Monetary and Fiscal Policy issues in the Euro Area

Thesis by LUIGI LANDOLFO

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Abstract

The present study analyzes the performance of different strategies of monetary and fiscal policy to be used as possible guidelines for the establishment of the EMU. To this end, the research focuses on two important concepts that, during the past decade, have played a central role in the economic policy debate: the transmission mechanism of monetary policy and the sustainability of the fiscal policy. The study can be divided into three parts. The dissertation consists of six chapters. The first two parts are focused on the monetary policy implementation in the Euro area. The third part is focused on the sustainability of the public debt and the estimation of optimal fiscal policy rule, for a Government with commitment. In particular, the first part, Chapters 1 to 2, is based on the member country's past evidence to evaluate how, in each country, monetary policy strategies are transmitted to the real economy. Such a study is of crucial importance when assessing the future performance of the European Monetary Union. The second part, Chapter 3 and 4, attempts to evaluate how monetary policy strategies are transmitted to real economy in the Euro Area as whole. In the final part, Chapter 5 implements a multivariate test for the sustainability of public debt while Chapter 6 proposes a procedure to estimate the optimal fiscal policy rule.
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Needless to say, all the remaining weakness and errors are my own responsibility.
Author's declaration

I declare that the first three chapters of this thesis are joint works with Carlo Altavilla. I contributed to the elaboration of Chapter I by implementing appropriate RATS code to estimate the model for each country. I constructed all tables and figures needed for the presentation of the results and I draft the part of the work describing the results for the efficiency frontiers as well as the concluding remarks. Chapter I has been submitted for publication in Journal of Applied Econometrics. Regarding the elaboration of Chapter II and Chapter III I declare to be the main contributor about any aspect. These chapters have been respectively published in International Review of Applied Economics and Applied Economics.

I finally declare to be the sole author of the remaining three chapters. Chapter IV is the outcome of four months internship at ECB. This is the true copy of the thesis as accepted by my supervisor.
0.1 General Introduction

The present study analyzes the performance of different strategies of monetary and fiscal policy to be used as possible guidelines for the establishment of the EMU. To this end, the research focuses on two important concepts that, during the past decade, have played a central role in the economic policy debate: the transmission mechanism of monetary policy and the sustainability of the fiscal policy.

The debate which has developed around those concepts may seem aged. On the contrary, the increasing literature on the strategies a central bank should adopt to set interest rates, as well as on the sustainability of the public finances, demonstrate the actuality of the problems that are concealed behind them. In fact, the rapid and persistent evolution in the tools used as well as in the objectives pursued by monetary and fiscal policy authorities, making necessary a faster adaptation of theories and practices to the new developments, continuously renews the actuality of the debate. The frequent change of macroeconomic variables used as policy instrument, intermediate target and institutional goals in the implementation of monetary and fiscal policy strategies being an example.

Regarding the channels through which the monetary policy affects the real economy, in fact, there is a huge amount of empirical evidence indicating that, in the last few years, there has been a strong change in the behavior of the central banks regarding their use of explicit targets for conducting monetary policy. The change has concerned both the effective existence of announced rules and the presence of international institutions able to influence the behavior of the monetary authorities.

Also the empirical literature on the finances of the public sector seems to show new approaches to solve the problem. Nevertheless, the two most significant questions about these issues, still remain, namely: "Whether, or not, the high public debt is becoming increasingly unsustainable" and "Which is the optimal strategy to utilize in order to achieve the public debt target?".

***
We now introduce the two main concepts the research focuses on. The first of these is related to the transmission mechanism of monetary policy and the rules a central bank should follow, in applying its strategy. The second, is related to the sustainability of the fiscal policy, and the estimation of a fiscal rule to be used as a guideline for the fiscal policy authority.

The increasing literature on the effectiveness of the monetary policy, and therefore on the channels through which this effectiveness is realized, finds an appropriate justification in the inadequacy of the theoretical schemes in developing models that maintain a good level of approximation towards the empirical reality. Besides, a working knowledge of how the transmission process operates in countries that have membership, in a currency union, appears of crucial importance for assessing the future performance of the monetary union. These are the reasons that have pushed many researchers to investigate the exact characteristics of the transmission process of the country members of the EMU.

In addition to an explanation of these issues a certain degree of analysis, of the possible reaction function that the new European monetary institution may adopt is required. The reaction function, summarizing how central bank alters monetary policy in response to economic developments, can be useful in predicting actual monetary actions and, therefore, in assessing the current stance and the future direction of monetary policy. The econometric evidence resulting from this kind of study, can also suggest which monetary rule ECB should adopt, in order, to achieve its primary institutional goal, namely, price stability. To this end, the monetary rules analyzed differ in their method of expectation formation, some being backward looking, and others being forward looking and in the variables they allow to enter into the monetary policy reaction functions.

The joint study of the systematic portion of policy behavior and of the possible effects that monetary shock may have on the real economy, exhaustively explains the causes of the policy-instrument variability. In other words, by analyzing both the systematic and the stochastic components of a central bank's behavior, it is possible to obtain an enhanced assessment of the implemented monetary strategy.
The study of the two components requires different econometric techniques. In fact, while the analysis of the effects of systematic reaction the central bank has in response to economic developments requires structural modeling in order to develop policy-invariant behavioral relationships, the shock component is evaluated through VAR procedures.

The second topic the research faces, is the uncertainty as well as the sustainability of a country's budgetary position. The unsustainable accumulation of public debt for a country may hinder the Central Bank, in complying with its primary objectives of internal monetary stability. Moreover, the negative externalities of lax budgetary policies include among other things, its impact on interest, inflation and exchange rate. The need for having a procedure to test the budgetary sustainability of a country, as well as the need for having a procedure to estimate the fiscal policy rule are then generally accepted.

The study can be divided into three parts. The dissertation consists of six chapters. The first two parts are focused on the monetary policy implementation in the Euro area. The third part is focused on the sustainability of the public debt and the estimation of optimal fiscal policy rule, for a Government with commitment.

In particular, the first part, Chapters 1 to 2, is based on the member country's past evidence to evaluate how, in each country, monetary policy strategies are transmitted to the real economy. Such a study is of crucial importance when assessing the future performance of the European Monetary Union.

The second part, Chapter 3 and 4, attempts to evaluate how monetary policy strategies are transmitted to real economy in the Euro Area as whole.

In the final part, Chapter 5 implements a multivariate test for the sustainability of public debt while Chapter 6 proposes a procedure to estimate the optimal fiscal policy rule.
Chapter 1 analyses the implications of the monetary rules efficiency for practical monetary policy making. Special emphasis is given on strategies for setting interest rates by the EMU participating economies. First, the structural model, thought to be representative of the main effects that the central bank has on inflation and output, is presented. Second, a set of instrument rules consistent with the inflation targeting regime are estimated. Third, a comparative analysis is made of the ability of the rules to correspond to the historical central bank behavior. Fourth, the study focuses on the volatility of output, inflation and interest rate changes that each rule implies. Finally, the study compares the efficiency frontiers across EMU countries. The results suggest that simple rules perform quite well and that the advantages obtained from adopting an optimal control-based rule are not so great. Moreover, the analysis emphasizes the success of the German model of central banking, during the sample period.

Chapter 2 analyzes monetary policy asymmetries in EMU participating countries. In particular, we use a structural dynamic modeling approach to investigate asymmetric monetary transmission in Europe. Asymmetries are investigated in two different ways. First, we restrict the estimated structural models reflecting the monetary constraints each country faced during the EMS period. We obtain well-behaved and comparable effects of monetary policy shocks. Second, efficiency frontiers for the selected EMU countries are estimated. In computing the optimal combinations of output gap and inflation volatility we use a weighted average of interest rate and exchange rate, i.e. the Monetary Condition Index (MCI), as a policy instrument. The impulse response analysis implemented with the MCI shows relatively small differences in the responses of the real economy, to monetary policy shocks. Altogether the results suggest that, no matter which policy instrument is used, output gap and inflation respond to identical monetary shocks with a similar speed and movement, albeit with a different degree of effects.
0.1. GENERAL INTRODUCTION

Chapter 3 attempts to exploit whether the monetary authorities present different behavior during recession and expansion. To this end, a multivariate extension of Hamilton's (1989) Markov Switching Model is adopted. First, regime dependent Taylor-type rules are estimated for the Euro Area and the United Kingdom, in order to capture the systematic behavior of central banks. Then, impulse response functions that account for the different phases of the business cycle are analyzed. In addition, a comparative analysis concerning the estimated rules as well as the different reaction of the real economy to monetary shocks is implemented. The study strongly suggests that central banks cannot neglect the regime where the monetary action takes place. The phase of the business cycle is an important matter in the monetary policy decision process.

***

Chapter 4 aims to model the main determinants of the confidence of economic agents in the Euro Area. In particular, it measures the impact of the labour market conditions, the stock market developments and interest rates on consumer and business confidence. The study is articulated in two parts. We estimate two models, respectively for the survey indicators of consumer confidence and of business confidence. The analysis relies on the use of multivariate econometric techniques. Specifically, we use a structural VAR and a dynamic simultaneous equations model. Empirical results show that models are well-behaving. Impulse response analysis suggests that the main determinants of consumer confidence are labour market conditions and interest rates. Empirical evidence for business confidence shows that this indicator is strongly affected by economic conditions in the United States. Industrial production and labour market conditions in the euro area play an important role in the short term.

***

Chapter 5 provides a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint, and derives conditions that determine whether a fiscal stance is
sustainable in the medium and long term. In contrast to previous studies, it uses a log-linearization of the public debt identity and generalizes the results obtained in literature, by using a multivariate test. The analysis is applied to the fiscal position of the United States and Euro Area. On the basis of infinite horizon-tests the broad conclusion is that, both countries have an unsustainable fiscal policy. The chapter also provides two procedures to construct forecasts of the future level of public debt. Forecasted values confirm the evaluation elaborated in the cointegration analysis.

***

Chapter 6 constructs a procedure to estimate the optimal fiscal policy rule in the presence of a public sector, with objectives of convergence for public debt and primary balance to GDP ratios. To this end, the study uses a stochastic simulation framework. In order to ensure the existence of converging paths towards the target values of fiscal variables, we introduce a simple fiscal policy rule. According to this rule, the primary balance ratio is adjusted in function of the distance between the current and the target level of the public debt, the current and the target level of the primary deficit and the output gap. The study gives interesting insights. First, it shows that the fiscal rule displaying time invariant parameters may produce non-linear dynamic processes of adjustment of the public debt. Second, it suggests a procedure to construct confidence intervals for the parameters characterizing the optimal fiscal policy rule. Finally, it estimates a stochastic efficiency frontier for the public sector. The analysis is first applied to a theoretical economy and then to Italy and Belgium.
Part I

Asymmetries across EMU countries
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Chapter 1

The Efficiency of Monetary Rules in the EMU countries

This chapter analyzes the implications of the efficiency of monetary rules for practical monetary policy-making. The efficiency analysis of alternative policy rules is based on the differences between actual and estimated reaction functions as well as on the ability of the rules to stabilize output and inflation. Special emphasis is given to strategies for setting interest rates by the EMU participating economies. First, the structural model, thought to be representative of the main effects that the central bank has on inflation and output, is presented. Second, a set of instrument rules consistent with the inflation-targeting regime are estimated. Third, a comparative analysis is made of the ability of the rules to correspond to historical central bank behavior. Fourth, the study focuses on the volatility of output, inflation and interest rate changes that each rule implies. Finally, the study compares efficiency frontiers across the EMU countries. The results suggest that simple rules perform quite well, and that the advantages obtained from adopting a complicated rule are not so great. Moreover, the analysis emphasizes the success of the German model of central banking during the sample period.
1.1 Introduction

The study analyzes the performances of different reaction functions to be used as possible guidelines, for the establishment of monetary policy in the EMU. The reaction function, summarizing how the central bank alters monetary policy in response to economic development, can be useful in predicting actual monetary actions and, therefore, in assessing the current stance, as well as the future direction, of monetary policy. The econometric evidence resulting from this kind of study, can also suggest which monetary rule the ECB should adopt in order to achieve its primary institutional goal, namely, price stability.

The monetary rules analyzed differ in their method of expectation formation, some being backward-looking, others being forward looking and in the variables they allow to enter into the monetary policy reaction functions. The rules studied include an optimal feedback rule, two different specifications of the Taylor rule, a forward-looking rule and two alternative inflation rules. All the rules are retrieved from an intertemporal optimization problem of a loss function penalizing the volatility of output, inflation and policy rates under the constraints given by a small, backward-looking, closed-economy structural model.

In order to evaluate the various results a central bank obtains from adopting a particular rule, a preliminary definition of what constitutes rule-based monetary policy in practice must be given. As no central bank will be bound to the prescription of any simple rule (or any optimal control algorithm), the distinction between rule-based and discretionary monetary policy is crucial. As stressed in McCallum (2000), while a discretionary monetary policy takes into account current macroeconomic conditions, ignoring past development in the economic system, a rule-based monetary policy is based on a "timeless perspective", i.e. the rule is constructed as if the current conditions were not known. According to this definition, when following a discretionary policy, the central bank re-optimizes its decision-making process periodically, while in a rule-based policy, monetary authorities implement a contingency formula chosen to be applied for an infinite number of time periods. Nev-
1.1. INTRODUCTION

ertheless, in the rule-based framework, the possibility of revising a rule is also contemplated, once the central bank gets new information on the state of the economy. In this sense, the inflation-targeting regime, although not restricting monetary authorities to select instrument settings according to a particular rule, can be considered an example of rule-based policymaking.

The reason why a central bank should adopt a monetary rule, instead of discretionary behavior, has a theoretical basis in time-consistency literature. In this literature, to which the seminal contribution was made by Kydland and Prescott (1977) and Barro and Gordon (1983), it is shown that if a central bank does not commit itself to a rule, the policymakers will be tempted to choose a suboptimal inflation policy. The contribution of Barro and Gordon is of particular interest for the issues analyzed in the paper because the "rules vs. discretion" dichotomy is separated from the debate on "activist vs. non-activist" central bank policy. This separation has resulted in the possibility for monetary policymaking to concentrate on the issue of policy rules.

Moreover, there are other advantages the central bank can obtain by limiting the range of possible policies, i.e. adopting a rule. The first is an increase in monetary policy credibility. A second motivation, related to the first, considers a policy rule as a helpful device for improving transparency, accountability and the clear communication of targets, characteristics that may be viewed either as desirable in themselves or as serving to make commitments more effective. Thirdly, a rule may be useful in providing a guide to policy that is robust to uncertainty. The acknowledgment of which policy rules are likely to work robustly across a range of possible models is able to decrease the uncertainties of market participants.

The remainder of the chapter proceeds as follows. Section 1.2 presents a structural model designed to show the main channel through which monetary policies affect inflation and output. Section 1.3 introduces a set of instrument rules consistent with the inflation-targeting regime. In Section 1.4, the study focuses on the historical analysis of monetary policy rules. Section 1.5 examines the efficiency of the estimated monetary rules. Section 1.6 ends the chapter with concluding remarks.
1.2 The Model

In empirical literature, several types of models have been used for evaluating monetary policy rules, including an optimizing model with representative agents, closed and open-economy models, and rational expectation models. Despite EMU countries have been commonly considered as small open economies in the Pre-Euro Area period, we model these countries as closed economies. This choice relies on the results stressed by Clarida, Jordi and Gartler (Clarida, Gali and Gertler, 2001). They showed that the monetary policy design problem for the small open economy is isomorphic to the problem of the closed economy. In particular, they conclude that qualitative results for a closed economy can be extended to the small open economy and vice versa. Openness only suggests quantitative implications. Regarding the price setting behaviour of the private sector, we prefer a backward-looking model. There is a large part of literature arguing that the behaviour of the private agents should be forward-looking and the forecast should be therefore an integral part of it. Nevertheless, a different strand of this literature (i.e. Rudebusch and Svensson 1999,2002; Ball 1999,2000), following in the spirit the VAR models popularized by Sims, focuses on ad-hoc backward-looking model. They argue that this kind of models are appreciable from at least two important aspects. They tend to offer a good fit of the data, and their dynamics closely resemble those filtered with structural VARs, an issue that pure forward-looking models have some troubles in dealing with (Estrella and Fuhrer, 2002). Altogether, the model used in this analysis is a backward-looking closed economy model.

The specification of equations is thought to be representative of the main effects that a central bank might have on inflation and output.

The model consists of an aggregate supply equation of the form:

$$\pi_{t+1} = \sum_{i=1}^{h} \alpha_{i} \pi_{t-1-h} + \alpha_{h+1} y_{t} + u_{t+1}^{\pi}$$ (1.1)

This autoregressive Phillips curve relates the CPI inflation rate ($\pi$) to its own lags and to a lagged output gap ($y$), measured as a percent gap between the actual real GDP and the potential GDP. The specification of
aggregate supply is consistent with an adaptive representation of inflation expectations. The expectations are treated implicitly by the inclusion of lagged values of the variables.

Equation (1.2) identifies the aggregate demand equation:

$$y_{t+1} = \sum_{j=1}^{k} \beta_j y_{t+1-k} - \beta_{k+1}(\bar{r}_t - \pi_t) + u_{t+1}^y$$ (1.2)

According to the above equation, the output gap is related to the real interest rate and to its own lags. The real interest rate is calculated as the difference between the short-term interest rate and the inflation rate. In the above equation, \(\bar{r}_t\) is the four-quarter average short-term interest rate, i.e. \(\frac{1}{4} \sum_{j=0}^{3} i_{t-j}\); \(\pi_t\) is the four-quarter inflation rate, i.e. \(\frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}\). This specification implies that interest rate is an exogenous variable under the complete control of monetary authorities. The particular specification of the model is a crucial issue. In fact, the conclusions reached in the empirical analysis are all obtained assuming that the economic structure implied by the proposed model is not grossly incorrect. Nevertheless, literature has not converged on a single interpretation of inflation and output dynamics. In particular, the disagreement concerns the effects of inflation on the real economy. Much of the empirical literature suggests a negative influence of inflation on the output gap. Yet many economic theories predict neutrality, or even a positive effect of average inflation on economic performance. The particular specification of the inflation process is also important. The absence of forward-looking variables in the estimated model is in line with the analysis of Fuhrer (1997), on the importance of future price expectations in explaining price and inflation behavior. He finds that the performance of a model, built for a purely forecasting purpose, with a forward-looking specification of the inflation is no better than a backward-looking model.

The timing of the model can be summarized as follows: an increase in monetary policy instrument \(i_t\) in period \(t\) affects output with one period lag; it takes another period, i.e. at time \(t+2\), for output to affect inflation. The timing in which inflation affects the real economy as well as the size of the impact is important, because only if inflation is sticky can monetary
authorities influence real economic performance through monetary policy.

The model has been estimated by applying the OLS technique and using quarterly data for the period 1979:1-1998:4\(^1\). Potential output has been computed by using the Hodrick-Prescott (HP) filter. As we have quarterly data, we set the smoothing parameter to 1600 as in Kydland and Prescott (1990). All variables were de-meaned prior to estimation.

The order of the autoregressive coefficients in equation (1.1) and equation (1.2) is tested, for each country, by implementing standard test statistics. Table 1.1 computes the Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC), Hannan-Quinn, and FPE for aggregate demand equations. These are all done in log form, so their values look fairly similar. For each line in the table, the selected lag is highlighted. The table suggests that for most of the countries, the order of autoregression in the output equation is equal to two. Only for Austria and the Netherlands do we apply a four-lag output gap structure. Concerning the inflation equation, we impose a fourth order autoregressive structure. In order to analyze the long-term properties of the model, we also test the hypothesis that the autoregressive coefficients of the inflation equation equal one. This test is of particular interest because its outcome implies a particular slope of the Phillips curve. Table 1.2 summarizes the results. As shown, the hypothesis that the sum of the lag coefficients of inflation is equal to unity is not rejected for all countries. This restriction is imposed in the estimation. This means that we assume the existence of a natural unemployment rate (NAIRU) when inflation is stable.

\(^1\)The data used in the empirical analysis are taken from the IFS statistics.
## 1.2. THE MODEL

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Table 1.1: Lag Analysis
CHAPTER 1. THE EFFICIENCY OF MONETARY RULES

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<td>Germany</td>
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Table 1.2: Test of Restrictions

The estimated equations are presented in Table 1.3 and Table 1.4.

T-statistics are given in square brackets. For each equation, the tables also provide the $R^2$, the Durbin-Watson statistics ($DW$), the standard error of estimates ($S.E.E.$) and the ARCH(1) test. For our purposes, the key coefficients in the tables are the $\beta_{k+1}'s$, which represent the effect of the real interest rate on output gap; the coefficients on the lagged aggregate supply curve that represent the inertia of the inflation process, i.e. $\alpha_h's$; and $\alpha_{h+1}$ which measures the effect of an increase in the output gap on the inflation rate, i.e. the slope of the Phillips curve. At first glance, the model seems to perform rather well. Almost all coefficients are significant at the 5% level. The $DW$ test as well as the $ARCH(1)$ test give evidence of the absence of serial correlation and heteroscedasticity, respectively. The parameter $\beta_{k+1}$ ranges from -0.07 in Finland to -0.18 in Ireland and Belgium. On the average, a one percentage-point increase in the interest rate reduces the output gap by almost 12 basis points. Concerning the effects of the output gap on inflation, the estimated coefficients vary from 0.1 in Spain to 0.18 in the Netherlands.
### Table 1.3: Estimated Models

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<th>$\pi_{t-3}$</th>
<th>$Y_t$</th>
<th>$Y_{t-1}$</th>
<th>$Y_{t-2}$</th>
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<th>$r_t$</th>
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## Table 1.4: Estimated Models

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1.3 Monetary Policy Rules

The class of rules considered in the analysis, are instrument rules that are consistent with the inflation-targeting regime. The inflation-targeting regime is modelled by using a quadratic loss function over policy targets. Following Rudebush and Svensson (1998) and Svensson (1998a, b), the central bank should minimize an intertemporal loss function that increases, if there is a deviation between a target variable and the target level for this variable.

The loss function takes the following general form:

\[ E_t \sum_{i=0}^{\infty} \vartheta^i L_{t+i} \]  \hspace{1cm} (1.3)

where \( E_t \) refers to expectations conditional upon the available information set at time \( t \), while \( \vartheta \) is a given discount factor, with \( 0 < \vartheta < 1 \).

The specific features of the loss function that must be considered raise some problems. Several authors have stressed the perverse attitude to risk of the quadratic loss function; by utilizing such a function, we are implicitly assuming the central bank treats symmetrically both positive and negative deviations from the target. Even so, as shown in Chadha and Shellekens (1999), conducting the analysis with a different attitude to risk through the introduction of an exponential (CARA) or isoelastic (CRRA) loss function does not produce, in a context of additive uncertainty, a richer description of policymaking behavior. In fact, also in these cases certainty equivalence applies, provided that the alternative loss function is symmetric. For these reasons, the analysis uses a quadratic loss function of the form:

\[ L_t = \lambda \pi_t^2 + \varphi y_t^2 + \gamma (i_t - i_{t-1})^2 \]  \hspace{1cm} (1.4)

Following the terminology introduced in Svensson (1997), the above expression describes a flexible inflation target where the goal variables describing central bank preferences are \( \pi_t \), i.e. the deviation of actual inflation from a constant, given inflation target; \( y_t \), i.e. the output gap and \( i_t - i_{t-1} \), an interest-rate smoothing term. Moreover, \( \lambda \), \( \varphi \) and \( \gamma \) are non-negative.
weights that the central bank attaches to stabilize output and interest-rate smoothing, respectively. If $\varphi$ and $\gamma$ are set to zero, we are in a situation of strict inflation-targeting. Something must be said about the variables that enter into the loss function. In real monetary policy-making, the inflation rate is usually preferred to the output gap as a formal target for monetary policy. The reasons are related to the specific features the inflation rate has in comparison with the output gap. From a theoretical point of view, the long-run neutrality of monetary policy on output capacity suggests that central banks should concentrate on the variables, like inflation, that they can influence on a long-term basis. From a practical point of view, the difficulty in measuring the output gap and public familiarity with the concept of inflation supports the choice of inflation for central bank communication and econometrics estimation purposes, respectively. Nevertheless, even if the central bank official target is expressed in terms of inflation, it is believed that output stabilization is still important to monetary authorities. Finally, the inclusion of the objective of interest rate smoothing is proposed to account for two phenomena. The first is the aversion that the central banks have to frequently changing the direction of their strategy. The second is related to the idea that central banks also care about financial stability: interest rate instability can lead to a destabilization of the financial system.

As shown in Rudebush and Svensson (1998), for $\vartheta = 1$, the optimization problem can be rewritten interpreting the intertemporal loss function as the unconditional mean of the period loss function; it means that the intertemporal loss function can be written as the weighted sum of the unconditional variances of goal variables:

$$E [L_t] = \lambda \text{Var} [\pi_t] + \varphi \text{Var} [y_t] + \gamma \text{Var} [i_t - i_{t-1}].$$

(1.5)

In the following, this loss function will be used, assuming, therefore, the limiting case $\vartheta = 1$.

1.3.1 State-Space Representation

The State space representation of the estimated model is:
$X_{t+1} = AX_t + Bi_t + v_{t+1}$ (1.6)

This compact form is helpful in summarizing the structure underlined by the dynamic model. More precisely, in the above equation the $(k + h + 3) \times 1$ vector $X$ contains the state variables, the $(k + h + 3) \times (k + h + 3)$ matrix $A$ and the $(k + h + 3) \times 1$ column vector $B$ contains the estimated parameters, and the $(k + h + 3) \times 1$ column vector $v_t$ is the disturbance term. This representation summarizes the dynamic structure of the economy and the uncertainty that the central banks face regarding this structure. The matrix $A$ and the vector $B$ govern the dynamics of the state vector. Uncertainty enters through the additive stochastic vector $v_{t+1}$. The terms in equation (1.6) can be written as:

$$A = \begin{bmatrix}
\sum_{i=1}^{h} \alpha_i e_i + \alpha_{h+1} e_{h+1} & e_1 \\
\vdots & \vdots \\
e_h & e_{h+1} \\
\vdots & \vdots \\
e_{h+k} & e_{h+k+1} \\
e_0 & e_{h+k+2}
\end{bmatrix}$$

where $e_i$ denotes a $1 \times (k + h + 3)$ row vector with all elements equal to zero and with the elements $i = 1, \ldots, k + h + 2$ equal to unity; and where $e_{i:k}$ ($i < k$) denotes a $1 \times (k + h + 3)$ row vector with elements $i, i+1, \ldots, k$ equal

$^2$Where $h$ and $k$ are the number of lags of the inflation and output equations respectively.
to \( \frac{1}{4} \) and all other elements equal to zero. Notice that all variables entering in the state-space representation are expressed as a function of lagged data only. This condition comes from the particular model considered in the analysis which is, in fact, a backward-looking model\(^3\).

\[
X_t = \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\vdots \\
\pi_{t-h} \\
y_t \\
y_{t-1} \\
\vdots \\
y_{t-k} \\
i_{t-1} \\
i_{t-2} \\
i_{t-3}
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
\vdots \\
\vdots \\
0
\end{bmatrix} \quad \text{and} \quad \nu_t = \begin{bmatrix}
u_t^x \\
u_t^y \\
0 \\
0 \\
\vdots \\
\vdots \\
0
\end{bmatrix}
\]

Writing the target variables, \( \pi_t, y_t \) and \( i_t - i_{t-1} \) as a function of the state variable \( X_t \) we get:

\[
Y_t = \begin{bmatrix}
\bar{\pi}_t \\
y_t \\
i_t - i_{t-1}
\end{bmatrix} = C_X X_t + C_i i_t, \quad \text{where} \quad C_X = \begin{bmatrix} e_{1:h} \\ e_{h+1} \\ -e_{h+k+1} \end{bmatrix} \quad \text{and} \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

The loss function can now be expressed as:

\[
L_t = E [Y_t'KY_t], \quad \text{where} \quad K = \begin{bmatrix}
\lambda & 0 & 0 \\
0 & \varphi & 0 \\
0 & 0 & \gamma
\end{bmatrix}. \quad (1.7)
\]

\(^3\)A forward-looking, open-economy model was used in Svensson (1998b). In this case, the state-space representation is much more complicated to derive.
The class of linear feedback rules considered here takes the following generic form:

\[ i_t = f X_t \]  \hspace{1cm} (1.8)

where \( f \) denotes a \( 1 \times (k + h + 3) \) vector. Using the foregoing relations, the dynamics of the model follow:

\[ X_{t+1} = MX_t + v_{t+1}, \quad M = A + Bf \]  \hspace{1cm} (1.9)

\[ Y_t = CX_t, \quad C = C_X + C_if \]  \hspace{1cm} (1.10)

The optimal linear feedback rule is an interest rate rule that, given the economic structure implied by the rule, is able to minimize the central bank loss function. Thus, the optimal linear feedback rule can be expressed as:

\[ f = - \left( R + B'VB \right)^{-1} \left( U' + B'VA \right) X_t \]  \hspace{1cm} (1.11)

where the matrix \( V \) satisfies the Riccati equation:

\[ V = Q + UF + f'U' + f'Rf + M'VM \]  \hspace{1cm} (1.12)

and where:

\[ Q = C'_X K C_X, \quad U = C'_X K C_i \] and \( R = C'_i K C_i \)

In this section, different specifications of instrument rules will be estimated. Within this class of rules, the monetary policy instrument is expressed as a function of the available information. The analysis considers six instrument rules. The first one is the unrestricted optimal control rule:

\[ i_t = f X_t \]  \hspace{1cm} (1.13)

where \( f \) denotes a \( 1 \times (k + h + 3) \) vector of response coefficients. In this case, monetary policy responds to changes in every state variable.

The second rule is the classical Taylor rule. Since the Taylor (1993) seminal paper, a great deal of literature has been written which aims at
explaining the stabilizing power of active interest rate rules. Recently, several authors including Taylor (1998) and Gerlach and Schnabel (1999) have underlined the usefulness of the Taylor rule as an informal benchmark for setting interest rates in the EMU area. The classic Taylor rule (henceforth TR), assumes that the interest rate is a function of the current values of both inflation and the output gap:

\[ i_t = f_\pi \pi_t + f_y y_t \]  

(1.14)

where \( f_\pi \) and \( f_y \) are the response coefficients of inflation and output gap respectively.\(^4\)

By adding an autoregressive term to the previous specification, thus allowing the central bank to react to a lagged interest rate, we get the Generalized Taylor Rule (GTR):

\[ i_t = f_\pi \pi_t + f_y y_t + f_i i_{t-1} \]  

(1.15)

The fourth and fifth rule are pure inflation rules with and without an interest rate smoothing term:

\[ i_t = f_\pi \pi_t \]  

(1.16)

\[ i_t = f_\pi \pi_t + f_i i_{t-1} \]  

(1.17)

Finally the sixth rule is a forward-looking rule. In monetary policy literature, there has been a great debate on the information set that the central banks should use to fix the interest rate. More precisely, the discussion has focused on the possibility and the relevance for monetary authorities to include some forward-looking variables, in the reaction function specification. The need for a forward-looking dimension in monetary policymaking has been stressed by several authors, among others Batini and Haldane (1998) and Svensson and Woodford (2000), as a necessary condition for a better representation of central bank behavior. Nevertheless, many economists

\(^4\)The Taylor rule restricts the vector \( f \) as:

\[ f = f_{\pi \epsilon_{1:t}} + f_{\epsilon_{t+1}} \]
are sceptical about the improvement that can be obtained from the inclusion of a forward-looking variable in a macroeconomic model of monetary policy and, in any case, they stress the need to incorporate a sort of history-dependence in a rule to be considered as optimal\(^5\). This scepticism is based on the consideration that, by allowing a central bank to react to forecasts of future inflation, we are not eliminating the backward-looking component in central bank behavior; as the forward-looking components are recovered from current and lagged data of the related variables, they are, in fact, backward-looking. The main advantage of the forward-looking rule then, is the inclusion of other variables besides the output gap and inflation that can help to forecast monetary actions.

In our specification, the central bank is allowed to respond to an inflation forecast rather than to current inflation.

\[
i_t = f_\pi \pi_{t+8|t}^e + f_y y_t + f_i i_{t-1} \quad (1.18)
\]

Where \(\pi_{t+8|t}^e\) is the 8-quarter ahead inflation forecast for a given interest rate and it is calculated as:

\[
\pi_{t+8|t}^e = e_{1.8}(AX_t + Bi)\quad (1.19)
\]

The forecasts are also computed conditional upon the current state variables \(X_t\)\(^6\).

Figure 1.1 shows, for each country, the forecast used to compute the forward-looking rule together with the actual inflation rate. As shown in the figure, the forecasts appear to capture the dynamic of the inflation rate.

Table 1.5 and Table 1.6 present the estimated response coefficients of the selected reaction functions. In principle, there are several factors affecting the particular specification of the rule that a central bank can follow. In fact, different values of the state variable, \(X\), different impacts of monetary policy, \(A\) and \(B\), and different central bank preferences over inflation, output and interest rate smoothing, \(K\), may result in a different interest rate policy,

\(^5\)See Woodford (2000) on this point.

\(^6\)This means that the restrictions on \(f\) are:

\[
f = f_\pi e_{1.8}(A + Bi)^8 + f_y e_{h+1} + f_t e_{h+k+1}.
\]
i.e. a different rule. The differences in the response coefficients reflect all those variables.

The tables give some interesting results. Consistent with a-priori beliefs, the first interest-rate smoothing coefficients are quite high, 0.6 on the average, while the third and fourth lag coefficients are much lower.

For the optimal control rule, the estimated coefficients of inflation are much more persistent than the output gap lag coefficients. The inflation response coefficients for the Taylor rule and the Generalized Taylor rule are well above the value of 1.5 suggested in Taylor (1993).

However, Table 1.5 and Table 1.6 show that the coefficients in the TR, GTR and FLR have an unambiguous theoretical meaning; they suggest that the central banks of the EMU countries have raised nominal interest rates by more than any increase in inflation, so that inflation has never spun out of control. In fact, the interest rate response coefficients of the inflation rate, i.e. $f_π$, are above the stability threshold of one. This evidence, as stressed by Taylor (1998), is a crucial feature for a dynamically stable monetary policy. In his paper, Taylor also gives a theoretical basis for this result. Essentially, he argues that having a response coefficient lower than one results in a positively-sloped aggregate demand curve and causes the output to decrease in response to an inflation shock, which is destabilizing.

From the tables we can also see that Germany is the country where a rise in expected inflation produces the largest response from the central bank in terms of real interest rate reaction; an increase of one percent induces the monetary authorities to raise the real rates by 176 basis points. More generally, in all the EMU countries the central banks have responded to inflationary pressures by raising the real rates.

Another interesting result regards the output gap estimated coefficients. In all countries, a rise in the output gap induces central banks to increase interest rates. According to the optimal feedback rule, a one-percent increase in the output gap in Italy, for example, should induce the Bank of Italy to increase nominal (and thus real) rates by almost 50 basis points.

We can conclude that over the sample period, the central banks of the EMU countries reacted to real economy pressures independently of their
concern about inflation.
Figure 1.1: Actual Inflation Rates (solid lines) vs. Inflation Forecasts
## 1.3. Monetary Policy Rules

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Table 1.5: Response Coefficients


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Table 1.6: Response Coefficients
1.4 The Historical Behaviour of EMU Member Central Banks

This section focuses on the historical analysis of monetary policy rules. An analysis of the historical behaviour of the EMU central banks can give useful insights for the conduct of monetary policy, in the European currency area. In fact, as shown in De Grauwe and Piskorski (2001), the monetary strategy of the ECB might be considered as a proxy of the optimal policy rule estimated by using the national data of the EMU member states.

In particular, the section analyzes the differences between the estimated policy action functions and the rule the EMU member central banks have been using during the past decades. Following the terminology introduced by Taylor (1998), the deviation of the actual policy rules, summarized by the short-term nominal interest rate, from the optimal rules is considered to be a "policy mistake". We can, then, assess whether the value of the interest rate implied by the six policy rules considered in the previous section are, significantly, different from the actual central bank behaviour.

Figures 1.2 to 1.7 show the value of the interest rate implied by the six estimated policy rules (dashed lines), versus the actual central bank behaviour (solid lines) for each EMU member.

Consistently across countries, we can characterize three periods of monetary policy history.

In the first period, covering most of the 1980s, the actual interest rates appear to be well below the estimated policy rules. This evidence suggests that the monetary strategy implemented by the central banks of the EMU economies was too loose. Optimal monetary policy rules would have implied higher level of the interest rate for majority of the EMU countries. This is true for all EMU countries with the exception of Spain. For this country, the Taylor rule and the pure inflation rule suggest a lower interest rate in the sample period. Nevertheless, when the smoothing term is included in the rules, the results for this country become consistent with the remaining EMU countries.

The second period, occurring in the early 1990s, embraces the European
monetary system crises. In this period, the behaviour of the EMU monetary authorities was quite dissimilar. The differences basically derive from the strategy each central bank followed in response to speculative attacks. In fact, the selected rules predict well central bank behaviour during the ERM crisis for Italy, France, Germany, Austria and Belgium, but fail to do so for Ireland, Finland, the Netherlands, Portugal and Spain.

The third period goes from the early 1990s to the end of the sample. In this period, the outcome of the estimated rules appears to be much closer to the actual interest rates. According to the estimated rules, the decrease in interest rates in the late 1990s was about the right magnitude. In fact, the convergence criteria stated in the Maastricht Treaty, permitted interest rates to be lowered by forcing the central banks to stabilize the inflation rate without making policy mistakes.

We can conclude that, over the last two decades, macroeconomic performance has improved markedly in the EMU countries. Better macroeconomic performance has not only resulted in lower inflation: it has also improved the stability of the inflation and real growth rates. Monetary policy authorities have become more skilful in implementing strategies to meet their stabilization objectives.

The ability of the various rules to reproduce the actual interest rate, is shown in Figures 1.2 to 1.7. From this analysis, it is clear that all the rules perform quite well in replicating actual interest rate movements. In contrast, it is not clear which is the best performing rule. Overall, the Generalized Taylor rule and the forward-looking rule seem to be consistently the most successful across the countries, in describing historical central bank behavior. The inclusion of a smoothing term for interest rates, and the possibility for the central bank to respond to forecasts about future inflation, are then to be considered as realistic features of policy-making. The worse performing rules are instead those that only rely on inflation. These rules neglect the dynamics of the output gap in the monetary policy decision process. A comparative analysis between the optimal interest rate implied by a simple Taylor rule and the optimal interest rate generated by a pure inflation rule, suggests that a monetary policy rule accounting only for the
inflation dynamics would have implied higher interest rates over the whole sample period and especially during the 1980's. This result is consistent with the theory. However, monetary policy authorities cannot neglect the output gap dynamics in practice. Therefore, the bad performance of the rules, relying only on inflation, was to some degree expected.
CHAPTER 1. THE EFFICIENCY OF MONETARY RULES

Figure 1.2: Estimated Optimal Feedback Rules (dashed lines) vs. Actual Interest Rates (solid lines)
Figure 1.3: Estimated Taylor Rules (dashed lines) vs. Actual Interest Rates (solid lines)
Figure 1.4: Estimated Generalized Taylor Rules (dashed lines) vs. Actual Interest Rates (solid lines)
1.4. THE HISTORICAL BEHAVIOUR

Figure 1.5: Estimated Pure Inflation Rules (dashed lines) vs. Actual Interest Rates (solid lines)
Figure 1.6: Estimated Smoothing Inflation Rules (dashed lines) vs. Actual Interest Rates (solid lines)
Figure 1.7: Estimated Forward-Looking Rules (dashed lines) vs. Actual Interest Rates (solid lines)
1.5 The Efficiency of the Monetary Rules

This section analyzes the efficiency of the estimated rules. An efficiency analysis of alternative policy rules cannot solely rely on the differences between actual and estimated reaction functions. Following the definition of Taylor (1994), a policy rule has to be considered optimal if it minimizes a weighted sum, where the weights are set by the tastes of policymakers, output variance and inflation variance. In our case, given the specification of the loss function in equation (1.5), a term in interest-rate smoothing is also taken into account. In other words, the efficiency of a rule results from its ability to stabilize output, inflation and interest rate changes around their target values for an infinite number of periods.

