Improving Software Model Inference by Combining State Merging and Markov Models

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Abstract

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Labelled-transition systems (LTS) are widely used by developers and testers to model software systems in terms of their sequential behaviour. They provide an overview of the behaviour of the system and their reaction to different inputs. LTS models are the foundation for various automated verification techniques such as model-checking and model-based testing. These techniques require up-to-date models to be meaningful. Unfortunately, software models are rare in practice. Due to the effort and time required to build these models manually, a software engineer would want to infer them automatically from traces (sequences of events or function calls).

Many techniques have focused on inferring LTS models from given traces of system execution, where these traces are produced by running a system on a series of tests. State-merging is the foundation of some of the most successful LTS inference techniques to construct LTS models. Passive inference approaches such as $k$-tail and Evidence-Driven State Merging (EDSM) can infer LTS models from these traces. Moreover, the best-performing methods of inferring LTS models rely on the availability of negatives, i.e. traces that are not permitted from specific states and such information is not usually available. The long-standing challenge for such inference approaches is constructing models well from very few traces and without negatives.

Active inference techniques such as Query-driven State Merging (QSM) can learn LTSs from traces by asking queries as tests to a system being learnt. It may lead to infer
inaccurate LTSs since the performance of QSM relies on the availability of traces. The challenge for such inference approaches is inferring LTSs well from very few traces and with fewer queries asked.

In this thesis, investigations of the existing techniques are presented to the challenge of inferring LTS models from few positive traces. These techniques fail to find correct LTS models in cases of insufficient training data. This thesis focuses on finding better solutions to this problem by using evidence obtained from the Markov models to bias the EDSM learner towards merging states that are more likely to correspond to the same state in a model.

Markov models are used to capture the dependencies between event sequences in the collected traces. Those dependencies rely on whether elements of event permitted or prohibited to follow short sequences appear in the traces. This thesis proposed EDSM-Markov a passive inference technique that aimed to improve the existing ones in the absence of negative traces and to prevent the over-generalization problem. In this thesis, improvements obtained by the proposed learners are demonstrated by a series of experiments using randomly-generated labelled-transition systems and case studies. The results obtained from the conducted experiments showed that EDSM-Markov can infer better LTSs compared to other techniques.

This thesis also proposes modifications to the QSM learner to improve the accuracy of the inferred LTSs. This results in a new learner, which is named ModifiedQSM. This includes considering more tests to the system being inferred in order to avoid the over-generalization problem. It includes investigations of using Markov models to reduce the number of queries consumed by the ModifiedQSM learner. Hence, this thesis introduces a new LTS inference technique, which is called MarkovQSM. Moreover, enhancements of LTSs inferred by ModifiedQSM and MarkovQSM learners are demonstrated by a series of experiments. The results from the experiments demonstrate that ModifiedQSM can infer better LTSs compared to other techniques. Moreover, MarkovQSM has proven to significantly reduce the number of membership queries consumed compared to ModifiedQSM with a very small loss of accuracy.
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Software specifications are vital at varying stages during the development of software systems. A software specification is a description of the behaviours of the system under development. Specifications can be formal and informal. Formal specifications are based on a mathematical basis, represented in formal methods such as Z notations [1]. Informal specifications are usually presented in a readable form such as natural language or visual descriptions, and they are included to ease the comprehension of software systems.

In practice, specifications are difficult to write and to modify manually [2, 3]. Brooks [4] claimed that the hardest part during the development of a system is identifying a complete specification.
1.1 The Importance of Specification Inference

The importance of complete and up-to-date specifications is becoming necessary for program comprehension, validation, maintenance, and verification techniques [5, 6]. Maintenance costs can be high if specification missing or outdated [7]. Hence, the existence of up-to-date specifications can reduce maintenance costs [6].

Indeed, complete specifications can aid test generation techniques [8]. Tests can be generated from specifications. However, tests may be worthless if the quality of specifications are poor [9]. Therefore, testing strategies require the complete specification of a system to understand its behaviours and to run meaningful tests that can detect failures easily [8]. Thus, the correctness and reliability of the system are increased.

Today, most software systems are developed with incomplete specifications [20] since developers focus on developing software rather than keeping complete and up-to-date documentations [6]. This negatively affects the program comprehension needed by software engineers to understand the correct behaviours. Therefore, software maintenance can be costly if specifications are outdated or incomplete [2, 21].

To resolve the issue of imprecise and out-dated specifications, the term specification mining (inference) has been introduced to increase the program comprehension [22]. Specification mining can be defined as the automatic process of inferring (extracting) specification as rules [23–26] or behavioural models [22, 27, 28] for a software system. In general, specifications can be inferred from source code [29–31], test cases [32, 33], or execution traces [22, 27, 28].

Ammons et al. [22] stated that automatically extracting specifications can aid verification and enhance the quality of software. However, existing specification inference approaches may produce imprecise specifications [22].

1.1.1 State Machine Inference

In the previous section, the importance of inferring specification is described. In this section, a finite state machine (FSM) and labelled transitions systems (LTS) are introduced. After that, state-based specification inference is described. LTS models are widely used for
verification and validation techniques. In this thesis, we focus on inferring state machine specifications, especially LTS, using the state-merging strategy.

A FSM [10] is a model that is often used to represent a software system, and provides a high-level overview of a system. A FSM is used widely to represent specifications [11]. The state-machine model of a system consists of a set of states and transitions. Each state is represented visually by a circled node where a system may be in. Transitions are linking states to each other, so the system can change its state by moving from its current state to another one if there is a transition between them and this trigged by a specific event [10]. Transitions are shown as edges (arrows).

LTS [12] are an instance of a state machine often used to model system behaviour, and are relied upon by many verification and testing techniques. An LTS model is a simple structure of state machine consisting of states, transitions, and action labels. Behaviours of software systems are often ordered sequences of events or function calls, and can be represented using LTS models [13].

The importance of state-machine models arises in various stages during software development. Testing is one of the most crucial phases to ensure the quality of software systems during their development. It is well known that state machine models play a vital role in testing software system. For instance, model-based testing generation techniques benefit from behavioural models such as FSMs, which represent the intended behaviour of a system, to derive tests from these models, and thus increase the integration and reliability of the system under test. The majority of model-based testing techniques [14–16] rely upon state-based models that describe the behaviour of a system to generate tests from them. Tretmans [17], for instance, used LTS models as a base for model-based testing.

Additionally, model checking [18] is another verification technique that requires representing a system as a state-machine model to check whether it satisfies defined properties as temporal logic [19].

Despite the importance of those models, they can be incomplete in practice, since they require much time and effort to generate manually [34, 35]. To reduce the time and effort needed to generate models, developers have been focusing on inferring state machine models from software behaviours [28, 36].
The automatic inference (or learning) of state-machine models has been studied well in the domain of machine learning, especially grammar inference. *Grammar inference* or *induction* refers to the process of learning a formal grammar using machine-learning techniques from observations, and it is an instance of inductive inference. The problem of grammar inference is concerned with the process of identifying a language from positive (valid) sequences that belong to the language and negative (invalid) sequences that do not \[37, 38\]. Therefore, the problem of state machines inference has been solved using the means of grammar inference.

Several inference techniques have been developed to reduce human effort in generating state machine models automatically. State-machine inference from examples of software behaviours is widely used by software engineers. These examples can either be in the form of scenarios extracted from other models during the development of a software system, or execution traces from the current implementation of a program. Furthermore, the inference of state-machine models can be achieved with the help of machine-learning techniques, especially grammar inference approaches.

The task of inferring state-machine models has been well studied for a variety of reasons. It is generally agreed that today out-dated and incomplete specification leads to difficulties in program comprehension \[24\]. One of the well-known importance of state-based specification inference is software understanding \[6, 59, 60\]. Reiss and Renieris \[61\] stated that software comprehension can be achieved by the inferring of their behaviours.

Another motivation for specification inference is detecting bugs \[62\]. Finding and locating software bugs without specifications is hard \[6\]. Weimer and Mishra \[63\] stated that specification inference in the form of state machines can be used to find bugs. Tonella et al. \[53\] suggested that test cases can be generated from the inferred models in order to reveal bugs.

Additionally, improving test generation techniques is another motivation of inferring state-machine specifications. Walkinshaw \[64\] stated testing a black-box system without specifications is challenging, since there is no basis to estimate the adequacy of test sets. Subsequently, software model inference has become popular in the community of testing to overcome the lack of software models to generate effective test cases \[65–67\] and to reduce the effort of generating them \[6\].
Chapter 1. Introduction

There are many research studies that have attempted to combine the idea of inferring state machine and testing. For instance, Paiva et al. [68] presented a process to reverse-engineer behavioural models for a model-based testing of a GUI application. Other works attempted to infer models from test sets using the concept of inductive inference to find further test cases [64, 69].

1.1.2 Passive Inference and Active Inference

There are many approaches to inferring (or synthesizing) software models from their observations, either passively by reverse engineering (or inferring) models from logs or execution traces using techniques such as state merging, or actively where a human or oracle runs tests to optimize the quality of the mined models.

Passive inference of state machine models from traces have been investigated widely by software engineers [28, 39–42]. Passive approaches of inferring state-machine models have primarily been applied using the state-merging strategy [28, 39]. State merging [43] is the foundation of some of the most successful techniques in inferring state machines from examples.

The EDSM algorithm [44] is a state merging approach that was originally used to learn LTSs that recognize a regular language. Walkinshaw and Bogdanov [45] adapted grammar inference techniques such as EDSM [44] to infer state machine models from execution traces.

Active inference requires interacting with the system under inference to collect observations by asking queries. For instance, QSM [36] is an active inference algorithm to learn state machine models from traces or scenarios. It can be used to control the over-generalization by asking queries during the state-merging process. Passive and active approaches, discussed in detail in chapters 2 and 3, aim to infer state-machine models from provided traces using the idea of state merging.

In practice, inferring of state-machine models from program traces tends to be useless since it may require a large number of traces depending on the complexity of the system being inferred [46]. Besides, it is difficult to collect those execution traces [46, 47]. Indeed, it is unrealistic to gather all possible execution traces to obtain the exact models [47].
Besides, the inferred state machines can be incomplete or inaccurate if the supplied traces are insufficient [46, 47].

Smeenk et al. [48] used the concept of automata learning to infer a state machine model of Engine Status Manager (ESM), which is a software that is used in copies and printers. Smeenk et al. [48] showed that learning a model of ESM requires about 60 million queries to infer a model of ESM. The inferred model has 3,410 states and 77 alphabets. In the ESM case study, the main practical issue is finding the appropriate counterexamples that help the learner to construct the exact model.

1.2 Research Motivation

In this section, the problem of over-generalization is introduced. The main motivation of this thesis is to overcome on the over-generalization issue.

One of the most significant challenges during the inference of state machine is avoiding over-generalization [52]. The inferred models are said to be over-generalized if they permit impossible behaviours. In other words, allowing sequences of event calls that should not be permitted by a software system [53, 54].

In the grammar inference context, the over-generalized state-machine models are those that accept strings that should be rejected [52, 55]. Over-generalization is likely to happen when there are no negative examples, or when there are so few of them that an exact state machine cannot be inferred. Cook and Wolf [49] stated that the problem of identifying DFA from only positive examples is that the learner cannot determine when the over-generalization will occur.

In passive learning, over-generalization is likely to occur when there are no negative traces. Walkinshaw et al. [47] stated that inferred state-machine models are likely to be over-generalized if the negative traces are missed. Overcoming the over-generalization problem using passive inference methods requires a substantial amount of negative traces. Besides, finding an exact model without negative traces is difficult [56]. Despite the significance of negative samples (examples) in avoiding over-generalization of the inferred models, however in practice they are very rare [57, 58].
The current passive inference techniques are likely to over-generalize the inferred models. Lo et al. [70] claimed that verification and validation methods are adversely affected as a result of over-generalization. This raises the need to find a method that can infer exact or good approximation models that avoids the problem of over-generalization. Hence, verification and validation techniques can benefit from the inferred models. Despite this, the current passive inference methods failed at inferring state-machine models well with very few training data.

Active inference techniques of state machine models that represent a software system can tackle the difficulties faced by passive inference. They allow asking queries as tests to the system being inferred. Active inference algorithms such as QSM [36] can be used to learn state machine models. The idea of active learning is very effective in dealing with the over-generalization problem.

As the inferred models can be used for generating test cases [53], they are likely to be overgeneralized. Therefore, over-generalizations may hamper the process of generating test cases. Tonella et al. [53] stated that over-generalized models are not suitable for generating test cases since they would be invalid [53].

It is vital to automatically infer a correct model for different purposes. For instance, the inferred models can be used to assess test sets adequacies [71]. Given a test set, if the inference engine is able to infer a correct model from test executions, then the test set is considered adequate [71].

The main motivation for this research is to find better solutions to the problem of this thesis. The inference of accurate models will help model-based testing techniques to generate valid test cases.

1.3 Aims and Objectives

As mentioned in the previous section, the long-standing challenge for state-machine model inference approaches is in constructing good hypothesis models from very little data. In addition, finding the exact model without negative information is an intractable task. The main objective of this thesis is to improve the state-merging strategy to infer state-machine models in cases where negative traces are not provided.
In computer science, the Markov model is a well-known principle and is widely used to capture dependencies between events that appear in event sequences [49]. It is the simplest model of natural language. In general, the aim of a statistical language model such as the Markov chain models is to highlight likely event sequences by assigning high probabilities to the most probable sequences, and giving (allocating) low probabilities to unlikely ones [50].

Cook and Wolf [49] presented a method that uses Markov models to find the most probable FSM based on the probability of event sequences in the provided samples. Bogdanov and Walkinshaw [51] showed that FSMs obtained using Markov models can be closer to the target FSMs compared to those obtained using reverse-engineering techniques. The study made by Bogdanov and Walkinshaw [51] motivate us to study the influence of incorporating the Markov model and the state merging strategy. In this thesis, the major focus is on taking advantage of a Markov model to capture event dependencies from long high-level traces alongside the idea of inferring LTS models to optimize the quality of inferred models. This is due to the fact that the Markov model can capture the sequential dependencies between events, as described by Cook and Wolf [137]. The trained Markov models Thus, we used the sequential dependencies in the proposed work to identify whether the inferred models introduce inconsistencies (contradictions) with respect to the initial traces.

This thesis focuses on finding solutions to the above-mentioned challenges. Therefore, the concept of Markov model is used to capture event dependencies and improve the accuracy of the inferred LTSs. In other words, we focused on information obtained from Markov models to constraint the process of inferring LTS models. The extracted constraints from the trained Markov models aimed to prevent the over-generalization problem and hence infer an accurate model. The captured dependencies can be used to guide the idea of state-merging towards merging states correctly during the inference of LTS models. Intuitively, improving the inference techniques that rely on the generalization of the traces would enhance program understanding, and other software engineering tasks.

The following list summarizes the aims of this research:

- To study existing techniques of inference of LTS from few positive traces.
- To adapt the state-of-the-art approaches to solve the problem of inferring LTS from few traces where no negative traces are provided.
• To evaluate the proposed methods both on the type of problems they aim to solve and in a more general setting.

1.4 Contributions

1. An improvement to the EDSM learner, resulting in a new inference method, which is named EDSM-Markov. It benefits from both the trained Markov models and state-merging techniques in order to improve the accuracy of the inferred models.

2. An evaluation of the performance of the EDSM-Markov inference technique at inferring good LTSs from only positive traces, and demonstrating the improvement made by EDSM-Markov compared to SiccoN. The evaluation was performed using randomly-generated LTSs and case studies.

3. An improvement to the QSM learning algorithm, resulting in a new inference method, which is called ModifiedQSM. This introduces a new generator of membership queries in order to avoid the problem of over-generalization, benefiting from the idea of active learning.

4. An extension of the ModifiedQSM by incorporating heuristic based on the Markov model in order to reduce the number of membership queries consumed by ModifiedQSM. This results in a new LTS inference technique, which is called MarkovQSM.

5. Evaluation of the performance of the ModifiedQSM and MarkovQSM inference techniques, and showing the impact made by both learners on the accuracy of the inferred models and the number of membership queries.

1.5 Research Questions

The following research question will be answered in the concluding chapter.

1. How effective are Markov models at capturing dependencies between events in realistic software?

2. How effective are Markov models as a source of prohibited events in the inference of models from realistic software using EDSM?
3. Under which conditions does \textit{EDSM} with Markov models improve over \textit{EDSM} without Markov models?

4. To what extent are the developed inference algorithms able to generate exact models and avoid the over-generalization problem?

5. Under which conditions does \textit{QSM} with Markov models improve over \textit{QSM} without Markov models?

6. With respect to the concept of active inference, what is the reduction of the number of queries obtained by using Markov models, compared to \textit{QSM}?

1.6 Thesis Outline

This thesis is divided into different chapters as follows:

\textbf{Chapter 2.} This chapter describes the notation and types of models that are used in the thesis. It includes the basic idea of inferring LTS models in terms of state merging. This chapter also describes the methods to evaluate an inference algorithm from different perspectives.

\textit{State of the Art}

\textbf{Chapter 3.} This chapter reviews the related techniques and their drawbacks. In addition, it provides the theoretical and practical study of the applicability of existing algorithms to the thesis’s problem.

\textit{Contributions of this Thesis}

\textbf{Chapter 4.} This chapter describes the definition of the Markov model and introduces a solution to infer state-based models from very long sparse traces. In this chapter, the idea of Markov models is introduced to increase the accuracy of LTS models inferred by existing state-merging techniques. This chapter describes the \textit{EDSM-Markov} inference algorithm, which improves on an existing one.
Chapter 5. This chapter provides an evaluation of the performance of the *EDSM-Markov* inference algorithm.

Chapter 6. This chapter explores the inference technique with the aid of an automated Oracle in tackling the sparseness of data, and proposes an enhancement to minimize the efforts made by the automated Oracle. This chapter describes the *ModifiedQSM* and *MarkovQSM* inference algorithms, which improve on the original *QSM*.

Chapter 7. This chapter provides an evaluation of the performance of the *ModifiedQSM* and *MarkovQSM* inference algorithms.

*Conclusion and Future Work*

Chapter 8. This chapter provides conclusions and the findings of this research and proposes the direction for future work.
This chapter provides the basic definitions and notations related to model inference. It describes the learnability models that can be used as schemes of state machine inference. It also introduces an overview of the inference of state-machine models using the state-merging approach. At the end of this chapter, we present ways to evaluate model inference techniques.

### 2.1 Deterministic Finite State Automata

A deterministic finite state automaton (DFA) is one of the most widely used automata to represent software behaviours [35]. It can be defined with a 5-tuple as follows:

**Definition 2.1.** Following [34], a DFA can be represented with \((Q, \Sigma, F, \delta, q_0)\), where \(Q\) is a set of *states* with \(q_0\) the initial state and \(F\) the set of accept states, \(\Sigma\) is *alphabet* and \(\delta\) is the next state function \(\delta : Q \times \Sigma \rightarrow Q\). All sets are assumed finite and \(F \subseteq Q\).
Chapter 2. Definitions, Notations and Models

A DFA $A$ is called deterministic if, for a given state $q \in Q$ and a given label $\sigma \in \Sigma$, only at most one transition that is labelled with $\sigma$ can leave $q$ [72]. Otherwise, it is called non-deterministic.

2.2 Labelled Transition System

A labelled transition system (LTS) [12] is a basic form of state machine that summarizes all possible sequences of action labels [73]. LTS is used to model prefix-closed languages [35] and can be defined with a 4-tuple.

**Definition 2.2.** [13, 51] A deterministic Labelled Transition System (LTS) is a tuple $(Q, \Sigma, \delta, q_0)$, where $Q$ is the set of states with $q_0$ the initial state, $\Sigma$ is a alphabet and $\delta$ is the partial next state function $\delta : Q \times \Sigma \rightarrow Q$. All sets are assumed finite. All states are accepted.

The transition function $\delta$ is usually depicted using a diagram. Where $q, q' \in Q$, $\sigma \in \Sigma$ and $q' = \delta(q, \sigma)$, it is said that there is an arc labelled with $\sigma$ from $q$ to $q'$, usually denoted with $q \xrightarrow{\sigma} q'$. The behaviour is a set of sequences $L \subseteq \Sigma^*$, permitted by an LTS. Where there is not a transition with label $\sigma$ from $q$ such that $(q, \sigma) \notin \delta$, we write $\delta(q, \sigma) = \emptyset$.

Hopcroft et al. [74] introduced an extended transition function to process a sequence from any given state. In this way, the extended transition function, denoted by $\hat{\delta}$, is a mapping of $\hat{\delta} : Q \times \Sigma^* \rightarrow Q$.

The set of labels of the outgoing transitions for a given state $q \in Q$ is defined in Definition 2.3.

**Definition 2.3.** Given a state $q \in Q$ and the current automaton $(A)$. The set of labels of the outgoing transitions of $q$, denoted by $\Sigma^\text{out}_q$, is defined as follows: $\Sigma^\text{out}_q = \{ \sigma \in \Sigma | \exists q' \in Q \text{ such that } \delta(q, \sigma) = q' \}$.

2.2.1 LTS and Language

The language of an LTS $A$ is a set of sequences that are accepted by $A$. In other words, the language $L$, represented using an LTS $A$, accepts a sequence $w = \{ a_1 \ldots a_n \} \in \Sigma^*$, if there is a sequence of labels (path) from the initial state $q_0$ to any other state $q_1 \in Q$. 
Chapter 2. Definitions, Notations and Models

Given an LTS $A$ and a state $q \in Q$, the language of $A$ in the state $q$ denoted $L(A, q)$ can be defined as $L(A, q) = \{ w | \delta(q, w) \}$ [13]. Hence, the language of $A$, denoted by $L(A)$, is given by $L(A) = \{ w | \delta(q_0, w) \}$. For a given LTS $A$, the complement of a language $L(A)$ with respect to $\Sigma^*$ is the set of sequences that is not part of $L(A)$. This set is denoted by $\overline{L(A)}$ [13, 75].

**Definition 2.4.** [76] A prefix-closed language $L$ is a language that $\forall w \in L$, then every prefix $y$ of $w$ also belong to $L$.

### 2.2.2 Partial Labelled Transition System

A Partial Labelled transition system (PLTS) can be defined with a 5-tuple.

**Definition 2.5.** A Partial Labelled Transition System (PLTS) is a tuple $(\Sigma, Q, \delta, F^+, F^-, q_0)$, where $\Sigma$ is the finite alphabet, $Q$ is the set of states (with $q_0$ the initial state), and $\delta$ is the partial next state function $\delta : F^+ \times \Sigma \rightarrow Q$. So, there are not transitions leaving a rejected state. $F^+$ is a set of accepting states, and $F^-$ is a set of rejected states. $F^+ \cap F^- = \emptyset$, $F^+ \cup F^- = Q$.

A PLTS is introduced in this thesis because the learning of LTS models for a prefix-closed language can begin with negative traces or acquiring them during the active learning. Hence, the resulting machine is a PLTS. In this case, once the learner finishes, the PLTS is converted to an LTS.

### 2.2.3 Traces

A trace is a finite sequence of events or function calls. In this thesis, a trace is a sequence of alphabet elements to be an input to the inference process in this thesis. A trace is written formally $\langle e_1, e_2, \cdots, e_n \rangle$. The empty sequence is denoted by $\epsilon$ such that $\epsilon \in \Sigma^*$.

Let $x$, $y$, and $z$ denote sequences belongs to $\Sigma^*$. The concatenation of two sequences $y$ and $z$ is expressed as $y \cdot z$ or $yz$. We say that $y$ is the prefix of a sequence $x = yz$ and $z$ is the suffix of $x$. Let $|x|$ denote the length of the sequence $x$. 
Let $x = \langle e_1, e_2, e_3 \rangle$ and $y = \langle e_4, e_5, e_6 \rangle$. We write $z = x \cdot y$ to denote the concatenation of two sequences. In this case, $z = \langle e_1, e_2, e_3, e_4, e_5, e_6 \rangle$. The term traces and sequences are used interchangeably.

### 2.2.4 Example of Text Editor

Consider the text editor example introduced in [77], in which documents are initially loaded to be ready for editing. They can be closed after they have been loaded on the condition that no editing has been done to them. Once documents are edited, they can be saved. Documents can then be closed to load other documents. The text editor can be exited at any time. Figure 2.1 illustrates an LTS of a simple text editor. This example will be used through chapters 2 and 3.

![Figure 2.1: An LTS of a text editor](image)

In the text editor, examples of positive traces to state $D$ are as follows: \{\langle Load, Edit \rangle, \langle Load, Close, Load, Edit \rangle, \langle Load, Close, Load, Edit, Save, Edit \rangle\}.

### 2.3 Three Learning-Model Frameworks

This thesis focuses on the study of LTS model identification, which is widely used in verification techniques as we mentioned in the early sections in chapter 1. Synthesis of behavioural models can automatically follow one of the following model-learning schemes.
• **Identification in the limit** (Gold’s model): The learnability of state-machine models was studied originally by Gold [56], and it was shown that learning a DFA from samples is very difficult to solve [78].

• **Query learning** (Angluin’s model): It is a very common model to infer a DFA model to improve Gold’s identification of DFA [79]. It aimed to learn a correct hypothesis (LTS in our context) with the aid of a teacher to answer specific questions (queries).

• **PAC identification**: Valiant [80] introduced a probably approximately correct (PAC) model aimed at inferring a good approximation of the target DFA models.

### 2.3.1 Identification in the Limit

In computational learning theory, Gold [56, 78] presented a basic paradigm of inductive inference for language learnability, which is called *identification in the limit*, also known as *Gold’s model*. Gold [56, 78] investigated the ability to learn a model $M$ in terms of its language $L$, and it was the first attempt to identify the problem of language learnability using grammar inference methods. In Gold’s framework, the learner is given a sequence of positive information compatible with the target language or model. At each time step $i$ the learner must return a hypothesis $h_i$ representing the current guessing at the step $i$ based on the current representation of data [81, 82]. As the presented samples increased, the learner infers new guesses (hypothesis) [81, 82]. The target language $L$ is *identified in the limit* if, after a finite number of steps, all solutions (hypotheses) remain stable without any changes on the condition that the language of guesses (hypotheses) are the same.

Gold [56] showed that a language will be *learnable* if there is a *learner* to identify the correct language in a limit. The term *identification in the limit* has therefore become the most important concept to study in language acquisition and inductive inference. The meaning of *limit* is that a language is *identified or learnable* in a finite number of steps to guess the correct hypothesis model whenever a new sequence is provided [56]. In other words, Gold [56] concluded that the language is *learnable* if there is a learner to decide which strings belong to the language and which of them do not. However, in some cases, the learning process is never ending as information continues to grow, meaning the hypothesis is updating continuously [56, 78, 83]. Hence, the learner will never be confident enough
about the current hypothesis to decide whether the learning process can find the target concept or not.

In this thesis, passive inference techniques such as $k$-tails and EDSM follow the identification in the limit model. These techniques assume that there is a learner that is given examples and its role is to infer a model from the provided examples.

### 2.3.2 Angluin’s Model

One of the most successful models in the learning theory is the query model, active learning also known as Angluin’s model, which was originally studied by Angluin [79, 84] to tackle the difficulty of language identification in the Gold-style model. Angluin [79, 84, 85] assumed the existence of a person or machine called a teacher (oracle) who knows the hidden grammar of the target language (concept). Moreover, Angluin’s [84, 84, 85] model focuses on learning an unknown concept in a finite number of steps, whereas a learner interacts with a teacher to build an exact hypothesis. The learner asks questions to receive more information about the target concept and the teacher answers them.

This model is proven to return a hypothesis that correctly represents the target concept [81]. The effectiveness of Angluin’s model comes from the usage of equivalence queries to decide when to stop the learning process.

In this thesis, active inference techniques such as QSM follow Angluin’s model. The QSM algorithm assumes that there is a teacher where the QSM learner is given examples by the teacher. The QSM learner can interact with the teacher to infer a correct model.

### 2.3.3 PAC Identification Model

Valiant [80] proposed the probably approximately correct (PAC) framework that aimed to find an approximation hypothesis to the target concept with high probability. It differs from both identification in the limit and query learning models, and presents language learnability in a probabilistic perspective to identify a hypothesis with a low probability of errors. In a DFA inference setting, a PAC learner attempts to obtain a DFA (hypothesis) that approximates to the target DFA (concept) [37].
2.4 Finite Automata Inference

In this section, preliminaries of finite automata inference are given in section 2.4.1. We then describe the problem of inferring (finding) DFA using the aid of grammar inference techniques in section 2.4.2. The basic idea of state merging is described in section 2.4.3.

2.4.1 Preliminaries of finite automata inference

Let $Pr(x)$ denote the set of all possible prefixes of $x$. The set $Pr(L) = \{x \mid xy \in L\}$ is the set of prefixes of the language and the set $Suff(x) = \{y \mid xy \in L\}$ is the set of suffixes of $x$ in $L$.

The set of short prefixes $S_p(L)$ of a language $L$ is defined as $S_p(L) = \{x \in Pr(L) \mid \not\exists y \in \Sigma^* \text{ such that } Suff(x) = Suff(y) \text{ and } y < x\}$ [36, 37]. In the automaton $A(L)$ that identifies the language $L$, the $S_p(L)$ set contains sequences in which for each specific state $q$ in $Q$, there is a sequence $x \in S_p(L)$ leads to $q$. In the text editor example shown in Figure 2.1, the $S_p(L) = \{\epsilon, \langle Load \rangle, \langle Exit \rangle, \langle Load, Edit \rangle\}$. The kernel $N(L)$ of a language $L$ is defined as $N(L) = \{\epsilon\} \cup \{xa \mid x \in S_p(L), a \in \Sigma, xa \in Pr(L)\}$ [36, 37]. So, $S_p(L) \subseteq N(L)$ [36]. Let us consider the text editor illustrated in Figure 2.1, the $N(L) = \{\epsilon, \langle Load \rangle, \langle Exit \rangle, \langle Load, Edit \rangle, \langle Load, Edit, Save \rangle, \langle Load, Edit, Edit \rangle, \langle Load, Edit, Exit \rangle\}$.

2.4.2 The problem of LTS Inference Using Grammar Inference

Essentially, grammar inference methods focus on identification of the grammar of a language $G(L)$ from a given set of samples. Those samples contain positive samples $S^+$ that belong to the language $L$, and possibly some negative samples $S^-$ that do not belong to the language $L$. In other words, the problem of grammar inference includes constructing a model that describes the grammar such as LTS models. The problem of grammar inference is defined as follows:

**Definition 2.6.** Given a sample of positive and negative sequences $S = S^+ \cup S^-$ over a subset of alphabet $\Sigma^*$ such that $S^+ \in L$ and $S^- \notin L$, find a LTS $A$ which can accept all $S^+$ and reject all $S^-$. 
For any regular language \( L \), different DFAs might represent \( L \), and there exists the smallest DFA that accepts the positive sequences and rejects the negative ones \([55]\). The positive and negative samples are the starting point for DFA inference. DFA inference techniques are divided into two overall methods. First, passive learning, this is where a DFA is inferred in one shot from a finite set of positive and negative samples. Second, active learning algorithms use queries to a system being learnt to overcome missing information.

The problem of inferring DFA/LTS is re-investigated in the inductive-inference concept as the attempt to find a hypothesis (DFA) about a hidden concept (hidden regular language). It has aimed to find the smallest DFA/LTS that is consistent with the given training data. The problem of finding the smallest DFA/LTS has been shown to be a difficult task \([56, 86]\). The DFA hypothesis obtained by the learner needs to be very small in comparison to other possible hypotheses. The simplicity of the inferred hypothesis is important to achieve Occam’s razor principle, which states that the simpler explanation (representation) is the best \([87]\). In other words, given two DFA \( A, A’ \) consistent with the training data, the smaller DFA is preferable.

Unfortunately, the task of inferring the smallest LTS/DFA is very difficult. It has been shown that learning a DFA from samples is NP-hard \([78]\). Despite these difficulties, a number of approaches are developed to deal with the problem of inferring a DFA from positive and negative samples. In the following section, we describe the important solutions to the problem using state-merging techniques. In Chapter 3, we discuss possible algorithms of finding a DFA using idea state merging (Section 3.1) and other algorithms based on query learning in Section 3.2.

### 2.4.3 State Merging

In this section, we discuss one of the most important state machine model learning strategies, which is called state merging. The state-merging technique is the foundation for most successful techniques in inferring LTS from samples. Many passive inference methods rely on the idea of state merging; they begin by constructing a tree-shaped state machine built from the provided samples, and iteratively merging the states in the tree to construct an automaton. This tree-shaped state machine is called a prefix tree acceptor (PTA) if it is built from only positive samples \( S^+ \), where there is a unique path from the root state \( q_0 \).
to an accepting state for each sample in $S^+$ [88]. Formally, PTA is defined in the same way as a LTS, except that it cannot contain any loops.

**Definition 2.7.** A prefix tree acceptor is a tuple $(Q, \Sigma, \delta, q_0)$, where $Q$, $\Sigma$, $q_0$, and $\delta$ are defined as a LTS.

The PTA is called *augmented* prefix tree acceptor (APTA) if it is constructed from both positive and negative samples. An APTA is a PLTS built from positive and negative traces. It is defined formally in Definition 2.8.

**Definition 2.8.** An augmented prefix tree acceptor is a tuple $(Q, \Sigma, \delta, q_0, F^+, F^-)$, where $Q = F^+ \cup F^-$, $\Sigma$, and $\delta$ are defined as a LTS. $q_0$ is the root node in the tree. $F^+$ is the final nodes of the accepted sequences, and $F^-$ is the final nodes of the rejected sequences.

Consider the text editor example described above and introduced in [77], where the training sample could be $S^+ = \{\langle \text{Load, Edit, Edit, Save, Close} \rangle, \langle \text{Load, Edit, Save, Close} \rangle, \langle \text{Load, Close, Load} \rangle\}$ and $S^- = \{\langle \text{Load, Close, Edit} \rangle\}$. The constructed PTA from the training sample is as shown in Figure 2.2. The corresponding APTA is highlighted in Figure 2.3 where the grey state is a rejecting state, and the other states are accepting states.

The merging of two states $(q_1, q_2)$ means collapsing them into one and all outgoing and incoming transitions of $q_2$ are added into $q_1$. In other words, there is the construction of a new state (a merged state) that all outgoing and incoming transitions of both states $(q_1, q_2)$ are assigned to. Figure 2.4 illustrates an example of state merging. A merger of a pair of states is acceptable if they are *compatible*, this means that both of them must be either accepting or rejecting (see the first condition in Definition 2.9). In the text editor example which is illustrated in Figure 2.3, the state that is labelled with $N$ cannot be merged with any other states in the text editor PTA. Unless however, there are other rejecting states to merge with.
Definition 2.9. Given a pair of states \((q_1, q_2) \in Q\) and \(APTA(A)\). A merge of \((q_1, q_2)\) is said to be compatible if both of the following conditions are satisfied:

1. \((q_1 \in F^+ \land q_2 \in F^+) \lor (q_1 \in F^- \land q_2 \in F^-)\).
2. \(\forall \sigma\) such that \(q_1 \xrightarrow{\sigma} q_1'\), \(q_2 \xrightarrow{\sigma} q_2'\), \(q_1'\) and \(q_2'\) are compatible.

The second condition in Definition 2.9 implies that if there are outgoing transitions with the same label leaving both states, their target states must be compatible. For example, in the text editor example shown in Figure 2.3, states \(B\) and \(D\) are not compatible because there is a transition with input \(Edit\) from \(B\) leading to the accepting state \(C\). Also, the transition with the same input from \(D\) leading to the rejecting state \(N\). It is worth noting that states \(B\) and \(D\) satisfy the first condition but not the second one.

It is important to highlight that a merger may introduce a non-determinism. Hence, children of a pair of states are merged to remove non-determinism on the condition that those children nodes are compatible as well. The whole body of the state-merging function is provided in Algorithm 1. It begins by checking the compatibility of the given pair of
input: \( q_1, q_2, A \)

/* a pair of states \((q_1, q_2)\) and \(A\) is an APTA */

result: \(\text{mergeable}\) is a boolean value indicates whether a pair of states \((q_1, q_2)\) is mergeable or not

1. \(\text{compatible} \leftarrow \text{checkMergeCompatibility}(A, q_1, q_2)\);
2. if \(\text{compatible}\) then
   3. \(A_{\text{new}} \leftarrow \text{merge}(A, q_1, q_2)\);
   4. while \((q'_1, q'_2) \leftarrow \text{FindNonDeterministic}(A_{\text{new}}, q_1, q_2)\) do
      5. \(A_{\text{new}} \leftarrow \text{merge}(A_{\text{new}}, q'_1, q'_2)\);
      6. \(\text{compatible} \leftarrow \text{checkMergeCompatibility}(A_{\text{new}}, q'_1, q'_2)\);
      7. if \(\text{compatible}\) then
         8. \(\text{mergeable} \leftarrow \text{true}\);
      9. else
         10. \(\text{mergeable} \leftarrow \text{false}\);
         11. return \(\text{mergeable}\)
   12. end
3. end
4. else
   15. \(\text{mergeable} \leftarrow \text{false}\);
5. end
6. return \(\text{mergeable}\)

Algorithm 1: The state merging algorithm

states using the \(\text{checkMergeCompatibility}(A, q_1, q_2)\) function as shown in line 1. The pair of states \((q_1, q_2)\) are said to be \(\text{compatible}\) if both states are either accepting or rejecting. If the given pair of states are compatible, then the \(\text{merge}(A, q_1, q_2)\) function is invoked to merge states.

The loop in lines 4-13 is the procedure of the recursive state-merging. If there is a non-determinism, then target states of transitions causes a non-determinism, these nodes must be merged as shown in line 5. Moreover, the compatibility of these target states are checked again as shown in line 6.

A merge of a pair of states may produce a non-deterministic machine. Merging of states \(G\) and \(C\) in the text editor example leads to non-deterministic automata as shown in Figure 2.5. This is where two transitions are triggered with the same label \(\text{Save}\) from the state that is labelled with \(CG\). In this case, the target states \((H\) and \(E)\) of transitions labelled with \(\text{Save}\) are merged as well.
2.4.4 RPNI Algorithm

The idea of state merging was originally developed by Trakhtenbrot and Barzdin [89] to generate an acceptor of a language. Their algorithm was shown to infer a correct DFA on the condition that the provided samples were complete. Oncina and Garcia [43] proposed a refinement to Trakhtenbrot and Barzdin’s [89] algorithm called the Regular Positive and Negative Inference (RPNI). They claimed that samples should be characteristic to construct the exact identification of DFA. Characteristic samples include all paths that cover transitions between every pair of states as well as paths to distinguish between every pair of states.

Definition 2.10. Following [36], given an LTS $A$ and positive samples $S^+$ such that $S^+ \in L$, $S^+$ is considered structurally complete with respect to $A$ if all transition of $A$ are visited at least once during the collection of samples.

Definition 2.11. Given an LTS, a sample $S = S^+ \cup S^-$ is said to be characteristic [36, 43] if:

1. $\forall x \in N(L)$, if $x \in L$ then $x \in S^+$ else $\exists u \in \Sigma^*$ such that $xu \in S^+$. This implies the structural completeness [36].

   $\forall y \in N(L), \forall x \in S_p(L)$ if $Suff(x) \neq Suff(y)$ then $\exists u \in \Sigma^*$ such that $(xu \in S^+ \land yu \in S^-) \lor (xu \in S^- \land yu \in S^+)$.

There are two conditions that must be satisfied to imply that the provided samples $S$ are characteristic, and they are described in Definition 2.11. The first condition says that each
sequence $x$ in the kernel $N$ belongs to the correct language $L$, it also belongs to the set of positive samples $S^+$, otherwise the sequence $x$ can be a prefix for other suffixes $u$ in which the sequence $xu$ belongs to $S^+$. The second condition implies that a suffix $u$ would distinguish states whenever a sequence $x$ in the set of short prefixes $S_p(L)$ and $y$ belong to the kernel $N$ if they do not have the same set of suffixes $\text{Suff}(x) \neq \text{Suff}(y)$.

Before we describe the \textit{RPNI} algorithm, the following notation is used: suppose $A=(Q, \Sigma, F, \delta, q_0)$ is a finite state machine, and let $\pi$ be a partition of states $Q$ of $A$. A subset of elements of a partition is called block $B$. Provided mergers of all states in each block are valid, a quotient automaton $A/\pi$ is obtained by merging states that belong to the same block of $\pi$.

The whole body of the \textit{RPNI} algorithm is provided in Algorithm 2. The \textit{RPNI} takes a finite set of positive and negative samples and constructs the corresponding PTA from the positive samples. The negative samples are introduced to stop merging states if the resulting automaton leads to accept negative samples. It performs a breadth-first search to identify pairs of states to merge. After constructing the APTA from the positive and negative samples, an initial partition $\pi$ is determined. At each step of the generalization, two blocks $(B_i, B_j)$ of the partition $\pi$ are selected for merging. During the merging of blocks, a non-deterministic automaton might be obtained, and the partition is then updated to solve the non-determinism. After merging them, a new intermediate hypothesis automaton is obtained $PTA/\pi_{\text{new}}$. Once the new hypothesis solution is compatible with the negative samples, the partition $\pi$ is updated with $\pi_{\text{new}}$ as shown in line 6, otherwise the solution is rejected. The generalization process continues by selecting other candidates of blocks to merge until no more states can be merged.

```
input: $S^+$ and $S^-$ /* Sets of accepted and rejected sequences */
result: $A$ is a DFA that is compatible with $S^+$ and $S^-$
1 $PTA \leftarrow \text{Initialize}(S^+)$
2 $\pi \leftarrow \{0\}, \{1\}, \ldots, \{N-1\}$
3 while $(B_i, B_j) \leftarrow \text{SelectPairOfBlocks}(\pi)$ do
4     $\pi_{\text{new}} \leftarrow \text{Merge}(\pi, B_i, B_j)$
5     if $\text{Compatible}(PTA/\pi_{\text{new}}, S^-)$ then
6         $\pi \leftarrow \pi_{\text{new}}$
7     end
8 end
```

\textbf{Algorithm 2:} RPNI algorithm [90]
2.4.5 Example of RPNI

As an example to demonstrate the RPNI algorithm, let us consider that the following positive and negative samples of the text editor example described in Section 2.2.4 are given respectively $S^+ = \{\langle \text{Load}, \text{Edit}, \text{Edit}, \text{Save}, \text{Close} \rangle, \langle \text{Load}, \text{Edit}, \text{Save}, \text{Close} \rangle, \langle \text{Load}, \text{Close}, \text{Load} \rangle\}$, $S^- = \{\langle \text{Load}, \text{Save} \rangle, \langle \text{Load}, \text{Close}, \text{Edit} \rangle\}$. The RPNI algorithm constructs the initial PTA from the positive samples as illustrated in Figure 2.6. In this case, the initial partition is $\pi_0 = \{\{A\}, \{B\}, \{C\}, \{D\}, \{G\}, \{E\}, \{H\}, \{I\}, \{F\}, \{K\}\}$ where each state is added to a specific block $B$.

![Figure 2.6: An example PTA for a text editor.](image)

The RPNI algorithm then tries to merge the block $B_i = \{A\}$ that contains only $A$ state and the block $B_i = \{B\}$ resulting in the new quotient automaton $A/\pi$ as shown in Figure 2.7.

The partition $\pi_0$ is then updated yielding a new partition $\pi_1 = \{\{A, B\}, \{C\}, \{D\}, \{G\}, \{E\}, \{H\}, \{I\}, \{F\}, \{K\}\}$.

![Figure 2.7: An automaton after the merging of states A and B.](image)
2.5 Evaluation of Software Models

This section describes the methods of evaluating state-machine inference algorithms from different points of view. It includes methods that rely on generating test sequences using the W-method or random walks for evaluating the inferred models.

It is difficult to evaluate and compare state-machine inference techniques since there is no standardized way to accomplish this. Pradel et al. [97] stated that the task of evaluating different specification miners is difficult since there are no common methods to assess the quality of mined specifications. In general, the common way is to evaluate and compare the inferred state machines against their reference models. They are intended to represent software being reverse engineered.

2.5.1 The W-method

The W-method is the most common method of generating tests from an automaton. It and was originally proposed by Chow [91], Vasilevskii [92] for generating test cases from FSMs. The W-method has been investigated to generate tests from different kinds of state machines. For instance, Bogdanov et al. [93] used the W-method to construct test sequences from X-machines models. Whereas, Ipat and Banica [94] adapted the W-method to generate test sequences from hierarchical FSMs. Moreover, variations of the W-method have been developed, for example, Fujiwara et al. [95] proposed the partial W-method (Wp-method) to reduce the length of test sets.

To describe the W-method in the context of comparing two different LTSs, given a specification LTS $S$ and an implementation LTS $I$, the aim of the W-method is to construct a test set, which is a finite set of sequences (test cases), from $S$ to measure the conformity of $I$ against $S$. The test set $TS$ should cover each state of $S$ in order to find contradictions between $I$ and $S$ by passing $TS$ as tests to $I$. Once the implementation $I$ generates the corresponding outputs to the test sequences $TS$, those outputs would be compared to the correct ones of $S$.

The task of checking the conformance between an implementation LTS $I$ and a specification LTS $S$ can be unsolved as $I$ may contain extra faulty states that cannot be visited by the generated test set [13]. In a testing context, it is difficult to know the maximum number
of states \( m \) in the implementation \( I \). Also, a tester may not have access to the correct implementation \([13, 91]\). In this case, a tester has to estimate \( m \). This phase is critical where an incorrect estimation of \( m \) may cause the generation of inadequate test sets.

The \( W \)-method begins by estimating the number of states in the implementation \( I \). Once the maximum number of states \( m \) is known or estimated, the \( W \)-method makes some assumptions about the specification \( S \) and the implementation \( I \). The specification \( S \) should be minimal, completely specified and deterministic. Such assumptions are necessary in order to generate a finite set of test sets to ensure that the implementation \( I \) is correctly implemented against the specification \( S \). The \( W \)-method assumes that the number of states \( m \) in \( I \) may be larger than the number of states \( n \) in \( S \).

- **Construction of state cover set** \( C \).

  A state cover set \( C \) of a specification LTS \( S \) is a prefixed-closed set of sequences that are required to visit each state of an LTS \( S \) from the initial state \( q_0 \in Q \) at least once.

**Definition 2.12.** \( C \subseteq \Sigma^* \) is said to be a state cover of an LTS \( S \) such that \( C \subseteq L(S) \) if \( \epsilon \in C \) and \( \forall q \in Q \setminus \{q_0\}, \exists c \in C \) such that \( \hat{\delta}(q_0, c) = q \).

