Quantitative Policy Analysis for Sustainable Development in Water-stressed Developing Countries

A Case Study of Morocco

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by

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Abstract

Sustainable development has become a widely acclaimed objective in the global policy arena, but its operational principles have not yet been established. The main objective of this thesis is to contribute to operationalising the concept of sustainable development through conducting quantitative analysis of sustainable development policies. A special attention is paid to water crisis in water-stressed developing countries and a case study of Morocco is conducted.

Before developing an applied model for policy simulations, an analytic model based on Ramsey-Cass-Koopmans type growth model is constructed. The analytic model not only serves as a model platform of the applied model but also provides crucial insights to solutions of the applied model. The analytic model is studied both analytically and numerically and important policy insights are obtained. In addition, this combined analysis highlights the importance of following up qualitative analysis by quantitative simulations.

Based on the implications provided by the combined analysis, the applied model is constructed by incorporating various stylised facts observed in water-stressed developing countries. This model is calibrated based on the Moroccan data for the year 1994, and also dynamically validated with time-series data of selected endogenous variables. The validation has been successful.

Numerical simulations of sustainable development policies are conducted on the validated model. Due to stochastic elements in agricultural production and foreign exchange rates there exist gaps between expectations and realised outcomes. The policy simulations explicitly deal with these gaps and provide interesting policy insights. Major findings include (i) importance of public investment in both safe water access provision and the irrigation sector, (ii) impacts of exogenous drivers on sustainable development policies, and (iii) importance of international aid flows to implement sustainable development through mitigating severe water consumption suppression caused by optimal pricing schedules.
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I DEDICATE THIS THESIS TO MY WIFE

MICHIYO SUZUKI

"Individual happiness is impossible until the entire world gains happiness."

Kenji Miyazawa (1926) An Outline Survey of Peasant Art
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Author's Declaration

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Chapter 1

Introduction

1.1 Background

1.1.1 Motivation

This thesis is an attempt to operationalise sustainable development in the spirit of the Brundtland Report (WCED 1987).

Sustainable development has been widely celebrated as a common orientation of the global community for more than a decade. This wide consensus across rich and poor countries largely stems from the Brundtland Report in which sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generation to meet their own needs" (WCED 1987; p.43).\(^1\)

The Brundtland Report explicitly recognises the reciprocal linkages between environmental problems and poverty in which poverty is "a major cause and effect of global environmental problems" (WCED 1987; p.3). Sustainable development

\(^1\) There are criticisms against vagueness of this definition of sustainable development (e.g. O’Riordan 1988, Daly 1990, Lélé 1991). These criticisms appear to underestimate the fact that the Brundtland Report was a political document at high level of international politics where nothing can be approved without political compromise. See the next subsection 1.1.2 for more discussion on this issue.
in the Brundtland Report primarily signifies prompt actions of all members of
global community to alleviate poverty all over the world. At the same time, it
imposes a restriction on the solutions such that the future generation should not be
deprived opportunities to meet their own needs.

The appeal of this concept was demonstrated by the Rio de Janeiro Earth Summit in
1992, which mobilised 172 countries with 108 leaders of these countries and
managed to produce Agenda 21, a blueprint of promoting sustainable development
(United Nations 1997). It was an aspiration encouraged by the Brundtland Report
that enabled such huge attention to be drawn to this summit and official
commitment to pursue sustainable development.

A remarkable feature of Brundtlandian sustainable development is its recognition
that the poverty alleviation (intra-generational equity) is not only an end but also a
means (or more precisely a precondition) of sustainable development. Unfortunately, the correctness of this recognition has been proved by rapidly rising
security concern in many parts of the world. People start recognising the present
gross global inequity as the principal threat to global security.

These facts indicate that the concept of sustainable development established by the
Brundtland Report is not only the right direction of the global community but also a
precious political asset in terms of global consensus. The necessity to implement
sustainable development is thus high, but there seems a significant gap between
necessary information to operationalise this concept and what the previous
sustainable development studies have provided.

Most of the previous studies concerning sustainable development interpret
sustainability as some variation of non-declining conditions such as maintaining
capital intact, non-declining per capita consumption, and so on (e.g. Pezzy 1992,
Solow 1993, Toman et al. 1995). The basic principle underlying various
sustainability indicators is the same, except for Ecological Footprints (Rees 1992)
that are based on the ecological carrying capacity concept. These works are designed to tell us whether the current situation is sustainable or not in a narrow sense, but they are not designed to tell us whether the current situation is worth sustaining or not. We must keep in mind that "development that perpetuates today's inequalities is neither sustainable nor worth sustaining" (UNDP 1996; p.4).

The danger of narrowly interpreting sustainable development as maintaining the current situation is real. We must draw a lesson from the Johannesburg Sustainable Development Summit in 2002. What rich countries wanted to discuss there were issues of global warming or biodiversity loss directly affecting sustainability of the current situation of these countries, while poor countries strongly demanded to discuss poverty alleviation issues. The Summit was destined for failure when the global community could not be united in committing to the Brundtlandian sustainable development. The Summit was not able to produce an equivalent of Agenda 21, and people realised that both the misery of poverty and the threatening environmental degradation have continuously worsened since the Rio Summit and that the global community has failed to tackle these urgent problems effectively despite of the official commitment made in the Rio Summit.

The lesson is clear. We must urgently articulate and operationalise the concept of sustainable development in the spirit of the Brundtland Report, and try to provide information necessary to implement it. This is a tough challenge but the way forward for the global community. The motivation of this thesis is to contribute to this challenge through establishing a relevant policy analysis framework consistent with Brundtlandian sustainable development.

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2 The most well known sustainable indicators include Genuine Savings (Pearce and Atkinson 1993) and Green National Accounting (Asheim 1994, Hamilton 1994).

3 Immediately after the Johannesburg Summit, the special envoy of the UN Secretary General to the Johannesburg Summit told to BBC that the summit was "close to collapse" and that "there's a huge gap between what the delegates have managed to achieve and people's expectations of them" (BBC online, 11 September 2002).
1.1.2 Articulation of the concept of sustainable development

The definition of sustainable development in the Brundtland Report is deliberately phrased and in a sense vague.\(^4\) It worked out to achieve the primary task of the Brundtland Report; the task to establish a worldwide consensus on basic principles to deal with poverty and environmental issues, which can be summarised that the global community is neither sustainable nor worth sustaining unless intragenerational equity is achieved. More rigorous and narrow definition of sustainable development could not have gained such a success in this challenging task (Lélé 1991).\(^5\)

The operational definition of sustainable development must be a subject of political consensus of stakeholders and it has a case specific nature. For instance the operational definition of needs for a whole nation may differ from that for one local community. Thus, the task to establish the operational definition must be, by nature, undertaken by individual projects/researches.

This task requires clarification of both the meaning of ‘needs’ and the necessary conditions not to ‘compromise the ability of future generation to meet their own needs’, for which one cannot avoid making one’s value judgement explicit.\(^6\) Implementation of sustainable development without such clarification may allow persons who actually do not care sustainability to abuse it (O’Riordan 1988).

This thesis articulates the study-specific concept of sustainable development as follows.

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\(^4\) My position does not contradict Lélé’s (1991) that “clarification and articulation is necessary if SD [sustainable development] is to avoid either being dismissed as another development fad or being coopted by forces opposed to changes in the status quo” (p.618). I believe that such clarification and articulation must be done at a project level, based on a fundamental consensus achieved by more vague concept presented by the Brundtland Report.

\(^5\) Jacobs (1991) pointed out that sustainable development is a contestable concept such as liberty, social justice and democracy which “have basic meanings and almost everyone is in favour of them, but deep conflicts remain about how they should be understood and what they imply for policy” (p. 60).

\(^6\) Common (1995) emphasises the importance to make one’s value judgement explicit in order to make the sustainability debate more productive.
The meaning of needs

Two different approaches to human needs may be useful to understand and operationalise sustainable development; one is the basic human needs approach and the other is the fundamental human needs approach.

Basic human needs approach is widely adopted by international developing agencies. Basic human needs are defined as food, clothing, shelter, safe water access, primary education, and so on. Poverty is defined as lack of opportunity to meet these basic human needs. This approach is conceptually simple and easy to operationalise when these needs are not satisfied, because both the ends and the means of development are visible.

Fundamental human needs approach views human needs from broader perspectives. Among others, Max-Neef (1992) proposes an interesting and powerful theory of fundamental human needs. He classifies ten fundamental human needs by distinguishing needs from satisfiers that are means to satisfy the needs. He argues that fundamental human needs do not have hierarchical structure except for the situation where subsistence needs are insecure, and that they are common in all cultures and in all historical periods, with exception of evolutionary change of human species. Consequently, “any fundamental human need that is not adequately satisfied reveals a human poverty” (p.200). On the other hand, selection of satisfiers varies across cultures and historical periods. He classifies five types of satisfiers of which three types (violator, pseudo-satisfier, and inhibiting satisfier) do not satisfy the needs.

Max-Neef’s theory helps us to distinguish satisfaction of needs from material consumption. Jackson and Marks (1999) argue that a large part of the current prodigal consumption in the rich nations is likely to be associated with satisfaction of non-material needs, and “material consumption may offer at best a pseudo-satisfaction of non-material needs and at worst may actually inhibit or..."
violate the satisfaction of those needs” (p.439). They refer to mismatch theory proposed by evolutionary psychologists that the mismatch between material consumption and the nature of human needs satisfaction may account for the symptom of poverties in rich countries such as high incidence rates of suicide and depression.

The fundamental needs approach helps to recognise that poverty exists in both rich and poor countries whenever any fundamental human needs, either material or non-material, are not satisfied. This recognition is crucial for sustainable development in rich countries where overconsumption is one of the principal threats to sustainable development and shifting social values is urgently called for in order to fill the gap between current prodigal consumption patterns and the nature of fundamental needs satisfaction. Sustainable development policy in rich countries must primarily address this challenge to make consumption patterns be consistent with fundamental needs satisfaction (Ekins 1992, Dodds 1997, Jackson and Marks 1999). In addition, the established consumption patterns must be such that all members of global community can pursue them without compromising ability of future generations.

On the other hand, in underdeveloped countries where subsistence needs are not adequately satisfied, the basic human needs approach seems effective and operational. Moreover, it is recognised that in developing countries the connection between material consumption and satisfaction of needs is more direct than in rich countries (Ekins 1992, Jackson and Marks 1999). The conventional idea of economic growth oriented social welfare improvement seems still relevant to developing countries to a certain extent.

(2) Constraints not to compromise ability of future generation

Many sustainable development studies interpret sustainability constraints imposed by sustainable development as non-declination of either proxies of welfare or determinants of welfare such as per capita consumption, production capacity or capital stock. For example, increase of Genuine Savings is interpreted as sustainability of the current development path. It is obvious, however, that it cannot
be true if the needs of the present generation are not satisfied currently. To operationalise sustainable development in the spirit of Brundtland, another way must be sought.

Recall that the Brundtlandian sustainable development employs a negative form of constraint: 'without compromising the ability of future generation to meet their own needs'. It is apparently more natural to define sustainability as a negative form, in which sustainable means 'not unsustainable' (Becker et al. 1999). This negative form definition of sustainability is easily associated with the resilience of a dynamic system.

Holling (1973) defines resilience as the propensity of a system to retain its organisational structures after perturbations. Based on this concept, Perrings and Dalmazzone (1997) define the resilience of a system as the maximum perturbation of the system that does not cause the system to leave the original stability domain. According to this definition, resilience at time $t$ is determined not only by system parameters but also by distance between the system's location at time $t$ and the boundary of the stability domain. Once dynamic behaviour of a system is modelled, sustainability constraints can be represented by the condition that the perturbations caused by development should be less than the system's resilience at any $t$.

Rigorously, loss of resilience is not equal to unsustainability of the system, since leaving the original stability domain does not necessarily mean collapse of the system. Nevertheless, the fact that the system outside the original stability domain is associated with high degree of uncertainty makes sense of employing resilience conditions as sustainability constraints.

It is apparently nonsense to impose sustainability constraints on all the systems surrounding us. It seems reasonable to confine the scope of sustainability constraints to the ecosystems functioning as life-support systems, e.g. hydrological cycle, atmospheric constituents stabilisation, oxygen cycle, nutrient cycles, soil regeneration, food chains, etc. There may be no objection against the fact that the
loss of resilience of these life-supporting ecosystems will compromise future generation's ability to meet their needs.\textsuperscript{8}

Implementation of sustainability constraints based on the resilience concept is not an easy task due to non-linearity, path-dependence, discontinuity, and uncertainty associated with ecosystems (Perrings et al. 1995). The extreme difficulty to predict outcomes of irreversible environmental changes requires a precautionary approach. It might be realistic to establish safe minimum standards for each target ecosystem based on precautionary principles with the currently available scientific knowledge/information.\textsuperscript{9} This is what this thesis assumes.

(3) Operational principles of sustainable development in this thesis

Based on the above discussion, this thesis articulates the study-specific operational principles of sustainable development as follows:

(a) The primary task of sustainable development is to meet the basic human needs all over the world, particularly in poor developing countries. This task requires coordination between adequate domestic policies of poor countries for poverty alleviation and proper international aid policies.

(b) In most poor developing countries economic growth towards higher material consumption is required to meet fundamental human needs including basic human needs.

(c) In rich developed countries sustainable patterns of consumption must be realised both for meeting fundamental human needs and for mitigating environmental impacts of the current prodigal material consumption patterns. Developed countries have to seek patterns of consumption that all members of

\textsuperscript{8} My definition of sustainable development is close to Choucri's (1999): "Sustainable development means meeting the needs and demands of human populations without undermining the resilience of life-supporting properties" (p.143-144). Also see Common (1995) for this definition.

\textsuperscript{9} Any costs accrued by adopting safe minimum standards should not be regarded as unacceptable in this case, because keeping resilience of ecosystem underpinning life-support systems is a precondition of human well-being and must be prior to anything else.
global community can pursue without loosing resilience of ecosystems underpinning life-support systems.

(d) Resilience of ecosystems underpinning life-support systems must be kept as a precondition of sustainable development. For this purpose, sustainability constraints must be set and observed. Determination of sustainability constraints must follow the precautionary principle and scientific uncertainty cannot be an excuse not to implement them. Sustainability constraints may be imposed on, e.g., water withdrawal, pollutant emissions, fish catch, forest clearance, etc.

(e) When sustainability constraints are irreconcilable with consumer sovereignty, consumer sovereignty must be restrained (cf. Common and Perrings 1992). Suppose that meeting basic human needs all over the world without violating sustainability constraints requires massive capital transfer from rich to poor countries, and that its implementation and consumer sovereignty of rich countries are incompatible. According to the operational principles of sustainable development in this thesis, the rich countries are still responsible for implementing such capital transfer.

1.2 Basic methodology

1.2.1 Study scope

(1) Focusing on the water crisis

This thesis exclusively focuses on the water crisis in water-stressed developing countries. The choice of water crisis is motivated by the recent growing concern of the global community over this issue, which is symbolised by the famous statement of Ismail Serageldin, then vice president of the World Bank; "the wars of the next century will be about water". The same concern urged the United Nations to declare the decade from 2005 to 2015 as the 'International Decade for Action — Water for Life'. 
Chapter 1  Introduction

Water is essential for meeting basic human needs through various pathways. First of all, clean water is indispensable for drinking, cooking, and hygiene purposes. The reality is, however, that 1.1 billion people all over the world do not have access to clean and safe water (WHO and UNICEF 2000). The lack of safe water access causes various waterborne diseases, which account for around 80% of infections in developing countries. It is estimated that nearly 4 billion cases of diarrhea cause 2 million deaths each year in developing countries, and most of the victims are young children under the age of five (WHO and UNICEF 2000). 200 million people are infected with schistosomiasis, and 10% of the people living in developing countries are infected with intestinal worms. Moreover, it is estimated that trachoma blinds 6 million people and risks sights of 500 million people each year (Hoffman 2004).

Food production requires adequate amount of water at the adequate point of time, which can be achieved only by introducing irrigation in many water-scarce regions around the world. It is acknowledged that global food demand cannot have been satisfied without drastic increase of irrigated area, nearly a fivefold increase during the last century (Rosegrant et al. 2002a). On the other hand, this rapid irrigation expansion results in massive water consumption that often accounts for around 80 to 90 percent of total water consumption (Rosegrant et al. 2002a, 2002b).

Water is essential for industrial production as well. Various industries, in particular energy, textiles, and paper industries, require a vast amount of water, and their products constitute a part of subsistence goods.

Lastly, but surely not least, the hydrological cycle forms the backbone of ecosystems. Its disruption often triggers irreversible collapse of life-supporting ecosystems. Desertification might be the most conspicuous case.

Against increasing water demands for meeting basic human needs, the available amount of water per person is decreasing. During the period between 1970 and 1990 the average water supply worldwide per person dropped by a third, and this tendency is expected to continue for the future due to world population growth.
(United Nations 2003). This means that water-stressed developing countries will face further difficulty in pursuing sustainable development.

The costs of exclusively focusing on the water crisis must be mentioned. Recently integrity between water and energy has been recognised. In his opening speech to the International Conference on Freshwater held in Bonn in 2001, the Executive Director of UNEP asserted: "there are only two issues that are so intensively interrelated and important for development and they are water and energy".

Hoffman (2004) describes various direct and indirect linkages between water and energy. As direct linkages, extraction, treatment and transportation of water requires large quantities of energy inputs, while energy production requires large quantities of water such as cooling water for thermal power plants, water injection to oil wells, and so on. As indirect linkages, he counts water pollution caused by energy production and use. In addition, Rosenfeld (2000) warns that emissions from fossil fuel combustion may suppress precipitation. All of these linkages reveal the importance of addressing water and energy issues in an integrated manner. Nevertheless, it is likely that inclusion of energy into the study scope would make the problem intractable. I leave this challenge of dealing with water and energy issues in an integrated way as an important future research topic.

(2) Focusing on economic aspects

This thesis assumes that sustainability constraints necessary for keeping resilience of ecosystems underpinning life-support systems are given. This does not mean that I underestimate importance or difficulty to formulate them. On the contrary, I believe such an important and difficult task must be rendered by proper interdisciplinary collaboration across ecology, environmental science, engineering, and economics, at least. Moreover, a high degree of uncertainties and severe knowledge limitation concerning this issue may rule out clear-cut solution. Instead, it might be necessary to establish them in an operational way to prepare several alternative versions of sustainability constraints and to build political consensus on the choice among them.
This thesis sets aside this genuinely interdisciplinary task and focuses on economic aspects of sustainable development. Instead of asking what are sustainability constraints or how they are formulated, this thesis asks how they can be incorporated and utilised in sustainable development policies. It is also expected that answers to the latter question will be helpful to address the former questions. Nevertheless, it must be re-emphasised here that best efforts to establish appropriate sustainability constraints through both interdisciplinary collaboration and adequate political process are essential to operationalise sustainable development.

(3) Single country framework

This thesis adopts a single-country framework due to its advantages in both modelling and data collection. Although the policy analysis in a multi-country framework has recently attracted research interests, the complex nature of problems imposes severe restrictions on selection of analytical methods.\(^\text{10}\) It seems wiser to establish a policy analysis framework relevant to the Brundtlandian sustainable development in a single-country framework focusing on developing countries, and to extend the scope in future.

On the other hand, a severe drawback of the single developing country framework is that actions of developed countries cannot be properly incorporated. In this regard this thesis simply assumes that developed countries can afford significant amount of capital transfer if it is required to implement sustainable development of the case study country. Although the volume of international aid flows is currently reducing, I hope that demonstration of positive impacts of such capital transfer on sustainable development could help to convince developed countries to increase international aid flows.

\(^{10}\) For multi-country policy analysis, see Turnovsky (1995, 1997).
1.2.2 Adoption of quantitative policy analysis

This thesis adopts the method of quantitative policy analysis based on scenario simulations. The notion of policies in this thesis is thus narrowly referred to as government quantitative policies defined as "the changing, within the qualitative framework of the given structure, of certain political parameters or political instruments" (Tinbergen 1952; p.2).

Focusing on quantitative policies does not mean that qualitative policies are less important than quantitative policies. Ciriacy-Wantrup (1967) argues that water policy is better defined as "a set of decision rules in a multistage decision process" consisting of three different levels of decision-making (p.179). At the lowest level decisions mean choosing proper values of control variables such as amount of water withdrawal. At the second level the decision process controls the institutional framework within which the lowest level decisions are made. For example, selection of property right regime or type of water management organisation might be the subjects of decision at this level. At the highest level the second level decision-making process itself is the subject of decision.

The rationale of his argument is that the institutional conditions are rather the subjects of decision than the given constraint in social decision-making. He claims that "the second level is the most significant one for the study of water policy" and that it is necessary to seek "criteria that could serve as conceptually and operationally meaningful proxies for the functional construct of optimizing welfare" (p.183). From this stand the most relevant analysis is not quantitative optimisation itself but comparative analysis of institutional setting in which such an optimisation is pursued.

Nevertheless, the benefits from applying quantitative policy analysis seem to fully compensate the costs. Government policies to be studied in this thesis, such as setting the rates of water charges and the levels of public investments, will have the complex and often interdependent direct and indirect effects on the various sectors of economy. Quantitative policy analysis provides the most effective way, and often the only way, to reveal overall consequences with reasonable accountability
(Sadoulet and de Janvry 1995). When the policy target consists of multiple objectives that cannot be aggregated in any unique objective function, quantitative policy analysis enables policy makers to understand tradeoffs between each objective associated with each policy scenario. Obviously, the sustainable development policy is the case.

1.2.3 Policy analysis procedure

Quantitative policy analysis consists of two steps; model construction and policy simulation (Sadoulet and de Janvry 1995).

Before explaining each step, it is useful to classify models into analytic models, stylised numerical models, and applied models (Robinson 1989).

Analytic models help to extract important assumptions and causal mechanisms that can rarely be revealed by more realistic and complicated models.

Stylised numerical models are a generalised version of analytic models in terms of relaxing strong assumptions that enable analytic models to have closed form solutions. Stylised numerical models contribute, through numerical simulations, both to quantifying various complicating effects and to investigating causal mechanisms of which implications depend on functional specification or parameter values.

Applied models are designed to conduct policy simulations. Although the final goal of model construction is to construct an applied model, starting from an analytic model or a stylised numerical model facilitates model construction and improves usefulness of the analysis.

Construction of an applied model is further divided into three parts, i.e. specification, calibration (or estimation, in case of econometric models), and validation of the model. Successful validation is essential for applied models to generate reasonably realistic base run results that can be used as a benchmark for policy evaluation. As a part of validation process, sensitivity analyses help to clarify important assumptions and parameter values that severely affect outcomes.
Policy simulations start from preparation of alternative policy scenarios by combining relevant policy instruments with particular values. An applied model then simulates each policy scenario.

Hence, the study procedure of this thesis consists of the following steps;
- to construct an analytic model,
- to conduct numerical simulation of the analytic model,
- to construct an applied model,
- to calibrate and validate the applied model with sensitivity analysis, and
- to conduct policy simulations.

1.3 Research objectives

The research objectives of this thesis are twofold. One is to establish a new policy analysis framework with the capability to implement sustainable development. The other is to clarify policy implications of sustainable development policies by conducting quantitative policy analysis based on the proposed framework. These two aspects seem to be equally important to fill the existing information gap in implementing sustainable development.

For the former aspect, the basic requirements for the proposed framework are as follows:
- Performance of sustainable development policies can be evaluated in terms of both social welfare and poverty alleviation.
- Sustainability constraints can be anchored in resilience of ecosystems essential to maintain life-supporting systems.
- Multi-dimensionality of water crisis can be represented. In particular, various pathways through which water affects basic human needs can be modelled.

For the latter aspect, interesting research questions are as follows:
- What do sustainable development policies look like?
- Are sustainable development policies feasible?
- How does exogenous environment affect sustainable development policies?
- What would be the expected role of international aid flows in implementing sustainable development policies?

1.4 Overview of this thesis

This introductory chapter formulates the research objectives to be addressed in the remaining chapters, which consist of literature review (Chapter 2), model construction (Chapter 3 and Chapter 4), data description and model calibration (Chapter 5), policy simulations (Chapter 6), and conclusions (Chapter 7).

Chapter 2 establishes the basic analytical framework of this thesis through literature review. This chapter reviews various alternative analytical frameworks from the viewpoint of compatibility with the articulated concept of sustainable development. Further, relevance of several original ideas is examined by locating them in the existing literature.

The policy simulation models are constructed in two steps described respectively in Chapter 3 and Chapter 4.

In Chapter 3 an analytic model for investigating interactions between water and economy is developed. Policy implications obtained by theoretical analysis based on the analytic model are summarised in propositions. These policy implications are further investigated through numerical simulations. The analytic model constructed in this chapter provides model platform of an applied model constructed in Chapter 4. Moreover, some insights obtained in this chapter are indispensable to solve the applied model numerically.

Chapter 4 presents the applied model for policy simulations. The applied model is developed by incorporating stylised facts commonly observed in water-stressed developing countries, such as vulnerability of rainfed agriculture, high incidence of lack of safe water access, etc. into the analytic model. The applied model departs from deterministic world of the analytic model by introducing several uncertainties
such as production risks. Furthermore, tax and trade issues are incorporated in order to accommodate empirical data such as an existing social accounting matrix described in Chapter 5.

Chapter 5 reports calibration and validation of the applied model along with data description. In addition to detailed description of datasets, general features of Morocco, the case study country, are also described in this chapter. A detailed explanation of calibration procedure, which seems useful but rarely found in the existing literature, is also provided.

Chapter 6 reports policy simulations. Since the model involves uncertainty, policy planning and policy implementation must be clearly distinguished. This chapter addresses this issue in detail, and establishes simulation procedure and policy evaluation criteria. Policy scenarios cover not only policy alternatives but also alternative environment (or exogenous drivers) such as tax regime, climate change, and so on. In particular, different levels of both a society's safe minimum standards and international aid flows are represented as policy scenarios. This wide coverage of policy scenarios demonstrates applicability of the applied model developed in this thesis.

Chapter 7 concludes this thesis with both assessing how properly the research objectives are fulfilled and pointing out future tasks.
Chapter 2

Formulation of Basic Framework

2.1 Introduction

The objective of this chapter is to establish a basic analytical framework through reviewing the existing literature. Literature review in this chapter means two processes; one is to seek useful prototypes of the framework in the existing literature, and the other is to elucidate original and novel elements of my framework with assistance from the existing literature.

This chapter is organised as follows. In the next Section 2.2 several candidates for the framework are reviewed with the main focus on studies directly relevant to either water problems or sustainable development. In Section 2.3, the basic analytical framework of this thesis is developed. The reviewed literature in this section covers broader scope than that in the previous section in order to connect innovative elements of this thesis with the existing literature. Lastly, the established analytical framework of this thesis is summarised in Section 2.4.

2.2 Review of existing analytical frameworks

Among the vast literature dealing with either water or sustainable development issues, the following four models seem to provide highly relevant analytical frameworks for this thesis: i) hydrology-based optimum allocation models, ii)
growth models, iii) linear multisector models, and iv) computable general equilibrium models. This section reviews these models in this order.

2.2.1 Hydrology-based optimum allocation models

Economists have regarded the optimum water allocation as one of the central objectives of water policy. Development of proper analytical frameworks for this purpose, in particular for the applied purpose, has been associated with several difficulties. Among such difficulties the multi-objective nature of water resources management and the complexity of hydrological systems might be the most important ones. The former issue is real when non-commensurability exists among the objectives. Though there are some optimising techniques to deal with this case, for example goal programming and method of constraints (cf. Dauer and Krueger 1980), a more common treatment is to make objectives commensurable by translating them into net benefits. The calculation of net benefits requires a set of production functions and damage functions which increases model complexity further. Only the recent rapid improvement in computational capacities enabled hydrology-based optimum allocation models to be applied to empirical water policy analysis at basin-wide or nationwide scale.

McKinney and Cai (1997) develop an integrated economic-hydrologic model for Amu Darya and Syr Darya river basins in Kazakhstan. The hydrological component is a node-link network model in which nodes represent physical entities such as rivers, reservoir aquifers, irrigation fields, industrial plants, etc. This hydrological model and a crop-water simulation model are incorporated into the economic optimisation model in which net benefits derived from each water use are maximised. This model has also been applied to the Maipo river basin in Chile (Rosegrant et al. 2000).

Bouhia (1998) proposes a further integrated framework for analysing links between sustainable water management and socio-economic development, the potential of which is demonstrated by a case study in Morocco. Her analytical tool integrates McKinney-Cai type hydrology-based water management model and an input-output type economic model. The former component maximises the discounted
sum of aggregate net benefits of water use by allocating water resource to river basins and economic sectors under a set of political and sustainability constraints. The economic component evaluates nationwide economic impacts of the optimal water allocation, which is the output of the water management model, based on the water-extended input-output table. This general equilibrium setting makes it possible to capture interdependence among institutions that is a source of externality. The limitation of this approach is, as pointed by the author herself, inability to accommodate the dynamic nature of development. Furthermore, a linearity assumption of the input-output model precludes substitutability between water and other production factors, which plays a key role in welfare analysis.

The hydrology-based optimum allocation models have ability to capture detail of water resources systems. As Bouhia (1998) suggests, combining a hydrology-based water management model and a computable general equilibrium model seems a promising approach.

2.2.2 Growth models

An extensive body of economic growth literature has proved the congeniality of growth models to the sustainability issue.1

(1) The first generation sustainable growth models

The pioneering works of this literature are those of Dasgupta and Heal (1974), Mäler (1974), Solow (1974), and Stiglitz (1974a, 1974b) all of which apply the Ramsey-Cass-Koopmans (RCK) growth model (Ramsey 1928, Cass 1965, Koopmans 1965). It is by no means coincidence that all of them appeared in 1974. The research motivation shared by these authors was to challenge the doomsday prophecy of the ‘World Model’ in which resource depletion causes an economic catastrophe (Meadows et al. 1972). Except for the model of Mäler (1974), which will be separately reviewed in the next section due to its particular importance to this thesis, all of their models, for convenience labelled as ‘neoclassical sustainable sustainable'.
The growth models in this thesis, have very similar specifications, such as adoption of an infinite time horizon and CES (constant elasticity of substitution) production technology, in particular Cobb-Douglas production technology, which allow them to draw their main conclusion that it is possible for an economy to have an infinitely sustained optimal growth with the existence of exhaustible resources. To borrow Romer's words as to his celebrated endogenous growth model, their main conclusion seems "more like an assumption than a result of the model" (Romer 1990; p. S84). Nevertheless, the neoclassical sustainable growth models have long served as a solid point of departure for this literature.

As a direct extension of Solow (1974), Hartwick (1977) shows that the necessary condition to achieve constant consumption over an infinite time horizon in the Solow model is to invest all scarcity rents from exhaustible natural resources, so called the Hotelling scarcity rents (Hotelling 1931), into manmade capital. This condition is often called Hartwick's rule.

Following this work many researchers have investigated analogues of Hartwick's rule with more general assumptions. For example, Hartwick (1978) introduces CES production technology with many exhaustible resources, and Dixit et al. (1980) investigate the case of further generalised economy of which production possibility is assumed to be stationary and convex. This line of research helps to clarify the working of the neoclassical sustainable growth models but does not provide additional insights about the importance of environmental aspects in the sustainability issue.

(2) Environment-extended models

Another strand of this literature has incorporated various environmental aspects into the neoclassical sustainable growth model. This strand can be regarded as an amalgam of the neoclassical sustainable growth models and the model of Mäler (1974) in which environmental quality constitutes an argument of utility function.

Krautkraemer (1985) introduces natural resource stock as an argument of utility function to the neoclassical sustainable growth model in order to investigate
sustainability implications of including amenity aspects of natural resources that were first discussed by Krutilla (1967). He shows that permanent environmental preservation could be compatible with constant growth in consumption only if (i) substitutability between manmade capital and natural resources is high enough, or (ii) individual preference is sufficiently environment-oriented.

Tahvonen and Kuuluvainen (1991) investigate simultaneous optimal control of renewable resources, pollution emission and capital accumulation based on a generalised version of the neoclassical sustainable growth model with an equation of motion of pollution stock and a regeneration function of renewable resource, which employs a weaker assumption of production function than CES. They show that sustainable optimal solution exists only if the government can set the rate of emission tax at the social optimal.

(3) Environment-extended endogenous growth models

The emergence of endogenous growth models in late 1980's further boosted this literature. Among many, the work of Bovenberg and Smulders (1995) seems to exemplify this literature.

Bovenberg and Smulders (1995) introduce two-sector endogenous growth model developed by Rebelo (1991), which consists of a final-goods sector and an environmental R&D sector, into this literature. Their model employs the Tahvonen and Kuuluvainen type equation of motion of pollution stock in more general form but it lacks a regeneration function of renewable resource. It should be noted that Bovenberg and Smulders (1995) regard environmental quality as renewable resource while Tahvonen and Kuuluvainen (1991) deal with environmental quality (pollution stock) and renewable resource separately. The key issue of Bovenberg and Smulders (1995) is simultaneous optimal control of knowledge accumulation in the environmental R&D sector, pollution emission and capital accumulation. Their main conclusion, which is analogous to Tahvonen and Kuuluvainen (1991), is that a sustainable optimal solution can exist only if the government sets both the

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2 Influential pioneering works in this literature include Romer (1986, 1990) and Lucas (1988).
rate of emission tax and the subsidy to the environmental R&D sector at the social optimal, on condition that the production function of each sector satisfies several requirements. They also show that both the social optimal emission tax rate and the social optimal subsidy to the environmental R&D sector must grow at positive constant rates but the growth rate of the former must exceed that of the latter.

(4) Stylised numerical growth models

Oueslati (2002) examines both the short-run and the long-run effects of environmental policy on economic growth by endogenising labour-leisure choice in a variant of two-sector endogenous growth model in which human capital is produced by schooling. The constructed model is used as a stylised numerical model rather than an analytic model, with two-stage calibration based on the average environmental protection expenses in OECD countries. It allows investigation not only of the optimal steady-state but also of transitional dynamics in spite of relatively high complexity. Her main conclusion is that a sharp rise of environmental tax rate results in net welfare costs in the short-run due to negative crowding-out effects on consumption and investment, but in the long-run it will turn to overall welfare benefits because of neutralised crowding-out effects owing to factor substitution and positive effects of time allocation shift from leisure to schooling time.

This conclusion gives an interesting contrast to that of Bartolini and Bonatti (2003) who also introduce endogenous labour-leisure choice into a similar growth model. The major differences of their model from Oueslati’s model are absence of human capital production process and introduction of substitutability between manmade goods and natural resources in household production of ‘satisfaction’ embedded in utility function. Their main conclusion is that the optimal steady-state under laissez-faire is characterised by an inefficiently high level of production with an excessively high proportion of labour hours relative to leisure hours, and an inefficiently low level of environmental resource stock, which collectively reduce households’ welfare.
Anderies (2003) provides another very interesting example of a stylised numerical growth model. He investigates dynamic interactions between population dynamics, changes in renewable resources, and economic growth simultaneously based on a model in which endogenous population dynamics is incorporated into a two-sector version of Solow-Swan growth model (Solow 1956, Swan 1956) with a fixed saving rate, consisting of agricultural and manufacturing sectors. It means that the optimisation process of this model is not dynamic optimisation but a sequence of static optimisation interlinked by equations of motion. This specification reflects his choice of dynamical systems approach. Anderies (2003) advocates that a dynamical systems approach with computer-based bifurcation analysis has strong advantages in studying qualitative properties of growth paths over simulation analysis or optimal control based dynamic optimisation analysis. Obviously a two-sector version of RCK model with endogenous fertility is intractably complex with six endogenous (i.e. three state and three control) variables. Further, he claims that policy analysis based on RCK growth models has limited applicability in practice because the policy implication of RCK growth models “rests on many assumptions including omniscient households with perfect foresight” while the dynamical systems approach rests on less assumptions (Anderies 2003; p.238). This important point will be discussed in the next section. His analysis starts from obtaining the optimal steady-state solution numerically given the base run parameter values. Then effects of changing parameter values on the qualitative properties of growth paths, either converging on an optimal steady state or oscillating indefinitely (overshoot and collapse), are depicted in bifurcation diagrams. His main conclusion is that the overshoot and collapse type trajectories become more likely when the rate of resource regeneration is lower than that of economic growth, and that demographic factors seem relatively more important than technological factors in avoiding undesirable overshoot and collapse trajectories.

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3 For dynamical systems approach based on bifurcation analysis, see Kuznetsov (1995).
(5) Water-economy growth model

Recently Barbier (2004) proposes a growth model addressing the interaction between water scarcity and economic growth. This work is, according to him, the first attempt to deal with water crisis issue in this literature. His model is developed based on congestible public good models proposed by Barro and Sala-i-Martin (1992, 1995) in which aggregate public services subject to congestion are equal to a share of aggregate economic output.

Barbier claims that utilised water itself is a public good subject to congestion and that aggregate amount of utilised water is equal to "a share (...) of aggregate economic output that is specifically devoted to water supply (e.g. dams, irrigation networks, water pipes, pumping stations etc.)" (Barbier 2004; p.5). As Barbier himself notes in his footnote 3, however, the analogue of aggregate public services in Barro and Sala-i-Martin (1992) is not water itself but 'water system' in terms of water supply facilities. If one attempts to sort out this confusion without changing the model specification, it is necessary to assume equality between water supply facilities and utilised water, or at least constant marginal product of water with respect to water supply facilities. This assumption is not only rejected by empirical evidence suggesting a diminishing marginal product of water production (Rosegrant et al. 2002a) but also inconsistent with his own statement; "as water becomes increasingly scarce in the economy, the government must exploit less accessible sources of fresh water through appropriating and purchasing a greater share of aggregate economic output" (Barbier 2004; p.2).

Even if this constant marginal product assumption were accepted, his main hypothesis, an inverted-U relationship between growth and water utilisation, could not be derived from his theoretical model.5

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4 In theoretical work measurement unit can be freely chosen. Thus any linear relationship in the real world can be transformed into equality in theory by choosing proper measurement unit.

5 He derives this hypothesis by applying the first order conditions, which are valid only along the optimal paths, to sub-optimal paths. Along the optimal paths, his model predicts a unique combination, depicted as a point on graph, of economic growth and water utilisation rates.
Although this work contains interesting discussion and information relevant to sustainable water policy issue, his main conclusions that his theoretical model predicts an inverted-U relationship between economic growth and water utilisation and that it is consistent with cross-country data have no foundation.

(6) **Towards sustainable development growth model**

It has been seen that growth models have high relevance to the sustainability issue, in particular to sustainable growth. In order to apply the growth models to quantitative policy analysis for sustainable development, however, it seems to be essential to tailor them for this particular purpose.

The vast majority of the existing growth model literature has almost exclusively focused on finding the first-best optimal outcomes without scrutinising their feasibility. In fact, all of the reviewed RCK models assume perfect foresight of households that can hardly be assumed in the real world. From the policy perspective, the first-best outcomes are important only if they are achievable by some policies, for instance Pigouvian tax. Finding such a policy requires analysis in the second-best world representing the reality.\(^6\) If there is no such a policy available, the first-best outcomes are meaningless not only in practice but also in theory due to the fact that they are surely not generated by key mechanisms underlying the real economic phenomena.

This ‘first-best world bias’ of the literature does not necessarily mean that growth models have limited applicability as tools of quantitative policy analysis. Considering its high ability to conduct dynamic analysis it is worth trying to tailor the growth models applicable to quantitative policy analysis rather than to seek a new analytical framework.

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\(^6\) The word ‘second-best’ was first coined by Meade (1955) in which the problems of second-best optimality were discussed in empirical background. Lipsey and Lancaster (1956) investigate theoretical implication of this issue.
2.2.3 Linear multisector models

Multisector models play an important role when structural changes in production and consumption activities matter (Robinson 1989, Duchin 1998).

Early linear multisector models, static and dynamic input-output models, have provided useful analytical frameworks mainly in development economics where the main research subject is development policies that induce structural changes (Robinson 1989). These models are also widely used in environmental economics literature mainly because of their ability both to capture externalities related to interdependence of sectors and to accommodate material balance analysis.

In this literature, one of the most relevant studies to this thesis is Duchin and Lange (1994) in which policy implications of sustainable development proposed by the Brundtland Report are investigated based on a dynamic input-output model and database of the world economy constructed by Leontief et al. (1977). They conclude that the key policy recommendations of the Brundtland Report (WCED 1987) such as more extensive use of environmentally friendly technologies or recycling could not achieve the environmental objectives required for sustainable development.

Another major linear multisector model is a social accounting matrix (SAM), which was developed as an integration of the national income and product accounts into the input-output model (Robinson 1989). The SAM is an extension of the input-output model in which all transactions among institutions such as households and firms are described.\(^7\) The SAM provides not only an analytical framework particularly suitable to deal with distributional issues but also an input database for computable general equilibrium models.

Keuning (1994) proposes an integrated data framework of economic, environmental and social statistics by combining a SAM and the environmentally

\(^7\) For the detail of social accounting matrix, see the revised 1993 System of National Accounts (United Nations 1993).
extended national accounts so called NAMEA (Keuning 1993), as a useful data framework for sustainable development studies.

Xie (2000) proposes an environmentally extended SAM (ESAM) and illustrates how to construct ESAM as well as how to use ESAM for structural path analysis (Defourny and Thorbecke 1984) with an ESAM compiled for the Chinese economy. The SAM has recently attracted significant attention in sustainable consumption literature as well.

These linear multisector models provide a suitable analytical framework to capture economic structures and direct and indirect linkages among activities of each constituent institute. On the other hand, their inherent linearity severely limits their applicability as an analytical framework to price-incentive type policy analysis where price adjustment mechanisms through substitution are important (Robinson 1989). For this kind of analysis computable general equilibrium models are regarded as a standard tool.

2.2.4 Computable general equilibrium models

Along with the rapid expansion of computational capacities as well as development of handy modelling packages over the last few decades, computable general equilibrium (CGE) models have become the most popular tool for applied work on a wide range of issues, such as tax/subsidy policies, public investment policies, changes in economic and social structures, terms of trade and exchange policies, etc. In particular, when the economic policies subject to analysis involve inter-sectoral or macroeconomic effects and income redistribution, CGE models are often regarded as the most powerful analytical framework.

CGE models require specifications of (i) behavioural rules of all the relevant economic agents such as households' utility maximisation, (ii) institutional structures of economy such as a perfect capital market, and (iii) equilibrium conditions (Robinson 1989).
Chapter 2 Basic Framework

The simplest CGE models are static neoclassical CGE models representing numerical solution of Walrasian general equilibrium. Static neoclassical CGE models are rarely applicable to developing economies because of their inability to incorporate reality of developing economies. Most of the applied CGE models for developing countries, in which 'applied' means that they are designed for policy analysis in practice, incorporate both microeconomic constraints such as imperfect markets and macroeconomic constraints such as balance of payments in a rather ad hoc manner. ⁸

Treatment of dynamics in applied CGE models is far more problematic than that in theoretical works. Recall that most RCK growth models employ a strong assumption of households' perfect foresight. Although this assumption seems justifiable in the first-best optimal policy analysis, it seems too unrealistic for applied purposes (Robinson 1989). It is thus much common for applied CGE models to accommodate economic dynamics as a succession of temporary equilibria, which are solutions of static optimisation, linked by dynamic adjustment mechanisms such as exogenous growth of capital and labour supply, etc. These types of CGE models are called recursive dynamic models, or sometimes called quasi-dynamic models because of their lack of dynamic optimisation.

For analysing policies in which intertemporal aspects are essential, we may need dynamic optimisation CGE models rather than recursive dynamic CGE models.⁹ Sustainable development policy is certainly in this category.

(1) The first generation water policy CGE models

Despite the rapidly growing number of CGE studies, there are not many works in this literature dealing with either water policy or sustainable development.

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⁸ Dervis et al. (1982) and Robinson (1989) provide a detailed account of this topic.

⁹ The model of Jorgenson and Wilcoxen (1993) exemplifies this line of works. Devarajan and Go (1998) provide a simplified version of dynamic optimisation CGE model.
A pioneering work in this regard is that of Berck et al. (1991). They construct a static regional CGE model for the purpose of demonstrating applicability of CGE models for regional water policy evaluation. With this model they simulate the hypothetical reduction of available water quantity for agriculture in the San Joaquin Valley, California. An important drawback of their model is lack of substitutability between water and other factors such as fertilisers and pesticides. Further, it is assumed that only the agricultural sector uses water.

Goldin and Roland-Holst (1995) investigate the effects of irrigation water pricing and trade liberalisation on sustainable water use and macro indicators such as GDP, government budget balance, etc. based on a static nationwide CGE model of Morocco. Their model consists of four production sectors (rainfed agriculture, irrigated agriculture, manufactures, and services), two types of households (urban and rural) and the government. Publicly supplied water is a factor input for all the production sectors other than rainfed agriculture but is not consumed by households. It is assumed that water demands at any given water price are met by public water supply with fixed unit supply costs and that water transfer between rural and urban areas as well as between rainfed and irrigated agriculture is determined by relative water prices fixed by the government. Based on this model Goldin and Roland-Holst conducted policy experiments with three policy scenarios, i.e. doubling irrigation water price, complete removal of nominal tariffs, and a combination of both, and showed that the last scenario results in higher GDP with less water use than the base run. In ‘Discussion’ as a part of this work, Jaime de Melo argues that the assumption of infinitely elastic water supply at a fixed price cannot capture implications of water shortage, which is one of their research motivations. De Melo also suggests inclusion of water in the utility function with further disaggregation of households to capture effects of water pricing policies on economy wide income distribution.

More sophisticated water policy CGE models

Löfgren et al. (1997) construct a recursive dynamic CGE model to analyse rural development of Morocco. This model is characterised by highly disaggregate agricultural sector with 33 activities, Harris-Todaro type rural-urban migration
mechanism (Harris and Todaro 1970) which allows urban unemployment to persist, potential excess supply of water with zero rent, quality differences between exports, imports and domestic commercial outputs as well as between the outputs of different domestic activities such as irrigated or rainfed agriculture, and various policy targets with corresponding policy instruments, all of which contribute to capture stylised facts of the Moroccan economy. Most of parameter values are derived from either a disaggregated SAM for the year 1994 or data available from the other existing studies on Morocco, and only the total factor productivity is calibrated to generate the target level of real GDP. The policy scenarios cover major water policies such as irrigation water pricing, tradable water rights, and irrigation expansion by the government investment. Based on the simulation results several candidate win-win scenarios for rural development are proposed. This work exemplifies capabilities of applied CGE models for quantitative policy analysis.

Decaluwé et al. (1999) demonstrate another usage of CGE models. They use a static CGE model to study irrigation water tariff reforms of Morocco, incorporating inter-sectoral impacts of such reforms on resource allocation and social welfare. The most interesting feature of their model is its elaborate treatment of water production that explicitly reflects the water scarce situation of Morocco. They distinguish two types of water production technologies; one is water production using the existing reservoirs which is characterised by decreasing marginal costs, and the other is that by pumping groundwater or by retrieving surface water with more sophisticated technology which is characterised by increasing marginal costs. The whole country is divided into two regions, i.e. the relatively water rich northern region and the arid southern region. The model is calibrated based on a SAM for the year 1985. Water pricing scenarios are a combination of alternative water pricing, i.e. Boiteux-Ramsey pricing, marginal cost pricing and an arbitrary 10% increase in irrigation water price, with several tax policies. Their simulation results show that a combination of Boiteux-Ramsey pricing with a reduction in production taxes generates the most desirable outcome associated with less water consumption and higher social welfare measured by equivalent variation without subsidising the water producer.
(3) Towards sustainable development CGE models

CGE models have an immense ability to investigate a wide range of issues across not only economic but also social and environmental problems in a coherent way. On the other hand, incorporating various problems in a model makes interpretation of output intractable (Sadoulet and de Janvry 1995). The following conventional wisdom holds for CGE models as well; the success of the model depends upon a balance between realism and analytical tractability.

Successful application of CGE models to sustainable development issues requires adequate treatment of dynamic elements. Recall that the currently available options are either recursive dynamic models that are criticised for lack of dynamic optimisation or forward looking dynamic optimisation models that are criticised for the unrealistic perfect foresight assumption. I wonder why there is no forward looking dynamic optimisation CGE model without the perfect foresight assumption. If we could liberate RCK growth models from the perfect foresight assumption, constructing such a CGE model as a generalised version of the RCK growth model would be rather straightforward. This modelling strategy seems worth pursuing in this thesis.

2.3 Development of analytical framework

2.3.1 Necessity of dynamic models

The choice between the static and the dynamic models seems a useful first step towards selecting the analytical framework. Mäler (1974) illustrates their comparative advantages by locating each model in a general analytical framework for investigating interactions between the economic system and its environment.

First he sketches a general equilibrium model incorporating both the flows of materials and energy, which play a central role in the material balance approach (Ayres and Kneese 1969, Kneese et al. 1972), and the intertemporal aspects such as capital and waste accumulation, population growth, exhaustion of non-renewable resources, and so on. Because such a model is too complex to obtain useful
implications, Mäler constructs two models, i.e. a static general equilibrium model with materials balances approach and environmental effects of waste discharges, and a highly aggregate RCK growth model for a finite time horizon.

In the former static model he introduces several assumptions, e.g. convexity of the production possibility set and consumer preferences, such that the model becomes a special case of Arrow-Debreu type general equilibrium models, and the existence of a unique equilibrium that is at the same time Pareto optimal is proved by appealing to the proof of Debreu (Debreu 1959; p.19). For the latter dynamic model he studies four variations of an aggregate RCK growth model that incorporates environmental quality as an argument of instantaneous utility function and exhaustible natural resources as a material balance constraint. Four variations differ each other in controllability of population, number of environmental qualities (single or multiple), and possibility of recycling. He proves that all of four variant models are associated with the turnpike property, with which the optimal trajectories tend to approach the optimal steady-state and stay most of planning period at the optimal steady state if the time horizon is enough long (Samuelson 1965). This turnpike property enables the optimal trajectories to be approximated by the optimal steady state. Hence, Mäler claims that the former static general equilibrium model could be regarded as a good approximation of the dynamic general equilibrium model along the optimal growth path.

This might not be the case, however, for economies away from the optimal growth path, which many developing countries are often assumed to have. In addition, an absolute speed of convergence towards the optimal steady state matters from the practical viewpoint. If it takes, say, decades for an economy to realise the optimal steady state, such an approximation would provide misleading information. Because my main concern is sustainable development of water-stressed developing countries where optimality can hardly be assumed, I choose the dynamic optimisation models in this thesis.

2.3.2 Necessity of decentralised setting

One very important aspect of Mäler’s dynamic optimisation models, which differentiates his models from other neoclassical sustainable growth models of
Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974a, 1974b), is the explicit role assigned to the government. In his models aggregate social preferences are "represented by the government" (Mäler 1974; p.60). Introduction of the government as a decision maker makes it more plausible that the decision maker controls aggregate pollution discharge. On the other hand, it is less convincing that the decision maker can control individual consumption levels. It is impossible for the government to freely control individual consumption levels, except for the case of totally centralised economy where a central planner can dictate everything.

It must be emphasised here that an unrealistic hypothetical setting itself is not necessarily a weakness of models. Barro and Sala-i-Martin (1995) succinctly state that this setting, they call "a benevolent social planner" setting, is "useful in many circumstances for finding the economy's first-best outcomes" (p.71). Nevertheless, when we go further to seek optimal policy that is effective and feasible in the real world, as this thesis attempts, this setting is of little relevance.

The importance of decentralised setting in seeking public policy is clearly recognised by Arrow and Kurz (1970). They argue that the government in a decentralised society does not have discretion to make all allocative decisions but "has the choice of values of a limited range of instruments" (p.115). This understanding, which they regard as the dynamic version of the theory of economic policy proposed by Tinbergen (1952), leads them to set out a theory of controllability of public policy of which a central issue is the necessary conditions for the government to achieve the publicly optimal path, and a dynamic theory of the second-best policy which is defined as the best policy in noncontrollable situations with the given set of instruments.

In order to study these issues, Arrow and Kurz (1970) formulate dynamic optimisation models in which private sector optimisation and that of public sector are separately dealt with. In the private sector optimisation, the representative individual maximises the total net present value of all individuals' welfare in an economy, taking the government policy such as tax rate and stock of public capital as exogenously given. In the public sector optimisation, the government selects only the rates of public investment, taxes and government borrowing subject to a budget limitation. It is assumed that the representative individual who lives forever
and "looks ahead infinitely far with perfect foresight" and that the 'felicity function' and the rate of time preference of the government are different from those of the representative individuals (Arrow and Kurz 1970; p.153)\(^\text{10}\).

Because their main interest is to investigate general properties of controllability and stability of the public fiscal policy, they do not explicitly solve the public optimisation problem. Instead, they introduce several boundedness assumptions on convergence of variables and show the implications of controllability and stability with different combinations of policy instruments of the government. The decentralised setting investigated by Arrow and Kurz, in more general form, can be formalised as follows.

Let \(V^P\) and \(V^G\) denote the objective functions and \(f^P\) and \(f^G\) denote the constraint sets of the private and the public sectors, respectively. Each sector maximises (or minimises, depending on model specification) its objective function by choosing the values of its instruments (control variables) subject to the given constraint sets. Let vectors \(x^P, x^G, y^P\) and \(y^G\) denote the control variables and the state variables of the private and the public sectors, respectively.

The private sector problem is

\[
\max_{x^P} V^P(x^P, y^P; x^G, y^G) \text{ subject to } f^P(x^P, y^P; x^G, y^G) = 0. \tag{2.1}
\]

The private optimal solution can be described as functions of exogenous variables, which are denoted as \(x^P(x^G, y^G)\) and \(y^P(x^G, y^G)\).

Now the public sector problem can be expressed as

\[
\max_{x^G} V^G(x^G, y^G, x^P, y^P, x^P, y^P) \text{ subject to } f^G(x^P, y^P, x^G, y^G, x^P, y^P) = 0. \tag{2.2}
\]

\(^{10}\) Arrow and Kurz (1970) attribute the terminology 'felicity function' to Gorman (1957). I deliberately adopt this terminology instead of more common term 'instantaneous utility function'. See the next subsection for more discussion on this issue.

\(^{11}\) The semicolon in the argument of functions separates the endogenous (left) and the exogenous variables (right).
Note that the optimal solution of this public sector problem is in general the second best policy, which is the optimal policy from practical viewpoint. If we insist on the first-best policy, first we need to set up the following benevolent social planner problem in order to find the first best outcome.

\[
\begin{align*}
\max_{x^p, y^p, x^G, y^G} & \quad v^G(x^p, y^p, x^G, y^G) \\
\text{subject to} & \quad f^p(x^p, y^p, x^G, y^G) = 0 \quad \text{and} \quad f^G(x^p, y^p, x^G, y^G) = 0
\end{align*}
\]

(2.3)

Then, we have to find the solution of Eq. (2.2) resulting in the first best outcome. Whether there exists such a solution or not depends on the controllability. In any case we need to employ such a decentralised setting, if we want to find the feasible optimal policy. This thesis adopts the Arrow-Kurz type decentralised setting, which is referred to as 'two-stage optimisation' setting.

### 2.3.3 Objective function of the private sector

Arrow and Kurz (1970) advocate that even if the decision-making unit is the single representative individual the objective function has to include population, i.e.

\[
V^p = \int_0^\infty e^{-\rho t} L(t) u(c(t)) dt
\]

(2.4)

where \( \rho \): rate of pure time preference, \( L(t) \): population at time \( t \), \( c(t) \): per capita consumption at time \( t \), and \( u() \): the felicity function.

They show that if population is not included in Eq. (2.4) the optimal solution requires allocating less per capita consumption to the more populous generations. It is, however, not clear why the representative (average) individual, who is assumed to be ‘selfish’, has to care such social equity issue. In this respect their treatment of private setting is not fully convincing.

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12 More precisely, this is the definition of controllability (see Arrow and Kurz 1970; xv).

13 Distinction between felicity and utility in this thesis is analogous to that between ophelimity and utility in the works of Pareto. The assumption that \( u() \) solely depends on consumption is nothing controversial but the very definition if it is corresponding to 'ophelimity' a la Pareto of which meaning is satisfaction derived from current activities. For further discussion of ophelimity/utility distinction in Pareto, see Tarascio (1969).
The specification (2.4) is widely adopted in the recent growth literature, but from different perspective. In this regard Barro and Sala-i-Martin (1995) provide the following lucid explanation. It is assumed that the decision-making unit is not the representative individual but the representative household, and that the population growth takes the form of increase of the household size. Then the household felicity is assumed to be the sum of each household member's felicity.

This reasoning is simple and intuitive if we accept the underlying assumptions that a household is the basic decision unit and that the number of households in the economy is constant during the whole time horizon. Though it is a standard approach in microeconomics literature to regard the household as a single decision making unit, it has been pointed out that this approach violates the basic rule underpinning neoclassical microeconomic theory. Bourguignon and Chiappori (1992) argue that aggregating individual preference "within the ad-hoc fiction of a collective decision unit" cannot meet the principle of individualism (p.356). Moreover, it is claimed that incorporating intra-household decision-making process in policy analysis gives us significantly different implications concerning individual's welfare from those provided by the standard approach.14

Nevertheless, I follow the standard approach in this thesis, mainly because it provides suitable framework to address chronic high urban unemployment rate induced by rural-urban migration, which is explained in Chapter 4. The latter assumption of constant household number is clearly a fiction for the analytical purpose but it seems quite harmless and useful to clarify the main issue. This thesis employs the specification (2.4) for the private sector objective function in line with reasoning provided by Barro and Sala-i-Martin (1995).

Now let us investigate the meaning of the specification (2.4) in more depth. First question is what $V^p$ means for the representative household. It is common practice to call $V^p$ as utility function, which is defined as a measure of personal 'happiness'. The question is whether this utility function represents lifetime utility or utility at

14 Haddad and Kanbur (1992) list up the empirical evidence of this claim.
the particular moment of planning.\textsuperscript{15} My interpretation is the latter. Notice that we are not talking about social welfare but about utility of a private agent. It is highly likely that our current happiness is affected not only by the current felicity that is solely determined by the actions or situations at this particular moment but also by the expected felicities in future or even by the expected felicities of our descendents.

Based on my interpretation, $V^P$ is utility function of the present generation at the present moment, which is in a sense ‘instantaneous’ despite its involvement of intertemporal components. This is why I prefer the terminology ‘felicity function’ to more common ‘instantaneous utility function’ as the notation of the integrand $u(\cdot)$ in Eq. (2.4). In this thesis $V^P$ represents the household’s utility of the present generation, which is denoted by $U(0)$.

In light of the above interpretation of $V^P$, the choice of the rate of time preference for researchers is far less controversial and basically a matter of observation. The rate of time preference is nothing more than a psychological parameter (or a personal taste) in transforming the expected flow of future felicities into the utility at the present time, which is not necessarily purely inherent but could be formulated by economic and social contexts as well as by education to a certain degree.\textsuperscript{16}

\subsection*{2.3.4 Objective function of the public sector}

One of the advantages of two-stage optimisation setting would be its capacity to accommodate both the positive and the normative approaches in harmonised way. In this setting the modelling of the private optimisation is positive task in the sense that its success depends on how well the private sector’s response to the government policy can be predicted, while modelling of the public optimisation is essentially normative task in the sense that its success depends on how well

\textsuperscript{15} A celebrated book by Aghion and Howitt (1998) explicitly define $V^P$ as “lifetime utility function” (p.18). Arrow and Kurz (1970) also interpret in this way. Majority of the literature does not seem to have considered this issue carefully.

\textsuperscript{16} For the roles of economic/social contexts and education on time preference formation, see Becker and Mulligan (1997). For general debates on time preference issues, see Arrow et al. (1996).
collective general interests can be represented in the objective function $V^G$ as well as the constraint sets.

This normative aspect means on the one hand some freedom from representing 'reality' but on the other hand the lack of operational principles in order to have convincing model of the public sector's problem.

In his famous paper on the social welfare function, Bergson states that

"The number of sets [of value propositions which is sufficient for the evaluation of all alternatives] is infinite, and in any particular case the selection of one of them must be determined by its compatibility with the values prevailing in the community the welfare of which is being studied. For only if the welfare principles are based upon prevailing values, can they be relevant to the activity of the community in question" (Bergson 1938; p. 323).

To capture "the values prevailing in the community" in economic models is obviously far from easy. Nevertheless, there seems to be an agreement, at least in the growth literature, to adopt utilitarian social welfare functions for this purpose, in which social welfare is defined as an aggregate of individual utilities.

It seems highly interesting that it has been proved that Bergson-Samuelson social welfare function must have, with some additional assumptions, the form of weighted mean of all members' individual utilities (Harsanyi 1955) or of unweighted sum of them (Ng 1975), though Bergson explicitly disfavours unweighted sum of individual utilities as the social welfare function with claiming that it "is not a useful tool for welfare economics" (Bergson 1938; p. 327). As there is no a priori reason to deny the claims of Harsanyi and Ng, I follow the tradition of the literature to adopt the social welfare function defined as arithmetic mean (or unweighted sum) of all members' individual utilities as the maximand of the public sector.

---

17 In Harsanyi (1955) the expected utility property is the key assumption, while finite sensibility of individual and the Weak Majority Preference Criterion are the key assumptions in Ng (1975).
Now let's consider the following social welfare function.

$$V^G = \sum_i U^i(0) = \sum_i \int_0^\infty e^{-\rho s} l'(t)u(c'(t))dt,$$

(2.5)

in which the same notation as in Eq. (2.4) holds with the superscript $i$ denoting the $i^{th}$ household.

It represents the social welfare of the present generation at the present moment, which can be referred to as 'intra-generational' social welfare function of the present generation. Mäler (1974) has correctly pointed out that this social welfare specification "reflects the choices of the present government and does not involve those of future generations (whose preferences we obviously do not know)" (p.61). This is strength, not weakness, as a social welfare function, because it avoids unsolvable problem of representing welfare of the unborn future generations. I adopt this intra-generational social welfare function as the public sector objective function since it is perfectly consistent with my definition of sustainable development of which main goal is improving intra-generational social welfare in terms of poverty alleviation without violating the given sustainability constraints. Inter-generational aspects of sustainable development are represented by the sustainability constraints in my framework, not by the social welfare function.¹⁸

It does not seem worth trying to incorporate intergenerational aspect into the social welfare function itself in a way logically consistent with my approach, which would take the following form.

$$V^G = \int_0^T e^{-\gamma s} \left( \sum_i U^i(s) \right) ds = \int_0^T e^{-\gamma s} \left( \sum_i \int_0^\infty e^{-\rho s} l'(t)u(c'(t))dt \right) ds,$$

(2.6)

in which $T$: the time horizon of the public sector, $s$: generations, and $\gamma$: the discount rate of future social welfare.¹⁹

¹⁸ Toman et al. (1995) argue that it would be desirable to incorporate "sustainability function" into the social welfare function on the ground that sustainability constraint approach does not allow "tradeoffs between intergenerational concern and other social goods" (Toman et al. 1995; p.142). In my opinion, the benefit from allowing such 'substitutability' is not enough to compensate the difficulty in representing social welfare of unborn future generation.

