it is very difficult, if not impossible, to obtain an analytical global solution for maximizing $SINR$ at every user node for this transceiver beamforming scenario.

As an alternative, we propose an iterative algorithm which can achieve a desirable sub-optimal SINR, while being performed locally at each user node.

At the beginning, we initialize the beamforming vectors $c_i$ and $d_i$ as unity vectors $[\delta_M \delta_M \cdots \delta_M] \in \mathbb{C}^{1 \times N}$, where $\delta_M = \sqrt{M}$. Note that in practice, this initialization step may not be necessary, since the update process can always continue as long as the transmission keeps going, and when the channel state
changes slowly, the iteration number required to achieve convergence can be further reduced.

Then, we update \( a_i \) and \( b_i \) based on maximizing power of the desired signal received at each user node, which is also the numerator of the \( SINR \) expression. For user node \( X_{a,i} \), the SINR expression is given in (4.15) and (4.16), and the case for user node \( X_{b,i} \) is similar. Applying the individual transmit power constraint, after some simple derivations, we can express the updated values for the two transmit beamforming vectors as

\[
a_i = \lambda_{a,i} \cdot F_i^H G_i^* d_i, \quad b_i = \lambda_{b,i} \cdot G_i^H F_i^* c_i,
\]

which are the same as (4.13) in the earlier scheme, and \( \lambda_{a,i} \) and \( \lambda_{b,i} \) have been defined in (4.14). Next, the following SINR optimization problem for user node \( X_{a,i} \) can be solved locally to obtain the receive beamforming vector \( c_i \).

\[
\max_{c_i} \quad SINR_{a,i} = c_i^H \Theta_{a,i} c_i,
\]

\[
s.t. \quad ||c_i||^2 = 1,
\]

where

\[
\Theta_{a,i} = (\Xi_{a,i})^{-1} F_i^T Q_{a,i}^{(S)} F_i^*,
\]

\[
\Xi_{a,i} = \sigma_u^2 I_N + F_i^T Q_{a,i}^{(N)} F_i^* + F_i^T Q_{a,i}^{(I)} F_i^*.
\]

Then, the closed-form solution to this eigenvector problem leads to the updated value for \( c_i \), and similarly for \( d_i \) as well, as expressed in the following

\[
c_i = \rho\{\Theta_{a,i}\}, \quad d_i = \rho\{\Theta_{b,i}\},
\]
where \( \rho \{ \cdot \} \) denotes the principle eigenvector of a matrix.

As summarized in **Iteration Algorithm Summary I** and **Iteration Algorithm Summary II**, this iteration is repeated until a preset maximum iteration number is reached or convergence is achieved, and the relay nodes weights with the power-control scalar \( \lambda_R \) is decided from (4.11) and (4.12) at the final step.

### Iteration Algorithm Summary I

**Iterative Zero-Forcing:**

1) Initialization: \( c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N} \), where \( \delta_N = \sqrt{N} \), and set \( t=1 \).

2) Update \( a_i \) and \( b_i \) based on (4.13) and (4.14).

3) Decide \( W \) based on (4.11) and (4.12).

4) Update \( c_i \) and \( d_i \) based on \( U_{X_{a,i}}^{(0)} \) and \( U_{X_{b,i}}^{(0)} \) in (4.9).

5) If \( |x_i^{(t)} - x_i^{(t-1)|/x_i^{(t)} < \varepsilon \) or \( t > n \) (\( \varepsilon \) is a predetermined value for convergence check of the iterative process, \( x \leftarrow c \) for users from group \( X_a \) and \( x \leftarrow d \) for users from group \( X_b \)), go to the next step. Otherwise, \( t = t + 1 \) and go back to step 2).
**Iteration Algorithm Summary II**

**Iterative SINR Optimization:**

1) Initialization: $c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N}$, where $\delta_N = \sqrt{N}$, and set $t=1$.

2) Update $a_i$ and $b_i$ based on (4.17) and (4.14).

3) Decide $W$ based on (4.11) and (4.12).

4) Update $c_i$ and $d_i$ based on (4.19) and (4.20).

5) If $\frac{|x_i^{(t)} - x_i^{(t-1)}|}{x_i^{(t)}} < \varepsilon$ or $t > n$ ($\varepsilon$ is a predetermined value for convergence check of the iterative process, $x \leftarrow c$ for users from group $X_a$ and $x \leftarrow d$ for users from group $X_b$), go to the next step. Otherwise, $t = t + 1$ and go back to step 2).

**Note:** For both algorithms, the knowledge of all the receive beamforming vector $c_i$ and $d_i$ is required to update $a_j$ and $b_j$ for the $j$th user pair ($j = 1, \ldots, K$). They can all be calculated at the $j$th user pair (extra calculations needed), or shared within each user group using limited backhaul resources to reduce the computational complexity. Another way to reduce the computational complexity is to utilize a central processor (it can be one of the users) within each user group to perform all the computations and inform each user the updates of its beamforming vectors. The computational complexity of our second algorithm is higher than the first one, since the calculations of $c_i$ and $d_i$ updates are more complicated for it. The approximate flops needed for updating $c_i$ or $d_i$ using the two algorithms are $N(2K - 2)(2M - 1) + O(N(2K - 2)^2)$.
and $6M^2 N + 6MN^2 - 3MN - 2N^2 + \mathcal{O}(N^3)$, respectively. In order to demonstrate a clearer comparison of the two algorithms, we consider $M = N = 2K - 2 = \Lambda$ as an example. The approximate flops needed to update $c_i$ (or $d_i$) are $(2\Lambda^3 - 2\Lambda^2 + \mathcal{O}(\Lambda^3))$ and $(12\Lambda^3 - 5\Lambda^2 + \mathcal{O}(\Lambda^3))$, for the iterative ZF algorithm and the SINR optimizing algorithm, respectively.

### 4.2.3 Steps Reduced Iteration Algorithm

Supported by the simulation results, the 3rd step in our Iteration Algorithm Summary III that updates the relay scalar $\lambda_R$ and restricts the relay output power, can be moved to the last, which is outside the iteration loops. And the updated $\lambda_R$ will be used as the initializing value for $\lambda_R$ in the next round of iteration. By doing this, two instead of three steps are required in the iteration procedure, which can reduce the computational complexity furthermore. Note that, this process would possibly degrade the performance. However, when the value of $\lambda_R$ is distributed in a well concentrated region (as the simulations imply), the degradation will not be so significant.

The iteration steps are summarized in Iteration Algorithm Summary III as follows.
Iteration Algorithm Summary III

1) Initialization: \( c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N} \), where \( \delta_N = \sqrt{N} \), 
\( \lambda_R = \sqrt{P_R/(M + M\sigma_r^2)} \) (derived from the expectation of relay output power), and set \( t=1 \). Initialize \( W \) based on (4.11) and (4.12) where the values of \( a_i, b_i, c_i \) and \( d_i \) are set as their initial values.

2) Update \( a_i \) and \( b_i \) based on (4.17) and (4.14).

3) Update \( c_i \) and \( d_i \) based on (4.19) and (4.20).

4) If \( |x_i^{(t)} - x_i^{(t-1)}|^2 < \delta' \) or \( t > n_{\text{max}}' \) (\( x \leftarrow c \) for the user \( X_{a;i} \) and \( x \leftarrow d \) for the user \( X_{b;i} \)), go to the next step. Otherwise, \( t = t + 1 \) and go back to step 2).

5) Decide \( W \) based on (4.11) and (4.12).

4.3 Simulation results

In this section, numerical results are provided to demonstrate the performance of the two proposed transceiver beamforming strategies for multipair two-way distributed relay networks. The channels are assumed to be i.i.d. Rayleigh fading, i.e., the elements of each channel vector are complex Gaussian random variables with zero mean and unit variance. We also assume that the transmit power \( P_S \) is normalized to 1 (compensating the unconsidered path-loss), and the noise powers at all nodes are identical to 1 (\( \sigma_r^2 = \sigma_u^2 = 1 \)). The \( \text{SNR}_R \) is defined to be the ratio of relay node output power to the noise variance, i.e., \( \text{SNR}_R = P_R/\sigma_r^2 \). The value of \( \varepsilon = 0.01 \) is chosen to determine the convergence...
Fig. 4.2: SINR performance of ZF, IZF and ISINR methods with different iteration number (M=6, N=5, K=2, 3).

In Fig. 4.2, we present the average SINR performance of the proposed iterative ZF method (denoted by “IZF”) and the iterative SINR optimization method (denoted by “ISINR”), with $M = 6$, $N = 5$ and $K = 2, 3$. The performance of the two methods with different iteration numbers are provided in comparison with the non-iterative ZF method (denoted by “ZF”). As can be seen, our proposed iterative methods have outperformed the non-iterative ZF method with only 2 iterations, especially for the iterative SINR optimization method, where the improvement is more significant. Clearly, although the iterative SINR optimization method will not necessarily result in the optimum SINR, performance
improvement has been achieved for all iteration number settings; moreover, when the iteration number is increased to 10, the SINR performance is further enhanced. However, further increase of the iteration number leads to much less gain in the result and considering the associated cost for each iteration, the iteration process can then be stopped.

Fig. 4.3 shows the kernel density estimation results demonstrating the distribution of the value of $\lambda_R$ after each round of iteration steps. In statistics, kernel density estimation is a non-parametric way to estimate the probability density function of a random variable [120]. The results suggest that as $SNR_R$ (decided by relay power constraint $P_r$) changes, the majority of $\lambda_R$ values always stay
in the $\pm 20\%$ region of their mean value. That is to say, the value of $\lambda_R$ is well concentrated as long as $SNR_R$ is fixed in the transmission. This result supports us to propose the steps-reduced iteration algorithm in Section 4.2.3 (referred to as 2-step iSINR method later on. We also notice that the distribution of the value of $\lambda_R$ remains similar when the maximum iteration rounds $n$ changes. The performance of the 2-step rSINR method is further tested in the following simulations.

Fig. 4.4 provides the SINR performance versus $SNR_R$ of the proposed 2-step iSINR method with different iteration numbers, where the original iSINR is used as comparison. The result indicates that the 2-step method does not
introduce any noticeable degradation. That is to say, in our iSINR method, the updates of the relay scalar $\lambda_R$ can be moved out of the loops without having noticeable influence on the SINR performance of each user.

In Fig. 4.5, the average SINR performance of the iSINR method with different settings of user pair number and user antenna number ($n_{\text{max}} = 10$, $M = 6$) is presented. The number of iteration rounds is set as 10. Compared to Fig. 4.2, we can see that when the user pair number $K$ increases to satisfy $2K - 2 > N$, the degradation of SINR performance is very severe. The reason is, as described before, when $2K - 2 > N$, the IPI part is impossible to be eliminated by the iZF method. Although our SINR method is designed to suppress the IPI, not to eliminate it,
when the user pair number satisfies $2K - 2 > N$, the lack of ability to suppress the IPI still affects the performance greatly. That is to say, the requirement of $N > 2K - 2$ is essential in our iSINR method.

Then, we present the simulation results when different number of relays is involved in the network, and the maximum number of iteration rounds is set as 10. As can be seen from Fig. 4.6, when the total relay output power is low, the increase of relay number will improve the SINR. However, the improvement becomes very limited as the total relay output power becomes higher. Therefore, in our iSINR method, the relay node does not contribute much to the SINR.
Fig. 4.7: SINR performance of the proposed methods with different iteration rounds (M=6, N=5, K=3).

Next, the simulation results of the proposed methods with different number of iteration rounds are provided in Fig. 4.7. It shows that the performance of our proposed iSINR method and the 2-step iSINR method has no noticeable difference at any number of iteration rounds. We can also notice that although the proposed method does not have the best performance immediately after the initialization step, the average SINR will quickly approach its asymptotic value only after a few rounds of iterations. This pattern applies to different relay number settings and different total relay power budgets.
Fig. 4.8: Convergence performance of the IZF and ISINR methods with different iteration number (M=6, N=5, K=3).

At last, we study the convergence performance of the two schemes in Fig. 4.8, where the convergence probability (the percentage of samples that reaches convergence among a large number of simulations) with different preset maximum iteration number and $SNR_R$ is illustrated. From the figure, we can see that the convergence probability of the iterative SINR optimization scheme is always better than the corresponding IZF scheme, especially when the $SNR_R$ is low. As $SNR_R$ increases, the convergence probability of the second scheme decreases; meanwhile the iterative ZF scheme is not much affected. When the iteration number is large enough, the influence of $SNR_R$ becomes less significant for the iterative SINR optimization scheme. Combined with Fig. 4.2, it
also indicates that the improvement of SINR performance does not necessarily require the scheme to converge. Moreover, in some cases, the beamforming vectors will keep swapping between two values, both of which will lead to a similar and desirable SINR.

4.4 Summary

In this chapter, the transceiver beamforming problem for multipair two-way distributed relay networks has been presented, where the relay nodes are employed with very simple settings and all the computation processes and the main signal processing procedures are performed at the user nodes. In order to achieve a desirable performance, the transmit and receive beamforming vectors from the two separated user groups can be coordinated using iterative methods, where the first iterative method we propose aims to eliminate the IPI and the second one considers maximizing SINR at each user node. Both of them can be performed locally at each user node; however, if data exchange within the same group is allowed, utilization of limited backhaul resource can lead to reduction of the computational complexity. Simulations have been provided to evaluate the performance of the two transceiver beamforming designs in terms of both SINR and convergence speed, and the results indicate that both work effectively and can achieve a better performance with a small iteration number compared to the existing ZF scheme, and the second iterative method outperforms the first proposed method with a higher computational complexity.
Chapter 5

Distributed Iterative Transceiver Beamforming Algorithm for Multipair Two-Way Relay Networks

Reducing IPI and noise in a multipair two-way relay network requires relatively heavy task of computation. In many designs, the computation tasks are globally performed, and assigned to either the users side, or a central relay node. In some of the schemes, the same computation process has to be repeated at each user. It will lead to significant reduction in computational efficiency.

Motivated by this issue, in this chapter we considered the potential of our transceiver beamforming design to distribute the iteration process to all the users and the relay nodes, where the main computation tasks are assigned to each of the user nodes. More specifically, at each node, the iteration process only performs once before forwarding their updated vector to the next node in the iterative algorithm, and thus the computation redundancy can be totally avoided.
It can be predicted that this arrangement will lead to performance degradation at each user node, and the degradation is surely related to the channel stationarity. In this chapter, we will use several simulation results to demonstrate the relation of channel stationarity, SINR performance at each user node and the iterative algorithm.

Moreover, unlike using the most basic uniformly-amplify-and-forward strategy at the relay nodes in the previous design, a carefully designed AF strategy is proposed in this chapter for the relay nodes which allows them to decide their own weights with simple computation process using their local CSI only.

This chapter is organized as follows. In Section 5.1, the system model is introduced. The distributed iterative beamforming algorithm for SINR optimization scheme is presented in Section 5.2. Simulation results and relevant discussions are provided in Section 5.3 and conclusions are drawn in Section 5.4.