In the text editor example shown in Figure 2.1, the state cover set is \( S = \{\epsilon, \langle \text{Load} \rangle, \langle \text{Exit} \rangle, \langle \text{Load, Edit} \rangle\} \).

- **Construction of characterization set** \( W \).

**Definition 2.13.** Given a set of input sequences \( W \) such that \( W \subseteq \Sigma^* \) and two states \( q_1, q_2 \in Q \). So, \( q_1, q_2 \) is said \( W \)-distinguishable if \( (L(S, q_1) \cap W) \neq (L(S, q_2) \cap W) \)[13].

The set of input sequences that can distinguish between any two states in \( S \) is called a characterization or separation set.

**Definition 2.14.** Given a set of input sequences \( W \) such that \( W \subseteq \Sigma^* \). \( W \) is called a characterization set \([96]\) of \( S \) if any two distinct states of \( S \) are \( W \)-distinguishable \([13]\).

In the text editor example shown in Figure 2.1, the \( W \) set is \( W = \{\langle \text{Exit} \rangle, \langle \text{Save} \rangle, \langle \text{Close} \rangle\} \).
• Construction of transition cover set $P$.

In the traditional W-method, a transition cover set $P$ for a specification LTS $S$ is a finite set which contains all the sequences of inputs that visits each transition in $S$.

**Definition 2.15.** A transition cover $P$ is a prefixed-closed set containing all sequences of inputs needed to visit every transition of an LTS $S$ from the initial state $q_0$. That is, for each state $\forall q \in Q$ and for each element of an alphabet $\forall a \in \Sigma$ there exists a $p \in P$ such that $\delta(q, p \cdot a) = q_1$ for some $q_1 \in Q$.

Walkinshaw and Bogdanov [13] defined $P$ in terms of state cover set as follows:

$$P = (C \cup C\Sigma)$$  \hspace{1cm} (2.1)

In the text editor example shown in Figure 2.1, the transition cover set is $P = \{\epsilon, \langle\text{Exit}\rangle, \langle\text{Load}\rangle, \langle\text{Load, Edit}\rangle, \langle\text{Load, Close}\rangle, \langle\text{Load, Edit, Save}\rangle, \langle\text{Load, Edit, Edit}\rangle, \langle\text{Load, Edit, Close}\rangle, \langle\text{Load, Edit, Exit}\rangle\}$.

• Construction of distinguishing set $Z$.

$$Z = (\{\epsilon\} \cup \Sigma \cdots \Sigma^{m-n})W$$  \hspace{1cm} (2.2)

In the text editor example shown in Figure 2.1, the Z Set is $Z = \{\langle\text{Exit}\rangle, \langle\text{Save}\rangle, \langle\text{Close}\rangle, \langle\text{Load, Exit}\rangle, \langle\text{Load, Save}\rangle, \langle\text{Load, Close}\rangle, \langle\text{Exit, Exit}\rangle, \langle\text{Exit, Save}\rangle, \langle\text{Exit, Close}\rangle, \langle\text{Close, Exit}\rangle, \langle\text{Close, Save}\rangle, \langle\text{Close, Close}\rangle, \langle\text{Edit, Exit}\rangle, \langle\text{Edit, Save}\rangle, \langle\text{Edit, Close}\rangle, \langle\text{Save, Exit}\rangle, \langle\text{Save, Save}\rangle, \langle\text{Save, Close}\rangle\}$.

• Construction of test set $TS$.

The test set $TS$ is obtained by computing the cross product of two sets $P$ and $Z$:

$$TS = P \times Z$$  \hspace{1cm} (2.3)

$$TS = (C \cup C\Sigma)(\{\epsilon\} \cup \Sigma \cdots \Sigma^{m-n})W$$  \hspace{1cm} (2.4)
In the text editor example shown in Figure 2.1, the test set is

\[ TS = C(\{\epsilon\} \cup \Sigma)W \]

\[ \sum \] \[ W \] (2.5)

\[ TS = C(\{\epsilon\} \cup \Sigma \cdots \Sigma^{m-n})W \] (2.6)

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2.5.2 Comparing Two Models in Terms of Language

In the grammar inference domain, the process of evaluating the inferred models was originally proposed by Lang et al. [44] for the Abbadingo-One competition. The process involves generating random samples as test sequences for evaluation purposes, and then the number of test sequences that are correctly classified by the inferred model are counted [44]. Walkinshaw and Bogdanov [13] stated that the set of random tests needs to be diverse, where half of the samples belong to the language of the reference model, and the other half do not.

Walkinshaw and Bogdanov [13] claimed that the approach of taking random samples of test sequences that was applied by Lang et al. [44] causes two problems. The first issue is that obtaining randomly representative test sequences is impossible. This is because they will be biased towards some paths that are easily reached whenever sequences are generated randomly from the transition structure of a state machine [13]. The second problem relates to the metric that is used by Lang et al. [44] to compute the number of sequences that are classified correctly by both models (the inferred and reference LTSs or machines). This metric does not specify qualitative perceptions about differences between two models in terms of their languages [13]. For example, a low value of the metric such as 50% does not determine whether the inferred model tends to accept all sequences, or it equally correctly classifies [13].

On the other hand, Lo and Khoo [98] presented a Quality Assurance Framework (QUARK) to assess a specification inference. In QUARK [98], different dimensions were introduced to be considered for evaluating the quality of specification miners. The first factor is the scalability of a specification miner that measures its ability to infer accurately larger models. The accuracy also needs to be taken into account to measure the extent to which the specification inference can infer a model that is representative of the actual correct specification. In our context, the inferred models are said accurate if it is representative of the software being inferred. The robustness concerns the sensitivity of an inference algorithm to errors in training data. A specification miner is considered good among other miners if it can infer a very accurate specification compared to the reference specification on the condition that training data is characteristic [98].
In the QUARK framework [98], a specification miner and a simulator automaton that is defined by the user (reference model) are first given to QUARK. In addition, the percentage of the errors in the simulator is identified in order to examine the robustness of the miner. The QUARK framework then generates different sets of traces, each trace is a sequence of method calls, from the simulator model including erroneous traces. Those sets are fed into the specification miner to infer an automaton. It compares the behaviour of the inferred automaton against that of the reference model.

In the QUARK framework, Lo and Khoo [98] suggested generating two sets of traces for measuring the accuracy of a specification miner: the first set is obtained from the given reference model $R$ and the second set is generated from the inferred model $I$. Lo and Khoo [98] proposed two metrics to evaluate the similarities between $R$ and $I$. The metrics are computed based on the two generated sets of traces. The first metric measures the proportion of traces that are generated by $R$ and accepted by $I$. This metric represents the ratio of correctly inferred information by the mined model [98]. In the information retrieval community, this metric is also known as recall [99]. The second measurement computes the percentage of sequences that are generated by $I$, and accepted by $R$ [98]. This summarizes the quantity of correctly produced information by $I$. The second metric is also known as precision in the information retrieval domain [99]. In this chapter, we denote this manner of computing the precision and recall metrics as the conventional precision-recall.

Walkinshaw et al. [100] illustrated the conventional approach of computing precision-recall that is used in the QUARK framework [98] and explored the related issues. The computation of the conventional precision-recall is achieved by generating random samples from the target and inferred models to find the overlap between them. The random sequences are classified into retrieved ($RET$) and relevant ($REL$) sets depending on what is retrieved and relevant as illustrated in Table 2.1. If a sequence $e$ is classified as accepted for both machines such that $e \in L(R) \land e \in L(I)$, then $e$ is added to both $RET$ and $REL$ sets. If the sample $e$ is classified only as accepted by the inferred model such that $e \notin L(R) \land e \in L(I)$, then $e$ is only added to the $RET$ set, and so on. The precision and recall are then computed based on $RET$ and $REL$ sets after classifying each sequence in the test set. The precision and recall are computed as follows:
Chapter 2. Definitions, Notations and Models

\[ Precision = \frac{|REL \cap RET|}{|RET|} \quad (2.7) \]

\[ Recall = \frac{|REL \cap RET|}{|REL|} \quad (2.8) \]

<table>
<thead>
<tr>
<th>I Machine (Inferred)</th>
<th>R Machine (Reference)</th>
<th>RET</th>
<th>REL</th>
</tr>
</thead>
<tbody>
<tr>
<td>accept</td>
<td>accept</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>reject</td>
<td>reject</td>
<td></td>
<td></td>
</tr>
<tr>
<td>reject</td>
<td>accept</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>reject</td>
<td>reject</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Conventional manner of classifying sequences into relevant and retrieved sets

Walkinshaw et al. [100] claimed that the use of precision and recall measurements in the conventional method as described (above) by Lo and Khoo [98] to evaluate the accuracy of the mined models can be problematic. The conventional method has two shortcomings. First, random-positive test sets that are generated to detect the differences between the mined and reference models may ignore some sequences that are less likely to be generated. It only covers the disagreement between the models based on sequences that are easy to reach when they are randomly generated [100]. Hence, computing precision and recall based on such test sets will result in unreliable scores. Second, the computation of precision and recall in the conventional way is biased towards accepting sequences (behaviours) of the mined and reference models [100]. Therefore, it is important to include invalid sequences. However, even if they are included in the conventional manner, it will not make sense because the precision and recall rely on positive samples only. For instance, if a sequence \( e \) is rejected by both models, the RET and REL sets do not count for this case as shown in the last row in Table 2.1.

Walkinshaw et al. [100] addressed this first problem by applying the idea of conformance testing methods in order to obtain positive and negative sequences. They [100] suggested using techniques such as the W-method to generate a test set, which contains valid and invalid sequences that are not biased towards accepting behaviours of the reference machine. However, the test set tends to be very large making the execution of the whole test set unfeasible, since it is highly costly in practice [100]. Nevertheless, the balance that is required between positive and negative test sequences in the test set will be missed; this is because the vast majority of sequences that can be generated using W-method are
rejected [13]. This would lead a high score to the inferred state machine that rejects all sequences because it does not count for the balance assessment for the accuracy of the inferred state machines [35, 100].

Instead of applying Lo and Khoo’s [98] idea of generating random samples from both the mined and target machines, Walkinshaw et al. [100] proposed using the concept of model-based testing approaches in which a test set is generated from the target machine. This would highlight the disagreement between the grammars of the inferred and target state-machine models. Walkinshaw et al. [100] refined the conventional precision-recall in order to use them for classifying accepting and rejecting sequences. It is suggested to compute a specific precision for positive sequences and another one for negative ones. Additionally, they also [100] compute the recall for both categories (negatives and positives) of sequences. To achieve this, instead of categorizing test sequences into RET and REL sets, Walkinshaw et al. [100] divided both sets into $RET^+$ $RET^-$, $REL^+$, and $REL^-$ sub-sets. Thus, test sequences are added to those sub-sets as shown in Table 2.2. The refined precision and recall metrics are shown in Table 2.3.

<table>
<thead>
<tr>
<th>$I$ Machine (Inferred)</th>
<th>$R$ Machine (Reference)</th>
<th>$RET^+$</th>
<th>$REL^+$</th>
<th>$RET^-$</th>
<th>$REL^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>accept</td>
<td>accept</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>accept</td>
<td>reject</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>reject</td>
<td>accept</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>reject</td>
<td>reject</td>
<td></td>
<td></td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Table 2.2: Refined-way of classifying sequences into relevant and retrieved sets

\[
\text{Precision}^+ = \frac{|REL^+ \cap RET^+|}{|RET^+|} \quad \text{Precision}^- = \frac{|REL^- \cap RET^-|}{|RET^-|}
\]

\[
\text{Recall}^+ = \frac{|REL^+ \cap RET^+|}{|REL^+|} \quad \text{Recall}^- = \frac{|REL^- \cap RET^-|}{|REL^-|}
\]

Table 2.3: Refined-way of computing the precision and recall

A high value of $\text{Precision}^+$ indicates that the majority of the returned positive sequences are correctly represented (classified) by the inferred (mined) as positive with respect to the reference model (correctness). Moreover, a high value of $\text{Recall}^+$ indicates that a large number of relevant accepted test sequences are correctly retrieved by the inferred (hypothesis) model (completeness); and vice versa for $\text{Precision}^-$ and $\text{Recall}^-$. 

Walkinshaw and Bogdanov [13] showed how to compare the similarities between two LTSs in terms of the language. To achieve this, test sequences are generated using the W-method.
(described in 2.5.1) from the reference LTS $R$ and it compares how well the inferred LTS $I$ classifies the generated test sets. Those are organised in a confusion matrix, which is introduced in binary-classification tasks in the machine-learning domain\cite{101}. The confusion matrix is shown in Table 2.4. The true positive includes the number of sequences that are classified as accepted by both languages of $I$ and $R$, the number of sequences that are recognized by both $I$ and $R$ machines as rejected is included in the true negative set. The false positive refers to the number of sequences that are classified as rejected by $I$ but accepted by $R$. The number of sequences that are accepted by $I$ and rejected by $R$ is included in the false negative set.

<table>
<thead>
<tr>
<th>Reference LTS $R$</th>
<th>Inferred LTS $I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall t \in TS$</td>
<td>$t \in L(I)$</td>
</tr>
<tr>
<td>$t \in L(R)$</td>
<td>True Positive ($TP$)</td>
</tr>
<tr>
<td>$t \in \overline{L(R)}$</td>
<td>False Positive ($FP$)</td>
</tr>
</tbody>
</table>

Table 2.4: Confusion matrix for binary classification of sequences

Rijsbergen\cite{99}, Sokolova and Lapalme\cite{101} showed that the measures such as recall, precision, F-measure, and BCR\cite{13, 34} can be obtained based on the confusion matrix. The precision measure is defined as the percentage of sequences (tests) that belong to the language of the inferred LTS that also belong to the language of the reference LTS\cite{13}. A low precision value (for example below 0.4) means that the percentage of sequences that are accepted by the language of the inferred LTS and the reference LTS is small. In other words, many positive sequences that must be accepted by the inferred LTS are rejected.

The recall (sensitivity) metric is the proportion of tests that belong to the language of the reference LTS and are also classified as accepting sequences by the language of the inferred LTS\cite{13}. A large value of recall (for example 0.8) denotes that a large ratio of test cases is accepted by the reference and the inferred LTSs. The key difference between the precision and recall measurements is that the former reflects how well sequences that are accepted by the inferred machine are accurate. Moreover, the latter computes how many sequences that are accepted by the reference LTS are miss-accepted (rejected) by the inferred machine.
Chapter 2. Definitions, Notations and Models

<table>
<thead>
<tr>
<th>Measure</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision</td>
<td>$\frac{</td>
</tr>
<tr>
<td>Recall (Sensitivity)</td>
<td>$\frac{</td>
</tr>
<tr>
<td>Specificity</td>
<td>$\frac{</td>
</tr>
<tr>
<td>F-score</td>
<td>$\frac{2 \ast Precision \ast Recall}{Precision + Recall}$</td>
</tr>
<tr>
<td>Balanced Classification Rate (BCR)</td>
<td>$\frac{Specificity + Sensitivity}{2}$</td>
</tr>
<tr>
<td>Classification accuracy</td>
<td>$1 - \frac{FP + FN}{FP + FN + TP + TN}$</td>
</tr>
</tbody>
</table>

Table 2.5: Different metrics for comparing two LTS in terms of their languages

The specificity measure is concerned with the efficiency of the inferred LTS in classifying negative sequences correctly [13]. In other words, the specificity metric is the percentage of negative sequences in the language of the reference LTS that are also rejected by the inferred LTS [13]. The BCR is a well-known metric to evaluate the inferred automaton on test sequences where the automaton is treated as a classifier of sequences into $TP$, $TN$, $FP$, and $FN$ as shown in the confusion matrix in Table 2.4.

2.5.3 An Example of a Comparison of the Language of the Inferred Machine to a Reference One

In this section, we present an example to illustrate how the metrics that are shown in Table 2.5 are computed. For this purpose let us return to the example of the simple text editor illustrated in Figure 2.8(a), and assume this LTS is the reference machine. Suppose that the inferred LTS shown in Figure 2.8(b) is obtained using any specification miner.

In order to compute the metrics presented in Table 2.5, the test sequences are initially generated using the W-method as described in section 2.5.1 from the reference (correct) LTS. The number of states in the reference LTS $n = 4$ is equal to the number of states in the inferred LTS $m = 4$. The value of $m - n + 1$ is passed to the W-method representing the
Figure 2.8: The reference LTS and the mined one of the text editor example

The number of extra states in the inferred LTS. The number of tests generated using the W-method is 63. Table A.1 that is shown in appendix A provides the tests that are generated using the W-method and their classifications by both the reference and inferred LTSs.

<table>
<thead>
<tr>
<th>Reference LTS $R$</th>
<th>Inferred LTS $I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall t \in TS$</td>
<td>$t \in L(I)$</td>
</tr>
<tr>
<td>$t \in L(R)$</td>
<td>$TP = 2$</td>
</tr>
<tr>
<td>$t \in \overline{L(R)}$</td>
<td>$FP = 0$</td>
</tr>
</tbody>
</table>

Table 2.6: Confusion matrix for binary classification of sequences

The confusion matrix is built from the classification of the test sets as shown in Table 2.6. The outcomes of computing different metrics are presented in Table 2.7. The high score of precision means that all sequences that are classified by the inferred LTS as positive are also classified as positive by the reference LTS. On the other hand, the low value of recall (0.14) indicates that the inferred LTS rejects (classified as negative) a large number of sequences that must be accepted (classified as positive) according to the language of the reference LTS. This is because twelve out of fourteen positive-sequences are classified by the inferred LTS as negatives. The F-measure score (0.25) tells us the harmonic mean that combines
the precision and recall measures and indicates that the inferred automaton incorrectly classifies positive test sequences. The F-measure does not account for the accuracy of the inferred automaton in terms of its language complements. The BCR score of 0.57 is obtained by computing the average of recall and specificity which tells us the accuracy of the inferred automaton at classifying positive and negative.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision</td>
<td>1.0</td>
</tr>
<tr>
<td>Recall (Sensitivity)</td>
<td>0.14</td>
</tr>
<tr>
<td>Specificity</td>
<td>1.0</td>
</tr>
<tr>
<td>F-score</td>
<td>0.25</td>
</tr>
<tr>
<td>Balanced Classification Rate (BCR)</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 2.7: Metrics scores obtained from confusion matrix

2.5.4 Comparing Two Models in Terms of Structure

It is important to consider the structure of the inferred models when a software engineer compares them to their target models. Walkinshaw and Bogdanov [13] stated that the comparison of two state machines in terms of their structures can provide complementary insights into the dissimilarity of two state machines that cannot be obtained by comparing their languages. However, comparing two different LTSs in terms of structure is not necessary an easy task [51].

Walkinshaw and Bogdanov [13] proposed the $LTSDiff$ algorithm to accomplish the idea of comparing two LTSs in term of their structures. The idea of comparing the inferred LTS $I$ and the reference LTS $R$ revolves around determining which states and transitions are deemed to be equivalent in both LTSs, and then finding states and transitions in the $I$ that are considered as extra or missing in comparison to $R$ [13].
2.5.4.1 LTSDiff Algorithm

Walkinshaw and Bogdanov [13] presented the LTSDiff algorithm to capture the difference between two state machine models, especially LTSs in terms of their structure. In the LTSDiff algorithm, the comparison of two LTSs is established by measuring the similarity scores of pairs of states with respect to the surrounding states and transitions. The similarity score for a pair of states encodes the overlap of the surrounding states and transitions. Moreover, it can be computed locally based on the immediate transitions of two states, and globally based on the target and source states of those transitions.

The LTSDiff algorithm begins by the given two automata $I, R$. $R$ denotes the reference LTS and $I$ denotes the inferred LTS. It aims with a high level of confidence to identify pairs of states $(s_1, s_2)$ such that $s_1 \in Q_I \land s_2 \in Q_R$ that are most likely be equivalent. Those pairs are called key pairs and considered as common landmarks that exist in both automata. The dissimilarity between $I, R$ can be detecting based on landmarks, where states and transition comparisons in both automata rely on landmarks [13, 51].

In general, the idea of comparing two LTS in terms of their structures used in the LTSDiff algorithm can be seen as a human pointing to an unknown landscape in a map and looking for an identifiable place (landmark) near to the landscape and attempting to locate it on the map [13]. In the LTSDiff algorithm, easily recognizable states are those that are surrounded by distinctive states and transitions; such identifiable states are used as landmarks for further comparison. The LTSDiff algorithm is provided in Algorithm 3 and is summarized in the following phases:

**First Identifying Key Pairs.** In the LTSDiff algorithm, this is denoted by computeScores. The LTSDiff algorithm identifies the key pairs by computing the similarity scores (described later in this section) for any given pair of states ($A, C$) in terms of its transitions. The pairs of states that have the highest similarity score are selected to be reference points and are considered as key pairs. The computation of the structural differences between two LTSs starts from the reference point, and then compares the surrounding pairs of states and transitions until no further comparison can be found.
• Local similarity score

Essentially, the similarity (matching) score of the pair of states \((A, C)\) is obtained by computing the average of common transitions between two states. This is called a local similarity score. As we are interesting in a deterministic machine, the similarity score is computed by \(S_{AC} = \frac{\Sigma(A) \cap \Sigma(C)}{\Sigma(A) \cup \Sigma(C)}\). Table 2.8 shows four examples for computing the similarity score of two deterministic states \((A, C)\). In Figure(a) in Table 2.8, outgoing transitions of the states \((A, C)\) share the same Load label and this yields a score of 1. In Figure(b), there is no outgoing transitions with same labels, which makes the similarity score zero.

\[
\begin{align*}
S_{ac} &= 1 \\
S_{ac} &= 0
\end{align*}
\]

(a) (b)

\[
\begin{align*}
S_{ac} &= 0.5 \\
S_{ac} &= 0.3
\end{align*}
\]

(c) (d)

Table 2.8: Example of the similarity score computation

The computation of the local similarity score is defined and re-written [13] to count for non-deterministic state machine. The equation of computing the local similarity score is shown in Equation 2.9 for the given two states \((A, C)\) such that \(A \in Q_I, C \in Q_R\).

\[
S_{Succ}^{L}(A, C)[13] = \frac{|Succ_{A,C}|}{|\Sigma_{I}(A) - \Sigma_{R}(C)| + |\Sigma_{I}(C) - \Sigma_{R}(A)| + |Succ_{A,C}|}
\]  
(2.9)
In Equation 2.9, two states are given \((A, C)\) such that \(A \in Q_I, C \in Q_R\). The \(|\text{Succ}_{A,C}|\) denotes to the set of target states pairs that can be reached using the common transitions for each possible label \(\sigma \in (\Sigma_I \cup \Sigma_R)\). Definition 2.16 is defined formally in [13].

**Definition 2.16.** Following [13] Let \(B \in Q_I, D \in Q_R,\) and \(\sigma \in (\Sigma_I \cup \Sigma_R)\)

\[
\text{Succ}_{A,C} = \{ (B, D, \sigma) \mid \delta(A, \sigma) = B \land \delta(C, \sigma) = D \}
\]

The notation \(\Sigma_I^{\text{out}}(A)\) in Equation 2.9 denotes the labels of outgoing transitions (alphabets) from state \(A\) of LTS \(I\). Therefore, the expression \(|\Sigma_I^{\text{out}}(A) - \Sigma_R^{\text{out}}(C)|\) returns the number of elements of an alphabet corresponding unmatched outgoing transitions from state \(A\) compared to that from state \(C\), and vice versa for the expression \(|\Sigma_I^{\text{out}}(C) - \Sigma_R^{\text{out}}(A)|\).

For example, in Figure(C) in Table 2.8, the number of unmatched outgoing transitions between the state \(C\) compared to the state \(A\) is one (written as \(|\Sigma_I^{\text{out}}(C) - \Sigma_R^{\text{out}}(A)| = 1\) ). In this example, the expression \(|\Sigma_I^{\text{out}}(A) - \Sigma_R^{\text{out}}(C)|\) returns zero because only one outgoing transition from state \(A\), which is \(\text{Load}\), is matched with the outgoing transitions from state \(C\). In addition, the expression \(|\Sigma_I^{\text{out}}(C) - \Sigma_R^{\text{out}}(A)|\) returns one because the outgoing transition from state \(C\) that is labelled with \(\text{Edit}\) does not match with any other outgoing transitions from state \(A\). So, \(S_{\text{Succ}}^L(A, C) = \frac{1}{0+1+1} = 0.5\).

The equation 2.9 is concerned with computing the local similarity with respect to pairs of states that share the outgoing transitions. Walkinshaw and Bogdanov [13] defined the local similarity for incoming transitions as shown formally in Definition 2.10.

\[
S_{\text{Prev}}^L(A, C) [13] = \frac{|\text{Prev}_{A,C}|}{|\Sigma_I^{\text{inc}}(A) - \Sigma_R^{\text{inc}}(C)| + |\Sigma_I^{\text{inc}}(C) - \Sigma_R^{\text{inc}}(A)| + |\text{Prev}_{A,C}|}
\]

Following [13], the set of matching transition for a given states \((A, C)\).

**Definition 2.17.** Following [13] Let \(B \in Q_I, D \in Q_R,\) and \(\sigma \in (\Sigma_I \cup \Sigma_R)\)

\[
\text{Prev}_{A,C} = \{ (B, D, \sigma) \mid \delta(B, \sigma) = A \land \delta(D, \sigma) = C \}
\]
The notation $\Sigma^{inc}_I(A)$ denotes the incoming transitions to state $A$. Other expressions are defined in equation Equation 2.10 in the same way as in Equation 2.9.

- **Global similarity score**

The above computations of local similarity scores that are shown in equations 2.9 and 2.10 focus on measuring the similarity of states based on the neighbouring transitions. To this regard it is necessary to also consider the wider context. Walkinshaw and Bogdanov [13] showed the global similarity score that intends to measure the transition similarity for source and target states of adjacent transitions as well.

The computation of global similarity aimed to assign a higher score if the source and target states of the matched transitions are similar, and a lower score if they are different [13]. The computation procedure of the global similarity score $S^G_{Succ}(A, C)$ extends the local similarity score $S^L_{Succ}(A, C)$ that is illustrated in equation 2.9. The following equation computes recursively the similarity for the target states of a pair of states, and this is not considered in the local similarity score.

$$S^G_{Succ}(A, C) = \frac{1}{2} \frac{\Sigma_{(B,D,\sigma) \in Succ_{A,C}}(1 + S^G_{Succ}(B, D))}{|\Sigma^{out}_I(A) - \Sigma^{out}_R(C)| + |\Sigma^{out}_I(C) - \Sigma^{out}_R(A)| + |Succ_{A,C}|}$$

(2.11)

Walkinshaw and Bogdanov [13] stated that the procedure of computing the global similarity score that is introduced in equation 2.11 can lead to unintuitive scores. This is because for any given pair of states $(A, C)$, the recursive computation of the score can count for sequential target pairs of states that are not closer to $(A, C)$. To give priority to pairs of states that are closer to the pair of interest, Walkinshaw and Bogdanov [13] introduced an attenuation ratio $K$ as shown in the following equation 2.12.

$$S^G_{Succ}(A, C) \ [13] = \frac{1}{2} \frac{\Sigma_{(B,D,\sigma) \in Succ_{A,C}}(1 + kS^G_{Succ}(B, D))}{|\Sigma^{out}_I(A) - \Sigma^{out}_R(C)| + |\Sigma^{out}_I(C) - \Sigma^{out}_R(A)| + |Succ_{A,C}|}$$

(2.12)
In the same way, Walkinshaw and Bogdanov [13] defined $S_{\text{Prev}}^G(A, C)$ to compute the global similarity score in terms of the incoming transitions as follows:

$$S_{\text{Prev}}^G(A, C) [13] = \frac{1}{2} \left| \sum_{(B,D,\sigma) \in \text{Prev}_{A,C}} (1 + kS_{\text{Prev}}^G(B, D)) \right|$$

(2.13)

The final global score for each pair of states is computed in terms of incoming and outgoing transitions by taking the average of two notations $S_{\text{Succ}}^G(A, C)$ and $S_{\text{Prev}}^G(A, C)$ as follows:

$$S(A, C) [13] = \frac{S_{\text{Succ}}^G(A, C) + S_{\text{Prev}}^G(A, C)}{2}$$

(2.14)

The above computation of scores is denoted by the *computeScores* ($LTS_I, LTS_R, k$) function in the *LTSDiff* Algorithm 3.

Once the scores of state pairs are computed, the *LTSDiff* algorithm selects state pairs that have the greatest possibility to be equivalent in the two-stage approach as follows:

1. Introduce a threshold parameter $t$ where pairs of states with scores above $t$ are considered for the process of identifying key pairs. Walkinshaw and Bogdanov [13] stated that a state may be matched to multiple states, although this ambiguity is not preferable.

2. To reduce the ambiguities made in the previous step, Walkinshaw and Bogdanov [13] suggested selecting only one pair of states to be a key pair if its score is better than any other pairs beyond any doubt. In this case, a ratio $r$ is introduced as a second criterion to select only pairs that are suggested to be the best match $r$ times compared to other matches. Thus, the best matched pairs are added to the key pairs.

In the *LTSDiff* algorithm shown in Algorithm 3, the above strategy of selecting the best pairs of states is denoted by *identifyLandmarks* that is working as a filtering process by passing the set of pairs, the threshold $t$, and $r$ to collect the key pairs. The *LTSDiff* algorithm then collects the set of matched state pairs (NPairs) that surround each key pair. The surrounding pairs of states for a given pair $(A, C)$ are defined as follows: $\text{Surr}_{A,C} = \{(B, D) \in Q_I \times Q_R \mid \exists \sigma \in (\Sigma_I \cup \Sigma_R): ((A \xrightarrow{\sigma} B) \wedge (A \xrightarrow{\sigma} C))\}$. 


\[ B \land C \xrightarrow{\sigma} D \lor (B \xrightarrow{\sigma} A \land D \xrightarrow{\sigma} C) \] \}

The computation of \( \text{Surr}_{A,C} \) is shown in line 6 in Algorithm 3 and the matched surrounding pairs of states are added to the NPairs set.

The procedure in the loop in the lines 7-14 in Algorithm 3 focuses on selecting the matching pairs from the set of NPairs. It selects the pairs with the highest score from the set of NPairs using pickHighest function. Once the pair with the highest similarity has been selected, it is added to the list of KPairs.

**Second Computing a Patch.** Once the KPairs have been computed in lines 1-14, those pairs of states are used to collect the differences between the two \( I,R \) LTSs; this is referred to as a patch. For an inferred LTS \( I \), the patch contains two sets of transitions: The Removed set contains transitions that are removed from LTS \( I \) and the Added set contains those transitions that are added to LTS \( I \) with comparison with reference LTS \( R \).

The computed patched can be used to compute the structural-similarity score. To illustrate how the computed patch can be used to measure the structural difference between the reference LTS \( R \) and the inferred LTS \( I \) in this thesis, \( R_{\text{edge}} \) is used to denote the number of edges in \( R \), and \( I_{\text{edge}} \) refers to the number of edges in \( I \). Removed\(_{\text{edges}} \) denotes the number of edges that are missing from \( I \), while Added\(_{\text{edges}} \) means the number of edges that are extra to \( I \). The metric *structural similarity* is defined as an average of two measurements \((A,B)\), where the former computes how many transitions have not been inferred by a learner that generated \( I \), and the latter computes how many transitions of \( I \) that are new to \( I \). The pair \((A,B)\) is computed using the equations (2.15, 2.16) respectively, and the *structural-similarity score* is computed as shown in equation (2.17).

\[
A = \frac{R_{\text{edge}} - \text{Removed}_{\text{edges}}}{R_{\text{edge}}} \quad (2.15)
\]

\[
B = \frac{I_{\text{edge}} - \text{Added}_{\text{edges}}}{I_{\text{edge}}} \quad (2.16)
\]

\[
\text{structural-similarity score} = \frac{A + B}{2} \quad (2.17)
\]
Ch​apte​r 2. De​fine​tions, Nota​tions and Models

**input**: \( \text{LTS}_I, \text{LTS}_R, k, t, \text{and} r \)

/* LTSs are the two machines, k is the attenuation value, t is the threshold parameter, and r is the ratio of the best match to the second-best score */

**data**: \( KPairs, \text{PairsToScores}, NPairs \)

**result**: \( \text{Added}, \text{Removed}, \text{Renamed} \)

/* two sets of transitions and a relabelling */

1. \( \text{PairsToScores} \leftarrow \text{computeScores}(\text{LTS}_I, \text{LTS}_R, k); \)
2. \( \text{KPairs} \leftarrow \text{identifyLandmarks}(\text{PairsToScores}, t, r); \)
3. **if** \( \text{KPairs} = \emptyset \) and \( S(p_0, q_0) \) **then**
4.     \( \text{KPairs} \leftarrow (p_0, q_0); \)
      /* \( p_0 \) is the initial state in \( \text{LTS}_I \), \( q_0 \) is the initial state in \( \text{LTS}_R \) */
5. **end**
6. \( NPairs \leftarrow \bigcup_{a, b \in \text{KPairs}} Surr(a, b) - \text{KPairs}; \)
7. **while** \( NPairs \neq \emptyset \) **do**
8.     **while** \( NPairs \neq \emptyset \) **do**
9.         \( (a, b) \leftarrow \text{pickHighest}(NPairs, \text{PairsToScores}); \)
10. \( \text{KPairs} \leftarrow \text{KPairs} \cup (a, b); \)
11. \( NPairs \leftarrow \text{removeConflicts}(NPairs, (a, b)); \)
12. **end**
13. \( NPairs \leftarrow \bigcup_{a, b \in \text{KPairs}} Surr(a, b) - \text{KPairs}; \)
14. **end**
15. \( \text{Added} \leftarrow \{ b_1 \xrightarrow{\sigma} b_2 \in \delta_R \mid \nexists (a_1 \xrightarrow{\sigma} a_2 \in \delta_I \land (a_1, b_1) \in \text{KPairs} \land (a_2, b_2) \in \text{KPairs}) \}; \)
16. \( \text{Removed} \leftarrow \{ a_1 \xrightarrow{\sigma} a_2 \in \delta_I \mid \nexists (b_1 \xrightarrow{\sigma} b_2 \in \delta_R \land (a_1, b_1) \in \text{KPairs} \land (a_2, b_2) \in \text{KPairs}) \}; \)
17. \( \text{Renamed} \leftarrow \text{KPairs}; \)
18. **return** \( (\text{Added}, \text{Removed}, \text{Renamed}); \)

Algorithm 3: The LTSDiff Algorithm [13]

**Example 2.1.** For instance, consider the reference LTS of the text editor example shown in Figure 2.9(a), and the inferred models of the text editor as depicted in Figure 2.9(b).

The output of the comparison using LTSDiff is given in Figure 2.10 where dashed (green) transitions are missing and solid (red) transitions are incorrectly added to the inferred model. \( A = \frac{9-2}{9} = 0.778 \), and \( B = \frac{10-3}{10} = 0.7 \). So, structural similarity is computed using the equation 2.17.
Chapter 2. Definitions, Notations and Models

(a) The reference LTS of a text editor

(b) The inferred LTS of a text editor

Figure 2.9: Comparing the reference LTS and the mined one of the text editor example using the LTSDiff Algorithm

Figure 2.10: The output of LTSDiff between the reference LTS 2.9(a) and the inferred LTS 2.9(b) of a text editor example
2.6 The Evaluation Technique in the Statechum Framework

In this section, the evaluation technique that will be used throughout the thesis is explained. Figure 2.11 demonstrates the general overview of the evaluation processes of the inferred models in Statechum. It consists of four main phases. The first phase is called a reference model generator, which is responsible for the generation of a reference LTS either randomly generated LTSs or a real-world case studies LTSs. This phase is flexible because it allows an analyst to manually provide their own reference or a real-world case studies LTSs in different formats such as XML, which represents the states and transitions using XML elements.

In the Statechum framework, the Forest Fire algorithm \[102\] is used to generate directed random LTSs. The algorithm was developed by Walkinshaw and Bogdanov \[13\] to generate random LTSs for the STAMINA competition \[35\]. A random LTS must be deterministic, minimal, and every state must be reachable from the initial state.

Leskovec et al. \[102\] proposed the Forest-Fire algorithm to generate directed graphs that represent complex networks. It is an iterative process that aims to add new nodes that are connected to the other closest nodes in the graph. The Forest-Fire algorithm \[102\] initially defines three parameters: the number of nodes (vertices) \(n\), a backwards-burning probability \(b\), forward-burning ratio \(f\), and self-looping probability \(s\).

In the Forest-Fire model \[102\], nodes attach to a graph \(G\) at a time \(t\) and form outgoing edges to the earlier nodes. Given the graph \(G\), and considering that a node \(v\) arrives at a time \(t\) to be attached to \(G\). The current node \(v\) creates an outgoing arc to different nodes in the graph \(G\) at a time \(t\) as follows: First, \(v\) chooses uniformly a random ambassador node \(w\) where \(w \neq v\), and create an edge from \(v\) to \(w\). Second, two random numbers \(x\) and \(y\) are generated and geometrically distributed such that \(x = \frac{f}{1-f}\) and \(y = \frac{fb}{1-fb}\). Third, node \(v\) chooses outgoing and in-coming edges of node \(w\) to the non-visiting nodes in the graph. Finally, node \(v\) generates outgoing edges to the target nodes of the selected edges from and to node \(w\), and repeats the second and third processes.

In this thesis, the Forest-Fire algorithm is used to generate random graphs with the following configurations: \(f = 0.31\), \(b = 0.385\) and \(s = 0.2\). Those configurations were used in \[13\] for the generation of random LTSs.
The second phase in the evaluation framework is the *trace generator*. Once a random reference LTS is automatically generated, the training data are produced using random walks from the target (reference) automata. The developer defines a number of parameters as follows: the number of traces $T$ and the length of the traces $l$.

Once the traces are generated from the reference automaton, one or more inference algorithms takes those traces as inputs to learn automata using different state-merging strategies. The synthesised LTS can be visualised for the analyst (software engineer). This is the third phase in the evaluation process. Finally, the mined LTSs are used to compute the variety of metrics such as precision, recall, F-measure, BCR. It also includes a computation of the *structural similarity* metric that is discussed in the *LTSDiff* 2.5.4.1 section. In addition, statistical tests such as the Wilcoxon signed-rank test can be computed for the metrics.
2.7 DFA Inference Competitions

As mentioned earlier in this chapter, the task of evaluating the performance of state machines inference algorithms is a difficult task. Therefore, it is important to provide a practical and scientific basis for comparing different techniques of inferring DFA models. For this reason, researchers have organized a variety of competitions to compare and evaluate different inference techniques on the same DFA models. The following sub-sections provide insights into different competitions for the comparison of inference techniques in terms of their performance and capabilities to solve the different inference problems.

2.7.1 Abbadingo-One Competition

In the domain of grammar inference, numerous competitions were organised to allow competitors to evaluate their DFA inference algorithms. In general, organisers generated a set of random target models as problems to be solved by the competitors participating in the competition. The competitors were only provided with training data to start learning DFA. Sets of training samples were used for learning state-machine models and different sets of tests were used to evaluate the performance of the inferred models. Lang et al. [44] presented the first competition, which was named Abbadingo-One competition. The competition was organised by Kevin J. Lang and Barak A. Pearlmutter. They posted sixteen randomly generated DFAs as challenge problems.

The alphabet size used in Abbadingo-One competition was only two $\Sigma = \{0, 1\}$, implying that the maximum number of transitions for each state was two. The process of generating target DFAs of $n$ size in this competition by construction of a directed graph of $\frac{5}{4}n$ nodes. The depth of the generated target machines was $2\log_2 n - 2$. The set of training data for a target DFA of size $n$ was randomly drawn from a uniform distribution over $16n^2 - 1$ strings and their length was between 0 and $2\log_2 n + 3$. The remaining strings were used as a testing set. The winner in the competition was the Blue-Fringe algorithm [44] by Rod Price and Hugues Juille.
2.7.2 Gowachin Competition

Kevin J. Lang \cite{103} organized the Gowachin competition, which began in 1998. The DFAs were generated randomly based on the criteria used in the Abbadingo-One competition except that the DFA’s depth was not constrained to $2 \log_2 n - 2$. This allows the competitors to create their own DFA challenges by choosing the size of the DFA and the number of training sets. A competitor can also create a task where noise can be added to the training data.

2.7.3 GECCO Competition

The GECCO competition was held in 2004, focusing on inferring small DFA ranging between 10 and 50 from noisy data. The level of noisy data was 10%, and 5000 training data contained noisy ones. The winner of this the competition was the Hybrid Adaptive Evolutionary Algorithm (HAEA) by Gomez \cite{104}.

2.7.4 STAMINA Competition

Walkinshaw et al. \cite{35} organized a competition called StaMInA, which aimed to find the best inference algorithm for software models. Precisely, it aimed at finding the best technique to learn accurate DFAs that have a relatively large alphabet and infer correct DFAs from a sparse set of examples. In this competition, twenty sets of DFA problems with varying level of difficulty were provided to the competitors, where each set consists of five randomly generated DFAs. Only training sets were provided to the competitors and they did not have access to any of the DFAs. Similar to other competitions, competitors were then given a set of test sequences to measure how well the inferred DFAs classified test sequences.

The Blue-Fringe algorithm \cite{44} (described in section 3.1.4) was selected as the baseline for the StaMInA competition so that a competitor had to improve on it in order to have been recorded as having solved a problem of a specific size. The winner in the StaMInA competition was the DFASAT algorithm that is described in 3.1.8.
2.7.5 Zulu Competition

Combe et al. [105] launched a competition called Zulu that encouraged competitors to learn DFAs using the idea of Angluin’s algorithm [79]. A web-based server was provided to run the competition and allow users to create challenging tasks in order to interact with the Oracle during the learning of DFAs.

The Zulu server allows a player to create an account and ask for challenges after providing the number of states and alphabet size. The server returns the allowed number of membership queries $mb$ to be asked for each task, the player can interact with the Oracle to learn a specific DFA using membership queries up to $mb$. Once the learning process finished, the server computes the score of the task by measuring how well the inferred DFA classifies a test set which is a set of unlabelled strings. The winner in this competition was Howar et al. [106].

To sum up, it is difficult to assess the performance of the inference techniques because there is no common approach to accomplishing this. In this thesis, the BCR and structural-similarity metrics are chosen for evaluating the performance of the proposed inference techniques in the following chapters. The reason behind selecting the BCR score is that it is concerned with the accuracy of classifying both positive and negative test sequences. Additionally, it is used in the latest competition in the domain of grammar inference. A LTS model is considered to be inferred accurately if its BCR score is higher than or equal to 0.99 [34]. The structural-similarity score, on the other hand, is selected to examine the capability of miners (learner) in inferring LTSs that keep the state and transition structure.

It was interesting to study the various competitions in the domain of grammar inference. The intuition behind studying those competitions is to find the most appropriate algorithm as a baseline. In this thesis, the most relevant baseline algorithms are EDSM and DFASAT. Unfortunately, the DFASAT implementation is not publicly available. Moreover, DFASAT relies on both positive and negative traces to infer a DFA model. Due to the non-availability of the DFASAT implementation, it was difficult to compare the proposed algorithm against DFASAT. Moreover, the inference process using DFASAT begins with several steps of EDSM and it is known that EDSM performs effectively if there is enough positive and negative (characteristic) traces. It is critical to run the EDSM learner if the traces are not characteristic and without negative traces. Hence, it is necessary to
improve the EDSM learner (the first part of DFASAT) where only positive traces are available.
In this chapter, existing inference techniques are described. In addition, we investigate the performance of the most relevant techniques based on the problem statement considered in this thesis.

3.1 Passive Learning

The passive learning techniques rely on traces to infer state machine models. They aim to construct, identify, infer, mine, or synthesize state-machine models from traces without using queries to a system being inferred. The majority of such techniques are based on the idea of state merging as described in Section 2.4.3 where the solution starts with the most general hypothesis model to make it as specific as possible. In this section, we describe various techniques for passively inferring an LTS, and evaluate the performance of each technique by running a series of experiments using randomly generated LTSs.
3.1.1 $k$-tails Algorithm

Biermann and Feldman [107] proposed one of the most popular algorithms to mine state-machine specifications, which is named $k$-tails. In this section, an adopting $K$-tails that is proposed by Walkinshaw and Bogdanov [45] is described.

Given an LTS $A$ and a state $q \in Q$, the set of tails (sequence of event labels) of length $k$ that leave a state $q$ is denoted $L(A, q, k)$ can be defined as $L(A, q, k) = \{ w | \delta(q, w) \land |w| = k \}$. Two states $q$ and $q'$ are said to be $k$-equivalent, denoting $q \equiv_k q'$, if $L(A, q, k) = L(A, q', k)$.

In the $k$-tails algorithm, states are merged if and only if they are $k$-equivalent. It differs from the RPNI algorithm in the way that a pair of states are considered equivalent.

The inference process using the $k$-tails algorithm starts by building a PTA from the provided positive samples; this process is denoted with the $generatePTA(S^+)$ function in Algorithm 4. The $k$-tails learner iteratively merges states in the PTA tree if they are $k$-equivalent. In other words, a pair of states are merged on the condition that they have identical suffixes of length $k$. In line 2 in Algorithm 4, the constructed PTA $A$ and the value of $k$ are given to the $obtainPairofStates(A, k)$ to identify the first pair of states such that $L(A, q, k) = L(A, q', k)$. Once the first pair is obtained, the $Merge(A, (q, q'))$ function is responsible for merging the pair and updating the tree. The inference process is iterated until no further nodes in the tree can be merged. The whole body of the $k$-tails algorithm is provided in Algorithm 4.

```
input: $S^+$, and $k$
/* Sets of accepted sequences $S^+$ */
/* A $k$-value of the $K$-tail $k$ */
result: $A$ is a DFA that is compatible with $S^+$

1 $A \leftarrow generatePTA(S^+)$;
2 while $(q, q') \leftarrow obtainPairofStates(A, k)$ do
3      $A \leftarrow Merge(A, (q, q'))$
4 end
5 return $A$

Algorithm 4: The $k$-tails Algorithm
```

Example 3.1. Considering the text editor example that is described in section 2.2.4. Suppose the following positive samples are given: $S^+ = \{ \langle Load, Edit, Edit, Save, Close \rangle, \langle Load, Edit, Save, Close \rangle, \langle Load, Close, Load \rangle \}$ and the corresponding PTA is illustrated in Figure 3.1. Consider the value of $k$ parameter is one; it is obvious that states $H$ and $E$ have the same
future path (Close) of length $k$. In addition, there are two states $A$ and $D$ with the same future sequence of length one. However, merging them will result in a non-deterministic machine which is illustrated in Figure 3.2, because there are two outgoing transitions labelled with Close leave the merged state named $EH$: one is a transition to $I$ and another transition with the same input Close departing $EH$ to $F$. In addition, there are two outgoing transitions labelled with Load leaving the merged state $AD$. To make the machine deterministic, $E$ is merged with $H$ and $L$ must be merged with $B$, as shown in Figure 3.3, and this is the final deterministic machine after merging all possible pairs of states when $k$ equals one. In other words, the merging process will terminate only if the obtainPairofStates($A,k$) function cannot find any pair of states having the same tails of length one.