¹⁹ Note that both the 'normative' and the 'positive' discount rates of Arrow et al. (1996) coexist as $\gamma$ and $\rho$, respectively, in Eq.(2.6). Ramsey's famous aphorism that any nonzero discount rate is "ethically indefensible" sounds persuasive in case of $\gamma$ (Ramsey 1928; p.543).
This specification might be helpful to sort out the controversy over choice of discount rates in the social objective function, but might not be applicable to practical policy analysis due to the real difficulty in representing social welfare of the unborn future generation.

2.3.5 Expectation formation

One of the strong assumptions widely employed in dynamic optimisation models is perfect foresight of the households (or the individuals) in forming their expectations of time paths of price variables such as rental rates of capital, wage rates and exogenous prices.

Analogous to the benevolent social planner setting, this apparently unrealistic assumption is helpful and justifiable to seek the first best outcomes. It is, hence, not very surprising that the existing RCK growth model literature, which seems to focus exclusively on the first best world, tends to take this assumption as granted. When one goes further and seeks the optimal policy feasible with the given sets of instrument, however, a careful reconsideration of the expectation formation process becomes necessary.

A systematic investigation of alternative rationality concepts in economic theory was started by Simon (1955). He defines his task as replacing omniscient rationality, as often assumed in economics, with “a kind of rational behavior that is consistent with the access to information and the computational capacities that are actually possessed by organisms, including man” (Simon 1955; p.99). He suggests the concept of ‘bounded rationality’, a kind of conditional rationality given the limited ability of decision makers to gather and process information. It seems an adequate concept consistent with my modelling approach taking into account the fact that any information is not enough for the households to perfectly predict entire time paths of exogenous prices unless the households themselves make public decisions as in the benevolent social planner setting.

It is important to notice that the implication of bounded rationality is not confined in the way of information processing, such as selection of input information.
Rather, awareness of bounded rationality would urge decision makers to tailor decision-making procedure itself to fit their cognitive ability.\(^{20}\) For instance, recall that the existing RCK growth model literature assumes that households make a decision only once, since unbounded (omniscient) rationality enable them to find, with full of confidence, the first best optimal consumption path for entire period at the beginning of the planning period. Intuitively, replacing unbounded rationality with bounded rationality in this framework would likely introduce frequent monitoring-feedback process in the decision procedure. In fact, if we assume that monitoring-feedback process is costless, as assumed in this thesis, logical consequence of bounded rationality will be continuous monitoring-feedback for entire planning period.

Now let's turn to investigate decision input with bounded rationality. In the two-stage optimisation setting the central issue is how well the government can approximate the households' expectation in a way that the government can predict the response of the households to policies with adequate accuracy. The task is not to model the actual expectation formation mechanisms of the households in detail but to assume some simple but 'reasonable' approximation of the households' expectation based on which the government can predict the household response to the policy reasonably accurate within its cognitive limitation.

Stiglitz (1974b) identifies basic properties necessary for expectation formation models to be 'reasonable'. He expresses, in discrete time, the expectation formation as

\[
p_{t+1}^e = \varphi(p_t, p_{t-1}, \ldots, p_{t-n}, \ldots; t),
\]

where \(p_{t+1}^e\) is expected price at period \(t + 1\).

Stiglitz claims that this function must satisfy (i) linear homogeneity with respect to all arguments, (ii) stationarity (or time independency), and (iii) convergence of the expectation into the real price at the steady-state. Even after screening candidates

\(^{20}\) The tailored decision procedure is not necessarily 'optimal'. Lipman (1991) points out logical inconsistency to assume optimal decision procedure in modelling bounded rationality.
based on these three conditions, there still exist numerous valid alternatives of expectation formation process.

The final selection of decision input from these alternatives has to rely on one's intuition of which correctness can be judged only by empirical tests. In his pioneering work of rational expectation hypothesis, Muth (1961) compares his rational expectation model with other alternatives such as cobweb type models ($p_{t+1}^e = p_t$) and adaptive expectation models by testing proximities of model prediction to the observation of pig price cycle between 1911 and 1931. His conclusion is, with a caveat of potential bias due to serial correlation, that the rational expectation model, which is equivalent to perfect foresight in the deterministic world, would have relative advantages over its rivals. Although his empirical test is too sketchy to guide the choice of expectation formation process in this thesis, its implication that highly unrealistic hypothesis could generate better proxies of real phenomena is important.

The expectation formation process assumed in this thesis is a combination of continuous monitoring-feedback embedded decision procedure and the simplest decision input that requires the current values only, which means that the households expect that the future trajectories of exogenous variables are constant at the current values but next moment they update this expectation based on realised values of exogenous variables. It can be easily seen that this expectation process satisfies three necessary conditions of Stiglitz (1974b). Furthermore, this specification can be regarded as an example of "consistent planning" strategy of Strotz (1956) which he defines as the strategy "to find the best plan among those that he will actually follow" (p.173). Note that the estimated optimal consumption path at each moment in my specification is not a plan but mere decision input for households to find the best plan of instantaneous consumption at that moment which households actually follow.
2.4 Established analytical framework

This thesis adopts quantitative policy analysis based on policy simulations because of its capability to reveal overall consequences of complex and often interdependent effects of sustainable development policy. Due to the significance of dynamic aspects in sustainable development, the RCK growth model is chosen.

The existing economic growth literature tends to focus on the first-best optimal outcomes without considering their controllability or achievability in the second-best world. Although the first-best optimal studies have contributed not only in establishing theoretical framework but also in providing useful benchmark, it might be misleading to apply these outcomes directly to policy analysis. Their usefulness is maximised when they are utilised as intermediate knowledge inputs to establish proper analytical framework for studying the second-best optimal world.

I tailor the RCK growth model to analyse the second-best policy by introducing two novel features. Firstly it employs two-stage optimisation consisting of the private and the public optimisation processes. Secondly, it employs continuous monitoring-feedback in the household expectation formation process instead of the perfect foresight assumption. Liberating the RCK models from the perfect foresight assumption, which is criticised by proponent of static or recursive dynamic CGE models as unacceptably unrealistic, facilitates development of dynamic CGE models based on the RCK growth model.

In sum, the established analytical framework consists of an analytic model and an applied model. Note that this thesis aims at providing not a fully applied model ready to conduct policy analysis in practice but a prototype based on which such a fully applied model can be developed. The analytic model is a stylised RCK growth model with two-stage optimisation and continuous monitoring-feedback in households' expectation formation. Based on the analytic model, the applied model, a highly aggregate dynamic CGE model, is developed by incorporating key stylised facts of water-scarce developing countries.
Chapter 3

Analytic Model of Water-Economy Interaction

3.1 Introduction

In this chapter a stylised Ramsey-Cass-Koopmans (RCK) growth model is developed as an analytic model. The analytic model provides a model platform based on which an applied model for policy simulations is constructed. Furthermore, it helps to clarify impacts of supply constraints of water on sustainable development by abstracting from other stylised facts commonly observed in water-stressed developing countries, such as vulnerability of rainfed agriculture and high incidence of lack of safe water access, particularly in the rural areas. These stylised facts will be incorporated into the applied model as explained in Chapter 4.

The structure of this chapter is as follows. Section 3.2 explains general features of the analytic model, in particular the specification of two-stage dynamic optimisation without perfect foresight assumption. Section 3.3 shows the results of the first-stage optimisation in which households and private firms optimise their objective functions. Section 3.4 shows the results of the second-stage optimisation by the government. Section 3.5 presents the qualitative analysis of the predicted optimal time paths. Section 3.6 shows the numerical simulation of the analytic model, which not only provides further insights regarding to the results of
qualitative analysis but also clarifies the properties of the optimal trajectories. Section 3.7 provides conclusions of this chapter.

3.2 Outline of the analytic model

The analytic RCK growth model is designed so as to be compatible with the applied model described in Chapter 4. Like the applied model, the analytic model solves for the optimal consumption levels of the market good and domestic water for the household, factor inputs including water for the private firms, public investment and the water price for the public water producer.

The analytic model assumes a closed economy consisting of numerous identical households and identical competitive firms of which output is the numeraire of the economy. The population grows at a constant rate $\nu$ and capital depreciates at a constant rate $\delta$. Further, it is assumed that a budget neutral government provides water to households and private producers and collects a volumetric water charge.

The social optimisation process consists of two stages. At the first stage, households maximise utility by choosing consumption levels and private firms maximise profits by choosing the amounts of factor inputs taking the rate of water charge as given. At the second stage, the government maximises social welfare by choosing the rate of water charge and by investing the collected water charge in public capital that is the sole factor input of water supply service. In addition to the rationales mentioned in the previous chapter, this specification allows government intervention to be dynamically efficient.

Households form expectations without perfect foresight. The conventional RCK growth models assume that households determine the optimal consumption trajectory by deriving optimal conditions of instantaneous rates of change of consumption, with determining the optimal initial consumption. The latter is determined based on the consumption function derived from the intertemporal budget constraint, with an assumption that households can precisely predict the trajectories of the wage rate, the water price, and the interest rate. In this model
households make their decision of consumption level based on their expectation of the future trajectories of those exogenous variables, but they continuously modify their expectation based on the realised levels of the exogenous variables. The realised consumption trajectory satisfies the second-best optimality discussed in Chapter 2. At the optimal steady state the second-best outcomes coincide the first-best outcomes derived from the perfect foresight assumption.

3.3 First-stage optimisation

The first-stage optimisation consists of households' utility maximisation and firms' profit maximisation taking government policy in terms of water price as exogenously given.

3.3.1 The household's problem

(1) Problem formulation

It is assumed that households hold assets as equity shares of the private capital stock. Population is defined as the labour force population and each person supplies one unit of labour services per unit of time. Households earn wage and capital income, purchase publicly supplied water and manufactured goods for consumption, and invest in private capital stock. As a result the per capita budget constraint of the representative household is \( w + rm = c_M + pq_H + I \), where \( w \) is the wage rate, \( r \) is the real rate of return to equity shares, \( m \) is the household assets, \( c_M \) is per capita consumption of the manufactured good, \( q_H \) is per capita domestic water consumption, \( p \) is the rate of water charge (water price), and \( I \) is the household investment in equity shares.\(^2\)

---

\(^1\) It means that we assume the same proportion between consumption of labour force age person and that of his/her dependents such as young children and elderly people throughout time horizon. In other words, a person in our model consists of one labour force age person and his/her dependents. In empirical analysis this assumption is important.

\(^2\) All variables are time variant, i.e. \( w(t) \), \( r(t) \), etc., but time is omitted for notational simplicity.
The equation of motion of per capita equity shares owned by a household is
\[ \dot{m} = I - \gamma m. \]  
3 Superimposed dot means time derivatives. Note that the equity shares do not depreciate though the corresponding private capital docs.

The latter term corresponds to "dilution" due to growth of household size (Aghion and Howitt 1998; p.14).

These two equations merge into
\[ \dot{m} = w + (r - \nu)m - cM - pqH. \]  

It is assumed that households' utility at time \( t \) is determined by the discounted sum of felicities for certain length of period \( T \). 4 The felicity function is assumed to be CIES (constant intertemporal elasticity of substitution) type. 5

\[ U(t) = \int_t^{t+T} e^{-\rho(s)} u(c(s)) ds + V(t+T), \quad u(c(t)) = \frac{c(t)^{1-\sigma}}{1-\sigma}, \]

where \( c(t) \): the consumption of flow of satisfaction produced by the household itself at time \( t \), \( T \): the length of planning period, \( \rho \): the rate of pure time preference, \( \sigma \): the elasticity of marginal felicity with respect to consumption, and \( V(t+T) \): the value of the household assets at the terminal time. 6

The underlying assumption is that an immortal household consists of continuously distributed age groups and that the terminal time of planning horizon continuously shifts forward. This is a straightforward extension of the utility maximisation problem of an individual. We have little idea about the length \( T \) that might be formed by economic and social circumstances as well as education in the real world. 7 When we employ a finite \( T \), say 20 years, we have to specify the value function \( V \) as well. Both the choice of \( T \) and that of the value function are of quite arbitrary nature. As Aronsson et al. (2004) have done, we specify \( V(t + T) \) as the

\[ \text{(3.1)} \]

---

3 Superimposed dot means time derivatives. Note that the equity shares do not depreciate though the corresponding private capital docs.

4 The common notion of 'instantaneous utility' is avoided. See Footnote 13 in Chapter 2.

5 CIES functions are functionally identical with CRRA (constant relative risk aversion) functions. This specification is advantageous since later risks will be introduced in the applied model.

6 In this thesis \( \rho - \nu > 0 \) and \( \sigma > 1 \) are assumed. Arrow et al. (1996) report that majority of studies use values in the range of 1 to 2 for \( \sigma \).

7 Perrings (1989) argues that poverty may drive up poor farmers extremely myopic such that "all that matters is consumption today" (p.20).
discounted utility that the household can derive from those assets. This specification makes the utility function take the form

$$U(t) = \int_t^\infty e^{-(\rho - \gamma)s} u(c(s)) \, ds.$$ 

It is assumed that households produce a flow of satisfaction by consuming the manufacturing good and water.

$$c(c_M, q_H) = c_M^{1-\varphi} q_H^{\varphi}, \quad 0 < \varphi < 1,$$

where $\varphi$ is a weight of manufacturing good in this household production process.

Each household maximises its utility subject to budget constraint taking the water price as given. The representative household's optimisation problem at time $t$ is

$$\max_{c_M, q_H} U(t) = \int_t^\infty e^{-(\rho - \gamma)s} c(c_M(s), q_H(s)) \left(1 - \sigma\right) \, ds,$$

subject to

$$m = w + (r - \varphi)m - c_M - pq_H,$$

and

the initial assets $m(t)$ is historically determined at time $t$.

(2) Optimal growth rate of consumption

The current value Hamiltonian of this problem is

$$\tilde{H} = \frac{c(c_M, q_H)\left(1 - \sigma\right)}{1 - \sigma} + \lambda \left\{w + (r - \varphi)m - c_M - pq_H\right\},$$

where $\lambda$ is the Lagrange multiplier associated with household assets $m$, i.e. the marginal utility of those assets.

Assuming an interior solution, the necessary and sufficient conditions are as follows.\(^8\)

$$\frac{\partial \tilde{H}}{\partial c_M} = 0 \quad \Rightarrow \quad \lambda = \varphi \frac{c_M^{1-\sigma}}{c_M},$$

---

\(^8\) Since each of the objective function and the constraint is a concave function associated with a negative semidefinite Hessian matrix, the Mangasarian Sufficiency Theorem (Mangasarian 1966) holds for this problem.
\[ \frac{\partial \tilde{H}}{\partial q_H} = 0 \Rightarrow p \lambda = (1 - \varphi) \frac{c^{1 - \sigma}}{q_H} \quad (3.3b) \]

\[ \dot{\lambda} - (\rho - \nu) \lambda = - \frac{\partial \tilde{H}}{\partial m} \Rightarrow \frac{\dot{\lambda}}{\lambda} = -(\rho - \rho) \quad (3.3c) \]

In addition, the transversality condition is \[ \lim_{s \to \infty} \left[ e^{-(\rho - \nu)(s - t)} \tilde{\lambda}(s) \cdot \tilde{m}(s) \right] = 0. \]

From (3.2), (3.3a) and (3.3b) we derive

\[ q_H = \left( \frac{\varphi}{1 - \varphi} \right)^{-\varphi} \rho^{-\varphi} c, \text{ and } c_M = \left( \frac{\varphi}{1 - \varphi} \right)^{1 - \varphi} \rho^{1 - \varphi} c. \quad (3.4) \]

By putting (3.4) into (3.3b) we have \[ \lambda = \varphi \rho (1 - \varphi) \rho^{1 - \varphi} \rho^{-\varphi} c^{-\varphi}. \] By taking time derivatives of both sides with logarithmic transformation, we obtain

\[ \frac{\dot{\lambda}}{\lambda} = -(1 - \varphi) \frac{\dot{p}}{p} - \sigma \frac{\dot{c}}{c}. \]

From this equation and Eq. (3.3c) we derive the optimal growth rate of consumption as

\[ \frac{\dot{c}}{c} = \frac{1}{\sigma} \left[ (r - \rho) - (1 - \varphi) \frac{\dot{p}}{p} \right]. \quad (3.5) \]

The difference between this result and that of the standard RCK model lies in the far right term. There exists some negative effect of the water price rise on the consumption growth. The larger the weight of water consumption in household production \((1 - \varphi)\) is, the larger this effect is. It might be plausible that developing economies would be more sensitive to this negative impact of water price rise on consumption growth, due to higher share of water expenditure among the total household expenditure.

(3) Optimal consumption level

In order to determine the optimal consumption level, we need to construct the consumption function based on the intertemporal budget constraint. With the

\[ ^* \quad \wedge \quad \text{denotes the first stage solution.} \]
optimality conditions (3.4), the equation of motion of the household's assets becomes
\[ \dot{m} = w + (r - \nu)m - b_1 p^{-\sigma} c, \]
where \( b_1 = \phi^{-\sigma} (1 - \phi)^{-1 - \sigma} > 0 \).\(^{10}\)

The solution of this differential equation for the period between \( t \) and \( t + T \) is
\[ m(t + T)e^{-\int^t_{t+T} \{r(\tau) - \nu\} d\tau} = m(t) + \int_{t}^{t+T} \left[ w(s) - b_1 \{ p(s) \}^{1 - \sigma} c(s) \right] e^{-\int_{t}^{s} \{r(\tau) - \nu\} d\tau} ds. \]

When we take the limit as \( T \) approaches infinity, the left hand side becomes zero from the transversality condition. Thus, the intertemporal budget constraint becomes
\[ b_1 \int_{t}^{\infty} \{ p(s) \}^{1 - \sigma} c(s) e^{-\int_{t}^{s} \{r(\tau) - \nu\} d\tau} ds = m(t) + \int_{t}^{\infty} w(s) e^{-\int_{t}^{s} \{r(\tau) - \nu\} d\tau} ds. \]

The left hand side is the present value of the household's total spending, while the right hand side is wealth defined as the sum of the disposable assets and the present value of wage income. The following expression of consumption is obtained by solving Eq. (3.5) for the period between \( t \) and \( s \).
\[ c(s) = c(t) \left( \frac{p(t)}{p(s)} \right)^{b_2} \exp \left[ \frac{1}{\sigma} \int_{t}^{s} \{r(\tau) - \nu\} d\tau \right], \]
where \( b_2 = \frac{1 - \phi}{\sigma} > 0 \).

From these two equations we derive the following consumption function of the 'clairvoyant' household who can predict the future trajectories of \( w, r, \) and \( p \) perfectly.
\[ c(t) = \eta(t) \left[ m(t) + \int_{t}^{\infty} w(s) e^{-\int_{t}^{s} \{r(\tau) - \nu\} d\tau} ds \right], \]
where
\[ \eta(t) = \left[ b_1 \{ p(t) \}^{-b_2} \left( \int_{t}^{\infty} \{ p(s) \}^{-b_2} e^{-\int_{t}^{s} \{b_3 - b_4 r(\tau)\} d\tau} ds \right)^{-1} \right], \]
in which \( b_3 = \frac{(\sigma - 1)(1 - \phi)}{\sigma} > 0 \) and \( b_4 = \nu - \frac{\rho}{\sigma} \) and \( b_5 = \frac{\sigma - 1}{\sigma} > 0 \).

The term \( \eta(t) \) is the propensity to consume out of wealth at period \( t \). It is noted that the clairvoyant households need to use the consumption function only once when

\(^{10}\) These abridged constants (\( b_i \)'s) are listed in Appendix A0.
they choose the initial consumption at $t = 0$, then they just need to change the level of consumption based on the optimal consumption growth rate expressed as Eq. (3.5) in order to achieve the first-best outcome.

Now let's relax the perfect foresight assumption. Instead, it is assumed that the households' expectation about the trajectories of exogenous variables is that they are constant at their current values. With this assumption the following proposition is derived.

**Proposition 3.1: Optimal consumption level**

If $r(t) > \nu$ is satisfied, the optimal consumption is given as

$$
\hat{c}(t) = \varphi^p \left( \frac{1-\varphi}{p(t)} \right)^{1-p} \left( \varphi (t) - \nu - \frac{r(t)-\rho}{\sigma} \right) \left( m(t) + \frac{w(t)}{r(t)-\nu} \right).
$$

Otherwise, $\hat{c}(t)$ diverges towards either negative or positive infinity.

**Proof:** See Appendix A1.

In the following analysis, the real rate of return is assumed to be strictly greater than the population growth rate ($r > \nu$). Now $\hat{c}(t)$ is determined solely by the contemporaneous values of exogenous variables. Note that this optimal consumption coincides with the first-best optimal consumption derived from the conventional RCK model with perfect foresight assumption, provided an economy is at the steady-state. Otherwise, $\hat{c}(t)$ is larger or smaller than the first-best optimal solution depending on the differences between the steady-state values and the realised values of $p$, $r$, and $w$. For instance, if the actual wage rate is higher than the steady-state value and other prices take the steady-state values, $\hat{c}(t)$ is

---

11 It is also possible to incorporate past information in the expectation formation process. For instance, the expected trajectory might grow at constant rate estimated based on the past growth rates. I feel, however, that this kind of sophistication is of ad-hoc nature anyway and its rewards might not be enough to compensate its costs. Also see Subsection 2.3.5 in Chapter 2.
unambiguously greater than the first-best solution. Nevertheless, it is the second-best optimal consumption given the expectation formation process.

3.3.2 The firm’s problem

We assume that the representative firm’s production technology may be described by the following Cobb-Douglas production function with constant returns to scale.

\[ Y = F(K, Q_M, L) = K^\beta_K Q_M^\beta_Q L^\beta_L \]

where \( Y \): output, \( K \): private capital stock, and \( Q_M \): water input, \( L \): labour input, and \( \beta_K, \beta_Q, \text{ and } \beta_L \): factor shares of private capital, water and labour with \( \beta_K, \beta_Q, \beta_L \in (0,1) \) and \( \beta_K + \beta_Q + \beta_L = 1 \).

The constant return to scale assumption enables us to express the above production technology in the following intensive form.

\[ y = f(k, q_M) = k^\beta_K q_M^\beta_Q, \]

where \( y \): per worker output, \( k \): per worker private capital stock, and \( q_M \): per worker water input.

The firm’s per worker profit is \( \pi = y - (r + \delta)k - pq_M - w \), where \( \pi \) is per worker profit and \( p \) is the water price.\(^{12}\)

The representative firm maximises per worker profit by setting the partial derivatives of \( \pi \) with respect to \( k \) and \( q_M \) at zero, taking \( r \) and \( p \) as exogenously given. Thus the optimality conditions are

\[ \frac{\partial \pi}{\partial k} = 0 \Rightarrow \beta_K y = (r + \delta)k, \quad \text{and} \quad \frac{\partial \pi}{\partial q_M} = 0 \Rightarrow \beta_Q y = pq_M. \]

From the per worker production function and the above optimality conditions we can express \( y \) and \( q_M \) as a function of \( k \) and \( p \).

\(^{12}\) See Footnote 3 of this chapter. To compensate the depreciation of capital the rental price of capital must be \( r + \delta \) (see, e.g. Barro and Sala-i-Martin 1995: p.69).
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\[ y = (\beta Q/p)^{b_6} k^{b_7}, \text{ and } q_M = (\beta Q/p)^{\frac{1}{1-\beta Q}} k^{b_7}, \]  

(3.6)

where \( b_6 = \frac{\beta K}{1-\beta Q} > 0 \) and \( b_7 = \frac{\beta K}{1-\beta Q} > 0 \).

By putting the optimality conditions into the per worker profit function with Eq. (3.6), we obtain the optimal per worker profit function as a function of wage rate \( w \) and capital stock \( k \), i.e. \( \pi^* = \beta_L y^* - w = \beta_L p Q^{b_6} p^{1-\beta Q} k^{b_7} - w \). Thus \( w \) determines the sign of per worker profit as

\[ \pi^* \begin{cases} 
> 0 & \text{if } w \leq \beta_L \left( \frac{\beta Q}{p} \right)^{b_6} k^{b_7} \\
\leq 0 & \text{if } w \geq \beta_L \left( \frac{\beta Q}{p} \right)^{b_6} k^{b_7}.
\end{cases} \]  

(3.7)

The optimal capital input is also derived from the optimality conditions and Eq. (3.6) as

\[ k^* = \left( \frac{\beta K}{r+\delta} \right)^{\frac{1-\beta Q}{p Q}} \left( \frac{\beta Q}{p} \right)^{b_6} k^{b_7}. \]  

(3.8)

3.3.3 Market equilibrium

Equilibrium in the labour, capital and goods markets is achieved by a set of prices \( r^* \) and \( w^* \) such that these markets clear.\(^{13}\)

The competitive firm assumption drives the optimal profit towards zero, which results in the equilibrium wage rate \( w^* = \beta_L (\beta Q/p)^{b_6} k^{b_7} \) from Eq. (3.7). With this equilibrium wage rate the labour market clears such that total labour force population equals number of total workers, which means per capita values and per worker values coincide. Under this circumstance the capital market clearance condition is given as \( k^* = m \). From Eq. (3.8) this equality is realised only if the rate of return to private capital \( r \) takes the value

\[ r^* = \beta_K \left( \frac{\beta Q}{p} \right)^{b_6} \left( \frac{1}{m} \right)^{b_6} - \delta, \text{ where } b_6 = \frac{\beta L}{1-\beta Q} > 0. \]

\(^{13}\) Since the market good is numeraire its equilibrium price is always unity.
With this equilibrium rate of return \( r^* \) the equilibrium per worker stock of private capital is given as \( \hat{k} = m \), and we can finally express the equilibrium prices as

\[
w = \beta_L \left( \frac{\rho_o}{p} \right)^{b_k^{b_k}} \hat{k}^{b_k^7} \text{ and } \quad r^* = \beta_K \left( \frac{\rho_o}{p} \right)^{b_k^{b_k}} \left( \frac{1}{\hat{k}} \right)^{b_k} - \delta.
\]

### 3.3.4 First-stage solution

By putting \( w^* \) and \( r^* \) into the optimal consumption level (Proposition 1) and by substituting \( m \) with \( \hat{k} \), the optimal consumption can be expressed as follows.

\[
\hat{c} = \frac{\hat{k}}{p^{1-\rho}} \left( b_{10} - b_9 p^{b_k^{b_k}} \hat{k}^{b_k} \right) \left( \beta_K + \beta_L - b_{11} p^{b_k^{b_k}} \hat{k}^{b_k} \right) \frac{b_{13}}{b_{11} p^{b_k^{b_k}} \hat{k}^{b_k}} \left( \beta_K - b_{11} p^{b_k^{b_k}} \hat{k}^{b_k} \right),
\]

where \( b_9 = \delta + \sigma + \rho - \sigma \), \( b_{10} = \frac{(\sigma - 1) \beta_K \rho_o \sigma^{1-\rho_o}}{\sigma} > 0 \), and

\[
b_{11} = (\delta + \sigma) \beta_Q \rho_o^{1-\rho_o} > 0.
\]

The following equation of motion of the private capital is derived from equations (3.1), (3.4) and (3.7).

\[
\dot{k} = (1 - \beta_Q) \hat{y} - (\delta + \sigma) \hat{k} - b_1 p^{1-\rho} \hat{c}, \quad \hat{k}(0) = k_0 (= m_0, \text{ given}).
\]

From the above equation of motion with the equations (3.8) and (3.9) the optimal growth rate of private capital is as follows.

\[
\frac{\dot{k}}{k} = b_{12} \left( p^{b_k^{b_k}} \hat{k}^{b_k} \right)^2 - b_{13} p^{b_k^{b_k}} \hat{k}^{b_k} + b_{14} = \phi^k(\hat{k}, p), \quad \hat{k}(0) = k_0.
\]

where \( b_{12} = (\delta + \sigma)(\delta + \rho) \beta_Q \rho_o^{1-\rho_o} > 0 \),

\[
b_{13} = (\delta + \sigma) \beta_K + (\delta + \rho)(1 - \beta_Q) > 0, \quad \text{and} \quad b_{14} = \beta_K (1 - \beta_Q) \rho_o^{1-\rho_o} > 0.
\]

Taking the trajectory of the water price as exogenously given, the private firms determine the optimal stock level of private capital based on this equation. It is
convenient to introduce $\xi = p^b_k \tilde{k}^b_q$ and rewrite Eq. (3.10) as

$$\phi^k(\xi) = \frac{b_{12} \xi^2 - b_{13} \xi + b_{14}}{\sigma(\beta_K - b_{11} \xi)}.$$

The condition $r > v$ and $r^* = \beta_K (\beta_q^{-1} - \beta_o^{-1}) \xi^{-1} - \delta$ determine the domain of $\phi^k(\xi)$ as

$$0 < \xi < \frac{\beta_K}{\delta + v} \beta_q^{-1} - \beta_o^{-1} = \xi_{\text{max}}.  \quad (3.11)$$

Within this domain, the following lemma holds.

**Lemma 3.1:**

The sign of growth rate of capital is determined by the following rule.

$$\phi^k(\xi) > 0, \quad \text{if and only if} \quad \xi < \frac{\beta_K}{\delta + \rho} \beta_q^{-1} - \beta_o^{-1}.$$

Moreover, $\lim_{\xi \to \xi_{\text{max}}^-} \phi^k(\xi) = -\infty$.

**Proof:** See Appendix A2.

Now we examine the effects of the water price on the trajectory of $\tilde{k}$. Recall that $p(t)$ does not affect $\tilde{k}(t)$ although $p(t)$ does affect the growth rate of $\tilde{k}(t)$. It means that we can change $\xi(t)$ by setting $p(t)$ regardless of the level of $\tilde{k}(t)$, providing there is no supply side constraint which means physical and financial constraints imposed on the government in providing water and setting rate of water charge. Based on this fact, we derive the following two propositions from Lemma 3.1.

**Proposition 3.2:** Operational principle of controlling water price

If there is no supply side constraint, the government can achieve any desirable growth rate of the private capital stock by controlling the water price based on the following operational principle.

---

14 We exclude $\xi = 0$, which requires either $k$ or $p$ is zero, from the domain.
\[
\frac{\dot{k}(t)}{\dot{k}(t)} \leq 0, \quad \text{if and only if} \quad p(t) \leq \frac{1}{\xi} \left( \frac{1}{\frac{1}{\dot{k}(t)}} \right) \frac{\rho_k}{\rho_0}.
\]

**Proof:** Since we can freely change \(\xi(t)\) by setting proper \(p(t)\) regardless of the level of \(\dot{k}(t)\), Lemma 3.1 guarantees that we can control the growth rate of \(\dot{k}(t)\) from negative infinity to positive infinity by choosing \(\xi(t)\) by setting \(p(t)\), regardless of the level of \(\dot{k}(t)\).

*Q.E.D.*

**Proposition 3.3: Stability of the optimal steady-state without supply side constraint**

The optimal steady-state in a world without supply side constraint is globally stable within the domain of \(\phi(\xi)\), i.e. once any constant water price is set, \(\dot{k}(t)\) and \(\dot{c}(t)\) always converge towards the steady-state.

**Proof:** Assume that the constant water price is such that \(\xi(t) < \bar{\xi}\). Then Lemma 3.1 tells us that \(\dot{k}(t)\) grows at positive rate and consequently \(\xi(t)\) increases towards \(\bar{\xi}\), and vice versa. Thus the steady-state is globally stable within the domain of \(\phi(\xi)\). Since \(\dot{c}(t)\) is a function of \(\dot{k}(t)\) and \(p(t)\) only, at this steady-state \(\dot{c}(t)\) becomes constant.

*Q.E.D.*

Proposition 3.2 means that if the water price \(p\) is very low then the household expenditure is small enough for the household to increase its assets by investment (saving), in spite of a relatively high optimal consumption induced by the low water price. Proposition 3.3 is an interesting result of no-perfect foresight assumption. Recall that the optimal growth rate of consumption (3.5) is rewritten by incorporating the equilibrium real rate of rental as

\[
\frac{\dot{c}}{c} = \frac{1}{\sigma} \left[ \beta_k \left( \frac{\rho_\omega}{p} \right) \frac{1}{\dot{k}} \left( \frac{1}{\rho_\omega} \right) - (\delta + \rho) - (1 - \varphi) \frac{\dot{p}}{p} \right].
\]
This expression tells us that in the standard RCK model with perfect foresight assumption, a constant water price can drive an economy towards a steady-state only if the optimal capital stock converges on the steady-state value corresponding to \( \xi = \overline{\xi} \). Intuitively this happens only if the water price takes a certain particular value, if any. By contrast, Proposition 3.3 implies that any constant water price can drive an economy towards a steady-state in the second-best world under the no-perfect foresight assumption.

Finally these two propositions establish the following proposition.

\textbf{Proposition 3.4: Steady-state optimal consumption without supply side constraint}

If there is no supply side constraint, the government can induce any desirable level of steady-state optimal households' consumption by setting the water price.

\textit{Proof}: Recall that the steady-state level of the optimal consumption is given by Eq. (3.9) as

\[
\hat{c} = -\frac{k}{p^{1-\varphi}} \left( \frac{b_{10} - b_{2S}}{b_{1S}} \right) \left( \rho_K + \rho_L - b_{11S} \right) = b_{15} \left( \frac{1}{p} \right)^{1-\varphi} \rho_i \beta_i, \text{ where}
\]

\[
b_{15} = \varphi \left( 1-\varphi \right)^{1-\varphi} (\delta + \rho) \rho_i \beta_K \beta_i \beta_Q \left( \beta_{K} + (\delta + \rho) \beta_{L} \right) > 0. 
\]

Since the exponent of \( 1/p \) is strictly positive, any positive steady-state optimal level of \( \hat{c} \) can be induced by setting a constant price for water.

\textit{Q.E.D.}

These three propositions are of policy interest. For instance, if the economy is free from supply side constraints in water provision and the government has discretion in setting the water price, Proposition 3.4 tells us that the government can induce any desirable household consumption level by setting the water price based on this proposition. Needless to say, these 'desirable' results are largely due to strong assumptions such as no supply side constraint, full employment and full market equilibrium. These assumptions are rarely found in the real world, and never in developing economies. Nevertheless, these propositions provide a useful
benchmark against which we can investigate the implications of relaxing each assumption. Moreover, they facilitate analysis of supply side problems that are particularly important in the water-stressed developing economies.

3.4 Second-stage optimisation

3.4.1 Water production

As in the case of private goods production, we drastically simplify the actual processes consisting of water production, e.g. harnessing raw water from the natural hydrological cycle, water purification, transmission, and so forth into the aggregate water production function \( Q = F^W(G) \), where \( Q \) is an aggregate water production and \( G \) is an aggregate public capital stock.

Weitzman (1970) argues that social overhead capital including sanitation facilities, irrigation and drainage facilities, and water supply facilities belong to 'the \( \beta \) sector' characterised by very high capital intensity.\(^{15} \) This might justify the above specification. Note that here we use an aggregate production function because the water constraint is manifested not in per capita but in absolute terms.

Another basic feature of the \( \beta \) sector discussed by Weitzman (1970) is substantial economies of scale due to indivisibility and cost lumpiness in this sector. This is perfectly relevant to the case of water production, which is often associated with large-scale facilities such as dams, treatment plants and pipelines (Young and Haveman 1985). Hence the shape of the water production function might not be smooth but kinked at several points as illustrated in Figure 1.

\(^{15} \) World Bank (2004) reports that "the fixed costs of water supply are typically high relative to variable costs, more so than for other utilities such as electricity. For example, fixed costs account for more than 80 percent of water supply costs in the United Kingdom" (p.223).
Here some clarification may be necessary. In our model setting it is necessary to interpret this function as the relationship between public capital stock and capacity to produce clean water on condition that water production and consumption do not endanger sustainability. As discussed in Chapter 1, I define sustainability as keeping resilience of ecosystems underpinning life-support systems. This sustainability condition prohibits, for instance, exploiting surface water or groundwater more than its sustainable yield and discharging pollutants or wastewater to water bodies more than their assimilating capacities. Consequently, the technological choice in ‘sustainable production function’ is limited and the required amount of public capital covers not only narrowly defined water supply costs but also wastewater treatment costs.\(^{16}\)

When the supplied amount of water is smaller than \(Q_1\), harnessing water from hydrological cycle is technically easy and water related capital might be divisible. To supply more than \(Q_1\) we might need to install some large-scale facility, e.g. a large dam, construction of which requires certain amount of capital corresponding to \(G_2 - G_1\). Only after the installation of this facility it is possible to increase water supply capacity up to \(Q_3\) by further capital accumulation (section between \(G_2\) and \(G_3\)). In other words, the marginal product of water with respect to \(G\) is zero between \(G_1\) and \(G_2\) (i.e. \(dF_{W}/dG = 0\)). After repeating this process several times we will reach certain upper limit of water quantity (\(Q_5\)) harnessed from natural hydrological cycle without violating sustainability of ecosystem. Though it is possible to increase water supply capacity above this quantity by introducing water
recycling, desalination of sea water, water import, and so on, it is highly likely that the marginal water product of these technologies be very low, as is shown in Figure 1. Note that we assume the water production function is twice continuously differentiable, at least from left side, at any point.

The water balance constraint can be expressed as

\[ N_0 e^{rt}(q_H + q_M) \leq R^{r'}(G) \], where \( N_0 \) is the initial population. (3.11)

The left hand side is an aggregate water demand, while the right hand side is the water supply capacity.

### 3.4.2 The government

It is assumed that the budget neutral government collects volumetric water charges from both households and firms and it spends all the collected charges on public capital investment (\( I^G \)).

\[ p(q_H + q_M)N_0 e^{rt} = I^G. \] (3.12)

The assumption that the government undertakes water service is not only justifiable considering the natural monopolistic feature of water provision but also realistic in most developing countries. In the context of closed economy assumption the assumption of budget neutral government is sensible. From this assumption the equation of motion of the public capital becomes

\[ \dot{G} = I^G - \delta G = p(q_H + q_M)N_0 e^{rt} - \delta G. \] (3.13)

The aim of the government is to maximise the 'intra-generational' social welfare of the current generation by choosing a water price whilst observing the sustainability condition.\(^{17}\) Since we assume identical households, the government problem can be expressed as

---

\(^{16}\) The concept of this 'sustainable production function' approach will be discussed later in Section 5.6 of Chapter 5.

\(^{17}\) See Subsection 2.3.4 in Chapter 2 for the specification of the social welfare function.
\[ \begin{align*}
\text{Max} \int_0^\infty e^{-(\rho - \nu)t} \frac{\partial \hat{c}(\hat{k}, p)(t)}{1-\sigma} \, dt, \text{ subject to} \\
\dot{\hat{k}} = \hat{k} \phi^k(\hat{k}, p), \quad \hat{k}(0) = k_0, \text{ and} \\
\hat{G} = p\hat{q}_H(\hat{k}, p) + \hat{q}_M(\hat{k}, p) \right) N e^{\nu t} - \delta G, \quad G(0) = G_0, \\
F^W(G) - \left\{ \hat{q}_H(\hat{k}, p) + \hat{q}_M(\hat{k}, p) \right\} N e^{\nu t} \geq 0.
\end{align*} \] (3.14a-b-c-d)

The corresponding Lagrangian consisting of the current value Hamiltonian and the water balance constraint is

\[ L^c = \frac{\hat{c}(\hat{k}, p)(t)}{1-\sigma} + \mu^k \hat{k} \phi^k(\hat{k}, p) + \mu^G \left[ p \left\{ \hat{q}_H(\hat{k}, p) + \hat{q}_M(\hat{k}, p) \right\} N e^{\nu t} - \delta G \right] + \Theta \left\{ F^W(G) - \left\{ \hat{q}_H(\hat{k}, p) + \hat{q}_M(\hat{k}, p) \right\} N e^{\nu t} \right\}, \]

where \( \mu^G, \mu^k \) and \( \Theta \) are the Lagrange multipliers associated with \( G, \hat{k} \) and the water constraint, respectively.

Assuming an interior solution, the necessary conditions are as follows. \(^{18}\)

\[ \frac{\partial L^c}{\partial p} = 0 \Rightarrow \hat{c}^\sigma \frac{\partial \hat{c}}{\partial p} + \left\{ \mu^G \left( \hat{q}_H + \hat{q}_M \right) + \left( \mu^G p - \Theta \right) \left( \frac{\partial \hat{q}_H}{\partial p} + \frac{\partial \hat{q}_M}{\partial p} \right) \right\} N e^{\nu t} \]

\[ + \mu^k \hat{k} \frac{\partial \phi^k}{\partial p} = 0 \] (3.15a)

\[ \dot{\mu}^k - (\rho - \nu) \mu^k = -\frac{\partial L^c}{\partial \hat{k}} \Rightarrow \dot{\mu}^k = \mu^k \left( \rho - \nu - \phi^k - \hat{k} \frac{\partial \phi^k}{\partial k} \right) \frac{\sigma^\sigma}{\hat{k}} \frac{\partial \hat{c}}{\partial k} \]

\[ - \left( \mu^G p - \Theta \left( \frac{\partial \hat{q}_H}{\partial k} + \frac{\partial \hat{q}_M}{\partial k} \right) \right) N e^{\nu t} \] (3.15b)

\[ \mu^G - (\rho - \nu) \mu^G = -\frac{\partial L^c}{\partial \hat{G}} \Rightarrow \frac{\dot{\mu}^G}{\mu^G} = \rho + \nu - \frac{dF^W}{dG} \frac{\Theta}{\mu^G} \] (3.15c)

The Kuhn-Tucker condition for the water constraint is

\[ \Theta \geq 0, \quad F^W - \left( \hat{q}_H + \hat{q}_M \right) N e^{\nu t} \geq 0, \quad \Theta \left\{ F^W - \left( \hat{q}_H + \hat{q}_M \right) N e^{\nu t} \right\} = 0. \] (3.15d)

\(^{18}\) This problem does not satisfy the Mangasarian Sufficiency Theorem.
Note that $\Theta$ is zero only if the water constraint is satisfied with strict inequality. The transversality condition is

$$\lim_{t \to \infty} e^{-(\rho - \nu)t} \mu^G(t) \cdot G(t) = 0.$$  \hspace{1cm} (3.15e)

### 3.4.3 The optimal trajectories

It is known that each Lagrange multiplier represents the shadow price of its corresponding constraint. Since $G$ and $k$ are the same good, the ratio of their shadow prices in aggregate term is unity along the optimal trajectories. Otherwise it is possible to achieve higher social welfare by allocating more capital good to invest in either capital with higher shadow price. Hence the optimality requires $\mu^G = \frac{\mu^k}{N_0e^{\nu t}} = \mu$.

The necessary conditions for the optimal trajectories are depending upon whether water supply capacity exceeds water demand (i.e. $\Theta = 0$) or not. These two cases are separately analysed.

#### (1) Case 1: $\Theta = 0$ (Water supply capacity exceeds the demand)

In this case the optimality condition (3.15c) determines the optimal value of $\mu$ as $\mu(t) = \mu(0)e^{(\delta - \nu + \rho)t}$. This provides the following proposition.

**Proposition 3.5: Violation of transversality condition when $\Theta = 0$**

The transversality condition (3.15e) is not satisfied when $\Theta = 0$.

**Proof**: By taking integral of the both hand sides of the equation of motion of public capital, we obtain

$$G(t) = G_0e^{-\lambda t} + N_0e^{-\lambda t} \int_0^t p(s)\{\hat{g}_H(s) + \hat{g}_M(s)e^{(\delta + \nu)s}\}ds$$
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Putting this and \( \mu(t) = \mu(0)e^{(\delta - \nu + \rho)t} \) into the transversality condition, we have

\[
\lim_{t \to 0} \left[ \mu(0)e^{t} \cdot \left\{ G_{0}e^{-\alpha} + N_{0}e^{-\alpha} \int_{0}^{t} p(\hat{q}_{H} + \hat{q}_{M})e^{(\delta + \nu)t}ds \right\} \right] \\
= \mu(0)G_{0} + \mu(0)N_{0} \lim_{t \to 0} \left[ \int_{0}^{t} p(\hat{q}_{H} + \hat{q}_{M})e^{(\delta + \nu)t}ds \right] = 0.
\]

As the integrand is always non-negative, this condition cannot be satisfied unless \( \mu(0) \) is zero which requires either zero consumption or zero marginal consumption with respect to price from (3.15a). The former cannot be the socially optimal while the latter case seems a trivial exceptional case where water pricing does not affect water demand. On this rationale the case \( \mu(0) = 0 \) is precluded from the analysis.

Q.E.D.

Although Proposition 3.5 rules out the possibility of satisfying the transversality condition under \( \Theta = 0 \), it might be of practical interest to find the trajectories satisfying the remaining necessary conditions (3.15a) - (3.15d), which could be regarded as a candidate of the optimal trajectories. Recall the fact that the transversality conditions are derived from the assumption that the value function at the terminal point \( T \) takes the particular form, that is discounted utility at time \( T \). If we employ some alternative assumption about the value function, optimality does not require the transversality condition. Such a candidate of the optimal trajectories, which I referred to as 'the excess-supply trajectories', is summarised in the following proposition.

**Proposition 3.6: The excess-supply trajectories**

The excess-supply trajectories are determined by the following system;

\[
\begin{align*}
\dot{\bar{c}} &= \bar{c} - p(\hat{q}_{H} + \hat{q}_{M}) - (\phi^{k} + \delta + \nu)\bar{k} = 0, \\
\frac{\dot{G}}{G} &= \frac{p}{G}(\hat{q}_{H} + \hat{q}_{M})N_{0}e^{\alpha t} - \delta = \phi^{G} (\hat{G}, \hat{k}, p),
\end{align*}
\]

Another transversality condition \( \lim_{t \to 0} \left[ e^{-(\rho - \nu)t}\mu^{k}(t) \cdot \dot{\bar{k}}(t) \right] = 0 \) is automatically satisfied because the optimal household consumption \( \bar{c} \) is derived such that this condition is satisfied.
In addition, the rate of change of water price along the trajectories is given as

\[ \frac{\dot{p}}{p} = -\left( \frac{\delta + \rho + \sigma \varepsilon_k \phi^k}{\sigma \varepsilon_p} \right) = \phi^P_{ES} (p, \hat{k}). \]

**Proof**: See Appendix A3.

Proposition 3.6 tells us that there always exist such trajectories. Notice that the excess-supply trajectories do not necessarily satisfy the sustainability constraint (3.15d) which is not used to derive the equations constituting the system. Whether the excess-supply trajectories satisfy the sustainability constraint (3.15d) or not can be observed only with numerical simulation.

(2) **Case 2**: \( \Theta > 0 \) (Water supply capacity equals to the demand)

When \( \Theta > 0 \) we have the following equality.

\[ F^W (G) = \left\{ \hat{q}_H (p, \hat{k}) + \hat{q}_M (p, \hat{k}) \right\} N_0 e^{vt} \tag{3.16} \]

Given the state variables \( \hat{k} \) and \( G \) at any moment \( s \), \( p(s) \) is completely determined by this equation, and \( p(s) \), in turn, determines the rate of change of each state variables, and so on. It means that the trajectories, which are termed "market-clearing trajectories", are completely determined by the sustainability condition (3.16). This set of trajectories are summarised in the following proposition.

**Proposition 3.7: The market-clearing trajectories**

The market-clearing trajectories are determined by the following system;

\[ f^{MC} (p, \hat{k}, G) = F^W (G) - \left\{ \hat{q}_H (p, \hat{k}) + \hat{q}_M (p, \hat{k}) \right\} N_0 e^{vt} = 0 \]

\[ \frac{\dot{G}}{G} = \frac{dF^W}{G} - \delta = \phi^G_{MC} (G, p), \quad G(0) = G_0 \quad \text{given, and} \]

\[ \hat{k} = \phi^k (\hat{k}, p). \]
\[
\frac{\dot{k}}{k} = \varphi^k \left( \dot{k}, p \right), \quad \dot{k}(0) = k_0 \text{ given.}
\]

The rate of change of water price along the trajectories is expressed as

\[
\frac{\dot{p}}{p} = \frac{\varepsilon_G F^w \phi^G_{MC} - N_0 e^w \left\{ \nu \left( \partial q + \partial q_M \right) + \dot{k} \left( \partial q_H / \partial p + \partial q_M / \partial q_H \right) \right\} \varphi^k \left( p, \dot{k}, G \right)}{N_0 e^w p \left( \partial q_H / \partial p + \partial q_M / \partial p \right)} = \phi^G_{MC} \left( p, \dot{k}, G \right), \quad \text{in}
\]

which \( \varepsilon_G = \frac{d F^w}{d G} \frac{G}{p^w} \): an elasticity of water production with respect to public capital.

**Proof**: By putting Eq. (3.16) into Eq. (3.14c) the differential equation of \( G \) becomes

\[
\frac{\dot{G}}{G} = \frac{p F^w}{G} - \delta.
\]

Other two equations determining the trajectories are self-evident.

The differential equation of water price is obtained by taking time derivative of Eq. (3.16).

\section*{Q.E.D.}

Proposition 3.7 tells us that there always exist such trajectories, as in the case of excess-supply trajectories. Obviously it is entirely possible that these trajectories may not satisfy either the optimality conditions (3.15a) - (3.15c) or the transversality condition (which has not been used to derive them). Still, they are always the optimal trajectories that achieve the maximum social welfare with satisfying all the constraint, except for the situation where the excess-supply trajectories satisfy the sustainability constraint (3.15d) and at the same time achieve higher social welfare. Whether the market-clearing trajectories really maximise social welfare or not is investigated later using numerical simulations. Our intuition tells that having excess supply capacity should not confer any advantage on the government in this problem setting.

For practical purpose it is enough to derive the market-clearing trajectories and to confirm whether the attained social welfare is higher than those attained by the
excess-supply trajectories. For theoretical purpose, however, it might be interesting to derive the 'first-best optimal' trajectories as an interior solution of the government problem satisfying all the necessary conditions (3.15a) - (3.15e). Such an attempt leads me to the following proposition.

**Proposition 3.8: Non-existence of the interior-solution when \( \Theta > 0 \)**

When sustainable water supply capacity equals to the optimal water demand, the government problem (3.14a)-(3.14d) cannot have an interior solution.

*Proof*: See Appendix A4.

It is not clear why the government problem (3.14a) - (3.14d) cannot have any interior solution when the sustainability condition is an equality constraint. Although this problem clearly has an inherent nature of rigidity due to the budget neutral government assumption, still it is surprising that any combination of parameter values cannot generate an economically sensible interior solution.\(^{20}\)

### 3.5 Qualitative analysis of the optimal trajectories

Before conducting quantitative analysis with numerical simulation, let us investigate qualitative properties of the optimal trajectories such as stability of the steady-state and structural stability of the system.

#### 3.5.1 Optimal steady-state

The steady-state of each candidate of the optimal trajectories must satisfy \( \phi_i^p = \phi_i^O = \phi_i^k = 0 \) for \( i = ES \) and \( MC \). For any cases the steady-state must be associated with zero population growth (\( \nu = 0 \)) from the condition \( \phi_i^O = 0 \). Hence zero population growth is assumed throughout Section 3.5.

---

\(^{20}\) If it were a pure mathematical problem (without restrictions on signs of marginal felicities, etc.) some combinations of parameter values could generate an interior solution.
The results of investigation on the optimal steady-state are summarised in the following two propositions.

**Proposition 3.9: Non-existence of the excess-supply steady-state**

The excess-supply trajectories cannot have any steady-state.

*Proof:* The condition \( \frac{\partial E_s}{\partial E_s} = 0 \) at a steady-state becomes \(-\left(\frac{\delta + \rho}{\sigma}\right)\frac{1}{E_p} = 0\), which can hold if and only if \(1/E_p\) is zero at the steady-state. Let \(\bar{E}_p\) denote the steady-state value of \(E_p\). Due to the fact that the steady state value of \(\xi\) is given as

\[
\bar{E}_p = \frac{\frac{\beta_g}{\sigma} \left(\frac{\delta - \sigma}{\delta + \rho}\right)}{\left(1 - \beta_q\right) \rho \beta_l \left(\rho \beta_l + (\delta - \rho) \beta_k\right)}
\]

Since the denominators of the right hand side cannot be zero, \(1/\bar{E}_p\) cannot be zero.

*Q.E.D.*

**Proposition 3.10: The market-clearing steady-state**

When the population is constant, the government problem (3.14a)-(3.14d) can have a steady-state along the market-clearing trajectories if and only if all of the following conditions are satisfied;

1. \( k = \left(\frac{\beta_k}{\delta + \rho}\right)^{1-\beta_0} \left(\frac{\beta_0}{\mu} \right)^{\beta_0} \left(\frac{\mu}{p}\right)^{\mu_l} = \bar{k}(p), \)
2. \( G = pN_0 \{ q_H(p, \bar{k}(p)) + \hat{q}_M(p, \bar{k}(p))\}/\delta = \bar{G}(p) \), and
3. \( F^w(\bar{G}(p)) - N_0 \{ q_H(p, \bar{k}(p)) + \hat{q}_M(p, \bar{k}(p))\} = 0 \).
Proof: Lemma 3.1 tells that the condition (i) is the necessary and sufficient condition to have $\phi^k = 0$, and it is self-evident that the condition (ii) is the necessary and sufficient condition to have $\phi^O_{MC} = 0$ when $\phi^k = 0$. Since $\phi^k = \phi^O_{MC} = 0$ automatically results in $\phi^p_{MC} = 0$, the market-clearing steady-state exists if and only if $f^{MC} = 0$ is satisfied along with conditions (i) and (ii), which is summarised in condition (iii).

Q.E.D.

Propositions 3.9 and 3.10 reveal that if there exists the optimal steady-state it must be along with the market-clearing trajectories. Note that non-existence of excess-supply steady-state does not necessarily indicate sub-optimality of the excess-supply trajectories. If they achieve higher social welfare than the market-clearing trajectories whilst observing the sustainability condition, the steady-state along the market-clearing trajectories is not optimal. Keeping this reservation in mind, I refer to the market-clearing steady-state as the optimal steady-state.

3.5.2 Local stability analysis of the optimal steady-state

For studying stability of the optimal steady-state, first let us describe the market-clearing trajectories by the following vector differential equation.

$$
\frac{d}{dt} \begin{bmatrix} G \\ p \\ \hat{k} \end{bmatrix} = \begin{bmatrix} G\phi^O_{MC}(G, p) \\ p\phi^k_{MC}(G, p, \hat{k}) \\ \hat{k}\phi^k(\hat{k}, p) \end{bmatrix},
$$

(3.17)

$G(0) = G_0$, $\hat{k}(0) = k_0$, and $p(0)$ is such that $f^{MC}(p(0); G_0, k_0) = 0$.

Due to nonlinearity of the system defined by Eq.(3.17) it is difficult to analyse global stability or instability of the market-clearing steady-state. If it is partially stable, which is frequently observed in the optimal growth models, the Liapunov's second method does not work (Gandolfo 1997). In fact an application of the second method with a Euclidean distance function as a candidate of Liapunov function results that the time derivative of this candidate is neither positive nor negative.
definitive, which is consistent with partial stability. Hence we focus on analysing the local stability of the steady state of the original system determined by Eq. (3.17) with linearisation method.

As this system is autonomous, which means that time is not contained as an explicit argument, the following linearised system near steady state is certainly a uniformly good approximation to the original system around the optimal steady state (Gandolfo 1997).

\[
\frac{dx}{dt} = A(x - x^e), \quad \text{where} \quad x = \begin{bmatrix} G \\ \dot{p} \\ \dot{k} \end{bmatrix}, \quad x^e = \begin{bmatrix} \bar{G} \\ \bar{p} \\ \bar{k} \end{bmatrix}, \quad \text{and} \quad A \text{ is the Jacobian matrix of the original system evaluated at the optimal steady state, i.e.}
\]

\[
A = \begin{bmatrix}
\frac{\partial G}{\partial \dot{p}} & \frac{\partial G}{\partial \dot{k}} & \frac{\partial G}{\partial \dot{\theta}_N} \\
\frac{\partial \dot{p}}{\partial \dot{p}} & \frac{\partial \dot{p}}{\partial \dot{k}} & \frac{\partial \dot{p}}{\partial \dot{\theta}_N} \\
\frac{\partial \dot{k}}{\partial \dot{p}} & \frac{\partial \dot{k}}{\partial \dot{k}} & \frac{\partial \dot{k}}{\partial \dot{\theta}_N}
\end{bmatrix}(x = x^e)
\]

\[
A_{11} = \delta \left( \frac{\partial G}{\partial G} \right) - 1, \quad A_{12} = \frac{\partial G}{\partial \dot{p}}, \quad A_{21} = \frac{\partial \dot{p}}{\partial G}
\]

\[
A_{22} = \frac{\left( \frac{\partial \dot{p}}{\partial \dot{p}} \right) \left( \frac{\partial \dot{p}}{\partial \dot{p}} \right) \left( \frac{\partial \dot{p}}{\partial \dot{p}} \right)}{N_0 \dot{G} \dot{D}_1} + b_6 b_{16} \frac{\bar{D}_2}{\bar{D}_1}, \quad A_{23} = -b_6 b_{16} \frac{D_2}{D_1},
\]

\[A_{32} = -b_6 b_{16} \frac{\dot{k}}{\bar{p}}, \quad \text{and} \quad A_{33} = -b_6 b_{16}, \quad \text{in which} \quad b_{16} = \frac{(\delta + \rho)(\rho \beta_K + (\delta + \rho) \beta_L)}{\rho \sigma \beta_K},\]

\[D_1 = -\frac{\partial}{\partial p} \left( \dot{q}_H + \dot{q}_M \right) x^e > 0, \quad \text{and} \quad D_2 = \frac{\partial}{\partial k} \left( \dot{q}_H + \dot{q}_M \right) x^e > 0. \quad 21
\]

The following proposition summarises the local stability of the optimal steady-state.

\[21 \text{ For the derivation of the Jacobian matrix } A, \text{ see Appendix A5.}\]
Proposition 3.11: Local stability of the optimal steady-state

If \( \varepsilon_G(\bar{G}) < 1 \), the Jacobian matrix \( A \) has two stable and one unstable eigenvalues. If the economy is enough close to the steady-state and if it is allowed to adjust the initial value of either \( G \) or \( \hat{k} \) freely, it is possible to put the economy on a stable manifold towards the optimal steady-state by adjusting either \( G_0 \) or \( \hat{k}_0 \) properly.

If \( \varepsilon_G(\bar{G}) \geq 1 \), the Jacobian matrix \( A \) has one stable and two unstable eigenvalues. If the economy is enough close to the steady-state and if it is allowed to adjust the initial values of both \( G \) and \( \hat{k} \) freely, it is possible to put the economy on a stable manifold towards the optimal steady-state by adjusting both of \( G_0 \) and \( \hat{k}_0 \) properly.

**Proof:** See Appendix A6.

Proposition 3.11 makes a stark contrast to its counterpart in the no supply side constraint world, Proposition 3.3. Once supply side constraint in water provision is introduced, the global stability of the optimal steady-state in the no supply side constraint world is replaced by saddle path local stabilities with which either one or both of \( G_0 \) and \( \hat{k}_0 \) must be properly adjusted to achieve the optimal steady-state, even if the economy has reached the neighbourhood of the steady-state. The dominance of the water production elasticity \( \varepsilon_G \) on the local stability of the steady-state makes sense because this elasticity represents the impact of supply side constraint on the economy.

An interesting finding is that an elastic water production with \( \varepsilon_G(\bar{G}) \geq 1 \), with which an increase of water production capacity must be easier, is associated with more difficulty in achieving the optimal steady-state. It seems that a combination of the water market clearance condition and the budget neutral government assumption with which all the water charge revenue must be invested in water production makes an elastic water production case more sensitive to the change in the level of capital stock.
3.6 Numerical simulation of the optimal trajectories

3.6.1 Model specification

This section briefly describes the numerical simulation model, a discrete-time version of analytic model.

(1) First-stage optimisation in discrete-time setting

The representative household’s problem in discrete-time becomes

\[ M_{\min} U_t = \sum_{s=1}^{\infty} \left( \frac{1+r}{1+\rho} \right)^t \left( \frac{1}{1-\sigma} \right) f(c_t^H, q_t^H) \left[ m_{t+1} - m_t + \left( r_s - \nu \right) m_t - c_t^M - p_s q_t^H \right], \]

and the initial assets \( m_t \) is historically determined at time \( t \).

The discrete-time version of the optimal consumption, which is corresponding to Proposition 3.1 in continuous-time, is as follows.

**Proposition 3.12: Optimal consumption level in discrete-time**

If \( r > \nu \) is satisfied, the optimal consumption is given as

\[ \hat{c}_t = \phi^\rho \left( \frac{1-\phi}{P_t} \right)^{1-\phi} \left( 1 + r_t - (1+\nu) \left( \frac{1+r_t}{1+\rho} \right)^{1/\sigma} \right) \left( m_t + \frac{w_t}{r_t - \nu} \right). \]

Otherwise, \( \hat{c}_t \) cannot have a positive finite value.

**Proof**: See Appendix A7.

The difference between Proposition 3.12 and its continuous version Proposition 3.1 is the second term from the far right side. Recall that \((1+x)^t\) in discrete time corresponds to \(e^x\) in continuous time, or equivalently that \(\ln(1+x)\) in discrete time corresponds to \(x\) in continuous time, where \(x\) is any constant growth rate such as population growth rate, pure time preference, etc. Hence \(\nu + \frac{r(1)-\rho}{\sigma}\) in continuous...
time corresponds to \[ \ln \left( \frac{1 + r}{1 + \rho} \right) = \ln(1 + \nu) + \frac{\ln(1 + r) - \ln(1 + \rho)}{\sigma} \] in discrete time. This fact may illustrate the correspondence between two propositions.

The firms' problem is, due to its static nature, identical with the continuous-time case. To have more flexibility in numerical simulation, however, a technological parameter \( \tau \) is introduced into the firms' production function which, in per labour unit, is now \( y_t = \tau(k_t)\beta_t(k_t)^{\theta_t}/q_t^M \). This modification results in a new set of equilibrium prices \( r_t^* = \tau \beta_t(k_t)^{\theta_t}/q_t^M \), and \( w_t^* = \tau \beta_t(k_t)^{\theta_t}/q_t^M \).

Now the value of \( \xi \) becomes \( \tau \beta_t(k_t)^{\theta_t}/q_t^M \), instead of \( \beta_t(k_t)^{\theta_t}/q_t^M \).

(2) Second-stage optimisation in discrete-time setting

The discrete-time version of Propositions 3.6 and 3.7 is as follows.

