The Behaviour of Speculators in Foreign Exchange Markets

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Declaration

Chapters 1, 2, 3, 4, 7 and 8 are the result of my independent research. However, chapter 5 is a survey chapter linking the theory sections of chapters 2, 3 and 4 to the experimental section, chapter 6. It has been included in order to illustrate the importance of the experiments into informational cascades in the context of a thesis concerning currency crises.

Chapter 6 is the result of joint research undertaken with John Hey, University of York, UK. This research formed two papers. The first of these was presented at the 5th Italian Meeting of Experimental Economists in Trento, Italy in June 1997. The second paper was presented at the Public Choice Society and Economic Science Association Meeting in New Orleans, USA in March 1998.

Each of us contributed equal amounts in each of these papers.
Abstract

The 1992 currency crises in the foreign exchange markets saw the collapse of the Exchange Rate Mechanism. The Pound and the Lira left the system and a number of other currencies devalued voluntarily.

The purpose of this thesis is threefold. Firstly, I present three theoretical chapters which aim to explain particular features of currency crises and hence model certain aspects of the currency crises of 1992. Secondly, I perform two experiments to test a model of herd behaviour since it is argued that, in some circumstances, speculators may behave as if they are following a herd. Finally, I examine the empirical data of the currencies of Italy and the UK since comparatively little work has been produced for these currencies. The aim is to offer a model which successfully accounts for the periods both in and out of the EMS.
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Chapter 1

Introduction

The aim of this thesis is to model the events surrounding the currency crisis which led to the UK departure from the Exchange Rate Mechanism (ERM) in 1992. In order to do this, I draw from a wide literature in economics. I consider theoretical models of currency crises and also those models which have examined the empirical data.

In terms of the theory, there are two main schools of thought which seek to explain currency crises. Firstly, there are fundamentalist theories based on, for example, limited reserves of foreign currency (Krugman, 1979). Secondly, there are speculative theories which emphasise the role of self fulfilling speculative attacks (Obstfeld, 1986). However, neither of these approaches consider the timing of a speculative attack on a currency or the factors which determine the duration of a crisis.

In chapters 2-4 of this thesis I address these issues. Initially I provide a literature survey offering a background and justification to my research. I then set up a model of information externalities and search based on Caplin and Leahy (1998) which helps to explain the timing of a speculative attack. After consideration of the effect of a Tobin tax in such a context, I proceed in chapter 3 to examine a ‘war of attrition’ model based on the work of Alesina and Drazen (1991). In my thesis, this is used to explain the duration of a currency crisis.

In the next theory chapter, I provide an adaptation of the work of Morris
and Shin (1995). They argue that the ERM was ‘ripe for attack’ before the September 1992 crisis. If the state of the economy lies in a particular ‘ripe for attack’ region, each investor will sell his holdings of the currency if there is a lack of common knowledge among the investors regarding the state of the economy. However, in my adaptation, the state of the economy is known to all players. It is the value placed by the government in remaining in the ERM which is observed by each of the speculators with a degree of error. I show that a lack of common knowledge concerning government type leads to the same scenario as in the Morris and Shin case.

Recent literature in economics has related informational cascades and herding to financial data. This represents a growth area in economics and is also of particular relevance to this thesis. In chapter 5, I examine a model of informational cascades developed by Bikhchandani, Hirschleifer and Welch (1992). After setting up their framework, I show how this may apply to the events of 1992 in the foreign exchange markets. I further show the experiment designed by Anderson and Holt (1997) which tests this model.

With the exception of this paper by Anderson and Holt, there has been very little experimental work in this area. However, the work on informational cascades and herding lends itself very well to experimental testing. Therefore, in the following chapter, I describe an equally valuable model by Banerjee (1992). I argue that this may apply to currency crises and I set up an experiment to test its validity. While this model is not directly related to the mainstream currency crisis literature, it is still valid since it helps to analyse the behaviour of speculators. In particular, it can help to determine the timing of a crisis. A simulation of the actions of the subjects within the Banerjee framework can be used to establish the point in the sequence at which speculators are most likely to choose to follow the herd. This is then compared with the results of a laboratory experiment. A particular assumption is then removed from the Banerjee framework, and the actions of the subjects are simulated once more and a further experiment is carried out.

In chapter 7, I examine the empirical evidence. The aim is to model the exchange rate data before, during and after the 1992 crisis. As in previous chapters, I initially present the literature in this area. Much work has been
done on the French Franc against the German Deutschmark primarily because of the length of time in which it has been in the ERM. However, there is still a comparatively small literature on the British Pound or Italian Lira. Hence my work has focused on these two currencies.

The literature in this area has formed two basic strands. Many empirical economists have suggested GARCH models for the modelling of exchange rates across the period in question. However, Markov processes have also been used to model the switch from one exchange rate regime to another. I present the literature in these two areas and then apply the models to the Pound and Lira.

The final chapter contains my concluding remarks and also addresses some of the potential lines of inquiry.
Chapter 2

A Model to Explain the Timing of an Exchange Rate Collapse

2.1 Introduction

In January 1992, the ERM celebrated sixty months without a realignment. However, by the September, two of the currencies had left the system (the Pound and the Lira) and the Peseta and Escudo had devalued voluntarily. There has been a vast literature in this area which debates on the causes of the collapse. The arguments put forward are summarised in the paper by Eichengreen and Wyplosz (1993). There are four main explanations offered to explain this crisis.

Firstly, inflation differentials are blamed for the downfall of the ERM. However, the empirical evidence suggests that this reason is only applicable in the case of Italy. Secondly, it is argued that German unification brought about an asymmetric shock which required an appreciation of the Deutschmark. Germany was denied the right to revalue its currency so this implied that either inflation should rise in Germany or there needed to be a depreciation in the other currencies. It is argued that the crisis simply forced governments to accept this depreciation. However, this is not completely plausible since it does not explain the timing of the attack which was more than two years after unification and directly after the Danish referen-
The third category of explanation is based on Krugman’s theoretical discussion in 1979. He argued that a currency crisis is inevitable when a government’s stock of foreign exchange reserves become exhausted. He argues that a speculative attack occurs long before the reserves are exhausted as a result of maximising behaviour on the part of the speculators. It is argued that this may apply to the situation in Europe in 1992 since it is noted (Eichengreen and Wyplosz, (1993)) that the members were undergoing ‘deteriorating economic conditions’.

Finally, it is argued that the crisis was generated by a self fulfilling speculative attack. This can be seen in the pioneering paper by Obstfeld (1986). It is the fact that investors believe that monetary policy will be modified as a result of a speculative attack that makes the attack possible. Eichengreen and Wyplosz note that the incentives for such an attack were built into the Maastricht Treaty. A devaluation in a currency would disqualify it from EMU participation. This would mean there would no longer be a reason to maintain a tight monetary policy. Therefore, there is an incentive to launch an attack on a currency even when policy is consistent with balance of payments equilibrium. Eichengreen and Wyplosz found that the evidence supports this explanation of a crisis. They note that ‘The only mystery is how its outbreak was deferred for so long’. This is one of the issues which I wish to address in this chapter.

I explain the timing of a crisis in terms of an information externality in the foreign exchange market. Using a model originally developed by Caplin and Leahy (1998), I explain the long period without a realignment in the foreign exchange market and the sudden flurry of activity culminating in a series of speculative attacks.

This chapter will be organised as follows. The literature survey provides a background to this model and also that of the following chapter. I set out the model of information externalities and note its sequence of events. Having done this, I will solve the model. I will then consider the effect of a Tobin tax in deterring a crisis. In particular I consider the effect on the timing of the currency crisis. I will then add concluding comments on this
2.2 Literature Survey

There is a vast literature in the area of currency crises. In this section, I will highlight the most significant contributions in this area and explain their relevance to my research. There are two basic schools of thought explaining the collapse of an exchange rate regime. Firstly, there are the ‘fundamentalist’ theories. These are based on, for example, asymmetric shocks, competitiveness or, as in the case of Krugman (1979), limited reserves. In his paper, he develops a model in which exchange rate regime switches are the result of incompatible monetary and exchange rate policies. A balance of payment crisis is generated when a government operates a policy of domestic credit expansion while simultaneously fixing the exchange rate. Inevitably, reserves are depleted and so the fixed rate must be abandoned. His major finding is that with forward looking exchange markets, the final stage of the crisis involves a sudden discrete loss of reserves in a ‘speculative attack’. This is because speculators with foresight will attack the currency before the reserves are fully depleted and purchase all remaining reserves at a particular moment in time.

However, Krugman notes that there are limitations to such a framework. The analysis of those factors triggering a balance of payments crisis is very limited due to a highly simplified model. Furthermore, the model assumes that there are only two available assets. This places a constraint on the government since the only means of pegging its exchange rates is by selling its reserves. However, despite these drawbacks, this model pioneered much of the research into currency crises and a great deal of the literature in the area has grown out of this. Hence it is important to consider it here since it provides a starting point for much of the current work in currency crises.

In this and other models including the ones which I set up later in the chapter, it is assumed that the alternative regime to a fixed rate is a permanent float. However, this need not be the case. Agenor, Bhandari and Flood (1992) consider a temporary period of floating followed by a new peg
which was also seen in Obstfeld (1984). They set up a framework similar to that of Krugman. However, they extend his analysis to look at alternative post collapse regimes, uncertainty, the possibility of external borrowing and capital controls, sticky prices and asset substitutability. This paper is relevant to my research since it considers the issue of timing of an exchange rate collapse. In my research, I am concerned with the timing of the speculative attack and also the timing of the eventual abandonment of the exchange rate. They find that the timing of the collapse is linked to the size of the expected devaluation and also the length of time for which the currency is allowed to float.

The second basic school of thought is based around 'speculative theories'. These focus on the idea of self-fulfilling speculative attacks and was originally developed by Obstfeld (1986). He considers the possibility that a speculative attack is generated by private sector expectations of a loosening of monetary policy after the collapse of a fixed rate regime. In his model, it is the expectation of a devaluation which triggers the attack which exhausts reserves. This forces the authorities to abandon the fixed rate. As a result, if the authorities in fact loosen monetary policy, the exchange rate will depreciate and the expectations of speculators will be fulfilled. The important finding of this paper is that rational expectations equilibria can exist with a speculative attack even when the initial policy stance of the government is sustainable.

Eichengreen and Wyplosz (1993) argue that this model is appropriate for explaining the September 1992 crisis in the foreign exchange markets. Furthermore, the incentives for self fulfilling speculative attacks were built into the Maastricht Treaty. A speculative attack which forced a devaluation would prohibit a country from participating in the EMU and thus allow a shift towards a more accommodatory policy. They note that the knowledge that there is an incentive to change policy in the event of an attack provided the speculators with an incentive to attack.

However, in theory governments could have prevented the collapse of the regime by raising interest rates to a sufficiently high level. The fact that they choose not to do so implies something about their preferences. It follows that
the exchange rate switch was the result of an optimising decision and not a policy action forced upon the government through a lack of reserves as in the Krugman analysis.

This would suggest that the pioneering work of Krugman (1979) and Obstfeld (1986) provides only a partial analysis of the events of the 1992 currency crisis. What is omitted from their framework is the inclusion of an optimising policy maker.

The work of Ozkan and Sutherland (1995) introduces an optimising policy maker into the literature. The model stresses the relationship between the government and expectations of private sector agents. It is this interaction which determines the timing of an exchange rate collapse. This model is particularly significant in terms of my research since I focus on the determinants of the timing and duration of a crisis.

The government chooses to maximise its welfare function which depends on domestic output. It cannot influence domestic monetary policy since the domestic interest rate is set to maintain currency parity under a fixed regime. However, under a floating rate it can use its exchange rate to offset any adverse shocks. Within the model, the government has a once and for all option of switching to the floating rate. Having done so, it cannot return to the fixed rate system. They show that the optimal strategy for the government is to select a trigger level for the interest rate of the centre country. If the interest rate exceeds this then the domestic country should abandon the fixed rate system.

It is assumed that private agents know the preferences of the government. Therefore, it can calculate the trigger point of foreign interest rates and deduce the level at which the government will abandon the fixed regime. Therefore, as foreign interest rates approach the trigger level, expectations of a devaluation increase domestic interest rates. The gap between domestic and foreign interest rates increases thus influencing output.

However, Ozkan and Sutherland show that the government abandons the fixed rate earlier than it prefers. This is because the private sector is aware that the government wishes to influence expectations. Since the government is unable to pre commit to a trigger point, the private sector will not believe
a policy announcement that a devaluation is unlikely. Hence the timing of a crisis is the result of the interaction between an optimising policy maker and private sector agents.

A further paper which considers the interaction of private sector agents and an optimising policy maker is that of Obstfeld (1994). He argues that any explanations of crises based purely on limited foreign reserves are by implication, based on fiscal weakness. If the fiscal position were strong, it would be feasible to borrow sufficient funds in order to defend the currency. Within his two period model, he finds two factors determining the probability of an attack; the maturity structure of the government’s debt obligations and the currency composition of the overall debt. He assumes that foreign reserves can be borrowed on the world capital market subject only to the government’s intertemporal budget constraint.

Having set up his framework, Obstfeld shows that by assigning certain values to the variables, more than one equilibrium arises. The government then faces an inconsistency problem since its loss is lower in the low depreciation equilibria but there is no guarantee that the bond market will choose the corresponding low interest rate. He concludes that the inability to precommit to a policy together with the importance of private sector expectations in the budget constraint leads to multiple equilibria.

The model developed by Bensaid and Jeanne (1997) is similar to that of Ozkan and Sutherland (1995) and Obstfeld (1994) since expectations of a devaluation create a cost for the government. However, it differs in its treatment of the dynamics of the crisis. They model crises as having a specific beginning, duration and end which implies that the crisis may be a short or long lasting process. This paper is of particular importance in the context of my research since it addresses the issue of the duration of a crisis. Effectively, it models the length of time of the crisis before the collapse of a fixed rate regime. However, the method employed by the authors differs from my approach since I use a ‘war of attrition’ framework.

They assume that there is a trade off between the cost of leaving the fixed rate system and the cost involved in raising the nominal interest rate to defend the currency. It is optimal for the government to devalue when the
cost of defending the currency exceeds the opting out cost. However, they further assume that the market does not know the opting out cost of the government and so is always expecting a devaluation. They show that this generates self fulfilling crises since the nominal interest rate rises until the government abandons the peg.

It follows that the outcome of this scenario is always a devaluation. Furthermore, it demonstrates that a currency crisis can occur even when agents do not expect a more expansionary policy after the abandonment of the fixed rate. In considering the duration of the crisis, Bensaid and Jeanne show that it is longer when the opting out cost is higher and shorter when the government has a better reputation. The reasoning behind this is that the reputation of the government in this model is achieved at such a high cost that it does not translate into a corresponding increase in credibility. They note that this finding is similar to that of Drazen and Masson (1994) who show that an improvement in the reputation of the policy maker does not necessarily increase the credibility of a ‘tough’ policy.

2.3 A Model of Information Externalities and Search

A number of economists share the opinion that the downfall of the Pound in 1992 was the result of a self fulfilling speculative attack. However, the timing of the collapse is an issue which is yet to be explained. As noted earlier, the EMS had been characterised by considerable stability in exchange rates. The puzzle is why the attack should occur in September 1992 and not at some earlier date.

In this framework, the argument is as follows. Speculators in the market originally hold a given quantity of Pounds, although it is possible to hold their funds in other ways. There is a cost involved in switching away from Pounds which must be borne by the speculator. This cost need not be a tax but may represent the time and effort of switching funds from one currency
to another\textsuperscript{1}. It follows that, if he can learn about the best destination for his funds from observing the experiences of other speculators, he has an incentive to wait and see if someone else moves first. Therefore, it is optimal for each speculator to play a waiting game in the hope that one of the others will move from Pounds to another currency and thus reveal some valuable information regarding the market. This public information concerns the strength of the Pound and implies that speculators will be able to distinguish between a ‘wet’ or ‘dry’ central bank. Once this information is revealed, the remaining speculators can make a more informed decision of the best place to put their funds.

The salient feature of this model is the delay until the first speculator moves. This is generated by an information externality. When deciding whether to move their funds from one place to another, speculators do not take into account the value of the information revelation to others.

I consider a scenario in which there are a number of large speculators, $N$, in the foreign exchange market each holding a given amount of Pounds. I concentrate on those speculators who hold large amounts of the currency since they have a degree of influence in the foreign exchange market. The sale of a few Pounds is likely to have a very small effect on government policy. However, if a speculator moves a considerable amount of funds, this may induce the government to act. In so doing, it will reveal some information about its ability and willingness to remain in the ERM. It is the fact that a large speculator can cause information to be released that is the central feature of this model.

I assume that a speculator may hold his funds in Pounds or transfer to Deutschmarks (DMs) or, alternatively, he can invest in something which will give him a fixed return on his money. For the sake of argument, I am assuming that the commodity, gold, will give him a guaranteed level of earnings.

The task of a speculator is to discover the best place for his funds. I assume that in each time period, each speculator receives a private piece of news concerning the relative merits of holding Pounds or DMs. I assume that

\footnote{This cost is not modelled explicitly since its inclusion would not qualitatively change the results.}
p is uniformly distributed over \([0, 1]\). The larger the value of \(p\), the greater the perceived benefit to the speculator in moving from Pounds to DMs than for someone receiving a value of \(p\) close to 0. I am not necessarily assuming that this payoff is financial. The payoff could represent the benefit to the speculator in terms of peace of mind. Hence, a large value of \(p\) tells him that the benefit in switching immediately to DMs is greater for him than if he waited for further pieces of news. Once he receives this information, he decides whether to shift his funds into DMs. If he chooses to keep his funds in Pounds, he will receive another piece of news, \(p\), in the next period and the process is repeated. Note that the switching of funds from Pounds to DMs is irreversible in this model.

Each speculator must calculate the expected value of the payoff for each value of \(p\) from switching to DMs. Uncertainty arises here since he does not know the value of the public information, \(s\). I assume that \(s\) is uniformly distributed on \([0, 2]\) and that the payoff to a speculator receiving news, \(p\), from switching to DMs is \(p.s\).

The vital assumption in this analysis is that the public information can only be realised after the first speculator moves his funds into DMs. Once this becomes public knowledge, the speculators are aware of the payoffs in holding Pounds or DMs.

A further assumption I make involves the outside option. I assume that the return to holding gold takes on a fixed value, \(G > 0\). This alternative use is necessary since it means that the public information, \(s\), is relevant to the decisions of the speculators once one speculator has moved into DMs. If \(s\) takes on a value close to zero, once the first speculator moves, the remaining speculators can move their funds into gold which will yield a higher return than if they simply continued to hold Pounds. In addition, I include a discount rate, \(\delta \in (0, 1)\), since speculators would rather move sooner than later.

The above argument ensures that there are two main phases within this model. Firstly, there is the uninformed phase in which the speculators do not know \(s\). This terminates when one speculator moves into DMs. In the second phase \(s\) is known. The speculator is fully informed about the public
information and can decide whether to wait for a higher value of $p$ or to move into gold. If he decides to wait, he must choose those values of $p$ at which he is prepared to move into DMs.

This framework implies that there will be reservation levels of $p$ in each of the two phases. In addition, there will be a cut-off level for $s$ in the informed phase. Nash equilibrium implies that all decisions must be optimal given that every other speculator is operating using the same decision rules.

2.4 Sequence of Events

At the beginning of each period, each speculator receives a piece of private information about the government and the relative merits of holding Pounds or DMs. This information is uncorrelated across speculators and also over time. One must also consider the issue of how a devaluation and a speculative attack are defined in this framework.

In the literature, a speculative attack occurs when investors change the composition of their portfolios so that they reduce the proportion of the domestic currency which they hold and increase the share in foreign currency. There is pressure on the domestic exchange rate to devalue and the government’s foreign currency reserves begin to deplete. The government must then decide whether to continue to support the exchange rate which it could achieve through the acquisition of a loan. However, it may be compelled to devalue the currency.

In terms of this model, a speculative attack is defined as follows. If a speculator receives private information which tells him that there is a greater benefit in holding DMs, he will move out of the domestic currency. This reveals the public information concerning the government type. If the government proves to be ‘wet’ then other speculators will follow. Hence a speculative attack occurs as a result of an individual receiving a piece of information which induces him to move out of the domestic currency.

A devaluation is not modelled explicitly in this framework. It is assumed that the speculative attack does not have any effect on the probability of a devaluation. Furthermore, it is assumed that there is no correlation between
the probability of a devaluation and the action taken by the government which reveals its type. Clearly, these are simplifying assumptions which have been included in order to solve the model. A more realistic model would relax these assumptions.

If none of the speculators have moved from the domestic currency in this period, each speculator receives a piece of information in the following period. This process continues until an individual receives information which induces him to move.

2.5 Solution

I use dynamic programming to solve this two phase model. I start by examining the decision process for the speculator in the second phase. Notably, the public information is now known so the speculator can calculate the optimal reservation level in the informed phase. He can also compute the cut-off level of the devaluation below which it is optimal to choose the outside option, $G$. The speculator can use his optimal decision in the informed phase to decide the reservation level of $p$ in the uninformed phase.

The first step is to solve for the reservation level in the informed phase, $I(s)$. A speculator who has received the news, $p$, and knows the public information, $s$, has an optimal strategy whose value is given by $V_I(p, s)$. This translates into the Bellman equation:

$$V_I(p, s) = \max \left\{ p.s, \delta \int_0^1 V_I(p't, s)dp't \right\}$$

The first term on the right hand side represents the payoff from moving into DMs while the second denotes the discounted payoff from waiting and receiving a different piece of news, $p't$. It follows that there will be a certain reservation level, $I(s)$. Below this value, the speculator will find it optimal to wait for further news. Above this level, he will switch from Pounds to DMs. The value of the optimal strategy is therefore:
\[
V_I(p, s) = s \cdot p \quad if \quad p \geq I(s) \\
V_I(p, s) = s \cdot I(s) \quad if \quad p \leq I(s)
\]

This can also be seen graphically as follows:

Figure 2.1: Optimal Strategy of a Speculator

In figure 2.1, it is clear that if a speculator receives a value of \( p \) below the reservation level, \( I(s) \), he will prefer to remain in Pounds than to move into DMs. In short, he will not accept a payoff which is less than \( s \cdot I(s) \). At a value of \( p \) equal to \( I(s) \), the speculator will be indifferent between remaining in Pounds or switching to DMs. Combining this property with (2.1) and
(2.2) gives the following:

\[ I(s).s = \delta \int V_I(p, s)dp = s\delta \left( \frac{1 + I(s)^2}{2} \right) \]  

(2.3)

From (2.3), it follows that \( I(s) \equiv I \) is independent of \( s \). It is now possible to write:

\[ \delta (1 + I^2) = 2I \] 

(2.4)

which illustrates the point that for any value of the discount rate, there is a unique solution for the value of \( I \) in the interval \((0, 1)\). This is found by solving (2.4) for \( I \) so that:

\[ I = \frac{1 \pm \sqrt{1 - \delta^2}}{\delta} \]  

(2.5)

Notably, for any value of \( \delta \) between 0 and 1, there is a unique value for \( I \).

At this stage, the outside option may be added to discover whether it is optimal to continue to wait once the public information has been revealed. The value of holding out for a higher value of \( p \) is given by:

\[ \int V_I(p, s)dp = s \left( \frac{1 + I^2}{2} \right) \]  

(2.6)

The decision of whether to continue waiting or to switch into gold involves comparing (2.6) with \( G \). The speculator will move his funds into gold if the size of \( s \) falls below a certain level, \( \bar{s} \), where:

\[ \bar{s} = \frac{2G}{1 + I^2} \]  

(2.7)

At this stage, it is important to impose an assumption regarding the size of
$G$ to ensure that waiting occurs in the uninformed state. If $G$ were too large then it would never be optimal to wait since the speculator would receive a higher payoff from moving out of Pounds and into gold instead of DMs. The required condition is:

$$1 \geq \bar{s} = \frac{2G}{1 + I^2}$$

(2.8)

As in the Caplin and Leahy framework, I am including a transformation:

$$g \equiv \frac{G}{1 + I^2} \in \left(0, \frac{1}{2}\right)$$

(2.9)

The purpose of this is merely to simplify the derivation of the solution to the model.

Having set up the second phase and the decision rule of the speculators, it is now appropriate to examine the first phase. Dynamic programming is used once more to establish the decision rule. The optimal strategy for the speculator who receives the news, $p$, but does not know the public information, is given by $V_U(p)$. He compares the value of moving into DMs with what happens if he remains in Pounds. If none of the speculators move, then the first state is repeated. The probability of this occurring is $Pr(U)$. However, if one of the speculators switches, then the public information is revealed. The probability of this occurring is $Pr(I) = 1 - Pr(U)$. This translates into the following Bellman equation:

$$V_U(p) = max \left\{ p, \delta \left[ Pr(U) \cdot \int V_U(pt)dt + Pr(I)EV_I \right] \right\}$$

(2.10)

Once again, the first term represents the expected payoff from moving into DMs now while the second term denotes the value of waiting. The speculator chooses the larger of the two. Note that $EV_I$ is the ex ante expected value at the start of next period of being informed of $s$. This expected value is given
by:

\[
\int \left\{ \max \left[ g(1 + I^2), s \left( \frac{1 + I^2}{2} \right) \right] \right\} ds \quad (2.11)
\]

Note that the first term is the value of the outside option, gold. The second term represents the value of waiting. This reduces to:

\[
EV_I = g^2(1 + I^2) + \int_{2g}^{2} s \left( \frac{1 + I^2}{2} \right) \frac{ds}{2} \quad (2.12)
\]

Expansion of this gives:

\[
EV_I = (1 + g^2) \left( \frac{1 + I^2}{2} \right) \quad (2.13)
\]

The reservation level, \( U \), is defined by the point at which the speculator is indifferent between holding Pounds or DMs. Since \( \int V_U(p')dp' = \frac{1+U^2}{2} \), equation (2.10) may be used to show that:

\[
U = \delta \left[ Pr(U) \left( \frac{1 + U^2}{2} \right) + Pr(I) \left( \frac{1 + I^2}{2} \right) (1 + g^2) \right]
\]

\[
U = Pr(U)I. \left( \frac{1 + U^2}{1 + I^2} \right) + Pr(I)I. (1 + g^2) \quad (2.14)
\]

The right hand side of (2.14) shows the expected value of rejecting \( U \) in the hope of a higher value of \( p \) in the future. In order to remain uninformed next period, each of the other \( N-1 \) speculators should also receive a value of \( p \) below the reservation level. The probability of this occurring is denoted by \( Pr(U) = U^{N-1} \). Substituting this into (2.14) then gives:

\[
U = U^{N-1}.I. \left( \frac{1 + U^2}{1 + I^2} \right) + (1 - U^{N-1}).I.(1 + g^2) \quad (2.15)
\]
This is the solution for the reservation level in the uninformed phase. The main result of this analysis is that the reservation level is higher for the uninformed phase than for the informed phase. This is caused by the information externality and produces the delay in speculative behaviour. The intuition behind this is as follows. The speculators measure the value of moving into DMs against the value of remaining in Pounds. If they wait, another player may make the move into DMs and reveal the public information. The purpose of the outside option, gold, is to ensure that the speculator receives at least a certain payoff. It follows that the outside option has the effect of raising the expected value of the next period's value function. Hence, the benefit from another agent making the first move is increased relative to the value of moving straight away. It follows that the reservation level in the first phase is higher than in the second phase. Once one speculator has moved, there is increased activity on the foreign exchange market. See the appendix for a proof of the existence and uniqueness of the equilibrium. This is taken from the Caplin and Leahy paper (1998).

Having set up this framework, it can now be used to evaluate the effectiveness of a Tobin tax in deterring an attack.

2.6 Capital Controls - A Tobin Tax

The purpose of capital controls is to protect currencies from volatility in the foreign exchange market. In the case of the ERM, these controls safeguarded against speculative attacks on the currency. Eichengreen and Wyplosz (1993) note that they allowed countries a degree of policy autonomy for a period of time. Italy, for instance, desired a looser monetary policy than that of Germany and so it was apparent that the removal of capital controls would inevitably lead to a devaluation of the Lira. The controls therefore provided protection from foreign exchange market pressures.

As noted by Eichengreen and Wyplosz, these controls took on a variety of different forms. They included restrictions on banks' ability to lend abroad and also taxes on holdings of foreign currency assets. In 1988 the countries subject to the most restrictions were Italy, Ireland, Spain, Greece and Por-
tugal. However, the bulk of these controls were terminated in 1990 as part of the ‘1992 Program’, a move towards the completion of the internal market.

Notably, the removal of capital controls appeared successful initially since there were no speculative attacks on the currency. However, the time span between the removal of controls in 1990 and the crisis in 1992 may be explained by the presence of an information externality.

It is argued that the imposition of a control contradicts the spirit of a monetary union in Europe. However, Eichengreen and Wyplosz argue that while it is not a first best solution to impose a tax, the implementation of a suitable policy is not as cumbersome as people may think and would protect the EMU against further attacks.