The unconditional variances of inflation, output and interest rate are calculated using the method developed in Rudebush and Svensson (1998). More precisely, the $3 \times 3$ covariance matrix of the goal variables is given by:

$$
\sum_{YY} \equiv E \left[ Y_tY'_t \right] = C \sum_{XX} C'
$$

(1.19)

where the $(k + h + 3) \times (k + h + 3)$ matrix $\sum_{XX}$ represents the unconditional covariance matrix of the state variables and satisfies the following relationship:

$$
\sum_{XX} = E \left[ X_tX'_t \right] = M \sum_{XX} M' + \sum_{vv}
$$

(1.20)

In order to recover the covariance matrix of the state variables we can use:\

$$
vec \left( \sum_{XX} \right) = vec \left( M \sum_{XX} M' \right) + vec \left( \sum_{vv} \right) \\
= (M \otimes M) vec \left( \sum_{XX} \right) + vec \left( \sum_{vv} \right)
$$

Finally we can solve for $\left( \sum_{XX} \right)$:

$$
vec \left( \sum_{XX} \right) = \left[ I - (M \otimes M) \right]^{-1} vec \left( \sum_{vv} \right)
$$

(1.21)

Table 1.7 and Table 1.8 provide the results for the volatility of goal variables, measured as the unconditional variances, implied by the six estimated

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7The relationships used are: $vec(A + B) = vec(A) + vec(B)$ and $vec(ABC) = vec(C' \otimes A) + vec(B)$. 
1.5. THE EFFICIENCY OF THE MONETARY RULES

rules under the hypothesis that $\lambda = 0.4$, $\varphi = 0.4$ and $\gamma = 0.2$. With this assumption, the analysis is implicitly carried out under the hypothesis that, for the central bank, the volatility of output and inflation are equally undesirable ($\lambda = \varphi$) while the variability of nominal interest rate changes are much less costly ($\gamma = 0.2$).

These tables also report the loss implied by the rules and the relative ranking in terms of loss in the fourth and fifth columns of Tables 1.7 and 1.8, respectively. In all countries, the variability of optimal feedback rules outperforms, in terms of minimum losses, the other rules. It means that the volatility of the goal variables is minimized once the central bank adopts an optimal feedback rule. Moreover, the simple, forward-looking, Taylor-type rule is consistently, across the countries, the second, top-performing rule; the results in terms of the volatility of target variables and, therefore, in terms of losses are very close to those of the optimal feedback rule. We can conclude that the inclusion of a forward-looking dimension in a monetary authority decision process seems to improve the performance of the simple rule.

The Generalized Taylor rule outperforms, with the exception of Portugal, the classic Taylor rule. This is thought to be due mainly to the inclusion of an autoregressive term in the GTR. This result corroborates the evidence emerging from comparative analysis between actual and estimated rules; an interest-rate smoothing term then improves not only the ability of the rule to give a better representation of central bank behavior, but also the efficiency, measured in terms of volatility, of the rules.

Nevertheless, for many models, the volatility of interest rate changes is higher in the rule that reacts to the lagged interest rate.

However, the inclusion of an autoregressive term does not improve the performance of the rule in all cases. This is, in fact, not true for the pure inflation rule. The smoothing inflation rule performs badly relative to simple inflation-targeting in all countries with the exceptions of Germany, Spain and Portugal.

To augment the tables, Figure 1.8 and Figure 1.9 show the efficiency frontiers of each rule in all EMU countries. These curves illustrate the
trade-off between inflation variability and output gap variability. They are obtained by varying the weight on inflation from 0.01 to 0.8 and assuming $\gamma = 0.2$. The weight on inflation stabilization implicitly determines the weight on output stabilization since $\phi$ is set to be equal to $1 - \lambda$. Thus, as the weight on inflation stabilization is increased by 0.01, the weight on output stabilization is reduced by the same amount at each step. As $\lambda$ increases, the rules correspond to points further to the right on the curve.

The trade-off resulting from the optimal rule is shown as a solid line. The dashed lines show the remaining rules. The efficiency frontiers basically confirm the results of the tables. Moreover, they provide the following insights. First, the performances of the rules seem to be very close to one another even though the performances of the optimal and the forward-looking rules result in the best performance. This is particularly true for France and Portugal, as the Netherlands and Belgium show greater differences between the efficiency frontiers, with respect to the other EMU countries. Second, the performances of the rules become closer as the weight on inflation stabilization increases. This means that a higher weight on inflation reduces the differences in the efficiency frontiers of each rule.
### 1.5. THE EFFICIENCY OF THE MONETARY RULES

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Table 1.7: Results of Inflation and Output Volatility
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Table 1.8: Results of Inflation and Output Volatility
1.5. THE EFFICIENCY OF THE MONETARY RULES

Figure 1.8: The Efficiency Frontiers for the Six Rules in the EMU Countries (Inflation Variances on the horizontal axis - Output Gap Variances on the vertical axis)
Further conclusions arise from the comparison of the efficiency frontiers across the EMU countries. Figure 1.10 shows the efficiency frontiers for the
1.5. **THE EFFICIENCY OF THE MONETARY RULES**

OFR in the largest countries, i.e. Germany (DE), France (FR), Italy (IT) and Spain (ES). Figure 1.11 shows the efficiency frontiers for the remaining countries, i.e. Austria (AT), Belgium (BE), Netherlands (NL), Portugal (PT), Finland (FI) and Ireland (IR). In these graphs, we only report the optimal feedback rule for the following reasons. First, as the performances of the rules are quite similar, the results of the comparative analysis would remain unchanged. Thus, the results obtained for this rule can be easily extended to the other estimated rules. Second, the OFR has resulted to be the top-performing rule and thus the best candidate for this analysis. Figure 1.11 confirms the success of the Bundesbank strategy in stabilizing output and inflation during the sample period. The results also highlight the bad performance of the Italian and French models of central banking.

Figure 1.12 shows the good performance of Austria, Belgium and the Netherlands with respect to the other EMU countries. These countries, in fact, are relatively small neighbors of Germany and strongly influenced by its economic condition. Their monetary policy has been strongly affected by the Bundesbank decisions during the last few decades. A closer performance of the OFR in Austria, Belgium and Netherlands to that in Germany was, then, expected.

Altogether, these results support the choice of the Bundesbank framework as a role model in designing the ECB strategy.
Figure 1.10: The Efficiency Frontiers across EMU Countries

Figure 1.11: The Efficiency Frontiers across EMU Countries
1.6 Concluding Remarks

This chapter attempts to analyze the performance of different rules capable of modeling how the central banks of the EMU countries have made policy decisions affecting interest rates. In particular, the study focuses on six different rules relating the interest rate, which the central banks are assumed to control, to a set of variables thought to affect monetary authority behavior. The set of rules includes an optimal feedback rule, two different specifications of the Taylor rule, a forward-looking rule and two alternative inflation rules. Consistent with the inflation-targeting regime, all the rules are calculated from an intertemporal optimization problem of a loss function penalizing the volatility of output, inflation and policy rates under the constraints given by a small, backward-looking, closed-economy structural model. The estimated coefficients of the rules are used to detect the main differences they imply in terms of monetary policy strategy. Once the interest rate rules are estimated, we study their performance by using two different analyses.

The first one relies on the historical behavior of EMU member central banks. A comparison between the actual and optimal policy rules gives rise to some important observations. First, for most countries the estimated rules suggest that the actual policy followed by the central banks was looser than needed in the early and mid-1980s, while higher interest rates would have been advisable during the late 1980s and early 1990s. Second, the selected rules predict well central bank behavior during the European monetary system crisis for Italy, France, Germany, Austria and Belgium, but fail to do so for Ireland, Finland, Netherlands, Portugal and Spain. Another interesting observation that arises from the analysis, is that the actual policy rule has come closer to the optimal rule since 1993. This evidence suggests that monetary policy has indeed become more efficient during the 1990s. Another issue considered in the analysis, is the ability of the rules to replicate historical interest rate movements, i.e. central bank behavior. The results emerging from the study stress that simple rules perform quite well in following interest rate historical records. The ability to mimic inter-
est rate changes increases once an interest-rate smoothing term is included in the reaction function. This suggests that central bank behavior can be better explained by adding a lagged interest rate. Moreover, considering a forward-looking dimension that takes into account expectations of future inflation movements seems to give further improvement.

Finally, the study focuses on the efficiency of the estimated reaction functions. The analysis suggests that the rule obtained by solving an optimal control algorithm is the top-performing rule. Nevertheless, the performance of a simple, forward-looking rule with a smoothing term for the interest rate is almost as stabilizing as the optimal feedback rule. Thus, the gains obtained by a central bank in following a complicated rule are not so high. It follows that the European Central Bank should adopt a simple rule as a guideline for its monetary strategy. In fact, the easier communicability of the simple rule can also increase the transparency and thus the credibility of the central bank.

Further conclusions arise from the comparison of the efficiency frontiers across the EMU countries. The outcome confirms the success of the Bundesbank strategy in stabilizing output and inflation during the sample period. It also highlights the bad performance of the Italian and French models of central banking.

These results provide additional evidence on how the European monetary authorities should conduct and characterize their policy strategy. The analysis strengthens the choice of the Bundesbank framework as a role model in designing the ECB strategy.
Chapter 2

Monetary Shocks and Systematic Policy

This chapter analyzes monetary policy asymmetries within EMU participating countries. In particular, we use a structural dynamic modelling approach to investigate asymmetric monetary transmission in Europe. Asymmetries are investigated in two different ways. First, we estimate structural models reflecting the monetary constraints each country faced during the EMS period. We obtain well-behaved and comparable effects of monetary policy shocks. Second, we derive the efficiency frontiers for the selected EMU countries. In computing the optimal combinations of output gap and inflation volatility we use a weighted average of interest rate and exchange rate, i.e. the Monetary Condition Index (MCI), as a policy instrument. The impulse response analysis implemented with the MCI, shows relatively small differences among the EMU countries in the responses of the real economy to monetary policy shocks. Altogether the results suggest that, no matter which policy instrument is used, output gap and inflation respond to identical monetary shocks with a similar speed and movement, albeit with a different degree of effects.
2.1 Introduction

Since the beginning of 1999, the monetary policy regime in Europe has changed substantially. The centralization of the monetary policy of the EMU member countries has resulted in a single decision process that prevents national monetary authorities from pursuing systematic policies to offset country-specific shocks. In this scenario, understanding the transmission mechanism of the monetary policy is crucial to a successful implementation of the Eurosystem's monetary policy strategy.

The chapter analyzes monetary policy asymmetries in 10 EMU countries (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands and Spain) with a particular emphasis on the time lag involved between a policy change and its impact on inflation and output.

There exists a large quantity of empirical literature focusing on whether monetary policy asymmetries across EMU members will become less pronounced, over time, as a result of a single monetary orientation.

Most of these studies apply VAR methodology to evaluate the effects of monetary policy on output and prices, through the unsystematic component of monetary authority actions. Ramaswamy and Sloek (1998), for example, find that the effect on output in Germany, Austria, Belgium, Finland and the Netherlands takes longer to occur but is almost twice as great as that in France, Italy, Spain, Sweden, Portugal and Denmark. Gerlach and Smets (1995; 97) using long-run identifying restrictions find that the output response in Germany is larger than the one in France and Italy. Ehrmann (1998) detects a substantial heterogeneity in the magnitude of responses: small responses in small economies are opposed to large reactions in large countries. Finally, Kiler and Saaranheimo (1998) highlight that, given the strong dependence of the results on applied identification schemes, it is not
2.1. INTRODUCTION

possible to detect significant cross-country differences in the monetary transmission mechanisms in France, Germany and the United Kingdom. Other research explicitly models the ERM constraints the EMU members faced during the last two decades. Peersman and Mojon (2001) and Clements et al. (2001) propose imposing alternative identification restrictions on each EMU country in order to account for the EMS institutional requirements. In particular, the study of Clements et al. (2001) suggests that for France, Italy, Ireland and Finland, the average effect on output of a contractionary monetary policy shock is larger than for the other EMU economies.

Other studies, the research of BIS (1995) being an example, use large-scale macroeconometric models. The results of BIS (1995) reveal an almost identical response of output to a monetary shock in Germany, Italy and France. Also Hallett and Piscitelli (1999) using the IMF MULTIMOD follow this approach. They underline the lesser effect of monetary policy on output in Italy, with respect to the reaction of output in France and Germany.

Britton and Whitley (1997), instead, utilize a small-scale structural model. Their research detects non-significant differences of the output reactions in Germany and France. Moreover, the same study detects a larger response of the United Kingdom with respect to other EMU countries.

Finally, other authors use single equation models. Dornbush et al. (1998), for example, stress that the impact-effect of a change in monetary policy is similar in Germany, France and the United Kingdom and smaller in Sweden and Italy. However, they also suggest that the full effect of the co-ordinated monetary policy action is lower in the UK than in France and Germany.

The contribution of the study with respect to the existing literature on the asymmetric transmission mechanism in Euro Area countries is twofold.

First, we show that imposing restrictions on the estimated structural
models, reflecting the monetary constraints each country faced during the EMS period, obtains well-behaved and comparable effects of the monetary policy shocks in the selected economies. Moreover, following Peersman and Mojon (2001), our approach avoids the homogeneity of models that characterizes most of the literature on cross-country comparisons of the monetary transmission mechanism.

Second, we model policy behaviour with monetary rules that account for exchange rate changes. Such rules better describe the reaction functions each country used, in order to set interest rates during the Pre-EMU period.

An outline of the chapter is as follows. Section 2.2 presents an open-economy structural model, which is designed to summarize the main channels through which monetary policies affect the real economy. Section 2.3 studies the asymmetries in the output and inflation responses to monetary policy shocks across the EMU countries. Section 2.4 derives a set of efficient rules in a small open economy consistent with an inflation targeting regime. Section 2.5 analyzes the response of the real economy, when monetary authorities use the estimated MCI as a policy instrument. In Section 2.6, concluding remarks end the study.

2.2 A Structural Model of EMU Countries Economy

In the empirical analysis, we use a structural dynamic modelling approach to investigate asymmetric monetary transmission in Europe.

The model consists of an aggregate supply equation of the form:

$$\pi_t = \sum_{i=1}^{4} \gamma_i \pi_{t-i} + \rho y_{t-1} - \xi (q_t - q_{t-1}) + \eta_t$$  \hspace{1cm} (2.1)

This autoregressive Phillips curve relates the CPI inflation rate ($\pi$) to its
2.2. A STRUCTURAL MODEL OF EMU COUNTRIES ECONOMY

own lags, to a lagged output gap \( y \), measured as a percent gap between the actual real GDP and the potential GDP, and the change in the real effective exchange rate \( \Delta q_t \). The last term \( \Delta q_t \) reflects the pass-through of the exchange rate to the consumer price index. The specification of aggregate supply is consistent with an adaptive representation of inflation expectations. The expectations are treated implicitly by the inclusion of lagged values of the variables.

\[
y_t = \sum_{i=1}^{4} \alpha_i y_{t-i} - \beta q_{t-1} - \varphi (\tilde{q}_t - \overline{\pi}_t) + \varepsilon_t
\]  

(2.2)

According to equation (2.2), the output gap is related to the real interest rate and to its own lags. The real interest rate is calculated as the difference between a short-term interest rate and the inflation rate. In the above equation, \( \tilde{q}_t \) is the four-quarter short-term nominal interest rate, i.e. \( \frac{1}{4} \sum_{j=0}^{3} \tilde{i}_{t-j} \); \( \overline{\pi}_t \) is the four-quarter inflation rate, i.e. \( \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j} \). From equation (2.2) we can see that an increase in \( q_t \), representing an appreciation of the home currency, shifts the aggregate demand of the home country \( (\beta > 0) \).

The last equation of the model is a monetary policy reaction function. We explore the real effects of a monetary shock using alternative specifications of monetary authority behaviour. On each EMU member, we impose an explicit instrument rule, taking into account the predominant variables that are likely to have driven their monetary policy during the last two decades. Within this class of rules, the monetary policy instrument is expressed as a function of the available information. In the following, we consider three versions of open-economy Taylor rules.

\footnote{Since the Taylor (1993) seminal paper, a great amount of literature has been written which aims at explaining the stabilizing power of active interest rate rules. Recently, several authors including Taylor (1998) and Gerlach and Schnabel (1999) have underlined the usefulness of the Taylor rule as an informal benchmark for setting interest rates in the}
The first refers to the German economy. It assumes that the interest rate is a function of the lagged values of inflation, output gap and the real effective exchange rate. We also add a smoothing term for the interest rate.

\[ i_t = \delta_1 y_{t-1} + \delta_2 \pi_{t-1} + \delta_3 q_t + \delta_4 i_{t-1} + \omega_t \quad (2.3) \]

The second type of rule is estimated for Austria, Belgium and the Netherlands. As stressed in Peersman and Mojon (2001) these countries have strictly followed the monetary policy of the Bundesbank during the last decades. Thus, for those countries, the home interest rates are replaced with the German nominal rate \((i_t^{DE})\):

\[ i_t = \delta_1 y_{t-1} + \delta_2 \pi_{t-1} + \delta_3 q_t + \delta_4 i_{t-1}^{DE} + \omega_t \quad (2.4) \]

The third Taylor-type rule is designed for Finland, France, Ireland, Italy, Spain and Portugal. This rule takes into account the so called "German dominance" in the EMS in two different ways. First, the nominal interest rate of Germany is included in the reaction functions. Second, the real effective exchange rate is replaced with the real bilateral DM exchange rate \((q_t^{DE})\):

\[ i_t = \delta_1 y_{t-1} + \delta_2 \pi_{t-1} + \delta_3 q_t^{DE} + \delta_4 i_{t-1} + \delta_5 i_{t-1}^{DE} + \omega_t \quad (2.5) \]

Although we use structural models, the monetary policy rules we adopt are similar to the interest rate equations of the VAR considered in Peersman and Mojon (2001).

The model has been estimated by applying the OLS technique and using quarterly data for the period 1979:1-1998:4\(^2\). The length of the sample

\(^2\)The data used in the empirical analysis are taken from the IFS statistics.
period is chosen in order to have a single monetary policy regime involved in the estimations. Potential output has been computed by using the Hodrick-Prescott (HP) filter. As we have quarterly data, we set the smoothing parameter to 1600 as in Kydland and Prescott (1990). All variables were de-meaned prior to estimation.

The estimated equations are reported in table A2.1, A2.2 and A2.3 in Appendix. Most of the crucial coefficients have the expected sign and are significant at 5% (or 10%) level for the majority of the countries under examination. Among the explaining variables in the demand equation, special emphasis must be given to the coefficient on the real effective exchange rate. Only a few EMU countries show the right sign and have a significant coefficient. This might be due to the J-curve effect. A change in the domestic currency rarely shows its effect on the GDP after only one quarter. The Marshall-Learner condition only holds in the long run. Regarding the monetary policy functions, it should be stressed that several countries in the third group show not significant parameters. This is particularly true for the coefficients relating the domestic interest rate with the German variables. Overall, Spain and Portugal are the countries displaying the worse statistics. This is probably due to data limitation.

Having estimated the model, it is then possible to observe some properties of the monetary policy shock time series.

Figure 2.1 shows the monetary shocks. As the measure of monetary shock is by construction serially uncorrelated, the four-quarter moving average of the shock, i.e. \((\omega_t^i + \omega_{t+1}^i + \omega_{t+2}^i + \omega_{t+3}^i) / 4\), is reported.

In interpreting the results, we assume that a positive smoothed policy shock represents a tight monetary policy, while a negative shock corresponds to a loose monetary policy. Figure 2.1 shows that the selected measure of a policy shock, the monetary policy of almost all EMU countries, was con-
tractionary during the main recessionary episodes and became expansionary after depressions. This evidence is in line with the historical records of monetary policy actions. If we look, for instance, at the period 1992-93 we can see that all countries experienced a period of tight monetary policy.

Figure 2.1: Four Quarter Monetary Shock
2.3 Simulation Analysis

After estimating the model, it is now possible to apply the impulse response analysis. The objective of this analysis is to measure the time profile of the incremental effect of the innovation of the variables on the future state of the economy. In other words, once the monetary rule has been estimated, the analysis focuses on the response of the macroeconomic variables to a deviation from the rule.

The estimated responses to a 1% point increase in the nominal interest rate are reported in figures 2.2 and 2.3. Each response is provided with the associated asymptotic confidence bands. The patterns of the responses are similar in all the countries. In all the economies, a positive monetary policy shock decreases output gap. Moreover, after an initial delay, the response function shows a hump-shaped pattern that reaches the maximum decline after roughly a year to a year and a half. Figure 2.3 reports the inflation response to a monetary policy contractionary shock. Common to all member countries, inflation falls slightly after a monetary shock. The estimated responses corroborate the presence of a certain degree of rigidities in inflation dynamics.

Figure 2.4 outlines the average responses as well as the maximum impact of the monetary policy shocks on output gap and inflation. Despite some differences, figure 2.4 seems to suggest a similar quantitative response across countries. The largest responses are observed in Germany and Italy; in particular, the response of the German output gap reaches the minimum of minus thirty-six basis points after one year and a half, while the reaction

---

3Confidence bands are computed using Monte Carlo integration. See Sims and Zha(1999) for a discussion on these issues.
of the Italian output gap is quicker and smaller: minus thirty-three basis points after almost one year. This evidence, supported by some previous studies, is in line with the larger manufacturing sector in Germany and Italy that increases the sensitivity of the output to monetary policy shocks.

The asymmetric inflation responses to monetary shock across the countries are displayed in Figure 2.4. In almost all countries the inflation response is initially very low; this result is consistent with the presence of nominal rigidities. The maximum impact on inflation is observed in Finland, Italy and France. These countries also share the largest average inflation response. It means that in these countries, inflation rates appear to be more sensitive to the monetary policy shocks.

\footnote{Notice that the bar charts in Figure 2.4 denote negative impacts.}
2.3. SIMULATION ANALYSIS

Figure 2.2: Response of Output Gap to a 1% Positive Monetary Shock
Figure 2.3: Response of Inflation Rate to a 1% Positive Monetary Shock
Our analysis from individual country models provides evidence that the monetary policy transmission mechanisms of EMU participating economies are similar in many respects. Specifically, the speed and the nature of the adjustment occurring to the inflation and output gap in response to monetary policy shocks are reasonably similar. The size of those responses seems to be different across countries. However, when considering the asymptotic confidence bands those differences almost disappear.

Table 2.1 compares our results with previous studies. The studies differ in terms of the size of monetary shock they consider as well as for the model used in the impulse response analysis. Potentially, the differences across the selected economies may stem from different measurement of the monetary shock. In the present study, the monetary policy shock was set for all
countries equal to 1%. This may improve the cross-country comparability of the estimated responses. Other studies, for example the BIS (1995) research, analyze the responses of the real GDP and the GDP deflator to a 100 basis point increase in the policy rate for two years, followed by a return of the rate to the normal level. Finally, Peersman and Mojon (2001) assume a one standard deviation monetary policy shock.

<table>
<thead>
<tr>
<th>Study/Source</th>
<th>AT</th>
<th>BE</th>
<th>DE</th>
<th>ES</th>
<th>FI</th>
<th>FR</th>
<th>IR</th>
<th>IT</th>
<th>NL</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahavilla - Landolfi (iii)</td>
<td>-0.207</td>
<td>-0.214</td>
<td>-0.358</td>
<td>-0.204</td>
<td>-0.262</td>
<td>-0.206</td>
<td>-0.112</td>
<td>-0.332</td>
<td>-0.128</td>
<td>-0.019</td>
</tr>
<tr>
<td>Mojon and Peersman (2001) [iv]</td>
<td>-0.25</td>
<td>-0.32</td>
<td>-0.2</td>
<td>-0.14</td>
<td>-0.44</td>
<td>-0.2</td>
<td>-0.32</td>
<td>-0.12</td>
<td>-0.49</td>
<td>-0.08</td>
</tr>
<tr>
<td>Clemens et al. (2001) [iii]</td>
<td>-1</td>
<td>-1.4</td>
<td>-0.8</td>
<td>-1.3</td>
<td>-1.7</td>
<td>-2.2</td>
<td>-1.2</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.3</td>
</tr>
<tr>
<td>Peersman and Smets (2001) [iv]</td>
<td>-0.93</td>
<td>-0.87</td>
<td>-1.8</td>
<td>-1</td>
<td>-1.15</td>
<td>-1.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mihov (2001) [ii]</td>
<td>-0.55</td>
<td>-0.3</td>
<td>-0.35</td>
<td>-0.35</td>
<td>-0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ehrmann (2000) [i]</td>
<td>-0.05</td>
<td>-0.36</td>
<td>-0.9</td>
<td>-0.22</td>
<td>-0.6</td>
<td>-0.4</td>
<td>-0.42</td>
<td>-0.1</td>
<td>-0.4</td>
<td></td>
</tr>
<tr>
<td>Barran et al. (1997) [iv]</td>
<td>-0.35</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.36</td>
<td>-0.46</td>
<td>-0.3</td>
<td>-0.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramaswamy and Sloek (1997) [iv]</td>
<td>-0.7</td>
<td>-0.95</td>
<td>-0.75</td>
<td>-0.28</td>
<td>-0.85</td>
<td>-0.48</td>
<td>-0.5</td>
<td>-0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIS: National central banks (1995) [ii]</td>
<td>-0.18</td>
<td>-0.37</td>
<td>-0.25</td>
<td>-0.23</td>
<td>-0.36</td>
<td>-0.44</td>
<td>-0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gertach and Smets (1995) [iv]</td>
<td>-0.28</td>
<td>-0.19</td>
<td>-0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: [i]: effect of monetary policy on industrial production.
[ii]: effect of a 100 basis points, eight quarters sustained increase of the interest rate.
[iii]: effect of a 1 percentage point increase in the short-term rate.
[iv]: effect of a one-standard deviation increase in the short-term interest rate

Table 2.1. The maximum impact of Monetary Policy on Output: A Comparison with Previous Studies

As shown in table 2.1 the estimated effect on output gap are similar to the one obtained in Mojon and Persman (2001) and BIS (1995). We next explore the statistical significance of the estimated asymmetries by computing standard Wald tests. Under the null hypothesis, the average output response in the country $i$, i.e. $\mu_i^{IR}$, is equal to the average of the estimated impulse response function ($IR$) for country $j$.

$$H_0 : \mu_i^{IR} = \mu_j^{IR}$$
2.4. OPEN ECONOMY MONETARY RULE FOR EMU COUNTRIES

\[ \frac{\left[ \mu_i^{IR} - \mu_j^{IR} \right]^2}{\text{var} (IR_i) + \text{var} (IR_j) - 2\text{cov} (IR_i, IR_j)} \sim \chi^2 (1) \]

Table 2.2 investigates the degree of asymmetry in the interest rate channel across the three biggest EMU member countries, i.e. Italy, Germany and France. The table presents the test results regarding the significance of the existence of asymmetric transmission. Those results suggest that the differences in the policy impact across EMU members are not significant. Only in Ireland is the difference in the estimated output responses statistically significant.

<table>
<thead>
<tr>
<th>Country</th>
<th>Germany</th>
<th>Italy</th>
<th>France</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1.31</td>
<td>0.51</td>
<td>-1.25</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.13</td>
<td>1.33</td>
<td>-0.47</td>
</tr>
<tr>
<td>Finland</td>
<td>0.48</td>
<td>-0.36</td>
<td>-2.02</td>
</tr>
<tr>
<td>France</td>
<td>2.63</td>
<td>1.90</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>-</td>
<td>-0.82</td>
<td>-2.63</td>
</tr>
<tr>
<td>Ireland</td>
<td>24.10*</td>
<td>34.07*</td>
<td>10.72*</td>
</tr>
<tr>
<td>Italy</td>
<td>0.82</td>
<td>-</td>
<td>-1.90</td>
</tr>
<tr>
<td>Netherlands</td>
<td>3.18</td>
<td>3.88</td>
<td>0.88</td>
</tr>
<tr>
<td>Portugal</td>
<td>2.53</td>
<td>1.90</td>
<td>-0.15</td>
</tr>
<tr>
<td>Spain</td>
<td>1.07</td>
<td>2.68</td>
<td>3.05</td>
</tr>
</tbody>
</table>

Table 2.2: Testing for asymmetric interest rate effects on output gaps

2.4 Open Economy Monetary Rule for EMU Countries

In the previous section the monetary policy behavior has been modelled by imposing explicit instrument rules to the structural model. This section
derives the set of efficient rules in a small open economy consistent with the inflation targeting regime. We then use the estimated rules to compute the responses of output and inflation to a monetary policy shock under the estimated reaction functions. In order to compute implicit monetary rules for an open economy, we augment the model expressed in equation (2.2) and equation (2.1) with an equation linking the real exchange rate and the real interest rate.

\[ q_t = \theta r_t + v_t \]  

Equation (2.6) is similar to the one used in Ball (1998). It captures in spirit the Uncovered Interest Parity (UIP). In particular, this relationship expresses the idea that a rise in the interest rate makes domestic assets more attractive, leading to an appreciation of the exchange rate. The shock \( v_t \) captures other influences on the exchange rate, such as expectations, investor confidence and foreign interest rate. The model is similar to the more complicated macroeconometric models of many central banks.

By including equation (2.6) the timing of the model goes as follows. The interest rate takes two periods to work. A monetary tight raises the interest rate and the exchange rate contemporaneously, but it takes a period for these variables to affect output and another period for output to affect inflation. In contrast, an exchange rate change takes only one period to affect inflation. These assumptions express the common view that the exchange rate effect is the quickest channel through which monetary policy influences inflation.

Unlike Chapter I and the first part of the current chapter, the policy instrument used here is a weighted sum of the interest rate and the exchange rate. The resulting series is the so-called Monetary Condition Index (MCI). The ratio is as follows.
Many economists advocate the use of a Taylor rule as guideline for the monetary policy authorities. According to this rule, a central bank rises interest rates when inflation rises above its target, or when output rises above its long run level, and reduces interest rates in the opposite cases. The idea is "leaning against the wind". Under a closed economy assumption, this rule fits reasonably well with the macroeconomic scenario. However, if we assume a certain degree of openness, the monetary policy rule must be modified to account for the role of the exchange rate in the economy. Under this assumption, since the interest rate affects the exchange rate, central banks, choosing the interest rate, choose indirectly the exchange rate. Monetary policy affects spending through two channels: the interest rate and the exchange rate. The overall stimulus to spending depends on the average effect of these two variables with weights given by the IS coefficients. Hence, the intuition for putting the MCI in the rule is straightforward. The inclusion of this index provides a better assessment of the overall impact on the real economy as well as of the underlying monetary conditions. This is particularly true for small open economies, where the exchange rate has greater importance for economic developments.

Recently, there has been a long debate about whether the MCI should appear in the rule. It has been argued that the inclusion of the MCI in the policy rule is not absolutely necessary. The openness of the economy could be taken into account by adding the exchange rate on the right hand side of the Taylor rule. In this way, policymakers would adjust the interest rate in response to output and inflation dynamics as well as the exchange rate fluctuations. A strand of literature believes that, to some degree, the issue "Modified Taylor rule or MCI?" is merely semantic. Using trivial algebra can be shown, in fact, that the modification of the Taylor rule accounting for the exchange rate and the policy rule having the MCI as policy instrument
are equivalent. The only difference is that the latter looks like more radical. However there is a different strand of literature arguing that the difference is not merely semantic in reality. In practice, policymakers adjust their instrument slowly over time. Policy setting is held constant between policy reviews. Within a quarter an interest rate rule means that the interest rate is constant, and an MCI rule means that the interest rate is adjusted to offset exchange rate movements. Therefore, the rules produce different outcomes when there are shocks to the exchange rate and consequently, the choice of policy instrument does matter in practice (Ball, 2000).

In computing the efficiency frontier and impulse response analysis, the parameters of the model are calibrated using a set of base values. Several of these values are borrowed from the small open economy model described in Ball (1998). We assume that the total output loss from a one point rise in the interest rate is 1. In the current model, this means that \( \varphi + \beta \theta = 0.175 \). If we set \( \varphi / \beta \) equal to 3 (capturing a common rule-of-thumb about IS coefficients) and \( \theta \) equal to 0.5 (a one-point rise in the interest rate causes a two percent appreciation on a yearly basis), we implicitly assume that \( \varphi \) and \( \beta \) are, respectively, equal to 0.15 and 0.05. Finally, we also assume that the slope of the Philips curve, i.e. \( \rho \), is equal to 0.2 and the coefficient \( \xi \) is equal to 0.05 (a one percent appreciation reduces inflation by two tenths of a point on a yearly basis). Unlike the calibrated model used by Ball, we leave \( \alpha_i \) as free parameters, and since the analysis is based on quarterly data, we make the coefficients \( \gamma_i \) total 1 (i.e. \( \sum_{i=1}^{4} \gamma_i = 1 \)). This means that we assume the existence of a natural unemployment rate (NAIRU) when inflation is stable.

The approach followed in this study differs from the traditional approach in two aspects.

First, we use the Monetary Conditions Index (MCI) as a policy instru-
2.4. OPEN ECONOMY MONETARY RULE FOR EMU COUNTRIES

As mentioned above, the rationale for using an MCI\(^0\) is that it measures the overall policy stance, by considering both interest rate and exchange rate dynamics. Second, we assume as a measure of inflation the following term (\(\pi^*\)): 
\[
\sum_{i=1}^{4} \gamma_i \pi_{t-i+1} + \xi q_{t-1}
\]
This term can be interpreted as a long-run forecast of inflation under the hypothesis that output is kept at its natural level. There are two good reasons for targeting long run inflation \(\pi^*\) instead of ordinary inflation. First, policy makers tighten only when there is inflation increases that would otherwise be permanent. This approach allows some short run volatility in inflation, but it keeps inflation stable over the longer run and reduces output variability. Second, it allows to distinguish temporary from permanent shifts in the exchange rate. Of course, it is not easy in practice to tell whether a change in the exchange rate is permanent or temporary. A disadvantage in targeting \(\pi^*\) is that requires the estimation of the long run equilibrium exchange rate in the Phillips curve (\(\gamma\)). In contrast, measuring ordinary inflation does not require knowledge of equilibrium exchange rates or Philips curve parameters.

Following Ball (1998), we define the efficient rule as the rule that is optimal for some weights, or equivalently, a rule that puts the economy on the output variance/ inflation variance frontier.

Every rule is usually interpreted as a rule for setting the interest rate. Using equation (2.6), it is possible to turn a rule for setting the interest rate into a rule for setting the exchange rate. In particular, by substituting equation (2.6) into equation (2.2) we eliminate the interest rate from the model. Then, in order to show the effects of the current exchange rate on future output and inflation, we shift the lags of equation 2.1 and equation 2.2 one period ahead. This yields:

---

\(^5\)The MCI is a weighted sum of the interest rate and exchange rate.
A policy maker choosing the interest rate, and implicitly the current exchange rate, looks at the expressions corresponding to terms on the right sides of equation (2.7) and equation (2.8). The state variables of the model are obtained by using such expressions: \( \sum_{i=0}^{3} \alpha_i y_{t-i} - (\beta + \frac{\varphi}{\theta}) q_t - \frac{\varphi}{\theta} v_t + \varepsilon_t \) (2.7)

\[ \pi_{t+1} = \sum_{i=0}^{3} \gamma_i \pi_{t-i} + \rho y_t - \xi \Delta q_t + \eta_t \] (2.8)

The future paths of output and inflation are determined by these two expressions, the rule for choosing the exchange rate and future shocks. Since the model is linear quadratic, one can show the optimal rule is linear in the two state variables:

\[ q_t = m \left[ \sum_{i=1}^{4} \alpha_i y_{t-i} + (\varphi/\theta) v_t \right] + n \left[ \sum_{i=1}^{4} \gamma_i \pi_{t-i} + \rho y_t + \xi q_{t-1} \right] \] (2.9)

By equation (2.6) \( v_t \) can be replaced by \( q_t - \theta r_t \). Making this substitution and rearranging terms yields\(^6\):

\[ wr + (1 - w) q = a \sum_{i=1}^{4} y_{t-i} + b \left[ \sum_{i=1}^{4} \gamma_i \pi_{t-i} + \xi q_{t-1} \right] \] (2.10)

where \( w \) is the weight and it is equal to:

\[ w = m\varphi\theta/(\theta - m\varphi + m\varphi\theta) \] (2.11)

\(^6\)Notice that \( \sum_{i=1}^{4} \gamma_i = 1 \)
2.4. OPEN ECONOMY MONETARY RULE FOR EMU COUNTRIES

and $a$ and $b$ are the coefficients of the rule and are equal respectively to:

$$a = \theta \left( m \sum_{i=1}^{4} \alpha_i + n \rho \right) / (\theta - m \varphi + m \varphi \theta)$$

$$b = n \theta / (\theta - m \varphi + m \varphi \theta)$$

The coefficients of the rule depend upon the constants $m$ and $n$. Equation (2.10) states the optimal policy as a rule for a weighted average of interest rate and exchange rate. By substituting equation (2.2) and equation (2.1) into equation (2.9) yields an expression for the exchange rate:

$$q_t = \left[ m \alpha_1^2 + n \rho (1 + \alpha_1) \right] + \sum_{i=2}^{4} \left[ m \alpha_i (1 + \alpha_1) + n \rho \alpha_i \right] y_{t-i} + n \gamma_1^2 \pi_{t-1} + \sum_{i=2}^{4} \left[ m \gamma_i (1 + \gamma_1) \pi_{t-i} \right] +$$

$$- \left\{ (m \alpha_1 + n \rho) \left( \beta + \frac{\varphi \rho}{\theta} \right) \right\} q_{t-1} + \left[ m \alpha_1 + n \rho \right] \varepsilon_t + m \frac{\varphi \rho}{\theta} v_t + \left[ (m + n \rho) \frac{\varphi \rho}{\theta} \right] v_{t-1}$$

This equation, together with equation (2.2) and equation (2.1), represent the single equations of a VARMA$(4,1)$ process. Thus follows:

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + A_3 X_{t-3} + A_4 X_{t-4} + B_1 \varepsilon_t + B_2 \varepsilon_{t-1} \quad (2.12)$$

where $A_i$ are the $(3 \times 3)$ matrices of the restricted coefficients and $B_i$ are the structural matrices of the errors; $X_t$ is the vector of the endogenous variable $[y_t, \pi_t, q_t]^\prime$ and $\varepsilon_t$ is the vector of the shocks $[\varepsilon_t, \eta_t, v_t]^\prime$. The appendix shows the model in a more explicit form.

For a given parameter values and given values of the constants $m$ and $n$, one can numerically derive the variance of the vector $X$ using standard formulas. Thus follows:
where $A$ is the companion matrix and $\Sigma_\varepsilon$ is the variance covariance matrix.