**Proposition 3.13: The excess-supply and the market-clearing trajectories**

The excess-supply trajectories in discrete-time are determined by the following system:

\[
\begin{align*}
\left(1 + \frac{p_{t+1}^{1-\varphi}}{\varphi \rho (1-\varphi)^{1-\varphi} (1+\nu)}\right) \hat{c}_t - p_t \left(\hat{q}_{t}^{H} + \hat{q}_{t}^{M}\right) - \frac{\left(\alpha - \delta\right)\hat{k}_t + w_t^*}{1+\nu} &= 0, \\
G_{t+1} - G_t &= p_t \left(\hat{q}_{t}^{H} + \hat{q}_{t}^{M}\right) N_0 \left(1+\nu\right) - \delta G_t, \quad G_0 \text{ given, and} \\
\hat{k}_{t+1} - k_t &= \frac{1}{1+\nu} \left\{\left(\alpha - \delta\right)\hat{k}_t + w_t^* - b_1 p_t^{1-\varphi} \hat{c}_t\right\}, \quad k_0 \text{ given.}
\end{align*}
\]

\[\text{22 Rigorously, } \xi \text{ in discrete-time is a solution of } \varphi^{(1)}(\xi) = 0 \text{ which contains a term associated with } \xi^{1/\sigma}. \text{ It is numerically confirmed that the error of approximation of this solution to the continuous-time version } \xi \text{ is negligible at the order of } 10^{-15}. \text{ See Simulation 1 below.}\]
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The market-clearing trajectories are determined by the following system:

\[ f^{MC} \left( p_t; \hat{k}_t, G_t \right) = F^W \left( G_t \right) - \left\{ q^M \left( p_t; \hat{k}_t \right) + q^H \left( p_t; \hat{k}_t \right) \right\} N_0 \left( 1 + \nu \right)^t = 0 \]

\[ G_{t+1} - G_t = p_t F^W \left( G_t \right) - \delta G_t, G_0 \text{ given, and} \]

\[ \hat{k}_{t+1} - \hat{k}_t = \frac{1}{1 + \nu} \left\{ \left( r^* - \nu \right) \hat{k}_t + w^* - b_1 p_t \right\}, k_0 \text{ given.} \]

**Proof:** See Appendix A8.

(3) Specification of water production function

Depending on the purpose of the simulation, two sets of water production functions are constructed.

The first set for analysing stability of the optimal steady state is designed such that there exist three steady-states, as shown in Figure 2, of which only the middle one is associated with an elasticity greater than unity \( \left( \varepsilon_G \left( \overline{G} \right) \geq 1 \right) \).

![Figure 2 Water production function for numerical simulation](image)

From Proposition 3.10 the relationship \( p \overline{Q} = p F^W \left( \overline{G} \right) = \delta \overline{G} \) holds at a steady-state. It means that once the steady-state water price \( \overline{p} \) is fixed, the line \( \overline{Q} = \left( \delta / \overline{p} \right) \overline{G} \) is fixed. Three steady-state points \( \left( \overline{G}_i, \overline{Q}_i \right) \) are obtained by changing the population size \( N_i \) for \( i = 1, 2, 3 \), and the water production function \( Q = F^W \left( G \right) \) is calibrated such that it goes through these steady-state points. The employed functional forms are \( F^W \left( G \right) = a_i \ln G - b_i \) for the outer sections and \( F^W \left( G \right) = a_2 G^{b_2} - c_2 \) for the middle section, in which \( a_i, b_i, \) and \( c_i \) are production parameters to be calibrated.
The second set is constructed by scaling up the first set such that sustainable water supply capacity of the excess-supply trajectories exceeds the optimal water demand for a reasonably long period, since the first set with employing the steady-state capital stock of each state variable as its initial level is associated with water deficit throughout simulation period. The scaling-up factor is set at five.

(4) Calibration

Most of model parameters are exogenously given but the technological parameter \( \tau \), the steady-state private and public capital stock (\( \bar{k} \) and \( \bar{G}_i \)) and parameters in water production functions are endogenously calibrated.

The technological parameter \( \tau \) is calibrated as

\[
\tau = \frac{\bar{r} + \delta}{\rho + \delta}, \quad \text{where} \quad \bar{r} \quad \text{is the steady-state value of} \quad r^*. 
\]

The steady-state private capital stock \( \bar{k} \) is calibrated as

\[
\bar{k} = \left( \frac{\xi}{P^k} \right)^{1/\beta_k}, \quad \text{where} \quad \xi = \tau \beta_k \beta_Q \left( \delta + \rho \right) \quad \text{is the steady-state value of} \quad \xi_i.
\]

The steady-state public capital stock \( \bar{G}_i \) is calibrated as follows. First, the steady-state aggregate water demand \( \bar{Q}_i \) is obtained as

\[
\bar{Q}_i = (\bar{q}^H + \bar{q}^M)N_i, \quad \text{where} \quad \bar{q}^M = \tau \bar{k}^{b_q} \left( \beta_Q / P \right)^{1/(b_q - \rho)} \quad \text{and} \quad \bar{q}^H = \frac{1 - \varphi}{P} \left[ 1 + \bar{r} - (1 + \nu) \left( \frac{1 + \bar{r}}{1 + \rho} \right)^{\sigma} \right] \left( \bar{k} + \frac{\bar{W}}{\bar{F} - \nu} \right), \quad \text{in which} \]

\[
\bar{W} = \tau \beta_L \bar{k}^{b_L} \left( \beta_Q / P \right)^{b_L}.
\]

Then the steady-state public capital stock is calibrated as \( \bar{G}_i = \bar{p} \bar{Q}_i / \delta \). The parameters in water production functions are calibrated such that all the steady-state points \( (\bar{G}_i, \bar{Q}_i) \) are contained and the steady-state water production elasticities have desired values.
The following exogenous parameter values are chosen.

- Population growth rate: $\nu = 0.02$ (for Simulation 3, $\nu = 0$)
- Elasticity of marginal felicity: $\sigma = 3$
- Weight of market good consumption in satisfaction production: $\varphi = 0.98$
- Depreciation rate: $\delta = 0.05$
- Rate of pure time preference: $\rho = 0.075$
- Factor share of private capital in market commodity production: $\beta_k = 0.5$
- Factor share of water in market commodity production: $\beta_0 = 0.2$
- Factor share of labour in market commodity production: $\beta_L = 0.3$

The population sizes are set at $N_1 = 1,000$, $N_2 = 2,000$ and $N_3 = 3,000$. The water price and the real rate of return at the steady-state are set at $\bar{p} = 1$ and $\bar{r} = 0.1$ in calibration process. Calibrated water production parameters are $a_1 = -5,476.4$, $b_1 = -61,077.8$, $a_2 = 45,526.4$, $b_2 = 0.1$, $c_2 = 121,759.2$, $a_3 = 14,400.4$, and $b_3 = 143,107.1$.

(5) Solution algorithm

The economic situation to be studied is given as a set of $(N_i, G_0, \hat{k}_0)$ in which initial capital stock levels are specified as a proportion of their steady-state values, e.g. $G_0 = 0.8 \bar{G}_2$, $\hat{k}_0 = 1.1 \bar{k}$, and so on. Note that $\bar{k}$ is fixed once parameters and the steady-state water price $\bar{p}$ are given.

Given the initial values of state variables, the equilibrium water price at $t = 0$ is obtained for both the excess-supply and the market-clearing trajectories by solving the implicit functions $f^{ES}(p_0; \hat{k}_0) = 0$ and $f^{MC}(p_0; G_0, \hat{k}_0) = 0$, respectively, as a static problem. The obtained water price, in turn, determines the capital stock levels of the next period $(G_1, \hat{k}_1)$ through the equations of motion, and $f^{ES}(p_1; \hat{k}_1) = 0$ and $f^{MC}(p_1; G_1, \hat{k}_1) = 0$ are solved for these stock levels, and so on. This solution algorithm is specified as a mixed complementarity problem in GAMS and solved by MILES solver, and executed for 200 time periods for the most part of simulations. The employed GAMS code is attached as Appendix B1.
3.6.2 Simulation results

Simulation 1: No supply side constraint case

Simulation 1 tests the validity of Proposition 3.3 (global stability of the optimal steady-state without supply side constraints) and the accuracy of the approximation of $\xi$ in discrete-time to the continuous-time counterpart. For these purposes Simulation 1 covers only the first stage optimisation model in which there is no supply side constraints. Figure 3 shows the global stability of the steady-state in the no supply side constraint world as Proposition 3.3 predicts, when water price is fixed at unity.

![Figure 3 Global stability of the optimal steady-state without supply side constraint](image)

Proposition 3.3 predicts that $\hat{k}$ always converges to $\bar{k} = \xi \frac{1-\rho_Q}{\rho_T} \left( \frac{\rho_Q}{p} \right)^{\mu_T}$ from everywhere within its domain $\hat{k} \in (0, \hat{k}_{\max})$ which corresponds to $\xi \in (0, \xi_{\max})$. Figure 3 clearly demonstrates this global stability of the steady-state when water scarcity is not taken into account.

The accuracy of approximating discrete-time $\xi$ with continuous-time version is tested by setting the initial capital stock at $(1+1.0 \times 10^{-9}) \bar{k}$ and by simulating 1000 time periods. The difference between the values of $\xi$ derived from the value of $\hat{k}$
after 1000 time periods and its continuous-time counterpart becomes $2.22 \times 10^{-15}$. This illustrates the closeness of the continuous-time version approximation of $\bar{F}$.

**Simulation 2: Comparison between the market-clearing and the excess-supply trajectories**

One of the most interesting questions for the numerical simulation is whether the excess-supply trajectories satisfy the sustainability constraint (3.14d), and, if so, whether they could attain higher social welfare than that attained by the market-clearing trajectories.

As mentioned in the explanation of the water production function construction, the original version of water production function is designed to test the stability of the optimal steady-state along the market-clearing trajectories. It is found that it cannot have positive water balance along the excess-supply trajectories without significant scaling-up. This means that proportion of $Q_t$ to $G_t$ is one of major factors determining whether the sustainability condition can hold or not. Once the water production function is specified, the simulation reveals that the initial stock of the private capital mostly determines whether the sustainability condition can be satisfied or not, and how long this condition can hold. *Figure 4* illustrates this finding.

![Figure 4 Sustainability condition of the excess-supply trajectories](image)

To provide an answer to the latter question, social welfare as a discounted sum of felicity is evaluated for the period for which the sustainability condition is satisfied.
It is found that the market-clearing trajectories always yield higher social welfare than the excess-supply trajectories, and this is consistent with our intuition that excess supply capacity has no advantage for society under this problem setting. To confirm this finding, Figure 5 shows the comparison between the market-clearing trajectories (‘MC’ in the figure) and the excess-supply trajectories (‘ES’ in the figure) with two contrasting initial conditions of capital stock level, either \((G_0, k_0) = (\bar{G}, 0.5 \bar{k})\), which allows ‘ES’ trajectories to maintain excess water supply for entire simulation period, or \((G_0, k_0) = (10 \bar{G}, \bar{k})\), which represents more ‘developed’ situation in terms of capital accumulation. Note that trajectories are depicted only when they satisfy the sustainability condition.

Figure 5 illustrates two important properties. First, the ‘MC’ paths are always more favourable than the ‘ES’ paths. When the ‘ES’ paths approach to or just pass over the ‘MC’ paths, the sustainability condition is no longer satisfied. This observation leads us to the following corollary.

Figure 5 Comparison of two candidates of the optimal trajectories
Corollary 3.1: Optimality of the market-clearing trajectories

The market-clearing trajectories are the optimal trajectories for the government problem (3.14a) - (3.14d).

In the next chapter it will be clear that feasibility of the second-stage optimisation of the generalised model is underpinned by this crucial result.

Second, each ‘MC’ path tends to converge a unique path regardless of the initial conditions. This property is further investigated. Figure 6 shows the optimal trajectories generated by highly heterogeneous initial conditions.

These simulations clarify that the optimal trajectories has strong convergence tendency towards a unique set of paths that clearly reflect positive rate of population growth.

Simulation 3: Stability of the optimal steady-state

It is expected from Proposition 3.11 that the optimal steady-state is characterised by saddle-path local stability. This proposition also tells us that it is necessary to adjust
initial conditions of the both state variables, \( G_0 \) and \( k_0 \), for putting the economy on the stable manifold, when the elasticity of water production at the steady-state, \( \varepsilon_0(\bar{G}) \), is greater than unity. To investigate this proposition the steady-state associated with \( N_2 \), which is designed to have \( \varepsilon_0(\bar{G}) > 1 \), is chosen and wider variety of combination of capital stock levels are given. Note that for the steady-state analysis population growth rate is set at zero. The outcome of this simulation is shown in Figure 7.

These simulation results reveal that the optimal steady-state is globally stable, at least in practical sense. The reservation arises from the fact that convergence to the steady-state seems slow comparing with the rapid convergence observed in Figure 6. Recall that the steady-state always has at least one zero real eigenvalue that functions to keep distance between trajectories and the steady-state constant. In this sense it is highly likely that the steady-state observed here is asymptotically unstable at the local level, as Proposition 3.11 predicts. Nevertheless, it is clear from Figure 7 that the optimal trajectories can be regarded as stable at the global level, and exactly at this level policy has a role to bring the economy towards the optimal path.
At last the difference among three steady-states are investigated. To demonstrate the outcome I chose only the water price trajectory which exhibits most interesting patterns, as shown in Figure 8.

Contrary to what we may expect from Proposition 3.11, no qualitative difference between the trajectories associated with $\bar{E}_0 < 1$ and $\bar{E}_0 \geq 1$ is observed.

Simulation 3 reveals the limited relevance of local stability analysis where the non-linear system is globally stable which is not asymptotically stable but stable in a sense that any trajectories converge to certain neighbourhood of the steady-state. This finding highlights the usefulness and importance of numerical analysis to follow up the results obtained from theoretical analysis.

### 3.7 Conclusions

In this chapter an analytic model based on RCK growth model is developed. It includes two novel specifications for improving applicability of RCK growth model to quantitative policy analysis; (i) continuous monitoring-feedback in the households’ price expectation formation, and (ii) two-stage optimisation in which private optimisation and public optimisation processes are separated. This analytic model serves as a platform based on which an applied model, a highly aggregated dynamic CGE model, is constructed. The applied model is explained in the next chapter.
The solution of the first-stage optimisation, i.e. the private optimisation, shows that if there is no supply side constraint and the government can freely set the rate of water charge, then (i) any constant rate of water charge drives the economy towards the optimal steady-state (Proposition 3.3), and (ii) the government can induce any desirable level of social welfare as the optimal steady-state by setting appropriate constant rate of water charge (Proposition 3.4).

The solution of the second-stage optimisation, i.e. the public optimisation, has several policy implications.

The most important of these is the fact that the government problem cannot have an interior solution if water sustainability constraint is specified as a strict equality (Proposition 3.8), although it is mathematically possible to construct a system of differential equations which determines trajectories. Instead, there are two candidate optimal trajectories; one corresponds to an interior solution when water supply capacity exceeds the optimal water demand, which is termed as ‘excess-supply’ trajectories (Proposition 3.6), and the other is a set of trajectories solely determined by water market clearance condition, which is termed as ‘market-clearing’ trajectories (Proposition 3.7). Because of the non-existence of first-best solution, it is analytically unclear which of two candidates is the optimal solution. Although the excess-supply trajectories do not converge on any steady-states, this does not preclude the possibility that this set of trajectories yields the greater net benefits to society.

Local stability of the steady-state associated with the market-clearing trajectories is studied by linearisation method. The results show that this steady-state exhibits saddle-path stability which is commonly observed in RCK growth model literature. It is found that the Jacobian matrix evaluated at the steady-state has either two stable and one unstable eigenvalues, or one stable and two unstable eigenvalues, depending on whether the water production elasticity is greater than unity or not (Proposition 3.11). This is because this elasticity captures the impact of water scarcity on the economy.
Useful insights for policy are offered by numerical simulations of the analytic model. The simulations reveal that for reasonable parameter values the market-clearing trajectories do in fact form the optimal solution to the government’s problem (Corollary 3.1). This implies that the optimal solution for the problem to be tackled with policy simulation in the following chapters is likely to be associated with market clearance of publicly supplied goods (water). This greatly reduces the difficulty in solving the dynamic optimisation problem. Another interesting finding is the global stability of the optimal trajectories despite the fact that they are not necessarily locally stable. This illustrates the value of numerical simulation. In both cases numerical simulations have clarified the policy implications of the analytic model.
Chapter 4

The Applied Model

4.1 Introduction

In this chapter an applied model for the policy simulation is constructed by incorporating several key stylised facts of water scarce developing countries into the analytic model explained in the previous chapter. Furthermore, trade and intermediate goods flow are introduced into the model. The applied model can be regarded as a highly aggregated version of a forward-looking dynamic computable general equilibrium (CGE) model without the assumption of perfect foresight.

Following this introductory section, Section 4.2 describes the main features of the model and explains how to incorporate the key stylised facts into the model. Sections 4.3 and 4.4 explain the first-stage and the second-stage optimisation respectively. In these sections basic model set-up and notation are the same as in the previous chapter. Section 4.5 explains the issues arising from introducing trade and taxes. Although the role of trade and taxes in this research is mainly related to calibration and validation of the model, explicit treatment of them certainly widens the scope of policy simulation. Section 4.6 summarises the major outcomes and concludes this chapter.
4.2 Modelling the key stylised facts

The applied model is constructed based on the analytic model with incorporation of the following stylised facts commonly observed in many water-scarce developing countries.¹

- Irrigation accounts for the vast majority of total water use, often reaching 80 to 90% (Rosegrant et al. 2002b).

- Production risks in rainfed agriculture are one of the main causes of rural poverty and consequently of rural-urban migration (Fafchamps et al. 1998).

- Urban unemployment is high with considerable rural-urban migration in spite of priority public investments in urban modern sectors (Harris and Todaro 1970, Beladi and Yabuuchi 2001).

- A lack of safe water access, which is common in the rural areas or in the urban squatter areas, severely undermines the social welfare through various pathways, via direct and indirect health risks and higher medical and water expenditure, or via depriving children from educational opportunities (WHO and UNICEF 2000).

These key stylised facts are reflected in the applied model as follows.

Firstly, the single private production sector in the analytic model is disaggregated into two rural and one urban production sectors, i.e. the rainfed and the irrigated agricultural sectors and the urban modern sector. The outputs of all production sectors are assumed to be tradable under the small open economy assumption. On the other hand, labour and capital markets are assumed to be domestic.² Rainfed agriculture is regarded as a household activity that requires only internal resources such as family labour, owned machinery and owned farmland.

¹ See Chapter 5 for the relevance of these stylised facts to the case study country, Morocco.

² The share of foreign direct investment in gross fixed capital formation was around 3% between 1960 and 1990 and 9% during 1990's, but the latter high share is largely due to privatisation of Maroc Telecom (Bouoiyour 2003).
Secondly, a multiplicative risk factor with stochastic distribution is introduced into the rainfed production function in order to reflect the high production risks of the sector, while production risks in irrigated agricultural and the urban modern sectors are assumed to be zero.\(^3\) Although there exists empirical evidence that irrigation does not eliminate agricultural production risk in water abundant areas (e.g. rice production in Philippines reported in Roumasset 1976), the zero production risk assumption in irrigation helps to highlight the vulnerability of rainfed agriculture against erratic rainfall patterns in arid and semi-arid regions.

Thirdly, downward rigidity of the urban modern sector wage is assumed, which reflects the minimum wage legislation. The irrigated agricultural sector wage remains flexible. More precisely, it is assumed that unskilled labour wage in the urban modern sector is fixed at the minimum wage rate despite the presence of surplus labour, while flexible wage rates of urban skilled labour and of irrigated agricultural labour clear these two labour markets. This specification generates Harris-Todaro type rural-urban migration in which the wage gap between the rural and the urban sectors induces domestic migration (Todaro 1969, Harris and Todaro 1970).

Fourthly, it is assumed that some fraction of household members who are allocated to rural sectors lacks safe water access and that this fraction is determined by the stock level of public capital in water supply sector.\(^4\) Welfare impacts of lack of safe water access are represented by ‘penalty’ on household members who lack safe water access. In reality this ‘penalty’ might be direct or indirect health risks, some additional expenditure for bottled water, medical care, the cost of boiling raw water, etc. or a reduction in income due to less time allocation to wage labour or to education activity. In the applied model the ‘penalty’ is specified as a reduction in total working time that results in a reduction in wage income.

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\(^3\) The coefficient of variation for annual rainfall variability of rainfed regions in Morocco is estimated at 0.2 while that for cereal production reaches 0.4 ~ 0.5 (Kamky and Arndt 2002).

\(^4\) WHO/UNICEF (2001) estimates access to improved drinking water source (including 50% of protected spring and well water) at 98% in urban areas and at 56% in rural areas in the year 2000. Some other sources estimate rural access to safe water much less due to different definition of safe water, e.g. WHO/UNICEF (1996) estimates at 14%.
In order to incorporate the latter two stylised facts into the Ramsey-Cass-Koopmans (RCK) framework, it is assumed that a household’s decision is based on household pooled values (pooled income, pooled consumption, etc.) in spite of heterogeneity among its members. For example, if one member earns 10 units and the other member earns nothing, the household makes decisions as if two members would earn 5 units each. Though this assumption is a straightforward extension of the conventional view of the household as a single decision-making unit, it appears much stronger in this situation than under an identical individual assumption. Nevertheless, if we were to abandon the assumption that the household is the decision-making unit, we would be forced into an undesirable model specification in which individuals must be indexed based on their migration history and their asset accumulation history.

4.3 First-stage optimisation

4.3.1 The household’s problem

(1) Problem formulation

It is assumed that rainfed agricultural sector is defined as households’ farm activities. There are a fixed number (N) of identical households of which initial size (number of members) at \( t = 0 \) is normalised at unity and grows at constant rate \( \nu \). Each household is endowed with the same amount of rainfed farmland, whose productivity is assumed to be fixed at \( \bar{Y}_R \). More precisely, \( \bar{Y}_R \) represents the per household maximum productivity of rainfed farmland with the average rainfall. In

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5 In the model ‘one person’ consists of one labour force age person and his/her dependents. See Footnote 1 in Chapter 3.

6 Rainfed agricultural production data from Haute Chaouia region in Morocco (de Janvry et al. 1992) reveal no difference between the small and the medium farms’ per hectare crop productivity (1,453 Dirham per ha for the former and 1,433 Dirham per ha for the latter) in spite of three-fold difference in per labour productivity (1,219 Dirham per labour for the former and 3,572 Dirham per labour for the latter). Zagdouni and Betanya (1990) found that labour displacement due to mechanisation caused high rural-urban migration from Haute Chaouia to urban areas such as Casablanca. These empirical data support this assumption.
the household farm production an important factor input, water, is exogenously and randomly given by rainfall and is represented by a multiplicative risk factor with stochastic distribution. The rainfed agricultural production technology is specified as a Leontief function of intermediate goods and aggregate input of labour and capital, and labour-capital aggregate is specified as a Cobb-Douglas function. Hence the per household production function of rainfed agriculture is

\[ Y^R_t = \min \left[ \frac{s^R_{i,j}}{a_{Ri}}, \left( \frac{s^R_{i,u}}{a_{Ru}} \right) \right], \quad \bar{Y}_R = \tau_R \left( L^R_t \right)^{\beta_{RL}} \left( K^R_t \right)^{\beta_{RK}}, \quad (4.1) \]

where \( Y^R_t \): per household yields in year \( t \), \( \omega_t \): the production risk factor in year \( t \) with \( E[\omega_t] = 1 \) and \( \text{Var}[\omega_t] = \sigma^2_\omega \), \( \tau_R \): technological parameter in rainfed agriculture, \( L^R_t \): per household family labour input in year \( t \), \( K^R_t \): per household stock of rainfed capital in year \( t \), \( s^R_{i,j} \): input of intermediate goods produced by the sector \( j \), \( a_{Rj} \): input-output coefficient associated with \( s^R_{i,j} = a_{Rj} Y^R_i \), and \( \beta_{RL} \) and \( \beta_{RK} \): factor shares of family labour and rainfed capital with \( \beta_{RL} + \beta_{RK} = \beta_R < 1 \), which reflects the suppressed fixed factor input, rainfed land.

When a household optimises profits, the yield is determined by the term \( \omega_t \bar{Y}_R \) and the household adjusts input level of intermediate goods such that equalities between arguments of Leontief function hold. Consequently the per household optimal profit from the rainfed agricultural activity is given as

\[ \Pi^R_t = p^R_{t,R} \omega_t \bar{Y}_R - p_t^{le} a_{Ri} \omega_t \bar{Y}_R - p_t^{le} a_{Ru} \omega_t \bar{Y}_R - p_t^{le} \omega_t \bar{Y}_R - p_t^{le} I^R_t = \bar{P}_t^R \omega_t \bar{Y}_R - p_t^{le} I^R_t, \]

where \( p^R_{t,R} \): the domestic producer price of rainfed products, \( p_t^{le} \): the domestic consumer price of urban products which are used as both consumer goods and production capital, \( p_t^{li} \): the domestic consumer price of irrigated agricultural products, \( I^R_t \): the investment in rainfed capital, and \( \bar{P}_t^R \): the net producer price \( = p_t^{le} - p_t^{le} a_{Ri} - p_t^{le} a_{Ru} \).\(^7\)

\(^7\) The numeraire of the applied model is not urban products as in the analytic model but local currency. Throughout Chapters 4 to 6, \( p^{le} \) and \( p^R \) denotes a domestic consumer price and a domestic producer price of good \( i \), respectively. The relationship between producer and consumer prices will be discussed in Section 4.5.
A household allocates a fraction of its members $l_i$ to labour category $i$ where $i = R$: rainfed agricultural labour, $I$: irrigated agricultural labour, $U$: urban unskilled labour, or $S$: urban skilled labour, and $l_i^R + l_i^I + l_i^U + l_i^S = 1$. Further, the fixed labour allocation to urban skilled labour, $T^S$, is assumed.

To capture the productivity cost of lack of water supply, it is assumed that the members without safe water access (public water supply) can supply only $1 - z$ units of labour services per unit of time instead of $1$ unit with safe water access. The coverage of public water supply, which is determined by the level of public capital stock in terms of water supply facilities, is $\theta_i$ in the rural areas and the unity in the urban areas. In addition it is assumed that the members without public water supply collect an amount of water, $q_{no}$, without any money transaction. Hence the per capita average water consumption becomes

$$\bar{q}_i'' = \left\{l_i^U + T^S + (1 - l_i^U - T^S)\theta_i\right\} q_i'' + \left(1 - l_i^U - T^S\right)(1 - \theta_i)\bar{q}_{no},$$

and the per capita average water expenditure becomes

$$\left\{l_i^U + T^S + (1 - l_i^U - T^S)\theta_i\right\} p_i q_i'' ,$$

where $q_i''$ is per 'user' water consumption of publicly supplied water.

From these assumptions with Eq. (4.1), the labour allocation to the irrigated agriculture is expressed as

$$l_i^I = \left(1 - l_i^U - T^S\right) - \frac{1}{\{1 - (1 - \theta_i)\bar{z}\}} \left(\frac{1}{1 + \nu}\right) \left(\frac{\bar{Y}_R}{\tau_R}\right) \frac{1}{\rho_{xx}} \left(\frac{K^R}{\rho_{xx}}\right) .$$

(4.2)

The urban unskilled wage rate is assumed to be higher than that in irrigated agriculture, and to be downwardly rigid due to the existence of minimum wage legislation. 

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8 Morocco has legislated both the urban and the rural minimum wages (Economist Intelligence Unit 2002).
Analogous to the rural-urban migration model of Harris and Todaro (1970), the differences of wage rates as well as public water supply coverage between the urban and the rural areas induce chronic urban unemployment. I assume that households optimise their labour allocation between urban unskilled and irrigated agricultural labour such that allocating a member to either labour category generates the same level of indirect utility derived from expected income wage. This assumption can be regarded as a generalised version of Harris-Todaro model. In fact, the Harris-Todaro model corresponds to the special case when all the rural area is covered by public water service (θt=1). Let θtE denote the probability for the unskilled member migrating to urban area to be employed in the urban modern sector. The following lemma gives the equilibrium value of θtE as a result of the household decision problem.

**Lemma 4.1 Urban unemployment equilibrium**

Assume that θt is enough large to satisfy the condition

\[ \bar{z}w'_t(\theta_t)^2 + (1 - \bar{z})w'_t - p_t\bar{q}_{}\bar{q}_{} + w'_t(\theta_t)^1 - \bar{q}_{} + p_t\bar{q}_{} \leq 0, \]

where \( w'_t \) is the wage rate of the irrigated agricultural labour and \( w'_t(\theta_t)^1 - \bar{q}_{} \) is that of the urban unskilled labour fixed at the legislated minimum rate.

The generalised Harris-Todaro assumption (income-generated indirect utility equalisation assumption) gives the equilibrium value of θtE as

\[ \theta_t^{eq*} = \frac{(\theta_t)^{\bar{q}_{}}}{w'_t} \left[ w'_t \left\{1 - \theta_t(\bar{z})\right\} + p_t\bar{q}_{} \left(\frac{1}{\theta_t} - 1\right)\right]. \]

**Proof**: See Appendix A9.

It is easy to check that the Harris-Todaro employment probability, \( \theta_t^{eq*} = \frac{w'_t}{w'_t(\theta_t)^1} \), is obtained by setting \( \theta_t = 1 \) in Lemma 4.1. We expect that the higher \( \theta_t \), ceteris paribus, the more favourable is the irrigation sector relative to the urban modern sector.
sector, which results in the higher \( \theta_i^{E*} \). The expected sign \( \frac{\partial \theta_i^{E*}}{\partial \theta_i} > 0 \) is, however, not automatically established but requires
\[
P_t q_{\text{so}} < \frac{w_i^t \theta_i \left[ \phi_q + \left( \theta_i - (1 - \theta_i) \phi_q \right) z \right]}{1 - (1 - \theta_i) \phi_q}.
\]
This is because my model captures the welfare impact of lacking safe water access as sacrificed labour hours (represented by \( z \)) for fetching subsistence water (\( q_{\text{so}} \)) instead of purchasing water at price \( p_t \). Hence the utility generated by saved income by fetching water \( p_t q_{\text{so}} \) must be smaller than the disutility caused by forgone wage income as well as by limiting water consumption at the subsistence level.

Now, the per capita household income with incorporating Eq. (4.2) is expressed as
\[
\left( \frac{1}{1 + \nu} \right) \left( \mu_i^R R + r_t P_t^{uc} m_t + \bar{w}_i u_t^{E*} \theta_i^{E*} + w_s T_s \right)
\]
\[
+ \left( 1 - u_t^{E*} - T_s \right) \left( \frac{1}{1 + \nu} \right) \left( \frac{1}{\tau_i^R} \frac{1}{K_i^R} \right)
\]
where \( m_t \): household assets (equity shares of private capital), and equilibrium labour \( l_t^{u*} \): equilibrium labour allocation to the urban unskilled labour.

Households purchase and consume market commodities produced by three production sectors as well as publicly supplied water and invest the rest of income into equity shares of private capital \( m \). The per capita household expenditure is
\[
p_t^R c_i^R + p_t^I c_i^I + p_t^{uc} c_i^U + \left( \frac{\theta_i + \left( l_t^{u*} + T_s \right) (1 - \theta_i)}{\theta_i} \right) p_t q_i^H + p_t^{uc} I_t,
\]
where \( c_i \): per capita consumption of commodities produced by the sector \( i \) \((i=R, I, U)\), and \( I_t \): per capita investment in the equity shares of the private capital.

From the above income and the expenditure expressions, the household budget constraint in per capita terms becomes
By incorporating this equation into the equation of motion of rainfed capital

\[ K_{t+1}^R - K_t^R = I_t^R - \delta K_t^R, \]

in which \( \delta \) is the depreciation rate, we obtain

\[
K_{t+1}^R - K_t^R = (1 + \nu)^t \left[ r_t p_t^I m_t + \frac{w_t^I}{p_t^I} \left( 1 - l_t^U - \bar{T} \right) \left\{ 1 - (1 - \theta_t) \bar{c}_t \right\} - \frac{w_t^I}{(1 + \nu)^t} \left( \frac{\bar{Y}_R}{T_R} \right)^{\frac{1}{\beta_{MC}}} \left( \frac{1}{K_t^R} \right)^{\frac{\beta_{MC}}{\beta_{MC}}} \right]
\]

\[
+ \frac{w_t^U}{p_t^I} l_t^U \theta_t^U + \frac{w_t^I}{p_t^I} \left( \frac{1}{1 + \nu} \right)^t \left( \omega_t P_t^R Y_t^R - P_t^U I_t^R \right) = p_t^R c_t^R + p_t^I c_t^I + p_t^U c_t^U
\]

\[ + \left\{ \theta_t + \left( l_t^U + \bar{T} \right)(1 - \theta_t) \right\} p_t q_t^H + p_t^U I_t. \]

Another equation of motion is for private capital (equity), \( m_{t+1} - m_t = I_t - \frac{\gamma m_t}{1 + \nu} \).

The households' satisfaction depends on consumption as follows.

\[
c_t = \left( c_t^R \right)^{\varphi_t} \left( c_t^I \right)^{\varphi_t} \left( c_t^U \right)^{\varphi_t} \left( q_t^H \right)^{\varphi_t}, \varphi_t \in (0, 1) \text{ for all } i \text{ and } \sum_i \varphi_i = 1. \]

Assuming a CRRA (constant relative risk aversion) function \( u(c_t) = \left( \frac{c_t}{\gamma} \right)^{-\sigma} \)

in which \( \sigma \) is the coefficient of relative risk aversion, the representative household's problem at time 1 is expressed as the following utility maximisation problem given the household's expectation of exogenous variables.\(^9\)

\(^9\) The notation \( \cdot \) denotes expectation of exogenous variables.
Maximize
\[ U_t = \sum_{i=t}^{\infty} \left( \frac{1 + \nu}{1 + \rho} \right)^i u(c_i), \]
subject to
\[
K_{t+1}^R - K_t^R = (1 + \nu)^i \left[ \tilde{\mu}_t m_t + \frac{\tilde{w}_t^r}{\tilde{p}_t^u c_t} \left( 1 - \tilde{\theta}_t s_t - \tilde{T}_s \right) \left( 1 - \tilde{\theta}_t s_t - \tilde{T}_s \right) \left( 1 - \tilde{\theta}_t s_t - \tilde{T}_s \right) \right] + \frac{\tilde{w}_t^u}{\tilde{p}_t^u c_t} \theta_t s_t \tilde{u}_t^* + \frac{\tilde{w}_t^s c_t}{\tilde{p}_t^u c_t} \]
\[
- \frac{\tilde{w}_t^r}{\tilde{p}_t^u c_t} \left( \frac{1}{K_t^R} \right) \frac{\tilde{c}_t}{\tilde{r}_t} \left( \frac{1}{K_t^R} \right) \left( \frac{1}{K_t^R} \right) + \left( \tilde{\pi}_t^r - \tilde{\pi}_t^u a_{tR} - \tilde{\pi}_t^u a_{tU} \right) \frac{dR_t^r}{\tilde{p}_t^u c_t} \]
\[
- (1 + \nu)^i \left[ \tilde{\pi}_t^{r+} \tilde{p}_t^{u\sigma} c_t^{r+} + \tilde{\pi}_t^{t+} c_t^{t+} + \tilde{\pi}_t^{u+} q_{t^+} \left( \tilde{\epsilon}_t + \left( \tilde{\mu}_t^u + \tilde{T}_s \right) \left( 1 - \tilde{\theta}_t s_t \right) \right) \right] \delta K_t^R,
\]
\[
m_{t+1} - m_t = \frac{I_t - \nu m_t}{1 + \nu}, \text{ and the given initial values } K_0^R \text{ and } m_0.
\]

As before, \( \rho \) is the rate of pure time preference.

(2) The optimal consumption level

The current value Hamiltonian of the household's problem is
\[
\tilde{H}_t = c_t^{1-\sigma} + \lambda_{s+1}^R \left( 1 + \nu \right)^{s+1} \left[ \tilde{\mu}_t m_t + \frac{\tilde{w}_t^r}{\tilde{p}_t^u c_t} \left( 1 - \tilde{\theta}_t s_t - \tilde{T}_s \right) \left( 1 - \tilde{\theta}_t s_t - \tilde{T}_s \right) \right] + \frac{\tilde{w}_t^u}{\tilde{p}_t^u c_t} \theta_t s_t \tilde{u}_t^* + \frac{\tilde{w}_t^s c_t}{\tilde{p}_t^u c_t} \]
\[
+ \frac{\tilde{w}_t^r}{\tilde{p}_t^u c_t} \left( \frac{1}{K_t^R} \right) \frac{\tilde{c}_t}{\tilde{r}_t} \left( \frac{1}{K_t^R} \right) \left( \frac{1}{K_t^R} \right) \left( (1 + \nu)^i \right) \left( \frac{1}{K_t^R} \right) \frac{dR_t^r}{\tilde{p}_t^u c_t} \]
\[
- \frac{\tilde{\pi}_t^{r+}}{\tilde{p}_t^u c_t} \tilde{c}_t^{r+} - \frac{\tilde{\pi}_t^{t+}}{\tilde{p}_t^u c_t} \tilde{c}_t^{t+} + \frac{\tilde{\pi}_t^{u+}}{\tilde{p}_t^u c_t} q_{t^+} \left( \tilde{\epsilon}_t + \left( \tilde{\mu}_t^u + \tilde{T}_s \right) \left( 1 - \tilde{\theta}_t s_t \right) \right) \right] \delta K_t^R,
\]

where \( \lambda^R \) and \( \lambda^m \) are the Lagrange multipliers.

Assuming an interior solution, the necessary conditions are as follows.

\[
\frac{\partial \tilde{H}_t}{\partial c_t^{r+}} = 0 \quad \Rightarrow \quad c_t^{1-\sigma} \frac{\phi_t^r}{\tilde{p}_t^u c_t} = \left( 1 + \nu \right)^{s+1} \frac{\tilde{w}_t^r}{\tilde{p}_t^u c_t} \lambda_{s+1}^R \frac{\tilde{p}_t^{u\sigma}}{\tilde{p}_t^u c_t} \quad \text{for } i = R, I, U. \quad (4.5a)
\]
\[
\frac{\partial \tilde{H}_t}{\partial q_{t^+}} = 0 \quad \Rightarrow \quad c_t^{1-\sigma} \frac{\phi_t^u}{\tilde{p}_t^u c_t} = \left( 1 + \nu \right)^{s+1} \frac{\tilde{w}_t^u}{\tilde{p}_t^u c_t} \lambda_{s+1}^R \frac{\tilde{p}_t}{\tilde{p}_t^u c_t} \quad (4.5b)
\]
\[
\frac{\partial \tilde{H}_t}{\partial l_s} = 0 \quad \Rightarrow \quad \lambda_{s+1}^R \left( 1 + \nu \right)^{s+1} = \lambda_{s+1}^m \quad (4.5c)
\]
\[
\frac{1 + \nu}{1 + \rho} \lambda_{s+1}^r - \lambda_s^r = - \frac{\partial \tilde{H}_s}{\partial K_s^r} \Rightarrow \\
\frac{\lambda_s^r}{\lambda_{s+1}^r} = \frac{(1 - \delta)(1 + \nu)}{1 + \rho} + \frac{\tilde{u}_s^t}{\tilde{P}_s} \left( \frac{1 + \nu}{1 + \rho} \right) \beta_{RL} \left( \frac{\tilde{Y}_s}{\tilde{K}_s^r} \right) \left( \frac{1}{\beta_{RL}} \right) \beta_{RL} \lambda_{s+1}^r (1 + \nu)^{s+1} \\
\frac{1 + \nu}{1 + \rho} \lambda_{s+1}^m - \lambda_s^m = \frac{\partial \tilde{H}_s}{\partial m_s} \Rightarrow \frac{\lambda_s^m}{\lambda_{s+1}^m} = \frac{1}{1 + \rho} + \frac{\lambda_{s+1}^r (1 + \nu)^{s+1}}{\lambda_{s+1}^m} \left( \frac{\tilde{r}_s}{1 + \rho} \right) \\
\text{(4.5d)} \\
\text{(4.5e)}
\]

In addition, the transversality conditions are

\[
\lim_{s \to \infty} \left[ \frac{1}{(1 + \rho)^{t-s}} \lambda_s^r K_s^r \right] = 0 \text{ and } \lim_{s \to \infty} \left[ \frac{(1 + \nu)}{(1 + \rho)} \lambda_s^m m_s \right] = 0. \\
\text{(4.5f)}
\]

The derivation of the optimal consumption growth rate is similar to the analytic model case. From Eqs. (4.5a) - (4.5d) with (4.4) we derive

\[
c_s = c_s \frac{\varphi_1}{\tilde{P}_s} \left( \frac{\tilde{p}_s}{\varphi_0} \right)^{\phi_0} \prod_{i=R,I,U} \left( \frac{\tilde{p}_i}{\varphi_i} \right)^{\phi_i} \text{ for } i = R, I, U, \\
\text{(4.6a)} \\
\tilde{q}_s^m = c_s \frac{\varphi_0}{\tilde{p}_s} \left( \frac{\tilde{p}_s}{\varphi_0} \right)^{\phi_0} \prod_{i=R,I,U} \left( \frac{\tilde{p}_i}{\varphi_i} \right)^{\phi_i}. \\
\text{(4.6b)}
\]

By substituting (4.6a) into (4.5a) we have

\[
\frac{(1 + \nu)^{s+1}}{1 + \rho} \lambda_{s+1}^r = \tilde{p}_s^{uc} \left( \frac{\varphi_0}{\tilde{p}_s} \right)^{\phi_0} \prod_{i=R,I,U} \left( \frac{\varphi_i}{\tilde{p}_i} \right)^{\phi_i} \\
\text{.}
\]

Eq. (4.5c) and Eq. (4.5e) are combined into \( \lambda_s^r = \frac{(1 + \nu)(1 + \tilde{r})}{1 + \rho} \lambda_{s+1}^r \). From these two equations with some algebraic manipulation, we obtain the following equation for optimal per capita consumption.\(^{10}\)

\[
\hat{c}_{t+T} = \hat{c}_t \left[ \frac{\tilde{p}_{1+T}}{\tilde{P}_{1+T}} \left( \frac{\tilde{p}_t}{\tilde{p}_{1+T}} \right)^{\phi_0} \prod_{i=R,I,U} \left( \frac{\tilde{p}_i}{\tilde{p}_{1+T}} \right)^{\phi_i} \prod_{j=0}^{T-1} \left( \frac{1 + \tilde{r}_{1+T+j}}{1 + \rho} \right) \right]^{-\frac{1}{\sigma}} \\
\text{(4.7)}
\]

\(^{10}\) The notation ' ^ ' denotes the optimal solution.
Similarly, from Eq. (4.5d) and \( \lambda^R_s = \frac{(1 + \nu)(1 + \bar{r}_s)}{1 + \rho} \lambda^R_{s+1} \) we obtain

\[
\hat{K}^R_s = \left( \frac{Y_R}{\tau_R} \right)^{1/\rho} \left\{ \frac{\beta_{RK}w^I_t \bar{w}_s}{\beta_{RL}P_{lc}^I (\bar{r}_s + \delta)} \right\} \left( \frac{\beta_{RK}}{\beta_{RL}} \right)^{1/\rho} \hat{s}_t.
\]

(4.8)

This equation shows that the optimal rainfed capital stock at time \( s \) is solely determined by expectations \( \bar{w}_t^I \) and \( \bar{r}_s \). This condition can be satisfied by choosing the appropriate level of investment in \( K^R \) for any future time \( s \geq t + 1 \), given the value of \( K^R_t \).

Here the no-perfect-foresight expectation assumption is applied. It is assumed that households employ the current value of each exogenous variable in year \( t \) as their expectation for its future values for all \( s \geq t + 1 \), e.g., \( \bar{p}_s = p_t, \quad \bar{w}_t^I = w_t^I, \quad \bar{r}_s = r_t, \quad \bar{s}_s = \theta_t \), and so on, except for a random variable \( \omega_t \) of which the expected value is some constant \( \bar{\omega} \). This assumption makes \( \hat{K}^R_s \) constant for all \( s \geq t + 1 \). Therefore the optimal level of household investment in \( K^R \) is given as

\[
\hat{I}^R_s = \hat{K}^R_{s+1} - (1 - \delta) \hat{K}^R_s = \left\{ \frac{Y_R}{\tau_R} \right\}^{1/\rho} \left\{ \frac{\beta_{RK}w^I_t \bar{w}_s}{\beta_{RL}P_{lc}^I (\bar{r}_s + \delta)} \right\} \left( \frac{\beta_{RK}}{\beta_{RL}} \right)^{1/\rho} \hat{s}_t - (1 - \delta) K^R_s \quad \text{for } s = t, \quad \text{and}
\]

\[
\delta \left( \frac{Y_R}{\tau_R} \right)^{1/\rho} \left\{ \frac{\beta_{RK}w^I_t \bar{w}_s}{\beta_{RL}P_{lc}^I (\bar{r}_s + \delta)} \right\} \left( \frac{\beta_{RK}}{\beta_{RL}} \right)^{1/\rho} \hat{s}_t \quad \text{for } s \geq t + 1.
\]

Moreover, this assumption makes \( I^R \) and \( \theta^R \) constant and equal to \( I^R \) and \( \theta^R \). Given the above results, Eq. (4.3) and \( m_{s+1} - m_t = \frac{I_s - \nu m_t}{1 + \nu} \) produce the following equation of motion for the household assets.

\[
\hat{m}_{s+1} = \frac{1 + r^*}{1 + \nu} \hat{m}_t - B_1 \hat{c}_i + B_2 + \left( \frac{1}{1 + \nu} \right)^{t+1} B_3 - \left( \frac{1 + \nu}{1 + \nu} \right)^{t+1} w^I_t \bar{w}_s \left( \frac{Y_R}{\tau_R} \right)^{1/\rho} \hat{s}_t - (1 - \delta) K^R_s \quad \text{for } s \geq t + 1,
\]

and

\[
\left( \frac{1}{1 + \nu} \right)^{t+1} \left[ \frac{Y_R}{\tau_R} \right]^{1/\rho} \left\{ \frac{\beta_{RK}w^I_t \bar{w}_s}{\beta_{RL}P_{lc}^I (\bar{r}_s + \delta)} \right\} \left( \frac{\beta_{RK}}{\beta_{RL}} \right)^{1/\rho} \hat{s}_t - (1 - \delta) K^R_s
\]

, and
Chapter 4 Applied Model

The derivation of the optimal consumption from this equation is the same as in the case of the analytic model in the previous chapter. The result is summarised in the following proposition.

**Proposition 4.1: Optimal consumption level**

If \( r_t > \nu \) is satisfied, the optimal level of per capita consumption at time \( t \) is given as follows.

\[
\hat{c}_t = \left( \frac{\varphi_0}{p_t} \right)^{\varphi_t} \prod_{i=R,L,U} \left( \frac{\varphi_i}{p_t} \right)^{\eta_i} \left\{ 1 + r_t - \left( 1 + \nu \right) \frac{1 + r_t}{1 + \rho} \right\} \left[ p_t U_t m_t + \frac{1}{r_t - \nu} \left\{ \bar{w}_t^{U_t^{x_t} U_t^{x_t}} + w_t^{s} (1 - l_t^{U_t^{x_t} - s} (1 - \theta_t) \right\} + \left( \frac{1}{1 + \nu} \right) \left( \frac{\bar{w}_t^{R_t^{x_t} R_t^{x_t}} - \frac{1}{1 + \nu} \left( \frac{1}{r_t (1 + r_t)} \right) \left( \frac{1}{\beta_R \frac{\rho_{U_t^{x_t}}} {\beta_R} \left( \frac{1}{\beta_R \frac{\rho_{U_t^{x_t}}} {w_t^{U_t^{x_t}} (1 + r_t + \delta)} \right)} \right) \right) - \left( \frac{1}{1 + \nu} \right) \left( 1 + r_t \right) \left( \frac{1}{K_t^R} \right) - \left( 1 - \delta \right) p_t U_t K_t^R \right) \right. \]

Otherwise, \( \hat{c}_t \) cannot take positive and finite values.

**Proof:** See Appendix A10.

An important feature of this optimal consumption decision is its independence from the realised production risk factor \( \omega_t \). The optimal consumption decision is
deterministic, and the gap between planned and realised income due to the
difference between $\bar{\omega}$ and $\omega_t$ is assumed to be absorbed by investment $I_t$.

To compare Proposition 4.1 with Proposition 3.12, its counterpart in the
discrete-time analytic model, the following transformation is useful.

\[
\hat{c}_t = \left( \frac{\varphi_0}{p_t} \right)^{\rho_0} \prod_{l,R,L,U} \left( \frac{p_{lt}}{p_{lt}} \right)^{\rho_l} \left\{ 1 + r_l - (1 + \nu) \left( \frac{1 + \rho_l}{1 + \rho} \right)^{\nu} \right\} \\
\times \left\{ p_{lt}^{U^8} m_t + \frac{1}{r_l - \nu} \tilde{W}_t^{total} - \frac{1}{1 + r_l} \left( \frac{1}{1 + \nu} \right)^t p_{lt}^{U^8} I_t^R \right\},
\]

where

\[
\tilde{W}_t^{total} = W_t^{total} - \nu \left( \frac{1}{1 + \nu} \right)^t \bar{\omega} \bar{p}_t^R \bar{Y}_R, \text{ in which}
\]

\[
W_t^{total} = \left( 1 - \bar{U}_t^* - \bar{T}_S \right) \left\{ 1 - (1 - \theta_t) \bar{\nu} \right\} \left( \frac{1}{1 + \nu} \right)^t \left( \frac{\bar{Y}_R}{\tau_R} \right)^{\rho_a} \left( \frac{1}{K_R^{t_R}} \right)^{\rho_{a_k}} + \bar{W}_t^{U^8} \bar{U}_t^* + \bar{W}_t^{T_S} + \left( \frac{1}{1 + \nu} \right)^t \bar{\omega} \bar{p}_t^R \bar{Y}_R + p_t \bar{a}_m \left[ 1 - \bar{U}_t^* - \bar{T}_S \right] (1 - \theta_t), \text{ and }
\]

\[
\bar{I}_t^R = \bar{I}_t^R \left( \frac{\beta_B \bar{L}_t^R + \beta_R \delta}{\beta_R \bar{L}_t^R} \right) \left( \frac{1}{\tau_R} \right)^{\rho_a} \left\{ \frac{\beta_B \bar{K}_t^R \bar{W}_t^{U^8}}{\beta_R \bar{L}_t^R \bar{U}_t^8 (r_t + \delta)} \right\}^{\rho_{a_k}}.
\]

In the above expression, $\tilde{W}_t^{total}$ can be interpreted as the present value of expected
total per capita labour income, $W_t^{total}$, including rainfed agricultural profit and
income saved by fetching water instead of purchasing publicly supplied water. The
former can be regarded as income from rainfed labour and the latter can be regarded
as labour income from fetching water. Similarly, $p_{lt}^{U^8} \bar{I}_t^R$ can be interpreted as the
present value of rainfed capital investment per household. This transformation
makes it clear that the analytic model and the applied model share essentially the
same structure.
4.3.2 The firm's problem

(1) The irrigated agricultural sector

I assume that the production technology of the irrigated agricultural sector may be described by a Leontief function of intermediate goods, and a Cobb-Douglas function (with constant returns to scale) of factors of production. Note that a Leontief specification is compatible with the social accounting matrix which will serve as a database of the applied model. A Cobb-Douglas function, which is a CES function with substitution elasticities of unity, is adopted mainly due to its tractability. This specification may overestimate substitutability between water or land and other factors, but the benefit to employ more elaborate functional specification, a CES function with substitution elasticities of less than unity, does not seem to be enough to compensate its intractability. The employed production function is

\[
Y_t^I = \min \left[ F_t^I, \frac{s_t^{IR}}{a_{IR}}, \frac{s_t^{IU}}{a_{IU}} \right], \quad F_t^I = \tau_t \left( A_t^I \right)^{\beta_A} \left( K_t^I \right)^{\beta_K} \left( L_t^I \right)^{\beta_L} \left( Q_t^I \right)^{\beta_Q},
\]

where \( Y_t^I \) : nation wide aggregate product from this sector in year \( t \), \( s_t^j \) : input of intermediate goods produced by the sector \( j \), \( a_{ij} \) : input-output coefficient associated with \( s_t^j \) (\( s_t^j = a_{ij}Y_t^I \)), \( \tau_t \) : technological parameter in irrigated agriculture, \( A_t^I \) : aggregate irrigated land input in year \( t \), \( K_t^I \) : aggregate irrigation capital input in year \( t \), \( L_t^I \) : aggregate irrigation labour input in year \( t \), \( Q_t^I \) : aggregate irrigation water input in year \( t \), and \( \beta_j \) : the share of factor input \( j \) (\( j = A, K, L, Q \)) with \( \beta_j \in (0,1) \) for all \( j \) and \( \sum_j \beta_{ij} = 1 \).

There is a constraint concerning irrigation land capacity.

\[
A_t^I \leq \bar{A}_t = \bar{A}(G_t^I),
\]

where \( \bar{A}_t \) is the designated land to irrigation in year \( t \) determined by the public capital stock in the irrigation sector \( G_t^I \).
It means that the aggregate irrigation land use cannot exceed designated land to irrigated agriculture developed by the government. In this study it is assumed that the government sets the irrigated land charge $p_i^A$ such that this condition is satisfied.

Aggregate profits in the sector are defined as

$$
\Pi_i^j = p_i^w Y_i^j - p_i^A A_i^j - (r + \delta) p_i^w K_i^j - p_i^w Q_i^j - w_i^j L_i^j - p_i^w s_i^j - p_i^w s_i^U
$$

$$
= \bar{p}_i^j Y_i^j - p_i^A A_i^j - (r + \delta) p_i^w K_i^j - p_i^w Q_i^j - w_i^j L_i^j ,
$$

where $\bar{p}_i^j = p_i^w - a_{rt} P_i^R - a_{ju} P_i^U$, and $p_i^w$ is the irrigation water charge.

Aggregate profits are maximised by setting the partial derivatives of $\Pi_i^j$ with respect to $A_i^j$, $K_i^j$, $Q_i^j$ and $L_i^j$ at zero, taking all prices as exogenously given, i.e.

$$\tilde{p}_i^j \beta_{A_i^j} Y_i^j = p_i^A A_i^j , \tilde{p}_i^j \beta_{K_i^j} Y_i^j = (r_i + \delta) p_i^w K_i^j , \text{ etc.}$$

From the production function and the optimal conditions, we can rewrite the optimal aggregate profit function as a function of $K^j$ and the exogenous variables.

$$
\Pi_i^{\star} = \left[ \tau_i \tilde{p}_i^j \left( \frac{\beta_{K_i}^A}{p_i^A} \right)^{\beta_A} \left( \frac{\beta_{K_i}^R}{p_i^R} \right)^{\beta_R} \left( \frac{\beta_{K_i}^U}{p_i^U} \right)^{\beta_U} \left\{ \frac{p_i^w (r_i + \delta)}{\beta_{K_i}^w} \right\}^{1-\beta_{K_i}} - \frac{p_i^w (r_i + \delta)}{\beta_{K_i}^w} \right] K_i^j
$$

This expression is a short-run optimal profit function taking the level of capital stock $K^j$ and the interest rate $r$ as exogenously given. Whether aggregate profits are positive, zero or negative depends on the wage rate of this sector, that is

$$
\Pi_i^{\star} \begin{cases} > 0 & \text{if } w_i^j \leq \beta_{ul} \\ < 0 & \text{if } w_i^j \geq \beta_{ul} \end{cases} \left[ \tau_i \tilde{p}_i^j \left( \frac{\beta_{K_i}^A}{p_i^A} \right)^{\beta_A} \left( \frac{\beta_{K_i}^R}{p_i^R} \right)^{\beta_R} \left( \frac{\beta_{K_i}^U}{p_i^U} \right)^{\beta_U} \left\{ \frac{p_i^w (r_i + \delta)}{\beta_{K_i}^w} \right\}^{1-\beta_{K_i}} \right] . \quad (4.9)
$$

(2) The urban modern sector

The production technology of the urban modern sector is assumed to be described by a Leontief function of intermediate goods and a Cobb-Douglas function of factors of production with constant returns to scale.

$$
Y_i^U = \min \left[ \frac{I_i^U}{a_{UR}} , \frac{I_i^U}{a_{UI}} , \frac{s_i^U}{a_{UL}} , \frac{s_i^U}{a_{UL}} \right] , \quad F_i^U = \tau_U \left( K_i^U \right)^{\beta_K} \left( I_i^U \right)^{\beta_I} \left( I_i^U \right)^{\beta_I} \left( I_i^U \right)^{\beta_I} ,
$$
where \( Y^U \): nation wide aggregate product from this sector, \( s^U_j \): input of intermediate goods produced by the sector \( j \), \( \alpha^U_j \): input-output coefficient associated with \( s^U_j \) (\( s^U_j = \alpha^U_j Y^U \)), \( \tau^U \): technological parameter in urban modern sector, \( K^U \): aggregate urban modern capital, \( L^U \): aggregate unskilled labour input, \( L^S \): aggregate skilled labour input, and \( \beta^U_j \): the share of factor input \( j \) (\( j = K, L, S \)) with \( \beta^U_j \in (0,1) \) for all \( j \) and \( \sum_j \beta^U_j = 1 \).

The sector’s aggregate profits are

\[
\Pi^U_i = \Pi^U_i Y^U - \left( r_i + \delta \right) p^U_i K^U_i - \bar{w}^U_i L^U_i - \bar{w}^S_i L^S_i,
\]

where \( \Pi^U_i = \Pi^U_i - \alpha^U_i p^U_i - \alpha^U_i p^U_i \).

Aggregate profits are maximised by setting the partial derivatives of \( \Pi^U \) with respect to \( K^U, L^U \) and \( L^S \) at zero, taking \( r, \bar{w}^U \) and \( \bar{w}^S \) as exogenously given, i.e.

\[
\beta^U_{ik} \bar{p}^U_i Y^U_i = \left( r_i + \delta \right) p^U_i K^U_i, \quad \beta^U_{il} \bar{p}^U_i Y^U_i = \bar{w}^U_i L^U_i \quad \text{and} \quad \beta^U_{is} \bar{p}^U_i Y^U_i = \bar{w}^S_i L^S_i.
\]

From the production function and the optimal conditions, we can rewrite the optimal aggregate profit function as a function of \( L^U \) and the exogenous variables.

\[
\Pi^U_i^* = \left[ \frac{\beta^U_{ik} \beta^U_{il} \beta^U_{is}}{p^U_i (r_i + \delta)} \right]^{1/3} \frac{\bar{w}^U_i \bar{w}^S_i}{\beta^U_{ik}} - \delta.
\]

This expression is a short-run optimal profit function taking the labour input \( L^U \) and the wage rate \( \bar{w}^U \) as exogenously given. Whether aggregate profits are positive, zero or negative depends on the interest rate \( r \), that is

\[
\Pi^U_i^* > 0 \quad \text{if} \quad r_i < \frac{\beta^U_{ik} \beta^U_{il} \beta^U_{is}}{p^U_i (r_i + \delta)} \left[ \tau_i \bar{p}^U_i \left( \frac{\beta^U_{ik} \bar{w}^U_i}{p^U_i} \right)^{1/3} \left( \frac{\beta^U_{il} \bar{w}^S_i}{p^U_i} \right)^{1/3} \right]^{-1} - \delta.
\]

### 4.3.3 Market equilibrium

In the applied model, the markets for irrigation labour, urban skilled labour and private capital (excluding rainfed capital) are closed and perfectly competitive. The
equilibrium prices in these markets, $r^*$, $w^*$, and $w^S*$, are determined by three market clearance conditions.

First, competition drives optimal profits of both production sectors towards zero, which results in the following equilibrium price relationships from Eqs. (4.9) and (4.10).

\[
    r_t^* = \frac{\beta_{UK}}{p_t^{Uc}} \left\{ \tau_t \bar{p}^{Uc} \left( \frac{\beta_{UL}}{w_t^*} \right) \frac{\rho_a}{\rho_x} \left( \frac{1}{\beta_{UL}} \right) \right\} - \delta = r^*(w_t^*), \quad \text{and} \quad (4.11)
\]

\[
    w_t^* = \beta_{UL} \left[ \tau_t \bar{p}_t^{Uc} \left( \frac{\beta_{UL}}{p_t^{Uc}} \right) \frac{\rho_a}{\rho_x} \left( \frac{1}{\beta_{UL}} \right) \right] = w^*(w_t^*). \quad (4.12)
\]

The remaining equilibrium price $w_t^S*$ is determined by the following market clearance conditions along with the equilibrium labour allocation to urban unskilled labour $l_t^U$.

(i) **Capital market clearance**

Because of the wage rigidity of urban unskilled labour and the existence of positive urban unemployment the urban sector has discretion to determine the optimal unskilled labour input $L_t^U$. Given $L_t^U$ the optimal urban capital demand is expressed from the optimality condition as follows.\(^{11}\)

\[
    K_t^{UD} = \frac{\beta_{UK} \bar{w}_t^{Uc}}{\beta_{UL} p_t^{Uc} \left( r_t^* + \delta \right)} L_t^U = \frac{1}{\tau_t \bar{p}_t^{Uc} \left( \frac{\beta_{UL}}{\beta_{UL}} \right)} \left( \frac{w_t^U}{w_t^S} \right)^{\rho_a} \left( \frac{1}{\beta_{UL}} \right) \left( \frac{1}{\beta_{UL}} \right) L_t^U
\]

The supply of private capital is historically given by the household assets holding $m_t$ at time $t$. Hence the capital market clearance requires $K_t^{UD} + K_t^{ID} = N (1 + \nu)^t m_t$, and the equilibrium capital stock in the irrigated agricultural sector is given as

\(^{11}\) The additional superscript 'D' means the optimal demand and 'S' means the optimal supply in this section.
\[ K_i^* = N \left( 1 + \nu \right) \frac{1}{m_i - \frac{1}{1 + v} \frac{\bar{w}_i^U}{\beta_{UL}} \left( \frac{w_i^U}{\beta_{US}} \right)^{\frac{1}{\beta_{ps}}} L_i^*} \]

Recall the relationship \( L_i^U = N \left( 1 + \nu \right) \theta_i^U \) and the fact that \( \theta_i^U \) is a function of \( w_i^U \).