In terms of the model, a tax such as that proposed by Eichengreen and Wyplosz would imply an additional cost to the speculator in switching from Pounds to DMs. I have assumed that each of the speculators holds the same amount of Pounds and trades with that given amount. He cannot sell a fraction of it and hold on to the remaining portion. It follows that a tax will be a fixed cost which is the same for each speculator. The structure of the model does not hold a great deal of scope for including changes such as extra costs. However, one can imagine the tax being incorporated into the payoff which the speculator receives if he moves into DMs. This can be seen in equations below. Clearly, one would expect a fall in the payoff caused by the transactions tax to deter the speculator from moving into DMs. Thus, the effect of this capital control is to delay even further, the onset of a currency crisis.

\[
V_1(p, s) = \max \left\{ p.s(1 - t), \delta \int_0^1 V_1(p\ell, s) d\ell \right\}
\]

(2.16)

Equation (2.16) provides the new Bellman equation for this scenario. The optimal strategy is as below which can also be represented graphically in figure 2.2.
The impact of the tax can be seen by comparing figures 2.1 and 2.2. Notably, the reservation level, \( I(s) \), remains unchanged. However, the payoff from moving into DMs has fallen. At the point where \( p \) equals \( I(s) \), the speculator is indifferent between moving into DMs or remaining in Pounds. Combining (2.16) and (2.17) gives:

\[
V_t(p, s) = s.p(1 - t) \quad \text{if} \quad p \geq I(s)
\]
\[
V_t(p, s) = s.I(s)(1 - t) \quad \text{if} \quad p \leq I(s)
\]  

(2.17)
\[ I(s)s(1 - t) = \delta \int V_I(p, s)dp = s\delta(1 - t) \left( \frac{1 + I(s)^2}{2} \right) \quad (2.18) \]

Adding the outside option allows:

\[ \int V_I(p, s)dp = s(1 - t) \left( \frac{1 + I^2}{2} \right) \quad (2.19) \]

to be compared with \( G \) so that the speculator moves into gold if the public information falls below a particular level, \( \bar{s} \), where:

\[ \bar{s} = \frac{2G}{(1 - t)(1 + I^2)} \quad (2.20) \]

Proceeding as in the previous case with the same transformations gives:

\[ g = \frac{G}{(1 - t)(1 + I^2)} \quad (2.21) \]

Once again, this helps to simplify the derivation of the solution to the model.

Having set up the informed phase, it is now appropriate to examine the first phase. The Bellman equation is given by:

\[ V_U(p) = \max \left\{ p(1 - t), \delta \left[ Pr(U) \int V_U(p')dp' + Pr(I)EV_I \right] \right\} \quad (2.22) \]

in which \( EV_I \) is given as follows:

\[ EV_I = \int \left\{ \max \left[ g \left[ (1 - t)(1 + I^2) \right], s(1 - t) \left[ \frac{1 + I^2}{2} \right] \right] \right\} ds \quad (2.23) \]

Expanding this gives:
\[ EV_I = (1 + g^2) \left( \frac{(1 + I^2)(1 - t)}{2} \right) \]  

(2.24)

It is now possible to find the reservation level for the first phase.

\[(1 - t)U = \delta \left[ Pr(U)(1 - t) \left( \frac{1 + U^2}{2} \right) + Pr(I)(1 - t) \left( \frac{1 + I^2}{2} \right) (1 + g^2) \right] \]

(2.25)

which under a Nash equilibrium gives:

\[(1 - t)U = Pr(U).I.(1 - t) \left( \frac{1 + U^2}{1 + I^2} \right) + Pr(I).I.(1 + g^2)(1 - t) \]  

(2.26)

As one would expect, the imposition of a Tobin tax has had the effect of raising the reservation level in the first phase. However, there is no impact on the reservation level in the second phase. The logic behind this is as follows.

As mentioned earlier, a speculator weighs up the value of moving into DMs against the value of waiting in the hope that another player will move and reveal the information. The crucial factor here is the value of the outside option. In effect, the outside option rules out some of the lower outcomes (see figure 2.2) so that the expected value of next period’s value function is increased. Note that when a Tobin tax is imposed, the speculator moves into gold if the public information falls below:

\[ \bar{s} = \frac{2G}{(1 - t)(1 + I^2)} \]  

(2.27)

As the tax increases, so too does the value of \( \bar{s} \). Hence the tax has further
increased the expected value of next period's value function. The overall effect is seen in (2.26) in which the only difference is in the size of $g$. This has increased to:

$$ g = \frac{G}{(1 - t)(1 + T^2)} $$

(2.28)

which, in turn, raises the reservation level in the uninformed phase.

### 2.7 Conclusion

In this chapter, I have pinpointed a particular feature of a currency crisis i.e. the five year period of relative calm in the foreign exchange market followed by a flurry of activity culminating in a series of speculative attacks. I have shown how this might be explained by the existence of an information externality.

However, this is a highly simplified framework which does not claim to capture many of the other features associated with currency crises. For instance, it does not incorporate an optimising policy maker. In my model, the government may be thought of as an autonomous body which generates devaluations according to a random process. Nevertheless, within this limited framework I am still able to assess the impact of a transactions tax in the market. As discussed earlier, this delays further the speculative attack by raising the level at which speculators are indifferent between remaining in Pounds or moving into DMs. This would suggest that a Tobin tax would help in delaying the onset of a crisis. It has been suggested that, during this time, a government may intervene to fix the fundamentals.

Future lines of enquiry may include extensions to this model. When I clarified the sequence of events, I noted that there was no correlation between the private signal and public information. However, one can imagine how the two may be related. If a government is 'wet', then it might be more likely that a speculator receives a piece of news urging him to switch into DMs. As my model stands, the two are independent of each other. A second potential way
forward involves examining the relationship between the speculative attack and the devaluation. I have assumed that the attack does not have any effect on the likelihood of a devaluation. In the existing framework, devaluations are generated by the government according to a random process. It may be more appropriate to allow a speculative attack to generate a devaluation when the government is ‘wet’.

In conclusion, I stress that this analysis does not thoroughly account for the events of the 1992 crisis in the exchange markets. However it focuses on an area which is unexplored in the currency crisis literature. I have shown that an information externality can play a part in generating a speculative attack.
Appendix 2.1 - Existence and Uniqueness of Solution

It can be shown that for any $I \in (0, 1)$, $N \geq 2$ and $g \in (0, \frac{1}{2}]$ there exists a unique equilibrium. The task is to show that for any values of $I$, $N$ and $g$ in the specified ranges with $I(s) = I$, $s = 2g$ there is a unique value of $U \in (I, \min[1, (1 + g^2).I])$. To prove the above, it is also important to show that initially waiting for a higher value of $p$ is more beneficial than opting for the alternative of gold. It follows that the condition for this is:

$$\frac{1 + U^2}{2} \geq G = g(1 + I^2)$$

The value of $g$ is smaller than $\frac{1}{2}$ so it follows that $U \geq I$. This is an important characteristic which I have already discussed and which makes the delay until an attack possible. This implies that existence and uniqueness of a solution requires a proof that (2.15) has a unique solution for $U$ in the range $(I, 1)$. This can be done by rewriting (2.15) as:

$$A_1 U + A_{N-1} U^{N-1} = A_0 + A_{N+1} U^{N+1}$$

in which:

$$A_1 = 1 + I^2$$

$$A_{N-1} = [(1 + g^2)(1 + I^2) - 1] I$$

$$A_0 = (1 + g^2).I.(1 + I^2)$$

$$A_{N+1} = I$$

Note that when $U = 0$, the right hand side of the equation is greater then the left hand side. Thus, it follows that $A_0 > 0$. Conversely, when $U = 1$, the left hand side of the equation exceeds the right hand side since $A_1 + A_{N-1} > A_0 + A_{N+1}$. Both sides are continuous so this implies that a
unique solution exists. Furthermore, the value of $U$ must exceed that of $I$ since if they are set equal, the right hand side of (2.29) exceeds the value of the left hand side. This establishes the existence of a solution. The next step is to ensure that this solution is unique.

As in the Caplin and Leahy framework, I set out to prove that the difference between the left and right hand sides of (2.29) is quasi concave in $U$ over its range $[0,1]$. Note that:

$$D(U) = A_1U + A_{N-1}U^{N-1} - A_0 - A_{N+1}U^{N+1}$$

In order to satisfy the condition of quasi concavity, it must be shown that for any value, $\overline{U}$ in the range, $[0,1]$, where $D'(\overline{U}) = 0$, the second order derivative, $D''(\overline{U}) < 0$. The second order derivative takes on the value:

$$D''(\overline{U}) = (N-1)(N-2)A_{N-1}\overline{U}^{N-2} - (N+1)N.A_{N+1}\overline{U}^{N+1}$$

$$D''(\overline{U}) < \frac{(N-1)D'(\overline{U})}{\overline{U}} = 0$$

This proves that the function is locally concave at any point, $\overline{U}$ implying that there can only be one such critical point. This establishes the uniqueness of a solution.
Chapter 3

A Model to Explain the Duration of a Currency Crisis

3.1 Introduction

I have already shown how the timing of a speculative attack may be explained by the presence of an information externality within the market. However, the duration of the crisis and the subsequent collapse of the currency may also be explained using a ‘war of attrition’ model such as that described by Alesina and Drazen (1991) in the context of a fiscal stabilisation.

The chapter is set out as follows. Firstly, I will set out the ‘war of attrition’ model and solve it to show the optimal time of concession. I will then analyse the effect of changing the parameter values. In particular, I am concerned with the effect on the delay until one side concedes. In an extension to the model, I will introduce asymmetric post stabilisation utilities into the framework. I will then draw conclusions.

3.2 A ‘War of Attrition’ Model to Explain the Duration of a Crisis’

The logic of the Alesina and Drazen argument is as follows. If a stabilisation has particular implications i.e. a burden to be borne by the parties in ques-
tion, then each group will attempt to shift the burden onto the other. This leads to a 'war of attrition' in which each group attempts to hold out in the hope that the other will concede first and bear the larger share of the burden. In the Alesina and Drazen case, the initial shock reduces available tax revenues to pay off a budget deficit and each party tries to shift the resulting tax incidence onto the other.

The framework also lends itself to the issue of foreign aid. This has been shown by Casella and Eichengreen (1996) who investigate the importance of the timing of aid. They find that foreign aid decided upon and transferred earlier in the game can lead to an early stabilisation. However, if it is decided upon and transferred late in the game, the effect can be destabilising. This encourages the further postponement of reforms.

My model uses the Alesina and Drazen approach and applies it to a currency crisis. In this model, I consider two governments; Germany and the UK. A speculative attack is launched on the Pound at time $t = 0$. This has the effect of imposing a cost on each of the member governments. This can take the form of a political burden since an attack on a currency can jeopardise the future of the ERM and thus have serious consequences for all members. It may also be thought of as purely financial in terms of undesired movements in foreign currency reserves. The attack implies that investors sell their holdings of sterling and purchase Deutschmarks. In the absence of intervention, this would generate a decrease in the UK money supply and a corresponding increase in the German money supply. However, as a temporary measure, the UK and German authorities can overcome these movements in foreign currency reserves by the use of sterilised intervention. This instrument is costly for each government and hence it becomes increasingly difficult to maintain this situation. As a consequence, the governments endure a mounting pre stabilisation utility loss which is a function of the cost imposed by sterilisation.

The situation can only be restored by a fundamental change in policy undertaken by one of the governments. This could take the form of a change to fiscal policy, a change in interest rates or the abandonment of the fixed rate regime. This would halt an attack on the currency and hence resolve
the currency crisis. Thus, in this context, a stabilisation is brought about by a change in policy by one of the two governments. The government carrying out the change in policy is deemed the ‘losing’ country. It therefore bears a utility loss in excess of the ‘winning’ country but this loss is smaller than that which each country was enduring prior to stabilisation.

The crucial feature of this model is that the governments differ in terms of their welfare loss and that neither government knows the welfare loss of its opponent. Hence, as time passes prior to stabilisation, each government can only make deductions about the relative strength of its opponent. Given this scenario, it is possible to calculate the optimal time of concession for a government and hence the timing of the collapse of an exchange rate regime.

3.3 Model

Within this framework, I consider two governments, namely the UK and Germany. However, the model can be extended to include more than two players. These governments differ in the welfare loss they suffer as a result of an attack. This is private information i.e. each government knows its own welfare loss but does not know that of its opponents. It is assumed that reserves are undepleted before time $t = 0$. At this time, a speculative attack is launched on the Pound. This generates a cost of $c$ per period. I am assuming that this level is constant.\(^1\)

The utility loss for each government is proportional to the size of the cost but differs across governments. Each government’s utility loss is determined by the parameter, $\theta_i$ which lies between the values, $\underline{\theta}$ and $\bar{\theta}$. It is assumed that prior to a stabilisation, the flow utilities for each government are given by:

\(^1\)In the Alesina and Drazen framework, government spending before a stabilisation is financed through a combination of new bond issues and distortionary taxation. It follows that although initial expenditure is constant, there is an increase in interest payments over time caused by the rising stock of bonds. Casella and Eichengreen note that this is misleading since it implies that an increasing burden hastens the stabilisation. This is not, in fact, the case. Therefore, I present the model in a simple form with a constant cost which causes a welfare loss.
where $i = 1, 2$ denotes each of the two governments. Each government estimates the opponent's cost using the cumulative probability distribution function, $F(\theta)$ and the associated density function, $f(\theta)$. For simplicity, the distribution of $\theta$ is assumed to be uniform between $\underline{\theta}$ and $\bar{\theta}$.

A resolution of the crisis implies that there is no longer a political and financial cost imposed on the governments arising from the crisis. Thus, there is an incentive to concede and end the crisis. However, the resolution of the crisis involves costly policy changes which are divided unequally between the governments. The loser bears the larger portion of the burden, $\alpha$, which is assumed to be greater than $\frac{1}{2}$. The winner bears the smaller share of $(1 - \alpha)$. It is assumed that this share of the burden is not bargained over. Furthermore, the governments bear this cost forever. Notably, a value of $\alpha$ close to $\frac{1}{2}$ would indicate what Alesina and Drazen refer to as 'political cohesion' i.e. it would indicate a willingness for each country to bear approximately the same portion of the burden. I will develop this idea in the next section.

It follows that after a stabilisation, the utility losses borne by each government will be determined by the value of $\alpha$ and the cost, $c$, so that flow utility will become:

\begin{equation}
    u^L = -\alpha c
\end{equation}

for the loser and:

\begin{equation}
    u^W = -(1 - \alpha) c
\end{equation}

for the winner. The important point to note is that the flow utilities for the winner and loser are higher than the pre stabilisation utility. Before one side concedes, each government has a utility given by (3.1). The loss is determined
by the cost per period and each government’s value of \( \theta \). Following a stabilisation, the governments share the cost, \( c \), with the winner bearing a smaller burden. It follows that it would be better to be the losing government than to endure (3.1). However, the crucial point is that each government does not know the strength of its opponent as given by the opponent’s \( \theta \). This implies that it is optimal to wait in the hope that the opponent will concede first. The discounted lifetime utilities at the point of stabilisation are as follows:

\[
V^L = \frac{-\alpha c}{\delta} \tag{3.4}
\]

for the loser and:

\[
V^W = \frac{(1-\alpha)c}{\delta} \tag{3.5}
\]

for the winner. Note that \( \delta \) is the discount rate. It follows that the lifetime utility from the date at which the crisis begins of the winner and loser may be written as:

\[
U^j(T) = \int_0^T u^D(x)e^{-rx}dx + e^{-rT}V^j(T) \tag{3.6}
\]

where \( j = W, L \).

It is now possible to evaluate the expected utility as of time 0 as a function of the chosen concession time of a government, \( T_i \). This is the sum of \( U^W(X) \) multiplied by the probability of the opponent conceding at any time, \( X \leq T_i \) plus \( U^L(T_i) \) multiplied by the probability that the opponent has not conceded before \( T_i \). The solution of the game is the function, \( T(\theta_i) \) which maps the cost parameter, \( \theta_i \), onto its optimal time of concession, \( T_i \). The expected utility of a government can now be written as:
where \( H(T) \) is the distribution of the opponent’s optimal time of concession and \( h(T) \) is the density function. Substituting (3.6) into (3.7) gives the following:

\[
EU(T_i) = [1 - H(T_i)] \left[ \int_0^{T_i} u^D(x)e^{-rx}dx + e^{-rT_i}V^L(T_i) \right]
\]

\[
+ \int_{x=0}^{x=T_i} \left[ \int_0^x (z)e^{-rz}dz + e^{-rz}V^W(x) \right] h(x)dx
\]  

(3.8)

It is possible to find the optimal time of concession by finding the value of \( T_i \) which maximises (3.8). Differentiating (3.8) with respect to \( T_i \) and setting the resulting expression equal to zero gives:

\[
\frac{dEU}{dT_i} = h(T_i) \left[ V^W(T_i) - V^L(T_i) \right]
\]

\[
+ [1 - H(T_i)] \left[ u^D(T_i) - u^L(T_i) + \frac{dV^L(T_i)}{dT_i} \right] = 0
\]  

(3.9)

Substituting in the values of (3.1), (3.2), (3.4) and (3.5) gives:

\[
\frac{dEU}{dT_i} = h(T_i)(2\alpha - 1) \frac{c}{\delta} + [1 - H(T_i)] \left[ c \left( \alpha - \frac{1}{2} - \theta_i \right) \right] = 0
\]  

(3.10)

Differentiating with respect to \( \theta_i \) gives:
\[
\frac{d^2 EU}{dT_i d\theta_i} = -[1 - H(T_i)] \cdot c < 0 \tag{3.11}
\]

Hence the optimal concession time, \( T_i \), is monotonically decreasing in \( \theta_i \). This result is significant since it defines the relationship between \( H(T) \) which is unknown and \( F(\theta) \) which is known. This relationship is:

\[
1 - H(T(\theta)) = F(\theta) \tag{3.12}
\]

Differentiating this gives:

\[
-h[T(\theta)] T(\theta) = f(\theta) \tag{3.13}
\]

The Nash equilibrium is described by the function, \( T(\theta_i) \). This defines the optimal point of concession given that the opponent is following the same decision rule. Using (3.10), (3.12) and (3.13), the symmetric Nash equilibrium can be described as follows:

\[
T(\theta) = \frac{f(\theta)}{F(\theta)} \frac{2\alpha - 1}{\delta (\theta + \frac{1}{2} - \alpha)} \tag{3.14}
\]

It is assumed that a government with the highest possible cost of waiting will concede immediately. Hence this gives the boundary condition of:

\[
T(\bar{\theta}) = 0 \tag{3.15}
\]

The differential equation, (3.14), can now be solved to find the function, \( T(\theta) \). This is given by:
$T(\theta) = \frac{2\alpha - 1}{\delta (\theta + \frac{1}{2} - \alpha)} \left( \frac{\ln \theta + \frac{1}{2} - \alpha}{\theta + \frac{1}{2} - \alpha} - \frac{\ln \theta - \frac{\theta}{\theta - \theta}}{\theta - \theta} \right)$ \hspace{1cm} (3.16)

An additional assumption is imposed here. It is assumed that $\theta > \alpha - \frac{1}{2}$. This implies that a government will concede in finite time. If the government possessed a $\theta$ such that $\theta + \frac{1}{2} < \alpha$, it would never be optimal for the government to concede because before stabilisation, it is bearing a utility loss which is smaller than the utility loss of a loser.

In summary, the working of the game is as follows. At the outset there is a speculative attack the resolution of which imposes a large political and financial cost which is divided unequally between the governments. The winner takes on the smaller share while the loser adopts the larger part of the burden. They know what payoffs they will receive if they win or lose. At time 0 immediately following the speculative attack, there will be a probability that the opponent will concede i.e. a probability that the opponent has a $\theta = \overline{\theta}$. If it does not concede straight away, the government realises that its opponent is not of the ‘weakest’ type. As time progresses, if the opponent still does not concede, the government learns more about it. It learns that the opponent does not have a value of $\theta$ above a particular level. This process continues until the conditional probability of the opponent conceding is such that (3.16) holds. This denotes the optimal time for the government to give in and accept being a loser.

I argue that the speculative attacks on the Pound in 1992 may have generated a ‘war of attrition’ set up similar to the one described above. Each government was reluctant to accept the policy changes required to halt the crisis. The UK hoped for outside support while Germany had financial commitments elsewhere. The result was a ‘waiting game’ while each hoped the other side would concede.
3.4 The Effect of Different Parameter Values on the Expected Time of Stabilisation

In this section, I am concerned with the effect of changing certain parameter values on the solution to the model.

3.4.1 Political Cohesion

In the model, the value of \( \alpha \) is not bargained over. It is determined exogenously and both players know this value at the beginning of the game. It follows that if \( \alpha = \frac{1}{2} \), stabilisation will occur immediately since there is nothing to be gained from delaying. This is because \( V^W = V^L \) and since there are costs to not conceding, it is optimal to concede straight away. Conversely, where \( \alpha \) is close to 1, there is an incentive to wait in the hope that the opponent will concede first. Therefore, the closer is \( \alpha \) to 1, the larger is the delay, other things being equal.

This is an important result in terms of the ERM since it indicates the level of political cohesion within the community i.e. the willingness to share equally (or not) the burden of reserve depletion. Clearly, if there had been a great degree of political cohesion in 1992, a speculative attack on the Pound would have resulted in an immediate stabilisation with the UK and German governments sharing the burden. The fact that there was a delay in which the UK held out hoping for support indicates a lack of cohesion within the system. This then raises the issue of how political cohesion may be achieved. One possibility may be to require member governments to agree to share equally any burden arising from a currency crisis. Clearly, this introduces the idea of precommitment and may strengthen the credibility of the ERM.

This has serious implications for a future attempt at monetary union. It suggests that unless there is a willingness to share the costs incurred, a speculative attack on a currency will merely lead to a repeat performance of the 1992 crisis.
3.4.2 Size of the Political and Financial Cost

Significantly, a change in the size of $c$ has no effect on the optimal time of concession. This is the point stressed by Casella and Eichengreen (1996). It is not an increasing burden which causes stabilisation. Instead it is generated by groups who do not know the ‘type’ of opponent they are facing. It becomes individually rational for each to hold out in the hope that the other has a value of $\theta$ larger than its own. Consequently, the size of the costs to the government following a speculative attack do not affect the optimal time of its concession. In considering future monetary union, this would suggest that the size of the total burden is not the issue. What is important is the share of this cost apportioned to each of the players.

3.5 Extension to the Framework

In the above analysis, I made an assumption that the exchange rate mechanism would survive the attack on the currency. The ‘war of attrition’ was concerned with who bears the larger share of the burden. In this extension to the framework, I shall consider the effect of country dependent payoffs for the winner and loser. In particular, I shall assume that if the UK wins the ‘war of attrition’ then the system survives and the result is as before. However, if Germany is the winner, I shall assume that the UK leaves the system. This generates a lower payoff for both countries than in the previous scenario.

I assume that the pre stabilisation utility is (3.1) as before. However, following the stabilisation, the flow utility for Germany will be:

$$u^L_i = -\alpha e$$

(3.17)

if it loses or:
\[ u_g^W = -\gamma c \]  
where \( \gamma > 1 - \alpha \) if it is the winner. The payoff in being the winner is smaller than in the previous case since the UK has left the system. Conversely, the flow utility for the UK will be:

\[ u_{uk}^L = -\beta c \]  
if it loses, where \( \beta > \alpha \). This is also lower than in the initial case since, in this scenario, the UK will have to leave the exchange rate mechanism. If it wins the ‘war of attrition’ the flow utility will be:

\[ u_{uk}^W = -(1 - \alpha)c \]  

By including the discount rate in the above results one may arrive at the corresponding discounted lifetime utilities at the point of stabilisation. The optimal times of concession for each country may now be calculated as before. The above results are substituted into (3.9). For the UK, this gives:

\[
\frac{dEU}{dT_i} = h(T_i)(\alpha + \beta - 1)\frac{c}{\delta} + [1 - H(T_i)]\left[c\left(\beta - \frac{1}{2} - \theta_i\right)\right] = 0 \tag{3.21}
\]

As before, I use (3.12), (3.13) and (3.21) to arrive at the differential equation:

\[ T_l(\theta) = \frac{f(\theta)}{F(\theta)\delta} \frac{\alpha + \beta - 1}{\theta + \frac{1}{2} - \beta} \]  

Solving (3.22) given the initial boundary condition of (3.15) gives:
\[ T(\theta) = \frac{(\alpha + \beta - 1)}{\delta \left( \theta + \frac{1}{2} - \beta \right)} \left( \ln \frac{\theta + \frac{1}{2} - \beta}{\theta + \frac{1}{2} - \alpha} - \ln \frac{\theta - \theta}{\theta - \theta} \right) \] (3.23)

Firstly, if it is assumed that \( \theta > \beta - \frac{1}{2} \), then the government will concede in finite time. However, the main result here is that the optimal time of concession has increased. This is apparent when the term outside the brackets is examined. The value, \( \beta \), exceeds \( \alpha \). The significance of this is that the disparity between the winning and losing payoffs has increased. Therefore, there is a greater incentive to hold out in the hope that the opponent will concede.

For Germany, the opposite applies. Substituting into (3.9) gives:

\[
\frac{dEU}{dT_i} = h(T_i)(\alpha - \gamma) \frac{c}{\delta} + \left[ 1 - H(T_i) \right] \left[ c \left( \alpha - \frac{1}{2} - \theta_i \right) \right] = 0 \] (3.24)

which, together with (3.12) and (3.13) gives:

\[ T_\alpha(\theta) = -\frac{\frac{f(\theta)}{F(\theta)} \frac{\alpha - \gamma}{\delta \left( \theta + \frac{1}{2} - \alpha \right)}} \] (3.25)

Solving for (3.25) with the initial boundary condition, (3.15) gives:

\[
T'(\theta) = \frac{(\alpha - \gamma)}{\delta \left( \theta + \frac{1}{2} - \alpha \right)} \left( \ln \frac{\theta + \frac{1}{2} - \alpha}{\theta + \frac{1}{2} - \alpha} - \ln \frac{\theta - \theta}{\theta - \theta} \right) \] (3.26)

As in the initial case, an assumption that \( \theta > \alpha - \frac{1}{2} \) ensures that the government will concede in finite time. However, the important result here is that the optimal time of concession has decreased. The reason behind this is that the difference between the winning and losing payoffs has narrowed. Hence, the incentive to hold out for the opponent’s concession has been reduced. The greater the utility loss endured by the German government
as a result of the UK leaving the system, the more likely is the German government to concede.

The final step in this analysis is to compare (3.23) with (3.26). The first point to note is that each government assumes that the opponent is playing the same strategy as itself. Hence it believes that the only way in which its opponent will differ will be in its value of $\theta$. However, since each government receives a different flow utility according to whether it wins or loses, its optimal time of concession will also be governed by these factors. Hence, for a given value of $\theta$, the closer is $\gamma$ to $1 - \beta$, the smaller will be the difference between the countries’ optimal times of concession.

One may argue that this analysis is a more accurate description of the events of 1992 since the attack on the pound led to the UK leaving the exchange rate mechanism.

### 3.6 Conclusion

In conclusion, I would argue that this basic ‘war of attrition’ framework is extremely versatile. It lends itself, not only to the scenarios of foreign aid and tax distribution but also to the topical area of currency crises. My aim was to offer a possible explanation for the duration of a currency crisis and ultimately the timing of the UK exit from the regime. In the initial framework, it is assumed that the system remains intact. However, despite its simplicity, it has produced some interesting points with regard to political cohesion. Notably, in the case of the UK, there was a considerable delay until the decision was taken to leave the system. This delay would indicate a lack of political cohesion between countries in the ERM.

In the extension to this framework, I consider the effect of asymmetric payoffs for the winner and loser. I find that the larger the difference between the payoffs of winning and losing, the larger is the optimal time of concession.

I do not claim to have fully accounted for the events of the 1992 crisis. However, I have demonstrated, using a ‘war of attrition’ model, how the duration of the crisis may be explained. If each government knew its opponent’s ‘type’, then there would be no delay prior to a stabilisation. It is the fact
that neither government knows the strength of its opponent, that creates a situation in which each finds it optimal to hold out in the hope that the opponent will concede first.
Chapter 4

A Model of Informational Events which Triggers a Currency Crisis

In this chapter I show how informational events can trigger currency crises. In order to put my research into context, I outline two of the more influential papers in this area. Following this brief literature survey, I present a model based on that of Morris and Shin (1995). Firstly, I set up the model and show how it would work if all information were observed without error. I then set out the case of symmetric imperfect information and then the revealing scenario where differential information is noisy. Finally I discuss the results and draw conclusions.

4.1 Literature Survey

The literature concerning the aggregation of information in society is relevant to this thesis since it can be used to explain speculative behaviour in foreign exchange markets. In this section, I outline two of the key papers in the field.

One such paper is that of Lohmann (1994). She analyses political action prior to an election. She argues that by signing petitions or taking part in demonstrations, people signal their dissatisfaction with the current policy.
These actions can convey information about policy consequences to other voters and, therefore, influence other individuals' voting decisions. Hence information can be dispersed among society through political action.

In her model, policy preferences of individuals are correlated. If one person reveals his preferences, other peoples' preferences are influenced. Furthermore, voters believe that the incentive to take part in political action is weak. Hence, only a small number of political actions are necessary for there to be a decisive result.

Lohmann discovers that those individuals with moderate preferences will undertake political action to signal private information to other individuals. Those with extreme preferences will take action whatever their private information in order to influence other voters. Individuals make a voting decision based on the size of the political action they observe. They discount this for extremist political action and base their decision on the result. She finds that, ex ante, political action can decrease the likelihood of an incorrect voting outcome. However, political action can simply add noise to the system when, in the absence of pre election communication, voting could lead to the full information outcome.

Caplin and Leahy (1994) also consider the importance of information revelation. However, theirs is a three stage model. In the first stage, information of common interest is in private hands. Once this information reaches a trigger level, agents change their behaviour thereby releasing information. In the final stage, the market reacts to this revelation. They focus attention on industry investment but also note that the same analysis could be applied to political crises, bank runs and international debt crises.

The model is of irreversible investment in which there is a continuum of firms. Each can take part in production of a single unit of a consumption good. They must spread this production over T periods. In this time, they can accumulate information on the state of final demand of their good. They use this information to decide whether or not to make the additional investments to finish the project.