### 5.1 System Model

Consider the same time-slotted dual-hop multipair two-way distributed relay network as in previous chapter, which consists of $2K$ multi-antenna users (antenna number = $N$), forming $K$ communication pairs ($X_a$, $X_b$).
Similarly as an example, we here simply repeat the expression for the received signal at $X_{a,i}$ after the receive beamformer.

$$
y_{a,i} = c_i F_i^T W G_{i,b_i} x_{b,i} + c_i F_i^T W F_{i,a_i} x_{a,i} + c_i F_i^T W n_R
+ c_i n_{a,i} + c_i F_i^T W \sum_{j \neq i}^K (F_j a_j x_{a,j} + G_j b_j x_{b,j}), \quad (5.1)
$$

\[\text{Desired signal} \quad \text{Self Interference} \quad \text{IPI}\]

5.2 Distributed Iterative Beamforming Algorithm for SINR Optimization

In this section, motivated by the iSINR method proposed in Chapter 4, we propose the distributed iteration algorithm for SINR optimization (noted as distributed iSINR), where the iteration is divided into three parts: the transmitter part, the relay part and the receiver part. The computation performed at each user node and relay node will only update their own beamforming weights.
Therefore, the power usage for performing the required tasks is much more efficient.

Take user $X_{a,i}$ as an example. From (5.1), the SINR at this user can be expressed as follows,

$$SINR_{a,i} = \frac{c_i^H F_i^T Q^{(S)}_{a,i} F_i^* c_i}{\sigma_u^2 + c_i^H F_i^T Q^{(N)}_{a,i} F_i^* c_i + c_i^H F_i^T \underbrace{Q^{(I)}_{a,i} F_i^* c_i}_{\text{IPI}}}$$

(5.2)

where,

$$Q^{(I)}_{a,i} = P_S \cdot \sum_{j \neq i}^K (WF_j a_j a_j^H F_j^H W^H + WG_j b_j b_j^H G_j^H W^H),$$

$$Q^{(N)}_{a,i} = \sigma_r^2 \cdot WW^H, \quad Q^{(S)}_{a,i} = P_S \cdot WG_i b_i b_i^H G_i^H W^H.$$  (5.3)

As can be seen, if maximizing $SINR_{a,i}$ is the only objective, $a_j$ and $b_j$ ($j = 1 \cdots K, j \neq i$) could be carefully chosen to completely eliminate the IPI part, and the remaining part can be maximized by $c_i$ and $b_i$. However, the optimal choice of $a_j$ and $b_j$ for user $X_{a,i}$ will unlikely result in a sufficiently good SINR for other users, as the beamforming vectors of one user not only affects its own SINR, but also others. In fact, it is very difficult, if not impossible, to obtain an analytical solution for maximizing SINR at all user nodes for this transceiver beamforming scenario.

Therefore, as an alternative, an iterative process composed of the three parts mentioned earlier is employed to achieve a sub-optimal SINR.
5.2.1 Iteration Step on the Transmit Part

We assume that the CSI is either estimated at the user or fed back to it by the relay nodes via low rate feedback channels, so that the beamforming vectors can be decided at the user nodes.

The first iteration step is applied to the user nodes to decide their transmit beamforming vectors $\mathbf{a}_i$ and $\mathbf{b}_i$, for user $X_{a,i}$ and $X_{b,i}$, respectively. At this step, the receive beamforming vectors $\mathbf{c}_i$, $\mathbf{d}_i$ and relay weights $\mathbf{W}$ are fixed to an updated value through previous steps; otherwise, an initial value should be assigned to them. Then, we try to optimize $\mathbf{a}_i$ and $\mathbf{b}_i$ based on maximizing the power of the desired signal received at $X_{a,i}$ and $X_{b,i}$, respectively, under a transmit power constraint.

$$
\max_{\mathbf{b}_i} |\mathbf{c}_i^H \mathbf{F}_i^T \mathbf{W} \mathbf{G}_i \mathbf{b}_i|^2, \text{ s.t. } ||\mathbf{b}_i||^2 \leq P_S, \\
\max_{\mathbf{a}_i} |\mathbf{d}_i^H \mathbf{G}_i^T \mathbf{W} \mathbf{F}_i \mathbf{a}_i|^2, \text{ s.t. } ||\mathbf{a}_i||^2 \leq P_S. \quad (5.4)
$$

These two problems have closed-form solutions, given by

$$
\mathbf{a}_i = \lambda_{a,i} \cdot \mathbf{F}_i^H \mathbf{W}^H \mathbf{G}_i^* \mathbf{d}_i, \quad \mathbf{b}_i = \lambda_{b,i} \cdot \mathbf{G}_i^H \mathbf{W}^H \mathbf{F}_i^* \mathbf{c}_i, \quad (5.5)
$$

where $\lambda_{a,i}$ and $\lambda_{b,i}$ are the power-control scalars

$$
\lambda_{a,i} = \sqrt{\frac{P_S}{||\mathbf{F}_i^H \mathbf{W}^H \mathbf{G}_i^* \mathbf{d}_i||^2}}, \quad \lambda_{b,i} = \sqrt{\frac{P_S}{||\mathbf{G}_i^H \mathbf{W}^H \mathbf{F}_i^* \mathbf{c}_i||^2}}. \quad (5.6)
$$

The obtained transmit beamforming vectors should be forwarded to their user pairs through the relay nodes in order to perform the updates of the other beamforming weights. Until receiving an update for $\mathbf{c}_i$ and $\mathbf{d}_i$, the transmit beamforming vectors should remain constant.
5.2.2 Iteration Step on the Relay Part

The second step is applied to the relay nodes where \( c_i, d_i, a_i \) and \( b_i \) are fixed to their previously updated value. Let \( f_{i,m}, g_{i,m} \in \mathbb{C}^{1 \times N} \) represents the \( m \)-th row of \( F_i \) and \( G_i \), respectively. We propose the following phase rotating rule for the \( m \)-th relay node \((m = 1, \ldots, M)\).

\[
\begin{align*}
\lambda_m &= \lambda_m \left( \sum_{i=1}^{K} f_{i,m}^* c_i b_i^H g_{i,m}^* d_i a_i^H f_{i,m}^H \right) \\
&= \lambda_m \left( \sum_{i=1}^{K} \hat{u}_{i,m}^* v_{i,m}^* + \hat{v}_{i,m}^* u_{i,m}^* \right),
\end{align*}
\]

(5.7)

where \( \hat{u}_{i,m} \triangleq f_{i,m}^* c_i, \hat{v}_{i,m} \triangleq f_{i,m}^* a_i, \hat{u}_{i,m} \triangleq g_{i,m}^* d_i \) and \( v_{i,m} \triangleq g_{i,m}^* b_i \). \( \lambda_m \) is a power-control parameter which limits the output power of each relay node, given by

\[
\lambda_m = \sqrt{ \frac{P_{R,m}}{\sum_{i=1}^{K} \hat{u}_{i,m}^* v_{i,m}^* + \hat{v}_{i,m}^* u_{i,m}^*} \left( \frac{\sigma_r^2}{\sum_{i=1}^{K} |u_{i,m}|^2 + |v_{i,m}|^2} \right) },
\]

(5.8)

where \( P_{R,m} \) is the individual power budget at the \( m \)-th relay.

As will be observed from the updating process for \( c_i \) in Section III-C, \( c_i \) is not directly determined by \( f_{i,m} \) in our scenario, and in fact their correlation is very weak, especially when \( M \) and \( K \) are large. As a result, we can consider them as two independent variables. We have \( ||c_i||^2 = 1 \), and accordingly \( \hat{u}_{i,m} \) has the distribution of \( \mathcal{CN}(0, \Gamma_{i,m}^{u}) \), where \( \Gamma_{i,m}^{u} \) is a constant value decided by the value of \( c_i \) and the variance of \( f_{i,m} \). Similarly, \( v_{i,m}, u_{i,m} \) and \( \hat{v}_{i,m} \) have distribution of \( \mathcal{CN}(0, \Gamma_{i,m}^{v}) \), \( \mathcal{CN}(0, \Gamma_{i,m}^{u}) \) and \( \mathcal{CN}(0, \Gamma_{i,m}^{\hat{v}}) \), respectively.

In order to provide further insight for choosing the phase rotating coefficient on the relay node, we rewrite (5.1) after removing the self interference part, in terms of \( u_{i,m}, v_{i,m}, \hat{u}_{i,m} \) and \( \hat{v}_{i,m} \).
\[
\hat{y}_{a,i} = \sum_{m=1}^{M} \hat{u}_{i,m} w_m v_{i,m} x_{b,i} + \sum_{m=1}^{M} \hat{u}_{i,m} w_m n_{R,m} + n_{a,i} \\
\text{Desired signal} \\
+ \sum_{m=1}^{M} \sum_{j \neq i}^{K} \left( \hat{u}_{i,m} w_m u_{j,m} x_{a,j} + \hat{u}_{i,m} w_m v_{j,m} x_{b,j} \right) \\
\text{IPI} \\
= G_{a,i}^{(S)} x_{b,i} + G_{a,i}^{(Noise)} n_{R,m} + n_{a,i} \\
+ \sum_{j \neq i}^{K} (G_{ab,ij}^{(IPI)} x_{a,j} + G_{ba,ij}^{(IPI)} x_{b,j}), \\
(5.9)
\]

where \( G_{a,i}^{(S)} \), \( G_{a,i}^{(Noise)} \), \( G_{ab,ij}^{(IPI)} \) and \( G_{ba,ij}^{(IPI)} \) represents the gain of each component, \( n_{R,m} \) represents the complex Gaussian noise of the \( m \)-th relay node with the distribution \( CN(0, \sigma_r^2) \) and \( n_{a,i} = d_i n_{b,i} \). Since in our scheme, \( d_i \) is a normalized vector (\( ||d_i||^2 = 1 \)), \( n_{a,i} \) will have a distribution given by \( CN(0, \sigma_u^2) \).

Let \( \hat{y}_{a,i}^{(S)} \), \( \hat{y}_{a,i}^{(IPI)} \) and \( \hat{y}_{a,i}^{(Noise)} \) denote the desired signal, IPI and noise part in (5.9), respectively. We have

\[
\hat{y}_{a,i}^{(S)} = \sum_{m=1}^{M} \lambda_m \hat{u}_{i,m} (\sum_{i=1}^{K} \hat{u}_{i,m}^* v_{i,m}^* + \hat{v}_{i,m}^* u_{i,m}^*) v_{i,m} x_{b,i} \\
(5.10)
\]

\( \hat{u}_{i,m}, \hat{v}_{i,m}, \hat{u}_{i',m}(i' \neq i) \) and \( \hat{u}_{i,m'}(m' \neq m) \) can be considered as zero mean mutually uncorrelated random variables, with \( \mathbb{E}[x^2] = \sigma^2 \), where \( x \sim CN(0, \sigma^2) \).

As a result, we have

\[
\mathbb{E}[G_{a,i}^{(S)}] = \mathbb{E}\left[ \sum_{m=1}^{M} \lambda_m ||\hat{u}_{i,m}||^2 ||v_{i,m}||^2 \right] = \sum_{m=1}^{M} \lambda_m \\
(5.11)
\]
Denote $\gamma_{i,m} = \hat{u}_{i,m} w_m v_{i,m}$ for $m=1, \ldots, M$. As all $\gamma_{i,m}$ are independent random variables, we can apply the Tchebyshev’s inequality theorem [121], and for any constant $\zeta$ obtain

$$\Pr\left[ \left| \frac{G_{a,i}}{M} - \mathbb{E}[G_{a,i}] \right| \geq \zeta \right] \leq \frac{\text{Var}[\hat{y}_{a,i}^{(S)}]/M^2}{\zeta^2} \quad (5.12)$$

where $\Pr[\cdot]$ represents the probability operator. Apparently $\hat{y}_{a,i}^{(S)}/M$ will be more likely to approach $\mathbb{E}[G_{a,i}]/M = \lambda_{\Gamma,m} x_{a,i}$ ($\lambda_{\Gamma,m}$ denotes the average value of $\lambda_m \Gamma^\text{u}_{i,m} \Gamma^\text{v}_{i,m}$) as $M$ increases. As a result, the asymptotic value of $|\hat{y}_{a,i}^{(S)}|^2$ is proportional to $M^2$, when $M$ is large.

Similarly, we can derive that $\mathbb{E}[G_{a,i}^{(Noise)}] = 0$, $\mathbb{E}[G_{ab,ij}^{(IPI)}] = 0$ and $\mathbb{E}[G_{ba,ij}^{(IPI)}] = 0$, and when $M$ is large, $\hat{y}_{a,i}^{(IPI)}/M$ and $\hat{y}_{a,i}^{(Noise)}/M$ will have a high probability of taking a value around 0.

In another word, the $\lambda_m \hat{u}_{i,m} \hat{u}_{i,m}^* v_{i,m} v_{i,m} x_{b,i}$ part in $\hat{y}_{a,i}^{(S)}$ is the only component in $\hat{y}_{a,i}$ that can grow steadily through accumulation as $M$ increases; meanwhile, the other parts will grow much more slowly. The situation is similar for $\hat{y}_{b,i}$ (received signal at $X_{b,i}$).

### 5.2.3 Iteration Step on Receiver Part

In the third step, based on the updated values of $a_i$, $b_i$ and $W$, we determine the receive beamforming vector $c_i$ (similar process for $d_i$) by solving the following SINR optimization problem for user node $X_{a,i}$. From (5.2) and (5.3) we have

$$\max_{c_i} \quad \text{SINR}_{a,i} = c_i^H \Theta_{a,i} c_i, \quad s.t. \quad ||c_i||^2 = 1, \quad (5.13)$$
where

\[
\Theta_{a,i} = (\Xi_{a,i})^{-1} F_i^T Q^{(S)}_{a,i} F_i^*, \\
\Xi_{a,i} = \sigma^2 u I_N + F_i^T Q^{(N)}_{a,i} F_i^* + F_i^T Q^{(I)}_{a,i} F_i^*.
\]

This eigenvector problem can be solved locally at each user node with the closed-form solution given by

\[
c_i = \rho\{\Theta_{a,i}\}, \quad d_i = \rho\{\Theta_{b,i}\},
\]

where \( \rho\{\cdot\} \) denotes the principle eigenvector of a matrix.

It can be seen that in order to determine \( c_i \) at user \( X_{a,i} \), transmit beamforming vectors of all the other users are required. In our scheme, we assume this information is gathered at the relay node first, and then broadcast to all the users with the relay weights information.

### 5.2.4 Summary of the Distributed Iteration Algorithm

In the proposed distributed iteration algorithm, \( a_i \) and \( b_i \) are first decided, by assigning an initial value for the relay weights and the receive beamforming vectors, as indicated in **Summary of Iteration Steps**. Then, \( a_i \) and \( b_i \) remains fixed until the next round of iteration begins.

Each relay node updates its AF weight based on the proposed strategy, only when it has received the complete set of updated \( a_i \) and \( b_i \). Their updated weights should be broadcasted back to the user nodes, and until the updated values of \( c_i \) and \( d_i \) arrive, their weights remain unchanged. Note that the extra
information share of beamforming weights will require some overhead of bandwidth, and it can be transmitted either together with the information streams or independently.