![Figure 3.1: A PTA of text editor from positive samples](image1)

![Figure 3.2: A non-deterministic machine after merging pairs of states (A,D) and (H,E)](image2)

![Figure 3.3: A machine of text editor where K=1.](image3)
It is important to mention that the key factor in the \( k \)-tails algorithm is the value of \( k \). There is no systematic way to pick the value of \( k \), it is essentially based upon developer judgement \cite{42}. If the value of \( k \) is big, many states are not merged together. In contrast, if it is very low, the inferred machine will be over-generalized because it is likely to merge non-equivalent states \cite{42}. Lo et al. \cite{70} stated that if there is a very limited number of traces (sparseness), the value of \( k \) should be small in order to mine good generalization models.

Walkinshaw and Bogdanov \cite{45} stated that a pair of states may be inequivalent if they have the same suffixes of length \( k \). Moreover, a pair of states that do not match the same suffixes of length \( k \) may not be different, especially in situations where training data is sparse \cite{45}.

To sum up, the poor performance of the \( k \)-tails algorithm is not surprising since it is only working with positive traces, which are not adequate to infer an accurate LTSs \cite{45}. Moreover, inferring state-machine models from execution traces using the \( k \)-tails algorithm may produce imprecise models that contain wrong behaviours \cite{41, 98}. According to Walkinshaw and Bogdanov \cite{45} the absence of negative traces causes the learning process to fail to stop over-generalizing LTSs by avoiding invalid mergers \cite{45}.

### 3.1.2 Experiments Using \( k \)-tails

We ran a small experiment to express the effect of \( k \) values on the inferred LTSs using the \( k \)-tails algorithm. Series of random LTSs were generated for each number of states ranging between 5 and 50 in steps of five. In other words, 15 different LTSs were randomly generated for each chosen number of states (10 steps * 15 = 150 LTSs in total). The size of alphabet is given by \( \Sigma = |Q| \times 2 \). The Forest Fire algorithm that is described in Section 2.6 was used to generate random LTSs. The reason behind learning this number of LTSs is to assess the performance of various algorithms on random LTSs with different traces fed to each LTS. In addition, the randomly generated LTSs were connected.

For each LTS two sets of training data were generated, bringing the number of LTSs learnt per experiment to 300. The generated LTSs were initially connected, had alphabet size \( 2 \times \) the number of states. In this experiment, a set of positive traces (training sequences) was
randomly generated. The set of traces consisted of $|Q| \times 5$ random walks of length ranged between 2 and $d + 5$, where $d$ the reference graph diameter.

In addition, learning was aborted when inferred LTSs were reaching 300 red states and a zero was recorded as a score. In this experiment, the blue-fringe (blue-red framework) [44] was used to reduce the number of states that would be evaluated in terms of the possibility to merge them. The red-blue strategy is described in section 3.1.4

A variant of the original $k$-tails algorithm was considered in this experiment, where a pair of states is merged if the states share at least one tail (path) of length $k$, and this is denoted by $k$-tails (a). In the original $k$-tails, it is likely to block mergers that must be merged if training data is sparse. Hence, the reasoning behind introducing this variant is to deal with sparse training data where a pair of states is merged even if there is one matching of length $k$.

The boxplots of the structural-similarity scores obtained by variants of $k$-tails learners and multiple $k$ settings are shown in Figure 3.4. From Figure 3.4, it is clear that the $k$-tails(a) performs better than the $k$-tails algorithm if $k > 1$. The maximum average of the structural-similarity score attained when the $k$ value is two. It is obvious that $k$-tails when $k = 1$ performs better than the case when $k = 2$, as shown in Figure 3.4. This is because the shortest paths from a pair of states are more likely to match, but it may produce over-generalized state machines.

Interestingly, $k$-tails (a) when $k = 2$ performs better than $k$-tails, because a pair of states is more likely to share some of the paths between a pair of states. This agrees with the saying that if two states do not match the same tails this does not mean that they are inequivalent.

Another way of evaluating the performance of the $k$-tails algorithm by computing the BCR score for the inferred LTSs. Figure 3.5 illustrates BCR scores of the inferred LTSs using $k$-tails learners with multiple $k$ settings. In many cases, BCR scores were 0.5, which means the $k$-tails algorithm made random guesses at the process of state merging. As can be seen in Figure 3.5, the BCR scores of LTS inferred by $k$-tails(a) when $k = 2$ are higher than those obtained $k$-tails. The poor performance of $k$-tails is due to the fact that it requires a vast number of traces to infer good LTS models. Sometimes, states that must be merged
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**Figure 3.4**: Structural-similarity scores of LTSs inferred using the $k$-tails algorithm for different $k$ values

**Figure 3.5**: BCR scores of LTSs inferred by the $k$-tails algorithm for different $k$ values
using $k$-tails are ignored; this is because states do not have identical paths, as training data is not complete and some paths that should exist are missing.

Therefore, to sum up, there is a slight improvement made by $k$-tails(a) compared to $k$-tails. This is because training data is sparse and it is not likely to have identical paths of length $k$ leaving the equivalent states. In this way, $k$-tails(a) merges a pair of states if there is at least one match between paths of length $k$ leaving both states.

### 3.1.3 Variants of the $k$-tails

Variants of the $k$-tails algorithm have been investigated in the literature. Cook and Wolf [49] stated that the original $k$-tails may be unrolling a loop of a sequence of event $e = \langle e_1, e_2, e_3, e_4 \rangle$ and producing non-deterministic automaton, and this is due to the value of $k = 2$. Hence, Cook and Wolf [49] modified the algorithm to handle rolling loops by removing non-determinism. On the other hand, Miclet [108] introduced a technique to infer a regular language from positive samples, which is called tail-clustering and can be seen as a generalization version of $k$-tails [108]. In the tail-clustering algorithm, states are merged based on the similarities in their sets of $k$-tails. These similarities are determined using a distance metric where a pair of states is merged if the distance metric between the set of tails is less than a certain limit.

Several attempts have been made to adapt the $k$-tails algorithm to infer different kinds of automata. Lorenzoli et al. [39] described the GK-tail to infer Extended Finite State Machines (EFSMs) that have context variables between interaction traces, that can be used in EFSMs testing since it increases program comprehension and analysis [39]. This algorithm relies on many positive samples of traces, and generates an (EFSMs) in four steps.

First, the GK-tail algorithm receives a set of traces that are augments with variables, and called input-equivalent traces. Those traces are equivalents since they have the same (sharing) events calls or methods invocations, and differ in data value of inputs. In this case, the GK-tail learner merges those similar input-equivalent traces into a single one that is annotated with multiple values of data [40]. Second, GK-tail produces predicates as constraints that are derived from data values and are associated with traces. Third, GK-tail constructs the initial EFSM by building the tree from the merged traces that is achieved
in the first step. The initial EFSM is annotated with the derived predicates. Fourth, \textit{GK-tail} learner iteratively merges equivalent states to infer the consistent EFSM to the observed traces. For each pair of states, \textit{GK-tail} compares the future paths in the length \( k \) in the same way as the \( k \)-tails learner to decide which states are to be merged. Lorenzoli et al. [39] introduced an \textit{equivalent} criterion where pairs of states are merged if they have the same \( k \)-paths of events and the constraints allocated to transitions. Sometimes [39], the traces may be incomplete, and in this case two different merging criteria are proposed: \textit{weakly subsumes}, and \textit{strongly subsumes}.

Another variant of the \( k \)-tails learner was proposed by Raman et al. [109] to infer probabilistic finite state automata (PFSA), which is called \textit{sk-strings}. This is may be defined as strings that are considered as \( k \)-tails, but they do not have to end at terminated states, unless they are less than the specified \( k \) value [109]. In other words, the set of \( k \)-string is defined in Definition 3.1.

\textbf{Definition 3.1.} [109] The \( k \)-string is defined as \( \{w \mid w \in \Sigma^*, |w| = k \land \hat{\delta}(q, w) \subset Q \lor |w| < k \land \hat{\delta}(q, w) \cap F_c \neq \emptyset\} \). Where \( F_c \) is the set of leaves nodes in the PTA.

In \textit{sk-strings} [109], the notion of top \( s \% \) was introduced to denote the most probable \( k \)-strings that can be generated from a state. Thus, two states are merged on the condition that they are indistinguishable for the most probable \( k \)-string [98, 109]. Only \( k \)-strings with probabilities up to \( s \% \) will be considered during evaluating the decision of merging states. That is, two states are said to be mergeable if the sets of \( k \)-strings for both states share the top \( s \% \) of their \( k \)-strings. The resulting PFSA is over-generalized if \( s \% \) is small [109]. In addition, the \textit{Sk-strings} method requires all \( k \)-strings of both states to be the same if \( s = 100\% \).

\subsection*{3.1.4 Evidence-Driven State Merging}

This section introduces one of the most successful passive inference techniques known as \textit{evidence-driven state merging} (EDSM). The EDSM learner [44] won the Abbadingo one DFA learning competition. It can be seen as a refinement of the RPNI algorithm. The name of the EDSM algorithm reflects its purpose, it uses heuristic evidence (a score) to evaluate each potential pair of states before merging them.
In the EDSM learner, each pair of states is given a score, and is computed by counting the number of states that are merged if the merging process is performed. That is, the score that is assigned to a possible merge is obtained by counting the total number of states that would be merged with others states [36, 44, 110]. This score can be called a compatibility score or an EDSM score.

The idea behind computing the EDSM score is to measure the likelihood that a pair of states is equivalent. Hence, the EDSM learner gives preference to a pair of states with the highest score to be merged first [44, 111]. The key advantage of implementing EDSM is that possible pairs in a specific boundary are evaluated before selecting the most likely pair to be merged based on its score. Possible pairs of states are prioritized (ranked) from the highest to the lowest scores. A pair of states with the highest score is merged first. Unlike the k-tails algorithm that relies on merging the first pair of states that share the same set of tails of length k, the EDSM learner relies on computing scores for possible pairs of states, and then picks one of them for merging [45].

Additionally, the EDSM algorithm assigns a negative score to each pair of states that cannot be merged. Merging of an accepting state with a rejecting state is not allowed by the EDSM learner (see definition in 3.2). In this situation, EDSM assigns a negative score denoting that a pair of states is unmergeable. Moreover, any states that would be merged recursively during the determinization process must be compatible to avoid merging a rejecting state with an accepting one, and vice versa. That is, a merge of two states \((q_1, q_2)\) is rejected if there is a transition \(a\) from a state \(q_1\) leads to a rejecting state \(\delta(q_1, a) \in F^-\), and there is another transition with the same label \(a\) leaving a state \(q_2\) reaches an accepting state \(\delta(q_2, a) \in F^+\).

**Definition 3.2.** Given a pair of states \((q_1, q_2)\) \(\in Q\) and \(APTA(A)\). A merge of \((q_1, q_2)\) is rejected by the EDSM iff \(q_1 \in F^+, q_2 \in F^-\) and vice versa.

**Example 3.2.** Consider the text editor example that is illustrated in Figure 3.6; the EDSM learner would assign a score of 4 to the pair of \(B\) and \(C\), and a score of -1 to denote that the pair of \(B\) and \(D\) is not compatible because merging of an accepting state \(C\) with a non-accepting one \(N\) is blocked.

To reduce the search space of evaluating possible pairs of states that can be merged, the red-blue technique [44] can be applied with EDSM. The red-blue strategy is sometimes
called the blue-fringe strategy. Basically, the red-blue technique begins by colouring the root state of an APTA red and all adjoining nodes are made blue [44]. Then, the possibility of merging every blue state to the red states is measured using the EDSM score. If a red-blue pair of states cannot be merged, the EDSM assigns a negative score. Once a blue state cannot merge with any red state, it is coloured red and its children nodes then become blue, as illustrated in Figure 3.6. This process is performed until all nodes in the tree are coloured red. The whole body of the EDSM algorithm is provided in Algorithm 5.

The EDSM algorithm starts by invoking the generatePTA \((S^+, S^-)\) function to construct the initial APTA in line 1. In addition, the generalization threshold \(\pi\) should be initialized before starting the inference process. A pair of states with a EDSM score above or equal \(\pi\) is considered for merging. After that, the red-blue strategy is invoked to colour the root of the PTA red, and its children non-red states are coloured blue. The process of making the non-red state blue is performed using the ComputeBlue\((A, R)\) function in Algorithm 5. The computation of blue states is defined formally in Definition 3.3 where \(B\) denotes the set of blue states and \(R\) denotes the set of red states.

**Definition 3.3.**

\[
\text{ComputeBlue}(B) = \{q_1 \in Q_A \mid \text{for some } q \in R \text{ and } \sigma \in \Sigma, \text{ such that } q_1 = \delta(q, \sigma) \text{ and } q_1 \notin R\}
\]

The term blue boundary is also used to refer to the set of blue states that neighbour the red states. The EDSM algorithm then iterates through the set of blue states to evaluate the ability of merging them with red states as shown in lines 9-22. For each pair of red/blue
**Algorithm 5:** The EDSM inference algorithm

states, the EDSM learner calls the `computeEDSMScore(A, q_r, q_b)` function to count the number of states that may be eliminated if merging them is performed. Once the EDSM score is computed, the pair of states are added to the `PossiblePairs` set if the allocated score is higher or equal to $\pi$, as shown in line 14, and the blue state is marked as mergeable. Furthermore, if a blue state is unmergeable with any red state, it is added to the $R$ set, and its children states become blue. The process is then iterated to evaluate the new blue states with each red state.
The PickPair(PossiblePairs) function is responsible for picking the pair of states with the highest score, and it is passed to merge the states in the pair using the Merge(PairToMerge) function. The inference process is continued with the same procedure until all states are coloured red. In other words, the process is terminated when the PossiblePairs set is empty.

It is important to emphasise that the EDSM learner expected to have positive and negative sequences in order to generalize the LTS models under the control of negative sequences. Walkinshaw et al. [100] suggested variants of the EDSM learner by introducing the generalization (merging) threshold. It is introduced to block merging of a pair of states with a score below the generalization threshold.

In the absence of negative samples, the merging threshold can be used to mitigate the over-generalization problem. However, this solution tends to be useless since it is difficult to pick the appropriate threshold for the provided traces. In addition, it may arbitrarily block states that should be merged.

It is important to point out that the EDSM learner fails to infer an exact LTS model for two reasons. First, training data are often too sparse to accumulate sufficient evidence about correct merges of states [112, 113]. Second, the absence or the amount of negative samples does not help the inference process to avoid over-generalization. If negative samples are not available, false merges will be more likely to happen [114]. In addition, the only restriction to avoid bad mergers is the compatibility constraint (avoiding merging accepted states with rejecting ones and vice versa), which is not enough if there are few negative traces [110].

### 3.1.5 Experiments Using EDSM

In order to evaluate the efficiency of the EDSM algorithm when alphabet size is large, the same experiments that are described in Section 3.1.1 were conducted for different variants of the EDSM algorithm. In cases where only positive traces were considered, the total number of sequences is given by $|Q| \times 5$. In this section, another experiment was conducted to study the impact of negative traces. Therefore, the total number of traces is $|Q| \times 5$ where half of the sequences were positive and the other half were negative.
Figure 3.7 shows two groups of box-plots representing the BCR scores of the inferred LTSs in two cases. First, if only positive samples are included in the inference process (the right group of box-plots), and second if negative samples are provided with positive ones (the left group of box-plots).

![Box-plot diagram showing BCR scores for different EDSM thresholds](image)

**Figure 3.7:** BCR scores obtained using the EDSM algorithm for different EDSM threshold values

The right group consists of different box-plots representing the BCR scores of LTSs inferred using different EDSM learners from positive sequences only. From Figure 3.7 we can see that EDSM over-generalizes LTSs when the threshold is set to zero or one; this is due to the absence of negative samples that can control the generalization by preventing many incorrect mergers. The horizontal line in the right group in Figure 3.7 shows that the BCR values are 0.5, which indicates that the learner makes random guesses of states merging (over-generalization).

In the absence of negative samples, one may constrain the merging process by increasing the threshold to two, three, and four. Figure 3.7 illustrates that when the threshold is three, the average BCR is around 0.67. However, EDSM under-generalizes LTSs when the threshold is greater than 2; this means many states are blocked from being merged, which is considered bad during the generalization process.
As noted by Walkinshaw et al. [100], the accuracy of the inferred models becomes very low when the merging threshold is low compared to that with a high threshold. Moreover, with a very low threshold, the language of the inferred models accept many false positive sequences. The study in this section agrees with their findings [100] in which the BCR scores can be improved by increasing the EDSM threshold from two to three.

The left group of box-plots shown in Figure 3.7 summarizes the BCR scores attained by variants of the EDSM algorithm in cases where positive and negative sequences were supplied. In comparison to the case when only positive samples are provided, the figures show that EDSM performs better if negative samples are available and the merging threshold is one or two. For instance, the average BCR scores of LTSs inferred when negative samples are available and the threshold is zero is 0.60 compared to 0.5 in cases of positive samples only.

Additionally, the impact of the EDSM threshold on the structural-similarity scores of the inferred LTSs using EDSM is illustrated in Figure 3.8. In cases where only positive sequences are provided and the merging threshold is one or two, the average structural-similarity score of inferred LTSs are zero; this denotes that the models are over-generalized.
It is clear that the structural-similarity scores achieved by learners are very sensitive to the existence of negative samples and the settings of the $EDSM$ threshold. The average structural-similarity scores attained by $EDSM$ is nearly 0.5 when the threshold is three, which is higher than others obtained by different settings of the $EDSM$ threshold.

During the conducted experiments, the ratio of correctness for the number of states was computed as follows:

$$\text{ratio of correctness} = \frac{\text{The number of states of LTSs inferred using a learner}}{\text{The number of states of the target LTSs}} \quad (3.1)$$

The ratio of correctness for the number of states of LTSs inferred using different $EDSM$ learners are shown in Figure 3.9. It is apparent from Figure 3.9 that the number of states is affected by the setting of the $EDSM$ threshold. As shown in Figure 3.9, the $EDSM$ learner generates LTSs with the number of states close to those in the hidden target LTSs when the $EDSM$ threshold equals two. From Figure 3.9, the figures indicate that many mergers are not made that should be when the threshold is three or four. This indicates that the setting of the $EDSM$ threshold is critical.

In situations where positive and negative sequences are provided, the number of states is affected by the setting of the $EDSM$ threshold, as shown in Figure 3.10. It is apparent that the inferred LTSs have more states compared to the target LTSs if the threshold is set to three or four. On the other hand, the numbers of states of the inferred models are so close to the target LTSs when the $EDSM$ threshold is two.
Figure 3.9: Ratio of correctness for the number of states of learnt LTSs using different EDSM learners from positive samples only

3.1.6 Improvements on EDSM

Bugalho and Oliveira [115] stated that the EDSM algorithm outperforms the RPNI method because it uses statistical evidence to evaluate pairs of states before performing any merger. In contrast to this, the RPNI method performs the first admissible merger of states without making a preference between possible mergers [38]. The reason behind using the evidence measure in the EDSM algorithm is to avoid merging invalid pairs of states and to merge those that are most likely to be correct based on their scores [44].

As pointed out in [111], the original EDSM method suffers from weaknesses related to incomplete (sparse) training data. Hence, it is possible that an incorrect merge of inequivalent states can occur [116]. Moreover, it requires a correct merger at each iteration during the learning process to infer the exact target DFA, otherwise bad mergers can happen in early mergers [111]. In addition to the weakness related to the original EDSM method, there is no backtracking to undo an incorrect merge, and to identify when that occurred [111].
Various research studies [111, 115] have attempted to improve the performance of the EDSM learner with respect to the search procedure to the inference process. They [111, 115] aimed to increase the quality of the inferred models by exploring the tree (APTA) to consider other possible state merges alongside those determined by the greedy search of the EDSM learner. The following search techniques were used in the literature:

- The evidence-driven backtrack search (ED-BTS) is an improvement to the EDSM algorithm that applies the backtracking search to the last merger if there is another possible (alternative) merge [115]. Unfortunately, this backtrack method does not significantly improve the inferred machines compared to the normal EDSM as stated by Bugalho and Oliveira [115].

- Lang [117] proposed the evidence-driven beam search (ED-BEAM) algorithm, and developed it by combining the beam search techniques with the heuristic strategy used in the EDSM algorithm. The ED-BEAM algorithm starts by trying all possible merging choices close to the root of tree, and then applies a heuristic search to select
the best choice. The reason for using beam search strategy is to avoid earlier wrong merge states made by a heuristic search [115, 117]. Lang [117] stated that ED-BEAM is applicable for larger state machines inference.

- The evidence-driven stochastic search (ED-SS) [115] algorithm is designed to find the state-merge (decision) that is very likely to be a bad merge. It uses a special measurement that is computed to each possible merge of states, and this measurement calculates the effect of merging two states on alternative possible merges. The score is computed for each merge based on three scores [115].

To sum up, Bugalho and Oliveira [115] have investigated the performance of the above-mentioned three variates of EDSM and concluded that there is a slight improvement made by ED-BEAM and ED-SS compared to EDSM, which is not significant as claimed by Bugalho and Oliveira [115].

### 3.1.7 Other Improvements

Lang et al. [44] suggested considering only merging pairs of states that lie within a specific distance (window) from the root. This aimed to speed up the time of running the EDSM algorithm. The W-EDSM algorithm performs faster than EDSM since it reduce the search space during selecting pairs of states. Unfortunately, it leads to bad performance because it misses deep pairs of states that have the highest scores.

Cicchello and Kremer [113] described the windowed EDSM (W-EDSM) as follows. In the current hypothesis, make a window $w$ of states in breadth-first order starting from the root node. The size of the window $w$ is set to be twice the size of the target machine. Then, each pair of states within the distance $w$ is evaluated, and the pair of states that has the highest scores is merged. Once the size of the window is decreased after performing merges, further states will be considered in the windows to make sure its size becomes twice the target state machine. If there is no possible merge within the given distance $w$, the size of the window is increased by two. The process is terminated when no further merges are possible [112, 113]. Recently, Heule and Verwer [110] showed an improvement to the EDSM learner in order to improve the inferred LTSs by introducing a new constraint named a consistency check, which is described in the following sections.
3.1.8 Introduction of Satisfiability to the State-Merging Strategy

Heule and Verwer [110] proposed a novel technique to synthesize software models. Their algorithm is called DFASAT which is based on satisfiability (SAT) and a greedy technique which is represented by the EDSM learner. Heule and Verwer [118] proposed using an exact translation of DFA identification into SAT instances [119]. The SAT solver is then used to find optimal DFA solutions. Initially, the inference process using DFASAT begins with several steps of EDSM to minimize the inference problem before implementing the SAT solver. Section 3.1.9 describes the consistency constraint proposed by Heule and Verwer [110]. After that, the DFASAT algorithm is described in Section 3.1.11.

3.1.9 Heule and Verwer Constraint on State Merging

Heule and Verwer [110] showed a very interesting constraint during the inference method using EDSM, where mergers are blocked if the merging process step adds new transitions to a red state. This constraint is called consistency check, and developed with an assumption that the red states are identified as correct states in the hidden target LTSs [110]. In this way, a merge is not permitted to add new labels of the outgoing transitions from a blue state to a red state [110]. The reason for blocking such mergers is an assumption that considers red states to be correctly identified parts of the target model [110]. We refer to this idea as Sicco’s idea. The Sicco’s idea can be implemented for only the considered pair to merge, which is further referred to as SiccoN. It is important to highlight that SiccoN is a variant of the EDSM learner without any threshold. In this way, SiccoN only block mergers when the EDSM score is below zero.

Example 3.3. Consider a merging of A and B in the PTA shown in Figure 3.11 is assumed to be an invalid merge because the blue state B would add Close and Edit labels to the red state A.

3.1.10 Experiments Using SiccoN

In order to evaluate the benefit of adding Sicco’s idea to the EDSM learner, the same experiments that are described in Section 3.1.5 were conducted to measure the impact of
Sicco’s idea on the quality of the inferred LTSs. The aim of this experiment was to study the performance of SiccoN compared to various settings of the EDSM learner.

Figure 3.12 shows the BCR scores of the inferred LTSs, where only positive samples are provided. The results of the comparisons show a clear improvement in the accuracy of inferred LTSs using SiccoN compared to variants of the EDSM learner. The mean value of BCR scores attained by SiccoN is 0.75, which is better than any other learners. It is observed from Figure 3.12 that SiccoN reduced the problem of over-generalization of the inferred LTSs compared to different EDSM learners.
Figure 3.13 illustrates the BCR scores of the inferred LTSs using various learners, where both positive and negative sequences were used to complete the inference process. The obtained LTSs using SiccoN are good hypotheses compared to their target LTSs. The median value of BCR scores attained by SiccoN is about 0.72, which is higher than any other learners.

It is important to compare between SiccoN and EDSM learners in terms of the structural-similarity scores. A boxplot of the structural-similarity scores of LTSs inferred from entirely positive sequences using different learners are shown in Figure 3.14. It appears that structural-similarity scores obtained by SiccoN are higher than those attained by the EDSM learners. That is, SiccoN infers LTS models where their structures are closer to the target LTSs than EDSM. On the other hand, the structural-similarity scores of inferred LTSs from both positive and negative sequences are shown in Figure 3.15. The SiccoN learner achieves the highest average of the structural-similarity scores compared to other learners.

It is interesting to observe the number of states of the inferred LTSs from only positive traces using SiccoN and EDSM. Hence, the ratio of correctness for the number of states

\[
\text{BCR}_{\text{SiccoN}} = 0.72
\]
Figure 3.14: Structural-similarity scores achieved by SiccoN and different EDSM learners from positive sequences only.

Figure 3.15: Structural-similarity scores achieved by SiccoN and different EDSM learners from positive sequences and negative.
was computed using Equation 3.1. Figure 3.16 shows that the number of states of LTSs inferred using SiccoN is very close to the target LTSs. It is worth noting that the number of states of the inferred models using EDSM learner when the merging threshold is two is close to the target number of states. However, this does not mean that the inferred models are good with respect to the BCR and structural-similarity scores. In other words, the smallest models are not always better in terms of language and structure. On the contrary, given positive and negative samples, numbers of states of the inferred LTSs using SiccoN converge to the exact number of states in the target LTSs as shown in Figure 3.17.
3.1.11 DFASAT Algorithm

The earlier study by Heule and Verwer [118] suggested the translation of the DFA inference problem into satisfiability (SAT). This translation that has been used by Heule and Verwer [118] is inspired by the previous translation of the DFA identification problem into the graph colouring issue [120].

It is the problem of colouring nodes in the given graph where nodes connected with an edge have a different colour, and sometimes is known as state colouring. The DFA identification problem use the colouring graph such that compatible states in the same block are coloured with the same colour, and those that cannot be merged are given different colours [120].

Heule and Verwer [118] have focused on translating the graph colouring strategy into SAT. However, this translation can result in a huge number of clauses, which is too difficult for the existing SAT solver. This explain why the DFASAT algorithm attempts to run EDSM in the earlier steps before calling the SAT solver to complete the inference process and avoiding handling the large number of clauses.
Heule and Verwer [110] developed the DFASAT algorithm that attempts to find multiple DFA solutions inferred for each inference task. The number of solution is identified by the user by setting the parameter $n$. Heule and Verwer [110] stated that the early solutions obtained by the DFASAT algorithm can reach 99% accuracy if the training data is not sparse. However, multiple solutions can be combined to classify the test set during the StaMinA competition if the data is very sparse.

In general, DFASAT begins by running EDSM in the early steps in order to reduce the problem of inferring DFA to be solvable by the SAT solver. The resulting state machine from this stage is called a partial DFA. The reason behind incorporating the SAT solver is to solve the problem when the EDSM learner becomes very weak at finding good DFA solutions [110].

It is important to identify when to stop the EDSM learner and start running the SAT solver. Heule and Verwer [110] introduced the $m$ parameter to determine when to stop the traditional EDSM state merging and begin the SAT solving. The method stops the merging procedure when the number of states that are reachable by the positive examples obtained from the provided training samples is less than $m$. The parameter $m$ is set to 1000 in the StaMinA competition.

The DFASAT algorithm is illustrated in Algorithm 6. The DFASAT learner begins with the initialization of a parameter $t$ to infinity, this parameter is used later to indicate the target number of states for the inferred DFA. The benefit of setting the parameter $t$ is that if the number of red states in the current hypothesis DFA is larger than $t$, then the performed merges are assumed to be inefficient [110]. The setting of parameter $t$ is initially equal to infinity, and many merges are performed using the greedy procedure before calling the SAT solver when $|R| \leq t$ to reduce $t$ to the size of red states $R$ [110]. After initializing $t$, the DFASAT invokes generateAPTA ($S^+, S^-$) to generate the initial APTA from the provided samples. States are selected and merged using the EDSM algorithm for several steps as shown in lines 7-11.

The parameter $m$ is used as a boundary for a number of mergers to be performed using EDSM before starting the SAT solver. Once the number of states in $A$ that are reached by the positive examples is smaller than $m$, the SAT solving will begin to find the smallest DFA [110]. Otherwise, it continues learning LTSs using EDSM. A parameter $t$ is used
Chapter 3. Existing Inference Methods

Require: an input sample of sequences \( S = S^+ \cup S^- \), a test sample \( S_t \), merge bound \( m \), number of DFA solutions \( n \), accepting vote percentage \( avp \) between 0 and 1

Ensure: Label is a labelling for \( S_t \) aimed to give high accuracy for software models

1. Let \( t \leftarrow \infty \)
2. Let \( D \leftarrow \emptyset \) \hspace{1cm} // \( D \) is a set of multiple DFAs solutions
3. \( A = \text{GenerateAPTA}(S^+, S^-) \) \hspace{1cm} // generate the APTA \( A \) from sequences
4. \( \text{while} |D| < n \) do
5. \hspace{1cm} // while the number of DFA solution is less than \( n \)
6. \hspace{2cm} \( A' \leftarrow \text{copyAPTA}(A) \) \hspace{1cm} // create another copy of APTA \( A' \)
7. \hspace{2cm} \text{while} |A'|^p < m \ do
8. \hspace{4cm} // while the positive sequences reach more than \( m \) states in \( A' \)
9. \hspace{5cm} select \( q \) and \( q' \) in \( A' \) using random greedy ;
10. \hspace{5cm} \( A' = \text{merge}(A', q, q') \) \hspace{1cm} // merge states in \( A' \) using random greedy
11. \hspace{2cm} end
12. \hspace{1cm} // if \( A' \) has more than \( t \) red states
13. \hspace{2cm} if \( |R| > t \) \hspace{1cm} (\( R \) being the red states in \( A' \)) then
14. \hspace{3cm} // find a better partial DFA solution
15. \hspace{4cm} continue the next while loop iteration
16. \hspace{2cm} end
17. \hspace{2cm} set \( t \leftarrow |R| \) // else update \( t \) to the amount of resulting red states
18. \hspace{2cm} let \( i \leftarrow 0 \) // initialize the number of additional states to 0
19. \hspace{2cm} \( \text{while} \) true \ do
20. \hspace{3cm} translate \( A' \) to a SAT formula using \( |R| + i \) colours \hspace{1cm} // try to find an exact solution with \( i \) extra states
21. \hspace{3cm} solve the formula using a SAT-solver ;
22. \hspace{3cm} if \ the solver return a DFA solution \( A'' \) then
23. \hspace{4cm} // if the SAT solver finds a solution add it to \( D \)
24. \hspace{4cm} add \( A'' \) to \( D \) and \( \text{break} \)
25. \hspace{3cm} else if \ the solver used more than 300 seconds \( A'' \) then
26. \hspace{4cm} \( \text{break} \) \hspace{1cm} // try another partial solution if the problem is too hard
27. \hspace{3cm} else
28. \hspace{4cm} set \( i \leftarrow i + 1 \) \hspace{1cm} // else try to find a larger solution
29. \hspace{3cm} end
30. \hspace{2cm} end
31. \hspace{2cm} end
32. \hspace{2cm} let \( Label \) be an empty labeling \hspace{1cm} // initialize the test labeling
33. \hspace{2cm} \// iterating through test set \( S_t \)
34. \hspace{2cm} \( \text{forall} \) the \( s \in S_t \) \ do
35. \hspace{3cm} if \( |\{ A \in D | s \in L(A) \}| \geq avp \) then
36. \hspace{4cm} append ‘1’ to \( Label \) \hspace{1cm} // \( s \) is labelled as positive because at least \( avp \) % of the solutions accept \( s \)
37. \hspace{3cm} else
38. \hspace{4cm} append ‘0’ to \( Label \) \hspace{1cm} // label \( s \) as negative
39. \hspace{3cm} end
40. \hspace{2cm} return \( Label \)
41. end

Algorithm 6: The DFASAT Algorithm [110]
later to refer to a target size of a DFA [110]. Once the APTA becomes small, the APTA is translated to many clauses and they are passed to the solver to find a DFA as shown in lines 19-30. Every time a DFA is inferred, it will be added to the set $D$ as shown in line 24. The reason behind collecting all possible solutions is to find the optimal generalization of DFA using multiple DFA solutions using the ensemble method [121].

The $DFASAT$ algorithm attempts to generate many DFA solutions. When a number of DFA solutions are generated, the test sequences are passed to each DFA to decide which of them are rejected or accepted. [110] introduced accepting vote percentage (avp) such that if a test sequence is accepted by avp % of the generated DFA, then it is classified as positive, and otherwise, it is classified as negative. This idea is motivated by the ensemble method [121] to improve the classification accuracy and treating the problem of data sparseness in the StaMinA competition.

### 3.1.12 Inferring State-Machine Models by Mining Rules

Lo et al. [6] described rule mining as the process of identifying constraint between the pre-condition and post-condition of rules. In the last decade, rule mining from traces has gained attention from software engineers looking to understand how a program behave [7, 23].

Interestingly, rule mining techniques can be used to steer state machine inference strategies. Lo et al. [70] suggested the leveraging of two learning methods: rule mining and automata inference to avoid the over-generalization problem. There are two phases in their miner [70]. First, rule mining to statistically infer temporal properties between the events in traces [70]. Those properties in the forms of the rules identified the relation between the distant events in the traces [70]. The mined rules can be either future-time or past-time rules. Future-time rules determine the relations or dependencies between events, such that whenever a series of events occurs (appears in the traces), another sequence of events must happen subsequently [70]. Past-time rules determine relations between events such that whenever a sequence of events occurs, another sequence must happen before [70].

The second phase involves inferring automata with steering based on the mined rules in the first step. In the second phase, Lo et al. [70] states are merged only if all the mined rules are satisfied by the automata generated after the merging step. If the resulting automaton violates the mined rules, then states are not merged. The evaluation of their
approaches [70] showed that the accuracy of inferred automata using temporal rules can be increased in terms of the precision scores.

In this thesis, it is of limited value to evaluate the proposed learner with such rule-mining based approach. This is because the mined rules are represented as pre- and post-condition pairs where the post condition is known to be held if the pre-condition is held based on the confidence metric [70]. In the EDSM-Markov learner that is proposed in this thesis, EDSM-Markov does not have any knowledge as to whether the proprieties are held in the target system or not. Their approaches [70] allow users to modify, and delete the mined rules while EDSM-Markov prevents the users intervention. The inference of state machine by mining rules is applied with the $k$-tails algorithm. However, the performance can be poor at inferring LTSs where there are few traces.
3.2 Active Learning

In passive learning, the inference process attempts to generate an automaton model from the provided traces. Unfortunately, traces might not contain sufficient information about the behaviour of a system, and then it becomes difficult to learn an exact model from the traces. The reason for this difficulty is that the provided traces cannot distinguish every pair of states and are not able to identify equivalent states among a number of states.

Additionally, the performance of passive inference techniques is poor if the provided samples are sparse. An alternative approach was introduced to tackle the difficulties that have faced passive learning techniques, and it is called the active learning strategy. Angluin [79] introduced a powerful active learning algorithm named Lstar and it is denoted by $L^*$. It is widely used in the grammar inference field to learn DFA models representing a specific language from strings or sentences. Angluin [79] proved that an automaton can be identified in polynomial time if the learning algorithm asks queries to collect missing information to get exact models. In the $L^*$, there is a minimal adequate teacher (MAT) responsible to answer specific kinds of queries that are posed by the inference learner.

In Angluin’s algorithm, two kinds of queries are posed to a teacher or an oracle: membership queries and equivalence queries. In membership queries, the learner poses a sequence $S$ as a query to the oracle to decide whether it belongs to the language $L$. The answer of membership queries is either accepted denoted as 1 indicating that a sequence over $\Sigma^*$ belongs to the unknown language, or rejected denoted as 0 meaning that a sequence does not belong to the target language. In equivalence queries, a query is asked to decide whether an inferred model is isomorphic to the target model. If the answer of an equivalence query is yes, then the state machine model is conjectured. Otherwise, a counter example is returned in the form of membership queries. The answers are recorded as new observations in a table called an observation table (OT).

In software model inference, the $L^*$ algorithm is aimed at exploring the system being learnt by asking queries about its behaviour and returning the corresponding consistent state machine models. It requires an oracle (a software system being inferred) to answer queries. It requires interacting with the system under inference to collect observations by asking queries.
3.2.1 Observation Table

In Angluin’s algorithm, there is an assumption that the alphabet Σ is known. The $L^*$ incorporates the answered queries (sequences) in an observation table (OT). It is a specific representation of an automaton in a table. All gathered sequences that are classified by the posed queries are organized into the OT.

The OT is a 3-tuple $OT = (S, E, T)$ where rows $S$ is a prefix-closed set of sequences over $Σ$, columns $E$ is a suffix-closed set of sequences over $Σ$, and $T$ is a finite mapping function, which maps $((S \cup S \cdot Σ) \cdot E)$ to $\{0, 1\}$ [79]. All sets $(S, E)$ are assumed to be finite and non-empty. Rows in the OT are labelled with $(S \cup S \cdot Σ)$, and columns are labelled with $E$. A cell in the OT is labelled with $T(s \cdot e)$, where $s$ represents a row of the cell such that $s \in S \cup S \cdot Σ$, and $e$ is a column of the cell such that $e \in E$. Thus, $T(s \cdot e)$ is mapped to 1 if the sequence $s \cdot e$ belongs to the target DFA model, otherwise it is mapped to 0 to denote the sequence $s \cdot e$ does not belong to the language. Table 3.1 illustrates an example of the OT, where the set of alphabet is $a, b$.

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$\epsilon$</td>
</tr>
<tr>
<td>$S \cdot Σ$</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>b</td>
</tr>
</tbody>
</table>

Table 3.1: An example of the observation table

The equivalence of any two rows in the OT is identified based on the $E$ set. Let $s_1, s_2 \in S \cup S \cdot Σ$ be a pair of rows, then $s_1$ and $s_2$ are equivalent, denoted by $row(s_1) =_{eq} row(s_2)$, if and only if $T(s_1 \cdot e) = T(s_2 \cdot e)$, $\forall e \in E$. An OT is called closed if $\forall s_1 \in S \cdot Σ$ where there exists $s_2 \in S$ such that $row(s_1) =_{eq} row(s_2)$. The OT is considered consistent as long as $s_1, s_2 \in S$ such that $row(s_1) =_{eq} row(s_2)$ and $row(s_1 \cdot σ) =_{eq} row(s_2 \cdot σ)$, $\forall σ \in Σ$.

3.2.2 $L^*$ Algorithm

The $L^*$ algorithm first constructs the table and initializes $S = E = \{\epsilon\}$. Then, the algorithm fills the OT to ensure the closed and consistent conditions by asking membership queries for $\epsilon$ and each $σ \in Σ$. Once the OT is not consistent, the $L^*$ finds a pair of rows $s_1, s_2 \in S, σ \in Σ$, and $e \in E$ such that $row(s_1) =_{eq} row(s_2)$ where $T(s_1 \cdot σ \cdot e) \neq T(s_2 \cdot σ \cdot e)$.
The OT is extended by adding the sequence $\sigma \cdot e$ to $E$ and asking membership queries to fill missing information in $(S \in S \cdot \Sigma) \cdot (\sigma \cdot e)$ [79].

During the learning process, if the OT is not closed, then the $L^*$ algorithm attempts to find $s_1 \in S \cdot \Sigma$ such that $row(s_1) \neq_{eq} row(s_2)$ for all elements of $s_2 \in S$. The $L^*$ then adds $s_1$ to $S$. Then, the OT must be extended (expanded) by asking membership queries for missing elements. This process is repeated until the OT becomes closed and consistent [79]. Once the OT is known to be consistent and closed, $L^*$ constructs the corresponding DFA conjecture over $\Sigma$ as follows:

- $Q = \{row(s) : s \in S\}$
- $q_0 = row(\epsilon)$
- $F = \{row(s) : s \in S \text{ and } T(s) = 1\}$
- $\delta(row, \sigma) = row(s \cdot \sigma)$

The DFA conjecture may contain a small number of states in comparison with the target size of the correct DFA. The $L^*$ passes the resulting conjecture to an oracle to check its correctness against the target one. This is called an equivalent query and it requires an answer from the oracle. If it replies yes this indicates that the conjecture is correct or it returns a counterexample. The process will terminate if the answer is yes. However, if the oracle returns with a counterexample, then the returned counterexample and its prefixes are added into the set of $S$ to extend the OT. Then, the OT is filled by asking membership queries. The $L^*$ procedure is presented in Algorithm 7.
input: A finite set of the alphabet $\Sigma$
result: DFA conjecture $M$

1. $S \leftarrow \{ \epsilon \}$
2. $E \leftarrow \{ \epsilon \}$
3. $OT \leftarrow (S, E, T)$
4. repeat
5. while $OT$ is not closed or not consistent do
6. if $OT$ is not closed then
7. find $s_1 \in S \cdot \Sigma$ such that $row(s_1) \neq eq row(s), \forall s \in S$
8. Move $s_1$ to $S$;
9. add $s_1 \cdot a$ to $S \cdot \Sigma, \forall a \in \Sigma$;
10. Extend $T$ to $(S \cup S \cdot \Sigma) \cdot E$ by asking membership queries to fill the table
end
12. if $OT$ is not consistent then
13. find $s_1, s_1 \in S, \sigma \in \Sigma, \text{ and } e \in E$ such that $row(s_1) = eq row(s_2)$,
14. but $T(s_1 \cdot \sigma, e) \neq T(s_2 \cdot \sigma, e)$;
15. add $\sigma \cdot e$ to $E$;
16. Extend $T$ to $(S \cup S \cdot \Sigma) \cdot E$ by asking membership queries to fill the table;
end
18. end
19. $DFA \leftarrow$ conjecture ($OT$)
20. $CE \leftarrow$ FindEquivalenceQuery ($DFA$)
21. if $CE \neq \phi$ then
22. add $CE$ and all the prefixes of $CE$ to $S$
23. Extend $T$ to $(S \cup S \cdot \Sigma) \cdot E$
24. by asking membership queries to fill the table
end
26. until the oracle does not return any counterexample to $DFA$;

Algorithm 7: The L* Algorithm Following [79, 122]
3.2.3 Example of L*

In this section, an illustration of how the L* algorithm can infer DFA \( A \). The alphabet set \( \Sigma = \{ \text{Load, Edit, Save, Close, Exit} \} \) is known to the L* learner. To begin with inferring the LTS \( A \), the L* initializes the \( OT = (S, E, T) \) as follows: \( S = E = \{ \epsilon \} \), and \( S \cdot \Sigma = \{ \text{Load, Edit, Save, Close, Exit} \} \) as shown in Table 3.2a. Then, the L* learner asks the following membership queries \{\langle \epsilon \rangle, \langle \text{Load} \rangle, \langle \text{Edit} \rangle, \langle \text{Save} \rangle, \langle \text{Exit} \rangle, \langle \text{Close} \rangle \} \) to fill the \( OT_1 \) as shown in Table 3.2b.

It is clear from the \( OT_1 \) that the sequences \{\langle \text{Load} \rangle, \langle \text{Exit} \rangle \} belong to the target language and other sequences do not. The current \( OT_1 \) is consistent since only one sequence in the prefix-closed set \( S = \{ \epsilon \} \) but it is not closed because \( row(\text{Edit}) \in S \cdot \Sigma \neq row(\epsilon) \in S \).

To make the \( OT_1 \) closed, the \( row(\text{Edit}) \) is moved from \( S \cdot \Sigma \) to \( S \), and the set \( S \cdot \Sigma \) is updated by concatenating sequence \( \text{Edit} \) with each alphabet \( \sigma \in \Sigma \). To construct a completed observation table \( OT_2 \), new sequences \{\langle \text{Edit, Load} \rangle, \langle \text{Edit, Edit} \rangle, \langle \text{Edit, Save} \rangle, \langle \text{Edit, Close} \rangle, \langle \text{Edit, Exit} \rangle \} \)
\{Edit, Exit\} are added to \(S \cdot \Sigma\) as shown in Table 3.2c. The \(L^*\) asks membership queries to fill the new rows as illustrated in Table 3.2d.

### 3.2.4 Improvements of \(L^*\) in Terms of Handling Counterexamples

The important phase of the \(L^*\) is handling counterexamples obtained during the inference process. In the original \(L^*\) [79], the counterexample handler adds counterexamples and all their prefixes to the \(S\) set and leads to numerous membership queries [123]. Rivest and Schapire [123] suggested removing the consistency check for the OT. The inconsistencies can be avoided by making the \(S\) set distinct. In other words, it is not allowed to have equivalent rows in the \(S\) set. Rivest and Schapire [123] improved the counterexample handler using a binary search to identify a single distinguishing sequence (suffix) in a counterexample and adds the suffix to the \(E\) set.