There are three different sources of information. Firstly, the firm has a prior knowledge about the state of demand. There are two equally likely
states of demand; high or low. The second source of information comes from the private signal that each firm receives immediately after paying the initial fee. This concerns the state of demand. Thirdly, is the information the firm receives each period in which it is in active production. In the final period, each firm must decide whether to complete or abandon production. This type of approach may be applied to foreign exchange markets.

4.2 Informational Events which Trigger Currency Crises

The model presented by Morris and Shin shows how the ERM was ‘ripe for attack’ long before the September 1992 crisis. A currency is ‘ripe for attack’ in this framework when speculators need to coordinate to bring about a collapse by selling their holdings of sterling. There are three possible regions for the value of the fundamentals. In the stable region, a fixed rate regime will be maintained even if all investors sell their holdings of the currency since the cost of intervention falls short of the value of sustaining the regime. In the unstable region, the cost of maintaining the regime exceeds the benefit irrespective of the actions of the investors. In the ‘ripe for attack’ region, the decision to sustain a fixed rate regime depends crucially on the behaviour of the investors. If they all sell, it is optimal for the government to abandon the regime. However, if they all retain their holdings of the currency, the cost of intervening to support the currency is less than the benefit. It is then optimal to maintain the fixed rate regime. Morris and Shin argue that if the state of the economy lies in the ‘ripe for attack’ region each investor will sell his holdings of the currency if there is a lack of common knowledge among the investors concerning the state of the economy.

My contribution to this area is to adapt the Morris and Shin framework so as to allow the state of the economy to be known to all players. However, the value placed by the government in remaining in the ERM is observed by each of the speculators with a degree of error. The benefit from remaining in the system may arise, for example, from an improved political reputation
or a favourable impact on inflation. It is assumed that a tough government places a large value on sustaining the regime whereas a weak government places a small value on maintaining the parity. I show that a lack of common knowledge concerning government type leads to the same scenario as in the Morris and Shin case. Instead of three possible regions existing for the fundamentals, they exist for the value placed by the government on maintaining a fixed rate regime.

Morris and Shin note that imperfect information alone is not enough to guarantee the onset of a crisis. In their model, they show that if all speculators observe a public signal which gives the true state of the economy with error, the content of this signal is common knowledge. It follows that 'nothing more can be said beyond the existence of multiple equilibria'.

When each speculator receives differential information concerning the state of the economy, the outcome will be sensitive to this differential information. This is because the state of the economy is no longer common knowledge. Even if the degree of noise in the signal is very small, it is never common knowledge that the exchange rate parity will be maintained. Each speculator knows that his payoff depends on the actions of the others. In turn, their actions are determined by their beliefs. It follows that the speculator will be concerned about the beliefs of his counterparts. Despite receiving a message which rules out certain states, he may still have to consider these states of the world since they may contain information about his opponents' beliefs. His opponents face the same problem. It follows that although every player knows that the exchange rate can be maintained at its current parity, they must take into consideration what would happen if this parity was unsustainable because the actions of other speculators may make it unsustainable.

Within the framework, selling sterling yields a fixed, known payoff. However, holding on to the currency is risky. The returns are high if the speculators agree not to attack but low if the currency collapses. Unless these speculators can agree on how to interpret their news, remaining in the currency is not optimal. Therefore, when each speculator receives a signal, independently and uniformly distributed around the true state of the eco-
nomy, the target rate collapses in both the unstable and ‘ripe for attack’ regions.

In this adaptation of Morris and Shin’s framework, it is the value placed by the government on maintaining an exchange rate parity which is observed with error. It is uncertainty about how the speculator should interpret his information which makes it optimal to attack. I show that with the value of maintaining the parity unknown, the same situation arises as with the Morris and Shin case. When speculators receive noisy differential information concerning the value placed on maintaining the parity, this produces three distinct regions for the value. In the ‘ripe for attack’ region, the government will abandon the fixed rate providing that a sufficient number of speculators sell their sterling. In this region, a sufficiently high tax rate will deter speculators from abandoning sterling.

4.3 Model

As in the Morris and Shin framework, I consider the relationship between the UK government and a large number of speculators. The reason I assume that there are many speculators is to ensure that each one is small in relation to the total population. Demand for the currency is determined through the foreign exchange market and is denoted by:

\[ D(e) \]  

(4.1)

where \( e \), the exchange rate, is the number of Deutschmarks, for example, per Pound. A large value of \( e \) implies a strong Pound. It follows that demand for sterling is decreasing in \( e \). At the start of the game, each speculator holds sterling. In total, these holdings have been normalised to 1. The aggregate sale of sterling is denoted by \( s \). I assume that even under a floating regime there is some degree of government intervention denoted by \( I(v) \) where \( v \) is the value that the government places on maintaining the parity. I assume that a tough government will intervene to a greater degree under a floating
regime than will a weaker government.

If the currency is allowed to float, the exchange rate will be determined by the intersection of the supply and demand schedules. The point of equilibrium is denoted by:

\[ f(v, s) \]  

(4.2)

which is that value of the exchange rate at which demand for the currency equals supply so that:

\[ I(v) + D(e) = s \]  

(4.3)

\( f(v, s) \) is decreasing in \( s \) and increasing in \( v \).

It is further assumed that the government has an exchange rate target of \( e^* \). This represents the fixed rate which, it is assumed, exceeds the floating rate. This implies that intervention to support the target is necessary if the fixed rate regime is to be maintained. The government can achieve this through purchasing sterling and thereby incur a cost of \( c(x) \) where \( x \) denotes the number of Pounds to be bought. It is assumed that, firstly, this cost increases in \( x \) and that it is a one off cost. Secondly, the government will undertake some level of intervention. I also assume that the value placed on maintaining the target denoted by \( v \) is drawn from a uniform distribution on the interval \([0,1]\). If it is at its lowest point, 0, I assume that the cost of intervention exceeds the benefit from maintaining the exchange rate parity.

As in the original Morris and Shin framework, it is now possible to identify three regions for the value of \( v \). Firstly, there is \( v^* \). Beyond this point, even if all speculators sell their holdings of the currency, the cost of intervention falls short of the value of maintaining the parity. This is given by \( c(-D(e^*) + 1) = v^* \). Note that \(-D(e^*) + 1\) is the net supply of sterling which the government must purchase when all speculators sell their holdings of the currency. This is known as the stable region and is defined where
$v \in [v^*, 1]$. Here the exchange rate parity will be maintained and the actions of the speculators do not affect the decision by the government to remain in the exchange rate regime.

The value $v_*$ solves $c(-D(e^*)) = v_*$. Below this value of $v$, the cost of intervention is in excess of the benefit even if no speculator sells his currency. This is the unstable region defined for values of $v$ where $v \in [0, v_*]$. Once more, the actions of the speculators do not affect the decision by the government. This time it chooses not to maintain the exchange rate target.

For a value of $v$ in the interval, $v \in [v_*, v^*]$, the behaviour of the speculators determines the decision by the government to intervene. If all speculators choose to retain their holdings of sterling, then the cost of intervening is less than the value of maintaining the parity. However, if all speculators choose to sell their holdings, it is optimal for the government to abandon the target.

In this region, it follows that if more than a certain proportion of speculators sell their holdings of sterling, the government will be compelled to abandon the currency. This proportion of speculators required to trigger an exit from the system is called the trigger mass and is given by $\alpha(v)$. Notably, where $v$ is equal to $v_*$, $\alpha(v_*) = 0$ and for $v$ equal to $v^*$, $\alpha(v^*) = 1$.

As in the Morris and Shin framework, I shall impose the condition on the floating rate such that the stronger the government, the smaller will be the devaluation i.e. $f(v, \alpha(v))$ is weakly increasing in $v$.

I have already noted that there are a large number of speculators each of which is small relative to the size of the total population. I assume that each can sell all of his holdings or retain all his holdings. He does not sell a fraction of what he holds. In selling his share of sterling, he bears a fixed cost, $t > 0$. This can represent a transactions cost or tax.

His payoff is dependent on the government’s decision to maintain or abandon the target. If the target is maintained and the speculator retains his holdings of sterling his payoff is normalised to 0. If the government abandons the target rate and the speculator does not switch out of sterling, his payoff will be the difference between the floating rate and the target:
If he sells his holdings of sterling he can expect the fixed payoff of:

\[ f - e^* \]  \hspace{1cm} (4.4)

This is not related to the value of maintaining the target rate.

Having described the components of this model, the nature of the game can be explained. In the first instance, I assume that the speculators can observe \( v \) without error. The value to be placed on sustaining the target rate, \( e^* \) is determined. The speculators view this perfectly and then decide whether to retain their holdings of sterling. This then decides the aggregate sale of sterling, \( s \). Having observed this, the government then chooses whether to intervene in support of the currency. This determines the exchange rate and hence the speculators receive their payoffs.

This is a very basic scenario in which the speculators have perfect information. They know that if \( v \) falls in the stable region, the government will sustain the target rate irrespective of speculative behaviour. Therefore, it is optimal to retain their holdings of sterling. Conversely, if they observe a value of \( v \) in the unstable region, it is optimal to sell the currency since the government will not support the pound even if all speculators retain their holdings.

Within the ripe for attack region, the actions of the government will depend on speculative behaviour. This is a situation of multiple equilibria since if all speculators co-ordinate their actions and retain their holdings, the government will find it optimal to defend the target. However, if they all sell their currency then the government will find that the costs of defending the target exceed the benefit. Hence, it will devalue the Pound.

A similar result emerges even when it is assumed that \( v \) is observed with a degree of error. In this scenario, each speculator views a public signal. This is
known to be observed with an error which lies within $\epsilon$ of the true value, $v$, so $\epsilon$ represents a limit to the distribution. The speculators receive a message, $m$, which is conditional on $v$ and uniformly distributed over the interval, $[v - \epsilon, v + \epsilon]$. The significance of this is that the message is still common knowledge. Although any decision by the speculators will now be determined by the message, $m$, the three regions will still exist. Furthermore, multiple equilibria will still be a feature of the ripe for attack region. Therefore, the outcome is the same as in the perfect information case.

However, the result is different if the speculators observe differential information which is noisy. It is assumed that each speculator receives a message, $m$, from the interval, $[v - \epsilon, v + \epsilon]$. These messages are assumed to be independent across speculators and are uniformly distributed over the interval, $[v - \epsilon, v + \epsilon]$. As in the Morris and Shin framework, I consider the equilibria determined by a $k$ trigger strategy. The speculator sells his holding of sterling if he receives a message which is less than $k$. If the message, $m$, is in excess of $k$, then he retains his sterling. If each speculator behaves according to this decision rule then an equilibrium is known as a $k$ trigger equilibrium.

Payoffs for the speculators are as before. They receive $-t$ if they sell their holdings of sterling and 0 if they retain sterling and, subsequently, the target is maintained. If the target is abandoned by the government and the speculators are holding sterling, the payoff is $f - e^*$. The aggregate sale of sterling will depend upon the number of speculators who have received news less than $k$. The sale of sterling is denoted by:

$$s(v, k)$$

This is the aggregate sale of the currency when the value of sustaining the target rate is $v$ and all speculators employ the $k$ trigger strategy. Notably, when the true $v$ lies below $k - \epsilon$, all speculators will have received a message that falls short of $k$. Therefore, they will all choose to sell their sterling. Conversely, if the true value of $v$ exceeds $k + \epsilon$, all speculators will have
received a message greater than $k$. Therefore, none of them will choose to sell their sterling. It follows that $s(v, k)$ is given by:

$$s(v, k) = \begin{cases} 
0 & : v \geq k + \epsilon \\
1 & : v \leq k - \epsilon \\
\frac{k + \epsilon - v}{2\epsilon} & : k - \epsilon < v < k + \epsilon
\end{cases}$$

(4.7)

This is best illustrated graphically using figure 4.1.
Figure 4.1: The Decision to Abandon or Maintain Sterling
Notably, the floating exchange rate is increasing in \( v \), while \( s(v, k) \) is decreasing in \( v \). It follows that the function, \( f(v, s(v, k)) \) is increasing in \( v \).

When the speculators decide on whether to sell their currency, they take into account the likelihood of government intervention. As in the Morris and Shin framework, it is now possible to assign a number:

\[
\text{d}(k) \tag{4.8}
\]

for any value of \( k \in (v_*, \varepsilon, v^* - \varepsilon) \) which denotes the value of \( v \) at which the government is indifferent between sustaining the target rate and devaluing the currency. It is, therefore, the value of \( v \) which solves

\[
c(s(v, k) - D(e^*)) = v \tag{4.9}
\]

and also

\[
s(v, k) = \alpha(v) \tag{4.10}
\]

This is known as the \textit{devaluation point} for a \( k \) trigger strategy. Again this can be seen using figure 4.1.

The total intervention necessary to maintain the parity is given by \( s(v, k) - D(e^*) \). It follows that the cost of intervening is then \( c(s(v, k) - D(e^*)) \) which increases in \( s \). The trigger mass is that value of \( s \) which solves \( c(s(v, k) - D(e^*)) = v \) i.e. it is the intersection of the cost function with \( s(v, k) \). When \( v \) falls below this, the exchange rate at the end of the game i.e. the \textit{post intervention exchange rate} will be the floating rate. If it exceeds this point, the target rate will prevail. This can be seen in the diagram by selecting a point for \( v \) and then examining the respective values of \( c(s(v, k) - D(e^*)) \) and \( s(v, k) \). The post intervention exchange rate is denoted by:
$$\psi(v, k) = \begin{cases} e^* & : v \geq d(k) \\ f(v, s(v, k)) & : v < d(k) \end{cases}$$  (4.11)

where $\psi(v, k)$ is non decreasing in $v$. It follows that the payoff to holding sterling given $v$ is $\psi(v, k) - e^*$.

I shall now show that for any $k$ in the interval $k \in (v_*, \epsilon, v^* - \epsilon)$, there is a tax rate $t > 0$ which makes the $k$ trigger strategy a symmetric equilibrium strategy.

A speculator receiving a message in the above interval will know that the true value of $v$ falls in the ripe for attack region. The above statement says that for any message that tells him that $v$ is in the ripe for attack region, there exists a unique tax rate for which this is a trigger point. I shall now set out to prove the above.

It is assumed that all speculators use the same decision rule in deciding whether to hold the currency. If all but the $i$th speculator follow the $k$ trigger strategy and speculator $i$ receives the message, $k_t$, his expected payoff from holding sterling given that all others are following the $k$ trigger strategy is:

$$\Pi(k_t, k) = \frac{1}{2\epsilon} \int_{k_t-\epsilon}^{k_t+\epsilon} (\psi(v, k) - e^*) \, dv$$  (4.12)

Furthermore, speculator $i$’s conditional density over $v$ given the message, $k_t$ is uniform over $[k_t - \epsilon, k_t + \epsilon]$. I have already noted that the post intervention exchange rate is non decreasing in $v$. Therefore, it follows that speculator $i$’s expected payoff is also non decreasing in $v$ and is weakly increasing in $k_t$. It is now possible to examine the expected payoff for holding sterling when all other speculators are adopting the $k$ trigger strategy. Note that when $v \geq d(k)$, the post intervention exchange rate is equal to the target rate. This means that the payoff to a speculator who receives message $k$ when all other speculators are following a $k$ trigger strategy is:
\[ \Pi(k, k) = \frac{1}{2\epsilon} \int_{k-\epsilon}^{k+\epsilon} (\psi(v, k) - e^*) dv = \frac{1}{2\epsilon} \int_{k-\epsilon}^{d(k)} (f(v, s(v, k)) - e^*) dv < 0 \] (4.13)

Note that below the devaluation point, the post intervention rate is equal to the floating rate, hence the second half of the equation above. The reason that there is an inequality follows from the premise that for values of \( v \) less than the devaluation point, the payoff from retaining sterling is negative. Therefore, a tax rate set at the level, \( t = -\Pi(k, k) \) will ensure that a speculator is indifferent between holding sterling and selling his share. Since (4.12) is increasing in \( k \), it follows that a speculator receiving a message in excess of \( k \) will prefer to retain his holdings while if he receives a message below \( k \) he will want to sell sterling. Therefore, \( k \) is the optimal trigger point.

I shall now show that for any message, \( m \in (v_*, \epsilon, v^* - \epsilon) \), there is a tax level, \( t_0 > 0 \) below which all speculators will want to sell their holdings of the currency given \( m \). This is an important result since it suggests that when differential information is noisy, a low tax rate induces a speculator to sell his sterling if he knows \( v \) falls in the ripe for attack region. I shall now set out to prove that this is the case.

Firstly, when the value of maintaining the target \( v = d(k) \), the sale of sterling, \( s(v, k) = \alpha(v) \). Notably, \( s(v, k) \) decreases in \( v \) but \( \alpha(v) \) increases in \( v \). This implies that for a value of \( v \) smaller than or equal to \( d(k) \):

\[ f(v, s(v, k)) \leq f(v, \alpha(v)) \] (4.14)

Again this can be seen in figure 4.1 for a value of \( v \leq d(k) \). Furthermore, when \( k \in (v_*, \epsilon, v^* - \epsilon) \), it can be shown that \( k - \epsilon < d(k) < k + \epsilon \). This is because the devaluation point is defined by calculating the value of \( v \) which solves:
where $0 < \alpha(v) < 1$. For values of $k - \epsilon < v < k + \epsilon$, sale of sterling is given by $\frac{k + \epsilon - v}{2\epsilon}$. A devaluation will occur if the trigger mass is equal to this level so that: $\alpha(v) = \frac{k + \epsilon - v}{2\epsilon}$. Rearranging this yields (4.15). It follows that this value of $v$ must lie in the ripe for attack region since $k \in (v^* + \epsilon, v^* - \epsilon)$.

It can also be shown that within the interval $(v^* + \epsilon, v^* - \epsilon)$, $d(k)$ is invertible and increasing in $k$ and $d(k) - k$ is decreasing in $k$. In figure 4.1 this is apparent since an increase in $k$ within this interval leads to corresponding shifts in the cost function and $s(v, k)$. Algebraically, this can be seen by examining $k = v - \epsilon (1 - 2\alpha(v))$. Since $\alpha$ depends on $v$, so too does $k$. It is apparent that $d(k)$ is increasing in $k$ since its inverse is increasing in $v$ over the interval $(v^*, v^*)$. In considering $d(k) - k$, note that it is decreasing in $k$ since $\alpha$ and $d^{-1}$ are increasing functions in $d(k) - k = \epsilon (1 - 2\alpha(d^{-1}(k)))$.

If a speculator receives a message in the interval $m \in (v^* + \epsilon, v^* - \epsilon)$, then where $k \leq m$, the expected payoff produces the following:

$$
\Pi(k, k) = \frac{1}{2\epsilon} \int_{k-\epsilon}^{d(k)} (f(v, s(v, k)) - e^*) dv \leq \frac{1}{2\epsilon} \int_{k-\epsilon}^{d(k)} (f(v, \alpha(v)) - e^*) dv
$$

$$
\leq \frac{1}{2\epsilon} \int_{m-\epsilon}^{d(m)} (f(v, \alpha(v)) - e^*) dv \leq \frac{1}{2\epsilon} \int_{m-\epsilon}^{d(m)} (f(v, \alpha(v)) - e^*) dv < 0 \quad (4.16)
$$

This is a similar result to that found by Morris and Shin. The first inequality arises since $f(v, s(v, k))$ is bounded by $f(v, \alpha(v))$. The second inequality results from $f(v, \alpha(v))$ increasing in $v$ while $d(m) - m \leq d(k) - k$. It follows that the final integral must be negative since, in the ripe for attack region, $f(v, \alpha(v))$ falls short of the target rate. A tax level of:
\begin{equation}
  t_0 = -\frac{1}{2\epsilon} \int_{m-\epsilon}^{d(m)} (f(v,0) - c^*) \, dv
\end{equation}

implies that \(-t_0 \leq \pi(k,k)\) for all \(k \leq m\) so it is never optimal to retain sterling for any value of \(k \leq m\) and so it is not possible to find a trigger equilibrium.

### 4.4 Conclusion

In this chapter I have shown how Morris and Shin's model of currency crises may be extended to consider the case where speculators have differential information regarding government type. When there is a lack of common knowledge of the value a government places on maintaining a target rate, it becomes optimal for each speculator to attack the currency when this value lies in the 'ripe for attack' region. Conversely, when differential information is observed with an error in the form of a public signal, an attack need not take place in this region. This is because the news received is common knowledge and hence, speculators can agree on how to interpret it. It is only when this differential information becomes noisy that it becomes optimal for each speculator to attack in the ripe for attack region.

The results from the model would suggest two policy implications. Firstly, the government needs to put a sufficiently large value on maintaining the target rate. This value must not be so low that it puts the currency in the unstable region. Secondly, if \(v\) lies in the 'ripe for attack' region, the government can avoid a crisis by imposing a Tobin tax which ensures that no attack will be optimal. The debate on taxes in an exchange rate mechanism has been raised by a number of economists. It is regarded by some as contradicting the spirit of a united Europe. However, it is also noted (Eichengreen and Wyplosz (1993)) that while taxes represent a second best solution, they also represent a plausible way of achieving monetary union in Europe in which currencies are defended from speculative attacks.
Chapter 5

Informational Cascades as an Explanation For a Currency Crisis

In the last chapter, I was concerned with informational events which trigger currency crises. In this chapter, I show how informational cascades can develop in models where individuals move sequentially. These cascades can help to explain the timing of the crises.

An informational cascade takes place when an individual observes the actions which others have taken ahead of him and uses this information to formulate his own decision. In so doing, he abandons or pays little attention to his own private information. It can be shown that individuals converge on a particular action on the basis of very little information. Once an individual ignores his own information in favour of the previous actions of others, his decision has little informational value to the subsequent players.

It has been argued that this is the case in many financial markets. Cascades develop when depositors view other depositors' withdrawal behaviour and also withdraw their funds since they believe that the bank will become insolvent (Diamond and Dybvig, 1983). Banerjee (1992) also notes that asset markets display 'excess volatility' and this may be due to the herding behaviour of investors. More recently Chari and Kehoe (1997) discuss the role...
of herd behaviour in a debt-default setting. I apply the standard model of informational cascades developed by Bikhchandani, Hirschleifer and Welch to speculative behaviour in a foreign exchange market.

In this chapter, I review the extensive literature in the area of informational cascades and herding. I then set up the framework of Bikhchandani, Hirschleifer and Welch. Having established the model, it is easy to show firstly how cascades occur in a foreign exchange market. Also of importance is the likelihood of incorrect cascades emerging. In this scenario, initial investors observe an incorrect private signal and the subsequent investors follow suit. This has dire implications for a currency since even a strong government may find itself forced to devalue if a run is generated on sterling. Furthermore, it can be shown that despite the fact that a number of people converge on the same action, the ‘depth’ of the cascades does not increase. Once a cascade has developed, further adoptions of the same action become less informative to subsequent investors. This implies that cascades are ‘brittle’ to use Bikhchandani, Hirschleifer and Welch’s terminology since the arrival of some new public information can reverse a cascade.

I then describe the experiment of Anderson and Holt (1997) which tests this model. They find that in the twelve sessions, cascades developed in 87 of the 122 periods in which they were possible. Of most interest, however, was the tendency for subjects to use a simple counting rule of actions rather than to behave according to Bayes’ rule. Their results were of particular relevance to my thesis since they form the basis for the experiment which I set up in chapter 6.

5.1 Literature Survey

In this section, I focus on the literature which relates herding behaviour directly to financial markets. One such paper is that of Avery and Zemsky (1995) which argues that if prices aggregate public information, cascades can be avoided. This is based on the asset market model of Glosten and Milgrom (1985) and assumes that a market maker sets bid and ask prices equal to the expected value of the asset in question conditional on all the publicly
available information. Agents buy or sell a unit of the asset at the posted prices. There are two types of trader. The first buys or sells for liquidity purposes and is termed a noise trader. The second is an informed trader since he has some private information which he wishes to exploit for profit making reasons. The market maker must infer the information available to the informed traders. However, the informed traders have an advantage over him due to the actions of the noise traders. Eventually, the market maker will be able to tell whether his price is too high or too low from observing the trades occurring on one side of the market or another. He will then adjust it accordingly. The authors conclude that the inclusion of price as a continuous variable has the effect of aggregating public information so as to avoid an informational cascade.

However, Gale (1995) notes that this is not a critical observation for the theory. In Avery and Zemsky’s model, there are no gains from trade. Informed traders buy and sell since the value they place on the asset differs from that of the market maker. They will trade whatever the size of this difference. Furthermore, they will reveal some of their own private information irrespective of how much information has already been revealed. Gale notes that if traders had other motives such as hedging, the analysis would be different.

Scharfstein and Stein (1990) also examine herd behaviour with regard to investment. However, their paper is more concerned with reputational cascades. Each agent receives a signal concerning the value of a number of options but this signal may or may not be informative. The task of the individual is to maximise the probability that an outside agent will put on the chance that he is an informed agent. In other words, he wants to maximise his reputation. It is assumed that informed agents receive signals which have correlated errors. Therefore, it follows that subsequent agents maximise their appearance of being an informed agent by taking the same action as their predecessors. The crucial difference between this paper and those such as

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1A further example of reputational cascades is given by Trueman (1994) and also Froot et al. (1992) In this paper, agents accumulate information about what other agents know rather than about the fundamental in question.
Banerjee (1992) is that agents are rewarded for convincing a principal that they are correct. Thus there is a distortion in incentives.

The work of Gul and Lundholm (1995) demonstrates how agents’ decisions may become clustered together and have the appearance of an informational cascade. However, in this instance, information is being used efficiently. In their model, each of the two agents wants to predict the future value of a project and wishes to do this sooner rather than later. The value of the project is the sum of two independent random variables drawn from a uniform distribution between 0 and 1. Each agent receives some private information concerning the value of this project. More specifically, he receives the realisation of one of the random variables. The utility function for each agent shows a trade off between the cost of an error in the agent’s prediction and the cost of delaying the decision in order to get a more accurate prediction.

Gul and Lundholm show that a unique symmetric Nash equilibrium exists. They also note that the utility of each agent does not depend on anyone else’s perception of him. Therefore, there is no incentive to copy the behaviour of another agent. However, clustering of decisions still occurs. This is because the squared difference in agents’ predictions when forecasts are endogenously ordered has a smaller expectation than the squared difference when forecasts are exogenously ordered. The authors note that this is useful since it enables us to distinguish between a cascade and a clustering of decisions. The sign of a cascade is that the accuracy of decision making does not improve over time. However, when there is clustering, accuracy will improve since the second agent will know the previous signal and also that this exceeds his own. The authors note that, in effect, information has leaked due to the trade off between accuracy and delay and this generates clustering.

Vives (1995) also has some criticisms of the literature on herd behaviour on information cascades. He notes that the literature on herd behaviour focuses on market failure as a source of herding. However, the rational expectations literature stresses the role of the market mechanism as an aggregator of the information of agents. In his paper he presents a survey of some of the social learning models and examines the extent to which these cast a doubt

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on the workings of the market mechanism. In particular, he examines the speed at which full information equilibrium is reached and also the relevance of costly information.

He notes that there are two important assumptions underlying the basic herding models. Firstly, the models assume a discrete action space. If the action space were continuous and the reward received by the agents varied with their proximity to the true action, the actions would converge. As it stands, he argues that agents mimic the actions of others since they cannot 'fine tune' their actions according to the information. His second point is that the signals in these models have to be of bounded precision. If signals had unbounded precision, an incorrect herd could be terminated by one agent who has a signal which is sufficiently informative.

For these reasons, Vives argues that the models presented are not sufficiently robust. He argues that they do not correctly model fads, fashion or decision making. He also stresses that, as yet, none have attempted to model agents as acting simultaneously. For the reasons described above, he develops a model which includes smooth and noisy learning from others.

His main result concerns two properties of learning from others. Firstly, he shows that public precision accumulates over time but at a slow rate. He describes this as the self correcting policy of learning from others. However, he shows that a higher precision of public information induces a low response to private information which, in turn, gives a smaller increase in public precision. This is termed the self defeating property. According to his theory, herd behaviour occurs when the self defeating property is large and so the private signals of subjects are overwhelmed by public information.

Gale (1995) also draws together a considerable amount of the literature on social learning. Firstly, he sets up a basic model to describe the features of informational cascades. He then proceeds in examining the robustness of herd behaviour. He looks at the nature of the signals received and notes that, in a considerable number of models, there are only two signals. This implies that a cascade may be easily generated and the only way of breaking it would be for an individual to have a larger or better signal.

He stresses the importance of models such as the one by Gul and Lund-
holm (1995) in which the preferences for agents are defined over a continuous variable. It follows that information will eventually be fully revealed.

In examining endogenous sequencing, he notes that the timing of agents' decisions is assumed to be exogenous in much of the literature. However, if agents were allowed to choose their position in the queue, most would want to go last. Again he refers to Gul and Lundholm (1995) and also the work of Chamley and Gale (1994) since these papers study the importance of strategic delay. As a final point, he notes the type of equilibria encountered in the literature. In particular, the choice of symmetric or asymmetric equilibria makes a great difference to the result of the model.

He concludes that herd behaviour is robust but only subject to certain qualifications. Furthermore, he stresses the importance of building on the work of Vives in modelling the dynamics of information revelation.

Finally, I shall examine the recent literature which has evolved from the earlier work on herding behaviour. Firstly, there is the model by Chari and Kehoe (1997). This applies herding to a standard debt-default setting. Within this setting, fiscal crises and frictions in international markets can generate excess volatility of capital flows. The frictions in the financial market imply that information is not transmitted efficiently across investors. This, in turn, generates herding behaviour among investors which leads to debt-default problems for borrowing countries. This model also tackles the issue of reputation since these crises act as a test of the government's commitment to its fiscal responsibility. If it survives such a crisis, its reputation improves and future capital flows become more stable. However, if it fails, its reputation declines as does future foreign investment.