We derived the optimal efficiency rule by searching the combinations of $m$ and $n$ that put the economy on the output variance - inflation variance frontier for a given value of $\lambda$. The coefficient $\lambda$ is the non negative weight that we impose upon the variance of the inflation in the loss function. The coefficient $\lambda$ is a non-negative weight that the central bank attaches to stabilize inflation. The value of $\lambda$ implicitly determines the weight on the output stabilization since it is equal to $1 - \lambda$. Thus, as the weight on the inflation stabilization increases, the weight on the output stabilization is reduced by the same amount at each step. By applying this procedure, we derive the efficient frontiers for the EMU countries.

\[ (I_{9K^2} - A \otimes A)^{-1} \text{vec}(\Sigma_\varepsilon) \]
Figure 2.5: Efficiency frontiers for the largest EMU countries
Figures 2.5 and 2.6 show the efficiency frontiers for the selected EMU members. Two results are noteworthy. From figure 2.5, it emerges that Germany is the second top-performing country. Surprisingly, the first country is France. As the good performance of Germany is consistent with historical
records of the country, and thus justifies the choice of the German model as a benchmark model in the ECB monetary strategy, the good performance of France is inconsistent with results obtained in literature so far. This result could be due to the removal of the closed economy hypothesis or/and the calibration of the model.

On the other hand, the bad performance of Italy and Spain with respect to France and Germany are consistent with the results of most of the literature.

Another reasonable result concerns the performance of Austria, Belgium and the Netherlands. Figure 2.6 suggests that the performance of these countries is quite similar. This result is highlighted in figure 2.7 where the efficiency frontiers of these countries are compared to that of Germany.
Figure 2.7: Efficiency frontiers for Germany and Neighbours

This graph confirms the common belief that Austria, Belgium and the Netherlands are relatively small neighbors of Germany and strongly influenced by its economic condition. During the sample period, they followed monetary policy strategy similar to that conducted by the Bundesbank.
2.5 Impulse Response Analysis under the MCI

In this section we present the estimated dynamic effects of monetary policy shocks on output gap and inflation. In particular, we examine the similarity of monetary policy transmission mechanisms in each economy, when they operate according to the estimated reaction function and face flexible exchange rates. To this end, we analyze the effect of one percent shock to the MCI on the real economy in each country. This is accomplished by using impulse response functions with a structural decomposition of the variance covariance matrix explained below. A 25-quarter horizon is considered.

Impulse responses under the estimated monetary rules are depicted in figures 2.8 and 2.9, with dashed lines denoting the asymptotic bands around the estimated responses. The impulse responses for Germany, Italy and Finland are again slightly larger than those for the other countries, though it should be remembered that there are slight differences in the model specifications and thus comparisons should not necessarily be accepted at face value. With the caveats mentioned above, we observe some similarities in the response of each country to a monetary policy shock.

The effect of the monetary policy shock on the inflation rate persists in all economies for approximately 6 quarters. The inflation rate follows a similar path in all countries. Consistent with our priors, the CPI inflation in all countries does not respond immediately to the monetary policy shock. Moreover, we do not observe a price puzzle. We note that the speed of the adjustment of variables in the EMU members is similar. Some differences emerge in the sizes of adjustments. In Austria, France and Italy, the observed decrease of inflation is greater than in the other countries.
Figure 2.8: Response of Output Gap to a 1% Positive Monetary Shock
2.5. IMPULSE RESPONSE ANALYSIS UNDER THE MCI

Figure 2.9: Response of Inflation Rate to a 1% Positive Monetary Shock
The average response and the maximum impact of a contractionary monetary policy shock are shown in figure 2.10. The evidence emerging from these figures suggests that the timing of the inflation response is similar across the countries; the monetary shock exerts the maximum effect after more than two years in almost all countries. The largest responses are observed in Germany, Austria and Finland.

2.6 Concluding Remarks

Up to now, the large quantity of literature aimed at analyzing the possible asymmetries in output and price responses to the single monetary policy across the EMU countries, has not yet provided a consistent and unambigu-

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8Similarly to figure 2.4, the bar charts in Figure 2.10 denote a negative impact.
2.6. CONCLUDING REMARKS

ous picture of cross-country heterogeneity in monetary transmission. The aim of the chapter is to shed some light on how monetary transmission may heterogeneously work in different countries.

In this respect, the chapter has attempted to detect the different effects of monetary policy shock on the output gap and inflation rate, across EMU countries. The importance of this issue lies in the consideration that differences in transmission mechanisms can generate asymmetric behaviour among currency union partners, even when they experience the same monetary policy shock.

The analysis has been divided into two parts. First, we have shown that imposing restrictions on the estimated structural models according to the monetary constraints each country faced during the EMS period, obtained well-behaved and comparable effects of the monetary policy shocks in all the countries. Second, we have modelled policy behaviour with monetary rules which accounts for exchange rate changes. Such rules better describe the reaction functions each country used in order to set interest rates during the Pre-EMU period.

The impulse response analysis highlights the presence of nominal and real divergences across EMU members. These asymmetries, concerning the interest rate channel, will probably not be large enough to cause frictions for the EMU. Importantly, the results also suggest that the output gap and inflation in the selected economies respond to identical monetary policy shocks with similar speed and movement, albeit with different dimensions of the effects. From the empirical analysis applied in this chapter, we see that the effect of a monetary shock on output mostly depends on the output structure of the country.

Other channels, not considered here, could be a cause of concern. The study did not take into account the asymmetries that might arise from the
credit channel or from the stock market channel. Nevertheless, it seems plausible to imagine that the cross-country differences in monetary transmission across EMU countries could decrease over time, as a result of increasing financial structure homogeneity. This means that, in the long run, asymmetries in monetary transmission will not be a cause of concern. However, some differences are likely to persist; those divergences call for a better understanding and monitoring of the national monetary transmission mechanisms.
Part II

Monetary Policy issues in the Euro Area
Blank
In
Original
Chapter 3

Do Central Banks act asymmetrically?

This Chapter attempts to exploit whether the monetary authorities have a different behavior during recession and expansion. To this end, a multivariate extension of Hamilton's (1989) Markov Switching Model is adopted. First, regime dependent Taylor-type rules are estimated for the Euro Area and the United Kingdom in order to capture the systematic behavior of central banks. Then, impulse response functions that account for the different phases of the business cycle are analyzed. In addition, a comparative analysis concerning the estimated rules as well as the different reaction of real economy to monetary shocks is implemented. The study strongly suggests that central banks cannot neglect the regime where the monetary action takes place. The phase of the business cycle is an important matter in monetary policy decision process.

3.1 Introduction

The aim of this study is to shed some light on three important issues. First, it investigates whether the monetary policy authorities have an asymmetric
behaviour in the different phases of the business cycle. Second, it analyses whether a monetary policy action has a different effect on the real economy depending on the phase of the business cycle. Third, it observes whether there exist some asymmetries in the monetary policy strategies followed by the European Central Bank (ECB) and the Bank of England.

The empirical literature concerning monetary policy rules, as well as the strand of literature on the monetary transmission mechanisms, is quite extended. Most of these studies model the central bank behaviour with a linear reaction function that is by construction, symmetric over the different phases of the business cycle. Moreover, they neglect the asymmetric impact that a monetary policy action might have on the real economy according to the particular phase of the business cycle, where the action takes place.

The current study differs from the existing literature in three aspects. First, it accounts for non-linearity in the econometric model. Second, it derives regime dependent monetary policy rules that account for the different phases of the business cycle. Finally, it provides a quantitative analysis of the asymmetric impact of a monetary policy action on the real economy.

The analysis focuses on the Euro Area and the United Kingdom economies. The econometric technique used in the empirical analysis is a multivariate extension of Hamilton's (1989) Markov-switching model. In particular, we use a Markov Switching Vector Autoregressive model (henceforth MS-VAR).

The chapter is organized as follows. Section 3.2 investigates the presence of non-linear serial dependence in a trivariate linear VAR. Section 3.3
discusses the econometric methodology used in order to analyze the way through which monetary policy affects the real economy depending on the state of the economy. Section 3.4 provides the estimation and the comparison of state-dependent reaction functions for the ECB and the Bank of England. Section 3.5 presents the main results of the impulse response analysis. In particular, it shows the timing and the dimension of the likely effects the ECB and the Bank of England have on output gap and inflation during recessions and booms. In Section 3.6, concluding remarks end the chapter.

3.2 Linearity vs. Non-linearity hypothesis

Most of the literature on central banking applies VAR methodology to evaluate the effects of the monetary actions on output and prices. These studies, neglecting the effects of business cycle fluctuations on the monetary authority behaviour, rely on single state linear models.

However, the existence of asymmetric effects of unanticipated monetary policy changes and thus the use of non-linear methodologies can be motivated by a variety of theoretical models.

According to the Keynesian framework, for example, prices and wages are assumed to be more rigid downward than upward. The different sign of the shock might produce asymmetries concerning the effects of tight and loose monetary policies. In particular, negative shocks lead to a lower output and employment while positive shocks are thought to be neutral.
In the credit channel theory many studies, such as Bernanke and Blinder (1992), Gertler and Gilchrist (1993), Schmidt (1999), and Bernanke and Gertler (1995), assume that monetary policy has a greater impact on the real economy in low growth states. Finally, the size of the monetary authorities' actions is considered a further source of asymmetries in the menu cost frameworks. Those models, like Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987), predict that only small policy actions would have real effects, since a large shock is reflected in a change of the menu prices.

Nevertheless, the strand of empirical literature accounting for non-linearity is not so extended. Some studies utilize the Multiple Regime Smooth Transition Autoregressive model (MRSTAR) methodology to evaluate the non-linear effects of monetary policy on output and prices. These models, introduced by van Dijk and Franses (1999), generalize the STAR\(^1\) models that were extensively used in the literature. Dufrenot et al. (2002), for example, use this methodology to analyse the effect of monetary policy on the US real economy, assuming that output fluctuations are governed by regime-shift models.

In the present study, instead, we account for non-linearity in the economic structure by adopting a multivariate extension of Hamilton’s (1989) Markov-switching model. In particular, we analyse the asymmetric behaviour of the monetary authorities by estimating a regime-dependent reaction

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\(^1\)STAR (Smooth Transition Autoregressive) models were originally introduced by Teräsvirta and Anderson (1992). Their statistical properties are studied in Luukkonen et al. (1988), Luukkonen and Teräsvirta (1991), Granger and Teräsvirta (1993), Eitrheim and Teräsvirta (1996).
3.2. **LINEARITY VS. NON-LINEARITY HYPOTHESIS**

In order to test for non-linearity in the monetary policy behaviour we proceed as follows. First, we estimate a linear VAR with the OLS technique. Then, we check the non-linearity of the residuals by employing a battery of standard tests.

The sample period, for both the Euro Area and the United Kingdom, goes from 1980:1 to 2001:4. The data used in the empirical analysis for the UK are seasonally adjusted quarterly observations and were drawn from Datastream, which, in turn, takes the data from the OECD Main Economic Indicator Database. The aggregate variables for the Euro Area, instead, come from the dataset used by Fagan et al. (2001) to construct the Area-Wide Model for the Euro Area. As the last data set ends in 1998:4 we extend the time series by adding the values reported in the ECB Monthly Bulletin.

The estimated model is a trivariate VAR of the form:

$$X_t = A(L)X_{t-1} + \varepsilon_t$$ (3.1)

where the vector $X_t$ contains the following variables:

$$X_t' = [\tilde{y}_t \, \pi_t \, i_t]$$

In model (3.1), $A(L)$ is a polynomial lag of order four; $\tilde{y}_t$ is the output gap, measured as a percentage gap between actual real industrial production and potential industrial production\(^2\); $\pi_t$, the inflation rate, is the percentage

\(^2\)Potential output was computed by using the Hodrick-Prescott (HP) filter. As we
of the annual \( HICP \) inflation rate, i.e. \( 100 \left( \log HICP_t - \log HICP_{t-4} \right) \) and \( i_t \) is the short term interest rate.

The model is identified by assuming a recursive structure. The variables are ordered as output gap, inflation rate and nominal interest rate. As the monetary instrument is the last variable, it implies that monetary authorities can react to changes in output gap and inflation, but it is not able to affect the other variables within the same quarter. This structure is consistent with the general assumption that real markets do not instantaneously react to monetary policy actions. The timing of the model can be summarized as follows. A shock to monetary policy instruments in period \( t \) affects output with one period lag. In period \( t + 1 \) the aggregate supply change resulting from the slightly decreasing in inflation rate leads to a fall in output gap through the real interest rate channel\(^3\).

We check the non-linearity of the residuals by using three of the most popular tests. Specifically, we apply the BDS, the Engle and Tsay test to the residuals of each equation in the VAR system, i.e. the output gap, the inflation rate and the interest rate. The null hypothesis for these tests is that the residual generating process is linear. Table 3.1 to 3.3 show the results.

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\(^3\)We keep this particular causal order also in the MS-VAR analysis.
### Table 3.1: BDS Test statistics

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
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<tbody>
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<td>2</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.069</td>
<td>0.103</td>
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<tr>
<td>3</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.007</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
<td>0.005</td>
<td>0.000</td>
<td>0.001</td>
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</table>

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
<th>Asymptotic</th>
<th>Bootstrap</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.006</td>
<td>0.021</td>
<td>0.000</td>
<td>0.004</td>
<td>0.741</td>
<td>0.876</td>
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<tr>
<td>3</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.005</td>
<td>0.043</td>
<td>0.061</td>
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<tr>
<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.007</td>
<td>0.004</td>
<td>0.014</td>
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### Table 3.2: Engle Test

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<th>Using up to lag</th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.000</td>
<td>0.071</td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>0.000</td>
<td>0.027</td>
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<tr>
<td>3</td>
<td>0.002</td>
<td>0.000</td>
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</tr>
<tr>
<td>4</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Using up to lag</th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.408</td>
<td>0.221</td>
<td>0.588</td>
</tr>
<tr>
<td>2</td>
<td>0.055</td>
<td>0.045</td>
<td>0.068</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>0.067</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>0.634</td>
<td>0.001</td>
<td>0.381</td>
</tr>
</tbody>
</table>

### Table 3.3: Tsay Test

<table>
<thead>
<tr>
<th>Output Gap</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro Area</td>
<td>0.015</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.034</td>
</tr>
</tbody>
</table>
The tables report, for each equation from the VAR, the p-values under the null hypothesis that the corresponding residual is a serially i.i.d. process. Table 3.1 also reports the bootstrapped p-values for the BDS test statistic. All tests reject the null hypothesis of a linear generating mechanism for the residuals in the Euro Area. For the United Kingdom the BDS and the Tsay test fully reject the linear hypothesis while the Engle test rejects the null only for some lags.

The analysis altogether suggests the presence of non-linearity in the residuals. This evidence corroborates the decision of estimating the model in non-linear form.

3.3 Estimation Issues

The asymmetric behaviour of the monetary authorities is investigated by estimating a multivariate extension of Hamilton’s (1989) Markov-switching model. In the MS-VAR framework, the shocks to the interest rate policy rule followed by the monetary authorities are allowed to influence both the growth rate of output and the transition probabilities of moving from one phase to another.

The asymmetry of the effects is captured by allowing for state-dependent parameters where the latent state variable follows a Markov switching process. The idea behind this class of models, is that the parameters of the underlying data generating process of the observed time series vector $X_t$ depend
upon the unobservable regime variable \( s_t \), which represents the probability of being in a different state of the world.

This variable \( s_t \) is governed by a discrete state of a Markov stochastic process, which is defined by the following transition probabilities:

\[
p_{ij} = \Pr(s_{t+1} = j \mid s_t = i)
\]

\[
\hat{P} = \begin{bmatrix}
    p_{11} & p_{12} & \cdots & p_{1m} \\
    p_{21} & p_{22} & \cdots & p_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{m1} & p_{m2} & \cdots & p_{mm}
\end{bmatrix}
\]

where \( p_{ij} \) is the probability that state \( i \) is followed by state \( j \) and \( \hat{P} \) is the correspondent transition matrix.\(^4\)

The two states MS(m)-VAR(p) model can be represented as follows:

\[
X_t = \begin{cases} 
  v_1 + A_{11}X_{t-1} + \cdots + A_{p1}X_{t-p} + B_1u_1 & \text{if } s_t = 1 \\
  v_2 + A_{12}X_{t-1} + \cdots + A_{p2}X_{t-p} + B_2u_2 & \text{if } s_t = 2
\end{cases}
\]  

(3.2)

where \( X_t = [\pi_t \, \tau_t \, \iota_t]' \) is the vector of the endogenous variables; \( v_1 \) and \( v_2 \) are the vectors of intercepts respectively in state 1 and 2; \( A_{11} \cdots A_{p1} \) are the matrices of the coefficients in the state 1; \( A_{12} \cdots A_{p2} \) are the matrices of the coefficients in the state 2; \( B_1 \cdots B_2 \) are respectively the structural matrix\(^5\) in state 1 and in state 2; \( u_1 \) and \( u_2 \) are the errors respectively in the state 1 and state 2 distributed as \( N(0, \Omega_s) \).\(^6\)

When the process is in regime 1 (expansion), the observed vector of variables \( X_t \) is presumed to have been drawn from a \( N(\mu_1, \Omega_1) \) distribution.

---

\(^4\)Note that \( p_{11} + p_{12} + \cdots + p_{1m} = 1 \)

\(^5\)These matrices describe the relationship between endogenous variables within each regime.

\(^6\)\( \Omega_s \) is the Variance-Covariance matrix in the state \( s \) and \( m \) is the number of the states.
If the process is in regime 2 (recession), then it is drawn from a \(N(\mu_2, \Omega_2)\).

This means that, given the order of the endogenous variables, the last equation in the system corresponds to the backward-looking version of the regime-dependent Taylor rule.

In the empirical applications not all the parameters are conditioned on the state of the Markov chain. Usually, just some of them are regime dependent. Depending on which parameters are allowed to switch a great variety of specifications arise. These specifications are named in different ways. In this work a Markov switching intercept heteroskedasticity VAR-model (MSIH(m)-VAR(p)) is used. This means that, while the vector of the intercepts and the variance-covariance matrix are allowed to switch within regimes, the autoregressive coefficients are state invariant. In addition, the homoskedasticity assumption of the residuals is relaxed.

The optimal lag length is checked by computing three classical tests, i.e. AIC, SC, HC. The lag length is chosen, for both countries, to be one in order to ensure the non serially correlation of the residuals (see figure in appendix 3.1). This choice is supported by one (the SC criteria) of the three tests we implemented to assess the optimal lag length. Table 3.4 shows the results of the tests.
3.3. ESTIMATION ISSUES

We re-estimated the model with two lags in order to check the robustness of the results. The qualitative results of the analysis remain unchanged whether we use one or two lags.

The population of parameters in a MSIH-VAR is given by $\theta \equiv (\mu_1, \mu_2, \Omega_1, \Omega_2, \pi_1, \pi_2)$. In a n-dimensional MS(m)-VAR(p) model where all parameters are regime dependent the number of switching parameters is equal to $\Phi^7$ and the number of parameters $\theta$ to be estimated is then equal to $\Phi \times m$. In this case, the total number of parameters to be estimated is $27^8$ as $X_t$ is a three dimensional vector and the autoregressive coefficients are time-invariant.

Maximum likelihood estimation of $\theta$ is based on the implementation of

---

\[\Phi \text{ is equal to } n(\text{number of intercepts}) + (n \times n)p (\text{number of autoregressive coefficients}) + (n \times (n + 1)/2) \text{ number of elements in the variance covariance matrix}\]

\[\Phi \times m \text{ is equal to } [3(\text{number of intercepts}) + 6(\text{number of elements of the variance-covariance matrix})] \times 2(\text{the number of states}) + 9(\text{invariant autoregressive coefficients})\]
the Expectation Maximization problem proposed by Hamilton (1990). This algorithm, introduced by Dempster, Laird and Rubin (1977), is designed for a general class of models where the observed time series depend on some unobservable stochastic variables. For a MS-VAR model these are the regime variables $s_t$.

Once we have estimated the model we compute the transition probabilities of moving from one state to another for the UK and the Euro Area. As the estimated process follows a 2-state Markov chain, it is possible to collect the transition probabilities in a $(2 \times 2)$ transition matrix:

$$
\hat{P} = \begin{bmatrix}
0.828 & 0.049 \\
0.171 & 0.950
\end{bmatrix} \quad \hat{P} = \begin{bmatrix}
0.786 & 0.065 \\
0.213 & 0.934
\end{bmatrix}
$$

The elements on the main diagonal of the transition matrices suggest that the estimated regimes are very persistent. From the estimated transition probabilities one can easily derive the expected duration of an expansion as:

$$
\sum_{z=1}^{\infty} z p_{11}^{z-1}(1 - p_{11}) = (1 - p_{11})^{-1}
$$

Similarly, the expected duration of a recession is:

$$
\sum_{z=1}^{\infty} z p_{22}^{z-1}(1 - p_{22}) = (1 - p_{22})^{-1}
$$

\(^9\text{For further details about the maximization problem see appendix 3.2}\)

\(^{10}\text{For this class of model an overview on alternative numerical techniques for the maximum likelihood estimation of VAR(M)-MS(p) model is given in Krolzig(1997b).}\)

\(^{11}\text{All estimations were done with MSVAR package for OX.}\)
3.3. \textit{ESTIMATION ISSUES}

The expected duration of both recessions and expansions\textsuperscript{12} are reported in table 3.5.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
 & Euro Area & United Kingdom \\
\hline
Recession & 5.83 & 4.69 \\
Expansion & 20.24 & 15.33 \\
\hline
\end{tabular}
\caption{Expected Duration}
\end{table}

Table 3.5 suggests that the degree of inertia of the two regimes is very different. There is a significant asymmetry between the average length of expansions and recessions, the latter much shorter than the former, which is to be expected of classical cycles in a growing economy.

This result is consistent with the evidence coming from previous studies, such as Artis and Krolzig (1999), on the asymmetries between booms and recession. The average durations of expansion are 15.33 and 20.24 quarters for the United Kingdom and the Euro Area, respectively. This means that the Euro Area tends to have a longer cycle than the UK although, the differences are not great.

The smoothed probabilities together with the output growth are depicted in figure 3.1 and figure 3.2. The shaded area represents the time path of smoothed probabilities of being in recession, while the solid line is the quarterly output growth. The figures demonstrate the ability of the estimated

\textsuperscript{12}See appendix 3.3 for the proof.
model in capturing the historical business cycles features. Consistent with the results of the traditional business cycle dating analysis, the years 1986, 1990-92, 1996 and 1998-99 are identified as periods of recession. The consistency of the estimated business cycles can be also checked by looking at the time path of the two series in the figures. Specifically, when the probability of being in recession is relatively high the output growth is lower than zero.

Figure 3.1: Output Growth and Recession Probabilities in the Euro Area
3.3. ESTIMATION ISSUES

Figure 3.2: Output Growth and Recession Probabilities in United Kingdom
3.4 Regime-dependent Reaction Function

Most of the literature on monetary policy reaction function models the central bank behaviour with a linear reaction function, that is by construction symmetric over the different phases of the business cycle. As stressed above, this procedure neglects the possibility for the monetary authorities to calibrate their action on the state of the economies.

In order to fill this gap this chapter allows for switching monetary rules within regimes. The reaction function considered in the analysis is a classical Taylor rule of the following form:

\[ i_t = i^* + \alpha_1 (\pi_t - \pi^*) + \alpha_2 (y_t - y^*) + v_t \quad (3.3) \]

where \( i^* \) is the steady-state value of nominal interest rate, \( \pi_t \) is the current value of inflation rate, \( \pi^* \) is the target value for annual inflation rate while the term \( (y_t - y^*) \) represents the output gap, i.e. the deviation of the current output from its potential value. Usually this rule is estimated as:

\[ i_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \tilde{y}_t + v_t \quad (3.4) \]

where \( \alpha_0 = i^* - \alpha_1 \pi^* \), and \( \tilde{y}_t = (y_t - y^*) \)

Nevertheless, we consider a backward-looking version of the classical Taylor rule in order to derive the monetary policy reaction function from the MS-VAR\textsuperscript{13}:

\textsuperscript{13}In our model, the last equation in the system represents the Taylor rule, as we consider only one lag, i.e. \( n = 1 \).
\[ i_t = k + \sum_{j=1}^{n} \beta_j \pi_{t-j} + \sum_{j=1}^{n} \gamma_j \tilde{y}_{t-j} + \sum_{j=1}^{n} \rho_j i_{t-j} + u_t \quad (3.5) \]

The estimated monetary rules and related standard errors for the Euro Area are:

**Regime 1**

\[ i_t = 0.272 + 0.17\pi_{t-1} + 0.08\tilde{y}_{t-1} + 0.9i_{t-1} \]

[S. E 0.0182] [S. E 0.0912] [S. E 0.0290] [S. E 0.0738]

**Regime 2**

\[ i_t = 2.0 + 0.32\pi_{t-1} + 0.19\tilde{y}_{t-1} + 0.59i_{t-1} \]

[S. E 0.7341] [S. E 0.0797] [S. E 0.0475] [S. E 0.0101]

while the estimated reaction functions for United Kingdom are:

**Regime 1**

\[ i_t = 0.259 + 0.25\pi_{t-1} + 0.16\tilde{y}_{t-1} + 0.85i_{t-1} \]

[S. E 3.16E-01] [S. E 0.065] [S. E 0.098] [S. E 0.0733]

**Regime 2**

\[ i_t = 3.19 + 0.283\pi_{t-1} + 0.37\tilde{y}_{t-1} + 0.5i_{t-1} \]

[S. E 1.0137] [S. E 0.34E-02] [S. E 0.0113] [S. E 0.01285]

These rules capture the criteria used by the central banks to set the short-term interest rate depending on the state of the economy. Having estimated the regime-dependent reaction functions it is then possible to compute the long-run response coefficients as follows:

\[
\alpha_0 = \frac{k}{1 - \sum_{j=1}^{n} \rho_j}; \quad \alpha_1 = \frac{\sum_{j=1}^{n} \beta_j}{1 - \sum_{j=1}^{n} \rho_j}; \quad \alpha_2 = \frac{\sum_{j=1}^{n} \gamma_j}{1 - \sum_{j=1}^{n} \rho_j}
\]

The corresponding long-run response coefficients are shown in Table 3.6.
The differences concerning the dimension of the inflation and the output gap response coefficients might depend on a particular state of the economy the central bank faces. The results suggest that there is alternation between periods of aggressive reaction to the state of the economy and periods of less aggressive response. The aggressive type of policy seems to be more associated with periods of low output growth. By contrast, during expansions the monetary authorities seem to react to the economic developments, in a less aggressive way.

The estimated long-run response coefficients can be also used to compare the behaviour the two central banks demonstrate during contractions and booms. In fact, having estimated a regime dependent reaction function is possible to check the inflation target each rule implies.

As $\alpha_0 = i^* - \alpha_1 \pi^*$, and $i^* = r^* + \pi^*$, the implied inflation target can be recovered as follows:

$$\pi^* = \frac{r^* - \alpha_0}{\alpha_1 - 1}$$

where $r^*$ is the long run equilibrium real interest rate (if the sample is
sufficiently long we can use the sample average).

Table 3.7 reports the computed targets for the two central banks.

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation target</td>
<td>Inflation target</td>
</tr>
<tr>
<td>Euro Area</td>
<td>1.77</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Table 3.7: Comparison between Estimated Target Inflation Rate

Note: Estimates of $\pi^*$ for United Kingdom and Euro Area assume that long-run equilibrium real interest rates are equal to 3.87 and 4.35 respectively.

The estimated inflation targets can be better understood if we take into account the general framework followed by the ECB and the Bank of England. In fact, the monetary strategies of the two central banks adhere to different frameworks. In particular, the ECB adhere to a model of "narrow" central bank, i.e. the German model, rather than to a "broad" central bank, i.e. the Anglo-Saxon one. This means that the ECB designs its own strategy with the sole aim of achieving price stability. On the contrary, the Bank of England following the "broad" central bank framework can pursue several targets. Financial supervision, output and unemployment stabilization as well as exchange rate pegging are all targets a broad central bank may attempt to achieve.
It is then not surprising to observe the different size of the inflation targets described in table 3.7.

In particular, during recession both central banks seem to set the interest rate in order to achieve a lower level of inflation target, in comparison with the estimated targets the central banks pursue during expansion. In fact, while during booms the estimated targets are 3% and 5.1% for the Euro Area and the UK respectively, in recession times those coefficients almost halve: the ECB is estimated to set its inflation target at 1.77% while the Bank of England targets its inflation rate at 2.56%. Moreover, the results suggest that, independently from the phases of the business cycle, a narrow central bank achieves a lower level of inflation target.

3.5 Simulation

After estimating the model, it is now possible to apply the impulse response analysis. The natural object of this analysis is to measure the time profile of the incremental effect of variables' innovation on the future state of the economy. Unlike the linear case, we allow the impulse response functions to be regime dependent. This means that we capture the asymmetric effects that the central bank interventions might have on the real economy, conditional on a given regime dominant at the time the disturbances occurred.

The use of the impulse response functions in the analysis requires a priori identification of the fundamental disturbances in the system. The identification problem is faced by imposing a recursive structure to the economy. The
restricted model allows for a simultaneous feedback from monetary variables to macroeconomics variables but not the vice versa. This kind of identification implies that the shock to a variable has only contemporaneous effects on the variable itself, and the variables ordered below it. The resulting lower triangular matrix is exactly identified\textsuperscript{14}.

In the single state case, the matrix $B$ is obtained from the Choleski decomposition of the estimated variance-covariance matrix $\Omega$ of the unrestricted VAR:

$$
\Omega = E(Bu_t'u'_tB') = BE(u_t'u'_t)B' = BIkB' = BB'
$$

In a MS-VAR model the variance-covariance matrix is regime dependent and consequently the matrix $B$. The Choleski decomposition is then as follows:

$$
\Omega_s = E(B_su_t'u'_sB'_s) = B_sE(u_t'u'_t)B'_s = B_sIkB'_s = B_sB'_s
$$

In each regime the vectors of the fundamental disturbances are pre-multiplied by the regime dependent matrix $B_s$ which describes the relationship between the endogenous variables in the system.

The estimated responses to a 1%, i.e. contractionary, monetary policy shocks are reported in figure 3.3 and figure 3.4. In these figures the first column refers to regime 1 while the second column synthesizes the result of

\textsuperscript{14}This is achieved by imposing $K(K - 1)/2$ exclusion restrictions above the main diagonal of the structural matrix $B$. Where $K$ is the number of variables. As in the estimated system $K$ is equal to three we impose three restrictions.
regime 2. Each response is provided with the associated asymptotic confidence bands.

In both the UK and the Euro Area the patterns of the responses appear to be very similar. A positive monetary policy shock leads, in both regimes, to a decrease in output gap and inflation.

The output gap shows, after an initial delay, a hump-shaped response. This particular pattern of output gap reaction, is in line with many previous studies conducted with standard VAR models on the monetary policy transmission mechanism.\footnote{See Christiano et al. (1998) and Peersman and Smets (2001) for example.}

An important feature of the specified MS-VAR model consists of not generating the so-called price puzzle. This puzzle refers to a situation in which a contractionary monetary policy shock results in an increase in inflation. On the contrary, in our models, inflation slightly falls after a monetary shock. Moreover, consistent with the presence of some degree of nominal rigidities, the inflation response is initially very small.

The different timing and size concerning the real economy response across the Euro Area and the United Kingdom are depicted in Figure 3.3 and Figure 3.4. Table 3.8 summarizes the results.

The table describes the average response and the minimum level reached by output gap and inflation rate, after a monetary policy shock. In the third column it is also represented the number of periods output and inflation take to reach their minimum level, i.e. the period after shock where the maximum
3.5. SIMULATION

effect occurred.
Figure 3.3: Impulse Response Analysis for Euro Area
3.5. SIMULATION

Figure 3.4: Impulse Response Analysis for UK
Common to the UK and the Euro Area, the results document significant negative effects on output gap of a rise in the interest rate during periods of below-average growth.

Some differences arise from the timing of the responses. During recession, the maximum effect of the monetary tight is experienced in the Euro Area. Table 3.8 shows a decrease of $-0.649$ that occurs after 5 periods. In the United Kingdom the minimum level of the output gap is $-0.359$ and this happens after 6 quarters. The situation is different during expansion. In the Euro Area the output gap takes 10 quarter to reach its lowest level ($-0.408$), while in the UK the greatest effect on output ($-0.162$) happens
3.5. *SIMULATION*

after 2 periods.

Asymmetries arise also when considering the dynamic pattern of the inflation rate after monetary shock. In both countries positive shocks have a larger negative effect on inflation rate in time of expansion than during recession. However, in the Euro Area the decrease in inflation rate, $-0.22$ after 6 quarters, is greater and quicker than the one estimated for the United Kingdom ($-0.14$ after 9 periods). The situation is opposite when considering the expansionary periods: in the UK the minimum incremental effect ($-0.802$) happens after 7 quarters while in the Euro Area the effects are slower (14 periods) and smaller ($-0.788$).

The obvious implication that arises from the impulse response analysis is that an interest rate increase will have a larger (negative) effect on output gap during recessions than in booms. Consequently, when monetary authorities show an increase in the interest rate during expansion, it will not have the same recessionary effect on real economy. These results confirm those derived in theoretical models assuming price rigidities and implying a convex supply curve.

The size and the timing of those effects mostly depend on the strategy a central bank follows. In general, monetary authorities can follow two types of frameworks. In the first one, the central bank only aims at achieving price stability. This framework is followed by the ECB. In the second one, a central bank can pursue several objectives like price stability, output stability and low unemployment. The Bank of England adheres to the latter framework.
The choice of a particular model influences the way through which a central bank affects the real economy. In particular, a central bank that only targets inflation, like the ECB, has a greater and quicker change in output during recession than a central bank that targets several variables, like the Bank of England, that has quicker but smaller output reaction during expansion.

3.6 Conclusion

The chapter analysed whether monetary authorities asymmetrically affect the real economy depending on the phases of the business cycle.

Asymmetric effects are a common feature of many theoretical models. In the literature there are many different versions of such asymmetries. They might be related to the direction of monetary policy action, or to the size of the monetary change, or to the position of the business cycle. In this chapter, we investigated the asymmetries due to the position of the business cycle. In particular, we measured the likely effects of monetary policy actions on output and inflation using a multivariate extension of Hamilton's regime switching model.

The empirical analysis was organized as follows.

We estimated a regime dependent Taylor rule in order to account for the different phases of the business cycle; then, we analyzed how monetary policy shocks are transmitted to the output gap and inflation in the United Kingdom and the Euro Area.

The empirical evidence coming from the estimated models corroborated
3.6. CONCLUSION

The estimated Taylor rules for the UK and the Euro Area led to the following conclusions.

First, the two central banks have a more aggressive behaviour in recession rather than in expansion. In fact, the response coefficients in both countries are higher in regime 1 than in regime 2. This result was also confirmed by the estimated inflation targets. In both countries the implied targets are significantly higher in expansion than in recession.

Second, the reaction function of the Bank of England shows a higher output gap response coefficient. This supports the common belief that the Bank of England contemporaneously targets output and inflation. Moreover, the occurrence of a switch in regime, seems to affect both inflation and output gap response coefficients: they are greater in recession than in expansion.

Third, the ECB seems to be more sensitive to the inflation dynamic. The response coefficients of the inflation and output gap in expansion are greater in recession in comparison with the ones obtained in expansion. Nevertheless, the ECB output gap reaction seems to be lower with respect to the one estimated for the Bank of England. This confirms that the primary goal of the ECB consists of achieving price stability.

Some interesting results also came from the impulse response analysis.

First of all, the models appeared to perform quite well. No price puzzles have been detected in the simulations. In general, an interest rate shock
leads to a larger effect on the output gap during recession than during expansion. In other words the interest rate has a greater influence on output when past growth has been high than when past growth has been normal or negative.

We also provided a comparison of the impact of monetary policy shock in different regimes.

Common to the UK and the Euro Area, the results highlight significant negative effects of an increase in interest rate, during recessionary periods.

Some differences arise from the size and the timing of the responses. The patterns of the responses are supposed to be depending upon the particular strategy followed by the central bank. A central bank that primarily aims at achieving price stability, like the ECB, has a greater and quicker output response during recession than a central bank that targets several variables, like the Bank of England, that, in contrast, has quicker but smaller output reaction during expansion.

Altogether the analysis strongly suggests that the stage of the business cycle is important in the monetary policy decision process. A central bank cannot neglect the specific regime where the monetary action takes place.

The obvious implication that arises from the impulse response analysis is that an interest rate increase will have a larger (negative) effect on output gap during recessions than in booms. Consequently, a tight monetary policy will not have the same negative effect on the real economy during expansion.
Chapter 4

Modelling Confidence in the Euro Area

This chapter aims to model the main determinants of the confidence of economic agents in the euro area. In particular, it measures the impact of the labour market conditions, the stock market developments and interest rates on consumer and business confidence. The study is articulated in two parts. We estimate two models, respectively for the survey indicators of consumer confidence and of business confidence. The analysis relies on the use of multivariate econometric techniques. Specifically, we use a structural VAR and a dynamic simultaneous equations model. Empirical results show that models are well-behaving. Impulse response analysis suggests that the main determinants of consumer confidence are labour market conditions and interest rates. Empirical evidence for business confidence shows that this indicator is strongly affected by economic conditions in the United States. Industrial production and labour market conditions in the Euro Area play an important role in the short term.
4.1 Introduction

This chapter analyses how the general economic environment affects consumer and business confidence in the euro area, with a focus on the role of monetary policy.

The literature analyzing what drives economic confidence is rather poor. Among the few papers attempting to model confidence, most focus on the causal relationship from one factor to consumer confidence. Moreover, it seems that there has not been any published study attempting to model the business confidence indicator.

The contribution of the chapter with respect to the existing literature is two-fold. First, it expressly specifies a model for the business confidence indicator. Second, it uses a structural multivariate framework in order to account for the interactions of several factors a priori determining confidence indicators.

The chapter is articulated in two parts, respectively proposing a model for consumer and business confidence. Specifically, the models are a structural VAR for consumer confidence, and a dynamic simultaneous equation framework for business confidence. The models are estimated for the overall indices as well as for their sub-components (i.e. the indices corresponding to the individual questions in the surveys). In the absence of an existing theoretical framework, we select a priori the factors determining consumer and business confidence based on experience and previous analysis. The importance of these factors is then tested empirically, by analyzing their
4.1. INTRODUCTION

impact on the confidence indicators.

A priori analysis suggests labour market conditions, stock market developments and interest rates as possible determining factors for consumer confidence. In addition to these factors, industrial production is considered for the model of business confidence, together with variables capturing external developments. Such external factors are an indicator of business confidence in the United States, the price of oil and the exchange rate of the euro.

Regarding the results for consumer confidence, the study suggests that it is mainly driven by labour market developments in the Euro Area (measured by the change in the unemployment rate). The real short-term interest rate is likely to play a key role in the long run. In contrast to results from other studies, the chapter shows that the role of stock market developments is significant, albeit less important with respect to the other factors.

Regarding the results for business confidence, the dominating factor among the variables chosen in our model is business confidence in the United States. Among the endogenous variables, industrial production seems to have a significant effect in the short run, but negligible compared to the external factor just mentioned.

The chapter is organized as follows. Section 4.2 describes the motivation and the conceptual framework. Section 4.3 briefly describes the existing literature. Section 4.4 describes the applied econometric technique. Then, it estimates the model for the consumer confidence indicator and discusses
the results of the dynamic and the sub-components analysis. Section 4.5 applies the same analysis to the Business Confidence indicator. Section 4.6 concludes.

4.2 Motivation and conceptual framework

4.2.1 Motivation

The confidence of economic agents plays a key role for economic growth. Policy-makers often argue along these lines. In addition, business cycle analysts and forecasters closely monitor confidence indicators. Although economic confidence receives a wide interest in practice, the literature is quite scarce on this issue.