Maler and Pnueli [124] modified the counterexample handler by adding a counterexample and its suffixes to the \(E\) set to ensure that the OT is consistent and closed. Similar to Maler and Pnueli [124], Irfan et al. [125] adds the counterexample to the \(E\) set in the OT. However, Irfan et al. [125] proposed a refinement to the process of handing a counterexample. Irfan et al. [125] proposed a counterexample handler, which is called \(\text{Suffix1by1}\). It adds the suffixes of the counterexample under process to the columns \(E\) one by one. Once the distinguished sequence is found that makes improvements to the conjecture, it stops adding the remaining suffixes to the \(E\) set. Finding counterexample using random oracle can lead to the asking of many membership queries [125]. Irfan et al. [125] stated that \(\text{Suffix1by1}\) can reduce the number of membership queries that random oracle causes.

### 3.2.5 Complexity of \(L^*\)

Angluin [79] stated that the worst case of algorithm is filling all holes in the OT. The upper bound of membership queries is \(O(m|\Sigma|Q^2))\) [79, 126], where \(m\) represents the length of the longest received counterexample. For example, consider a DFA with 50 states and 10 alphabets. In addition, consider that the length of the longest counterexample is 50; the number of membership queries required to find the DFA in the worst case using the \(L^*\) algorithm \(10 \times 50 \times 50^2 = 1250000\) queries.
Since Angluin’s algorithm was proposed, much research has been carried out to reduce the number of membership queries. Rivest and Schapire [123] improved Angluin’s algorithm $L^*$ without resetting the machine and they [123] replaced the reset process with the idea of a homing sequence [127]. Rivest and Schapire [123] showed that the upper bound of the worst case in Angluin’s algorithm is reduced and can be given as follows: $O(|\Sigma|Q^2 + Q\log m)$. Kearns and Vazirani [128] used a binary discrimination (classification) tree to record answers, and their algorithm reduced the upper bound on the number of queries to $O(|\Sigma|Q^3 + Qm)$. In terms of learning prefix-closed language, as in our context, Berg et al. [126] stated that the number of membership queries with respect to the number of states and alphabet size is given as $k|\delta|$, where $|\delta| = |Q||\Sigma|$ and $k \approx 0.016$.

### 3.2.6 Query-Driven State Merging

The idea of state merging to infer state machine specifications may fail because the collected traces are insufficient to meet all of the behaviours of a system. Dupont et al. [36] stated that state-merging techniques can benefit from the concept of active learning to maximize the knowledge about the hidden system. Dupont et al. [36] developed a new algorithm called *query-driven state merging* (QSM) in order to adapt the RPNI algorithm to become active learning by posing membership queries to control the generalization of a DFA. The QSM algorithm is an incremental method, since the examples grow during the learning process.

In general, the inference process is similar to the EDSM learner, but the QSM asks queries after each step of state merging to verify a merger of two states. The available sequences are used alongside newly classified membership queries (new sequences) to control the generalization of a DFA.

The inference process using the QSM initially starts by generating an initial PTA from positive only or an APTA if there are negative sequences. Similar to EDSM, pairs of states are selected iteratively. Once a pair of states is chosen for merging, the Merge function constructs a new hypothesis model $A_{\text{new}}$ which is obtained by merging states.

After that, the Compatible function checks whether the new hypothesis model $A_{\text{new}}$ accepts all positive sequences and rejects all negative ones correctly. Once the intermediate hypothesis model $A_{\text{new}}'$ is compatible with the available traces, any new sequences obtained
Chapter 3. Existing Inference Methods

input: A non-empty initial scenario collection \( S_+ \) and \( S_- \)

result: \( A \) is a DFA that is consistent with \( S_+ \) and \( S_- \)

/* Sets of accepted and rejected sequences */

\[
1 A \leftarrow Initialize(S_+, S_-) \\
2 \textbf{while} (q, q') \leftarrow ChooseStatePairs(A) \textbf{do} \\
3 \quad A_{\text{new}} \leftarrow Merge(A, q, q') \\
4 \quad \textbf{if} \text {Compatible}(A_{\text{new}}, S_+, S_-) \textbf{then} \\
5 \quad \quad \textbf{while} \text{Query} \leftarrow GenerateQuery() \textbf{do} \\
6 \quad \quad \quad \text{Answer} \leftarrow \text{checkWithOracle}(q); \\
7 \quad \quad \quad \textbf{if} \text{Answer is true} \textbf{then} \\
8 \quad \quad \quad \quad S_+ \leftarrow S_+ \cup \text{Query} \\
9 \quad \quad \quad \quad \text{if} \neg \text{Compatible}(A_{\text{new}}, \{\text{Query}\}, \emptyset) \textbf{then} \\
10 \quad \quad \quad \quad \quad \text{return } QSM(S_+, S_-) \\
11 \quad \quad \quad \textbf{end} \\
12 \quad \textbf{else} \\
13 \quad \quad S_- \leftarrow S_- \cup \text{Query} \\
14 \quad \quad \textbf{if} \neg \text{Compatible}(A_{\text{new}}, \emptyset, \{\text{Query}\}) \textbf{then} \\
15 \quad \quad \quad \text{return } QSM(S_+, S_-) \\
16 \quad \quad \textbf{end} \\
17 \textbf{end} \\
18 \textbf{end} \\
19 \textbf{end} \\
20 A \leftarrow A_{\text{new}} \\
21 \textbf{end}
\]

Algorithm 8: The QSM algorithm

as a result of merging is a possible query for classification by an oracle into positive or negative. The process is restarted again if the merged automaton rejects sequences that answered as yes by the oracle as shown in line 10, and vice versa as shown in lines 15.

In Dupont et al. [36], the membership queries are generated by concatenating the shortest sequences from the initial states leading to the red state with suffix sequences of the blue state in the graph before merging. In other words, the membership queries are generated by adding all suffixes of the blue state to the shortest prefixes of the red node from the initial state in the current hypothesis. The resulting queries belong to the language of the merged graph but do not belong to the graph before merging. These membership queries are called Dupont’s queries in this thesis. More details about Dupont’s queries will be described in Chapter 7.

Example 3.4. Let us consider the PTA of the text editor example presented in Figure 3.18, and suppose that states \( B \) and \( C \) are considered for merging. The resulting merged graph
(hypothesis-machine) is shown in Figure 3.19, and the Dupont generator returns a list of queries as follows: Dupont’s queries = \{ (Load, Save) \}.

3.3 Applications of Active Inference of LTS Models From Traces

3.3.1 Reverse Engineering LTS Model From Low-Level Traces

Walkinshaw et al. [28] used dynamic analysis to generate a list of execution traces that can be served as an input for grammar inference techniques. Those low-level traces are required in an abstraction process to obtain high-level abstraction. They integrated the reverse-engineering technique represented in QSM into a testing framework. Their idea was performed in four activities as follows:

1. **Dynamic analysis**: This process generates a collection of system execution traces, which is considered as sequences of method calls.

2. **Abstraction**: This process focuses on generating a function that can use the low-level traces obtained in activity 1 as input and return equivalent sequences of functions at a level of abstraction as output.

3. **Trace abstraction**: The abstraction method in step 2 is applied to the set of traces derived in step 1. It returns a finite set of abstract function sequences, which is passed as input for the next step.
4. **QSM**: In this process QSM is applied to the function sequences. They \[28\] improved the QSM algorithm by modifying the questions generator, and adding a facility to add negative sequences to eliminate the invalid edges in the resulting machine.

Similar to the original QSM, Walkinshaw et al. \[28\] used the EDSM to select a pair of states to merge. In the QSM framework \[28\], a slight modification to the membership queries generator was implemented compared to the original QSM algorithm \[36\]. The improved generator generates membership queries from the merged graph, and the reason for introducing this method is that new sequences can appear as a result of the merging and determinism processes. The improved generator creates queries by concatenating the shortest prefixes to the red state with suffixes of the merged state in the graph after merging.

**Example 3.5.** Let us return to the example of the text editor in Figure 3.18, the merging of states B and C can result in a new machine as illustrated in Figure 1.3, and a new edge labelled **save** is added to the red states labelled with **BCG**. The improved generator returns a list of question as follows: Improved Queries = \{⟨Load, Save⟩, ⟨Load, Edit, Close, Load⟩\}.

### 3.3.2 Reverse Engineering LTS Model Using LTL Constraints

Walkinshaw and Bogdanov \[77\] proposed a technique to use temporal constraints in the model inference process. The main reason for introducing LTL constraints in DFA inference is to reduce the reliance upon the execution traces.

The technique that is proposed by Walkinshaw and Bogdanov \[77\] allows adding LTL constraints alongside the list of traces to infer a state machine. In addition, a model checker is used to ensure that the hypothesis machine does not violate any temporal rules. If the proposed machine violates defined rules, then counterexamples are generated from a model checker to feed them into the inference learner to start learning again.

Additionally, this technique \[77\] might be run in a passive or an active manner. In passive learning, LTL constraints are provided initially by the developer alongside traces. The inference process starts by generating APTA from the provided positive and negative traces. Iteratively, pairs of states are selected using the EDSM learner with the red-blue
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framework. The pair of states with the highest score is picked for merging. Once the hypothesis machine is obtained after merging a pair of states, it is passed to the model checker to ensure that it does not violate LTL constraints [77]. If there is any violation with the provided LTL properties, the model checker returns a counterexample, and the inference process is restarted [77].

On the other hand, [77] showed that the QSM learner can benefit from the integration of LTL constraints. Similar to the case of passive inference described above, the learning starts by augmenting sequences into APTA and merges states iteratively. It calls the model checker to find any contradiction with LTL constraint. In cases where no counterexamples are returned from the model checker, the active algorithm checks the correctness of a merger of two states by asking queries in the same manner in the QSM learner. This differs from the passive learning in that it continues to merge states if there are no counterexamples obtained from the model checker [77].

Besides, in the case of the active learning strategy, the advantage is that the QSM learner attempts to find undiscovered sequences by asking queries. Moreover, there is a possibility of adding a new LTL properties that can help to confirm or reject new scenarios that appear during the inference process [77].

Walkinshaw and Bogdanov [77] stated that LTL constraints are very helpful in reducing the amount of traces required to generate the exact machine. In addition, Walkinshaw and Bogdanov [77] stated that without such constraints a considerable number of traces are required to infer an accurate model. However, there are barriers related to identifying LTL constraints because it requires effort and a large numbers of traces [77]. The drawback of the inference of a state-machine model using the LTL constraints is the reliance still upon the developer to provide reasonable LTL constraints, which requires more effort Walkinshaw and Bogdanov [77].

Walkinshaw and Bogdanov [77] showed that a number of membership queries can be reduced with the aid of LTL constraints. To sum up, if a large number of constraints are supplied with traces, a large number of queries will be avoided during the inference process [77].
3.4 Tools of DFA Inference Using Grammar Inference

3.4.1 StateChum

StateChum [129] is an open-source Java-based framework developed by Kirill Bogdanov and Neil Walkinshaw. It has been developed to implement many regular grammar inference techniques such as QSM, K-tail, and EDSM. The inferred state-machine model can be visualized after learning a model successfully. The main objective of this framework is to reverse-engineer state-machine models from traces. In addition, it includes a possibility to show the structural difference between the generated model and the target model. Moreover, there is an option to generate test sets using the W-method. It contains a way to generate random FSM, and other features [129]. Our proposed techniques are implemented in this framework.

3.4.2 The LearnLib Tool

LearnLib [130, 131] is a free framework originally written in c++. Learnlib has been developed to implement Angluin’s algorithm to learn DFA and its extensions deriving Mealy machines. Recently, LearnLib has been re-written in Java and is still under-development.

3.4.3 Libalf

Libalf is an open-source framework for learning FSMs written in c++ and developed by Bollig et al. [132]. It includes several well-known algorithms to learn DFA and non-deterministic finite automata (NFA). Some of these algorithms can be run on-line, and others off-line. It has an independent feature that provides Java interfaces using the Java Native Interface (JNI) [133].

3.4.4 Gitoolbox

Akram et al. [134] presented an open-source framework to run some grammar inference algorithms in MATLAB [135]. It includes passive grammar inference algorithms such as RPNI and EDSM.
3.5 The Performance of Existing Techniques From Few Long Traces

This section investigates the problem of learning LTSs from few long training samples using the existing techniques. The reason behind studying this kind of problem is to estimate how well the existing techniques are at constructing good hypothesis models from few positive traces. In order to study the problem in instances of passive inference techniques, we compare them using variants EDSM, SiccoN, and variant of k-tails. Learning of LTSs was aborted when inferred LTSs were reaching 200 red states and a zero was recorded as a score.

Figure 3.20 shows that SiccoN and EDSM $\geq 3$ learners perform better than other settings of EDSM and k-tails. From Figure 3.20, the exact learning is very hard to achieve using the existing techniques. The exact learning means that inferring LTSs with BCR scores is higher than or equal to 0.99 [34]. This denotes that there is still some kind of bad generalization of LTSs by the studying techniques.

![Figure 3.20: BCR scores attained by different learners where the number of traces is 7 and the length of traces is given by $= 0.5 \times |Q| \times |\Sigma|$](image-url)
Figure 3.21 illustrates the structural-similarity score of LTSs inferred using learners that participated in the study. A low the structural-similarity score reflects how learners are not able to avoid bad generalization of LTSs. From Figure 3.21, it is clear \textit{SiccoN} and \textit{EDSM} $>= 3$ scores good compared to other learners, but it is still far from what we aimed to achieve in this thesis.

The first aim of this thesis is to improve the \textit{EDSM} learner benefiting from evidence obtained by training Markov models. This aims to capture the dependencies between events appearing in the traces. The dependencies can be used to help the \textit{EDSM} learner making decisions as to which pairs of states correspond to the same state in a target automaton. In particular, the study focuses on improving the performance of the \textit{EDSM} learner to tackle the case that no negative traces are provided. Chapter 5 and Chapter 6 show how Markov models can be used alongside the \textit{EDSM} learner to solve the problem of over-generalization.

On the other hand, one would consider applying active learning methods such as \textit{QSM} to learn a LTS from few positive traces. The reason behind this is to improve the accuracy of the inferred LTSs, benefiting from asking queries as tests to the LTSs being learnt. The
boxplots of the BCR scores attained by QSM are depicted in Figure 3.22. It is clear that the exact inference of LTSs cannot be achieved even though traces cover transitions by 80% when the number of traces is three.

The boxplots that are depicted in Figure 3.23 represent the structural-similarity scores attained by QSM. They indicate that an extra check using the membership queries can improve the quality of the inferred LTSs.

The boxplot of the number of queries for 10, 20, and 30 states are shown in Figure 3.24. It is clear that the number of membership queries increases while the number of traces is increased. It is interesting to improve the accuracy of the inferred LTSs with fewer membership queries. The second aim of this thesis is to improve the QSM learner at inferring LTSs from few positive traces. Chapter 7 investigates further membership queries that can be used to solve the problem of bad inference of LTS.

**Figure 3.22: BCR scores of LTSs inferred using QSM**

The boxplots that are depicted in Figure 3.23 represent the structural-similarity scores attained by QSM. They indicate that an extra check using the membership queries can improve the quality of the inferred LTSs.

The boxplot of the number of queries for 10, 20, and 30 states are shown in Figure 3.24. It is clear that the number of membership queries increases while the number of traces is increased. It is interesting to improve the accuracy of the inferred LTSs with fewer membership queries. The second aim of this thesis is to improve the QSM learner at inferring LTSs from few positive traces. Chapter 7 investigates further membership queries that can be used to solve the problem of bad inference of LTS.
Figure 3.23: Structural-similarity attained by QSM

Figure 3.24: Number of membership queries asked by QSM
As shown in Chapter 3, passive state-merging algorithms can successfully infer LTS models well if traces are characteristic or complete [45]. However, these algorithms fail to generate good models in many cases when training data are not complete. In particular, the problem arises if the inference process begins with few long representative traces; this does not mean the training data is not sufficient, but it denotes that inference algorithms fail to accumulate good evidence to guide the state-merging process.

This chapter describes the extension of the EDSM inference to handle relatively few long traces. It is motivated by the observation that in software models one would usually have a comparatively large alphabet and few transitions from each state. The idea in this chapter is to use evidence obtained by training Markov models to bias the EDSM learner towards merging states that are more likely to correspond to the same state in a model.
4.1 Introduction

The existing passive inference techniques are aimed at inferring of an LTS or FSM from accepted and possibly rejected sequences of events (abstract traces) without using queries to a system being learnt.

The sparseness of the training data and the absence of negative sequences are the most significant problems encountered in the grammar inference field, as mentioned in chapter 3. In practice, a software engineer might need to infer good state machines from a small subset of characteristic traces. In such cases, finding adequate models using passive algorithms is difficult, especially when there are no negative samples to avoid bad generalization of models.

Markov model is a well-known principle and is widely used to capture dependencies between events appear in logs or traces [49, 136]. Cook and Wolf [137] defined the sequential dependence between events in the event log, and it is based on the probability of an event to follow a sequence of events. Cook and Wolf [137] stated that one of the best techniques of capturing the dependencies is the Markov learner that was introduced by the same author [49].

In this chapter, Markov models are trained from sequences of events in order to capture sequential dependencies. For instance, in the text editor example, an event save is likely to follow edit. This kind of such dependencies is forward where an event can follow an event sequence of a specific length. Capturing forward dependencies can be used to aid the EDSM learner to decide whether an event is permitted or not to follow a sequence of events. In this chapter, the trained Markov models intended to determine elements of an alphabet that can follow a sequence of alphabet appears in a long trace.

The challenge considered in this chapter is to learn LTSs from a few long accepting sequences. Therefore, a new heuristic has been developed to learn LTSs from a few long positive traces. In general, the heuristic used the concept of Markov model alongside the EDSM heuristic. The proposed heuristic combined two scores: the first is the EDSM score reflecting evidence suggesting that a pair of states is considered equivalent, and the second is called inconsistency to compute evidence suggesting that the pair is different based on inconsistencies detected during merging states. Inconsistencies are defined as
contradictions with the trained Markov models that can be introduced during learning LTSs.

### 4.2 Cook and Wolf Markov Learner

Cook and Wolf [49] proposed a Markov method to learn an FSM from an event log. The idea of the Markov method that is proposed by Cook and Wolf [49] begins by computing probabilities of short sequences of events from the given event stream (training data) to build an event-sequence probability table. Each cell represents the average probability of a future event (column) with the current events (row). The table is then used to generate an automaton (FSM) that accepts only sequences whose probabilities of occurrence are higher than a user-identified threshold. It is worth mentioning that the proposed method by Cook and Wolf [49] does not rely on the state-merge strategy that builds a PTA and recursively merges states. The idea of the Markov method proceeds as follows.

- First, the probability tables of event sequences are built from the event stream (trace). It is achieved by tallying occurrence and computing the probabilities of sub-sequences. In [49], the first-order and second-order probability tables are obtained. For example, consider the following event stream (trace): \((\text{Load, Edit, Edit, Edit, Close, Load, Close, Load, Edit, Save, Edit, Save, Edit, Save, Exit, Load, Edit, Edit, Close, Load, Close, Load, Edit, Save})\) as an illustration. Table 4.1 shows the first-order and second-order probability table obtained from the above event stream.

- Second, the directed event graph is built from the first-order probability table. Each unique event (an element of alphabet) corresponds to a vertex (node) in the directed graph. For each event sequence of length \(n + 1\) (the order of the Markov model plus one) whose probability exceeds the user-specified threshold, an edge with a unique label is created from an event in the sequence to the following event in the same sequence.

**Example 4.1.** Let us consider the event sequence \(\langle \text{Load, Edit} \rangle\), which has a probability of 0.66 according to the first-order table. For a probability threshold \(\leq 0.1\), an edge is made from node \text{Load} to node \text{Edit} in the event graph. Figure 4.1 illustrates the event graph that is generated from the first-order table.
Chapter 4. Improvement of EDSM Inference Using Markov Models

<table>
<thead>
<tr>
<th>Current state</th>
<th>Load</th>
<th>Edit</th>
<th>Close</th>
<th>Save</th>
<th>Exit</th>
</tr>
</thead>
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</table>

Table 4.1: The First- and Second-order probability table of text editor example

![Event graph](image)

Figure 4.1: The event graph generated from the first-order table
4.3 The Proposed Markov Models

In this section the proposed Markov model (ML) is described. It relies on predicting one element of alphabet $\sigma \in \Sigma$ depending on the previous $k$ elements of alphabet.

As described in the previous section, Cook and Wolf [49] mapped entries of the Markov table into probabilities reflecting how frequent short event sequences of a specific length appeared in the training data. They [49] used a cut-off threshold to avoid noise in training data to identify the most probable event sequences.

In this thesis, the assumption is that training data is very sparse, and it is hard to use a non-zero threshold such as that used by Cook and Wolf [49] to identify the most probable sequences since there is no noise in training data. Moreover, sequence of events with low frequencies cannot be ignored because they can be indicators of valid predictions. Therefore, predictions are based on the presence or absence of specific sequences rather than the number of times they are observed.

4.3.1 Building the Markov Table

This section describes the way of training Markov model. It begins by creating the event-sequence table. In general, the event-sequence table is constructed in the same way as proposed by Cook and Wolf [49]. However, the entries in the event-sequence table are boolean values to denote whether an event is permitted or prohibited to follow sequences.

The process of building the Markov table ($MT$) initially requires a sample of positive and possibly few negative traces similar to those that feed into any state-merging technique. Each trace is a sequence of alphabet elements representing a sequence of events. After that, the construction of the $MT$ is performed by choosing a prefix length $k$ and recording elements of an alphabet (events) following a subsequence of length $k$ in any of the traces in the training data. Hence, $k$ can be seen as the order of the Markov model.

Given a training sequence $\sigma_1, \sigma_2, \ldots, \sigma_n$, one looks at subsequences $\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}$ and records them as pairs of two elements. The first part in the pair is called the prefix sequence of the current subsequence of length $k$ over $\Sigma^*$. The second part is a suffix which is an element of alphabet $\sigma \in \Sigma$. 
Table 4.2: The First- and Second-order event-sequence table of text editor example

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<thead>
<tr>
<th>Current state</th>
<th>Load</th>
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<th>Save</th>
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The next step is to record a pair \(\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k} \rangle\) as a *positive* if \(\sigma_{i+k}\) is a permitted event to follow the prefix sequence \(\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle\), a *negative* if it is not, and a *failure*, if for the same prefix sequence, both a positive and a negative occurrence of the same event were observed. Since the focus is on inference of LTSs which recognise prefix-closed languages, the only case where \(\sigma_{i+k}\) is a *negative* is if it is at the end of a trace from negative traces \(S^-\). For the purpose of predictions, failure entries are ignored. Definition 4.1 defined the \(MT\) table in the proposed \(ML\) method.

**Definition 4.1.** Let \(Markov = \{Pos, Neg, Fail\}\) be possible entries of Markov table.
A MT is mapping $MT : \Sigma^k \times \Sigma \mapsto \text{Markov}$. The domain of the MT function is given by $\text{dom}(MT) = \Sigma^k \times \Sigma$. The outcome from the Markov table for a pair $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \in \text{dom}(MT)$ is given by $MT((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$.

A Markov prediction is a label (an element of alphabet) that the trained Markov model suggested either to follow or not to follow a sequence $\sigma \in \Sigma^k$. In terms of execution traces, a prediction is a function or a method name that is either predicted to be called after invoking sequences of methods, or prohibited to after them. From Definition 4.1, we say that a label (an element of alphabet) $\sigma_{i+k}$ is predicted as permitted to follow $\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle$ if $MT(\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle, \sigma_{i+k}) = \text{Pos}$. On the other hand, a label (an element of alphabet) $\sigma_{i+k}$ is predicted as prohibited to follow $\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle$ if $MT(\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle, \sigma_{i+k}) = \text{Neg}$.

Algorithm 9 describes the process of building the Markov table from both positive and negative traces. The $\text{obtainSubsequence}$ is responsible for splitting a sequence into subsequences of elements of length $k + 1$. For example, for a trace $\sigma_1, \sigma_2, \sigma_3, \sigma_3, \sigma_5$, consider that $k = 2$ and $i = 1$; the $\sigma_1, \sigma_2, \sigma_3$ subsequence is returned. The process of constructing the Markov table begins with the positive sequences before the negative ones. The process of building the Markov table is terminated when all traces have been processed. It is important to highlight that $\oplus$ denote the override process on table entries.
Chapter 4. Improvement of EDSM Inference Using Markov Models

4.3.2 Markov Predictions for a Given State

This section describes the way of collecting Markov predictions for a given state \( q \in Q \) based on a given sequence \( \Sigma^k \) of length \( k \). Let \( \text{prefixpaths}(A, k, q) \) be a function to return a set of all paths of length \( k \) leading to state \( q \). The prefix path is formally defined

\[
\text{prefixpaths}(A, k, q) = \{ \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k} \mid \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k} \in A, i = 1, 2, \ldots, n-k \text{ and } A^i = q \}.
\]

The algorithm to construct the Markov table is as follows:

**Input:** \( S^+ \) and \( S^- \)

\( S^+ \) is the set of positive sequences, \( S^- \) is the set of negative sequences

**Result:** \( MT \)

\( MT \) is the Markov table

// Declare the prefix length \( k \)

Declare: \( k \leftarrow \text{Integer} \)

1. for For each positive sequence \( PosSeq \in S^+ \) of length \( n \) do
   2. for \( i = 1 \cdots n \) do
      3. \( \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k} \leftarrow \text{obtainSubsequence}(PosSeq, i, k); \)
      4. if \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \notin \text{dom}(MT) \) then
         5. Record a pair \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \) in \( MT \) as a positive subsequence.
         6. \( MT = MT \oplus ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}), \text{Pos}) \)
      7. end
   8. end
9. end

10. for For each negative sequence \( NeqSeq \in S^- \) of length \( n \) do
   11. for \( i = 1 \rightarrow n \) do
      12. \( \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k} \leftarrow \text{obtainSubsequence}(NeqSeq, i, k); \)
      13. if \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \notin \text{dom}(MT) \) then
         14. if \( i + k = n \) then
            15. Record a pair \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k}, \sigma_{i+k}) \) in \( MT \) as a negative subsequence.
            16. \( MT = MT \oplus ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}), \text{Neg}) \)
         17. else
            18. Record a pair \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \) in \( MT \) as a positive subsequence.
            19. \( MT = MT \oplus ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}), \text{Pos}) \)
         20. end
      21. else
         22. if \( i + k = n \) and \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \rightarrow true \in MT \) then
            23. Update a pair \( (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \) in \( MT \) as a failure subsequence.
            24. \( MT = MT \oplus ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}), \text{Fail}) \)
         25. end
      26. end
   27. end
28. end

return \( MT \)

**Algorithm 9:** The Construct-Markov algorithm
in Definition 4.2. It differs from the prefix sequence that takes a path from the initial state to \( q \).

**Definition 4.2.** Given a state \( q \in Q \) and LTS \( A \). The set of prefix paths that lead to \( q \) is given by: 
\[
\text{prefixpaths}(A, k, q) = \{ \sigma \in \Sigma^k \mid \exists q' \in Q. \delta(q', \sigma) = q \}
\]

**Definition 4.3.** Given a sequence of length \( k \) (prefix path), denoted by \( pr \) over \( \Sigma \) and the Markov table \( MT \). The set of permitted predictions for a given \( q \) after \( pr \) is given by: 
\[
ML_{\text{permitted}}(q, pr) = \{ \sigma \in \Sigma \mid MT(pr, \sigma) = \text{pos}, (pr, \sigma) \in \text{dom}(MT) \}
\]

**Definition 4.4.** Given a sequence of length \( k \) (prefix path), denoted by \( pr \) over \( \Sigma \) and the Markov table \( MT \). The set of prohibited predictions for a given \( q \) after \( pr \) is given by: 
\[
ML_{\text{prohibited}}(q, pr) = \{ \sigma \in \Sigma \mid MT(pr, \sigma) = \text{Neg}, (pr, \sigma) \in \text{dom}(MT) \}
\]

### 4.3.3 The Precision and Recall of the Markov Model

This section describes how to check the correctness of the trained Markov model with respect to the reference LTS model in terms of their outgoing transitions. It begins with definitions related to obtaining Markov predictions for a given state \( q \in Q \).

### 4.3.4 Definitions of Precision and Recall for Markov Models

For computing the precision and recall purposes, Definition 4.5 suggested that an element of alphabet \( \sigma \) is returned as permitted to follow the given state \( q \) on the following condition. If \( \sigma \) is predicted as permitted to follow \( q \) for at least one prefix path and there does not exist another prefix path suggesting that \( \sigma \) is prohibited to follow \( q \).

**Definition 4.5.** Given an element of alphabet \( \sigma \in \Sigma \), a state \( q \in Q \), Markov table (MT), prefix length \( k \) and LTS \( A \). We say that \( \sigma \) is returned as permitted prediction if:

1. \((\exists pr \in \text{prefixpaths}(A, k, q), \sigma \in ML_{\text{permitted}}(q, pr)) \land (\nexists pr' \in \text{prefixpaths}(A, k, q), \sigma \in ML_{\text{prohibited}}(q, pr'))\)

Definition 4.6 stated that an element of alphabet \( \sigma \) is returned as prohibited to follow the given state \( q \) on the following condition. If \( \sigma \) is predicted as prohibited to follow \( q \) for at least one prefix path and there does not exist another prefix path suggesting that \( \sigma \) is permitted to follow \( q \).
Definition 4.6. Given an element of alphabet $\sigma \in \Sigma$, a state $q \in Q$, Markov table (MT), prefix length $k$ and LTS $A$. We say that $\sigma$ is returned as prohibited prediction:

1. $(\exists pr \in \text{prefixpaths}(A, k, q), \sigma \in ML_{\text{prohibited}}(q, pr)) \land (\not\exists pr' \in \text{prefixpaths}(A, k, q), \sigma \in ML_{\text{permitted}}(q, pr'))$

The precision and recall metric is computed with respect to the outgoing transitions of reference LTSs. It is important to determine which of the returned predictions are relevant. Definition 4.7 shows how accurate (relevant) the returned predictions are with respect to the outgoing transitions in the reference LTS. A returned prediction $\sigma$ is said to be relevant in two cases. First, if $a$ is predicted as permitted to follow a given state $q$ and there is a transition labelled with $\sigma$ lead to $q' \in Q$. Second, if $a$ is predicted as prohibited to follow a given state $q$ and there does not exist a transition labelled with $\sigma$ lead to $q' \notin Q$.

It is worth-mentioning that all states in an LTS are accepted as described in Chapter 2

Definition 4.7. Given a state $q \in Q$ and LTS $A$. A returned Markov prediction from the $q$ is relevant if any of the following conditions are satisfied.

1. A returned prediction satisfying conditions in Definition 4.5, which element of alphabet $\sigma$ is said relevant as permitted from the given state $q$ in the reference LTS if:
   \[ \exists q' \in Q \text{ such that } \delta(q, \sigma) = q' \text{ and } MT(pr, \sigma) = Pos. \]

2. A returned prediction satisfying conditions in Definition 4.6, which element of alphabet $\sigma$ is said relevant as prohibited from the given state $q$ in the reference LTS if:
   \[ \delta(q, \sigma) = \emptyset \text{ and } MT(pr, \sigma) = Neg. \]

It is important to highlight that the returned and relevant predictions that are defined in this section are used to compute the precision and recall scores in the following sections.

4.3.5 Markov Precision and Recall

This section introduces the precision and recall metrics that are computed to measure the correctness of Markov model. Therefore, predictions made by the trained Markov models are measured against the transitions in the target reference graph. The correctness of Markov models are measured using the precision and recall metrics that are designed to
cope with predictions. This is inspired by the use of precision and recall metrics in the information retrieval context.

In an information retrieval context, the precision is defined as the proportion of retrieved documents that are relevant [99]. The precision of the Markov model can be defined as the proportion of returned predictions that are relevant (see Definition 4.7). It is used to evaluate how accurate the returned predictions by the Markov model are.

The precision summarizes the exactness of the predictions. A high precision score means that the trained Markov model captures the dependencies between events in the traces well.

\[ \text{Precision} = \frac{\mid \text{returned predictions} \cap \text{relevant predictions} \mid}{\mid \text{returned predictions} \mid} \quad (4.1) \]

Additionally, in an information retrieval context, the recall is defined as the proportion of relevant documents that are returned [99]. The recall of the trained Markov models is defined as the proportion of labels of outgoing transitions in the reference graph that are predicted correctly. It is introduced to measure how complete labels of the outgoing transitions in the reference graph are predicted correctly. Furthermore, the recall summarizes the completeness of predictions.

\[ \text{Recall} = \frac{\mid \text{returned predictions} \cap \text{relevant predictions} \mid}{\mid \text{relevant labels} \mid} \quad (4.2) \]

**Example 4.2.** Consider the reference LTS of the text editor shown in Figure 4.2. Table 4.3 illustrates the MT that was built from the following positive traces: \( S^+ = \{ \langle \text{Load, Edit, Close, Load, Edit, Edit, Edit} \rangle, \langle \text{Load, Edit, Close, Load, Close, Load, Edit} \rangle, \langle \text{Load, Close, Load, Close} \rangle \} \) where \( k = 1 \). Label Save is not predicted from the D state since the trained Markov model did not observe the subsequence \( \langle \text{Edit, Save} \rangle \). Moreover, label Exit is not predicted from A, B, and D states. Other transitions are predicted correctly. The precision is given by \( \frac{5}{5} = 1 \), and the recall is given by \( \frac{5}{9} = 0.55 \). In this example, no extra wrong predictions are made from each state. However, four relevant labels of outgoing transition are not predicted.
4.4 EDSM-Markov

This section introduces *EDSM-Markov*, a passive inference method that relies on the proposed Markov model. The idea behind using the Markov model is that whenever the resulting state machine introduces contradictions with the trained Markov models, they are recorded as inconsistencies. Those inconsistencies can be seen as introducing new behaviours that are not permitted by the software system being inferred.

This section describes the notion of inconsistencies in the idea of state merging. It begins by introducing the inconsistency score that is computed for a given state in an automaton as described in Section 4.4.1. The adaptation of the inconsistency score to the *EDSM* algorithm results in a new LTS learner called *EDSM-Markov*; this is described in Section 4.4.4.

### 4.4.1 Inconsistency Score (*Incons*)

In this section, the Markov model that is described above in Section 4.3 is used to compute what are called inconsistencies for a given state either in the current automata during inference or for an LTS model. The inconsistency score is defined as the number of
contradictions between the existing labels of the outgoing transitions against corresponding predictions, for a given state or for all states in an LTS.

### 4.4.1.1 Inconsistency Score for a Specific State

In this section, the process of finding and counting the inconsistency score for a given state is described. The inconsistency score for a given state is computed with respect to its outgoing transitions. It begins by collecting the set of prefix paths for a given state $\text{prefixpaths}(A, k, q)$ as described in Definition 4.2. Then, a set of labels of outgoing transitions are obtained, denoted $\Sigma_q^{out}$ and defined in Definition 2.3.

A label of an outgoing transition $\sigma \in \Sigma_q^{out}$ is said to be *consistent* if there is a match between the outgoing transition and the corresponding prediction as defined in Definition 4.8.

**Definition 4.8.** Given a state $q \in Q$, prefix length $k$, automaton $A$, a prefix path $pr \in \text{prefixpaths}(A, k, q)$, a label of outgoing transition $\sigma$, and the trained Markov model $MT$. An element of alphabet $\sigma$ is said to be *consistent* based on Markov predictions for a prefix path $pr$ if:

1. $(\exists q' \in F^+. \delta(q, \sigma) = q' \land \sigma \in ML_{\text{permitted}}(q, pr)) \lor (\exists q'' \in F^-. \delta(q, \sigma) = q'' \land \sigma \in ML_{\text{prohibited}}(q, pr))$

From Definition 4.8, a label of outgoing transition $\sigma \in \Sigma_q^{out}$ is said to be *consistent* with Markov predictions in two cases. First, if there is a transition labelled with $\sigma$ that leads to an accepting state from a state $q$ and the $\sigma$ is predicted as permitted to follow $pr$. The first case if the provided automaton is LTS or PTA. Second, if $\sigma$ is predicted by the Markov model as prohibited after $pr$ and there is an outgoing transition labelled with $\sigma$ leaving a state $q$ leading to a rejecting state $q' \in F^-$. The second case is included alongside the first case if the learner infers PLTS from positive and negative traces. Otherwise, $\sigma$ is said to be *inconsistent* for the current prefix path $pr \in \text{prefixpaths}(A, k, q)$.

Algorithm 10 summarized the process of computation of the inconsistency score for a given state $q \in Q$ in the current automaton $A$, and this score is denoted by $(\text{Incons}_q)$. It begins by collecting a set of all prefix paths of length $k$ leading to state $q$. The $\text{prefixpaths}(A, q, k)$ function in Algorithm 10 returns all prefix paths. For example, for
the automaton $A$ and state $B$ that is illustrated in Figure 4.3, the set of prefix path of length 1 is $\text{prefixpaths}(A, B, 1) = \{(\text{Load}), (\text{Close})\}$ and the set of prefix path of length 2 contains $\text{prefixpaths}(A, B, 2) = \{(\text{Load, Close}), (\text{Close, Close})\}$.

![Figure 4.3: Example of computing $Incons_q$](image)

**Algorithm 10:** The computation of inconsistency for a given state $q$

```plaintext
Input: $q$ and $k$
/* $S^+$ is the current state, $k$ is the prefix length $k$ */

Result: $Incons_q$
// $Incons_q$ is the number of inconsistencies

1 $Incons_q \leftarrow 0$;
2 for $pr \in \text{prefixpaths}(A, q, k)$ of length $k$ do
3     for $\sigma \in \Sigma^\text{out}_q$ do
4         if checkConsistencies($pr, \sigma, \text{MT}$) is false then
5             $\text{incons} \leftarrow \text{incons} \cup (pr, \sigma)$;
6         else
7             $\text{cons} \leftarrow \text{cons} \cup (pr, \sigma)$;
8         end
9     end
10 end
11 return $Incons_q \leftarrow |\text{incons}|$
```

The second step during the computation of the $Incons_q$ score is to check the consistency of each element in $\Sigma^\text{out}_q$ against Markov predictions. The aim is to find inconsistencies (contradictions) between the outgoing transitions and the corresponding predictions. Given a state $q$, the computation of the $Incons_q$ score is achieved by iterating through the set of prefix path leading to $q$, and checking the consistency of an element of alphabet $\sigma \in \Sigma^\text{out}_q$ for the current $pr$. It classifies each given prefix path and each label in $\Sigma^\text{out}_q$ into the inconsistent ($\text{incons}$) and consistent ($\text{cons}$) sets as shown in Table 4.4. If a label of an outgoing transition $\sigma \in \Sigma^\text{out}_q$ is consistent with respect to a prefix path $pr \in \text{prefixpaths}(A, q, k)$,
then the pair \((pr, \sigma)\) is added to the \emph{cons} set. Otherwise, it is added to the \emph{incons} set. It is important to highlight that the consistency (matching) of the label of outgoing transition \(\sigma\) with predictions is determined as described in the previous section (see Definition 4.8). The \(\text{Incons}_q\) score is the number of pairs that are added to the \emph{incons} set after the classification process.

<table>
<thead>
<tr>
<th>(pr \in \text{PrefixPaths}(A,q,k))</th>
<th>(\sigma \in \text{ML}_{\text{permitted}}(pr))</th>
<th>(\sigma \in \text{ML}_{\text{prohibited}}(pr))</th>
<th>((pr, \sigma) \notin \text{dom}(MT))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma \in \Sigma_{\text{out}}^+ \mid q' = \delta(q,\sigma), q' \in F^+)</td>
<td>\text{cons}</td>
<td>\text{incons}</td>
<td>\text{incons}</td>
</tr>
<tr>
<td>(\sigma \in \Sigma_{\text{out}}^- \mid q' = \delta(q,\sigma), q' \in F^-)</td>
<td>\text{incons}</td>
<td>\text{cons}</td>
<td>\text{incons}</td>
</tr>
</tbody>
</table>

**Table 4.4: Classification of inconsistency**

<table>
<thead>
<tr>
<th></th>
<th>Load</th>
<th>Edit</th>
<th>Save</th>
<th>Close</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load, Load</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Load, Edit</td>
<td>-</td>
<td>Pos</td>
<td>Pos</td>
<td>-</td>
</tr>
<tr>
<td>Load, Save</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Load, Close</td>
<td>Pos</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Edit, Load</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Edit, Edit</td>
<td>-</td>
<td>-</td>
<td>Pos</td>
<td>-</td>
</tr>
<tr>
<td>Edit, Save</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Pos</td>
</tr>
<tr>
<td>Edit, Close</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Save, Load</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Save, Edit</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Save, Save</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Save, Close</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Close, Load</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Close, Edit</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Close, Save</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Close, Close</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 4.5: The Markov Table where \(k = 2\)**

**Example 4.3.** Let us again consider the automaton shown in Figure 4.3, and the \(MT\) illustrated in Table 4.5 where \(k = 2\). The \(MT\) is built from the following positive traces: \(S^+ = \{\langle\text{Load},\text{Edit},\text{Edit},\text{Save},\text{Close}\rangle, \langle\text{Load},\text{Edit},\text{Save},\text{Close}\rangle, \langle\text{Load},\text{Close, Load}\rangle\}\). The set of labels of outgoing transitions of state \(B\) (\(\Sigma_{B}^{\text{out}}\)) contains the following labels: \{Load, Edit, Close\}. The \(\text{prefixpaths}(A,B,2)\) function returns the following prefix paths: \{\langle\text{Load, Close}\rangle, \langle\text{Close, Close}\rangle\}. The next step is to check consistency for each label in the \(\Sigma_{B}^{\text{out}}\) set against Markov predictions for each prefix path in \(\text{prefixpaths}(A,B,2)\). For instance, the \textit{Load} label is predicted as permitted to follow \(\langle\text{Load, Close}\rangle\) based on the Markov table that
is illustrated in Table 4.5. Thus, the Load label is considered consistent since there is an outgoing transition that leads to the accepting state labelled with Z. However, the Load label is not predicted after the prefix path \( \langle \text{Close, Close} \rangle \), and this is considered as inconsistency. Table 4.6 shows the inconsistency classification for each label of the outgoing transitions for the B state where the prefix path is \( \langle \text{Load, Close} \rangle \).

<table>
<thead>
<tr>
<th>( \langle \text{Load, Close} \rangle \in \text{collectPrefixPaths}(A,q,k) )</th>
<th>Markov Table (MT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma \in \text{ML}_{\text{permitted}}(\langle \text{Load, Close} \rangle) )</td>
<td>( \sigma \in \text{ML}_{\text{permitted}}(\langle \text{Load, Close} \rangle) )</td>
</tr>
<tr>
<td>Load ( \in \Sigma^* ) ( \delta(B, \sigma) = K \in F^+ )</td>
<td>cons</td>
</tr>
<tr>
<td>Edit ( \in \Sigma^* ) ( \delta(B, \sigma) = C \in F^+ )</td>
<td>-</td>
</tr>
<tr>
<td>Close ( \in \Sigma^* ) ( \delta(B, \sigma) = B \in F^+ )</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.6: Classification of inconsistency for the prefix path \( \langle \text{Load, Close} \rangle \) and state B

### 4.4.1.2 Inconsistency Score for an Automaton

The computation of Incons for an automaton is described formally in Definition 4.9, where \( \text{Incons}(A, MT, q) \) is the function that returns the inconsistency score for a given state and it is computed as described in the previous section.

**Definition 4.9.**

\[
\text{Incons}(A, MT) = \sum_{q \in Q_A} \text{Incons}(A, MT, q)
\]

### 4.4.2 Inconsistency Heuristic for State Merging

This section introduced the idea of incorporating the computation of inconsistency during the state-merging process. It is known that the EDSM heuristic is concerned with the amount of evidence suggesting that the pair of states are equivalent. Instead of focusing on computing the agreement evidence between a pair of states, one would compute the disagreement between them to measure how likely it is that the states are different. Since negative sequences are usually missing or not sufficient to prevent over-generalizing during the state-merging process, the computation of \( \text{Incons}(A, MT, q) \) described in the previous section can be used to determine whether a specific transition matches predictions or not from a particular state. In other words, inconsistencies that may appear as a result of merging states imply that the current hypothesis (LTS) accepts sequences of elements of length \( k + 1 \) where the Markov model does not, and vice versa.
This section presents an inconsistency score that is computed for a given pair of states during the state-merging process and this is denoted by $Im$. The $Im$ score is computing by taking the difference of two inconsistency scores. The first inconsistency is computed for a merged automaton and the second one for the automaton before merging the states. Given two states $q$ and $q'$ that are chosen for merging, the inconsistency $Im$ score is obtained by first computing the inconsistency of the current automaton, denoted by $Incons(A,MT)$. Where $q$ and $q'$ of $A$ are merged, a new inconsistency of the merged automaton $Incons(merge(A,q,q'),MT)$ is computed. The inconsistency score for a given pair of states $Im$ is obtained as follows:

$$Im = Incons(merge(A,q,q'),MT) - Incons(A,MT)$$  \hspace{1cm} (4.3)

The intention behind computing $Im$ for a given pair of states is to determine how many inconsistencies resulted from merging the states. It is important to highlight the fact that the $merge(A,q,q')$ function in Equation 4.3 is the merging procedure as described in Section 3.1.4. Moreover, the computation of the inconsistency for the current automaton $Incons(A,MT)$ is necessary to isolate the inconsistencies observed as a result of previous mergers from those detected from computing the current merger of states. Example 4.4 and 4.5 below illustrates the way of computing the $Im$ score.

**Example 4.4.** Consider a PTA of the text editor example that is shown in Figure 4.4 and assume that states $B$ and $C$ are chosen to be merged by a learner. In this way, the process of computation of $Im$ includes building the Markov table and getting predictions as described in Section 4.3. The merging process results in an LTS as shown in Figure 4.5 where a transition that is labelled with $close$ from $BCG$ state is not predicted after the following prefix path: $⟨Edit⟩$. This is considered as an inconsistency of merging $B$ and $C$ states. In addition, another inconsistency that occurred as a consequence of merging $B$ state with $C$ is that the outgoing transition labelled with $Save$ leaving $BCG$ state is not predicted by $MT$ after the following prefix sequence: $⟨Load⟩$. Hence, merging of $B$ and $C$ states resulted in an $Im$ score of two.
Example 4.5. Let us again consider the PTA that is illustrated in 4.4 and assume that state $K$ is chosen to merge with state $D$. The merged automaton is shown in Figure 4.6. The set of prefix paths of length $k = 1$ leading to the $DK$ state contains the following paths: $\{\langle Load \rangle, \langle Close \rangle \}$. The set of labels of outgoing transitions leaving the $DK$ state contains the following: $\Sigma_{out}^{DK} = \{Load, Edit\}$. The outgoing transition that is labelled with $Edit \in \Sigma_{out}^{DK}$ is predicted by the Markov model as permitted to follow, denoted by $MT(\langle Load \rangle, Edit) = pos$, and this is considered as an inconsistency, since a transition that is labelled with $Edit$ leads to the $N$ rejecting state from the $DK$ state.