The framework is based on that of Banerjee (1992) and also Bikhchandani et al. (1992) since there are $N$ lenders with a choice of committing to investment in either a domestic or borrowing country. The project in the borrowing country will only go ahead if a sufficient number of investors commit to its funding. If fewer than the required amount agree to invest, then the project

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2 Chamley and Gale develop a framework similar to that of Bikhchandani et al. However, agents choose their timing of actions strategically. Although they find that cascades do occur, these results rely on the finiteness of the choice set for agents.
is cancelled and the investors can commit to domestic investment. As in the aforementioned papers, the ordering of actions is exogenous and each investor receives a piece of private information concerning the state of the economy. Thus, he will base his decision on his private information, his prior views as to the competency of the government and also on the history of actions taken by his predecessors.

Again, this is a paper which illustrates that herding occurs as a result of market failure. In this circumstance, inefficient aggregation of information results in frictions in the financial markets which in turn, generates fiscal crises. Chari and Kehoe note that herding would not occur in this model if investors made their decisions simultaneously rather than sequentially. This contrasts with the work of Cole and Kehoe (1996) in which crises occur because agents move simultaneously.

Lee (1997) also explains booms and crashes in financial markets as failure in the aggregation of information. However, his model introduces a dynamic element into the literature. He defines market crashes as evolving through four basic stages. Firstly, there is the boom phase followed by a period of euphoria which is modelled as an information cascade. Following this there is a trigger from which panic sets in. This panic is modelled as what he terms, an information avalanche.

The inefficiency in the aggregation of information occurs because traders place a larger weight on the history of previous prices in their decision making i.e. they are putting a greater emphasis on this public information rather than their own signal. It follows that a trader with a signal containing bad news will tend to discount this when a number of his counterparts have good news signals. He will instead make an investment choice closer to that of the other traders. Lee notes that if this is combined with a transaction cost, the actions of this speculator may become indistinguishable from the others. Therefore, the price will remain high for this asset. This is based on the notion of informational cascades as in the earlier literature.

However, when there are a large number of these hidden bad news traders, a market crash can be imminent. Since they all have bad news as their private information, a trigger that implies that the true state is the bad
one can provoke a crash. This information avalanche is based on the notion developed by Bikhchandani et al. which suggests that cascades are fragile and thus may be reversed. This will be discussed in greater detail later in the chapter.

The result of Lee's paper is to demonstrate how a large change in the price of an asset can occur without there having to be a substantial piece of news. Furthermore, small errors due to information-generated frictions in the financial markets can lead to large errors in terms of the price of the asset.

\section{Model}

In the Bikhchandani, Hirschleifer and Welch framework, it is assumed that individuals move sequentially and that the order in which they move is exogenous. In terms of a foreign exchange market, the model can be interpreted as follows.

The task for each investor is to decide whether to retain his funds in sterling or whether to move out of sterling and into a different currency. For simplicity, I shall use the same letters as Bikhchandani, Hirschleifer and Welch to denote each of the variables. If the investor does retain his Pounds, he faces a fixed known cost of $C = \frac{1}{2}$. The gain from doing so is also the same for each individual and is denoted by $V$. This takes on the value of 0 or 1 with probability of $\frac{1}{2}$ and is determined by government type. If a government is weak then the value from remaining in sterling will be zero since a weak government is more likely to devalue its currency and leave the exchange rate system. Conversely, a strong government is more likely to remain in the system and thus the value for the investors from remaining in sterling is 1.

Each investor receives a private signal concerning $V$. This signal takes one of two forms. If $V = 1$ he receives signal, $H$ with probability $p_i > \frac{1}{2}$ and signal, $L$ with probability, $1 - p_i$. However, if $V = 0$ he receives the signal, $H$ with probability $1 - p_i$ and signal, $L$ with probability $p_i$. Furthermore, it is assumed that signals are identically distributed so that $p_i = p$ for all $i$. If an investor is indifferent between retaining sterling or moving from the
currency, he will toss a coin to determine his action and thus, the probability of remaining in Pounds will be $\frac{1}{2}$.

It follows that the first individual will remain in Pounds if he receives signal $H$, but will abandon the currency if he receives $L$. The second individual may view the action of the first investor and can use this information in addition to his own signal. If the first individual has remained in Pounds then the second investor will also retain sterling if he receives a signal, $H$. However, if he receives signal $L$, he will be indifferent between retaining sterling and abandoning the currency. This is because the probability that the first investor receives the correct signal is the same as that of the second investor. Therefore, he will toss a coin to decide the outcome.

There are three possible scenarios for the third investor. Firstly, individuals one and two may both have abandoned the currency. Secondly, the first two individuals may have each retained sterling. Finally, one may have abandoned the currency while the other has retained sterling. The third investor then receives his own private signal. In the first two scenarios, he will follow the actions of his predecessors. This is optimal for him since even if he receives a signal contrary to the actions of the other players, there is still a higher probability that they are correct and that his signal is incorrect. In the third scenario, the investor is in the same situation as the first individual to move. This is because his own signal determines his choice.

It is now possible to see a pattern emerging. As in the Bikhchandani, Hirschleifer and Welch paper, the ex ante probabilities of a cascade retaining sterling, abandoning sterling or no cascade at all after two investors have moved are given by:

$$\frac{1 - p + p^2}{2}, p - p^2, \frac{1 - p + p^2}{2}$$

After $n$ investors have moved where $n$ is an even number, these probabilities become:
\[
\frac{1 - (p - p^2)^{\frac{3}{2}}}{2}, \frac{p - p^2}{2}, \frac{1 - (p - p^2)^{\frac{3}{2}}}{2} \quad (5.1)
\]

Clearly, if \( p \) is allowed to vary, then the larger it becomes, the sooner a cascade is likely to set in. It is also possible to calculate the probabilities of the above cascades setting in given that the true value equals 1. After two investors have moved, these become:

\[
\frac{p(p + 1)}{2}, p(1 - p), \frac{(p - 2)(p - 1)}{2} \quad (5.2)
\]

Again, for an even number, \( n \) investors, this is:

\[
\frac{p(p + 1)}{2} \left[ 1 - (p - p^2)^{\frac{3}{2}} \right], (p - p^2)^{\frac{3}{2}}, \frac{(p - 2)(p - 1)}{2} \left[ 1 - (p - p^2)^{\frac{3}{2}} \right] \quad (5.3)
\]

Even for large values of \( p \), the likelihood of an incorrect cascade (the third term in (5.2)) is still very high. The reasoning behind this is that investors view the actions of previous investors but not the private signal received by these individuals. The implication is that when a cascade starts, no further information about the private signal received is conveyed. Thus actions do not improve the decision making of investors who move later on.

While this is an important result, the model can be generalised to demonstrate how a cascade may be fragile in that it is vulnerable to the release of some new public information. Again, Bikhchandani, Hirschleifer and Welch's model can be used to demonstrate the situation in a foreign exchange market.

There are a number of investors denoted \( i = 1, 2, \ldots, n, \ldots \), each faced with a decision of whether to retain their funds in sterling or switch to another currency. The cost of remaining in sterling is again equal to \( C \). However, the government type is no longer just equal to 0 or 1. It now has a finite set of possibilities where \( v_1 < v_2 < \ldots < v_s \). Furthermore, the cost of remaining in sterling falls within this interval so that \( v_1 < C < v_s \). The prior probability
that \( V = v_l \) is denoted by \( \mu_l \).

In this generalisation, there are more than two possible signals which the investors may receive. The signals are conditionally independent and identically distributed. They take on the values, \( x_1 < x_2 < x_3 < \ldots < x_R \). The probability that an investor receives signal, \( x_q \) when the true government type is \( v_l \) is \( p_{ql} \). The corresponding cumulative distribution of the signals given \( V = v_l \) is given by:

\[
P_{ql} \equiv Pr (X_i \leq x_q | V = v_l) = \sum_{j=1}^{q} p_{jl}
\]

\( J_i \) denotes the set of signal realisations which induce the investor to retain his funds in sterling.

A cascade will occur when an investor’s action is not a product of his own signal but of the actions of others. His decision to retain or reject sterling is based on the following.

The \( i \)th investor observes the history of actions adopted by his predecessors denoted by \( A_i = (a_1, a_2, \ldots, a_i) \). It is assumed that \( J_i (A_i-1, a_i) \) is the set of signal realisations that induce this investor to choose action \( a_i \) for the history \( A_i-1 \). It follows that an individual \( n+1 \) who has a signal \( x_q \) and a history of signals, \( A_n \) will have a conditional expectation of government type given by:

\[
V_{n+1} (x_q, A_n) \equiv E [V | X_{n+1} = x_q, X_i \in J_i (A_i-1, a_i), \text{ for all } i \leq n]
\]

He will remain in sterling if the value in doing so exceeds the cost i.e. if government type is sufficiently large so that \( V_{n+1} (x_q; A_n) \geq C \). The assumption here from the action of investor, \( n+1 \) is that \( X_{n+1} \in J_{n+1} (A_n, a_{n+1}) \). This implies that he will remain in sterling if:

\[
J_{n+1} (A_n, \text{remain}) = \{ x_q \text{ such that } V_{n+1} (x_q; A_n) \geq C \}
\]
and will switch currency if:

\[ J_{n+1}(A_n, \text{switch}) = \{x_q \text{ such that } V_{n+1}(x_q; A_n) < C \} \]

To complete the model, Bikhchandani, Hirschleifer and Welch impose two conditions. Firstly, it is assumed that the conditional distributions \( Pr(X_i | V = v_i) \) are ordered by the monotone likelihood ratio property. This implies that if an individual receives a high signal realisation he will conclude that the government type is high. Furthermore, this means that the conditional expectation for each investor increases in his signal realisation.

Secondly, it is assumed that if an investor can learn a sufficient amount about government type by observing his predecessors, he will no longer be indifferent between retaining and rejecting sterling. This implies that a cascade will inevitably set in.

However, Bikhchandani, Hirschleifer and Welch show that once a cascade has set in, it need not persist if new public information is revealed. It has already been argued that once a cascade has started, actions taken by future investors contain no new information regarding government type. It follows that a public signal will make him more informed without reducing the information conveyed to him through the actions of his predecessors.

Furthermore, a small amount of public information can break a cascade even when this signal is less informative than the private signal of the individual investor. This is best illustrated by using an example. Consider the decision of the third investor. The two predecessors may both have chosen to abandon sterling. This information tells him that either they both received an \( L \) signal or that the first investor received an \( L \) while the second received an \( H \) but tossed a coin. In the absence of public information, the investor will be induced to abandon sterling whatever his private information. However, just one public signal of \( H \) is enough to make him pay attention to his own signal. This new information now induces him to retain sterling if he receives an \( H \) signal or abandon sterling if he receives an \( L \). This has the effect of shattering a cascade even if the investor moves late in the sequence.
5.3 Policy Implications

In terms of the foreign exchange market, this model has some important implications. Firstly, there is the inevitability that a cascade will set in. This was seen in the generalised model. This implies that whatever form a future exchange rate system would take it would still be dogged by cascades. More significantly, is the likelihood that these cascades would be in the wrong direction. This would suggest that even a strong government with a substantial commitment to maintaining the parity, may find itself having to defend speculative attacks on sterling.

However, it has been shown that a cascade is very fragile in that a small piece of public information may have the effect of reversing it. This may be seen as an advantage in that an incorrect cascade may be reversed through the disclosure of public information regarding the government type. This may take the form of a government pledge to remain in the ERM and defend the parity. However, it is also possible that a correct cascade may be broken by a piece of public information. In terms of the foreign exchange market this may for instance be an unfavourable result in a referendum.

The speed with which cascades set in and also their vulnerability to public information has the overall effect of making such markets tremendously volatile. This feature of exchange markets has already been remarked upon by a number of economists.

5.4 Experiment

The model by Bikhchandani, Hirschleifer and Welch has been tested by Anderson and Holt using experimental techniques. Experimental economics provides a very successful way of evaluating such a model. In particular, it allows them to analyse the degree to which subjects deviate from the model and examine the strategy employed by subjects when they do not follow the Bayesian approach. This is a particularly valuable contribution since there has been a growing interest in cascades and their application to financial markets as shown in the literature survey presented earlier in the chapter.
It is also relevant to this thesis since I carry out my own experiment in the following chapter testing the assumptions of the Banerjee framework.

5.4.1 Experimental Design

It is assumed that an individual observes a private signal which reveals some information concerning the occurrence of one of two equally likely events. These events are denoted by $A$ and $B$. Each individual receives a signal which is either $a$ or $b$.

Each individual selects from one of two urns. These look identical but their contents differ. In urn $A$, two of the three balls are labelled $a$. The third ball is labelled $b$. In urn $B$, two of the balls are labelled $b$ and one is labelled $a$. A die is thrown to determine the urn that was used in each period.

Clearly, the posterior probability of event $A$ given that the individual has selected a ball labelled $a$ is $\frac{2}{3}$. Similarly, the posterior probability of event $B$ given that the individual has drawn a ball labelled $b$ is also $\frac{2}{3}$. The order in which individuals move is exogenous and each views the decision of his predecessors but, significantly, not the signals which these people received.

If each individual makes his decision assuming that everyone else is behaving according to Bayes’ rule, then he will use the following to calculate the posterior probability of events $A$ and $B$:

$$Pr(A \mid n, m) = \frac{Pr(n, m \mid A)Pr(A)}{Pr(n, m \mid A)Pr(A) + Pr(n, m \mid B)Pr(B)}$$

$$= \frac{\left(\frac{2}{3}\right)^n \left(\frac{1}{3}\right)^m \left(\frac{1}{2}\right)}{\left(\frac{2}{3}\right)^n \left(\frac{1}{3}\right)^m \left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)^n \left(\frac{2}{3}\right)^m \left(\frac{1}{2}\right)}$$

$$= \frac{2^n}{2^n + 2^m}$$

where $n$ is the number of $a$ signals and $m$, the number of $b$ signals. It is now possible to calculate the posterior probabilities of each event for any combination of $a$ and $b$ signals.
5.4.2 Experimental Procedure

There were 12 sessions in all with 6 subjects used in each session. Each session consisted of 15 periods. In each period, a die was thrown to establish which urn was to be used. The subjects were partitioned off from each other to prevent communication and each was approached at random and asked to select a ball from the urn. They were then asked to report their decision, A or B. This information was then conveyed to the other participants. Once all participants had made their choice the correct urn was revealed and the participants recorded their earnings. They each received a participation fee upon arrival and the subsequent payout had been calculated so that earnings were on average $20. The subjects were paid privately in cash when they were released from the experiment. This procedure was carried out for 3 of the sessions. In a further 3 sessions, Anderson and Holt included public information. In the remaining 6 sessions, they considered asymmetry in terms of the contents of the urns.

5.4.3 Results

Anderson and Holt show that an informational cascade can occur when an individual receives a signal which is contrary to the signals he believes are received by his predecessors. They find that in total there are 56 periods in which there is the potential for a cascade. Cascades occur in 41 of these 56 periods. In order to examine the data more closely, they consider the efficiency of their subjects' decisions together with observed biases in information processing. Furthermore, they consider the extent to which subjects make their decisions based on the number of signals in each direction. I shall summarise these results and then discuss the implications in terms of a foreign exchange market.

Efficiency

Anderson and Holt use the expected earnings from behaving according to different strategies to arrive at a method for measuring how well individuals
use their information. Payoffs are defined as follows. The optimal expected payoff is that which will be earned if the individual uses Bayes rule to establish his decision. The random choice expected payoff is that which is earned if the subject chooses randomly. The private information payoff is earned if the individual bases his decision on his private information alone. Finally, the actual expected payoff is the expected earnings for the individual's actual decision.

These payoffs are then used to derive how efficiently individuals use the information available to them. Actual efficiency is the difference between the actual expected payoff and the random choice payoff as a percentage of the difference between the optimal expected payoff and the random choice payoff i.e.

$$\text{actual efficiency} = \frac{100 (\pi_A - \pi_R)}{(\pi_O - \pi_R)}$$

where $\pi_O, \pi_R, \pi_p, \text{and} \pi_A$ refer respectively to the sum of the expected payoffs for all 15 periods for optimal, random choice, private information and actual expected payoffs. Private information efficiency is calculated as follows:

$$\text{private information efficiency} = \frac{100 (\pi_p - \pi_R)}{(\pi_O - \pi_R)}$$

Anderson and Holt found that over all subjects the actual efficiency averaged at 91.4%, while private information efficiency averaged at 72.1%. Significantly, two thirds of the participants attained actual efficiencies of 100% implying that they behaved according to Bayes’ rule. Furthermore, two thirds of the remaining subjects performed better than they would have done if they had used only their private information.

Biases

Anderson and Holt examined the data for evidence that there was a systematic bias in favour of following the decision of the previous subject. They
analysed the 6 sessions for which the content of the urns was symmetrical. They found that there were 68 cases in which the Bayes distribution was $\frac{1}{2}$ and the private signal received by the subject differed from the previous decision. In 57 out of the 68 cases, the subject did not follow the previous decision but, instead his behaviour was consistent with his own private information. Therefore, it would appear that there was little or no bias in favour of the actions of the previous subject.

They also examined bias in terms of 'representativeness'. This has been discussed by Grether (1980 and 1992) and suggests that subjects have a tendency to underweight prior probabilities and pay more attention to the comparison between their own sample and that of a particular population. Anderson and Holt introduce public draws in sessions 4 and 5 and this makes representativeness possible. The fifth and sixth subjects see two public draws plus their own private signal. Before this information is conveyed to them, the subjects form prior probabilities based on the decisions of their predecessors. They are then able to form posterior probabilities when the public information and their own private signal are made known to them.

Anderson and Holt found little evidence for representativeness since the subjects' behaviour was consistent with Bayes' rule rather than information conveyed from the public draws.

Do Subjects Use Simple Counting or Bayesian Decision Making?

Anderson and Holt set up 6 sessions in which the contents of the two urns is asymmetrical. The reason behind this is that when the contents are symmetrical, the optimal Bayesian decision is to predict the urn which has the highest number of inferred signals. In order to distinguish between simple counting and Bayesian behaviour, the contents must be asymmetrical. Thus, for these sessions, urn $A$ contains 6 $a$ balls and 1 $b$ ball whereas urn $B$ contains 5 $a$ balls and 2 $b$ balls. As before, the urns are equally likely to be chosen. The difference now is that merely counting the inferred signals will not necessarily provide the correct Bayesian decision.

In these sessions, cascades occurred in 46 out of the 66 periods in which
they were possible. Reverse cascades were higher in this set up. The effect of counting reduced the Bayesian cascades from 73% to 70%. Anderson and Holt found that when Bayes’ rule predicted one action and counting predicted a different action, the subjects made a Bayesian decision approximately 50% of the time. Over all 6 sessions, 115 out of the 540 decisions were inconsistent with Bayesian decision making. Over a third of these inconsistencies were due to counting.

The authors also reported figures for the expected gains and losses from applying different decision rules. They found that across the subjects in these 6 sessions, the actual efficiency was 67.6% while the private efficiency was 45.2%. These are both lower than in the original case. Furthermore, they define another form of efficiency. Counting efficiency is the percentage of expected gains over random decision making of using a counting rule. Algebraically this is denoted by:

\[
\text{counting efficiency} = \frac{100(\pi_C - \pi_R)}{(\pi_O - \pi_R)}
\]

where \(\pi_C\) is the expected gains from counting. Of the 36 subjects in the asymmetric design sessions, 21 performed better than counting since their actual efficiencies were greater than the counting efficiencies. However, across all subjects the counting and actual efficiencies were approximately equal. The reasoning behind this lay in the fact that a few subjects experienced low expected payoffs since they did not conform to either the counting or Bayesian rules.

### 5.5 Implications for a Foreign Exchange Market

Anderson and Holt found that across all 12 sessions, cascades occurred in 87 out of the 122 periods in which they were possible and where the incentive to follow the crowd was minimised. They found that subjects followed other
subjects' actions when it was rational to do so. In terms of a foreign exchange market this implies that cascades are to be expected since it proves to be rational to follow the crowd under certain circumstances. It is particularly disturbing that incorrect cascades were so prevalent. There were approximately half as many as correct cascades. This implies that a run can occur on a currency even when a government is committed to maintaining an exchange rate parity.

Anderson and Holt's public information results are also a concern. They found that there was little evidence to support the idea of 'representativeness'. Hence this suggests that subjects did not place a large weight on public information when forming their decisions. This implies that a public announcement stating a government's commitment to maintaining a parity would not be sufficient to deter a run on the currency.

5.6 Conclusion

Clearly, there a number of drawbacks with this model. One criticism which could be made is that individuals move sequentially and that in a foreign exchange market, this is not typically the case. Investors tend to move simultaneously. Furthermore, the order in which they move is not an exogenous process as explained by the model but may be determined by the amplitude of the signal. One may also argue that the choice of participants in the experiment was inappropriate since the behaviour of students could be very different from that of investors.

Given these limitations however, the experiment still shows that cascades occur as the theory predicts. However, the introduction of public information into the setting did not have the predicted effect in the experiment of reversing a cascade.

In terms of future research into this area, I consider two different avenues. Firstly, in the Anderson and Holt experiment, the individuals receive one of two signals. In reality there may be a large number of signals and a large number of actions which the individuals may be able to follow. I perform an experiment on a related paper by Banerjee (1992) in which there are a
large number of signals and actions. In addition, he considers the scenario
in which investors may not receive a signal at all. In this situation, their
actions depend crucially on the behaviour of their predecessors.

As a second possible way forward, it would be a valuable, although log-
istically tough, exercise to set up a stock exchange environment. My aim
here would be to allow subjects to move as and when they chose rather than
sequentially. It would also be possible to monitor their reactions to private
and public signals. I feel that this would come closer to testing speculative
behaviour. However, this is not attempted within this thesis.
Chapter 6

Two Experiments to Test a Model of Herd Behaviour

6.1 Introduction.

In chapters 2-4 of this thesis, I modelled particular features of currency crises. The currency crises of 1992 saw the UK and Italy leave the exchange rate mechanism and a number of currencies devalue. The Bank of England intervened in support of the currency earlier in the summer of 1992 yet speculators continued to sell their holdings of the Pound. It can be argued that herding or an informational cascade may be responsible for the ERM collapse.

There has been a considerable literature in the field of herd behaviour and informational cascades which has been used to explain certain behaviour in financial markets. Froot et.al (1992) and also Scharfstein and Stein (1992) provide examples. However, in terms of experimental economics this has remained largely untested. This represents a very significant omission since experimental work can be used to analyse the dynamics of speculative behaviour in foreign exchange markets.

In chapter 5, I discussed how the model developed by Bikhchandani, Hirschleifer and Welch (1992) can apply to a foreign exchange market. This was tested by Anderson and Holt (1997) who find that information cascades occur in 85 of the 117 periods even when incentives are provided to deter
herding. This would suggest a strong tendency to follow the crowd and ignore one's own information. However, while this is a valuable result, we feel that our experiment takes the analysis one step further. Unlike the Bikhchandani, Hirschleifer and Welch paper, the model that we test includes the possibility that people do not receive a signal. Furthermore, the winning option is drawn from a set of options given by a line segment. In the Bikhchandani, Hirschleifer and Welch model, the subjects face a simple choice between two possible outcomes. Therefore, it is argued that our experiment is a generalisation of that performed by Anderson and Holt.

The experiment is based on the model developed by Banerjee (1992). We have focused on this model for two reasons. Firstly, it provides a cornerstone to a great deal of the more recent literature in informational cascades. Secondly, we argue that, to an extent, it can be applied to a foreign exchange market. Banerjee shows that when people observe the actions of others and attempt to use this information in their own decision rule, each person's decision becomes less informative to other people. Following the optimal strategy implies that, firstly, the equilibrium can be inefficient in terms of welfare. Secondly, the probability that no one in the population chooses the winning option may be large. Finally, the pattern of decisions over a number of plays of the game are volatile. This is because the onset of the herd and the direction it takes will depend upon the signal received by the first few people.

Banerjee's model presents some very powerful results. Firstly, the probabilities of receiving a signal and of that signal being correct do not influence the optimal strategy. This seems counter intuitive since one would assume that these probabilities would play a role in an individual's choice of action. Secondly, individuals are privately optimising. However, the result is socially suboptimal. Thirdly, herds set in at an early stage in this model. If the first two individuals follow the same course of action then a herd cannot be broken. Finally, the probability of an incorrect herd may be high. This does depend on the values of receiving a signal and of that signal being correct.

In addition to performing two experiments to test the validity of this model, we stress an important theoretical point. Our investigation reveals
that a particular assumption of his model which he claims merely reduces
the probability of herding is crucial to the solution of the model. The Baner-
jee strategy produces an elegant result in which the decision rule remains
the same regardless of (a) an individual’s position in the sequence, (b) his
probability of receiving a signal and (c) the probability of that signal being
correct. We show that the removal of this seemingly innocuous assumption
generates an optimal decision rule differs according to each of these factors.

The aim of this chapter is to test the null hypothesis that individuals be-
have according to the optimal strategy given by Banerjee. This is important
since if they do behave in this manner, this implies a large degree of herding.
This is particularly relevant to markets of foreign exchange. The chapter is
set out as follows. Firstly, we set up Banerjee’s model showing the optimal
strategy. We then illustrate the importance of the assumption in the con-
text of his model. In order to do this, we show how the signals received by
the players generate a particular set of observations under the model with
the assumption. When this assumption is removed, the same combination
of signals generates a different set of observations. It thus follows that an
individual’s action differs according to whether the assumption is made. We
then simulate the actions of the individuals when the Banerjee rule is used
both with and without the assumption in question. We use this simulation to
arrive at an appropriate design and procedure for the experiment. We then
analyse the results from each of the two experiments and draw conclusions.
Finally, we suggest promising future lines of research.

6.2 Model.

The model has a relatively simple framework in which there is a set of options
given by a line segment. Within this set, there is one correct option. Subjects
must find the winning option and thereby gain an award, \( z \). All other subjects
receive a payment of 0. The population consists of \( N \) people who move
sequentially. The order in which they move is determined exogenously. If
an individual is informed, he receives a signal. This occurs with probability
\( \alpha \). However, this signal is only correct with probability \( \beta \). If an individual
is uninformed, he does not receive a signal. To solve the model, Banerjee analyses a ‘Bayesian-Nash equilibrium’ since he finds that, for this setting, the equilibrium decision rule holds for all parameter values.

Banerjee introduces three assumptions into the game. These are as follows:

Assumption A - If a player has received no signal and all other previous players have chosen 0, he must also choose 0.

Assumption B - If a player is indifferent between following his own signal and another player’s choice, he will follow his own signal.

Assumption C - If a player is indifferent between following more than one of the previous players, he will follow the one with the highest signal.

In terms of an experiment, B and C are difficult to impose since it is not easy to establish individuals’ indifference between options. Therefore, these have not been included in our experiment. In our first experiment, we include assumption A while in the second experiment, assumption A is dropped and subjects are allowed to make a guess at the winning option.

Given assumptions A, B and C, Banerjee offers a solution to the model in the form of an optimal decision rule for each individual. This optimal strategy is adopted by each individual irrespective of their order of play. This is a particularly interesting point since the optimal strategy is the same for each player despite the fact that they move sequentially. This rule is illustrated in the figure over the page reproduced from Banerjee’s paper.

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1 This is Banerjee’s terminology. However, we shall henceforth refer to this as the ‘optimal strategy’ since the game is a sequential one. Individuals base their decisions on the actions of previous players plus their own signal.

2 He states that ‘The nature of the equilibrium play, however, turns out to depend on certain critical assumptions. Some of these assumptions may be dispensed with by strengthening the equilibrium concept, but it seems more natural to introduce them as explicit assumptions.....the relevance of these assumptions will become clear in the appropriate context. It should also be possible to see that each of the assumptions is made to minimise the possibility of herding.’
Figure 6.1: The kth Decision Maker's Choice Problem ($k > 2$)
The first player follows his own signal if he receives one. If he does not get a signal, he is obliged by assumption A to choose $i = 0$. The subsequent individuals will adopt the following rules. They will follow their own signals either if and only if:

(i) the signal matches that of another player or if this does not hold

(ii) no option has been chosen by more than one person apart from $i = 0$.

If a player receives a signal which does not match the action of a previous player, he will choose the option chosen by more than one of the individuals. If the option with the highest value of $i$ has been chosen by more than one person, he will choose this option providing that no other option has been chosen by more than one person and no one else’s choice matches his own signal.

If the player does not receive a signal, then he will choose $i = 0$, due to assumption A, if everyone else has chosen this. If some option has been chosen by more than one person, then he will also choose this. However, if no option has been chosen more frequently than any of the others, he will choose the one with the highest value of $i$.

A crucial point to note here is that, according to his rule, the decision rule forming the optimal strategy holds irrespective of the values of $\alpha$ and $\beta$. If Assumption A is removed, this feature no longer holds. Furthermore, the results of the experiments revealed that our subjects appeared to use these values in their decision rule and modified their behaviour accordingly.

### 6.3 Importance of Assumption A.

The crucial difference between the model including assumption A and the model omitting assumption A is that the same sequence of signals will generate different sequences of observations. In discussing the experiment, we refer to assumption A as Rule A. The reason is that in the first experiment, this assumption is imposed whereas in the second experiment it is omitted.
A player bases his action on the signal he receives and the actions of the previous players. Thus, his own action depends on whether the rule is imposed or omitted. We demonstrate this through decision trees and a table which analyse the possible decisions facing player 3 in the sequence. The decision tree, figure 6.2, shows the combinations of signals which may produce a particular set of observations when assumption A is included. Figure 6.3 gives the decision tree when this assumption is omitted.