The user nodes perform the iteration step to decide $c_i$ and $d_i$ after they received the updates of all relay weights. After that, the new receive beamforming vectors are sent back to their user pairs through the relay nodes; however, this will not trigger the weight updating process of the relay nodes, which ensures that the relay nodes only update their weights once at each iteration round.

For user $X_{a,i}$, when it receives the transmit beamforming vector updates from its user pair, namely $c_i$, as well as all updated weights of the relay nodes, a new round of iteration begins. We assume that the channels are quasi-stationary for $t_{max}$ rounds of iterations. In detail, we denote $f_{m,n,i}^{(t)}$ and $g_{m,n,i}^{(t)}$, where $t \in (1, t_{max})$, as the channel coefficients of the $t$-th round of iteration, and we assume that $\Delta f_{m,n,i} = f_{m,n,i}^{(t+1)} - f_{m,n,i}^{(t)}$ and $\Delta g_{m,n,i} = g_{m,n,i}^{(t+1)} - g_{m,n,i}^{(t)}$ are i.i.d., and bounded by an upper value $\xi$. After $t_{max}$ rounds of iterations, the channel coefficients are assigned with newly estimated values. The values of $t_{max}$ and $\xi$ together define the level of stationarity of the channel.

Note that during the iteration, the instantaneous output power at some relays may exceed their budgets. However, it can be prevented if the individual power constraint is set well below their output power capability. In fact, supported by the simulation results, the required transmit power at each relay node is modest to give a satisfactory performance, especially when the relay number is large.

As can be seen, the computation task assigned for each user node only de-
terminates their own beamforming vectors, while in the iSINR method proposed in ([122]), each user node has to compute the beamforming vectors of its own and its user pair’s at least. Moreover, for the iSINR method, several iteration steps are required for determining the beamforming vectors before convergence is reached, which is not required in the proposed method. Therefore, the computational complexity of the iSINR method is at least $2t_{\text{conv}}$ times that of the proposed method ($t_{\text{conv}}$ denotes the iteration steps required to reach/approach convergence).

### Summary of Iteration Steps

1) Initialization: $c_i = d_i = [\delta_N \delta_N \cdots \delta_N]^T$, where $\delta_N = \sqrt{N}$, $w_m = \sqrt{\frac{P_{R,m}}{1 + \sigma_r^2}}$ (derived from the expectation of relay output power), and set $t = 1$.

2) Update $a_i$ and $b_i$ based on (5.5) and (5.6).

3) Update $w_m$ based on (5.7) and (5.8).

4) Update $c_i$ and $d_i$ based on (5.14) and (5.15).

5) Go to step 1) if $t \geq t_{\text{max}}$; otherwise, set $t = t + 1$ and go to step 2).

### 5.3 Simulation results

In this section, simulation results are provided for performance evaluation of the proposed method. For simplicity, we set $P_S = 1$ (compensating for the unconsidered path-loss); all relay nodes have the same output power budget of
$P_R/M$, to ensure the same total relay output power for different relay number settings. $P_R/M$ is determined by $SNR_R$, which is the ratio of relay output power constraint to the noise variance, i.e., $SNR_R = P_R/(M\sigma_r^2)$.

![Graph](image)

Fig. 5.2: SINR performance versus $SNR_R$ with different relay number settings ($t_{max}=10$, $N=5$, $K=3$, $\xi=0$).

Figs. 5.2 and 5.3 show the average received SINR versus $SNR_R$ with different number of relay nodes, where a perfect quasi-stationary channel is assumed ($\xi=0$). In Fig. 5.2, the iSINR method from [122] is used as a comparison. Moreover, results based on a non-iterative ZF method (denoted by “ZF”) used in [122] are also provided. Specifically, in this ZF method, the true CSI is considered, $\mathbf{a}_i$ and $\mathbf{b}_i$ are generated as the eigenvectors corresponding to the largest
Fig. 5.3: SINR performance versus SNR with “relay-strategy-only” method as comparison ($t_{\text{max}}=10$, $N=5$, $K=3$, $\xi=0$).

The eigenvalues of $F_i^H F_i$ and $G_i^H G_i$, respectively, and together with $c_i$ and $d_i$, the IPI parts are eliminated completely without any iteration. Both iteration based methods have outperformed the ZF method significantly and the performance of our proposed scheme is the best, at both the low-relay-power and high-relay-power regions. The improvement is more obvious when the relay number is large, and it can also increase the asymptotic SINR by employing more relay nodes in the network, while the original iSINR method can not achieve that. As can be seen from the gap between distributed iSINR ($M=10$) and distributed iSINR ($M=100$), the improvement is very significant, which demonstrates that our proposed relay strategy can well utilize the diversity gain introduced by the
In Fig. 5.3, a “relay-strategy-only” method is used as a comparison where the beamforming vectors $a_i$, $b_i$, $c_i$ and $d_i$ are fixed to their initial values. The figure shows that when only the relay strategy is used in our scheme, the average SINR increases as more relay nodes are employed in the network. However, without the iterative transceiver beamforming steps, the performance is very limited when the relay number is small and the SINR improvement introduced by the transceiver beamforming is significant with any relay number settings.

Fig. 5.4: SINR performance versus iteration rounds with different relay number settings ($N=5$, $K=3$, $\xi=0.1$).

Fig. 5.4 illustrates the average SINR of the proposed method after certain rounds of iterations. As can be seen, although the proposed method does not
have the best performance immediately after the initialization step, the average SINR will quickly approach its asymptotic value only after a few rounds of iterations. And this pattern applies for different relay number settings and different total relay power budgets.

Fig. 5.5: SINR performance of the proposed algorithms with different user pair number and user antenna number (M=20, N=5, K=3, $t_{max}$=10).

Then we present the SINR performance of the proposed method with different settings of user pair number and user antenna number in Fig. 5.5, where $M=20$ and $t_{max}=10$. This figure indicates that, just like the iSINR method, the performance of each user is still much affected by the relationship of user pair
number $K$ and user antenna number $N$, and when $2K-2 > N$, the degradation is still severe. However, with our new relay strategy applied, when the user pair number is large, a satisfactory average SINR performance is still guaranteed.

![Graph](image)

Fig. 5.6: SINR performance of the proposed method with channels of different stationarity level ($t_{\text{max}}=10, M=10, N=5, K=3$).

Fig. 5.6 shows the performance for channels with different stationarity levels. By introducing the random channel difference between different iteration rounds, variance of the global channel states will be affected, which will make the comparison unfair. Accordingly, the simulations are performed after compensating the variance changes. The results demonstrate that our proposed scheme will be affected by the channel stationarity level; however, the degradation is within an acceptable range. When the channel states change smoothly ($\xi=0.1$), the performance degradation is hardly noticeable, compared with the
perfect quasi-stationary channel $\xi=0$.

In Fig. 5.7, the performance of the distributed SINR scheme with different iteration rounds is depicted, with the number of relays $M$ being set as 20. We can see that the channel stationarity will affect the average SINR of each user, and the more rapidly the channel states change, the more the performance degrades. Although a larger number of iteration rounds should lead to a better SINR performance, we can see that the performance degradation is clearer when a large maximum number of iteration rounds is set. The reason is that the iteration rounds also define the frequency of updating the CSI, and when
the iteration number is small, the CSI will be more frequently updated and thus a larger number of iteration rounds will also make the mismatch of CSI more severe and degrade the performance more.

5.4 Summary

On the one hand, an iterative transceiver beamforming algorithm has been proposed for multipair two-way distributed relay networks, where the iteration steps are distributed among user nodes and relay nodes. As a result, the overall computational complexity can be effectively reduced. On the other hand, a relay strategy is designed for the relay nodes which can significantly increase the SINR performance without the need of extra total relay power, and it only requires simple signal processing operations and local CSI for each relay node. Simulation results indicate that the proposed method is quite robust to channel state changes between different rounds of iterations.
Chapter 6

Robust Iterative Transceiver Beamforming for Multipair Two-Way Distributed Relay Networks

In distributed relay networks, CSI is one of the very essential factors that can significantly affect the system scheme design and the performance of transmission. When CSI is not available at the relay nodes, distributed space-time coding and distributed space-time block coding can be used to obtain proper cooperative diversity gain [11, 44–47]. However, with available CSI estimated by the user nodes and/or the relay nodes, distributed relay networks can provide much better performance. In fact, in most of the aforementioned literatures of distributed relay networks, CSI of different transmission paths will directly decide the beamforming vectors of the relay nodes, and in some other distributed relay schemes, CSI will be used to guide relay selection, where some of all the available relay nodes are chosen to forward the information stream to achieve
best quality-of-service (QoS) or to avoid jamming [12, 123–126].

In another aspect, since CSI errors can potentially lead to significant performance degradation, and such errors can hardly be avoided in distributed relay networks, due to inaccurate channel estimation, mobility of relays, and quantization errors, much work has been done for robust designs in distributed relay networks [48–53, 53–56]. In [54], the robust distributed relay beamforming problem was investigated for single-pair one-way relay networks, and a robust relay scheme for multi-user single-destination one-way relay networks was proposed in [51] with the decode-and-forward protocol. In [55], a worst-case based distributed beamforming scheme was developed for a single communication pair with norm-bounded CSI errors. The filter and forward relay beamforming scheme was studied with spherical CSI uncertainties in [56], while in [52] ellipsoidal CSI uncertainties were considered for a multi-pair one-way communication network.

In our iterative transceiver beamforming designs, the quality-of-service (evaluated by SINR) of each user node is jointly determined by three beamforming vectors: transmit beamforming vector, relay beamforming vector and receive beamforming vector, and the overall beamforming problem becomes more difficult than the single-antenna-user case. In this chapter we will further illustrate the different roles of the three beamforming vectors in their contribution to the received SINR and how we use it to divide the overall beamforming problem into the three sub-problems, and we propose two different relay strategies, with consideration of sum relay power constraint and individual relay power con-
straint, respectively. Moreover, based on the structure, we also investigate the robustness of our proposed methods in the presence of CSI errors and propose worst-case based beamforming strategies for transmit beamformers and relay nodes, and as demonstrated by simulation results, the two proposed methods are extremely robust against CSI errors.

This chapter is organised as follows, in Section 6.1, our considered system model is presented. Then, Section 6.2 present our proposed worst-case based robust iterative beamforming algorithms for SINR optimization, where the three iteration steps are introduced. Simulation results are given in Section 6.3 and conclusion of this chapter is drawn in Section 6.4

6.1 System Model

We consider the same time-slotted dual-hop multipair two-way distributed relay network as in Chapter 4 and 5, in which communications between $K$ multi-antenna pairs ($X_a$, $X_b$) take place in two transmission phases aided by $M$ single-antenna relay nodes, as shown in Fig. 6.1.
We use $y_{a,i}$ and $y_{b,i}$ to represent the signal received by $X_{a,i}$ and $X_{b,i}$, respectively, and we repeat their expressions as follow,

\[
y_{a,i} = \underbrace{c_i F_i^T W G_i b_{i} x_{b,i}}_{\text{Desired signal}} + \underbrace{c_i F_i^T W F_i a_{i} x_{a,i} + c_i F_i^T W n_R}_{\text{Self Interference}}
+ \underbrace{c_i n_{a,i}}_{\text{IPI}} + \underbrace{c_i W F_i^T \sum_{j \neq i} (F_j a_j x_{a,j} + G_j b_j x_{b,j})}_{\text{IPI}}, \quad (6.1)
\]

\[
y_{b,i} = \underbrace{d_i G_i^T W F_i a_{i} x_{a,i}}_{\text{Desired signal}} + \underbrace{d_i G_i^T W G_i b_{i} x_{b,i}}_{\text{Self Interference}} + \underbrace{d_i G_i^T W n_R}_{\text{IPI}}
+ \underbrace{d_i n_{b,i}}_{\text{IPI}} + \underbrace{d_i G_i^T W \sum_{j \neq i} (F_j a_j x_{a,j} + G_j b_j x_{b,j})}_{\text{IPI}}, \quad (6.2)
\]

Fig. 6.1: The considered time-slotted dual-hop multipair two-way distributed relay network.
6.2 Worst-case Based Robust Iterative Beamforming Algorithm for SINR Optimization

Based on the iSINR method, in this section we propose two worst-case based robust iterative beamforming algorithms for SINR optimization, with two different relay strategies, where the relay nodes are involved in helping the multi-pair transmission, and later simulation results will demonstrate that the contribution of the relay nodes can be very significant when the relay number is large. In the proposed schemes, the objective is still to optimize the SINR at each user node under total or individual relay power constraint. Furthermore, we investigate the two systems at the worst case when CSI errors exist.

As an example, consider user $X_{a,i}$. From (6.1), the SINR at this user can be expressed as follows,

$$SINR_{a,i} = \frac{c_i^H F_i^T Q_{a,i}^{(S)} F_i^* c_i}{\sigma_u^2 + c_i^H F_i^T Q_{a,i}^{(N)} F_i^* c_i + \underbrace{c_i^H F_i^T Q_{a,i}^{(I)} F_i^* c_i}_{\text{IPI}}},$$

where,

$$Q_{a,i}^{(S)} = P \cdot WG_i b_i b_i^H G_i^H W^H,$$

$$Q_{a,i}^{(N)} = \sigma_r^2 \cdot WW^H,$$

$$Q_{a,i}^{(I)} = P \sum_{j \neq i}^K W(F_j a_j a_j^H F_j^H + G_j b_j b_j G_j^H) W^H.$$  

Similarly, we assume that the CSI is either estimated at the user or fed back to it by the relay via low rate feedback channels. Due to various reasons, such as
resolution of the feedback CSI and mobility of the users and relays, the obtained CSI is likely to be imperfect, modeled as

$$F_i = \hat{F}_i + \Delta F_i, \quad G_i = \hat{G}_i + \Delta G_i$$

(6.5)

where $\hat{F}_i$ and $\hat{G}_i$ are the estimated channel matrices at the user nodes, and $\Delta F_i$ and $\Delta G_i$ represent the CSI error matrices. Using the uncertainty model exploited in [49, 50, 53], we further assume that the norm of the errors are bounded by some known constants $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$, i.e,

$$|\Delta f_{m,n}^{(i)}| \leq \epsilon_{m,n}^{(i)}; \quad |\Delta g_{m,n}^{(i)}| \leq \beta_{m,n}^{(i)},$$

$$m \in \{1, ..., M\}, \quad n \in \{1, ..., N\},$$

(6.6)

where $\Delta f_{m,n}^{(i)}$ and $\Delta g_{m,n}^{(i)}$ are the $(m, n)$-th element of the channel matrices $\Delta F_i$ and $\Delta G_i$, respectively.

According to [127], the proper values of $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$ can be obtained using preliminary knowledge of the channel type. Note that even though there is an alternative way to model the uncertainty in $\hat{F}_i$ and $\hat{G}_i$, which is using a combined uncertainty model where the Euclidean norm of each row of $\hat{F}_i$ and $\hat{G}_i$ is bounded by some constant values, it will be seen that if we use this assumption in our optimization problem, the error terms will need to be decoupled and the knowledge of each $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$ will still be needed.