In Example 4.5, states $D$ and $K$ in Figure 4.4 can be merged using the EDSM learner since both are accepting states and there is no outgoing transitions from $K$ state in order to check the acceptance condition (Definition 2.9).

To sum up, the benefit of considering $Incons$ during the state-merging process is to avoid bad mergers. Moreover, if merging of a pair of states $(q, q')$ leads to an automaton where
one or more outgoing transitions from $q, q'$ are not predicted by the trained Markov model, this is not considered good if there is another possible pair of states such that all outgoing transitions from the pair match predictions. In other words, merging a pair of states among other possible pairs is considered to be a good decision if the merger leads to an automaton with the maximum matching of outgoing transitions with the predicted ones. The next section describes the way of incorporating the Incons value for each merger of states with the EDSM heuristic in order to infer good LTSs.

### 4.4.3 EDSM-Inconsistency Heuristic

The proposed $EDSM$-$inconsistency$ heuristic is introduced to demonstrate that the $EDSM$ algorithm can benefit from the computation of inconsistencies. The purpose of the $EDSM$-$inconsistency$ heuristic is to prevent merging of inconsistent pairs of states and to rank them using both $EDSM$ scores and inconsistencies. A pair with a high $EDSM$ score and low inconsistency score is considered to be the most likely pair to be merged. The presented heuristic approach is to compute an $EDSM$ score of a pair of states and subtract the inconsistency score $Im$ from it.

The computation of scores of pairs following the $EDSM$-$inconsistency$ heuristic is described in Definition 4.10.

**Definition 4.10.**

$$I\text{Score}(A, q, q', MT) = edsmScore(A, q, q') - (Incons(merge(A, q, q'), MT) - Incons(A, MT))$$
**Definition 4.11.**

\[ IScore(A, q, q', MT) = edsmScore(A, q, q') - Im \]

The score of the *EDSM-inconsistency* can be simply obtained via \( edsmScore(A, q, q') - Im \). For example, consider a merged automaton shown in Figure 4.5 resulting from merging \( B, C \) states. In this case, the *EDSM* score is 4 since the number of states in the automaton has dropped from 11 to 7. The inconsistency score of the automaton before merging states denoted \( Incons(A, MT) \) is 0 since the Markov model is trained from the original PTA and no mergers have been performed yet. After the merging of \( B \) and \( C \), the inconsistency score of merged graph \( Incons(merge(A, q, q')) \) is 2 since the outgoing transition labelled with *Save* is not predicted after the prefix path \( \langle Load \rangle \) and the same for the outgoing transition labelled with *Close* and the prefix path \( \langle Edit \rangle \). The final inconsistency score \( Im \) is 2. So, the *EDSM-inconsistency* score is 2 in this example, because it is given by subtracting the \( Im \) score from *EDSM*.

**Definition 4.12.**

\[ IScore(A, q, q', MT) = edsmScore(A, q, q') - (Im \times Incon) \]

We ran an experiment to vary the *Im* score by introducing the multiplier \( Incon \). Hence, the \( IScore(A, q, q', MT) \) score that is introduced in Definition 4.11 is rewritten as shown in Definition 4.12. The idea behind this experiment is to study the influence of the *Im* score compared to the *EDSM* score, justifying why those two heuristics are combined.

In the conducted experiment, random LTSs were generated using the Forest Fire algorithm that is described in Section 2.6. The number of states ranged between 10 and 40 in steps of 10. Fifteen LTSs were generated for each selected number of states. Hence, the total number of random LTSs is 4 steps \( \times \) 15 = 60 LTSs. For each LTS, 5 sets of training data were generated, bringing the number of LTSs learnt per experiment to 300. In addition, the randomly generated LTSs were connected, and had an alphabet size two times the number of states. Moreover, the influence of \( Incon \) were studied with different numbers of traces where it ranged from 1 to 7 traces, incrementing by 2. The length of traces is given by \(|Q| \times |\Sigma| = (2 \times |Q|^2)\). This figure is loosely motivated by the size of a *transition*.
cover set, which is the number of sequences to reach every state of an LTS and attempt every input in it.

The boxplots of the BCR scores obtained by EDSM-Markov for all various inconsistency multiplier $Incon$ considered are illustrated in Figure 4.7. The BCR scores attained by the EDSM-Markov learner are high when $Incon = 1$ compared to other settings of $Incon$. However, it is clear that the performance of EDSM-Markov learner tend to be worse when $Incon > 1$. The reason behind this is that the following expression ($Im \times Incon$) will exceed EDSM scores and forcing the EDSM-Markov learner to block mergers. Hence, the inferred models will be under-generalized.

![Boxplots of BCR scores obtained by EDSM-Markov for different inconsistency multiplier $Incon$.](image)

**Figure 4.7:** BCR scores obtained by EDSM-Markov for different inconsistency multiplier $Incon$

The boxplots of the structural-similarity scores obtained by EDSM-Markov for different settings of $Incon$ are depicted in Figure 4.8. With $Incon = 1$, the structural-similarity scores achieved by EDSM-Markov are the highest compared to other settings of $Incon$. As can be seen in Figure 4.8, the structural-similarity scores attained by EDSM-Markov are affected by different settings of $Incon$. The structural-similarity scores are decreased dramatically when $Incon > 1.0$ where the inferred models are under-generalized.
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Figure 4.8: Structural-similarity scores obtained by EDSM-Markov for different inconsistency multiplier $Incon$

Two examples of computing the $IScore$ score for different pairs of states compared to the EDSM score are shown below with an assumption that the length of $k$ was 1. They demonstrate the benefit of computing the EDSM-inconsistency score to block some mergers.

Example 4.6. Suppose the EDSM learner attempts to merge the $D$ state with the $Z$ state that is coloured blue as illustrated in Figure 4.9(a), the EDSM assigns a score of 1 since merging $D$ and $Z$ leads to a reduction of the number of states by 1, as shown in Figure 4.9(b). On the other hand, the EDSM-inconsistency heuristic first computes the inconsistency score $Incons(A, MT)$ based on the current PTA that is shown in Figure 4.9(a), and this yields $Incons(A, MT) = 0$ because no states were previously merged. The EDSM-inconsistency heuristic then measures how much inconsistency appears if the learner merges $D$ and $Z$. The inconsistency score on the merged graph shown in Figure 4.9(b) is 2 (denoted by $Incons(merge(A, D, Z), MT) = 2$) because a label close is not predicted after $⟨Close, Close⟩$ incrementing the inconsistency score by 1; the path $⟨Load, Edit⟩$ leads to $N$ rejecting state contradicting the Markov predictor that suggests that the path must lead to an accepting state and this raises the inconsistency score to 2. In this case, $Im = Incons(merge(A, D, Z), MT) - Incons(A, MT) = 2$. The EDSM-inconsistency
score is then computed as defined in Definition 4.10 and this yields \( IScore = -1 \) computed as follows: 
\[
IScore(A, D, Z, MT) = edsmScore(A, D, Z) - (Incons(merge(A, D, Z), MT) - Incons(A, MT)) = 1 - (2 - 0) = -1.
\]

\[\begin{array}{c}
A \quad B \quad D \\
\end{array}\]
\[\begin{array}{c}
N \\
\end{array}\]
\[\begin{array}{c}
Z \\
W \\
\end{array}\]
(a) before merging \( D \) and \( Z \)

\[\begin{array}{c}
A \quad B \quad D \quad Z \\
\end{array}\]
\[\begin{array}{c}
N \\
W \\
\end{array}\]
(b) after merging \( D \) and \( Z \)

**Figure 4.9:** The first example of inconsistency score computation

**Example 4.7.** The second example is shown in Figure 4.10(a). Suppose that the EDSM learner tries to merge \( A \) and \( D \) since both are accepting states, the EDSM score is 3 because merging them results in an automaton that is depicted in Figure 4.10(b) where the number of states is reduced from 6 to 3. In this example, the EDSM-inconsistency score agrees with the EDSM score since the following paths \{⟨Load, Close⟩, ⟨Close, Load⟩, ⟨Close, Edit⟩\} obtained from the merged PTA that is shown in Figure 4.10(a) match predicted paths.

\[\begin{array}{c}
A \quad B \quad D \\
\end{array}\]
\[\begin{array}{c}
N \\
\end{array}\]
\[\begin{array}{c}
Z \\
W \\
\end{array}\]
(a) before merging \( A \) and \( D \)

\[\begin{array}{c}
A \quad B \quad D \\
\end{array}\]
\[\begin{array}{c}
N \\
\end{array}\]
\[\begin{array}{c}
ADW \\
BZ \\
\end{array}\]
(b) after merging \( A \) and \( D \)

**Figure 4.10:** The second example of inconsistency score computation

### 4.4.4 EDSM-Markov Inference Algorithm

The inference process of an LTS using the EDSM-Markov method is described in Algorithm 11. The inference process starts by constructing a PTA from the provided positive samples of input sequences or an APTA if there are negative ones, and is denoted by the \( \text{generatePTA}(S^+, S^-) \) function in Line 1. After that, the \( \text{TrainMarkovModel}(S^+, S^-) \)
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The set of red states $R$ is initialized with the root state.

```
input: $S^+, S^-$  /* Sets of accepted $S^+$ and rejected $S^-$ sequences */
result: $A$ is an LTS that is compatible with $S^+$ and $S^-$

Data: $A$, $MT$, $R$, $B$, $PossiblePairs$

1 $A \leftarrow \text{generatePTA}(S^+, S^-);$  
2 $MT \leftarrow \text{TrainMarkovModel}(S^+, S^-);$  
3 $R \leftarrow \{q_0\};$  // $R$ is a set of red states  
4 do  
5 do  
6 $PossiblePairs \leftarrow \emptyset;$  // $PossiblePairs$ possible pairs to merge  
7 $R_{\text{extended}} \leftarrow \text{false};$  
8 $B \leftarrow \text{ComputeBlue}(A, R);$  // $B$ is a set of blue states  
9 for $q_b \in B$ do  
10 $mergeable \leftarrow \text{false};$  
11 for $q_r \in R$ do  
12 $IScore \leftarrow \text{ComputeIScore}(A, q_r, q_b, MT);$  
13 if $IScore \geq 0$ then  
14 $PossiblePairs \leftarrow PossiblePairs \cup \{(q_r, q_b)\};$  
15 $mergeable \leftarrow \text{true};$  
16 end  
17 end  
18 if $mergeable = \text{false}$ then  
19 $R \leftarrow R \cup \{q_b\};$  
20 $R_{\text{extended}} \leftarrow \text{true};$  
21 end  
22 end  
23 while $R_{\text{extended}} = \text{true};$  
24 if $PossiblePairs \neq \emptyset$ then  
25 $PairToMerge \leftarrow \text{PickPair}(PossiblePairs);$  
26 $A \leftarrow \text{Merge}(PairToMerge);$  
27 end  
28 while $PossiblePairs \neq \emptyset;$  
29 return $A$
```

**Algorithm 11:** The EDSM-Markov inference algorithm

The set of blue states $B$ is computed using the $\text{ComputeBlue}(A, R)$ function, where the uncoloured children states of the red nodes are coloured blue. Next, pairs of states for merging are selected iteratively after comparing possible red-blue pairs of states. It is worth noting that the comparison is based on both the $EDSM$ and $Im$ scores as shown
in Line 12. The selection procedure of pairs of states is described in Lines 9-22, where the idea of Blue-Fringe EDSM learner is applied to select possible pairs to merge and evaluate using $I\text{Score}(A, q_r, q_b, MT)$. The idea of the Blue-Fringe search was proven to reduce the number of evaluating pairs since red states will only be compared against blue ones. Each blue state in the $B$ set is evaluated to merge with each red state in the $R$ set using the $\text{ComputeIScore}(A, q_r, q_b, MT)$ function as described in Definition 4.10. Once it is computed, a score is assigned to each possible red-blue pair of states. If the score is zero or above, it is added to the ordered set of possible pairs, denoted by $\text{PossiblePairs}$. Otherwise, the pair of states is blocked whenever $I\text{Score}$ is below 0.

During the comparison of possible pairs, if any blue state cannot be merged with any of the red ones, it is added to the $R$ set, and its children states become blue. Moreover, each newly coloured state as blue is compared to each red state in the $R$ set. Algorithm 11 is terminated when all states have become red denoting that there are no further mergers that can be performed. Otherwise, the comparison between red and blue states is continued.

It is important to emphasize that pairs of states in the $\text{PossiblePairs}$ set are ordered based on their $I\text{Score}$ scores where the pair of states with the highest score becomes the top in the set. The pair of states with the highest score is picked and the $\text{Merge}$ function is called to merge the states.

The main difference between the idea of inconsistencies and techniques that rely on mining rules is that rules are expected to hold universally and the number of inconsistencies reflects the number of violated rules.

To the best of our knowledge, no such incorporation of a prediction model with the state merging strategy exists in the automata learning community to compute the inconsistency of merging states. However, other techniques [70, 138] rely on mining rules from the execution traces in the form $pre \rightarrow post$, and then use the mined rules to block state merges that contradict the rules. The principle presented in the $EDSM$-$Markov$ differs from those techniques that rely on rules [70, 138].
4.5 Summary of the Chapter

The chapter introduced the idea of training a non-probabilistic Markov model to predict an element of alphabet after a prefix sequence of length $k$. Predictions are made based on observations made from the provided traces and $k$ histories. In addition, the correctness of the trained Markov model can be measured using the most common metrics in the information retrieval domain.

Additionally, this chapter presented $EDSM-Markov$, a heuristic-based learner that relies on Markov predictions to avoid merging inconsistent pairs of states during the generalization process. It is introduced to overcome the over-generalization problem when negative traces are rare or not present. Hence, generalization processes that can lead to many inconsistencies are not preferable.

On comparing $EDSM-Markov$ against other passive learners in the grammar inference community, it tends not to only use the local similarity of the existing labels of the outgoing transitions but to use predictions as well to make decisions during the state-merging process. For instance, the $EDSM$ learner will only block merging states if an accepting state would be merged with a rejected one. The next chapter presents the evaluation of the performance of $EDSM-Markov$ in terms of the language and structure of the inferred LTSs.
5

Experimental Evaluation and Case Studies of EDSM-Markov

5.1 Introduction

In the previous chapter, the Markov-EDSM learner was presented in order to infer LTS models from a few long traces. This chapter studies and evaluates the performance of the proposed learner and describes the requirements to infer LTSs that recognize the hidden target language well.

There are many ways of comparing the performance of inference techniques such as those that are described in Section 2.5. Due to the difficulties in comparing the inference techniques to each other, the selection of software models to infer them from traces is still problematic [34]. Hence, to reduce these difficulties, it is important to use diverse reference models of different sizes and of various alphabet sizes. Since there are several
parameters such as the prefix length $k$ and alphabet size that may affect the performance of the \textit{EDSM-Markov} algorithm, the following section presents the experimental evaluation from different aspects. The improvement in learner performance is demonstrated by a series of experiments using both randomly generated labelled transition systems and case studies.

5.2 Experimental Evaluation of the EDSM-Markov Algorithms

An early empirical study of the existing techniques in relation to the problem of the thesis showed that the \textit{SiccoN} method performed better than other techniques used in the study, as shown in section 3.5. The objective of evaluating the efficiency of the \textit{EDSM-Markov} algorithm is to measure its performance in different settings.

The aim of this evaluation is to answer the following quantitative research questions:

1. What is the relationship between the number of traces and the performance of \textit{EDSM-Markov} and \textit{SiccoN}?

2. How much improvement would be made in terms of the quality of the inferred LTSs using \textit{EDSM-Markov} against those obtained using \textit{SiccoN} with a large-sized alphabet?

3. What is the impact of the length of the traces on the quality of the induced LTSs using \textit{EDSM-Markov} compared to \textit{SiccoN}?

4. What is the impact of a prefix length $k$ on the quality of the inferred LTSs using the \textit{EDSM-Markov} learner compared to \textit{SiccoN}?

5. Under which settings and conditions can the idea of the \textit{EDSM-Markov} produce the exact DFA hypotheses?

5.2.1 Methodology

In order to evaluate the performance of the \textit{EDSM-Markov} learner, a series of random LTSs were generated for each number of states ranging between 10 and 40 in steps of 10.
Fifteen LTSs were generated for each chosen number of states. Hence, the total number of random LTSs is 4 steps * 15 = 60 LTSs. For each LTS, 5 sets of training data were generated, bringing the number of LTSs learnt per experiment to 300. This was under an assumption that the same training data is passed for the learners for each inference task. The idea behind learning this number of LTSs is to assess the performance of the proposed algorithms on various random LTSs with different training data feed to each LTS. In addition, the randomly generated LTSs were connected, and had an alphabet size two times the number of states. The actual number of states varies because the random LTS generator produces reduced connected machines by adding states until the state number after reduction reaches the target value, plus/minus 20%. Each state contained around 3 outgoing transitions. Most of the states could be pairwise distinguished by single transitions, and around 36% of the states could be uniquely identified by a single element of an alphabet.

Initially, training data comprised of 5 random walks of the length $|Q| \times |\Sigma| = 2 \times |Q|^2$. This figure is loosely motivated by the size of a transition cover set, which is the number of sequences to reach every state of an LTS and attempt every input in it. Once LTSs are inferred by different learners, the same test sets are given to measure how well their inferred LTSs classify a test set. The accuracy of classification is represented using the BCR scores. In addition, $2 \times |Q|^2$ test sequences were generated of length $3 \times |Q|$. It is worth noting that the set of tests were diverse ensuring that half of them belonged to the language and the other half did not. In Section 5.2.3, the performance of EDSM-Markov will be measured with different numbers of traces where it ranges from 1 to 7 traces, incrementing by 2. In this experiment, two metrics were selected to score the performance of the algorithms; the former metric is a BCR, the latter is a structural similarity, and they are described in detail in Section 2.5.

The reason for selecting inferring LTS models that have a large size of alphabet is to be more representative to software models [34, 35]. Moreover, that the state-of-the-art methods focus on inferring such models. Additionally, the considered problem in this thesis is to infer LTSs that have large alphabets from only a few positive traces. In this experiment, SiccoN was selected to be a reference learner to compare it with the proposed learners to measure their performance since SiccoN performs reasonably well when the alphabet size is large and very little positive training data is provided. In other
settings, SiccoN has been proven to perform well if the training data is not sparse and there are sufficient negative traces.

The experiment was implemented using the extended version of the Statechum tool, available for clone via https://github.com/AbdullahUK/EDSM_QSM_MarkovPhd.git. For the following experiment, the launch configuration has to start statechum.analysis.learning.experiments.PairSelection.MarkovLearnerExperimentWithStatisticalAnalysis class.

In the conducted experiments, Java 7 was used with JVM arguments of -ea -Dthreadnum=1 -Djava.library.path=linear/.libs;"C:/Program Files/R/R-3.0.1/library/rJava/jri/x64" -Xmx26000m and environment variable R_HOME set to the location of R, such as C:/Program Files/R/R-3.0.1/lib64/R java. The R toolset was used for all analysis. The R tool has to have JavaGD, rJava and aplpack installed.

5.2.2 Main Results

The main results of the experiment are shown in Figures 5.1 and 5.2. The figures are a bag-plot (a bivariate boxplot), which is a generalization of a boxplot, introduced by Rousseeuw et al. [139]. The star denotes the average value, and the dark blue region (‘bag’) surrounding it contains 50% of the points. Figure 5.1 illustrates the BCR scores of LTSs inferred using EDSM-Markov compared to SiccoN. The BCR scores attained by EDSM-Markov are higher than those attained by SiccoN, where the average increases from 0.80 to 0.93 as shown in Figure 5.1. All points above the diagonal line in the bagplots are improvements in EDSM-Markov over SiccoN. In terms of structural-similarity measurement, the score raises on average from 0.41 for SiccoN to 0.76 for EDSM-Markov as shown in Figure 5.2 with nearly all dots above the diagonal line.

From Figure 5.1 and Figure 5.2, it is clear that EDSM-Markov performs better than SiccoN in the considered setting. The paired Wilcoxon signed-rank test was used to measure statistically the significant difference between both algorithms (EDSM-Markov and SiccoN). The null hypothesis $H_0$ in this case is that the BCR scores of EDSM-Markov and SiccoN learners are equal. The outcome of this test is a $p$-value, as shown in Table 5.1. The resulting $p$-value is less than the 0.05 significance level, indicating that there is a clear statistical difference between the BCR score achieved by EDSM-Markov and SiccoN learners. Hence, the $H_0$ is rejected.
In terms of measuring statistically the significance of the difference between structural-similarity scores attained by \textit{EDSM-Markov} and \textit{SiccoN}, the null hypothesis $H_0$ in this case is that the structural-similarity scores from \textit{EDSM-Markov} and \textit{SiccoN} are the same. The Wilcoxon signed-rank test reports a $p$-value of $3.76 \times 10^{-40}$. Therefore, the $H_0$ is rejected, denoting that there is a significant statistical difference between the structural-similarity scores obtained by the two learners.
Table 5.1: p-values obtained using the Wilcoxon signed-rank test for the main results

<table>
<thead>
<tr>
<th>EDSM-Markov v.s. SiccoN</th>
<th>BCR scores</th>
<th>Structural-similarity scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1.97 \times 10^{-43}$</td>
<td>$3.76 \times 10^{-49}$</td>
</tr>
</tbody>
</table>

5.2.3 The Impact of the Number of Traces on the Performance of EDSM-Markov

This section answers the first research question that is considered in Section 5.2. The number of traces ($T$) is an important parameter that should be considered in the evaluation of the EDSM-Markov learner. The objective of this investigation is to quantify the effect of $T$ on the performance of EDSM-Markov compared to SiccoN. Therefore, the EDSM-Markov learner was evaluated across different numbers of traces.

A boxplot of the BCR scores attained by EDSM-Markov and SiccoN learners across various settings of $T$ is shown in Figure 5.3. It is clear that the EDSM-Markov learner inferred LTSs with higher BCR scores compared to SiccoN when $T > 1$. The median value of BCR scores obtained by EDSM-Markov is 0.99 when $T = 7$; In this case, the improvements are reasonable. The reason behind these improvements is that Markov models were
trained from *structural complete* training data. However, the *EDSM-Markov* learner overgeneralized LTSs when $T < 3$; this is because the random generator of traces does not cover transitions well. When $T = 1$, the mean BCR scores is 0.58 for *EDSM-Markov*, and the mean BCR scores is 0.56 for *SiccoN*; this does not show a clear improvement made by *EDSM-Markov* in this case.

During the conducted experiments, the ratio of improvement was computed as follows:

$$\text{ratio of BCR} = \frac{\text{BCR score using } EDSM-Markov}{\text{BCR score using } SiccoN}$$

(5.1)

![Figure 5.4: Improvement ratio of BCR scores achieved by *EDSM-Markov* to *SiccoN*](image)

In order to measure the improvement made by *EDSM-Markov* compared to *SiccoN* in terms of language, the ratio of improvement was computed using Equation 5.1 for BCR scores. There are a clear improvements made by *EDSM-Markov over SiccoN* when $T > 1$, as can be seen in Figure 5.4. Besides this, it is apparent that *EDSM-Markov* does not show a clear improvement if $T = 1$. The improvements are affected by the setting of $T$. However, the ratio of improvements are small when $T > 3$. This can be attributed to the improvement of *SiccoN* for a larger number of traces.
Additionally, the small ratio of improvement does not mean that the \textit{EDSM-Markov} learner performed badly, but it is because \textit{SiccoN} inferred LTSs with BCR scores close to those obtained using \textit{EDSM-Markov}. The \textit{SiccoN} learner tends to block invalid mergers correctly in case where $T$ is large. This is intuitive because \textit{SiccoN} benefits from the performance of \textit{EDSM} that performs better on heavily branching traces.

![Figure 5.5: A boxplot of structural-similarity scores attained by \textit{EDSM-Markov} and \textit{SiccoN} for a different number of traces](image)

The effect of $T$ on the structural-similarity scores obtained using \textit{EDSM-Markov} and \textit{SiccoN} is shown in Figure 5.5. Judging by the boxplots shown above, it is clear that the structural-similarity scores achieved by \textit{EDSM-Markov} increase while $T$ increases. This implies that the Markov models were trained enough to identify inconsistencies during merging states. It can be seen from Figure 5.5 that \textit{EDSM-Markov}, at every setting of $T$, inferred LTSs with higher structural-similarity scores compared to \textit{SiccoN}. It is obvious that \textit{EDSM-Markov} achieves reasonable structural-similarity scores when $T > 5$.

\begin{equation}
\text{Ratio of structural difference} = \frac{\text{Structural-similarity score using } \textit{EDSM-Markov}}{\text{Structural-similarity score using } \textit{SiccoN}} \quad (5.2)
\end{equation}
The ratio of improvement of the structural-similarity scores achieved by EDSM-Markov over SiccoN learners was computed using Equation 5.2; this ratio is shown in Figure 5.6. It appears from Figure 5.6 that, the structural-similarity scores of LTSs inferred using EDSM-Markov are higher than those obtained using SiccoN. This is because SiccoN tends to prevent merging equivalent states that should be merged since training data is sparse.

Table 5.2 shows the p-values obtained using the paired Wilcoxon signed-rank test after comparing the BCR and structural-similarity scores attained by EDSM-Markov and SiccoN. The null hypothesis $H_0$ is that SiccoN produces similar results to EDSM-Markov. In all cases, the considered $H_0$ can be rejected because the p-values are less than 0.05. This denotes that there is a clear statistical difference between the scores obtained using EDSM-Markov compared to SiccoN. In addition, Table 5.2 provides the mean values for BCR and structural-similarity scores. The mean BCR score for EDSM-Markov is higher than the mean BCR score for SiccoN, as shown in Table 5.2.
Table 5.2: $p$-values obtained using the Wilcoxon signed-rank test of comparing EDSM-Markov v.s. SiccoN across different number of traces

<table>
<thead>
<tr>
<th>$T$</th>
<th>$p$-value of E-M vs SiccoN</th>
<th>Mean BCR</th>
<th>Mean structural similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>E-M</td>
<td>SiccoN</td>
</tr>
<tr>
<td>BCR</td>
<td>structural similarity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>$4.49 \times 10^{-07}$</td>
<td>0.58</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$7.33 \times 10^{-40}$</td>
<td>0.85</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$1.97 \times 10^{-43}$</td>
<td>0.93</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>$2.09 \times 10^{-39}$</td>
<td>0.96</td>
</tr>
</tbody>
</table>

5.2.4 The Impact of Alphabet Size on the Performance of EDSM-Markov

This section answers the second research question that is considered early in Section 5.2. In order to evaluate different types of LTS, an alphabet size is a significant factor to consider for evaluating the performance of the EDSM-Markov learner. In Section 5.2.3, the alphabet size in the experiment was two times the number of states $|\Sigma| = 2 \times Q$. Hence, experiments were conducted to measure the impact of various sizes of alphabet on the quality of the inferred LTSs. The size of the alphabet was modified in stages, and it ranged with values between $\frac{1}{4}$ and 4 times the alphabet size used in Section 5.2.2. In this way, an alphabet multiplier parameter $m$ was introduced to vary the alphabet size such that $|\Sigma| = m \times |Q|$, and $|\Sigma|$ ranged between $\frac{1}{4} \times |Q|$ and $8 \times |Q|$ in this experiment. Positive sequences of length $2 \times |Q|^2$ were used as training data. The number of traces ($T$) ranged from 1 to 7, incrementing by 2. Thus, the variance in this experiment is based on both $|\Sigma|$ and $T$.

The boxplots of the BCR scores obtained by EDSM-Markov and SiccoN for all different alphabet sizes considered are illustrated in Figure 5.7. The BCR scores attained by the EDSM-Markov learner appear to be optimal when $m > 1$ and $T \geq 5$. However, the EDSM-Markov learner over-generalized LTSs when $m = 0.5$ and $T < 7$. The reason behind the over-generalization is that whenever a pair of states are merged, new labels of outgoing transitions would be added to the merged node which are incorrectly predicted as permitted to follow it; in this case, inconsistencies are not detected.
Figure 5.7: BCR scores obtained by EDSM-Markov and SiccoN for different alphabet multiplier $m$ in $|\Sigma| = m \times |Q|$

The ratio of improvements in the BCR scores achieved by EDSM-Markov over SiccoN is shown in Figure 5.8, and was computed using Equation 5.1. As can be seen from Figure 5.8, SiccoN performs badly when the size of alphabet is small. This is because the number of blue states with labels of outgoing transitions similar to a red state is large and SiccoN allows them to be merged where they should be blocked. Additionally, SiccoN fails to block those mergers because training data is few, and distinct outgoing transitions from blue states compared to red states are missing as a result due to training data sparsity.

What is interesting in the BCR scores obtained by EDSM-Markov is that there is a relation between them and the alphabet size. Moreover, the performance of the EDSM-Markov learner improves as long as the alphabet size is increased in terms of BCR values. It is important to compute the precision and recall of the trained Markov models; this aimed to study the relationship between the accuracy of the Markov models and the BCR scores. It is important in this regard to mention that the precision and recall scores of the trained models were computed as described in the previous chapter. The precision values reflect
the accuracy of Markov predictions over the returned predictions. The recall values represent how accurately the trained Markov models at predicting the exiting labels of the outgoing transitions in the target LTSs.

The accuracy of the trained Markov models are shown in Figure 5.9 for different settings of $m$ and $T$. The precision scores increase whenever the size of the alphabet is raised, as shown in Figure 5.9. The recall scores are accurate in all cases, which means all outgoing transitions in the reference LTSs are predicted correctly. It is clear that the precision scores of Markov models are high when $m > 1$; this may explain why the BCR scores are high in such cases.

The boxplots of the structural-similarity scores obtained by EDSM-Markov and SiccoN for different settings of $m$ and $T$ are depicted in Figure 5.10. As can be seen in Figure 5.10, the structural-similarity scores attained by EDSM-Markov are affected by different settings of $m$. The structural-similarity scores are increased as long as the alphabet multiplier parameter $m$ increases. The structural-similarity scores achieved by EDSM-Markov are higher than those achieved by SiccoN, as shown in Figure 5.10.
Figure 5.9: Accuracy of Markov predictions for a different alphabet multiplier across various number of traces.

Figure 5.10: Structural-similarity scores of EDSM-Markov and SiccoN for different alphabet multiplier $m$ in $|\Sigma| = m \cdot |Q|$.
Figure 5.11: Improvement ratio of structural-similarity scores achieved by EDSM-Markov to SiccoN for different alphabet multiplier and various number of traces.

Figure 5.11 illustrates the ratio of structural-similarity scores obtained by EDSM-Markov to those attained by SiccoN. The results from Figure 5.11 demonstrate that SiccoN inferred LTSs with lower structural-similarity values compared to EDSM-Markov. Unsurprisingly, the improvement in the structural-similarity values is clear because the recall scores of the trained Markov models are very high in all settings of $m$; this denotes that the generated traces cover transitions well, particularly if $T > 3$. It appears that SiccoN allows merging states that it should not, especially if $m < 2$.

The paired Wilcoxon signed-rank test was carried out to statistically check the null hypothesis $H_0$ that SiccoN produces similar results to EDSM-Markov. Table 5.3 summarizes the $p$-values obtained by comparing the BCR and structural-similarity scores for both learners. The resulting $p$-values are less than 0.05, denoting that there is a clear statistical difference between the scores obtained by both learners. Therefore, the considered $H_0$ can be rejected. However, the $H_0$ can be accepted when $T = 1$ and $m = 0.5$ since the $p$-value is 0.09, indicating that there is no significant difference between the BCR scores attained by EDSM-Markov and SiccoN.
Chapter 5. Experimental Evaluation and Case Studies of EDSM-Markov

The third research question considered in Section 5.2 is to investigate the influence of the length of a few traces on the performance of the EDSM-Markov learner; the findings in this section answer this question. One of the most important factors to evaluate the efficiency of inference algorithms is the capability to generate good LTSs from different lengths of traces. In the previous sections, the length of traces was given by \( \text{length} = 2 \times |Q|^2 \). Therefore, experiments were carried out to measure the effect of different lengths of traces on the performance of the proposed learner. The length of traces was given by \( l \times 2 \times |Q|^2 \).

<table>
<thead>
<tr>
<th>( m )</th>
<th>( T )</th>
<th>( p )-value of E-M vs SiccoN</th>
<th>Mean BCR</th>
<th>Mean structural-similarity</th>
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<td></td>
<td>( m )</td>
<td>( T )</td>
<td>( BCR )</td>
</tr>
<tr>
<td>0.5</td>
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<tr>
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<td>1.97 ( \times 10^{-50} )</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1.24 ( \times 10^{-27} )</td>
<td>4.81 ( \times 10^{-50} )</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 5.3: Wilcoxon signed rank test with continuity correction of comparing EDSM-Markov v.s. SiccoN using various alphabet multiplier

5.2.5 The Impact of the Length of Traces on the Performance of EDSM-Markov

The third research question considered in Section 5.2 is to investigate the influence of the length of a few traces on the performance of the EDSM-Markov learner; the findings in this section answer this question. One of the most important factors to evaluate the efficiency of inference algorithms is the capability to generate good LTSs from different lengths of traces. In the previous sections, the length of traces was given by \( \text{length} = 2 \times |Q|^2 \). Therefore, experiments were carried out to measure the effect of different lengths of traces on the performance of the proposed learner. The length of traces was given by \( l \times 2 \times |Q|^2 \).
where the parameter $l$ denotes the length multiplier, and introduced to vary the length of traces. Besides, the *EDSM-Markov* learner on different lengths of traces and various alphabet sizes as well. Thus, the alphabet size was given by $|\Sigma| = m \times |Q|$, and $|\Sigma|$ ranged between $\frac{1}{2} \times |Q|$ and $2 \times |Q|$ in the conducted experiment.

### 5.2.5.1 When $m = 2.0$

Figure 5.12 shows the boxplots of the BCR scores obtained using *EDSM-Markov* and *SiccoN* when $m = 2$. As expected, the performance of *EDSM-Markov* is affected by the length of traces where long ones result in generating good LTSs; this is because transitions are covered well. The median value of the BCR scores obtained by *EDSM-Markov* is 0.99 when $l = 2$ and $T = 7$. It appears from Figure 5.12 that the exact LTSs can be inferred if the provided traces are very long. The *EDSM-Markov* learner inferred LTSs with higher BCR values compared to *Sicco* in the majority of cases as shown in Figure 5.12.

![Figure 5.12: Blots of BCR scores obtained by *EDSM-Markov* and *SiccoN* for different setting of $l$ and various numbers of traces where $m = 2.0$, the length of traces is given by $= l \times 2 \times |Q|^2$](image)

It can be seen from Figure 5.12 that the BCR scores obtained by *EDSM-Markov* are very low when $T = 1$ and $l < 2.0$. This is because the generated traces cover transitions well,
as shown in Figure 5.13. Moreover, the Markov models were not trained well to make predictions correctly. Thus, new prefix paths of length $k$ were added to the merged node during the state-merging process where Markov models did not see them; this caused inconsistency scores to be too large. Hence, many pairs of states that should be merged were blocked.

![Figure 5.13: Transition coverage for different setting of $l$ and various numbers of traces where $m = 2.0$ and the length of traces is given by $= l \times 2 \times |Q|^2$](image)

Figure 5.14 presents boxplots of the structural-similarity scores achieved by EDSM-Markov and SiccoN. As can be seen from Figure 5.14, the structural-similarity scores of the inferred LTSs using EDSM-Markov climbs steadily with the increase in $l$. The structural-similarity scores of LTSs inferred using EDSM-Markov are higher than those obtained using SiccoN.

Table 5.4 gives the $p$-values obtained by the paired Wilcoxon signed-rank test after comparing the BCR and structural-similarity scores of both algorithms. The null hypothesis $H_0$ to be tested in this study is that there is no difference between the scores attained by EDSM-Markov and SiccoN. The $p$-values show that there is clear evidence that EDSM-Markov inferred LTSs with structural-similarity scores higher than SiccoN. The resulting $p$-values are less than 0.05, supporting the clear improvement shown in Figure 5.14. Thus, the null hypothesis $H_0$ is rejected. However, when comparing the BCR scores attained by
both learners, the $H_0$ can be accepted when $l = 0.125$, denoting that there is no statistical difference between the scores. With $T = 1$ and $l = 0.25$, the $p$-value is 0.79 when comparing the BCR scores attained by both learners, and the $H_0$ can be accepted.
<table>
<thead>
<tr>
<th>1</th>
<th>T</th>
<th>( p )-value of E-M Vs.SiccoN</th>
<th>Mean BCR</th>
<th>Mean structural similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BCR</td>
<td>Structural similarity</td>
<td>E-M</td>
</tr>
<tr>
<td>0.125</td>
<td>1</td>
<td>0.98</td>
<td>(2.95 \times 10^{-08})</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.38</td>
<td>(1.43 \times 10^{-25})</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.33</td>
<td>(5.29 \times 10^{-30})</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.37</td>
<td>(6.48 \times 10^{-31})</td>
<td>0.65</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>0.79</td>
<td>(6.42 \times 10^{-27})</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.02 \times 10^{-07}</td>
<td>(2.73 \times 10^{-37})</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.79 \times 10^{-11}</td>
<td>(3.66 \times 10^{-44})</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3.68 \times 10^{-16}</td>
<td>(2.05 \times 10^{-45})</td>
<td>0.79</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.001</td>
<td>(3.35 \times 10^{-38})</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.71 \times 10^{-28}</td>
<td>(4.03 \times 10^{-46})</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.38 \times 10^{-32}</td>
<td>(8.41 \times 10^{-49})</td>
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</tr>
<tr>
<td></td>
<td>7</td>
<td>1.27 \times 10^{-36}</td>
<td>(5.69 \times 10^{-50})</td>
<td>0.90</td>
</tr>
<tr>
<td>1.0</td>
<td>1</td>
<td>4.49 \times 10^{-07}</td>
<td>(4.40 \times 10^{-46})</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.33 \times 10^{-40}</td>
<td>(2.38 \times 10^{-49})</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.97 \times 10^{-43}</td>
<td>(3.76 \times 10^{-49})</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>2.09 \times 10^{-39}</td>
<td>(2.07 \times 10^{-49})</td>
<td>0.96</td>
</tr>
<tr>
<td>2.0</td>
<td>1</td>
<td>2.69 \times 10^{-17}</td>
<td>(1.26 \times 10^{-47})</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.51 \times 10^{-43}</td>
<td>(2.05 \times 10^{-50})</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.36 \times 10^{-41}</td>
<td>(3.59 \times 10^{-50})</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5.74 \times 10^{-46}</td>
<td>(4.73 \times 10^{-50})</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 5.4: \( p \)-values obtained using the Wilcoxon signed-rank test by comparing \(EDSM-Markov\) v.s. \(SiccoN\) across different number of traces where \(m=2.0\)

5.2.5.2 When \(m=0.5\)

In the previous section, the \(m\) parameter was 2. In this section, the performance of \(EDSM-Markov\) is evaluated for different lengths when \(m=0.5\). Figure 5.15 shows the BCR scores obtained by \(EDSM-Markov\) and \(SiccoN\) when \(m=0.5\). Indeed, unlike when \(m=2.0\), the BCR scores achieved by \(EDSM-Markov\) are very low even for long traces, especially when \(T<5\). The precision scores of the trained Markov models are too low and this may contribute to the low BCR scores. Despite this, the BCR scores of the generated LTSs using \(EDSM-Markov\) are higher than \(SiccoN\) when \(T>3\). When \(T=7\), for instance, the average BCR scores attained by \(EDSM-Markov\) is 0.70 at \(l=2.0\) and 0.65 at \(l=0.5\). This
indicates that the length of the traces affects the performance of \textit{EDSM-Markov}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.15}
\caption{BCR scores obtained by \textit{EDSM-Markov} and \textit{SiccoN} for different $l$ where $m = 0.5$, $= l \times 2 \times |Q|^2$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.16}
\caption{Structural-similarity scores obtained by \textit{EDSM-Markov} and \textit{SiccoN} for different $l$ where $m = 0.5$, $= l \times 2 \times |Q|^2$.}
\end{figure}
It is necessary to provide insight about the impact of the length of traces on the structure of the inferred LTSs. Figure 5.16 illustrates the structural-similarity scores of LTSs inferred using EDSM-Markov and SiccoN. The scores achieved by EDSM-Markov are higher than those attained by SiccoN. The improvement made by EDSM-Markov over SiccoN in terms of the structural-similarity is clear even when $T = 1$ and $l = 0.125$. The average structural-similarity scores obtained by EDSM-Markov increased by 70.59% compared to SiccoN when $T = 1$ and $l = 0.125$.

Table 5.5 summarizes the $p$-values computed using the paired Wilcoxon signed-rank test for the BCR and structural-similarity scores. The null hypothesis $H_0$ in this investigation is that the scores attained by EDSM-Markov and SiccoN are the same. The $p$-values show that the scores obtained by learner are significant at the 0.05 level in the majority of cases. In such cases, the null hypothesis $H_0$ is rejected. When $T = 1$ and $l = 0.25$, $l = 0.5$, and $l = 1.0$, the $H_0$ can be accepted because the $p$-values are higher than 0.05, which means that the results are not significant in these cases.
<table>
<thead>
<tr>
<th>T</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>1</td>
<td>8.19 × 10^-04</td>
<td>1.16 × 10^-40</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.11 × 10^-08</td>
<td>3.24 × 10^-44</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7.04 × 10^-22</td>
<td>3.93 × 10^-43</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>1.79 × 10^-17</td>
<td>4.98 × 10^-43</td>
<td>0.57</td>
</tr>
<tr>
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<td>1</td>
<td>0.08</td>
<td>2.12 × 10^-42</td>
<td>0.5</td>
</tr>
<tr>
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<td>3</td>
<td>4.07 × 10^-07</td>
<td>2.57 × 10^-40</td>
<td>0.54</td>
</tr>
<tr>
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<td>5</td>
<td>1.78 × 10^-18</td>
<td>1.73 × 10^-35</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7.49 × 10^-22</td>
<td>4.07 × 10^-32</td>
<td>0.61</td>
</tr>
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<td>4.89 × 10^-48</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>6.20 × 10^-12</td>
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</tr>
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<td>1.05 × 10^-22</td>
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</tr>
<tr>
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<td>1.29 × 10^-32</td>
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<td>0.65</td>
</tr>
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<td>1</td>
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<td>2.74 × 10^-47</td>
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</tr>
<tr>
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<td>3.32 × 10^-24</td>
<td>3.87 × 10^-35</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.98 × 10^-37</td>
<td>9.34 × 10^-37</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3.27 × 10^-38</td>
<td>2.44 × 10^-31</td>
<td>0.69</td>
</tr>
<tr>
<td>2.0</td>
<td>1</td>
<td>2.68 × 10^-04</td>
<td>1.09 × 10^-40</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.60 × 10^-31</td>
<td>1.19 × 10^-29</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4.34 × 10^-38</td>
<td>2.70 × 10^-28</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3.67 × 10^-40</td>
<td>6.95 × 10^-31</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 5.5: p-values obtained using the Wilcoxon signed-rank test by comparing EDSM-Markov v.s. SiccoN across different numbers of traces where m=0.5.
5.2.5.3 When $m = 1.0$

In this section, the outcomes of evaluating the performance of the EDSM-Markov learner with different lengths of traces are presented. The size of the alphabet is given by $\Sigma = m \times |Q|$ where $m = 1.0$. Figure 5.17 shows the BCR scores obtained by EDSM-Markov and SiccoN when $m = 1.0$. The graph illustrates that there is a gradual increase in the BCR scores obtained using EDSM-Markov with the increase of the lengths of traces. In addition, the BCR scores obtained by EDSM-Markov are higher than those obtained by SiccoN when $T > 1$.

It is worth noting that the BCR scores attained by EDSM-Markov are higher than those obtained by the same learner when $l = 0.5$. This is because the precision scores of the Markov models when $m = 1.0$ are higher than those of Markov models when $m = 0.5$.

Figure 5.17: BCR scores obtained by EDSM-Markov and SiccoN for different setting of $l$ and various numbers of traces where $m = 1.0$ and the length of traces is given by $\ell = l \times 2 \times |Q|^2$.

Figure 5.18 shows boxplots of the structural-similarity scores obtained by EDSM-Markov and SiccoN. In Figure 5.14, there is a clear tendency for the structural-similarity scores
of the inferred LTSs using \textit{EDSM-Markov} to increase while \( l \) increases. The structural-similarity scores of the inferred LTSs using \textit{SiccoN} are very low compared to \textit{EDSM-Markov}.

Table 5.6 summarizes the statistical test results using the paired Wilcoxon signed-rank test for the BCR and structural-similarity scores. The null hypothesis \( H_0 \) considered in this research question is that the scores of the inferred LTS using \textit{EDSM-Markov} and \textit{SiccoN} are the same. When \( l = 0.125 \) and the number of traces is 1, \textit{EDSM-Markov} does not show a significant difference compared to \textit{SiccoN} in terms of BCR scores. However, the \( p \)-values are below the 0.05 significance level in cases where \( l > 0.125 \), so the considered null hypothesis can be rejected.

The fourth column in Table 5.6 summarizes the statistical test results obtained using the Wilcoxon signed-rank test for structural-similarity scores. It is clear that the resulted \( p \)-values are below the 0.05 significance level in all cases. Therefore, the \( H_0 \) is rejected since in the majority of cases, it denotes that the structural-similarity scores obtained by \textit{EDSM-Markov} are higher than scores obtained by \textit{SiccoN}.

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure5_18.png}
\caption{structural difference scores obtained by EDSM-Markov for trace length multiplier \( l \) setting the length of each of the 5 traces to \( l \cdot |Q| \cdot |\Sigma| = 2 \cdot l \cdot |Q|^2 \)}
\end{figure}
Table 5.6: \( p \)-values obtained using the Wilcoxon signed-rank test by comparing EDSM-Markov v.s. SiccoN across different numbers of traces where \( m=1.0 \)

5.2.6 The Impact of Prefix Length on the Performance of EDSM-Markov

As Markov predictions rely on a prefix length \( k \) of the trained Markov models, it is meaningful to study the influence of \( k \) on the accuracy of the inferred LTSs. Experiments were conducted on random LTSs to answer the fourth research question considered in Section 5.2.