In table 6.1, we have shown each possible combination of signals received by each of the three players. These are denoted by bold type and are given in the first three columns.

The signals of players 1 and 2 generate actions which player 3 observes. Notably, he observes their actions but not the signals which they have received. Columns 4 and 5 illustrate the actions of players 1 and 2 when assumption A is included. The sixth column indicates the relevant branch (or branches) in the decision tree of figure 6.2. Each of these points in the tree represents a combination of signals which could have produced the observed actions of players 1 and 2. The seventh column shows player 3’s optimal action given the signal he has received plus the observations he makes of previous players’ decisions. Actions of players are denoted by italics.

Columns 8 and 9 give the actions of players 1 and 2 when assumption A is omitted. The tenth column represents the corresponding branch (or branches) in the decision tree of figure 6.3. Again, these points represent possible combinations of signals producing the observed actions of players 1 and 2. Player 3 bases his decision on his observations and his own signal if he receives one. The result is seen in the eleventh column.
<table>
<thead>
<tr>
<th>Signals Received</th>
<th>Actions With Assumption A</th>
<th>Corresponding Branches In Decision Tree</th>
<th>Action of Player 3</th>
<th>Actions Without Assumption A</th>
<th>Corresponding Branches In Decision Tree</th>
<th>Action of Player 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>Player 2</td>
<td>Player 3</td>
<td>Player 1</td>
<td>Player 2</td>
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Table 6.1: Importance of Assumption A
The main point to note is that player 3’s optimal decision differs considerably when the assumption is removed. In the first nine rows of table 6.1, we show those sequences of signals for which the optimal strategy of player 3 is the same irrespective of assumption A.

In the first row, the signals received are \((0, x, x)\). In each scenario, player 3 finds it optimal to follow his own signal since it matches the action of a previous player. In the second row, the signals received are \((0, y, x)\). With assumption A, player 3 observes this sequence and follows his own signal. Without assumption A, he observes \((z, y, x)\) and again follows his own signal from assumption B.

In the third row, the signals received are \((x, x, x)\). In each scenario, player 3 notes that his signal matches the action of a previous player and finds it optimal to follow his signal. The same outcome arises when the signals received are \((x, 0, x)\) as seen in row 4.

In row 5, three different signals are received giving \((y, z, x)\). This sequence is observed by player 3 under each of the two scenarios. By assumption B, he follows his own signal. When the signals received are \((x, 0, 0)\), as in row 6, player 3 observes \((x, x, 0)\). He follows the herd irrespective of whether assumption A is included.

In row 7, the signals received are \((y, x, x)\). In each instance, player 3 finds it optimal to follow his own signal since it matches the action of another player. The same argument applies when the signals received are \((x, y, x)\) as in row 8. Finally, in row 9, the signals received are \((x, x, 0)\). Player 3’s decision is always to follow the herd.

We now review those sequences of signals for which the optimal strategy for player 3 differs depending on whether assumption A is made. Row 10 in the table shows the sequence of signals \((0, 0, 0)\). When assumption A is made, players 1, 2 and 3 are obliged to choose zero. However, on removal of this assumption, player 3 observes \((x, x, 0)\) and follows the incorrect herd \((x, x, x)\). This confirms Banerjee’s suggestion that imposing assumption A ‘minimises the possibility of herding’. Clearly, if the probability of receiving a signal is small, the omission of this assumption will imply a large proportion of incorrect herds.
Consider row 11 in the table for which the sequence of signals is \((0, x, 0)\). When assumption A is included, the third player knows that player 2 has received a signal. Therefore, it is optimal for him to follow this player’s action. When assumption A is omitted, player 3 observes player 1 choosing \(y\) and player 2 choosing \(x\). He is not indifferent between the two cases under this scenario since he knows that the first player may not have received a signal and has guessed at the winning option. However, he knows for sure that the player 2 has received a signal. Therefore he finds it optimal to choose \(x\) since:

\[
 P(y = \text{correct} | (y, x, 0)) = \frac{(\alpha \beta) \alpha (1 - \beta) (1 - \alpha)}{P(y, x, 0)}
\]

while

\[
 P(x = \text{correct} | (y, x, 0)) = \frac{(\alpha \beta) \alpha (1 - \beta) (1 - \alpha) + (1 - \alpha)^2 (\alpha \beta)}{P(y, x, 0)}
\]

Since the first term in each equation is the same, it is always optimal to choose \(x\).

However, when the signals received are \((x, y, 0)\), as in row 12, assumption C comes into play. When assumption A is included, player 3 observes \((x, y, 0)\), and is indifferent between following players 1 and 2. By assumption C, he chooses to follow the one with the higher signal.

By contrast, when assumption A is omitted, he will no longer be indifferent between the actions of players 1 and 2. He notes that player 1 may not have received a signal and thus may have guessed that \(x\) is the winning option. He knows for certain, that player 2 has received a signal since his action differs from that of player 1. Therefore, he finds it optimal to follow player 2 as shown in the above proof.

We now show three instances where omitting assumption A leads to ambiguity in the decision making process. Player 3’s decision to follow his own signal or the action of a previous player will depend on the value of \(\alpha\) and \(\beta\).

If the sequence of signals received is \((0, 0, x)\), players 1 and 2 do not
receive a signal but player 3 receives the signal, $x$. With assumption A, player 3 observes the fact that the previous players have not received a signal. He thus finds it optimal to choose $x$. However, on removal of this assumption, he observes $(y, y, x)$. His decision now depends on the relative sizes of $\alpha$ and $\beta$ since:

$$P(x = \text{correct}|(y, y, x)) = \frac{\alpha(1 - \beta)(1 - \alpha)\alpha\beta + (1 - \alpha)^2\alpha\beta}{P(y, y, x)}$$

$$P(y = \text{correct}|(y, y, x)) = \frac{\alpha(1 - \beta)(1 - \alpha)\alpha\beta + (\alpha\beta)^2\alpha(1 - \beta)}{P(y, y, x)}$$

This implies that the third player will only follow $y$ if:

$$\alpha^2\beta(1 - \beta) > (1 - \alpha)^2$$

i.e. if

$$\beta(1 - \beta) > \left(\frac{1 - \alpha}{\alpha}\right)^2$$

The same scenario emerges for the signal sequence $(y, 0, x)$ as given in row 14. When assumption A is included, player 3 observes $(y, y, x)$. He abandons his own signal irrespective of the sizes of $\alpha$ and $\beta$ since:

$$P(x = \text{correct}|(y, y, x)) = \frac{\alpha(1 - \beta)(1 - \alpha)\alpha\beta}{P(y, y, x)}$$

$$P(y = \text{correct}|(y, y, x)) = \frac{\alpha(1 - \beta)(1 - \alpha)\alpha\beta + (\alpha\beta)^2\alpha(1 - \beta)}{P(y, y, x)}$$

Since the first term in each equation is the same it follows that the prob-
ability that \( y \) is correct exceeds the probability that \( x \) is correct given the sequence \((y, y, x)\). The player will thus find it optimal to abandon his own signal and follow the herd.

However, when this assumption is omitted, the decision rests on the values of \( \alpha \) and \( \beta \) as in row 13.

In row 15, the signals received are \((y, y, x)\). When assumption A is included, player 3 finds it optimal to abandon his own signal. Once again, when this assumption is omitted, this decision depends on the relative sizes of \( \alpha \) and \( \beta \).

Therefore, we see problems emerging in the lower half of the decision tree. The fact that a player is allowed to make a guess at the winning option when he has no signal reduces the information available to others. We have shown that, as a result, player 3’s decision becomes more complex and depends on the relative sizes of \( \alpha \) and \( \beta \). This has repercussions for the decisions of players later in the sequence. They will use the sizes of \( \alpha \) and \( \beta \) to arrive at their own optimal decision. Furthermore, they will assume that previous players have also used these values in their decision rules and build this into their own decision. The model becomes very complex. It follows that there is no single strategy which applies to each player irrespective of his position in the sequence.
Winning No. (a)

\[ \alpha \beta \]

Losing No. (b)

\[ \alpha(1 - \beta) \]

1 - \alpha 0 - Follow 1st Player (c)

Winning No. (d)

\[ \alpha \beta \]

Losing No. (e)

\[ \alpha(1 - \beta) \]

1 - \alpha 0 - Follow 1st Player (f)

Winning No. (g)

\[ \alpha \beta \]

Losing No. (h)

\[ \alpha(1 - \beta) \]

1 - \alpha 0 (i)

Player 1

Player 2

Figure 6.2: Decision Tree For the Banerjee Rule With Assumption A
Figure 6.3: Decision Tree For the Banerjee Rule Without Assumption A
6.4 Simulations of the Banerjee Framework.

The aim of these experiments was to test the null hypothesis that the Banerjee strategy was being followed. If individuals followed the Banerjee rule, this would imply a large degree of herd behaviour. In order to test the hypothesis, we simulated the actions of individuals when they used the Banerjee rule. In the first simulation, we included assumption A and in the second case, we omitted this assumption. This established the number of subjects to be used and the number of times that each experiment should be performed. We were particularly interested in the role of each of the parameter values in the framework.

The simulations were designed so as to allow the user to insert the required number of iterations of the model, the values of $\alpha$ and $\beta$, the number of participating subjects and also the upper bound of the random variable used.\(^3\) The results were displayed so as to show the signal received by each subject together with the corresponding choice of that subject. It then reported the number of true and false herds for these parameter values plus the number of iterations in which no herd occurred.

A herd occurred when the first two subjects chose the same option. Alternatively, a herd also set in when two or more subjects chose the same option later in the sequence. For the purpose of our experiments, we distinguish between a 'run' and a 'herd'. A 'herd' is described above whereas a 'run' occurs when two or more subjects follow the same option but this is subsequently broken. Clearly, under the Banerjee rule, this only occurs when the run is incorrect and a subject earlier in the sequence has chosen an option which matches the signal received by the current subject.

We also reported the expected number of winners for each position of play. This became relevant later in the analysis when we examined the behaviour of the subjects in the experiment. In particular, we wanted to know if players in

\(^3\)Correspondence with Professor Banerjee revealed that the herding result would hold even if the set of options was not a continuum providing that the number of options was sufficiently large. This is necessary when the probabilities of true and false herds are considered. Even in situations when the probability of a false herd exceeds that of a true herd, players still find it optimal to herd. This is because the probability that the player's signal is correct is less than the probability of a true herd.
each position of play all adopted the same strategy or whether they modified their behaviour according to the order in which they played.

We used 7 subjects for each session since, according to each of the simulations, this gave a large amount of information concerning the probability and direction of herding. As can be seen in tables 6.3 and 6.9, this number of subjects implied that a herd would occur at least 85 per cent of the time. We allowed $\alpha$ and $\beta$ to take on each of the values of 0.25 and 0.75 since these combinations provided results which were significantly different from each other. Again, this can be seen in tables 6.3 and 6.9. The probabilities of true and false herds were significantly different from one another. Furthermore, these values were simple to implement in an experiment.

The results of the simulated Banerjee model for different values of $\alpha$ and $\beta$ are given in tables 6.3 and 6.9. As one would expect, large values of $\alpha$ and $\beta$ produced a high probability of a true herd and, conversely, small values of $\alpha$ and $\beta$ produced a high probability of a false herd.

### 6.5 Experimental Design.

In carrying out our experiment, the first task was to decide which form this should take. One possibility was to computerise the set up so as to allow a large number of iterations for each set of parameter values. This procedure would have been easy to implement and not time consuming for the subjects. However, the main drawback with this approach was that it would have been difficult to prove to the subjects that they were actually making a random draw. We wanted to avoid the scenario in which the subjects thought that the experiment was rigged in some way.\(^4\) In order to do this we settled on a less elegant but effective approach. We presented our subjects in turn with 16 bags.\(^5\) These appeared identical but their contents differed. Where the

\(^4\)In our pilot experiment, we used a spinner to determine whether a subject received a signal. If the pointer fell in a certain region, the subject was allowed to choose from one of the 4 bags available. However, we found this approach to be particularly awkward and time consuming. We decided to dispense with the spinner and allow the subjects to choose from one of the 16 available bags.

\(^5\)We are very grateful to Irene Griffiths for her assistance in the making of the bags.
probability of receiving a signal was 0.25, 12 of these bags contained discs which were blank. The contents of the remaining 4 bags depended on the value of $\beta$. If this equaled 0.25, 3 bags contained 10 discs numbered from 1 to 10 and 1 bag contained 10 discs each displaying the winning option. When the probability of receiving a signal was 0.75, 4 of the bags contained 10 blank discs. Of the 12 remaining bags, 3 contained 10 discs with the winning option and 9 contained the numbers 1 to 10 when $\beta$ took on the value of 0.25. When $\beta$ equals 0.75, 9 bags contained the winning option and 3 contained the discs numbered 1 to 10.

In order to establish the number of rounds that we wanted for each combination of $\alpha$ and $\beta$, we wanted there to be a sufficient number to reflect the findings of the simulation but not so many that the subjects would be becoming bored and disinterested. We found that 10 rounds for each combination giving a total of 40 rounds in the experiment proved to be satisfactory for each of the two experiments. Our pilot experiment revealed that each session lasted approximately one hour.

In order to arrive at an appropriate figure for payment when the subjects chose the winning option, we used the two simulations to calculate the expected number of winners per game. We based our calculations on a payout of approximately £7 per person per hour. This meant a payout of £4 for each successful subject in each round of session 1 where $\alpha$ and $\beta$ both took on the value of 0.25. In session 2 where $\alpha$ was 0.25 and $\beta$ was 0.75 the payout was £2 for each subject choosing the winning option in a round. For session 3 where $\alpha$ was 0.75 and $\beta$ was 0.25 the payout was also £2. In the final session in which $\alpha$ and $\beta$ were both 0.75, this payout was £1.

6.5.1 Experimental Procedure.

The subjects were students from a variety of disciplines. The number of subjects to be used raised another issue. There would clearly have been problems in using the same set of subjects throughout each session. Equally, it would have been impractical to use different subjects for each round since each group would have had to digest the instructions and understand the
rules. We decided to use a different set of subjects for each of the 4 sessions of 10 rounds.

At the start of each session, they were brought into the room and were seated between large dividing screens to prevent communication during the experiment. We handed each subject written instructions\(^6\) (as seen in the appendix) and also read these aloud so as to ensure that each group received the same information and understood what was expected of them.

We then approached each subject at random and asked them to select one of the 16 bags available. From this, they drew a disc which was either blank or had a number between 1 and 10 printed on it. They were instructed not to reveal this information to the other subjects. We made a note of the disc drawn for future reference. We then asked them for their guess at the winning option. We wrote this on the board at the front of the room for all to see and then approached the next subject.

When all 7 subjects had chosen the number they believed to be the correct option, we announced the winning option and awarded the cash prizes. The process was then repeated for another 9 rounds after which the session was completed. Notably, for each session the subjects were informed of the number of bags containing the winning option, the numbers 1 to 10 and the blank discs. We found the results of the experiment interesting since they used this information in their decision making process. According to Banerjee’s rule, the values of \(\alpha\) and \(\beta\) do not play a part in the subject’s choice.

### 6.6 Results.

Firstly, our experiment revealed that herding did not occur as frequently as the theory predicted under certain sets of parameter values for the experiment with rule A. When this rule was not imposed, the degree of herding was consistent with and sometimes exceeded that predicted by the Banerjee simulation. Secondly, contrary to Banerjee’s theory, the subjects appeared

\(^6\)In the experiment omitting Rule A, the paragraph placing restrictions on subjects if they chose a blank disc from the bag was dropped.
to use the values of $\alpha$ and $\beta$ in formulating their decision.

The results of each of the sessions are reported in the appendix. They are laid out so as to show the number drawn by each of the subjects together with their corresponding choice. In the final column, the winning option is stated. This has allowed us to analyse the behaviour of our subjects. Although we only have 10 rounds of observations for each set of parameter values, there are particular patterns emerging. In the first instance, we will set out the results for the experiment in which Banerjee’s rule A was enforced. We will then compare these results with the experiment in which this rule was relaxed.

6.6.1 Experiment Including Rule A.

Firstly, it was important to establish the relative frequency with which signals were received and the proportion which were correct. Table 6.2 illustrates the expected and actual number of signals for each session together with the expected and actual proportion of correct signals. It also illustrates a 95% confidence interval for the predicted values. These confidence intervals were defined as follows:

$$L, U = p \pm 1.96 \sqrt{\frac{p(1-p)}{n}}$$

in which $L$ is the lower boundary of the confidence interval, $U$ is the upper boundary, $n$ is the total number of decisions made in each session (in all cases this was 70) and $p$ is the predicted proportion of signals. Note that the normal approximation can be used here since $np > 5$ and $n(1-p) > 5$. 

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<table>
<thead>
<tr>
<th>Session</th>
<th>Predicted No. of Signals</th>
<th>Actual No. of Signals</th>
<th>Conf. Interval for Actual No. of Signals Received</th>
<th>Predicted No. of Correct Signals Given Actual No. Received</th>
<th>Actual No. of Correct Signals</th>
<th>Conf. Interval for No. of Correct Signals Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.5</td>
<td>14</td>
<td>10.4-24.6</td>
<td>3.5</td>
<td>4</td>
<td>-0.1-7.1</td>
</tr>
<tr>
<td>2</td>
<td>17.5</td>
<td>20</td>
<td>10.4-24.6</td>
<td>15</td>
<td>18</td>
<td>8.3-21.7</td>
</tr>
<tr>
<td>3</td>
<td>52.5</td>
<td>50</td>
<td>45.4-63.9</td>
<td>12.5</td>
<td>22</td>
<td>6.2-18.8</td>
</tr>
<tr>
<td>4</td>
<td>52.5</td>
<td>59</td>
<td>45.4-63.9</td>
<td>44.25</td>
<td>38</td>
<td>36.3-52.2</td>
</tr>
</tbody>
</table>

Table 6.2: Actual and Predicted Number of Signals Received For the Experiment Using the Original Banerjee Rule
In each session, the actual number of signals received fell within the 95% confidence interval for that which was predicted. In examining the actual number of correct signals received, all but those of session 3 lay within the 95% interval. In session 3, the actual number of correct signals received was 22 which exceeded the upper limit of the confidence interval. However, since this is a small sample, it does not have severe implications for the experimental results.
<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.25</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Actual</td>
<td>Predicted</td>
</tr>
<tr>
<td>0.25</td>
<td>True Run</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>True Herd</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>False Run</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>False Herd</td>
<td>0.1</td>
</tr>
<tr>
<td>0.75</td>
<td>True Run</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>True Herd</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>False Run</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>False Herd</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 6.3: Actual and Predicted Proportions of True and False Herds For the Experiment Using the Banerjee Rule Plus Actual Proportions of True and False Runs
Under Banerjee's definition, a herd occurs when 2 subjects choose the same action. However, while we find that herds started, inspection of the data in the appendix reveals that these were frequently broken. Therefore, we distinguish between a 'run' and a 'herd'. If 2 or more consecutive subjects follow the same number but this is subsequently broken by another subject choosing a different option, we denote this as a 'run'. However, if this is not broken we describe this as a 'herd'. Table 6.3 illustrates the actual and predicted proportions of true and false herds and also the proportions of true and false runs in each of the two experiments. For each of these predicted proportions we have included a confidence interval calculated as before. However, note that the n value is now equal to 10 since we are now considering the number of rounds in each session. This implies that np and n(1−p) are no longer greater than 5 and hence the proportions can no longer be approximated by a normal distribution. We have included this measure in the absence of a suitable alternative but the results must be viewed with a degree of caution.

In session 1, there are no true herds but one true run. This is not significantly different from that which the theory predicts. However, there were significantly fewer false herds setting in than the theory predicted. We found that the subjects displayed a much stronger tendency to follow their individual signals or appeared to choose randomly.

In session 2, the number of true herds was smaller than predicted by Banerjee. However, there were a number of true runs occurring. The same pattern emerged for the number of false herds. There was just one false herd in this session. However, there were three false runs.

In session 3, the proportion of true herds was close to that predicted by Banerjee. There were also two runs occurring. However, the number of false herds was significantly lower than predicted. However, there were a number of false runs emerging. If these had not been broken, the total number of false herds would have been close to that predicted by the Banerjee strategy.

In session 4, the actual proportions of true and false herds corresponds to that predicted by the Banerjee rule with just one true run occurring.

In table 6.4, we report the proportion of rounds in each session which
Table 6.4: Proportion of Rounds in which the Banerjee Strategy is Played For the Experiment Including Rule A

were compatible with the Banerjee strategy. This proves to be very revealing since his strategy is only closely followed in one of the four sessions. This would suggest that in the other sessions, subjects are adopting an alternative approach. This is examined in more detail in tables 6.6 and 6.7.

In table 6.5, we have compared the actual and predicted proportion of subjects choosing the winning option. Again, confidence intervals have been reported. Subjects in sessions 1 and 4 perform as predicted under the Banerjee rule. The same can be said for session 2 with the exception of player 6 who does not perform as well as predicted. In session 3, subjects perform as predicted in each position of play with the exception of player 2. His performance is statistically better than predicted by the Banerjee strategy.
<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Predicted</th>
<th>Conf. Interval</th>
<th></th>
<th>Actual</th>
<th>Predicted</th>
<th>Conf. Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
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<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>0.0825</td>
<td>-0.088-0.2530</td>
<td>Player 1</td>
<td>0.1</td>
<td>0.1925</td>
<td>-0.0519-0.4369</td>
</tr>
<tr>
<td>Player 2</td>
<td>0.1</td>
<td>0.1425</td>
<td>-0.0742-0.3592</td>
<td>Player 2</td>
<td>0.4</td>
<td>0.3394</td>
<td>0.0459-0.6329</td>
</tr>
<tr>
<td>Player 3</td>
<td>0.1</td>
<td>0.1891</td>
<td>-0.0536-0.4318</td>
<td>Player 3</td>
<td>0.4</td>
<td>0.4517</td>
<td>0.1432-0.7602</td>
</tr>
<tr>
<td>Player 4</td>
<td>0.1</td>
<td>0.2231</td>
<td>-0.0349-0.4811</td>
<td>Player 4</td>
<td>0.4</td>
<td>0.5369</td>
<td>0.2278-0.8459</td>
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<tr>
<td>Player 5</td>
<td>0.1</td>
<td>0.2519</td>
<td>-0.0172-0.5210</td>
<td>Player 5</td>
<td>0.4</td>
<td>0.6013</td>
<td>0.2978-0.9048</td>
</tr>
<tr>
<td>Player 6</td>
<td>0.1</td>
<td>0.2720</td>
<td>-0.0038-0.5478</td>
<td>Player 6</td>
<td>0.3</td>
<td>0.6501</td>
<td>0.3545-0.9457</td>
</tr>
<tr>
<td>Player 7</td>
<td>0.2</td>
<td>0.2884</td>
<td>0.0076-0.5692</td>
<td>Player 7</td>
<td>0.5</td>
<td>0.6887</td>
<td>0.4017-0.9757</td>
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<tr>
<td></td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Player 1</td>
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<td>-0.0231-0.5083</td>
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<td>Player 2</td>
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<td>0.0207-0.5923</td>
<td>Player 2</td>
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<td>0.4534-1.0044</td>
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<td>0.7336-1.0874</td>
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<td>0.4444</td>
<td>0.1364-0.7524</td>
<td>Player 7</td>
<td>0.9</td>
<td>0.9154</td>
<td>0.7429-1.0879</td>
</tr>
</tbody>
</table>

Table 6.5: Actual and Predicted Proportions of Winning Subjects in Each Position of Play In the Experiment Including Rule A
Table 6.6 examines the behaviour of subjects when they receive a signal. When \( \alpha \) and \( \beta \) both equal 0.25, subjects appear to follow their own signal if they get one. However, in session 2, with \( \beta \) equal to 0.75, there were 2 occasions in which subjects abandoned their own signal in favour of a previously chosen signal and on 1 occasion, a subject abandoned his signal and appeared to choose randomly. In session 3, the strategy appeared to change according to the position of play. For earlier players in the rounds, there was a tendency to follow their own signal. However, later players were equally likely to abandon their own signal and follow a previously chosen number. Session 4 was close to the Banerjee strategy. Subjects playing early in the round had a tendency to follow their own signal. However, later subjects were more willing to abandon their own signal if it did not match that of an existing herd.

In table 6.7, we examine the behaviour of those subjects not receiving a signal. Again there was a particular pattern of behaviour emerging. In session 1, subjects appeared to choose randomly more often than following the most frequently chosen number. This occurred for each position of play. In session 2, there was also a strong tendency to choose a number which had not already been chosen. However, for later rounds, subjects were equally likely to follow the most frequently chosen number.
<table>
<thead>
<tr>
<th></th>
<th>0.25</th>
<th></th>
<th></th>
<th>0.75</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Follow Own Signal</td>
<td>Follow Previously Chosen No.</td>
<td>Other</td>
<td>Follow Own Signal</td>
<td>Follow Previously Chosen No.</td>
<td>Other</td>
</tr>
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<td>-</td>
<td>-</td>
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<td>Player 2</td>
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</tr>
<tr>
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<td>-</td>
<td>-</td>
<td>Player 3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Player 4</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>Player 6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Player 7</td>
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<td>0</td>
<td>0</td>
<td>Player 7</td>
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<tr>
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<td>0</td>
<td>0.3333</td>
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<td>0.1111</td>
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<tr>
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<td>0.5</td>
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<td>Player 7</td>
<td>0.375</td>
</tr>
</tbody>
</table>

Table 6.6: Behaviour of Subjects Given That They Receive a Signal In the Experiment Including Rule A
For sessions 3 and 4 there were very few occasions in which a blank disc was drawn. In session 3, subjects were more likely to follow the most frequently chosen number. In session 4, subjects always followed the most frequently chosen number.

As a final point it is worth examining the extent to which subjects learned as the rounds progressed. By the term ‘learned’ we mean the degree to which they altered their behaviour throughout the experiment. In order to achieve this, we monitored the position of play of each individual in each round. In session 1, we could not detect any real evidence that subjects were changing their behaviour. In session 2 we discovered that behaviour did change. At round 4, two of the subjects switched their strategies from choosing apparently randomly to following the Banerjee rule. Two other subjects followed suit in round 8. Two further subjects followed in round 9. The final 2 subjects did not appear to learn.

In session 3, there was also some evidence that subjects conformed to the Banerjee rule. One subject appeared to behave according to Banerjee from the outset. A second subject followed from the second round onwards. A third followed Banerjee in round 3 onwards. Two more subjects followed suit in round 6. Again, there were 2 other subjects out of the 7 who did not appear to change their behaviour.

In session 4, behaviour was very similar to that predicted by Banerjee from the outset and all had adopted his strategy from round 2 onwards.
<table>
<thead>
<tr>
<th></th>
<th>Follow Most Frequently Chosen No.</th>
<th>Follow Another Chosen No.</th>
<th>Other</th>
<th>Follow Most Frequently Chosen No.</th>
<th>Follow Another Chosen No.</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>-</td>
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<td>-</td>
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<tr>
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<td>Player 7</td>
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</tbody>
</table>

Table 6.7: Behaviour of Subjects Given That They Do Not Receive a Signal In the Experiment Including Rule A
6.6.2 Experiment Without Rule A.

As in the previous experiment, it is important to establish the actual number of signals received and proportion of those which are correct compared with the predicted numbers. This is seen in table 6.8. In each session, the total number of signals received lay within the 95% confidence interval. However, in session 3, the actual number of correct signals lay slightly outside the confidence interval. Those of the other sessions fell within the confidence interval.

Table 6.9 shows the actual and predicted proportions of true and false herds for the experiment plus the actual proportion of true and false runs. Firstly, it is important to compare the actual results with those predicted. We have included the confidence interval in order to achieve this. However, it is also worthwhile comparing these results with those of the original experiment since this shows the effect of omitting rule A.

In the first session, the predicted proportion of false herds exceeded that of the original Banerjee strategy with assumption A. Conversely, the predicted proportion of true herds was less than under the original Banerjee strategy. This implies that the removal of assumption A leads to an increase in the proportion of incorrect herds when $\alpha$ and $\beta$ are small. In this session, the number of herds was close to that which was predicted. However, there were more runs occurring than under the original Banerjee rule.

In the second session, the predicted proportion of false herds also exceeded that of the original Banerjee strategy with assumption A. Again, the predicted proportion of true herds was less than under the original Banerjee strategy. The main feature of this session, however, was the large number of runs. If these had not been broken, they would have generated a far greater number of herds than predicted. This confirms Banerjee’s argument that assumption A reduces the possibility of herding.

In session 3, herding was consistent with that which was predicted. However, there was a large proportion of false herds. If these had not been broken they would have created a proportion of false herds significantly greater than predicted.