When \( L_i^U \) is given, the labour allocation to urban unskilled labour is equilibrated such that \( L_i^U = N \left( 1 + \nu \right) \theta_i^U \). With this relationship the equilibrium level of irrigation capital stock is given as

\[ K_i^* = N \left( 1 + \nu \right) \frac{1}{m_i - \frac{1}{1 + v} \frac{\bar{w}_i^U}{\beta_{UL}} \left( \frac{w_i^U}{\beta_{US}} \right)^{\frac{1}{\beta_{ps}}} \theta^* \left( w_i^U \right) L_i^*} \]

(ii) Irrigation labour market clearance

From the optimal condition, demand for irrigation labour is given as

\[ L_i^{DL} = \frac{\beta_{IL} p_i^{UL} \left( r_i^* + \delta \right)}{\beta_{IK} w_i^U} K_i^* \], while its supply is

\[ L_i^{DS} = N \left( 1 + \nu \right) \left( 1 - 1 - L_i^U - \bar{I} \right) \left( 1 - (1 - \theta) \bar{z} \right) - N \left( \frac{\bar{Y}_R}{\tau_R} \right) \frac{1}{\bar{K}_i} \frac{\frac{1}{\beta_{ps}}}{\beta_{ps}} \]

The market clearance condition \( L_i^{DL} = L_i^{DS} \) requires that the equilibrium labour allocation to urban unskilled labour must satisfy

\[ L_i^* = \frac{\left\{1 - (1 - \theta) \bar{z}\right\} \left(1 - \bar{I} \right) - \beta_{IL} p_i^{UL} \left( r_i^* \left( w_i^U \right) + \delta \right) \frac{1}{m_i - \frac{1}{1 + v} \frac{\bar{w}_i^U}{\beta_{UL}} \left( \frac{w_i^U}{\beta_{US}} \right)^{\frac{1}{\beta_{ps}}} \theta^* \left( w_i^U \right) \frac{1}{\bar{K}_i} \frac{1}{\beta_{ps}}}{\left\{1 - (1 - \theta) \bar{z}\right\} - \beta_{IL} p_i^{UL} \left( r_i^* \left( w_i^U \right) + \delta \right) \left[ \frac{1}{1 + v} \frac{\bar{w}_i^U}{\beta_{UL}} \left( \frac{w_i^U}{\beta_{US}} \right)^{\frac{1}{\beta_{ps}}} \theta^* \left( w_i^U \right) \right]} \]

\[ = f_i \left( w_i^U \right) \]
(iii) Urban skilled labour market clearance

The supply of urban skilled labour is given as \( L^S = N (1 + \nu)^T \), while the optimal demand is \( L^D = \frac{\beta^w \bar{w}^U \hat{L}^*}{\beta^w \bar{w}^U} N (1 + \nu)^T \theta^{E^*} \hat{L}^* \). The market clearance condition \( L^D = L^S \) requires \( \hat{L}^* = \frac{\beta^w \bar{w}^U \hat{L}^*}{\beta^w \bar{w}^U} \theta^{E^*} (w^S) = f_2 \left( w^S \right) \).

Finally, from two expressions of \( \hat{L}^* \), i.e. \( f_1 \) and \( f_2 \), the equilibrium wage rate of urban skilled labour \( w^S \) is obtained as a solution of an implicit function

\[
f_3 \left( w^S; m_t, K^R, \theta_t, \bar{w}^U, p^r, p^c, p^o, p^p, p^l, \right) = 0,
\]

where

\[
f_3 \left( w^S \right) = f_1 \left( w^S \right) - f_2 \left( w^S \right) = \frac{\Phi_1 - \Phi_2 \left( 1/w^S \right) \frac{\beta^w \beta^c}{\beta^c \beta^x}}{\Phi_1 - \Phi_2 \left( 1/w^S \right) \frac{\beta^w \beta^c}{\beta^c \beta^x}} - \frac{\Phi_3 \left( w^S \right)}{\Phi_1 + \Phi_2 \left( w^S \right) \frac{\beta^w \beta^c}{\beta^c \beta^x}},
\]

in which

\[
\Phi_1 = \left\{ 1 - \left( 1 - \theta_t \right) \right\} \left\{ 1 - T^S \right\} - \left( \frac{1}{1 + \nu} \right)^T \left( \frac{\bar{w}^U}{\tau R} \right) \frac{1}{\tau R} \left( \frac{1}{K^R} \right),
\]

\[
\Phi_2 = \left( \frac{1}{1 + \nu} \right)^T \frac{\beta^w m_t}{\beta^w} \left\{ \tau U \bar{p}^U \left( \frac{\beta^w}{\bar{w}^U} \right) \left( \beta^w \right)^{\beta^x} \right\}
\]

\[
\times \left( \frac{1}{\tau U \bar{p}^U} \left( \frac{p^r}{\beta^w} \right) \left( \frac{p^c}{\beta^w} \right) \left( \beta^w \right)^{\beta^x} \right),
\]

\[
\Phi_3 = \left\{ 1 - \left( 1 - \theta_t \right) \right\} \left\{ 1 - \beta^w \beta^c / \beta^c \beta^x \left( \theta_t \right)^{\beta^c} \right\},
\]

\[
\Phi_4 = \frac{\beta^w}{\beta^w \beta^c} \left( \theta_t \right)^{\beta^c} p_t \bar{q}_t \left( \frac{1}{\theta_t} - 1 \right) \left\{ \tau U \bar{p}^U \left( \frac{\beta^w}{\bar{w}^U} \right) \left( \beta^w \right)^{\beta^x} \right\}
\]

\[
\times \left( \frac{1}{\tau U \bar{p}^U} \left( \frac{p^r}{\beta^w} \right) \left( \frac{p^c}{\beta^w} \right) \left( \beta^w \right)^{\beta^x} \right),
\]

\[
\Phi_5 = \frac{\beta^w \bar{w}^U}{\beta^w \bar{w}^U}, \quad \Phi_6 = \frac{\left( \theta_t \right)^{\beta^c}}{\bar{w}^U} p_t \bar{q}_t \left( \frac{1}{\theta_t} - 1 \right),
\]

and
4.3.4 First-stage solution

The first-stage solution of the applied model is optimal per capita consumption as a function of the state variables \( (m, KR) \) and the exogenous variables, and the equations of motion for the state variables. It is summarised below.

From the proposition 4.1 and the equilibrium values the optimal expected per capita consumption is determined as follows.

\[
\mathcal{C}_t = \left( \frac{\varphi_0}{p_t} \right)^\nu \prod_{i=1}^{\nu} \left( \frac{\varphi_i}{p_{i_t}} \right)^{\nu_i} \left\{ 1 + r_t^* \left( 1 + \nu \right) \left( \frac{1 + r_t^*}{1 + \rho} \right) \right\} \left[ \frac{p_t^{\nu_t} m_t + 1}{r_t^* - \nu} \left( \frac{\beta_{\text{E}}}{\beta_{\text{K}}} \right) \right] \\
+ w_t^{*=*}{T^*} + w_t^{*} \left( 1 - l_t^{*} \right){T^*} \left( 1 - (1 - \theta_t) \right) + p_t \bar{\alpha}_m \left( 1 - l_t^{*} \right){T^*} \left( 1 - \theta_t \right) \right\} \\
+ \left( \frac{1}{1 + \nu} \right) \left( \frac{1}{1 + \nu} \right)^t \left( \frac{\beta_{\text{R}}}{\beta_{\text{K}}} \right) \left( \frac{w_t^{*} r_t^*}{\tau_R} \right) \left( \frac{\beta_{\text{E}}}{\beta_{\text{K}}} \right) \left( \frac{p_t^{\nu_t} \left( r_t^* + \delta \right)}{\beta_{\text{K}}} \right) \\
- \left( \frac{1}{1 + \nu} \right) \left( \frac{1}{1 + \nu} \right) \left( \frac{w_t^{*} r_t^*}{\tau_R} \right) \left( \frac{1}{\beta_{\text{K}}} \right) \left( \frac{1}{K_t^*} \right) \left( \frac{1 - \delta}{p_t^{\nu_t} K_t^*} \right), 
\]

in which \( r_t^* , w_t^{*} , \theta_t^{*} \) and \( l_t^{*} \) are determined by \( w_t^{*} \), and \( w_t^{*} \) is determined by an implicit function \( f_3 (w_t^{*}) = 0 \).

Investment is residually determined by the gap between the planned income and the realised income, which is given as

\[
I_t = r_t^* m_t - \mathcal{C}_t \left( \frac{p_t}{\varphi_0} \right)^\nu \prod_{i=1}^{\nu} \left( \frac{p_{i_t}}{\varphi_i} \right)^{\nu_i} + \left[ \frac{\beta_{\text{E}}}{\beta_{\text{K}}} \right] \left( \frac{w_t^{*} r_t^*}{\tau_R} \right) \left( \frac{w_t^{*} r_t^*}{\tau_R} \right) \left( \frac{1}{K_t^*} \right) \left( \frac{1 - \delta}{p_t^{\nu_t} K_t^*} \right) \\
+ w_t^{*}{T^*} + w_t^{*} \left( 1 - l_t^{*} \right){T^*} \left( 1 - (1 - \theta_t) \right) + p_t \bar{\alpha}_m \left( 1 - l_t^{*} \right){T^*} \left( 1 - \theta_t \right) \right\]
The equations of motion for \( m \) and \( K_R^e \) are

\[
m_{t+1} = \frac{1}{1+\nu} (m_t + I_t), \text{ and}
\]

\[
\hat{K}_{it+1} = \left( \frac{Y_R}{T_R} \right) \frac{1}{\rho_u} \left( \frac{1}{K_R^e} \right) - (1-\delta)K_R^e.
\]

Given the initial values \( m_0 \) and \( K_0^e \) as well as the exogenous variables, the above system uniquely determines the optimal trajectories of \( c, m \) and \( K^e \). The first-stage solutions of the remaining endogenous variables necessary for the second stage optimisation are as follows.

\[
\hat{\epsilon}_t^j = \hat{\epsilon}_t^j \frac{\varphi_j}{\rho_u} \left( \frac{P_A}{\varphi_j} \right) \prod_{i=R,I,U} \left( \frac{P_i^e}{\varphi_i} \right)^n, \text{ for } j = R, I, U,
\]

\[
\hat{q}_t^H = \frac{\hat{q}_t^H - \bar{q}_0 \left( 1 - l_{t}^U - T_s^e \right) (1-\theta_t)}{\theta_t + (l_{t}^U + T_s^e) (1-\theta_t)}, \text{ in which } \hat{q}_t^H = \hat{\epsilon}_t^j \left( \frac{\varphi_j}{P_i} \right)^{1-\rho_u} \prod_{i=R,I,U} \left( \frac{P_i^e}{\varphi_i} \right)^n,
\]

\[
\hat{y}_t^I = \frac{w_t^I \hat{L}_t^I}{\beta_{IL} P_t^I}, \quad \hat{y}_t^I = \frac{w_t^I \hat{L}_t^I}{\beta_{IL} P_t^I}, \quad \hat{K}_t^I = \frac{\beta_{IL} w_t^I \hat{L}_t^I}{\beta_{IL} P_t^I (r_t^I + \delta)}, \text{ and } \hat{\omega}_t^I = \frac{\beta_{IL} w_t^I \hat{L}_t^I}{\beta_{IL} P_t^I},
\]

in which \( \hat{L}_t^I = N (1+\nu)^\prime \left( 1 - l_{t}^U - T_s^e \right) (1-(1-\theta_t)z) - N \left( \frac{Y_R}{T_R} \right) \frac{1}{\rho_u} \left( \frac{1}{K_R^e} \right) \),

\[
\hat{y}_t^U = \frac{w_t^U \hat{L}_t^U}{\beta_{IU} P_t^U}, \quad \hat{K}_t^U = \frac{\beta_{IU} w_t^U \hat{L}_t^U}{\beta_{IU} P_t^U (r_t^I + \delta)}, \text{ and } \hat{L}_t^U = N (1+\nu)^\prime T_s^e.
\]

Considering the structural similarity between the analytic and the applied models as seen in Proposition 4.1, it is expected that Proposition 3.3, which tells us that the trajectories of the first-stage solution are globally stable under the constant prices of
public goods without supply side constraints, is also valid for the applied model. Properties of the first-stage solution without supply side constraints are investigated using numerical simulation in Chapter 6.

4.4 Second-stage optimisation

4.4.1 Outline

In the applied model it is assumed that the budget neutral government produces two types of water, i.e. untreated water \( R \) which is used as either irrigation water or raw water to the public water supply, and treated water \( Q \). Further, the government develops designated irrigation land \( A \).

As in the analytic model, each production process is represented by a simple production function. The aggregate untreated water production function is \( R_i = F^R_i (G^R_i) \) where \( G^R \) is stock of public capital for this production. The treated water production function is \( Q_t = F^D_t (G_t) \) where \( G \) is stock of public capital associated with water treatment and distribution. The level of \( G \) determines not only the treated water supply capacity \( F^D \) but also the coverage of public water supply in the rural area \( \theta \). Similarly, the aggregate area of designated land for irrigation is represented by \( A_i = F^A_i (G^I_i) \). As in the previous chapter these production functions are constrained by a requirement that production and consumption of goods do not endanger sustainability. All production processes involve large-scale facilities and have characteristics of Weitzman’s (1970) \( \beta \) sector as discussed in the previous chapter.

The government collects irrigation land charge \( p^A \) per unit area and two types of volumetric water charges, i.e. \( p \) for public water supply and \( p^w \) for irrigation water. All the collected public charges are invested in either \( G \), \( G^R \) or \( G^I \).

4.4.2 The government problem

The government revenue from the charges at year \( t \) is expressed as
The government budget constraint at year $t$ is $M_t^G = p_t^{\mathrm{Uc}} \left( I_t^G + I_t^{GR} + I_t^{GI} \right)$, where $I_t^G$, $I_t^{GR}$ and $I_t^{GI}$ are public investment in $G$, $G^R$ and $G^I$, respectively. Let $\theta^G$ and $\theta^R$ denote fractions of the government budget invested in $G$ and $G^R$, i.e. $p_t^{\mathrm{Uc}} I_t^G = \theta^G M_t^G$ and $p_t^{\mathrm{Uc}} I_t^{GR} = \theta^R M_t^G$. The public capital accumulation is described by the following three equations of motion.

$$G_{t+1} - G_t = \theta^G M_t^G / p_t^{\mathrm{Uc}} - \delta G_t,$$

$$G_{t+1}^R - G_t^R = \theta^R M_t^G / p_t^{\mathrm{Uc}} - \delta G_t^R,$$

and

$$G_{t+1}^I - G_t^I = \left( 1 - \theta^G - \theta^R \right) M_t^G / p_t^{\mathrm{Uc}} - \delta G_t^I.$$

The sustainability conditions are

$$\dot{q}_t^D (1 + \nu) \left\{ \theta_t + \left( I_t^* + \bar{T}_S \right) \left( 1 - \theta_t \right) \right\} \leq 0 = F^D \left( G_t \right),$$

$$\dot{q}_t^H (1 + \nu) \left\{ \theta_t + \left( I_t^* + \bar{T}_S \right) \left( 1 - \theta_t \right) \right\} + \dot{\lambda}_t^I \leq 0 = F^R \left( G_t^I \right),$$

and

$$\dot{\lambda}_t^I \leq A_t = F^A \left( G_t^I \right).^{12}$$

Note that the second sustainability condition captures a tradeoff between the domestic and the irrigation water demands given raw water production capacity. The objective of the government is to maximise social welfare of the current generation without violating sustainability conditions by choosing the values of policy instruments for the planning period $T$. In this chapter the policy instruments are confined to the rates of three public charges and the public investment allocation among $G$, $G^R$ and $G^I$. Other candidates such as the urban minimum wage $\overline{w}_t^U$, tariffs or taxes are exogenously fixed.\(^{13}\)

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\(^{12}\) As in the previous chapter these are sustainability conditions rather than mere supply-demand balance. See Section 5.6 in Chapter 5.

\(^{13}\) These policy instruments will be discussed in Chapter 6.
Social welfare is defined as the net present value of household utility of the representative household based on the identical households assumption.\textsuperscript{14}

It must be noted that this thesis clearly distinguishes between the decision-making process of the government and that of private agents. The government plans policies for the entire planning period at the planning moment (t = 0) based on expectation of exogenous variables. Households make their consumption plan for the planning moment only and their lack of perfect foresight means that their consumption plan is determined by currently realised information only. This is an important difference. In the government problem, $\hat{c}_t$ for any $t \geq 1$ contains price expectation $\bar{p}_t^{ie}$ and $\bar{p}_t^{ip}$ from the perspective of the planning moment $t = 0$. My assumption is that $\bar{p}_t^{ie} = \bar{p}_0^{ie}$ and $\bar{p}_t^{ip} = \bar{p}_0^{ip}$, which is exactly the same as the private expectation on surface, but its justification is totally different. For the household problem this 'myopic' expectation is compensated by the monitoring-feedback decision-making process with which the decision is made based on updated current information. This justification is not valid for the government decision for which a commitment of implementation is required at $t = 0$.\textsuperscript{15}

The justification for the government comes from the fact that commodity prices are partially controlled by the government through its foreign exchange policy as well as import taxation. This control is partial because there exist other exogenous external factors affecting the real exchange rates. Hence, the government designs policies with the expectation of constant commodity prices since this is one of the government policy targets, but there almost certainly exist gaps between planned and realised commodity prices. This expectation error will differentiate both planned and realised revenue and the planned and the actual budget required to implement the investment plan. The policy implementation issues arising from the

\textsuperscript{14} Recall the assumption that the household is the decision-making unit despite the heterogeneity among its members. See Section 4.2 of this chapter.

\textsuperscript{15} Recall that in practice infrastructure development is implemented based on large-scale contracts with private constructors, which require the government's commitment.
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expectation error will be discussed later in Chapter 6. At this moment I focus on the optimal policy planning.

With the assumption of identical households the government problem can be formally expressed as

$$\max_{\{p_t, \rho_t, \sigma_t, \theta_t, \phi_t\}} \sum_{t=0}^{T} \left( \frac{1}{1+p_t} \right) \frac{\hat{e}^{1-\sigma}_t}{1-\sigma},$$

subject to

$$G_{i+1} - G_i = \theta_i M_i^G / p_0^{Uc} - \delta G_i,$$

$$G_{i+1}^R - G_i^R = \theta_i^R M_i^G / p_0^{Uc} - \delta G_i^R,$$

$$G_{i+1}^I - G_i^I = (1 - \theta_i^G - \theta_i^R) M_i^G / p_0^{Uc} - \delta G_i^I,$$

$$m_{i+1} - m_i = \frac{r^*_i - \nu}{1+\nu} m_i - \frac{\hat{c}_t}{(1+\nu)p_0^{Uc}} \left( \sum_{i=R,I,U} \left( \frac{p_0^{Uc}}{q_0} \right) \prod_{i=R,I,U} \left( \frac{p_0^{Uc}}{q_0} \right)^{\nu_i} + \frac{1}{(1+\nu)p_0^{Uc}} \right) \left[ \bar{w}_t^{U} \theta_i^{*U} + w_t^{*T} \right]$$

$$+ w_t^{U} (1 - l_t^{U*} - T_t^S) (1 - (1 - \theta_i) \bar{e}) + p_i \bar{q}_n \left( 1 - l_t^{U*} - T_t^S \right) (1 - \theta_i) \bar{e} - \left( \frac{1}{1+\nu} \right)^{i+1} \alpha_{0i} \bar{P}_0^{R \bar{W}} \bar{P}_0^{Uc}$$

$$- \left( \frac{1}{1+\nu} \right)^{i+1} \left( \bar{Y}_R \right) \left( \frac{\beta_{R,R} w_t^{U*}}{\beta_{R,L} p_0^{Uc} (r_i^* + \delta)} \right) + \frac{w_t^{U*}}{p_0^{Uc} \left( \frac{1}{K_t^R} \right)} \left( \frac{1}{K_t^R} \right) - (1 - \delta) K_t^R,$$

$$\hat{K}_{i+1}^R = \left( \frac{\bar{Y}_R}{\bar{Y}_R} \right) \left( \frac{\beta_{R,R} w_t^{U*}}{\beta_{R,L} (r_i^* + \delta)} \right) \left( \frac{\beta_{R,L}}{\beta_{R,R}} \right),$$

$$\hat{q}_t^H (1 + \nu) \left( \theta_i + (l_t^{U*} + T_t^S) (1 - \theta_i) \right) \leq F^D (G_i),$$

$$\hat{Q}_t^I + \hat{q}_t^H (1 + \nu) \left( \theta_i + (l_t^{U*} + T_t^S) (1 - \theta_i) \right) \leq F^R (G_i^R),$$

and

$$\hat{A}_t \leq \bar{A}_i = F^A (G_i^I),$$

together with the initial and the terminal conditions of each state variable.

Due to the existence of stochastic elements, the solution differs between open-loop optimisation without feedbacks of the policy outcomes, and closed-loop optimisation adjusted for the realised situation at each time. The commitment to implement the policies at $t = 0$ implies the open-loop solution. A practical
implication of this is that the numerical solution in this thesis is obtained by directly maximising social welfare, and not by stochastic dynamic programming.

4.4.3 Solution of the second-stage optimisation

The government problem formalised in the previous subsection is structurally the same as the one in the analytic model. Its high degree of complexity, however, discourages application of the same technique, the maximum principle, to this problem. Furthermore, since the analytic model reveals non-existence of interior solution of the government problem, as stated in Proposition 3.8, this may also be expected in the applied model.

The employed solution algorithm to the second-stage optimisation of the applied model heavily relies on the market-clearing trajectories as stated in Corollary 3.1. The sustainability conditions are satisfied as strict equality constraints, and optimal public charges and investment can, at least conceptually, be determined as follows.

At the starting period \( t = 0 \) each of three types of public charges \((p_0, p_0^w, p_0^d)\) given initial stock levels of state variables \((m_0, K_0, G_0, G_R, G_I)\) is determined such that three 'markets' of untreated water, treated water and irrigation land clear simultaneously in each period. Note that this price determination process in the period 0, which is a static problem, is not affected by the choice of allocation share of public investment \( \theta_0^G \) and \( \theta_0^R \) which determines the levels of public capital stock in the period 1.

To ease explanation let us introduce the vector notation

\[
X_t = (m_t, K_t, G_t, G_R^t, G_I^t); \text{ a vector of all the state variables, and}
\]

\[
Y_t = (p_t, p_t^w, p_t^d, w_t^*, \ldots); \text{ a vector of all the endogenous variables.}
\]

Using this notation, the optimal values of endogenous variables can be expressed as

\[
\hat{Y}_0(X_0) = (\hat{p}_0(X_0), \hat{p}_0^w(X_0), \ldots).^{16}
\]

---

^{16} As before, '\^\*\' denotes the optimal solution.
Once a set of the rates of public charges as well as other endogenous variables are
determined, government revenue $M_o^G$ is determined, and a public investment
policy $(\theta_o^G, \theta_o^R)$ fixes the level of public capital stock in the next period $G_1, G_1^R$
and $G_1^T$. In addition, the level of private capital stock $m_1$ is uniquely determined by
$Y_0$. Given a vector of state variables $X_1$, the above price determination process
results in $\hat{Y}_1(X_1)$ and this cycle is repeated for the entire planning period.

It is also possible to optimise public investment policy $Z_t = (\theta_t^G, \theta_t^R)$ for entire
planning period, at least in theory, using dynamic programming. The solution
algorithm is as follows.

Let $A_n = (A_n^G, A_n^R)$ denote a vector of the optimal investment policy function for
the government optimisation problem when the planning period is $n$. This optimal
policy function vector is such that $\hat{Z}_n^T = A_n^T X_n$, which mean that $\theta_t^G = A_n^G (X_t)$ and
$\hat{\theta}_t^R = A_n^R (X_t)$.\textsuperscript{17} To illustrate the solution algorithm, let us start from $n=2$, because
when $n=1$, the optimal solution is immediately obtained as $\hat{Y}_1(X_0)$ regardless of
public investment policy.

The government problem for two periods, subsuming all constraints into the
relationship $\hat{Y}_t = \hat{Y}_1(X_0)$, is expressed as

$$\max_{z_0} V_2 = [u(\hat{\epsilon}(X_0)) + \left(\frac{1+\nu}{1+\rho}\right)u(\hat{\epsilon}(X_1(Z_0;X_0))].$$

Here a parsimonious notation is employed throughout this explanation, e.g.

$\hat{\epsilon}(X_1(Z_0;X_0))$ stands for $\hat{\epsilon}(Y_1(X_1(Z_0;Y_0(X_0),X_0)),X_1(Z_0;Y_0(X_0),X_0))$.

If there is an interior solution, the first order necessary condition is obtained as

$$\frac{\partial \hat{\epsilon}}{\partial \theta_o^G} = \frac{\partial \hat{\epsilon}(X_1(Z_0;X_0))}{\theta_o^G} = 0$$

\textsuperscript{17} Superscript $T$ stands for vector transposition.
The optimal solutions of these equations are $\hat{\theta}_0^Q = A^Q_2(X_0)$ and $\hat{\theta}_0^R = A^R_2(X_0)$, thus $A_2 = (A^Q_2, A^R_2)$ is obtained.

Then, let us think about the case $n = 3$. We have

$$\max_{Z_0, Z_1} V_3 = \left[ u_0 + \left( \frac{1+\nu}{1+\rho} \right) u\left( \hat{\theta}_1(X_1(Z_0; X_0)) \right) + \left( \frac{1+\nu}{1+\rho} \right)^2 u\left( \hat{\theta}_2(X_2(Z_1; X_1)) \right) \right],$$

in which $u_0 = u\left( \hat{\theta}_0(X_0) \right)$; a constant.

Here suppose, for the time being, $X_1$ is given. Since $u\left( \hat{\theta}_1(X_1) \right)$ is determined, this problem is identical to the $n = 2$ case. The optimality condition is

$$\frac{\partial \hat{\theta}_1(X_2(Z_1; X_1))}{\partial A^Q_1} = \frac{\partial \hat{\theta}_1(X_2(Z_1; X_1))}{\partial A^R_1} = 0.$$

Hence the optimal policy is given as $\hat{\theta}_1^Q = A^Q_2(X_1)$ and $\hat{\theta}_1^R = A^R_2(X_1)$, or in vector notation $\hat{Z}_1 = A_2(X_1)$. This corresponds to the principle of optimality (Bellman 1957). With this optimal policy the optimisation problem can be rewritten as

$$\max_{Z_0} V_3(Z_0) = u\left( \hat{\theta}_1(X_1(Z_0; X_0)) \right) + \left( \frac{1+\nu}{1+\rho} \right) u\left( \hat{\theta}_2(A_2(Z_0; X_0), Z_0; X_0) \right),$$

in which $u\left( \hat{\theta}_2(A_2(Z_0; X_0), Z_0; X_0) \right)$ stands for

$$u\left( \hat{\theta}_2(A_2(X_1(Z_0; X_0)); X_1(Z_0; X_0)) \right).$$

Since $V_3$ is a function of $Z_0$, we can derive the optimal policy $\hat{Z}_0 = A_3(X_0)$.

Repeating this algorithm, finally we can transform the original government problem corresponding to the case $n = T$ as

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18 This is so because deleting constant terms and dividing or multiplying by arbitrary positive constant do not change the solution of this problem.
Max \( V_T(Z_0) = u\left(\hat{c}(X_1(Z_0; X_0))\right) + \left(\frac{1 + \nu}{1 + \rho}\right)u\left(\hat{c}(X_2(A_{T-1}(Z_0; X_0)))\right) \\
+ \left(\frac{1 + \nu}{1 + \rho}\right)^2 u\left(\hat{c}(X_3(A_{T-2}(A_{T-1}(Z_0; X_0))))\right) + \cdots \\
+ \cdots + \left(\frac{1 + \nu}{1 + \rho}\right)^{T-1} u\left(\hat{c}(X_T(A_T(A_{T-1}(A_{T-2}(A_{T-3}(Z_0; X_0))))\right) \right). \\

This solution determines \( A_T = (A_T^G, A_T^R) \). The optimal public investment policy is sequentially obtained as \( \hat{Z}_0 = A_T(X_0), \hat{Z}_1 = A_{T-1}(\hat{X}_1), \) and so on, in which \( \hat{X}_t \) is obtained as \( \hat{X}_t(A_T(X_0), X_0), \hat{X}_2(A_{T-1}(\hat{X}_1), \hat{X}_1), \) etc. As a result, this algorithm can find the optimal trajectories of public investment policy \( \{\hat{\theta}_0^G, \hat{\theta}_1^G, \cdots, \hat{\theta}_{T-1}^G\}, \) \( \{\hat{\theta}_0^R, \hat{\theta}_1^R, \cdots, \hat{\theta}_{T-1}^R\}. \n
This algorithm is a special case of dynamic programming where \( n \)-period dynamic optimisation problem can be reduced to a static optimisation problem. This conceptually feasible solution algorithm is, however, difficult to implement due to the high degree of complexity of the government problem. Policy simulations to be explained in Chapter 6 thus employ the optimal setting of public charges along with exogenously given public investment policy as policy scenarios, and investigates whether policy scenarios can satisfy certain policy targets, such as reducing unemployment or lack of safe water access as well as growth of income, or not.

4.5 Trade and tax

4.5.1 General

Up to now issues related to trade and tax have been deliberately avoided. This is because, as exogenous variables, they do not change the model structure or solution. This section incorporates them into the model.

This study employs an assumption of imperfect substitutability between foreign and domestic goods: the so called Armington assumption (Armington 1969), for
both imports and exports. This assumption is very widely employed in applied studies, particularly in CGE literature.

In addition to import tax and export subsidy, commodity-specific sales tax $\tau^i (i = R, I, U)$ for producers and income tax $\tau_H$ for households are introduced.

To facilitate the following explanation, flows of commodity and money in the economy are schematically illustrated in Figure 9. The following notation is common throughout the remaining part of this dissertation.

$P^p_i$: Producer price of good $i$ in local currency,

$P^{lc}_i$: Consumer price of good $i$ in local currency,

$P^{Di}_i$: Domestic wholesale price of good $i$ in local currency,

$P^{lw}_i$: World price of good $i$ in international currency,

$R^e$: Real exchange rate, which is defined as a real price of international currency in terms of local currency,

$E^i$: Exports of good $i$ in physical term,

$I^i$: Imports of good $i$ in physical term,

$Y^{Di}_i$: Supply of domestic product $i$ to the domestic wholesale market,

$D^i$: Total demand for import-domestic produced composite good $i$,

$D^{j}_i$: Demand of institution $j$ for composite good $i$, in which $j = H$ (household), $G$ (government), and $S$ (saving account),

$s^{ik}_i$: Intermediate input demand of producers of good $k$ for composite good $i$,

$\tau^i$: Producer tax rate of good $i$,

$\tau^i_{ht}$: Import tax rate of good $i$, and

$s^i$: Export subsidy rate of good $i$. 
Note that Figure 9 does not contain income tax flow from households to the government which has no effect on commodity prices.

4.5.2 Trade and commodity price

(1) Imports

Imperfect substitution between domestically produced and imported goods is specified as the following CES (constant elasticity of substitution) function.

\[ D^i = \alpha^i_M \left\{ \delta^i_M \left( \frac{\sigma^i_{M-1}}{\sigma^i_M} \right) + \left( 1 - \delta^i_M \right) \left( \frac{\sigma^i_{M-1}}{\sigma^i_M} \right) \right\}, \text{ in which} \]

\[ \alpha^i_M : \text{scale parameter, } \delta^i_M : \text{preference parameter of import good, and } \sigma^i_M : \text{CES elasticity of substitution, with respect to good } i. \]

The solution to the consumer’s expenditure minimisation problem given disposable income results in the optimality condition
\[
\frac{M^i}{Y^{Di}} = \left( \frac{\delta_M^i}{1 - \delta_M^i} \right)^{\sigma_M^i} \left\{ \left( \frac{p_{Di}^{pD}}{1 + \tau_M^i} \right) p_{Wi}^{w} r_e \right\}^{\sigma_M^i}.
\]

From the commodity and monetary balance conditions, we have
\[
D_i = Y^{Di} + M^i, \quad \text{and}
\]
\[
p^{ic} D_i = p^{Di} Y^{Di} + (1 + \tau_M^i) p_{Wi}^{w} r_e M^i.
\]

From these four equations, the following relation among prices is derived.
\[
p^{ic} = p_{Wi}^{w} r_e (1 + \tau_M^i) \eta_M^i, \quad \text{in which}
\]
\[
\eta_M^i = \left( \frac{\delta_M^i}{1 - \delta_M^i} \right)^{\sigma_M^i} \left\{ \left( \frac{1 + \tau_M^i}{p_{Di}^{Di}} \right) \right\}^{\sigma_M^i-1}.
\]

(2) Exports

Similar to the import case, imperfect substitution between export supply and domestic market supply is specified as the following CET (constant elasticity of transformation) function.
\[
Y^i = \alpha_E^i \left\{ \delta_E^i \left( E^i \right)^{\sigma_E^i-1} + \left( 1 - \delta_E^i \right) \left( Y^{Di} \right)^{\sigma_E^i-1} \right\}, \quad \text{in which}
\]
\[
\alpha_E^i: \text{ scale parameter, } \delta_E^i: \text{ preference parameter of export good, and } \sigma_E^i: \text{ CET elasticity of substitution, with respect to good } i.
\]

The solution to the producers’ profit maximisation problem given quantity of total product results in the optimality condition
\[
\frac{E^i}{Y^{Di}} = \left( \frac{\delta_E^i}{1 - \delta_E^i} \right)^{\sigma_E^i} \left\{ \left( \frac{p_{Di}^{Di}}{1 + s^i} \right) p_{Wi}^{w} r_e \right\}^{\sigma_E^i}.
\]

From these commodity and monetary balance conditions, we have
\[ Y^i = Y^{Di} + E^i \], and
\[ (1 + t^i)p^{Di}Y^i = p^{Di}Y^{Di} + (1 + s^i)p^{iw}e^i. \]

From these four equations, the following price relationship is derived.

\[ p^{lp} = p^{iw}r^e\left(\frac{1 + s^i}{1 + \tau^i}\right)\eta^i_E, \]
in which

\[ \eta^i_E = \left(\frac{\delta^l_E}{\delta^l_E}\right)^{\sigma^l_E} + \left(1 - \delta^l_E\right)^{\sigma^l_E}\left\{\left(1 + s^i\right)p^{iw}r^e/p^{Di}\right\}^{\sigma^l_E-1}, \]

(3) Determination of domestic wholesale price

The last element to have operational significance for policy simulation is the relationship between domestic wholesale price and world price. The employed assumption is that the government subsidises export goods such that the producers are indifferent to export their products or to supply them to the domestic wholesale market, while the products can be sold in the rest of the world at the world price.\(^{19}\)

This assumption establishes the relationship

\[ p^{Di} = (1 + s^i)p^{iw}r^e. \]

With this relationship, consumer and producer prices are determined as

\[ p^{ic} = p^{iw}r^e\left(1 + \tau^i_M\right)\eta^i_M, \]
in which

\[ \eta^i_M = \left(\frac{\delta^i_M}{\delta^i_M}\right)^{\sigma^i_M} \left(1 + s^i\right)^{\sigma^i_M-1} + \left(1 - \delta^i_M\right)^{\sigma^i_M} \left(1 + \tau^i_M\right)^{\sigma^i_M-1} \], and

\[ p^{lp} = p^{iw}r^e\left(\frac{1 + s^i}{1 + \tau^i}\right), \text{ as } \eta^i_E = 1. \]

\(^{19}\) This is because my model cannot accommodate multiple products of one industry. See Sadoulet and de Janvry (1995; p.204-205) for the case when differentiation between export goods and domestically sold goods is allowed.
Optimal imports and exports can then be derived from the optimality conditions and commodity balance conditions as

\[ \hat{E}^i = \Omega_E^i \hat{y}^i, \]

in which \( \Omega_E^i \equiv \frac{\left( \delta_E^i \right)^{\sigma_E}}{\left( \delta_E^i \right)^{\sigma_E} + \left( 1 - \delta_E^i \right)^{\sigma_E}}, \) and

\[ \hat{M}^i = \left( \frac{1 + s^i}{1 + \tau_M^i} \right)^{\sigma_M} \left( \frac{-\delta_M^i}{1 - \delta_M^i} \right)^{\sigma_M} \left( 1 - \Omega_E^i \right) \hat{y}^i. \]

The former equation indicates that the proportion of exports to total production is constant at \( \Omega_E^i \) regardless of tax or subsidy policy. This is an outcome of the assumption that \( p^{di} = (1 + s^i) p^{w} r^e \), which implies that subsidy rate is automatically determined by world and domestic market. The role of the government in this case is purely passive as it merely follows the market signal. The reading of the latter equation must be, therefore, that the government can control the ratio of imports to total production by setting a rate of import tax, but cannot control it through subsidies.

Exports and imports can be simulated based on these equations, once the CES and CET parameters have been fixed.

### 4.5.3 Income tax

It is assumed that the government levies a uniform rate tax \( \tau_H \) on household income. This tax can be incorporated into the household problem by multiplying factor \( (1 - \tau_H) \) to all wage rates, real rate of return \( r_h \), and net producer price of rainfed product \( p^R_i \). As a result the optimal consumption is modified as follows.

\[ \hat{c}_i = \left( \frac{\varphi_2}{p_i} \right)^{\varphi_2} \prod_{t \in \{I, U\}} \left( \frac{\varphi_{1t}}{p_i^{\varphi_1}} \right)^{\varphi_{1t}} \left[ 1 + (1 - \tau_H) r^{*}_i - (1 + \nu) \left( \frac{1 + (1 - \tau_H) r^{*}_i}{1 + \rho} \right)^{\frac{1}{\sigma}} \right] \times p^{t_0} m_i \]

\[ + \frac{1 - \tau_H}{(1 - \tau_H) r^{*}_i - \nu} \left( \tilde{w}_{i}^{H} \theta^{*}_i s^{*}_i + w_{i}^{\tilde{H}} \nu^{*}_i + w_{i}^{L} \left( 1 - l^{*}_i - \tilde{s} \right) \left( 1 - (1 - \theta_i) z \right) \right) \]
\[ + \frac{p_t \eta_{mo} (1 - l_t^{u*} - r_i) (1 - \theta_i)}{(1 - \tau_H) r_i^{*} - \nu} + \left( \frac{1}{1 + \nu} \right)^{i} \tilde{\omega} \frac{p_t^R Y_R}{r_i^{*}} - \left( \frac{1}{1 + \nu} \right)^{i} \times \]

\[ \frac{\beta_R}{(1 - \tau_H) r_i^{*} \{1 + (1 - \tau_H) r_i^{*} \}} \left( \frac{Y_R}{\tau_R} \right)^{\frac{1}{\beta_k R}} \left( \frac{(1 - \tau_H) w_i^{l*}}{\beta_{RL}} \right) \frac{\beta_k R}{\beta_k} \left( \frac{p_{l^u} (1 - \tau_H) r_i^{*} + \delta}{\beta_{RL}} \right) \]

\[ - \left( \frac{1}{1 + \nu} \right)^{i} \frac{1}{1 + (1 - \tau_H) r_i^{*}} \left[ \left( \frac{1 - \tau_H}{\tau_R} \frac{Y_R}{Y_R} \right)^{\frac{1}{\beta_k R}} \left( \frac{1}{K_i} \right)^{\frac{\beta_k R}{\beta_k}} \right]. \]

The equation of motion of private assets is also modified as

\[ m_{i+1} = \frac{1}{1 + \nu} m_i - \frac{1}{(1 + \nu) p_t^{l^u}} \left( \frac{p_t^{l^u}}{q_i} \right)^{q_i} \prod_{l=1}^{l^u} \left( \frac{p_t^{l^u}}{q_i} \right)^{q_i} \left( 1 - \frac{1 - \tau_H}{1 + \nu} p_t^{l^u} \right) \times \]

\[ \left[ w_i^{l*} (1 - l_t^{u*} - r_i) (1 - (1 - \theta_i) \tilde{v}) + \overline{w_i^{l^u}} \theta_{l^{u*}} l_t^{u*} + w_i^{l^u} \tilde{v} \right] + \frac{p_t \eta_{mo} (1 - l_t^{u*} - r_i) (1 - \theta_i)}{(1 + \nu) p_t^{l^u}} \times \]

\[ + \left( \frac{1}{1 + \nu} \right)^{i} \tilde{\omega} \frac{p_t^R Y_R}{(1 + \nu) p_t^{l^u}} - \left( \frac{1}{1 + \nu} \right)^{i+1} \left( \frac{1 - \tau_H}{p_t^{l^u}} \right)^{1} \frac{1}{\beta_k R} \frac{1}{(1 + \nu) p_t^{l^u}} \left( \frac{Y_R}{\tau_R} \right)^{\frac{1}{\beta_k R}} \frac{\beta_k R}{\beta_k} \left( \frac{1}{K_i} \right)^{\frac{\beta_k R}{\beta_k}} \]

\[ - \left( \frac{1}{1 + \nu} \right)^{i+1} \left[ \left( \frac{Y_R}{\tau_R} \right)^{\frac{1}{\beta_k R}} \left( \frac{\beta_k R (1 - \tau_H) w_i^{l*}}{\beta_{RL} p_t^{l^u} (1 - \tau_H) r_i^{*} + \delta} \right) \frac{\beta_k R}{\beta_k} \right] \left( - (1 - \delta) K_i^R \right). \]

It should be noted that household decision is summarised in these two equations. The effects of income tax are, thus, fully covered by the above modification.

### 4.6 Conclusions

This chapter has addressed a way to incorporate the stylised facts of water-scarce developing countries into the analytic model developed in the previous chapter. Four key stylised facts are taken into account; (i) dominant water use share of irrigated agriculture, (ii) vulnerability of rainfed agriculture, (iii) a high urban unemployment rate, and (iv) poor safe water access in the rural areas. For this purpose the analytic model is modified in the following way.
Firstly, the production sector is disaggregated into three sectors; rainfed agricultural, irrigated agricultural and urban modern sectors. This specification allows the model not only to address irrigation-rainfed issues but also to pave a way to model rural-urban migration mechanism. Production risk is introduced in rainfed agriculture only in order to reflect its inherent environmental vulnerability. Note that the household problem is specified as a utility maximisation given household expectations of specific variables, instead of expected utility maximisation. This is because it is difficult to regard household expectations about prices and public policy variables as stochastic. The expected utility model is not suitable to deal with this case.

Secondly, labour market equilibrium for unskilled labourers is introduced. It is assumed that a household allocates its unskilled members to either irrigated agricultural or urban modern sectors such that indirect utility derived from wage income is indifferent in either case. The obtained equilibrium can be regarded as a generalised version of Harris-Todaro model. Indeed, when public water supply coverage is 100% in rural area, my model is reduced to Harris-Todaro model.

Thirdly, poor safe water access in rural area is modelled through a 'penalty’, which generates disutility for household members who lack safe water access. This penalty is represented as a reduction of labour supply which can be interpreted as a reduction of net wage income. This reflects the opportunity cost of forgone wage income from fetching water and additional expenditures on e.g. medical care caused by water-born diseases. An idea to assign higher price of water, such as price of bottled water, is abandoned because it requires introducing an additional production sector, private water sector, in this general equilibrium framework.

These modifications are implemented without losing tractability. It should be noted that this is underpinned by a crucial assumption that the representative household is a single decision-making unit in spite of heterogeneity of household members in terms of occupation and safe water access. This assumption is at odds with

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20 Division of skilled and unskilled labourers is purely empirical specification without which model calibration has difficulty.
distributional issues, which I believe one of key aspects of sustainable development.

It has been confirmed that the applied model maintains essentially the same structure as the analytic model despite its drastically increased complexity. In the second-stage government optimisation, however, stochastic elements are introduced which make much more careful treatment of the government problem necessary. In this chapter an explicit treatment of the government expectations is introduced. Issues of policy implementation under uncertainty are discussed in Chapter 6.

Insights provided by the analytic model are used to establish a solution algorithm. The solution algorithm optimises public charges, but does not optimise public investment. Instead, several patterns of public investment policy are exogenously set as policy scenarios, and are evaluated to see whether they can satisfy certain policy targets or not.

Issues arising from incorporating trade and tax into the model have also been discussed. Obtained results are highly important to calibrating and validating the model. Moreover, an explicit treatment of trade and tax enables the model to consider more policy scenarios.
Chapter 5

Calibration and Validation of the Applied Model

5.1 Introduction

Modelling requires, by definition, abstraction from certain aspects of reality. For this purpose some aspects relevant to research objectives are specified in some depth while other aspects are rather grossly simplified or ignored by assumption. If a model fails to be a useful and relevant tool for addressing research questions, this is not necessarily because of the simplification or ignorance of reality itself, as is often suggested, but may be because of its inability to replicate those aspects of reality relevant to the research. This can be evaluated only by model validation. Therefore, conducting simulations without model validation cannot provide any reliable answer to empirical questions.

This chapter reports the results of calibration and validation of the applied model along with detailed data description. Section 5.2 introduces Morocco as the case study country. Section 5.3 describes the data and explains construction of an aggregate version of social accounting matrix (SAM) based on a published highly disaggregated SAM. The aggregate version of SAM serves as the principle database of the applied model. Section 5.4 explains calibration process and reports the results, and Section 5.5 does the same for validation. In Section 5.6 sustainable production functions of water and irrigation land are constructed, and Section 5.7 concludes this chapter.
5.2 Case study country - Morocco

The applied model constructed in Chapter 4 is calibrated and validated based on Moroccan data. Morocco is chosen as the case study country for the following two reasons:

- Morocco is a water-scarce developing country where all the key stylised facts incorporated into the applied model are observed, and
- There exist several datasets on Morocco that are highly relevant to this dissertation. In particular, a Moroccan social accounting matrix for the year 1994 (Löfgren et al. 1997) provides an ideal dataset for this dissertation.

Some important facts about Morocco for this study are described below.

5.2.1 Geography and climate

Morocco is located in the North Africa bordered by both the Atlantic Ocean and the Mediterranean Sea. Moroccan territory is 71 million hectare, including the Western Sahara, and characterised by high degree of geographical heterogeneity that is well illustrated by the fact that both the broadest plains and the highest mountains in North Africa lie in Morocco.

As a consequence, climatic conditions vary across the country. In general, the plain regions along the Mediterranean and Atlantic coasts are associated with annual precipitation levels of 400 ~ 750 mm, while the annual precipitation level frequently exceeds 750 mm in the high and the middle Atlas regions which form a north-east part of the Atlas mountains. By contrast the Saharan regions have often under 100 mm precipitation per year. On average, total precipitation for the entire Moroccan territory is estimated at 150 billion m³ per year (World Bank 1995), which is equivalent to 211 mm per year. Table 1 shows spatial variation of mean precipitation in Morocco and Figure 10 shows the country map.
Chapter 5 Calibration and Validation

Table 1 Spatial variation of annual mean precipitation in Morocco

<table>
<thead>
<tr>
<th>City</th>
<th>Region</th>
<th>75-79</th>
<th>80-84</th>
<th>85-89</th>
<th>90-94</th>
<th>95-99</th>
<th>75-99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangier</td>
<td>North</td>
<td>839.1</td>
<td>600.9</td>
<td>676.1</td>
<td>624.3</td>
<td>804.0</td>
<td>708.9</td>
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<tr>
<td>Fes</td>
<td>North</td>
<td>572.2</td>
<td>407.3</td>
<td>468.2</td>
<td>391.7</td>
<td>401.8</td>
<td>448.2</td>
</tr>
<tr>
<td>Ifrane</td>
<td>North (Middle Atlas)</td>
<td>1034.3</td>
<td>785.9</td>
<td>944.8</td>
<td>774.9</td>
<td>968.2</td>
<td>901.6</td>
</tr>
<tr>
<td>Rabat</td>
<td>North (Atlantic coast)</td>
<td>632.2</td>
<td>453.0</td>
<td>494.8</td>
<td>347.7</td>
<td>511.1</td>
<td>487.8</td>
</tr>
<tr>
<td>Casablanca</td>
<td>North (Atlantic coast)</td>
<td>431.9</td>
<td>302.1</td>
<td>396.2</td>
<td>316.3</td>
<td>442.0</td>
<td>377.7</td>
</tr>
<tr>
<td>Marrakech</td>
<td>Central</td>
<td>223.8</td>
<td>197.7</td>
<td>228.2</td>
<td>198.0</td>
<td>277.5</td>
<td>225.0</td>
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<td>Ouarzazate</td>
<td>Central east</td>
<td>92.7</td>
<td>77.1</td>
<td>188.5</td>
<td>116.9</td>
<td>96.4</td>
<td>115.1</td>
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<td>Agadir</td>
<td>South (Atlantic coast)</td>
<td>214.7</td>
<td>260.1</td>
<td>322.8</td>
<td>156.7</td>
<td>328.9</td>
<td>256.6</td>
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<td>South (Western Sahara)</td>
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<td>45.2</td>
<td>78.1</td>
<td>73.4</td>
<td>N.A.</td>
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</table>


Figure 10 Country map of Morocco

During the past two decades drought occurs every two years on average, which is significantly higher than its average for the past 100 years (every 10 years). Recent drought years are 1992, 1993, 1995, 1999, and 2000. Although it is inconclusive whether this higher frequency of droughts is caused by anthropogenic factors such as greenhouse gas emissions or not, several climate change researchers predict further decrease of precipitation in this region.
Hulme et al. (1995) derive future climate change scenarios for Africa using a two-stage process. First, an energy-balance model predicts the transient response of the global system, but not the regional response. Second, general circulation models (GCMs) predict regional responses mostly based on equilibrium experiments corresponding to the steady-state with a fixed level of greenhouse gases. Simulation results of nine GCMs are combined for the second stage because of significant difference between model experiments. Although their scenarios show a huge range of possible future rainfall from decrease of tens of percent to increase of similar magnitude from the current level, they conclude that "it does appear that northern Africa may experience a further reduction in rainfall amounts over coming decades" (p.16).

Knippertz et al. (2003) statistically investigate the effects of the large-scale atmospheric circulation parameters, especially synoptic and baroclinic activities, on precipitation in Morocco based on monthly precipitation data of the Global Historical Climatology Network (GHCN). They utilise their findings to examine the consistency between simulation results of precipitation, which is "not always well reproduced in GCMs" (p.81), and other more reliable variables related to the large-scale atmospheric circulation, from a coupled atmosphere-ocean GCM, ECHAM4/OPYC3, developed in co-operation between the Max Planck Institute and the Deutsches Klimarechenzentrum (DKRZ). Their analysis consistently predicts a negative influence of increasing greenhouse gas emissions on precipitation in the regions along the Atlantic and the Mediterranean coasts, but it cannot provide conclusive prediction of precipitation in the regions south of the Atlas Mountains where local and orographic influences are dominating.

5.2.2 Demography

The total population of Morocco was estimated at 29.6 million in 2002. Population growth rate has gradually dropped from 2.0 % per year during the 1980's to 1.6 % per year during the 1990's. This drop of population growth is clearly reflected in decreasing youth population share of age under 15 from 39.8 % in the year 1990 to 32.3 % in the year 2000 (Royaume du Maroc 1982 - 1985, 1990 - 2003). Table 2 summarises the demographic data of Morocco.
Table 2 Demographics of Morocco

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</thead>
<tbody>
<tr>
<td>Total population [million persons]</td>
<td>17.2</td>
<td>20.1</td>
<td>22.1</td>
<td>24.5</td>
<td>26.4</td>
<td>28.7</td>
</tr>
<tr>
<td>Youth share of age under 15 group</td>
<td>46.0%</td>
<td>44.4%</td>
<td>41.6%</td>
<td>39.8%</td>
<td>36.2%</td>
<td>32.3%</td>
</tr>
<tr>
<td>10-year mean annual growth rate</td>
<td>2.6%</td>
<td>2.8%</td>
<td>2.5%</td>
<td>2.0%</td>
<td>1.8%</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

* Interpolation based on 1979 and 1982 data.


Note that Morocco is one of the major labour exporting countries to the European Union. The net emigration rate in 2004 is estimated at 0.98 persons per thousand people (Central Intelligence Agency 2005).

5.2.3 Economy

The Moroccan economy has been transformed from a French colonial economy which exported agricultural products and phosphate rocks, into more diversified economy in which the service sector plays a key role at least in the urban areas. Although Morocco is still a leading producer of phosphate, the role of mining industry in exports has drastically reduced from 32 % of total value of exports in 1981 to 8 % in 1991, due to both diversification of economic activity and declining world price of phosphates (World Bank 1995).

Morocco is an upper middle-income country with national gross domestic product of 32.9 billion US dollar, or 3,597 US dollar per capita based on purchased power parity (PPP) in 2000 (Economist Intelligence Unit 2002). Based on the Human Development Index, Morocco was 112th among 162 countries in 1999 and 126th out of 175 countries in 2001 (UNDP 2000, 2003).

(1) Agricultural sector

The role of agriculture has been more and more limited in terms of GDP share, 18.5 % in 1994, but is still highly significant in labour absorption as it provides 45 % of total employment in the same year. The rural economy provides nearly 80 % of total rural employment and produces 60 % of total rural value-added (Löfgren et al. 1999).
Furthermore, it has been observed that high vulnerability of rainfed agriculture to draught has sometimes severely hampered nationwide economic development. Figure 11 shows relationship between year-to-year variation of nationwide real GDP and rainfed agricultural production in physical term.¹

![Figure 11 Variation of rainfed agricultural production and real GDP](image)

Note the magnitude of variation of rainfed agricultural production represented by non-industrial crop production in the figure. It is obviously a tough business for the farmers to cope with such a huge production fluctuation. Furthermore, several climate change researchers predict higher frequency of extreme climatic events which may significantly worsen this fluctuation (Hulme et al. 1995, Parish and Funnell 1999).

Against this background the Moroccan government has focused on irrigation infrastructure development as a core of rural development strategy. The government has allocated more than 60 % of public investment in agriculture to irrigation development since 1965 (Kadi 2002). It is estimated that the large-scale and medium-scale irrigation areas administered by the government has increased from 640 thousand hectares in 1984 to 860 thousand hectares in 1994 (Académie du Royaume du Maroc 2000). Although the share of irrigation areas is estimated at 10 - 15 % of the total cultivated land including small-scale private irrigation areas, irrigated agriculture currently contributes around 45 % of the agricultural value-added and 75 % of agricultural exports (Kadi 2002).

¹ Rainfed agricultural production is approximated by total non-industrial crop production.
(2) Urban unemployment

One of the major challenges of the Moroccan economy is a chronically high rate of urban unemployment, underpinned by significant rural-urban migration. This migration-induced urban unemployment mechanism is observed by a contrast of unemployment rates between rural and urban areas as shown in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>1995</th>
<th>1999</th>
<th>2000</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban unemployment</td>
<td>22.3%</td>
<td>21.8%</td>
<td>21.2%</td>
<td>18.2%</td>
</tr>
<tr>
<td>Rural unemployment</td>
<td>7.6%</td>
<td>5.0%</td>
<td>4.6%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>


The main driving force of rural-urban migration is not only lower income or higher poverty incidence in the rural areas but also poorer public services in the rural areas mainly in terms of safe water, electricity, and education (Löfgren et al. 1999). The World Bank (1995) reports that a lack of safe water access remains “the primary cause of ill-health in the rural areas” (p.2).

(3) Trade and foreign direct investment

The trade balance of Morocco is chronically negative with the ratio of exports to imports around 70 - 80%. Recently the leading export goods are textiles followed by food products, in particular fish products, while the leading import goods are machinery and equipment for industrial purposes (Economic Intelligent Unit 2002). The government has launched a trade liberalisation programme since the early 1980’s under the Structural Adjustment Programme led by the International Monetary Fund. Since then, trade liberalisation has gradually been implemented through elimination of quantitative restrictions and reductions of import tariffs (Löfgren et al. 1999). There still remains, however, a high degree of trade protection for domestic agriculture.

The government maintains relatively tight foreign exchange rate control scheme as a principle policy tool to deal with inflation, despite increasing pressure to devalue mainly from exporters. The Moroccan local currency dirham is regarded as overvalued which obviously contributes to the chronic trade deficit. The share of
the foreign direct investment to the gross fixed capital formation is small. It is estimated at 3.2% in 1980’s and 9.7% in 1990’s, and the large part of the latter is associated with the privatisation of Maroc Telecom (Bouoïyour 2003).

5.2.4 Water sector and water policy

(1) Key institution in the water sector

The major part of water sector in Morocco is directly controlled by the government. Key players are DGH (the Direction Générale de l’Hydraulique), ONEP (the Office National de l’Eau Potable), and ORMVA (the Office Régional de Mise en Valeur Agricole). DGH is responsible for water resource development, including mobilising raw water for large-scale irrigation perimeters. ONEP is mainly responsible for drinking water supply including planning, constructing and managing water treatment and supply facilities except for the large cities where autonomous water providers (Régies) supply drinking water. ORMVA is responsible for management of the major irrigation perimeters. There exist nine ORMVAs; Loukkos, Moulouya, Gharb, Doukkala, Tadla, Haouz, Souss-Massa, Ouarzazate, and Tafilalet. Overall management of national water policy is provided by the National Water and Climate Council (World Bank 1995).

(2) Water resource development

Of 150 billion m³ of average annual nationwide rainfall, about 120 billion m³ is evaporated and 30 billion m³ remains as a potential water resource of which 22.5 billion m³ flows as surface water and 7.5 billion m³ replenishes groundwater.

The rainy season is between December and February and most surface flow is concentrated in this period. Due to this highly seasonal nature of the hydrological cycle, surface water resource development in Morocco has heavily relied on dam construction. Moroccan water resource development plan anticipates that annual mobilisation of surface water will be increased from 8.26 billion m³ in 1990 to 13.78 billion m³ in 2020 by 5.52 billion m³, of which 2.78 billion m³ is to be covered by the Al Wahida dam and the remaining 2.74 billion m³ will be covered by the other 51 dams.
Of 7.5 billion m$^3$ annual groundwater recharge the total renewable groundwater resource available for use is estimated at between 3 and 4.5 billion m$^3$. Due to very uneven spatial distribution of the renewable groundwater, i.e. the Middle and the High Atlas regions are endowed with 70% of the renewable groundwater resources, mining of non-renewable groundwater is undertaken in some regions. Nationwide total extraction is estimated at 2.73 billion m$^3$ which is less than the total renewable groundwater resources available for use. Groundwater resource development plan anticipates 10% increase of groundwater harvest to 2.99 billion m$^3$ in 2020.

Other water resources, such as desalination of seawater or recycling of wastewater, are rarely utilised in Morocco. For instance desalinisation and demineralisation facilities in the southern regions currently produce 0.015 billion m$^3$ as a whole, which is merely 0.01% of total domestic water supply (World Bank 1995).

The World Bank (1995) reports that the average total public investments in dams and transfers for the period between 1990 and 1994 was 0.7 billion 1993 DH per year, and was expected to reach 3 billion 1993 DH per year for the period from 1995 to 2000.

(3) Water use

Nationwide water use is dominated by irrigation whose share was estimated at 85% in 1990 (World Bank 1995) and 88% in the early 2000's (Kadi 2002). The former estimate is based on an assumption that some part of return flow from irrigation, a proportion of irrigation water returning to aquifers, could be recycled. Based on more conventional estimate of gross consumptive use, the share of irrigation water in the same year is estimated at 92% (World Bank 1995; p.9). The shares of domestic and industrial water uses are estimated at 8% and 4% respectively in the early 2000's (Kadi 2002).

Water consumption in 1990 is estimated at 9,190 million m$^3$ for irrigation use and 1,210 million m$^3$ for domestic and industrial uses. The government has traditionally attached high priority to satisfying domestic and industrial water demand. The future projection of water demands is thus based on estimation of the
future domestic and industrial water demands, and the difference between projected water supply capacity and the forecast domestic and industrial demands is defined as the irrigation water supply potential. The future domestic and industrial water demands in 2020 are forecast at 2,720 million m³, which is equivalent to 2.7 % annual growth, based on the following assumptions.²

- Per capita domestic consumption rises gradually from the current estimate of 100 l/cap.day to 150 l/cap.day in the urban areas and from the current estimate of 7.5 l/cap.day to 40 l/cap.day in the rural areas,
- Industrial demand grows annually at a rate of 4 %,
- Population grows annually at a rate of 2.5 % in the urban areas and 1.5 % in the rural areas, taking into rural-urban migration into account, and
- Water supply delivery loss drops from the current level of 30 % to 20 %.

As a result, with an additional assumption that 310 million m³ must be kept in water courses to flush solid wastes and other debris to the sea, the potential irrigation water supply is forecast at 13,610 million m³, which is equivalent to 1.3 % annual growth. This potential irrigation water supply shares 81 % of the forecast water supply capacity in 2020 (World Bank 1995).

(4) **Irrigation infrastructure development**

As mentioned before, irrigation infrastructure development has been given high priority in rural development strategy. The Moroccan government has prepared the irrigation investment plan as the National Irrigation Programme (Plan National d'Irrigation) in 1992 of which main goal is to expand irrigation areas to exceed 1 million hectares by 2000 and thereafter reach 1.2 million hectares by the year 2010. The total expenditures under the National Irrigation Programme are estimated at over 40 billion 1993 Dirham (DH) including the capital costs of new facilities, the rehabilitation costs of existing facilities, operation and maintenance costs, recurrent

² This forecast is corresponding to the base case, and the lower and upper bound cases are also prepared by changing the assumptions, e.g. lower or higher industrial demand growth, status quo delivery loss or further reduction to 10 %, etc. The lower bound forecast is 1,949 million m³ and the upper bound is 6,276 million m³ (World Bank 1995; p.14).
costs, and debt service. The required annual public spending is estimated at around 2 billion 1993 DH over the period 1995 - 2000, which is approximately twice more than the investment level over the period 1988 - 1994 of 0.9 billion 1993 DH (World Bank 1995).

Löfgren et al. (1997) provide more detailed information about the National Irrigation Programme for the period between 1993 and 2001, as shown in Table 4.

<table>
<thead>
<tr>
<th>Table 4 National Irrigation Programme</th>
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<tbody>
<tr>
<td>---</td>
</tr>
<tr>
<td>Investment [Billion 1994 DH]</td>
</tr>
<tr>
<td>Area [Thousand ha]</td>
</tr>
<tr>
<td>Water [Million m³]</td>
</tr>
</tbody>
</table>

Source: Löfgren et al. (1997)

According to this table, the forecast irrigation expansion is 0.09 million hectare during this period, which is too small to achieve the original goal. This discrepancy seems mainly due to the exclusion of low probability future projects of which total costs is estimated at 8.0 billion 1994 DH.

Löfgren et al. (1997) reveal that the National Irrigation Programme envisages unchanged per hectare irrigation water use. The forecast irrigation water supply potential requires, however, significant reduction of per hectare irrigation water use. The World Bank (1995) reports that expected irrigation area expansion to 830,000 hectares of large-scale and 470,000 hectares of small-medium scale irrigation areas will result in reduction of available irrigation water from 14,140 m³ per hectare in 1990 to 10,000 m³ per hectare in 2020. Such a reduction seems to be "not likely to be feasible with current technology and has important consequences for future investment and for water management" (p. 15).

(5) Safe water access

While most statistics agree with almost 100 % coverage of public water service in the Moroccan urban areas, the estimate in the rural areas significantly varies. For instance, Löfgren et al. (1999) refer to an estimate that only 4 % of rural households
have safe water access in 1994, while the World Bank (1995) employs an estimate that 14% of rural households have secured water supplies in 1990, 2% being household connection, 6% standpipes and 6% modernised wells.


<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>Safe water access</td>
<td>87.2</td>
<td>94.1</td>
<td>96.8</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Private tap</td>
<td>69.7</td>
<td>76.4</td>
<td>85.6</td>
<td>84.0</td>
</tr>
<tr>
<td></td>
<td>Standpipe</td>
<td>17.5</td>
<td>17.7</td>
<td>11.2</td>
<td>16.0*</td>
</tr>
<tr>
<td></td>
<td>Improved sanitation</td>
<td>91.0</td>
<td>83.0</td>
<td>90.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Rural</td>
<td>Safe water access</td>
<td>25.4</td>
<td>17.5</td>
<td>19.8</td>
<td>38.0</td>
</tr>
<tr>
<td></td>
<td>Private tap</td>
<td>7.9</td>
<td>9.1</td>
<td>10.6</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>Standpipe</td>
<td>17.5</td>
<td>8.4</td>
<td>9.2</td>
<td>32.0*</td>
</tr>
<tr>
<td></td>
<td>Improved sanitation</td>
<td>28.0</td>
<td>31.0</td>
<td>39.0</td>
<td>36.0</td>
</tr>
</tbody>
</table>

* Defined as population with reasonable access to a public water point.