The boxplots of the BCR scores of the inferred LTSs using EDSM-Markov and SiccoN with different values assigned to \( k \) are illustrated in Figure 5.19. It is noticed that the EDSM-Markov learner inferred LTSs that are closer to the target ones, especially if \( k = 2 \) and the number of traces is 5 and 7, as shown in Figure 5.19.
As can be seen in Figure 5.19, a prefix length of two ($k = 2$) appears to be optimal compared to $k = 1$ and 3. It is clear that the EDSM-Markov learner performs better than SiccoN when $k = 1$ and 2 and the number of traces is more than 1. This is because EDSM-Markov detects inconsistencies accurately during the comparison of pairs of states to prevent merging invalid ones using the $IScore$ heuristic. Moreover, the inferred LTSs using EDSM-Markov when $k = 2$ are better than those inferred if $k = 1$, and this is because the precision scores of the trained Markov model if $K = 2$ are higher than the precision scores if $k = 1$, as shown in Figure 5.20.

It is important to highlight that, whenever two states are considered for merging, new labels of transitions might be added to the merged node; if they are incorrectly predicted to follow the node, then inconsistencies will not be detected during the computation of the $IScore$ score. In the conducted experiments, this occurred when $k = 1$ since the Markov precision is very low in this case, as shown in Figure 5.20. With the number of trace being one, the BCR scores are not as good as when the number of traces are 5 and 7. This may be because many predictions of labels of transitions are missed out due to the sparsity of data. Since a complete table predicting transitions based on the history of $k$ transitions
has \(|\Sigma|^k\) entries, one would expect larger values of \(k\) to correspond to fewer predicted transitions and thus lower recall values.

Figure 5.21 illustrates the ratio of improvement of BCR scores for LTSs inferred using EDSM-Markov to SiccoN. It appears that the BCR scores improved when \(K < 3\), as shown in Figure 5.21. For a prefix length of 3, many Markov predictions are missed to prevent merging of an inequivalent pair of states. Despite the high values of the precision and recall when \(k = 3\), the performance of EDSM-Markov is reduced as shown by a reduction in the BCR scores of the inferred LTSs. This is due to the sparsity of training data, which means that most prefix paths of length 3 leading to states in an LTS will not have any predictions. Hence, all labels of outgoing transitions would be seen as inconsistencies with respect to the trained Markov models, forcing EDSM-Markov to merge relatively few states.

Figure 5.22 shows the number of inconsistencies computed for the reference LTSs after training Markov models. A very low inconsistency score means that a Markov table is trained well with respect to subsequences of length \(k + 1\). One can observe that when \(k = 3\), the mean value of the BCR scores for the inferred LTSs using EDSM-Markov is
below (say 0.9) in the majority of cases, except when the number of traces is 7. Besides, low BCR scores of the learnt LTSs when the number of traces is very small is due to the number of inconsistencies being very high. In general, a large inconsistency score denotes that Markov models need to have extra training data to visit states using different paths in order to infer LTSs with high BCR scores.

It is clear that EDSM-Markov inferred well-structured LTSs when \( k = 2 \), as shown in Figure 5.23. The inferred LTSs using EDSM-Markov are over-generalized when \( k = 1 \), and this is due to the precision scores of the Markov models are very low, as shown in Figure 5.20.

Table 5.7 summarizes the \( p \)-values obtained using the Wilcoxon signed-rank test. The null hypothesis in this research question is that the BCR and structural-similarity scores do not show clear improvements. The third column in Table 5.7 shows the statistical test results for BCR scores. The reported \( p \)-values are higher than 0.05 (significance level) when \( K = 3 \) and the number of traces is 5 or 7, so the considered null hypothesis can be accepted. This proves that the BCR scores of the inferred LTSs using EDSM-Markov do not show an improvement compared to SiccoN. However, the \( p \)-values are lower than 0.05 when \( K < 3 \), so the null hypothesis in this case is rejected. This demonstrates that
**Figure 5.22:** Number of inconsistency of the trained Markov with comparison to the target model

**Figure 5.23:** Structural difference scores attained by EDSM-Markov for a different prefix length and various numbers of traces
BCR scores obtained by *EDSM-Markov* are higher than those obtained using *SiccoN* in the majority of random LTSs used in the experiments.

<table>
<thead>
<tr>
<th>K</th>
<th>T</th>
<th>$p$-value of E-M Vs.SiccoN</th>
<th>Mean BCR Structural similarity</th>
<th>Mean structural similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>BCR</td>
<td>SiccoN</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$4.47 \times 10^{-07}$</td>
<td>0.34</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$9.77 \times 10^{-40}$</td>
<td>$1.62 \times 10^{-08}$</td>
<td>0.79</td>
</tr>
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<td>$1.52 \times 10^{-07}$</td>
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</tr>
<tr>
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<td>7</td>
<td>$6.02 \times 10^{-08}$</td>
<td>$6.98 \times 10^{-06}$</td>
<td>0.89</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
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<td>$4.40 \times 10^{-46}$</td>
<td>0.58</td>
</tr>
<tr>
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<td>$7.33 \times 10^{-40}$</td>
<td>$2.38 \times 10^{-49}$</td>
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</tr>
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</tr>
<tr>
<td></td>
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<td>$2.07 \times 10^{-49}$</td>
<td>0.96</td>
</tr>
<tr>
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<td>1</td>
<td>$4.60 \times 10^{-08}$</td>
<td>0.34</td>
<td>0.54</td>
</tr>
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<td>$1.72 \times 10^{-08}$</td>
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</tr>
<tr>
<td></td>
<td>7</td>
<td>0.97</td>
<td>$4.93 \times 10^{-12}$</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 5.7: $p$-values obtained using the Wilcoxon signed rank test for different prefix length

Additionally, the fourth column in Table 5.7 summarizes the statistical test results using the Wilcoxon signed-rank test for structural-similarity scores. The obtained $p$-values are below 0.05 in the majority settings of $K$. Hence, the null hypothesis can be rejected. This proves that the structural-similarity scores of the inferred LTSs using *EDSM-Markov* are higher than those obtained using *SiccoN*. Furthermore, the null hypothesis is accepted when the number of traces is one and $k = 1$ or $k = 3$. This indicates that there is no significant difference between the scores obtained using both learners.

### 5.3 Case Studies

In the previous section, the performance of the *EDSM-Markov* learner was evaluated on randomly generated LTSs. In this section, the performance of *EDSM-Markov* was evaluated on a number of case studies. For each of the following case studies, the number of traces ranged from 1 to 8, and the length of traces was given by $l \cdot |Q| \cdot |\Sigma|$, where $l$ is a parameter to vary the length of generated traces. In the conducted experiment, 30 different random traces were generated. In addition, $2 \cdot |Q|^2$ test sequences were generated.
of length $3 \times |Q|$. This was chosen to match the settings used in the conducted experiments using random LTSs.

### 5.3.1 Case Study: SSH Protocol

The first case study is the Secure Shell (SSH) protocol that is used for secure network connect and login services [140]. Poll and Schubert [141] showed a formal state-machine specification of SSH protocol that can be used for evaluating specification inference methods. In this case study, the number of states is 13 and alphabet size is 9, and number of transitions is 17.

The outcomes that are shown in Figure 5.24 represent the BCR scores achieved using different learners when $l = 0.3, 0.5, \text{ and } 1.0$ respectively for the SSH case study. As can be seen in Figure 5.24, the inferred LTSs using the EDSM-Markov learner are close to the reference LTSs in terms of their language. The BCR scores increase whenever the number of traces is increased and $k = 2 \text{ or } 3$; this is because the Markov tables tend to be complete and hence all labels of outgoing transitions that should be predicted are returned. It is clear that the EDSM-Markov learner performs badly when $k = 1$, especially if the number of traces is 4 or 8. In addition, in terms of language comparison, SiccoN inferred better LTSs compared to the EDSM-Markov $k = 1$ learner.

<table>
<thead>
<tr>
<th>$l$</th>
<th>Trace Number</th>
<th></th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.015</td>
<td>0.01</td>
<td>0.64</td>
<td>$4.04 \times 10^{-05}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.002</td>
<td>0.002</td>
<td>$1.82 \times 10^{-06}$</td>
<td>$2.63 \times 10^{-05}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.74</td>
<td>0.005</td>
<td>$9.75 \times 10^{-06}$</td>
<td>$2.27 \times 10^{-05}$</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.01</td>
<td>0.05</td>
<td>0.31</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>$3.48 \times 10^{-04}$</td>
<td>$1.02 \times 10^{-05}$</td>
<td>$3.01 \times 10^{-06}$</td>
<td>$1.76 \times 10^{-06}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.002</td>
<td>$3.04 \times 10^{-04}$</td>
<td>$4.04 \times 10^{-06}$</td>
<td>$1.56 \times 10^{-06}$</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.35</td>
<td>0.05</td>
<td>0.76</td>
<td>$3.60 \times 10^{-06}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>$3.98 \times 10^{-06}$</td>
<td>$1.60 \times 10^{-04}$</td>
<td>$2.73 \times 10^{-06}$</td>
<td>$1.72 \times 10^{-06}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>$6.31 \times 10^{-05}$</td>
<td>$1.01 \times 10^{-04}$</td>
<td>$1.99 \times 10^{-06}$</td>
<td>$1.80 \times 10^{-06}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: $p$-values obtained using the Wilcoxon signed-rank test of SSH protocol case study for BCR scores

Table 5.8 summarizes the resulting $p$-values from the Wilcoxon signed-rank statistical test for the BCR scores attained by learners. The null hypothesis $H_0$ states that the BCR scores
obtained by \textit{EDSM-Markov} and \textit{SiccoN} are the same. The \( p \)-values suggest rejection of
the \( H_0 \) when the number of traces is larger than 2 and \( k > 1 \); because the \( p \)-values are less
than 0.05, indicating that there is a statistically significant difference between them. As
mentioned in Table 5.8, when comparing \textit{EDSM-Markov} \( k=1 \) and \textit{SiccoN}, the \( H_0 \) can be
accepted when the number of traces is 4. This indicates there is no significant difference
between both learners.

The evaluation of \textit{EDSM-Markov} using random LTSs in Section 5.2 was shown to signifi-
cantly improve the structural-similarity scores of the inferred LTSs. In the SSH case study,
the structural-similarity scores obtained by \textit{EDSM-Markov} when \( k = 2 \) and 3 were higher
than those obtained by \textit{SiccoN} as shown in Figure 5.25. The low values of the structural-
similarity scores of LTSs inferred using \textit{SiccoN} means that the synthesized LTSs had extra
transitions that should be removed. In addition, the structural-similarity scores attained
by the \textit{EDSM-Markov} learner when \( k = 1 \) are worse than other learners. This contributed
to the low precision scores of the trained Markov model whereas many inconsistencies
are not detected by the \textit{EDSM-Markov} \( k = 1 \) learner. Hence, it is obvious that \textit{SiccoN}
generated LTSs better than \textit{EDSM-Markov} if \( k = 1 \). The reason behind this is that whenever
the \textit{EDSM-Markov} \( k = 1 \) learner merged a pair of states, new labels of the outgoing

\textbf{Figure 5.24:} BCR scores of SSH Protocol case study.
transitions were introduced where the Markov models predicted them wrongly.

In order to statistically measure the significant difference between the structural-similarity scores attained by EDSM-Markov and SiccoN, the null hypothesis $H_0$ is that the scores of LTSs obtained using EDSM-Markov and SiccoN are the same. The Wilcoxon signed-rank statistical test reported $p$-values less than 0.05 for structural-similarity scores in all considered comparisons as shown in Table 5.9. The $H_0$ can be rejected because the $p$-values are less than 0.05.

Figure 5.26 shows the accuracy of the trained Markov models using the precision/recall scores for different settings of prefix length $k$ and various numbers of traces. It is clear that the precision scores when $k = 1$ are very low compared to other settings of $k$. This explains why the EDSM-Markov learner performs worse than SiccoN when $k = 1$. Moreover, the precision scores are very high (above 0.8) when $k = 2$ or 3, and it significantly affects the BCR and structural-similarity scores. The inferred LTSs using EDSM-Markov learner are overgeneralized whenever the precision scores are very low (say below 0.5), and this occurs when $k = 1$. 
### Table 5.9: p-values obtained using the Wilcoxon signed-rank test of the structural-similarity scores for the SSH protocol case study

<table>
<thead>
<tr>
<th>$l$</th>
<th>EDSM-Markov k=1 vs. SiccoN</th>
<th>EDSM-Markov k=2 vs. SiccoN</th>
<th>EDSM-Markov k=3 vs. SiccoN</th>
<th>EDSM-Markov k=1 vs. SiccoN</th>
<th>EDSM-Markov k=2 vs. SiccoN</th>
<th>EDSM-Markov k=3 vs. SiccoN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>$1.81 \times 10^{-6}$</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$1.80 \times 10^{-6}$</td>
<td>$1.81 \times 10^{-6}$</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$1.80 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$6.37 \times 10^{-7}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$1.81 \times 10^{-6}$</td>
<td>$1.81 \times 10^{-6}$</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$1.80 \times 10^{-6}$</td>
<td>$1.80 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$6.34 \times 10^{-7}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>$1.79 \times 10^{-6}$</td>
<td>$2.69 \times 10^{-6}$</td>
<td>$1.72 \times 10^{-6}$</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$1.77 \times 10^{-6}$</td>
<td>$8.85 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.26:** Markov precision and recall scores of SSH Protocol case study

It was interesting to compute inconsistency scores for reference LTSs based on the trained Markov models using Equation 4.9. Figure 5.27 shows the boxplots of the number of inconsistencies computed after training Markov models.

With 4 traces, the performance of $EDSM-Markov k=3$ is not good as when the number of traces is 8. The BCR scores attained by $EDSM-Markov k=3$ are very high when the
number of traces is 4 or 8. This is because the number of inconsistencies computed for reference graphs was below 10. The number of inconsistencies was very high when \( k = 3 \) and the number of traces is 1 and 2, denoting that Markov models need to have extra training data to visit states using different paths in order to get higher BCR scores.

There is a relationship between the number of inconsistencies in reference graphs and the quality of the inferred LTS models. Generally, a very high inconsistency score means that a Markov table does not train well with respect to subsequences of events of length \( k + 1 \).
5.3.2 Case Study: Mine Pump

The second case study is the mine pump system that is introduced by Damas et al. [142] for the following requirement: the pump must be switched off whenever the water level is below a low threshold. Damas et al. [142] showed a simplified LTS specification of mine pump that can be used for evaluating LTS inference methods. In this case study, the number of states is 10, alphabet size is 8, and the number of transitions is 13.

![Figure 5.28: BCR scores of water mine pump case study](image)

Figure 5.28 illustrates the BCR scores of the inferred LTSs for the mine pump case study using EDSM-Markov and SiccoN, where different numbers of traces were considered. It is obvious from Figure 5.28 that the EDSM-Markov learner inferred LTSs with higher BCR scores in the majority of cases, especially when the number of traces was higher than 1 and $k > 1$. The EDSM-Markov $k = 1$ learner did not learn LTSs well compared to SiccoN if the number of traces was 4 or 8. This was because the accuracy of the trained Markov model when $k = 1$ was not good compared to $k = 2$ or $k = 3$. It is apparent from Figure 5.28 that the SiccoN learner performs well on heavily-branching traces compared to the EDSM-Markov learner when $k = 1$. It is interesting to note that EDSM-Markov $k=2$ and EDSM-Markov $k=3$ learners inferred LTSs with BCR scores much higher than those obtained...
using *SiccoN* even if there are 1 or 2 traces considered. This indicates that the trained Markov models predicted labels of outgoing transitions well during the process of merging states.

Additionally, *SiccoN* performed well when the number of traces was 8, in contrast to when it was 1 or 2. This is because *SiccoN* is going to infer LTSs well whenever the traces are heavily branched, and this interprets why *SiccoN* generates LTSs of the mine pump case study with BCR scores close to those inferred by *EDSM-Markov k=2* and *EDSM-Markov k=3*.

Table 5.10 shows the reported *p*-values of BCR scores obtained from the Wilcoxon signed-rank statistical test. The null hypothesis $H_0$ is that there is no significant difference between the BCR scores of the inferred LTS using *EDSM-Markov* and *SiccoN*. The resulting *p*-values were less than 0.05. Therefore, the $H_0$ could be rejected. It is clear that there was a significant difference between *SiccoN* and *EDSM-Markov* when $k = 1$ if the number of traces was 8, indicating that *SiccoN* performed better than *EDSM-Markov k=1*. On the other hand, the null hypothesis was accepted if the number of traces was 4, denoting that there was no significant difference between *SiccoN* and *EDSM-Markov k=1*.

<table>
<thead>
<tr>
<th>$l$</th>
<th>Trace Number</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.52</td>
<td>0.03</td>
<td>0.75</td>
<td>2.18 × 10^{-05}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.06</td>
<td>0.002</td>
<td>7.33 × 10^{-04}</td>
<td>9.55 × 10^{-07}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.017</td>
<td>8.40 × 10^{-04}</td>
<td>0.003</td>
<td>9.55 × 10^{-07}</td>
</tr>
<tr>
<td>0.5</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.002</td>
<td>5.82 × 10^{-04}</td>
<td>0.40</td>
<td>7.76 × 10^{-04}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.003</td>
<td>2.58 × 10^{-05}</td>
<td>5.33 × 10^{-05}</td>
<td>1.16 × 10^{-06}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.06</td>
<td>0.003</td>
<td>7.08 × 10^{-05}</td>
<td>1.16 × 10^{-06}</td>
</tr>
<tr>
<td>1.0</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>0.003</td>
<td>0.024</td>
<td>0.39</td>
<td>2.98 × 10^{-05}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>2.13 × 10^{-04}</td>
<td>5.03 × 10^{-05}</td>
<td>2.60 × 10^{-06}</td>
<td>1.14 × 10^{-06}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>1.29 × 10^{-04}</td>
<td>1.81 × 10^{-06}</td>
<td>2.58 × 10^{-06}</td>
<td>1.14 × 10^{-06}</td>
</tr>
</tbody>
</table>

Table 5.10: *p*-values of Wilcoxon signed rank test of water mine case study for BCR scores

Figure 5.29 shows the structural-similarity scores of the mined LTSs for the water mine pump case study. The outcomes that are shown in Figure 5.29 support the hypothesis that *EDSM-Markov* generates LTSs models that are structurally very similar to the reference LTS compared to those models inferred using *SiccoN* when $k = 2$ and 3. The
structural-similarity scores of the inferred LTSs using SiccoN were low, denoting that the synthesized LTSs were over-generalized. Furthermore, the structural-similarity scores of the inferred LTSs using the EDSM-Markov learner were worse than other learners when $k = 1$; this is attributed to the low precision scores of the trained Markov model.

Besides, it is apparent from Figure 5.29 that SiccoN inferred LTSs better than EDSM-Markov if $k = 1$, and this was because the EDSM-Markov learner predicted labels of outgoing transitions incorrectly. In terms of measuring the performance of the EDSM-Markov when $k = 2$ and 3 on structural-similarity scores of the inferred models, it is evident that EDSM-Markov identifies LTSs of higher structural-similarity scores, as shown in Figure 5.29.

Table 5.11 summarizes the $p$-values obtained from the Wilcoxon signed-rank statistical test for the mine pump case study. The null hypothesis $H_0$ is that the structural-similarity values of EDSM-Markov and SiccoN are not significantly different. The test reported $p$-values for structural-similarity values less than the 0.05 significance level in all numbers of traces considered. Therefore, the $H_0$ could be rejected, and this means that the structural-similarity values of both EDSM-Markov and SiccoN were significantly different. However,
The $H_0$ was accepted if the number of traces was 2 and $l = 0.5$ when the structural-similarity scores for the mined LTSs using $EDSM$-Markov $k=1$ were compared to the scores attained by Sicco$N$, and this suggested that there was no significant difference between the structural-similarity scores.

<table>
<thead>
<tr>
<th>$l$</th>
<th>Trace Number</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>$1.25 \times 10^{-05}$</td>
<td>$1.04 \times 10^{-04}$</td>
<td>0.004</td>
<td>$1.78 \times 10^{-04}$</td>
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<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.04</td>
<td>$3.09 \times 10^{-04}$</td>
<td>$1.79 \times 10^{-06}$</td>
<td>$1.03 \times 10^{-06}$</td>
</tr>
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<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.001</td>
<td>$1.16 \times 10^{-05}$</td>
<td>$1.78 \times 10^{-06}$</td>
<td>$1.03 \times 10^{-06}$</td>
</tr>
<tr>
<td>0.5</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>$1.13 \times 10^{-05}$</td>
<td>0.10</td>
<td>0.01</td>
<td>$1.52 \times 10^{-05}$</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>$3.40 \times 10^{-05}$</td>
<td>$1.82 \times 10^{-06}$</td>
<td>$1.80 \times 10^{-06}$</td>
<td>$1.17 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>$1.17 \times 10^{-05}$</td>
<td>$2.47 \times 10^{-06}$</td>
<td>$1.80 \times 10^{-06}$</td>
<td>$1.17 \times 10^{-06}$</td>
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<td>1.0</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>$1.53 \times 10^{-05}$</td>
<td>$1.91 \times 10^{-04}$</td>
<td>0.002</td>
<td>$8.12 \times 10^{-05}$</td>
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<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>$1.07 \times 10^{-05}$</td>
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<td>$1.77 \times 10^{-06}$</td>
<td>$1.16 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>$4.00 \times 10^{-06}$</td>
<td>$1.81 \times 10^{-06}$</td>
<td>$1.77 \times 10^{-06}$</td>
<td>$1.16 \times 10^{-06}$</td>
</tr>
</tbody>
</table>

Table 5.11: p-values of Wilcoxon signed rank test of water mine case study for structural-similarity Scores

The precision and recall scores of the Markov models were computed during the conducted experiments. The intention behind computing this is to study the influence of Markov models on the accuracy of the inferred LTSs. Figure 5.30 illustrates the precision/recall scores of the trained Markov models for different settings of prefix length $k$, and a varied number of traces were considered. It can be seen from Figure 5.30 that the precision scores of the trained Markov models when $k = 1$ were very low compared to other settings of $k$, and this explains why the $EDSM$-Markov learner performed worse than Sicco$N$ when $k = 1$. The $EDSM$-Markov learner over-generalized whenever the precision score was very low (say below 0.5), and this happened if $k = 1$. It is noticed that the precision scores were very high (above 0.8) when $k = 2$ or 3 and it had a significantly positive effect on the BCR and structural-similarity scores.

Figure 5.31 shows the number of inconsistencies computed for the reference LTS of the water mine case study after training Markov models. In case where $k = 3$, the mean value of the BCR scores of the inferred LTSs using $EDSM$-Markov was higher than (say 0.95) when the number of traces was 4 or 8. This can be attributed to the low inconsistency score in this case, as shown in Figure 5.31. In contrast, low BCR scores of the inferred LTSs using $EDSM$-Markov were achieved if the number of traces was very small and $k = 3$;
5.3.3 Case Study: CVS Client

The third case study is concurrent versions system (CVS) protocol that is proposed by Lo and cheng Khoo [143]. They used it to evaluate their state machine inference approach. The functionalities of CVS implementation are incorporated with the FTP package that is included by Jakarta Commons Net library [143]. In this case study, the number of states is 15, and the alphabet size is 15, and the number of transitions is 27.

The BCR results of LTSs that were inferred using the considered miners are shown in Figure 5.32. It is interesting to note that $EDSM$-$Markov$ $k=1$ inferred LTSs with better BCR scores compared to other learners if the number of traces was 1 or 2. Moreover, the $EDSM$-$Markov$ $k=3$ performed badly since Markov models did not train well and many labels of outgoing transitions were miss-predicted after merging states, which resulted in
large inconsistency scores that led to block state-merge wrongly. In this case study, SiccoN performed well as long as the number of traces increased. The BCR scores of the inferred models using EDSM-Markov $k=2$ were very similar to SiccoN.

Table 5.12: $p$-values of Wilcoxon signed rank test of CVS case study for BCR scores

<table>
<thead>
<tr>
<th>$L$</th>
<th>Trace Number</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>EDSM-Markov $k=1$ vs. SiccoN</td>
<td>$1.03 \times 10^{-04}$</td>
<td>0.01</td>
<td>0.005</td>
<td>0.503</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov $k=2$ vs. SiccoN</td>
<td>0.856</td>
<td>0.466</td>
<td>0.51</td>
<td>0.473</td>
</tr>
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<td>EDSM-Markov $k=3$ vs. SiccoN</td>
<td>0.02</td>
<td>$3.25 \times 10^{-03}$</td>
<td>$1.20 \times 10^{-04}$</td>
<td>0.004</td>
</tr>
<tr>
<td>0.5</td>
<td>EDSM-Markov $k=1$ vs. SiccoN</td>
<td>$3.18 \times 10^{-04}$</td>
<td>0.012</td>
<td>0.96</td>
<td>$3.85 \times 10^{-04}$</td>
</tr>
<tr>
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<td>EDSM-Markov $k=2$ vs. SiccoN</td>
<td>0.68</td>
<td>0.88</td>
<td>0.005</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov $k=3$ vs. SiccoN</td>
<td>$4.25 \times 10^{-05}$</td>
<td>$1.48 \times 10^{-04}$</td>
<td>$2.02 \times 10^{-06}$</td>
<td>0.001</td>
</tr>
<tr>
<td>1.0</td>
<td>EDSM-Markov $k=1$ vs. SiccoN</td>
<td>0.001</td>
<td>0.08</td>
<td>0.10</td>
<td>$4.92 \times 10^{-06}$</td>
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<tr>
<td></td>
<td>EDSM-Markov $k=2$ vs. SiccoN</td>
<td>0.57</td>
<td>0.20</td>
<td>0.35</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov $k=3$ vs. SiccoN</td>
<td>$7.14 \times 10^{-04}$</td>
<td>0.002</td>
<td>0.016</td>
<td>$6.06 \times 10^{-04}$</td>
</tr>
</tbody>
</table>

Table 5.12 shows the $p$-values resulting from the Wilcoxon signed-rank statistical test for
Figure 5.32: BCR scores of CVS protocol case study

BCR scores. The null hypothesis $H_0$ is that the BCR scores of $EDSM$-$Markov$ and $SiccoN$ are the same. The resulting $p$-values obtained by comparing the BCR scores of $EDSM$-$Markov$ learners and $SiccoN$ were larger than 0.05, so the $H_0$ could be accepted as shown in Table 5.12. In cases where the $p$-value was higher than 0.5, it denotes that there was no significant difference between learners. For instance, $EDSM$-$Markov$ $k=1$ did not show significant improvement when it compared with $SiccoN$ in the case where the number of traces was 8 and $l = 0.3$.

Figure 5.33 shows the structural-similarity scores of the inferred LTSs for CVS case study. The findings that are shown in Figure 5.33 show that $EDSM$-$Markov$ $k=1$ generated LTS models with very low structural-similarity scores compared to other learners. This is because $EDSM$-$Markov$ $k=1$ inferred over-generalized LTSs. In addition, the structural-similarity scores of the inferred LTSs using the $EDSM$-$Markov$ learner when $k = 1$ were
worse than other learners. It is clear from Figure 5.33 that SiccoN inferred LTSs better than EDSM-Markov if \( k = 1 \). This was because the EDSM-Markov learner predicted labels of outgoing transitions incorrectly. Moreover, the structural-similarity scores of the inferred models EDSM-Markov learner when \( k = 2 \) and 3 were higher than SiccoN in some cases, especially when the number of traces was 1 or 2, as shown in Figure 5.33.

Figure 5.34 illustrates the precision/recall scores of the trained Markov models for CVS case study. It is clear from Figure 5.34 that the precision scores of the trained Markov models were very high in all settings of \( k \). This may explain why the EDSM-Markov learner generated LTSs with high BCR scores when \( k = 1 \).

Figure 5.35 shows the number of inconsistencies computed for the reference LTS of CVS case study after training Markov models. In general, a very low inconsistency score means that a Markov table does not train well with respect to subsequences of length \( k + 1 \). One
Table 5.13: p-values of Wilcoxon signed rank test of CVS case study for structural-similarity scores

<table>
<thead>
<tr>
<th>l</th>
<th>Trace Number</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>6.93 \times 10^{-05}</td>
<td>1.42 \times 10^{-04}</td>
<td>2.97 \times 10^{-05}</td>
<td>1.82 \times 10^{-06}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.001</td>
<td>9.22 \times 10^{-06}</td>
<td>0.04</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.008</td>
<td>0.134</td>
<td>0.018</td>
<td>0.21</td>
</tr>
<tr>
<td>0.5</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>1.68 \times 10^{-06}</td>
<td>6.15 \times 10^{-08}</td>
<td>2.61 \times 10^{-08}</td>
<td>1.86 \times 10^{-09}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>0.001</td>
<td>0.003</td>
<td>0.114</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.35</td>
<td>0.10</td>
<td>0.90</td>
<td>0.55</td>
</tr>
<tr>
<td>1.0</td>
<td>EDSM-Markov k=1 vs. SiccoN</td>
<td>1.22 \times 10^{-05}</td>
<td>2.61 \times 10^{-08}</td>
<td>3.73 \times 10^{-09}</td>
<td>1.82 \times 10^{-06}</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=2 vs. SiccoN</td>
<td>4.97 \times 10^{-05}</td>
<td>0.01</td>
<td>0.39</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>EDSM-Markov k=3 vs. SiccoN</td>
<td>0.001</td>
<td>0.04</td>
<td>0.83</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Figure 5.34: Markov precision and recall scores of water mine case study

can observe that when $k = 3$, the mean value of the BCR scores for the inferred LTSs using $EDSM$-$Markov$ was below 0.95, for example, in the majority cases, except when $l = 1.0$ and the number of traces was 8; low BCR scores of the learnt LTSs when $k = 3$ is due to that the number of inconsistencies were very high.
Chapter 5. Experimental Evaluation and Case Studies of EDSM-Markov

Figure 5.35: Inconsistencies of CVS case study
5.4 Discussion

As mentioned in the introduction chapter, one of the aims in this thesis is to improve the existing techniques to tackle the over-generalization problem. In this chapter, experimental assessment of the performance of EDSM-Markov has been achieved using random LTSs and case studies. The performance of EDSM-Markov has been studied from different aspects.

In Section 5.2, different questions were introduced in order to answer them after evaluating the performance of the proposed learner. The first question is about the effect of the number of traces on the performance of EDSM-Markov. The findings obtained from the conducted experiments based on both random LTSs and case studies demonstrate that the efficiency of the EDSM-Markov learner can be improved while the number of traces increases. This is intuitive because the quality of the inferred state machine models depend on the number of the provided traces, as stated by Walkinshaw and Bogdanov [77]. With respect to our findings, EDSM-Markov fails to generalize LTSs well if the provided traces are insufficient.

Another important finding is that EDSM-Markov improves the accuracy of the inferred LTSs when the alphabet size are very large. This is because Markov precisions tend to be high in such settings and this helps to make correct mergers during the inference process. The results from the conducted experiments using the random LTSs demonstrate that with few long traces available, the Markov-EDSM learner can improve the accuracy of the inferred LTSs with large alphabets. This addresses the research question that concerned about how well EDSM-Markov infers LTSs with large alphabets.

One of the more significant findings in this chapter is reasonable results were obtained if the supplied traces were covered transitions in the reference LTSs well. Moreover, with shorter traces provided, a poorer performance of the EDSM-Markov learner is expected and this is because inferring the exact state-machine models from such traces are difficult.

It is important to highlight that the performance of EDSM-Markov are affected by the setting of prefix length \( k \). In the conducted experiments with random LTSs, EDSM-Markov performed good when \( k = 2 \) compared to \( k = 3 \). The bad performance of EDSM-Markov when \( k = 3 \) is because the provided traces are very sparse and Markov models
do not predict labels of outgoing transitions from a state to follow prefix paths of length 3. Surprisingly, for the water mine pump and ssh case studies, \textit{EDSM-Markov} tends to perform well if $k = 3$ and the number of traces is 8.

One interesting finding is that whenever the computed inconsistency scores for the reference LTSs after training the Markov models are too small, the \textit{EDSM-Markov} learner can infer reasonable LTSs on the condition that the precision scores of the trained Markov models are high.

It was observed that \textit{EDSM-Markov} learner performed better than \textit{SiccoN} in the majority of cases. This is possibly due to the clever idea of \textit{EDSM-Markov} that relies on training Markov models and the \textit{IScore} heuristic that, unlike \textit{SiccoN} that blocks mergers. Moreover, \textit{EDSM-Markov} calculates an inconsistency score based on global view of dependencies between events (elements of an alphabet) in the traces in order to block mergers, and this differs from the way that \textit{SiccoN} blocks mergers based on the local similarity of labels of outgoing transitions.

Over-generalization is a known problem in the grammar inference domain. In their study, Lo et al. [58] stated that the over-generalization during the inference of state machines should be controlled using the negative traces. However, the findings in this chapter prove that \textit{EDSM-Markov} can control the over-generalization. This is due to the idea of inconsistencies that help \textit{EDSM-Markov} to determine when a merger of states introduces new labels of outgoing transitions that are prohibited to follow states based on Markov predictions.

An approximate identification of state machines from sparsely training data is possible as claimed by [113]. The \textit{EDSM-Markov} learner has proven to generate good LTSs on the condition that the alphabet size is very large.

To conclude, the conducted experiments agreed to the fact that the inference of automaton from only positive traces is a difficult task. As stated by Chen and Roşu [144], the inference of state-machine specification from positive traces is hard, it has been shown as a major limitation of their works. In our study using random LTSs, a similar difficulty was observed in the conducted experiments when few traces were provided and whenever the alphabet size was $|\Sigma| = 0.5 \times |Q|$. 
5.5 Threats to Validity

1. **The randomly generated LTSs may not represent real-world models.**
   
   This threat is mitigated by evaluating the performance of EDSM-Markov on case studies that are used in the literature review and the survey of stamina competition [35].

2. **The selection of training data may not be representative.**
   
   One possible threat to validity is the selection of training data where they might not be representative. For random LTSs, this threat is mitigated against it, each LTS was attempted to infer 5 times with different paths generated each time since random traces may follow the same paths many times. When the performance of EDSM-Markov was evaluated on case studies, 30 different sets of random traces were used in order to mitigate this threat.

3. **The size of case studies may be limited.** One possible threat to validity is that the sizes of case studies are small. The performance of EDSM-Markov was evaluated using case studies that are widely used in state machine inference papers. This threat is mitigated by evaluating EDSM-Markov using random LTSs of different sizes.

4. **The parameters settings may bias the results.** In the conducted experiments, there are many parameters such as the number and the length of traces, alphabet size, and prefix length. Another potential threat to validity is that the variance of such parameters may bias the results. This threat is mitigated by choosing a different multiplier to vary the parameters. For instance, the length multiplier $l$ was introduced to assess the performance of EDSM-Markov on different length of traces. Moreover, the alphabet multiplier $m$ was chosen to vary the alphabet size to assess the performance of EDSM-Markov on different types of LTSs.

5.6 Conclusions

The EDSM-Markov learner was proposed in the previous chapter, and it is a new method of LTS inference using state merging based on computing inconsistencies scores from the Markov models. This chapter evaluated the performance of the EDSM-Markov learner
from different dimensions such as the ability to infer LTSs from various lengths of traces and sizes of alphabets. The purpose of the practical investigation was to determine the effect of EDSM-Markov on solving the research problem, which is inferring LTSs from few positive training data.

The evaluation of the performance of EDSM-Markov has demonstrated the capability to improve upon SiccoN on the condition that the alphabet size is very large and the provided traces cover transitions well.

The Markov learner that was proposed by Cook and Wolf [49] are not publicly available. There is no information in the literature about the algorithm of the Markov learner. Therefore, the comparison against their algorithm [49] are not difficult. Their ideas rely on predictions using the first-order and second-order Markov models to build the event graph. In the conducted experiments, the mean precision score of the second-order Markov models was 65.6 when the number of traces was five. This showed that there is a kind of over-generalisation in the inferred model using the Markov learner that was proposed by Cook and Wolf [49]. However, the inferred models using EDSM-Markov prevents the over-generalisation problem as shown in the earlier sections.
6

Improvements to the QSM Algorithm

This chapter focuses on inferring LTS models from abstracted positive traces with the aid of the active learning concept. In the previous chapter, the inference of LTS models using passive methods was limited in some cases. This is due to the fact that passive techniques require a high coverage of the system to infer LTS models well. This chapter presents ModifiedQSM and MarkovQSM algorithms that are designed to infer LTS models from positive training data using membership queries only. The basic idea behind this is to improve the accuracy of the inferred LTSs using membership queries that are asked to avoid bad state mergers.

6.1 Introduction

Passive LTSs inference from an incomplete set of samples cannot guarantee the generation of complete models. This is because the prior set of samples may not cover every aspect of
the system under inference (learn), and this explains why passive techniques fail to infer the exact models. Besides, collecting all requisite samples in advance to infer a correct state machine can be expensive [100].

As stated in Chapter 3, active learning techniques of state machine models are very efficient to accurately learn them. In the context of inferring LTS models from the samples, Dupont et al. [36] showed that the idea of state-merging techniques can be integrated with the concept of active learning. Thus, the QSM learner was developed by Dupont et al. [36]. It relies upon the provided samples that may not be characteristic and aims to infer the exact models.

The QSM learner infers LTS models by asking membership queries to an oracle. The submitted questions are considered as new samples (abstracted traces) that explore more behaviours of the system under inference. Once the answers are obtained, the answered queries are added to the initial collection of samples during the inference process. Moreover, the learning process is restarted whenever the answered query is contradicted with the merged automaton.

The improvement that can be achieved by the QSM learner against passive state-merging techniques is due to the fact that QSM asks queries to an oracle to gather information about the language of the target LTSs. Furthermore, the queries that are asked during the learning session of LTSs are aimed at preventing bad generalizations (merging non-equivalent states in the hidden LTS model). Those queries extend the prior knowledge about the behaviour of the system being learnt. Therefore, the answered queries are fed again into the current automaton to explore how a system under inference behaves.

The performance of the QSM learner was studied earlier in this thesis in section 3.5, we noticed that the accuracy of the learnt machines was not good enough if the alphabet size was very large. In this chapter, we developed the ModifiedQSM and MarkovQSM learners to improve the accuracy of the inferred LTSs. The performance of ModifiedQSM and MarkovQSM were evaluated by a series of experiments using randomly generated labelled-transition systems.
6.2 The Proposed Query Generators

In this section, the membership query generator of the ModifiedQSM and MarkovQSM algorithms is described. It contains two sub-generators that are designed to work together as one generator. The two sub-generators are described in Sections 6.2.1 and 6.2.2. The following definitions are introduced before describing the generators of membership queries.

The set of sequences that lead to a state $q$ from the initial state $q_0$ is defined in Definition 6.1.

**Definition 6.1.** Given a state $q \in Q$ and the current automaton $(A)$. $\text{Seq}(q) = \{ w \in L(A) | \hat{\delta}(q_0, w) = q \}$.

The shortest sequences that lead to a state $q$ from the initial state $q_0$ are defined in Definition 6.2, denoted by $S_p(q)$. The shortest sequences of the state $S_p(q)$ are a subset of the short prefixes of the language $S_p(L)$ that is identified by the automaton $A$.

**Definition 6.2.** Given a state $q \in Q$, let $\text{Seq}(q)$ denote the set of sequences that lead to a state $q$ from the initial state $q_0$, and the current automaton $(A)$. A sequence $w$ that belongs to the $\text{Seq}(q)$ is said to be the shortest sequence if there is no other sequence $y \in \text{Seq}(q)$ where the length of $y$ is shorter than $w$. $S_p(q) = \{ w \in \text{Seq}(q) | \exists y \in \text{Seq}(q) \backslash \{ w \} \text{ such that } |w| > |y| \}$.

6.2.1 Dupont’s QSM Queries

The main generator of membership queries in the QSM algorithm was introduced by Dupont et al. [36] and is called the Dupont generator in this thesis. The Dupont generator is responsible for the generation of queries about new scenarios (sequences) that appear as a consequence of merging states. In other words, it is asked about sequences that belong to the language of the merged automaton but do not belong to the language of the current solution (LTS hypothesis). The objective of asking these queries is to prevent bad generalizations (state merging) of the inferred models [36]. Hence, it is considered as an essential (main) generator in ModifiedQSM and MarkovQSM.

Let $\text{Suff}(q_b, A)$ denotes the set of suffixes of the blue state $q_b$ in the current automaton $(A)$. The Dupont generator constructs the membership queries by first collecting the shortest
sequences that lead to the red state $q_r$ from the root state $(q_0)$ in $A$, denoted by $S_p(q_r)$. The membership queries are generated by concatenating each sequence belonging to $S_p(q_r)$ with each suffix belonging to $Suff(q_b, A)$ and not to $Suff(q_r, A)$. A generated membership query is a sequence obtained by concatenating two sequences $s \cdot y$ such that $s \in S_p(q_r)$ and $y \in Suff(q_b, A)$. Thus, the generated query $s \cdot y$ belongs to $L(A')$ and does not belong to $L(A)$. The way of constructing Dupont queries is defined in Definition 6.3.

**Definition 6.3.** Given a pair of red/blue states $(q_r, q_b) \in Q$, the current automaton $(A)$, and the merged automaton $(A')$. The Dupont queries is defined by:

$$\text{Dupont queries} = \{ s \cdot y \mid s \in S_p(q_r), y \in Suff(q_b, A) \} \text{ such that } s \cdot y \in L(A') \setminus L(A).$$

The following two examples show how to construct the membership query for a recursive and non-recursive merge of states respectively.

![Diagram](Figure 6.1)

**Figure 6.1:** The first example of computing the Dupont queries

**Example 6.1.** Figure 6.1(a) shows the current automaton during the induction process. Let us consider that the $B$ state is chosen to merge with the $A$ state. The shortest sequence that leads to the red state $A$ are empty, denoted by $S_p(q_r) = \{\epsilon\}$. The $Suff(q_b, A)$ set contains the following sequences: $\{\langle Edit, Edit, Save, Close \rangle, \langle Edit, Save \rangle, \langle Close, Load \rangle\}$. The Dupont queries queries are generated by concatenating the $S_p(q_r)$ with each suffix in the $Suff(q_b, A)$ set as described in Definition 6.3. In this way, the following membership
queries are generated: \( \text{Dupontqueries} = \{ \langle \text{Edit, Edit, Save, Close} \rangle, \langle \text{Edit, Save} \rangle, \langle \text{Close, Load} \rangle \} \). It is clear that the generated queries belong to \( L(A') \), which is shown in Figure 6.1(b), and do not belong to \( L(A) \).

**Example 6.2.** Figure 6.2(a) illustrates the current automaton during the inference process. Consider that the \( G \) state is chosen to merge with the \( C \) state. Figure 6.2(a) shows the merged automaton \( (A') \) computed by merging the chosen pair of states. The shortest sequence that leads to the red state \( C \) is \( S_{p}(q_{r}) = \langle \text{Load, Edit} \rangle \). The \( \text{Suff}(q_{b}, A) \) set contains the following sequences: \( \{ \langle \text{Save, Close} \rangle \} \). The \( \text{Dupontqueries} \) queries are generated by concatenating the \( S_{p}(q_{r}) \) with each suffix in the \( \text{Suff}(q_{b}, A) \) set as described in Definition 6.3. In this way, the following membership queries are generated: \( \text{Dupontqueries} = \{ \langle \text{Load, Edit, Save, Close} \rangle \} \). It is noticed that the query belongs to \( L(A') \) and does not belong to \( L(A) \).

![Diagram of the current automaton](image-a)

![Diagram of the merged automaton](image-b)

**Figure 6.2:** The second example of computing the \( \text{Dupontqueries} \)

### 6.2.2 One-step Generator

The second generator of membership queries is called *one-step*. It is motivated by the observation that the membership queries that are constructed using the \( \text{Dupont} \) generator are insufficient to prevent merging inequivalent pairs of states. The example below shows...
that the Dupont generator does not generate any query. It is important to highlight that the one-step queries are only present in the ModifiedQSM and MarkovQSM.

**Example 6.3.** Consider the automaton that is shown in Figure 6.3, and suppose that the C state is chosen to merge with the B state. The Dupont generator will not generate any queries since merging of states will not add new scenarios to the merged (red) node. There is no label that will be added to the red state if the EDSM merges them. In this case, the set of Dupont queries is empty. The following example shows that the pair of states (B, C) are compatible for merging using the QSM learner because they are both accepting states. However, they are inequivalent with respect to the language of the reference LTS.

![Figure 6.3: An example of computing the one-step generator](image)

It is interesting to consider extra membership queries in order to detect incompatible pairs of states to avoid merging them. Thus, one way is to ask about the labels of the outgoing transitions of a red state that lead to an accepting state; where those labels are not overlapped with the labels of outgoing transitions of a blue state. In other words, labels (elements of alphabet) of the outgoing transitions that lead to an accepting state where they belong to \( \Sigma_{q_r}^{out} \) and do not belong to \( \Sigma_{q_b}^{out} \) are asked from the blue state. It is inspired by the notion of the k-tails algorithm in which a pair of states are deemed to be equivalent if they share the same suffixes of length \( k \). It is worth mentioning that k-tails suffixes are leading to accepting states.

The one-step generator constructs queries by collecting the shortest sequences from the root state to the blue node \( S_p(q_b) \) to pick one of them. Then, the shortest sequence \( c \in S_p(q_b) \) is concatenated with each label of the outgoing transitions of the red state that lead to an accepting state, but there is no transition to emerge from the blue state with the same label. The construction of the one-step queries is defined formally in Definition 6.4.
Definition 6.4. Given a pair of red/blue states \((q_r, q_b) \in Q\) and the current automaton \((A)\). The one-step queries is defined by \((oq) = \{s \cdot \langle \sigma \rangle \mid s \in S_p(q_b), \sigma \in \Sigma_{q_b}^{\text{out}} \setminus \Sigma_{q_r}^{\text{out}} \land q' \in \delta(q_r, \sigma)\) such that \(q' \in F^+\).