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In session 4, herding was close to that predicted but this time there was a large proportion of true runs. The following tables break this information down so as to show why these patterns emerged.
<table>
<thead>
<tr>
<th>Session</th>
<th>Predicted No. of Signals</th>
<th>Actual No. of Signals</th>
<th>Conf. Interval for Actual No. of Signals Received</th>
<th>Predicted No. of Correct Signals Given Actual No. Received</th>
<th>Actual No. of Correct Signals</th>
<th>Conf. Interval for No. of Correct Signals Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>19</td>
<td>10.4-24.6</td>
<td>4.75</td>
<td>7</td>
<td>0-13</td>
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<tr>
<td>2</td>
<td>17.5</td>
<td>23</td>
<td>10.4-24.6</td>
<td>17.25</td>
<td>18</td>
<td>10.2-24.3</td>
</tr>
<tr>
<td>3</td>
<td>52.5</td>
<td>54</td>
<td>45.4-63.9</td>
<td>13.5</td>
<td>20</td>
<td>7.0-19.2</td>
</tr>
<tr>
<td>4</td>
<td>52.5</td>
<td>62</td>
<td>45.4-63.9</td>
<td>46.5</td>
<td>48</td>
<td>38.8-54.2</td>
</tr>
</tbody>
</table>

Table 6.8: Actual and Predicted Number of Signals Received For the Experiment Without Rule A
<table>
<thead>
<tr>
<th>α</th>
<th>0.25</th>
<th>0.75</th>
<th>β</th>
<th>Actual</th>
<th>Predicted</th>
<th>Conf. Interval</th>
<th>Actual</th>
<th>Predicted</th>
<th>Conf. Interval</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>True Run</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>True Run</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>True Herd</td>
<td>0.1</td>
<td>0.1895</td>
<td>-0.0534-0.4324</td>
<td>True Herd</td>
<td>0.5</td>
<td>0.3753</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>False Run</td>
<td>0.1</td>
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<td>False Run</td>
<td>0.7</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>False Herd</td>
<td>0.7</td>
<td>0.8023</td>
<td>0.5554-1.0490</td>
<td>False Herd</td>
<td>0.6</td>
<td>0.6158</td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td></td>
<td>True Run</td>
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<td>True Run</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>True Herd</td>
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<td>0.0744-0.6744</td>
<td>True Herd</td>
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<td></td>
<td></td>
<td>False Run</td>
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<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>False Herd</td>
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<td>0.5003</td>
<td>0.1903-0.8102</td>
<td>False Herd</td>
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<td>0.1336</td>
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</tbody>
</table>

Table 6.9: Actual and Predicted Proportions of True and False Herds For the Experiment Without Rule A Plus Actual Proportions of True and False Runs
Table 6.10 shows the proportion of rounds in which the Banerjee strategy was played throughout. The main point to note there is that his strategy was played in only a small proportion of rounds. Also note how this compares with the original experiment. Session 2 is the same for each experiment and session 1 is very close. However, the Banerjee strategy is played much more frequently for sessions 3 and 4 under the original experiment with rule A. The probability of receiving a signal here is large at 0.75. Therefore, it is less likely that subjects earlier in the experiment are not receiving signals and thus guessing thereby generating false herds. Hence, it is more sensible to follow the Banerjee strategy.

Table 6.11 examines the actual and predicted proportions of winning subjects in each position of play in the experiment. In each session, the players perform as predicted and none of the results are statistically significantly different from that which is predicted. However, it should be noted that due to a small sample size (i.e. 10 rounds), the power of the statistical tests is weak.
<table>
<thead>
<tr>
<th></th>
<th>0.25</th>
<th></th>
<th>0.75</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Predicted</td>
<td>Conf. Interval</td>
<td>Actual</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 1</td>
<td>0.2</td>
<td>0.1550</td>
<td>-0.0693-0.3793</td>
<td>Player 1</td>
</tr>
<tr>
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<td>0.1551</td>
<td>-0.0693-0.3795</td>
<td>Player 2</td>
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<tr>
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<td>-0.0696-0.3780</td>
<td>Player 3</td>
</tr>
<tr>
<td>Player 4</td>
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<td>0.1558</td>
<td>-0.0690-0.3806</td>
<td>Player 4</td>
</tr>
<tr>
<td>Player 5</td>
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<td>-0.0688-0.3812</td>
<td>Player 5</td>
</tr>
<tr>
<td>Player 6</td>
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<td>0.1579</td>
<td>-0.0681-0.3839</td>
<td>Player 6</td>
</tr>
<tr>
<td>Player 7</td>
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<td>0.1557</td>
<td>-0.0690-0.3804</td>
<td>Player 7</td>
</tr>
<tr>
<td>0.75</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.2714</td>
<td>-0.0042-0.5470</td>
<td>Player 1</td>
</tr>
<tr>
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<td>0.2636</td>
<td>-0.0095-0.5367</td>
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</tr>
<tr>
<td>Player 3</td>
<td>0.4</td>
<td>0.3252</td>
<td>0.0349-0.6155</td>
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</tr>
<tr>
<td>Player 4</td>
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<td>0.3614</td>
<td>0.0636-0.6592</td>
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</tr>
<tr>
<td>Player 5</td>
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<td>0.0924-0.6986</td>
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</tr>
<tr>
<td>Player 6</td>
<td>0.6</td>
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<td>0.1152-0.7272</td>
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<tr>
<td>Player 7</td>
<td>0.4</td>
<td>0.4382</td>
<td>0.1307-0.7457</td>
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</tbody>
</table>

Table 6.11: Actual and Predicted Proportions of Winning Subjects in Each Position of Play In the Experiment Without Rule A
Tables 6.12 and 6.13 show the behaviour of subjects conditional on whether they receive a signal. Table 6.12 shows the behaviour of those which receive a signal. In session 1 with the exception of one person, all subjects followed their own signal if they received one despite the fact that they knew that this was only correct with probability, 0.25. We suggest that this feature may be due to an endowment effect. The subject has selected a disc from a bag and feels that this number belongs to him. He is therefore reluctant to abandon it in favour of some other course of action.

In session 2 there was also a strong tendency to follow the signal received. Players 1 and 2 always follow their own signal. Players 3 to 7 follow their own signal for a large proportion of instances but also follow a previously chosen number for a small proportion of cases.

For session 3, players 1 and 2 follow their own signal predominantly. In those cases when they do not follow their own signal, they appear to choose randomly. Players 3, 4 and 5 also choose to follow their own signal for the majority of cases and when they do not follow their signal, they follow a previously chosen option. Players 6 and 7 predominantly choose to follow a previously chosen option with only a small proportion choosing to follow their own signal. Thus in this session we see a pattern emerging. Earlier players favour their own signal while players later in the sequence are more willing to abandon their own signal.

In session 4, there is once again a strong tendency to follow one’s own signal. This is intuitively appealing since the probability that this signal is correct is high. However, this line of thinking is known as the ‘gambler’s fallacy’. Individuals who adopt this approach neglect to consider the information available in the actions of previous individuals. These earlier players have also faced a large $\alpha$ and $\beta$. If their actions are different from those of the current player and he fails to consider their actions, he is more likely to choose the incorrect course of action. However, there is a small percentage, particularly later in the session who do follow a previously chosen number.
<table>
<thead>
<tr>
<th></th>
<th>0.25</th>
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</tr>
</thead>
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<td>Follow</td>
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<td>Follow</td>
</tr>
<tr>
<td></td>
<td>Own</td>
<td>Previously</td>
<td></td>
<td>Own</td>
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<td>0</td>
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<td>Player 4</td>
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</tr>
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<td>0</td>
<td>Player 6</td>
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Table 6.12: Behaviour of Subjects Given That They Receive a Signal In the Experiment Without Rule A
Table 6.13 shows the behaviour of those subjects who do not receive a signal. In session 1, we cannot find an observable pattern emerging. Players 2 and 3 have a tendency to choose randomly and follow the most frequently chosen number less often. Player 4 follows the most frequently chosen number 50% of the time, follows another chosen number 20% of the time and appears to choose randomly 30% of the time. Player 5 chooses randomly 50% of the time and follows the most frequently chosen number or another chosen number 25% of the time. Players 6 and 7 follow the most frequently chosen number a third of the time with player 6 following another chosen number 50% of the time and player 7 showing a tendency to follow another chosen number or choose randomly.

There is a pattern emerging in session 2 in that there is an increasing tendency to follow the most frequently chosen number as the session progresses. Note that in sessions 3 and 4 there are only a few subjects who do not receive a signal so there is only a small amount of data available here. In session 3, Players 3, 6 and 7 predominantly follow the most frequently chosen number. Player 2 only does this 50% of the time and appears to choose randomly for the remaining 50%. Player 4 displays a tendency to choose randomly rather than follow the most frequently chosen number. In session 4, subjects always follow the most frequently chosen option when they do not receive a signal.
<table>
<thead>
<tr>
<th></th>
<th>Follow Most Frequently Chosen No.</th>
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<th>Other</th>
<th>Follow Most Frequently Chosen No.</th>
<th>Follow Another Chosen No.</th>
<th>Other</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>Player 5</td>
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<td>Player 6</td>
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<td>0</td>
</tr>
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<td>0.25</td>
<td>0</td>
<td>Player 7</td>
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</table>

Table 6.13: Behaviour of Subjects Given That They Do Not Receive a Signal In the Experiment Without Rule A
6.7 Conclusion.

Our concluding comments fall into two categories. Firstly there are the implications of our theoretical findings. We show that, without the assumption A which Banerjee claims merely reduces herding, the whole decision making process becomes dependent on (a) the position in the sequence and (b) the probabilities, $\alpha$ and $\beta$.

In terms of future lines of research, this implies the ambitious task of solving the model without assumption A.

When carrying out the experiments, we referred to this as rule A. The main result of our experiment with rule A was that herding occurred less frequently than predicted by the Banerjee framework. The behaviour of subjects was far more individual than the theory suggested with subjects using the information on $\alpha$ and $\beta$ and also their position of play in the rounds to formulate their decisions. In his model, Banerjee found that there was tremendous volatility in the pattern of decision making over a number of plays of the game. This was because the onset of herding and its direction depended upon the signal received by the first few individuals. However, we found that this volatility occurred within rather than between rounds. When a run set in, it did not necessarily continue. With certain parameter values, players were inclined to break the run using either their own signal or by appearing to choose randomly.

However, in analysing individual behaviour we discovered that there was some evidence in sessions 2, 3 and 4 to suggest that individuals may have followed the Banerjee strategy provided that there were enough rounds in which learning could take place. One may argue however, that in real life scenarios, people do not always have the benefit of experience from a repeated situation.

In the experiment without rule A, herding was more prevalent than under the original experiment with assumption A. Significantly, we also witnessed a large number of runs which were subsequently broken. These would indicate a willingness on the part of some subjects to follow a herd and then the opposite behaviour from other subjects breaking that run. These appeared
to occur regardless of the values of $\alpha$ and $\beta$ and generated volatility within rather than between rounds.

In this experiment, subjects were even less inclined to follow the Banerjee strategy. In sessions 1 and 2, there was a strong tendency to apparently choose randomly rather than the most frequently chosen number when no signal was received. Given this type of behaviour, it would be revealing to circulate a questionnaire to each subject following each session asking them about their strategies in a future experiment to test herding.
Appendix 6.1 - Instructions for the Subjects.

Welcome to the Experiment!
Firstly, you will notice that you have been partitioned off from the other players. There is nothing sinister here: one of the few rules that I am imposing is that you do not communicate with the others.

- I will be running the experiment 10 times and will be awarding a cash prize of £4 to each player who chooses the winning number in each round. The aim of the exercise is to find this winning number.

- For each game that will be played, the winning number and the order in which you play have been chosen at random. I put discs numbered 1 to 10 into a bag and picked a disc from the bag. This is the winning number for the first game. I then replaced the disc and repeated this to determine the winning numbers for the other 9 rounds.

- I will present each of you, in turn, with 16 bags and you will be asked to pick one. The bags all look the same but their contents differ. Each contains 10 discs. Twelve bags contain blank discs. Three of the bags contain the numbers 1 to 10. The other contains 10 discs with the winning number.

- You will then draw a number from your chosen bag. Do not disclose this to anyone.

- I will then ask you to write your chosen number on my clip board. This may or may not be the number appearing on your disc. You are not obliged to stick with the number which is written on your disc if you think you know better.

- The only rule I make regarding your choice of number is that if you pick a blank disc from the bag, you are not allowed to choose a number if:
  (a) You are the first person to move in the game or
  (b) No one who has moved before you has chosen a number.
• I will then write your chosen number on the flip chart for the other players to see.

• When all the players have chosen the number which they think is the winner, I will announce the winning number and award the cash prizes. Good Luck!
Appendix 6.2 - Data from the Experiment Including Rule A

Session 1 - $\alpha$ and $\beta$ equal 0.25

<table>
<thead>
<tr>
<th>Nos Picked From Bags</th>
<th>Nos Chosen By Subjects</th>
<th>Winning Option</th>
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<tbody>
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<td>10</td>
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<td>- - 2 - - 1</td>
<td>- - 6 6 8 1</td>
<td>1</td>
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Session 2 - $\alpha$ equals 0.25 and $\beta$ equals 0.75

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**Session 3 - \( \alpha \) equals 0.75 and \( \beta \) equals 0.25**

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**Session 4 - \( \alpha \) and \( \beta \) equal 0.75**

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Appendix 6.3 - Data from the Experiment Without Rule A

Session 1 - $\alpha$ and $\beta$ equal 0.25

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Session 2 - $\alpha$ equals 0.25 and $\beta$ equals 0.75

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Session 3 - $\alpha$ equals 0.75 and $\beta$ equals 0.25

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Session 4 - $\alpha$ and $\beta$ equal 0.75

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Chapter 7

GARCH and Markov Regime Switching Models of Exchange Rate Data

There has been a great deal of literature discussing the empirical evidence surrounding European exchange rates. I survey some of these papers in the next section. However, many economists have been solely concerned with the data contained in the ERM period. This implies that the literature has mainly studied the currencies of long standing members of the European Exchange Rate Mechanism (ERM). Alternatively, they have examined the data leading up to or following the collapse of the ERM.

I argue that this represents an important omission in the literature to date especially since a number of currencies have spent a large amount of time outside the ERM. In this chapter, I seek to examine the behaviour of the Pound and the Lira before, during and after their membership in the ERM. In particular, I attempt to provide models which explain their behaviour over the entire period. In the case of the Lira, the data stretches from 1978 to 1997 and in the case of the Pound, the data covers the period 1975-1997.

The aim of this study is twofold. Firstly, I examine the behaviour of each exchange rate when it is restricted to a currency band and compare this with the periods outside the band. In order to do this, I employ a GARCH
model. I show that a currency’s position in the band has an impact upon the mean value of the change in the exchange rate and also on the variance function within a GARCH setting. Furthermore, I show that shocks to the currency, such as monetary policy announcements, have an impact on the variance function of the change in the exchange rate. In the case of the Lira, I further show that shocks in terms of exchange rate realignments also impact upon the variance function. Secondly, I compare the results obtained from using GARCH and regime switching models in order to see whether the two modelling techniques are interchangeable.

The chapter is set out as follows. Firstly, I review some of the relevant literature in the area. I start by examining those GARCH models which focus on studies of floating rates. I then move on to consider those which examine the EMS period. Secondly, I examine Markov regime switching models since I also use this framework in my analysis of the Pound and Lira. Finally, I discuss the jump diffusion literature. Having set out the literature in the area, I examine the raw data. I provide plots of each exchange rate and summary statistics. I then proceed to look at the EMS period in more detail and how the events of this period impacted upon the Lira and Pound. I provide a small history of events and explain, in the context of the Lira, why each realignment took place.

In the next section, I set up a GARCH model for the entire period for each exchange rate and discuss the suitability of each of these models. I then discuss the importance of including into the analysis, each exchange rate’s position in the band. I re-estimate the models with this modification. The next step is to include the realignment dummy for the Lira into the original GARCH model. Having examined this, I look at the importance of policy shocks to the each of the currencies. For each currency, I set up a dummy variable explaining the major shocks over the period. I re-estimate each model taking these into account. I then include a model which incorporates each of the features of position in the band and policy change dummy. Finally, I examine the regime switching model as an alternative approach and examine its suitability. I set out the results of this framework and then compare the results with the GARCH model.
In my concluding section, I draw together all my results and suggest possible extensions to this work.

7.1 Literature Survey

In examining the empirical evidence behind the events of the 1992 crisis, the literature has formed two basic strands. Firstly, GARCH models have been suggested as suitable for modelling exchange rates across the period in question. Secondly, Markov processes have been used to model the switch from one exchange rate regime to another. An extension to this notion is the jump diffusion literature.

In this section to the chapter, I will consider each of these strands of literature and also show how they have been combined to describe exchange rate movements.

7.1.1 GARCH Models

Firstly, I will show why GARCH models have proved to be so popular in modelling exchange rate behaviour. It was Engle (1982) who first showed that it is possible to model the mean and variance of a series simultaneously. The recent literature has grown out of this pioneering work.

The first point to note is that conditional forecasts are superior to unconditional forecasts. This is seen by examining an ARMA process such as \( y_t = a_0 + a_1 y_{t-1} + \epsilon_t \) in which the aim is to forecast \( y_{t+1} \). This produces a conditional forecast of \( y_{t+1} \) given by:

\[
E_t y_{t+1} = a_0 + a_1 y_t
\]  

(7.1)

In forecasting \( y_{t+1} \), the forecast error variance is \( E_t[(y_{t+1} - a_0 - a_1 y_t)^2] = E_t \epsilon_{t+1}^2 = \sigma^2 \). However, an unconditional forecast produces a forecast error variance which is \( 1/(1-a_1^2) > 1 \) and is therefore greater than the conditional forecast. It follows that conditional forecasts are more suitable since they
take into account the known current and past realisations of a series.

A similar process is used when the variance of $\epsilon_t$ is not constant. However, when the conditional variance is not constant, Engle shows that this may be modelled as an $AR(q)$ process using the square of the estimated residuals so that:

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + \ldots + \alpha_q \hat{\epsilon}_{t-q}^2 + \nu_t$$

(7.2)

where $\nu_t$ is a white noise process. If the above values of $\alpha$ are non zero then the conditional variance evolves according to an autoregressive process. This implies that the above equation can be used to forecast the conditional variance as at time $t+1$ as:

$$E_{t}\hat{\epsilon}_{t+1}^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_t^2 + \alpha_2 \hat{\epsilon}_{t-1}^2 + \ldots + \alpha_q \hat{\epsilon}_{t+1-q}^2$$

(7.3)

This is an autoregressive conditional heteroskedastic model (ARCH) and has formed the basis for a vast amount of literature.

Bollerslev (1986) took this one step further and allowed the conditional variance to be not solely an $AR(q)$ process but an ARMA process. Crucially, this model allows for the heteroskedastic variance to be both autoregressive and a moving average. He defines the error process as:

$$\epsilon_t = \nu_t \sqrt{h_t}$$

(7.4)

with $\sigma^2 = 1$ and

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$$

(7.5)

The conditional and unconditional means of $\epsilon_t$ are equal to zero. This can be seen by taking the expected value of $\epsilon_t$ so that $E \epsilon_t = E \nu_t \sqrt{h_t} = 0$. The
conditional variance of $\epsilon_t$ is given by $E_{t-1} \epsilon_t^2 = h_t$. The important point to note is that the conditional variance of $\epsilon_t$ is given by $h_t$ in equation (7.5). This is known as a generalised ARCH model or GARCH model.

This type of approach has proved to be very popular in describing financial data. This is due to the ability of ARCH and GARCH frameworks to capture particular features of this data. Floating exchange rates have been shown to exhibit clustering volatility and a fat tailed distribution. This feature has been shown by both Baillie and Bollerslev (1989) and Diebold and Nerlove (1989). However, in addition, it has been shown (Engle and Gau (1997)) that some EMS rates can share these floating rate characteristics. Hence I have chosen to use a GARCH framework to model the Lira and Pound exchange rates for the periods in and out of the ERM.

7.1.2 GARCH Models of Floating Exchange Rate Regimes

In this subsection, I will consider those studies which have been devoted largely to floating exchange rates. There is a large amount of literature in this area. An excellent review of the progress up to 1992 can be found in Bollerslev, Chou and Kroner (1992).

Firstly, I consider the work of Diebold and Nerlove (1989) since this represents an early work in the area which uses an ARCH model. They examine the Canadian Dollar, French Franc, Deutschmark, Italian Lira, Japanese Yen, Swiss Franc and British Pound using weekly spot rates. The interval chosen is July 1973 to August 1985 and the day of the week which they report is the observation on a Wednesday. This is consistent with my analysis since there are very few holidays emerging on that day and there is no 'weekend effect'. They argue that ARCH is important since it provides a means of modelling the idea that large changes in exchange rate tend to be followed by further large changes. Conversely, small changes are followed by further small changes. This generates a clustering of prediction error variances. They further argue that ARCH effects are consistent with unconditional leptokurtosis in exchange rate changes.
However, exchange rates within the EMS, are subject to currency realignments. When these occur, a large change in the exchange rate generated by a realignment is not generally followed by further large changes in the exchange rate. This is because the realignment has the effect of reducing the effect of the lagged error term in a GARCH setting. Hence, I take into account realignment dates when modelling the Lira and Pound exchange rates.

Importantly, ARCH can be used to provide a meaningful measure of exchange rate volatility. Economists are concerned with volatility in exchange rates since it generates uncertainty for prices of imports and exports and also for values of international reserves. It follows that ARCH is useful since it provides a way of modelling an evolving conditional variance.

Diebold and Nerlove set up a multivariate time series model which contains a latent variable displaying ARCH tendencies. In effect this is a particular type of martingale process which appears to provide a useful description of exchange rate movements. They show that nominal Dollar spot rates are well described by an approximate vector random walk with ARCH innovations. As they note, ‘The ‘state of the world’ is a serially correlated thing; hence we find ARCH.’

Hsieh (1988) uses daily data. He examines five different nominal US Dollar rates and shows that the conditional distributions of the daily nominal returns are changing through time. This is seen through significant autocorrelations for the squared returns. An ARCH(12) model with a linearly declining lag structure captures most of the nonlinear stochastic dependencies present.

Baillie and Bollerslev (1991) set up a seasonal GARCH model with hourly dummy variables. Their data is hourly and taken from four major floating rates against the US Dollar. They use the British Pound, Deutschmark, Swiss Franc and Japanese Yen since these rates are the most actively traded currencies in the New York and London markets. The spot rate series are taken on an hourly basis over 6 months in 1986.

The reason for taking data of such short intervals is so that they can distinguish between currency specific movements in the exchange rates and market specific factors. Clearly, their data is of much shorter intervals than
that which I use in my analysis because their aims are very different from mine. However, it would be interesting to apply their analysis to the 6 months leading up to the UK and Italian departure from the ERM to more closely analyse speculative behaviour in an EMS setting.

They apply LM tests and find that the GARCH model is a good representation of the data. However, their most interesting find concerns volatility which they describe as ‘distinctive’. They find that volatility is relatively short lived and there is very little persistence found between different days. However, they show that traders with diverse information exploit their private information at the same time as other traders are active. This creates a greater volume of trade and price volatility. This is apparent from examination of each individual market.

The US market appeared the most volatile on average with the European market in second place. The Yen was not volatile. They found an increase in this volatility occurring around and immediately preceding the times of openings of the London and New York markets. There was a much smaller increase associated with the opening of the Tokyo market. There was also a drop in volatility occurring around the lunch hours in London and Tokyo. The authors suggest that this may be due to a systematic release of news at opening time.

The GARCH model is believed to be very successful at capturing changes in volatility caused by variations in the arrival of information. This was the focus of the work of Lamoureux and Lastrapes (1990). This line of argument was confirmed by the work of Engle, Ito and Lin (1990). As in the work of Baillie and Bollerslev, they define four separate market locations, Europe, New York, Pacific and Tokyo. Using a model of GARCH, they aim to show that information processing is the source of volatility clustering.

Their test works as follows. Information arrives in one market. If this is uncorrelated with information arriving in another market, then a test of whether volatility increases in one market causes volatility increases in another market is equivalent to testing if information processing is a source of clustering.

They find for intra day observations, with the exception of Tokyo, each
market's volatility is significantly affected by changes in the volatility of other markets. This is known as a ' meteor shower'.

I close this subsection by discussing the work of Nelson (1992). He investigates those properties of conditional covariance estimates which are generated by a misspecified ARCH model. This is a particularly relevant paper since it helps to explain my choice of frequency of data in my own contribution to the area.

He notes that as the frequency of the data increases, the difference between the conditional covariance estimates and the true conditional covariance estimates approaches zero. He notes that this may account for the success of ARCH models in short term forecasting using high frequency data. Even misspecified ARCH models can produce 'good' estimates of volatility. This was one of the reasons that I chose to use weekly and not daily data.

The literature in this section has argued that GARCH modelling is useful for a number of reasons. Floating exchange rates exhibit certain characteristics for which GARCH modelling is most applicable. Typically these are clustering volatility, insignificant serial correlation and fat-tailed distributions. Furthermore, this type of model is very useful in capturing changes in exchange rate caused by variations in the arrival of information. It is therefore, highly suited for applying to the movement of exchange rates over time. As Bollerslev, Chou and Kroner argue 'The linear GARCH(p,q) is a natural candidate for modeling exchange rate dynamics'.

7.1.3 GARCH Models of the Exchange Rate Mechanism

In this subsection, I examine those models which have focused on the EMS period in particular. The first of these is by Engle and Gau (1997). They find that the EMS rates display some of the characteristics described in the previous subsection. In particular they note the presence of fatter tailed distributions and clustering volatility.

Their model differs from mine in that it uses an MA(1)-GARCH(1,1) model to capture mean reversion and the GARCH effect. A test on the
residuals from the regression, $\Delta \log s_t = a_0 + \epsilon_t$ did not reveal substantial serial correlation. Hence, I model the change in the mean of the exchange rate as a serially uncorrelated variable and the variance as a GARCH(1,1) process.

Krugman (1991) argued that exchange rate changes exhibit a nonlinear form of mean reversion. The presence of a currency band and central bank intervention drive exchange rates away from the edges of the band towards its centre.

Koedijk, Stork and de Vries (1998) find support for this S shaped relation between the exchange rate and fundamentals. Their analysis uses weekly spot rates of the Belgian Franc, Danish Mark, Dutch Guilder, French Franc, Irish Punt and Italian Lira against the Deutschmark.

Therefore, Engle and Gau introduce a variable showing the deviation of exchange rates from the central parity. In my contribution, this is achieved by including the position in the actual band into the mean and conditional variance functions. However, the authors choose to use the 'effective band'. This comes from the work of Pill (1994). He argues that it is more appropriate to look at the EMS parity grid as a whole rather than solely the DM central parity. Engle and Gau adopt this approach and use it to examine the relationship between the position of the exchange rate and its volatility. However, likelihood ratio tests show that one cannot reject the null hypothesis of zero on the coefficient of this new variable. Therefore, I have chosen to estimate using the actual bands for each exchange rate.

A further difference between my estimation and that of Engle and Gau is that they examine a multilateral framework. Their reasoning is that they wish to examine the co-movement of exchange rates among member countries. In order to do this, they include a vector of all ERM currencies' positions into the GARCH equation. As a result, they find that other currencies' positions do affect the conditional volatility of a specific currency.

They use daily data from January 1986 to July 1993 from the Belgian Franc, Danish Mark, French Franc, Italian Lira and Dutch Guilder. As a comparison, they also examine the British Pound, Dollar and Yen. Their findings are consistent with what one may expect from a GARCH model.
the non ERM rates, large exchange rate changes are followed by further large changes. Small changes are followed by further small changes. They observed that these series were characterised by periods of tranquility followed by periods of high volatility. In my contribution, much of the data set occurs outside the ERM period, hence this type of model is ideal for capturing these types of features.

For ERM rates, they find what they describe as ‘abrupt’ changes and note that many of these correspond to realignments within the ERM. To conclude, the authors note that in all six ERM currencies there was mean reversion and a GARCH effect. However, they note that an interesting inclusion would be the introduction of realignments into conditional volatility.

This analysis confirms the theory that a realignment generates a large change in the exchange rate and a consequent increase in the variance but that this is not followed by equally large exchange rate changes and high variance since the lagged squared error term no longer plays a role in the conditional variance of the series. Note that this is not consistent with volatility clustering which is observed in floating exchange rate analysis. This needs to be taken into account when forming a model to capture exchange rate behaviour both in and out of the ERM. For this reason, I have included a realignment dummy in my analysis when considering the Italian Lira which appears in the conditional variance function in order to model these effects.

Lastrapes (1989) has made progress in this area. He includes dummy variables in the conditional variance to allow for policy changes by the FED. He shows that this has the effect of reducing the degree of leptokurtosis in the standardised residuals. This is in line with what I have done. I have included a dummy for each of the two currencies I consider. These dummies account for large movements in the exchange rates which are as a result of significant policy changes.

A similar approach has been adopted by Koedijk, Stork and de Vries (1992). In their GARCH model of the EMS, they ‘dummy out’ realignments. The reasoning for this is to avoid mixing discontinuous changes in the particular distribution with the GARCH process. However, a number of criticisms have been made about this type of approach. Firstly, in examining
the history of the ERM it is noted that not all jumps in exchange rates were necessarily realignments. Secondly, not all realignments were actually jumps in exchange rates. Although I report the results for the GARCH model including the realignment dummy, I place more emphasis on those results for the model including the ‘policy change’ dummy. As a further justification for my approach I note the comments of Bollerslev et al., ‘Further work trying to endogenously determine the timing of major exchange rate movements and changes in regimes would be interesting’.

7.1.4 Markov Regime-Switching Processes

Running concurrently with this line of approach has been the strand of literature concerned with Markov switching models. This type of model has been used extensively by the likes of Hamilton\(^1\) and stems from the work on Markov chains as in Baum et al. (1970). Hamilton (1989) argues that changes in a regime are not the result of some past realisations of the variable in question. In fact, they may be due to processes which are unobservable. He therefore argues that in many circumstance, the approach of Tong (1983) is inappropriate since this describes changes as deterministic i.e. they depend on past realisations of the variable. Instead he recommends Markov chains for analysing time series which are subject to changes in regime. I shall briefly discuss the progress made in this area.

In an examination of US stock market returns, Ceccetti, Lam and Mark (1990) use an equilibrium model with regime switching in dividends. They find that this is successful in modelling some of the previously documented features of the stock market returns. In particular, it models the observed skewness and mean reversion tendencies. van Norden and Schaller (1993) develop this work to consider monthly stock market data from the Centre for Research in Securities Prices for the period 1926-89. They simulate a model in which fundamentals are switching and then estimate the switching regression by using the artificial data from the simulations. They then compare the timing of the actual stock market crashes with that of switches in dividend

\(^1\)In particular I refer to his 1990 paper and also Engel and Hamilton (1990).
growth. They find that both speculative and switching fundamentals are responsible for the large swings in stock market prices.