Then, without loss of generality, again consider user $X_{a,i}$ as an example. From the expression of $y_{a,i}$, the receive SINR of $X_{a,i}$ can be derived as expressed in (6.3) and (6.4).
With the CSI errors, to maximize the minimum SINR at the user $X_{a,i}$ side, we have the following problem based on the worst-case scenario.

$$\begin{align*}
\max_{a_k, b_k, c_i, W} \quad & \min_{\Delta F_k, \Delta G_k} \quad \text{SINR}_{a,i}, \\
\text{s.t.} & \quad ||c_i||^2 = 1, \\
& \quad ||a_k||^2 \leq P_S, \quad ||b_k||^2 \leq P_S,
\end{align*}$$

(6.7)

$$P_{\text{relay}} \leq P_r \quad \text{or} \quad P_{\text{relay}} \leq P_r,$$

$$||[\Delta F_i]_{mn}|| \leq \epsilon^{(i)}_{m,n}, \quad ||[\Delta G_i]_{mn}|| \leq \beta^{(i)}_{m,n},$$

where $P_{\text{relay}}$ and $P_r$ represent the sum relay output power and the sum relay power constraint, respectively. $\textbf{P}_{\text{relay}} = [P_{\text{relay},1} \ P_{\text{relay},2} \cdots \ P_{\text{relay},M}]^T$ and $\textbf{P}_r = [P_{r,1} \ P_{r,2} \cdots \ P_{r,M}]^T$ are the individual relay output power and the individual relay power constraint, respectively. The two relay power constraints will be discussed in Section 6.2.1 and Section 6.2.2, respectively.

As we can see from (6.7), the transmit beamforming vectors $a_i$ and $b_i$ have very different roles with the receive beamforming vectors $c_i$ and $d_i$, in maximizing the SINR. For example, by carefully choosing their coefficients, $c_i$ and $d_i$ can effectively reduce the IPI and propagation noise of the $i$-th user, but the same task is hard for $a_i$ and $b_i$, since they contribute to the IPI of all the other users except for its own. However, carefully designed $a_i$ and $b_i$ can directly lead to an optimal desired signal power (numerator of the SINR expression) of user $X_{a,i}$. Therefore, we decide not to jointly solve problem (6.7) and other $2K - 1$
similar problems (for other users), where the global solution is extremely difficult, if not impossible, to obtain. As an alternative we decompose the problem into three sub-problems, each of which is carefully designed based on the role of the transceiver beamforming vectors and the relay coefficients in the SINR expression, and the three sub-problems are solved in three iteration steps. Note that, although the solution to the three sub-problems will very unlikely be the actual global solution of problem (6.7), it can provide a rather satisfactory performance. Such a strategy will also help us find a solution that can mitigate the quality-of-service reduction caused by channel errors as well as meeting the power constraint.

6.2.1 Iteration Step I: Maximizing the Overall Gain

In the first step of our iterative design, \( c_i, d_i \) and \( W \) are fixed to either initial values or previously updated values and we try to optimize \( a_i \) and \( b_i \) to maximize the overall gain of the desired signal, which is also the power of the desired signal, under a transmit power constraint in the case of imperfectly known CSI. We will also demonstrate that in our designed scheme, the choice of \( a_i \) and \( b_i \) in the first iteration step leads to an optimal desired signal power not only when the CSI is precisely measured, but also in the worst-case situation. Now we
formulate the transmit beamforming problem with CSI errors as follows

$$\max_{b_i} \min_{\Delta F_i, \Delta G_i} |c_i^H F_i^T W G_i b_i|^2,$$

s.t. $$||b_i||^2 \leq P_S,$$

$$||[\Delta F_i]_{mn}|| \leq \epsilon_m^{(i)} n, \quad ||[\Delta G_i]_{mn}|| \leq \beta_m^{(i)},$$

$$\quad (m \in \{1, \ldots, M\}, \ n \in \{1, \ldots, N\})$$(6.8)

$$\max_{a_i} \min_{\Delta F_i, \Delta G_i} |d_i^H G_i^T W F_i a_i|^2,$$

s.t. $$||a_i||^2 \leq P_S.$$

$$||[\Delta F_i]_{mn}|| \leq \epsilon_m^{(i)} n, \quad ||[\Delta G_i]_{mn}|| \leq \beta_m^{(i)},$$

$$\quad (m \in \{1, \ldots, M\}, \ n \in \{1, \ldots, N\})$$

Denote $$f_i = W^T F_i c_i^*$$ and $$g_i = W^T G_i b_i^*$$, where $$f_i, g_i \in \mathbb{C}^{M \times 1}$$. We have

$$|c_i^H F_i^T W G_i b_i|^2 = |f_i^T G_i b_i|^2,$$

$$|d_i^H G_i^T W F_i a_i|^2 = |g_i^T F_i a_i|^2. \quad (6.9)$$

From the CSI uncertainty expression (6.5), we can rewrite the two vectors as

$$f_i = \hat{f}_i + \Delta f_i = W^T \hat{F}_i c_i^* + W^T \Delta F_i c_i^*,$$

$$g_i = \hat{g}_i + \Delta g_i = W^T \hat{G}_i d_i^* + W^T \Delta G_i d_i^*. \quad (6.10)$$

Using $$\hat{f}_{m}^{(i)}$$ and $$\hat{g}_{m}^{(i)}$$ to represent the $$m$$-th element of $$\hat{f}_i$$ and $$\hat{g}_i$$, respectively, we have

$$\hat{f}_{m}^{(i)} = \sum_{n=1}^{N} c_{i,n}^* \hat{f}_{m,n}^{(i)} w_m + c_{i,n}^* \Delta f_{m,n}^{(i)} w_m,$$

$$\hat{g}_{m}^{(i)} = \sum_{n=1}^{N} d_{i,n}^* \hat{g}_{m,n}^{(i)} w_m + d_{i,n}^* \Delta g_{m,n}^{(i)} w_m. \quad (6.11)$$

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where $\hat{g}_{m,n}^{(i)}$, $\hat{f}_{m,n}^{(i)}$, $\Delta f_{m,n}^{(i)}$ and $\Delta g_{m,n}^{(i)}$ are the $(m, n)$-th element of the channel matrices $\hat{G}_i$, $\hat{F}_i$, $\Delta G_i$ and $\Delta F_i$, respectively. And $c_{i,n}$ and $d_{i,n}$ represents the $n$-th element of $c_i$ and $d_i$, respectively.

Without loss of generality, consider user $X_{a;i}$ as an example. From (6.11) and the channel error constraint, the absolute value of the $m$-th element of $\Delta f_i$ can be expressed by

$$|\Delta f_{m}^{(i)}| = \left| \sum_{n=1}^{N} c_{i,n}^{*} \Delta f_{m,n}^{(i)} w_{m} \right| \leq \sum_{n=1}^{N} \epsilon_{m,n}^{(i)} |c_{i,n}^{*} w_{m}| \triangleq \xi_{m}^{(i)}. \quad (6.12)$$

The upper bound of $|\Delta f_{m}^{(i)}|$ is reached when $|\Delta f_{m,n}^{(i)}| = \epsilon_{m,n}^{(i)}$ for $n = 1, \ldots, N$, and all the values of $c_{i,n}^{*} \Delta f_{m,n}^{(i)} w_{m}$ have the same phases. From the expression we can also notice that the phase of $\Delta f_{m}^{(i)}$ can be arbitrary.

Now denote the matrix product of $c_i^{H} F_{i}^{T} W G_{i}$ in (6.8) by $h_{FG}^{(i)} = \hat{h}_{FG}^{(i)} + \Delta h_{FG}^{(i)}$, $\in \mathbb{C}^{1 \times N}$, where $\hat{h}_{FG}^{(i)}$ is related to the estimated value of the channel matrix, and

$$\Delta h_{FG}^{(i)} = \hat{f}_{i}^{T} \Delta G_{i} + \Delta f_{i}^{T} \hat{G}_{i} + \Delta f_{i}^{T} \Delta G_{i}, \quad (6.13)$$

is the error. Then, the absolute value of the $n$-th element of $\Delta h_{FG}^{(i)}$ is given by

$$|\Delta h_{FG,n}^{(i)}| = \left| \sum_{m=1}^{M} (\hat{f}_{m}^{(i)} \Delta g_{m,n}^{(i)} + \Delta f_{m}^{(i)} \hat{g}_{m,n}^{(i)} + \Delta f_{m}^{(i)} \Delta g_{m,n}^{(i)}) \right|$$

$$\leq \sum_{m=1}^{M} (|\hat{f}_{m}^{(i)}| \beta_{m,n}^{(i)} + \xi_{m,n}^{(i)} |\hat{g}_{m,n}^{(i)}| + \xi_{m,n}^{(i)} \beta_{m,n}^{(i)}) \triangleq \phi_{FG,n}^{(i)}. \quad (6.14)$$

The equality holds when all the $\hat{f}_{m}^{(i)} \Delta g_{m,n}^{(i)}$, $\Delta f_{m}^{(i)} \hat{g}_{m,n}^{(i)}$ and $\Delta f_{m}^{(i)} \Delta g_{m,n}^{(i)}$ have the same phase. Moreover, $\Delta h_{FG,n}^{(i)}$ can have arbitrary phase. As a result, the error
vector $\Delta h_{FG}^{(i)}$ has an upper norm bound as

$$
||\Delta h_{FG}^{(i)}|| = \left(\sum_{n=1}^{N} |\Delta h_{FG,n}^{(i)}|^2\right)^{\frac{1}{2}}
\leq \left(\sum_{n=1}^{N} \varphi_{FG,n}^{(i)}\right)^{\frac{1}{2}} \triangleq \varphi_{FG}.
$$

(6.15)

Now, we can rewrite the worst-case based sub-problem (6.8) for user $X_{a,i}$ using $h_{FG}^{(i)}$.

$$
\max_{b_i} \min_{\Delta h_{FG}^{(i)}} \lambda_R^2 |(\hat{h}_{FG}^{(i)} + \Delta h_{FG}^{(i)})b_i|^2,
\text{s.t. } ||b_i||^2 \leq P_S,
\varphi_{FG}
$$

(6.16)

Using triangle inequality and Cauchy-Schwarz inequality, we have

$$
|(|\hat{h}_{FG}^{(i)} + \Delta h_{FG}^{(i)})b_i|^2 \geq (|\hat{h}_{FG}^{(i)}b_i| - ||\Delta h_{FG}^{(i)}|| \cdot ||b_i||)^2
\geq (|\hat{h}_{FG}^{(i)}b_i| - \varphi_{FG}||b_i||)^2,
\text{ where we have made a reasonable assumption of } |\hat{h}_{FG}^{(i)}b_i| > \varphi_{FG}||b_i||. \text{ It can be derived that the particular } \Delta h_{FG}^{(i)} \text{ for the equality to hold is}
$$

$$
\Delta h_{FG}^{(i)} = -\varphi_{FG} b_i ||b_i||^{-1} e^{j\theta}, \quad \theta \triangleq \text{angle}(\hat{h}_{FG}^{(i)}b_i).
$$

(6.17)

Therefore, the worst-case optimization sub-problem (6.16) for user $X_{a,i}$ can be rewritten as

$$
\max_{b_i} \left(|\hat{h}_{FG}^{(i)}b_i| - \varphi_{FG}||b_i||\right)^2,
\text{s.t. } ||b_i||^2 \leq P_S.
\text{ (6.19)}
$$
It can be proved that the optimal solution of $b_i$ will always satisfy the upper bound determined by $P_S$. We prove it by contradiction. Assume the optimal $b_{i,\text{opt}}$ does not satisfy the upper bound, i.e., $||b_{i,\text{opt}}||^2 = P'_S = P_S/\rho$, $\rho > 1$. Then, there must exist a $b'_i = \sqrt{k}b_{i,\text{opt}}$ which satisfies $||b'_i||^2 = P_S$, leading to

$$
(||\hat{h}^{(i)}_{FG}b'_i - \varphi^{(i)}_{FG}||b'_i||)^2 = \rho(||\hat{h}^{(i)}_{FG}b_{i,\text{opt}} - \varphi^{(i)}_{FG}||b_{i,\text{opt}}||)^2 \\
> (||\hat{h}^{(i)}_{FG}b_{i,\text{opt}} - \varphi^{(i)}_{FG}||b_{i,\text{opt}}||)^2, \quad (6.20)
$$

which contradicts the assumption that $b_{i,\text{opt}}$ is the optimal solution. Therefore, the problem (6.19) becomes

$$
\max_{b_i} \quad (||\hat{h}^{(i)}_{FG}b_i - \varphi^{(i)}_{FG}\sqrt{P_S})^2, \\
\text{s.t.} \quad ||b_i||^2 = P_S. \quad (6.21)
$$

Notice that the above problem has a closed-form solution, which is the same as the solution of (6.8) when $\epsilon^{(i)}_{m,n} = \beta^{(i)}_{m,n} = 0$. That is to say, the solution of our first iteration step applies to both the optimal case and the worst case of the first sub-problem. Similarly, the first sub-problem for user $X_{b,i}$ can be solved using the same procedure. The solutions lead to the updated values of $a_i$ and $b_i$ as

$$
a_i = P^\frac{1}{2}_S \hat{F}_i^H \hat{G}_i^* d_i, \quad b_i = P^\frac{1}{2}_S \hat{G}_i^H \hat{F}_i^* c_i. \quad (6.22)
$$

6.2.2 Iteration Step II-1: Relay Strategy 1 (Sum-Relay Power Constraint)

In the second iteration step of our design, the relay weights are decided based on fixed values (either initialized or updated) of $a_i$, $b_i$, $c_i$ and $d_i$. And we propose two different relay strategies based on two different relay power assumptions.
In this subsection, we consider our first relay strategy when a total relay power budget is applied to the network, where the designed beamforming coefficients enable the relay nodes to jointly construct a stream transmission environment that can help the users to obtain a better QoS.

Firstly, considering perfect CSI, the following formulation is adopted to find the relay weights that optimizes the sum desired signal power received by all the user nodes.

\[
\begin{align*}
\max_{\mathbf{w}} & \quad \sum_{i=1}^{K} (|c_i^H \mathbf{F}_i^T \mathbf{W} \mathbf{G}_i \mathbf{b}_i|^2 + |d_i^H \mathbf{G}_i^T \mathbf{WF}_i \mathbf{a}_i|^2), \\
\text{s.t.} & \quad P_{\text{relay}} \leq P_r. \quad (6.23)
\end{align*}
\]

Denote \( \mathbf{g}_i' = \mathbf{G}_i \mathbf{d}_i^* \), \( \mathbf{f}_i' = \mathbf{F}_i \mathbf{c}_i^* \), where \( \mathbf{f}_i' \), \( \mathbf{g}_i' \in \mathbb{C}^{M \times 1} \), and \( \mathbf{w} = [w_1 w_2 \ldots w_M]^H \).