This holds especially true for the impact of monetary policy decisions on confidence. The literature on the transmission mechanism of monetary policy typically ignores the possibility of a "confidence channel", through which central banks' interest rate decisions would also channel to eventually affect economic activity. Even though it is possible to argue for the existence of such a channel in addition to those commonly reviewed in the literature.¹

However, it may not be surprising that the literature does not consider a specific "confidence channel", given that the effectiveness of this channel

¹For instance, P. McAdam and J. Morgan (2003) identified an income/cash flow channel; a wealth channel; a direct interest rate channel on consumption; a cost of capital channel; and an exchange rate channel. Other papers also identify an "expectations channel", which is related to confidence but only to a small extent. In this channel, a commitment by the policy-maker to alter the level of future interest rates in response to a current shock affects current spending, so that the current interest rate needs to be changed less in order to stabilise output and inflation (Bean, Larsen and Nikolov, 2003).
4.2. MOTIVATION AND CONCEPTUAL FRAMEWORK

is a priori not clear. In particular, it is not necessarily the case that a cut in interest rates would boost confidence, and thereby have positive effects on economic growth. For example, a reduction in policy interest rates can be perceived as a signal of low confidence in the economic outlook, on the part of the monetary policy authorities. This would affect negatively the confidence of households and investors. Such counter-intuitive effects could take place under special circumstances. Furthermore, a policy of keeping interest rates unchanged (policy of the "steady-hand") may contribute to a monetary environment that is favorable for economic confidence.

Empirical studies produced by various kinds of institutions (including central banks) are numerous concerning the role of confidence indicators for economic activity. In this respect, it is generally recognized that survey indicators of confidence have a leading property for economic activity. In particular, consumer confidence has a leading role for private consumption,

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2 The study of non-linearities in the impact of monetary policy decisions on confidence, and of the role of a "steady hand" attitude from monetary policy authorities for confidence are both left for future research.

3 Non-linear effects may also be related to the credibility of the central bank. For instance, a cut in policy rates may dampen the confidence in the central bank's commitment to preserving price stability, while an increase in policy interest rates may be seen as helping to restore the agents' confidence in the central banks' commitment to pursue price stability.

4 Central banks, and especially the ECB, have often argued that monetary policy contributes to economic stability by keeping a steady hand on interest rates (i.e. keeping interest rates unchanged). It is obviously difficult to measure empirically the effects of such a policy, given the resulting limited variability in the series of interest rates. However, this problem may be partly overcome by using the deviation of the real interest rate from its 'natural' or 'equilibrium' level. This real interest rate gap provides a measure of the monetary policy stance, which could fruitfully replace the actual real interest rate in a model designed to analyze the impact of monetary policy on confidence. Obviously, the variability in the series of the real rate gap depends on that of the estimate chosen for the natural level of interest, which is typically highly model-dependent.
and business confidence has a leading role for industrial production, which is even stronger than the latter.

The literature analyzing confidence from the opposite perspective, i.e. what drives economic confidence instead of what is the effect of confidence on economic activity, is rather poor. Among the few papers attempting at modelling confidence, most are focused on the causal relationship from one factor (usually developments in stock markets, and more rarely in the labour market) to consumer confidence. Moreover, as already mentioned, it seems that there has not been any published study attempting at modelling the business confidence indicator.

This chapter attempts at contributing to filling this gap. Understanding what drives confidence is indeed crucial for interpreting the confidence indicators themselves, and hence for concluding on their explanatory power for future economic activity. A better understanding of the determinants of confidence is also crucial for policy decision-making. In particular, a better understanding of the impact of monetary policy on the confidence of economic agents can help policy-makers in taking decisions on interest rates and in their communication with the public.

While the literature on confidence is poor in the field of monetary policy, the fiscal policy literature has addressed the role of confidence to a large extent. The confidence of economic agents in the sustainability of public finances plays a key role in the non-linearities sometimes observed in the way fiscal policies affect economic activity (e.g. a budgetary consolidation may not necessarily lead to a decline in output).
4.2. MOTIVATION AND CONCEPTUAL FRAMEWORK

4.2.2 Conceptual framework

The first issue addressed in this chapter is to determine the main factors possibly influencing confidence. While recognizing an influential role from political factors (such as terrorist attacks and speeches of policy-makers) on economic confidence, this chapter focuses on the possible economic determinants. Given the absence of a theoretical framework describing what drives economic confidence, we use several means to identify the possible key determinants of confidence. These include the theory on consumption and investment, a review of the literature on economic confidence, and an examination of the questionnaire used to construct the confidence indicators. In addition, a graphical analysis of the possible factors identified in this way help check their potential relevance.

In this process, we select the labour market conditions, the developments in stock markets and interest rates as the potential key determinants of confidence, especially consumer confidence.

Labour market conditions are a traditional indicator of the economic health of the country. A high unemployment rate is usually associated with a low level of consumer confidence. Consumers who lose their jobs curtail spending in response to the loss of income, while the others who remain employed may curtail spending in anticipation of future job losses. As a result, even small increases in unemployment may lead to a significant decline in

\[\text{See Alan Garner, "Consumer confidence after September 11", for a description of the effects on consumer confidence in United States after the terrorist attack on 11th September, 2001}\]
consumer confidence. We measure labour market conditions with the change in the unemployment rate rather than its level for two reasons. The first is its higher correlation compared to the level variable with the consumer confidence index. The second is the non-stationarity of the variable in level. Figure 4.1 illustrates the relationship between Confidence and the remaining variables in the system.

![Chart 1](image1)

![Chart 2](image2)

![Chart 3](image3)

![Chart 4](image4)

Figure 4.1. The plots of the variables in the consumer confidence model

Each chart plots the consumer confidence index (values on the left-hand side) against one of the variables in the model (dotted line; corresponding scale on the right-hand side). Specifically, chart 2 draws the Consumer

\[ D_{URX} \rightarrow \text{The change of the unemployment rate} \]
Confidence index against the first difference of the unemployment rate. The graphical analysis clearly implies a negative relationship between the two variables, which is particularly strong at the beginning and at the end of the sample.

The linkage between the (real) rate of interest rate and confidence seems to be more uncertain. This may be related to the fact that cuts (respectively increases in) policy interest rates might not necessarily lead to a drop (respectively an increase) in the confidence of economic agents, as it theoretically does on economic activity. As discussed in the introductory section, decisions to keep policy rates unchanged might also have an impact on confidence, while counter-intuitive (or non-linear) effects may also take place in special circumstances. In addition, it cannot be expected that the relationship between interest rates and confidence is as clear as in the case of economic activity, given the absence of a very close link between consumer (respectively business) confidence and consumption (respectively business investment or production). In other terms, such an assumption may be too strong to draw any clear conclusions on the expected impact of interest rates on confidence from their theoretical impact on economic activity.

However, in the absence of other relevant framework, the effects of interest rates as implied by standard theory of consumption and growth might be the best departure point for studying the impact of interest rates on

\[
R_{STir} \rightarrow \text{Real Short Term interest rate} \\
PER \rightarrow \text{Price-earning ratio} \\
EC_{cons} \rightarrow \text{Consumer Confidence Indicator}
\]
In this respect, it is commonly argued that a rise in the real interest rate implies that consumption and investment (economic activity) are ceteris paribus discouraged and saving is encouraged. The opposite holds true for a decline in the real rate. More specifically, in an inter-temporal consumption model, an increase in the rate of interest leads to a decrease in the level of consumption. (In a model with two periods (e.g. Fisher model), an increase in the rate of interest rate typically leads to a postponement of consumption decisions to time \( t + 2 \)). It may be argued that this effects channels through consumer confidence, at least in part (see Section 4.4). Similarly, an increase in interest rates should affect industrial confidence negatively, since it raises the cost of borrowing, thereby reducing the firms' profit margins. It may be argued that this lowers the level of the firms' confidence and ultimately, investment and production (see Section 4.5). For the opposite reason, a decrease in interest rates can normally be expected to affect (consumer and business) confidence positively.

In our model, monetary policy is captured by a measure of the real short-term interest rate, which is the main variable through which the monetary authority affects the macro-economy, through its decisions on policy rates. In a preliminary analysis, we verified that, among other interest rates (long term interest rate, real long-term interest rate, nominal short-term interest rate), the real short term interest rate performs better in terms of the magnitude and expected sign of its impact on confidence indicators. Chart
4.2. MOTIVATION AND CONCEPTUAL FRAMEWORK

3 in figure 4.1 plots the consumer confidence index against the real short-term interest rate. Over the whole sample, the chart shows only a moderate inverse relationship between the two variables. The negative correlation is strong until the end of 1995, but becomes less clear since then.

In principle, stock market developments can affect consumption directly via three main channels, namely that of wealth, balance sheets and consumer confidence. The idea of a possible confidence channel in the transmission of stock market developments to consumption has been proposed by Poterba and Samwick. The triangular relationship between stock market developments, consumer confidence and consumption is frequently found in the press, in particular at times when stock markets experience sudden slumps (Kremer and Westermann, 2004). The link is twofold. Households perceive the changes in the stock market developments as implying changes in their personal finances and in the general economic environment. In our model, stock market developments are captured by the price-earnings ratio in the Euro Area stock market rather than by a stock index measuring share prices, which is clearly non-stationary (and would therefore lead to bias the estimates in our models of confidence). The graphical analysis suggests a positive relationship between the price earnings ratio and both consumer and business confidence indicators, as well as a predictive power of the price earnings ratio.

For the business confidence model, additional variables have been used in comparison to the consumer confidence model, which include the growth rate
of industrial production (smoothed over a quarter) as well as external factors, such as a measure of business confidence in the United States (the Chicago PMI), the price of oil and the euro exchange rate (in nominal effective terms).

The variables are plotted in Figure 4.2:

Figure 4.2. The plots of the variables against the Business Confidence Indicator

Each chart shows the plot of one variable versus the Business Confidence

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8 We label with:

- D_IP → the growth rate of industrial production excluding constructions;
- EC_man → Industrial Confidence Indicator
- OILP → Oil Price expressed in dollar
- EEN → Nominal Effective Exchange Rate
- PMI_US → NAPM production expectations in manufacturing.

The other variables keep the above mentioned meaning.
4.3. BRIEF LITERATURE REVIEW

Indicator (BCI)\(^9\).

Overall, the graphical analysis suggests negative relationships between the BCI and the first difference in the unemployment rate, the real short-term interest rate and the price of oil. The relationships are positive in the cases of the growth rate of industrial production, the expectation of manufacturing and the price earnings ratio. The relationship between the nominal effective exchange rate and BCI is less clear.

4.3 Brief literature review

As briefly argued above, the literature is quite poor on the modelling of economic confidence. While there seems to be no study addressing the determinants of business confidence, a few papers analyse the relationship between consumer confidence and other (usually one) economic factors.

Regarding the relationship between confidence and the unemployment rate, Zaretsky (1992) found a weak link between these two variables for the United States. The study of the link between interest rate and consumer confidence is also scarce. Bernanke, Boivin and Eliasz (2003) found that a rise in the Fed funds rate leads to a drop in consumers' expectations in the first 18 months, while the longer-term impact is less clear. Basuchoudhary, Guttermuth and Sen (1996) find a relationship between consumer confidence and the real interest rate by using Granger causality tests. Berg and Bergstrom (1997) found that the real after tax interest rate and the change in the

---

\(^9\)The vertical axis on the left-hand side corresponds to the BCI, while the axis on the right-hand side corresponds to the other variable.
inflation are important determinants confidence indices in Sweden. More exhaustive seems to be the literature explaining the link between confidence and stock prices. The BIS (2001) reports some partial evidence on the relationship between consumer confidence and movements in the NASDAQ. Jansen and Nahuis (2003) show that the relationship between stock prices and consumer confidence essentially reflects a link with expectations about general economic conditions rather than with, expectations about personal finances. Kremer-Westermann (2002) found a significant causal relationship from stock market developments to consumer confidence in the Euro Area. A less direct relationship is instead highlighted from the following studies. Morck et al (1990) stress a predictive content of stock prices for consumption. Campbell (1996), in line with other findings such as Stock and Watson (2001), provides mixed and not conclusive results about the relationship between the price earnings ratio and consumption growth rate. Katona (1976), on the base of a psychological approach, proposed the idea that confidence is one of the most important determinants of household spending. Romer (1990) uses stock market volatility as a proxy for consumer uncertainty or confidence. Mustaka and Sbordone (1995), as well as Schiller (2000), argue that stock market developments are important because they provide daily information affecting the attitude of the consumers. Desroches and Gosseling (2002) and Garner (2002) found that consumer confidence on its own, has some forecasting power for private consumption, which decreases or vanishes, when controlling for fundamental variables such as income and stock
4.3. BRIEF LITERATURE REVIEW

prices.
4.4 Consumer Confidence model: a Structural VAR model

In a multivariate framework, where there is no a priori knowledge about the theoretical relationships between variables, the use of a VAR model appears to be particularly appealing. In particular, we estimate the $K$-structural VAR\(^\text{10}\) as follows:

\[
A_0 y_t = \sum_{i=1}^{k} A_i y_{t-i} + B v_t
\]

where $y_t = [D_{URX}, PER, DR_{STir}, EC_\text{cons}]'$ is a $p$ dimensional vector of endogenous variables\(^{11}\), $A_0$ is an invertible $(p \times p)$\(^{12}\) matrix describing the instantaneous relationships between the variables and $B$ is a $(p \times p)$ diagonal matrix. The vector of errors $v_t$ is distributed as a Normal process with zero mean and Variance Covariance matrix $\Sigma$.

The order of the autoregressive coefficients in the VAR system is tested by implementing standard tests statistics. Table 4.1 provides results for several information criteria:

\(^{10}\)See G. Amisano and C. Giannini, 1997 for the definition.

\(^{11}\)We label with $DR_{STir}$ the first difference of the real short-term interest rate while the other variables keep the above mentioned meaning.

\(^{12}\)In our case $p$ is equal to four.
4.4. **CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL**

**VAR Lag Order Selection Criteria**

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL.</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HO</th>
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<td>1047.507</td>
<td>6.34E-05*</td>
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<td>1.874848*</td>
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<td>4.008275</td>
<td>2.843139</td>
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<td>0.000107</td>
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<tr>
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<tr>
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</tbody>
</table>

* LR: sequential modified LR test statistic (each test at 5% level)
  FPE: Final prediction error
  AIC: Akaike Information criterion
  SC: Schwarz Information criterion
  HO: Hannan-Quinn Information criterion

* Indicates lag order selected by the criterion

Table 4.1. Optimal Lag Length
All tests select one as the optimal lag length. In a more explicit form, with one lag, the model becomes as follows:

\[ y_t = A_1 y_{t-1} + A_0^{-1} v_t \]

In the matrix \( A_0 \) we identify a structure for the instantaneous relationships between variables. The basis for our assumptions on contemporaneous relationships is the lower triangular matrix used in the simple Choleski decomposition. In this framework, the ordering of the variables determines the level of endogeneity of each variable. In our model we put the change in the unemployment rate first as we consider it, a priori, as the most exogenous variable. On the other hand, consumer confidence comes last in the ordering of the system as we expect it to be the most endogenous variable. Based on reasonable assumptions about the economic relationships in the system, we model \( A_0 \) as follows:

\[
A_0 = \begin{bmatrix}
1.0 & 0 & 0 & 0 \\
0 & 1.0 & a_{12} & 0 \\
0 & 0 & 1.0 & 0 \\
a_{31} & a_{32} & a_{33} & 1.0
\end{bmatrix}
\]

Unlike the Choleski decomposition, we assume no instantaneous relationship between the first difference of the unemployment rate and the first difference of the real short term interest rate. This is a safe assumption given the long lags in the adjustment of the economy, and especially the labour market, to interest rate changes. Moreover, we impose an instantaneous relationship between the the change of the real short term interest rate
4.4. CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL

and the price earnings ratio reflecting the direct link between stock market valuation and interest rates. The estimated model is overidentified. The over-identifying restrictions are tested and not rejected at 10% confidence.

The variance covariance matrix of the model is obtained by maximizing the following concentrated likelihood function:

\[
\frac{T}{2} \{\log |A^2| - \log |B^2| - \sum \log (B^{-1}ASA'B'^{-1})_{ii}\}
\]

where S is the estimated Variance covariance matrix from an unstructured VAR model.

The maximization algorithm is the one proposed by Broyden, Fletcher, Goldfarb and Shannon (BFGS described in Press et al, 1988).

4.4.1 Data Description

The study uses monthly aggregated data for the period between 1993:1 and 2003:12. The model is estimated by using the OLS estimator. For the consumer confidence indicator (CCI) we use Survey data provided by the European Commission. This index is defined as the arithmetic average of four forward looking questions about the expected, personal and general, economic conditions.

---

\(\chi^2\) result follows:

\(\chi^2(1) = 3.41757324\) Significance Level 0.06450577

BFGS starts with a diagonal matrix. At each iteration, it is updated based upon the change in parameters and in the gradient in an attempt to determine the curvature of the function. The basic theoretical result governing this is that function is truly quadratic, and if exact line searches are used, then in n iterations, G will be equal to \(-H^{-1}\). If the function is not quadratic, G will be an approximation of \(-H^{-1}\).

Specifically, it is the arithmetic average of the results from the following questions (percentage balances):
The unemployment rate is the one published for the Euro Area by Eurostat. The real short term interest rate is computed as the difference between the three month Euribor and the annual percentage change in the Harmonized Index of Consumer Prices (later referred as inflation).

The price-earnings ratio (P/E ratio) is calculated by Thomson Financial as total market capitalization divided by total earnings of constituent companies (which is equivalent to the ratio of average share prices over average earnings per share).

A first specification test involves determining the time series properties of the data. The Dicky Fuller test rejects the hypothesis of a deterministic trend in the data but, it does not reject the null hypothesis of a unit root for our sample. Results of the test are reported in table A4.1 in appendix 16.

The choice between a model in levels with near unit-root properties and a model in the first differences of variables basically, entails a trade-off between the risk of drawing invalid statistical inference due to the non-standard

1. Financial situation of the household over the next 12 months;
2. General economic situation over the next 12 months;
3. Unemployment expectations over the next 12 months (with inverted sign);
4. Savings over the next 12 months.

The survey is conducted among 20,000 individuals in the EU. To ensure data are representative, National institutes use different random sampling together with stratification criteria. Data for the euro area are available from January 1985 onwards. (Until October 2000, the questions included in the survey were somewhat different, but consumer confidence data have been calculated back to correct for this).

16 The table shows the statistics, as well as the quantiles of their asymptotic distribution, for each variable in level. The null hypotheses under which the asymptotic distributions are tabulated are always joint hypotheses concerning the coefficient ($\rho$), the mean ($\mu$), and the trend ($\beta$). The reason is clearly explained in Hamilton (1994). The first three statistics are referred to the model without trend, while the remaining statistics to model with trend. The asymptotic distribution of all the tests on the coefficients of this regression is not standard, but is known and tabulated. The table, in fact, also reports the references of the asymptotic distribution which the tests refer to.
distributions of the estimated coefficients, and the risk of losing important information contained in the levels. It remains an open issue in literature whether the variables in a VAR system need to be stationary. Sims (1980) and Sims, Stock and Watson (1990) recommend against differencing even if the variables contain a unit root. They argue that a goal of a VAR analysis is to determine the interrelationships among the variables, not to estimate the parameters of these relationships. The main argument against differencing is that it discards information concerning the co-movements in the data (such as the possibility of cointegrating relationships). The appropriate way to estimate a VAR model containing non-stationary variables is then the Vector Error Correction Model (VECM).

Given the specific focus of this chapter, and the difficulty to consider cointegration relationships in such an empirical model, we prefer to estimate our VAR in levels. Nevertheless, we take the first difference of the unemployment rate and the real short term interest rate due to a higher correlation that the changes in these variables have with the consumer confidence indicator. Furthermore, the unemployment rate is the only variable, which for the hypothesis of a unit root is accepted on the practical and theoretical ground and, therefore, the first difference is theoretically needed. As regards, the price earnings-ratio and the consumer confidence indicator we keep these variables in levels. This choice is supported by the common belief that those variables are theoretically stationary. The price-earnings ratio, for instance, should be a stationary process as stock prices and earn-
ings form a cointegrated relationship with unitary coefficient (Campbell and Shiller, 1987). The consumer confidence index, as stressed by Kremer and Westermann (2002), is stationary on the business cycle horizons. Further arguments arise from the specification analysis. Table A4.2 in appendix 4 reports the roots of the model, when including all variables in level, against the roots of the model when including the changes of the unemployment rate and the real short interest rate (i.e. the estimated model). The table shows that the estimated model does not contain explosive roots. Residuals analysis is reported in table A4.3 in appendix. Both Jarque-Bera normality test and Mardia Multivariate test strongly accept the hypothesis of normality. Furthermore, the residuals of our model do not exhibit autocorrelation. The permanentau test rejects the hypothesis of autocorrelation at 95% significance level. Overall the specification analysis suggests that the introduction of the level of price-earnings ratio, and the level of consumer confidence indicator in the system does not affect the diagnostic tests of the model: the system is stable and statistically correct.

4.4.2 Impulse response analysis

The econometric tool used here to investigate the reaction of consumer confidence to the various shocks is the impulse response analysis. A stationary stable VAR(p) process can be written in the moving average (MA) representation as follows:
4.4. CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL

\[ y_t = X_t \beta + \sum_{s=0}^{\infty} \Psi_s u_{t-s} \]

where \( \Psi_s \) is the matrix coefficient at time \( s \) and \( u_{t-s} \) is the error distributed as \( N(0, \Sigma) \).

The response at \( t = k \) to an initial shock \( z \) in the \( u \) process is \( \Psi_k z \). For instance, the response at step \( k \) to a unit shock in equation \( i \) at \( t = 0 \) is just the \( ith \) column of the \( \Psi_k \) matrix\(^\text{17}\). The responses are computed with the following formula:

\[ \Psi_s = J M^s J^t G \]

where \( J \) is the extraction matrix, \( M \) is the companion matrix obtained by starting from the \( A_i \) values and \( G \) is the decomposition factor.

The analysis is applied to the consumer confidence index and its subcomponents. In particular, we re-estimate the model by replacing the aggregate index of consumer confidence with the results of individual constituent questions (i.e. UPE, ECOE, SAVE and FINE)\(^\text{18}\).

Plotting the impulse response function is a practical way to visually represent the behaviour of one variable in response to various shocks. Figure 4.3 shows a complete picture of the size and time profile of the reaction

\(^{17}\) For an orthogonalized innovations \( \text{Var}(u) = \Sigma = GG^t \) and then \( u = Gv \) where \( \text{Var}(v) = I \)

\(^{18}\) We label with UPE the index describing the expectations of the labour market condition; with ECOE the expectations about the general economic conditions; with SAVE the expectations about the possibility to save over the next twelve months and finally with FINE, the index describing the personal financial situation over the next twelve months.
of consumer confidence\(^{19}\) (and its subcomponents) to shocks in the other variables forty periods ahead.

![Graphs showing the response of the CCI to various shocks](image)

**Figure 4.3. Response of the CCI to various shocks**

The impulse response analysis suggests that the model is well behaving, meaning that the responses of CCI (and its sub-components) to the shocks in the other variables have the expected sign and a reasonable timing. Moreover, the responses are significant, according to the estimated confidence bands, which are computed using Monte Carlo integration\(^{20}\). Given that the sign of the responses remain in all cases unchanged over time, the average of these responses provide useful information on the magnitude of the reactions.

\(^{19}\)The time profile of the responses of its sub-components are reported in figure A4.1 in the Appendix.

\(^{20}\)See Sims and Zha(1999) for a discussion of these issues.
4.4. CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL

Figure 4.4 compares the average impact on CCI (and its sub-components) of a one standard deviation shock to the other variables.\(^{21}\)

\(^{21}\)Figure 4.4 is composed by two columns. On the left hand side, the graphs compute the average impact over twenty periods and, on the right hand side, the average impact over forty periods. Black columns indicate a negative impact as grey columns indicate a positive impact. We keep this notation for the rest of the chapter.
Results suggest that the main driving force of CCI is the change in the unemployment rate over twenty periods, followed by real short-term interest rates and the price earnings ratio. In the long run the order of the main driving forces reverses. The first difference of the real short-term interest rate becomes the prime mover. At the sub-component level, developments in the labour market are the key factor affecting unemployment expectations (UPE), the assessment of the general economic outlook (ECOE) and the financial situation over the next twelve months (FINE). As expected, interest rates have, in relative terms, a more prominent role in determining the expectations about the possibility to save over the next twelve months (SAVE). In the long run, the real short term interest rate becomes the main driving force for all subcomponents.

In a different way, Figure 4.5 compares the impact on the different sub-components of the various shocks.
Figure 4.5. Cross-Comparison analysis of the average impact on the sub-components of the CCI

This highlights in particular, that the unemployment rate expectations (UPE) is the most reactive component of confidence, and this with respect to all shocks accounted for in the model. The assessment of the general economic climate (ECOE) is also strongly affected by various shocks. This is particularly true for the price earnings ratio. A shock in this variable has an impact on the assessment of the general economic conditions (ECOE) as strong as the one on the unemployment rate expectations (UPE). Overall,
confidence is influenced by interest rates through unemployment expectations (UPE) and the assessment of the general economic climate (ECOE). Unlike the other variables, a shock in the first difference of the real short term interest rate has a significant effect on the assessment of the consumer’s financial situation (FINE, SAVE).

Figure 4.6 shows the maximum impact and the timing of the responses.

Figure 4.6. Maximum impact and Timing of the model

The same analysis is illustrated for the sub-components of the consumer confidence index in Figure 4.7.
The maximum impact on consumer confidence following an unemployment rate shock is reached after 8 months. The real short-term interest rate and the price-earnings ratio achieve the maximum impact on CCI respectively after 15 and 18 months. The timing of the model for the interest rate is consistent with the results obtained by most of the literature on the monetary transmission mechanism.\textsuperscript{22}

Overall, the impulse response analysis suggests that the first difference of the unemployment rate, has the quickest impact on confidence and the strongest in the first twenty periods. A shock to the change in the unemployment rate affects the CCI mainly through the expectations of future

\textsuperscript{22}See Angeloni et al.
labour market conditions and the general economic outlook. The reaction to a shock in the first difference of the real short-term interest rate, is slightly lower than to a change in the unemployment rate but becomes the prime mover in the long run. Overall, it appears that the real short-term interest rate affects consumer confidence mainly through the expectations of future labour market conditions. An interest rate shock has a similar effect on the unemployment rate. This would indicate that the channel of interaction between confidence and interest rates mainly goes through real economic developments. The unemployment rate is, in fact, the only variable capturing the real side of the economy in the system. The expectation about the financial situation over the next twelve months (FINE) and the expectations about savings (SAVE), also play an important role in the transmission of interest rate shocks to confidence. The reactions of SAVE and FINE to an increase in the real short-term interest rate are negative. This result could be explained by the fact that consumers are both borrowers and lenders. Even though they are net lenders on average, the negative effect of an increase in the real short-term interest rate on borrowers’ expectations seem to more than compensates for the positive effect on the lenders’ expectations.

Finally, a change in the price earning ratio mainly impacts confidence through the expectations about the general economic conditions. Consistently with the results provided by Jansen and Nahuis (2003), households do not seem to perceive an increase of the price-earnings ratio, as an improvement in their personal finances (as indicated by the response of the
4.4. CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL

sub-component FINE). This could be explained by the fact that a small proportion of households' detain stock market wealth.

4.4.3 Forecast Error Decomposition

In this sub-section, we compute the forecast error variance decomposition for the VAR model. This analysis provides the proportion of the movements in a variable which is due to its own shocks, versus shocks to the other variables. At each point in time, the forecast error variance is decomposed as follows:

\[
\sum_{s=0}^{k-1} \Psi_s GG' \Psi_s'
\]

where \( G \) is the decomposition factor and \( \Psi_s \) keeps the above mentioned meaning.

This analysis also helps to test the results of the previous impulse response analysis. In particular, it checks whether or not the shocks of the variables have a trivial effect on the responses. Table 4.2 decomposes the forecast error into the part due to each innovation process. Specifically, it provides the decomposition due to each variable at each step. For instance, the first value in the first column provides the percentage of the one step forecast variance, which is due to the innovation in the change of the unemployment rate.
The more interesting information is at longer steps, where the interactions among the variables start to become felt. Figure 4.8 plots the decomposition of forecast variance due to each variable. Over the first 20 periods, the Variance of the consumer confidence Index is primarily explained by the change in the unemployment rate. An innovation in this variable takes almost 3 periods to have an effect but becomes quickly the prime mover. The importance of the real short term interest rate is increasing and becomes as important as the change in the unemployment rate after twenty-three periods. The explanatory power of the price-earnings ratio is increasing but

---

23 Each curve corresponds to one of the columns in table 4.3.
remains constantly lower with respect the other variables across the range.

Figure 4.8 shows graphically the decomposition.

Overall, the analysis mostly confirms the results of the impulse response analysis, namely the main driving force of Consumer confidence is the change of the unemployment rate. Nevertheless, in contrast with the results of the impulse response analysis, the real short term interest rate explains the smallest part of the Variance for the consumer confidence indicator.

Here, we also decompose for each variable the historical values of its variance into a base projection and the cumulated effect of current and past innovations. The historical decomposition is based upon the following partition of the moving average representation:
The first sum represents that part of $y_{t+j}$ which is due to innovations in periods $T+1$ to $T+j$. The second sum is the forecast of $y_{t+j}$ based on information available at time $T$.

If $u$ has $N$ components then the historical decomposition has $N+1$ parts. The forecast of $y_{t+j}$ based on the information at time $T$, is the term in brackets. For each of the $N$ components of $u$, the part of the first term is due to the time path of that component. Figure 4.9 shows the effect of the change in the unemployment rate, the price-earnings ratio and the real short term interest rate on the consumer confidence index.

Figure 4.9. CCI: Historical Variance decomposition
4.4. CONSUMER CONFIDENCE MODEL: A STRUCTURAL VAR MODEL

We use the historical Variance Decomposition in order to make a counterfactual analysis of recent developments in confidence. The shaded area in the charts shows the historical variance decomposition of the Consumer Confidence after the introduction of the euro. The analysis suggests that, in the pre-euro period, almost the entire difference between the actual consumer confidence index and the base forecast, with the exception of the period 1997-1999, can be attributed to the real short term interest rate. After the introduction of the euro, the change of the unemployment rate becomes the variable explaining most of this difference. This result is consistent with data relating the change of the unemployment rate one year ahead to the Consumer Confidence. Since 1999 a positive rate of the annual change in the unemployment rate flows with the behavior of Consumer Confidence.

4.4.4 Concluding Remarks

In this part, we have attempted to model the main driving force of consumer confidence in the Euro Area. To this end, we used a multivariate framework. The analysis suggests that the change of unemployment rate is the principal factor driving Consumer Confidence. A shock to the change in the unemployment rate affects the consumer confidence index mainly through two channels, respectively the expectations concerning the labour market and the general economic conditions. On the contrary, such a shock has a relative small effect on the other components. The developments in the stock market, measured by the price-earnings ratio, seem to be a minor
driving force. Similarly to the change in the unemployment rate it affects the consumer confidence index through the labour market and the general economic conditions. Consistently with the results of Jansen and Nahuis (2003), a change in the price-earnings ratio is not really perceived as an improvement in the personal financial situation. This could be due to the way the index has been constructed as well as to the fact that the shock is not perceived as permanent, because of the high volatility of price-earnings ratio. The change in the real short-term interest rate becomes the main driving force on the longer horizon. This variable affects the consumer confidence index mainly through the labour market conditions. A positive monetary policy shock is basically perceived by the households, as implying a future increase of the unemployment rate. Nevertheless, the effect on the expectations about their finances and savings over the twelve months is also quite strong.

4.5 Business Confidence model: a DSE model

The business confidence model has a broadly similar structure to the one for consumer confidence. In addition to the set of variables included in the previous model, we add here the growth rate of industrial production. This variable is understood as a good proxy for expected future production developments, which is an explicit question included in the surveys. Moreover, while consumer confidence is mainly affected by domestic factors, business confidence is influenced by external developments as well. Therefore, we ac-
count for the presence of key exogenous variables such as US manufacturing confidence, the effective exchange rate and oil price.

In this case the use of a VAR methodology is inappropriate. In a structural VAR model no distinction is drawn between endogenous and exogenous variables and the constraints are usually imposed on the simultaneous relationships matrix $A_0$. In this context we use a Dynamic Simultaneous Equations Model (DSEM). The structural form of the model follows:

$$
\begin{bmatrix}
  A_0 & B_0 \\
  0 & I_q
\end{bmatrix}
\begin{bmatrix}
  y_t \\
  x_t
\end{bmatrix}
= 
\begin{bmatrix}
  A(L) & B(L) \\
  0 & C(L)
\end{bmatrix}
\begin{bmatrix}
  y_t \\
  x_t
\end{bmatrix}
+ 
\begin{bmatrix}
  u_t \\
  v_t
\end{bmatrix}
$$

where $y_t = [D_{URX}, D_{IP}, PER, DR_{Stir}, EC_{man}]'$ is a $p$ dimensional vector of endogenous variables, $x_t = [D_{OILP}, PMI_{US}, EENJ]'$ is a $q$ dimensional vector of exogenous variables, and $u_t$ and $v_t$ are white noises processes. The $A_0$ and $B_0$ are $(p \times p)$ and $(p \times q)$, respectively describing the instantaneous relationships among the endogenous variables and between the exogenous and endogenous variables. The matrices $A(L), B(L)$ and $C(L)$ are the lag polynomials describing the relationships between variables at time $t$ and their on lags.

The selection of the VAR order is based on information criteria as for the consumer confidence model. Two of the tests reach their minimum for $k = 1$. Table 4.3 shows the results.

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24 The variables keep the above mentioned meaning.
25 We label with $D_{OILP}$ the first difference of Oil Price expressed in dollar while the other variables keep the above mentioned meaning.
26 In our model $p$ and $q$ are respectively equal to 5 and 3.
### VAR Lag Order Selection Criteria

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
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* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akake Information criterion
SC: Schwarz Information criterion
HQ: Hannan-Quinn Information criterion

Table 4.3. BCI: Optimal lag length

The order of the VAR is therefore the same as for the consumer confidence model. In a more explicit form, with one lag, the model can be represented as follows:

\[
\begin{bmatrix}
A_0 & B_0 \\
0 & I_q
\end{bmatrix}
\begin{bmatrix}
y_t \\
x_{t-1}
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
0 & C_1
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
x_{t-1}
\end{bmatrix} +
\begin{bmatrix}
u_t \\
v_t
\end{bmatrix}
\]
where $C_1$ is a $(q \times q)$ diagonal matrices modelling the $AR(1)$ exogenous processes. The $(p \times p)$ structural sub-matrix $A_0$ is equal to the structural matrix of the previous model. In addition, we impose some zero restrictions modelling the non-instantaneous relationship between the change in the unemployment rate, and the growth rate of the industrial production. The $(p \times q)$ sub-matrix $B_0$ imposing the instantaneous relationships between exogenous and endogenous variable is the following:

$$
B_0 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
b_1 & b_2 & b_3
\end{bmatrix}
$$

It reflects the assumption that all exogenous variables have a direct impact on the Business Confidence Indicator.

Given that the structural partitioned matrix and the submatrix $A_0$ are not singular, the reduced form of model can be written as follows\(^{27}\):

$$
\begin{bmatrix}
y_t \\
x_t
\end{bmatrix} = \begin{bmatrix}
A_0 & B_0 \\
0 & I_q
\end{bmatrix}^{-1} \begin{bmatrix}
A_1 & B_1 \\
0 & C_1
\end{bmatrix} \begin{bmatrix}
y_{t-1} \\
x_{t-1}
\end{bmatrix} + \begin{bmatrix}
A_0^{-1}u_t + A_0^{-1}B_0v_t \\
v_t
\end{bmatrix}
$$

Tests for the exogeneity of the variables are reported in table 4.5.

\(^{27}\)Notice that: $\begin{bmatrix} A_0 & B_0 \\ 0 & I \end{bmatrix}^{-1} = \begin{bmatrix} A_0^{-1} & A_0^{-1}B_0 \\ 0 & I \end{bmatrix}$
<table>
<thead>
<tr>
<th>OILP equation</th>
<th>PMI_US equation</th>
<th>EEN equation</th>
</tr>
</thead>
<tbody>
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<td>D_URX Lag(s) 1</td>
<td>D_URX Lag(s) 1</td>
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<tr>
<td>D_IP Lag(s) 1</td>
<td>D_IP Lag(s) 1</td>
<td>D_IP Lag(s) 1</td>
</tr>
<tr>
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<td>STIR Lag(s) 1</td>
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<tr>
<td>PER Lag(s) 1</td>
<td>PER Lag(s) 1</td>
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<td>EC_MAN Lag(s) 1</td>
<td>EC_MAN Lag(s) 1</td>
<td>EC_MAN Lag(s) 1</td>
</tr>
<tr>
<td>PMI_US Lag(s) 1</td>
<td>OILP Lag(s) 1</td>
<td>OILP Lag(s) 1</td>
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<td>EEN Lag(s) 1</td>
<td>EEN Lag(s) 1</td>
</tr>
<tr>
<td>Chi-Squared(7)= 9.563506 with Significance Level 0.21468743</td>
<td>Chi-Squared(7)= 12.423783 with Significance Level 0.03745576</td>
<td>Chi-Squared(7)= 12.466184 with Significance Level 0.08623327</td>
</tr>
</tbody>
</table>

Table 4.4 Testing the exogeneity of the variables in the Dynamic equation models

The coefficients are estimated by GLS. In particular, we use the estimator described in Theil(1971). The GLS estimator is:

$$\beta = (X' (\Sigma^{-1} \otimes I) X^{-1}) (X' (\Sigma^{-1} \otimes I) y)$$

where $\beta$ and $y$ are formed by stacking vectors from the N equations, and $X$ is formed by stacking augmented $X_i$ matrices: matrices with columns of zeros for explanatory variables in the other equations. The covariance matrix of the estimates is:
4.5. BUSINESS CONFIDENCE MODEL: A DSE MODEL

\[ V(\beta) = [X'(\Sigma^{-1} \otimes I)X]^{-1} \]

The maximization algorithm remains the one used for the CCI model (i.e. BFGS described in Press at all.). The model is estimated over the sample 1993:01-2003:12, on monthly data.

4.5.1 Data description

Similarly to the consumer confidence model, we use survey data provided by the European Commission for the Business Confidence indicator (BCI). The BCI is the arithmetic average of the balances (in the percentage points) of the answers to the questions related to production expectations, assessment of order books and assessment of stocks of finished goods (with an inverted sign).

The survey in manufacturing industry includes other questions (on selling prices expectations, employment expectations and production trend in recent months) which are not part of the business confidence indicator.

The real short term interest rate is computed as the difference between the three month Euribor and the annual percentage of producer price inflation. We use the three-month on three-month growth rate of industrial production excluding constructions given its strong correlation with BCI.

Regarding the exogenous variables, the Chicago PMI index provides a proxy for production expectation in US manufacturing. In addition, the price of oil refers to dated Brent and is expressed in euros, while the euro
exchange rate is measured in nominal effective terms.

Tests for stationarity of the variables are reported in table A4.4. The table shows that, with the exception of Chicago PMI index, the Dicky fuller test does not reject the unit root hypothesis for all variables. Nevertheless, we just take the first difference of the industrial production and the oil price due to the high correlation with the BCI index. For the Business confidence index and the nominal effective exchange rate we keep the variables in level. Regarding the first variable, we recall the argument described for the CCI index while for the second one, we believe that firms are more interested in the level of the nominal effective exchange rate than its change.

The roots analysis, reported in table A4.5 in appendix, shows that the estimated model does not contain explosive roots. Similarly to Consumer Confidence Model, residuals analysis in table A4.6 does not reject the normality of the residuals while the Permanenat test rejects the hypothesis of autocorrelation.