Example 6.4. Let us consider the pair of states that is shown in Figure 6.3 above, and suppose that the \(C\) state is chosen to merge with the \(B\) state. The shortest path to the blue state from the root state is \(S_p(C) = \langle \text{Load}, \text{Edit} \rangle\). The \(\Sigma_{q_r}^{\text{out}} \setminus \Sigma_{q_b}^{\text{out}}\) contains only one label as follows: \(\{\text{Close}\}\). In this example, the one-step generator results in only one membership by concatenating \(S_p(C)\) with the \(\text{Close}\) label. This yields the following query: \(\text{onestep queries} = \{(\text{Load}, \text{Edit}, \text{Close})\}\)

6.3 The Modified QSM

In the original QSM [36], the RPNI learner computes a quotient automaton \((A_{\text{new}})\) that is obtained by merging the chosen pair of states from the current automaton \((A)\). After that, the QSM algorithm asks membership queries about new scenarios that are considered new for \(A\), but not for \(A_{\text{new}}\). Dupont et al. [36] modified the strategy of selecting pairs of states by adapting the EDSM and blue-fringe methods. In addition, the incorporation of the EDSM and blue-fringe methods in QSM leads to a reduction in the number of membership queries consumed by QSM compared to the states-selection procedure using the RPNI learner [36]. The QSM is described in detail in chapter 3. In this section, the ModifiedQSM algorithm is presented, and aims to tackle the large number of queries produced by the QSM and increases the accuracy of the inferred LTS. In other words, it aims to obtain better generalization of LTS compared to the QSM learner.

The ModifiedQSM is an adaptive algorithm of the EDSM learner and is designed to make it an active learning. Membership queries are asked during the computation of the EDSM score for possible pairs of states. The reason behind this is to detect incompatible pairs of states and prevent merging them. In comparison with the original QSM, the benefit of asking membership queries at that stage is that no restart of the learning process is required. It differs from the original QSM that restarts learning an LTS model whenever a membership query is answered as negative.
input: \( S^+, S^- \)

/* Sets of accepted \( S^+ \) and rejected \( S^- \) sequences */

result: \( A \) is an LTS that is compatible with \( S^+, S^- \), and generated queries

\[
\begin{align*}
A &\leftarrow \text{generatePTA}(S^+, S^-); \\
R &\leftarrow \{q_0\}; \\
\text{do} &
\begin{array}{l}
\text{do} &
\begin{array}{l}
\text{PossiblePairs} \leftarrow \emptyset; \\
\text{Rextended} \leftarrow \text{false}; \\
B &\leftarrow \text{ComputeBlue}(A, R); \\
\text{for } q_b \in B \\
\text{mergeable} &\leftarrow \text{false}; \\
\text{compatible} &\leftarrow \text{false}; \\
\text{for } q_r \in R \\
\text{compatible} &\leftarrow \text{checkMergeCompatibility}(A, q_r, q_b); \\
\text{if } \text{compatible} \\
\text{Queries} &\leftarrow \text{generateDupontQueries}(A, q_r, q_b); \\
\text{Queries} &\leftarrow \text{Queries} \cup \text{generateOneStepQuery}(A, q_r, q_b); \\
\text{if } \text{EDSMScore}(A, q_r, q_b) \geq 0 \\
\text{PossiblePairs} &\leftarrow \text{PossiblePairs} \cup \{(q_r, q_b)\}; \\
\text{mergeable} &\leftarrow \text{true}; \\
&\end{array}
\end{array}
\text{end}
\text{end}
\text{end}
\text{while } \text{Rextended} = \text{true};
\text{if } \text{PossiblePairs} \neq \emptyset \\
(q_r, q_b) &\leftarrow \text{PickPair}(\text{PossiblePairs}); \\
\text{if } \text{EDSMScore}(A, q_r, q_b) \geq 0 \\
A &\leftarrow \text{merge}(A, q_r, q_b); \\
&\end{array}
\text{end}
\text{while } \text{PossiblePairs} \neq \emptyset;
\text{return } A
\end{align*}
\]

Algorithm 12: The ModifiedQSM algorithm

The inference process of an LTS using ModifiedQSM is described in Algorithm 12. Similar to the original QSM, ModifiedQSM first constructs a PTA from the provided positive samples of input sequences, and this process is denoted by the \text{generatePTA}(S^+, S^-) function in Line 1. Then, the traditional blue-fringe strategy is called to start the inference process
by colouring the root state red and all neighbouring states blue. The $\text{ComputeBlue}(A, R)$ function is called to colour the adjacent states of the red states blue.

The loop in Lines 8-27 is the selection of pairs of states in the $\text{ModifiedQSM}$ algorithm based on the blue-fringe strategy. It starts by iterating through the current blue $B$ states in order to evaluate their suitability for merging with the red states. Next, for each possible pair of states, the compatibility of the pair is checked using the $\text{checkMergeCompatibility}(A, r, b)$ function as shown in Line 12. The pair of states $(r, b)$ is said to be compatible if both states are either accepting or rejecting. Moreover, the $\text{checkMergeCompatibility}(A, r, b)$ function checks the compatibility of states that would be merged recursively as well. If the pair of states are incompatible, then no queries will be asked in this case. Otherwise, membership queries are generated to avoid bad state merges.

The next stage is to construct membership queries in order to check the compatibility of the pair of states based on queries to detect the incompatible ones and avoid merging them. A list of membership queries is generated as described earlier in this chapter, and this is denoted by the $\text{generateDupontQueries}(A, r, b)$ and $\text{generateOneStepQuery}(A, r, b)$ functions.

Having generated a list of membership queries for a pair of states in Lines 14-15, the $\text{processQueries}(A, r, b, \text{Queries})$ function is called to answer queries one by one by submitting them to an oracle. Once a query is answered, it is added to the current automaton $A$, and the compatibility of pairs is checked by computing the $\text{EDSM}$ score. It is important to say that the process of asking and answering queries can be terminated when the pair of states is proven to be incompatible, even if there are remaining ones that have not been answered yet. The process of answering membership queries is discussed in depth later in Section 6.3.1. The pair of states is added to the $\text{PossiblePairs}$ set if the $\text{EDSM}$ score is higher or equal to zero, denoting that the pair of states is compatible. The $\text{EDSM}$ score is computed for the current pair of states based on the updated automaton $A'$.

During the inference process, if the current blue state is mergeable with any red state, then the pair $(r, b)$ is added to the $\text{PossiblePairs}$ set and the blue state is marked as mergeable. For each blue state in the $B$ set, it is promoted to red if it cannot merge with any of the red states, and this is what $\text{EDSM}$ does, as shown in Lines 23-26. The process is iterated to colour the adjacent states of the red states blue.
The process of merging the pairs of states (generalization) is performed in Lines 29-34. The PossiblePairs set is passed to the pickPair(PossiblePairs) function to pick the pair with the highest score first. The function Merge(PairToMerge) is called to merge the pair of states. The inference of LTS models using the ModifiedQSM algorithm is terminated when all blue states are coloured red.

6.3.1 Processing Membership Queries

The idea of processing the membership queries includes two phases. First, it answers the queries by submitting them to an automatic oracle that knows the target language of the hidden LTS model. Second, the current automaton is updated by augmenting the answered queries. Therefore, the automaton is extended by extra information. Third, the EDSM score is computed for the pair of states after answering each submitted query. The reason for computing the EDSM score is stop asking the remaining queries if the score is below zero, which indicates that the pair of states is incompatible, and there is no benefit in asking the remaining queries. It is important to highlight that queries are only asked if there no path from the initial state to any existing state in the current automaton or (PTA in the initial iteration).

```
Input: A, q_r, q_b, Queries
/* A is a current automaton, Queries */
Result: The score of the pair of states, updated automaton A’
1 while q ← Queries do
2     Answer ← checkWithOracle(q);
3     A’ ← updateAutomaton(A, Answer);
4     score ← computeEDSMScore(A’, q_r, q_b);
5     if score < 0 then
6         /* Terminate asking queries */
7             Break
8     end
9 end
10 return A’
```

Function processQueries(A, q_r, q_b, Queries)

The strategy of processing membership queries is described above in the processQueries function. It begins by iterating over the generated queries to answer them. Once the oracle answers each query, then it is added to the current automaton. The function that is responsible for adding the answered query into the automaton is called UpdateAutomaton.
Chapter 6. *Improvements to the QSM Algorithm*

The automaton is updated so that the answered query may provide additional information about the behaviour of the system under inference and helps the generalization of LTSs to avoid merging incompatible pairs of states.

The process of updating the automaton after answering the membership query is summarized in the `Updateautomaton` function below. The function receives the current automaton `A`, the current query, and the corresponding answer. It begins updating the automaton by pointing to the root state `q_0` as shown in Line 1, where `q_{pointer}` denotes the current state under processing. Then, an iteration over each alphabet element `σ_i` in the answered query is performed in order to make a transition for new elements. If there is no outgoing transition labelled with `σ_i`, then the function `createNewState(A)` is called to create a new vertex `q_{new}` in `A`. After that, a transition is added where the source state is `q_{pointer}` and its target is `q_{new}`. The next step is to mark the target state either as an accepting or rejecting state depending on the answer of the membership query as shown in Lines 6-10.

It is important to mention that elements of the query may already exist in the current automaton. In this case, the pointer is moved to the target state of the current transition.
in order to process the next alphabet element in the answered query as shown in Line 13.

**Example 6.5.** Consider that the current automaton \( A \) is shown in Figure 6.4(a) and the one-step generator generates the following query:\( \langle \text{Load}, \text{Load} \rangle \). Suppose that the oracle answers the query as \( \text{no} \) in the second element. According to the \text{Updateautomaton} function described above, \( n = 2 \) since there are two elements in the query under processing. The function begins by pointing to the root state, which is the \( A \) state. In this way, \( q_{\text{pointer}} = A \). Since there is a transition labelled with \( \sigma_1 = \text{Load} \), the \( B \) state becomes the pointer for the next iteration as shown in Line 13 in the \text{Updateautomaton} function. In the next step, the second element in the query is selected for updating the automaton. There is no transition labelled with \( \sigma_2 = \text{Load} \) from the current pointer. The \text{createNewState}(A) function creates the state labelled with \( F \), denoted by \( q_{\text{new}} = F \). After that, a transition is made where its source state is \( q_{\text{pointer}} = B \) and the target state is \( F \). The newly added transition is shown in Figure 6.4(b) as a dashed arrow. Next, the added state \( q_{\text{new}} \) is marked as a rejecting state because the query is answered as \( \text{no} \). Finally, the \text{Updateautomaton} function is terminated since all elements in the query are processed.

![PTA before and after augmentation](image-url)

(a) The PTA before augmenting the query  
(b) The PTA after augmenting the query

**Figure 6.4:** An example of updating a PTA

### 6.4 Introduction of Markov Predictions to the ModifiedQSM Algorithm

In this section, the incorporation of Markov model predictions to the ModifiedQSM algorithm is introduced, resulting in a new algorithm called MarkovQSM. The notion behind this is to study the impact of inconsistency heuristic \( \text{Incons} \) presented in Chapter 5 to reduce the cost of queries consumed by the ModifiedQSM algorithm.
Chapter 6. *Improvements to the QSM Algorithm*

In general, *MarkovQSM* constructs the initial Markov model from the collected abstracted traces as described in Chapter 5. For each pair of states, the *Incons* is computed based on the current Markov model. The pair of states are considered for merging without asking membership queries whenever the *EDSM* score is higher than or equal to the *Incons* score. In this case no queries are generated since there is evidence suggesting the pair of states are equivalent. *MarkovQSM* is an active learner of LTS models in which it obtains new information during the inference process after asking membership queries. Hence, the Markov model should be updated since additional analysis of the system under inference is obtained.

The following subsections discuss the usage of the Markov model with the *Markov QSM* algorithm in detail. Section 6.4.1 discusses the strategy of updating the Markov model after answering membership queries. Finally, Section 6.4.2 presents the *Markov QSM* algorithm that is designed to reduce the number of membership queries that are answered by an oracle.

### 6.4.1 Updating the Markov Matrix

This section introduces a method of updating the Markov matrix (*MT*) after answering membership queries. During the inference process, using the *MarkovQSM* algorithm, which will be described later in this chapter, the trained *MT* that is built based on the initial traces should be updated since membership queries that are asked may exercise a new aspect of a system under inference.

The idea of updating the *MT* begins by given a membership query $\sigma_1, \sigma_2, \ldots, \sigma_n$ that is answered by an oracle, and the value of prefix length $k$ that is used to construct the initial Markov model. Then, it calls the automatic Markov updater (*MU*) that looks at subsequences of length $k + 1$ in the membership query to update the Markov matrix as follows:

- If the membership query is answered as *yes* denoting that the query belongs to the language of LTS, the *MU* records a pair $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k})$ as a *positive* if it is seen for the first time. We write $ML : ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \mapsto Pos$ to...
denote that $\sigma_{i+k}$ is predicted by the Markov prediction function $ML$ as a permitted element of alphabet to follow the sequence $\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle$.

- The pair $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k})$ is recorded as negative if it is never seen by the Markov model and observed at the end of a membership query that is answered as no. In this case, $i + k = n$. We write $ML : ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \mapsto \text{Neg}$ to denote that $\sigma_{i+k}$ is predicted by the Markov prediction function $ML$ as not permitted to follow the prefix sequence of length $k$ $\langle \sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1} \rangle$.

- The pair $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k})$ is updated to failure if it is already observed in the Markov model as a positive and it is at the end of a membership query that is answered as no such that $i + k = n$. On the other hand, if the oracle answers the membership query as yes where $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k})$ is already observed in the Markov model as a negative subsequence, it is updated to failure. In this case, we write $ML : ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \mapsto \text{Fail}$ to denote that $\sigma_{i+k}$ is not predicted by the Markov prediction function $ML$ during the computation of the inconsistency score. This differs from the non-observed subsequence, and we write $ML : ((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \notin \text{dom}(MT)$ to denote that $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k})$ has not been seen so far.

The algorithm of updating the Markov model is called $UpdateMarkov$ and is shown below in Algorithm 13. It starts by providing the current Markov table $MT$, the query, and its answer. The provided query is a sequence of alphabet elements. If the query is answered as no, it is passed to the algorithm up to the element of alphabet that caused the oracle to reject the query. For example, if a given query $\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n$ is answered as no at the element $\sigma_2$, then $\sigma_1, \sigma_2$ is passed to the $UpdateMarkov$ algorithm. On the other hand, all alphabet elements of the query that are answered as yes are passed to the algorithm.

Algorithm 13 then obtains subsequences of length $k + 1$ iteratively from the provided query. The $obtainSubsequence(query, i, k)$ function is responsible for obtaining the current subsequence from $\sigma_i$ to $\sigma_{k+i}$ of length $k + 1$. If the obtained subsequence does not belong to the $MT$, denoted by $((\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}), \sigma_{i+k}) \notin \text{dom}(MT)$, then the procedure in Lines 4-8 is performed to record the newly observed subsequences. On the other hand, if the subsequence belongs to the $MT$, the updating procedure of the $MT$ is shown in Lines 10-16.
Input: $MT$, query $= (\sigma_1, \sigma_2, \ldots, \sigma_n)$, Answer

/* $MT$ is the Markov table, the answered query, and the answer of the query either true or false */

// The result is the updated Markov table

Result: $MT'$

// Declare the prefix length $K$

Declare: $K \leftarrow \text{Integer}$

for $i = 1 \cdots n$ do

\[ (\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \leftarrow \text{obtainSubsequence}(\text{query}, i, K); \]

/* if the subsequence $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$ is seen for the first time */

if $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \notin \text{dom}(MT)$ then

if Answer is false and $i + k = n$ then

Record a pair $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k})$ in $MT$ as a negative subsequence.

$MT' = MT \oplus \{(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{Neg}\}$

else

Record a pair $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$ in $MT$ as a positive subsequence.

$MT' = MT \oplus \{(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{Pos}\}$

end

else /* Otherwise the subsequence $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$ is already seen in $MT$ */

if Answer is false and $i + k = n$ and $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{true} \in \text{ML}$ then

Update a pair $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$ in $MT$ to be a failure subsequence.

$MT' = MT \oplus \{(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{Fail}\}$

else

if Answer is yes and $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{false} \in \text{ML}$ then

Update a pair $(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k})$ in $MT$ to be a failure subsequence.

$MT' = MT \oplus \{(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+k-1}, \sigma_{i+k}) \mapsto \text{Fail}\}$

end

end

end

return $MT$

Algorithm 13: The UpdateMarkov algorithm

Example 6.6. Suppose that the current automaton during the induction process is shown in Figure 6.5, and consider that the corresponding $MT$ is shown in Table 6.1a where $k = 1$. Let us consider the $D$ state is chosen to merge with the $B$ state. Assume that the oracle answers the following membership query: $(\text{Load}, \text{Load})$, which is generated using
the Dupont generator, as no. In this example, there is one subsequence: \{\langle \text{Load, Load} \rangle \} of length \( k + 1 \) observed in the membership query. Following the above description of updating the MT, the subsequence \{\langle \text{Load, Load} \rangle \} is considered as a new subsequence for the MT that is shown in Table 6.1a. The Markov model is then updated by recording \{\langle \text{Load, Load} \rangle \} as negative, as shown in Table 6.1b.

\[
\begin{array}{c|cccc}
\text{Load} & \text{Edit} & \text{Save} & \text{Close} \\
\hline
\text{Load} & - & \text{Pos} & - & \text{Pos} \\
\text{Edit} & - & \text{Pos} & \text{Pos} & - \\
\text{Save} & - & - & - & - \\
\text{Close} & \text{Pos} & - & - & - \\
\end{array}
\]

(a) The Markov matrix before asking queries where \( k = 1 \)

\[
\begin{array}{c|cccc}
\text{Load} & \text{Edit} & \text{Save} & \text{Close} \\
\hline
\text{Load} & \text{Neg} & \text{Pos} & - & \text{Pos} \\
\text{Edit} & - & \text{Pos} & \text{Pos} & - \\
\text{Save} & - & - & - & \text{Pos} \\
\text{Close} & \text{Pos} & - & - & - \\
\end{array}
\]

(b) The Markov matrix after asking queries where \( k = 1 \)

Table 6.1: An example of updating the Markov table when \( k = 1 \)

Example 6.7. Figure 6.7 shows the current automaton during the inference process. Table 6.2a illustrates the corresponding MT where \( k = 2 \). Let us consider the pair of states \((G, C)\) is considered to compute its score. Assume that the oracle answers the following membership query: \langle \text{Load, Edit, Edit, Edit} \rangle, which is generated using the one-step generator, as yes. In this example, there are two subsequences: \{\langle \text{Load, Edit, Edit} \rangle, \langle \text{Edit, Edit, Edit} \rangle \} of length \( k + 1 \) observed in the membership query. Following the above description of updating the Markov model, the subsequence \{\langle \text{Load, Edit, Edit} \rangle \} is already seen in the MT,
and the subsequence \( \langle \text{Edit, Edit, Edit} \rangle \) is considered as a new subsequence for the MT that is shown in Table 6.2a. The MT is then updated by recording \( \langle \text{Edit, Edit, Edit} \rangle \) as positive, as shown in Table 6.2b.

\[ \text{Figure 6.7: The automaton before asking queries} \]

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & \text{Load} & \text{Edit} & \text{Save} & \text{Close} \\
\hline
\text{Load, Load} & - & - & - & - \\
\text{Load, Edit} & - & \text{Pos} & \text{Pos} & - \\
\text{Load, Save} & - & - & - & - \\
\text{Load, Close} & \text{Pos} & - & - & - \\
\text{Edit, Load} & - & - & - & - \\
\text{Edit, Edit} & - & - & \text{Pos} & - \\
\text{Edit, Save} & - & - & - & \text{Pos} \\
\text{Edit, Close} & - & - & - & - \\
\text{Save, Load} & - & - & - & - \\
\text{Save, Edit} & - & - & - & - \\
\text{Save, Save} & - & - & - & - \\
\text{Save, Close} & - & - & - & - \\
\text{Close, Load} & - & - & - & - \\
\text{Close, Edit} & - & - & - & - \\
\text{Close, Save} & - & - & - & - \\
\text{Close, Close} & - & - & - & - \\
\hline
\end{array}
\]

(A) The Markov matrix before asking queries where \( k = 2 \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & \text{Load} & \text{Edit} & \text{Save} & \text{Close} \\
\hline
\text{Load, Load} & - & - & - & - \\
\text{Load, Edit} & - & \text{Pos} & \text{Pos} & - \\
\text{Load, Save} & - & - & - & - \\
\text{Load, Close} & \text{Pos} & - & - & - \\
\text{Edit, Load} & - & - & - & - \\
\text{Edit, Edit} & - & - & \text{Pos} & - \\
\text{Edit, Save} & - & - & - & \text{Pos} \\
\text{Edit, Close} & - & - & - & - \\
\text{Save, Load} & - & - & - & - \\
\text{Save, Edit} & - & - & - & - \\
\text{Save, Save} & - & - & - & - \\
\text{Save, Close} & - & - & - & - \\
\text{Close, Load} & - & - & - & - \\
\text{Close, Edit} & - & - & - & - \\
\text{Close, Save} & - & - & - & - \\
\text{Close, Close} & - & - & - & - \\
\hline
\end{array}
\]

(B) The Markov matrix after asking queries where \( k = 2 \)

\begin{table}
\caption{An example of updating the Markov table when \( k = 2 \)}
\end{table}
6.4.2 The ModifiedQSM With Markov Predictions

This section presents the MarkovQSM algorithm, which is an extension to the ModifiedQSM algorithm. The objective of the MarkovQSM algorithm is to study the influence of computation of inconsistency on the accuracy of the inferred models and on the number of membership queries that are consumed using the ModifiedQSM algorithm. Hence, the MarkovQSM algorithm incorporates Markov predictions and the computation of the inconsistency during the evaluation of each pair of states for merging.

The induction process of an LTS using the MarkovQSM is summarized in Algorithm 14. Similar to the ModifiedQSM learner, MarkovQSM begins by constructing the initial PTA from the positive traces. The Markov matrix is trained from the same traces as shown in line 2. The Markov matrix is built in the same way that is used in the EDSM-Markov algorithm, which is described in Chapter 5. The process continues in the same way as in ModifiedQSM except that membership queries are only generated if the inconsistency score $Im = (\text{Incons}(\text{merge}(A, q, q'), ML) - \text{Incons}(A, ML))$ is greater than the EDSM score as shown in Line 15, as shown in Algorithm 14. The idea behind asking queries in this case is to measure and determine whether the pair of states are equivalent or not.

It is important to highlight that a pair of states are added to the PossiblePairs set if the EDSM score is higher than or equal to the inconsistency score (see Lines 24-25); this denotes that there is evidence suggesting that states in the pair are equivalent and it is not necessary to ask membership queries in this stage.

The strategy of processing membership queries in the MarkovQSM is performed in the same way in ModifiedQSM, except that the Markov model is updated after answering each membership query as described in Section 6.4.1. The step of updating the Markov table is performed using the updateMarkovTable($MT, query, Answer$) function in Line 4, as illustrated in the processQuerieswithMarkov function. The inference of LTS models using the MarkovQSM is terminated if all states in the current automaton are coloured red.
6.5 Conclusion

This chapter introduced the ModifiedQSM state-merging inference algorithm that improves the accuracy of the inferred models in comparison with QSM. The one-step generator is introduced to help the proposed learners to avoid the over-generalization issue.

An alternative extension of the ModifiedQSM learner has been introduced, known as MarkovQSM. It relies upon training Markov models from the provided traces and updating Markov models after asking each query. It allows the MarkovQSM learner posing membership queries only if an inconsistency score $Im$ is greater than an EDSM score.
input: \( S^+, S^- \)
/* Sets of accepted \( S^+ \) and rejected \( S^- \) sequences */
result: \( A \) is an LTS that is compatible with \( S^+, S^- \), and generated queries

1 \( A \leftarrow \text{generatePTA}(S^+, S^-); \)
2 \( MT \leftarrow \text{trainMarkovTable}(S^+, S^-); \)
3 \( R \leftarrow \{q_0\}; \) // \( R \) is a set of red states

4 do
5    do
6       PossiblePairs \( \leftarrow \emptyset \); // PossiblePairs possible pairs to merge
7       Rextended \( \leftarrow \text{false} \);
8       B \( \leftarrow \text{ComputeBlue}(A, R) \); // \( B \) is a set of blue states
9          for \( q_b \in B \) do
10             mergeable \( \leftarrow \text{false} \);
11             compatible \( \leftarrow \text{false} \);
12                for \( q_r \in R \) do
13                   compatible \( \leftarrow \text{checkMergeCompatibility}(A, q_r, q_b) \);
14                   if compatible then
15                      if \( Im \) is greater than \( \text{EDSM} \) score then
16                         Queries \( \leftarrow \text{generateDupontQueries}(A, q_r, q_b) \);
17                         Queries \( \leftarrow \text{Queries} \cup \text{generateOneStepQuery}(A, q_r, q_b) \);
18                         /* update automaton \( A' \) after asking queries */
19                         A \( \leftarrow \text{processQueries}(A, q_r, q_b, \text{Queries}) \);
20                         if \( \text{EDSM} \text{Score}(A, q_r, q_b) \geq 0 \) then
21                           PossiblePairs \( \leftarrow \text{PossiblePairs} \cup \{(q_r, q_b)\} \);
22                           mergeable \( \leftarrow \text{true} \);
23                      end
24                   else
25                      PossiblePairs \( \leftarrow \text{PossiblePairs} \cup \{(q_r, q_b)\} \);
26                      mergeable \( \leftarrow \text{true} \);
27                end
28          end
29          if mergeable = false then
30             R \( \leftarrow R \cup \{q_b\}; \)
31             Rextended \( \leftarrow \text{true} \);
32       end
33     end
34 while Rextended = true:
35       if PossiblePairs \( \neq \emptyset \) then
36          \( (q_r, q_b) \leftarrow \text{PickPair}(\text{PossiblePairs}) ; \)
37          if \( \text{EDSM} \text{Score}(A, q_r, q_b) \geq 0 \) then
38             A \( \leftarrow \text{merge}(A, q_r, q_b) \);
39       end
40 end
41 while PossiblePairs \( \neq \emptyset \);
42 return \( A \)

Algorithm 14: The Markov QSM algorithm
**Input:** $A, q_r, q_b, \text{Queries}, MT$

/* $A$ is a current PTA hypothesis, \text{Queries} */

(Result: The updated PTA $A'$)

1. while \( \text{query} \leftarrow \text{Queries} \) do
2. \( \text{Answer} \leftarrow \text{checkWithOracle}(\text{query}); \)
3. \( A' \leftarrow \text{updateAutomaton}(A, \text{Answer}); \)
4. \( MT \leftarrow \text{updateMarkovTable}(MT, \text{query}, \text{Answer}); \)
5. \( \text{score} \leftarrow \text{ComputeEDSMScore}(A', q_r, q_b); \)
6. if \( \text{score} < 0 \) then
7. /* Terminate */
8. Break
9. end
10. end
11. return $A'$

**Function** processQuerieswithMarkov($A, q_r, q_b, \text{Queries}, MT$)
7

Experimental Evaluation of ModifiedQSM and MarkovQSM

7.1 Introduction

In the previous chapter, the ModifiedQSM and MarkovQSM learners were introduced. The main purpose of establishing them is to infer LTS models from a few traces. This chapter evaluates the performance of the proposed learners using both randomly generated LTSs and case studies.

The following section presents the experimental evaluation of the proposed algorithms. The aim of running experiments is to assess the capability of the ModifiedQSM and MarkovQSM learners at improving the accuracy of the inferred models.
7.2 Experimental Setup and Evaluation

This section presents the investigation and studies the performance of the *ModifiedQSM* and *MarkovQSM* learners. It includes a comparison of the developed algorithms compared to *QSM*. The measurements were performed based on different criteria. The experiment aimed to address and investigate the following research questions:

1. Do the *ModifiedQSM* and *MarkovQSM* infer LTS models better than *QSM*?

2. What is the impact of introducing the inconsistency computation in *MarkovQSM* on the accuracy of the inferred models and on the number of membership queries compared to other algorithms?

In order to evaluate the performance of the *ModifiedQSM* and *MarkovQSM*, a series of random LTSs were generated in different sizes (number of states): 10, 20 and 30. The randomly-generated LTSs were connected and had different alphabet sizes to vary their complexity. In this way, an alphabet multiplier parameter $m$ was introduced to vary the alphabet size so that $|\Sigma| = m \times |Q|$, where $m$ was set to 0.5, 1, and 2 in this experiment.

Furthermore, the performance of *ModifiedQSM* and *MarkovQSM* were evaluated with different numbers of traces $T$: 3 and 5 that were obtained by a random walk of the generated LTSs. The length of the traces is $|Q| \times |\Sigma| = 0.125 \times |Q|^2 \times 2$. This length was chosen since the passive inference in the previous chapter failed to learn good LTSs in this setting. So, it is interesting to select this length of traces to compare between different learners. The mean value of the BCR scores in the passive experiment was around 0.55 where the number of traces is 5. Hence, it is better to study the improvement made by the concept of active learning with the worst case in the passive experiment in Chapter 6.

Specifically, ten LTSs were randomly generated for each chosen size of state and each alphabet multiplier. Thus, 10-30 states is 3 steps $\times$ 10 LTSs = 30 LTSs in total for each $m$ setting. For each LTS, three sets of training data were generated, bringing the number of LTSs learnt per experiment to 90 for each setting of $T$, under an assumption that the same training data is passed for the learners for each evaluation task. The idea behind learning this number of LTSs is to assess the performance of the proposed algorithms on various numbers of random LTSs with different training data fed to each LTS.
In the original study of QSM by Dupont et al. [36], the evaluation of the QSM is made using random LTSs where the alphabet size was two; both positive and negative samples were supplied to the learner. Walkinshaw et al. [28] evaluated the efficiency of their QSM using random LTS where the alphabet size is six, and only positive traces were provided to start the inference process. In this experiment, ModifiedQSM and MarkovQSM learners will be evaluated against Dupont’s QSM where the size of alphabet is large, and only a few positive traces are provided.

In this experiment, BCR and structural-similarity metrics were selected to score the performance of the algorithms. In addition, the number of membership queries was chosen to measure the efficiency of the algorithms.

The implementation of this experiment is available for clone via https://github.com/AbdullahUK/EDSM_QSM_MarkovPhd.git. For the conducted experiment, the launch configuration has to start statechum.analysis.learning.experiments.PairSelection.MarkovActiveExperiment class.

In the conducted experiments, Java 7 was used with JVM arguments of `-ea -Dthreadnum=1 -Djava.library.path=linear/.libs;"C:/Program Files/R/R-3.0.1/library/rJava/jri/x64" -Xmx26000m` and environment variable R_HOME set to the location of R, such as `C:/Program Files/R/R-3.0.1/lib64/R java`. The R toolset was used for all analysis. The R tool has to have JavaGD, rJava and aplpack installed.

### 7.2.1 Evaluating the Performance of ModifiedQSM and MarkovQSM in Terms of BCR Scores

The boxplots of the BCR values produced by MarkovQSM, ModifiedQSM, and QSM are depicted in Figure 7.1. It is clear that the BCR scores of LTSs inferred using the ModifiedQSM learner are the highest compared to both MarkovQSM and ModifiedQSM. It can be seen from Figure 7.1 that MarkovQSM, at every number of traces, inferred LTSs with higher BCR score compared to QSM.

Table 7.1 summarizes the median values of the BCR scores of LTSs inferred achieved by MarkovQSM, ModifiedQSM and QSM. In addition, there is an obvious reduction in the
Chapter 7. Experimental Evaluation of ModifiedQSM and MarkovQSM

![Boxplots of BCR scores achieved by various learners for different setting of m and T](image)

**Figure 7.1:** Boxplots of BCR scores achieved by various learners for different setting of \( m \) and \( T \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>Trace Number</th>
<th>( T )</th>
<th>ModifiedQSM</th>
<th>MarkovQSM</th>
<th>QSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3</td>
<td>0.95</td>
<td>0.91</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.98</td>
<td>0.95</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>0.89</td>
<td>0.88</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.96</td>
<td>0.95</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>3</td>
<td>0.85</td>
<td>0.85</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.94</td>
<td>0.95</td>
<td>0.89</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.1:** The median values of BCR scores obtained by ModifiedQSM, MarkovQSM, and QSM

BCR scores attained by MarkovQSM compared to ModifiedQSM when \( m = 0.5 \). Additionally, the average BCR scores of LTSs inferred using MarkovQSM decreased by 6.52% compared to the scores attained by ModifiedQSM when the number of traces is 3 and when \( m = 0.5 \).

In order to statistically measure the significant difference between the resulting BCR scores of LTSs obtained using the proposed algorithms against QSM, the paired Wilcoxon signed-rank test was conducted between the BCR scores of the three algorithms at the significance level of 0.05 \((\alpha = 0.05)\). Table 7.2 summarizes the statistical test results using the
Chapter 7. Experimental Evaluation of ModifiedQSM and MarkovQSM

<table>
<thead>
<tr>
<th>m</th>
<th>T</th>
<th>ModifiedQSM vs. QSM</th>
<th>MarkovQSM vs. QSM</th>
<th>MarkovQSM vs. ModifiedQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3</td>
<td>$6.10 \times 10^{-36}$</td>
<td>$1.66 \times 10^{-17}$</td>
<td>$5.15 \times 10^{-13}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$1.16 \times 10^{-34}$</td>
<td>$1.42 \times 10^{-13}$</td>
<td>$4.50 \times 10^{-13}$</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>$3.11 \times 10^{-37}$</td>
<td>$1.23 \times 10^{-28}$</td>
<td>$2.04 \times 10^{-04}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$4.45 \times 10^{-35}$</td>
<td>$6.11 \times 10^{-28}$</td>
<td>$7.08 \times 10^{-06}$</td>
</tr>
<tr>
<td>2.0</td>
<td>3</td>
<td>$2.59 \times 10^{-33}$</td>
<td>$1.47 \times 10^{-29}$</td>
<td>$0.51$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$2.71 \times 10^{-28}$</td>
<td>$1.71 \times 10^{-26}$</td>
<td>$0.52$</td>
</tr>
</tbody>
</table>

Table 7.2: The $p$-values obtained using the Wilcoxon signed-rank test for different comparisons of the BCR scores attained by ModifiedQSM, MarkovQSM, and QSM.

Paired Wilcoxon signed-rank test for the BCR scores achieved by different learners. In the first column, the null hypothesis $H_0$ is that the BCR scores of the inferred LTS using ModifiedQSM and QSM are the same. In all cases the resulting $p$-values were less than $\alpha = 0.05$, indicating that the BCR results were statistically significant. Hence, the $H_0$ was rejected.

In the second column in Table 7.2, the $p$-values were less than 0.05 in all cases suggesting that the null hypothesis, that the BCR values of the MarkovQSM and QSM are the same, could be rejected. The findings from the paired Wilcoxon signed-rank test indicate that the BCR scores of the inferred LTSs using MarkovQSM were higher than the BCR score of the inferred LTSs using QSM. The third column summarizes the $p$-values that were obtained after comparing the BCR scores of the inferred LTSs using ModifiedQSM and MarkovQSM. The $p$-values were less than 0.05 in the majority of settings of $m$, denoting that there was a significant difference between the ModifiedQSM and MarkovQSM. However, the null hypothesis $H_0$, which is that the BCR scores of the inferred LTS using ModifiedQSM and MarkovQSM are identical, cannot be rejected when $m = 2$. In this case, it is possible to say that the BCR scores of the induced LTSs using ModifiedQSM and MarkovQSM were not significantly different.

### 7.2.2 Evaluating the Performance of ModifiedQSM and MarkovQSM in Terms of Structural-Similarity Scores

The boxplots of the structural-similarity scores of LTSs inferred using MarkovQSM, ModifiedQSM, and QSM are illustrated in Figure 7.2. The structural-similarity scores obtained by ModifiedQSM are the highest compared to other learners in this experiment. As can
be seen in Figure 7.2, MarkovQSM inferred LTSs with poor structural-similarity scores in many cases, especially when $m = 0.5$. This is due to earlier incorrect mergers that are allowed which should do not happened (over-generalization). This denotes queries should be asked in those cases. The median values of the structural-similarity scores of the learnt LTSs using MarkovQSM, ModifiedQSM and QSM are summarized in Table 7.3.

![Boxplots of structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM learners for different setting of $m$ and $T$](image)

**Figure 7.2:** Boxplots of structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM learners for different setting of $m$ and $T$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$T$</th>
<th>ModifiedQSM</th>
<th>MarkovQSM</th>
<th>QSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3</td>
<td>0.97</td>
<td>0.92</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.99</td>
<td>0.96</td>
<td>0.90</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>0.96</td>
<td>0.95</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.98</td>
<td>0.98</td>
<td>0.92</td>
</tr>
<tr>
<td>2.0</td>
<td>3</td>
<td>0.95</td>
<td>0.94</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.98</td>
<td>0.97</td>
<td>0.94</td>
</tr>
</tbody>
</table>

**Table 7.3:** The median values of structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM
The results, which are illustrated in Figure 7.2, show that the structural-similarity scores of LTSs inferred using the MarkovQSM when \(m = 0.5\) are worse than those inferred using QSM in some cases. Hence, it was necessary to measure the significant difference between the structural-similarity scores attained by different learners. This was measured using the paired Wilcoxon signed-rank test for the structural-similarity scores achieved using various algorithms. When comparing the structural-similarity scores of the inferred models using QSM against ModifiedQSM, the null hypothesis \(H_0\), that the structural-similarity scores between both algorithms are the same, was rejected. This was because the \(p\)-values were less than 0.05 in all cases, as shown in the first column in Table 7.4.

<table>
<thead>
<tr>
<th>(m)</th>
<th>(T)</th>
<th>ModifiedQSM vs. QSM</th>
<th>MarkovQSM vs. QSM</th>
<th>MarkovQSM vs. ModifiedQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3</td>
<td>3.26 × 10^{-38}</td>
<td>0.06</td>
<td>1.58 × 10^{-21}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9.81 × 10^{-35}</td>
<td>0.21</td>
<td>1.42 × 10^{-20}</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>4.31 × 10^{-37}</td>
<td>1.50 × 10^{-13}</td>
<td>1.61 × 10^{-10}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7.60 × 10^{-35}</td>
<td>1.31 × 10^{-12}</td>
<td>7.45 × 10^{-08}</td>
</tr>
<tr>
<td>2.0</td>
<td>3</td>
<td>4.91 × 10^{-34}</td>
<td>1.42 × 10^{-18}</td>
<td>9.70 × 10^{-04}</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.95 × 10^{-29}</td>
<td>1.27 × 10^{-17}</td>
<td>2.58 × 10^{-05}</td>
</tr>
</tbody>
</table>

Table 7.4: The \(p\)-values obtained using the Wilcoxon signed-rank test for different comparisons of the structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM

The second column in Table 7.4 summarizes the resulting \(p\)-values when comparing the structural-similarity scores of the inferred LTSs using QSM and MarkovQSM. In this study, the null hypothesis \(H_0\) states that there is no significant difference between the structural-similarity scores of the inferred LTS model using MarkovQSM and QSM. The \(p\)-values were less than 0.05 when \(m \geq 1\), indicating that MarkovQSM inferred LTSs with higher structural-similarity scores compared to QSM in the majority of inferred LTSs. Hence, the \(H_0\) can be rejected. However, the \(p\)-values were higher than 0.05 when \(m = 0.5\), so the \(H_0\) cannot be rejected.

Additionally, the third column in Table 7.4 reports the resulting \(p\)-values after comparing the structural-similarity scores of the inferred LTSs using MarkovQSM and ModifiedQSM. The null hypothesis \(H_0\) in this comparison states that there is no significant difference between MarkovQSM and ModifiedQSM in terms of the structural-similarity scores. In all cases, the \(p\)-values were less than 0.05, the null hypothesis could be rejected.
7.2.3 Number of Membership Queries

An important factor that must be taken into consideration while evaluating the performance of ModifiedQSM, MarkovQSM, and QSM is the number of membership queries that are submitted to the oracle. Figure 7.3 illustrates the number of membership queries submitted to the oracle when $m = 0.5$. Interestingly, when the number of states was 30, the average number of membership queries that were asked by MarkovQSM decreased by 1.43% compared to QSM, and reduced by 11.63% compared to ModifiedQSM.

![Number of Membership Queries](image)

**Figure 7.3:** The number of membership queries that were asked by different learners when $m = 0.5$

Table 7.5 shows the median values of the number of membership queries when $m = 0.5$. When the number of states was 30, the median value of the number of membership queries that were asked by MarkovQSM were less than the median value of the consumed membership queries using other algorithms. Otherwise, the smallest median value of the number of membership queries was observed for the QSM algorithm.

The paired Wilcoxon signed-rank statistical test was used to statistically measure the significant difference between the number of membership queries that were asked by different learners. The null hypothesis $H_0$ states that there is no significant difference in
Chapter 7. Experimental Evaluation of ModifiedQSM and MarkovQSM

<table>
<thead>
<tr>
<th>$m$</th>
<th>$T$</th>
<th>Number of states</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10</td>
<td>95</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>375</td>
<td>321</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>801</td>
<td>704</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>107</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>440</td>
<td>316</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>914</td>
<td>715</td>
</tr>
</tbody>
</table>

Table 7.5: The median values of number of membership queries when $m = 0.5$

the number of membership queries asked by different learners. Table 7.6 shows the resulting $p$-values using the Wilcoxon test. When comparing ModifiedQSM against QSM, the reported $p$-values were less than 0.05 and the null hypothesis $H_0$ could be rejected.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$T$</th>
<th>QSM vs. ModifiedQSM</th>
<th>QSM vs. MarkovQSM</th>
<th>MarkovQSM vs. ModifiedQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3</td>
<td>$1.47 \times 10^{-18}$</td>
<td>$4.18 \times 10^{-06}$</td>
<td>$5.77 \times 10^{-13}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$6.72 \times 10^{-23}$</td>
<td>0.36</td>
<td>$8.30 \times 10^{-16}$</td>
</tr>
</tbody>
</table>

Table 7.6: The $p$-values obtained using the Wilcoxon signed-rank test for different comparisons of the number of membership queries when $m = 0.5$

Figure 7.4 illustrates the number of membership queries submitted to the oracle when $m = 1.0$. It is clear that ModifiedQSM and MarkovQSM asked more membership queries...
than QSM. When the number of traces was 5 and \( l = 0.3 \), the average of the number of membership queries posed by MarkovQSM decreased by 7.26% in comparison with those asked by QSM.

Table 7.7 summarizes the median values of the number of membership queries submitted to the oracle using various learners when \( m = 1.0 \). It is obvious that the median values of the number of membership queries that were asked by QSM were less than the median value of the consumed membership queries using other algorithms when the number of states was 10 or 20.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( T )</th>
<th>Number of states</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3</td>
<td>10</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>284</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>825</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>348</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>117</td>
<td>914</td>
</tr>
<tr>
<td></td>
<td></td>
<td>482</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1064</td>
<td>928</td>
</tr>
</tbody>
</table>

Table 7.7: The median values of number of membership queries when \( m = 1.0 \)

Table 7.8 shows the resulting \( p \)-values using the paired Wilcoxon signed-rank statistical test. When comparing the number of membership queries asked by ModifiedQSM and QSM, the reported \( p \)-values were less than 0.05. Therefore, the null hypothesis \( H_0 \), that stated there is no significant difference, could be rejected.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( T )</th>
<th>QSM vs. ModifiedQSM</th>
<th>QSM vs. MarkovQSM</th>
<th>MarkovQSM vs. ModifiedQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3</td>
<td>( 1.15 \times 10^{-10} )</td>
<td>0.03</td>
<td>( 4.40 \times 10^{-24} )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( 2.14 \times 10^{-22} )</td>
<td>0.15</td>
<td>( 5.44 \times 10^{-27} )</td>
</tr>
</tbody>
</table>

Table 7.8: The \( p \)-values obtained using the Wilcoxon signed-rank test for different comparisons of the number of membership queries when \( m = 1.0 \)

Figure 7.5 illustrates the number of membership queries that were generated to the oracle when \( m = 2.0 \). It can be seen that MarkovQSM and ModifiedQSM asked more queries if the number of states were below 30. When the number of states was 30 and the number of traces was 5 MarkovQSM asked fewer queries compared to other learners.

Table 7.9 shows the median values of the number of membership queries for each setting of \( m \) and \( T \). In the majority of cases, the mean value of the number of membership queries that were asked by QSM was less than the mean value of the consumed membership queries.
using other algorithms. The highest mean value of the number of membership queries was observed for the ModifiedQSM algorithm.

The resulting \( p \)-values were less than 0.05 when comparing the number of queries submitted to the oracle using MarkovQSM and ModifiedQSM, as shown in Table 7.10. There was a clear evidence that MarkovQSM asked fewer membership queries than ModifiedQSM.

**Table 7.9**: The median values of number of membership queries

<table>
<thead>
<tr>
<th>( m )</th>
<th>( T )</th>
<th>Number of states</th>
<th>ModifiedQSM</th>
<th>MarkovQSM</th>
<th>QSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>3</td>
<td>10</td>
<td>103</td>
<td>87</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>455</td>
<td>396</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>1080</td>
<td>953</td>
<td>1043</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10</td>
<td>124</td>
<td>94</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>524</td>
<td>424</td>
<td>358</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>1219</td>
<td>981</td>
<td>1214</td>
</tr>
</tbody>
</table>

**Table 7.10**: The \( p \)-values obtained using the Wilcoxon signed-rank test for different comparisons of the number of membership queries

<table>
<thead>
<tr>
<th>( m )</th>
<th>( T )</th>
<th>QSM vs. ModifiedQSM</th>
<th>QSM vs. MarkovQSM</th>
<th>MarkovQSM vs. ModifiedQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3</td>
<td>( 1.61 \times 10^{-10} )</td>
<td>( 3.43 \times 10^{-04} )</td>
<td>( 6.89 \times 10^{-25} )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( 6.01 \times 10^{-11} )</td>
<td>0.91</td>
<td>( 1.14 \times 10^{-32} )</td>
</tr>
</tbody>
</table>
Figure 7.6 illustrates the transition cover that was collected from the training data during the conducted experiment. It is worth noting that the ModifiedQSM and MarkovQSM generated better LTSs compared to the QSM even if many of those transitions in the target LTSs were not covered. This is the advantage of considering one-step queries that guide the learners to avoid merging states that are not equivalent in the target hidden LTSs.

![Transition Coverage](image)

**Figure 7.6:** The transition cover of the generated traces

During the conducted experiments using random LTSs, it was interesting to compute the Markov precision and recall scores of the trained Markov models. In this experiment, a low precision was observed when $m = 0.5$ denoting that the trained Markov models predicted transitions wrongly as shown in Figure 7.7. This may explains why the MarkovQSM did perform well when $m = 0.5$ compared to other settings of $m$.