Together with (6.10), we can rewrite the objective function in (6.23) as

\[
\sum_{i=1}^{K} (|\mathbf{w}^H \mathbf{G}_i \mathbf{f}_i'|^2 + |\mathbf{w}^H \mathbf{F}_i \mathbf{g}_i'|^2) = \mathbf{w}^H \mathbf{Q}_R \mathbf{w}, \quad (6.24)
\]

where \( \mathbf{G}_i \) and \( \mathbf{F}_i \in \mathbb{C}^{M \times M} \) are diagonal matrices in which the entries of their main diagonal correspond to \( \mathbf{G}_i \mathbf{b}_i \) and \( \mathbf{F}_i \mathbf{a}_i \), respectively, and

\[
\mathbf{Q}_R = \sum_{i=1}^{K} (\mathbf{G}_i \mathbf{f}_i' \mathbf{f}_i'^H \mathbf{G}_i^H + \mathbf{F}_i \mathbf{g}_i' \mathbf{g}_i'^H \mathbf{F}_i^H). \quad (6.25)
\]

Now the sum relay power \( P_{\text{relay}} \) is given by

\[
P_{\text{relay}} = \mathbf{w}^H (\sigma_r^2 \mathbf{I}_M + \sum_{i=1}^{K} \mathbf{G}_i \mathbf{G}_i^H + \sum_{i=1}^{K} \mathbf{F}_i \mathbf{F}_i^H) \mathbf{w} = \mathbf{w}^H \mathbf{Q}_P \mathbf{w}, \quad (6.26)
\]
where $Q_P$ is a diagonal matrix. The problem (6.23) can now be rewritten as

$$\max_{\lambda_R} \quad w^H Q_R w,$$

$$s.t. \quad w^H Q_P w \leq P_r. \quad (6.27)$$

It can be transformed to an eigenvector problem with a closed-form solution, which leads to the following updated value for $w$ when CSI errors are not presented

$$w = \lambda \rho \{Q_P^{-1} Q_R\}, \quad (6.28)$$

where $\lambda$ is a power control scalar decided by $P_r$.

On the one hand, in the presence of CSI errors, we propose to maintain the power constraint of the relay system for all possible CSI errors. On the other hand, since the worst case of maximizing $|w^H \hat{G}_i \hat{y}_i'|^2$ and $|w^H \hat{F}_i \hat{g}_i'|^2$ for each individual user node is already considered in our first iteration step, setting the objective function here with worst-case scenario again will not be necessary, and it will lead to performance degradation. Accordingly, we keep the objective function of (6.23) with the channel matrices replaced by their estimated values, and transform (6.23) to the following problem

$$\max_{w} \quad \sum_{i=1}^{K} (|w^H \hat{G}_i \hat{y}_i'|^2 + |w^H \hat{F}_i \hat{g}_i'|^2),$$

$$s.t. \quad \max_{\Delta F_i, \Delta G_i} P_{\text{Relay}} \leq P_r,$$

$$||[\Delta F_i]_{mn}|| \leq \epsilon^{(i)}_{m,n}, \quad ||[\Delta G_i]_{mn}|| \leq \beta^{(i)}_{m,n}.$$

$$(m \in \{1, \ldots, M\}, \ n \in \{1, \ldots, N\}) \quad (6.29)$$
According to (6.26), the maximum value of $P_{\text{relay}}$ for all $[\Delta F_i]_{mn}$ and $[\Delta G_i]_{mn}$ is obtained when all the diagonal elements of the matrix $Q_P$ take their maximum values. Denote $Q_{p,m}$, $G_{i,m}$ and $F_{i,m}$ as the $m$-th entry of the main diagonal of $Q_P$, $G_i$ and $F_i$, respectively, and we have

$$Q_{p,m} = \hat{Q}_{p,m} + \Delta Q_{p,m}$$  

$$= \sigma_r^2 + \sum_{i=1}^{K} (|\hat{G}_{i,m} + \Delta G_{i,m}|^2 + |\hat{F}_{i,m} + \Delta F_{i,m}|^2),$$  

(6.30)

where

$$|\Delta G_{i,m}| = \left| \sum_{n=1}^{N} \Delta g_{m,n} b_{i,n} \right| \leq \sum_{n=1}^{N} \beta_{m,n} |b_{i,n}| = \xi_{G_m}^{(i)}. \quad (6.31)$$

The upper bound is reached when $|\Delta g_{m,n}^{(i)}| = \beta_{m,n}^{(i)}$ for $n = 1, ..., N$, and all $\Delta g_{m,n} b_{i,n}$ have the same phase. Similarly, we can derive the upper bound, denoted as $\xi_{F_m}^{(i)}$, for $|\Delta F_{i,m}|$.

Then, (6.30) becomes

$$Q_{p,m} \leq \sigma_r^2 + \sum_{i=1}^{K} (|\hat{G}_{i,m}|^2 + 2|\hat{G}_{i,m}|\xi_{G_m}^{(i)} + \xi_{G_m}^{2(i)} + |\hat{F}_{i,m}|^2$$

$$+ 2|\hat{F}_{i,m}|\xi_{F_m}^{(i)} + \xi_{F_m}^{2(i)}) = Q'_{p,m}. \quad (6.32)$$

Now construct an $M \times M$ diagonal matrix $Q'_P$ with the $m$-th diagonal entries being $Q'_{p,m}$. The maximum value of $P_{\text{relay}}$, denoted as $P'_{\text{relay}}$, can be expressed by

$$P'_{\text{relay}} = w^H Q'_P w. \quad (6.33)$$
(6.29) can be rewritten as
\[
\max_w \sum_{i=1}^{K} \left( |w^H \hat{G}_i \hat{G}'_i|^2 + |w^H \hat{F}_i \hat{G}'_i|^2 \right),
\]
\[
s.t. \quad \max_{\Delta F, \Delta G} w^H Q'_P w \leq P_r,
\]
\[
|[\Delta F_i]_{mn}| \leq \epsilon^{(i)}_{m,n}, \quad |[\Delta G_i]_{mn}| \leq \beta^{(i)}_{m,n}.
\]
\[
(m \in \{1, ..., M\}, \ n \in \{1, ..., N\}) \quad (6.34)
\]

Let \( \hat{Q}_R \) denote the estimated value of \( Q_R \), and the closed-form solution to (6.34) becomes
\[
w = \lambda' \rho \{ Q'_P^{-1} \hat{Q}_R \} = \lambda' \bar{w}, \quad (6.35)
\]
where we use \( \bar{w} \) to represent the normalized principle eigenvector of \( Q'_P^{-1} \hat{Q}_R \) and the power control scalar \( \lambda' \) can be obtained by
\[
\lambda' = \sqrt{\frac{P_r}{\bar{w}^H Q'_P \bar{w}}}. \quad (6.36)
\]

6.2.3 Iteration Step II-2: Relay Strategy 2 (Individual-Relay Power Constraint)

In this subsection, we propose our second relay strategy in the second iteration step for the case that each relay node has its own power budget. This strategy is preliminarily introduced in the previous chapter, and it mainly utilizes the fundamental result from [128] that when a large number of relay nodes are involved in the network, the channels between the users and relays could be pairwisely nearly orthogonal, and accordingly, the contribution of the relay nodes in our
second scheme is expected to reveal when the number of relay nodes is large. In this chapter, we do not repeat the theoretical foundation of this strategy, and we just recall how we decide the relay beamforming vector.

Here we also consider the case when perfect CSI is obtained at first. Let $f_{i,m}, g_{i,m} \in \mathbb{C}^{1 \times N}$ represent the $m$-th row of $F_i$ and $G_i$, respectively. We propose the following phase rotating rule for the $m$-th relay node ($m = 1, ..., M$).

$$w_m = \lambda_m \left( \sum_{i=1}^{K} f_{i,m}^* c_i b_i^H g_{i,m}^H + g_{i,m}^* d_i^H f_{i,m}^H \right)$$

$$= \lambda_m \left( \sum_{i=1}^{K} \bar{u}_{i,m}^* \bar{v}_{i,m} + \bar{v}_{i,m}^* \bar{u}_{i,m} \right), \quad (6.37)$$

where $\bar{u}_{i,m} \triangleq f_{i,m}^* c_i$, $u_{i,m} \triangleq f_{i,m} a_i$, $\bar{v}_{i,m} \triangleq g_{i,m}^* d_i$, and $v_{i,m} \triangleq g_{i,m} b_i$, and as discussed in Chapter 4, they have distributions of $\mathcal{CN}(0, \Gamma_{\bar{u}_{i,m}})$, $\mathcal{CN}(0, \Gamma_{u_{i,m}})$, $\mathcal{CN}(0, \Gamma_{\bar{v}_{i,m}})$, and $\mathcal{CN}(0, \Gamma_{v_{i,m}})$, respectively. $\lambda_m$ is a power-control parameter which limits the output power of each relay node, given by

$$\lambda_m = \sqrt{\frac{P_r,m/\left| \sum_{i=1}^{K} \bar{u}_{i,m}^* \bar{v}_{i,m} + \bar{v}_{i,m}^* \bar{u}_{i,m} \right|^2}{\sigma_r^2 + \sum_{i=1}^{K} \left| u_{i,m} \right|^2 + \left| v_{i,m} \right|^2}}, \quad (6.38)$$

where $P_r,m$ is the individual power budget at the $m$-th relay.

Let us rewrite (6.1) after removing the self interference part, in terms of $u_{i,m}$, $v_{i,m}$, $\bar{u}_{i,m}$ and $\bar{v}_{i,m}$.
\[ \tilde{y}_{a,i} = \sum_{m=1}^{M} \tilde{u}_{i,m} w_m v_{i,m} x_{b,i} + \sum_{m=1}^{M} \tilde{u}_{i,m} w_m n_{R,m} + n_{a,i} \]

<table>
<thead>
<tr>
<th>Desired signal</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ \sum_{m=1}^{M} \sum_{j \neq i}^{K} \left( \tilde{u}<em>{i,m} w_m u</em>{j,m} x_{a,j} + \tilde{u}<em>{i,m} w_m v</em>{j,m} x_{b,j} \right)</td>
<td></td>
</tr>
</tbody>
</table>

\[ = \mathcal{G}_{a,i}^{(S)} x_{b,i} + \mathcal{G}_{a,i}^{(Noise)} n_{R,m} + n_{a,i} \]

\[ + \sum_{j \neq i}^{K} \left( \mathcal{G}_{ab,ij}^{(IPI)} x_{a,j} + \mathcal{G}_{ba,ij}^{(IPI)} x_{b,j} \right), \quad (6.39) \]

where \( \mathcal{G}_{a,i}^{(S)} \), \( \mathcal{G}_{a,i}^{(Noise)} \), \( \mathcal{G}_{ab,ij}^{(IPI)} \) and \( \mathcal{G}_{ba,ij}^{(IPI)} \) represents the gain of each component, \( n_{R,m} \) represents the complex Gaussian noise of the \( m \)-th relay node with the distribution \( \mathcal{CN}(0, \sigma_r^2) \) and \( n_{a,i} \sim \mathcal{D}_i n_{b,i} \). Due to the fact that in our scheme, \( d_i \) is a normalized vector (\( ||d_i||^2 = 1 \)), \( n_{a,i} \) will have a distribution given by \( \mathcal{CN}(0, \sigma_u^2) \).

Let \( \tilde{y}_{a,i}^{(S)} \), \( \tilde{y}_{a,i}^{(IPI)} \) and \( \tilde{y}_{a,i}^{(Noise)} \) denote the desired signal, IPI and noise part in (6.39), respectively. We have

\[ \tilde{y}_{a,i}^{(S)} = \sum_{m=1}^{M} \lambda_m \tilde{u}_{i,m} \left( \sum_{i=1}^{K} \tilde{u}_{i,m}^* v_{i,m}^* + \tilde{v}_{i,m}^* \tilde{u}_{i,m}^* \right) v_{i,m} x_{b,i}. \quad (6.40) \]

Since \( \tilde{u}_{i,m}, \tilde{v}_{i,m}, \tilde{u}_{i',m}(i' \neq i) \) and \( \tilde{u}_{i,m'}(m' \neq m) \) can be considered as zero mean mutually uncorrelated random variables, with \( \mathbb{E}[x^2] = \sigma^2 \), where \( x \sim \mathcal{CN}(0, \sigma^2) \), we have

\[ \mathbb{E}[\mathcal{G}_{a,i}^{(S)}] = \mathbb{E} \left[ \sum_{m=1}^{M} \lambda_m ||\tilde{u}_{i,m}||^2 ||v_{i,m}||^2 \right] = \sum_{m=1}^{M} \lambda_m \Gamma_{i,m} \Gamma_{i,m}^v. \quad (6.41) \]

Now, considering the presence of CSI errors, we propose to use the same phase rotating rule for the \( m \)-th relay node with a new power scalar \( \lambda_m' \) to restrict
the output power of each relay node in the worst case.

\[ w_m = \lambda_m' \left( \sum_{i=1}^{K} \hat{u}_{i,m}^* \hat{v}_{i,m}^* + \hat{v}_{i,m}^* \hat{u}_{i,m}^* \right), \]  

(6.42)

where the new notations are: \( \hat{u}_{i,m} \triangleq \hat{f}_{i,m}^* c_i \), \( \hat{u}_{i,m} \triangleq \hat{f}_{i,m}^* a_i \), \( \hat{v}_{i,m} \triangleq \hat{g}_{i,m}^* d_i \) and \( \hat{v}_{i,m} \triangleq \hat{g}_{i,m}^* b_i \).

Similarly, using user \( X_{a,i} \) as an example, \((6.40)\) can be rewritten as

\[ \tilde{y}_a^{(S)} = \sum_{m=1}^{M} \lambda_m' (\hat{u}_{i,m} + \Delta \bar{u}_{i,m}) \]

\[ \times \left( \sum_{i=1}^{K} \bar{u}_{i,m}^* v_{i,m}^* + \bar{v}_{i,m}^* u_{i,m}^* \right) (\hat{v}_{i,m} + \Delta v_{i,m}) x_{b,i}, \]  

(6.43)

where we have \( \Delta \bar{u}_{i,m} \triangleq \Delta f_{i,m}^* c_i \) and \( \Delta v_{i,m} \triangleq \Delta g_{i,m}^* b_i \). If we assume that \( \mathbb{E}[\Delta f_{i,m}] = \mathbb{E}[\Delta g_{i,m}] = 0 \), we have \( \mathbb{E}[\Delta \bar{u}_{i,m}] = \mathbb{E}[\Delta v_{i,m}] = 0 \), and as a result, \( \mathbb{E}[G_{a,i}^{(S)}] \) will stay the same as in \((6.41)\). It demonstrates that our phase rotating rule will remain effective in the presence of CSI, and now we will derive the choice of \( \lambda_m' \) in the worst case scenario.