According to the World Bank (1995), the average total public investments for the period between 1990 and 1994 was 0.2 billion 1993 DH per year in domestic water distribution and 0.15 billion DH per year in sanitation and sewerage, and is forecast to reach 1 billion 1993 DH per year each in domestic water distribution and in sanitation and sewerage for the period from 1995 to 2000.

(6) Water and irrigation land pricing

In Morocco, volumetric domestic water charge schemes with multiple price ranges (or tiers) have been implemented for cost recovery of the domestic water service providers (Régies), and the rates of domestic water charges significantly vary among the Régies. These rates have been revised at almost annual basis. Table 6 shows a time series of real-term domestic water tariffs of the average of 10 cities

Table 6 Average of real water tariff of 10 cities in Morocco

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1st range rate [1994 DH/m³]</td>
<td>1.14</td>
<td>1.18</td>
<td>1.75</td>
<td>1.71</td>
</tr>
<tr>
<td>5-year mean annual change [%]</td>
<td>-2.9</td>
<td>0.6</td>
<td>8.3</td>
<td>-0.5</td>
</tr>
<tr>
<td>2nd range rate [1994 DH/m³]</td>
<td>2.25</td>
<td>2.41</td>
<td>3.60</td>
<td>4.88</td>
</tr>
<tr>
<td>5-year mean annual change [%]</td>
<td>3.4</td>
<td>1.4</td>
<td>8.3</td>
<td>6.3</td>
</tr>
<tr>
<td>3rd range rate [1994 DH/m³]</td>
<td>3.07</td>
<td>3.33</td>
<td>4.97</td>
<td>7.13</td>
</tr>
<tr>
<td>5-year mean annual change [%]</td>
<td>10.1</td>
<td>1.6</td>
<td>8.4</td>
<td>7.5</td>
</tr>
</tbody>
</table>

1st range 0-30 m³ 0-24 m³ 0-8 m³
2nd range 30-60 m³ 24-60 m³ 8-20 m³
3rd range > 60 m³ > 60 m³ 20-40 m³
4th range - - > 40 m³


Although the recent drastic real price escalation of domestic water appears surprising, it is consistent with the Moroccan government policy to achieve full cost recovery of the urban water distribution systems as well as the operation and maintenance costs of the sewerage systems by the year 2000 (World Bank 1995; p.31).

According to Kadi (2002), irrigation land and water pricing for the large-scale irrigation areas is legislated by the Code of Agricultural Investment in 1969 as follows:

- Irrigation land charge (land improvement tax), which can be paid by yearly instalments for 17 years with 4% annual interest rate, covers 30% of capital investment costs. First five hectares are exempted for the farmers with less than 20 hectares land holdings.

- Volumetric irrigation water charge covers 10% of capital investment costs, full operation and maintenance costs plus 40% of replacement costs.

³ 10 cities are; Agadir, El Jadida, Essaouira, Fes, Kenitra, Marrakech, Meknes, Safi, Tangier and Tetouan. Casablanca and Rabat are excluded due to lack of data since 2000. The rate of the 4th range is slightly higher than that of the 3rd range, e.g. 7.81 versus 7.76 current DH/m³ in Fes in 2000.
In addition, irrigation farmers are responsible for the energy costs for pumping, if necessary. The government is responsible for the remaining capital investment and replacement costs. In practice, however, volumetric water charges are set far below the legislated level, usually only 25% - 50% of the level calculated based on the Code. The main reason is the necessity for the government both to encourage farmers to irrigate and to equalise rate of volumetric charges across different irrigation areas. The cost recovery through irrigation land charges is worse due to the fact that 80% of farmers hold less than 5 hectares of land each.

Coping with this reality, the revision of pricing scheme for irrigation land and water are ongoing (Kadi 2002). Although I have not found reliable nationwide statistics for the irrigation land and water charges, there is little doubt that rates are drastically rising as in the case of domestic water charges.

5.3 Data description

5.3.1 Data sources

The input datasets for calibration and validation of the applied model are constructed based on the following data/information.

(1) Moroccan social accounting matrix for the year 1994

Löfgren et al. (1999) construct a disaggregated social accounting matrix (SAM), a 104 × 104 matrix, for the year 1994. Because this SAM is designed to capture both the urban-rural and the rainfed-irrigated agriculture dichotomies, it is an ideal and indispensable database for my research in which these dichotomies play important roles.

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4 International Food Policy Research Institute (IFPRI) provides this SAM in Excel format on request.
Several studies have provided Moroccan SAMs for different years. They include; Sadoulet and de Janvry (1995) for the year 1980; Mateus (1988), Goldin and Roland-Holst (1995), Martens (1995), and Decaluwe et al. (1999) for the year 1985; and Roland-Holst (1996) for the year 1990 and 1994. They are not used in this study because none of them contain the information necessary to divide rainfed and irrigation agricultures in a satisfactory manner.

(2) Moroccan national statistical yearbook

The following data are collected from a series of Moroccan national statistical yearbooks (Annuaire Statistique du Maroc).5

- Demography,
- Climatic conditions,
- Crop production,
- Minimum wage rates,
- Domestic water tariff, and
- Urban and rural labour force and employment.

(3) International Financial Statistics (IFS)

The following data are collected from a series of International Financial Statistics.

- Nominal effective exchange rate,
- Total value of export,
- Total value of import, and
- GDP deflators for Morocco and USA.

(4) Others

The other data sources are as follows.

- The real interest rate data are collected from a series of World Development Indicators (WDI).

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5 In England the collections of the SOAS library and the LSE library cover series of Annuaire Statistique du Maroc for several years during 1980’s and all years since 1990.
Chapter 5 Calibration and Validation

- Nationwide total irrigation area data are from Académie du Royaume du Maroc (2000).
- Public capital investment is based on data from the World Bank (1995).
- Proportion of skilled labour in 1994 is from Löfgren et al. (1999).

5.3.2 Construction of aggregate social accounting matrix

The applied model requires a SAM consisting of;
- 5 factors - land, capital, unskilled labour, skilled labour and irrigation water,
- the government and a homogeneous group of households,
- the rest of the world and a saving account,
- 3 private production sectors - rainfed agriculture, irrigation agriculture and urban modern sector, and
- accounts representing taxes and subsidies.

This aggregate SAM is constructed based on the $104 \times 104$ Moroccan SAM as follows.\(^6\)

(1) Reclassification by summation

Most parts of aggregation can be easily done by summation of the accounts that can be regarded as breakdowns of an account. By this method the original SAM is reclassified as follows. Note that codes within round brackets are used in GAMS coding as well as in the aggregate SAM.

- Unskilled labour (L): A sum of [1] - [4]. Note that rural skilled labour, which represents 8.7% of labour force, is classified into this category due to the assumption in my model that all rural labour is unskilled.
- Private capital (K): A sum of [10] - [14].

\(^6\) In this subsection the $k^{th}$ account in the original $104 \times 104$ Moroccan SAM is referred to as $[k]$. 
- Rest of the World (ROW): Either [21] or a sum of [22] and [23], because the original SAM sometimes breaks down the rest of the world into EU [22] and non-EU [23] countries.

- Irrigation agricultural sector (I): A sum of [25] - [39], [53] - [55], and [59] - [61], which represent all the rural production activities except for the rainfed agricultural activities, which includes rural manufacturing, rural construction, etc.\(^7\)

- Rainfed agricultural sector (R): A sum of [40] - [52].

- Urban modern sector (U): A sum of [56] - [58] and [62] - [65].

- Direct and indirect taxes (TAX): A sum of [100] and [101].

- Tariffs and non-tariff barriers (TNT): A sum of [15], [103], and [104].

Skilled labour (S) [5], irrigation water (Q) [9], government (GOV) [20], saving account (SAV) [24], and subsidies (SUB) [102] are unchanged.

(2) Mapping commodity accounts to production activity accounts

The original 104 x 104 Moroccan SAM employs a commodity-by-industry account framework to represent private production activities, which allows better treatment of secondary products of production activities (Miller and Blair 1985). In the commodity-by-industry framework, consumers purchase commodities supplied by various industries (production activities) as well as by imports. Despite its advantage of representing the actual economic transactions, this framework is not compatible with my model in which each production sector produces only one type of commodity. To have compatibility it is necessary to map each of 34 commodities recorded in the original SAM to one of 3 private production sectors. This mapping has been done based on an assumption that a composition of a commodity output in terms of each industry’s product supply is fixed, which is

\(^7\) Recall that the purpose of rainfed-irrigation division in my model is to capture vulnerability of rainfed agriculture. Hence irrigation agriculture stands for non-rainfed rural activities.
often referred to as the industry-based technology assumption (Miller and Blair 1985; p.165).^8

The mapping processes are as follows (cf. Miller and Blair 1985). First, let me introduce the following matrix notation.

- The make matrix \((V)\); a \(3 \times 34\) matrix of which rows are production activities and columns are commodities.
- The use matrix \((U)\); a \(34 \times 3\) matrix of which rows are commodities and the columns are production activities.
- The final commodity demand matrix \((E)\); a \(34 \times 5\) matrix of which rows are commodities and the columns are institutions. Note that subsidy account, which pays money to commodity accounts, is treated as an institution in addition to household, government, rest of the world and saving account.
- The vectors of commodity-based trade-and-tax \((C_i)\); row vectors with 34 elements represent transactions from each commodity account to \(i^{th}\) account, where \(i = \text{ROW, TAX, SUB, and TNT}\).

All of these matrices and vectors are immediately obtained from the reclassified SAM as its parts.

Let \(v_{ij}\) denote \(i^{th}\) row-\(j^{th}\) column of \(V\). The vector of total production of commodity \((Q)\) is obtained as a row vector with 34 elements which represent total output of commodities, i.e. \(q_j = \sum_i v_{ij}\). Similarly, the vector of industry total input \((X)\) is obtained as a row vector with three elements representing total input of production activities, i.e. \(x_j\) is a column sum of all elements of \(j^{th}\) production activity account of the reclassified SAM.

---

^8 The alternative assumption is that a composition of an industry’s product in terms of commodity input is fixed, which is referred to as the commodity-based technology assumption. For the discussion on choice between these two technology assumptions, see p.166 of Miller and Blair (1985), which is based on Stone (1961).
Now we can derive the following two matrices which serve as mapping operators between commodities and industries. Note that in this subsection 'A' indicates a diagonal matrix of a vector.

\[ B = U \hat{X}^{-1} : \text{a } 34 \times 3 \text{ matrix represents the value of commodity required as intermediate input to produce one unit value of output of industry, and} \]

\[ D = V \hat{Q}^{-1} : \text{a } 3 \times 34 \text{ matrix represents a fraction of output value of industry to produce one unit value of commodity.} \]

It can be interpreted that the \( D \) matrix is an operator to map commodity demand to industry demand and the \( B \) matrix is the commodity-by-industry version of technological matrix. These operators provide all the necessary information.

First, \( A = DB \), a \( 3 \times 3 \) matrix, functions as a conventional technological matrix which represents the value of industrial output as intermediate input to produce one unit value of industries' output. Similar to conventional input-output table, the intermediate goods flow among industries are given as \( Z = AX \) \( (3 \times 3) \).

The final industry product demand matrix is derived as \( Y = DE \) \( (3 \times 5) \).

Transactions from each industry account to \( i^{th} \) account, where \( i = \text{ROW, TAX, SUB, and TNT} \), are derived as \( F_i^T = DC_i^T \) in which superscript \( T \) denotes transposed vectors. The derived \( F_i \) is a 3-element row vector. Note that indirect taxes appear both in production activity and commodity accounts in the original SAM and these two accounts must be combined after this mapping operation.

(3) The aggregate version SAM

The aggregate version SAM is shown in Table 7, in which intra-sectoral intermediate goods flows (such as a transaction from \( R \) to \( R \)) are eliminated.
Table 7 Aggregate version of Moroccan SAM for 1994

<table>
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<tr>
<th></th>
<th>L</th>
<th>S</th>
<th>A</th>
<th>Q</th>
<th>K</th>
<th>HH</th>
<th>GOV</th>
<th>ROW</th>
<th>SAV</th>
<th>R</th>
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<th>U</th>
<th>TAX</th>
<th>SUB</th>
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<td>Total</td>
<td>42.1</td>
<td>74.0</td>
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<td>262.1</td>
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<td>116.7</td>
<td>322.9</td>
<td>38.9</td>
<td>3.2</td>
<td>28.4</td>
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</tr>
</tbody>
</table>

Note: Codes are explained in (1) of this subsection.

5.3.3 Other datasets

All the other input data for the model calibration are as follows.

- Total population [person]: 26,590,000
- Rural labour force [person]: 5,024,400
- Urban unskilled employment [person]: 2,279,666
- Urban skilled employment [person]: 1,590,734
- Urban unemployment rate* [%]: 22.3
- Public water service coverage in rural areas* [%]: 20
- Water delivery loss in domestic water supply [%]: 30
- Real interest rate [% per year]: 9.55
- Irrigation water charge [1994 DH / m³]: 0.347
- Irrigation area [ha]: 863,800
- Total water supply [million m³ / year]: 11,377

* Based on the value in 1995.

All the exogenous input data for the model validation are shown in Table 8.
### Table 8 Exogenous input data for model validation

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</tr>
</thead>
<tbody>
<tr>
<td>Production risk factor [average of '75~'02 = 1]</td>
<td>1.60</td>
<td>0.44</td>
<td>1.71</td>
<td>0.85</td>
<td>1.13</td>
<td>0.75</td>
<td>0.38</td>
<td>0.90</td>
<td>1.08</td>
</tr>
<tr>
<td>Urban minimum wage [1994 DH/year]</td>
<td>7,626</td>
<td>7,098</td>
<td>7,634</td>
<td>7,549</td>
<td>7,522</td>
<td>7,481</td>
<td>8,108</td>
<td>7,966</td>
<td>7,919</td>
</tr>
<tr>
<td>Domestic water charge [1994 DH/m³]</td>
<td>2.50</td>
<td>2.58</td>
<td>2.55</td>
<td>2.50</td>
<td>2.88</td>
<td>2.98</td>
<td>5.20</td>
<td>5.11</td>
<td>5.08</td>
</tr>
<tr>
<td>Irrigation water charge [1994 DH/m³]</td>
<td>0.347</td>
<td>0.364</td>
<td>0.383</td>
<td>0.402</td>
<td>0.422</td>
<td>0.443</td>
<td>0.465</td>
<td>0.488</td>
<td>0.513</td>
</tr>
<tr>
<td>Irrigation land charge [1994 DH/m³]</td>
<td>3,373</td>
<td>3,542</td>
<td>3,719</td>
<td>3,905</td>
<td>4,100</td>
<td>4,305</td>
<td>4,520</td>
<td>4,746</td>
<td>4,983</td>
</tr>
<tr>
<td>Domestic water service coverage [%]</td>
<td>20.0</td>
<td>24.0</td>
<td>27.7</td>
<td>31.3</td>
<td>34.7</td>
<td>37.9</td>
<td>41.0</td>
<td>43.9</td>
<td>46.7</td>
</tr>
</tbody>
</table>

Real exchange rates are obtained based on the nominal exchange rates with eliminating inflation of both the local currency (DH) and the international currency (USD) using GDP deflators.

Production risk factors are given as a proportion of each year’s non-industrial crop production in physical term to its average for the period 1975 – 2002.

Urban minimum wage rates are based on the actual rates with deflating by GDP deflator. Number of working hours per year is assumed to be constant at the value estimated by dividing calibrated annual minimum wage (to be explained later) by the actual minimum wage rate in 1994.

Domestic water charge is estimated by assuming per capita domestic water consumption of 120 lcd during the 1990’s and 100 lcd since 2000 and applying an average of 10 city’s water tariff in real term.9

Irrigation water charge in the base year is based on the estimated total irrigation water cost by Löfgren et al. (1997). A 5 % annual price escalation is then applied.

---

9 The value in 1994 is calibrated as explained later. It is assumed that a drastic (more than 70 %) price rise in 2000 induces reduction of domestic water consumption.
Irrigation land charge in the base year is estimated by dividing irrigation land factor payment recorded in SAM by the estimate of total irrigation area of 0.864 million hectares. A 5% annual price escalation is then applied.

Domestic water service coverage is estimated using the calibrated production function such that the value becomes 20% in 1994 and 38% in 1999.

The observed data for the model validation are shown in Table 9.

**Table 9 Observed data for validation**

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Real interest rate [% per year]</td>
<td>9.6</td>
<td>-</td>
<td>9.6</td>
<td>-</td>
<td>-</td>
<td>12.5</td>
<td>11.6</td>
<td>10.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Urban unemployment rate [%]</td>
<td>-</td>
<td>22.3</td>
<td>17.8</td>
<td>16.7</td>
<td>-</td>
<td>21.8</td>
<td>21.2</td>
<td>-</td>
<td>18.2</td>
</tr>
<tr>
<td>Total value of export (FOB) [billion 1994 DH]</td>
<td>51.0</td>
<td>54.6</td>
<td>55.0</td>
<td>60.3</td>
<td>61.5</td>
<td>65.6</td>
<td>69.2</td>
<td>69.6</td>
<td>74.1</td>
</tr>
<tr>
<td>Total value of import (FOB) [billion 1994 DH]</td>
<td>70.4</td>
<td>74.3</td>
<td>72.6</td>
<td>76.3</td>
<td>81.4</td>
<td>87.0</td>
<td>99.4</td>
<td>99.1</td>
<td>103.0</td>
</tr>
</tbody>
</table>

5.4 Model calibration

5.4.1 Overall procedure

The applied model has been calibrated based on the above data for the year 1994, the base year of this study. It should be noted that only the first-stage optimisation is subject to be calibrated since the government decision does not require any calibration. This is another advantage to separate decision-making process of the government from that of the private agents. Calibration requires an assumption that the observed decisions satisfy optimality conditions, which seems much less strong for the private agents’ decisions than for those of the government. Calibration procedure for this model requires particular attention to the following two issues.

One is the fact that ‘one person’ in the model represents one labour force age person and his/her dependents. This means that the number of households $N$ is not equal to total population in the base year, even though initial household size is normalised at unity. Moreover, this is important to set, or to judge relevance of, per capita values, particularly per capita consumption.
The other issue is the physical unit of commodity or, as its dual concept, the price of commodity. One unit of, say, urban modern product in the model is defined as a composite of highly heterogeneous commodities such as cars, houses, business consulting services, and so on. Obviously there is no information about the price of such a composite commodity, but there must be a particular relationship between the price and the physical unit of commodity that can be compatible with the model. In my model this relationship is represented by the technological parameters in the production function. For each commodity, technological parameter is calibrated given arbitrarily set commodity price.

The overall calibration procedure is as follows. Calibration equations are derived from either the optimal conditions or the optimal solutions of the first-stage optimisation explained in Chapter 4. In the explanation $SAM(i, j)$ denotes the $i$th row-$j$th column of the aggregate SAM, and super impose "^" indicates either observed or calibrated values.

**Step 1:** Calibrate the number of households and urban labour allocation:

$$\hat{N} = \hat{L}_{rural} + \left(\hat{L}^U + \hat{L}^S\right)/(1 - \hat{\theta}^U),$$  

in which $L^\text{rural} \equiv L^R + L^I$, $\hat{\theta}^E = \frac{(1 - \hat{\theta}^U)\hat{L}^U}{\hat{L}^U + \hat{\theta}^U\hat{L}^S}$,  

$$\hat{L}^S = \frac{\hat{L}^S}{\hat{N}},$$  

and $\hat{L}^U = \frac{\hat{L}^U}{\hat{N}\hat{\theta}^E}$.

**Step 2:** Calibrate 'per user' domestic water consumption $\hat{q}^H$ and average per capita water consumption $\hat{\bar{q}}^H$:

$$\hat{q}^H = \frac{\alpha^O SAM(\text{GOV}, HH)}{\hat{p}\hat{N} \left\{ \hat{\theta} + (\hat{L}^U + \hat{L}^S)(1 - \hat{\theta}) \right\}},$$  

in which $\alpha^O$ is a fraction of water expenditure to total transaction from HH to GOV, and

$$\hat{\bar{q}}^H = \hat{q}^H \left\{ \hat{\theta} + (\hat{L}^U + \hat{L}^S)(1 - \hat{\theta}) \right\} + \alpha_{in}(1 - \hat{L}^U - \hat{L}^S)(1 - \hat{\theta}).$$

---

10 The distinction between "per user" and "per capita" is important throughout calibration and validation processes. A part of population is not covered by public water supply service.
Step 3: Calibrate parameters $\phi_i$ and $\beta_{ik}$:

$$\hat{\phi}_i = \sum_{k=R,I,U} \frac{SAM(i,HH)}{SAM(k,HH) + \eta^Q \cdot SAM(GOV,HH)}$$

for $i = R, I, U$, where

$$\eta^Q = \frac{\hat{q}^H}{q^H \left( \hat{\theta} + \left( \hat{r}_U + \hat{r}_S \right) \left( 1 - \hat{\theta} \right) \right)}$$

and

$$\hat{\phi}_Q = \sum_{k=R,I,U} \frac{\eta^Q \cdot SAM(GOV,HH)}{SAM(k,HH) + \eta^Q \cdot SAM(GOV,HH)}.$$

$$\hat{\beta}_{ik} = \sum_{j=Factors} \frac{SAM(k,i)}{SAM(j,i)}$$

for $i = R, I, U$, in which Factors = L, S, K, A, Q.

Step 4: Calibrate wage rates and rural labour allocation:

$$\hat{\omega}^U = \frac{SAM(L,U)}{L^U}, \quad \hat{\omega}^S = \frac{SAM(S,U)}{L^S},$$

and

$$\hat{\omega}^L = \frac{\hat{\theta}^E \cdot \hat{\omega}^U / \hat{\theta}^\varphi_0 - \hat{\varphi}_{q_{inn}} \cdot \left( \hat{N}_{pop} / \hat{N} \right) \left( 1 / \hat{\theta} - 1 \right)}{1 - \left( 1 - \hat{\theta} \right) \cdot \varr}$$

where $N_{pop}$ is total population, and

$\varr = 3.65 \ [m^3/\text{person/year}]$, which is equivalent to 10 [led], and $\varr = 0.2$ are chosen.\(^{12}\)

$$\hat{\omega}^L = \frac{SAM(L,I)}{\hat{N} \cdot \hat{\omega}^L},$$

and

$$\hat{r} = \frac{\hat{I}_{rural}}{\hat{N}} \cdot \hat{I}^L, \quad \omega^R = \frac{\hat{L}_{rural}}{\hat{N}} - \hat{I}^L.$$

Step 5: Calibrate rates of import, product and income taxes and export subsidy:

\(^{11}\) An underlying assumption is that the fraction $\alpha^Q$ of the transaction from household account to the government account in the SAM represents domestic water expenditure. $\alpha^Q$ will be calibrated in Step 8 by trial and error.

\(^{12}\) Subsistence amount of water $q_{no} = 10$ led seems reasonable judging from the fact that the rural domestic water demand is estimated at 7.5 led in 1990 (World Bank 1995; p.55). Note that not only a labour force age person but also his/her dependent is counted as 'one capita' for the figures measured in terms of led. 'Penalty parameter' of 0.2 is a rough guess.
\[ \hat{\tau}_M^I = \frac{\text{SAM}(\text{TNT},i)}{\text{SAM}(\text{ROW},i)} \quad \text{and} \quad \hat{\tau}^I = \frac{\text{SAM}(\text{TAX},i)}{\sum_{j=\text{Prod}} \text{SAM}(j,i)} \quad \text{for } i = R, I, U, \text{ in which } \text{Prod} = L, S, K, A, Q, R, I, U, \]

\[ \hat{s}^I = \frac{\text{SAM}(i,\text{SUB})}{\text{SAM}(i,\text{ROW})} \quad \text{for } i = I, U, \text{ and} \]

\[ \hat{\tau}_H = \frac{\text{SAM}(\text{TAX},HH)}{\sum_{\forall l} \text{SAM}(i,HH) - \text{SAM}(\text{TAX},HH)}. \]

**Step 6**: Calibrate trade parameters (see Subsection 5.4.2),

**Step 7**: Calibrate technological parameters (see Subsection 5.4.3), and

**Step 8**: Calibrate remaining parameters such as \( \rho, \sigma \), rates of water loss, etc. as well as domestic water consumption and so on. (see Subsection 5.4.4).

Below the last three steps are explained.

### 5.4.2 Calibration of trade parameters

Recall the optimal imports and exports derived in Chapter 4.

\[ \hat{E}^I = \Omega_E^I \hat{\tau}^I \quad \text{and} \quad \hat{M}^I = \left( \frac{1 + s^I}{1 + \hat{\tau}_M^I} \right)^{\sigma_M^I} \left( \frac{\delta_M^I}{1 - \delta_M^I} \right)^{\sigma_M^I} \left( 1 - \Omega_E^I \right) \hat{\tau}^I. \]

First we derive \( \Omega_E^I = \frac{\hat{E}^I}{\hat{\tau}^I} = \frac{\text{SAM}(i,\text{ROW})}{\sum_{k=\text{Prod}} \text{SAM}(k,i)} \left( \frac{1 + \hat{s}^I}{1 + \hat{\tau}^I} \right) \).

The parameters of the CES function for import composite goods are then calibrated as

\[ \hat{\delta}_M^I = \frac{\Omega_M^I}{1 + \Omega_M^I}, \quad \text{in which} \]

\[ \Omega_M^I = \left( \frac{1 + \hat{\tau}_M^I}{1 + \hat{s}^I} \right)^{\sigma_M^I} \left[ \frac{\text{SAM}(\text{ROW},i)}{\sum_{k=\text{Prod}} \text{SAM}(k,i) - \text{SAM}(i,\text{ROW})} \right]^{1/\sigma_M^I}. \]
CES elasticity of substitution $\sigma_M$ are determined as $\sigma_M^R = 2.0$, $\sigma_M^I = 3.0$, $\sigma_M^U = 5.0$ using (a) values reported in literature, and (b) the proportion of import to commodity production (7.1% for rainfed agriculture, 15.1% for irrigation agriculture and 32.5% for urban modern products) as a proxy of substitutability between traded goods and domestically produced goods.\(^\text{13}\)

5.4.3 Calibration of technological parameters in production functions

It is assumed that the world price of each commodity in terms of 1994 USD ($p^w$) is constant. Since the choice of these commodity prices does not affect the simulation results, they are normalised at unity. Note that the unit of these prices are 1994 USD per physical unit of commodity $i$, in which $i = R, I, U$.

Given the world commodity prices, the rates of import and product taxes and export subsidy as well as trade parameters, the technological parameters are calibrated as follows. Note that the capital depreciation rate is set at 0.07, which is widely used in literature.\(^\text{14}\)

First, notice the producer and net producer prices are calibrated as

\[ \hat{p}_{i}^{p} = \left(1 + \frac{r}{1 + n_i} \right) p^w r^e \quad \text{and} \quad \hat{p}^{i} = \frac{\sum_{k=\text{Factor}} \text{SAM}(k, i)}{\sum_{j=\text{Prod}} \text{SAM}(j, i)} \hat{p}_{i}^{p}, \text{in which} \]


Similarly, the consumer prices are calibrated as

\[ \hat{p}_{j}^{c} = \hat{p}^w \left(1 + \frac{n_i}{n_M} \right) \eta_M, \text{in which} \]

\(^{13}\) Löfgren et al. (1999) assign values between 2 and 7 to $\sigma_M$ of each disaggregated commodity, with higher value for grains. Agénor and Aynaoui (2003) employ $\sigma_M = 0.8$ for agricultural products and $\sigma_M = 1.0$ for urban products.

\(^{14}\) King and Levine (1994) employ $\delta = 0.07$ to construct cross-country dataset of physical capital stock. Agénor and Aynaoui (2003) choose $\delta = 0.08$ for Moroccan private capital.
\[
\eta_M = \frac{(1 + \delta^l)^{\sigma_M} (1 + \delta^l)^{\sigma_{M^{-1}}} + (1 - \delta^l)^{\sigma_M} (1 + \delta_M^l)^{\sigma_{M^{-1}}}}{(\delta_M^l)^{\sigma_M} (1 + \delta^l)^{\sigma_{M}} + (1 - \delta_M^l)^{\sigma_M} (1 + \delta_M^l)^{\sigma_{M}}}
\]

From these price expressions, the equilibrium real interest rate expression, Eq. (4.11), are modified into

\[
\hat{\tau} + \hat{\delta} = \frac{\hat{\beta}_{U^K}}{\hat{\beta}_{U^C}} \left\{ \tau_U \hat{\beta}_{U^S} \left( \frac{\hat{\beta}_{U^L}}{\hat{w}^U} \right) \right\} \left\{ \tau_U \hat{\beta}_{U^S} \left( \frac{\hat{w}^U}{\hat{w}^S} \right) \right\}
\]

This gives the calibration equation of \( \hat{\tau} \) as

\[
\hat{\tau}_U = \frac{1}{\hat{\beta}_U} \left\{ \frac{\hat{\beta}_{U^C} (\hat{\tau} + \hat{\delta})}{\hat{\beta}_{U^C}} \right\} \left\{ \frac{\hat{\beta}_{U^L} (\hat{\tau} + \hat{\delta})}{\hat{\beta}_{U^L}} \right\}
\]

Similarly, \( \hat{\tau}^l \) and \( \tau^R \) are calibrated by the following equations derived from the equilibrium irrigation sector wage and urban unskilled labour allocation solutions.

\[
\hat{\tau}^l = \frac{1}{\hat{\beta}_U} \left\{ \frac{\hat{\beta}_{A^C} (\hat{\tau} + \hat{\delta})}{\hat{\beta}_{A^C}} \right\} \left\{ \frac{\hat{\beta}_{A^L} (\hat{\tau} + \hat{\delta})}{\hat{\beta}_{A^L}} \right\}, \text{ and}
\]

\[
\hat{\tau}^R = \frac{\sum_i \text{SAM (i, R)} \left\{ \hat{N} (\hat{\tau} + \hat{\delta}) \hat{p}_{U^C} \right\}^{\beta_{xx}}}{\text{SAM (K, R)}} / D^{\tau R}, \text{ in which}
\]

\[
D^{\tau R} = \left\{ \frac{1 - (1 - \delta)^{\tau}}{1 - \tau_U - \tau^S} \right\} \frac{\hat{\beta}_{IL} \{\text{SAM (K, U) + SAM (K, I)}\}}{\hat{\beta}_{IL} \hat{w}^I \hat{N}}
\]

\[
+ \frac{\hat{\beta}_{IL} \hat{p}_{U^C} (\hat{\tau} + \hat{\delta})^{\hat{\beta}_{IL}} \left\{ \frac{1}{\hat{\tau}_U \hat{p}_U} \left( \frac{\hat{w}^U}{\hat{w}^S} \right) \right\}^{\beta_{xx}}}{\hat{\beta}_{IL} \hat{w}^I \hat{N}}
\]

5.4.4 Calibration of remaining parameters

The last step in the calibration of the model deals with all the remaining parameters (\( \phi, \rho \) and \( \sigma \)) as well as the rates of domestic water charge, water expenditure fraction \( \alpha^Q \), and water delivery loss in irrigation. They are calibrated such that (a) the computed optimal household expenditure and total water supply are close to the
observed values, and (b) the computed per user domestic water consumption is close to an estimate based on which the rate of domestic water charge is determined. The latter may require some explanation. Recall that the domestic water tariffs in Morocco employ a tiered-structure in which unit water price differs depending on water consumption. Consumption levels of, say, 100 lcd and that of 150 lcd result in different rates of domestic water charge, which in turn affect the computed value of optimal domestic water consumption. Therefore this last step in the calibration has been done by trial and error. Once preference parameters and the rate of water charge are calibrated, total water supply is adjusted to coincide with the observed value by calibrating water delivery loss in irrigation.

Regarding the last step of the calibration process, two caveats must be made: One is the treatment of household income that is not covered by the model. The SAM records transactions from GOV, ROW, and TNT to the household account of which share to total household income reaches 14.2 %. It cannot be excluded since it definitely affects households' decision of the optimal consumption. This extra income is thus treated, both in the calibration and validation, as a part of household income subject to the no-perfect foresight household expectations, but it is excluded from policy simulations. The other is the fact that the computed optimal household expenditure with plausible values of $\tilde{\omega}$ and other preference parameters has been bound to underestimate the observed value. One possible explanation is that the households' expectation of rainfed production would be significantly affected by the current production. Nevertheless, the assumption of risk-averse forward-looking farmers, which is represented by $\tilde{\omega} < 1$, is maintained. The degree of underestimation, 6.4 % in the base case, seems acceptable judging from the complexity of the model.

5.4.5 Calibration results

The results of the calibration process are as follows.

---

15 Remind that the base year (1994) was one of the best crop years with $\omega_r = 1.6$.

16 The GAMS code for calibration is included in the code for validation shown in Appendix B2.
Number of households: $N = 9,621,410$.

Pure rate of time preference: $\rho = 0.075$.

Depreciation rate: $\delta = 0.07$.

Elasticity of marginal felicity: $\sigma = 10$.

Expected value of production risk factor: $\bar{\sigma} = 0.9$.

Commodity weight in satisfaction production:

$$\varphi_R = 0.119, \varphi_I = 0.240, \varphi_U = 0.631, \varphi_2 = 0.010.$$  

Factor share of production function: $\beta_{RK} = 0.348, \beta_{RL} = 0.383, \beta_{IK} = 0.409$,  
$\beta_{IL} = 0.455, \beta_{IA} = 0.074, \beta_{IQ} = 0.061, \beta_{UK} = 0.438, \beta_{UL} = 0.107, \beta_{US} = 0.455$.

Technological parameter in production function:

$$\tau_R = 140.5, \tau_I = 63.3, \tau_U = 70.1.$$  

Input-output coefficient:

$$a_{RU} = 0.146, a_{RU} = 0.174, a_{JR} = 0.080, a_{RU} = 0.453, a_{UR} = 0.075, a_{UI} = 0.136.$$  

Consumer commodity price [1994 DH/unit]:

$$p_R^e = 9.849, \quad p_I^e = 10.617, \quad p_U^e = 10.097.$$  

Producer commodity price [1994 DH/unit]:


Initial levels of private capital stock [unit]: $m_0 = 5,445.3, K_0 = 385.7$.

Rates of income tax: $\tau_H = 0.062$.

Rates of product tax: $\tau^R = 0, \tau^I = 0.031, \tau^U = 0.096$.

Rates of import tax: $\tau^R_K = 1.024, \tau^I_K = 0.675, \tau^U_K = 0.229$.

Rates of export subsidy: $s^R = 0, s^I = 0.061, s^U = 0.043$.

CES elasticity of substitution:

$$\sigma^R_M = 2.0, \quad \sigma^I_M = 3.0, \quad \sigma^U_M = 5.0,$$

CES import preference parameter:

$$\delta^R_M = 0.354, \quad \delta^I_M = 0.470, \quad \delta^U_M = 0.496.$$  

Fraction of water expenditure to SAM(GOV, HH): $\alpha^O = 0.86$.

Domestic water charge rate [1994 DH/ m³]: $p = 2.497$.

Irrigation water charge rate [1994 DH/ m³]: $p^I = 0.347$.


Per user domestic water consumption [l/day]: $q^I = 120$.

Water delivery loss in domestic water supply [%]: 30.

Water delivery loss in irrigation [%]: 32.6.
5.5 Validation

5.5.1 Base case

The model calibration described in the previous section was based on data/information in single year. The statically calibrated model is then dynamically validated with time-series data of the following variables as exogenous input.\(^{17}\)

- Production risk factor
- Real exchange rate
- Urban minimum wage
- Rates of public charges (domestic and irrigation water, irrigation land)
- Public water service coverage in the rural areas

The following four variables are selected as validation criteria and their observed values and the simulated values will be compared.

- Real interest rate
- Urban unemployment rate
- Total value of exports
- Total value of imports

Trade data reported in IFS are significantly smaller than those recorded in the SAM (export: 28\%, import: 18\%). These data are proportionally scaled-up such that the absolute level of trade data in 1994 coincides. Taxes, subsidies and import taxes are fixed at the 1994 level.

As the base case, a 5\% annual price escalation of irrigation water and land charges in real term is introduced. Judging from recent revision of domestic water tariff table and the government policy towards full-cost recovery in irrigation agriculture, this assumption is not unrealistic. The base case results are shown in Figure 12.

---

\(^{17}\) Berentsen et al. (1996) distinguish three types of model validation process; technical validation which covers logical consistency, data selection, and methodological issues; operational validation which concerns reproducibility of reality; and dynamic validation which concerns durability of the model through its life cycle. This section explains operational validation of the applied model.
Except for the failure to capture fluctuation of real interest rates, the simulated values quite well reproduce the observed trends. Considering the rather wide scope and consequent complexity of the model, the validation seems quite successful.

5.5.2 Sensitivity analysis

The robustness against parameter values is tested by sensitivity analysis in which each parameter value is changed upward and downward by 50%. One of the most important endogenous variables, the optimal consumption of satisfaction that determines the social welfare, is added into the sensitivity analysis.

Before discussing the results, it might be worth mentioning that each sensitivity run affects the simulated trajectories in a way that the trajectories pivot around the initial point.\(^{18}\) Figure 13 illustrates this fact.

---

\(^{18}\) Only the exception is \(c\). Changing \(\delta\) or \(\sigma\) causes a parallel shift, while changing other parameters results in pivot around, or converging to, the terminal point.
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Urbanunemploymentrate' 20%
15%
-ý - Base case (rho= 0.075)
- - Low (rho = 0.0375)
- ( - High (rho = 0.1125)
10%

Figure 13 Illustration of effects of changing parameter values

The results of sensitivity analysis are summarised in Table 10.

Table 10 Results of sensitivity analysis
[Percentage change of mean value from the base case]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario</th>
<th>c</th>
<th>r</th>
<th>ρU</th>
<th>E (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escalation rate of</td>
<td>50% Low</td>
<td>0.4 %</td>
<td>1.0 %</td>
<td>−7.3 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>p^A, p^W</td>
<td>50% High</td>
<td>−0.4 %</td>
<td>−0.9 %</td>
<td>7.1 %</td>
<td>−0.4 %</td>
</tr>
<tr>
<td>β</td>
<td>50% Low</td>
<td>−1.8 %</td>
<td>−2.9 %</td>
<td>−4.2 %</td>
<td>1.8 %</td>
</tr>
<tr>
<td></td>
<td>50% High</td>
<td>1.8 %</td>
<td>3.2 %</td>
<td>4.3 %</td>
<td>−1.8 %</td>
</tr>
<tr>
<td>σ</td>
<td>50% Low</td>
<td>−1.3 %</td>
<td>−1.9 %</td>
<td>−2.8 %</td>
<td>1.2 %</td>
</tr>
<tr>
<td></td>
<td>50% High</td>
<td>0.5 %</td>
<td>0.7 %</td>
<td>1.0 %</td>
<td>−0.4 %</td>
</tr>
<tr>
<td>ρ</td>
<td>50% Low</td>
<td>−3.6 %</td>
<td>−5.4 %</td>
<td>−7.9 %</td>
<td>3.3 %</td>
</tr>
<tr>
<td></td>
<td>50% High</td>
<td>2.8 %</td>
<td>5.3 %</td>
<td>7.2 %</td>
<td>−3.0 %</td>
</tr>
<tr>
<td>δ</td>
<td>50% Low</td>
<td>5.6 %</td>
<td>0.1 %</td>
<td>1.2 %</td>
<td>−0.5 %</td>
</tr>
<tr>
<td></td>
<td>50% High</td>
<td>−3.6 %</td>
<td>−0.4 %</td>
<td>−1.1 %</td>
<td>0.4 %</td>
</tr>
</tbody>
</table>

E and M are both proportional to total product and respond exactly the same way.

These results show the robustness of the model against the variation of parameter values. The most parameter-sensitive variable seems urban unemployment rate, in particular against ρ and price escalation rate of irrigation water and land charges.

As a whole the pure rate of time preference ρ is the most influential parameter. The importance of ρ in dynamic optimisation is understandable. The welfare improvement effect of higher ρ, which means more myopic preference, is simply due to the short period of validation, only 8 years. It is expected that for the longer period higher ρ will result in lower welfare as shown in Figure 14.
The observed relatively significant impact of changing depreciation rate on welfare illuminates the importance of private capital stock. This shift is expected to hold for the entire planning horizon.

5.6 Construction of sustainable production functions

5.6.1 Concept of sustainable production function

As explained in Chapter 3, the public sector production functions represent the relationship between public capital stock and production capacity on condition that production and consumption of publicly supplied goods do not endanger sustainability. What I demonstrate here is admittedly rudimentary application of this concept, but the potential of this approach should not be underestimated.

Construction of sustainable production functions in reliable and operational manner requires knowledge of impacts of production activity on ecosystem resilience, consensus on safe minimum standards to maintain the resilience, and technological and engineering knowledge to achieve it. This challenge provides an excellent opportunity to implement truly interdisciplinary study in which each discipline plays a distinct and indispensable role.

In this thesis sustainable production functions are constructed by multiplying the required level of capital stock based on the present engineering practice by a factor
(termed as ‘sustainability coefficient’). Even such a simplified method could provide, I hope, some useful insight about sustainable development policy.

5.6.2 Sustainable raw water production function

The World Bank (1995) reports the expected public expenditure in water sector and the detailed information of dams in Morocco (p.56-58). Based on this information a time series of both raw water supply capacity and public capital stock in raw water production is constructed as shown in Table 11.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>11,072</td>
<td>71.5</td>
<td>6.60</td>
<td>0.7</td>
</tr>
<tr>
<td>1991</td>
<td>11,272</td>
<td>200.9</td>
<td>6.28</td>
<td>0.7</td>
</tr>
<tr>
<td>1992</td>
<td>11,279</td>
<td>6.4</td>
<td>6.54</td>
<td>0.7</td>
</tr>
<tr>
<td>1993</td>
<td>11,291</td>
<td>12.1</td>
<td>6.78</td>
<td>0.7</td>
</tr>
<tr>
<td>1994</td>
<td>11,464</td>
<td>173.3</td>
<td>7.01</td>
<td>0.7</td>
</tr>
<tr>
<td>1995</td>
<td>11,764</td>
<td>300.0</td>
<td>7.22</td>
<td>3.0</td>
</tr>
<tr>
<td>1996</td>
<td>11,816</td>
<td>52.0</td>
<td>9.71</td>
<td>3.0</td>
</tr>
<tr>
<td>1997</td>
<td>13,565</td>
<td>1,749.1</td>
<td>12.03</td>
<td>3.0</td>
</tr>
<tr>
<td>1998</td>
<td>13,573</td>
<td>8.2</td>
<td>14.19</td>
<td>3.0</td>
</tr>
<tr>
<td>1999</td>
<td>13,954</td>
<td>380.7</td>
<td>16.20</td>
<td>3.0</td>
</tr>
<tr>
<td>2000</td>
<td>14,043</td>
<td>88.9</td>
<td>18.06</td>
<td>3.0</td>
</tr>
<tr>
<td>2001</td>
<td>14,182</td>
<td>139.0</td>
<td>19.80</td>
<td>3.0</td>
</tr>
<tr>
<td>2002</td>
<td>14,267</td>
<td>85.0</td>
<td>21.41</td>
<td>3.0</td>
</tr>
<tr>
<td>2003</td>
<td>14,440</td>
<td>172.5</td>
<td>22.91</td>
<td>3.0</td>
</tr>
<tr>
<td>2004</td>
<td>14,732</td>
<td>292.5</td>
<td>24.31</td>
<td>3.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>14,740</td>
<td>8.0</td>
<td>25.61</td>
<td>3.0</td>
</tr>
<tr>
<td>2006</td>
<td>14,841</td>
<td>100.6</td>
<td>26.82</td>
<td>5.0</td>
</tr>
<tr>
<td>2007</td>
<td>14,968</td>
<td>127.4</td>
<td>29.94</td>
<td>5.0</td>
</tr>
<tr>
<td>2008</td>
<td>15,107</td>
<td>138.5</td>
<td>32.84</td>
<td>5.0</td>
</tr>
<tr>
<td>2009</td>
<td>15,315</td>
<td>206.5</td>
<td>35.54</td>
<td>5.0</td>
</tr>
<tr>
<td>2010</td>
<td>15,370</td>
<td>57.0</td>
<td>38.06</td>
<td>5.0</td>
</tr>
<tr>
<td>2011</td>
<td>15,573</td>
<td>203.0</td>
<td>40.39</td>
<td>5.0</td>
</tr>
<tr>
<td>2012</td>
<td>15,593</td>
<td>20.0</td>
<td>42.56</td>
<td>5.0</td>
</tr>
<tr>
<td>2013</td>
<td>15,730</td>
<td>137.0</td>
<td>44.58</td>
<td>5.0</td>
</tr>
<tr>
<td>2014</td>
<td>15,730</td>
<td>339.0</td>
<td>46.46</td>
<td>5.0</td>
</tr>
<tr>
<td>2015</td>
<td>16,069</td>
<td>339.0</td>
<td>48.21</td>
<td>5.0</td>
</tr>
<tr>
<td>2016</td>
<td>16,069</td>
<td>49.84</td>
<td>51.35</td>
<td>5.0</td>
</tr>
<tr>
<td>2017</td>
<td>16,102</td>
<td>32.5</td>
<td>52.75</td>
<td>5.0</td>
</tr>
<tr>
<td>2018</td>
<td>16,102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2019</td>
<td>16,244</td>
<td>142.0</td>
<td>54.06</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Source: Adopted from World Bank (1995)

The annual depreciation rate is assumed to be 7%, the same as for the private capital stock, and the value of initial capital stock is assumed to be 6 billion 1993 DH such that public capital stock does not decrease between 1990 and 1995. Annual public investments in raw water production are based on the World Bank (1995) up to the year 2000, and those after the year 2000 are assumed to be 3 billion 1993 DH until 2005 and 5 billion 1993 DH from 2006.

Based on this table, public capital stock is converted into physical terms based on the calibrated consumer price of capital good, and a sustainability coefficient of 1.3 is applied based on a crude assumption that raw water supply development without
loosing ecosystem resilience requires 30% more capital stock input than the present engineering practice.\textsuperscript{19} This assumption makes, for example, the sustainable production capacity in 1994 be 90% of the observed production capacity with the present engineering practice. The sustainable production function is calibrated as shown in Figure 15.

![Figure 15 Sustainable raw water production function](image)

The calibrated production function consists of following equations.

\( R_t = 11,829.2 \times \left(1 - 45.53e^{-0.0085GR^R}\right) \) for \( GR^R < 1,250 \),

\( R_t = 13,591.0 \times \left(1 - 2,876.45e^{-0.008GR^R}\right) \) for \( 1,250 \leq GR^R < 1,827 \),

\( R_t = 14,342.2 \times \left(1 - 5.16e^{-0.0025GR^R}\right) \) for \( 1,827 \leq GR^R < 2,757 \),

\( R_t = 15,135.3 \times \left(1 - 3.59e^{-0.0015GR^R}\right) \) for \( 2,757 \leq GR^R < 3,855 \),

\( R_t = 17,020.9 \times \left(1 - 0.38e^{-0.0003GR^R}\right) \) for \( 3,855 \leq GR^R < 6,417 \), and

\( R_t = 9,911.2 \times (GR^R)^{0.1} - 7,746.36 \) for \( GR^R \geq 6,417 \),

in which \( R_t \) : sustainable raw water production [million m\textsuperscript{3} /year], and \( GR^R \) : public capital stock in raw water production [million unit].

\textsuperscript{19} The model sensitivity to sustainability coefficients is tested in Chapter 6.
5.6.3 Sustainable treated water production function

Based on treated water production data in the ASM with treated water supply public expenditure reported in the World Bank (1995), a time series of treated water supply capacity and public capital stock in treated water production is constructed as shown in Table 12, with assumptions that the annual depreciation rate is 7%, that the value of initial capital stock is 1 billion 1993 DH, and that delivery loss is 30% in 1990 with decreasing 0.5% per year when the annual investment is 0.2 billion 1993 DH and 1.35% per year after 1995.

<table>
<thead>
<tr>
<th>Year</th>
<th>Water production [mil. m³]</th>
<th>ONEP</th>
<th>Regies</th>
<th>Delivery Loss</th>
<th>Total supply [mil. m³]</th>
<th>Capital stock [Billion 1993DH]</th>
<th>Investment [Billion 1993DH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>387.0</td>
<td>107.0</td>
<td></td>
<td>32.5%</td>
<td>333.5</td>
<td>1.00</td>
<td>0.2</td>
</tr>
<tr>
<td>1986</td>
<td>393.0</td>
<td>109.0</td>
<td></td>
<td>32.0%</td>
<td>341.4</td>
<td>1.13</td>
<td>0.2</td>
</tr>
<tr>
<td>1987</td>
<td>423.0</td>
<td>116.0</td>
<td></td>
<td>31.5%</td>
<td>369.2</td>
<td>1.25</td>
<td>0.2</td>
</tr>
<tr>
<td>1988</td>
<td>461.0</td>
<td>117.0</td>
<td></td>
<td>31.0%</td>
<td>398.8</td>
<td>1.36</td>
<td>0.2</td>
</tr>
<tr>
<td>1989</td>
<td>478.0</td>
<td>122.0</td>
<td></td>
<td>30.5%</td>
<td>417.0</td>
<td>1.47</td>
<td>0.2</td>
</tr>
<tr>
<td>1990</td>
<td>525.0</td>
<td>128.0</td>
<td></td>
<td>30.0%</td>
<td>457.1</td>
<td>1.57</td>
<td>0.2</td>
</tr>
<tr>
<td>1991</td>
<td>562.0</td>
<td>127.0</td>
<td></td>
<td>29.5%</td>
<td>485.7</td>
<td>1.66</td>
<td>0.2</td>
</tr>
<tr>
<td>1992</td>
<td>587.0</td>
<td>140.0</td>
<td></td>
<td>29.0%</td>
<td>516.2</td>
<td>1.74</td>
<td>0.2</td>
</tr>
<tr>
<td>1993</td>
<td>560.0</td>
<td>126.0</td>
<td></td>
<td>28.5%</td>
<td>490.5</td>
<td>1.82</td>
<td>0.2</td>
</tr>
<tr>
<td>1994</td>
<td>593.0</td>
<td>119.0</td>
<td></td>
<td>28.0%</td>
<td>512.6</td>
<td>1.89</td>
<td>0.2</td>
</tr>
<tr>
<td>1995</td>
<td>583.0</td>
<td>96.0</td>
<td></td>
<td>26.7%</td>
<td>497.9</td>
<td>1.96</td>
<td>1.0</td>
</tr>
<tr>
<td>1996</td>
<td>611.0</td>
<td>91.0</td>
<td></td>
<td>25.3%</td>
<td>524.2</td>
<td>2.82</td>
<td>1.0</td>
</tr>
<tr>
<td>1997</td>
<td>615.0</td>
<td>114.0</td>
<td></td>
<td>24.0%</td>
<td>554.0</td>
<td>3.62</td>
<td>1.0</td>
</tr>
<tr>
<td>1998</td>
<td>630.0</td>
<td>111.0</td>
<td></td>
<td>22.7%</td>
<td>573.0</td>
<td>4.37</td>
<td>1.0</td>
</tr>
<tr>
<td>1999</td>
<td>649.0</td>
<td>105.0</td>
<td></td>
<td>21.3%</td>
<td>593.1</td>
<td>5.06</td>
<td>1.0</td>
</tr>
<tr>
<td>2000</td>
<td>650.0</td>
<td>120.0</td>
<td></td>
<td>20.0%</td>
<td>616.0</td>
<td>5.71</td>
<td>1.0</td>
</tr>
<tr>
<td>2001</td>
<td>658.0</td>
<td>127.0</td>
<td></td>
<td>18.7%</td>
<td>638.5</td>
<td>6.31</td>
<td>1.0</td>
</tr>
</tbody>
</table>


Based on this table, public capital stock is converted into physical terms based on the calibrated consumer price of capital good, and a sustainability coefficient of 1.8 is applied based on a crude assumption that sustainable supply of treated water including high level of wastewater treatment requires 80% more capital stock input than the present engineering practice.\(^\text{20}\) This assumption makes, for example, the

\(^\text{20}\) Remind that the Moroccan government plans to invest one billion 1993 DH annually for the sewerage facilities in addition to one billion 1993 DH investment for the drinking water supply system between 1994 and 2000 (World Bank 1995), and that the current sewerage service is far from satisfactory.
sustainable production capacity in 1994 be 69 % of the observed production capacity with the present engineering practice. The calibrated sustainable production function is shown in Figure 16.

![Figure 16 Sustainable treated water production function](image)

The calibrated production function consists of following equations.

\[ Q_t = 528.1 \times \left(1 - 3.13e^{-0.012G_t}\right) \text{ for } G_t < 503, \]
\[ Q_t = 622.9 \times \left(1 - 0.72e^{-0.003G_t}\right) \text{ for } 503 \leq G_t < 903, \text{ and} \]
\[ Q_t = 82.9(G_t)^{0.3} - 45.29 \text{ for } G_t \geq 903, \]

in which \( Q_t \) : sustainable treated water production [million m\(^3\) /year], and \( G_t \) : public capital stock in treated water production [million unit].

In addition, it is assumed that both the public water supply service coverage in the rural areas and the delivery loss of treated water are determined by the following function of public capital stock \( G \):

\[ \theta_t = 1 - 0.93e^{-0.0008G_t}, \text{ and } Loss-H_t = 0.08 + 0.25e^{-0.0013G_t}, \]

in which \( \theta_t \) : rural water service coverage [dimensionless], and \( Loss-H_t \) : Domestic water delivery loss.

These functions are calibrated such that delivery loss rates of 30 % and 20 % correspond to the capital stock in 1990 and 2000 respectively, and such that rural water service coverage of 20 % and 38 % corresponds to the capital stock in 1994 and 1999.
5.6.4 Sustainable irrigation land production function

Based on irrigation land data (Académie du Royaume du Maroc 2000) with the National Irrigation Development Programme (Löfgren et al. 1997) for the irrigation area expansion after the year 1996 and public expenditure data in irrigation infrastructure (World Bank 1995), a time series of irrigation area and public capital stock in irrigation infrastructure is constructed as shown in Table 13 with assumptions that the annual depreciation rate is 7% and that the value of initial capital stock is 3.5 billion 1993 DH.

Table 13 Irrigation land development

<table>
<thead>
<tr>
<th>Year</th>
<th>Irrigation area (1000 ha)</th>
<th>Capital stock (Billion DH)</th>
<th>Investment (Billion DH)</th>
<th>Year</th>
<th>Irrigation area (1000 ha)</th>
<th>Capital stock (Billion DH)</th>
<th>Investment (Billion DH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>578</td>
<td>3.5</td>
<td>0.5</td>
<td>1990</td>
<td>763</td>
<td>6.2</td>
<td>0.9</td>
</tr>
<tr>
<td>1981</td>
<td>596</td>
<td>3.8</td>
<td>0.5</td>
<td>1991</td>
<td>802</td>
<td>6.6</td>
<td>0.9</td>
</tr>
<tr>
<td>1982</td>
<td>612</td>
<td>4.0</td>
<td>0.5</td>
<td>1992</td>
<td>835</td>
<td>7.1</td>
<td>0.9</td>
</tr>
<tr>
<td>1983</td>
<td>620</td>
<td>4.2</td>
<td>0.5</td>
<td>1993</td>
<td>856</td>
<td>7.5</td>
<td>0.9</td>
</tr>
<tr>
<td>1984</td>
<td>644</td>
<td>4.4</td>
<td>0.5</td>
<td>1994</td>
<td>864</td>
<td>7.8</td>
<td>0.9</td>
</tr>
<tr>
<td>1985</td>
<td>653</td>
<td>4.6</td>
<td>0.5</td>
<td>1995</td>
<td>875</td>
<td>8.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1986</td>
<td>670</td>
<td>4.8</td>
<td>0.5</td>
<td>1996</td>
<td>891</td>
<td>9.6</td>
<td>2.0</td>
</tr>
<tr>
<td>1987</td>
<td>693</td>
<td>5.0</td>
<td>0.5</td>
<td>1997</td>
<td>916</td>
<td>10.9</td>
<td>2.0</td>
</tr>
<tr>
<td>1988</td>
<td>709</td>
<td>5.1</td>
<td>0.9</td>
<td>1998</td>
<td>922</td>
<td>12.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1989</td>
<td>734</td>
<td>5.6</td>
<td>0.9</td>
<td>1999</td>
<td>942</td>
<td>13.3</td>
<td>2.0</td>
</tr>
</tbody>
</table>


Based on this table, public capital stock is converted into physical terms based on the calibrated consumer price of capital good, and a sustainability coefficient of 1.5 is applied based on a crude assumption that sustainable supply of irrigation land requires 50% more capital stock input than the present engineering practice. This assumption makes, for example, the sustainable production capacity in 1994 be around 78% of the observed production capacity with the present engineering practice. The calibrated sustainable production function is shown in Figure 17.
The calibrated production function consists of following equations.

\[ A_i = 770.1 \times \left(1 - 1.19e^{-0.003G_i} \right) \text{ for } G_i :< 685, \]

\[ A_i = 790.9 \times \left(1 - 21.09e^{-0.007G_i} \right) \text{ for } 685 \leq G_i :< 914, \text{ and} \]

\[ A_i = 894.1 \times \left(1 - 87.95e^{-0.007G_i} \right) \text{ for } 914 \leq G_i :< 1,429, \text{ and} \]

\[ A_i = 751.5 (G_i)^{0.1} - 663.4 \text{ for } G_i :\geq 1,429, \]

in which \( A_i \): sustainable irrigation land provision [1000 ha /year], and \( G_i \): public capital stock in irrigation land production [million unit].

In addition, the following relationship between public capital stock \( G_i \) and delivery loss of irrigation water is assumed.

\[ Loss-IR_i = 0.15 + 0.3043 e^{-0.007G_i}, \] in which \( Loss-IR_i \): irrigation water conveyance loss.

### 5.7 Conclusions

This chapter provides the necessary information to conduct policy simulations. The case study country, Morocco, is introduced with taking care to explain the specification and key assumptions of the applied model.
The employed data and model calibration procedure is explained in depth in order to ensure reproducibility of this research. Although a calibration procedure itself has highly study specific nature, a detailed explanation of this process, which is rarely found in literature, may be useful for other researchers.

The result of model validation seems successful. Sensitivity analysis reveals parameter-robustness of the applied model. The effects of changes in parameter values are consistent with economic theory as well as our intuition.

Finally sustainable production functions that play a key role in policy simulations are constructed based on a time-series of public capital accumulation and supply capacity development that represents the relationship between supply capacity and public capital stock with present engineering practice that is not necessarily sustainable. Therefore sustainability coefficients, which are rather arbitrarily chosen for illustrative purpose and are subject to sensitivity analysis, are multiplied to the required level of public capital stock.
Chapter 6

Policy Simulation

6.1 Introduction

This chapter reports methodology and results of the policy simulations. The objectives of policy simulations of this thesis are;

- to establish a policy analysis procedure under uncertainty,
- to investigate performance of sustainable development policy alternatives in each policy environment which is determined by exogenous drivers,
- to see policy implications of different level of sustainability coefficients in sustainable production functions, which represent safe minimum standards of a society, and
- to see policy implications of international aid flows that represent a sustainable development policy of the global community.

Concerning the first objective, this study proposes a policy analysis procedure with a clear distinction between the planning and the implementation of policies. Policies are planned based on expectations of uncertain exogenous variables but they must be implemented with realised values of those variables that do not, in general, satisfy the expectations. The policy-making process must accommodate uncertainty not only in optimisation process but also in simulating the implementation process when the expectations are not satisfied. This thesis explicitly incorporates uncertainty into both parts of the policy-making process,
using Monte-Carlo simulation technique. This approach allows policy simulations to deal with uncertainty in a more flexible and practical way than the stochastic dynamic optimisation approach. The remaining objectives are achieved by simulating policy scenarios in which each policy alternative consisting of endogenous policy variables is located in alternative environments determined by exogenous drivers.

This chapter is organised as follows. Before conducting policy simulations, properties of the first-stage solution without supply side constraints are investigated in Section 6.2. This section illuminates both similarity and difference between the analytic and the applied models. It also provides a control against which implications of supply side constraints in the applied model are clarified. Section 6.3 establishes the simulation procedure with a detailed explanation of the policy implementation process under uncertainty. Section 6.4 formulates policy scenarios. Each policy scenario represents a particular combination of endogenous policy variables with a particular environment. The main objectives of policy simulations are (i) to demonstrate potential of the proposed methodology, and (ii) to provide rough idea of policy implications, rather than to provide practical policy advice. In this sense my model is not a fully applied model, but a platform to construct such a model. Section 6.5 reports the simulation results, and discussion on the results is provided in Section 6.6.

6.2 Numerical simulation without supply side constraints

6.2.1 Simulation setting

The numerical simulations in this section correspond to Simulation 1 in Chapter 3 that tests the validity of Proposition 3.3. As in Simulation 1, the simulation model consists of only the first stage optimisation with constant rates of public charges.

The model is almost identical to that for validation, except for the exogenous input data. The following exogenous inputs are given.
- Public charges are fixed at the levels in 1994 and rural water service coverage is set at 100%.
- Urban minimum wage is fixed at the levels in 1994.
- Production risk factors and foreign exchange rates are randomly disturbed around their expected values with standard deviations of 0.366 for the production risk factor and 0.09 for the real exchange rate.¹
- 3 different levels of initial private capital stock are given by multiplying the calibrated private capital stocks in 1994 by 1, 0.5, and 2, respectively.

In order to investigate convergence property of the trajectories, the planning period is set at 100 years. Taking random factors into account, Monte Carlo simulations with 5 trials are employed and the mean values are evaluated.

### 6.2.2 Results

Figure 18 shows the simulated trajectories of consumption and capital stock levels.

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¹ These values are exactly the same in policy simulations. See Subsection 6.3.2 of this chapter.
These trajectories clearly exhibit a convergence tendency, as observed in the analytic model. This convergence property, which precludes the existence of threshold values of initial capital stock below which the economy is driven into a poverty trap, is the most important similarity between the analytic and the applied models. By analogy it is expected that introduction of supply side constraints does not change this convergence tendency.

A distinct difference between the analytic and the applied models is lack of steady-states in the applied model, which reflects the fact that the nationwide productivity of rainfed agriculture in the applied model is fixed and its per capita value decreases with population growth. It is an introduction of trade that enables the applied model to have positive growth in the per capita consumption of satisfaction in spite of the decrease of per capita rainfed agricultural productivity.

It appears prima facie surprising that consumption level of public supplied water, of which price is constant at unity, is disturbed by random shocks more severely than the level of satisfaction consumption. This is so because it is the relative price, not the nominal price, of commodities that directly affects the optimal consumption levels. In terms of relative price the price of publicly supplied water fluctuates the most because other three commodity prices follow the same fluctuation of real exchange rates.