Besides, the performance of MarkovQSM are very close to ModifiedQSM when $m = 2.0$, because both learners inferred LTSs with very similar BCR and structural-similarity scores. This is due to the high precision value of the trained Markov model where $m = 2.0$. In this case, inconsistencies were detected well whenever a merger added labels of outgoing transitions that predicted incorrectly with respect to the trained Markov models.
In general, $\textit{ModifiedQSM}$ inferred LTSs with higher structural-similarity scores than $\textit{MarkovQSM}$. The structural-similarity scores of the inferred models using $\textit{MarkovQSM}$ when $m = 0.5$ was poor compared to $m = 1, 2$. The discrepancy in the structural-similarity scores with different $m$ settings could be attributed to the accuracy of Markov model predictions. In the conducted experiment, a low precision was observed when $m = 0.5$ denoting that the trained Markov model predicted transitions wrongly as shown in Figure 7.7. This demonstrates that the computation of inconsistency scores had a negative effect since many inconsistencies were not observed during the inference process, and this leads to ignore checking the state merges using the membership queries.
7.3 Case Studies

In the previous section, the efficiency of the ModifiedQSM and MarkovQSM learners was evaluated on randomly-generated LTSs. It was clear that the ModifiedQSM learner performed well compared to other learners. In this section, a number of case studies are used to evaluate the efficiency of different learners to infer good LTSs. The performance of the ModifiedQSM, MarkovQSM, and QSM learners was evaluated on a number of case studies. For each of the following case studies, the number of traces ranged from two to four, and the length of traces was given by \( l \times |Q| \times |\Sigma| \), where \( l \) is a parameter to control the length of generated traces. In the conducted experiment, 30 different random traces were generated for each number and length of traces. In addition, \( 2 \times |Q| \times |Q| \) test sequences were generated of length \( 3 \times |Q| \). This was chosen to match the settings used in the conducted experiments using random LTSs.

7.3.1 Case Study: SSH Protocol

The findings of the SSH case study are shown in Figure 7.8, and summarize the BCR scores of the inferred models using different learners when \( l = 0.3 \) and 0.5 respectively. From Figure 7.8, it is apparent that the inferred LTSs using ModifiedQSM and MarkovQSM
learners were close to the reference LTSs in terms of their language. It was noticed that the QSM learner performed badly when $l = 0.1$ compared to other learners. This is due to that Dupont’s QSM queries are insufficient to avoid merging incompatible pairs of states, especially if few traces are provided. The improvement made by ModifiedQSM and MarkovQSM learners over QSM was caused by the one-step queries and this made it possible to detect incorrect states merges and avoid merging them.

Table 7.11 summarizes the $p$-values of BCR scores obtained from the Wilcoxon signed-rank statistical test. The null hypothesis $H_0$ states that the BCR scores of the inferred LTS using the learners are the same. The resulting $p$-values suggest rejecting the $H_0$ when comparing ModifiedQSM and MarkovQSM against QSM since the $p$-values are less than 0.05 (significance level), and this indicates that there is a statistically significant difference between them. In other words, there is strong evidence to support the alternative hypothesis which stated the BCR scores of the inferred LTSs using ModifiedQSM and QSM are not same. Besides, the $H_0$ is accepted if the trace number is 4 and $l = 0.3$, which means that there is no statistically significant difference between the three learners. However, when comparing ModifiedQSM and MarkovQSM, the $H_0$ is accepted.

<table>
<thead>
<tr>
<th>$l$</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>ModifiedQSM vs. QSM</td>
<td>$1.77 \times 10^{-06}$</td>
<td>$3.91 \times 10^{-06}$</td>
<td>$1.76 \times 10^{-05}$</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>$2.61 \times 10^{-06}$</td>
<td>$3.91 \times 10^{-06}$</td>
<td>$1.76 \times 10^{-05}$</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>$2.53 \times 10^{-04}$</td>
<td>0.004</td>
<td>0.08</td>
</tr>
<tr>
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<td>MarkovQSM vs. QSM</td>
<td>$2.53 \times 10^{-04}$</td>
<td>0.004</td>
<td>0.08</td>
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<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.11: $p$-values obtained using the Wilcoxon signed-rank test after comparing the BCR scores attained by ModifiedQSM, MarkovQSM, and QSM for the SSH protocol case study

The performance of the ModifiedQSM learner was evaluated in Section 7.2 using randomly-generated LTSs and was shown to significantly improve the structural-similarity scores of the inferred LTSs compared to MarkovQSM and QSM learners. In this case study, the structural-similarity scores of the inferred LTSs using both ModifiedQSM and MarkovQSM were higher than QSM as illustrated in Figure 7.9. The average scores attained by ModifiedQSM increased by 23.75% compared to the scores attained by QSM when $l = 0.1$ and the number
of trace was 2. The structure of the inferred LTSs using MarkovQSM and ModifiedQSM were similar to the structure of reference LTS, unlike those models inferred using QSM as shown in Figure 7.9. It is apparent that the performance of both the ModifiedQSM and MarkovQSM learners are the same.

![Figure 7.9: The structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM for the SSH protocol case study](image)

Table 7.12: *p*-values obtained using the Wilcoxon signed-rank test after comparing the structural-similarity scores attained by ModifiedQSM, MarkovQSM, and QSM for the SSH protocol case study

<table>
<thead>
<tr>
<th>l</th>
<th>Trace Number</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>ModifiedQSM vs. QSM</td>
<td>1.72 × 10⁻⁶⁶</td>
<td>3.56 × 10⁻⁶⁶</td>
<td>1.40 × 10⁻⁵⁵</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>1.72 × 10⁻⁶⁶</td>
<td>3.56 × 10⁻⁶⁶</td>
<td>1.40E − 0⁵</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>1</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>1.17 × 10⁻⁴⁴</td>
<td>0.002</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>1.17 × 10⁻⁴⁴</td>
<td>0.002</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Table 7.12 shows the resulting *p*-values using the paired Wilcoxon signed-rank statistical test. When comparing the developed learners against QSM, the reported *p*-values are less than 0.05 when *l* = 0.1 and we reject the null hypothesis *H₀* that stated the structural-similarity scores of LTSs obtained using learners are the same. Thus, there is strong
evidence to claim that ModifiedQSM and MarkovQSM outperformed QSM for the SSH protocol case study. In addition, the $H_0$ can be accepted in case where the number of traces is 4 and $l = 0.3$. Furthermore, in case of comparing the structural-similarity scores of the inferred LTSs using ModifiedQSM and MarkovQSM, the $H_0$ is accepted since both learners generated LTSs with similar structural-similarity scores.

Figure 7.10 shows the number of membership queries posed to the Oracle using various algorithms. It shows that ModifiedQSM and MarkovQSM learners asked more queries than QSM. This is due to the fact that ModifiedQSM and MarkovQSM learners posed the one-step queries, unlike QSM that only asked Dupont’s queries; however, both learners improved the BCR and structural-similarity scores of the inferred models. Numbers of membership queries that are posed using MarkovQSM was 1.96% slightly less than those posed using ModifiedQSM when $l = 0.1$ and the number of traces is two. In addition, numbers of membership queries were decreased by 5.45% when $L = 0.3$. Furthermore, MarkovQSM asked fewer membership queries compared to ModifiedQSM when $L = 0.3$, due to the way that queries are only asked if the $Im$ score is higher than the $EDSM$ score.

![Figure 7.10: The number of membership queries of different learners](image)

To compare the number of membership queries that are posed using various learners, the paired Wilcoxon signed-rank statistical test was used. Table 7.13 summarizes the resulting $p$-values using the paired Wilcoxon signed-rank statistical test. The null hypothesis $H_0$
states that the number of membership queries posed using the learners are the same. In case of comparing the proposed learners against QSM, the reported $p$-values are less than 0.05 when $l = 0.1$. Thus, the null hypothesis $H_0$ can be rejected, denoting that there is strong evidence to say that QSM asked fewer queries than ModifiedQSM and MarkovQSM. However, the $H_0$ can be accepted in case of comparing MarkovQSM and QSM if the number of traces is four and $l = 0.3$. When comparing the number of membership queries that were asked by ModifiedQSM and MarkovQSM, the $H_0$ is rejected since MarkovQSM asked fewer queries compared to QSM.

<table>
<thead>
<tr>
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<th>Trace Number</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>ModifiedQSM vs. QSM</td>
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<td>$2.47 \times 10^{-06}$</td>
<td>$2.97 \times 10^{-05}$</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>$1.82 \times 10^{-06}$</td>
<td>$3.65 \times 10^{-06}$</td>
<td>$8.32 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>0.002</td>
<td>0.47</td>
<td>0.60</td>
</tr>
<tr>
<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>$2.24 \times 10^{-06}$</td>
<td>$4.43 \times 10^{-05}$</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>$4.04 \times 10^{-06}$</td>
<td>$2.60 \times 10^{-04}$</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>$3.46 \times 10^{-06}$</td>
<td>$7.32 \times 10^{-04}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 7.13: $p$-values obtained by the Wilcoxon signed-rank test of structural-similarity scores for SSH protocol case study

Figure 7.11 shows the transition coverage which was computed as the ratio of the transitions that were visited by the traces in the conducted experiments. From Figure 7.11, it is noticed that the QSM learner performed well on the condition that all transitions were visited once by the generated traces. For instance, when the number of traces is four and $l = 0.3$, the median value of the BCR scores of inferred models using the QSM learner is 1.0, and this happened when the transition cover was 100%. It is interesting to note that ModifiedQSM and MarkovQSM performed well even when the transition cover was 80%.

Figure 7.12 illustrates the accuracy of the trained Markov models that were computed using the precision/recall scores. It can be seen that the recall scores of the Markov models are very low, denoting that many existing transitions of the reference graph were not predicted by the Markov models. Moreover, it is noticed that the precision score is very high (above 0.8) and significantly affects the BCR and structural-similarity scores to detect inconsistencies.
In general, a very low inconsistency score means that the Markov table is trained well with respect to subsequences of length $k + 1$. The boxplot of the inconsistency scores computed for the reference LTS of the SSH protocol case study after training the Markov models are shown in Figure 7.13. The BCR and structural-similarity scores of the inferred LTSs using the MarkovQSM learner were very high even if the number of inconsistencies
was higher than 10. This is attributed to the idea of asking queries on the condition that the $Im$ score is higher than the $EDSM$ score. Therefore, whenever the $Im$ score exceeds the $EDSM$ score, merging of two states is checked using membership queries.

![Figure 7.13: Inconsistencies of SSH protocol case study](image)

### 7.3.2 Case Study: Mine Pump

Figure 7.14 depicts the BCR scores of the inferred LTSs using $MarkovQSM$, $ModifiedQSM$ and $QSM$ respectively for the mine pump case study, where different numbers of traces were considered. In Figure 7.14 there is apparent that $MarkovQSM$ and $ModifiedQSM$ learners inferred LTSs with higher BCR scores in the majority of cases. The $QSM$ learner, however, did not learn LTSs well compared to other learners, especially when the number of traces was below 4. This is because the membership queries posed by the $QSM$ learner allow bad generalizations of LTSs. In addition, the findings in Figure 7.14 shows that $MarkovQSM$ and $ModifiedQSM$ inferred models well, and this is because the one-step queries provide additional information that can prevent the merging of inequivalent pairs of states. Additionally, $QSM$ generated LTSs well when the number of traces was four and $l = 0.3$, unlike if it was two or three.

Table 7.14 reports the $p$-values of BCR scores obtained from the Wilcoxon signed-rank statistical test. The considered null hypothesis $H_0$ for this study is that there is no
significant difference between the BCR scores of the inferred LTS using learners. When comparing \textit{ModifiedQSM} and \textit{QSM} in terms of the BCR scores, the \(H_0\) can be rejected, indicating that there is a statistically significant difference between them. However, if the number of traces is four and \(l = 0.3\), there is no statistically significant difference between \textit{ModifiedQSM} and \textit{QSM} since the \(p\)-value exceeded the significance level (0.05). This is noticeable as the mean value of the BCR scores of the inferred models using \textit{ModifiedQSM} and \textit{QSM} are the same. The \(H_0\) that stated there is no difference between the \textit{ModifiedQSM} and \textit{MarkovQSM} learners is accepted because the \(p\)-values are larger than 0.05.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\(l\) & Trace Number & \multicolumn{3}{c|}{Trace Number} \\
& & 2 & 3 & 4 \\
\hline
0.1 & ModifiedQSM vs. QSM & \(8.58 \times 10^{-06}\) & \(2.70 \times 10^{-04}\) & 0.001 \\
& MarkovQSM vs. QSM & \(1.27 \times 10^{-05}\) & \(1.84 \times 10^{-04}\) & 0.001 \\
& ModifiedQSM vs. MarkovQSM & 1 & 1 & 1 \\
0.3 & ModifiedQSM vs. QSM & 0.003 & 0.008 & 0.37 \\
& MarkovQSM vs. QSM & 0.001 & 0.008 & 1 \\
& ModifiedQSM vs. MarkovQSM & 0.34 & – & 1 \\
\hline
\end{tabular}
\caption{\(p\)-values of the Wilcoxon signed-rank test of BCR scores for water mine case study}
\end{table}
Figure 7.15: Structural-similarity scores of water mine pump case study

Figure 7.15 illustrates boxplots of the structural-similarity scores of the mined LTSs using MarkovQSM, ModifiedQSM and QSM respectively. The outcomes that are shown in Figure 7.15 demonstrate that the ModifiedQSM and MarkovQSM learners can infer LTSs with higher structural-similarity scores compared to QSM for the water mine pump case study. It is noticed that QSM performed well if $l = 0.3$ since the generated traces cover more paths in the reference graph compared to $l = 0.1$. However, both MarkovQSM and ModifiedQSM inferred good LTSs compared to QSM even if $l = 0.1$.

<table>
<thead>
<tr>
<th>$l$</th>
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<th>4</th>
</tr>
</thead>
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<td>0.1</td>
<td>ModifiedQSM vs. QSM</td>
<td>$5.70 \times 10^{-04}$</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
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<td>MarkovQSM vs. QSM</td>
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<td>0.03</td>
<td>0.01</td>
</tr>
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<td>ModifiedQSM vs. MarkovQSM</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>0.003</td>
<td>0.008</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>0.001</td>
<td>0.008</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>0.34</td>
<td>Nan</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7.15: $p$-values of Wilcoxon signed rank test of water mine case study for structural-similarity Scores
Table 7.15 summarizes the $p$-values obtained from the Wilcoxon signed-rank statistical test after comparing the structural-similarity scores attained by different learners. In this context, the null hypothesis $H_0$ is stated as the structural-similarity scores of the inferred models using the two learners are identical. When comparing ModifiedQSM and QSM, the test reported $p$-values less than 0.05, and this led to the $H_0$ being rejected, which means that ModifiedQSM inferred models with higher structural-similarity scores. With the null hypothesis $H_0$ which stated ModifiedQSM and MarkovQSM learners infer models with the same structural-similarity scores, it can be accepted since the $p$-values are greater than 0.05; this proves that there is no significant difference between the structural-similarity scores of both learners.

Figure 7.16 shows the number of membership queries that were asked by various algorithms. Clearly, ModifiedQSM and MarkovQSM learners asked queries more than QSM as illustrated in Figure 7.16. Despite the similarity between MarkovQSM and ModifiedQSM in terms of the BCR and structural-similarity scores, the MarkovQSM learner asked fewer membership queries compared to ModifiedQSM when $L = 0.3$, due to the idea that MarkovQSM asks membership queries if the $Im$ score is higher than the EDSM score. Moreover, the number of membership queries that posed using the MarkovQSM learner was 8.2% less than those posed using ModifiedQSM when $l = 0.3$. Thus, a possible
explanation for this reduction might be that the collected traces does not cover paths in the reference LTS well.

It is important to assess the performance of the proposed algorithms in terms of the number of membership queries posed to the oracle. As shown in Figure 7.16, QSM asked fewer queries than other learners. Table 7.16 provides summary of the $p$-values obtained from the Wilcoxon signed-rank statistical test for the number of membership queries. The null hypothesis $H_0$ is stated as the number of membership queries posed by the learners are the same. The test reported $p$-values less than 0.05 when comparing ModifiedQSM and QSM. In this case, the $H_0$ is rejected, which means that ModifiedQSM consumed more queries as shown in Figure 7.16. With the number of traces three or four, the null hypothesis $H_0$ which stated ModifiedQSM and MarkovQSM learners posed the same number of membership queries can be rejected since the $p$-values are greater than 0.05; this means that MarkovQSM asked fewer queries compared to ModifiedQSM.

<table>
<thead>
<tr>
<th></th>
<th>Trace Number</th>
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</tr>
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<tbody>
<tr>
<td></td>
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<td>4</td>
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<td>0.1</td>
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<td>$1.75 \times 10^{-06}$</td>
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<td>$1.74 \times 10^{-06}$</td>
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<tr>
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<td>ModifiedQSM vs. MarkovQSM</td>
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<td>0.49</td>
</tr>
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<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>$1.80 \times 10^{-06}$</td>
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<td>$1.79 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>0.15</td>
<td>$1.56 \times 10^{-04}$</td>
</tr>
</tbody>
</table>

Table 7.16: $p$-values obtained by the Wilcoxon signed-rank test of number of membership queries for water mine case study

Figure 7.17 shows the percentage of transitions that were covered by the traces in the conducted experiments for this case study. It turned out that the QSM learner performed poorly when the generated traces did not visit all transitions. It is important to highlight that ModifiedQSM and MarkovQSM performed better than OSM if the transition cover was 80%. It was noticed early in this section, the BCR scores achieved by MarkovQSM when $l = 0.1$ were lower than the attained scores when $l = 0.3$. This is because the transition cover increased when $l = 0.3$ in comparison with $l = 0.1$, as seen in Figure 7.17.
Figure 7.17: Transition coverage of water mine case study

Figure 7.18: Markov precision and recall scores of water mine case study

Figure 7.18 shows the precision/recall scores of the trained Markov models for various numbers of traces that were considered in the conducted experiments. The precision scores of the trained Markov model were above 0.80, and this led to the detection of
inconsistencies whenever state merges were performed. However, the recall is very low when the number of traces is two and \( l=0.1 \) since the transitions were not covered well in this case, as shown in Figure 7.17.

![Figure 7.19: Inconsistencies of water mine case study](image)

During the conducted experiment, it was interesting to compute the number of inconsistencies with respect to the reference LTS. Figure 7.19 shows the number of inconsistencies that were found in the reference LTS based on the trained Markov models. The idea behind computing the inconsistency score for the reference graph is to measure how accurately the MarkovQSM learner can detect inconsistencies. In addition, there is a relationship between the number of inconsistencies computed for the reference model based on the trained Markov model and the performance of the learners. One may notice that the highest BCR scores of the inferred models using QSM happens when the collected traces cause the smallest inconsistencies.

### 7.3.3 Case Study: CVS Client

The BCR scores of the mined LTSs using different algorithms are shown in Figure 7.20. ModifiedQSM learned LTSs with better BCR scores compared to other learners. Moreover, the BCR scores of the inferred LTSs using the MarkovQSM learner were not good enough as ModifiedQSM, especially when \( L = 0.3 \). When the number of traces were three and
\( l = 0.3 \), the average reduction in the BCR scores attained by MarkovQSM was 5.43\% in comparison with ModifiedQSM. In addition, there is a decrease of 5.21\% of the BCR scores obtained by MarkovQSM compared to ModifiedQSM. The reason behind this drawback in the MarkovQSM learner is that early merging of states was not correct due to EDSM scores being higher than \( Im \) scores; hence, states were merged without asking queries. Similar to ModifiedQSM, MarkovQSM outperformed upon QSM and the credit goes to one-step queries.

![BCR scores of CVS protocol case study](image)

**Figure 7.20:** BCR scores of CVS protocol case study

<table>
<thead>
<tr>
<th>( l )</th>
<th>Trace Number</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
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<tr>
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<td>ModifiedQSM vs. QSM</td>
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<td>( 1.05 \times 10^{-04} )</td>
<td>( 2.88 \times 10^{-05} )</td>
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<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>( 1.96 \times 10^{-04} )</td>
<td>( 3.73 \times 10^{-04} )</td>
<td>( 2.88 \times 10^{-05} )</td>
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<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
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<td>ModifiedQSM vs. MarkovQSM</td>
<td>0.01</td>
<td>( 4.81 \times 10^{-04} )</td>
<td>( 4.81 \times 10^{-04} )</td>
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</table>

**Table 7.17:** \( p \)-values of Wilcoxon signed-rank test of BCR scores for the CVS case study
A summary of statistical analysis for the CVS case study is given in Table 7.17 where the \( p \)-values resulted from comparing BCR scores. In this study, the considered null hypothesis \( H_0 \) stated that the BCR scores of any two learners are the same. The \( p \)-values obtained by comparing the BCR scores of ModifiedQSM and QSM suggested rejection of the \( H_0 \) since the \( p \)-values were less than 0.05, and this indicates that there is a statistically significant difference between both learners. This means that the BCR scores attained by ModifiedQSM were higher than QSM as shown in Figure 7.20. When comparing MarkovQSM and QSM when \( l = 0.1 \), the \( p \)-values were too small and showed a statistically significant difference between both learners. In other words, the performance of MarkovQSM was better than QSM. However, when the number of traces is four and \( l = 0.3 \), MarkovQSM did not perform better than QSM since the \( p \)-value was 0.27. The \( p \)-values were above 0.05 when \( l = 0.1 \) when comparing the BCR scores of ModifiedQSM and MarkovQSM learners, indicating that there was no significant difference between the two learners. Moreover, the Wilcoxon \( p \)-values were below 0.05 when \( l = 0.3 \), implying that ModifiedQSM inferred LTSs with better BCR scores compared to MarkovQSM.

![Figure 7.21: Structural-similarity scores of CVS protocol case study](image)

Figure 7.21 depicts the structural-similarity scores of the inferred LTSs using MarkovQSM, ModifiedQSM, and QSM respectively for the CVS case study. From the boxplots that are
shown in Figure 7.21, it is apparent that ModifiedQSM generated LTSs models with the highest structural-similarity scores compared to other learners. In addition, the structural-similarity scores achieved by MarkovQSM was not good, as compared with the structural-similarity scores of the inferred models using ModifiedQSM. It is worth mentioning that the structural-similarity scores of the inferred LTSs using QSM were much lower than those attained by other learners. This proves that membership queries asked by QSM are insufficient to learn LTSs well.

Table 7.18 summarizes the $p$-values that resulted from the Wilcoxon signed-rank statistical test after comparing the structural-similarity scores of learners. The null hypothesis $H_0$ is that the structural-similarity scores of the learners are the same. The $H_0$ is rejected when comparing ModifiedQSM and QSM because the $p$-value is less than 0.05, and this is a strong evidence that there is a statistically significant difference between both learners. Moreover, when comparing structural-similarity scores obtained by MarkovQSM and QSM learners, the $p$-values show that there is a significant difference between both learners when $l = 0.1$. However, the $H_0$ is accepted when $l = 0.3$ and the number of traces are four; this means that the performance of MarkovQSM and QSM are the same in terms of the structural-similarity scores as shown in Figure 7.21.

<table>
<thead>
<tr>
<th>$l$</th>
<th>Trace Number</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>ModifiedQSM vs. QSM</td>
<td>$4.00 \times 10^{-06}$</td>
<td>$4.00 \times 10^{-06}$</td>
<td>$2.70 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>$6.18 \times 10^{-06}$</td>
<td>$1.16 \times 10^{-05}$</td>
<td>$2.70 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>0.059</td>
<td>0.016</td>
<td>0.04</td>
</tr>
<tr>
<td>0.3</td>
<td>ModifiedQSM vs. QSM</td>
<td>$1.03 \times 10^{-05}$</td>
<td>$1.82 \times 10^{-06}$</td>
<td>$2.64 \times 10^{-06}$</td>
</tr>
<tr>
<td></td>
<td>MarkovQSM vs. QSM</td>
<td>0.06</td>
<td>0.049</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>ModifiedQSM vs. MarkovQSM</td>
<td>$1.30 \times 10^{-04}$</td>
<td>$8.69 \times 10^{-06}$</td>
<td>$1.92 \times 10^{-05}$</td>
</tr>
</tbody>
</table>

Table 7.18: $p$-values of Wilcoxon signed rank test of CVS case study for structural-similarity scores

The number of membership queries that were submitted to the oracle using various algorithms is shown in Figure 7.22. As the figure demonstrates, the MarkovQSM learner asked fewer queries compared to ModifiedQSM when $l = 0.3$. This demonstrates that the EDSM scores were higher than the Im scores, and this led to queries being skipped, which was not accurate. It is worth mentioning that ModifiedQSM and MarkovQSM learners
asked more queries than QSM, as illustrated in Figure 7.22. There is a slight decline of 1.71% of the average of numbers of membership queries posed by MarkovQSM compared to ModifiedQSM when $l = 0.3$ and the number of traces is two. In addition, when the number of traces is three, the number of membership queries asked by the MarkovQSM learner was decreased by 12.04% compared to ModifiedQSM.

It is vital to determine whether any of the learners asked fewer queries than other learners. Table 7.19 provides a summary of the $p$-values obtained from the Wilcoxon signed-rank statistical test for the number of membership queries. The null hypothesis $H_0$ is stated as the number of membership queries posed by the learners are the same. The test reported $p$-values less than 0.05 when comparing ModifiedQSM and QSM. In this case, the $H_0$ is rejected, which means that ModifiedQSM consumed more queries, as shown in Figure 7.22. With $l = 0.3$, the null hypothesis $H_0$ which stated ModifiedQSM and MarkovQSM learners posed the same number of membership queries can be rejected if the number of traces is four; this means that there is strong evidence MarkovQSM asked fewer queries compared to ModifiedQSM. However, there is no significant difference between the ModifiedQSM and MarkovQSM learners in terms of the number of membership queries when the number of traces is three since the $p$-values > 0.05.
Figure 7.23 shows the transition coverage that was computed based on the traces in the conducted experiments for this case study. It is obvious that only the ModifiedQSM learner performed well when not all transitions were visited by the generated traces. On the other hand, the performance of QSM was worse than both ModifiedQSM and MarkovQSM learners.

The precision and recall scores of the trained Markov models computed for all traces considered in the conducted experiments are illustrated in Figure 7.24. The precision
scores of the trained Markov model were above 100% and this led to accurate predictions being made. However, many predictions were missed since the recall was very low when \( l = 0.1 \) and the transitions were not covered well in this case as shown in Figure 7.23. The performance of MarkovQSM was not affected by the low value of the recall since MarkovQSM asks queries whenever the Im score exceeds the EDSM.

The number of inconsistencies computed for the reference LTS of CVS case study after training Markov models are illustrated in Figure 7.25. It is obvious that the number of inconsistencies was high when \( l = 0.1 \), and this meant that the MarkovQSM learner did not skip asking membership queries during the early mergers of states.
Chapter 7. Experimental Evaluation of ModifiedQSM and MarkovQSM

7.4 Discussion

The approaches presented in this chapter designed to infer LTS models of a software system from positive traces. The idea of asking membership queries in QSM, ModifiedQSM, and MarkovQSM is inspired by the Angluin’s learner [79] and is aimed at avoiding poor generalizations of LTSs.

The comparison of the performance of ModifiedQSM and MarkovQSM against QSM shows that in the majority of cases ModifiedQSM able to infer LTSs with higher BCR scores even if the number of traces was small. In addition, MarkovQSM has proven to infer LTS with higher BCR scores compared to QSM. This can answer the first question that is described in Section 7.2 which is about whether ModifiedQSM and MarkovQSM can infer LTSs with high accuracy better than QSM. The outcomes obtained from the conducted experiments based on both random LTSs and case studies showed that the performance of the ModifiedQSM and MarkovQSM learner outperforms QSM.

In terms of the structural-similarity scores, the findings from the conducted experiment using random LTSs shows that the performance ModifiedQSM is superior to QSM. In cases
where $m > 0.5$, ModifiedQSM shows a slight improvement compared to MarkovQSM in terms of structural-similarity scores. This is due to the MarkovQSM learner finding inconsistencies accurately, and allows asking membership queries when $Im$ scores exceed EDSM scores.

The efficiency in the performance of MarkovQSM against ModifiedQSM shows that both learners can infer LTSs with very similar BCR and structural-similarity scores when $m = 2.0$. This is due to the high precision value of the trained Markov model. In this case, MarkovQSM allowing merging states without asking membership queries accurately compared to the cases where $m < 2.0$. Therefore, it appears that the performance of the MarkovQSM learner depends on the accuracy of the trained Markov model.

It was noticed during the conducted experiments that the ModifiedQSM algorithm asked more queries compared to other algorithms. Since both ModifiedQSM and MarkovQSM asks membership queries using the Dupont and one-step generators, it is expected to ask more membership queries compared to the QSM learner. Although ModifiedQSM and MarkovQSM asked membership queries more than QSM, they inferred LTSs with higher accuracy. This is due to the extra check made by the one-step queries that help the ModifiedQSM and MarkovQSM learners to prevent bad state merges during the inference process.

Another important finding was that MarkovQSM able to reduce the number of membership queries submitted to the oracle compared to ModifiedQSM. For the conducted experiment using random LTSs, MarkovQSM has shown to reduce the number of membership queries with slight losing in the accuracy of the inferred LTSs when $m = 1.0$ or $2.0$. Surprisingly, for water mine pump and SSH case studies, MarkovQSM performed very similar to ModifiedQSM. Moreover, MarkovQSM asked fewer queries compared to ModifiedQSM in both case studies, especially when $l = 0.3$. This is because the initial Markov models were trained with enough traces. Hence, MarkovQSM was able to skip asking membership queries accurately.

To conclude, the conducted experiment has proven that the quality of the inferred LTS can be improved using the concept of active learning. Walkinshaw and Bogdanov [45] stated that the QSM learner can infer an accurate LTSs if the provided traces is structurally complete (see Definition 2.10). Sometimes, the ModifiedQSM learner can infer a correct automaton even if the traces are not structurally complete. Therefore, it is important to
emphasize that the ModifiedQSM learner is able to infer a correct automaton if they ask queries that prevent bad mergers.

7.5 Threats to Validity

1. **The randomly generated LTSs may not represent real-world models.**

   This threat is mitigated by evaluating the performance of ModifiedQSM and MarkovQSM on case studies that are used in the literature review and the survey of stamina competition [35].

2. **The selection of training data may not be representative.**

   One possible threat to validity is the selection of training data where they might not be representative. For random LTSs, this threat is mitigated against it, each LTS was attempted to infer 5 times with different paths generated each time since random traces may follow the same paths many times. When the performance of ModifiedQSM and MarkovQSM were evaluated on case studies, 30 different sets of random traces were used in order to mitigate this threat.

3. **The size of case studies may not representative of all state machines.** One possible threat to validity is that the sizes of case studies where their sizes are small. The reason behind evaluating the performance of ModifiedQSM and MarkovQSM on such case studies are that they are widely used in state machine inference papers.

4. **The parameters settings could bias the results.** In the conducted experiments, there are many parameters such as the number of traces, alphabet size, and prefix length. Another possible threat to validity is that the variance of such parameters may bias the results. This threat is mitigated by choosing a different multiplier to vary the parameters. For instance, the alphabet multiplier $m$ intended to vary the alphabet size to assess the performance of ModifiedQSM and MarkovQSM on different types of LTSs. In addition, different number of traces were selected to assess the performance of the proposed algorithms.
7.6 Conclusion

This chapter evaluates *ModifiedQSM* and *MarkovQSM* learners. The performance of the *ModifiedQSM* was evaluated using randomly generated LTSs and case studies, the outcomes showing that despite the cost of the number of membership queries was high, the accuracy of the inferred LTSs was improved significantly.

On the other hand, the *MarkovQSM* learner showed its ability to improve the accuracy of the inferred LTSs compared to *QSM*. Moreover, the computation of inconsistencies can aid the *MarkovQSM* learner to reduce the cost of membership queries that are asked by *ModifiedQSM*. However, the performance of *MarkovQSM* is limited when the alphabet size is not very high.
8 Conclusion and Future Work

8.1 Introduction

Inferring state-machine models is an approach to supporting verification and validation techniques that is gaining interest among software engineers. The existing inference techniques suffer from a poor ability to infer good models when only positive traces are available. This thesis has focused on the inference of LTS models in cases where the negative traces are sparse or completely missing. In this research, new LTS learning algorithms have been proposed to improve the existing ones. The first inference algorithm relies on the idea of heuristic-based state merging representing in the EDSM learner and the trained Markov models to work together to prevent bad generalization of models. The second learning algorithm utilizes the idea of active learning to overcome bad generalization of LTSs, which is named ModifiedQSM. The third inference algorithm is called MarkovQSM, which is an extension to ModifiedQSM. MarkovQSM depends on information extracted
from the Markov models to reduce the number of membership queries consumed by other learners.

8.2 Summary of Thesis and Achievements

The introduction to the thesis is described in Chapter 1, including the motivation and problem of the thesis. In addition, it provides an overview of the problem of specification inference and its impact on verification and validation techniques. Chapter 2 introduced the definitions and notations related to LTS models. Besides, it includes introduction to LTS learning in terms of state merging strategy. Chapter 2 addressed the ways of evaluating state-machine inference techniques from different perspectives, providing the evolution framework used in this thesis.

Chapter 3 described the existing inference techniques that rely on state merging and identified problems to be solved in the following chapters. In addition, it provides other relevant research studies of LTS inference.

Chapter 4 and Chapter 5 demonstrated the possibility of using the Markov model alongside the EDSM learner to help the inference process in order to infer good LTS models. This combination technique is called EDSM-Markov, which is described in Chapter 4. In addition to this, ways to evaluate the correctness of the trained Markov models are described using the precision and recall.

The quality of the inferred LTSs by EDSM-Markov was compared to SiccoN inference using metrics, and it was shown to obtain reasonable results (see Chapter 5) in the majority of cases. Although inferring good LTSs using the EDSM-Markov learner was achieved when the size of the alphabet was large. However, there were different cases that did not achieve good BCR scores such as where $|\Sigma| = 0.5 \times |Q|$.

Chapter 6 proposed two different LTS inference algorithms that aimed to improve performance of the QSM learner [36] to infer better LTSs. The efficiency of ModifiedQSM and MarkovQSM learners was evaluated as demonstrated in Chapter 7. to measure the capability of both learners using different randomly-generated LTSs and case studies. The outcomes from the experimental evaluation can be summarized by saying that
the ModifiedQSM learner outperformed other learners in terms of the quality of the inferred LTSs, however it also asked more membership queries. The MarkovQSM learner has been proven to reduce the number of membership queries compared to ModifiedQSM.

8.3 Contributions

The study in this thesis has focused on the well-known problem of inferring LTSs from only positive traces and introduced improvements to the existing state-merging inference algorithms. The present study contributes evidence that suggests that the inference of LTSs can benefit from a predictive model in order to overcome difficulties of inferring LTSs from positive traces only.

The EDSM-Markov learner makes it possible to improve an existing EDSM inference method in the context of LTSs encountered in software models. As stated in Chapter 1, passive state-machine learning based on the state-merging strategy is known to cause over-generalization, especially if the provided traces are sparse or negative traces are insufficient to prevent over-generalization. The empirical finding in this thesis showed that it is possible to reduce the problem of over-generalization. This is based on building Markov models that can then be used in a scoring heuristic, permitting effective learning from a few entirely positive traces. It has proven to perform well for learning LTSs when the alphabet size is very large.

One of the important contributions of the described work is that the EDSM-Markov learner has the ability to assess its own performance. Where the inconsistency score computed for an inferred LTS is very different to the one computed for the reference LTS based on the initial PTA (Traces), this usually indicates that the inferred graph is very far from what might be expected. Furthermore, in the conducted experiment with random LTSs, if the inconsistency score computed for the initial models was above a few hundred, this indicates that the EDSM-Markov learner has to collect more traces.

Interestingly, the empirical findings in this thesis provide a new route of investigating using an inconsistency measurement as a coverage metric for a training set. This is based on the experimental observation that where an inconsistency of a reference graph based on training data is above 100, it is unlikely that such training data will be useful in inference.
The present study in Chapter 7 contains several contributions to the original QSM learner. It extends the membership queries by proposing new generator of queries that are designed to avoid bad state mergers. In addition, both ModifiedQSM and MarkovQSM learners avoid restarting the learning process of an LTS in the original QSM that was proposed by Dupont et al. [36]. The findings of using the inconsistency computation to the concept of active learning contributes additional evidence that the number of queries can be reduced. This is represented in the MarkovQSM learner described in Chapter 7.

8.4 Research Questions

1. **How effective are Markov models at capturing dependencies between events in realistic software?**

   The precision scores of the trained Markov models indicates how correctly they can capture dependencies between events that appeared in the provided traces. During the conducted experiments, the precision scores were high if the alphabet size was large, denoting that Markov models can capture dependencies between events appears in traces.

2. **How effective are Markov models as a source of prohibited events in the inference of models from realistic software using EDSM?**

   The use of Markov models to compute inconsistencies during the inference has been shown to aid the state-merging strategy to avoid bad mergers. Thus, events that are predicted as prohibited to follow a sequence of events of length $k$ can be used to detect inconsistencies accurately. Events that are not predicted either as permitted or prohibited by a Markov model can be used as a source of prohibited events if the generated traces cover transitions well. The evaluation of the EDSM-Markov learner proved that it benefits from computing inconsistencies based on such prohibited events. It was clear that the accuracy of the inferred LTSs models the EDSM-Markov learner was increased, and this was because inconsistencies were detected correctly.

3. **Under which conditions does EDSM with Markov models improve over EDSM without Markov models?**

   The findings in the experiments demonstrate that inferring LTSs using EDSM-Markov can achieve good results if the traces are covering the system being inferred well. In
addition, it was obvious that the quality of the inferred models is improved when
the alphabet size is very large. The proposed heuristic that relies upon the Markov
models helps the EDSM-Markov learner to identify inconsistencies resulting from
merging states. Those inconsistencies showed to help in blocking invalid mergers of
states.

The precision of the trained Markov model plays a vital role in detecting inconsis-
tences during the state merging process. In cases where the precision scores of the
trained Markov models are small, the EDSM-Markov learner fails to collect incon-
sistencies, and this leads to merge states that should not be merged. Hence, the
improvements were not good if the precision scores of the trained Markov models
were small.

4. To what extent are the developed inference algorithms able to generate
exact models and avoid over-generalization problem?

Inferring the exact models from only positive traces is difficult [56]. The developed
inference method that passively infers LTS models showed that obtaining the exact
model still hard to be accomplish. In some cases, the experimental outcomes proved
that it is possible to infer the exact model on the condition that the provided positive
traces covering the system well and the alphabet size is very large.

Besides this, over-generalization is a well-known problem that is encountered in the
domain of grammar inference and specification mining in a wider context. Over-
generalization occurs in case of learning from only positive training data [52, 145,
146]. The investigation in this thesis demonstrates that the problem of over-generalization
can be reduced either using the IScore heuristic or utilizing active learning, which is
more costly because it relies membership queries.

5. Under which conditions does QSM with Markov models improve over
QSM without Markov models?

The outcomes from the conducted experiments showed that inferring LTSs using MarkovQSM
can improve over QSM if the alphabet size is large. This is due to the precision of the
trained Markov models being high enabling identification of questionable mergers.
In cases where alphabet size is very large, the BCR scores of the inferred LTS using
MarkovQSM were higher than those obtained using QSM.
6. With respect to the concept of active inference, what is the reduction of the number of queries obtained by using Markov models, compared to \textit{QSM}?

Since the concept of active learning was proposed by Angluin [79], researchers have worked on reducing the number of queries submitted to the oracle. Unfortunately, the \textit{ModifiedQSM} learner has proven to ask more queries compared to the \textit{QSM} learner. This is intuitive and it is possible to say that gaining an improvement regarding the accuracy of the inferred models takes more effort in terms of the number of asked queries.

\section*{8.5 Limitations and Future Work}

One of the major limitations of the present study is that the idea of finding inconsistencies based on the Markov models is not suitable for learning LTSs with small alphabets. Another limitation of the proposed learner that relies on the concept of passive learning (\textit{EDSM-Markov}) is the collection of traces. Where achieving high-approximation LTSs is not an easy task, the performance of \textit{EDSM-Markov} learner is limited if the number of traces is small or they did not cover the system under inference. Further improvements can be introduced in the future work as will be described in Section 8.5.1.

Another weakness in the proposed techniques that rely on the concept of active learning is that the exact identification of LTS models is difficult if the number of traces is very small; this is because the performance of those techniques depends on the provided traces. Possible improvements can be made to improve the quality of the inferred LTSs and to reduce the number of membership queries submitted to an oracle.

\subsection*{8.5.1 Possible Improvements to \textit{EDSM-Markov}}

Future work could aims to improve the inference technique in terms of the accuracy of their outcomes. For instance, one may use Markov models to predict labels of transitions leaving a state, based on other sequences of labels of outgoing transitions leaving the same state, or predicting labels of incoming transitions based on the existence of outgoing ones.
A significant part of future work will be making it easy to adjust heuristical scoring for specific kinds of automata. In addition, experiments of the EDSM-Markov learner showed that it was not easy to find the right balance between the EDSM score and Im score. One direction of future improvement is to use the inconsistency score as an estimator to identify pairs of states that cause the smallest inconsistency to give them preference to be merged first.

A future study could investigate starting the inference process using merging states not from the root but anywhere in a PTA. The aim is to avoid the problem of early mergers where limited choices of pair of states exist to pick the best one. This may be achieved by collecting states that share labels or a sequence of labels of outgoing transitions where merging such states causes the smallest inconsistencies based on the Im score.

Sometimes, the EDSM-Markov learner fails to find a good LTS even if the computed inconsistency for the initial PTA is small. Based on a high inconsistency score of the learnt LTS, the EDSM-Markov learner could then restart the inference process, with different heuristics or with rules mined from the traces, aiming to reduce the perceived inconsistency.

Finally, it is known that an effective test set can be used to infer a model from which it was derived, and in a similar way, one possible investigation of future work could be to see to what extent a measure of inconsistency could be good as a metric reflecting test set adequacy.

8.5.1.1 Finding Multiple Solutions

The proposed techniques of LTS inference from positive traces presented in this thesis attempted to infer only one LTS per specific problem to be solved. In their paper [110], multiple DFA solutions were inferred for each specific inference task. In addition, Heule and Verwer [110] used an ensemble method [121] to generalize those solutions by finding an average DFA language. In a similar manner, further research might investigate the possibility of inferring multiple LTSs for each specific inference problem to solve the sparseness of the traces. Besides, the computation of inconsistency introduced in Chapter 5 can be used to determine which one of those solutions causes the smallest inconsistency score. Further studies could examine the relationship between BCR scores and inconsistency scores. Given multiple LTSs inferred for a specific task, study the following problem:
How likely is it that the inferred LTS with the highest BCR score has the smallest inconsistency scores among other solutions?

The performance of EDSM-Markov becomes weak if the alphabet size is not too large and it tends to merge states incorrectly, which yields over-generalized LTSs. Future work may include introducing more constrains such those used by Heule and Verwer [110].

8.5.1.2 Mining Rules from the Traces

Another method that might be implemented to improve the proposed algorithms is to incorporate rule-mining techniques with the state-merging strategy. This is inspired by previous works [70, 138] where mined rules were used to block merging states if there is a contradiction with these rules. In other words, the mined rules capture dependencies between events in the collected traces, and the rules which can be used in our context to block merging of states if any of those rules are violated.

8.5.2 Possible Improvements to ModifiedQSM and MarkovQSM

The idea behind ModifiedQSM and MarkovQSM is to ask the membership queries during the computation of scores to give preference to states in the pair that are most likely to be equivalent. Unfortunately, similar to the QSM learner, both learners ModifiedQSM and MarkovQSM require training data that should have high coverage to infer the exact models. To avoid the sparseness of the training data, using counterexamples in the same way that they are used by Angluin [79] can help to overcome the low coverage of training data. However, this can be challenging because a restart of the learning is required after each counterexample.

Another line of improvement would be to investigate the effect of re-ordering the list of membership queries on the number of submitted queries to an oracle. The aim of re-ordering is to decide which membership queries to ask first, this may reduce the number of membership queries. To achieve this, a trained Markov model can be used to evaluate membership queries based upon the appearance of subsequences, where a query that has the maximum number of subsequences never seen so far will take a higher priority to be asked first.
As described in Chapter 7, in the one-step generator, labels of outgoing transitions leaving a red state that lead to an accepting state are asked from a blue state. Future work may include asking membership queries of making two or three steps. A future study investigating the use of rule-mining techniques to block merging of states before asking the membership queries would be very interesting.

8.6 Thesis Conclusion

This thesis has investigated the possibility of integrating the concept of Markov models to the problem of inferring LTSs from positive traces. There are different solutions that have been introduced to tackle this problem, and their efficiency is assessed using practical experiments.

The inference of LTS models from only positive traces has been demonstrated to be a hard task; however, the difficulty can be reduced by searching for further solutions, especially integrating domain-specific information such as temporal rules. The sparsity of the training data is known to be a challenge in different domains, and the concept of grammar inference is one of them. While passive inference methods may be enhanced by improving heuristics or integrating domain knowledge, the sparsity of training data will be problematic. This can be tackled using the concept of active inference, but it still needs further investigation to reduce the cost of asking and answering queries.
Appendix of inferred model evaluation

A.1 Test sequences generated for the text editor example

In the following table, the test sets that are generated from the reference LTS of the text editor example. The first column represents the list of tests, the second column represents the correct classification of each test whether the test is accepted (true) or rejected (false) by the reference LTS. For each test, the third column represents the corresponding classification that is obtained from the inferred LTS.

<table>
<thead>
<tr>
<th>Test</th>
<th>Reference LTS</th>
<th>Inferred LTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Close</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Save</td>
<td>false</td>
<td>false</td>
</tr>
</tbody>
</table>

*Table A.1: The set of tests and the corresponding classification using the reference LTS and the inferred LTS*

*Continued on next page*
Table A.1 – Continued from previous page

<table>
<thead>
<tr>
<th>Test</th>
<th>Reference LTS</th>
<th>Inferred LTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edit</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Exit, Close</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Exit, Save</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Exit, Exit</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Exit, Load</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Exit, Edit</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Save</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Load</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Edit, Load</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Close, Close</td>
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<td>false</td>
</tr>
<tr>
<td>Load, Close, Save</td>
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<td>false</td>
</tr>
<tr>
<td>Load, Close, Edit</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Exit, Close</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>Load, Exit, Save</td>
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<tr>
<td>Load, Exit, Exit</td>
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<td>false</td>
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</tr>
<tr>
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<td>false</td>
</tr>
<tr>
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<tr>
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<td>false</td>
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<td>Load, Edit, Exit, Close</td>
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<td>false</td>
</tr>
<tr>
<td>Load, Edit, Exit, Edit</td>
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<td>false</td>
</tr>
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