From the definition of \( \Delta u_{i,m} \) and \( \Delta v_{i,m} \), we have

\[ \Delta u_{i,m} = \sum_{n=1}^{N} \Delta f_{m,n} (\hat{u}_{i,m}^* a_{i,n} \leq \sum_{n=1}^{N} e_{m,n} |a_{i,n}| = \xi_{u_{i,m}}^{(i)}, \]

\[ \Delta v_{i,m} = \sum_{n=1}^{N} \Delta f_{m,n} (\hat{v}_{i,m}^* b_{i,n} \leq \sum_{n=1}^{N} \beta_{m,n} |b_{i,n}| = \xi_{v_{i,m}}^{(i)}. \]  

(6.44)

The power control scalar \( \lambda_m' \) that can restrict the maximum output power of the \( m-th \) relay in the worst case can now be derived from \((6.38)\), and it is given as

\[ \lambda_m' = \sqrt{\frac{P_{r,m}/ \left| \sum_{i=1}^{K} \hat{u}_{i,m}^* \hat{v}_{i,m}^* + \hat{v}_{i,m}^* \hat{u}_{i,m}^* \right|^2}{\sigma_r^2 + \sum_{i=1}^{K} (|\hat{u}_{i,m}| + \xi_{u_{i,m}}^{(i)})^2 + (|\hat{v}_{i,m}| + \xi_{v_{i,m}}^{(i)})^2}}. \]  

(6.45)
6.2.4 Iteration Step III: Maximizing User SINR

Now we have updated the values of the two transmit beamforming vectors \( a_i \) and \( b_i \), and the relay coefficients \( w \). Next are the two beamforming vectors \( c_i \) for user \( X_{a,i} \) and \( d_i \) for user \( X_{b,i} \).

In our first two iteration steps, the power of the desired signal at each user node and the relay output power has been considered in the worst case with specific values of \( \Delta F_i \) and \( \Delta G_i \). As a result, the first two iteration steps would have sufficiently compensated for the user SINR in extreme cases (worst case for desired signal power) along with guaranteeing that the power constraints are satisfied in the worst case. The part that remains unconsidered in the SINR expression (6.3) and (6.4) is mainly the IPI and propagation noise in the denominator of the SINR expression. However, it can be observed that the IPI part is jointly decided by the transmission channel matrices of all the other user pairs apart from user \( X_{a,i} \), and thus the worst-case formulation will be too conservative and has much poorer performance. Due to these reasons, finding the lower bound on the cost function of (6.7) would not be as important as in the two earlier steps, and we formulate the following problem for user \( X_{a,i} \) to decide its receive beamformer vector (expressions for user \( X_{b,i} \) can be similarly derived).

\[
\begin{align*}
\max_{c_i} & \quad c_i^H \hat{F}^T_i \hat{Q}_{a,i}^{(S)} \hat{F}_i^* c_i \\
& \quad \sigma_u^2 + c_i^H \hat{F}^T_i \hat{Q}_{a,i}^{(N)} \hat{F}_i^* c_i + c_i^H \hat{F}^T_i \hat{Q}_{a,i}^{(I)} \hat{F}_i^* c_i, \\
\text{s.t.} & \quad ||c_i||^2 = 1
\end{align*}
\]
where,

\[
\hat{Q}_{a,i}^{(S)} = P_s \cdot W \hat{G}_i b_i b_i^H \hat{G}_i^H W^H,
\]

\[
\hat{Q}_{a,i}^{(N)} = \sigma_r^2 \cdot W W^H,
\]

\[
\hat{Q}_{a,i}^{(I)} = P_s \cdot \sum_{j \neq i} W (\hat{F}_j a_j a_j^H \hat{F}_j^H + \hat{G}_j b_j b_j^H \hat{G}_j^H) W^H.
\]  \quad (6.47)

In the objective function of the above formulation, the channel matrices are replaced by their estimated values. The solution \(c_i\) to this sub-problem can very possibly provide a satisfactory user SINR even at the presence of CSI errors. The optimization problem can be transformed to an eigenvector problem with a closed-form solution. The results are

\[
c_i = \rho\{\Theta_{a,i}\}, \quad d_i = \rho\{\Theta_{b,i}\},
\]  \quad (6.48)

where

\[
\Theta_{a,i} = (\Xi_{a,i})^{-1} \hat{F}_i^T Q_{a,i}^{(S)} \hat{F}_i^*,
\]

\[
\Xi_{a,i} = \sigma_u^2 I_N + \hat{F}_i^T Q_{a,i}^{(N)} \hat{F}_i^* + \hat{F}_i^T Q_{a,i}^{(I)} \hat{F}_i^*.
\]  \quad (6.49)

### 6.2.5 Summary of the Proposed Iteration Algorithm

In our proposed algorithms, the SINR of each user node is collaboratively maximized by the transmit beamformer, receive beamformer and relay nodes together. The iteration process with the above three steps is repeated until reaching the stopping criterion, which is defined by a preset maximum iteration number \((n_i')\) or some convergence requirement (defined by a preset small positive
real number $\delta')$. In fact, supported by simulation results, our proposed algorithm does not necessarily require convergence to achieve good SINR performance, and a proper $n'_t$ could be set with trade-off between better performance and lower computational complexity.

**Iteration Steps: Sum-Relay Power Constraint**

1) Initialization: $c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N}$, where $\delta_N = 1/\sqrt{N}$, $w = [\delta_M \delta_M \cdots \delta_M]$, where $\delta_M = 1/\sqrt{M}$, and set $t=1$.

2) Update $a_i$ and $b_i$ based on (6.22).

3) Update $w$ based on (6.35) and (6.36).

4) Update $c_i$ and $d_i$ based on (6.48) and (6.49).

5) If $|x_i^{(t)} - x_i^{(t-1)}|^2 < \delta'$ (considered to be converged) or $t > n'_t$ ($x \leftarrow c$ for user $X_{a,i}$ and $x \leftarrow d$ for user $X_{b,i}$), iteration stops; otherwise, set $t = t + 1$ and go to step 2).
**Iteration Steps: Individual-Relay Power Constraints**

1) Initialization: \( c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N}, \) where \( \delta_N = 1/\sqrt{N}, \)
\( w = [\delta_M \delta_M \cdots \delta_M], \) where \( \delta_M = 1/\sqrt{M}, \) and set \( t=1. \)

2) Update \( a_i \) and \( b_i \) based on (6.22).

3) Update \( w \) based on (6.42) and (6.45).

4) Update \( c_i \) and \( d_i \) based on (6.48) and (6.49).

5) If \( |x_i^{(t)} - x_i^{(t-1)}|^2 < \delta' \) (considered to be converged) or \( t > n_t \) (\( x \leftarrow c \) for user \( X_{a,i} \) and \( x \leftarrow d \) for user \( X_{b,i} \)), iteration stops; otherwise, set \( t = t + 1 \) and go to step 2).

In practice, for continuous transmission, the initialization step is only needed at the very beginning. When the channel states change slowly, the iteration number required for good performance can be further reduced.

For one user to apply the iteration algorithm locally to determine its transmit and receive beamforming vectors, knowledge of the received beamforming vectors of other users is required. This can either be calculated on this user node (assuming the initialization settings are known by each user) or shared by users from the same user group through limited backhaul resource before the next iteration loop begins. The former choice has higher computational complexity and the latter one requires intra-group communication resource. There is another way to reduce the computational complexity and the required intra-group communication resources in this scenario, which is employing a central node (it can be one of the users) for each side to perform the iteration processes and
calculate the beamforming vectors for each user node in the group, and then inform them of the results.

6.3 Simulation Results

In this section, simulation results are provided to evaluate the performance of the proposed method. The channels are of i.i.d. Rayleigh fading; the noise variance at any node is set at 1 ($\sigma_r^2 = \sigma_u^2 = 1$) and we set the source power at 0 dB ($P_S = 1$, compensating the unconsidered large-scale fading) and $P_r$ is determined by $SNR_R = P_r/\sigma_r^2$. Our proposed scheme with relay strategy 1 and relay strategy 2 are referred to as $r\text{SINR-1}$ and $r\text{SINR-2}$ in all the figures, respectively. As for $r\text{SINR-2}$, we use $P_{r,m} = P_r/M$ as the individual power constraint. For a fair comparison, the sum-relay output powers of all schemes are kept the same.

Fig. 6.2 shows the SINR performance versus $SNR_R$ of the proposed methods with different numbers of relay nodes, where the iSINR method in [122] and results based on a non-iterative ZF method (denoted by “ZF”) used in [122] are provided for comparison. Specifically, in this ZF method, real CSI is considered, $\mathbf{a}_i$ and $\mathbf{b}_i$ are generated as the eigenvectors corresponding to the largest eigenvalues of $\mathbf{F}_i^H\mathbf{F}_i$ and $\mathbf{G}_i^H\mathbf{G}_i$, respectively, and together with $\mathbf{c}_i$ and $\mathbf{d}_i$, the IPI parts are eliminated completely without iterations. We can see from the figure that both of our proposed methods have significantly outperformed the non-iterative ZF method. We can also see that the iSINR method cannot benefit
Fig. 6.2: SINR performance of the proposed algorithms with different relay number settings \((\epsilon^{(i)}_{m,n} = \beta^{(i)}_{m,n} = 0, N=5, K=3, n_t=5)\).

much from the number increase of the relay nodes when relay output power is large, while both of our methods yield significant SINR improvement as the relay number increases. It is also noteworthy that to achieve a certain average SINR, the total relaying power required is reduced when the number of relays increases and thus the per-relay output power decreases even more.

Fig. 6.3 demonstrates the effect of our proposed transceiver beamforming scheme, where two relay-only methods are used as comparison where \(a_i, b_i, c_i\) and \(d_i\) are fixed to their initial values. It shows that when only the two relay
Fig. 6.3: SINR performance of the proposed algorithms with relay-only methods as comparisons ($\epsilon^{(i)}_{m,n}=\beta^{(i)}_{m,n}=0$, $N=5$, $K=3$, $n_t=5$).

strategies are used in our scheme, the average SINR increases as more relay nodes are employed in the network, and our first proposed relay method has a better performance than the second one for all relay number settings. However, without the iterative transceiver beamforming steps, the performance is very limited when the relay number is small and the SINR improvement introduced by the transceiver beamforming process is significant for all relay number settings, and the advantage becomes clearer when $P_r$ is larger.

Now we investigate the performance in terms of the CSI uncertainty bounds. Fig. 6.4 and 6.5 present the results for rSINR-1 and rSINR-2, respectively. The
Fig. 6.4: SINR performance of the proposed algorithms (with the 1st relay strategy) with different uncertainty bounds (M=30, N=5, $n_t=5$).

situations are similar for both methods; by maintaining power constraints in their worst-case scenarios, the conservative relay strategies together with the mismatch of CSI, lead to certain degradation in SINR performance. However, the performance reduction is very limited (within 1.5dB for any $P_r$ settings, even when $\epsilon = \beta = 0.20$), which indicates that the robustness of both of our proposed schemes is very high.

To demonstrate the cooperative performance of our proposed schemes through iteration steps, Fig. 6.6 illustrates the average SINR of the proposed methods after certain rounds of iterations. When the iteration round is set at 1, the three
beamforming vectors can be considered as uncoordinated. As can be seen, the proposed method does not have the best performance immediately after the initialization step. However, the average SINR will quickly approach its asymptotic value after only a few rounds of iterations, and then the performance improvement becomes rather limited with further iteration. This pattern applies to different relay number settings and different total relay power budgets.

Then in Fig. 6.7, we study the performance of the two schemes in terms of iteration steps under channels of different uncertainty, where the relay number $M$ is fixed at 20. The figure indicates that the CSI uncertainty will introduce cer-
tain degradation to the SINR performance of each user at any iteration rounds settings. And the degradation introduced by the CSI uncertainty remains similar for both methods. We can also notice that the rSINR-2 method will have advantage in lower iteration rounds settings, while the SINR performance of the rSINR-1 method surpasses the rSINR-2 method when iteration rounds are high.
Fig. 6.7: SINR performance versus iteration rounds ($\epsilon_{m,n}^{(i)} = \gamma_{m,n}^{(i)} = 0$, $N=5$, $K=3$)

6.4 Summary

The transceiver beamforming problem has been studied for multipair two-way distributed relay networks and in particular in the presence of CSI errors. Iterative algorithms have been proposed where the SINR performance of each user is collaboratively optimized by the transceiver beamformers and relay nodes. For the imperfect CSI case, the robust worst-case based formulation was considered mainly in our first two iteration steps, and two different worst-case based relay strategies are proposed for the situation when total and individual relay output power is restricted, respectively. As demonstrated by simulation results, a satisfactory SINR performance has been achieved, especially when the number of relay nodes is large.
Chapter 7

Iterative Transceiver Beamforming of Distributed Relay Networks in Cognitive Radio

7.1 Introduction

Typical cognitive radio (CR) networks have been preliminarily introduced in Chapter 3. In CR networks, one or several SUs are allowed to opportunistically access the spectrum resources licensed to the PUs under limitations such as interference perceived at PUs being regulated below a predetermined level. The dynamic access strategy of SUs can provide great efficiency enhancement to the communication networks. As demonstrated in Chapter 3, with properly designed beamforming strategy in CR networks, both PU and SU sources can have simultaneous communications to their destinations in the same channel. However, when multiple SUs instead of one are accessing the same spectrum resources of the PUs [129–133], it becomes more challenging to cooperate all
the SUs.

In this chapter, we consider multi-pair communications between SUs in cognitive radio networks, where multiple user pairs access the spectrum resources of the PUs for their own two-way communications with the assistance of multiple relay nodes. Compared with the system network in Chapter 3, additional users are included in the secondary transmission links and thus extra leakage interference will be introduced to the primary user node. In order to keep the leakage interference under a predefined level while maintaining the transmission quality between user pairs in the secondary transmission link, we investigate the application of our iterative transceiver beamforming schemes in this scenario. In our considered two-stage communication network, we assume that the PUs are having one-way communication and accordingly only the PU receiver are interfered by the communication of SUs. We only use the spectrum overlay techniques in the first communication stage; more specifically, in our considered scenario, the SUs only broadcast signals to the relays when the PU receiver is idle, while the relay nodes transmit irrespective of whether primary link is idle or not.