From the viewpoint of social welfare, positive growth of per capita consumption level shown in Figure 18 contributes to welfare improvement. To complete the picture, however, information on both the safe water access rate, which is fixed by assumption in this experiment, and the unemployment rate is also important. Figure 19 shows the optimal trajectories of urban unemployment rate and irrigation labour wage rate, which further endorse the convergence property of the optimal trajectories of the applied model.

---

2 Despite the perfect safe water access, the mean level of satisfaction consumption in the first year is nearly 20% less than the calibrated consumption level in 1994. This is mainly due to elimination of extra income such as transactions from GOV, ROW, and TNT to households (see Subsection 5.4.4 in Chapter 5).
government commits to implement and how the government absorbs the expectation errors.

Among many alternatives, I assume the following implementation rule. The government commits to implement the planned pricing schedules of public charges and to invest (accumulate) the planned amount of public capital in physical terms. This assumption aims at capturing inflexible aspects of public pricing and infrastructure investment policies in practice.

The gap between the plan and the reality is absorbed by introducing safety margins of both the supply capacity and the financial position. In the policy planning process the sustainability constraints and the equations of motion of public capital are modified into, for instance,

\[
\hat{q}_i^t + \hat{q}_i^H N(1 + \nu)^{-1} \left[ \theta_i + \left( H^* + f^* \right)(1 - \theta_i) \right] = \chi^{SC} F^R G_i^R,
\]

and

\[
G_{it+1} - G_i^R = \chi^{OB} \xi_i^R M_i^R / \rho - \delta G_i^R,
\]

in which \(\chi^{SC} \in (0,1)\) and \(\chi^{GB} \in (0,1)\) are safety factors regarding to the sustainability constraints and the government budget, respectively. ³

The lower values of safety factors are associated with the larger safety margins. There is a trade-off between satisfying the sustainability constraints or keeping positive budget balance and the attainable social welfare. The larger safety margins are associated with the lower probability of violating sustainability constraints or running budget deficit and at the same time with the lower social welfare. My model could capture this trade-off by introducing the Monte Carlo method for the policy implementation simulations. ⁴

³ Needless to say, in the simulation of policy implementation the original equations of motion without safety factors are used.

⁴ Monte Carlo simulation of this study is a common application of this technique. For wider application of Monte Carlo simulation, see Rubinstein (1981).
6.3.2 Simulation procedure

The established simulation procedure is as follows.

(i) Set the exogenous policy variables, e.g., investment allocation, tax rates, etc.

(ii) Based on the expectation of production risk factors and foreign exchange rates, endogenously determine the optimal rates of public charges \( p_n, p_t, p_A \), as well as consequent levels of public investment \( I_t^G, I_t^{GR}, \) and \( I_t^{GI} \).

(iii) Introduce random disturbances around the expected value of the production risk factor and foreign exchange rates, which are given as normally distributed random numbers generated by GAMS. Standard deviations are set at 0.366 for the production risk factor based on a coefficient of variation of non-industrial crop production between 1975 and 2002, and 0.09 for the real exchange rate based on a coefficient of variation of real exchange rates for the decade after 1993 when the structural adjustment program finished.

(iv) Conduct Monte Carlo simulations of policy implementation process given randomly disturbed production risk factors and foreign exchange rates. In addition to all the exogenous policy variables, the optimal pricing and investment schedules determined in (ii) are given exogenously.

In order to absorb imbalance between the revenue and the expenditure of the government, it is assumed that the government has a saving account in which surplus budget is saved and deficit is covered by, if possible, dissaving. Only the year in which the government saving account runs a deficit is regarded as the budget deficit year. Based on preliminary Monte Carlo simulations with this assumption, safety factors are set at \( \chi^{SC} = 0.85 \) and \( \chi^{GB} = 0.98 \) such that the probabilities to violate sustainability constraints and to run a government budget deficit become around 10 to 15% for the former and around 5% for the latter under the current environment.
In addition, it is assumed that 20% of import tax revenues is earmarked to water sector public investment.\(^5\) This specification aims at capturing the relatively high dependence of the revenue of the Moroccan government on trade taxes (Economist Intelligence Unit 2002). Without taking into account negative effects of trade liberalisation in terms of reduction of disposable budget, any trade liberalisation proposals are hardly convincing for the policy-makers.\(^6\)

A number of trials of Monte Carlo simulations is set at 100. For demonstrative purpose it seems large enough. The planning period is set at 15 years.

### 6.3.3 Evaluation criteria

Separation of the planning and the implementation processes with introducing Monte Carlo simulation for the latter allows the policy makers to evaluate policy scenarios from wider perspectives. It is expected that some policy could attain better mean values but more risky in terms of larger dispersion and some performs other way around, as the most clearly demonstrated by the choice of safety margins. There is no unique optimal policy but a range of alternative satisficing policies, and a choice of ‘the best’ policy necessarily depends on tastes, for instance the degree of risk averseness, of the policy makers and the stakeholders in the policy-making process.\(^7\) For such a policy making procedure the following evaluation indicators are employed.

a. Social welfare
b. Probability to satisfy the sustainability constraints
c. Probability to satisfy the government budget constraint

---

\(^5\) 1994 SAM records 28.4 billion 1994 DH of import tax revenue. Judging from the fact that the total public expenditure for the water sector during the period 1995-2000 is approximately 7 billion DH according to World Bank (1995), this assumption seems reasonable.

\(^6\) For example, Goldin and Roland-Holst (1995) conduct policy simulations of trade liberalisation without incorporating the effects of revenue reduction for the whole economy. They celebrate the positive effects of combination of trade liberalisation and irrigation water price doubling policy, and simply add that “[t]he government budget still declines appreciably with tariff revenues, but this might be offset by alternative, nondistortionary sources of revenue” (p.190).

\(^7\) This policy making process matches an aphorism of Simon (1982): “Optimizing in a simplified world is an important means for satisficing in the real world” (p.xx).
The indicators (a) - (c) measure the performance of policy from the viewpoint of the original government optimisation problem, and the indicator (d) measures the severity of the least favourable event. In addition to the original utility metric social welfare, money-metric social welfare is derived in order to grasp the magnitude of welfare difference between policy scenarios. Money-metric social welfare is measured by equivalent variations (EV) defined as the amount of income which must be given up to make felicity levels under the base case and under the alternative policy scenarios indifferent. More precisely, the EV in the year $t$ is such a 'constant income stream' for the entire time horizon starting from the year $t$, and it is obtained as

$$EV_t = \frac{\left(1 - \tau_H\right) r_t^* - \nu \left(\hat{c}_t - \hat{c}_t^{base}\right)}{1 + \left(1 - \tau_H\right) r_t^* - (1 + \nu) \left(1 + \left(1 - \tau_H\right) r_t^*\right)} \left(\frac{p_t}{\phi_t}\right)^{\rho_0} \prod_{i = R, I, U} \left(\frac{p_t^{ei}}{\phi_i}\right)^{\rho_i},$$

in which $\hat{c}_t^{base}$ is the realised consumption of satisfaction under the base case scenario, and all the other variables are corresponding to each alternative policy scenario.

The indicators (e) - (f) represent the terminal conditions of state variables that form a part of the original problem, although they have not explicitly been accounted. The indicators (g) and (h) are included as proxies of poverty alleviation in terms of reduction of socially disadvantaged population.

These indicators are obtained for each trial of Monte Carlo simulation, and then the mean and the standard deviation across trials are computed.  

---

8 The GAMS code for policy simulation is attached as Appendix B3.
6.4 Policy scenarios

6.4.1 Policy alternatives

The applied model contains two types of endogenous policy variables: (i) rates of public charges, and (ii) public investment allocation among three types of public capital. The policy alternatives are constructed as their combinations.

(1) Pricing schedule of public charges

There are several CGE studies dealing with water pricing policy in Morocco.

Goldin and Roland-Holst (1995) investigate the effects of doubling the price of irrigation water. As de Melo comments in the discussion section, the model set-up such as perfectly elastic water supply at a fixed cost cannot capture implications of water shortage and does not provide useful insight.

Löfgren et al. (1997) conduct policy simulation in which irrigation water pricing policy is located within rural development policy. They prepare two alternatives of irrigation water tariff; one covers the operation and maintenance cost and the other covers full costs of irrigation water supply. This study also does not take into account the effects of water scarcity on water supply or water price.

The most interesting study in this regard is Decaluwe et al. (1999). They explicitly incorporate the effects of water scarcity into two types of water production function, of which the first type represents the water produced by the existing dams and the second type does the combination of retrieving both surface water and groundwater. The high degree of spatial heterogeneity of water availability in Morocco is captured by differentiating water production technology between the northern and the southern regions with different segmentation of water markets. Then they
Chapter 6 Policy Simulation

conducted policy simulations with Boiteux-Ramsey pricing and marginal cost pricing along with tax reform policies.  

Although the Boiteux-Ramsey pricing is not applicable to my model due to lack of proper cost function, it is possible to derive a marginal cost as a cost of hypothetical additional unit of investment entailed by an infinitesimal additional demand. Considering the importance of marginal cost pricing in the literature, marginal cost pricing was envisaged as an alternative of pricing policy. A test trial of simulation immediately revealed, however, that the marginal costs in my model fluctuate drastically and the magnitude of prices could reach several hundreds times higher than the calibrated rates of charges. As a result, only one pricing policy, the optimal pricing policy such that the sustainable supplies and the optimal demands coincide, is employed.

(2) Public investment allocation

According to the World Bank (1995) the current public expenditure of the Moroccan government is summarised in Table 14.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share</td>
<td>million 1993 DH</td>
<td>Share</td>
<td>million 1993 DH</td>
</tr>
<tr>
<td>Water resource development</td>
<td>36.0%</td>
<td>700</td>
<td>43.0%</td>
<td>3,000</td>
</tr>
<tr>
<td>Drinking water and sewerage</td>
<td>18.0%</td>
<td>350</td>
<td>28.5%</td>
<td>2,000</td>
</tr>
<tr>
<td>Irrigation development</td>
<td>46.0%</td>
<td>900</td>
<td>28.5%</td>
<td>2,000</td>
</tr>
<tr>
<td>Total</td>
<td>100.0%</td>
<td>1,950</td>
<td>100.0%</td>
<td>7,000</td>
</tr>
</tbody>
</table>


9 Boiteux-Ramsey pricing rule is obtained as an interior solution of a consumer surplus maximisation problem under certain budgetary constraint (Boiteux 1956; p.35). Although this rule is designed to achieve the second-best optimality, realisation of the second-best optimality along Boiteux-Ramsey pricing requires several restrictive assumptions regarding to elasticity of supply at marginal cost, convexity of demand in regard to the budgetary requirements, and so on (Dicker 1991).

10 This is corresponding to the long-run marginal cost. In my model the short-run marginal cost is zero because of no variable costs.
The base case of infrastructure investment policy is set at the investment allocation pattern during the period 1995 - 2000. The alternatives of investment policy are prepared as follows.

Base (status-quo): \((\theta^R, \theta^G, \theta^I) = (0.43, 0.285, 0.285)\)

Alt. 1 (domestic water priority): \((\theta^R, \theta^G, \theta^I) = (0.285, 0.43, 0.285)\)

Alt. 2 (irrigation priority): \((\theta^R, \theta^G, \theta^I) = (0.285, 0.285, 0.43)\)

Alt. 3 (raw and domestic water priority): \((\theta^R, \theta^G, \theta^I) = (0.43, 0.43, 0.14)\)

Alt. 4 (domestic water and irrigation priority): \((\theta^R, \theta^G, \theta^I) = (0.14, 0.43, 0.43)\)

Alt. 5 (raw water and irrigation priority): \((\theta^R, \theta^G, \theta^I) = (0.43, 0.14, 0.43)\)

6.4.2 Policy environments

In addition to the abovementioned two types of policy variables, the applied model contains several variables which could be treated as policy variables. For instance the applied model has potential to investigate impacts of revising import tariffs, which are key trade policy variables in the real world. Nevertheless, this thesis regards these variables as exogenous drivers that determine policy environments. This treatment is consistent with the original model design while it is still possible to see the influence of these variables on sustainable development policies. Import tax rates and minimum wage of urban unskilled labour are selected as such exogenous drivers to be investigated, due to their importance in the real policy and relevance to sustainable development.

Further, climate change is also selected as another exogenous driver considering its importance in sustainable development studies.

Each exogenous driver provides alternative environments as follows.

(1) Climate change

The most part of Africa including Morocco has experienced secular climate change in terms of annual rainfall reduction and temperature rise, and several studies warn that climate change in Africa could accelerate in the future (Hulme et al. 1995, Parish and Funnell 1999, Knippertz 2003).
Assessment of climate change impacts entails so high degree of uncertainty that we could not conclude whether annual rainfall will increase or decrease (Hulme et al. 1995, Knippertz 2003). Nevertheless, there seems a consensus that climate change would be manifested as more frequent extreme events such as droughts and floods (Hulme et al. 1995). It is expected that a larger coefficient of variation of rainfed productivity could represent such a climate change scenario.\(^\text{11}\) The status quo value of coefficient of variation (0.366) is selected as the base case (CCbase), and 50% higher value (0.55) is chosen as an alternative environment (CC1).

(2) Trade regime

Since the introduction of the structural adjustment programme in early 1980’s the trade liberalisation has been one of key political issues in Morocco. While Morocco is undertaking gradual trade liberalisation under WTO rules and the Association Agreement with the European Union and most non tariff barriers have been eliminated as a result, there remain considerably high import tariffs which aim at protecting domestic production sectors, in particular agricultural sectors, as well as keeping this important revenue source for the government (Löfgren et al. 1999, Economists Intelligence Unit 2002). This reluctance of the government to voluntarily implement further trade liberalisation supports the idea of the trade regime as an exogenous driver.

The status quo trade regime in 1994 is selected as the base case (TRbase). As an alternative trade regime (TR1), 20% reduction of import tax rate resulting in no earmarked tax revenue to water sector is chosen consulting with Löfgren et al. (1999), as they provide the most detailed account of Moroccan trade liberalisation policy among Moroccan CGE studies introduced so far. Their simulations cover four policy alternatives combining tariff unification at two different levels (29% and 10%) and non-tariff barriers elimination. In their model the government saving level is kept constant by adjusting the rate of value added tax. This specification enables the model to capture the welfare effects of revenue reduction

\(^{11}\) Possibility that climate change would affect water production process is, however, not incorporated.
due to trade liberalisation. Studies without taking into account these negative effects of trade liberalisation cannot provide practically meaningful insights.

(3) Urban minimum wage

A wage gap between urban and rural areas is the principal determinants of urban unemployment rate in Harris-Todaro type migration equilibrium model, which is employed in the applied model. Considering the importance of urban unemployment problem from the social welfare viewpoint, it is interesting to see how wage reduction of urban unskilled labour affects sustainable development policy via reduction of urban unemployment. A motivation to regard the urban minimum wage as an exogenous driver is the fact that it is often determined by lobbying competitions between business organisations and labour unions.

The status quo minimum wage for urban unskilled labour in 1994 is selected as the base case (MWbase), and 10% reduction of the minimum wage is chosen as an alternative case (MW1) consulting with Agénor and Aynaoui (2003) who analyse economic impacts of Moroccan labour market policy using a CGE model. They explicitly introduce not only urban-rural differentiation but also formal-informal dichotomy of urban production sector in order to capture potential impacts of labour market reforms in realistic manner. Their policy experiments consist of 5% minimum wage reduction, which affects only unskilled workers in the urban informal sector, and 5% payroll tax rate reduction, which affects urban formal employers of unskilled workers. The main message of their study is that labour-market reform such as minimum wage cut or payroll tax cut alone cannot have long lasting positive effects on the labour market but have to be located in wider economic policy such as public investment policy aiming at economic growth. This is exactly what my model can do, although the specifications related to the labour market policy in my model are much cruder than those in their model.

6.4.3 Sustainability coefficients

Sustainability coefficients employed in construction of sustainable production functions are chosen in ad-hoc manner and it is necessary to conduct sensitivity analysis of the applied model to them. Moreover, it is possible to interpret this
sensitivity analysis as a policy experiment of society’s safe minimum standards that are necessarily embodied in sustainability coefficients. In reality, any sustainable production function cannot eliminate risks of violating sustainability constraints or, in other word, losing ecosystem resilience due to high degree of uncertainty. Although it is not always possible to assign probability of losing ecosystem resilience to particular values of sustainability coefficients, there generally exists a tradeoff between reducing risks of violating sustainability constraints and reducing costs in choosing sustainability coefficients. Policy simulations with different values of sustainability coefficients provide information for such a decision and/or policy implications of different degrees of risk averseness of a society.

The base case scenario (SCbase) employs sustainability coefficients of 1.3 for raw water, 1.8 for treated water, and 1.5 for irrigation land, as explained in Chapter 5. These values are increased by 20% for the alternative scenario (SC1), i.e., 1.56 for raw water, 2.16 for treated water, and 1.8 for irrigation land.

6.4.4 International aid flows

In developing countries insufficient public capital stock is one of the most challenging problems which often triggers a vicious circle in which insufficient public capital stock results in low service quality which hinders sufficient investment in public capital due to inadequate level of cost recovery. Apart from low service quality, it is often observed that full cost recovery pricing, which is beneficial in the long run, is not feasible due to negative welfare effects in the short run. For these cases international aid flows, either as grants or as loans, can play a vital role in pursuing sustainable development. The last policy scenarios investigate the policy implications of international aid flows.

The volume of international aid flows is mostly determined by donor countries, and it is natural to regard it as one of exogenous drivers. The reason why I separately investigate the international aid flows lies in my belief that international aid flows are key instruments of sustainable development policies of the global community. It is interesting to see how the international aid flows affect an individual nation’s sustainable development policies, in particular how they facilitate the policy implementation.
The base case scenario (IA_base) assumes no international aid flow, and the alternative scenario (IA1) assumes availability of external loans such that initial public capital stocks increase 20%. Repayment conditions for loans are set as follows.

- Interest rate: 7% per annum including grace period
- Grace period: 5 years with interest payment
- Maturity period: 15 years including grace period

### 6.5 Simulation results

#### 6.5.1 Sustainable development policies in various policy environments

(1) Optimal pricing schedules

Given the investment allocation policy as a policy alternative, sustainable development policies in this thesis are presented as the optimal pricing schedules of three public charges.

*Figure 20 shows the optimal pricing schedules of domestic water charges.*

Note that there is no difference between policies in the base and CC1 environments because the difference in the coefficient of variation of production risk between two environments plays a role only in the policy implementation process, not in the planning process. In addition, note that Alt.1, Alt.3 and Alt.4 have almost identical optimal domestic water pricing schedules, as well as Base and Alt.2.
Figure 20 shows that the policy alternatives associated with the same investment allocation $\theta^2$ have almost identical optimal domestic water pricing schedules. This fact suggests that there is little difference between policy alternatives in total public capital accumulation in spite of heterogeneous composition of $G^R$, $G$, and $G'$, otherwise the accumulated $G$ significantly varies in spite of the same investment allocation $\theta^2$.

Figures 21 and 22 show the optimal pricing schedules of irrigation water charges and irrigation land charges, respectively.

These two figures show more complicated pricing schedules than those of domestic water charges. It is observed that minimum wage reduction (MW1 environment) has very little effects on optimal pricing schedules while that tariff reduction (TR1 environment) significantly affects them.
(2) Social welfare

*Figures 23 and 24* show the simulated trajectories of mean per capita consumption of satisfaction and mean per user consumption of publicly supplied water.\(^{12}\)

---

\(^{12}\) For the per capita water consumption calculation not only a labour force age person but also his/her dependent is counted as 'one capita', otherwise the term 'per capita' always means per one labour force age person. See Footnote 12 in Chapter 5.
It is observed that the publicly supplied water consumption is severely suppressed in the first year, less than 40% of the level with status quo public charges (90 lcd). This suppression is considerably mitigated in the next year and further diminished throughout the planning period, but the status quo level of 90 lcd is not achieved even in the terminal year. This severe suppression of water consumption has only marginal impacts on the level of satisfaction consumption of which reduction from the level with status quo public charges (600 unit/capita/year) is less than 1.6% in the first year, and in most cases the levels of satisfaction consumption exceed this status quo level in the second year.

The simulated money-metric social welfare in terms of the net present values of equivalent variations (EV) under various environments is shown in Table 15.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base ((\varphi^b, \varphi'^b, \varphi''^b) = (0.43, 0.285, 0.285))</td>
<td>Mean [DH/capita]</td>
<td></td>
<td>-21.9</td>
<td>192.1</td>
<td>1,961.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>0.43</td>
<td>0.285</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td>Alt. 1 ((\varphi^1, \varphi'^1, \varphi''^1) = (0.285, 0.43, 0.285))</td>
<td>Mean [DH/capita]</td>
<td>486.8</td>
<td>464.8</td>
<td>781.1</td>
<td>2,449.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>2.9</td>
<td>11.4</td>
<td>2.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Alt. 2 ((\varphi^2, \varphi'^2, \varphi''^2) = (0.285, 0.285, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>130.9</td>
<td>109.0</td>
<td>330.0</td>
<td>2,091.9</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>3.9</td>
<td>11.4</td>
<td>6.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Alt. 3 ((\varphi^3, \varphi'^3, \varphi''^3) = (0.43, 0.43, 0.14))</td>
<td>Mean [DH/capita]</td>
<td>127.8</td>
<td>105.9</td>
<td>404.5</td>
<td>2,092.2</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>6.0</td>
<td>9.7</td>
<td>4.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Alt. 4 ((\varphi^4, \varphi'^4, \varphi''^4) = (0.14, 0.43, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>490.7</td>
<td>468.7</td>
<td>740.4</td>
<td>2,452.7</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>2.8</td>
<td>11.3</td>
<td>2.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Alt. 5 ((\varphi^5, \varphi'^5, \varphi''^5) = (0.43, 0.14, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>-988.5</td>
<td>-1,010.3</td>
<td>-662.0</td>
<td>975.8</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>3.1</td>
<td>52.2</td>
<td>6.5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment. The abbreviation “c.v.” stands for coefficients of variation. Policy environments “base” stands for \((CC_{base}-TR_{base}-MW_{base}-SC_{base}-IA_{base})\), “CC1” stands for \((CC1-TR_{base}-MW_{base}-SC_{base}-IA_{base})\), and so on.

Good performance of Alt. 1 and Alt. 4 and poor performance of Alt. 5 collectively imply the importance of investment in treated water production sector, while relatively poor performance of Alt. 3 indicates that excessively low irrigation investment could hamper welfare improvement effects of treated water production investment.
Another interesting finding is the effects of policy environments (exogenous drivers) on sustainable development policies. Alt. 4 achieves the highest social welfare in the base, CC1, and MW1 environments, while Alt. 1 does in TR1 environment.

Welfare implications of each policy alternative are more clearly illustrated by the trajectories of mean annual EV shown in Figure 25.

A common pattern for all environments is that Alt. 4 achieves the highest EV at the beginning but after several years Alt. 1 overtakes it. Whether Alt. 4 overtakes Alt. 1 once again or not depends on the policy environment. It can be inferred that higher social welfare could be achieved by altering policy between Alt. 1 and Alt. 4 appropriately. Figure 25 provides useful information for designing the optimal policy.

Figure 25 also shows that the trajectories in the base, CC1 and MW1 environments have essentially a similar shape, while those in TR1 environment have a different
shape. In particular trajectories under the base and CC1 environments have an almost identical shape. These findings well correspond to the shape of optimal pricing schedules.

The main implication of future climate change represented by CC1 environment is not revealed in mean values but in drastically increased coefficients of variation shown in Table 15. This implication is graphically illustrated in Figure 26, which depicts trajectories of annual EV of every fifth trials (5th, 10th, 15th, ...) among 100 trials associated with Alt.4 under the base and CC1 environments.

Figure 26 shows that drastically amplified uncertainty makes comparison between policy alternatives almost impossible under CC1 environment. This amplified uncertainty is also reflected in the minimum trajectories of satisfaction consumption among 100 trials shown in Figure 27.

Figure 27 Effects of climate change on minimum consumption
Figure 27 shows that the amplified uncertainty significantly pulls down the minimum values of satisfaction consumption among 100 trials but that it does not change the relativity among policy alternatives.

(3) Constraints observance

Table 16 shows how likely policy alternatives violate sustainability constraints.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Probability to violate sustainability constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Policy environment</td>
</tr>
<tr>
<td>Mean</td>
<td>[%]</td>
</tr>
<tr>
<td>c.v.</td>
<td>[%]</td>
</tr>
<tr>
<td>Alt.1</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.2</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.3</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.4</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.5</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
</tbody>
</table>

The results show no significant difference between policy alternatives. It is interesting that high uncertainty under CC1 environment has no effect on observance of sustainability constraints.

Table 17 shows that policy alternatives have almost no effect on another constraint of the original optimisation problem, the government budget constraint, although policy environments have some effects.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Probability of budget deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Policy environment</td>
</tr>
<tr>
<td>Mean</td>
<td>c.v. [%]</td>
</tr>
<tr>
<td></td>
<td>[%]</td>
</tr>
<tr>
<td>Alt.1</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.2</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.3</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.4</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
<tr>
<td>Alt.5</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
</tr>
</tbody>
</table>
Tables 16 and 17 show that both probabilities are associated with large coefficients of variation. If these constraints are strict, it is necessary to apply lower safety factors, which also pulls down the attainable levels of social welfare.

(4) Terminal capital stock levels

The terminal values of both public and private capital stocks have important implications for social welfare after the planning period.

Tables 18 and 19 show terminal values of per capita private capital stock consisting of household assets ($m$) and rainfed capital ($K^R$) and terminal values of total public capital stock, respectively.

**Table 18 Terminal values of per capita private capital stock**

<table>
<thead>
<tr>
<th>Policy</th>
<th>Mean [unit/capita]</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>6,799.8 (3.0)</td>
<td>6,799.8 (4.4)</td>
<td>6,810.7 (3.0)</td>
<td>6,828.6 (3.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt.1</td>
<td>6,805.5 (3.0)</td>
<td>6,805.5 (4.4)</td>
<td>6,818.9 (3.0)</td>
<td>6,834.3 (2.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt.2</td>
<td>6,802.0 (3.0)</td>
<td>6,802.0 (4.4)</td>
<td>6,813.1 (3.0)</td>
<td>6,830.8 (3.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt.3</td>
<td>6,799.3 (3.0)</td>
<td>6,799.3 (4.4)</td>
<td>6,812.5 (3.0)</td>
<td>6,828.2 (3.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt.4</td>
<td>6,805.6 (3.0)</td>
<td>6,805.6 (4.4)</td>
<td>6,817.9 (3.0)</td>
<td>6,834.4 (2.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alt.5</td>
<td>6,786.2 (3.0)</td>
<td>6,786.2 (4.4)</td>
<td>6,798.5 (3.0)</td>
<td>6,815.3 (3.0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

**Table 19 Terminal values of total public capital stock**

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>[million unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base and CC1</td>
<td>TR1</td>
</tr>
<tr>
<td>Base</td>
<td>12,697.4</td>
<td>7,142.6</td>
</tr>
<tr>
<td>Alt.1</td>
<td>12,803.3</td>
<td>7,293.6</td>
</tr>
<tr>
<td>Alt.2</td>
<td>12,752.8</td>
<td>7,191.9</td>
</tr>
<tr>
<td>Alt.3</td>
<td>12,651.0</td>
<td>7,170.7</td>
</tr>
<tr>
<td>Alt.4</td>
<td>12,807.5</td>
<td>7,268.6</td>
</tr>
<tr>
<td>Alt.5</td>
<td>12,394.3</td>
<td>6,896.9</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

The difference of capital accumulation between policy alternatives is much less than 1% of the stock for private capital and less than 2% for public capital, if we
Tables 16 and 17 show that both probabilities are associated with large coefficients of variation. If these constraints are strict, it is necessary to apply lower safety factors, which also pulls down the attainable levels of social welfare.

(4) Terminal capital stock levels

The terminal values of both public and private capital stocks have important implications for social welfare after the planning period.

Tables 18 and 19 show terminal values of per capita private capital stock consisting of household assets ($m$) and rainfed capital ($K^R$) and terminal values of total public capital stock, respectively.

### Table 18: Terminal values of per capita private capital stock

<table>
<thead>
<tr>
<th>Policy</th>
<th>Mean [unit/capita]</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>6,799.8</td>
<td>6,799.8</td>
<td>6,810.7</td>
<td>6,828.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(3.0)</td>
<td></td>
</tr>
<tr>
<td>Alt.1</td>
<td>6,805.5</td>
<td>6,805.5</td>
<td>6,818.9</td>
<td>6,834.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(2.9)</td>
<td></td>
</tr>
<tr>
<td>Alt.2</td>
<td>6,802.0</td>
<td>6,802.0</td>
<td>6,813.1</td>
<td>6,830.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(3.0)</td>
<td></td>
</tr>
<tr>
<td>Alt.3</td>
<td>6,799.3</td>
<td>6,799.3</td>
<td>6,812.5</td>
<td>6,828.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(3.0)</td>
<td></td>
</tr>
<tr>
<td>Alt.4</td>
<td>6,805.6</td>
<td>6,805.6</td>
<td>6,817.9</td>
<td>6,834.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(2.9)</td>
<td></td>
</tr>
<tr>
<td>Alt.5</td>
<td>6,786.2</td>
<td>6,786.2</td>
<td>6,798.5</td>
<td>6,815.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(4.4)</td>
<td>(3.0)</td>
<td>(3.0)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

### Table 19: Terminal values of total public capital stock

<table>
<thead>
<tr>
<th>Policy</th>
<th>Base and CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>12,697.4</td>
<td>7,142.6</td>
<td>12,611.9</td>
</tr>
<tr>
<td>Alt.1</td>
<td>12,803.3</td>
<td>7,293.6</td>
<td>12,718.5</td>
</tr>
<tr>
<td>Alt.2</td>
<td>12,752.8</td>
<td>7,191.9</td>
<td>12,667.2</td>
</tr>
<tr>
<td>Alt.3</td>
<td>12,651.0</td>
<td>7,170.7</td>
<td>12,567.0</td>
</tr>
<tr>
<td>Alt.4</td>
<td>12,807.5</td>
<td>7,268.6</td>
<td>12,722.5</td>
</tr>
<tr>
<td>Alt.5</td>
<td>12,394.3</td>
<td>6,896.9</td>
<td>12,310.4</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

The difference of capital accumulation between policy alternatives is much less than 1% of the stock for private capital and less than 2% for public capital, if we
exclude Alt. 5. This observation is consistent with what domestic water optimal pricing schedules imply.

A policy alternative associated with the largest terminal values in each environment coincides with the best policy alternative in terms of net present value of EV. This fact precludes trade-offs between social welfare during planning period and that after planning period and makes it easy to identify the best policy alternative in each environment.

Note that the coincidence of order between net present value of EV and terminal values of capital stock is not entirely general although 1st, 2nd and 6th policy alternatives always coincide. For example, under the base environment Alt. 3 ranks 4th in the net present value of EV but it ranks 5th in the terminal values of either private or public capital stock.

(5) Unemployment and safe water access

As explained in Chapter 1, meeting basic human needs is at the heart of sustainable development. In this thesis basic human needs are represented by safe water access and employment. Figure 28 shows the mean values of nationwide safe water access rates.

![Figure 28 Mean values of nationwide safe water access rates](image)
Similar to the optimal pricing schedules of domestic water charges, the policy alternatives associated with the same investment allocation $\theta^2$ (e.g. Base and Alt.2) have almost identical paths of the nationwide safe water access rates. The achievement of sustainable development policies, especially policy alternatives with domestic water priority, is remarkable in safe water access provision.

*Figure 29* shows the mean values of nationwide unemployment rates.

In contrast to the safe water access provision, performance of sustainable development policies in unemployment reduction is disappointing. Nationwide unemployment rates rapidly decrease for first several years, but then they start rising again. This poor performance is mainly due to population concentration into urban areas with positive population growth, because the probability for the urban unskilled labourers to get jobs in urban modern sector ($\theta^5$) generally increases throughout the planning period, as shown in *Figure 30*. 
Recall Lemma 4.1 in p.105. The fact that the shape of trajectories in Figure 30 is similar to that of safe water access rate indicates that rise of urban job getting probability is mainly due to improvement of rural safe water access. This is further endorsed by the fact that no policy alternative is enough effective for raising irrigation labour wage in the long run as shown in Figure 31.
For further achievement in unemployment reduction, some additional policy may be necessary. Relatively high irrigation labour wages associated with Alt.5 may suggest the possibility to raise irrigation labour wage by investing more in both the raw water and the irrigation land production.

### 6.5.2 Policy implication of sustainability coefficients

This subsection reports the simulation results under policy scenarios associated with SCI environment in which sustainability coefficients are raised by 20%. With modified sustainability coefficients each sustainable production function is recalibrated as follows.

#### [Raw water production]

\[
R_t = \begin{cases} 
11,821.9 \times \left(1 - 167.75e^{-0.0085G_t^R}\right) & \text{for } G_t^R < 1,500, \\
13,580.4 \times \left(1 - 21,143.18e^{-0.0085G_t^R}\right) & \text{for } 1,500 \leq G_t^R < 2,193, \\
14,312.4 \times \left(1 - 12.41e^{-0.0025G_t^R}\right) & \text{for } 2,193 \leq G_t^R < 3,309, \\
15,080.9 \times \left(1 - 7.72e^{-0.0015G_t^R}\right) & \text{for } 3,309 \leq G_t^R < 4,626, \\
16,795.9 \times \left(1 - 0.44e^{-0.0003G_t^R}\right) & \text{for } 4,626 \leq G_t^R < 7,700, \text{ and} \\
7,884.7 \times (G_t^R)^{0.1} - 3,225.51 & \text{for } G_t^R \geq 7,700.
\end{cases}
\]

#### [Treated water production]

\[
Q_t = \begin{cases} 
525.9 \times \left(1 - 4.77e^{-0.012G_t^T}\right) & \text{for } G_t^T < 604, \\
614.6 \times \left(1 - 0.90e^{-0.003G_t^T}\right) & \text{for } 604 \leq G_t^T < 1,084, \text{ and} \\
79.9(G_t^T)^{0.3} - 56.92 & \text{for } G_t^T \geq 1,084.
\end{cases}
\]

#### [Irrigation land provision]

\[
A_t = \begin{cases} 
728.9 \times \left(1 - 1.35e^{-0.003G_t^I}\right) & \text{for } G_t^I < 853, \\
787.6 \times \left(1 - 66.96e^{-0.007G_t^I}\right) & \text{for } 853 \leq G_t^I < 1,097, \text{ and} \\
892.2 \times \left(1 - 312.80e^{-0.007G_t^I}\right) & \text{for } 1,097 \leq G_t^I < 1,715, \text{ and} \\
738.0(G_t^I)^{0.1} - 663.58 & \text{for } G_t^I \geq 1,715.
\end{cases}
\]

As a result, the proportions of the current sustainable production capacities to the current capacity with the present engineering practice become 55.7% for raw water production, 50.9% for treated water production, and 72.3% for irrigation land provision. It is the raw water production that is most severely affected by the
increase of sustainability coefficients. Judging from this fact, it is anticipated that more public investment in raw water production may have favourable effects on social welfare.

(1) Optimal pricing schedules

Figures 32 to 34 show the optimal pricing schedules.

As observed in Figure 20, the policy alternatives associated with the same $\theta^G$ have almost identical domestic water pricing schedules. The increased sustainability coefficients significantly affect the shapes of optimal pricing schedules. In all policy scenarios, the optimal prices are significantly higher in the first year but tend
to converge on the counterpart pricing schedules under SCbase environment. It seems that a 20% increase of sustainability coefficients does not induce qualitative change in sustainable development policies.

(2) Social welfare

*Figures 35 and 36 show the paths of mean per capita satisfaction consumption and mean per user publicly supplied water consumption in SC1 environment.*

![Figure 35 Mean satisfaction consumption in SC1](image1)

![Figure 36 Mean publicly supplied water consumption in SC1](image2)
Although the suppression of domestic water consumption in the first year is much severer in SC1 environment than in SCbase environment, the effects on satisfaction consumption is small, and there is almost no difference of the terminal values of satisfaction consumption after 15 years between SCbase and SC1 environments. This result corroborates our expectation of convergence property of optimal trajectories observed in the experiments without supply side constraints.

The simulated money-metric social welfare in terms of the net present values of EV under increased sustainability coefficients is shown in Table 20.

### Table 20 Net present value of EV in SC1 environment

<table>
<thead>
<tr>
<th>Policy environment</th>
<th>SC1</th>
<th>SC1-CC1</th>
<th>SC1-TR1</th>
<th>SC1-MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base ((\theta, \theta, \theta) = (0.43, 0.285, 0.285))</td>
<td>Mean [DH/capita]</td>
<td>-498.2 (4.5)</td>
<td>-520.1 (101.7)</td>
<td>-385.3 (5.9)</td>
</tr>
<tr>
<td>c.v. [%]</td>
<td>(4.5)</td>
<td>(101.7)</td>
<td>(5.9)</td>
<td>(2.1)</td>
</tr>
<tr>
<td>Alt.1 ((\theta, \theta, \theta) = (0.285, 0.43, 0.285))</td>
<td>Mean [DH/capita]</td>
<td>-23.0 (88.7)</td>
<td>-44.9 (1,178.4)</td>
<td>138.1 (10.4)</td>
</tr>
<tr>
<td>c.v. [%]</td>
<td>(88.7)</td>
<td>(1,178.4)</td>
<td>(10.4)</td>
<td>(2.3)</td>
</tr>
<tr>
<td>Alt.2 ((\theta, \theta, \theta) = (0.285, 0.285, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>-356.4 (5.8)</td>
<td>-378.3 (140.1)</td>
<td>-259.6 (7.8)</td>
</tr>
<tr>
<td>c.v. [%]</td>
<td>(5.8)</td>
<td>(140.1)</td>
<td>(7.8)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Alt.3 ((\theta, \theta, \theta) = (0.43, 0.43, 0.14))</td>
<td>Mean [DH/capita]</td>
<td>-433.7 (5.0)</td>
<td>-455.6 (115.5)</td>
<td>-400.6 (5.6)</td>
</tr>
<tr>
<td>c.v. [%]</td>
<td>(5.0)</td>
<td>(115.5)</td>
<td>(5.6)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Alt.4 ((\theta, \theta, \theta) = (0.14, 0.43, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>-82.9 (24.7)</td>
<td>-104.8 (504.6)</td>
<td>-24.7 (64.8)</td>
</tr>
<tr>
<td>c.v. [%]</td>
<td>(24.7)</td>
<td>(504.6)</td>
<td>(64.8)</td>
<td>(2.3)</td>
</tr>
<tr>
<td>Alt.5 ((\theta, \theta, \theta) = (0.43, 0.14, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>-1,453.4 (3.0)</td>
<td>-1,475.1 (35.8)</td>
<td>-1,160.1 (4.0)</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.
Policy environments "SC!" stands for (SC1-IAbase-CCbase-TRbase-MWbase), etc.

Comparison between SCbase results (Table 15) and SC1 results (Table 20) shows that in terms of net present value of EV the most severely affected policy alternatives by increase of sustainability coefficients are always Alt.3 and Alt.4, in which Alt.4 is the most affected one under TRbase environment while Alt.3 is under TR1 environment. It means not only policy alternatives with low investment allocation to raw water sector as expected but also those with low investment allocation to irrigation land are mostly damaged by SC1 environment, in terms of social welfare.

More conspicuous difference between SCbase and SC1 environments is drastic increase of coefficients of variation associated with Alt.1 and Alt.4 in SC1
environment. This does not mean, however, dispersion of entire trajectories as caused by CC1 environment. Instead, it is caused by dispersion of annual EV only in the first year as shown in Figure 37.

Figure 37 Effects of larger sustainability coefficients on annual EV paths

This observation is confirmed by the fact that elimination of the first year values from the calculation of net present value of EV reduces the coefficients of variation from 88.7% to 4.9% for Alt.1 and from 24.7% to 7.2% for Alt.4 in SC1-IAbase-CCbase-TRbase-MWbase environment.

Figure 38 shows the trajectories of mean annual EV in SC1 environment.

Figure 38 Mean annual EV in SC1 environment
Except for the very low values recorded in the first year, the shape of trajectories associated with each policy alternative is very similar to SCbase counterparts. It suggests that a 20% rise of sustainability coefficients does not structurally change the welfare effects of sustainable development policies. This observation is further endorsed by Figure 39 showing that the increase of sustainability coefficients only marginally affects minimum values of satisfaction consumption among 100 trials, except for the first year.

(3) Constraints observance

Tables 21 and 22 show the probabilities to violate sustainability constraints and to run deficit of the government saving accounts in SC1 environment.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.4</td>
<td>13.7</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.1)</td>
<td>(66.3)</td>
<td>(66.6)</td>
</tr>
<tr>
<td>Alt.1</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.3</td>
<td>13.4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.2)</td>
<td>(66.6)</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt.2</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.3</td>
<td>13.5</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.2)</td>
<td>(65.9)</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt.3</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.4</td>
<td>13.8</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.1)</td>
<td>(66.0)</td>
<td>(67.7)</td>
</tr>
<tr>
<td>Alt.4</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.3</td>
<td>13.5</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.2)</td>
<td>(66.6)</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt.5</td>
<td>Mean [%]</td>
<td>13.3</td>
<td>13.5</td>
<td>13.6</td>
<td>13.6</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(67.5)</td>
<td>(65.7)</td>
<td>(66.1)</td>
<td>(66.1)</td>
</tr>
</tbody>
</table>
Increase of sustainability coefficients barely affects these probabilities. It implies that these probabilities are mostly determined by safety factors, not by sustainability coefficients.

(4) Terminal capital stock levels

Tables 23 and 24 show terminal values of per capita private capital stock consisting of household assets (m) and rainfed capital (K'^{r}) and terminal values of total public capital stock, in SC1 environment.
Table 24 Terminal values of total public capital stock in SC1

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base and CC1</td>
</tr>
<tr>
<td>Base</td>
<td>12,579.7</td>
</tr>
<tr>
<td>Alt.1</td>
<td>12,685.9</td>
</tr>
<tr>
<td>Alt.2</td>
<td>12,637.1</td>
</tr>
<tr>
<td>Alt.3</td>
<td>12,523.6</td>
</tr>
<tr>
<td>Alt.4</td>
<td>12,668.2</td>
</tr>
<tr>
<td>Alt.5</td>
<td>12,280.6</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

There is no significant difference between SCbase and SC1 environments in terms of terminal capital stock. The previous observation that a policy alternative associated with the largest terminal values in each environment achieves the highest net present value of EV is still valid.

(5) Unemployment and safe water access

Figures 40 and 41 show the mean values of nationwide safe water access rates and nationwide unemployment rates in SC1 environment.13

---

13 The trajectories in CC1 environment are omitted because they are almost identical with those in CCbase environment. As observed in Figure 28, the policy alternatives associated with the same $\theta^0$ have almost identical paths of safe water access rates.
The influence of higher sustainability coefficients is not negligible in nationwide unemployment rates. In the first year the nationwide unemployment rates associated with each policy environment in SC1 environment are 1.4% higher than their counterparts in SCbase environment. Except for the first year, the differences in unemployment rates between SCbase and SC1 environments for each policy alternative in average for the planning period are 0.2% to 0.3% in TRbase environment, and this difference for Alt.3 in TR1 environment reaches 0.54%. Not only the reduction of EV but also this considerable increase of unemployment rates must be counted as social costs of higher sustainability coefficients.

The noticeable rise of unemployment rates coupled with almost unchanged safe water access rates in SC1 environment indicates downward effects of increased sustainability coefficients on irrigation labour wages. This irrigation labour wage reduction in SC1 environment is demonstrated by Figure 42.

\[ \text{Figure 42 Mean values of irrigation labour wage in SC1} \]

It is clearly observed that the wage rate trajectory of Alt.3 in TR1 environment goes down most conspicuously.

6.5.3 Policy implications of international aid flows

The last set of policy simulations investigates policy implications of international aid flows.

(1) Optimal pricing schedules

\text{Figures} 43 to 45 show the optimal pricing schedules under increased initial public capital stock by 20% provided by international aid flows in terms of external loans.
Note that the convergence of the optimal domestic water pricing schedules of the policy alternatives with the same $\theta^G$ is observed as in Figures 20 and 32. The increased initial public capital stock significantly changes the shapes of optimal pricing schedules, but after several years the optimal prices tend to converge on the counterpart pricing schedules under IA$\text{base}$ environment. It seems that 20% increase of initial public capital stock financed by external loans does not induce qualitative change of sustainable development policies.

(2) Social welfare

Figures 46 and 47 show the paths of mean per capita consumption of satisfaction and mean per user consumption of publicly supplied water in IA1 environment.
Figure 46 Mean satisfaction consumption in IA1

Figure 47 Mean publicly supplied water consumption in IA1

The suppression of publicly supplied water consumption in the first year is mitigated by the increased initial public capital stock. Although the first year consumption of 42 lcd is still less than a half of the status quo level, this mitigation of water consumption suppression improves feasibility of sustainable development policy. The convergence property of optimal trajectories is observed again and
there is almost no difference of the terminal values of satisfaction consumption after 15 years between IAbase and IA1 environments.

The simulated net present values of equivalent variations (EV) under increased initial public capital stock due to international aid flows are shown in Table 25.

### Table 25 Net present value of EV in IA1 environment

<table>
<thead>
<tr>
<th>Policy environment</th>
<th>IA1</th>
<th>IA1-CC1</th>
<th>IA1-TR1</th>
<th>IA1-MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base ((\theta^g, \theta^o, \theta) = (0.43, 0.285, 0.285))</td>
<td>Mean [DH/capita]</td>
<td>134.8</td>
<td>112.9</td>
<td>363.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(8.1)</td>
<td>(469.1)</td>
<td>(8.5)</td>
</tr>
<tr>
<td>Alt.1 ((\theta^g, \theta^o, \theta) = (0.285, 0.43, 0.285))</td>
<td>Mean [DH/capita]</td>
<td>644.9</td>
<td>622.9</td>
<td>963.7</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(3.1)</td>
<td>(85.1)</td>
<td>(3.3)</td>
</tr>
<tr>
<td>Alt.2 ((\theta^g, \theta^o, \theta) = (0.285, 0.285, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>272.3</td>
<td>250.3</td>
<td>479.1</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(4.3)</td>
<td>(211.9)</td>
<td>(6.3)</td>
</tr>
<tr>
<td>Alt.3 ((\theta^g, \theta^o, \theta) = (0.43, 0.43, 0.14))</td>
<td>Mean [DH/capita]</td>
<td>321.8</td>
<td>299.9</td>
<td>686.4</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(5.2)</td>
<td>(175.9)</td>
<td>(4.3)</td>
</tr>
<tr>
<td>Alt.4 ((\theta^g, \theta^o, \theta) = (0.14, 0.43, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>643.7</td>
<td>621.7</td>
<td>903.3</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(3.0)</td>
<td>(85.3)</td>
<td>(3.5)</td>
</tr>
<tr>
<td>Alt.5 ((\theta^g, \theta^o, \theta) = (0.43, 0.14, 0.43))</td>
<td>Mean [DH/capita]</td>
<td>-858.4</td>
<td>-880.2</td>
<td>-499.5</td>
</tr>
<tr>
<td></td>
<td>c.v. [%]</td>
<td>(3.7)</td>
<td>(59.9)</td>
<td>(9.8)</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

Comparison between IAbase results (Table 15) and IA1 results (Table 25) shows that Alt.3 enjoys the largest benefits in terms of the higher net present values of EV from the increased initial public capital stock, and Alt.1 and Alt.4 follow it, all of which share \(\theta^G = 0.43\). This observation indicates the effectiveness of public investment in treated water sector for improving social welfare. To confirm this, an additional alternative environment (IA2) is prepared, in which the same amount of external loans as IA1 environment is exclusively invested in \(G\). Figure 48 shows the comparison of mean annual values of EV between IA1 and IA2 environments.
The figure illustrates unambiguous advantages of concentrating external loans on the investment into treated water production capital $G$. In fact, the net present values of EV increase more than 20% and reach 794.8 DH per capita with Alt. 1 and 797.8 DH per capita with Alt. 4 under IA2 environment.

The effects of IA1 environment are more clearly observed in Figure 49.

It is observed that trajectories of mean annual EV generally shift upward in IA1 environment but tend to converge on those in IAbase environment towards the end of planning period. Furthermore, the relation between different policy alternatives, such as overtaking point between Alt. 1 and Alt. 4, is not affected by the initial public capital increase. It suggests that a 20% increase in initial public capital stock does not structurally change the welfare effects of sustainable development policies.

(3) Constraints observance

Tables 26 and 27 show the probabilities to violate sustainability constraints and to run deficit of the government saving accounts in IA1 environment.
### Table 26 Probability to violate sustainability constraints in IA1

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.3</td>
<td>13.4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>67.5</td>
<td>(65.2)</td>
<td>66.6</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt. 1</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>66.9</td>
<td>(65.3)</td>
<td>67.5</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt. 2</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.3</td>
<td>13.4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>66.9</td>
<td>(65.2)</td>
<td>66.6</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt. 3</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.3</td>
<td>13.4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>67.5</td>
<td>(65.2)</td>
<td>66.6</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt. 4</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>66.9</td>
<td>(65.3)</td>
<td>67.5</td>
<td>(67.5)</td>
</tr>
<tr>
<td>Alt. 5</td>
<td>Mean (%)</td>
<td>13.3</td>
<td>13.5</td>
<td>13.5</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>67.5</td>
<td>(65.7)</td>
<td>66.3</td>
<td>(66.9)</td>
</tr>
</tbody>
</table>

### Table 27 Probability of budget deficit in IA1

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Mean (%)</td>
<td>4.5</td>
<td>7.7</td>
<td>9.2</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(204.7)</td>
<td>(221.7)</td>
<td>(151.6)</td>
<td>(204.7)</td>
</tr>
<tr>
<td>Alt. 1</td>
<td>Mean (%)</td>
<td>4.5</td>
<td>7.7</td>
<td>9.2</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(204.7)</td>
<td>(222.2)</td>
<td>(151.6)</td>
<td>(204.7)</td>
</tr>
<tr>
<td>Alt. 2</td>
<td>Mean (%)</td>
<td>4.5</td>
<td>7.7</td>
<td>9.2</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(204.7)</td>
<td>(221.7)</td>
<td>(151.6)</td>
<td>(204.7)</td>
</tr>
<tr>
<td>Alt. 3</td>
<td>Mean (%)</td>
<td>4.5</td>
<td>7.7</td>
<td>9.2</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(204.7)</td>
<td>(222.2)</td>
<td>(151.6)</td>
<td>(204.7)</td>
</tr>
<tr>
<td>Alt. 4</td>
<td>Mean (%)</td>
<td>4.5</td>
<td>7.7</td>
<td>9.2</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(204.7)</td>
<td>(222.2)</td>
<td>(151.6)</td>
<td>(204.7)</td>
</tr>
<tr>
<td>Alt. 5</td>
<td>Mean (%)</td>
<td>4.6</td>
<td>7.8</td>
<td>9.4</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>c. v (%)</td>
<td>(202.6)</td>
<td>(221.6)</td>
<td>(150.5)</td>
<td>(202.6)</td>
</tr>
</tbody>
</table>

Increase of initial public capital stock barely affects these probabilities. It further suggests that safety factors are almost sole determinants of these probabilities.

(4) Terminal capital stock levels

*Tables 28 and 29 show terminal values of per capita private capital stock consisting of household assets ($m_i$) and rainfed capital ($K^R_i$) and terminal values of total public capital stock, in IA1 environment.*
Table 28 Terminal values of per capita private capital stock in IA1

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base</th>
<th>CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,801.2 (2.9)</td>
<td>6,801.2 (4.4)</td>
<td>6,812.5 (3.0)</td>
<td>6,830.0 (2.9)</td>
</tr>
<tr>
<td>Alt.1</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,807.2 (2.9)</td>
<td>6,807.3 (4.4)</td>
<td>6,820.9 (3.0)</td>
<td>6,836.0 (2.9)</td>
</tr>
<tr>
<td>Alt.2</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,803.5 (2.9)</td>
<td>6,803.5 (4.4)</td>
<td>6,814.7 (3.0)</td>
<td>6,832.3 (2.9)</td>
</tr>
<tr>
<td>Alt.3</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,801.6 (2.9)</td>
<td>6,801.6 (4.4)</td>
<td>6,816.1 (3.0)</td>
<td>6,830.5 (2.9)</td>
</tr>
<tr>
<td>Alt.4</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,807.4 (2.9)</td>
<td>6,807.4 (4.4)</td>
<td>6,819.6 (3.0)</td>
<td>6,836.1 (2.9)</td>
</tr>
<tr>
<td>Alt.5</td>
<td>Mean c.v. [unit/capita] [%]</td>
<td>6,787.6 (2.9)</td>
<td>6,787.6 (4.4)</td>
<td>6,800.2 (3.0)</td>
<td>6,816.6 (2.9)</td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

Table 29 Terminal values of total public capital stock in IA1 [million unit]

<table>
<thead>
<tr>
<th>Policy</th>
<th>Policy environment</th>
<th>Base and CC1</th>
<th>TR1</th>
<th>MW1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>12,494.7</td>
<td>6,937.2</td>
<td>12,408.8</td>
<td></td>
</tr>
<tr>
<td>Alt.1</td>
<td>12,607.9</td>
<td>7,093.1</td>
<td>12,522.6</td>
<td></td>
</tr>
<tr>
<td>Alt.2</td>
<td>12,550.9</td>
<td>6,982.4</td>
<td>12,464.8</td>
<td></td>
</tr>
<tr>
<td>Alt.3</td>
<td>12,466.7</td>
<td>6,997.5</td>
<td>12,382.3</td>
<td></td>
</tr>
<tr>
<td>Alt.4</td>
<td>12,613.6</td>
<td>7,062.8</td>
<td>12,328.2</td>
<td></td>
</tr>
<tr>
<td>Alt.5</td>
<td>12,189.1</td>
<td>6,690.5</td>
<td>12,104.7</td>
<td></td>
</tr>
</tbody>
</table>

Note: Bold italic indicates the best policy alternative in each environment.

These results are of high interest from two aspects. First, reduction of terminal public capital stock due to loan repayment is small at around 1.5 to 3.0 %, in spite of relatively tough repayment conditions in terms of a short maturity period and an interest payment obligation during the grace period. If a portion of international aid flows can be given as grants, it is fully possible that international aid flows even mainly consisting of external loans could increase terminal capital stock.

Second, coincidence between the best policy alternative in terms of social welfare and that in terms of terminal capital stock in each environment is no more valid, except for TR1 environment. This case poses a tough decision to policy-makers between social welfare during the planning period and the source of future social welfare after the planning period.
(5) Unemployment and safe water access

Figures 50 and 51 show the mean values of nationwide safe water access rates and nationwide unemployment rates in IA1 environment.

Although the benefits of increased initial public capital stock in meeting basic human needs are marginal, they are at least positive as a mean of planning period. This fact guarantees the positive effects of international aid flows.

Lastly, Figure 52 illustrates the effects of exclusively investing external loans into treated water production capital $G$ in meeting basic human needs.
Note that the paths of nationwide safe water access rates of Alt.1 and Alt.4 are almost identical in each environment. Only disadvantage of exclusively investing external loans into treated water production capital seems 0.5% higher nationwide unemployment rate in the first year, while there is almost no difference of period average nationwide unemployment rates in IA1 and IA2 environments. In safe water access provision there is clear advantage of concentrating external loans on the investment into \( G \).

6.6 Discussion

Before discussing the simulation results it might be appropriate to reconfirm what this thesis tries to demonstrate through policy simulations and what it does not.

This thesis tries to demonstrate the potential of the proposed methodology with providing concrete images of policy analysis based on that methodology. In addition, it aims to provide rough ideas of policy implications of sustainable development policies.

This thesis does not, however, try to provide any materials directly applicable to the real policy-making. Neither does it aim to seek the best sustainable development policies. For such purposes much more accurate sustainable production functions, at least, must be prepared based on highly interdisciplinary research efforts. The following discussions must read with this caveat.

6.6.1 Advantages of proposed methodology

Policy simulations of this thesis provide information across multiple aspects of sustainable development policies including (i) social welfare during the planning period, (ii) future ability to generate social welfare after the planning period, (iii) observance of sustainability constraints, and (iv) basic human needs satisfaction. This rich information, which is rarely found in the existing literature, not only facilitates policy-makers to make decisions but also improves accountability of sustainable development policies. The simulation results contain interesting cases (IA1, IA1-CC1, and IA1-MW1) in which the best policy alternatives from the aspect of the present social welfare and of the future ability to generate social
welfare are different. Based on my methodology it is not necessary to integrate these incommensurable results in some unique indicator, but to imagine as if the policies were implemented and to choose more 'preferable' ones.

The advantages to separate the planning and the implementation processes are also well demonstrated by policy simulation results. It enables policy makers to see the implications of policies not only in terms of the expected values but also the minimum values or degree of dispersion of the simulated trajectories. The comparison between CCbase and CCI environments most clearly demonstrates its merits, in which the effects of increased variability of production risk factor presumably caused by climate change are manifested as both drastically dispersed annual EV trajectories and considerable reductions of minimum consumption values across 100 trials. These policy implications cannot be obtained without separating policy planning simulation based on expectations from Monte Carlo simulations of policy implementation process.

6.6.2 General implications of sustainable development policies

The general implications of simulated outcomes of sustainable development policies can be described as follows.

a. A slight suppression of satisfaction consumption in the first year followed by a steady growth at around 0.5 % annually throughout the planning period.

b. A severe suppression of publicly supplied water consumption in the first year, which is more than 60 % reduction from the simulated consumption level with status quo public charges. The consumption level considerably rises in the second year and continues to grow, but it remains around 95% of the status quo level in the terminal year even in the most preferable cases.

c. Total private capital stock gradually grows at around 1 % a year, while total public capital stock explosively grows at around 15 % a year on average.

d. Safe water access is quickly and drastically improved. In the terminal year nationwide safe water access rates often approach to 100 %.

e. Nationwide unemployment rates drop quickly in the first 5 to 8 years by 2 to 3 % in the best case, but gradually increase afterward.
Overall impression of sustainable development policies is successful considering the fact that these outcomes are achieved under very severe sustainability constraints. On the other hand, a severe suppression of water consumption and a mediocre performance in fighting against unemployment suggest that some additional policies might be necessary to realise sustainable development.

Throughout the policy simulations the sustainable development trajectories show a tendency to converge towards the end of planning period, and this tendency is shared by both the analytic and the applied models. It precludes the possibility for these models to generate poverty traps, which are of high empirical interests. There are several potential sources of this convergence property.

The most plausible one is a particular specification of expectation formation process of households assumed in this thesis. Because households always assume zero growth rates of exogenous prices (constant prices) regardless of their actual growth rates in every moment, it functions to absorb the fluctuations of exogenous variables.

Another strong candidate is the lack of human capital accumulation mechanisms, which are often regarded as a generator of poverty traps. For example, it might be plausible that there exists some threshold value of a society’s wealth above which a better access to education increases a proportion of skilled workers to unskilled workers of which consequence is higher productivity, while below which such an education access cannot be realised and productivity drops as a result of less skilled workers. This kind of mechanism is not incorporated in the applied model.

6.6.3 Implications of policy alternatives

Comparison among six policy alternatives reveals that either Alt.1 or Alt.4 always achieves the best outcomes in many aspects, and Alt.5 is always the worst. The main implication is the importance of improving safe water access for sustainable

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See Kremer and Chen (2002) or De la Croix and Doepke (2003) for this issue.
development. Although it is undeniably a product of the employed model specifications, I believe its relevance to the real problems.

Relatively poor performance of Alt.3, particularly in terms of nationwide unemployment rates, implies the necessity of maintaining appropriate level of irrigation investment. Irrigation is often regarded as one of main environmental threats, but it is also widely recognised that irrigation is indispensable not only to feed global population but also establish sound rural economy in many developing countries. The simulation results support the latter view.

Another interesting finding is that the best policy alternatives switch depending upon policy environment. Its policy implication is that the government has to forecast policy environment in order to determine the strategy. In addition, it is observed that the policy alternatives associated with the highest mean annual EV alternate between Alt.1 and Alt.4 as time goes by. It provides useful information how to find better policy alternatives in which investment allocation is allowed to be time variant.

6.6.4 Implications of policy environments

The simulation results also provide us several policy implications of each policy environment. The major implications are as follows.

- Severer climate change (CC1 environment) drastically disperse consumption trajectories of which mean values are hardly affected. It makes the worst events more devastating and decision-making more difficult.

- Trade liberalisation in terms of 20 % tariff reduction (TR1 environment) has positive effects on social welfare in terms EV due to lowering consumer prices. On the other hand, it results in much lower public capital accumulation, by more than 40 %, than in TRbase environment. These negative effects result in poorer basic needs satisfaction performance in terms of both the nationwide safe water access and unemployment rates.

- 10 % reduction of urban unskilled labour wage (MW1 environment) improves the performance of sustainable development policy in every aspect.
Chapter 6 Policy Simulation

- Adoption of safer sustainability coefficients (SC1 environment) has significant negative welfare impacts only in the first year. These negative impacts remain throughout the planning period but their magnitude is small.

- 20% increase of initial public capital stock financed by external loans (IA1 environment) greatly mitigates negative welfare impacts of sustainable development policies. The negative effects of loan repayment on public capital stock are small. It is observed that reductions in terminal total public capital stock from IAbase environment are no more than 1.7%.

It should be noted that lack of distributional aspects in the model results in the all-round triumph of minimum wage reduction (MW1) despite the fact that urban unskilled labourers are highly likely to be losers. Introducing distributional issue into sustainable development policy analysis must be realised in future to have directly relevant implications to the real policy.

6.6.5 Feasibility of sustainable development policies

The biggest threat for the feasibility of sustainable development policies is identified as severely suppressed water consumption, particularly in the first year. This drawback of the sustainable development policies can be mitigated by increasing initial public capital stock to a certain degree, as demonstrated in IA1 environment. In fact, if the government exclusively invests the same amount of external loans as in IA1 environment into treated water production capital G (IA2 environment), per user consumption of publicly supplied water in the first year becomes 52 l/cd, which is nearly 50% more than the value in IAbase environment. Nevertheless, it seems not enough to secure feasibility of sustainable development policies.

In order to improve the feasibility of sustainable development policies, it might be necessary not only to increase international aid flows but also to set transitional periods to apply sustainable production functions. For instance, it might be practical to gradually increase the values of sustainability coefficients from unity to the target levels for, say, 5 years.
Chapter 7

Conclusions

7.1 General conclusion

This thesis presents a coherent body of research work encompassing conceptual clarification, theoretical analysis and empirical analysis with a case study, for which motivation is to make a contribution to operationalising the concept of sustainable development and to providing necessary information to implement it. To make this challenge somehow tractable, the study scope is confined in water crisis in water-stressed developing countries from a wide range of issues relevant to sustainable development.