Much of the earlier work on regime-switching models of interest rates has been inspired by the behaviour of the Federal Reserve in the period 1979-82. During this time, it deviated from the targeting of interest rates and instead used non-borrowed reserves as the targeted instrument. The effect was to create volatility in the interest rate. Similar volatility was experienced during the crash of 1987, the OPEC oil crisis and also during war time. This motivated the work of Cai (1994) and also that of Hamilton (1988).

A similar approach has also been applied to exchange rates. Vigfusson (1996) bases his work on that of Frankel and Froot (1988). They develop an exchange rate forecasting model which includes both fundamentalist and chartist behaviour. Fundamentalists base their expectations of changes in the exchange rate on economic fundamentals whereas chartists form their expectations by relying on the past behaviour of the exchange rate. Vigfusson argues that a model which combines these two behaviours successfully models an exchange rate. In particular, he looks at the US Dollar in the 1980s. He stresses the importance of the inclusion of chartism since this is used on a day to day basis in financial markets.

Taylor and Allen (1992) survey the participants in the London foreign exchange market and find that 90% of the participants use chartism on short term prices and 60% consider this technique to be at least as important as fundamentals. However, when forecasting long term prices, the majority of those surveyed (85%) thought fundamentals were more important than charts. Therefore, a model encompassing the two techniques would appear to be favourable.

Vigfusson notes that empirical testing on this type of approach has not taken place since it is difficult to measure the relative importance of each technique over time. Furthermore, the extent to which each technique is used is not directly observable. It is therefore, difficult to estimate the model using standard statistics. His work attempts to overcome this through adopting a Markov regime-switching model. In particular, he includes two forecasting equations in the model each corresponding to the two types of speculative
behaviour. Secondly, he puts a time-varying weight on each of these two forecasts. In effect, he reinterprets the Frankel and Froot model as a Markov regime switching model.

The data used is the Canada-US daily exchange rate from 1983 to 1992. Vigfusson finds that the evidence is ‘favourable though inconclusive’. He encounters different variances in each of the two regimes which proved to be a dilemma since the different periods may be determined by high and low variance regimes rather than by chartist-fundamentalist behaviour. He suggests that this may be overcome by implementing a two regime Markov switching process in which each regime shares a common ARCH effect. This, he argues, would have the effect of distinguishing the chartist-fundamentalist effects on the exchange rate since the variance induced influence would be eliminated.

The Markov regime switching approach has also been considered by Kamincky (1993) and also Engel (1994). Engel uses this type of model since he observes that the Dollar exchange rate appears to follow ‘long swings’. Typically, it drifts upwards for a time and then switches to a downward drift. These regimes are not explicitly connected with changes in policy within the US. In my regime switching model, I assume that a change in regime is associated with a major change in economic policy. Engel uses quarterly and also monthly data for 18 exchange rates for the period 1973-1986. Of these exchange rates, 11 are non US rates. The period 1986-1991 is used for post sample forecasting. The reason that he chooses to use both quarterly and monthly data is that there is no particular frequency which measures changes in exchange rates. He notes that a particular Markov process may successfully model a certain frequency of data but there is no guarantee that it will fit the data for a different frequency. For these reasons, he examines the results of two different frequencies.

The results are disappointing in that the models are outperformed by forecasts of a random walk or of a forward rate. This is a surprising result since the Markov model outperforms generalised versions of the random walk in-sample. However, Engel finds that the model does not tend to have a lower mean squared error than either of these alternative models. In its favour,
the Markov model does prove to be superior in predicting the direction of exchange rate changes.

In considering further research in this area, Engel suggests a third state for the process. He notes that the Louvre accord of March 1987 had the effect of stabilising all exchange rates and this occurred just one year after the end of the period used for estimation. He argues that a regime switch at this point may be appropriate. The new state would be characterised by low variances and only small changes in exchange rates. If this were taken into account, it may be the Markov model would perform better than in this original study.

In spite of this result, the modelling of Markov processes remains popular. It has been shown that Markov processes can be positively serially correlated which is an advantage since they can model the exchange rate feature of volatility clustering. In the initial work on Markov processes, it was assumed that the probability of switching from one regime to another was constant throughout. However, Gray (1996) notes that this is no longer the case. He points to the work of Diebold, Lee and Weinbach (1994), Ghysels (1993) and Filardo (1993, 1994) as examples in which regime switching probabilities are allowed to change. Furthermore, Markov processes are flexible in that they can model data which has arisen due to different economic mechanisms. Gray notes that a single regime model may assume that the short rate is mean reverting. This implies that in the long run, the speed of reversion will remain the same throughout the period. However, in a regime switching model, the speed of reversion may differ in the long run. In effect, he argues that regime switching models allow for non-linearities but, at the same time, are easy to estimate and implement.

Jump Diffusion Models

A related area of study has been the literature on jump diffusion models. This has grown from the work developed by Jorion (1989) who models jump processes in the foreign exchange and stock markets. In this model, changes in the exchange rate are assumed to be drawn from a combination of distri-
butions. The number of jumps in each period is drawn from a Poisson or Benoulli distribution. He discovered evidence of discrete jumps in US Dollar exchange rates in addition to ARCH effects. In fact, such discontinuities appeared to be typical of the US exchange rate.

There are two crucial differences between this type of model and the Markov model. Firstly, it is assumed that jumps occur independently over time. Secondly, the conditional variance is increased each period by a constant amount due to the chance of discontinuous jumps in the exchange rate.

It is argued that this type of approach is especially suitable for modelling exchange rates within target zones. This is because they model the discontinuities of realignments associated with the data. Neely (1994) notes that a number of papers have explored ERM data using an ARCH framework. However, the work such as that of Diebold and Pauly (1988) has overlooked the problem of modelling realignments of the bands in which the exchange rates are allowed to move. Their paper concludes that the ERM had the effect of reducing the conditional volatility of the ARCH residuals of an AR(2) process for both the French Franc-Deutschmark and Lira-Deutschmark exchange rates. Neely argues that it is therefore appropriate to include a jump diffusion framework when considering ERM exchange rates.

This has been considered by Nieuwland, Verschoor and Wolff (1994) who use a Poisson specification of Jorion. They focus attention on the mean reverting properties of ERM exchange rates and also seek to explain the excess kurtosis observed in the data. They conclude that the model which offers the best fit for the data is a jump-GARCH model with conditionally $t$-distributed innovations. Vlaar and Palm (1993) also apply jump diffusion. They discuss the relative merits of the Bernoulli and Poisson distributions and conclude that there is very little between each of these approaches. In fact, they choose a normal and multivariate normal distributions for their jump diffusion models.

I shall now consider the recent literature which combines Markov processes with a GARCH framework. These are considered to be an improvement on the existing literature in modelling exchange rate movements. They also form the cornerstone to my own investigation into the data.
Of particular interest is the paper by Dueker (1994) who develops a compound model of GARCH and Markov switching in order to make the existing GARCH framework more flexible in analysing exchange rates in target zones. He, initially addresses some of the drawbacks with the existing literature. Firstly, not all jumps in European exchange rates have been of the same magnitude. Therefore, he proposes a model in which the size of the jump is endogenous. Furthermore, these changes in exchange rate are not limited to purely realignments. Secondly, he argues that models such as the one developed by Engel and Hakkio (1993) with a mean variance relationship assume that all periods of high variance are also periods of skewness. However, he argues that the large variance may be due to a large dispersion in the GARCH process with no switch in the Markov process.

As a third point he notes that a large amount of the literature assumes that the innovations are student-t rather than normally distributed and the number of degrees of freedom remain constant over time. However, his model allows these degrees of freedom in the student-t to change over time. Finally, he observes that previous models have taken the extreme views that realignments are innovations or are not innovations to the GARCH process. Dueker allows his model to endogenise the extent to which a realignment can be regarded an innovation in the process.

His model considers weekly data for EMS currencies with respect to the Deutschmark commencing in 1980. The bands in the exchange rate are defined in terms of the Ecu basket currency. He finds that this type of model is a good fit for the exchange rate data. He notes the valuable contribution of the Markov switching process in the student-t degrees of freedom in endogenising the weight placed on the previous period’s squared residual in the GARCH process. He also notes that the use of the Markov process is more appropriate than dummying out the EMS realignments from the GARCH conditional variances. Furthermore, it allows large shifts in the conditional variance so that volatility may return to normal levels within a short time after a large jump.

In estimating goodness of fit, he observes that when the student-t degrees of freedom parameter is allowed to switch, four out of the six currencies tested
pass the specification test.

This model raises an important issue which I consider in the latter part of this chapter. Dueker finds the Markov process of endogenising regime switches is superior to dummying out realignments in a standard GARCH model. I compare these two approaches by forming a correlation coefficient between a policy change dummy seen in the GARCH model and the smoothed probabilities of being in one regime as opposed to an alternative. The results reveal that the two are not highly correlated thereby indicating that each approach provides a very different type of information.

The work by Neely (1994) has already received a mention. This is another influential paper in this area. He develops a model which is a modified jump diffusion framework. He combines a time varying jump probability with a GARCH model in order to improve upon the existing literature in the jump diffusion area when considering target zones.

He claims that his paper improves upon previous models in three basic ways. Firstly, the issue of credibility is handled more thoroughly since information regarding this is incorporated into the model. This is achieved through a time varying probability of realignment for the ERM exchange rate. Secondly, he includes the absolute value GARCH models as used by Taylor (1986) and Schwert (1989) in an attempt to provide a better estimate of the forecast conditional variance. These are alternative types of GARCH model from the ones considered in the previous section and used in my analysis. Finally, he observes the conditional volatility at the times of realignment and finds evidence that this is higher in the weeks surrounding these periods.

Neely considers both exchange rates and interest rates from seven ERM countries and four non-ERM countries for the period from 1973 to 1992. All exchange rates were with respect to the Deutschmark and the sample period was chosen so as not to include the speculative attacks of September 1992.

Bekaert and Gray (1998) take a different approach to the problem. They argue that their analysis is more flexible than the original target zone literature pioneered by Krugman (1991). Within this literature, the exchange rate depends on fundamentals and the expected exchange rate. One of the fundamentals is controlled in order to keep the exchange rate within a certain
band while the other is assumed to follow a Brownian motion. The result is that the target zone is perfectly credible and is defended with only marginal interventions. However, tests of this model in Smith and Spencer (1991) and De Jong (1994) have not been favourable. As a result, a great many extensions have been made to the Krugman model. Bekaert and Gray propose a model which is a generalisation of Krugman's approach.

This paper is particularly relevant to my work since I adopt a number of its salient features. They estimate a single equation reduced form model for changes in the French Franc-Deutschmark exchange rate using maximum likelihood. They then compare this with the Deutschmark-Dollar rate. They use weekly data for the EMS period from 1979 until 1993.

They argue that Krugman's model can be restrictive and hence they offer a model which offers a more 'rich' characterisation of the conditional distribution of the exchange rate. They note that, during the EMS period, there was evidence of two different types of jump in exchange rate. Firstly, there are realignment jumps which occur at the time of realignment of a target zone. Secondly, there are jumps within the band for which there is no realignment of the target zone. I use this idea in setting up my GARCH model. I introduce realignment jumps and also jumps in exchange rate due to major policy shifts into the conditional variance function.

They further argue that they provide a more satisfactory measure of target zone credibility. When no jump occurs, the target zone is said to be credible. They note that, in the work of Rose and Svensson (1993) and Chen and Giovannini (1993), the authors rely on uncovered interest rate parity and interest rate differentials to make inferences about expectations of movements outside a band. However, since Bekaert and Gray condition the distribution of the exchange rate changes on a jump variable, they argue that they have a more appropriate measure of the credibility of a target zone. The size and probability of this jump are determined by macroeconomic variables.

Their final contribution is to examine the implied currency risk premia. They are critical of the commonly imposed assumption of uncovered interest rate parity since the empirical evidence is not entirely favourable.

They find that, contrary to previous literature in the area, the exchange
rate in question is non-linear and realignments can be predicted. Furthermore, they conclude that the credibility of the system did not increase after 1987. They also observe that the foreign exchange risk premium increases during a currency crisis. They compare these results with the Dollar-Deutschmark rate to examine the effect of a target zone on exchange rate behaviour. They conclude that target zones do have a substantial effect on the behaviour of exchange rates over time.

As in Bekaert and Gray, I distinguish between realignment jumps and within the band jumps. I also include the 'position in the band' variable in the mean and conditional variance. This allows me to compare the persistence of a standard GARCH model with one incorporating dependence on position in the band and realignment or policy change jumps. As future research, it would also be valuable to follow their analysis and actually model the exchange rate jumps.

Gray (1996) also develops a generalised regime-switching model of the short term interest rate. He notes that in much of the literature, the mean and variance of the short rate are typically held constant. However, in his approach, he allows for mean reversion and conditional heteroskedasticity. It is an extension of the typical Markov ARCH approach adopted by Cai (1994) and also Hamilton and Susmel (1994) since the conditional variances are allowed to incorporate the persistence of GARCH effects. Notably, each of the parameters in the GARCH process is regime dependent. He argues that this is an important feature since it is often observed that the persistence of individual shocks is typically lower in periods of extreme volatility. This is consistent with the scenario in which a large shock has the effect of reducing pressure on the system.

Furthermore, the conditional variance in each regime depends upon the level of the interest rate. The implication is that the conditional variance accommodates not only volatility clustering but also dependence on the interest rate. In addition, the Markov switching probabilities are state dependent and depend on the level of the interest rate. Gray argues that this model "delivers sensible results". He finds that it is more applicable to short term interest data than previous models. He further finds that the performance of out of
sample forecasting is good.

7.2 Data

The data has been obtained from the PACIFIC exchange rate service (Policy Analysis Computing and Information Facility in Commerce). This service provides daily exchange rates through an online database retrieval system. It is provided by Professor Werner Antweiler from the University of British Columbia, Vancouver, Canada.

In my analysis, I use weekly data. This is consistent with the studies of other economists. It is also known that ARCH effects weaken with less frequently sampled data. Baillie and Bollerslev (1989) use a Ljung Box portmanteau test to illustrate this. They show that this test result decreases gradually for the first ten autocorrelations for the squared logarithmic first difference of exchange rates averaged across six currencies. For daily data, this is a highly significant 130.6 whereas for monthly data this is insignificant at 10.6. I have chosen to report weekly data from Wednesday in each week. When this data was not available due to a holiday falling on that day, I have taken the next available day’s observation.

In figures 7.1-7.4, I plot the weekly data for each of the exchange rates to be considered. Firstly, consider the Pound. Figure 7.1 illustrates a steady depreciation against the Deutschmark until the end of 1995 followed by an appreciation of the currency. Table 7.9 provides a calendar of economic events corresponding to each of the large changes in the exchange rate. Figure 7.2 illustrates the Pound/Dollar exchange rate. This series is characterised by a large appreciation in the early 1980s followed by a large depreciation in the mid 1980s. Following 1992, volatility in this rate was greatly reduced with only small exchange rate movements. Table 7.10 provides the events corresponding to large exchange rate changes for this series.

Figure 7.3 shows the Lira/DM rate. Again this is characterised by a steady depreciation in the exchange rate until the end of 1995 followed by an appreciation. Note the period between 1987 and 1992 shows very little change in the exchange rate. By 1992, the EMS had celebrated 60 months without
a realignment. However, following 16th September, there is considerable movement and the exchange rate depreciates rapidly. Table 7.7 describes the economic events surrounding this period. By contrast, the Lira/Dollar rate given in figure 7.4, shows great volatility in the exchange rate until 1992 followed by only small exchange rate changes. The events are described in table 7.8.

The Lira spent the period between 1979 and 1992 in the ERM but the exchange rate floated thereafter. By contrast, the Pound spent only a small period of time in the ERM. It entered in 1990 and left in 1992. My aim is to provide a model which incorporates each of the states of being in and out of the ERM. I aim to capture the volatility clustering and fat tail distribution typical of a floating rate and also the ‘abrupt’ changes of realignment present in the ERM period.

For each of the exchange rates, I have included a table of summary statistics. This reports the mean and standard deviation of the data plus the maximum and minimum values. In addition, I have included measures of skewness and kurtosis. For a null hypothesis of i.i.d. normally distributed standardised residuals, $\frac{\hat{e}_t}{\sigma_t}$, has a mean of zero and a variance of $\frac{6}{\sqrt{n}}$ for skewness. When the reported value for skewness is significantly greater than zero, this indicates skewness to the right. When this value is significantly less than zero, this indicates skewness to the left. For each exchange rate, the results suggest skewness to the left. Kurtosis is a measure of curvature and for a null hypothesis of i.i.d. normally distributed standardised residuals, $\frac{\hat{e}_t}{\sqrt{\sigma_t^2}}$, has a mean of zero and a variance of $\frac{24}{\sqrt{n}}$. When the reported value is significantly greater than zero, this suggests thicker tails than normal. If the tails are thinner than normal, this value is significantly less than zero. In all cases, this reveals considerable kurtosis as is typical of this type of data. This is particularly noticeable in the Lira rates.

The Jarque-Bera test is also a test of the normality of the regression residuals and combines the measures of skewness and kurtosis. It is given by $\chi^2(2) = n(\frac{1}{6}\kappa_1 + \frac{1}{24}\kappa_2^2)$ where $\kappa_1$ is the squared reported result for skewness and $\kappa_2$ is the reported result for kurtosis. Clearly, the data suggests that the residuals are non normal and that most of this results from kurtosis.
Table 7.1: Summary Statistics for Lira and Pound Exchange Rates

<table>
<thead>
<tr>
<th></th>
<th>Lira/DM</th>
<th>Lira/Dollar</th>
<th>Pound/DM</th>
<th>Pound/Dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0893</td>
<td>-0.0698</td>
<td>-0.0607</td>
<td>-0.0318</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.8252</td>
<td>1.4786</td>
<td>1.1058</td>
<td>1.4699</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.5852</td>
<td>-1.0641</td>
<td>-0.5834</td>
<td>-0.2529</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>16.0301</td>
<td>10.7920</td>
<td>3.2321</td>
<td>3.3917</td>
</tr>
<tr>
<td>Jarque-Bera $\chi^2(2)$</td>
<td>10936.5</td>
<td>4955.8</td>
<td>572.1956</td>
<td>569.8450</td>
</tr>
<tr>
<td>Max</td>
<td>4.4639</td>
<td>6.7823</td>
<td>3.7004</td>
<td>7.3905</td>
</tr>
<tr>
<td>Min</td>
<td>-7.6227</td>
<td>-14.8593</td>
<td>-6.8262</td>
<td>-8.6422</td>
</tr>
</tbody>
</table>

Table 7.1: Summary Statistics for Lira and Pound Exchange Rates
Figure 7.1: The Pound/Deutschmark Exchange Rate
Figure 7.2: The Pound/Dollar Exchange Rate
Figure 7.3: The Lira/Deutsche Mark Exchange Rate

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Events of the EMS Period and their Impact on the Pound and Lira

Firstly, it is relevant to examine each of the realignments experienced by the Lira during this period and offer an explanation for why these were considered necessary. The UK did not enter the ERM until October 1990 and left in September 1992. It follows that the only realignments experienced during this period were those from entering and leaving the ERM.

<table>
<thead>
<tr>
<th>Realignment Date</th>
<th>Lira/DM Central Parities</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Mar 1979</td>
<td>457.314</td>
</tr>
<tr>
<td>24 Sept 1979</td>
<td>466.460</td>
</tr>
<tr>
<td>30 Nov 1979</td>
<td>466.460</td>
</tr>
<tr>
<td>23 Mar 1981</td>
<td>496.232</td>
</tr>
<tr>
<td>5 Oct 1981</td>
<td>539.722</td>
</tr>
<tr>
<td>22 Feb 1982</td>
<td>539.722</td>
</tr>
<tr>
<td>14 Jun 1982</td>
<td>578.574</td>
</tr>
<tr>
<td>21 Mar 1983</td>
<td>626.043</td>
</tr>
<tr>
<td>22 Jul 1985</td>
<td>679.325</td>
</tr>
<tr>
<td>7 Apr 1986</td>
<td>699.706</td>
</tr>
<tr>
<td>4 Aug 1986</td>
<td>699.706</td>
</tr>
<tr>
<td>12 Jan 1987</td>
<td>720.699</td>
</tr>
<tr>
<td>8 Jan 1990</td>
<td>748.217</td>
</tr>
<tr>
<td>16 Sep 1992</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.2: Realignment Dates and Central Parities for the Lira/DM Exchange Rate

The exchange rates given in this table are expressed in terms of spot ecus, which are recorded at a daily central bank conference at 2.30pm Swiss time.

The first point to note from these realignments is that the exchange rate did not jump at every realignment date. These realignments occur without a corresponding devaluation.

Secondly, it has been argued (Eichengreen and Wyplosz, 1993), that much of Italy’s financial problems were due to a competitiveness problem.
Hence the devaluations were necessary so as to maintain a degree of competition amongst its counterparts. At this stage, I will take each realignment in turn and explain the events surrounding it.

**June 1979** - The DM rose against the other currencies affecting the weaker members of the ERM. Central banks sold DM in an attempt to keep the Danish Krone from falling through its lower limit. Sterling was strengthened by North Sea Oil and appreciated against the dollar. There were concerns that the British economy would become uncompetitive as a result. Pressure on the Irish Punt was seen because of the strong economic ties between Ireland and Britain.

**September 1979** - The Danish Krone fell through its lower limit due to upward pressure on the DM against the dollar.

**November 1979** - Denmark devalued its currency by 5%.

**March 1981** - Italy realigned its currency due to competitiveness problems. In addition, Denmark was suffering a sizeable current account deficit and there were implications for the Belgian Franc.

**October 1981** - France and Italy devalued their currencies while the DM and Dutch Guilder were revalued. This follows the decline of the Franc in May 1981 which occurred as a result of an increase in US interest rates. In September of the same year, the Danish Krone was weakened as a result of Sweden’s actions to devalue its currency against a basket of currencies.

**February 1982** - Belgium devalued its currency by 8.9% due to high unemployment and a large current account deficit. This led to Denmark devaluing the Krone by 3%.

**June 1982** - The French Franc was devalued by 5.75%. This was triggered by the strong Dollar and speculation against the currency. As a result, France lost in excess of two thirds of its foreign exchange reserves.

**March 1983** - There had been fears dating back to December 1982 over the strength of the Irish Punt since sterling was weakening. Sterling then fell to
a record low in March 1983. In France, expansionary policies put in place by the socialist government were creating concerns on the currency markets. France threatened to leave the EMS but opted for a realignment instead.

**July 1985** - A further realignment in the system. The Lira effectively devalued by 8% within the EMS.

**April 1986** - The French Franc was in trouble again. It was devalued by 3% on 7th April 1986. However, there were concerns that this was insufficient. The 0.25% reduction in the Bank of France’s discount rate confirmed these suspicions.

**August 1986** - The Punt was devalued by 8% in August due to concerns of lower oil prices and their impact on sterling.

**January 1987** - The French Franc fell to the bottom of its band. This was triggered by student riots and strikes. At the same time, Italy announced plans to liberalise exchange controls. The Danish Krone also devalued by 3% and the Belgian Franc by 2%.

**January 1990** - A further realignment in the system. This followed political upheaval in Central and Eastern Europe.

**June 1992** - Denmark rejected the Maastricht treaty. At the end of the month, sterling was approaching the bottom of its band since speculators sold it in the belief that the monetary union may be delayed.

**August 1992** - There was further heavy selling of sterling following the statement of the Bundesbank president that the Pound should be devalued.

**September 1992** - Sterling and Lira left the ERM. The Spanish Peseta was forced to devalue and the Irish Punt appeared on the point of devaluation. The French Franc was also weak and hence the Belgian Franc was also in danger. Following this period, the Lira continued to be under pressure of depreciation as was sterling.
7.4 GARCH Model and its Extensions

The aim of this chapter is to provide a model which incorporates each of the states of being in and out of the ERM. Therefore, a model is needed which captures the volatility clustering and fat tail distribution typical of a floating rate and also the 'abrupt' changes of realignment present in the ERM period. Engel and Gau (1997) find that an MA(1)-GARCH(1,1) provides a suitable fit for the data.

In order to decide which was the more appropriate model for the mean of the change in the exchange rate, I tested it for evidence of serially correlated errors. This involved collecting the residuals from the regression of

\[ \Delta \log s_t = a_0 + \epsilon_t \]  \hspace{1cm} (7.6)

and then performing the regression below:

\[ \epsilon_t = \alpha_0 + \alpha_1 \epsilon_{t-1} \]

where the \( \epsilon_t \)s are estimated residuals. This was then tested by comparing \( nR^2 \) with a \( \chi^2(1) \) statistic. In table 7.3, I show the \( R^2 \) value, the test statistic and also the 5% critical value. Notably, the only exchange rate for which there is some evidence of serially correlated errors is the Lira/DM rate. For this reason, I have used a simple mean function as shown below.

\[ \Delta \log s_t = a_0 + \epsilon_t \]  \hspace{1cm} (7.7)

where

\[ \epsilon_t \sim N(0, \sigma^2_t) \]  \hspace{1cm} (7.8)

conditional on \( I_{t-1} \) observable information.

For each of the exchange rates, I tested for ARCH(1) and ARCH(4) as a preliminary investigation of the data. The results shown in table 7.4 indicated
Table 7.3: A Test of Serial Correlation in the Mean Exchange Rate

that long lags in the variance were required and hence, this prompted me to model the data in terms of a GARCH process.

Table 7.4: ARCH Tests of the Exchange Rate Data

Therefore, the conditional variance was modelled as follows:

\[ \sigma_t^2 = b_0 + b_1 \sigma_{t-1}^2 + b_2 \sigma_{t-2}^2 \]  

(7.9)

The results are reported in table 7.5 and are set out to show the parameter estimate, its standard error and also the associated t statistic where the statistic is calculated as follows:

\[ t_0 = \frac{\hat{\beta} - \beta}{SE(\hat{\beta})} \]

This is then compared with the critical value. For the 5% level this is |1.645|. For the 10% level this is |1.282| and for the 25% level it is |0.674|.

In addition, I have performed a Ljung Box portmanteau test for model misspecification. This tests up to 15th order serial correlation in the squared standardised residuals, \( \frac{\hat{c}^2}{\sigma_t^2} \). If the model has been correctly specified, then
the null of independent residuals cannot be rejected and the LB statistic will not be significantly different from zero. For all exchange rates tested here, this null cannot be rejected at the 5% level. I also measure the degree of skewness and kurtosis in the residuals. For a null hypothesis of i.i.d. normally distributed standardised residuals, \( \frac{\hat{\kappa}_1}{\sqrt{\sigma^2}} \), has a mean of zero and a variance of \( \frac{6}{\sqrt{n}} \) for skewness and \( \frac{24}{\sqrt{n}} \) for kurtosis. Note that in each case, while skewness does not represent a large problem, there is considerable evidence of kurtosis especially in the Lira rates.

This is supported by the Jarque-Bera test for normality in the residuals. As described earlier, this is given by: \( \chi^2(2) = n\left( \frac{6}{\hat{\kappa}_1} + \frac{1}{24}\hat{\kappa}_2^2 \right) \) where \( \hat{\kappa}_1 \) is the squared reported result for skewness and \( \hat{\kappa}_2 \) is the reported result for kurtosis. In each case, there is considerable evidence of non-normality. Much of this is due to excess kurtosis.

In examining the results I shall discuss each parameter in the model. Firstly there is the constant term, \( a_0 \). The sign on this indicates whether there has been an appreciation or depreciation in the currency over the time period. Clearly for the Pound and Lira, this suggests a depreciation of the currency against both the Deutschmark and the Dollar. Examining the estimates, all are significant at the 25% level but only the estimate for the Lira/DM is significant at the most commonly used 5% level.

The \( b_0 \) term is the constant term in the conditional variance function. For the Pound, this is greater for the Pound/DM exchange rate than it is for the Pound/Dollar rate. However, for the Lira the opposite scenario holds. All results are statistically significant at the 5% level.

The \( b_1 \) term is the parameter on the lagged squared errors in the conditional variance function. This is the moving average component of the model and indicates the extent to which past errors determine the error variance of the data. The Pound/DM and Pound/Dollar rates display very similar values of this parameter. However, the estimate on the parameter for the Lira/Dollar is far greater than that on the Lira/DM. Once again, all results are statistically significant at the 5% level.

The final parameter, \( b_2 \), is the parameter on the lagged error variances and forms the autoregressive component of the model. A significantly large
value here indicates that the error variance is heavily dependent on its past values. Notably, each result is significantly greater than zero.

Within this framework, persistence of the GARCH effect can be examined by considering $b_1 + b_2$. If this is not significantly different from 1, then one cannot reject the hypothesis that there is complete persistence in the conditional variance function. This would suggest possible integration in the variance and thus point to IGARCH as a suitable model. However, in the context of this analysis I do not pursue this possibility. Note that, for each rate, the results indicate a large amount of persistence.

### 7.4.1 Position in the Band

It has been shown (Bekaert and Gray, (1998)) that including a currency’s position in the band into the conditional mean encapsulates the idea of mean reversion in the exchange rates. It is argued that this type of model is consistent with Krugman (1991) since the expected change in the exchange rate is larger when the exchange rate is close to the edge of its band.