This chapter will be organized as follows. In Section 7.2, the system model is introduced. Then, the iterative transceiver beamforming algorithm is derived in Section 7.3. Following that, simulation results are demonstrated in Section 7.4. Finally, Section 7.5 concludes this chapter.
7.2 System Model

As shown in Fig 7.1, we consider a time-slotted dual-hop distributed CR relay network with multipair two-way communications between $K$ multiple-antenna SU nodes ($X_a$, $X_b$), where multiple ($R$) single-antenna CR relay nodes help forward the information stream, and we also assume that the direct link between source and destination nodes does not exist. The transmission is divided into two time-slotted stages. In the first stage, when the primary source is idle, the SUs broadcast their information streams to all CR relay nodes with transmit beamforming and their weights are denoted by $a_i$ and $b_i$ ($\in \mathbb{C}^{N \times 1}, i = 1, ..., K$), and in the second stage, each CR relay node forward the received signal back to all the SUs with relay beamforming, which assures that no impermissible interference be caused at the primary destination. Following that, the received signal undergoes receive beamforming, denoted by $c_i$ and $d_i$ ($\in \mathbb{C}^{N \times 1}, i = 1, ..., K$), at $X_{a,i}$ and $X_{b,i}$ sides, respectively.
We denote the SU-source-to-relay channel (fore-channel) from $X_{a,i}$ and $X_{b,i}$ to the relay nodes by $F_i, G_i \in \mathbb{C}^{M \times N}$, respectively. We further assume the transmission channels are reciprocal and quasi-stationary, so that the channel gains remain unchanged during the two time slot phases. And the received signal at the relay nodes can be represented by $r \in \mathbb{C}^{M \times 1}$,

$$r = \sum_{i=1}^{K} F_i a_i x_{a,i} + \sum_{i=1}^{K} G_i b_i x_{b,i} + n_R, \quad (7.1)$$

where the complex Gaussian noise vector of relay nodes are represented by $n_R \in \mathbb{C}^{M \times 1}$ with the distribution $\mathcal{CN}(0, \sigma_r^2 I)$. Then, each relay node amplifies
the received signal to generate the transmit signal $r_T$ as

$$r_T = Wr,$$  \(7.2\)

where the relay weights matrix $W \in \mathbb{C}^{M \times M}$ is diagonal and we use an $M \times 1$ vector $w = [w_1 w_2 \ldots w_M]^H$ to denote its diagonal entries. Next, in the second time slot, the relay nodes broadcast $r_T$ to all the SUs. We use $y_{a,i}$ and $y_{b,i}$ to represent the signal received by $X_{a,i}$ and $X_{b,i}$, respectively, with

$$y_{a,i} = c_i F_i^T W G_i b_i x_{b,i} + c_i F_i^T W F_i a_i x_{a,i} + c_i F_i^T W n_R$$

\(\text{Desired signal} + \text{Self Interference}\)

\(\text{IPI}\)

$$+ c_i n_{a,i} + c_i W F_i^T \sum_{j \neq i} (F_j a_j x_{a,j} + G_j b_j x_{b,j}), \hspace{1cm} (7.3)$$

$$y_{b,i} = d_i G_i^T W F_i a_i x_{a,i} + d_i G_i^T W G_i b_i x_{b,i} + d_i G_i^T W n_R$$

\(\text{Desired signal} + \text{Self Interference}\)

\(\text{IPI}\)

$$+ d_i n_{b,i} + d_i G_i^T W \sum_{j \neq i} (F_j a_j x_{a,j} + G_j b_j x_{b,j}), \hspace{1cm} (7.4)$$

where $n_{a,i}, n_{b,i} \in \mathbb{C}^{N \times 1}$ are the additive white complex Gaussian noise vector at the user node, with the distribution $\mathcal{CN}(0, \sigma_u^2 I)$. The receive beamforming vectors $c_i$ and $d_i$ are assumed to be unit vectors ($||c_i||^2 = 1, ||d_i||^2 = 1$).

Since its own transmitted signal is known by each user node, the self interference (SI) in (7.3) and (7.4) can be removed through some standard adaptive filtering techniques [134]. For simplicity, they are ignored in the following derivation.
Then, we use $y^{(PU)}$ to denote the leakage signal introduced by CR relays at the primary receiver,

$$y^{(PU)} = t_P \mathbf{W}_r$$

$$= t_P \sum_{i=1}^{K} \mathbf{W}_f \mathbf{a}_i x_{a,i} + t_P \sum_{i=1}^{K} \mathbf{W}_g \mathbf{b}_i x_{b,i} + t_P \mathbf{n}_R$$  \hspace{1cm} (7.5)$$

where $t_P \in \mathbb{C}^{M \times 1}$ represents the relay-to-PU channel (interference channel).

### 7.3 Iterative Beamforming Algorithm for Cognitive Networks

In the following, two transceiver beamforming schemes will be considered for this multipair two-way cognitive network with distributed relays. In our first scheme, the aim is to maximize the SINR at each SU node, while ensuring the leakage signal introduced by CR relays at the primary receiver does not exceed a predefined threshold level. In the second one, a total relay output power constraint is added.

Taking user $X_{a,i}$ as an example. From (5.1), the SINR at this user can be expressed as follows,

$$SINR_{a,i} = \frac{c_i^H \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(S)} \mathbf{F}_i c_i}{\sigma_u^2 + c_i^H \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(N)} \mathbf{F}_i c_i + c_i^H \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(I)} \mathbf{F}_i c_i},$$  \hspace{1cm} (7.6)$$

where,

$$\mathbf{Q}_{a,i}^{(I)} = P_s \sum_{j \neq i}^{K} (\mathbf{W}_f \mathbf{a}_j \mathbf{a}_j^H \mathbf{F}_j^H \mathbf{W}^H + \mathbf{W}_g \mathbf{b}_j \mathbf{b}_j^H \mathbf{G}_j^H \mathbf{W}^H),$$

$$\mathbf{Q}_{a,i}^{(N)} = \sigma_r^2 \cdot \mathbf{W} \mathbf{W}^H, \quad \mathbf{Q}_{a,i}^{(S)} = P_s \cdot \mathbf{W}_g \mathbf{b}_i \mathbf{b}_i^H \mathbf{G}_i^H \mathbf{W}^H.$$  \hspace{1cm} (7.7)
In our design, we propose to maximize the SINR of each user node while suppressing the interference that is introduced to the primary user node, under a sum relay output power constraint. Therefore, we can write the overall system formulation for the $i$th user as follows,

$$\max_{a_k, b_k, c_i, w} \ SINR_{a,i},$$

s.t. $||c_i||^2 = 1,$
$$||a_k||^2 \leq P_S, \ ||b_k||^2 \leq P_S,$$
$$P_{relay} \leq P_r,$$
$$\mathbb{E}[y^{(PU)}] \leq P_{\text{leak}}$$

(7.8)

where $P_{\text{relay}}$ represents the sum relay output power. As discussed before, in order to solve the complicated global SINR maximization problem when every user node is considered altogether, we decompose it into three sub-problems which are associated with the decisions of transmit beamforming vector, relay beamforming vector and receive beamforming vector, respectively. The iteration steps will be discussed in the following.

### 7.3.1 Iteration Step on the Transmit Part

In our design, when we decide the transmit beamforming vectors at the first iteration steps, we do not consider their contribution to the leakage interference introduced to the primary receiver. The first reason is that the transmissions between SUs and relays at the first transmission stage will not cause any QoS
degradation to the PUs, since they are idle. Secondly, although the transmit beamforming vectors do affect the leakage interference introduced by the relay nodes at the second transmission stage, proper designs of the transmit beamformers to reduce the leakage interference will result in significant performance degradation at each user node. The reason is that in our transceiver design, the transmit beamforming vectors of one user pair are directly related to the desired signal power of their own transmission. Accordingly, in our design, the leakage interference introduced to the PU receiver is only considered in our second iteration step where the relay beamforming vectors are decided.

In the first iteration step, the receive beamforming vectors $c_i$, $d_i$ and relay weights $W$ are fixed to an updated value through previous steps; otherwise, an initial value should be assigned to them. Then, we optimize $a_i$ and $b_i$ based on maximizing the power of the desired signal received at $X_{a,i}$ and $X_{b,i}$, respectively, under a transmit power constraint.

$$\max_{b_i} |c_i^H F_i^T W G_i b_i|^2, \ s.t. \ |b_i|^2 \leq P_S,$$

$$\max_{a_i} |d_i^H G_i^T W F_i a_i|^2, \ s.t. \ |a_i|^2 \leq P_S. \tag{7.9}$$

These two problems have closed-form solutions, given by

$$a_i = \lambda_{a,i} \cdot F_i^H W G_i^* d_i, \quad b_i = \lambda_{b,i} \cdot G_i^H W^H F_i^* c_i, \tag{7.10}$$

where $\lambda_{a,i}$ and $\lambda_{b,i}$ are the power-control scalars

$$\lambda_{a,i} = \sqrt{\frac{P_S}{\|F_i^H W G_i^* d_i\|^2}}, \quad \lambda_{b,i} = \sqrt{\frac{P_S}{\|G_i^H W^H F_i^* c_i\|^2}}. \tag{7.11}$$

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7.3.2 Iteration Step on the Relays - Maximization of SINR at the Secondary Destination

In the second iteration step of our design, the relay weights are decided based on fixed values (either initialized or updated) of $a_i$, $b_i$, $c_i$ and $d_i$. We propose two relay strategies associated with our two considered schemes, where the first one aims at suppressing the leakage interference power at the PU receiver, and the second one adds a total relay output power constraint.

In this subsection, we consider our first relay strategy when the designed relay beamforming coefficients aims at enabling the relay nodes to jointly construct a stream transmission environment that can help the users to obtain a better QoS while suppressing the leakage interference power at the PU receiver.

Constructing a diagonal matrix $T_P$ with its diagonal entries being the elements of $t_P$, together with (7.1) and (7.2) we can derive the expression of the leakage interference power,

$$
\mathbb{E}[y^{(PU)}] = w^H Q_{leak} w \quad (7.12)
$$

where

$$
Q_{leak} = \sum_{i=1}^{K} T_P F_i a_i a_i^H F_i^H T_P^H + \sum_{i=1}^{K} T_P G_i b_i b_i^H G_i^H T_P^H \quad (7.13)
$$

Using the above results, the following formulation is adopted to find the relay weights that optimizes the sum desired signal power received by all the
user nodes.

\[
\max_w \sum_{i=1}^{K} (|c_i^H F_i^T W G_i b_i|^2 + |d_i^H G_i^T W F_i a_i|^2),
\]

\[\text{s.t. } w^H Q_{\text{leak}} w \leq P_{\text{leak}}. \tag{7.14}\]

where \(P_{\text{leak}}\) is the pre-defined threshold for the leakage signal power at the primary receiver. Similarly, we can transform this problem into an eigenvalue problem where closed-form solutions can be obtained as,

\[
w = \lambda \rho \{Q_{\text{leak}}^{-1}Q_R\}, \tag{7.15}\]

where \(\lambda\) is a power control scalar decided by \(P_{\text{leak}}\), and \(Q_R\) has the same definition as in Chapter 6.

\[
Q_R = \sum_{i=1}^{K} (G_i g_i' g_i'^H G_i^H + F_i f_i' f_i'^H F_i^H). \tag{7.16}\]

where \(G_i\) and \(F_i\) \(\in \mathbb{C}^{M \times M}\) are diagonal matrices with their main diagonal entries corresponding to \(G_i b_i\) and \(F_i a_i\), respectively, \(g_i' = G_i d_i^*\) and \(f_i' = F_i c_i^*\).

We use \(\bar{w}\) to represent the normalized principle eigenvector of \(Q_{\text{leak}}^{-1}Q_R\) and the power control scalar \(\lambda\) can be obtained by

\[
\lambda = \sqrt{\frac{P_{\text{leak}}}{\bar{w}^H Q_{\text{leak}} \bar{w}}}. \tag{7.17}\]

7.3.3 Iteration Step on the Relays - Maximization of SINR at the Secondary Destination with Total Relay Output Power Constraint

Now consider the relay beamforming problem with total relay output power constraint in this subsection. Using user \(X_{a,i}\) as an example, from (7.2) and
(7.3), we can write the sum relay power $P_{\text{relay}}$ as,

$$P_{\text{relay}} = w^H (\sigma_r^2 I_M + \sum_{i=1}^{K} G_i G_i^H + \sum_{i=1}^{K} \mathcal{F}_i \mathcal{F}_i^H) w = w^H Q_P w,$$

(7.18)

where $Q_P$ is a diagonal matrix. The beamforming problem can now be represented as,

$$\max_w \sum_{i=1}^{K} (|c_i^H F_i^T W G_i b_i|^2 + |d_i^H G_i^T W F_i a_i|^2),$$

s.t. $w^H Q_{\text{leak}} w \leq P_{\text{leak}}$

$$w^H Q_P w \leq P_r$$

(7.19)

where $P_r$ represents the sum relay power constraint.

Taking user $X_{a,i}$ as an example, we rewrite (7.6) and (7.7) with respect to the definition of $G_i, F_i, g'_i$ and $f'_i$.

$$\text{SINR}_{a,i} = \frac{w_i^H Q_{a,i}^{(S)} w_i}{\sigma_r^2 + w_i^H Q_{a,i}^{(N)} w_i + w_i^H Q_{a,i}^{(I)} w_i},$$

(7.20)

where,

$$Q_{a,i}^{(I)} = Ps \cdot \sum_{j \neq i} (\mathcal{F}_j f'_i f'_i^H \mathcal{F}_j^H + G_j f'_i f'_i G_j^H),$$

$$Q_{a,i}^{(N)} = \sigma_r^2 f'_i f'_i, \quad Q_{a,i}^{(S)} = Ps \cdot G_j f'_i f'_i G_j^H.$$  

(7.21)

Then, similarly as in Chapter 3, using (7.23) and (7.24), and introducing an
auxiliary variable $\mu < 0$ [113], (7.22) can be transformed into

$$
\begin{align*}
\max_{w, \mu} & \quad \mu \\
\text{s.t.} & \quad \frac{w_i^H \bar{Q}_{a,i}^{(S)} w_i}{\sigma_w^2 + w_i^H \bar{Q}_{a,i}^{(N)} w_i + w_i^H \bar{Q}_{a,i}^{(I)} w_i} \geq \mu^2 \\
& \quad w^H Q_{\text{leak}} w \leq P_{\text{leak}} \\
& \quad w^H Q_P w \leq P_r
\end{align*}
$$

(7.22)

Denoting $h = \sqrt{P_s \cdot G_i f_i}$, (7.22) can be changed into a standard SOCP as

$$
\begin{align*}
\max_{w, \mu} & \quad \mu \\
\text{s.t.} & \quad \mu ||U\tilde{w}|| \leq \sqrt{P_s} \tilde{w}^H \tilde{h} \\
& \quad ||\tilde{V}_Q \tilde{w}|| \leq P_N \\
& \quad ||\tilde{V}_P \tilde{w}|| \leq P_0 \\
& \quad \tilde{w}_{\text{first}} = 1
\end{align*}
$$

(7.23)

where $\tilde{w} = [1, w^T]^T$, $\tilde{V}_P = [0_{M \times 1}, V_P]$, $\tilde{V}_Q = [0_{M \times 1}, V_Q]$, $\tilde{h} = [0, h^T]^T$, and $\tilde{w}_{\text{first}}$ denotes the first element of $\tilde{w}$, with

$$
\tilde{Q} = \begin{bmatrix}
\sigma_v^2 & 0_{1 \times M} \\
0_{M \times 1} & Q_{a,i}^{(I)} + Q_{a,i}^{(N)}
\end{bmatrix} = U^H U
$$

$$
Q_{\text{leak}} = V_Q^H V_Q
$$

$$
Q_P = V_P^H V_P
$$

(7.24)

Note that $U, V_Q$ and $V_P$ are the Cholesky factorization product of matrix $\tilde{Q}$, $D$ and $Q_{\text{leak}}$, respectively. The SOCP (3.37) can be solved by firstly reducing it to
a SOCP feasibility problem by assigning a value of \( \mu \) using the bisection search procedure [117] and then the interior point method [113] or some other advanced interior-point-based methods, such as the SeDuMi package [118], which produces a feasibility certificate if the problem is feasible.

Using the bisection search procedure and interior-point-based methods to solve problem (7.24) requires several rounds of iteration and thus requires relatively complicated computation resources. However, under particular conditions, the problem can be reduced to one of the following sub-schemes, and as our simulation results will demonstrate, with particular settings in our considered network those conditions can be met with high probabilities. The two sub-schemes are given as,

\[
\max_w \quad \text{SINR} \\
\text{s.t.} \quad w^H Q_{\text{leak}} w \leq P_{\text{leak}} 
\]

(7.25)

and

\[
\max_w \quad \text{SINR} \\
\text{s.t.} \quad w^H Q_P w \leq P_r
\]

(7.26)

Both of them can be solved using the same approach as in Section 7.3.2. Denote the solution to problem (7.25) and (7.26) as \( w_{\text{opt1}} \) and \( w_{\text{opt2}} \), respectively. Under the following conditions, problem (7.23) can be transformed into either of the above sub-schemes.