4.5.2 Impulse Response Analysis

Similarly to the consumer confidence model we implement an impulse response analysis\textsuperscript{28}. Figure 4.10 shows the reaction of the BCI and its subcomponents to various shocks. The reaction of the variables have the expected sign and die out within forty periods.

\textsuperscript{28}Technical details about the impulse response analysis have already been given in the previous part of the chapter.
The magnitudes of the responses are summarized in figure 4.11. It shows the average impact on the BCI and its sub-components in response to a shock in the other variables of the system.
Figure 4.11. Average responses of BCI to various shocks

The US production expectation in manufacturing appears to have the strongest impact on Business Confidence. This is also true when the BCI is replaced with one of the sub-components. The nominal effective exchange
rate and the price of oil affect Business Confidence to a smaller extent.

The timing of the response are represented in Figure 4.12.

Figure 4.12. BCI: Maximum impact and Timing

The first difference in the growth rate of the industrial production and unemployment rate reach the maximum impact on Industrial Confidence respectively after three and six months. The responses of the BCI following a shock to US production expectations in manufacturing and the first difference of Oil Price peaks after eleven months. The price-earnings ratio and the nominal effective exchange rate achieve the maximum effect around two years. The change in the real short term interest rate reach the maximum impact, after two months. The timing of the model slightly changes when replacing the BCI by its subcomponents. Among the other things it must be stressed the slower impact of the nominal effective exchange rate,
the price-earnings ratio and the oil price on the Business confidence and its subcomponents.

The first remarkable result of the impulse response analysis is that external conditions appear to affect considerably business confidence in the euro area. In particular, the US PMI is the single most important factor driving confidence in the model. Beyond the importance of the US economy as a trading partner, the strong correlation of euro area business confidence with the US PMI could also reflect the crucial role of US in determining global economic trends and the usual lead of the US business cycle with respect to that in Europe. The peak of the effect of the US PMI is after about a year, which broadly corresponds to the average gap between the US and European cycle. Oil price and exchange rate developments also seem to have a significant role in driving business confidence, albeit significantly less so than US economic conditions. The negative impact of an appreciation of the euro exchange rate appears to have a stronger and more lasting effect on confidence than an increase in oil prices.

Among domestic variables, actual industrial production seems to be the most important driver of business confidence in the short term, followed by the changes in the real short term interest rates and the unemployment rate.

As regards the sub-components of the BCI, from the magnitude of the responses, it appears that the assessment of order books is the most reactive component of business confidence, which reflects the fact that it most directly captures firm's demand expectations. It is particularly sensitive to the
US PMI, euro area industrial production and the exchange rate. Changes in the unemployment rate also seem to be an important factor affecting the assessment of order books, probably as a proxy for households' disposable income developments. Among other components of confidence, production expectations seems relatively more reactive than others to changes in stock-market valuation. Changes in stockmarket valuation can be viewed as supply side shocks affecting firms' financing conditions, and therefore changing the optimal level of production irrespective of demand expectations. Interest rates seem to affect confidence equally through its three sub-components: production expectations, assessment of stocks and order books. Finally, a relatively small effect of oil price shocks seems to come mainly via order books and the assessment of stocks, while production expectations seem little affected.
4.5.3 Forecast Error Decomposition

In the same way as for the consumer confidence model, we then proceed with a forecast variance decomposition. Table 4.5 decomposes the forecast error into the parts due to each of the innovation processes:

![Table 4.5 Forecast Error Variance Decomposition](image)

In particular, it provides the decomposition due to each variable at each step. The graphical representation of the forecast variance decomposition is
The variance of the business confidence index is primarily explained by the US production expectation of manufacturing. Innovation in this process quickly becomes the prime mover.

The growth rate of the industrial production and the change in the real short term interest rate show their effect mainly in the first months. Then, their effect remain fairly constant. Even though the explanatory power of the price-earnings ratio is slightly higher than the previous variable, it is still quite low. The nominal effective exchange rate explains an increasing

---

29 Each curve corresponds to one of the columns in table 4.5.
part of the variance while the other variables seem to explain just a small part of it.

The forecast Error Decomposition basically confirms the results of the impulse response analysis. The dominating key factor of the industrial confidence indicator is the US production expectation of manufacturing. The growth rate of industrial production affects confidence basically in the short run. The nominal effective exchange rate seems to show its effects on the longer horizons.

4.5.4 Concluding remarks

This part of the chapter attempted to detect the main driving force of Business Confidence in the Euro Area.

Similarly to the Consumer Confidence Model, we used a multivariate framework. The analysis suggests that, the among the variables of our model, the production expectation in US manufacturing is the principal factor driving the Business Confidence Indicator in the Euro Area. The US confidence indicator basically leads the Industrial Confidence Indicator. This result is supported by the magnitude and the timing of the responses. The Forecast Variance Decomposition confirms the results of the impulse response analysis.

The cross comparison analysis of the average responses gives further insights. The most reactive component of the Industrial Indicator is the index measuring the assessment of the order books. The result is consistent among all sub-components with the exception of the index measuring the
4.6. **GENERAL CONCLUSION**

In this chapter, we have analysed the key determinants of economic confidence in the Euro Area since 1993. We set up two multivariate frameworks to model respectively consumer and business confidence.

The results suggest that consumer confidence is mainly driven by labour market developments in the Euro Area (measured by the change in the unemployment rate). The real short-term interest rate seems to play a key role in the long run. Stock market developments play a significant, but smaller role than the other factors, and smaller than the available literature seems to suggest. Stock market developments affect the consumer confidence index essentially, through its component measuring the way households perceive general economic conditions.

Regarding the results for business confidence, the dominating factor is a measure of business confidence in the United States, which suggests a rather strong role for the world economic cycle on Euro Area business confidence. Among the endogenous variables, industrial production and real short term interest rate seem to have a significant effect in the short run, but this effect is of a lesser magnitude than the external factor just mentioned. In addition, stock market developments are assessed to have a larger effect on the confidence of firms compared to households. Regarding the breakdown of the business confidence indicator into its sub-components, the most reactive
component is the index describing the assessment of order books.

Looking at the impact of monetary policy more in detail, it appears that changes in interest rates have the expected negative effect both on consumer and business confidence. A comforting result also concerns the timing of these effects, which is similar to the transmission lag of monetary policy on economic activity commonly found in the literature. These results provide some evidence for the presence of a "confidence channel" in the transmission of monetary policy on real activity. The models could be extended to include price variables to also analyse further, the transmission of monetary policy on inflation. However, the fact that the effect of interest rates on consumer confidence has a similar timing as the effect on real economic activity leads to a cautious interpretation. Indeed, the model of consumer confidence does not include any variable of economic activity (other than the change in the unemployment rate), meaning that the confidence indicator itself could be argued to capture, in fact, the effects on economic activity. The results of the business confidence model tend to rule out the presence of such a problem. Indeed, the timing of the response to a shock in interest rates is similar to that in the consumer confidence model, while the business confidence model includes a variable controlling for economic activity (the industrial production variable). Thus, interest rates can be argued to affect business confidence beyond their impact on economic activity.

However, further analysis is needed before concluding about the relative influence of monetary policy on the confidence of economic agents in the
4.6. GENERAL CONCLUSION

Euro Area compared to other factors.

First, it cannot be ruled out that some variables may have been omitted in the models proposed in this chapter. Their omission may lead to bias the results.

Second, the analysis of monetary policy shocks could be refined. In this respect, shocks are simply defined in the multivariate frameworks proposed in this chapter as a one standard deviation change in a given variable. Alternative ways of measuring shocks would need to be used to address the impact of monetary policy on confidence. In particular, policy shocks can be measured as the deviations of the actual policy rates decided by monetary policy-makers compared to the expected interest rates. Monetary policy shocks can also be seen as indicators of the monetary policy stance. This policy stance can be measured by the deviation of actual real interest rates from their natural level. Overall, alternative ways of measuring policy shocks may lead to different assessments of the impact of monetary policy on confidence.

Finally, central banks do not affect confidence only through changes in interest rates. It is commonly argued that central banks pursuing a policy of the "steady-hand" (i.e. of keeping interest rates unchanged) make a positive contribution to confidence.
Part III

Fiscal policy issues
Chapter 5

The sustainability of the fiscal policy: Empirical Evidence in Euro Area and United States

This Chapter provides a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint, and derives conditions that determine whether a fiscal stance is sustainable in the medium and long term. In contrast to previous studies, it uses a log-linearization of the public debt identity and generalizes the results obtained in literature by using a multivariate test. The analysis is applied to the fiscal position of the United States and Euro Area. On the basis of infinite horizon-tests the broad conclusion is that both countries have an unsustainable fiscal policy. The chapter also provides two procedures to construct forecasts of the future level of public debt. Forecasted values confirm the evaluation proposed in the cointegration analysis.
5.1 Introduction

The sustainability of fiscal policy has been receiving increasing attention from economists. Several developed countries have been facing significant fiscal deficits in the last few decades. In the U.S, for instance, the fiscal year 2000 ended with the highest dollar debt in the country's history - despite more claims of a 'surplus', fiscal year 2001 debt was higher than that and, 2002 debt was $420 billion higher. The same is true for some of the EMU countries where, despite the imposition of deficit and debt ceilings, the public debt is higher than 100% i.e. Italy and Belgium. The main question about this issue is whether or not the high public debt is becoming more and more unsustainable.

This chapter provides a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint, and derives conditions that determine whether a fiscal stance is sustainable. This analysis is then applied to the fiscal position of the United States and the Euro Area.

Most of the literature on this topic uses deflated variables and is focused on the peculiarities of the process generating public debt series in ratio to GDP. In particular, fiscal policy is declared sustainable if public debt, or the first difference, are found to be stationary. The stationarity of this variable (or the first difference) is usually conditioned to the stationarity of the explanatory variables, or to the presence of a cointegrated relationship between them. Most of these studies use univariate tests.
The theoretical framework provided in this chapter differs from the existing literature, in several important aspects. In contrast to previous studies, it uses a log-linearization of the public debt identity and generalizes the results obtained in literature by using a multivariate test.

In particular, the contribution of this study is twofold. First, it shows that, under the assumption of a stochastic interest rate, the sustainability of fiscal policy can be considered as a linear problem and then becomes suitable to be studied through a VAR technique. Second, it provides two different procedures to compute forecasts of the public debt level conditioned to given processes generating primary deficit and interest rate. The former is related with the public debt identity in a VAR form and the latter arises from the iteration n-periods ahead of the public debt identity in non-linear form.

The use of a multivariate test is justified by several considerations. First, by using a multivariate test, it is possible to apply specific tests for VAR models to the sustainability of the fiscal policy i.e. roots analysis. Second, under a Vector Error Correction representation, the VAR model permits to apply a cointegrated analysis to the data involving all variables in the system.

The sustainability of fiscal policy has been studied under different scenarios. In the former case, we assume both primary deficit and interest rate as exogenous variables. In the latter case, we assume the public debt and the primary deficit as endogenous variables and the interest rate as the only exogenous variable.
The analysis suggests that the condition for sustainability depends upon the stability of the VAR system. If the system is globally stationary, and then stable, the fiscal policy is said to be sustainable. On the contrary, if the system is not globally stationary, and then not stable, the fiscal policy is said to be unsustainable.

The chapter is organized as follows. Section 5.2 provides a short review of the fiscal deficit arithmetic; it derives the government’s intertemporal budget constraint and the condition of sustainability in discrete time. Section 5.3 looks at the way in which fiscal sustainability has been assessed in the literature so far. Section 5.4 discusses the public debt identity in a VAR framework, and provides the conditions for sustainability. Two cases are presented in which different assumptions about the stochastic processes generating the primary deficit and interest rate are examined. Section 5.5 applies this test to the U.S and Euro Area economies. Section 5.6 constructs forecasts of the public debt level conditioned to given processes generating primary deficit and interest rate. Finally Section 5.7 contains concluding comments.

5.2 The Government intertemporal budget constraint.

Previous to the sustainability issue, a short review of the fiscal deficit arithmetic seems to be necessary in order, to set the theoretical framework for the subsequent sustainability analysis.
5.2. **THE GOVERNMENT INTERTEMPORAL BUDGET CONSTRAINT.**

The government borrowing constraint (IBC) for period $t$ may be written with the following dynamic equation:

$$ B_t = G_t - T_t + (1 + \rho_t)B_{t-1} - (M_t - M_{t-1}) $$

(5.1)

where the variables are defined as follows: $G$ - government spending excluding interest payment; $T$ - government revenues; $B$ - public debt; $M$ - nominal monetary base; $\rho$ - real interest rate.

Under the assumption of a constant money growth rate the identity (5.1) becomes:

$$ B_t = G_{t-1} - T_{t-1} + (1 + \rho_{t-1})B_{t-1} $$

(5.2)

The IBC is normally presented with variables as ratios of GDP. This is due to the common belief that the level of the public debt in nominal terms, is a misleading measure of the real indebtedness of the Government. In terms of ratio to GDP equation (5.2) can be expressed as follows:

$$ b_t = g_t - \tau_t + (1 + \rho_t - \eta_t)b_{t-1} $$

where the lower case letters $g$, $b$, $\tau$ denote the ratio of the corresponding upper-case variables to nominal GDP, $\rho_t$ is the real interest rate at time $t$ and $\eta_t$ represents the output growth rate. In a more compact form, this identity can be rewritten as:
\[ \Delta b_t = d_t + r_{t-1} b_{t-1} \] (5.3)

where \( d_t = g_t - \tau_t \) is the primary government deficit expressed as proportion of nominal GDP and \( r_t \) is the real interest rate adjusted for output growth. Equation (5.3) states that the change in the stock of public debt from one period to the next must cover the budget deficit inclusive of interest payments \( r_b_{t-1} \).

The traditional approach in investigating the sustainability of countries' fiscal policy is based on the intertemporal budget constraint of the government. This is obtained solving the equation of the debt forward for periods \( t + 1, t + 2, t + 3, \ldots, t + n \):

\[
\begin{align*}
  b_t = & \sum_{s=1}^{\infty} \frac{d_s}{1 + r_{t+s}} + \prod_{j=1}^{s} \frac{b_{t+s}}{(1 + r_{t+j})}
\end{align*}
\]

In the long term fiscal policy is said to be sustainable if the present value of the existing stock of public debt is identical to the present value of future primary surpluses:

\[
  b_t = \sum_{s=1}^{\infty} \frac{d_s}{1 + r_{t+s}}
\]

Thus, a necessary and sufficient condition for sustainability is that the right hand side of the intertemporal equation is zero as \( n \to \infty \):

\[
  \lim_{s \to \infty} \prod_{j=1}^{s} \frac{b_{t+s}}{(1 + r_{t+j})} = 0
\]
This also known as transversality condition and implies no Ponzi games, meaning no new debt is issued to meet new interest rate payments.

This condition does not mean that debt should go to zero at any point in time. The debt can also grow at a positive rate. Of course a permanent positive growth rate is inconsistent with the above equation. A deficit at any point in time (or over a period of time) has to be offset by a surplus at another point in time (Uctum-Wickens, 1997).

In the medium term, fiscal policy is sustainable, or intertemporally consistent, if it is able to achieve a given target level of the Debt-GDP ratio $b^*$. The Government borrowing constraint becomes:

$$b_t - b^* = \sum_{s=1}^{T^*} \frac{dt}{\Pi_j (1 + rt+j)}$$

where $b^*$ denotes the desired level of debt-GDP ratio at the end of the planning period $T^*$.

The transversality condition, then, slightly changes in:

$$\lim_{s \to T^*} \left( \prod_{j=1}^{T^*} \frac{b_{t+j}}{(1 + rt+j)} \right) = b^*$$

The inclusion of a time constraint in the analysis leads to further considerations. In the previous case, as long as the output growth is higher than real interest rate ($\rho < \eta$), the discounted value of debt approaches zero as time progresses; this happens independently of the initial level of the debt.

In this case the possibility of achieving the target level also depends upon the gap between the current and the target level of debt ($b_t - b^*_t$) as a
time constraint to be respected exists. Thus, fiscal policy can be said to be sustainable if the previous conditions are respected, and if the government is able to generate surpluses capable of compensating for such a gap and for the deficits that have arisen in the interval $T^* - t$.

The previous considerations rely on the assumption of a constant interest rate and a constant output growth rate. In the following we try to derive conditions for the sustainability of the fiscal policy, when a stochastic interest rate is taken into account.

5.3 Literature review


In these works the main tools used to analyse the sustainability of budget deficits seem to be stationarity tests for the stock of public debt, and cointegration tests between government expenditures and government revenues (or alternatively stock of public debt and public deficit). Some results of
5.3. LITERATURE REVIEW

these works follow.

Hamilton and Flavin(1986) suggest that a sufficient condition for the Present value of the Budget constraint (PVBC) to hold is that the primary balance, and therefore that public debt, is a stationary series. It should be noted that this is a sufficient but not necessary condition for sustainability; fiscal policy could be sustainable even if debt is non-stationary. They find that non stationarity can be rejected and that the PVBC therefore is not violated.

Trehan and Walsh(1988) argue that if debt is integrated of order 1, and if the (expected) real interest rate is constant, a necessary and sufficient condition for the PVBC to hold is that debt and the primary fiscal balance \((d_{t-1}, b_{t-1})\) are cointegrated. Looking at the equation (5.3) one can see that if \(b\) is I(1), the change in debt must be stationary by definition; the overall deficit, \(d_{t-1} + rb_{t-1}\), is I(0). Thus, if \(r\) is constant, \(d_{t-1}\) and \(b_{t-1}\) are cointegrated with a cointegrating vector \((1, r)\).

Three years later the same authors(Trehan and Walsh,1991) end up with conclusion that if the (expected) real interest rate is not constant: sustainability no longer implies that \((d_{t-1}, b_{t-1})\) are cointegrated. A sufficient condition for the PVBC to hold is that the overall deficit, \(d_{t-1} + rtb_{t-1}\), is stationary.

The approach of Hakkio and Rush(1991) is to test the cointegrated relationship between public spending and level of taxes. Their work relies on the hypothesis that the real interest rate is a stationary variable with mean
The authors assert that when there is cointegration between these variables, the fiscal deficit is not sustainable; when there is cointegration with coefficient $\beta = 1$ the deficit is sustainable; when there is cointegration with $\beta < 1$, government expenditures are growing faster than the revenues, and the deficit may not be sustainable.

Wilcox (1989) shows that when the transversality condition holds, the present value of government debt is stationary and has an unconditional mean of zero. For the U.S. he finds mixed evidence on stationarity and rejects an unconditional mean of zero, thus concluding that postwar U.S. fiscal policy has been unsustainable.

Wickens and Uctum (1997), by extending the results of Wilcox (1989) to the case where the discount rate is stochastic and time varying, and assuming the discounted primary deficit either exogenous or endogenous, show that a necessary and sufficient condition for sustainability is that the discounted debt-GDP ratio should be a stationary zero-mean process.

Lastly, Roberds (1991) found that the transversality condition implies cross-equation restrictions on the stochastic processes generating debt and deficits in a VAR framework.

The test implemented in the rest of the chapter, similarly the approach proposed by Wilcox (1989), Trehan-Walsh (1991) and Wickens and Uctum (1997), accounts for a stochastic interest rate. However, this study differs from the previous works in two important aspects. It uses a log-linear form of the
5.4 A VAR FRAMEWORK

The main aim of this section is to assess the sustainability of fiscal policy by using a VAR framework.

The sustainability of public deficits is determined by the dynamics of the public debt. The factors which influence such dynamics can be derived from the intertemporal budget constraint (IBC). At time \( t \) it is as follows:

\[
b_t = d_{t-1} + (1 + r_{t-1})b_{t-1}
\]

where the variables keep the above mentioned meaning.

In this form the IBC is not suitable to be studied by linear models. Under the assumption of a stochastic interest rate, the public debt is a non-linear combination of the explanatory variables. In order to express the IBC in a linear form, we take the log linear approximation of equation (5.2). This solves the non-linearity of the model and allows us to express the IBC in terms of a VAR(1) model.

In log linear form the IBC becomes:

\[
\ln b_t = \kappa + \beta^{-1} \ln b_{t-1} + \frac{d}{b} \ln d_{t-1} + \beta^{-1} r_{t-1}
\]  

(5.4)

\(^1\) As we mentioned in the introduction, to express the IBC in a log-linear form allows for estimating the model in a VAR framework.

\(^2\) See appendix 5.1 for the proof.
where \(\beta\) is the discount factor \((1/(1 + \tau))\) and \(\bar{b}, \bar{d}, \bar{r}\) are respectively the sample means of public debt, primary deficit and real interest rate adjusted for the output growth rate.

Equation (5.4)\(^3\) represents one of the equations of a three dimensional VAR(1) model having respectively the logarithm of the deflated public debt, the logarithm of the deflated primary deficit and the real interest rate as endogenous variables. The VAR(1) system can be represented as follows:

\[
y_t = K + \Pi_y y_{t-1} + \epsilon_t
\]  

(5.5)

where \(y_t\) is the vector of the endogenous variables \([\ln b_t, \ln d_t, r_t]\), \(\Pi_y\) is the \((3 \times 3)\) matrix of the coefficients and \(K\) is the \((3 \times 1)\) vector of constants.

The sustainability of fiscal policy depends upon the stability of model (5.5). In particular, if model (5.5) is globally stationary, fiscal policy is said to be sustainable. If model (5.5) is not globally stationary, fiscal policy is said to be unsustainable. The stationarity of the VAR system is investigated in two steps. First, we apply a roots analysis and, then, if necessary, we conduct a cointegration analysis. The analysis is conducted under two scenarios. We assume the interest rate to be exogenous with respect to the remaining variables in the system. Under this hypothesis, following the approach proposed by Wickens and Uctum (1997), we leave the discounted primary deficit to

---

\(^3\)A stochastic error term has been added in order to capture the error due to the log linear approximation.
be either exogenous or endogenous with respect to the public debt. The former case reflects a situation in which the Government, in setting public spending, neglects both the level of public debt and the level of real interest rate. On the contrary, the latter case describes a situation in which the government, in setting the public spending at time $t$, takes into account both the level of the interest rate and the level of public debt at time $t-1$. For a Government devoted to reducing the level of public indebtedness, negative feedbacks from discounted debt and level of the interest rate are expected. Government reacts to an increase of the interest rate and or an increase of the public debt at time $t-1$, by reducing the primary deficit or by generating primary surpluses over time.

\[ a) \text{Exogenous interest rate and exogenous primary deficit:} \]

A VAR(p) model including exogenous variables can be represented as follows:

\[ y_t = \sum_{i=1}^{k} \Pi_{y,i}y_{t-i} + \sum_{i=0}^{k} \Pi_{z,i}z_{t-i} + \varepsilon_t \quad (5.6) \]

where $\Pi_y$ is $(p \times p)$ matrix, $\Pi_z$ is $(p \times q)$ matrix, $z_t$ is the vector of the exogenous variables and $\varepsilon_t$ is a Gaussian process with zero mean and constant variance\(^4\). The other symbols keep the above mentioned meaning.

If the system is a first order VAR, model(5.6) can be represented:

\[ ^4 \text{p and q represent respectively the number of endogenous and exogenous variables.} \]
\[ y_t = \Pi_{y,i} y_{t-1} + \Pi_{z,i} z_{t-1} + \varepsilon_t \] (5.7)

In general, the estimated roots of the autoregressive polynomial in model (5.7) are obtained by the solutions of the equation:

\[ |I_{P_y} - \sum_{i=1}^{k} \Pi_{y,i} z^{-i}| = 0 \] (5.8)

The necessary and sufficient condition for stability is that the modulus of the roots in reverse characteristic equation (5.8) are smaller than one. The system is said to be unstable if the system contains roots equal to one. Nevertheless, if the variables are cointegrated, the system is globally stationary, and then stable, even though the system contains unit roots. The system is said to be explosive if one of the roots is greater than one.\(^5\)

Under the assumption of exogenous interest rate and exogenous primary deficit, model (5.6) includes only an endogenous variable: \( \Pi_{y,i} \) is just a scalar. The VAR is reduced to a univariate process. The stability of the system only depends upon the root of the public debt series. If \( \Pi_{y,i} \) is smaller than one, the system is said to be stable. The root of the system, that we know to be equal to \((1 + \bar{r})\) from the IBC in log-linear form, depends upon the

\(^5\)The condition for the stability of the VAR can be stated in terms of the roots of characteristic and reverse characteristic polynomial and can be confusing if it is not clear which polynomial is being referred to. In terms of the characteristic polynomials \( (\Pi_1 - zI) \) the stability condition is that all eigenvalues (roots) have modulus less than one. In terms of the reverse characteristic polynomials \( (I - \Pi_1 z) \) the stability condition is that none of its eigenvalues (roots) have modulus less than one. The former case is the one applied in this paper.
sample mean of the interest rate. If the sample mean of the real interest rate adjusted for the output growth rate is negative then $\Pi_{y,t}$ is less than one and the fiscal policy is said to be sustainable. If the sample mean $\bar{r}$ is positive, the coefficient $\Pi_{y,t}$ is greater than zero the system is explosive and the fiscal policy is said to be unsustainable. Finally, if the sample mean of the interest rate (adjusted for the output growth rate) is equal to zero (or close to zero), $\Pi_{y,t}$ is equal to one and cointegration analysis is required in order to assess the sustainability of the fiscal policy.

Thus, the system is globally stationary if the processes generating the exogenous variables (i.e. primary deficit and interest rate) and the public debt are cointegrated. The cointegration analysis follows\footnote{Notice that, since the remaining parameters involved in model(5.6) are not considered in this analysis, stationarity is meant as stationary around some deterministic components or stationary around, some possibly non stationary pattern determined by exogenous variables.}.

As VECM, model(5.6) may be reparameterised as follows:

$$\Delta y_t = \sum_{i=1}^{k-1} \Gamma_{y,i} \Delta y_{t-i} + \sum_{i=0}^{k-1} \Gamma_{z,i} \Delta z_{t-i} + \Pi_{y,i} y_{t-1} + \Pi_{z,i} z_{t-1} + \varepsilon_t \quad (5.9)$$

where the $\Gamma_{y,i}$ 's, the $\Gamma_{z,i}$ 's, $\Pi_y$ and $\Pi_z$ are obtained from $\Pi_{y,i}$ 's and the $\Pi_{z,i}$ 's by solving:

$$I_{P_y} - \sum_{i=1}^{k-1} \Pi_{y,i} L^i = (I_{P_y} - \sum_{i=1}^{k-1} \Gamma_{y,i} L^i)(1 - L) - \Pi_y L$$

$$\sum_{i=0}^{k-1} \Pi_{z,i} L^i = \sum_{i=0}^{k-1} \Gamma_{z,i} L^i(1 - L) - \Pi_z L$$
When the exogenous variables $z_t$ are included, the dynamic properties as well as the interpretation of (5.9) will depend on the assumption about the full model for $y_t$ and $z_t$. There are basically two alternative assumptions in the literature. In the first (Johansen, 1992a, Harbo at all, 1998) $z_t$ is assumed to be determined by a model like:

$$
\Delta z_t = \sum_{i=1}^{k-1} G_{y,i} \Delta y_{t-i} + \sum_{i=0}^{k-1} G_{z,i} \Delta z_{t-i}
$$

In the second (Rahbek-Mosconi, 1998, Mosconi-Rahbek, 1996), $z_t$ is assumed to be determined by a model of the type:

$$
\Delta z_t = \sum_{i=1}^{k-1} G_{y,i} \Delta y_{t-i} + \sum_{i=0}^{k-1} ab' z_{t-i}
$$

and the stationary linear combinations of the levels of $z_t$ is allowed to enter the conditional subsystem without any restrictions. We have chosen the model proposed by Johansen(1992a).

Under this assumption, $\Pi_y$ and $\Pi_z$ are assumed to share the same column space i.e.

$$
\Pi_y = \alpha \beta'_y , \Pi_z = \alpha \beta'_z
$$

so that model(5.9) may be rewritten in the form:

$$
\Delta y_t = \sum_{i=1}^{k-1} \Gamma_{y,i} \Delta y_{t-i} + \sum_{i=0}^{k-1} \Gamma_{z,i} \Delta z_{t-i} + \alpha \beta'_y y_{t-1} + \alpha \beta'_z z_{t-1} + \epsilon_t
$$

(5.10)
where $\alpha$ is the loading matrix and $\beta'$ is the matrix of the cointegrated relationship.

Since model (5.5) is a VAR(1), equation (5.9) becomes:

$$\Delta y_t = \alpha_y \beta'_y y_{t-1} + \alpha_z \beta'_z z_{t-1} + \varepsilon_t$$

(5.11)

where $\alpha_y$ is $(p \times p)$ matrix and $\alpha_z$ is $(p \times q)$ matrix.

As mentioned above, if the system contains unit roots the system is said to be unstable. Nevertheless, if the variables are cointegrated, the system is said to be globally stationary and then stable, even though the system contains unit roots. In this case, the system is said to be stationary if the only endogenous variable (i.e. $b_t$) and the remaining $I(1)$ processes in the system are cointegrated. If the interest rate is assumed to be a stationary process, the cointegrated vector is between the public debt and the primary deficit. It is as follows:

$$\xi_{t-1} = (1, \beta_1, 0) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}$$

If the primary deficit is a stationary process the cointegrated vector is between the public debt and the interest rate. It is as follows:

$$\xi_{t-1} = (1, 0, \beta_1) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}$$

The stationarity of the real interest rate is an open issue and is strictly related with time series properties of the consumer price index (CPI). If the CPI is an $I(1)$ variable, the real interest rate is not stationary as the inflation rate is $I(0)$. If the CPI is an $I(2)$, the inflation rate is $I(1)$ and then nominal interest rate and inflation rate cointegrate with cointegrating vector $(1, 1)$, in the long run (St-Amant, 1996).
If the exogenous variables (i.e. primary deficit and interest rate) are both $I(1)$ processes, a cointegrated relationship among all the variables is necessary in order for the system to be globally stationary. The cointegrated vector is:

$$\xi_{t-1} = (1, \beta_1, \beta_2) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ \pi_{t-1} \end{bmatrix}$$

b) Endogenous primary deficit and exogenous interest rate

Under the assumption of an endogenous primary deficit, system (5.5) includes two endogenous variables and one exogenous variable. In this case, the matrix $\Pi_{y,t}$ is a $(2 \times 2)$ matrix and $\Pi_{z,t}$ is a $(2 \times 1)$ vector. In a more explicit form, model (5.5) can be expressed as follows:

$$\begin{bmatrix} \ln b_t \\ \ln d_t \end{bmatrix} = \begin{bmatrix} (1+\bar{\pi}) & \bar{d} \\ \bar{b} & \bar{\pi}_2 \end{bmatrix} \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \end{bmatrix} + \begin{bmatrix} (1+\bar{\pi}) \\ \bar{\pi}_2 \end{bmatrix} \pi_{t-1} + \begin{bmatrix} \varepsilon_{t1} \\ \varepsilon_{t2} \end{bmatrix}$$

Unlike the previous case, the roots of the system will depend not only on the sample mean of the real interest rate ($\bar{\pi}$) but also on the sample mean of the primary deficit (i.e. $\bar{d}$), the sample mean of the public debt (i.e. $\bar{b}$), the parameter measuring the feedback of the government with respect the public debt (i.e. $\pi_{21}$) and the autoregressive coefficient of the primary deficit (i.e.$\pi_{22}$).

The system is stable and the fiscal policy sustainable, if the system contains both roots smaller than one and the process generating the only exogenous variable (i.e. interest rate) is stationary. The system is explosive,
and the fiscal policy is unsustainable, if one or both roots are greater than one. Finally, the system is unstable if it contains unit roots. In this case, a Cointegration analysis is necessary in order to assess the stability of the system and, thus, the sustainability of the fiscal policy.

The Vector Error Correction representation of model (5.12) is as follows:

\[
\begin{bmatrix}
\Delta \ln b_t \\
\Delta \ln d_t
\end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22} \end{bmatrix} \beta' \begin{bmatrix} \ln b_{t-1} \\
\ln d_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{13} \\
\alpha_{23} \end{bmatrix} \beta'_z r_{t-1} + \begin{bmatrix} \varepsilon_{t1} \\
\varepsilon_{t2} \end{bmatrix} (5.13)
\]

where \( \alpha_{ij} \) are the coefficients of the loading matrix. \( \beta'_y \) and \( \beta'_z \) represent the sub-matrices of the partitioned matrix \( \beta' \). The dimension of \( \beta' \) depends upon the number of cointegrated relationships necessary, in order, for the VAR model to be stationary. If the system contains two unit roots, there is just one cointegrated relationship. In this case, the matrix of cointegrated relationship \( \beta' \) is reduced to be a vector. It follows:

\[ \beta' = (\xi_{1t-1}) \]

If the interest rate is stationary, the cointegrated vector is between the public debt and the primary deficit. It is as follows:

\[ \xi_{1t-1} = (1, \beta_1, 0) \begin{bmatrix} \ln b_{t-1} \\
\ln d_{t-1} \\
r_{t-1} \end{bmatrix} \]

If the system contains a unit root and the process generating the exogenous variable is not stationary, the cointegrated vector is between the public debt and the interest rate\(^9\). It is as follows:

\(^9\)The cointegrated vectors have been both normalised with respect to the public debt.
When the system (5.12) contains two unit roots and the process generating the exogenous variable is not stationary, there must be two cointegrated relationships in order for the system to be globally stationary. In this case the matrix of the cointegrated relationship $\beta'$ is the following:

$$
\beta' = \begin{pmatrix}
\xi_{1t-1} \\
\xi_{1t-2}
\end{pmatrix}
$$

where $\xi_{1t-1}$ and $\xi_{1t-2}$ are the two cointegrated vectors.

The first cointegrated relationship is between the endogenous variables (i.e. $b_t$ and $d_t$) in the system. It is as follows:\(^{10}\)

$$
\xi_{1t-1} = (1, \beta_1, 0) \begin{bmatrix}
\ln b_{t-1} \\
\ln d_{t-1} \\
\tau_{t-1}
\end{bmatrix}
$$

The second cointegrated relationship is between one or both of the endogenous variables (i.e. $b_t$ or $d_t$) and the only exogenous variable (i.e. $r_t$).

If the primary deficit and the interest rate are cointegrated, the cointegrated vector $\xi_{1t-2}\(^{11}\)$ is:

$$
\xi_{1t-2} = (0, 1, \beta_1) \begin{bmatrix}
\ln b_{t-1} \\
\ln d_{t-1} \\
\tau_{t-1}
\end{bmatrix}
$$

If the public debt and the interest rate are cointegrated the cointegrated vector $\xi_{1t-2}\(^{12}\)$ is:

---

\(^{10}\)The vector has been normalized with respect to the public debt.

\(^{11}\)The CV has been normalized with respect to primary deficit.

\(^{12}\)The CV has been normalized with respect to public debt.
If the second cointegrated relationship involves all variables in the system, the cointegrated vector $\xi_{1t-2}$ is:

$$\xi_{1t-2} = (1, 0, \beta_1) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}$$

5.5 Empirical evidence

In this section, we describe the data and the econometric methodology used in this chapter. The data used in the analysis are quarterly and are taken respectively from NIPA and ECB website. The U.S sample covers the period 1966 to 2002. The Euro Area sample, due to unavailability of aggregate data, covers the shorter period 1977 to 1997.

The econometric methodology follows in three steps: in the first step, we study the stationarity properties of the time series by using unit root tests; in the second step, we conduct classic tests aiming to test the specification of the model and the stability of the system (i.e. Lag length, exogeneity test and roots analysis). In the final step, given that the variables are not stationary, we apply a cointegration analysis.

5.5.1 Univariate analysis

The unit root test applied is the Augmented Dickey Fuller (ADF) test (see for example Said-Dickey, 1984). This test is performed to determine the

---

13 The CV has been normalized with respect to public debt.
series order of integration. The order of integration of a series is the number of times a series needs to be differenced to achieve stationarity. The test is described in the appendix 5.2.

If the null hypothesis is not rejected the series is non-stationary. In particular, if the first difference is stationary, the series is integrated of order one. For testing the stationarity of the first difference of the variable we apply again the ADF test to the first difference of the series. If the null of a unit root is rejected, the series in differences is stationary and thus the series in level is I(1). If the null hypothesis is not rejected, the series of the first differences is not stationary and one performs the ADF test on the second differences and so on.

Tables 5.1 and 5.2 show the results of the Dicky Fuller test for both countries. Specifically, it shows the statistics, as well as the quantiles of their asymptotic distribution, for each variable in level. The null hypotheses under which the asymptotic distributions are tabulated are always joint hypotheses concerning the coefficient(ρ), the mean(μ), and the trend(β). The reason is clearly explained in Hamilton(1994). The first three statistics are referred to the model without trend, while the remaining statistics to model with trend. The asymptotic distribution of all tests on the coefficients of this regression is not standard, but is known and tabulated. Thus, the tables

\[ \text{See appendix 5.2 for further details about the Dicky Fuller and the Augmented Dicky Fuller Test.} \]
\[ \text{See model A 5.2.1 in the Appendix 5.2.} \]
\[ \text{See model A 5.2.3 in the Appendix 5.2.} \]
\[ \text{The test statistic does not have a normal distribution so that it would be inappropriate to use conventional normal or 't' tables to look up the critical values. Appropriate, critical} \]
also report the references of the asymptotic distribution which the tests refer to\textsuperscript{18}. The shaded areas highlight the tests not rejecting the unit root hypothesis. All tests suggest that variables are non-stationary.

Table(5.3) and table(5.4) show the same tests for the first differences of the variables. All tests reject the null hypothesis. The first differences of the variables are stationary. This proves that the variables in level are integrated of order one I(1).