In this thesis the concept of sustainable development is articulated with paying special attention to its principal objective and the meaning of sustainability. The primary objective is specified as meeting the basic human needs all over the world, and the meaning of sustainability is defined as not undermining the resilience of ecosystems functioning as life-support systems.

The proposed methodology is built on the foundation of microeconomic theory, in particular Ramsey-Cass-Koopmans (RCK) type neoclassical growth theory, but its practical applicability and relevance to the articulated concept of sustainable development are at the same time pursued. This is why I dare to replace several basic constituents of the neoclassical growth theory with innovative ideas in spite of
high risks associated with deviating from a well-established framework. The important innovative ideas include:

- To separate optimisation processes of the private and the public sectors, instead of a prevailing benevolent social planner setting.

- To replace the perfect foresight assumption with the no-perfect foresight assumption in which households expect that the future trajectories of exogenous variables are constant at the current values but every moment they update this expectation based on the realised values at that moment.

- To separate policy simulations of the policy planning phase from those of the policy implementation phase in order to deal with exogenous variables associated with uncertainty.

The advantages of the proposed methodology are not a few. Firstly, it paves a way to develop dynamic optimisation CGE models free from the perfect foresight assumption. Secondly, it enables policy analysis to reflect the second-best nature, or controllability, of public policy. A little thought tells us that the first-best outcomes obtained by the conventional approach with social benevolent planner setting and perfect foresight assumption are of little relevance to empirical policy analysis. Thirdly, it can provide rich information useful for both policy-making and justification purposes. In addition, it can accommodate various uncertainties in a straightforward manner, in which the uncertainties without probability distribution can also be dealt with. These advantages are demonstrated by a case study of Morocco.

The model construction strategy of this thesis is to start from a simple analytic model and then to develop an applied numerical model. The former model, the analytic model, is thus assigned to intermediate roles, but its contribution to overall achievements of this thesis should not be underestimated. The analytic model itself is a pioneering water-extended RCK growth model free from both benevolent social planner setting and perfect foresight assumption. The main contributions of the analytic model are (i) to serve as an indispensable model platform of the applied model, and (ii) to provide key implications only based on which the applied model can be solved numerically. Furthermore, comparison between predictions of local
stability analysis with linearisation method and the results of numerical simulations reveals the severely limited applicability of the linearised local stability analysis to highly non-linear systems.

The applied model employed in the case study is a highly aggregate dynamic optimisation CGE model, which is developed based on the analytic model with incorporating four key stylised facts of water-stressed developing economies; (i) dominant water use share of irrigated agriculture, (ii) high production risks of rainfed agriculture, (iii) high urban unemployment rates, and (iv) poor safe water access in the rural areas. The latter two stylised facts are modelled as a generalised version of Harris-Todaro rural-urban migration equilibrium in which indirect utilities derived from the expected income in the rural and the urban areas are equalised. The applied model is successfully calibrated and validated based on empirical data of Morocco between 1994 and 2002.

The policy simulations generate highly interesting results. The major findings include:

a. The importance of public investment in safe water access provision and irrigation sector is unambiguously demonstrated;

b. The best policy alternatives switch in different policy environments;

c. Severe consumption suppression of publicly supplied water is identified as a major threat to feasibility of sustainable development policy; and

d. Contribution of international aid flow (even without grant element) to implement sustainable development is demonstrated.

Although it does not aim at providing any materials directly applicable to the real world, this thesis is expected both to fill the existing information gap in operationalising sustainable development to a certain extent and to stimulate research efforts sharing the same motivation, which will hopefully facilitate implementation of sustainable development in the real world.
Chapter 7 Conclusions

7.2 Fulfilment of research objectives

This thesis has two equally important research objectives; one is to establish a policy analysis framework relevant to sustainable development, and the other is to clarify policy implications of sustainable development policies.

For the former objective, the following advantages of the proposed framework prove its relevance to the articulated concept of sustainable development.

\begin{itemize}
  \item Sustainable development policies can be evaluated in various aspects encompassing social welfare during the planning period, terminal capital stock as a basis of future social welfare after the planning period, observance of sustainability constraints, safe water access rates and unemployment rates as measures of basic human needs satisfaction.
  \item Sustainability constraints are translated into sustainable production functions that are defined as the relationship between public capital stock and production capacity on condition that production and consumption of publicly supplied goods do not endanger resilience of ecosystems functioning as life-supporting systems. Although the employed sustainable production functions are hypothetical, sustainable constraints are conceptually anchored in ecosystem resilience.
  \item The model addresses negative welfare impacts of lack of safe water access, competition between domestic water use and irrigation water use, and effects of public investment in safe water access provision and irrigation on urban unemployment. These issues represent various pathways through which water affects basic human needs.
\end{itemize}

For the latter objective, the policy simulations provide rich information to see outcomes of sustainable development policies in various policy environments. It is observed that the best policy alternatives switch according to policy environments. Further, the policy simulations identify severe consumption suppression of publicly supplied water as a potential threat to feasibility of sustainable development, and also demonstrate the positive impacts of international aid flows on this feasibility.
Although there are several important issues left over by this thesis as mentioned in the next section, it can be concluded that this thesis has successfully fulfilled the research objectives.

7.3 Future work

7.3.1 Distributional issue

Distributional issue is at the heart of sustainable development mainly in two aspects. One is its implication for poverty, and the other is its importance for feasibility of policy.

For the former, Drenowski (1977) discusses the concept of relative poverty based on which the poor are those who gain when income becomes more evenly distributed and the non-poor are those who lose. Furthermore, Lintott (1998) asserts that “once basic material needs are satisfied, it is an individual’s relative, not absolute, consumption that counts for his or her welfare” (p.242). It implies that intra-generational equity matters in sustainable development of which principal objectives include poverty alleviation.

For the latter, it is well recognised that Pareto optimality criterion is hardly applicable to the real policy because policies always generate winners and losers. A little thought tells us that feasibility of policies crucially depends on the proportion of losers and the magnitude of their loss. Evaluation of feasibility based on the average of a society is thus neither reliable nor convincing.

The main reason why I gave up to incorporate such an important issue into the applied model is the fact that departure from a crucial assumption of the representative household would result in an undesirable model specification in which indexing individuals or households based on their migration and asset accumulation history is required. Incorporating distributional issue will drastically increase model complexity and make it difficult to keep analytical tractability. Nevertheless, this is daunting but attractive challenge and the payoff for success must be immense.
7.3.2 Sustainable production functions

Construction of sustainable production functions in a reliable and operational manner may require truly interdisciplinary research projects in which ecologists contribute to understand impacts of production activity on ecosystem resilience, economists and sociologists play a crucial role in forming consensus on certain safe minimum standards to maintain this resilience, and scientists and engineers provide technological knowledge to achieve it. It is not necessarily a barrier to implement sustainable development but can be regarded as an excellent opportunity to conduct such an interdisciplinary project.

Successful construction of sustainable production functions not only improves the direct applicability of the proposed methodology to real policy analysis but also promotes genuinely interdisciplinary activities of which synergy effects could be substantial.

7.3.3 Integrated treatment of water and energy

Water and energy are acknowledged as the most important issues for sustainable development. Furthermore, there are various direct and indirect linkages between water and energy, as described in Chapter 1. An example of linkages is energy input in water production. Suppose that desalination of seawater is indispensable to increase water production capacity, and necessary energy input exceeds sustainable energy consumption level. Lacking energy from the analysis may provide us fallacious prescriptions for sustainable development in such a case. It is thus highly desirable to address water and energy issues in an integrated manner.

An integrated treatment of water and energy issues is, however, expected to be costly in terms of analytical tractability. This is mainly why I set aside this integration issue. Nevertheless, this integration of water and energy issues is another rewarding challenge to go one step forward to realise sustainable development.
Appendix A Mathematical Appendices

Appendix A0 List of abridged constants in Chapter 3

\[
\begin{align*}
    b_1 &= \varphi^{-(1-\varphi)-(t-\varphi)}, \quad b_2 = \frac{1-\varphi}{\sigma}, \quad b_3 = \frac{(\sigma-1)(1-\varphi)}{\sigma}, \quad b_4 = \nu - \frac{\rho}{\sigma}, \\
    b_5 &= \frac{\sigma-1}{\sigma}, \quad b_6 = \frac{\beta_Q}{1-\beta_Q}, \quad b_7 = \frac{\beta_K}{1-\beta_Q}, \quad b_8 = \frac{\beta_L}{1-\beta_Q}, \quad b_9 = \delta + \nu - \frac{\delta + \rho}{\sigma}, \\
    b_{10} &= \frac{(\sigma-1)\beta_K \beta_Q}{\sigma} \frac{\beta_0}{1-\beta_Q}, \quad b_{11} = (\delta + \nu) \beta_Q \frac{\beta_0}{1-\beta_Q}, \quad b_{12} = (\delta + \nu) (\delta + \rho) \beta_Q \frac{\beta_0}{1-\beta_Q}, \\
    b_{13} &= (\delta + \nu) \beta_K + (\delta + \rho) (1 - \beta_Q), \quad b_{14} = \beta_K (1 - \beta_Q) \frac{\beta_0}{1-\beta_Q}, \\
    b_{15} &= \varphi^{\nu} (1 - \varphi)^{1-\nu} (\delta + \rho) \frac{\beta_Q}{\beta_K} \frac{\beta_0}{\beta_Q} \frac{\beta_0}{1-\beta_Q} (\nu - \beta_K + (\delta + \rho) \beta_L), \quad \text{and} \\
    b_{16} &= \frac{(\delta + \rho) \beta_K + (\delta + \rho) \beta_L}{\rho \sigma \beta_K}.
\end{align*}
\]

Appendix A1 Proof of Proposition 3.1

Recall the "clairvoyant" consumption function at period \(t\).

\[
c(t) = \eta(t) \left[ m(t) + \int_t^\infty w(s) e^{-\int_t^s \{r(t) - \nu\} ds} ds \right], \quad (A1.1)
\]

where \(\eta(t) = \left[ b_1 \{p(t)\}^{b_2} \int_t^\infty \{p(s)\}^{b_3} e^{\int_t^s \{b_4 - b_5 r(t)\} ds} ds \right]^{-1} \).

If we substitute true trajectories of exogenous variables \(w, r,\) and \(p\) with the households' expectations about them, which we assume constant at the current values at period \(t\), we obtain

\[
\int_t^\infty w(t) e^{-\int_t^s \{r(t) - \nu\} ds} ds = w(t) \int_t^\infty e^{-\int_t^s \{r(t) - \nu\} ds} ds = w(t) \frac{d}{ds} \left[ \frac{1}{\{r(t) - \nu\} \int_0^\infty e^{-\int_t^\infty \{r(t) - \nu\} ds} ds} \right] ds
\]

\[
= \frac{w(t)}{\{r(t) - \nu\} \int_0^\infty e^{-\int_t^\infty \{r(t) - \nu\} ds} ds} \lim_{s \rightarrow \infty} e^{-\int_t^s \{r(t) - \nu\} ds} = \frac{w(t)}{\{r(t) - \nu\} \int_0^\infty e^{-\int_t^\infty \{r(t) - \nu\} ds} ds} \left[ 1 - \lim_{s \rightarrow \infty} e^{-\int_t^s \{r(t) - \nu\} ds} \right], \quad \text{(A1.2)}
\]
By putting (A1.2) and (A1.3) into (A1.1), the consumption function becomes

\[ c(t) = \frac{b_2 r(t) - b_4}{b_1 \{ p(t) \}^{1-p}} \left[ m(t) + \frac{w(t)}{r(t) - \nu} \right] \left\{ 1 - \lim_{s \to \infty} e^{b_4 - b_5 r(t) s} \right\} \]

(A1.4)

In order to converge two limits towards zero in (A1.4) we need the following two conditions.

\[ r(t) - \nu > 0 \Rightarrow r(t) > \nu, \quad \text{and} \]

(A1.5)

\[ b_3 r(t) - b_4 > 0 \Rightarrow r(t) > \frac{b_4}{b_3} = \frac{\nu \sigma - \rho}{\sigma - 1}. \]

(A1.6)

First we check the relativity between two critical values as follows.

\[ \nu - \frac{\nu \sigma - \rho}{\sigma - 1} = \frac{\nu (\sigma - 1) - (\nu \sigma - \rho)}{\sigma - 1} = \frac{\rho - \nu}{\sigma - 1} > 0. \]

(A1.7)

Here recall that we previously assume that both the numerator and the denominator of the far right hand side are positive. Now we can examine the following 3 cases.

(1) Case 1: \( r > \nu \)

Since (A1.5) and (A1.6) are satisfied, the above two limits converge to zero, and we obtain

\[ c(t) = \frac{\{ b_3 r(t) - b_4 \}}{b_1 \{ p(t) \}^{1-p}} \left[ m(t) + \frac{w(t)}{r(t) - \nu} \right]. \]

(A1.8)

(2) Case 2: \( b_3 / b_5 < r \leq \nu \)

If \( r(t) = \nu \), \( c(t) \) becomes positive infinity from (A1.1). Otherwise the limit corresponding to (A1.5) diverges to positive infinity and that of (A1.6) converges to zero. As a result, \( c(t) \) becomes negative infinity.
(3) Case 3: \( r \leq b_4/b_5 \)

If \( r(t) = b_4/b_5 \), \( c(t) \) becomes positive infinity from (A1.1). Otherwise, we need to modify (A1.4) into the following form to examine this case.

\[
c = \frac{(b_4r - b_4)m}{b_1p^{1-\rho}(r - \nu)^s} \left[ 1 - \lim_{s \to \infty} e^{-(b_5r-b_4)s} \right] + \frac{w}{b_1p^{1-\rho}(r - \nu)^s} \lim_{s \to \infty} \frac{1 - e^{-(r-\nu)s}}{b_1p^{1-\rho}(r - \nu)^s} \quad \text{(A1.9)}
\]

Further, the far right limiting term is modified as

\[
\lim_{s \to \infty} \frac{1 - e^{-(r-\nu)s}}{1 - e^{-(b_5r-b_4)s}} = \lim_{s \to \infty} \frac{e^{(b_5r-b_4)s} - e^{(b_5r-b_4)s-(r-\nu)s}}{e^{(b_5r-b_4)s} - 1} = \frac{0 - \lim_{s \to \infty} \exp \left\{ \frac{1}{\sigma} (\rho - r)^s \right\}}{0 - 1}
\]

\[
= \lim_{s \to \infty} \exp \left\{ \frac{1}{\sigma} (\rho - r)^s \right\} . \quad \text{(A1.10)}
\]

Since \( r < \nu \) we have the inequality \( \rho - r > \rho - \nu > 0 \).

Hence the far right hand side of (A1.10) diverges to positive infinity. Since the first term of (A1.9) converges to zero and the second term diverges to positive infinity, \( c(t) \) diverges to positive infinity.

In sum, we need the condition \( r > \nu \) to have sensible consumption decision. When this condition is satisfied, the optimal consumption level is determined by (A1.8).

\[ Q.E.D. \]

Appendix A2  Proof of Lemma 3.1

Recall

\[
\phi^k(\xi) = \frac{b_{12}^2 \xi^2 - b_{13} \xi + b_{14}}{\sigma(\beta_K - b_{11} \xi)^\xi} \equiv \frac{\Omega(\xi)}{\sigma(\beta_K - b_{11} \xi)^\xi} \quad \text{(A2.1)}
\]

for \( 0 < \xi < \frac{\beta_K}{\delta + \nu} \beta_2 \frac{\rho}{\delta - \rho} \equiv \xi_{\text{max}} \).

Note that the denominator of the right hand side of (A2.1) is always positive for the given domain. Since the numerator is a quadratic function of \( \xi \) with positive
intercept and the denominator converge to positive infinitesimal, we have
\[ \lim_{\xi \to 0^+} \phi^k(\xi) = \infty. \] Now we prove \( \lim_{\xi \to \xi_{\text{max}}^-} \phi^k(\xi) = -\infty \) as follows.

First, rewrite (A2.1) as
\[
\phi^k(\xi) = \frac{(\delta + \nu)(\delta + \rho)z^{-1}\xi^2 - \left[ (\delta + \rho)(1 - \beta_Q) + (\delta + \nu)\beta_K \right] \xi}{\sigma(\beta_K \xi - (\delta + \nu)z^{-1}\xi^2)} + \frac{b_{14}}{\sigma(\beta_K - b_{11})\xi} \\
= \frac{(\delta + \rho)[(\delta + \nu)z^{-1}\xi^2 - \beta_K \xi] - \left[ (\delta + \rho)(1 - \beta_Q) - (\rho - \nu)\beta_K \right] \xi}{\sigma(\beta_K \xi - (\delta + \nu)z^{-1}\xi^2)} + \frac{\beta_K (1 - \beta_Q)z}{\sigma(\beta_K - b_{11})\xi} \\
= \frac{\beta_K (1 - \beta_Q)z - \left[ (\delta + \rho)(1 - \beta_Q) - (\rho - \nu)\beta_K \right] \xi}{\sigma(\beta_K - b_{11})\xi} - \frac{\delta + \rho}{\sigma}, \quad \text{(A2.2)}
\]
where \( z \equiv \beta_Q^{-1} \beta_Q \).

Let \( D(\xi) \) denote the numerator of the left term of (A2.2). The limit of \( D(\xi) \) as \( \xi \) to \( \xi_{\text{max}} \) is as follows.
\[
\lim_{\xi \to \xi_{\text{max}}^-} D(\xi) = \beta_K (1 - \beta_Q)z - \left[ (\delta + \rho)(1 - \beta_Q) - (\rho - \nu)\beta_K \right] \frac{\beta_K z}{\delta + \nu} \\
= \frac{(\delta + \nu)\beta_K (1 - \beta_Q)z - \left[ (\delta + \rho)(1 - \beta_Q) - (\rho - \nu)\beta_K \right] \beta_K z}{\delta + \nu} \\
= \frac{(\rho - \nu)(\beta_Q - 1)\beta_K z + (\rho - \nu)\beta_K^2 z}{\delta + \nu} = \frac{(\rho - \nu)(\beta_Q + \beta_K - 1)\beta_K z}{\delta + \nu} = -\frac{(\rho - \nu)\beta_K \beta_{12} z}{\delta + \nu} < 0
\]
Since the corresponding denominator is positive infinitesimal, we have proven
\[ \lim_{\xi \to \xi_{\text{max}}^-} \phi^k(\xi) = -\infty. \]

Due to the fact that the denominator of (A2.1) is positive and the sign of limit towards upper bound is negative, it is necessary that \( \Omega(\xi_{\text{max}}) < 0 \). Since \( \Omega(0) = b_{14} > 0 \), there exists a unique \( \xi \) such that \( \Omega(\xi) = 0 \). \( \xi \) is the smaller root of the equation \( \Omega = 0 \), i.e.
\[ \xi = \frac{b_{13} - \sqrt{b_{13}^2 - 4b_{12}b_{14}}}{2b_{12}} = \frac{\beta_K}{\delta + \rho} \beta_Q^{-1} \beta_Q \]. It is easy to see \( \xi < \xi_{\text{max}} \).
Hence, \( \Omega(\xi) \geq 0 \), if and only if \( \xi \leq \bar{\xi} \).

Because the denominator is positive, we have proved that

\[
\phi^k(\xi) \geq 0, \text{ if and only if } \xi \leq \bar{\xi} = \frac{\beta_k}{\delta + \rho} \frac{\beta_q}{Q_{1-a}Q}.
\]

Moreover, we have shown \( \lim_{\xi \to 0^+} \phi^k(\xi) = \infty \) and \( \lim_{\xi \to \xi_{\text{max}}^-} \phi^k(\xi) = -\infty. \)

Q.E.D.

Appendix A3 Proof of Proposition 3.6

Two optimality conditions (3.15a) and (3.15b) are transformed into the following two equations with \( \mu(t) = \mu(0)e^{(\delta+\rho)t} \) and \( \Theta = 0. \)

\[
\begin{align*}
\varepsilon - \frac{\partial \hat{c}}{\partial p} &= -\mu(0)N_0e^{(\delta+\rho)t}\left\{\left(\hat{q}_H + \hat{q}_M\right) + p\left(\frac{\partial \hat{q}_H}{\partial p} + \frac{\partial \hat{q}_M}{\partial p}\right) + \hat{k}\frac{\partial \phi^k}{\partial p}\right\}, \quad (A3.1a) \\
\varepsilon - \frac{\partial \hat{c}}{\partial k} &= -\mu(0)N_0e^{(\delta+\rho)t}\left\{p\left(\frac{\partial \hat{q}_H}{\partial k} + \frac{\partial \hat{q}_M}{\partial k}\right) + \phi^k + \hat{k}\frac{\partial \phi^k}{\partial k} + \delta + \nu\right\}, \quad (A3.1b)
\end{align*}
\]

From these two equations we can derive the following optimality condition,

\[
\frac{\partial \hat{c}/\partial p}{\partial \hat{c}/\partial k} = \left\{\left(\hat{q}_H + \hat{q}_M\right) + p\left(\frac{\partial \hat{q}_H}{\partial p} + \frac{\partial \hat{q}_M}{\partial p}\right) + \hat{k}\frac{\partial \phi^k}{\partial p}\right\}, \text{ which is equivalent to}
\]

\[
\Gamma\left(p; k\right) = 0, \text{ where}
\]

\[
\Gamma\left(p; k\right) = \frac{\partial}{\partial p}\left[\hat{c}(p; k)\right] - \frac{\partial}{\partial k}\left[\hat{c}(p; k)\right] + \left\{\phi^k(p; k) + (\delta + \nu)\right\}k
\]

This implicit function determines water price \( p(t) \), given \( k(t) \). Once \( p(t) \) is determined, the level of \( k \) at the next moment is determined by the equation of motion, and it determines water price at that moment, and so on, while the time path
of $G$ is determined by that of $p$. Hence $\Gamma(p; k) = 0$ and the equations of motion of $\dot{k}$ and $G$ determine a set of trajectories.

Now let us derive the rate of change of water price along this set of trajectories. By taking time derivative of the equations (A3.1a) and (A3.1b) with logarithmic transformation, we derive

$$
\frac{\dot{p}}{p} = \frac{\delta + \rho - \left\{ p \frac{\partial}{\partial k} (\dot{c} - \Omega) - \sigma e_k \right\} \phi^k}{p \frac{\partial}{\partial p} (\dot{c} - \Omega) - \sigma e_p} = \phi_{ES1}^p, \quad \text{and}
$$

$$
\frac{\dot{p}}{p} = \frac{\delta + \rho - \left\{ k \frac{\partial}{\partial k} (\dot{c} - \Omega) - \sigma e_k \right\} \phi^k}{k \frac{\partial}{\partial p} (\dot{c} - \Omega) - \sigma e_p} = \phi_{ES2}^p, \quad \text{where}
$$

$$
\varepsilon_p = \frac{\partial \hat{c}}{\partial p} p, \quad \varepsilon_k = \frac{\partial \hat{k}}{\partial k} c, \quad \text{and} \quad \Omega = p(\hat{q}_H + \hat{q}_M) + (\phi^k + \delta + \nu) \dot{k}.
$$

It is clear that the necessary and sufficient condition to have a unique trajectory of water price is $\dot{c} - \Omega = 0$, and this satisfies $\Gamma(p; k) = 0$.

As a result, the system determining the trajectories becomes

$$
f^{ES}(p; k) = \dot{c} - p(\hat{q}_H + \hat{q}_M) - (\phi^k + \delta + \nu) \dot{k} = 0, $$

$$
\frac{\dot{G}}{G} = \frac{p}{G} (\hat{q}_H + \hat{q}_M) N_6 e^{\nu} - \delta = \phi_{ES}^G (G, k, \dot{k}), \quad \text{and}
$$

$$
\frac{\dot{k}}{k} = \phi^k (k, p).
$$

In addition, the differential equation of water price becomes

$$
\frac{\dot{p}}{p} = -\left( \frac{\delta + \rho + \sigma e_k \phi^k}{\sigma e_p} \right) = \phi_{ES}^p (p, k). \quad \text{Q.E.D.}
$$
Appendix A4  Proof of Proposition 3.8

When $\Theta > 0$ the relationship $\Theta = p\mu$ holds, since the shadow price of relaxing water balance consumption means the social benefit of providing additional water of which relative price to the capital good is $p$.

With $\Theta = p\mu$, sustainability condition (3.15d), and (3.15c), the optimality conditions (3.15a) and (3.15b) are transformed into

\[
\frac{\partial c}{\partial p} = -\mu \left( F^w + N_0 e^v \hat{k} \frac{\partial \phi^k}{\partial p} \right), \quad \text{and} \quad (A4.1a)
\]

\[
\frac{\partial c}{\partial \hat{k}} = -\mu N_0 e^v \left( \delta + \nu - p \frac{\partial F^w}{\partial G} + \phi^k + \hat{k} \frac{\partial \phi^k}{\partial p} \right). \quad (A4.1b)
\]

From these two optimality conditions, we derive the following equation.

\[
f^{ls} \left( p, \hat{k}, G \right) = \frac{\partial \hat{c}}{\partial \hat{k}} - \frac{\left\{ F^w + \hat{K} \left( \partial \phi^k / \partial p \right) \right\}}{N_0 e^v \left\{ \delta + \nu - p \frac{\partial F^w}{\partial G} + \phi^k + \hat{k} \left( \partial \phi^k / \partial p \right) \right\}} = 0,
\]

in which $\hat{K} = N_0 e^v \hat{k}$

By taking time derivative of both hand sides of Eq. (A4.1a) with logarithmic transformation, the following differential equation is derived.

\[
\frac{\dot{p}}{p} = \frac{(\delta + \rho)}{(\hat{c} - \sigma)} e_p - \frac{\left\{ F^w + \hat{K} \left( \partial \phi^k / \partial p \right) \right\}}{N_0 e^v \left\{ \delta + \nu - p \frac{\partial F^w}{\partial G} + \phi^k + \hat{k} \left( \partial \phi^k / \partial p \right) \right\}}
\]

\[
\left\{ F^w + \hat{K} \left( \partial \phi^k / \partial p \right) \right\} \left( \frac{\partial \phi^k / \partial \hat{k}}{\phi_{MC}} \right)
\]

\[
\left\{ F^w + \hat{K} \left( \partial \phi^k / \partial p \right) \right\} \left( \frac{\partial \phi^k / \partial \hat{k}}{\phi_{MC}} \right) = \phi_{MC} \left( p, \hat{k}, G \right).
\]

Due to the fact that the interior solution is a special case of market clear solutions, the rate of change of the water price has to follow that of the market-clear optimal trajectories. Recall the market-clear optimal counterpart is
Appendix A

\[ \phi_{\text{MC}}^{p} = \frac{\varepsilon_{a} F^{w} \phi_{\text{MC}}^{a} - N_{6} e^{w} \left\{ \nu \left( \partial_{H} + \partial_{M} \right) + \kappa \left( \partial_{H} / \partial p + \partial_{M} / \partial p \right) \right\} \phi^{k}}{N_{6} e^{w} p \left( \partial_{H} / \partial p + \partial_{M} / \partial p \right)}. \]

It is obvious that one of necessary conditions to have \( \phi_{\text{IS}}^{p} = \phi_{\text{MC}}^{p} \) is that the term \( r (a + \phi) \) in \( \phi_{\text{IS}}^{p} \) has to vanish, which requires either \( F^{w} + \kappa \left( \partial \phi^{k} / \partial p \right) \) is zero or \( \left| \partial^{2} \phi^{k} / \partial p^{2} \right| = \infty \), because it is economically insensible to associate an infinite value with \( \hat{c} \), \( \hat{k} \), \( p \) or \( \varepsilon_{p} \).

If the former is the case, \( \partial c / \partial p \) is required to satisfy \( f^{\infty} = 0 \). This cannot be true since neither \( \partial c / \partial p = 0 \) nor \( \partial c / \partial k = \infty \) can make economic sense.

If the latter is the case, \( \phi_{\text{IS}}^{p} \) must always be zero and this requires \( \phi_{\text{MC}}^{a} = \phi^{k} = 0 \) as well, which means a steady-state, due to the fact that \( \phi_{\text{IS}}^{p} = \phi_{\text{MC}}^{p} \). At the steady-state \( \partial^{2} \phi^{k} / \partial p^{2} \) is evaluated as

\[ \frac{b_{6} (\delta + \rho) \{ \rho \beta_{K} + (\delta + \rho) \beta_{L} \} + 2 \delta / \rho + 2 \delta \beta_{K}}{\rho \sigma \beta_{K} \rho^{2}} \left[ 1 - \beta_{Q} \right] \left[ \rho \beta_{K} + (\delta + \rho) \beta_{L} \right] \],

which cannot be neither positive nor negative infinite since zero water price does not make sense which requires infinite water supply capacity. As a result, when \( \Theta > 0 \) there is no interior solution satisfying necessary conditions (3.15a) - (3.15d).

Q.E.D.

Appendix A5 Derivation of A matrix

Each element of the matrix A is derived as follows. Note that the derivation often utilises the fact that \( \overline{\phi}_{\text{MC}}^{a} = \overline{\phi}_{\text{MC}}^{p} = \overline{\phi}^{k} = 0 \), in which the overline denotes the value evaluated at the steady-state.
(1) \( A_{11} = \frac{\partial G \phi_{\varepsilon}^G}{\partial G} \bigg| x^e = \frac{\partial}{\partial G} \left( p F^w - \delta G \right) \bigg| x^e = \frac{\partial}{\partial G} \left( F^w (\bar{G}) \right) = \frac{\delta \bar{G}}{\bar{p}} \).

(2) \( A_{12} = \frac{\partial G \phi_{\varepsilon}^p}{\partial p} \bigg| x^e = \frac{\partial}{\partial p} \left( p F^w - \delta G \right) \bigg| x^e = F^w (\bar{G}) = \frac{\delta \bar{G}}{\bar{p}} \).

(3) \( A_{13} = \frac{\partial G \phi_{\varepsilon}^G}{\partial \kappa} \bigg| x^e = 0 \).

(4) \( A_{21} = \frac{\partial \phi_{\varepsilon}^p}{\partial p} \bigg| x^e = \frac{\partial \phi_{\varepsilon}^p}{\partial G} \bigg| x^e \). Here, \( \frac{\partial \phi_{\varepsilon}^p}{\partial G} \bigg| x^e = \)

\[
\frac{\partial}{\partial G} \left[ \frac{\varepsilon_0 F^w (\bar{G}) \left( \delta G (\bar{G}) - 1 \right)}{N_0 \bar{p} \left( \delta q_H / \partial p + \delta q_M / \partial p \right)} \right] x^e
\]

\[
\frac{\phi_{\varepsilon}^p}{\partial G} \left( \varepsilon_0 F^w (\bar{G}) \left( \delta G (\bar{G}) - 1 \right) \right)
\]

in which \( D_1 = -\frac{\partial}{\partial p} \left( \delta q_H + \delta q_M \right) \). \( A_{21} = \)

\[
\frac{\delta F^w (\bar{G}) \varepsilon_0 (\bar{G}) \left( \varepsilon_0 (\bar{G}) - 1 \right)}{N_0 \bar{p}} e_{G} D_1.
\]

(5) \( A_{22} = \frac{\partial \phi_{\varepsilon}^p}{\partial p} \bigg| x^e = \frac{\partial \phi_{\varepsilon}^p}{\partial G} \bigg| x^e = \frac{\partial \phi_{\varepsilon}^p}{\partial G} \bigg| x^e \). Here, \( \frac{\partial \phi_{\varepsilon}^p}{\partial p} \bigg| x^e = \)

\[
\frac{\partial}{\partial p} \left[ \frac{\varepsilon_0 F^w \phi_{\varepsilon}^G}{N_0 \bar{p} \left( \delta q_H / \partial p + \delta q_M / \partial p \right)} \right] x^e
\]

\[
\frac{\varepsilon_0 F^w \phi_{\varepsilon}^G}{N_0 \bar{p} \left( \delta q_H / \partial p + \delta q_M / \partial p \right)} x^e
\]

\[
\frac{\varepsilon_0 F^w \phi_{\varepsilon}^G}{N_0 \bar{p} \left( \delta q_H / \partial p + \delta q_M / \partial p \right)} x^e
\]

\[
\frac{\varepsilon_0 F^w \phi_{\varepsilon}^G}{N_0 \bar{p} \left( \delta q_H / \partial p + \delta q_M / \partial p \right)} x^e
\]
\[
\begin{align*}
\epsilon_a(\bar{G}) \left\{ \frac{F''(\bar{G})}{G} \right\}^2 - N_0 \phi_k \left\{ \frac{\partial}{\partial p} \left( \frac{\partial \hat{\theta}_H + \partial \hat{\theta}_M}{\partial k} \right) \phi^k + \left( \frac{\partial \hat{\theta}_H + \partial \hat{\theta}_M}{\partial k} \right) \frac{\partial \phi^k}{\partial p} \right\} x^e \\
= -N_0 \bar{p} D_1
\end{align*}
\]

\[
\epsilon_a(\bar{G}) \left\{ \frac{F''(\bar{G})}{G} \right\}^2 - N_0 \frac{\partial \phi^k}{\partial p} \left\{ x^e \right\} , \text{ in which } D_2 = \frac{\partial}{\partial k} \left( \hat{\theta}_H + \hat{\theta}_M \right)x^e > 0.
\]

Here, \( \frac{\partial \phi^k}{\partial p} \left\{ x^e \right\} = \frac{d \phi^k}{d \xi} \frac{\partial x^e}{\partial p} \left\{ x^e \right\} , \) where

\[
\frac{d \phi^k}{d \xi} \left\{ x^e \right\} = \frac{2b_{12} \bar{\xi} - b_{13}}{\sigma (\beta_K - b_{11} \bar{\xi}) \bar{\xi}} - \frac{\partial}{\partial \xi} \left\{ \frac{\sigma (\beta_K - b_{11} \bar{\xi}) \bar{\xi}}{\beta (\beta_K - b_{11} \bar{\xi}) \bar{\xi}} \phi^k \right\} x^e
\]

\[
= \frac{(\delta + \rho) \left\{ \beta \bar{\xi} + (\delta + \rho) \beta \right\}}{\rho \sigma \beta K \bar{\xi}} = \frac{b_{16}}{\bar{\xi}}, \text{ in which}
\]

\[
b_{16} = \frac{(\delta + \rho) \left\{ \beta \bar{\xi} + (\delta + \rho) \beta \right\}}{\rho \sigma \beta K \bar{\xi}} > 0. \text{ Hence } \frac{\partial \phi^k}{\partial p} \left\{ x^e \right\} = -\frac{b_{16}}{\bar{p}} .
\]

\[
\therefore A_{22} = -\left[ \epsilon_a(\bar{G}) \left\{ \frac{F''(\bar{G})}{G} \right\}^2 + b_6 b_{16} \frac{k \bar{D}_2}{\bar{p} D_1} \right] .
\]

\[
(6) A_{23} = \frac{\partial \phi^p}{\partial k} \left\{ x^e = \bar{p} \frac{\partial \phi^p}{\partial k} \right\} \left\{ x^e = -\bar{p} \frac{\partial}{\partial k} \left\{ \frac{k (\hat{\theta}_H / \hat{\theta}_M) + \hat{\theta}_M / \hat{\theta}_H \phi^k}{p (\hat{\theta}_H / \hat{\theta}_M + \hat{\theta}_M / \hat{\theta}_H) \phi^k} \right\} x^e
\]

\[
= -\bar{p} \frac{\partial}{\partial k} \left\{ \frac{k (\hat{\theta}_H + \hat{\theta}_M)}{\hat{\theta}_H + \hat{\theta}_M} \phi^k + \bar{k} \frac{\partial \phi^k}{\partial k} \right\} x^e + \frac{\bar{p} k \bar{D}_2 \phi^k}{(-\bar{p} D_1)^2} = \frac{k D_2 \phi^k}{D_1 \phi^k} x^e .
\]

Here, \( \frac{\partial \phi^k}{\partial k} \left\{ x^e = \frac{d \phi^k}{d \xi} \right\} \left\{ x^e = \frac{b_8 \bar{\xi}}{k} \right\} \left\{ x^e = -\frac{b_{16}}{k} \right\} .
\]

\[
\therefore A_{23} = -b_8 b_{16} \frac{D_2}{D_1} .
\]

\[
(7) A_{31} = \frac{\partial \phi^k}{\partial G} \left\{ x^e = 0 .
\]

\[
(8) A_{32} = \frac{\partial \phi^k}{\partial p} \left\{ x^e = -\frac{\bar{p}}{\phi^k} \right\} \left\{ x^e = -b_6 b_{16} \frac{\bar{p}}{\phi^k} .
\]
Appendix A

Proof of Proposition 3.11

First of all, the local stability condition is valid only for the neighbourhood of the steady-state. Hence the economy must be enough close to the optimal steady-state to apply this proposition.

Let \( g(r) \) denote the characteristic polynomial of \( A \). We have

\[
g(r) = -r^3 + \text{trace } A \times r^2 + \Phi(A) \times r + \det A,
\]

where

\[
\text{trace } A = A_{11} + A_{22} + A_{33},
\]

\[
\Phi(A) = A_{12}A_{21} - A_{11} (A_{22} + A_{33}), \quad \text{and } \det A = A_{11} (A_{22}A_{33} - A_{23}A_{32}) - A_{12}A_{21}A_{33}.
\]

The coefficients of \( g(r) \) are evaluated as

\[
\text{trace } A = \delta \left( e_o(G) - 1 \right) - \frac{e_o(G) \left( G^{\text{inv}} (G) \right)^2}{N_0 G D_1} - \frac{b_6 b_{16} \bar{k} D_2}{\bar{p} D_1} - b_8 b_{16},
\]

\[
\Phi(A) = \left( e_o(G) - 1 \right) \left\{ b_6 b_{16} \frac{\bar{k} D_2}{G D_1} + b_8 b_{16} \right\}, \quad \text{and } \det A = 0.
\]

As \( \det A = 0 \) the characteristic equation \( g(r) = 0 \) has one zero real eigenvalue which is unstable. The remaining two eigenvalues are expressed as

\[
r = \frac{\text{trace } A \pm \sqrt{(\text{trace } A)^2 + 4 \Phi(A)}}{2}.
\]

Their signs are determined as follows.

**Case 1**: \( e_o(G) < 1 \)

Because both of \( \text{trace } A \) and \( \Phi(A) \) are strictly negative, two eigenvalues are either two negative real numbers or two conjugate complex numbers with negative real part. In both cases they are stable eigenvalues.
Case 2: \( \varepsilon_G(\bar{G}) > 1 \)

The sign of \( \Phi(A) \) is strictly positive and \( (\text{trace } A)^2 + 4\Phi(A) > (\text{trace } A)^2 \) holds regardless of the sign of trace \( A \). Thus one of the remaining two eigenvalues is positive real number and the other is negative real number, which means the former is an unstable and the latter is a stable eigenvalue.

Case 3: \( \varepsilon_G(\bar{G}) = 1 \)

\( \Phi(A) = \text{zero} \) is associated with one zero real eigenvalue and the other one is \( r = \text{trace } A \) which is negative with \( \varepsilon_G(\bar{G}) = 1 \). Hence this case is associated with one stable and two unstable eigenvalues.

Recall that the linearised system around the optimal steady-state is unstable unless as many initial conditions as the number of unstable eigenvalues are freely chosen (Gandolfo 1997; Theorem 18.3). Along the market-clear trajectories the initial water price is completely determined by the condition \( f^{MC} (p, G_0, k_0) = 0 \). Therefore the initial values of the same number of state variables as that of unstable eigenvalues must be freely chosen to achieve the optimal steady-state along the optimal trajectories.

Q.E.D.

Appendix A7 Proof of Proposition 3.12

The current value Hamiltonian of the household problem in discrete time is

\[
\bar{H}_s = \frac{c_s^{1-\sigma}}{1-\sigma} + \frac{1}{1+\rho} \lambda_{s+1} \left\{ (r_s - \nu) m_s + w_s - c_s^H - p_s q_s^H \right\}.
\]

The necessary conditions to have an interior solution are

\[
\frac{\partial \bar{H}_s}{\partial c_s^M} = 0 \Rightarrow \frac{1}{1+\rho} \lambda_{s+1} = \phi \frac{c_s^{1-\sigma}}{c_s^M}
\]

(A7.1a)

\[
\frac{\partial \bar{H}_s}{\partial q_s^H} = 0 \Rightarrow \frac{p_s}{1+\rho} \lambda_{s+1} = (1-\phi) \frac{c_s^{1-\sigma}}{q_s^H}
\]

(A7.1b)
\[
\left(\frac{1+\nu}{1+\rho}\right)\lambda_{s+1} - \lambda_s = \frac{\partial \tilde{H}_s}{\partial m_s} \implies \lambda_{s+1} = \frac{1+\rho}{1+r_s} \lambda_s \tag{A7.1c}
\]

\[
\lim_{s \to \infty} \left[ \left(\frac{1+\nu}{1+\rho}\right)^{(s-t)} \lambda_s \cdot m_s \right] = 0. \tag{A7.1d}
\]

The following optimal conditions are exactly the same as in continuous-time.

\[q_s^H = \left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} p_s^{-\varphi} c_s, \quad \text{and} \quad c_s^M = \left(\frac{\varphi}{1-\varphi}\right)^{1-\varphi} p_s^{1-\varphi} c_s. \tag{A7.2}\]

By putting (A7.2) into (A7.1a) we have \(\lambda_{s+1}/(1+\rho) = 1/(b_1 p_s^{1-\varphi} c_s^\sigma)\), and \(\lambda_s/(1+\rho) = 1/(b_1 p_{s-1}^{1-\varphi} c_{s-1}^\sigma)\) since the same relation must hold in period \(s-1\) as well. From these relations with Eq. (A7.1c), we derive

\[
c_{t+j} = c_t \left(\frac{p_t}{p_{t+j}}\right)^{1-\varphi} \prod_{i=0}^{j-1} \left(1+\frac{r_{t+i}}{1+\rho}\right)^{\frac{1}{\sigma}}, \quad \text{and with no-perfect foresight assumption}
\]

\[
c_{t+j} = c_t \left(\frac{1+\nu}{1+\rho}\right)^{\frac{j}{\sigma}}. \tag{A7.3}\]

Recall the equation of motion of the household assets with incorporating relations (A7.2) is expressed as

\[
m_{s+1} = \frac{1+r_s}{1+\nu} m_s + \frac{1}{1+\nu} \left(w_s - b_1 p_s^{-\varphi} c_s\right). \quad \text{From this with no-perfect foresight assumption, the inter-temporal budget constraint from period } t \text{ to } T \text{ is derived as follows.}
\]

\[
m_{t+T} \left(\frac{1+\nu}{1+r_t}\right)^T = m_t + \frac{w_t}{1+r_t} \sum_{i=0}^{T-1} \left(1+\nu\right)^i \left(\frac{1+r_{t+i}}{1+r_t}\right)^{\frac{1}{\sigma}} - b_1 \sum_{i=0}^{T-1} \left(1+r_{t+i}\right)^{\frac{1}{\sigma}} \left(1+\nu\right)^i \frac{(1+r_t)^{1-\sigma}}{1+r_t}. \tag{A7.4}\]

Recall that the transversality condition with Eq. (A7.1c) becomes

\[
\lim_{s \to \infty} \left[ \left(\frac{1+\nu}{1+\rho}\right)^{(s-t)} \left(\frac{1+\rho}{1+r_t}\right)^{(s-t)} \lambda_s \cdot m_s \right] = \lambda_t \lim_{s \to \infty} \left[ m_s \left(\frac{1+\nu}{1+r_t}\right)^{(s-t)} \right] = 0.
\]

It means that when \(T\) goes to infinity, \(m_{t+T} \left(\frac{1+\nu}{1+r_t}\right)^T\) becomes zero. Now we can derive the discrete-time consumption function as
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\[ \hat{c}_t = \eta_t \left\{ m_t + \frac{w_t}{1+r_t} \sum_{i=0}^{\infty} \left( \frac{1+r_t}{1+r_{t+1}} \right)^i \right\}, \text{ in which} \]

\[ \eta_t = \left[ b_1 p_{t+1}^{1-\rho} \sum_{i=0}^{\infty} \left( \frac{1+r_t}{1+\rho} \right)^{1/\sigma} \left( \frac{1+r_t}{1+r_{t+1}} \right)^{i-1} \right]. \]

In order to have sensible optimal consumption, it is required to have convergence for each summation, and it is so when \( \left( \frac{1+r_t}{1+r_{t+1}} \right) \) and \( \left( \frac{1+r_t}{1+\rho} \right)^{1/\sigma} \left( \frac{1+r_t}{1+r_{t+1}} \right)^{i-1} \) are strictly less than unity. It is easily found that its necessary and sufficient condition is \( r_t > \nu \) due to the fact that \( \nu - \left( \frac{(1+r_t)^{\sigma(\sigma-1)}}{(1+\rho)^{\sigma(\sigma-1)}} - 1 \right) = (1+\nu) \left[ 1 - \left( \frac{1+r_t}{1+\rho} \right)^{\sigma(\sigma-1)} \right] > 0 \), and the corresponding optimal consumption becomes as Proposition 3.12 states. When \( r_t \) does not satisfy this condition, \( \hat{c}_t \) cannot be a positive finite value.

Q.E.D.

Appendix A8 Proof of Proposition 3.13

The equation of motion of private capital in discrete-time is expressed as

\[ \dot{k}_{s+1} = \left( r_s - \nu \right) \hat{k}_s + w_s - b_1 p_{s+1}^{1-\rho} \hat{c}_s \right\} / (1+\nu) = \hat{k}_s \hat{d}_s. \]

Except for this slight modification, the discrete-time version of the system determining market-clear trajectories is exactly the same as the continuous-time version.

Now let us consider the excess-supply trajectories. The Lagrangian of the government problem in discrete time is

\[ L_s^G = \frac{1-\sigma}{1-\sigma} + \frac{1+\nu}{1+\rho} \hat{r}_s \hat{k}_s + \frac{1+\nu}{1+\rho} \lambda_{s+1}^G \left\{ p_s \left( \hat{q}_s^H + \hat{q}_s^M \right) + \hat{G}_s \right\} 

+ \Theta_s \left\{ F^w \left( G_s \right) - \left( \hat{q}_s^H + \hat{q}_s^M \right) N_0 (1+\nu)^{\nu} \right\}. \]

When the sustainable water supply capacity exceeds the optimal water demand, \( \Theta_s = 0 \). In addition, the following relationship among the other two Lagrange multipliers holds.
\[
\frac{1+v}{1+\rho} \lambda_{s+1}^G = \mu_s^G = \frac{1}{N_0 (1+v)^s} \mu_s^k = \frac{1}{N_0 (1+v)^s} \frac{1+v}{1+\rho} \lambda_{s+1}^k ,
\]
in which \( \mu_s^G \) is the shadow price of additional unit of investment in public capital, and \( \mu_s^k \) is the shadow price of additional unit of investment in private capital.

The first equality holds because the shadow price of additional unit of investment in public capital and the shadow price of allowing additional unit of public capital in the next year must be the same along the optimal trajectories. Note that \( \lambda_{s+1}^G \) represents the value of additional unit of public capital in year \( s+1 \) from the perspective of year \( s+1 \). The same value from the perspective of year \( s \) is \[ \frac{1+v}{1+\rho} \lambda_{s+1}^G . \] The same logic leads to \[ \mu_s^k = \frac{1+v}{1+\rho} \lambda_{s+1}^k , \] and the second equality \[ \mu_s^G = \frac{1}{N_0 (1+v)^s} \mu_s^k \] holds because the same capital good is invested in public capital and private capital and consequently the ratio of their shadow price along the optimal trajectories must be unity, as explained for the continuous-time case.

Let define \( \mu_s = \mu_s^G = \frac{1}{N_0 (1+v)^s} \mu_s^k \). The necessary conditions to have an interior solution under \( \theta_s = 0 \) are

\[
\hat{\epsilon}_s \sigma \frac{\partial \mu_s}{\partial p_s} = -N_0 (1+v)^s \mu_s \frac{\partial}{\partial p_s} \left\{ \hat{\kappa}_s \hat{\phi}_s^k + p_s \left( \hat{\theta}_s^H + \hat{\theta}_s^M \right) \right\} , \tag{A8.1a}
\]

\[
\hat{\epsilon}_s \sigma \frac{\partial \mu_s}{\partial \hat{k}_s} = -N_0 (1+v)^s \mu_s \frac{\partial}{\partial \hat{k}_s} \left\{ \hat{\kappa}_s \hat{\phi}_s^k + p_s \left( \hat{\theta}_s^H + \hat{\theta}_s^M \right) \right\} - N_0 (1+v)^s \mu_s + \frac{1+\rho}{1+v} N_0 (1+v)^{s-1} \mu_{s-1} , \tag{A8.1b}
\]

\[
\mu_s \frac{1+\rho}{1+v} \mu_{s-1} = \mu_s \delta . \tag{A8.1c}
\]

Equations (A8.1b) and (A8.1c) are combined into

\[
\hat{\epsilon}_s \sigma \frac{\partial \mu_s}{\partial \hat{k}_s} = -N_0 (1+v)^s \mu_s \left[ \frac{\delta +v}{1+v} + \frac{\partial}{\partial \hat{k}_s} \left\{ \hat{\kappa}_s \hat{\phi}_s^k + p_s \left( \hat{\theta}_s^H + \hat{\theta}_s^M \right) \right\} \right] . \tag{A8.2}
\]

From Equations (A8.1a) and (A8.2), the discrete-time version of \( \Gamma \) becomes
\[
\frac{\partial \tilde{e}_s}{\partial p_s} = \frac{\partial \partial p_s}{\partial \tilde{c}_s} \left\{ \tilde{c}_s \phi_k^m + p_s \left( \tilde{q}_s^H + \tilde{q}_s^M \right) \right\}, \text{ or equivalently }
\frac{\partial \tilde{e}_s}{\partial \tilde{c}_s} \left\{ \frac{\delta + v}{1 + v} + \partial \tilde{c}_s \left\{ \tilde{c}_s \phi_k^m + p_s \left( \tilde{q}_s^H + \tilde{q}_s^M \right) \right\} \right\}
\]

Hence the discrete-time version of \( f^{ES} \) is expressed as

\[
f^{ES} = \tilde{c}_s - \tilde{c}_s \left( \frac{\delta + v + \phi_k}{1 + v} \right) - p_s \left( \tilde{q}_s^H + \tilde{q}_s^M \right) = 0.
\]

By substituting \( \tilde{c}_s \phi_k^m \) with \( \frac{\left( r_s - \nu \right) \tilde{c}_s + w_s^* - b_1 p_s \tilde{c}_s}{1 + v} \), we have

\[
f^{ES} \left( p_s, \tilde{c}_s \right) \equiv \left\{ 1 + \frac{p_s \tilde{c}_s}{\phi^m (1 - \phi)^{1 - \phi} (1 + v)} \right\} \tilde{c}_s - p_s \left( \tilde{q}_s^H + \tilde{q}_s^M \right) - \frac{\left( \nu_s + \delta \right) \tilde{c}_s + w_s^*}{1 + v}.
\]

The remaining equations of motion are self-evident.

Q.E.D.

Appendix A9 Proof of Lemma 4.1

Let \( M^u_i \) denote the expected wage income derived from allocating one unskilled labourer to urban areas, and \( M^l_i \) that to the irrigation agriculture sector, respectively. Let \( \theta^c_i \) denote the probability for the unskilled member migrating to urban area to be employed in the urban modern sector. We have \( M^u_i = \theta^c_i \tilde{w}_u \) and \( M^l_i = w_i^t \left\{ 1 - (1 - \theta_i) \tilde{z} \right\} \).

Due to the poor access to safe water in rural areas, the expected water consumption is expressed as \( \tilde{q}_i^H = q_i^H \) for the urban residents and \( \tilde{q}_i^M = \theta_i q_i^H + (1 - \theta_i) \tilde{q}_i^M \) for the irrigation labourers.

Let \( V^u_i \) denote indirect felicity derived from expected income wage when one unskilled labourer is allocated to urban areas and \( V^l_i \) denote that to the irrigation
agriculture sector. The indirect felicity is derived by solving the following felicity maximisation problem.

\[
\max_{\{c^R, \bar{c}^U, \bar{c}^L\}} u(c_i), \quad c_i = (c^R_i)^{\varphi_R}(c^U_i)^{\varphi_U}(c^L_i)^{\varphi_L},
\]

subject to budget constraint \( p^R c^R_i + p^U c^U_i + p^L c^L_i + p_i q^H_i \leq M_i \), for \( i = U, I \).

Assuming an interior solution, we derive the Marshallian demands and consequently the following indirect felicity for each case.\(^1\)

\[
V^U_i = \theta^E w_i \prod_{\mu=R, I, U, Q} \left( \frac{\varphi_{R,i}}{p_{R,i}} \right)^{v_{R,i}}, \quad \text{and}
\]

\[
V^I_i = (\theta_i)^{v_0} \left[ \frac{w_i}{1} \left\{ 1 - (1 - \theta_i) \right\} + p_i q_{no} \left( \frac{1}{\theta_i} - 1 \right) \right] \prod_{\mu=R, I, U, Q} \left( \frac{\varphi_{R,i}}{p_{R,i}} \right)^{v_{R,i}}.
\]

The following equilibrium probability \( \theta^E_i \) is easily obtained by solving \( V^U_i = V^I_i \),

\[
\theta^E_i = \left( \frac{\theta_i}{w_i} \right)^{v_0} \left[ \frac{w_i}{1} \left\{ 1 - (1 - \theta_i) \right\} + p_i q_{no} \left( \frac{1}{\theta_i} - 1 \right) \right].
\]

In addition, there is a restriction that \( 0 < \theta^E \leq 1 \), which imposes a condition

\[
\bar{w}_i (\theta_i)^2 + \left\{ (1 - \bar{w}_i w_i - p_i q_{no}) \theta_i - \bar{w}_i (\theta_i)^{-v_0} + p_i q_{no} \right\} \leq 0.
\]

Q.E.D.

Appendix A10  Proof of Proposition 4.1

Recall that the equation of motion of household assets in terms of equity share is

\[
m_{t+1} = \frac{1+r_t}{1+v} m_t - B_t c_t + B_2 + \left( \frac{1}{1+v} \right) B_3 - \left( \frac{1}{1+v} \right) \frac{w_{t+1}}{p_{R,i}} \left( K_t^R \right)^{\frac{1}{\rho_{R,i}}} \frac{1}{p_{R,i}} \left( K_t^R \right)^{\frac{1}{\rho_{R,i}}}
\]

\[
- \left( \frac{1}{1+v} \right)^{\frac{1}{\rho_{R,i}}} \left( \frac{\beta_R w_{t+1}^{R,i}}{\beta_{R,i} p_{R,i}^{R,i} (r_t + \delta)} \right)^{\frac{1}{\rho_{R,i}}} - (1 - \delta) K_t^R,
\]

and \( \text{(A10.1)} \)

\(^1\) For the purpose of this analysis we can apply any positive monotonic transformation to the felicity function. Thus we use the simplest case, \( u(c) = c \).
m_{s+1} = \frac{1+r_i}{1+\nu} m_s - B_i c_i + B_2 + \left( \frac{1}{1+\nu} \right)^s \left( B_3 - B_4 \right), \text{ for } s \geq t+1, \quad (A10.2)

where \( B_1 = \frac{1}{(1+\nu) p^{\text{UC}} i} \), \( B_2 = \frac{1}{(1+\nu) p^{\text{UC}} i} \), and \( B_3 = \frac{1}{(1+\nu) p^{\text{UC}} i} \). The optimal consumption at the year \( t+T \) with the constant price expectation is obtained from Eq. (4.7) as follows.

\[ \hat{c}_{t+T} = \hat{c}_t \left( \frac{1+r_i}{1+\rho} \right)^{T/\sigma}. \quad (A10.3) \]

From Eq. (A10.2) and Eq. (A10.3), we obtain the intertemporal budget constraint between the years \( t+1 \) and \( t+1+T \) as follows.

\[ m_{t+1+T} \left( \frac{1+\nu}{1+r_i} \right)^T = m_{t+1} - B_i \hat{c}_t \sum_{i=1}^T \beta_i \sum_{j=1}^T \beta_j + B_2 \sum_{i=1}^T \beta_i + \left( \frac{1}{1+\nu} \right)^s \left( B_3 - B_4 \right) \sum_{i=1}^T \beta_i, \quad (A10.4) \]

where \( \beta_1 = \frac{1+\nu}{1+r_i} \), \( \beta_2 = \frac{1+\nu}{1+r_i} \), and \( \beta_3 = \frac{1}{1+r_i} \).