**Condition 1:** If \( w_{\text{opt1}}^H Q_P w_{\text{opt1}} \leq P_r \) and \( w_{\text{opt2}}^H Q_{\text{leak}} w_{\text{opt2}} > P_{\text{leak}} \), (7.23) is transformed to sub-scheme (7.25), and the solution is \( w_{\text{opt1}} \).
Condition 2: If $w_{opt1}^H Q_P w_{opt1} > P_r$ and $w_{opt2}^H Q_{leak} w_{opt2} \leq P_{leak}$, (7.23) is transformed to sub-scheme (7.26), and the solution is $w_{opt2}$.

Condition 3: $w_{opt1}^H Q_P w_{opt1} \leq P_r$ and $w_{opt2}^H Q_{leak} w_{opt2} \leq P_{leak}$ can only be satisfied when $w_{opt1}^H Q_P w_{opt1} = P_r$, and $w_{opt2}^H Q_{leak} w_{opt2} = P_{leak}$. In this case, $w_{opt1}$ and $w_{opt2}$ are identical.

Condition 4: If $w_{opt1}^H Q_P w_{opt1} > P_r$ and $w_{opt2}^H Q_{leak} w_{opt2} > P_{leak}$, (7.23) cannot be transformed into either of the sub-schemes, and it remains being solved as SOCP.

7.3.4 Iteration Step on the Receive Part

Now the values of the two transmit beamforming vectors $a_i$, $b_i$ and the relay coefficients $w$ are all updated. Next are the two beamforming vectors $c_i$ for user $X_{a,i}$ and $d_i$ for user $X_{b,i}$. Since the receive beamforming vectors will not produce any leakage interference to the PU receiver, it remains to be decided by the same rules as in 6.2.4 of Chapter 6.

For convenience, we repeat the SINR expression in (7.6) and (7.7) in the following,

$$SINR_{a,i} = \frac{c_i^H F_i^T Q_{a,i}^{(S)} F^*_i c_i}{\sigma^2 + c_i^H F_i^T Q_{a,i}^{(N)} F^*_i c_i + \underbrace{c_i^H F_i^T Q_{a,i}^{(I)} F^*_i c_i}_{\text{IPI}}},$$

(7.27)

where,

$$Q_{a,i}^{(I)} = P_{S} \cdot \sum_{j \neq i}^{K} (W F_j a_j a_j^H F_j^H W^H + W G_j b_j b_j^H G_j^H W^H),$$

$$Q_{a,i}^{(N)} = \sigma^2 \cdot W W^H,$$

$$Q_{a,i}^{(S)} = P_{S} \cdot W G_i b_i b_i^H G_i^H W^H.$$  

(7.28)
The receive beamforming vector that optimizes SINR of each user is given as,

\[ c_i = \rho \{ \Theta_{a,i} \}, \quad d_i = \rho \{ \Theta_{b,i} \}, \quad (7.29) \]

where

\[ \Theta_{a,i} = (\Xi_{a,i})^{-1} F_i^T Q^{(S)}_{a,i} F_i^* , \]
\[ \Xi_{a,i} = \sigma_u^2 I_N + F_i^T Q^{(N)}_{a,i} F_i^* + F_i^T Q^{(I)}_{a,i} F_i^*. \quad (7.30) \]

### 7.3.5 Iteration Algorithm Summary

In our proposed algorithms, we have collaboratively maximized the SINR of each SU by the transmit beamformer, receive beamformer and relay nodes together, while the leakage interference signal introduced to the PU receiver is reduced by carefully designed relay beamforming vectors. The three above iteration steps are repeated until reaching the stopping criterion, which is defined by a preset maximum iteration number \( n_t \) or some convergence requirement (defined by a preset small positive real number \( \delta \)).

The iteration steps of the two proposed schemes are summarized in **Iteration Algorithm Summary A** and **Iteration Algorithm Summary B** for the two considered schemes as follows.
Iteration Algorithm Summary A

1) Initialization: \( c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N} \), where \( \delta_N = 1/\sqrt{N} \), \( w = [\delta_M \delta_M \cdots \delta_M] \), where \( \delta_M = 1/\sqrt{M} \), and set \( t=1 \).

2) Update \( a_i \) and \( b_i \) based on (7.10) and (7.11).

3) Update \( w \) based on (7.15) and (7.17).

4) Update \( c_i \) and \( d_i \) based on (7.29) and (7.30).

5) If \( |x_i^{(t)} - x_i^{(t-1)}|^2 < \delta \) (considered to be converged) or \( t > n_t \) (\( x \leftarrow c \) for user \( X_{a,i} \) and \( x \leftarrow d \) for user \( X_{b,i} \)), iteration stops; otherwise, set \( t = t + 1 \) and go to step 2).

---

Iteration Algorithm Summary B

1) Initialization: \( c_i, d_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1 \times N} \), where \( \delta_N = 1/\sqrt{N} \), \( w = [\delta_M \delta_M \cdots \delta_M] \), where \( \delta_M = 1/\sqrt{M} \), and set \( t=1 \).

2) Update \( a_i \) and \( b_i \) based on (7.10) and (7.11).

3) Obtain \( w_{opt1} \) and \( w_{opt2} \) by solving 7.25 and 7.26, respectively. Update \( w \) based on which of the four conditions that \( w_{opt1} \) and \( w_{opt2} \) satisfy.

4) Update \( c_i \) and \( d_i \) based on (7.29) and (7.30).

5) If \( |x_i^{(t)} - x_i^{(t-1)}|^2 < \delta \) (considered to be converged) or \( t > n_t \) (\( x \leftarrow c \) for user \( X_{a,i} \) and \( x \leftarrow d \) for user \( X_{b,i} \)), iteration stops; otherwise, set \( t = t + 1 \) and go to step 2).
7.4 Simulation Results

In our simulations, we consider the cognitive network with multi-pair communication between SUs, with the number of user pairs being set as $K = 3$. The transmission channels between SUs to relays and relays to PU are quasi-static i.i.d. Rayleigh fading channels; the noise variance at any node is set as 1 ($\sigma^2_r = \sigma^2_u = 1$) and we set the source power at 0 dB ($PS = 1$, compensating the unconsidered large-scale fading). The leakage threshold $P_{\text{leak}}$ is determined by $SNR_L = P_{\text{leak}}/\sigma^2_r$, while the total relay output power constraint is determined by $SNR_R = P_r/\sigma^2_r$. The value of $\epsilon = 0.01$ is chosen to determine the convergence of the iterative process, and $n_t$ represents the maximum number of iteration rounds. And in our simulations, we consider the very worst situation when the PU receiver is located close to the relays, and thus the additional path loss between relays and PU receiver is assumed to be 0 dB.

In Fig. 7.2, we present the average SINR performance of the first proposed approach, versus the leakage interference power threshold at the primary receiver, for different number of relays. It can be seen that the SINR performance is very satisfactory considering that the PU receiver is located close to the relays. When the number of relays increases, the SINR is improved and the improvement is especially significant when the leakage interference power threshold is low. As the threshold $SNR_L$ is set higher, the performance gets better; however, when a large number of relays are included in the network ($M = 10$), the SINR difference at $SNR_L = -10dB$ and $SNR_L = 10dB$ is only 2 dB,
Fig. 7.2: SINR performance versus the leakage power threshold and number of relays (N=5, K=3, n_t=5).

which indicates that increasing the number of relays can dramatically reduce the interference introduced to the PU receiver, when total relay output power is not restricted. The reason is that, as indicated before, in our iterative transceiver algorithm, when the number of relays increases, the total relay output power required to achieve the same SINR can be greatly reduced and so is the total interference introduced to the PU receiver.

Fig. 7.3 demonstrates the SINR performance of the first proposed approach in terms of number of iteration rounds. As can be seen, the SINR performance
Fig. 7.3: SINR performance versus the number of iteration rounds ($P_{\text{leak}} = 0\text{dB}, N=5, K=3$).

versus number of iteration rounds remains in the same pattern as in the previous chapters, where in the first few iteration rounds the improvement is very significant. It shows that our proposed iterative transceiver beamforming algorithm can still effectively coordinate the users and relay transmissions and improve their transmission QoS in the cognitive radio networks.
Fig. 7.4: Probability of Condition 1, second scheme (N=5, K=3).

Fig. 7.5: Probability of Condition 2, second scheme (N=5, K=3).
Figs. 7.4-7.6 illustrate the probability of the relay beamforming vectors satisfying each decision condition of our second scheme, versus $SNR_R$ with different $SNR_L$ and number of relays settings. Note that, since Condition 3 is proved to be only valid when it is a special case of Condition 1 and Condition 2, it is not considered here. It can be seen from Fig. 7.4 that Condition 1 can be satisfied with high probability when a large number of relays are involved in the network and the total relay output power is low. However, in order to achieve good SINR performance at each user, some value of $SNR_R$ is required. Accordingly, unless the relay number is large and we sacrifice some SINR performance by lowering the total output power, the problem cannot be transformed into sub-scheme (7.25) with high probability.
For Condition 2, Fig. 7.5 indicates that in order to transform the scheme into sub-scheme (7.26) and avoid solving the problem as an SOCP, the networks should include less relays; otherwise, the relay nodes should have sufficient output power budget. In Fig. 7.6, the probability of our second scheme satisfying Condition 4 is depicted, in which the relay beamforming vectors can only be obtained by solving an SOCP with several iterations using the bisection search procedure. As shown in the figure, the probability peak is always 1 where the original scheme can not be transformed at all, and the peak is shifted as the number of relays changes. Carefully choosing the network settings to avoid the probability peak of Condition 4 can help reduce the computational complexity of our scenario.

7.5 Summary

In this chapter, the distributed beamforming problem in a cognitive network with multi-pair communication between SUs has been studied. Our previously proposed iterative beamforming algorithm was extended in this network to suppress the leakage interference received at the PU receiver and optimize the average SINR performance of each SU. When a total relay output power constraint is considered, the transceiver beamforming problem can be either solved as an SOCP or transformed into two simpler sub-schemes under some specific conditions. The probability of performing such a transformation is investigated and simulation results are provided to guide the network settings.
Chapter 8

Conclusions and Future Plan

8.1 Conclusions

In this thesis, the FF relay beamforming scheme has been extended in the context of cognitive radio networks in Chapter 3 and the simulations have illustrated that even with the extra task of suppressing interference introduced to the primary user, the FF relay nodes still can suppress the ISI properly. Besides, we have also proved that with certain settings of the network, the problem of designing the FF relay nodes can be reduced to solving two simpler sub-problem with less computational complexity.

Then in Chapter 4, the multi-pair relay network with multi-antenna user nodes has been studied where we proposed an iterative transceiver beamforming scheme to take advantage of the multi-antenna setting of user pairs and shift the main computation tasks from relay nodes to the user nodes. In our proposed scheme, the global solution is very difficult, if not impossible, to obtain, and accordingly we proposed to iteratively obtain a sub-optimal solution by solv-
ing the transmit beamforming vector, relay beamforming vectors and receive beamforming vectors with proper division of the original problem. From the simulation results we can conclude that our scheme can well enhance the average received SINR of each user node with the coordination of the transmit and receive beamformers. We also conclude from the simulation results of kernel density of relay scalar, that the iteration step that decides the relay scalar can be take out of the loop and move to the last step. This conclusion is further proved by our other simulation results in that chapter.

Then, in Chapter 5 we further extended the iterative transceiver beamforming scheme where the iteration steps are completely distributed among user nodes and relay nodes to increase the computational efficiency of the system. Following that, a relay strategy is designed for the relay nodes, which from the results has proved to be able to significantly increase the SINR performance compared to simply using an uniformly AF protocol on the relay nodes. Simulation results also indicate that the proposed method is quite robust to channel state changes between different rounds of iterations.

Robust design of the iterative transceiver beamforming scheme is studied in Chapter 6, where two relay strategies were proposed considering sum relay output power and individual relay output power constraint, respectively. The robust worst-case based formulation was derived in this chapter and the problem introduced by CSI uncertainty was also discussed, where our proposed schemes make some trade-off between guaranteeing the worst-case performance and improving the average performance of each user. The simulation indicates that
both of our proposed schemes are very robust against CSI errors while remaining significant improvement of average user SINR.

In Chapter 7, we considered the transceiver beamforming problem in the context of cognitive radio networks. We demonstrate by the simulation results that with our amended iterative transceiver beamforming scheme applied on the secondary user nodes, the interference introduced to the primary user can always be kept under a predefined interference threshold, while keeping the SINR of the transmissions between secondary user pairs at a satisfactory level.

8.2 Future Work

For future work, there exist some remaining problems and potentials of improvement in our proposed schemes. For the FF relay beamforming scheme, it is studied in the context of cognitive radio networks and it demonstrates its ability to combat the frequency selective channels. However, in our considered networks, the channel states are assumed to be quasi-static, which means that the CSI remains static over a frame time. But in practice, the communication channels are constantly changing, and just like how we studied our iterative transceiver beamforming schemes with different channel stationarity level, similar research could be done for the FF relay beamforming schemes, where proper changes should be applied to the original system models.

In our studied multi-pair two-way relay network with multi-antenna user nodes, the original beamforming problem of maximizing individual SINR of
each user is solved by dividing the problem into three sub-problems, each of which has a closed-form solution. We proposed this alternative way to solve this problem where each sub-problem can be potentially solved on each node with some proper shared information; however, the original problem still seems to remain too difficult to solve, since the SINR of each user is jointly decided by not only its own beamforming vectors, but also beamforming vectors of other users and the relay beamforming vectors. In our simulations, we find that sometimes during the iteration processes, there are some intermediate values of beamforming vectors that can lead to better SINR than the convergence values, which means that the solution of our proposed scheme is indeed sub-optimal.

So the main question is that, for the problem presented in the thesis, is it solvable? If it is, the global solution must require the global information of every user nodes, will the computational complexity be too high and are there any other ways to reduce it? What is the trade-off between reducing the complexity and obtaining the real optimal solution? Is there any other iterative algorithm that can lead to closer solutions to the optimal one?

Since our proposed schemes still require the CSI to be shared among users and relay nodes, another reasonable way to improve our algorithms is to use the statistics of the CSI, and thus the complexity of our schemes can be further reduced. However, this requires a total different system model to be established and this could be done in the future work. There must exist some methods to further reduce the complexity of our schemes by introducing some properly designed reference signals to jointly carry the information of channel states and
updated beamforming vectors, and together with a modified iterative algorithm, our transceiver beamforming schemes could potentially be further improved.

And in Chapter 6, we investigated the application of our iterative transceiver beamforming schemes in cognitive radio networks. However, it was only a simple start with assumption that the PU receiver will not be interfered at the first transmission stage. Further work could be done with respect to suppressing the interference introduced at the first transmission stage, where the decision of transmit beamforming vectors in our first iteration step should be adjusted with proper designs.
Bibliography


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