\footnotesize
\textsuperscript{18}For instance, the first row of the table gives the critical value tabulated by Fuller(1976, p.373, Block I) while the third row reports the critical value tabulated by Dicky-Fuller(1981, tab.1)
### Table 5.1 United States: The ADF test for the variables in level

<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0, μ=0</td>
<td>-0.88</td>
<td>-19.8</td>
<td>-16.3</td>
<td>-13.7</td>
<td>-11</td>
</tr>
<tr>
<td>p=0, μ=0</td>
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<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>p=0, μ=0</td>
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<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
<td>3.86</td>
</tr>
<tr>
<td>p=0, β=0</td>
<td>-0.65</td>
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<td>-23.6</td>
<td>-20.7</td>
<td>-17.5</td>
</tr>
<tr>
<td>p=0, μ=0, β=0</td>
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<td>-3.73</td>
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<td>p=0, β=0</td>
<td>2.81</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

**Notes:**
- Variable = InB
- Max lag in the AR corr. = 1
- Actual Sample size = 145
### AUGMENTED DICKEY-FULLER INTEGRATION TESTS

**Variable = \( \ln B \)**  
Max lag in the AR corr. = 1  
Actual Sample size = 82

<table>
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<tr>
<th>TEST Statistic</th>
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<th>2.5% value</th>
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<td>-19.3</td>
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<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
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<td>-20.7</td>
</tr>
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<td>6.5</td>
<td>5.59</td>
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<tr>
<td>( p=0, \mu=0 )</td>
<td>5.87</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
</tr>
</tbody>
</table>

**Variable = \( \ln B \)**  
Max lag in the AR corr. = 1  
Actual Sample size = 82

<table>
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<tr>
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<td>3.13</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
</tr>
</tbody>
</table>

**Variable = \( r \)**  
Max lag in the AR corr. = 1  
Actual Sample size = 82

<table>
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<th>TEST Statistic</th>
<th>1% value</th>
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<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
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<td>-0.87</td>
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<td>-19.3</td>
<td>-13.7</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
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<td>( p=0, \mu=0 )</td>
<td>0.54</td>
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<td>( p=0, \mu=0 )</td>
<td>1.71</td>
<td>6.5</td>
<td>5.59</td>
<td>4.88</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>2.23</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Table 5.2 Euro Area: The ADF test for the variables in level
<table>
<thead>
<tr>
<th>TEST Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0, µ=0</td>
<td>-74.05</td>
<td>-19.8</td>
<td>-16.3</td>
<td>-13.7</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>-5.43</td>
<td>-3.51</td>
<td>-3.17</td>
<td>-2.89</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>14.75</td>
<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>-76.39</td>
<td>-27.4</td>
<td>-23.6</td>
<td>-20.7</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>-5.52</td>
<td>-4.04</td>
<td>-3.73</td>
<td>-3.45</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>10.18</td>
<td>6.5</td>
<td>5.59</td>
<td>4.88</td>
</tr>
<tr>
<td>p=0, µ=0</td>
<td>15.26</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Table 5.3 United States: The ADF test for the first difference of the variables
### Augmented Dickey-Fuller Integration Tests

**Variable = \( \ln B \)**

Max lag in the AR corr. = 1

Actual Sample size = 82

<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-13.42</td>
<td>-19.8</td>
<td>-16.3</td>
<td>-13.7</td>
<td>-11</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-3.91</td>
<td>-3.51</td>
<td>-3.17</td>
<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>7.72</td>
<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
<td>3.86</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-14.66</td>
<td>-27.4</td>
<td>-23.6</td>
<td>-20.7</td>
<td>-17.5</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-4.18</td>
<td>-4.04</td>
<td>-3.73</td>
<td>-3.45</td>
<td>-3.15</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>5.88</td>
<td>6.5</td>
<td>5.59</td>
<td>4.88</td>
<td>4.16</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>8.74</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

**Variable = \( \ln d \)**

Max lag in the AR corr. = 1

Actual Sample size = 82

<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-17.2</td>
<td>-19.8</td>
<td>-16.3</td>
<td>-13.7</td>
<td>-11</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-2.92</td>
<td>-3.51</td>
<td>-3.17</td>
<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>4.59</td>
<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
<td>3.86</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-21.22</td>
<td>-27.4</td>
<td>-23.6</td>
<td>-20.7</td>
<td>-17.5</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-3.42</td>
<td>-4.04</td>
<td>-3.73</td>
<td>-3.45</td>
<td>-3.15</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>4.27</td>
<td>6.5</td>
<td>5.59</td>
<td>4.88</td>
<td>4.16</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>6.05</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

**Variable = \( r \)**

Max lag in the AR corr. = 1

Actual Sample size = 82

<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-24.39</td>
<td>-19.8</td>
<td>-16.3</td>
<td>-13.7</td>
<td>-11</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-3.45</td>
<td>-3.51</td>
<td>-3.17</td>
<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>6.24</td>
<td>6.7</td>
<td>5.57</td>
<td>4.71</td>
<td>3.86</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-33.32</td>
<td>-27.4</td>
<td>-23.6</td>
<td>-20.7</td>
<td>-17.5</td>
</tr>
<tr>
<td>( p=0, \mu=0 )</td>
<td>-4.37</td>
<td>-4.04</td>
<td>-3.73</td>
<td>-3.45</td>
<td>-3.15</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>6.76</td>
<td>6.5</td>
<td>5.59</td>
<td>4.88</td>
<td>4.16</td>
</tr>
<tr>
<td>( p=0, \mu=0, \theta=0 )</td>
<td>9.81</td>
<td>8.73</td>
<td>7.44</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

Table 5.4 Euro Area: The ADF test for the first difference of the Variables
5.5.2 Specification analysis

This subsection is aimed at testing the specification of the model. In the following, the results of the maximum lag test, the exogeneity test and the roots analysis are reported.

The optimal lag length has been checked by classical tests like Akaike (AIC), Schwartz(BIC) and Hannan and Quinn(HQ) criterias \(^9\). Table (5.5) shows the results:

![Maximum Lag Analysis Table]

The model selected is the one where the statistic reaches its minimum.

The optimal lag length is one for both countries.

\(^{10}\)Maximum lag analysis in VAR models is discussed at length in Lutkepohl(1991) and Reimers(1993). The formulae for the information criteria, i.e. Akaike (AIC), Schwartz(BIC), Hannan and Quinn(HQ) may be found in Lutkepol(1991).
Regarding the exogeneity test, we use three consecutive F-tests to check for the exogeneity of the real interest rate with respect to the public debt and primary deficit. First, we test whether the lag of real interest rate enters into both public debt and primary deficit equations. Then, we test whether the lags on public debt and primary deficit are jointly equal to zero into real interest rate equation. Table 5.6 reports the results for the Euro Area and United States:

<table>
<thead>
<tr>
<th>United States</th>
<th>Euro Area</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inb equation</strong></td>
<td><strong>Inb equation</strong></td>
</tr>
<tr>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
</tr>
<tr>
<td>$r_{Lag(s)}$</td>
<td>$r_{Lag(s)}$</td>
</tr>
<tr>
<td>$F(1,142) = 13.31774$ with Significance Level 0.00036854</td>
<td>$F(1,79) = 3.377$ with Significance Level 0.049</td>
</tr>
<tr>
<td><strong>Ind equation</strong></td>
<td><strong>Ind equation</strong></td>
</tr>
<tr>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
</tr>
<tr>
<td>$r_{Lag(s)}$</td>
<td>$r_{Lag(s)}$</td>
</tr>
<tr>
<td>$F(1,142) = 3.81432$ with Significance Level 0.042</td>
<td>$F(1,79) = 5.9197$ with Significance Level 0.017</td>
</tr>
<tr>
<td><strong>r equation</strong></td>
<td><strong>r equation</strong></td>
</tr>
<tr>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
<td><strong>Null Hypothesis</strong>: The Following Coefficients are Zero</td>
</tr>
<tr>
<td>$Inb_{Lag(s)}$</td>
<td>$Inb_{Lag(s)}$</td>
</tr>
<tr>
<td>$Ind_{Lag(s)}$</td>
<td>$Ind_{Lag(s)}$</td>
</tr>
<tr>
<td>$F(2,142) = 2.30491$ with Significance Level 0.106</td>
<td>$F(2,79) = 1.83012$ with Significance Level 0.169</td>
</tr>
</tbody>
</table>

Table 5.6. Testing for the exogeneity of the real interest rate.

F-tests reject the hypothesis\(^{20}\) that the lagged coefficient on the interest rate is zero for the first two equations. On the contrary, the F-test for the interest rate equation rejects the hypothesis that the lagged coefficients on the public debt and primary deficit are jointly equal to zero. Overall,

\(^{20}\)The hypothesis is accepted at 95% confidence when the significance level is greater than 0.05.
empirical evidence does not reject the exogeneity of the real interest rate for both countries. The real interest rate affects both primary deficit and public debt but these variables do not affect the real interest rate.

Given that real interest rate is exogenous, we conduct the roots analysis under the scenario (b) of section 5.4. Specifically, we conduct the analysis in terms of the characteristic polynomial. The stability condition is that all eigenvalues (roots) have modulus less than one.²¹

The roots of the model (5.7) are reported below:

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>ROOT</th>
<th>ABSOLUTE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.97453, 0.00000)</td>
<td>0.97453</td>
</tr>
<tr>
<td>2</td>
<td>(0.96070, 0.00000)</td>
<td>0.9607</td>
</tr>
</tbody>
</table>

Table 5.7 The roots of the model

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>ROOT</th>
<th>ABSOLUTE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.98625, 0.03903)</td>
<td>0.98625</td>
</tr>
<tr>
<td>2</td>
<td>(0.98625, -0.03903)</td>
<td>0.98625</td>
</tr>
</tbody>
</table>

where the first number represents the real part and the second number represents the imaginary part. In both countries the roots are very close to one. Despite they are slightly lower in United States, the system seems to be unstable for both countries.

²¹In the case of complex roots, say \( v_j = v'_j + v''_j i \), where \( v'_j \) and \( v''_j i \) are the real part and imaginary parts of \( v_j \), the modulus is defined as \( \text{mod}(v_j) = \left( (v'_j)^2 + (v''_j)^2 \right)^{0.5} \).
5.5. EMPIRICAL EVIDENCE

5.5.3 Cointegration analysis

Having established the empirical model, the next stage is to determine the cointegration rank corresponding to the number of equilibrium relationships among the variables in the VAR. The procedure for assessing the cointegration rank of the I(1) model is represented by a sequence of Likelihood ratio tests, as shown in Johansen (1996). Specifically, only the so called trace test is computed since the lambda-max test, although easily computable, does not give rise to a coherent testing strategy as illustrated in Johansen (1994) based on Pantula (1989). The tests are reported, together with the critical values\(^{22}\) in Table (5.8) and Table (5.9)\(^{23}\). The problem of testing for the cointegration rank \(r\) is deeply connected with the problem of determining the appropriate trend polynomial. In fact, the test has been conducted in each of the five models proposed in Johansen (1994) (i.e. from M1 to M5).\(^{24}\)

\(^{22}\)The critical values are tabulated in Johansen (1996),

\(^{23}\)The hypothesis is accepted when the calculated value is greater than the tabulated value. The shaded area highlights the first type error probability at which the hypothesis is not rejected.

\(^{24}\)The specification of the models is as follows:

- **M1. Model 1:** \(m_0 = 0\) and \(m_1 = 0\) (i.e. \(\mu_0 = 0\) and \(\mu_1 = 0\)) Zero mean in the I(0) components, non-zero mean in the I(1) components.
- **M2. Model 2:** \(m_0 = \mu_0\) and \(m_1 = 0\) (i.e. \(\mu_0 = \mu_0\) and \(\mu_1 = 0\)) Non-zero mean in both the I(0) and the I(1) components.
- **M3. Model 3:** \(m_0 = m_0\) and \(m_1 = 0\) (i.e. \(\mu_0 = \text{unrestricted}\) and \(\mu_1 = 0\)) Non-zero mean in the I(0) components, linear trend in the I(1) components.
- **M4. Model 4:** \(m_0 = m_0\) and \(m_1 = \mu_1\) (i.e. \(\mu_0 = \text{unrestricted}\) and \(\mu_1 = \mu_1\)) Linear Trend in both the I(0) and the I(1) components.
- **M5. Model 5:** \(m_0 = m_0\) and \(m_1 = m_1\) (i.e. \(\mu_0 = \text{unrestricted}\) and \(\mu_1 = \text{unrestricted}\)) Linear Trend in the I(0) components, quadratic trend in the I(1) components.
- **M6. Model 6:** Model 2 and Model 3 jointly
- **M7. Model 7:** Model 4 and Model 5 jointly

The meaning of the various models, except for Model 6 and 7, is self-explained in parenthesis. For instance, the part of the table corresponding to Model 1(M1) reports the results of the test when both constant and trend (i.e. \(\mu_0 = 0\) and \(\mu_1 = 0\)) are restricted to be zero. The part of the table marked with Model 2 shows the results of the test when
Moreover, the strategy for jointly determining the cointegration rank and the trend polynomial discussed in Johansen (1992), is implemented (i.e. M6 and M7).

the constant is restricted to be equal to $\alpha b_0$ (i.e. $\mu_0 = \alpha b_0$) and the trend to be equal to zero (i.e. $\mu_1 = 0$) and so on. Model 6 and 7 implement the strategy for jointly determining the cointegration rank and the trend polynomial discussed in Johansen (1992).
5.5. EMPIRICAL EVIDENCE

### Table 5.8 United States: Testing for cointegration rank

<table>
<thead>
<tr>
<th>r Const. Trend Statistic</th>
<th>50%</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>97.50%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 ab 0</td>
<td>74.88</td>
<td>29.53</td>
<td>35.56</td>
<td>39.06</td>
<td>42.44</td>
<td>45.42</td>
</tr>
<tr>
<td>1 ab 0</td>
<td>17.29</td>
<td>15.59</td>
<td>13.33</td>
<td>15.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 ab 0</td>
<td>6.6</td>
<td>5.55</td>
<td>8.65</td>
<td>10.49</td>
<td>12.25</td>
<td>14.21</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 ab 0</td>
<td>74.88</td>
<td>29.53</td>
<td>35.56</td>
<td>39.06</td>
<td>42.44</td>
<td>45.42</td>
</tr>
<tr>
<td>1 ab 0</td>
<td>17.29</td>
<td>15.59</td>
<td>13.33</td>
<td>15.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 ab 0</td>
<td>6.6</td>
<td>5.55</td>
<td>8.65</td>
<td>10.49</td>
<td>12.25</td>
<td>14.21</td>
</tr>
<tr>
<td>M3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 m0</td>
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<td>22.66</td>
<td>28.13</td>
<td>31.42</td>
<td>34.55</td>
<td>36.94</td>
</tr>
<tr>
<td>1 m0</td>
<td>12.75</td>
<td>9.68</td>
<td>13.56</td>
<td>16.06</td>
<td>18.17</td>
<td>20.13</td>
</tr>
<tr>
<td>2 m0</td>
<td>2.9</td>
<td>0.45</td>
<td>1.61</td>
<td>2.57</td>
<td>3.74</td>
<td>4.85</td>
</tr>
<tr>
<td>M4</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0 m0 abl</td>
<td>74.88</td>
<td>29.53</td>
<td>35.56</td>
<td>39.06</td>
<td>42.44</td>
<td>45.42</td>
</tr>
<tr>
<td>1 m0 abl</td>
<td>17.29</td>
<td>15.59</td>
<td>13.33</td>
<td>15.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 m0 abl</td>
<td>6.6</td>
<td>5.55</td>
<td>8.65</td>
<td>10.49</td>
<td>12.25</td>
<td>14.21</td>
</tr>
<tr>
<td>M5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 m0 ml</td>
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<td>22.66</td>
<td>28.13</td>
<td>31.42</td>
<td>34.55</td>
<td>36.94</td>
</tr>
<tr>
<td>1 m0 ml</td>
<td>12.75</td>
<td>9.68</td>
<td>13.56</td>
<td>16.06</td>
<td>18.17</td>
<td>20.13</td>
</tr>
<tr>
<td>2 m0 ml</td>
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<td>0.45</td>
<td>1.61</td>
<td>2.57</td>
<td>3.74</td>
<td>4.85</td>
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<tr>
<td>M6</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0 ab 0</td>
<td>74.88</td>
<td>29.53</td>
<td>35.56</td>
<td>39.06</td>
<td>42.44</td>
<td>45.42</td>
</tr>
<tr>
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<td>15.59</td>
<td>13.33</td>
<td>15.41</td>
<td></td>
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</tr>
<tr>
<td>2 ab 0</td>
<td>6.6</td>
<td>5.55</td>
<td>8.65</td>
<td>10.49</td>
<td>12.25</td>
<td>14.21</td>
</tr>
<tr>
<td>M7</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 m0 abl</td>
<td>74.22</td>
<td>29.53</td>
<td>35.56</td>
<td>39.06</td>
<td>42.44</td>
<td>45.42</td>
</tr>
<tr>
<td>1 m0 ml</td>
<td>17.29</td>
<td>15.59</td>
<td>13.33</td>
<td>15.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 m0 ml</td>
<td>6.6</td>
<td>5.55</td>
<td>8.65</td>
<td>10.49</td>
<td>12.25</td>
<td>14.21</td>
</tr>
</tbody>
</table>

NOTE: THE HYPOTHESIS IS ACCEPTED WHEN CALC. VALUE < TAB. VALUE
### Table 5.9 Euro Area: Testing for cointegration rank

<table>
<thead>
<tr>
<th>r</th>
<th>Const. Trend Statistic</th>
<th>50%</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>97.50%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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*NOTE: THE HYPOTHESIS IS ACCEPTED IF CALC. VALUE < TAB. VALUE.*
As illustrated in Johansen (1992), in order to control for the first type error probability, the tests have to be read from top to bottom, comparing the statistics with appropriate quantiles. The selected rank is the first for which the hypothesis is accepted.

The tables show that, when the model with or without unrestricted constant and restricted trend is used, and the first type error probability is fixed to 5%, the selected rank is equal respectively to 0 for Euro Area and 1 for United States in each model.

The inference on the cointegration rank for the I(1) model strongly rejects the hypothesis of a cointegrated relationship for the Euro Area while does not reject the hypothesis of a cointegrated relationship between the endogenous variables for United States. Nevertheless, for this country we still have a possible source of non-stationarity represented by the real interest rate. In this case, the problem of the fiscal policy sustainability relies on the time series properties of this variable. If the U.S. real interest rate is stationary, fiscal policy is said to be sustainable. If the U.S. real interest rate is not-stationary, fiscal policy is said to be unsustainable. The stationarity of the real interest rate is an open issue and is strictly related with time series properties of the consumer price index (CPI). If the CPI is an

\[ \xi = (1, -2.5511, 0) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix} \]

The model is estimated by using the Maximum likelihood algorithm illustrated in Johansen (1988).
I(1) variable, the real interest rate is not stationary as the inflation rate is I(0). If the CPI is an I(2), the inflation rate is I(1). Therefore, the nominal interest rate and inflation rate cointegrate with cointegrating vector(1,1). Consistently with a part of literature arguing that U.S CPI is I(2), evidence from our sample suggests that the U.S real interest rate is not stationary. Therefore, fiscal policy in United States is said to be unsustainable. However, there is a different strand of literature (St-Amaunt, 1996) arguing that the real interest rate is stationary in the long run. If we take this result as true, then fiscal policy in United States should be considered as sustainable in the longer period.

Overall, the analysis seems to suggest the instability of the system and the unsustainability of the fiscal policy in the Euro Area. An higher degree of uncertainty regards the fiscal position of United States. The sustainability of the fiscal policy for this country becomes an issue strictly related with the debate about the stationarity of the real interest rate. However, empirical evidence from our sample shows that the real interest rate is not stationarity and thus fiscal policy is not sustainable.

5.6 Forecasting public debt

The purpose of this section is to provide two different methods to construct forecasts of the future level of public debt. The two approaches differ in the assumption about the process generating primary deficit and in the

\[^{26}\text{See table 5.1 for the results of the Dicky Fuller Test.}\]
5.6. FORECASTING PUBLIC DEBT

technique utilized to compute forecasts. The former is based upon the VAR estimates and does not make any assumption about the process generating the primary deficit. The latter uses an iterative procedure and assume the condition for sustainability proposed by Wickens and Uctum i.e. a zero mean primary deficit. The two approaches follow:

a) Forecasts from a VAR model

The public debt identity in VAR form has several advantages. One of them is related to the possibility to make forecasts. Under the assumption of an exogenous AR(1) interest rate, model(5.4) can be represented as follows:

\[
\begin{bmatrix}
\ln b_t \\
\ln d_t \\
r_t
\end{bmatrix}
= \begin{bmatrix}
\kappa_1 \\
\kappa_2 \\
0
\end{bmatrix}
+ \begin{bmatrix}
(1 + \bar{r}) \\
\frac{\sigma^2}{\nu} \\
0
\end{bmatrix}
\begin{bmatrix}
\pi_{21} \\
\pi_{22} \\
0
\end{bmatrix}
\begin{bmatrix}
\ln b_{t-1} \\
\ln d_{t-1} \\
r_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{t1} \\
\varepsilon_{t2} \\
\varepsilon_{t3}
\end{bmatrix}
\tag{5.14}
\]

In this case, forecasted values of the public debt level are constructed using the optimality of the conditional expectation. Provided that the errors are independent white noise, the optimal predictor for a generic VAR(p) process is the following:

\[
E_t(y_{t+h}) = v + A_1 E_t(y_{t+h-1}) + \ldots + A_p E_t(y_{t+h-p})
\tag{5.15}
\]

The formula (5.15) can be used for recursively computing the h-step predictors starting with h=1:
By these recursions we get for the VAR(1) the formula for computing the forecasts h-periods ahead. It follows:

$$E_t(y_{t+h}) = (I_k + A_1 + \cdots + A_h^0)v + A^h_1$$

(5.16)

where $A_1$ is the $(3 \times 3)$ matrix in model(5.14).

b) Forecasts from an iterative procedure

The previous method constructs forecasts based upon the assumption that the processes generating primary Deficit and interest rate will continue into the future. In practice, however, it may be necessary to alter fiscal policy to achieve sustainability. This suggests the need to analyze sustainability allowing for expected future changes in fiscal policy. This method, in fact, aims to provide a procedure to construct forecasts conditioned to a given process of the primary deficit.

For given values of public debt, primary deficit and interest rate at time t, at time $t+1$ the public debt identity in non-linear form is as follows:

At time $t=1$ it is

$$B_1 = D_0 + (1 + r_0)B_0$$
A time \( t=2 \) it is:

\[
B_2 = D_1 + (1 + r_1)B_1
\]

At time \( t=3 \) it is:

\[
B_3 = D_2 + (1 + r_2)B_2
\]

In general at time \( t \)

\[
B_t = D_{t-1} + (1 + r_{t-1})B_{t-1}
\]

By substituting \( B_{t-1} \) in \( B_t \) recursively and assuming an initial value of public debt greater than zero \( (i.e. B_0 > 0) \) the above identity can be rewritten in terms of a geometric series of deficits/surpluses and the initial value of public debt:

\[
B_t = D_{t-1} + D_{t-2}(1 + r_{t-1}) + D_{t-3}(1 + r_{t-1})(1 + r_{t-2}) + \ldots + D_0(1 + r_{t-1})(1 + r_{t-2})\ldots(1 + r_{t-(t-1)}) + B_0(1 + r_{t-1})(1 + r_{t-2})\ldots(1 + r_{t-t})
\]

or equivalently:

\[
B_t = \sum_{i=1}^{t} \left[ D_{t-i} \left( \prod_{j=1}^{i-1} (1 + r_{t-i+j}) \right) \right] + B_0 \left( \prod_{i=1}^{t} (1 + r_{t-i}) \right)
\]

If the interest rate \( r \) is a stationary process, in particular a white noise with drift \( (i.e. r_t = \mu + \nu_t) \), and the primary deficit is a zero mean process the above equation becomes:
\[ B_t = \sum_{i=1}^{t-1} \left[ \varepsilon_{t-i} \left( \prod_{j=1}^{i} (1 + \mu + v_{t-i+j}) \right) \right] + B_0 \left[ \prod_{i=1}^{t} (1 + \mu + v_{t-i}) \right] \]

The expected value of this process is as follows:

\[ E(B_t) = (1 + \mu)^t B_0 \]

If we assume \( \mu \) to be equal to the sample mean of the interest rate (i.e. \( \bar{\mu} \)) and impose \( B_0 \) equal to the value of public debt at the end of the sample period (i.e. \( B_{es} \)) the forecast of public debt at time \( t + s \) can be obtained as follows:

\[ E(B_{t+s}) = (1 + \bar{\mu})^s B_{es} \quad (5.17) \]

Equation (5.17) allows to computation of public debt forecasts between period \( t \) and period \( t + s \).

Figure 5.1 shows the computed forecasts and the actual values of the public debt series in both countries.
5.6. FORECASTING PUBLIC DEBT

Figure 5.1 Forecasted values of Public Debt

The solid line represents the actual values of the variable while the dashed lines represent the forecasts; $F_a$ and $F_b$ specify the kind of forecasts. The
series $F_a$ represents the forecasts computed by method $a$ and $F_b$ represents
the forecasts computed by method $b$. Forecasts fifteen periods ahead start
from 2003:04 in the United States and from 1998:01 in the Euro Area. Both
procedures give arise to consistent estimates. This is particularly true for the
Euro Area where the two forecasting methods produce similar results. For
the U.S the forecasted public debt with the VAR model seems to be initially
decreasing, but then it increases on the path of the forecasts computed with
alternative method. Forecasts constructed under the case $b$ are coherently
lower than the forecasts constructed under the case in the Euro Area. This
is probably due to the fact that, in the second case, we assume a zero mean
primary deficit and a stationary real interest rate. On the contrary, in the
first case, forecasts are conditioned to the past evidence of the primary deficit
(that is negative) and interest rate.

The forecasted levels of public debt diverge towards higher values in both
countries. Forecasts seem to confirm the empirical results obtained in the
previous part of this chapter. They also verify the sustainability condition
proposed by Wickens Uctum(1997).

Forecasts evaluation is implemented by looking at the out-of-sample fore-
casts. Figure 5.2 compares our forecasts(dashed lines) with the actual value
of the U.S public debt six quarters ahead(solid line).
Chart 1 also reports 95% forecast interval computed with the VAR model (grey lines). These intervals are computed as $b(h) \pm z(\alpha/2)\sigma_b(h)$ where $b(h)$ is the point forecast, $\sigma_b(h)$ is the square root of the first diagonal element of the forecast MSE matrix and $z(\alpha)$ is the upper percentage point of the standard normal distribution (Lutkepohl, 1991). Notice that prediction goes from one to six.

\[\text{Figure 5.2 United States: Forecast Evaluation}\]
Forecast interval include the actual values of the public debt. Moreover, both methods perform relatively well. The observed out-sample forecast errors\(^{29}\) for both procedures are shown in chart 2, figure 5.2.

A comparative analysis suggests that method \(b\) generates positive forecast errors while method \(a\) gives rise to negative forecast errors.

Consistently with the theory, the out-of-sample forecast errors tend to get larger as the horizons gets longer. This is due to the fact that errors, at each time interval, build up in a cumulative way. Overall, the analysis shows evidence of forecast success for United States. Method \(b\) seems to have a better performance. It generates lower out-of-sample forecast errors (in absolute value) and captures the increasing level of public debt.

Regarding the Euro Area, out-of-sample forecasts are not observable due

\(^{28}\)If a model in logs is used to compute point and interval forecasts for future values of the original variables, then, a re-transformation of the forecasts and prediction intervals in their original units is required.

However, if the analysts takes logs of a variable, finds point forecast of the logs and assumes that they are unbiased, and then takes antilogs to get point forecasts of the original variable, then the latter forecasts will no longer be unbiased. Suppose that the analyst finds a 95% prediction interval (i.e. P.I) for the logarithm of the variable. If one takes the antilogs of the upper and lower limits of this P.I. to get re-transformed P.I. for the original variable, then it can be easily shown that there will still be a 95% probability that this interval will include the future value of the original variable. Nevertheless, this re-transformed P.I. will generally be asymmetric, as it reflects the asymmetry in the errors.

\(^{29}\)Forecast errors are computed as follows: \(\frac{(B(k) - \hat{B}(h))}{\sigma}\).
to data limitation. Nevertheless, a comparative analysis between our forecasts and OECD forecasts for the sample period 1998:01-1999:02 can still give interesting insights about the goodness of the two forecasting procedures. Figure 5.3 plots the predicted values with method a ($F_a$) and method $b(F_b)$ against OECD forecasts(Chart 1). Moreover, it reports the deviation of our forecasts with respect to the OECD predicted values in percentage point(Chart 2).
Figure 5.3 Euro Area: Forecasts evaluation

Chart 1 (figure 5.3) shows that 95% forecast interval covers the OECD forecasts. Nevertheless, while our forecasts predict an increasing level of public debt, the OECD forecast predict a decreasing level. A proxy of the out-of-sample forecast error is given in chart 2 by taking the difference between our forecasts and the predicted values from OECD\textsuperscript{30}. The two methods yields

\textsuperscript{30}This approximation relies on the assumption that OECD forecast do not differ significantly from the actual values.
similar results. Method a performs slightly better. Forecast analysis for the Euro Area shows that our forecasts do not differ significantly from the OECD forecasts. However, it should be born in mind that, while our forecast procedures predicted an increasing level of public debt, the OECD model forecasted a decreasing level.

Altogether, the section seems to support the goodness of our forecasts and the hypothesis of unsustainability of fiscal policy.

5.7 Conclusion

In this chapter, we have log-linearized the public debt identity and expressed it in terms of a multivariate system. By using the Vector Error Correction Model (VECM) and applying a cointegration analysis to data, we have derived conditions suitable for determining whether or not fiscal policies are sustainable in the long run. It arises that the condition for sustainability depends upon the stationarity of the VAR system. In particular, if the VAR model is globally stationary, fiscal policy is said to be sustainable. If the VAR model is not globally stationary, fiscal policy is said to be unsustainable.

The stationarity of the system depends upon the number of cointegrated relationships between the variables. A three dimensional VAR, where all variables are endogenous and integrated of order one, is stationary if it contains two cointegrated relationships. In our case there are two endogenous variables and an exogenous variable. Hence, if there are not cointegrated
relationships, or there is only a cointegrated relationship but the exogenous variable is not stationary, fiscal policy is said to be unsustainable.

We have applied these conditions to the fiscal stances of the United States and Euro Area. The results of the ADF test show that public debt, primary deficit and interest rate are integrated processes of order one, in both countries. The following cointegration analysis suggests that there are not cointegrated relationships in the estimated VAR for the Euro Area and one cointegrated relationship between the endogenous variables in the estimated VAR for the United States. Unlike the Euro Area, U.S fiscal position seems to be in a better situation. This is probably due to the higher growth rate of the GDP shown from the U.S with respect to the Euro Area in the sample period. The broad conclusion is that Euro Area has an unsustainable fiscal policy in the long run if the processes generating primary deficit will continue into the future. An higher degree of uncertainty regards the Fiscal position of United States. The sustainability of the fiscal policy for this country depends on time series properties of the real interest rate. Empirical evidence from our sample suggests the real interest rate is not stationarity and thus the fiscal policy is unsustainability. This result is consistent with a part of literature. However, there is a different strand of literature arguing that the real interest rate is stationary in the long run. If we take this result as true then, the fiscal policy in United States is said to be sustainable in the long run.

We have also constructed forecasts for the public debt level in two dif-
ferent ways. The forecasted values of public debt, obtained with both procedures, show a diverging path. These values are consistent with the evaluation elaborated in the cointegration analysis. Altogether the study rejects the hypothesis of sustainability of fiscal policy and stresses the need of an increasing fiscal austerity in the coming years, in order to achieve the sustainability in these countries. Inspection of the data reveals that an attempt in this direction has been made by the U.S. in the decade 1991-2001. In this period the U.S. federal government was running a large budget surplus. This surplus arose from various sources. The two tax increases in 1990 and 1993, signed respectively by the elder George Bush and Bill Clinton, and the productivity acceleration in the late 1990s pushed the federal government’s budget from deficit to surplus. Unfortunately, shocks to economy can alter the government’s revenue and spending. In fact, only a few months after George W. Bush moved into the White House, the terrorist attacks in September 2001 induced an increase in the government spending that brought the public debt back on the diverging path. The same effort has been made in the Euro Area where most of the EMU countries have shown a converging path toward the value of the public debt, set up by the stability growth and pact. Nevertheless, there are countries like France, Germany and Portugal that have recently broken the stability growth and pact by generating deficit greater than the deficit ceiling and countries, like Italy and Belgium, where the public debt is much higher than the target level. In the next chapter, the fiscal position of these countries is examined and the optimal fiscal rule
ensuring the convergence towards the public debt target, is estimated.
Chapter 6

Optimal Fiscal policy rules: Empirical Evidence in Italy and Belgium

This chapter attempts to characterize the optimal fiscal policy rule in the presence of a public sector with objectives of convergence for public debt and primary balance to GDP ratios. To this end, the study uses a stochastic simulation framework. In order to ensure the existence of converging paths towards the target values of fiscal variables, we introduce a simple fiscal policy rule. According to this rule, the primary balance ratio is adjusted in relation to the distance between the current and the target level of the public debt, the current and the target level of the primary deficit and the output gap. The study gives interesting insights. First, it shows that the fiscal rule displaying time invariant parameters may produce non linear dynamic processes of adjustment of the public debt. Second, it suggests a procedure to estimate the parameters, and related confidence intervals, characterizing
the optimal fiscal policy rule. Finally, it constructs a stochastic efficiency frontier illustrating the fiscal trade-off for the public sector. The analysis is first applied to a theoretical economy and then to Italy and Belgium.

6.1 Introduction

Sustainability of public finances and sound fiscal policies are at the core of the European Monetary Union (EMU). The Treaty of Maastricht and the Stability and Growth Pact set up precise objectives of fiscal convergence, for the public sectors of each member country. According to the protocol of the Treaty, the general government deficit to GDP ratio should in fact not exceed 3% and the public debt to GDP ratio should be lower than 60%. In addition, the Stability and Growth Pact requires member countries to reach a close to balance position over the medium term. The question of whether a given level of public debt is sustainable under specific economic conditions has, therefore acquired growing importance in the EMU, where governments can finance current deficits with higher taxes and/or lower public expenditures, without any recourse to seigniorage. This chapter illustrates the dynamics of debt and primary balance ratios under the hypothesis that the public sector presents well-defined objectives of convergence in a neoclassical model of analysis with three types of agents: consumers, firms and government. The main aim of the study is to fully characterize a fiscal policy rule ensuring stability for different initial fiscal positions. Moreover, the analytical framework makes it possible to estimate the optimal fiscal policy rule that
minimizes, for given weights, a quadratic loss function accounting for the
distance between the level of the public debt and a benchmark profile, the
level of primary deficit and its lower bound and the output gap. The analy-
sis is then applied to the EMU countries that have shown the highest public
debt to GDP ratio, in the last three decades (i.e. Italy and Belgium).

The study differs from the existing literature in different aspects. First,
we use a neoclassical model instead of an endogenous growth model with
overlapping generations, that is commonly considered the most appropriate
tool of analysis to explore the influence of the public debt in the economy.
Second, we consider a policy reaction function of the government explicitly
accounting for the output gap. Third, we include an exogenous stochastic
interest rate that is completely under the control of the monetary policy
authorities.

We prefer a neoclassical model for two reasons. First, the overlapping
generation model (OLG) studies the government debt convergence in the
long run\(^1\), while this chapter aims to analyse the public debt convergence
on a short/medium horizon. Second, we need a theoretical model explicitly
accounting for the output gap. The main goals of this chapter are, in fact,
to introduce a stochastic interest rate under the complete control of the
monetary policy authority and estimate the fiscal authority trade-off. For
both purposes the output gap plays a central role.

This analysis provides interesting insights into the linkages between fiscal

\(^1\) An unfortunate feature of the two period OLG model is, in fact, to consider each
period as being composed by thirty years, that is commonly considered as long period.
variables (i.e. debt and primary deficit ratios) and the dynamics of the interest rate. Moreover, the simulation exercise allows us to explore different path of adjustments depending on initial conditions and numerical values assigned to the parameters characterizing the fiscal policy rule governing the public sector.

The chapter is organized as follows. Section 6.2 relates the present paper to the economic literature, while Section 6.3 introduces the reference model. Section 6.4 calibrates a theoretical economy and performs some simulation exercises. In particular, it derives the parameter space compatible with the sustainability of public finances and estimates the optimal fiscal policy rule. It also constructs the efficiency frontier showing the fiscal trade-off of the Government. Section 6.5 applies the procedure used for the theoretical economy to Italy and Belgium. Section 6.6 concludes summarizing the main results of the chapter.

6.2 Related Literature

In addition to the studies already described in the previous chapter, there are parts of literature studying the sustainability of fiscal policy and related issues from different perspectives. Before turning to recent stochastic simulation model-based studies, we highlight some examples of recent works that have addressed similar fiscal policy issues using a deterministic, or non-stochastic, framework.

One of the most familiar projection exercises is McCallum (1996). In this
study, McCallum uses a projection model and estimates that, over the period 2003-2018, there could be enough federal fiscal room to cut the federal personal income tax in half or increase program spending by a third (relative to GDP). McCallum assumes that the federal government maintains a balanced budget over this period and then projects federal fiscal room given constant growth and interest rate assumptions. While the author acknowledges that there is some uncertainty associated with his projections, he does not attempt to quantify the sensitivity of his results. McCallum (1999) again projects fiscal dividends, however, in this exercise his estimates of fiscal room are reduced by prudence factors “in view of the major uncertainties involved in any future projections”. Over the period 1999 to 2007, McCallum estimates that if all of his revised estimates of federal fiscal room were devoted to tax reduction, federal personal income taxes could be cut by 20 per cent after adjusting for prudence factors. While McCallum attempts to account for uncertainty in his estimates, there is no way of determining whether the prudence factors he uses are sufficient to ensure that his fiscal objectives will be achieved.

An alternative approach is based on the overlapping generations models, in which the breakdown of the Ricardian equivalence makes the government debt be net wealth for households. These models, considered the most appropriate tool of analysis to explore the influence of public debt on the economy, are proposed in Diamond (1965). He analyzes the effects of a positive stock of debt on the long-run competitive equilibrium of an economy,
with neoclassical technology. The author shows that government debt causes a decline in the utility level when the equilibrium is dynamically efficient, but may increase the utility when the economy is dynamically inefficient. Ihori (1978) studies the effects of government debt on the long-run optimal conditions and analyzes the growth paths corresponding to alternative government policies in a life-cycle economy. Other recent contributions, shifting away from a present budget balance perspective, conduct the analysis into a life-cycle model. Chalk (2000) analyzes the sustainability of permanent bond-financed deficits and shows the conditions under which a growth rate larger than the interest rate is a necessary, but not a sufficient condition to ensure the sustainability of a permanent budget deficit. De la Croix and Michel (2002) investigate the effects of the introduction of public debt on the dynamic properties of a two-period overlapping generations model, and derive the conditions for ensuring sustainability. Marin (2002) studies how a simple primary surplus budgetary rule can ensure the sustainability of public finances and provide automatic stabilization in a small open economy.

A final approach is based on stochastic simulation methods. A number of recent studies (like the current work) have applied these techniques, in order to examine how much "fiscal prudence" would be required to insure against the risk of running a deficit. Boothe and Reid (1998) examine the case where the fiscal authority seeks to balance the budget, on average, over a number of years. Their simulation results indicate that a contingency reserve of between $6 and $9 billion would be sufficient to avoid a deficit.
outcome with 90 per cent probability over a budget period of two to four years. Robson and Scarth (1999) examine the case where the fiscal authority plans a budget surplus equal to one per cent of GDP, each fiscal year (which is currently about $10 billion). Their simulations indicate that aiming for a surplus of this magnitude would entail a negligible risk of running a deficit. Hermanutz and Matier (2000) examine a fiscal planning framework characterized by rolling two-year budget plans. They find that a prudence level of $3.3 billion in the first year and $7.2 billion in the second year would provide insurance against a deficit outcome, with 90 per cent probability in each year. Dalsgaard and De Serres (1999) examine how member countries of the European Monetary Union (EMU), can avoid running budget deficits in excess of 3 per cent of GDP as specified in The Stability and Growth Pact. Their simulation results show that aiming for a cyclically adjusted budget deficit of between 1.0 and 1.5 per cent of GDP would achieve this objective with 90 per cent confidence. This implies a level of prudence for the cyclically-adjusted budget balance of between 1.5 and 2.0 per cent of GDP. These studies focus on how uncertainty influences the budget balance, but not the level of debt or the debt-to-GDP ratio.

This study tries to keep together the strengths of each approach. To this end, it differs from the existing literature in several important aspects. First, we focus on the level of debt in ratio to GDP. Second, we use a neo-classical model instead of an endogenous growth model with overlapping generations. Third, we consider a policy reaction function of the govern-
ment explicitly accounting for the output gap. Fourth, we use a stochastic simulation framework including an exogenous stochastic interest rate, that is completely under the control of the monetary policy authorities. The description of the model follows in the next section.

6.3 Reference Model

The theoretical framework used in the analysis is a simple model with three agents: government, firms and consumers. In particular, the model is a backward-looking\(^2\), closed-economy model\(^3\) accounting for the impact that fiscal policy has on output gap.