When \( T \) approaches to infinity, the left hand side becomes zero from the transversality condition and \( \lambda_{t+1}^m = \lambda_t^m \left( \frac{1+\rho}{1+r_i} \right) \) obtained from the optimal conditions (4.5d) and (4.5e).

\[ \lim_{T \to \infty} \left[ \left( \frac{1+\nu}{1+\rho} \right)^T \cdot \lambda_{t+T}^m \cdot m_{t+T} \right] = \lim_{T \to \infty} \left[ \left( \frac{1+\nu}{1+\rho} \right)^T \cdot \lambda_t^m \left( \frac{1+\rho}{1+r_i} \right)^T \cdot m_{t+T} \right] = \lambda_t^m \lim_{T \to \infty} \left[ m_{t+T} \left( \frac{1+\nu}{1+r_i} \right)^T \right] = 0. \]

At the same time the boundedness of the right hand side requires that all the common ratios \( \beta \)'s must be less than unity, which is guaranteed if \( r_i > \nu \) due to the
fact that $v > \left( \frac{1+v}{1+\rho} \right)^{\gamma(\sigma-1)} - 1$. When this condition is satisfied Eq. (A10.4) becomes

$$m_{i+1} = B_i \frac{\beta_1}{1-\beta_1} - B_2 \frac{\beta_2}{1-\beta_2} - \left( \frac{1}{1+v} \right) (B_3 - B_4) \frac{\beta_3}{1-\beta_3}. \tag{A10.5}$$

Eq. (A10.5) is incorporated into Eq. (A10.1) and finally the optimal consumption at time $t$ is obtained as

$$\hat{c}_i = \left( \frac{\varphi_Q}{p_i} \right)^{\gamma_i} \prod_{i=1}^{n} \left( \frac{\varphi_i}{p_i^C} \right)^{\eta_i} \left[ 1 + r_i - (1+v) \left( \frac{1+r_i}{1+\rho} \right) \right]^{\gamma_i} \left[ p_i^{inc} m_i + \frac{1}{r_i - v} \left\{ \bar{w}^U \theta^U t^U + \bar{w}^l \bar{v} \right\} \right]$$

$$+ \bar{w}^U \tilde{T}^U + \bar{w}^l \left( 1 - l^U \tilde{T}^U \left( 1 - (1-\theta) \bar{z} \right) + p_i \bar{a}_n \left( 1 - l^U \tilde{T}^U \right) (1-\theta) \right)$$

$$+ \left( \frac{1}{1+v} \right)^{t'} \left[ \frac{\beta_8}{r_t^R} \frac{Y_R}{r_t} \right]^{\gamma_i} \left[ \frac{1}{\beta_i} \left( \frac{w_i}{r_t} \right)^{\beta_{Rk}} \left( p_i^{inc} \left( r_t + \delta \right) \right) \right]^{\beta_{EK}}$$

$$- \left( \frac{1}{1+v} \right)^{t'} \left[ \frac{1}{1+r_i} w_i \left( r_t \right)^{\beta_{Rk}} \left( \frac{1}{K_t^R} \right) - (1-\delta) p_i^{inc} K_t^R \right].$$

\textit{Q.E.D.}
Appendix B  GAMS Code

Appendix B1  GAMS code for simulations of the analytic model

$Ontext
Numerical simulations of analytic model
Comparison between MC and ES trajectories
Satoshi Kojima, University of York,
$Offtext

****Calibration of MC steady-state ****

Sets
  t time period /1*200 /
  i /1*3 /
  first(t) starting year;
  first(t) = YES$(ORD(t) EQ 1);

Scalars
  policy_k multiplicative factor to initial private capital stock /1/
  policy_G multiplicative factor to initial public capital stock /1/
  factor scaling-up factor /5/
  tau technological parameter [dnnl]
  wbar steady-state wage [unit per labourer per time period]
  qHbar steady-state domestic water consumption [water unit per HH per time period]
  qMbar steady-state industrial water consumption [water unit per labourer per time period]
  Qbar steady-state aggregate water demand [water unit per time period]
  rbar steady-state interest rate [per year] / .1/
  pbar steady-state water price [unit per water unit] / 1/
  kbar steady-state per capita private capital stock [unit per household]
  nyu population growth rate [per time period] /. 02 /
  delta depreciation rate /0.05 /
  rho time preference /. 075/
  phi weight of market goods in household production /0.98/
  sigma elasticity of marginal felicity / 3 /
  beta_K factor share of private capital / .5 /
  beta_Q factor share of industrial water / .2 /
  beta_L factor share of labour / .3 /
  b1, b6, b7, b8, b9, b10, b11, b12, b13, b14, d1, d2, d3, xbar2;
  * Calibration of technological parameter
  tau = (rbar+delta)/(rho+delta);

Parameters
  Nbar(i) steady state population [ person] / 1=1000, 2=2000, 3=3000 /
  Gbar(i) steady-state public capital stock [ unit]
  FWbar(i) steady-state water production [ water unit ];
  b1 =1/(1-phi)**(1-phi)/phi**phi;
  b6 = beta_Q / (1-beta_Q);
  b7 = beta_K / (1-beta_Q);
  b8 = beta_L / (1-beta_Q);
  b9 = delta+nu - (delta + rho)/sigma;
  b10 = (sigma-1)*beta_K*beta_Q**b6/sigma;
  b11 = (delta+nu)/beta_Q**b6;
  b12 = (delta+nu)(delta + rho)/beta_Q**b6;
  b13 = (delta+nu)*beta_K+(delta + rho)(1-beta_Q);
  b14 = beta_K*(1-beta_Q)*beta_Q**b6;
  d1=beta_K*beta_Q**b6;
  d2=(1-beta_Q)*beta_Q**b6;
Appendix B

\[ d^3 = \frac{1}{1 - \beta_Q}; \]
\[ x_{bar2} = \tau \beta_K \beta_Q^{**b6} \delta + \rho; \]
\[ k_{bar} = x_{bar2}^{**b1} \beta_K \beta_Q^{*b6}; \]
\[ w_{bar} = \tau \beta_L k_{bar}^{**b7} \delta + \rho; \]
\[ q_{Hbar} = (1 - \phi) \delta + (1 + \eta)(1 + \rho); \]
\[ q_{Mbar} = \tau k_{bar}^{**b7} \beta_Q^{*1/(1 - \beta_Q)}; \]
\[ F_{\bar{W}bar(i)} = (g_{Hbar} + g_{Mbar}) \bar{W}bar(i); \]
\[ G_{\bar{W}bar(i)} = \delta F_{\bar{W}bar(i)} / \delta; \]

**Calibration of water production parameters *****

Scalars

FW1

G1 1st threshold of G / 28000 /

FW2

G2 2nd threshold of G / 44000 /

a_1, a_2, a_3, b_1, b_3, c_2

b_2 / 0.1 /;

FW1 = factor*1000;

Parameters

e_G(i) water production elasticity at steady state [dmln];

\*model1

a_1 = (factor*FV1bar("1")) - FW1; 

b_1 = a_1*log(G1) - FW1;

e_G("1") = 1/(log(G1) - b_1/a_1);

\*model2

a_2 = (factor*FV1bar("2")) - FW1; 

b_2 = a_2*log(G2) - FW2;

e_G("2") = b_2/(1 - c_2/(a_1*G2"**b_2); 

FW2 = a_2*G2"**b_2-c_2; 

\*model3

a_3 = (factor*FV1bar("3")) - FW2; 

b_3 = a_3*log(G3) - FW2;

e_G("3") = 1/(log(G3) - b_3/a_3);

**** Calibration end *****

Scalars

N initial population

epsi small number / 0.0001 /

HHsize household size in equation

kz k(t) in equation [unit per person]

Gz G(t) in equation [unit]

Qz water production [water unit per time period]

GO initial public capital stock [unit]

k0 initial per capita private capital stock [unit per person];

*choose steady-state among i=1,2,3

N = Nbar("1");

GO = Gbar("1") * policy_G;

k0 = kbar * policy_K;

Parameters

k(t) optimal private capital stock at time t [unit per capita]

G(t) public capital stock at time t [unit]

c(t) per capita optimal consumption [unit per capita per time period]

w(t) wage [unit per labourer per time period]

r(t) interest rate [per time period]

x(t) latent variable

IG(t) public investment [unit per time period]

IP(t) per capita private investment [unit per capita per time period]

p(t) water price at time t [unit per water unit]

qH(t) optimal domestic water consumption [water unit per capita per time period]

qM(t) optimal industrial water consumption [water unit per labourer per time period]

Q(t) water production [water unit per time period]

S(t) water balance [water unit per time period]
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\[ u_{MC}(t) \text{ felicity at time } t \text{ for MC} \]
\[ u_{ES}(t) \text{ felicity at time } t \text{ for ES} \]

Positive variables
- \( wz \) wage
- \( rz \) interest rate
- \( pz \) water price
- \( qHz \) domestic water consumption
- \( qMz \) industrial water consumption

Equations
- Interest 'calc \( rz \)'
- Wage 'calc \( wz \)'
- Dom\_wat 'calc \( qHz \)'
- Ind\_wat 'calc \( qMz \)'
- MC 'market-clear solution'
- ES 'excess-supply solution'

\[ \text{Interest.. } rz = e^{\tau \beta K^{(\beta Q/pz)^{b6}}(1/kz)^{b7}}; \]
\[ \text{Wage.. } wz = e^{\tau \beta_L^{(\beta Q/pz)^{b6}}kz^{b7}}; \]
\[ \text{Dom\_wat.. } qHz = \frac{(1-\phi)/pz(1+rz-(1+nyu))^{((1+rz)/(1+\rho))^{(1/\sigma)}}(kz+wz/(rz-nyu)); \]
\[ \text{Ind\_wat.. } qMz = \frac{\tau k^{b7}(\beta_Q/pz)^{(1/(1-\beta_Q)); \]

Model MCpath / Interest, rz, Wage, wz, Dom\_wat.qHz, Ind\_wat.qMz, MC, pz / ;
Model ESpath / Interest, rz, Wage, wz, Dom\_wat.qHz, Ind\_wat.qMz, ES, pz / ;

***** Start of Excess-supply solution *****

Loop(t,)
  Initial guess and range *****
  \( G(first)=G0; \)
  \( k(first)=k0; \)
  \( qHz.I=qHbar; \)
  \( qMz.I=qMbar; \)
  \( pz.I=pbar; \)
  \( rz.I=rbar; \)
  \( wz.I=wbar; \)
  \( pz.lo = epsl; \)
  \( rz.lo = nu+epsi; \)
  \( HHsize=(1+nyu)^{(ord(t)-1)}; \)
  \( Gz=G(t); \)
  \( kz=k(t); \)
  \( Q(t)=(Gz \geq G1) \times (a_1 \times \log(Gz-b_1)+(Gz \geq G2) \times (a_2 \times \log(Gz-b_2)))\]
  \( \text{Solve ESpath using MCP; \}
  \( IP(t)=rz.I^{(1-kz+wz.I)}(1-\phi)qHz.I; \)
  \( k(t+1)=(IP(t)+k(t))/(1+nyu); \)
  \( IG(t)=pz.I^{(1-qHz.I+qMz.I)^N}HHsize; \)
  \( G.(t+1)=G(t)^{(1-\delta)}G(t); \)
  \( S(t)=Q(t) - (qHz.I+qMz.I)^N(1+nyu)^{(ord(t)-1)}; \)
  \( p(t)=pz.I; \)
  \( qH(t)=qHz.I; \)
  \( qM(t)=qMz.I; \)
  \};

***End Loop ***
\[ c(t) = qH(t)^*p(t)^*\phi((1-\phi))^{\phi}; \]
\[ u_{ES(t)} = 1/(\sigma-1)c(t)^{\sigma-1}; \]
\$libinclude XLDUMP p ESpath.xls p \$
\$libinclude XLDUMP Q ESpath.xls Q \$
Appendix B

GAMS code for validation of the applied model

$OnText Validation of the applied model.
Satōshi Kojima, University of York, 5 January 2005
$Offtext

***** Static calibration start *********
******************************************************************************
*Calibrating parameters
Scalars
omega_bar expectation of production risk factor [dmnl] / 0.9 /
sigma coefficient of relative risk aversion [dmnl] / 10 /
delta depreciation rate [per year] / 0.105 /
rho time preference [per year] / 0.075 /
nyu population growth rate [per year] / 0.02 /
pr_esc price escalation for irrigation water and land charge [per year] / 0.05 /;
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Set
i {account / A,K,L,S,Q,HH,ROW,GOV,SAV,R,I,U,TAX,SUB,TNT};
alias(i,j);

Set
jOT(j) subset for OT / Q, GOV, ROW, TNT /
jDG(j) subset for DG / HH, GOV, SAV, R, I, U /
iprod(i) subset for total revenue / A, K, L, S, Q, R, I, U /
fac(i) subset for total factor payment / A, K, L, S, Q /
sec(i) production sectors / R, I, U /

Table rawsam(i,j) raw data of Social Accounting Matrix [billion DH]
* SAM is omitted due to limited space

Parameters
SAM(i,j) social accounting matrix [DH];
SAM(i,j) = 1000000000*rawsam(i,j);

Scalars
HHsize Household size in loop statement
pop1994 total population in 1994 / 26590000 /
HHsize_c Household size in 1994
omega_c production risk factor in 1994 / 1.601 /
ex_wat share of water expenditure among payment to the government [dmnl] .66 /
theta_U urban unemployment rate .223 /
qno subsistence amount of water [m3 per "labour person unit" per year]
qno_lcd qno in lcd term [lcd] / 10 /
wcov_c public water supply coverage in rural area [dmnl] / 0.2/
rr_c real interest rate [ 1 per year ] / 0.095 /
*Employment data (Lofgren et al. 1999)
L_rur rural employment [labourer-year] / 4640200 /
LS_tc urban skilled employment [labourer-year] / 1590734 /
LU_tc urban unskilled employment [labourer-year] / 2279666 /
*Price data (ASM and Lofgren et al. 1997)
* p_c is calculated from mean range tariff with assuming that qH_lcd = 120.
pw_c irrigation water charge in 1994 [ DH per m3 ] / 2.4965 /
* pw_c is corresponding to irrigation water total cost in Lofgren (1997; p.A-11)
pw_c irrigation water charge in 1994 [ DH per m3] / 0.347 /
*pA_c is calibrated such that AI = 863831 ha in 1994
pA_c irrigation land charge in 1994 [ DH per ha] / 3372.84 /
*exchange rate from WDI
re_c official exchange rate in 1994 [ DH per USD] / 9.2027 /
oth_inc per capita income excluded from the model assumption [ DH per capita ]
epsi small number / 0.0001/
N household number [household]
Ybar average value product of rainfed sector [ DH per HH per year]
YRbar average productivity of rainfed sector [ unit per HH per year]
qH_c per capita publicly supplied water consumption in 1994 [m3 per capita per year]
qH_lcd per user publicly supplied water consumption [lcd]
qHbar_c per capita average water consumption in 1994 [m3 per capita per year]
qHbar_lcd per capita average water consumption [lcd]
Q_j total water supply [million m3 per year]
WU minimum wage in urban sector [DH per labour per year]
WS_c wage for skilled labourer in 1994 [DH per labour per year]
wl_c wage for irrigation labourer in 1994 [DH per labour per year]
th_E_c urban job success rate in 1994 [ dmnl]
LR_tc rainfed employment in 1994 [labourer per year]
IL_tc urban skilled employment in 1994 [labourer per year]
QL_tc aggregate irrigation water input in 1994 [million m3 per year ]
AI_tc irrigation land input in 1994 [ha]
IR_c labour allocation to RF sector in 1994 [dmnl]
IL_c labour allocation to IR sector in 1994 [dmnl]
IU_c unskilled labour allocation to urban sector in 1994 [dmnl]
IS skilled labour allocation to urban sector [dmnl]

*water loss rate (calibrated such that total water supply = 11,377 million m3)

loss_Hc water loss of domestic water supply in 1994 [dmnl]/0.3/

loss_IRc water loss of irrigation water supply in 1994 [dmnl]/0.326/

*Initial values

m0 per capita initial private capital stock [unit per capita]

KR0 per household initial rainfed capital stock [unit per HH]

x, y, R, c;

N = L_rur+(LS_tc +LU_tc)/(1-theta_U);

qno = qno_Lcd*pop1994/N*365/1000;

Ybar = sum(iprod, SAM(iprod, "R"))/N*omega_c;

qH_c = ex_wat*SAM("GOV","HH")/p_c/(wcov_c*L_rur+(LS_tc +LU_tc)/(1-theta_U));

qH_Lcd = qH_c/365*1000*N/pop1994;

th_E_c = (1-theta_U)*L_rur/LU_tc;

IS = LS_tc/N;

IU_c = LU_tc/N*th_E_c;

A_L_tc = SAM("A","R")/pA_c;

QI_tc = SAM("Q","I")/pW_c/1000000;

qHbar_c = qH_c*(wcov_c+(IU_c+IS)*(1-wcov_c))+qno*(1-IU_c-IS)*(1-wcov_c);

qHbar_Lcd = qHbar_c/365*1000*N/pop1994;

oth_inc = sum(UOT, SAM("HH", "IOT"))/N;

HHsize_c = pop1994/N;

Parameters

phi(sec) weight In satisfaction production [dmnl]

phi_Q weight of water consumption in satisfaction production [dmnl]

DH_sec HH expenditure except for water [DH]

beta(sec,fac) factor share [dmnl]

fac_t(sec) total factor payment [DH]

rev_t(sec) total revenue [DH]

tar_c(sec) import tariff [dmnl]

tax_p_c(sec) product tax [dmnl]

tax_h_c income tax [dmnl]

subs_c(sec) export subsity [dmnl]

tau(sec) technological parameter [dmnl]

p_bar(sec) world product price [USD per unit] / R=1, l=1, U=1 /

p_d(sec) world product price in local currency [DH per unit]

p_d(sec) = re_c*p_bar(sec);

DH_sec = sum(sec, SAM(sec,"HH");

phi(sec) = SAM(sec,"HH")/(DH_sec+ex_wat*SAM("GOV","HH");

phi_Q = (ex_wat*SAM("GOV","HH");

DH_sec+ex_wat*SAM("GOV","HH");

rev_t(sec) = sum(iprod, SAM(iprod, sec));

fatt tc = sum(fac, SAM(fac, sec));

beta(R,"sec") = SAM(fac, sec)/fatt_t(sec);

beta(R,"sec") = SAM(fac, sec)*omega_c;

beta(R,"A") = 1-beta(R,"R")-beta(R,"L");

tar_c(sec) = SAM("TNT",sec)/ SAM("ROW",sec);

tax_p_c(sec) = SAM("TAX",sec)/ rev_t(sec);

tax_h_c = SAM("TAX","HH")/sum(i, SAM(i,"HH")-SAM("TAX","HH");

subs_c(sec) = SAM(sec,"SUB")/SAM(sec,"ROW");

*Wage calculation

wU = SAM("L","U")/LU_tc;

wS_c = SAM("S","U")/LS_tc;

wl_c = (th_E_c*nu*wcov_c+phi_Q-p_c*qno*(1-wcov_c-1))/(1-(1-wcov_c)*zbar);

L1_c = SAM("L","I")/w1_c;

LR_c = L_rur/L1_c;

ll_c = L1_c/N;

IR_c = LR_c/N;

b_R = beta(R,"K")*beta(R,"L");

* Derive tau("UR") as a function of p_d("UR")
tau("U") = (1+tax_p_c("U"))*rev_t("U")/(1+subs_c("U"))*
( (rr_c+delta)\{1+subs_c("U")\})\{SAM("ROW","U")\}*
(1+subs_c("U")\{SAM("ROW","U")\})
* beta("U","K")/p_d("U")**(1-beta("U","K"));

* Derive y= tau("IR")*pp("IR") as a function of p_d("UR")

y=p_d("U")**beta("I","K")*(1+tax_p_c("I"))\{rev_t("I")\}/
( (1+subs_c("I"))\{SAM("ROW","I")\})**beta("I","K")/p_d("U")

** Calibration of trade parameters and technological coefficient aij's ***

Parameters
s_M(sec) Armington import elasticity 
R=2, I=3, U=5 /
d_M(sec) Armington share parameter ;
d_M(sec) = ( (1+tax_p_c(sec))/(1+subs_c(sec)))/( (1+tax_p_c(sec))/(1+subs_c(sec)))

Parameters
pc_c(sec) consumer price [DH per unit]
pp_c(sec) producer price [DH per unit]

Q_t=(qH_c*N*(wcov_c+(IU_c+IS)*(1-wcov_c))/(1-loss_Hc)+beta("I","Q")*wl_c*LI_t/
beta("L","R")*(beta("R","L")**(1/KRO)+beta("R","L")**(1/KRO))**beta("R","L")**(1/KRO));

O_t=(qH_c*N*(wcov_c)(1-loss_Hc)+beta("I","Q")*wl_c*LI_t/}
beta("L","R")**(1-loss_Rc));

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***** Static calibration end **********

***** Validation start ***********

Set
t total time period / 1994*2002 /
first(t) starting year ;
first(t) = YES$(ORD(t) EQ 1);

Scalars
pz domestic water price [DH per m3]
pwz irrigation water price [DH per m3]
pAz irrigation land price [DH per ha]
wUz wUmin(t) in equation [DH per labourer per year]
wcovz wcov(t) in equation [dmnl]
mz m(t) in equation [ unit per capita ]
KRz KR(t) in equation [ unit per household ] ;

*Exogenous variables
Parameter
omega(t) producition risk [dmnl]
/ 1994=1.601, 1995=0.444, 1996=1.708, 1997=0.847,
1998=1.129, 1999=0.750, 2000=0.384, 2001=0.905, 2002=1.079 /
re(t) real exchange rate [1994 constant DH per 1994 constant USD] / 1994=9.2030,
2001=11.5496, 2002=11.1444 /
p(t) domestic water price [DH per m3]/ 1994=2.4965, 1995= 2.585, 1996= 2.549,
pw(t) irrigation water price [DH per m3]
pA(t) irrigation land price [DH per ha]
wcov(t) public water supply coverage in rural area [dmnl]
/1994=0.2000, 1995=0.2396,
1996=0.2773, 1997=0.3131, 1998= 0.3471, 1999=0.3795, 2000= 0.4102, 2001=0.4394, 2002=0.4672 /
wUmin(t) urban minimum wage [DH per labourer per year] /1994=7625.7250, 1995=7097.6804,
2001=7966.3741, 2002=7919.1427 /
tar(sec) import tariff [dmnl]
tax_p(sec) product tax [dmnl]
tax_h income tax [dmnl]
subs(sec) export subsidy [dmnl]
eta_M(sec) consumer price coefficient [dmnl]
pp(sec,t) producer price of product [ DH per unit]
pc(sec,t) consumer price of product [ DH per unit]
ppz(sec) pp(sec t) in equation
pcz(sec) pc(sec t) in equation
qHbar(t) per capita average water consumption [m3 per capita per year]
qH(t) per capita piped water consumption [m3 per user per year]
rr(t) interst rate [per year]
IU(t) labour allocation to urban sector
wl(t) irrigation sector wage [DH per labour per year]
th_E(t) probability to get urban job
ws(t) skilled labour wage [DH per labourer per year]
L(t) aggregate Irrigation labourer [ person per year]
m(t) optimal private capital stock in year t [ unit per capita ]
K(t) rainfed capital stock per household [ unit per household ]
IP(t) per capita private capital stock investment [ unit per capita ]
cu_h(t) optimal consumption of urban product [ unit per year ]
c hat(t) optimal consumption of satisfaction [ unit per year ]
AI(t) aggregate irrigation land input [ 1000 ha ]
QI(t) aggregate irrigation water input [ million m3 per year ]
IR_ratio(t) share of irrigation usage [dmnl]
qHcd(t) per capita domestic water consumption [ lcd ]
Appendix B

*validation report variables

unemploy(t) unemployment rate [per cent]
E_tot(t) total exports [million 1994 DH]
M_tot(t) total imports [million 1994 DH]
real_in(t) real interest rate [per cent];

*Status quo tax and subsidies

tar(sec)=tar_c(sec);
tax_p(sec)=tax_p_c(sec);
tax_h=tax_h_c;
subs(sec)=subs_c(sec);

eta_M(sec) = (1+subs(sec))*(d_M(sec)*s_M(sec)*(1+subs(sec))**(s_M(sec)-1)+
(1-d_M(sec))*s_M(sec)*(1+tar(sec))**(s_M(sec)-1))/
(d_M(sec)*s_M(sec)+(1+subs(sec))*s_M(sec)+(1-d_M(sec))*s_M(sec)*
(1+tar(sec))**s_M(sec));
pp(sec,t) = (1+subs/sec) *( (1+tax_p/sec) * p_bar/sec * re(t));
pc(sec,t) = (1+tax/sec) * eta_M(sec) * p_bar/sec * re(t);

*5% price escalation schedule

pw(t) = (1+presc)**(ord(t)-1)*pw_c;
P(A,t) = (1+presc)**(ord(t)-1)*P_A_c;

Positive variables

th_Ez probability to get urban job [dmnl]
rrz real interest rate [per year]
wlz irrigation sector wage rate [DH per year]
wlz skilled labour wage rate [DH per year]
IUz labour allocation to urban sector [dmnl]
q1_barz per capita average water consumption [m3 per capita per year]
q1_hz per capita piped water consumption [m3 per user per year]
Ltz aggregate irrigation labourer [person per year];

Equations

interest evaluate rrz
wage evaluate wlz
HT_model Generalised Harris-Todaro model
ws_star solving equilibrium wage of skilled labour
urb_lab evaluate IUz
dom_wat evaluate qH_barz
qH evaluate qHz
ir_labour evaluate Ltz;

***** Declaration end *******

interest.. rrz+delta += beta("U", "K")/pcz("U")*
( tau(U)*(ppz(U)-aij("U", R)*pcz(R)-aij("U", P)))(beta(U, "L")/wUz)"beta("U, "S")/
wSz)**(beta("U", "S")**(1/beta("U", "K"));

wage.. wlz += beta("I", "Q") ( (tau(T))*(ppz(T)-aij(T,"R")"pcz("R")"beta(T,"A")"pAz)"beta(T,"A")*

HT_model.. th_Ez =e= wcovz/phi Q/wUz*(wlz*
(1-

ws_star.. (1+1-wcovz)*zbar)+pz*Qn0*(1-wcovz-1);
Model equilb / interest, rrz, wage, wiz, HT_model, th_Ez, ws_star, wSz, urb_lab, lUz, dom_wat, qHbarz, qHz, qno, L_tz Lo = epsl;

m(first) = m0;
KR(first) = KR0;

*** loop start *****

Loop(t,
HHsize=(1+nyu)**(ord(t)-1);
mz=m(t);
KRz=KR(t);
wUz = wUmin(t);
wcovz = wcov(t);
pz=p(t);
pwz=pw(t);
pAz=pA(t);
pz(sec)=pc(sec,t);
ppz(sec)=ppp(sec,t);

Solve equilib using MCP;

IP(t) = (1-tax_h)'(rrz. i'm(t)+oth inc/pc("U",t)+(wUz*th_Ez.
(IUlIZL+SI+S+wiz.t*"(1+Uz.I.S)*1(1-wcovz)zbar)}/pc("U",t) -
qHbarz.1*pz/phi Q/pc("U",t)+pz*"(1-1-Uz.HS)"(1-wcovz)"pc("U",t)+
(1-tax_h)"/HHsize"*omega(t)*YRbar/pc("U",t)"(pp("R",t)-aij("R","I")*
"pc("T",t)-aij("R","U")"pc("U",t)-)
/1/HHsize*

( (YRbar/tau("R"))"(1/b_R)"(beta("R","K")(1-tax_h)wiz.I/beta("R","L")/
pc("U",t)"(rrz.1+delta)"(beta("R","L")/b_R)+
(1-tax_h)"wz.I/pc("U","I")"YRbar/tau("R")"(1/beta("R","L")"(1/KR(t))**
(beta("R","K")/beta("R","L")"(1-delta)*KR(t) );
m(t+1) =((IP(t)+m(t)"/(1+nyu);
KR(t+1) = (YRbar/tau("R"))"(1/b_R)"(beta("R","K")(1-tax_h)*
wiz.I/beta("R","L")"pc("U",t)"(1-tax_h)"/rrz.1+delta) )**
(beta("R","L")/b_R);
th_E(t)=th_Ez.1;
I(t)=Iulz.1;
qu_H(t)=qHz.1;
qHbar(t) = qHbarz.1;
rr(t)=rrz.1;
w(t)=wiz.1;
wS(t)=wSz.1;
L_tz.1 = L_tz.1;
);***** loop end *****

cu_hat(t) = qHbar.1*phi("U")/pc("U",t);
c_hat(t) = qHbar.1*phi("U")/pc("U",t);
A_1(t)= beta("T","A")\cdot w(t)\cdot L_tz.1/pc("U",t)"*"1000;
Q1.1 = beta("I","Q")\cdot w(t)\cdot L_tz.1/pc("U",t)"*"1000000;
cU(t) = qH(t)\cdot 365*1000\cdot N/1994;
real int(t)=r(t)"*"100;
unemploy(t)=(1-(th_E(t)"/I(t)+I.S)/(I(t)+I.S))"*"100;
M_tot(t)=

((1+subs("R"))"/(1+tax("R")))"*s_M("R")*/(1-d_M("R"))(1-d_M("R"))"*s_M("R")*
(1-SAM("R","ROW")"/rev_1("R")/(1+tax_p_c("R")))*omega(t)*YRbar*N)p_bar("R")+
((1+subs("T"))"/(1+tax("T")))"*s_M("T")*/(1-d_M("T"))(1-d_M("T"))"*s_M("T")*
(1-SAM("T","ROW")"/rev_1("T")/(1+tax_p_c("T")))*"
Appendix B3  GAMS code for simulations of the applied model

$Ontext
Policy simulation of the applied model
Satoshi Kojima, University of York
$Offtext

Set
  t total time period / 1*15 /
  cv critical values / 1*7 /
  first(t) starting year
  gsec government sector / gr, gt, gi /
  first(t) = YES$(ORD(t) EQ 1);

*** Policy variables ****
Parameters
  theta(gsec, t) budget allocation [dmnl]
  safe_S(gsec) safety factor for supply capacity [dmnl] / gr = .85, gi = .85, gr = .85 /
  safe_G safety factor for government budget [dmnl] / .98 /
  *** International aid ***
  aid(gsec) multiplier to G0 [dmnl] / gt = 5, gi = 1.5, gr = 1.5 /
  *** Trade regime ***
  tariff tariff reduction rate [dmnl] / 1 /
  taxrev fraction of Import tax revenue invested in public capital [dmnl] / 0.2 /
  *** Climate change ***
  CC_cv coefficient of variation of production risk factor [dmnl] / .366 /
  *** Minimum wage ***
  MW_coef reduction rate of minimum wage [dmnl] / 1 /
  G0(gsec) Initial stock of public capital [million unit in 1994] / gr=694.0, gt=187.3, gi=776.6 /
  multi(gsec) multiplier for initial price [dmnl] / gr =2.1, gt = 2.6, gi =2.1 /
  multi_r multiplier for initial rr [dmnl] / 1 /
  *** Investment policy ***
  theta("gr", t) = 0.43;
  theta("gt", t) = 0.285;

$libinclude XLDUMP c_hat dyn_opt2.xls c
$libinclude XLDUMP real_int dyn_opt2.xls re
$libinclude XLDUMP unemploy dyn_opt2.xls un
$libinclude XLDUMP E_tot dyn_opt2.xls E
$libinclude XLDUMP M_tot dyn_opt2.xls M

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\[
\theta(\text{"g}^t, t) = 1 - \theta(\text{"g}^t, t) - \theta(\text{"g}^t, t);
\]

*** Static calibration part is omitted (see Appendix B) ***

**** 1st part: Policy planning ******

**Parameters**

- \( \text{tar(\text{sec})} \): Import tariff [dmnl]
- \( \text{tax}_p(\text{sec}) \): Product tax [dmnl]
- \( \text{tax}_h \): Income tax [dmnl]
- \( \text{subs(\text{sec})} \): Export subsidy [dmnl]
- \( \text{wUmin(t)} \): Urban minimum wage [DH per labourer per year]
- \( \text{G0mod(gsec)} \): Increased initial stock [million unit in 1994]

*** Tax and subsidy policy ***:

\[
\text{tar(\text{sec})} = \text{tar}_c(\text{sec}); \\
\text{tax}_p(\text{sec}) = \text{tax}_p_c(\text{sec}); \\
\text{tax}_h = \text{tax}_h_c; \\
\text{subs(\text{sec})} = \text{subs}_c(\text{sec});
\]

*** Minimum wage policy ***:

\[\text{wUmin(t)} = \text{MW}_c\text{wU}_c;\]

*** International aid policy ***:

\[\text{G0mod(gsec)} = \text{aid(gsec)} \times \text{G0(gsec)};\]

**Scalars**

- \( \text{HHsize} \): Normalised household size in loop statement \((t = 1 \rightarrow \text{HHsize} = 1)\)
- \( \text{omega_bar} \): Expectation of production risk factor / 0.9 /
- \( \text{wUz} \): \(\text{wUmin(t)}\) in equation [DH per labourer per year]
- \( \text{wcovz} \): \(\text{wcov(t)}\) in equation [dmnl]
- \( \text{mz} \): \(\text{m(t)}\) in equation [unit per capita]
- \( \text{KRz} \): \(\text{KR(t)}\) in equation [unit per household]
- \( \text{Rz} \): Raw water production \(\text{R(t)}\) in equation [million m3]
- \( \text{Qz} \): Treated water production \(\text{Q(t)}\) in equation [million m3]
- \( \text{Az} \): \(\text{A(t)}\) in equation [1000 ha]
- \( \text{loan_pr} \): External loan principal in foreign currency [million 1994 USD]
- \( \text{i_rate} \): Loan interest rate [per annum] / 0.07 /
- \( \text{loss_Hz} \): \(\text{loss-H(t)}\) in equation [dmnl]
- \( \text{loss_IRz} \): \(\text{loss_IR(t)}\) in equation [dmnl]

*** Sustainable production function construction ******

**** for SCbase ***

*Syntax*

**Parameters**

- \( \text{GR(cv)} \): [million unit] / 1 = 772.51, 2 = 1250, 3 = 1827, 4 = 2757, 5 = 3855, 6 = 6417, 7 = 8000 /
- \( \text{FR(cv)} \): [million m3 per year] / 1 = 11071.05, 2 = 11816.16, 3 = 13573.46, 4 = 14267.06, 5 = 14968.06, 6 = 16069.06, 7 = 16600 /

\[ \text{aR(cv)} / 6 = 0.01 / \]
\[ \text{bR(cv)} / 1 = 0.0085, 2 = 0.008, 3 = 0.0025, 4 = 0.0015, 5 = 0.0003 / \]
\[ \text{GT(cv)} \): [million unit] / 1 = 178.27, 2 = 503, 3 = 903, 4 = 1200, 5 = 10000, 6 = 10000, 7 = 10000 /
\[ \text{FD(cv)} \): [million m3 per year] / 1 = 333.45, 2 = 524.16, 3 = 593.15, 4 = 650 /
\[ \text{aT(cv)} / 4 = 0.3 / \]
\[ \text{bT(cv)} / 1 = 0.012, 2 = 0.003 / \]
\[ \text{GI(cv)} \): [million unit] / 1 = 519.96, 2 = 685, 3 = 914, 4 = 1429, 5 = 1979.81, 6 = 10000, 7 = 10000 /
\[ \text{FA(cv)} \): [1000 ha] / 1 = 578, 2 = 653.01, 3 = 763.18, 4 = 890.54, 5 = 942.04 /
\[ \text{ai(cv)} / 4 = 0.1 / \]
\[ \text{bi(cv)} / 1 = 0.003, 2 = 0.007, 3 = 0.007 / \]
\[ \text{a_wcov} \): Parameter of \(\text{wcov(t)}\) / 0.931194 /
\[ \text{b_wcov} \): Parameter of \(\text{wcov(t)}\) / 0.000811 /
Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR(cv) [million unit]</td>
<td>1 = 927, 2 = 1500, 3 = 2193, 4 = 3309, 5 = 4626, 6 = 7700, 7 = 10000 /</td>
</tr>
<tr>
<td>FR(cv) [million m3 per year]</td>
<td>1 = 11071.05, 2 = 11816.16, 3 = 13573.46, 4 = 14267.06, 5 = 14968.06, 6 = 16069.06, 7 = 16580 /</td>
</tr>
<tr>
<td>aR(cv)</td>
<td>6 = 0.01 /</td>
</tr>
<tr>
<td>bR(cv)</td>
<td>6 = 0.1 /</td>
</tr>
<tr>
<td>cR(cv)</td>
<td>6 = 0.007 /</td>
</tr>
<tr>
<td>GT(cv) [million unit]</td>
<td>1 = 213.92, 2 = 504, 3 = 1084, 4 = 1400, 5 = 10000, 6 = 10000, 7 = 10000 /</td>
</tr>
<tr>
<td>FD(cv) [million m3 per year]</td>
<td>1 = 333.45, 2 = 524.16, 3 = 593.15, 4 = 645 /</td>
</tr>
<tr>
<td>aT(cv)</td>
<td>3 = 0.1 /</td>
</tr>
<tr>
<td>bT(cv)</td>
<td>3 = 0.3 /</td>
</tr>
<tr>
<td>cT(cv)</td>
<td>1 = 0.012, 2 = 0.003 /</td>
</tr>
<tr>
<td>Gl(cv) [million unit]</td>
<td>1 = 624, 2 = 853, 3 = 1097, 4 = 1715, 5 = 2376, 6 = 10000, 7 = 10000 /</td>
</tr>
<tr>
<td>FA(cv) [1000 ha]</td>
<td>1 = 578, 2 = 653.01, 3 = 763.18, 4 = 890.54, 5 = 942.04 /</td>
</tr>
<tr>
<td>aL(cv)</td>
<td>3 = 0.1 /</td>
</tr>
<tr>
<td>bL(cv)</td>
<td>3 = 0.1 /</td>
</tr>
<tr>
<td>cL(cv)</td>
<td>1 = 0.003, 2 = 0.007, 3 = 0.007 /</td>
</tr>
</tbody>
</table>

a IossH parameter of loss H(t) / 0.250217 /

b IossH parameter of loss_H(t) / 0.001299 /

a IossIR parameter of loss IR(t) / 0.304277 /

b IosslR parameter of loss IR(t) / 0.000705 /

Loop(cv$(GR(cv) < 6417),
 bR(cv)=(FR(cv)-FR(cv+1)) / (FR(cv)*exp(-1*cR(cv)*GR(cv+1))-FR(cv+1)*
 exp(-1*cR(cv)*GR(cv)));
 aR(cv)=FR(cv)/(1-bR(cv)*exp(-1*cR(cv)*GR(cv))); );

aR(6) = (FR(7)-FR(6))/((GR(7)**bR(6)-GR(6)**bR(6));
 cR(6) = aR(5)**GR(6)**bR(6)-FR(6);

a IossH parameter of loss H(t) / 0.250217 /

b IossH parameter of loss_H(t) / 0.001299 /

a IossIR parameter of loss IR(t) / 0.304277 /

b IosslR parameter of loss IR(t) / 0.000705 /

Loop(cv$(GR(cv) < 6417),
 bR(cv)=(FR(cv)-FR(cv+1)) / (FR(cv)*exp(-1*cR(cv)*GR(cv+1))-FR(cv+1)*
 exp(-1*cR(cv)*GR(cv)));
 aR(cv)=FR(cv)/(1-bR(cv)*exp(-1*cR(cv)*GR(cv))); );

aR(6) = (FR(7)-FR(6))/((GR(7)**bR(6)-GR(6)**bR(6));
 cR(6) = aR(5)**GR(6)**bR(6)-FR(6);

Loop(cv$(GT(cv) < 903),
 bT(cv)=(FD(cv)-FD(cv+1)) / (FD(cv)*exp(-1*cT(cv)*GT(cv+1))-FD(cv+1)*
 exp(-1*cT(cv)*GT(cv)));
 aT(cv)=FD(cv)/(1-bT(cv)*exp(-1*cT(cv)*GT(cv))); );

aT(3) = (FD(4)-FD(3))/((GT(4)**bT(3)-GT(3)**bT(3)));
 cT(3) = aT(3)**GT(3)**bT(3)-FD(3);

Loop(cv$(GI(cv) < 1429),
 bl(cv)=(FA(cv)-FA(cv+1)) / (FA(cv)*exp(-1*cL(cv)*GI(cv+1))-FA(cv+1)*
 exp(-1*cL(cv)*GI(cv)));
 al(cv)=FA(cv)/(1-bl(cv)*exp(-1*cL(cv)*GI(cv))); );

aL(4) = (FA(5)-FA(4))/((GI(5)**bl(4)-GI(4)**bl(4)));
 cl(4) = aL(4)**GI(4)**bl(4)-FA(4);
Appendix B

\[ bT(cv) = \frac{(FD(cv) - FD(cv+1))}{FD(cv) \cdot \exp(-1 \cdot cT(cv) \cdot GT(cv+1)) - FD(cv+1)} \] 
\[ aT(cv) = \frac{FD(cv)}{1 - bT(cv) \cdot \exp(-1 \cdot cT(cv) \cdot GT(cv))} \]
\[ aT(3, 4) = \frac{(FD(4) - FD(3))}{GI(3) \cdot bl(3) - GI(3) \cdot bl(4)} \]
\[ cT(3) = aT(3) \cdot GI(3) \cdot bT(3) - FD(3) \]

Loop(cv, (GI(cv) < 1429),
\[ bI(cv) = \frac{(FA(cv) - FA(cv+1))}{FA(cv) \cdot \exp(-1 \cdot cI(cv) \cdot GI(cv+1)) - FA(cv+1)} \]
\[ aI(cv) = \frac{FA(cv)}{1 - bI(cv) \cdot \exp(-1 \cdot cI(cv) \cdot GI(cv))} \]
\[ aI(4) = \frac{(FA(5) - FA(4))}{GI(5) \cdot bl(4) - GI(4) \cdot bl(4)} \]
\[ cI(4) = aI(4) \cdot GI(4) \cdot bI(4) - FA(4) \]

**End of sustainable production function parameters **

Parameters
- \( p(t) \) domestic water price [DH per m3]
- \( pw(t) \) irrigation water price [DH per m3]
- \( pA(t) \) irrigation land price [DH per ha]
- \( th_E(t) \) probability to get urban job [dmnl]
- \( rr(t) \) real interest rate [per year]
- \(wl(t) \) irrigation sector wage rate [DH per year]
- \( wS(t) \) skilled labour wage rate [DH per year]
- \( IU(t) \) labour allocation to urban sector [dmnl]
- \( eta_M(sec) \) consumer price coefficient [dmnl]
- \( pp1(sec, t) \) producer price of product [DH per unit]
- \( pc1(sec, t) \) consumer price of product [DH per unit]
- \( pxz(sec) \) \( pp(sec t) \) in equation
- \( pgz(sec) \) \( pc(sec t) \) in equation
- \( qHbar(t) \) per capita average water consumption [m3 per capita per year]
- \( Q(t) \) per user piped water consumption [m3 per user per year]
- \( L_I(t) \) aggregate irrigation labourer [person per year]
- \( m(t) \) optimal private capital stock in year t [unit per capita]
- \( KR(t) \) rainfed capital stock per household [unit per household]
- \( IP(t) \) per capita private capital stock investment [unit per capita per year]
- \( G(gsec, t) \) public capital stock in year t [million unit]
- \( MG(t) \) total government revenue [million DH per year]
- \( MG_HH(t) \) government revenue from income tax [million DH per year]
- \( MG_trade(t) \) government revenue from trade tax [million DH per year]
- \( MG_total(t) \) government revenue from production tax [million DH per year]
- \( IG(gsec, t) \) public capital investment [million unit per year]
- \( A(t) \) aggregate irrigation land supply [1000 ha]
- \( R(t) \) raw water supply capacity [million m3 per year]
- \( Q(t) \) treated water supply capacity [million m3 per year]
- \( Loss_H(t) \) domestic water loss rate [dmnl]
- \( wcov(t) \) public water supply coverage in rural area [dmnl]
- \( loss_IR(t) \) irrigation water loss rate [dmnl]
- \( repay(t) \) repayment of external loan [million DH per year]

\[ eta_M(sec) = \frac{(1+subs/sec) \cdot (d_M/sec)^{s_M/sec} \cdot (1+subs/sec)^{s_M/sec-1} + (1-d_M/sec)^{s_M/sec} \cdot (1+tar/sec)^{s_M/sec-1}}{(1-d_M/sec)^{s_M/sec} \cdot (1+subs/sec)^{s_M/sec} \cdot (1-d_M/sec)^{s_M/sec}} \]
\[ ppl1/sec, t = (1+subs/sec)/(1+tax_p/sec) \cdot p_bar/sec \cdot re_c \]
\[ psc1/sec, t = (1+tar/sec) \cdot eta_M/sec \cdot p_bar/sec \cdot re_c \]

\[ loan_prf = sum( gsec, (aid(gsec)-1) \cdot pc1("U", t)/re_c \cdot G0(gsec) ) \]
\[ repay(t) = l_rate \cdot loan_prf \cdot re_c \cdot (ord(t) le 5) + (0.1 \cdot loan_prf \cdot re_c \cdot (15-ord(t)) \cdot l_rate) \cdot (ord(t) gt 5) \]

Positive variables
- \( th_Ez \) probability to get urban job [dmnl]
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rrz real interest rate \[ \text{per year} \]
wiz irrigation sector wage rate \[ \text{DH per year} \]
wSz skilled labour wage rate \[ \text{DH per year} \]
IuZ labour allocation to urban sector \[ \text{dmnl} \]
qHbarz per capita average water consumption \[ \text{m3 per capita per year} \]
qHz per capita piped water consumption \[ \text{m3 per user per year} \]
LIz aggregate irrigation labour \[ \text{person per year} \]
qHbarz domestic water price \[ \text{DH per m3} \]
pwz irrigation water price \[ \text{DH per m3} \]
pAz irrigation land price \[ \text{DH per ha} \]

Equations

interest evaluate rrz
wage evaluate wiz
HT_model Generalised Harris-Todaro model
ws_star solving equilibrium wage of skilled labour
urb_lab evaluate IuZ
dom_wat evaluate qHbarz
qH evaluate qHz
irr_lab evaluate LIz
raw raw water balance in million m3 per year
treat treated water balance in million m3 per year
irrig irrigation land balance in 1000 ha

interest.. rrz+delta =e= beta(U,"K")/pcz(U)*
(tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*
wxz)+beta(U,"S")+(1/beta(U,"K"));
wage.. wiz =e= beta(T,"L")*(tau(T))*(ppz(T)-aij(T,"R")*
pcz(T)-aij(T,"U")*pcz(U))*/beta(T,"A")/pzAz**beta(T,"A")*
(beta(T,"Q")/pwz)**beta(T,"Q")/(beta(T,"K")/pcz(U))/(rrz+delta)**
(beta(T,"K")**(1/beta(T,"L"));
HT_model.. th_Ez =e= w covz**phi_Q/wUz*( wiz*
(1-(1-wcovz)*zbar)+pz*zno*(1/wcovz-1)));

ws_star.. ((1-(1-wcovz)*zbar)/(1-IS)-
1/Hsize(YRbar/tau(R))*/(1/beta(R,"L")*(1/KRz)**
(beta(R,"K")/beta(R,"L")-
(1/wSz)**(beta(U,"S")/beta(T,"K")+beta(T,"L")/beta(T,"L")/
beta(U,"K")/Hsize*/beta(U,"K")/beta(T,"K")*sz/tau(U)*
(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*/beta(U,"L")/wUz)**
(beta(T,"K")/beta(U,"K")/beta(T,"L")/beta(U,"K")/
(1/tau(T))*(ppz(T)-aij(T,"R")*pcz(T)-aij(T,"U")*pcz(U))*)/**((beta(T,"Q")/pzAz)**beta(T,"Q")*)
(beta(T,"K")*/beta(T,"K")*)**
(1/beta(T,"L")*)
/((1-(1-wcovz)*zbar)/(1-beta(U,"K")/beta(U,"L")*beta(T,"K")/beta(T,"L")/
wcovz**phi_Q)**(1-wSz)**(beta(U,"K")/beta(T,"K")*beta(T,"L")/beta(U,"K")*
(1/wcovz-1)*(tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*/beta(U,"L")/wUz)**
(beta(T,"K")/beta(U,"K")/beta(T,"L")/beta(U,"K")/
(1/tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*)/**((beta(T,"Q")/pzAz)**beta(T,"Q")*)
(beta(T,"K")*/beta(T,"K")*)**
(1/beta(T,"L")*)
/((1-(1-wcovz)*zbar)/(1-beta(U,"K")/beta(U,"L")*beta(T,"K")/beta(T,"L")/
wcovz**phi_Q)**(1-wSz)**(beta(U,"K")/beta(T,"K")*beta(T,"L")/beta(U,"K")*
(1/wcovz-1)*(tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*/beta(U,"L")/wUz)**
(beta(T,"K")/beta(U,"K")/beta(T,"L")/beta(U,"K")/
(1/tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*)/**((beta(T,"Q")/pzAz)**beta(T,"Q")*)
(beta(T,"K")*/beta(T,"K")*)**
(1/beta(T,"L")*)
/((1-(1-wcovz)*zbar)/(1-beta(U,"K")/beta(U,"L")*beta(T,"K")/beta(T,"L")/
wcovz**phi_Q)**(1-wSz)**(beta(U,"K")/beta(T,"K")*beta(T,"L")/beta(U,"K")*
(1/wcovz-1)*(tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*/beta(U,"L")/wUz)**
(beta(T,"K")/beta(U,"K")/beta(T,"L")/beta(U,"K")/
(1/tau(U))*(ppz(U)-aij(U,"R")*pcz(U)-aij(U,"I")*pcz(T))*)/**((beta(T,"Q")/pzAz)**beta(T,"Q")*)
(beta(T,"K")*/beta(T,"K")*)**
(1/beta(T,"L")*)
/((1-(1-wcovz)*zbar)/(1-beta(U,"K")/beta(U,"L")*beta(T,"K")/beta(T,"L")/
wcovz**phil_Q)*
Appendix B

\[
\beta(T, K)^* \beta(T', K')^* (1/\beta(T', L)) \]
\( \eta = 0; \)

\[
\text{urb_lab.} \ L_{UZ} = e = \left( (1-(1-wcovz)*zbar)^* (1-IS) \right)^* (1-IS) \beta(I', L)^* \]
\[
\text{pcz}(U)^* (rrz+delta) / hlsize / (YRbar/tau(R))^* \]
\[
(1/\beta(R', L))^* (1/KRz)^* (beta(R', K) / beta(R', L)) \]
\[
= e \]
\[
\text{urb_lab..} \ I_{UZ} = e = \left( (1-(1-wcovz)*zbar)^* (1-IS) \right)^* (1-IS) \beta(I', L)^* \]
\[
\text{pcz}(U)^* (rrz+delta) / hlsize / (YRbar/tau(R))^* \]
\[
(1/\beta(R', L))^* (1/KRz)^* (beta(R', K) / beta(R', L)) \]
\[
= e \]

\[
\text{dom wat.} \ qHbarz = e = \phi Q / pz^* \]
\[
(1+(1-tax_h)*rrz-(1 +nu)*((l+(1-tax_h)*rrz)/(1+rho))**(1/sigma))^* \]
\[
(\text{pcz}(U)^* (rrz+delta) / hlsize / (YRbar/tau(R))^* \]
\[
(1/\beta(R', L))^* (1/KRz)^* (beta(R', K) / beta(R', L)) \]
\[
= e \]

\[
\text{it labour.} \ L_{TZ} = e = N^* hlsize / (1-IS-I_UZ)^* \]
\[
\text{raw.} \ safe S gr' = Rz^* \]
\[
\text{raw.} \ safe S gr' / (1-loss HZ)^* \]
\[
\text{treat.} \ safe S gr' / (1-loss HZ)^* \]
\[
\text{irrig.} \ safe S gr' / (1-loss HZ)^* \]

*** initial guesses and ranges ****

\[
\text{th_Ez.I} = \text{th_E_c}; \]
\[
\text{th_Ez.lo} = \text{epsi}; \]
\[
\text{th_Ez.up} = 1-epsi; \]
\[
\text{I_UZ.I} = 1*I_c; \]
\[
\text{I_UZ.lo} = \text{epsi}; \]
\[
\text{I_UZ.up} = 1-epsi; \]
\[
\text{wSz.I} = \text{wS_c}; \]
\[
\text{wSz.lo} = \text{wU_c}; \]
\[
\text{wiz.I} = \text{wI_c}; \]
\[
\text{wiz.lo} = 0.1*wI_c; \]
\[
\text{rrz.I} = \text{multi_r*rr_c}; \]
\[
\text{rrz.lo} = \text{epsi}; \]
\[
\text{rrz.up} = 1-epsi; \]
\[
\text{qHbarz.I} = 1*qHbar_c; \]
\[
\text{qHbarz.lo} = \text{qno}; \]
\[
\text{qHbarz.up} = 20*qHbar_c; \]
\[
\text{qIz.I} = \text{qI_c}; \]
\[
\text{qIz.lo} = \text{qno}; \]
\[
\text{L_tz.I} = \text{L_tz_c}; \]
\[
\text{L_tz.lo} = \text{epsi}; \]
\[
\text{pwz.lo} = 0.1*pw_c; \]
\[
\text{pz.lo} = 1*p_c; \]
Appendix B

Model equilib / interest, rrz, wage, wiz, HT_model, th_Ez, ws_star, wsZ, urb_lab, lUz, dom_wat, qHbarz, qqH, qHz, ir_labour, LL_tz, raw, pwz, treat, pz, irrig, pAz /;

r(first) = m0;
KR(first) = KRO;
G(gsec, first) = G0mod(gsec);

**** loop start ******

Loop(t,
HHsize=(1+nyu)**(ord(t)-1);
wcov(t) = 1-a_wcov/exp(b_wcov*G('gt', t));
loss_H(t) = 0.08+a_lossH/exp(b_lossH*G('gt', t));
loss_IR(t) = 0.15+a_lossIR/exp(b_lossIR*G('gt', t));
*for SCbase
*R(t) = aR('1')*(1-bR('1')/exp(cR('1')*G('ge', t)))$(G('ge', t) lt 1250)+
(G('gt', t) ge 1250)*aR('2')*(1-bR('2')/exp(cR('2')*G('gt', t)))$(G('gt', t) lt 1827)+
(G('gt', t) ge 1827)*aR('3')*(1-bR('3')/exp(cR('3')*G('gt', t)))$(G('gt', t) lt 3855)+
(G('gt', t) ge 3855)*aR('4')*(1-bR('4')/exp(cR('4')*G('gt', t)))$(G('gt', t) lt 6417)+
(G('gt', t) ge 6417)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+
(G('gt', t) ge 1250)*aR('5')*(1-bR('5')/exp(cR('5')*G('gt', t)))$(G('gt', t) lt 1250)+

*for SC1

Solve equilib using MCP;
Appendix B

\[\text{th}_E(t) = \text{th}_Ez. l;\]
\[\text{IU}(t) = \text{IUz. l;}\]
\[q_H(t) = q_Hlz. l;\]
\[q_Hbar(t) = q_Hbarz. l;\]
\[\text{rr}(t) = \text{rrz. l};\]
\[w(t) = wIz. l;\]
\[wS(t) = wSz. i;\]
\[LI(t) = LI tz. l;\]
\[p(t) = pz. l;\]
\[pw(t) = pwz. l;\]
\[pA(t) = pAz. I;\]

* trade tax revenue

\[\text{MG}_\text{trade}1(t) = \left(\frac{\text{tar}(R)\left((1 + \text{subs}(R)) / (1 + \text{tar}(R))\right)}{\text{s}_M(R)\left((d_M(R)/(1 - d_M(R)))^2\right)} + \frac{\text{tar}(T)\left((1 + \text{subs}(T)) / (1 + \text{tar}(T))\right)}{\text{s}_M(T)\left((d_M(T)/(1 - d_M(T)))^2\right)}\right)\]

\[\times \left\{\frac{wI(t)\beta(I; L)}{\text{pcz}(U)} - \frac{(\text{pcz}(R) - \text{pcz}(U))\text{pcz}(U)}{\text{pcz}(U)}\right\} - \frac{\text{pcz}(R)\text{pcz}(U)}{\text{pcz}(U)}\left(1 + (1 + \text{tax}_H)\right)\]

\[\text{IP}(t) = (1 - \text{tax}_h)\left(\frac{\text{rrz. l}m(t) + wIz. l + wS. l + \text{pcz}(U)}{1 - (1 - \text{wcovz})} + \frac{\text{pcz}(U)}{\text{pcz}(U)}\right) - \frac{\text{pcz}(R)}{\text{pcz}(U)}\]

\[\text{KR}(t+1) = \left(\frac{\text{YRbar}\text{tau}(R)}{\text{beta}(R, K)}\right)\left(1 + \text{delta}\right)\]

\[\text{MG}(t) = \left(\frac{\text{pcz}(U)}{\text{pcz}(U)}\right)\theta_{\text{gsec}, t};\]

\[\text{G}(\text{gsec}, t + 1) = \text{G}(\text{gsec}, t) + (1 - \text{delta})\theta_{\text{gsec}, t};\]

Set

\[\text{run Monte-Carlo trial / 1 * 100 I};\]

Scalar

\[\text{n_run} \text{ total number of trial / 100 I};\]

Table

\[\text{base value for EV calculation}\]

*** Base case result table is omitted due to limited space ***

*** Generate random shocks ***

Parameter

\[\text{omega}(t, r) \text{ random production risk [dnnl]}\]

\[\text{rd_re}(t, r) \text{ random shock for exchange rate [dnnl]}\]
re(t, run) real exchange rate [1994 constant DH per 1994 constant USD]
pp(sec, t, run) producer price of product [DH per unit]
pc(sec, t, run) consumer price of product [DH per unit]
wUs(t, run) urban unskilled labour wage rate [DH per year]
repays(t, run) repayment of external loan [million 1994 DH per year]

\[
\text{Loop}(\text{run}, \text{Loop}(t), \\
\text{omega}(t, \text{run}) = \text{normal}(1, \text{CC_cv}); \\
\text{rd}_re(t, \text{run}) = \text{normal}(1, .09); \\
) ; \\
\text{re}(t, \text{run}) = \text{re}_c \times \text{rd}_re(t, \text{run}); \\
\text{pp}(\text{sec}, t, \text{run}) = (1+\text{subs}(\text{sec}))/(1+\text{tax}_p(\text{sec})) \times \text{p_bar}(\text{sec}) \times \text{re}(t, \text{run}); \\
\text{pc}(\text{sec}, t, \text{run}) = (1+\text{tar}(\text{sec})) \times \eta_M(\text{sec}) \times \text{p_bar}(\text{sec}) \times \text{re}(t, \text{run}); \\
\text{wUs}(t, \text{run}) = \text{wUmin}(t) \times \text{rd}_re(t, \text{run}); \\
\text{repays}(t, \text{run}) = \text{i_rate} \times \text{Ioan_prc} \times \text{re}(t, \text{run}) \times (\text{ord}(t) \leq 5) + \\
(0.1 \times \text{loan_prc} \times \text{re}(t, \text{run}) \times (1+(15-\text{ord}(t)) \times \text{i_rate})) \times (\text{ord}(t) > 5);
\]

Parameters
th_Es(t, run) probability to get urban job [dmnl]
rss(t, run) real interest rate [per year]
wls(t, run) irrigation sector wage rate [DH per year]
wSs(t, run) skilled labour wage rate [DH per year]
IUs(t, run) labour allocation to urban sector [dmnl]
LI_ts(t, run) aggregate irrigation labourer [person per year]
LU_ts(t, run) aggregate urban labourer [person per year]
LR_ts(t, run) aggregate rainfed labourer [person per year]
LS_ts(t) aggregate skilled labourer [person per year]
ms(t, run) optimal private capital stock in year t [unit per capita]
KR_s(t, run) rainfed capital stock per household [unit per household]
IPs(t, run) per capita private capital stock investment [unit per capita per year]
MG_trade(t, run) government revenue from trade tax [million DH per year]
GSAV(t, run) government saving account [million DH per year]
R_dem(t, run) raw water demand [million m3 per year]
Q_dem(t, run) treated water demand [million m3 per year]
A_dem(t, run) irrigation land demand [1000 ha per year]
Bal_R(t, run) surplus supply of raw water [million m3 per year]
Bal_Q(t, run) surplus supply of treated water [million m3 per year]
Bal_A(t, run) surplus supply of irrigation land [1000 ha per year]
deficit(t, run) counter of GSAV deficit
unsust(t, run) counter of nonsustainable year
ratio_def(run) probability to fall in deficit
ratio_uns(run) probability to violate SC
c_hat(t, run) per capita consumption of satisfaction [unit per capita per year]
qHlcd(t, run) per user public supplied water consumption [l/cd]
cR_hat(t, run) per capita consumption of rainfed product [unit per capita per year]
cI_hat(t, run) per capita consumption of irrigation product [unit per capita per year]
cU_hat(t, run) per capita consumption of urban product [unit per capita per year]
qHbars(t, run) per capita average water consumption [m3 per capita per year]
qHs(t, run) per user piped water consumption [m3 per user per year]
YR(t, run) total rainfed output [unit per year]
YI(t, run) total irrigation output [unit per year]
YU(t, run) total urban output [unit per year]
ppR(t, run) producer price of rainfed product [DH per unit]
ppI(t, run) producer price of irrigation product [DH per unit]
ppU(t, run) producer price of urban product [DH per unit]
pcR(t, run) consumer price of rainfed product [DH per unit]
pcI(t, run) consumer price of irrigation product [DH per unit]
pcU(t, run) consumer price of urban product [DH per unit]
unemploy(t, run) nationwide unemployment rate [dmnl]
access(t, run) nationwide safe water access [dmnl]
EV(t, run) equivalent variation [DH per capita per year]
NPV_EV(run) net present value of EV(t run)
Gtot(t) total public capital stock [million DH]
Appendix B

Model implement / interest.rrz, wage.wlz, HT_model.th_Ez, ws_star.wSz, urb_lab.lUz, dom_wat.qHbarz, qqH.qHz, ir_labour.lLtz /

**** Outer loop start *****
Loop(run,
  *counters reset
deficit(t,run)=0;
unsust(t,run)=0;
*** initial guesses and ranges ****
qHbarz.l = 1*qHbar_c;
qHbarz.lo = qno;
qHbarz.up = 20*qHbar_c;
qHz.l = qHz_c;
qHz.lo = qno;
Ltz.l = Ltz_c;
Ltz.lo = epsi;
pwz.lo = .1*pw_c;
pz.lo = .1*p_c;
pAz.lo = .1*pA_c;
pz=multi("gr")*p_c;
pwz=multi("gr")*pw_c;
pAz=multi("gi")*pA_c;
ms("1",run) = m0;
KRz="1",run) = KR0;
G(gsec,"1") = G0mod(gsec);

**** Inner loop start *****
Loop(t,
  HHsize=(1+nyu)**(ord(t)-1);
pz.fx = p(t);
pwz.fx = pw(t);
pAz.fx = pA(t);
mz=ms(t,run);
KRz=KRz(t,run);
wUz = wUs(t,run);
w covz = wcov(t);
loss_Hz = loss_H(t);
loss_IRz = loss_IR(t);
Rz = R(t);
Qz = Q(t);
Az = A(t);
pcz(sec)=pc(sec,t,run);
ppz(sec)=pp(sec,t,run);
  th_Ez.l = th_E_c;
th_Ez.lo = epsi;
th_Ez.up = 1-epsi;
lUz.l = lU_c;
lUz.lo = epsi;
lUz.up = 1-epsi;
wSz.l = wS_c;
wSz.lo = wU_c;
wz.l = wL_c;
wz.lo = .1*wl_c;
rrz.l = multi_r*rr_c;
rrz.lo = epsi;
rrz.up = 1-epsi;
Solve implement using MCP;
trade tax revenue

\[ \text{MG\_trade}(t, \text{run}) = \left( \frac{\text{tar}(R) + (1 + \text{subs}(R))}{(1 + \text{tax\_p}(R))} \right) \times \frac{\text{omega}(t, \text{run}) \times \text{YRbar} \times N \times \text{p_bar}(R)}{1000000}; \]

\[ \text{IPs}(t, \text{run}) = (1 - \text{tax}_h) \times \frac{\text{rrz}(I) \times \text{pcz}(U) + (\text{wUz} \times \text{th\_Ez} \times \text{IUz} \times \text{wSz} \times \text{wSs} \times \text{LI tz}(t) / \beta(I, L) / \text{pA}(t))}{1000000} + \text{taxrev} \times \text{MG\_trade}(t, \text{run}) - \text{pcz}(U) \times \text{sum}(\text{gsec}, \text{IG}(\text{gsec}, t)) - \text{repays}(t, \text{run}); \]

\[ \text{GSAV}(t, \text{run}) \text{ when } (\text{ord}(t) = 1) = \frac{\text{p}(t) \times \text{gHz} \times N \times \text{HHsize} \times (\text{wcovz} + (\text{IUz} + \text{IS}) \times (1 - \text{wcovz}))}{(1 - \text{loss Hz})} + \frac{\beta(I, Q) \times \text{wlz} \times \text{LI tz}(t)}{1000000}; \]

\[ \text{unsust}(t + 1, \text{run}) = \text{unsust}(t, \text{run}) + 1 \times \left( \frac{\text{Bal}_R(t, \text{run})}{10} + \frac{\text{Bal}_Q(t, \text{run})}{10} + \frac{\text{Bal}_A(t, \text{run})}{10} \right); \]
\[ c_{R, \hat{t}, \text{run}} = q_{Hbars(t, run)} \times \phi(\text{"R"})/p(t)/\phi_Q/pc(\text{"R"}, t, \text{run}); \]
\[ c_{I, \hat{t}, \text{run}} = q_{Hbars(t, run)} \times \phi(\text{"I"})/p(t)/\phi_Q/pc(\text{"I"}, t, \text{run}); \]
\[ c_{U, \hat{t}, \text{run}} = q_{Hbars(t, run)} \times \phi(\text{"U"})/p(t)/\phi_Q/pc(\text{"U"}, t, \text{run}); \]
\[ E_V(t, \text{run}) = (c_{\hat{t}, \text{run}} - c\text{base}(\text{run}, t)) * ((1 - \text{tax}_h) * \text{rrs}(t, \text{run}) - \text{nyu}) * \text{prod}(\text{sec}, (pc(\text{sec}, t, \text{run})/\phi(\text{sec}))^{\phi(\text{sec})} / (1 + (1 - \text{tax}_h) * \text{rrs}(t, \text{run}) - \text{nyu})^{(1/\sigma)}); \]
\[ Y_R(t, \text{run}) = \omega(t, \text{run}) \times Y_{Rbar} \times N; \]
\[ Y_I(t, \text{run}) = w_{ls}(t, \text{run}) \times L_{ts}(t, \text{run})/\beta(\text{"I"}, \text{"L"})/pp(\text{"I"}, t, \text{run}) - a_{ij}(\text{"I"}, \text{"R"}) \]
\[ Y_U(t, \text{run}) = w_{Ss}(t, \text{run}) \times N \times (1 + \text{nyu})^{(\text{ord}(t) - 1)} \times I_{Us}(t, \text{run})/\beta(\text{"U"}, \text{"S"})/pp(\text{"U"}, t, \text{run}) - a_{ij}(\text{"U"}, \text{"R"}) \times pc(\text{"R"}, t, \text{run}) - a_{ij}(\text{"U"}, \text{"I"}) \times pc(\text{"I"}, t, \text{run}); \]
\[ pp_R(t, \text{run}) = pp(\text{"R"}, t, \text{run}); \]
\[ pp_I(t, \text{run}) = pp(\text{"I"}, t, \text{run}); \]
\[ pp_U(t, \text{run}) = pp(\text{"U"}, t, \text{run}); \]
\[ pc_R(t, \text{run}) = pc(\text{"R"}, t, \text{run}); \]
\[ pc_I(t, \text{run}) = pc(\text{"I"}, t, \text{run}); \]
\[ pc_U(t, \text{run}) = pc(\text{"U"}, t, \text{run}); \]
\[ LU_{ts}(t, \text{run}) = N \times (1 + \text{nyu})^{(\text{ord}(t) - 1)} \times I_{Us}(t, \text{run})/\text{th}_E(t, \text{run}); \]
\[ LS_{ts}(t) = N \times (1 + \text{nyu})^{(\text{ord}(t) - 1)} \times IS; \]
\[ LR_{ts}(t, \text{run}) = N \times (Y_{Rbar}/\text{tau}(\text{"R"})/\text{KRs}(t, \text{run}) * \beta(\text{"R"}, \text{"K"})^{(1/\beta(\text{"R"}, \text{"L"}))}; \]
\[ access(t, \text{run}) = w_{cov}(t) + (I_{Us}(t, \text{run}) + IS) * (1 - w_{cov}(t)); \]
\[ \text{unemploy}(t, \text{run}) = I_{Us}(t, \text{run}) * (1 - \text{th}_E(t, \text{run})); \]
\[ G\text{tot}(t) = \text{sum}(g\text{sec}, G(g\text{sec}, t)); \]

\$libinclude XLDUMP c_hat rawtab.xls c
\$libinclude XLDUMP qHicd rawtab.xls qHicd
\$libinclude XLDUMP ms rawtab.xls m
\$libinclude XLDUMP KR rawtab.xls KR
\$libinclude XLDUMP wg rawtab.xls w
\$libinclude XLDUMP wSs rawtab.xls wS
\$libinclude XLDUMP GSAV rawtab.xls GSAV
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