The supply side is described as an autoregressive Phillips curve:

\[
\pi_t = \alpha^\pi \pi_{t-1} + \beta^\pi y_{t-1} + u_t^\pi
\]  

(6.1)

This equation relates the CPI inflation rate (i.e. \(\pi_t\)) to its own lag (i.e. \(\pi_{t-1}\)) and to a lagged output gap (i.e. \(y_{t-1}\)) measured as a percent gap between the

\(^2\) As stresses in chapter I, there is a large part of literature arguing that the behaviour of the private agents should be forward-looking and the forecast should be therefore an integral part of the price setting behaviour. Nevertheless, a different strand of this literature (i.e. Rudebusch and Svensson 1999, 2002; Ball 1999, 2000), following in the spirit the VAR models popularized by Sims, focuses on ad-hoc backward-looking model. They argue that this kind of models are appreciable from at least two important aspects. They tend to offer a good fit of the data, and their dynamics closely resemble those filtered with structural VARs, an issue that pure forward-looking models have some troubles in dealing with (Estrella and Fuhrer, 2002).

\(^3\) The main goal of this chapter is to provide a procedure to characterize the optimal fiscal policy rule for a government with commitment. The initial choice of the model for a virtual economy is then marginally important. Section 6.5 will show how this procedure can be easily extended to small open economies by changing the theoretical framework.
actual real GDP and the potential GDP. The specification of aggregate supply is consistent with an adaptive representation of inflation expectations. The expectations are treated implicitly by the inclusion of lagged values of the variables.

The demand side is described by the following equation:

\[ y_t = \alpha y_{t-1} - \beta (i_{t-1} - \pi_{t-1}) + u_t^y \]  (6.2)

According to equation (6.2), the output gap is related to the annual real interest rate and its own lag. The real interest rate is calculated as the difference between the short-term interest rate and the inflation rate.

In the specified model the interest rate is an exogenous variable under the complete control of monetary authorities. In particular, we assume that the monetary authorities use the interest rate as policy instrument. Thus, the interest rate is most realistically interpreted as an overnight rate. Policymakers set the interest rate after observing the current shocks \( u_t^\pi \) and \( u_t^\pi \). Notice that we assume policymakers to set the real interest rate. In practice, the interest rates controlled directly by policymakers are nominal rates. However, policymakers can move the real interest rate to their desired level by setting the nominal rate equal to the desired real rate plus inflation.

The monetary policy rule followed by the monetary authorities in order to

---

4 Notice that, with respect to chapter 3, we have switched back the notation for the output gap from \( \bar{y} \) to \( y \).

5 It is, of course, a simplification to assume that spending depends directly on the overnight interest rate. Future work might consider an extension in which spending depends on a longer-term interest rate, which is linked to the overnight rate through the term structure.
stabilize output gap and inflation bias is a simple Taylor rule. It follows:

\[ i_t = \alpha^i(\pi_t - \pi^*) + \beta^i(y_t - y^*) + \psi_t \]  

(6.3)

where the inflation target (i.e. \(\pi^*\)) is set to be equal to the target announced by the European central Bank in the Maastricht treaty (i.e. 2%).

The public sector is described by an infinite lived government whose flow budget constraint is:

\[ b_t = s_{t-1} + (1 + r_{t-1})b_{t-1} \]  

(6.4)

where \(b_t\) is the government debt expressed as proportion of nominal GDP, \(r_{t-1}\) is the real interest rate adjusted for the output growth rate, \(s_{t-1}\) is the primary budget surplus expressed as proportion of nominal GDP and is defined as the difference between the total revenues and total expenditures.

Equation (6.4) states that the changes in the stock of public debt from one period to the next, must cover the budget surplus/deficit inclusive of interest payments (\(r_{t-1}b_{t-1}\)).

The public sector presents clear objectives of convergence for public debt and primary balance. In order to ensure the existence of converging paths towards the target values of fiscal variables, the government follows the following simple fiscal policy rule:

\[ s_t = a_1(s_{t-1} - b^*) + a_2(b_{t-1} - b^*) + a_3(y_{t-1} - y^*) \]  

(6.5)
6.3. REFERENCE MODEL

where \( d^* \) and \( b^* \) are respectively the targets set up by the stability growth pact for primary deficit and public debt (\( i.e. 3\% \) and 60\%).

According to this reaction function, the primary balance ratio is adjusted in response to the distance between the current and the target level of the public debt to GDP ratio, to the distance between the current and the lower bound of the primary surplus(deficit) to GDP ratio and to the output gap. This rule differs from the one proposed by Marin(2002) for the term accounting for the output gap. This term aims to catch the stabilizing power of the Government on the economy. When the economy goes into a recession taxes automatically fall, and transfers automatically rise. When economy goes into expansion taxes increase and transfers automatically fall. Here, we consider the case in which the government uses only the benefit rate as the instrument to control the dynamics of the primary surplus. Theoretically, the government has two instruments: the tax rate, \( \tau \), and the benefit rate, \( G \). In the simulation, we keep the tax rate constant over time.

We consider a policy framework wherein the fiscal authority sets the program spending at the beginning of each fiscal year. No tax or spending changes are made during the year. The fiscal authority revises its budget in the subsequent year taking the outcome from the previous period into account. As stressed in the stability growth pact, the Government cannot run primary deficits exceeding the 3\% of GDP. On the other hand, the Government can decide to devote the whole amount of the tax revenues to the reduction of the public debt meaning to run a surplus equal to the level
of taxes \( i.e. G = 0 \).

In our framework the fiscal authority faces a fundamental conflict between its debt control objective and its other objectives; namely policy smoothing and economic stabilization. Among these three goals, the debt control objective has clearly the most important implications for the credibility of the debt reduction plan. If the fiscal authority demonstrates that it can keep the debt to GDP ratio close to the announced profile, agents and the other EMU countries will come to believe that the debt reduction plan will be successful. This would give credibility to the government in two ways: at national level with respect to its own electorate and at international level with respect to the other EMU counties. However, the government is also interested in minimizing the malcontent of consumers and firms of the current generation. It acknowledges that consumers and firms perceive the surpluses generated in order to reduce the level of public debt to its target level, as an unfair redistribution of wealth from the current to the past generation. An excessive aggressive behavior of the fiscal authority would cause unpopularity for the Government, among the voters that would cost the no re-election at the end of the mandate. Last but not least, the government is also interested in the fiscal policy as an instrument to stabilize the economy. This issue has a long tradition in macroeconomics, dating back to Keynes 'The General Theory of Employment, Interest and Money(1936)'. The fiscal authorities, through a balanced Budget Rule, seek to dampen business cyclical fluctuations by implementing a counter cyclical overall policy stance. This
entails moving the budget balance toward a surplus (deficit) position during expansionary (contractionary) periods. Therefore, the higher the variance of the output gap the higher is the cost associated with an unstable economy. In order to examine the nature of the fiscal trade-off and evaluate alternative fiscal strategies, we use the following loss function:

\[
L_t = \sum_{i=1}^{n} \lambda_1 (B_{t-1} - BP^*)^2 + \lambda_2 (D_{t-1} - D^*)^2 + \lambda_3 (y_{t-1} - y^*)^2 \tag{6.6}
\]

where the difference between the public debt and the benchmark profile \((BP^*)^6\) measures the cost in terms of credibility for the government. The distance between the primary deficit and the deficit ceiling measures the malcontent of the consumers and firms of the current generation and the third term measures the loss attached to the difference between the GDP and the natural rate.

The parameters \(\lambda_1, \lambda_2, \text{ and } \lambda_3\), are respectively the non-negative weights attached to each goal and are determined by policymakers' tastes. We use a quadratic loss function as we treat symmetrically both positive and negative deviations from the targets.

The joint model captures several phenomena that are important for policymaking in practice. Both output and inflation are subject to unforecastable shocks. Output movements are persistent, because lagged output

\[\text{footnote}{In order to set a loss function for the Government we assume that the fiscal authority initially determines a benchmark profile in the debt-to-GDP ratio. Specifically, we assume a Government with clear objectives of convergence that planned to generate a surplus equal to 4\% plus interest on the public debt at time } t - 1. \text{ It then seeks to minimize stochastic fluctuations in the debt-to-GDP ratio relative to the benchmark profile.} \]
enters into equation (6.2). Inflation is inertial: once it rises, it stays high unless output falls. Finally, policy can offset shocks only with a lag. It takes a period for policy to affect output, and a period for output to affect inflation; combining these lags, it takes two periods for policy to affect inflation. This structure captures the stylized fact that policy affects output more quickly than it affects inflation (e.g., McCallum, 1995). In addition, it characterizes a fiscal policy rule for the Government accounting for the fiscal policy trade-off. The main virtues of the model are simplicity and realism. An important limitation is that the model ignores the Lucas critique. However, there are several studies supporting the statistical unimportance of this critique for the Philips curve when the analysis relies on a backward-looking model. This statement, to some degree, is even more real for a closed economy (Estrella and Fuhrer, 2002). Thus, our results still remain of interest.

Two more limitations regard the fact that the Government only has the public spending instrument to control the public debt dynamics and the assumption that equation (6.1) and equation (6.2) remain unchanged across different policy regimes; this may be untrue. We must keep this possibility in mind when interpreting the results.

6.4 Calibration and Simulation

Once we set up the theoretical framework we can simulate the model. Following the approach proposed by Annichiarico and Giammarioli (Annichiarico-Giammarioli, 2004), we study the dynamics of the public debt for different
families of parameters in the reaction function. Specifically, we search for the combinations of parameters \((a_1, a_2, a_3)\), in the interval \([0,1]\), that characterizes the optimal fiscal policy rule. We define as optimal the fiscal rule that minimizes the quadratic loss function expressed in equation(6.6)

In order to perform some simulation exercises, we interpret each period as a year. A year is a roughly realistic time lag for the effects of policy on output, and two years is about right for inflation (Christiano et al., 1996). In the simulations, we take the initial conditions as given and we assume that the fiscal rule, described with equation(6.5), becomes operative at time \(t = 1\). For simplicity, we assume the tax rate to be constant at its long-run level\((10\%)\) and the primary balance surplus to be equal to zero at time \(t = 1\) \((s_1 = 0)\). We also set the initial level of the public debt to be equal to 100%. For equation(6.1) and equation(6.2) the analysis uses a set of parameters base borrowed from the closed-economy model in Ball(1997) reported in table 6.1.

<table>
<thead>
<tr>
<th>Calibrated Model</th>
<th>(Y_{t+1})</th>
<th>(\pi_{t+1})</th>
<th>(r_{t+1})</th>
</tr>
</thead>
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<tr>
<td>(y_t)</td>
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<td>-</td>
<td>-1.0</td>
</tr>
<tr>
<td>(\pi_t)</td>
<td>0.4</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>(r_t)</td>
<td>0.5</td>
<td>1.5</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1 Calibrated values

\(^7\)Unlike this study, we operate in a three dimensional space.
Based on the evidence discussed there, we assume that the coefficient on lagged output (i.e. $\alpha^y$) to be equal to 0.8; we set the parameter $\beta^y$ to be equal to 1.0 (meaning that a one-point rise in the interest rate reduces output by one percent); we assume the Phillips curve to be vertical in the long run meaning that the coefficient $\sigma^\pi$ is equal to 1.0 and finally we set the Phillips curve slope (i.e. $\beta^\pi$) equal to 0.4. For equation (6.3) we borrow the parameters estimated by Taylor in the seminal paper of 1989. Specifically, we assume that the coefficient measuring the reactivity of the interest rate to the output gap $\alpha^i$ to be equal to 0.5 and the coefficient measuring the reactivity of the interest rate to the inflation rate to be equal to 1.5.

The stochastic simulation procedure operates as follows. Stochastic shocks displace all endogenous variables in the model each period. Mutually independent random shocks are drawn from a zero mean normal distribution, with variances calibrated to be roughly consistent with the historical data.

On the base of the calibrated coefficients we simulate the model 50 years ahead. Figure 6.1 shows the dynamics of the public debt for given combinations of parameters. Each chart plots the dynamic of the public debt for

---

8In the aggregate-spending equation, the coefficient on lagged output (i.e. $\alpha^y$) should be large, as output fluctuations are persistent.

9This value is close to the effect estimated in the macro-econometric framework based on VAR models proposed by Rudebusch (1995). With $\beta^y = 1.0$, a three-point rise in the interest rate reduces annual growth from 2.5 percent to -0.5 percent.

10The choice of this value relies on the fact that the sacrifice ratio for reducing inflation is $1/\beta^\pi$. Evidence suggests that a plausible ratio is 2.5, implying that $\beta^\pi$ must be equal to 0.4 (Ball, 1994b)

11The stochastic simulation process is repeated 1000 times.
the values of $a_1$, $a_3$ reported at the top of the graph and $a_2$ assuming the values: 0.1, 0.5 and 1.0. The grey horizontal line represents the public debt target (i.e. $B_{star} = 60\%$).
Figure 6.1 Simulated Public debt for different combinations of parameters
This figure gives interesting insights. First, it arises that, for some combinations of parameters, the public debt is divergent.\footnote{With the exception of chart G.1, this is true for each combinations of parameters displaying a coefficient $a_2$ equal to 0.1.} Second, there are families of parameters failing to reach the target even though they show converging paths.\footnote{For instance, the combination of parameters $a_1 = 0.5$, $a_2 = 0.5$, $a_3 = 0.5$ in chart G.4 or the combination $a_1 = 1.0$, $a_2 = 1.0$ and $a_3 = 1.0$ in chart G.6.} This is due to the fact that the Government, at some point, starts generating surpluses equal to the interest paid on debt at time $t-1$.\footnote{In chart G.4, for example, the government starts generating surpluses approximately equal to the interest paid on debt at time $t-1$ after 15 years; and after 8 years in chart G.6.} This makes the public debt stationary around a value of public debt that is different from the target.\footnote{In chart G.4 and chart G.6, the value of public debt around which the simulated public debt becomes stationary ($B^{star}$) is 70% that is 10% higher than the target level (i.e.60%).} In this case we say that the steady state is not the same as the target. Finally, some combinations of parameters give rise to the overshooting of the target level.\footnote{For instance, in chart G.1 the combination of parameters $a_1 = 0.1$, $a_2 = 1.0$ and $a_3 = 0.1$ leads to an overshooting of the public debt target.}

The parameter space$(a_1, a_2, a_3)$ of the coefficients ensuring the convergence paths toward the Public debt target is reported in figure 6.2. This is obtained by excluding the explosive combinations from the domain of the coefficients. The shaded area represents the whole combinations of parameters determining diverging paths for debt, while any other combination of those parameters(not shaded area) represents a fiscal policy rule guaranteeing never increasing debt.
The figure does not provide a clear relationship among the parameters. Nevertheless, it gives interesting insights. In particular, the shaded area suggests that the condition for the convergence is that the parameter governing the adjustment of the public debt towards its target, $a_2$, should be approximately greater than 0.4. However, if the parameter regulating the speed of the adjustment, $a_1$, and the parameter responding to the output gap movements, $a_3$, are both low (let's say less than 0.2) then a value of $a_2$ smaller than 0.4 still ensures the convergence toward the public debt target.

Among the converging values (not shaded area) we search for the fiscal rule that minimizes equation (6.6). In order to do that we perform the simulation $n = 1000$ times for given weights in the loss function. The three
6.4. CALIBRATION AND SIMULATION

dimensional scatter plot of the combinations \((a_1, a_2, a_3)\) for the following weights \((\lambda_1 = 0.4, \lambda_2 = 0.2, \lambda_3 = 0.4)\) is reported in Figure 6.3.

Figure 6.3 The 3-D scatter plot of the whole combinations minimizing the loss function

Each point displays the fiscal rule \((a_1, a_2, a_3)\) that minimizes the loss function after each iteration. The number attached to each point represents the frequency with which that combination of parameters is resulted to be the minimizing rule.\(^{17}\) The point with highest frequency \((i.e. 156)\) represents

\(^{17}\)In order to avoid the overcrowding of the figure, we only report the families of parameters displaying the highest frequencies. Notice that if we had reported the frequencies for each combination, the summation of such frequencies would have been 1000.
We repeat the same procedure for different weights in the loss function. Specifically, we repeat the same simulation exercise as the weight on the difference between the public debt and its benchmark profile increases by 0.1 at each step. The change in $\lambda_1$ implicitly determines the weight on the term related to the output gap since $\lambda_3$ is set to be equal to $1 - \lambda_1$ and $\lambda_2$ is assumed to be equal to a constant value (i.e. 0.2). Thus, as the weight on debt increases by 0.1, the weight on the output gap reduces by the same amount at each step.

Among other things, figure 6.3 also suggests some skewness in the outcome of simulation. In fact, the optimal fiscal rule seems not to be perfectly centered. This is the first signal about the fact that the distribution of the vector $(a_1, a_2, a_3)$ is probably not normal. This guess is confirmed by univariate normality tests conducted on each single distribution of the parameters. The results of the tests are reported in tables 6.2 and 6.3:

---

18Notice that 156 out of 1000 simulations is not an extremely high frequency. Nevertheless, since all families of parameters showing the highest frequencies are distributed around that point, we can assess, with a certain degree of confidence, that the optimal fiscal policy rule is characterized by those parameters.
### Table 6.2 Univariate Normality Tests

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<th>Signif Level (Sk=0)</th>
<th>Kurtosis</th>
<th>Signif Level (Ku=0)</th>
<th>Jarque-Bera</th>
<th>Signif Level (JB=0)</th>
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<tr>
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<tr>
<td>Skewness</td>
<td>1.29824</td>
<td>0</td>
<td>Kurtosis</td>
<td>1.17057</td>
<td>338.00795</td>
<td>0</td>
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<td>Kurtosis</td>
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<td>338.00795</td>
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<tr>
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<tr>
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<td>Signif Level (Sk=0) 0</td>
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<td>Jarque-Bera 272.0694</td>
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<td>Jarque-Bera 245.8726</td>
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<td>Jarque-Bera 993.3963</td>
<td>Signif Level (JB=0) 0</td>
<td></td>
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</table>

Table 6.3 Univariate Normality tests
The Jarque-Bera test and the tests on the skewness and kurtosis strongly reject the normality of the process generating the single parameters and therefore, the normality of the process generating the vectors of the optimal fiscal policy rules. The actual distribution of the parameters is investigated graphically in figure 6.4 and 6.5. Each graph shows the histogram of the outcome simulation for each single parameter. The skewness of the distributions is clearly displayed in each graph. In theory there is only one distribution that is defined on the domain \([0,1]\) and is right or left skewed depending on the parameters characterizing the distribution. This variate is called Beta distribution\(^{19}\). We use this distribution in order to approximate the outcome of our simulation with a continuous distribution. The goodness of fit is shown in figures 6.4 and 6.5:

\(^{19}\)For further details about Beta distribution see appendix 6.1.
Figure 6.3 Beta approximation
Figure 6.5 Beta approximation
The black line represents the beta distribution for given parameters ($\alpha, \beta$). The maximum likelihood estimation of the values ($\alpha, \beta$) used in each approximation, are reported, together with 95% confidence intervals, in the following table:

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$\alpha$ MLE</th>
<th>$\beta$ Lower</th>
<th>$\beta$ Upper</th>
<th>$\alpha$ MLE</th>
<th>$\beta$ Lower</th>
<th>$\beta$ Upper</th>
<th>$\alpha$ MLE</th>
<th>$\beta$ Lower</th>
<th>$\beta$ Upper</th>
</tr>
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<tbody>
<tr>
<td>2.473</td>
<td>2.1421</td>
<td>18.1618</td>
<td>3.4573</td>
<td>3.1134</td>
<td>2.2604</td>
<td>5.6522</td>
<td>5.196</td>
<td>35.104</td>
<td>5.196</td>
</tr>
<tr>
<td>9.2236</td>
<td>2.6038</td>
<td>10.285</td>
<td>2.3876</td>
<td>3.7941</td>
<td>2.5148</td>
<td>38.422</td>
<td>6.5448</td>
<td>41.739</td>
<td>35.104</td>
</tr>
</tbody>
</table>

Table 6.4 Maximum likelihood estimation of $\alpha$ and $\beta$ with related 95 percent confidence intervals

Once we have obtained the theoretical process generating the coefficients, we replace the parameter value, produced by the simulation, with the mode of the Beta distribution that we used for the approximation. Then, we compute the quantiles of the distribution in order to construct the confidence interval for each parameter. These intervals, together with the parameters and the values of $\lambda_1$ at which these parameters are obtained, are shown in
Table 6.5 suggests that optimal fiscal rules are characterized by a high parameter on the debt gap (i.e. $a_2$). On the contrary, the parameters measuring respectively the reactivity of the Government to the output gap and to the distance between the level of primary deficit and its lower bound are both very low. Specifically, they are both close to 0.1. This means that, given the initial conditions and the benchmark profile for the government, the optimal fiscal rule shows a higher reactivity to the debt gap with respect to the other objectives. This result is consistent with the fact that, among the assumptions underlying the model, we modeled a Government with a clear objective of convergence. Thus, a higher reactivity of the fis-
cal authority to the debt control objective was then expected. Moreover, the outcome of the simulation also highlights the highly conflicting nature of the three goals. This trade-off is partially illustrated by the efficiency frontiers. We assess the capacity of each fiscal policy rule to provide economic stabilization by measuring the extent to which fiscal policy influences cyclical fluctuations in output. In particular, we try to quantify the trade-off between the debt control objective and the economic stabilization by constructing the efficiency frontier for the government. This is obtained by reporting the standard deviation of the output gap and the standard deviation of the simulated public debt from its target respectively on the vertical and horizontal ax. The curve is illustrated in figure 6.6.
6.4. CALIBRATION AND SIMULATION

At each point on the black line corresponds to the optimal fiscal rule and the weight $\lambda_1$ at which it is obtained. The figure shows the extent to which a policy maker could sacrifice its debt control objective, in favour of a higher economic stabilization objective. It shows that strict debt rules are needed as the weight attached to the public debt objective increases. Nevertheless, strict debt rules would provide more debt control, but at the cost of less economic stabilization. For example, when the weight $\lambda_1$ is equal to 0.1 the standard deviation of output gap is approximately equal to 0.17%. But when the value of $\lambda_1$ increases to 0.7 the standard deviation of output becomes equal to 0.29%.
6.5 Public Debt Dynamics in Italy and Belgium

The procedure followed for a theoretical economy in the previous section is now applied to the EMU countries with highest public debt in the Euro Area (i.e. Italy and Belgium). However, for these countries we do not apply the backward-looking closed economy model used in the previous section. In the Pre-Euro period Italy and Belgium were commonly considered as small open economies. The estimation of a theoretical framework for these countries, therefore, cannot neglect the effect of the exchange rate on output gap, as well as the role played by the exchange rate in the monetary transmission mechanism. In order to fill these gaps we modify the model by adding the real exchange \( e \) to equation (6.8). The model turns out to be the Ball’s model proposed in chapter 2. Unlike the second chapter, here we use annual data and thus consider only one lag. The model is as follows:

\[
\pi_t = \alpha^\pi \pi_{t-1} + \beta^\pi y_{t-1} + u^\pi_{t-1} \tag{6.7}
\]

\[
y_t = \alpha^\nu y_{t-1} - \beta^\nu (i_{t-1} - \pi_{t-1}) - \xi^\nu e_{t-1} + u^\nu_{t-1} \tag{6.8}
\]

\[
w r_t + (1 - w) e_t = a y_{t-1} + b [\pi_{t-1} + \xi^\nu e_{t-1}] \tag{6.9}
\]

where \( \xi^\nu \) is the parameter measuring the reactivity of the output gap to the real exchange rate and \( w \) is the weight attached to the interest rate.
6.5. PUBLIC DEBT DYNAMICS IN ITALY AND BELGIUM

in the MCI\(^{20}\)(i.e \(we_t + (1 - w)e_t\)). All the other symbols keep the above mentioned meaning. The inclusion of the real exchange rate changes the Taylor rule in two aspects. First, the policy variable becomes a combination of \(r\) and \(e\)(i.e MCI). Second, the inflation rate is replaced by a combination of inflation and lagged exchange rate\(^{21}\).

Once we have implemented the reference model we proceed as follows. First, we estimate the equations (6.7), (6.8), and (6.9). Then, we simulate the model including the identity budget constraint and the reaction function for the government, on the base of the estimated parameters. In the simulation we assume the initial level of public debt to be equal to the level of indebtedness for the Government under examination, at the end of the sample period(i.e.2003). Specifically, we set the initial public debt for Italy to be equal to 106.2% and for Belgium to be equal to 100.5%. In addition, we keep the tax rate to the long run value that we assumed to be equal to 10%. For the estimation we use annual data provided by the European commission for the sample period 1983-2003\(^{22}\). All variables are demeaned and the model is estimated by using the OLS technique. We use, for the two countries, a bilateral exchange rate against the dollar. Table 6.6 and table 6.7 show the estimates respectively for Italy and Belgium.

\(^{20}\)Following Ball(1998) we assume \(w\) to be equal to 0.75. We do not exclude the possibility that a different value of \(w\) could affect the results of the simulation.

\(^{21}\)For further details about the MCI see section 2.3 Chapter 2.

\(^{22}\)We uses a short sample due to the limitation of the data on the yearly base.
The tables suggest that coefficients have the right sign in both countries. They are lower in Italy than in Belgium.

The results of the simulation are described in the following part of this section. For these countries we only report the main tables and graphs. The outcome of the simulation determining the parameter space is illustrated in figure 6.7 and figure 6.8:

### Table 6.6 Estimated coefficients for Belgium

<table>
<thead>
<tr>
<th></th>
<th>$y_{t-1}$</th>
<th>$\pi_{t-1}$</th>
<th>$r_{t-1}$</th>
<th>$c_{t-1}$</th>
<th>$\pi_{t-1} + \xi c_{t-1}$</th>
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<td>0.65</td>
<td>-</td>
<td>-0.38</td>
<td>-0.20</td>
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<tr>
<td></td>
<td>(s.e 0.14)</td>
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<td>(s.e 0.07)</td>
<td>(s.e 0.04)</td>
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<tr>
<td>$\pi_t$</td>
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<td>-</td>
<td>-</td>
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<td>(s.e 0.33)</td>
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<td></td>
<td></td>
<td>(s.e 0.38)</td>
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### Table 6.7 Estimated coefficients for Italy

<table>
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<th>$\pi_{t-1}$</th>
<th>$r_{t-1}$</th>
<th>$c_{t-1}$</th>
<th>$\pi_{t-1} + \xi c_{t-1}$</th>
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<tr>
<td></td>
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<td>(s.e 0.04)</td>
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</tr>
<tr>
<td>$\pi_t$</td>
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<td>-</td>
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</tr>
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<td>(s.e 0.06)</td>
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<td>(s.e 0.22)</td>
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<td>-</td>
<td>-</td>
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<td>(s.e 0.47)</td>
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</table>
6.5. PUBLIC DEBT DYNAMICS IN ITALY AND BELGIUM

Figure 6.7 Belgium: Explosive Area

Figure 6.8 Italy: Explosive Area
Similarly to the previous section, the shaded areas represent the whole combination of parameters \((a1, a2, a3)\) leading to diverging paths for public debt. A comparative analysis suggests that the explosive area is more extended in Italy than in Belgium\(^{23}\). In addition, both countries show greater explosive areas than the theoretical economy. This can be explained as follows. Both countries, but especially Belgium, have a starting fiscal position higher than the initial value that we assumed for the theoretical economy. Moreover, the overall effect of the exchange rate and the interest rate on the output gap for Italy and Belgium is lower than the impact, of the real interest rate on the same variable, in the first model. Finally, the monetary policy authorities, in both countries, have shown a less aggressive behavior with respect to a virtual central bank having as reactivity function the rule estimated by Taylor(1967). These factors jointly caused a higher interest rate and, therefore, a higher growth rate for public debt during the simulation. Consequently, a more aggressive behavior of the fiscal policy authority on the debt objective, seems to be needed in order to avoid diverging paths for the simulated public debt.

The optimal fiscal policy rules for the two countries and for different weights are reported respectively in table 6.8 and table 6.9.

\(^{23}\)From Figure 6.7 and 6.8, the explosive areas seem to be very similar in both countries. Nevertheless, looking at the dimension of the files obtained as outcome of the simulation, it arises that Italy is clearly characterized by a larger explosive area.
### Table 6.8 Belgium: Modes and related 95 per cent confidence intervals from the Beta distributions

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<tr>
<th>( \lambda_i )</th>
<th>Lower</th>
<th>( a_1 )</th>
<th>Upper</th>
<th>Lower</th>
<th>( a_2 )</th>
<th>Upper</th>
<th>Lower</th>
<th>( a_3 )</th>
<th>Upper</th>
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<td>0.29479</td>
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<td>0.141</td>
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<td>0.754</td>
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<td>0.99</td>
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</tr>
<tr>
<td>7</td>
<td>0.11041</td>
<td>0.113</td>
<td>0.19053</td>
<td>0.996</td>
<td>0.999</td>
<td>0.99</td>
<td>0.12841</td>
<td>0.131</td>
<td>0.20853</td>
</tr>
</tbody>
</table>

### Table 6.9 Italy: Modes and related 95 per cent confidence intervals from the Beta distributions

<table>
<thead>
<tr>
<th>( \lambda_i )</th>
<th>Lower</th>
<th>( a_1 )</th>
<th>Upper</th>
<th>Lower</th>
<th>( a_2 )</th>
<th>Upper</th>
<th>Lower</th>
<th>( a_3 )</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09391</td>
<td>0.1</td>
<td>0.26379</td>
<td>0.773</td>
<td>0.776</td>
<td>0.88518</td>
<td>0.10391</td>
<td>0.11</td>
<td>0.27379</td>
</tr>
<tr>
<td>2</td>
<td>0.11614</td>
<td>0.122</td>
<td>0.28167</td>
<td>0.878</td>
<td>0.881</td>
<td>0.99333</td>
<td>0.10214</td>
<td>0.108</td>
<td>0.26767</td>
</tr>
<tr>
<td>3</td>
<td>0.0965</td>
<td>0.101</td>
<td>0.20967</td>
<td>0.885</td>
<td>0.888</td>
<td>0.99</td>
<td>0.1185</td>
<td>0.123</td>
<td>0.23167</td>
</tr>
<tr>
<td>4</td>
<td>0.09941</td>
<td>0.104</td>
<td>0.21475</td>
<td>0.91</td>
<td>0.913</td>
<td>0.99</td>
<td>0.12941</td>
<td>0.134</td>
<td>0.24475</td>
</tr>
<tr>
<td>5</td>
<td>0.10711</td>
<td>0.11</td>
<td>0.19734</td>
<td>0.996</td>
<td>0.999</td>
<td>0.99</td>
<td>0.11811</td>
<td>0.121</td>
<td>0.20834</td>
</tr>
<tr>
<td>6</td>
<td>0.11921</td>
<td>0.122</td>
<td>0.20352</td>
<td>0.996</td>
<td>0.999</td>
<td>0.99</td>
<td>0.12821</td>
<td>0.131</td>
<td>0.21252</td>
</tr>
<tr>
<td>7</td>
<td>0.12841</td>
<td>0.131</td>
<td>0.20853</td>
<td>0.996</td>
<td>0.999</td>
<td>0.99</td>
<td>0.11941</td>
<td>0.122</td>
<td>0.19953</td>
</tr>
</tbody>
</table>

These tables also show the 95% confidence intervals for such parameters.
Similarly to the theoretical economy, strict debt rules result to be optimal in both countries. The simulation suggest that a very aggressive behaviour of the fiscal authorities in the these two countries is needed in order to hit the target. For Italy the optimal fiscal rule is characterized by the maximum value of $a_2$ for every value of $\lambda_1$ greater than 0.4. The same applies to Belgium for every value of $\lambda_1$ greater than 0.5. Unlike Italy, the optimal fiscal rules for Belgium seem to show a slightly higher reactivity of the primary surplus to the output gap. The value of $a_3$ for this country is, in fact, greater than 0.2 for $\lambda_1$ equal respectively to 0.2 and 0.3. This means that Belgian government, in setting the public spending, has a higher margin of control for stabilizing the economy and a less tight fiscal trade-off. This result is confirmed by the efficiency frontiers for the two countries that are reported in figure 6.9:
The figure shows an Italian efficiency frontier slightly shifted to the right with respect to the Belgian one. For every value of $\lambda_1$ the standard deviations of the two objectives are greater in Italy than in Belgium. This means that, despite a smaller fiscal position, the Italian government, in the simulation exercise, performed worse, in the achievement of its goals, with respect to the Belgian fiscal authority. This could be due to the differences in the economic and financial structure of the two economies highlighted with the estimated models. Despite these little asymmetries, the performances of the rules in Italy and Belgium seem to be very close to one another. Altogether the outcome of the simulation seems to highlight the need for an aggressive behavior on the part of the fiscal authorities in Italy and Belgium. In order to ensure the convergence of the public debt toward the target level the gov-
ernments, of the two EMU countries, should set the public spending mainly on the base of the distance between the public debt level and its target.

6.6 Conclusion

This chapter evaluated a set of alternative strategies aiming to reduce the public debt toward its target in the presence of uncertainty. In order to evaluate the performance of a simple fiscal policy rule in terms of its capability of guaranteeing convergence and sustainability of public finances, we simulated a neoclassical model with an exogenous stochastic interest rate under the complete control of the monetary policy authorities. The policy reaction rule considered in the theoretical framework linked the primary balance ratio to the distance between the current and the target level of public debt, the distance between the primary surplus and the deficit ceiling, and the output gap. With the objective of satisfying the rule, the government has only the benefit rate as the fiscal instrument. Furthermore, with the aim to stabilize the public debt at its target level, the government must take into account the trade-off that a fiscal framework usually arises. The simulation exercise made it possible to explore a different path of adjustments depending on numerical values assigned to the parameters characterizing the fiscal policy rule governing the public sector. In particular, it constructed a procedure to estimate the optimal fiscal policy rule given some initial conditions.

This analysis provided interesting insights into the linkages between the fiscal rule parameters and the dynamics of the public debt. It emerged that
the implementation of a fiscal rule characterized by time invariant parameters may trigger a non linear process of adjustment towards the objective of convergence. Specifically, the smaller the parameter regulating the reactivity of the primary surplus to the distance between the public debt and its target, the higher is the probability that the government can generate an explosive path for public debt. When the analysis is applied to Italy and Belgium it arises that the explosive area is slightly greater, in Italy, than in Belgium. These differences, as well as the differences in the parameters characterizing the optimal fiscal policy rules, are mainly due to the asymmetries in the economic and financial structure of the two economies, highlighted with the estimated models.

Among other things, the study has also provided the procedure to construct confidence intervals for the parameters characterizing the optimal fiscal policy rule and a procedure to estimate an efficiency frontier illustrating the fiscal trade-off for the Government. The efficiency frontier gave further insights. Strict debt rules provide more debt control, but at the cost of less economic stabilization. Regarding the two countries under examination, Belgium performed better than Italy in the achievement of the fiscal objectives.

The model is relatively simple, tractable and easy to understand. Some of our results may depend, at least in their quantitative details, on the choices we have made for the model’s parameters. We would argue, however, that our calibration of the macroeconomic model is largely accepted
from a part of the literature. Furthermore, the study shows that, once we estimated the macroeconomic model, the procedure is applicable to different economies. An example of such an applicability is given for Belgium and Italy.

The estimated rules are not intended to serve as an explicit guide in setting fiscal policy. Nevertheless, the whole analysis represents a simple transparent way to characterize the behavior of the fiscal authority in a stochastic simulation framework. Moreover, it could be used to evaluate a wide range of fiscal policy issues. The study is not the last word on the debt strategy problem. More work is required in a variety of directions. Future work is definitely needed to enhance the richness of our stochastic environment.
Part IV

General Conclusions
This dissertation focused on the rules and the transmission mechanism of monetary policy in the Euro Area. In the first part (Chapters 1 to 2) we studied the main features of the economic structures of the EMU countries emerging from the single country evidence. In particular, we analyzed the asymmetries in monetary policy transmission and monetary rules performance across EMU members. In the second part (Chapters 3 and 4) we considered Euro Area as a whole. The third chapter studied asymmetric behavior of the ECB and the Bank of England during the different phases of the business cycle, while the fourth chapter focused on the confidence channel in the monetary policy transmission mechanism. To this end it constructed a model for the Consumer and Business confidence. In the final part (Chapter 5 and 6) the thesis faced fiscal policy issues. Specifically, Chapter 5 proposes a multivariate test for sustainability of fiscal policy, while Chapter 6 estimated the optimal fiscal rule for a Government with commitment.

The main conclusions the dissertation reached can be summarized as follows.

***

Chapter 1 attempted to analyze different rules capable of modeling how the central banks of EMU countries have made policy choices affecting interest rates. In particular, the study focused on six different rules relating to the interest rate, which the central banks are assumed to control, to a set of variables thought to affect monetary authority behavior. This kind of study provided insight into how the new European monetary institution
should conduct and characterize its policy strategy. In other words, it can suggest how the ECB should move interest rates once a change in real output, inflation occurs. The first step of the analysis was the construction of a macroeconomics model to use as a basis for the comparison of estimated reaction functions. The features of the model are very important because the conclusions obtained depend, of course, on the belief that the economic structure implied by the proposed model is not grossly incorrect. The econometrics analysis considered the main properties of six different rules: three different specifications of the Taylor rule, an optimal feedback rule and a forward-looking rule. The first question considered in the comparative analysis was the ability of the rules to replicate historical interest rate movements, that is the central bank behavior. The results emerging from the study stressed that simple rules perform quite well in following interest rates historical records. The ability to mimic interest rate changes increases once an interest rate smoothing term is included in the reaction function. This suggested that central bank behavior can be better explained by adding a lagged interest rate. Moreover, considering a forward-looking dimension that takes into account expectations of future inflation movements, seems to give further improvement. The second issue is related to the ability of the rules to minimize the volatility of the variables the central bank considers as targets and, therefore, to stabilize the economy. The analysis suggests that even if the rule obtained by solving an optimal control algorithm is consistently, across the EMU countries, the top-performing rule, the per-
formance of a simple forward-looking rule with a smoothing term for the interest rate is almost as stabilizing as the optimal feedback rule. Then, it can be concluded that the gains a central bank can obtain by following a complicated rule are not so great. In addition, the easier communicability of the simple rule can also increase the transparency and thus the credibility of the central bank. The problem of transparency is of particular interest, once the problem of the possible rules the European Central Bank should adopt is considered. In fact, the inability of the ECB to communicate with the agents about its strategy is one of the main problems the new monetary institution is facing. It follows that the ECB should use simple rules as guidelines for its monetary strategy.

***

The aim of chapter 2 was to shed some light on how each channel of transmission may, heterogeneously, work in different countries and how those differences may change by the introduction of the euro. The analysis has been divided in two parts. In the first part, the possible asymmetries in output and prices responses to a monetary policy shock, across EMU countries, have been analyzed. We have shown that imposing restrictions on the estimated structural models according to the monetary constraints each country faced during the EMS period, obtains well-behaved and quantitative comparable effects of the monetary policy shocks in all the countries. In the second part, we have modelled policy behavior with monetary rules that accounts for exchange rate changes. After estimating the efficiency frontiers for each EMU
country, we have implemented the impulse response analysis in the light of the new instrument of monetary policy. The impulse response analysis highlights that output gap and inflation in the selected economies, respond to identical monetary policy shocks with a similar speed and movement, albeit with different size of effect. From the empirical analysis applied in the chapter, it emerges that the effect of a monetary shock on output depends mostly on the output structure of the countries. The results suggest the presence of small divergences across EMU members. Those asymmetries, concerning the interest rate channel, will probably not be so large to determine frictions for the EMU. Other channels, not considered here, could be a cause of concern. The chapter did not take into account the asymmetries that might arise from the credit channel or from the stock market channel. Nevertheless, it seems plausible to imagine that the cross-country differences in monetary transmission across EMU countries, could decrease over time as a result of an increasing financial structures homogeneity. This means that, in the long run, asymmetries in monetary transmission will not be a cause of concern. However, some differences are likely to persist; those divergences call for a better understanding and monitoring of the national monetary transmission mechanisms.

***

Chapter 3 analyzed whether monetary authorities asymmetrically affect real economy depending on the phases of the business cycle. In particular, the chapter measured the likely effects of monetary policy actions on output
and inflation, using a multivariate extension of Hamilton's regime switching model. The empirical analysis suggested that the two central banks display more aggressive behavior in recession rather than in expansion. In both countries the implied targets were significantly higher in expansion than in recession. The estimated regime dependent Taylor's rules, as well as the estimated inflation targets, also confirmed that, while the Bank of England contemporaneously targets output and inflation, the primary goal of the ECB consists of achieving price stability. Some interesting results also came from the impulse response analysis. The model appeared to perform quite well. No price puzzles have been detected in the simulations. In general, an interest rate shock leads to a larger effect on the output gap during recession than during expansion. Common to the UK and the Euro Area, the results highlight significant negative effects of an increase in interest rate during recessionary periods. Some differences arise from the size and the timing of the responses. The patterns of the responses are supposed to be dependent upon the particular strategy followed by the central bank. A central bank that primarily aims at achieving price stability, like the ECB, has a greater and quicker output response during recession than a central bank that targets several variables, like the Bank of England, that, in contrast, has quicker but smaller output reaction during expansion. Altogether the analysis strongly suggests that the stage of the business cycle is important in the monetary policy decision process. A central bank cannot neglect the specific regime where the monetary action takes place.
Chapter 4 analyzed the key determinants of economic confidence in the Euro Area since 1993. We set up two multivariate frameworks to model respectively consumer and business confidence.

The results suggest that consumer confidence is mainly driven by labour market developments in the euro area (measured by the change in the unemployment rate). The real short-term interest rate seems to play a key role in the long run. Stock market developments play a significant, but smaller role than the other factors, and smaller than the available literature seems to suggest. Stock market developments affect the consumer confidence index, essentially, through its component measuring the way households perceive general economic conditions.

Regarding the results for business confidence, the dominating factor is a measure of business confidence in the United States, which suggests a rather strong role for the world economic cycle on Euro Area business confidence. Among the endogenous variables, industrial production seems to have a significant effect in the short run, but this effect is of a lesser magnitude than the external factor just mentioned. In addition, stock market developments are assessed to have a larger effect on the confidence of firms compared to households. Regarding the breakdown of the business confidence indicator into its sub-components, the most reactive component is the index describing the assessment of order books.

Looking at the impact of monetary policy more in detail, it appears that
changes in interest rates have the expected negative effect both on consumer and business confidence. A comforting result also concerns the timing of these effects, which is similar to the transmission lag of monetary policy on economic activity commonly found in the literature. These results provide some evidence for the presence of a "confidence channel" in the transmission of monetary policy on real activity. The models could be extended to include price variables to also analyze further the transmission of monetary policy on inflation. However, the fact that the effect of interest rates on consumer confidence has a similar timing as the effect on real economic activity leads to a cautious interpretation. Indeed, the model of consumer confidence does not include any variable of economic activity (other than the change in the unemployment rate), meaning that the confidence indicator itself could be argued to capture, in fact, the effects on economic activity. The results of the business confidence model tend to rule out the presence of such a problem. Indeed, the timing of the response to a shock in interest rates is similar to that in the consumer confidence model, while the business confidence model includes a variable controlling for economic activity (the industrial production variable). Thus, interest rates can be argued to affect business confidence beyond their impact on economic activity.

However, further analysis is needed before concluding on the relative influence of monetary policy on the confidence of economic agents in the Euro Area compared to other factors. First, it cannot be ruled out that some variables may have been omitted in the models proposed in this chapter. Their
omission may lead to bias in the results. Second, the analysis of monetary policy shocks could be refined. In this respect, shocks are simply defined in the multivariate frameworks proposed, in this chapter, as a one standard deviation change in a given variable. Alternative ways of measuring shocks would need to be used to address the impact of monetary policy on confidence. In particular, policy shocks can be measured as the deviations of the actual policy rates decided by monetary policy-makers compared to the expected interest rates. Monetary policy shocks can also be seen as indicators of the monetary policy stance. This policy stance can be measured by the deviation of actual real interest rates from their natural level. Overall, alternative ways of measuring policy shocks may lead to different assessments of the impact of monetary policy on confidence. Finally, central banks do not affect confidence only through changes in interest rates. It is commonly argued that central banks pursuing a policy of the "steady-hand" (i.e. of keeping interest rates unchanged) make a positive contribution to confidence.

***

Chapter 5 studied the sustainability of the fiscal policy in the Euro Area and the United States. By using the Vector Error Correction Model (VECM) and applying a cointegration analysis to data, it derived conditions suitable for determining whether or not fiscal policies are sustainable in the long run. It arose that the condition for sustainability depends upon the stationarity of the VAR system. In particular, if the VAR model is globally stationary,
fiscal policy is said to be sustainable. If the VAR model is not globally stationary, fiscal policy is said to be unsustainable.

The broad conclusion is that both countries have an unsustainable fiscal policy. Unlikely the Euro Area, U.S fiscal position seems to be less worrying. This is probably due to the higher growth rate of the GDP shown from the U.S with respect to the Euro Area in the sample period. The chapter also constructed forecasts for the public debt level in two different ways. The forecasted values of public debt show a convergence towards lower values. These values are consistent with the evaluation elaborated in the cointegration analysis. Altogether the study rejected the hypothesis of the sustainability of fiscal policy and confirmed the results obtained from a strand of literature.

***

Chapter 6 evaluated a set of alternative strategies aiming to reduce the public debt toward its target, in the presence of uncertainty. In order to evaluate the performance of a simple fiscal policy rule in terms of its capability of guaranteeing convergence and sustainability of public finances, we simulated a neoclassical model with an exogenous stochastic interest rate under the complete control of the monetary policy authorities. The policy reaction rule considered in the theoretical framework linked the primary balance ratio to the distance between the current and the target level of public debt, the distance between the primary surplus and the upper limit
of the primary deficit, and the output gap. With the objective of satisfy-
ing the rule, the government has only the benefit rate as fiscal instrument. Furthermore, with the aim to stabilize the public debt at its target level, the government must take into account the trade-off that a fiscal frame-
work usually arises. The simulation exercise made it possible to explore a different path of adjustments depending on numerical values assigned to the parameters characterizing the fiscal policy rule governing the public sector. In particular, it constructed a procedure to estimate the optimal fiscal policy rule given some initial conditions.

This analysis provided interesting insights into the linkages between the fiscal rule parameters and the dynamics of the public debt. It emerged that the implementation of a fiscal rule characterized by time invariant parame-
ters may trigger a non linear process of adjustment towards the objective of convergence. Specifically, the smaller the parameter regulating the reactiv-
ity of the primary surplus to the distance between the public debt and its target, the higher is the probability that the government can generate an explosive path for public debt. When the analysis is applied to Italy and Belgium it arises that the explosive area is slightly higher, in Italy, than in Belgium. These differences, as well as the differences in the parameters characterizing the optimal fiscal policy rules, are mainly due to the asymmetries in the economic and financial structure of the two economies highlighted with the estimated models.

Among other things, the study has also provided the procedure to con-
struct confidence intervals for the parameters characterizing the optimal fiscal policy rule and a procedure to estimate an efficiency frontier illustrating the fiscal trade-off for the Government. The efficiency frontier gave further insights. Strict debt rules provide more debt control, but at the cost of less economic stabilization. Regarding the two countries under examination, Belgium performed better than Italy in the achievement of the fiscal objectives.
Part V

APPENDIX
Table A2.1: Estimated coefficients for the aggregate demand equation

<table>
<thead>
<tr>
<th>Yt</th>
<th>Yt-1</th>
<th>Yt-2</th>
<th>Yt-3</th>
<th>Yt-4</th>
<th>( \pi_t - \pi_{t-1} )</th>
<th>Q_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.767</td>
<td>0.081</td>
<td>0.033</td>
<td>-0.195</td>
<td>-0.151</td>
<td>-0.208</td>
</tr>
<tr>
<td>Austria</td>
<td>1.179</td>
<td>-0.642</td>
<td>0.213</td>
<td>0.127</td>
<td>-0.182</td>
<td>0.259</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.784</td>
<td>-0.074</td>
<td>-0.018</td>
<td>0.056</td>
<td>-0.206</td>
<td>0.172</td>
</tr>
<tr>
<td>Netherland</td>
<td>0.457</td>
<td>0.102</td>
<td>0.093</td>
<td>0.044</td>
<td>-0.157</td>
<td>-0.070</td>
</tr>
<tr>
<td>France</td>
<td>0.783</td>
<td>0.050</td>
<td>-0.025</td>
<td>-0.084</td>
<td>-0.171</td>
<td>0.059</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.133</td>
<td>0.278</td>
<td>-0.119</td>
<td>-0.155</td>
<td>0.029</td>
<td>-0.210</td>
</tr>
<tr>
<td>Italy</td>
<td>0.794</td>
<td>-0.223</td>
<td>0.247</td>
<td>-0.137</td>
<td>-0.143</td>
<td>0.135</td>
</tr>
<tr>
<td>Spain</td>
<td>0.709</td>
<td>0.250</td>
<td>0.169</td>
<td>-0.412</td>
<td>0.039</td>
<td>-0.265</td>
</tr>
<tr>
<td>Finland</td>
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<td>-0.093</td>
<td>0.027</td>
<td>-0.113</td>
<td>-0.145</td>
<td>0.302</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.824</td>
<td>0.083</td>
<td>0.064</td>
<td>-0.237</td>
<td>-0.004</td>
<td>-0.015</td>
</tr>
</tbody>
</table>

Table A2.2: Estimated coefficients for the aggregate supply equation

<table>
<thead>
<tr>
<th>( \pi_t )</th>
<th>( \pi_{t-1} )</th>
<th>( \pi_{t-2} )</th>
<th>( \pi_{t-3} )</th>
<th>( \pi_{t-4} )</th>
<th>Yt+1</th>
<th>DQ_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.398</td>
<td>0.023</td>
<td>0.144</td>
<td>0.435</td>
<td>0.070</td>
<td>1.876</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.052</td>
<td>0.149</td>
<td>0.100</td>
<td>0.803</td>
<td>0.182</td>
<td>3.582</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.050</td>
<td>0.263</td>
<td>0.176</td>
<td>0.431</td>
<td>0.146</td>
<td>4.631</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.141</td>
<td>0.322</td>
<td>0.000</td>
<td>0.537</td>
<td>0.165</td>
<td>4.717</td>
</tr>
<tr>
<td>France</td>
<td>0.599</td>
<td>0.198</td>
<td>0.086</td>
<td>0.207</td>
<td>0.016</td>
<td>-0.930</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.137</td>
<td>0.358</td>
<td>0.502</td>
<td>0.003</td>
<td>0.148</td>
<td>0.000</td>
</tr>
<tr>
<td>Italy</td>
<td>0.794</td>
<td>-0.205</td>
<td>0.256</td>
<td>-0.142</td>
<td>-0.185</td>
<td>0.096</td>
</tr>
<tr>
<td>Spain</td>
<td>0.030</td>
<td>0.225</td>
<td>0.147</td>
<td>0.592</td>
<td>0.105</td>
<td>1.074</td>
</tr>
<tr>
<td>Finland</td>
<td>0.499</td>
<td>-0.116</td>
<td>0.211</td>
<td>0.473</td>
<td>0.203</td>
<td>4.585</td>
</tr>
<tr>
<td>Portugal</td>
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<td>0.111</td>
<td>0.146</td>
<td>0.376</td>
<td>0.039</td>
<td>-0.832</td>
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</table>
Table A2.3: Estimated coefficients for the monetary policy function

<table>
<thead>
<tr>
<th>Country</th>
<th>$l_1$</th>
<th>$l_{1,t}$</th>
<th>$\pi_{t,t}$</th>
<th>$\gamma_{t,1}$</th>
<th>$q_{t}$</th>
<th>$I_{1,DE}$</th>
<th>$q_{DE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
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<td>0.034</td>
<td>0.125</td>
<td>1.163</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>-</td>
<td>0.053</td>
<td>0.179</td>
<td>0.410</td>
<td>0.813</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>-</td>
<td>0.605</td>
<td>0.103</td>
<td>-0.056</td>
<td>0.083</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>-</td>
<td>0.131</td>
<td>0.271</td>
<td>1.054</td>
<td>0.760</td>
<td></td>
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</tr>
<tr>
<td>France</td>
<td>0.791</td>
<td>0.116</td>
<td>0.095</td>
<td>-</td>
<td>0.062</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>0.78</td>
<td>0.173</td>
<td>0.06</td>
<td>-</td>
<td>0.03</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.885</td>
<td>0.097</td>
<td>0.108</td>
<td>-</td>
<td>0.079</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.459</td>
<td>0.465</td>
<td>0.026</td>
<td>-</td>
<td>0.069</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>0.952</td>
<td>0.073</td>
<td>0.021</td>
<td>-</td>
<td>-0.024</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>0.509</td>
<td>0.055</td>
<td>0.066</td>
<td>-</td>
<td>0.079</td>
<td>0.037</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 
- $l_1$ is the lagged dependent variable. 
- $l_{1,t}$ is the lagged independent variable. 
- $\pi_{t,t}$ is the inflation rate. 
- $\gamma_{t,1}$ is the slope coefficient. 
- $q_{t}$ is the error term. 
- $I_{1,DE}$ and $q_{DE}$ are additional variables.
Appendix 2.2

A preliminary descriptive study, following Bjorksten N. and Syrjanen M. (1999), is performed using the convergence barometer. The use of this graphic analysis helps to recognize the possible sources of an asymmetric effect of monetary policy. For each country the last observation available at the end of the year 2000 of concerning inflation, credit growth, GDP growth, unemployment rate, fiscal balance and the debt/GDP ratio is compared with statistics on the same variables referred to a weighted Euro Area average. The values in the Figure A2.1 to A2.4 refer to observations available from the OECD Economic Outlook. These six key variables show some structural and cyclical divergences among the European countries. The GDP growth differences are quite large: the good performance in the European periphery tends to amplify those asymmetries. In particular, the positive trend of real growth in Finland, the Netherlands and Ireland seems to be offset by lower GDP growth in Italy. Asymmetries emerge also in the inflation rate: the high-growth countries like Ireland, Spain, Portugal and the Netherlands are experiencing a higher inflation rate than the core European countries such as Germany or France. Finally, some considerations concerning the unemployment and public debt dynamics have to be given. Here, the difficulties of Italy and Spain in recovering a high employment rate together with the pressure for a downward trend of public debt in Belgium and Italy, represent the major explicative phenomena of the differences in the macroeconomic performance across EMU countries.
Figure A2.1: Convergence Barometer for Euro-Area Countries
Figure A2.2: Convergence Barometer for Euro-Area Countries
Figure A2.3: Convergence Barometer for Euro-Area Countries
Figure A2.4: Convergence Barometer for Spain
Appendix 2.3

Here we describe the methodology utilized to compute the variances.

Model (2.12) can be represented in a more explicit form as follows:

\[
\begin{align*}
\begin{bmatrix}
y_t \\
\pi_t \\
qt 
\end{bmatrix} &= \begin{bmatrix}
\alpha_1 & 0 & -\left(\beta + \frac{\omega}{\delta}\right) \\
\rho_2 & \gamma_1 & -\xi \\
2 & 0 & 0 \\
z_1 & z_5 & z_9 
\end{bmatrix} \begin{bmatrix}
y_{t-1} \\
\pi_{t-1} \\
qt 
\end{bmatrix} + \begin{bmatrix}
\alpha_2 & 0 & 0 \\
0 & \gamma_2 & \xi \\
0 & 0 & 0 \\
z_5 & z_6 & z_{10} 
\end{bmatrix} \begin{bmatrix}
y_{t-2} \\
\pi_{t-2} \\
qt 
\end{bmatrix} + \\
+ \begin{bmatrix}
\alpha_3 & 0 & 0 \\
0 & \gamma_3 & \xi \\
0 & 0 & 0 \\
z_3 & z_7 & 0 
\end{bmatrix} \begin{bmatrix}
y_{t-3} \\
\pi_{t-3} \\
qt 
\end{bmatrix} + \begin{bmatrix}
\alpha_4 & 0 & 0 \\
0 & \gamma_4 & \xi \\
0 & 0 & 0 \\
z_4 & z_8 & 0 
\end{bmatrix} \begin{bmatrix}
y_{t-4} \\
\pi_{t-4} \\
qt 
\end{bmatrix} + \\
+ \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & \xi \\
z_{11} & z_{14} & z_{12} 
\end{bmatrix} \begin{bmatrix}
\epsilon_t \\
\eta_t \\
\nu_t 
\end{bmatrix} + \begin{bmatrix}
1 & 0 & \frac{\omega}{\delta} \\
0 & 1 & 0 \\
0 & 0 & z_3 
\end{bmatrix} \begin{bmatrix}
\epsilon_{t-1} \\
\eta_{t-1} \\
\nu_{t-1} 
\end{bmatrix}
\end{align*}
\]

where \( z_i \) are respectively equal to:

\[
\begin{align*}
z_1 &= m\alpha_1^2 + n\rho(1 + \alpha_1); \\
z_2 &= m\alpha_2(1 + \alpha_1) + n\rho\alpha_2; \\
z_3 &= m\alpha_3(1 + \alpha_1) + n\rho\alpha_3; \\
z_4 &= m\alpha_4(1 + \alpha_1) + n\rho\alpha_4; \\
z_5 &= n\gamma_1^2; \\
z_6 &= m\gamma_2(1 + \gamma_1); \\
z_7 &= m\gamma_3(1 + \gamma_1); \\
z_8 &= m\gamma_4(1 + \gamma_1); \\
z_9 &= -(m\alpha_1 + n\rho)(\beta + \frac{\omega}{\delta}); \\
z_{10} &= m\alpha_1 + n\rho; \\
z_{11} &= m\frac{\omega}{\delta}z_{12} = (m + n\rho)\frac{\omega}{\delta}
\end{align*}
\]

For given parameters and given value of \( m \) and \( n \), it is possible to compute the variances of the vector \( X \) using the following standard formulas:

\[
(I_{9K^2} - A \otimes A)^{-1}vec(\Sigma_\epsilon)
\]

where \( A \) is the companion matrix and \( \Sigma_\epsilon \) is the variance covariance matrix\(^{24}\)

\(^{24}\)For details see Lutkepol, chapter 6
In order to construct the set of efficiency frontiers, we search, over a wide range of \( m \) and \( n \), the combinations that minimize the following loss function:

\[
Loss = (1 - \lambda) \text{Var}(y) + \lambda \text{Var}(\pi)
\]

where \( \lambda \) is the weight that we impose upon the variance of the inflation in the loss function and \( (1 - \lambda) \) is, consequently, the weight that we impose upon the variance of the output.

Every point of the efficiency frontier represents the combination of \( m \) and \( n \) that minimizes the loss function for a given value of \( \lambda \).
Appendix 3.1

**Figure A3.1**: Statistical properties of the normalized residuals
Appendix 3.2

Let denote with $X_t$ the vector of the endogenous variables and with $s_t$ the regime variable. The density of $X_t$ in state $j$ is equal to:

$$f(X_t|s_t = j; \theta) = (2\pi)^{-n/2}|\Omega_j^{-1}|^{-1/2} \exp \left[ (-1/2)(X_t - \Pi X_t)\Omega_j^{-1}(X_t - \Pi X_t) \right]$$

The joint multivariate distribution function of $X_t$ and $s_t$ is then equal to:

$$P(X_t, s_t = j; \theta) = \pi_j \cdot f(X_t|s_t = j; \theta) \quad (A\;3.2.1)$$

where $\pi_j$ is the unconditional probability to be in state $j$ and $m$ is the number of states.

The unconditional multivariate normal density is given by the sum of equation ($A\;3.2.1$)

$$f(y_t; \theta) = \sum_{j=1}^{m} p(X_t, s_t = j; \theta)$$

This result follows from the fact that the conditional probability of A given B is:

$$P\{A|B\} = \frac{P\{A \text{ and } B\}}{P\{B\}}$$

This expression implies that joint probability of A and B occurring together can be calculated as:

$$P\{A \text{ and } B\} = P\{A|B\} \cdot P\{B\}$$
The estimates of the parameters are obtained maximizing the log likelihood unconditional multivariate normal density:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t; \theta)$$  \hspace{1cm} (A 3.2.2)

subject to the constraints that $\sum_{j=1}^{m} \pi_j = 1$

In a more explicit form the maximization problem (A 3.2.2) can be rewritten as:

$$L(\theta) = \sum_{t=1}^{T} \log \left\{ \sum_{j=1}^{m} \pi_j \left\{ (2\pi)^{-n/2} |\Omega_j^{-1}|^{-1/2} \exp \left[ \frac{-1}{2} (X_t - \Pi X_t)^T \Omega_j^{-1} (X_t - \Pi X_t) \right] \right\} \right\}$$

subject to the constraints that $\sum_{j=1}^{m} \pi_j = 1$

Maximum likelihood estimation of $\theta$ is based on the implementation of the Expectation Maximization problem proposed by Hamilton (1990). This algorithm, introduced by Dempster, Laird and Rubin (1977), is designed for a general class of models where the observed time series depend on some unobservable stochastic variables. For a MS-VAR model these are the regime variables $s_t$. Each iteration of the EM algorithm consists of two steps. The expectation step involves a pass through the filtering and smoothing algorithm, using the estimated parameters vector $\theta^{j-1}$ of the last maximization step in place of the unknown true parameter vector. This delivers an estimate of the smoothed probabilities $Pr(s_t|Y, \theta^{j-1})$ of the unobserved states.
$s_t$ where $s$ records the history of the Markov chain. In the maximization step, an estimate of the parameter vector $\theta$ is derived as a solution $\hat{\theta}$ of the first order conditions associated with likelihood function, where the conditional regime probabilities $Pr(s|Y,\theta)$ are replaced with the smoothed probabilities $Pr(s|Y,\theta^{j-1})$ that are derived in the last expectation step. Equipped with a new parameter vector $\theta$ the filtered and smoothed probabilities are updated in the next expectation step, and so on, guaranteeing an increase in the value of the likelihood function at each step.
Appendix 3.3

This appendix aims to prove that the expected duration of a phase can be computed as \((1 - p_{\text{ii}})^{-1}\). In particular, we want to prove the expected duration to be in regime 1 is actually equal to \((1 - p_{\text{ii}})^{-1}\):

\[
\sum_{z=1}^{\infty} zp_{11}^{z-1}(1 - p_{\text{ii}}) = (1 - p_{\text{ii}})^{-1}
\]  
(A 3.3.1)

The first member of the above equation can be rewritten as follows:

\[
(1 - p_{\text{ii}}) \sum_{z=1}^{\infty} zp_{11}^{z-1}
\]
(A 3.3.2)

It is known that:

\[
\sum_{z=1}^{\infty} p_{11}^{z-1} = \frac{1}{1 - p_{\text{ii}}}
\]

and consequently that:

\[
p_{\text{ii}} \sum_{z=1}^{\infty} p_{11}^{z-1} = \sum_{z=1}^{\infty} p_{11}^{z} = \frac{p_{\text{ii}}}{1 - p_{\text{ii}}}
\]

The first derivative of \(\sum_{z=1}^{\infty} p_{11}^{z}\) with respect to \(p_{\text{ii}}\) is:

\[
\frac{\partial}{\partial p_{\text{ii}}} \sum_{z=1}^{\infty} p_{11}^{z} = \frac{1}{(1 - p_{\text{ii}})^2}
\]
(A 3.3.3)

By expressing the first derivative as a function of \(z\), the equation (A 3.3.2) can be written as follows:
By equalizing the second member of equation (A3.3.4) to the second member of equation (A 3.3.3) we obtain:

\[
\frac{\partial}{\partial p_{11}} \sum_{z=1}^{\infty} p_{11}^z = \sum_{z=1}^{\infty} z p_{11}^{z-1} \tag{A 3.3.4}
\]

\[
\sum_{z=1}^{\infty} z p_{11}^{z-1} = \frac{1}{(1 - p_{11})^2} \tag{A 3.3.5}
\]

The proof follows by plugging equation (A3.3.5) into equation (A3.3.1):

\[
(1 - p_{11}) \sum_{z=1}^{\infty} z p_{11}^{z-1} = \frac{(1 - p_{11})}{(1 - p_{11})^2} = (1 - p_{11})^{-1}
\]
### AUGMENTED DICKEY-FULLER INTEGRATION TESTS

#### Variable: UR

<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
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<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
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<td>3.671</td>
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</tr>
<tr>
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<tr>
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#### Variable: FEX

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<td>3.142</td>
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<td>2.593</td>
<td>3.128</td>
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<tr>
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<td>2.256</td>
<td>2.614</td>
<td>3.159</td>
<td>3.714</td>
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#### Variable: ECU

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<td>p=0, p=0</td>
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<td>3.142</td>
<td>3.695</td>
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<tr>
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<td>3.068</td>
<td>3.611</td>
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<td>p=0, p=0</td>
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<tr>
<td>p=0, p=0</td>
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<td>3.184</td>
<td>3.747</td>
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Table A4.1 CCI: Unit root tests
## Roots analysis

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<th>Levels</th>
<th>Estimated Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0839</td>
<td>0.9451</td>
</tr>
<tr>
<td>0.9553</td>
<td>0.9451</td>
</tr>
<tr>
<td>0.9553</td>
<td>0.7062</td>
</tr>
<tr>
<td>0.9229</td>
<td>0.0488</td>
</tr>
</tbody>
</table>

Table A4.2 Roots of the CCI model

### JARQUE-BERA NORMALITY TEST

<table>
<thead>
<tr>
<th>EQUATION</th>
<th>SKEWNESS</th>
<th>p-value</th>
<th>KURTOSIS</th>
<th>p-value</th>
<th>SKEW.&amp;KURT.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.281</td>
<td>0.131</td>
<td>2.04</td>
<td>0.153</td>
<td>4.321</td>
<td>0.115</td>
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<tr>
<td>2</td>
<td>0.918</td>
<td>0.338</td>
<td>0.021</td>
<td>0.896</td>
<td>0.039</td>
<td>0.025</td>
</tr>
<tr>
<td>3</td>
<td>6.105</td>
<td>0.013</td>
<td>4.857</td>
<td>0.029</td>
<td>10.962</td>
<td>0.004</td>
</tr>
<tr>
<td>4</td>
<td>0.668</td>
<td>0.414</td>
<td>0.417</td>
<td>0.519</td>
<td>1.084</td>
<td>0.581</td>
</tr>
</tbody>
</table>

System: 9.412 | 0.062 | 0.068 | 0.108 | 1.008 | 0.042

NOTE: NORMALITY IS ACCEPTED WHEN p-value>0.05

### MARDIA MULTIVARIATE NORMALITY TEST

<table>
<thead>
<tr>
<th>SKEWNESS</th>
<th>p-value</th>
<th>KURTOSIS</th>
<th>p-value</th>
<th>SKEW.&amp;KURT.</th>
<th>p-value</th>
</tr>
</thead>
</table>

System: 1.067 | 0.407 | 3.062 | 0.088 | 24.2 | 0.203

NOTE: NORMALITY IS ACCEPTED WHEN p-value>0.05

Table A4.3 Residuals Analysis in the CCI model
<table>
<thead>
<tr>
<th>TEST</th>
<th>Statistic</th>
<th>1% value</th>
<th>2.5% value</th>
<th>5% value</th>
<th>10% value</th>
</tr>
</thead>
<tbody>
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<td>-3.2</td>
<td>-2.8</td>
<td>-2.6</td>
<td>-2.3</td>
<td>-2.0</td>
</tr>
<tr>
<td>test 2</td>
<td>-3.2</td>
<td>-2.8</td>
<td>-2.6</td>
<td>-2.3</td>
<td>-2.0</td>
</tr>
<tr>
<td>test 3</td>
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<td>-2.6</td>
<td>-2.3</td>
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</table>

Table A4.4 BCI: Unit root tests
### Roots of the model

<table>
<thead>
<tr>
<th>Levels</th>
<th>1.0247</th>
<th>0.9654</th>
<th>0.964</th>
<th>0.9617</th>
<th>0.9445</th>
<th>0.9445</th>
<th>0.8516</th>
<th>0.7999</th>
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</thead>
<tbody>
<tr>
<td>Estimated Model</td>
<td>0.964</td>
<td>0.956</td>
<td>0.956</td>
<td>0.8415</td>
<td>0.76</td>
<td>0.6169</td>
<td>0.2982</td>
<td>0.2982</td>
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</tbody>
</table>

Table A4.5 BCI: Roots analysis

### JARQUE-BERA NORMALITY TEST

<table>
<thead>
<tr>
<th>EQUATION</th>
<th>SKEWNESS</th>
<th>p-value</th>
<th>KURTOSIS</th>
<th>p-value</th>
<th>SKEW.&amp;KURT.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.421</td>
<td>0.12</td>
<td>1.249</td>
<td>0.264</td>
<td>3.671</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.193</td>
<td>0.661</td>
<td>2.685</td>
<td>0.101</td>
<td>2.878</td>
<td>0.237</td>
</tr>
<tr>
<td>3</td>
<td>0.701</td>
<td>0.402</td>
<td>0.102</td>
<td>0.749</td>
<td>0.804</td>
<td>0.669</td>
</tr>
<tr>
<td>4</td>
<td>0.344</td>
<td>0.558</td>
<td>0.341</td>
<td>0.559</td>
<td>0.685</td>
<td>0.71</td>
</tr>
<tr>
<td>5</td>
<td>0.272</td>
<td>0.602</td>
<td>0.109</td>
<td>0.742</td>
<td>0.361</td>
<td>0.827</td>
</tr>
<tr>
<td>6</td>
<td>0.091</td>
<td>0.763</td>
<td>0.62</td>
<td>0.365</td>
<td>0.911</td>
<td>0.634</td>
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<tr>
<td>7</td>
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<td>0.78</td>
<td>0.377</td>
<td>3.487</td>
<td>0.175</td>
</tr>
<tr>
<td>8</td>
<td>2.667</td>
<td>0.102</td>
<td>1.341</td>
<td>0.247</td>
<td>4.009</td>
<td>0.135</td>
</tr>
<tr>
<td>SYSTEM</td>
<td>8.629</td>
<td>0.357</td>
<td>9.619</td>
<td>0.3</td>
<td>18.348</td>
<td>0.304</td>
</tr>
</tbody>
</table>

NOTE: NORMALITY IS ACCEPTED WHEN p-value>0.05

### MARDIA MULTIVARIATE NORMALITY TEST

<table>
<thead>
<tr>
<th>SKEWNESS</th>
<th>p-value</th>
<th>KURTOSIS</th>
<th>p-value</th>
<th>SKEW.&amp;KURT.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM</td>
<td>0.822</td>
<td>0.705</td>
<td>0.402</td>
<td>0.628</td>
<td>0.651</td>
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</table>

NOTE: NORMALITY IS ACCEPTED WHEN p-value>0.05

Table A4.6 Residuals Analysis in the BCI model
Figure A4.1 Responses of the CCI and its subcomponents to various shocks
Figure A4.2 Responses of EC_manOB to various shocks
Figure A4.3 Responses of EC_manST to various shocks
Figure A4.4 Responses of EC_manPE to various shocks
Appendix 5.1

The public debt identity at time $t$ is as follows:

$$b_t = d_{t-1} + (1 + r_{t-1})b_{t-1} \quad (A\ 5.1.1)$$

or alternatively it is:

$$e^z = e^x + e^y$$

where:

$$z = \ln b_t$$
$$x = \ln d_t$$
$$y = \ln[(1 + r_{t-1})b_{t-1}]$$

The first order Taylor polynomial approximation of $f(x, y, z) = e^z - [e^x + e^y] = 0$ about $x = \bar{x} = \ln \bar{d}$, $y = \bar{y} = \ln[(1 + \bar{r})\bar{b}]$ and $z = \bar{z} = \ln \bar{b}$ follows:

$$e^{\bar{z}} + e^{\bar{z}}(z - \bar{z}) - [e^{\bar{x}} + e^{\bar{x}}(x - \bar{x}) + e^{\bar{y}} + e^{\bar{y}}(y - \bar{y})] = 0 \quad (A\ 5.1.2)$$
By substituting $z, x$ and $y$ with their respective values, equation (A 5.1.2) becomes:

\[
e^{\ln b} + e^{\ln b}(\ln b_t - \ln b) = e^{\ln d} +
\]

\[
+ e^{\ln \bar{d}}(\ln d_{t-1} - \ln \bar{d}) + e^{\ln ((1+\bar{r})\bar{b})} +
\]

\[
+ e^{\ln ((1+\bar{r})\bar{b})}(\ln((1+r_{t-1})b_{t-1}) - \ln((1+\bar{r})\bar{b}))
\]

or equivalently:

\[
\bar{b} + \bar{b}(\ln b_t - \ln \bar{b}) = \bar{d} +
\]

\[
+ \bar{d}(\ln d_t - \ln \bar{d}) + (1 + \bar{r})\bar{b} +
\]

\[
+ (1 + \bar{r})\bar{b} \left[ (\ln(1 + r_{t-1}) + \ln b_{t-1}) - \ln((1 + \bar{r})\bar{b}) \right]
\]

By rearranging:

\[
\bar{b}(1 + \ln b_t - \ln \bar{b}) = \bar{d}\ln d_t + \bar{d}(1 - \ln \bar{d}) +
\]

\[
(1 + \bar{r})\bar{b} \left[ (1 + \ln(1 + r_{t-1}) + \ln b_{t-1}) - \ln((1 + \bar{r})\bar{b}) \right]
\]

By dividing both sides by $\bar{b}(1 + \bar{r})$:

\[
\beta(1 + \ln b_t - \ln \bar{b}) = \beta\left(\frac{\bar{d}}{\bar{b}}\ln d_t + \frac{\bar{d}}{\bar{b}}(1 - \ln \bar{d}) +
\right)
\]

\[
+ 1 + \ln(1 + r_{t-1}) + \ln b_{t-1} - \ln((1 + \bar{r})\bar{b}) \right]
\]
where $\beta$ is the discount factor and it is equal to $(1 + \bar{r})^{-1}$.

By solving with respect to $\beta \ln b_t$:

$$\beta \ln b_t = -\beta + \beta \ln \bar{b} + \beta \left[ \frac{\bar{d}}{\bar{b}} (1 - \ln \bar{d}) \right] +$$

$$+ 1 - \ln((1 + \bar{r})\bar{b}) + \ln b_{t-1} + \frac{\bar{d}}{\bar{b}} \beta \ln d_t + \ln(1 + r_{t-1})$$

By replacing $\ln(1 + r_{t-1})$ with $r_{t-1}$:

$$\beta \ln b_t = -\beta + \beta \ln \bar{b} + \beta \left[ \frac{\bar{d}}{\bar{b}} (1 - \ln \bar{d}) \right] +$$

$$+ 1 - \ln((1 + \bar{r})\bar{b}) + \ln b_{t-1} + \frac{\bar{d}}{\bar{b}} \beta \ln d_{t-1} + r_{t-1}$$

and dividing both sides by $\beta$ we obtain the log linear approximation of the public debt identity:

$$\ln b_t = \kappa + \beta^{-1} \ln b_{t-1} + \frac{\bar{d}}{\bar{b}} \ln d_{t-1} + \beta^{-1} r_{t-1}$$

where $\kappa$ is a constant equal to:

$$\kappa = -1 + \ln \bar{b} + \left[ \frac{\bar{d}}{\bar{b}} (1 - \ln \bar{d}) \right] + \beta^{-1} - \beta^{-1} \ln((1 + \bar{r})\bar{b})$$
Appendix 5.2

The Dickey fuller (DF) test is based on the estimation of the following equation:

\[ \Delta y_t = \mu + \rho y_{t-1} + \epsilon_t \]  \hspace{1cm} (A 5.2.1)

where \( \epsilon_t \) \((t = 1, 2, \ldots, T)\) are the error terms assumed to be stationary and uncorrelated. The null hypothesis \( H_0 \) states that the series has a unit root (i.e. the series is not stationary). Formally, the null hypothesis is \( \rho = 0 \) versus the alternative \( \rho < 0 \). For testing this hypothesis we compute the \( t \)-statistic in the usual way but the distribution of this statistic is not standard. Critical values were supplied by Dickey and Fuller (1979, 1981). If \( H_0 \) is rejected, the series \( y_t \) is stationary and if \( H_0 \) is not rejected the series is not stationary.

If the error terms are correlated, lagged values of the dependent variable are added until the errors are not correlated and we have then the Augmented Dickey - Fuller (ADF) test (Said Dicky, 1984). For non trending series this test is performed by estimating the following equation:

\[ \Delta y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{k-1} \Delta y_{t-i} + \epsilon_t \]  \hspace{1cm} (A 5.2.1)

and testing \( \rho = 0 \) versus the alternative \( \rho < 0 \), where \( k \) is the number of lags of the dependent variable.

Conversely, for trending series it is convenient to use the models:
\[ \Delta y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{k-1} \Delta y_{t-i} + \epsilon_t \]  (A 5.2.2)

\[ \Delta y_t = \mu + \beta(t - T/2) + \rho y_{t-1} + \sum_{i=1}^{k-1} \Delta y_{t-i} + \epsilon_t \]  (A 5.2.3)

where \( \beta \) is the coefficient of the linear trend. If the null hypothesis is not rejected the series is non-stationary.
Appendix 6.1

The Beta distribution is a general type of statistical distribution which is related to the gamma distribution. It has two free parameters, which are labeled, according to one of two notational conventions, respectively with $\alpha$ and $\beta$.

The domain of the distribution is included in the interval $]0,1[$. The probability function $P(x)$ and distribution function $D(x)$ are respectively given by:

$$P(x) = \frac{(1-x)^{\beta-1}x^{\alpha-1}}{B(\alpha, \beta)} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (1-x)^{\beta-1}x^{\alpha-1}$$

$$D(x) = I(x; a, b)$$

where $B(a, b)$ is the beta function, $I(x; a, b)$ is the regularized beta function and $\alpha, \beta > 0$.

The following plots are for various values of $(\alpha, \beta)$ with $\alpha = 1$ and $\beta$ ranging from 0.25 to 3.00.
The distribution is normalized since:

\[ \int_0^1 P(x)dx = 1 \]

The characteristic function is:

\[ \int_0^1 \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)} e^{-2\pi i xt}dx = \, _1F_1(a; b; z) \]

where \( _1F_1(a; b; z) \) is a confluent hypergeometric function.

The raw moments are given by:

\[ \mu_r = \int_0^1 P(x)(x - \mu)^r dx = \frac{\Gamma(\alpha + \beta)\Gamma(\alpha + r)}{\Gamma(\alpha + \beta + r)\Gamma(\alpha)} \]

(Papoulis 1984, p. 147), and the central moments by:

\[ \mu_t = \left( -\frac{\alpha}{\alpha + \beta} \right)^r _2F_1(-r, \alpha; \alpha + \beta; \frac{\alpha + \beta}{\alpha} ) \]

where \( _2F_1(a; b; c; x) \) is a hypergeometric function.

The mean, variance, skewness, and kurtosis are given by:

\[ \mu = \frac{\alpha}{\alpha + \beta} \]

\[ \sigma^2 = \frac{\alpha \beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \]

\[ \gamma_1 = \frac{2(\beta - \alpha)(1 + \alpha + \beta)^{1/2}}{(\alpha \beta)^{1/2}(2 + \alpha + \beta)} \]
The mode of a variate distributed as a Beta ($\alpha, \beta$) is:

$$
\hat{x} = \frac{\alpha - 1}{\alpha + \beta - 2}
$$
Bibliography


BIBLIOGRAPHY


anismen der Geldpolitik, Schriften des Vereins für Socialpolitik, Neue Folge Band 264, Berlin: Duncker und Humblot, 111-141.


BIBLIOGRAPHY