The original GARCH model is adapted as follows:

\[
\Delta \log s_t = a_0 + a_1 PB_t + \epsilon_t \quad (7.10)
\]

where

\[
\epsilon_t \sim N(0, \sigma_t^2) \quad (7.11)
\]

conditional on $I_{t-1}$ observable information and

\[
\sigma_t^2 = b_0 + b_1 \epsilon_{t-1}^2 + b_2 \sigma_{t-1}^2 + b_3 |PB_{t-1}| \quad (7.12)
\]

The position in the band takes on a value in the interval, $[-1, 1]$. When the exchange rate is at the centre of its band, $PB_t = 0$. When the exchange rate is at its upper boundary, $PB_t = 1$ and at its lower boundary it is $PB_t = -1$. It is calculated using the following:
\[ \frac{s_t - c_t}{\frac{1}{2}(u_t - l_t)} \]

where \( s_t \) is the exchange rate, \( c_t \) is the centre of the target zone, \( u_t \) is the upper boundary of the target zone and \( l_t \) is the lower boundary of the target zone.

I applied this to the data sets for the Lira and Pound. Plots of each currency’s position in the band are given in figures 7.5 and 7.6.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pound/DM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>-0.0390</td>
<td>0.0325</td>
<td>-1.2</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.1274</td>
<td>0.0206</td>
<td>6.1845</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0907</td>
<td>0.0170</td>
<td>5.3353</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.8081</td>
<td>0.0295</td>
<td>27.3932</td>
</tr>
</tbody>
</table>

Mean Log Likelihood = -1.49332
Jarque-Bera Test $\chi^2(2) = 1031.7897$
Skewness = -0.7370 Kurtosis = 4.3726
Ljung-Box (1) = 0.12323 Ljung-Box (15) = 7.8280

| **Pound/Dollar** | | | |
| $a_0$ | -0.0288 | 0.0388 | -0.7423 |
| $b_0$ | 0.0905 | 0.0196 | 4.6173 |
| $b_1$ | 0.0831 | 0.0139 | 5.9784 |
| $b_2$ | 0.8760 | 0.0214 | 40.9346 |

Mean Log Likelihood = -1.74565
Jarque-Bera Test $\chi^2(2) = 594.67818$
Skewness = 0.53018 Kurtosis = 3.3388
Ljung-Box (1) = 0.29620 Ljung-Box (15) = 6.9052

| **Lira/DM** | | | |
| $a_0$ | -0.0456 | 0.0256 | -1.7813 |
| $b_0$ | 0.0397 | 0.0022 | 18.0455 |
| $b_1$ | 0.1322 | 0.0098 | 13.4898 |
| $b_2$ | 0.8168 | 0.0094 | 86.8936 |

Mean Log Likelihood = -1.05242
Jarque-Bera Test $\chi^2(2) = 111143.31$
Skewness = -3.8056 Kurtosis = 51.5329
Ljung-Box (1) = 0.003629 Ljung-Box (15) = 0.69991

| **Lira/Dollar** | | | |
| $a_0$ | -0.0374 | 0.0409 | -0.9144 |
| $b_0$ | 0.1747 | 0.0484 | 3.6095 |
| $b_1$ | 0.2386 | 0.0291 | 8.1993 |
| $b_2$ | 0.7182 | 0.0411 | 17.4745 |

Mean Log Likelihood = -1.76288
Jarque-Bera Test $\chi^2(2) = 13559.29$
Skewness = -1.5298 Kurtosis = 17.9356
Ljung-Box (1) = 0.49882 Ljung-Box (15) = 6.6839

Table 7.5: The GARCH Model Using Pound And Lira Exchange Rate Data

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Figure 7.5: The Position of Sterling in the ERM Band
Figure 7.6: The Position of the Lira in the ERM Band
The results of the modified GARCH framework may be found in table 7.6. The interpretation of the variables is as before with the addition of the position in the band variable in the mean and the lagged absolute value of the position in the band variable in the conditional variance. I shall take each exchange rate in turn and also compare the results with the standard GARCH model.

Firstly, there is the Pound/DM rate. In the mean function, the parameter estimate on the constant has dropped slightly but is still negative indicating an overall depreciation of the currency. The t statistic has dropped and the result is insignificant at the 5% level but significant at the 25% level. The parameter on $PB_t$ is large but so too is the standard error. The t statistic suggests that this is not significant at the 5% level but again, it is significant at the 25% level.

Within the conditional variance function, the constant is still statistically significant although lower than in the original framework. The parameter on the lagged squared error term is also slightly lower as is its standard error. The parameter on the lagged conditional variance remains highly significant. The final parameter is that of the absolute position in the band variable. This is significantly different from zero. The overall result suggests that the standard GARCH model can be improved upon by including the lagged absolute value of the position in the band variable. However, the current position in the band does not have a significant effect upon the mean function. The persistence of the system has been reduced by the inclusion of the position in the band variable. This is in line with the findings of Bekaert and Gray in their examination of the French Franc against the Deutschmark.

The LB statistics are statistically insignificant and the degree of skewness and kurtosis are smaller than under the original GARCH framework. However, the Jarque-Bera test still indicates non normality in the residuals.

I now consider the Pound/Dollar rate. The parameter on the constant in the mean function is still negative indicating an overall depreciation but this is less than under the original GARCH framework and insignificant at the 5% level. The parameter on the position in the band variable is large but its standard error is also large as in the Pound/DM case producing a result.
which is not significant at the 5% level.

In examining the conditional variance function, the parameter on the constant is still significant and positive. The parameter on the lagged squared error term is statistically significant. The same may be said of the parameter on the lagged conditional variance. As in the case of the Pound/DM rate, the parameter on the absolute lagged value of the position in the band is large and significant.

Therefore, a pattern is emerging for the Pound. In each case, the position in the band did not play a significant part in the mean function. However, the lagged absolute value did play a part in the conditional variance function. This is not surprising since I have examined a large data set and the UK was only a member of the ERM for a very small proportion of this time (approximately 100 observations out of 1164). Furthermore, the persistence given by $b_1 + b_2$ has been reduced by including this extra variable.

The LB(15) statistic is insignificant at the 5% level. As with the Pound/DM, skewness and kurtosis have also been reduced but the Jarque-Bera test indicates non normality in the residuals.

This correlation between the Pound/DM and Pound/Dollar rates may be emerging because of the link between the DM and the Dollar. If these currencies are strongly related, then it is no surprise that events in Europe will ultimately impact on the Dollar and vice versa.

I now consider the Lira/DM rate and compare with the standard GARCH framework. Firstly, the parameter on the constant in the mean function is negative as one would expect from a depreciating currency but less than under the standard model. It is still statistically significant at the 5% level. The parameter on the position in the band is significant contrary to the results of the Pound rates. This is consistent with my findings in table 7.3 indicating serial correlation in the errors when the simple mean function given by equation 7.7 is used.

The parameter on the constant in the variance function is considerably smaller than under the original model but still significant and positive. Notably, the parameter on the lagged squared error term is larger than under standard GARCH as is that on the lagged squared variance. Both are sig-
significant at the 5% level. The parameter on the absolute lagged value of the position in the band is small but still significant.

In contrast to the Pound exchange rates, the persistence found in the Lira/DM rate has actually increased as a result of including the position in the band variable.

The LB(15) statistic is insignificant at the 5% level. More importantly, is that kurtosis has dropped considerably as has the degree of skewness. However, the Jarque-Bera test still shows considerable evidence of non normality.

A different story may be told for the Lira/Dollar rate. As before, the parameter on the constant in the mean function has a negative sign and is less than under the standard GARCH model. The parameter on the position in the band variable is less than under the Lira/DM rate. Again, this is no surprise since one would expect the position in the band variable to be more relevant to the Lira against an EMS currency then against a non-EMS currency such as the dollar. It is significant at the 10% level but not at the 5% level.

The parameter on the constant in the conditional variance function is smaller than under the standard GARCH but significant at the 5% level. The parameter on the lagged squared error term is still significant but less than under the original framework. However, the parameter on the lagged conditional variance is larger than under the original framework and highly significant. For this exchange rate, the parameter on the lagged absolute value of the position in the band is large and significant. The persistence has been reduced slightly by including the position in the band.

Again, the LB(15) statistic is insignificant at the 5% level and the degree of kurtosis has dropped considerably as has skewness. The Jarque-Bera test, however, shows evidence of non normality in the residuals.

These results suggest that the position in the band plays a role in both the Lira/Dollar and Lira/DM rates although the results are more pronounced for the Lira/DM.
7.4.2 Shocks to the Currency

It may be argued that GARCH effects are often detected in financial data because this data includes a number of outliers. In effect, a model of GARCH is a model of these outliers. Therefore, it seems more appropriate to ‘dummy out’ these shocks and then to test for evidence of GARCH. This is what I achieve in the following modified GARCH model. I include dummies for realignments and monetary policy changes into the conditional variance function. As a result, I still find evidence of GARCH and also find strong evidence to support these shocks.

I set up two dummies for the Italian Lira. The first indicates realignment dates within the ERM. These have been given in table 7.2. The second captures those announcements or policy changes which cause a significant change in the exchange rate.

For the case of the UK, I set up the one dummy to take into account those policy changes or announcements which generate a significant movement in the exchange rate.

In tables 7.7-7.10, I show the dates at which the exchange rates have moved by a significant amount and indicate the possible cause. This information has been obtained from the National Institute Economic Review (1975-97). It is from this information that I build the dummy variables described above.

Note that 1992 did not represent the first currency crisis experienced by the UK. 1976 also proved to be a year of sterling crises. As noted in the National Institute Economic Review (1977), ‘It was the year of sterling crises, of worsening unemployment, of public expenditure cuts and of the IMF visit’. This helps to explain the need for a dummy variable to capture large policy changes which occurred as a result of this and other events.

I incorporate these shocks into the GARCH model as follows.

\[
\Delta \log s_t = a_0 + \epsilon_t
\]  
(7.13)
where

$$\epsilon_t \sim N(0, \sigma^2_t)$$  \hspace{1cm} (7.14)

conditional on \(I_{t-1}\) observable information and

$$\sigma^2_t = b_0 + b_1 \epsilon^2_{t-1} + b_2 \sigma^2_{t-1} + d_0.Dum$$  \hspace{1cm} (7.15)

where \(Dum\) denotes the dummy for large policy shocks. In its place, one may also test for the realignment dummy on the Italian exchange rate data.

The results may be seen in tables 7.11 and 7.12. I firstly discuss the results of including the policy change dummy and compare with the standard GARCH model. For the Pound/DM, the parameter on the constant in the mean function is still negative and insignificant at the 5% level whereas the parameter on the constant in the conditional variance function is large and significant at the 5% level. The same may be said of the parameter on the lagged squared error term. The parameter on the lagged conditional variance is still significant but considerably smaller than originally. Finally the parameter on the dummy variable is large and the result is significant at the 5% level. Note also that by including the dummy into the conditional variance function, the persistence has reduced dramatically.

The degree of skewness and kurtosis is smaller than under original GARCH although the Jarque-Bera test still indicates non normality. However, the LB(15) statistic is still insignificant at the 5% level.

In considering the Pound/Dollar rate, the parameter on the constant in the mean function is still negative as expected but insignificant and smaller than under original GARCH. The parameter on the constant in the variance function is larger than in the original model as is that on the lagged squared error term. Both are statistically significant. The parameter on the lagged conditional variance is highly significant at the 5% level. Finally, the parameter on the policy change dummy is large and significant with a considerably smaller standard error than for the Pound/Dollar. Persistence has been reduced but not by as much as for the Pound/DM rate.
As with Pound/DM rates, the LB(15) statistic is not significant at the 5% level. Skewness and kurtosis are smaller than under the original GARCH setup but the Jarque-Bera test indicates non normality in the residuals.

I now examine the Lira exchange rates. I firstly consider the Lira/DM rate. The parameter on the constant in the mean function is still negative in sign and significant whereas the constant in the variance function is less than under the original GARCH model. However, it remains statistically significant. The parameter on the lagged squared error term is larger than originally but the parameter on the lagged conditional variance is slightly smaller although still statistically significant. The parameter on the dummy variable is again large and significant. Persistence is only slightly lower than under the original GARCH model.

The LB(15) statistic is insignificant at the 5% level. However, as before, the Jarque-Bera test indicates non normality in the residuals.

For the Lira/Dollar rate, the parameter on the constant in the mean function is still negative and insignificant at the 5% level but significant at the 25% level. The parameter on the constant in the variance function is considerably larger than under the original GARCH model and also for the Lira/DM rate. The parameters on the lagged squared error terms and lagged conditional variance are considerably smaller than with the original GARCH model. However, they are still statistically significant. The parameter on the dummy is large and significant at the 5% level. Including this policy change dummy has had the effect of reducing persistence considerably.

Here, the LB(15) statistic is now significant at the 5% level. However, skewness and kurtosis are now negligible and the Jarque-Bera test suggests that the residuals are normal.

I now consider the results of including a realignment dummy in place of a policy shock dummy for the Lira. I compare the results with the original GARCH model and of the model including the policy shift dummy. Firstly, I take the case of the Lira/DM rate. The parameter on the constant in the mean function is statistically insignificant. Furthermore, it is much smaller than under the original GARCH and policy change models. The parameter on the constant in the conditional variance function is larger than that of
the original GARCH and policy change models. It is also significant at the 5% level. The parameter on the lagged squared error term is considerably larger than under each of these alternative models. Notably, the parameter on the lagged conditional variance is still significant but considerably smaller than under original GARCH or policy change models. The parameter on the realignment dummy is significant at the 5% level. Persistence as measured by $b_1 + b_2$ is greater than under the original GARCH framework.

The LB(15) statistic is insignificant at the 5% level. Kurtosis has dropped from 51.5329 to 10.0624 and skewness has also fallen. However, the Jarque-Bera test still indicates considerable evidence of non normality.

In considering the Lira/Dollar rate, a similar pattern emerges for the mean function. The parameter on the constant is insignificant and smaller than under the previous two models. The parameter on the constant in the variance function is smaller than with the policy shift dummy but greater than with the original GARCH model and statistically significant. The parameter on the lagged squared error term is smaller than with original GARCH and policy change models but still significant at the 5% level. The parameter on the lagged conditional variance is greater than for the model containing the policy change dummy but smaller than that for the original GARCH framework. The parameter on the realignment dummy is smaller than for the policy change dummy but equally its standard error is also smaller. The result is a t statistic which far exceeds the 5% critical level. Note also that persistence is less than under the original GARCH model.

Skewness and kurtosis are both smaller than under original GARCH and the LB(15) statistic remains insignificant at the 5% level. However, the Jarque-Bera test still indicates non normal residuals.

### 7.4.3 A GARCH Model Including Position in the Band and Policy Changes

In this section, I incorporate both the position in the band and policy change variables into the standard GARCH framework. As before, the lagged absolute value of the position in the band appears in the conditional variance
function. The current position in the band appears in the mean function. However, there is now a dummy variable in the conditional variance function for major shifts in policy. The model becomes:

\[ \Delta \log s_t = a_0 + a_1 PB_t + \epsilon_t \quad (7.16) \]

where

\[ \epsilon_t \sim N(0, \sigma_t^2) \quad (7.17) \]

conditional on \( I_{t-1} \) observable information and

\[ \sigma_t^2 = b_0 + d_0 Dum + b_1 \epsilon_{t-1}^2 + b_2 \sigma_{t-1}^2 + b_3 |PB_{t-1}| \quad (7.18) \]

As before, I consider each exchange rate in turn. The results may be found in table 7.13. For the Pound/DM, the constant in the mean is still statistically insignificant while the parameter on the position in the band is significant at the 10% level. In the conditional variance function, the constant is highly significant at the 5% level as is the parameter estimate on the lagged squared error term and also the lagged conditional variance. The lagged position in the band, however, appears to play no role in this model. The parameter on the policy change dummy is significant at the 5% level. The key feature of this model involves the persistence of the system. Note that \( b_1 + b_2 \) is smaller than under the original GARCH framework. This supports the work of Bekaert and Gray (1998) and Jorion (1988) who each show that allowing for jumps greatly reduces the degree of persistence of shocks to exchange rates.

In looking at model misspecification, the LB(15) statistic is insignificant at the 5% level. Skewness and kurtosis are also considerably reduced although the Jarque-Bera test suggests some non normality in the residuals.

A different story may be said of the Pound/Dollar. The constant in the mean function is still insignificant at the 5% level as is the position in the band. However, the constant in the conditional variance function is signific-
tant as are the parameters on the lagged squared errors and lagged conditional variance. The estimate of the lagged absolute value of the position in the band is only significant at the 25% level. The parameter on the policy change dummy is highly significant. Note that in this instance, there is still considerable persistence as exhibited by $b_1 + b_2$.

Again the Ljung Box statistic is statistically insignificant. Measures of skewness and kurtosis are again, smaller than under original GARCH. However, the Jarque-Bera test suggests some non normality in the residuals but much less than under any of the other models considered.

I now consider the Lira exchange rates. For the Lira/DM, the constant in the mean function is significant at the 10% level. The parameter estimate on the position in the band variable is significant at the 5% level. Within the conditional variance function, the constant is highly significant at the 5% level as are the parameter estimates of the lagged squared errors and lagged squared variance. However, in this model, there is no role for the lagged position in the band variable. The policy change dummy is also significant. There is still considerable persistence exhibited in this model.

The LB(15) statistic is still insignificant at the 5% level and skewness and kurtosis are considerably smaller than under the original GARCH model although the Jarque-Bera test suggests that the residuals are still non normal.

For the Lira/Dollar rate, the constant in the mean function is only significant at the 25% level and there is no role for the position in the band within this function. This is not unexpected given the results of the Pound/Dollar and the fact that one would not expect the position in the band to play a large part in a non-EMS rate.

However, each of the parameter estimates in the conditional variance function are significant at the 5% level. Furthermore, as in Bekaert and Gray, the inclusion of the absolute value of the position in the band and the policy change variable significantly reduces the persistence.

Measures of skewness and kurtosis are far less than under original GARCH and insignificant at the 5% level. The Jarque-Bera test indicates normal residuals and the LB(15) statistic is also statistically insignificant at this level.
7.5 Summary of GARCH Results

The standard GARCH model provides an adequate description of the data. For each of the exchange rates considered, there was considerable evidence of persistence as indicated by $b_1 + b_2$. However, I argue that for the time period in which the currency was inside the ERM band, the position in the band played an important part in exchange rate determination. This was apparent from the results of the modified GARCH model allowing for the position in the band in mean and variance functions.

I also argue that 'news' or major policy announcements have an impact on the dynamics of exchange rate changes. It is argued that this type of announcement manifests itself as an outlier in the data and hence GARCH effects are merely the results of attempting to model these outliers. For these reasons, I introduce a dummy variable for policy changes and another for ERM realignments into the conditional variance function. Significantly, I still find evidence of GARCH. I also find that the parameter on the dummy variable is significant.

I then incorporate policy changes and position in the band information into the standard GARCH framework. This is a better model of the exchange rates considered. This is supported by the Ljung-Box portmanteau tests and also the tests for skewness and kurtosis.

7.6 Regime Switching Model

The regime switching model has proved popular in modelling financial data since it captures switches in the direction of movement of the variable in question. Engle observes that the dollar exchange rate appears to follow ‘long swings’ in that it drifts upwards for a period of time and then switches to a downward drift. Similarly, this type of approach has been suitable for modelling US interest rate behaviour between 1979 and 1982. During this time, policy changes by the Fed led to volatility in the interest rate.

Therefore, this model is appropriate for analysing the Lira and Pound exchange rates both in and out of the ERM. During this period, there have
been major policy changes for each of the countries in question. My aim in this part of the chapter is to find if the regime switching model is better at detecting these changes than the GARCH framework incorporating a policy change dummy. If it is no better, then it would be simpler to use a modified GARCH model in examining exchange rate data.

I assume that there are two possible exchange rate regimes for the Pound and Lira. In regime 1, the exchange rate is low and in regime 2 it is high. The intuition here is that the first regime corresponds with a depreciating currency and the second with an appreciating currency. A switch in this model implies a movement from one regime to another.

This model calculates regime-switching estimates for a two regime Hamilton Markov model. It is assumed here that there is constant mean and variance in each regime. Conditional normality is assumed for each regime so that the conditional distribution of $\Delta s_t$ is a combination of distributions depending on the regime. In regime 1, it is $N(\mu_1, \sigma_1^2)$ and in regime 2 it is $N(\mu_2, \sigma_2^2)$. The first regime occurs with probability $p_{1t}$ and the second with $1 - p_{1t}$. This can be written as:

$$\Delta s_t | I_{t-1} \sim \begin{cases} N(\mu_1, \sigma_1^2) & \text{w.p. } p_{1t} \\ N(\mu_2, \sigma_2^2) & \text{w.p. } 1 - p_{1t} \end{cases}$$  \hspace{1cm} (7.19)$$

where:

$$p_{1t} = Pr(S_t = 1|I_{t-1})$$  \hspace{1cm} (7.20)$$

so that:

$$p_{1t} = Pr(S_t = 1|s_{t-1}, s_{t-2}, \ldots)$$  \hspace{1cm} (7.21)$$

The regime indicator variable, $S_t$ is parameterised as a first order Markov process so that:
\[
\begin{align*}
Pr(S_t = 1|S_{t-1} = 1) &= P \\
Pr(S_t = 2|S_{t-1} = 1) &= (1 - P) \\
Pr(S_t = 2|S_{t-1} = 2) &= Q \\
Pr(S_t = 1|S_{t-1} = 2) &= (1 - Q)
\end{align*}
\] (7.22)

It follows that \( p_{1t} \) is given by:

\[
p_{1t} = (1 - Q) \left[ \frac{g_{2t-1}(1 - p_{1t-1})}{g_{1t-1}p_{1t-1} + g_{2t-1}(1 - p_{1t-1})} \right] + P \left[ \frac{g_{1t-1}p_{1t-1}}{g_{1t-1}p_{1t-1} + g_{2t-1}(1 - p_{1t-1})} \right]
\] (7.23)

in which:

\[
g_{1t} = f(\Delta s_t|S_t = 1)
\]

and

\[
g_{2t} = f(\Delta s_t|S_t = 2)
\]

The results are reported in table 7.14. For each set of exchange rate data, the likelihood function is constructed so as to obtain estimates for the conditional probabilities, \( P \) and \( Q \), the means in each regime, \( \mu_1 \) and \( \mu_2 \) and the variances in each regime, \( \sigma_1^2 \) and \( \sigma_2^2 \). I report the standard errors and also the relevant t-statistics in order to evaluate the suitability of the model.

The first point to note is that for each set of exchange rate data, the estimate of \( \mu_1 \) is negative indicating a depreciation in the currency in regime 1. In the case of UK data, this is particularly marked for the Pound/DM. The t statistic shows that this is significant at the 5% level. However, this effect is not so profound for the Pound/Dollar which is only significant at the 25% level. For the Italian Lira, this estimate is large both for the Lira/DM
and the Lira/Dollar. The former is significant at the 5% level while the latter is significant at the 10% level.

The estimate of the mean in regime 2, is small and positive for each of the Pound/DM and Pound/Dollar indicating an appreciation in the currency which one would expect following a realignment in the currency. However, for the Pound/Dollar, this is statistically insignificant whereas for the Pound/DM, this is significant at the 5% level.

For each of the Lira rates, these values are negative which indicates a depreciation in regime 2. The Lira/DM rate is statistically significant at the 5% level while the Lira/Dollar is not.

I now consider the variances for each regime. For the Pound, the significant estimate on the variance of regime 1 is almost the same for the Pound/DM and Pound/Dollar both of which are highly significant at the 5% level. For the Lira, this figure is particularly large for the Lira/Dollar rate.

In all cases, the estimate on the variance of regime 2 is smaller than for regime 1. This is consistent with the findings of Vigfusson (1996). In trying to analyse chartist-fundamentalist behaviour in Canada-US daily exchange rates, he discovered that the two regimes were instead characterised by high and low variance regimes. This is an important feature of this model. It has shown that for the regime in which the currency is depreciating, there is a large conditional variance. However, in the second regime, where there is a smaller degree of depreciation or an appreciation in the currency, the conditional variance is smaller.

In examining the conditional probabilities of being in each regime, the estimates on $P$ and $Q$ are in every case greater than 0.85. This suggests a large probability of the current regime being the same as that which prevailed in the previous period. For the Lira, $Q$ exceeds $P$ suggesting that if regime 2 was followed in the previous period, it is more likely to continue in the current period than if regime 1 had been followed in the last period.

The same scenario exists for the Pound/DM but curiously not for the Pound/Dollar rate. Here, $P$ exceeds $Q$ suggesting that if regime 1 has been followed in the past, it is more likely to continue than if regime 2 had been followed in previous periods.
7.6.1 A Comparison of the Regime Switching Probabilities with the Policy Change Dummy

In running the regime switching model, I also constructed the smoothed probabilities associated with the probability of being in regime 1. Plots of these are given in figures 7.7-7.10. I shall take each of these in turn and attempt to interpret the information shown. By examining the dates at which the probability of being in regime 1 is close to or equal to 1, it is possible to form a comparison with the policy change dummy formed in the previous section. It may be argued that if a policy change dummy can perform the job of a regime switching model in predicting a policy switch, then it would be easier to use a GARCH model with a dummy included as shown in the previous section. Following this, I will use a correlation coefficient to test the correlation between these two variables.

Figure 7.7 shows the smoothed probability for regime 1 for the Pound/DM rate. This series is characterised by a series of peaks and troughs indicating a regular change in regime.

The regime switching model detects a high probability of being in regime 1 for each of the major policy events indicated by the policy change dummy. In addition to this, it also highlights other events for which there was a smaller movement in the exchange rate regime. Thus, I argue that the regime switching model is sensitive to small policy changes and effective for the modelling of the Pound/DM rate. In addition to those events given by the dummy it detects the following:


15th and 29th September 1976 - I can find no concrete evidence to suggest a policy change in this period.

1st November 1978 - Minimum lending rate increased to 12.5%.

19th September 1979 and 22nd October 1980 - Again, I can find no concrete
evidence to suggest a policy change here.

5th November 1980 - Lead up to election for leadership of labour party.

10th December 1980 - A number of increases in US banks prime lending rate has impact on European rates.

15th June 1983 - The Conservative Party wins the General Election with an overall majority of 144. EC finance ministers to allow the European Commission to borrow the equivalent of 1500 million Ecus on international currency markets for project financing in member states. Clearing banks cut base lending rates from 10 to 9.5%.

27th March 1985 - Nigel Lawson announces the Budget. Leading banks reduce base lending rates by half percentage point to 13.5%.

11th December 1985 - Again, I can find no concrete evidence to suggest a policy change here.

27th January 1993 - Irish Punt devalued by 10 per cent. UK base rates reduced to 6% - lowest level for 15 years.

Invariably, this ‘news’ is bad and thus indicates a depreciating currency.

For the Pound/Dollar rate, the model detects a high probability of being in regime 1 for each of the dates given by the policy change dummy. Again, these policy changes were predominantly bad news and thus the model indicates that during these periods, there was a high probability of being in the lower mean, higher variance state. In addition to this, it detects smaller policy changes for which the movement in the exchange rate was not so marked. Figure 7.8 illustrates these events. I list them as before:

7th November 1984 - Ronald Reagan is elected as US President for a second term.

17th April 1985 - Barclays and Midland banks cut base lending rates by 0.5% to 12.75%.
11th September 1985 - Government announces the launch of a 2.5 billion dollar floating rate note issue to increase foreign exchange reserves.

14th June 1989 - US cuts prime rate to 11% but dollar stays strong.

20th March 1991 - In UK interest rates are cut to 12.5%.

15th January 1992 - Again, I can find no concrete evidence to suggest a policy change here.

30th September 1992 - IMF and World Bank meet in Washington. USA, Germany and Italy encouraged to cut their deficits. Further 1% base rate cut in UK. This is the lowest since 1988.

7th October 1992 - UK base rate cut to 8%.

Note that following the start of 1994, the smoothed probability of being in regime I was very small. As can be seen by examining the raw exchange rate data, this corresponds with a time when the Pound was appreciating against the dollar.

For the Lira/DM rate, the regime switching model produces 82 dates at which the probability of being in regime I is equal to 1! However, many of these are clustered around particular economic events. I describe those events for which there is a clustering of high probabilities of being in the depreciating currency regime. The first point to note is that, in comparison with the policy change dummy, all but two realignments (16th April 1986 and 14th January 1987) are detected by this approach.

October-November 1978 - The first cluster of dates for which there is a high probability starts on 18th October 1978 and finishes on 1st November 1978. I can find no concrete evidence to suggest a major change in policy in this period.

September-November 1992 - This corresponds with the ERM crisis of 1992. Following the Italian departure on 16th September, the Lira depreciated further. This was accompanied by public sector pay freezes and the introduction
of new taxes in Italy. In the meantime, the German central bank was intervening massively in the foreign exchange markets to defend the French Franc. In addition, the German cabinet approved plans to support East German industry.

Late November -December 1992 - German economic advisors give a pessimistic prediction for the country’s economy in 1993. Sweden floats the Krone thereby abandoning plans to peg it to the ECU. Spain and Portugal devalue by 6%. Figures indicate a sharp rise in German money supply.

January 1993 - Single European Market established. Reduction in German short term money interest rates helps to ease pressure in ERM.

Mid February-Early March 1993 - Unemployment reaches 3.5 million in Germany. Share prices across the world rise in the hope of the beginning of EC recovery. G7 finance ministers meet in London.

April 1993 - In Italy, two former prime ministers are under suspicion of illicitly financing Christian Democratic Party. Giuliano Amato resigns as Italian Prime Minister. Carlo Azeglio Ciampi is asked to form government.

July 1993 - Spain, Portugal and Denmark struggle within ERM and Germany props up the French Franc.

February-March 1994 - Germany cuts discount rate by $\frac{1}{2}$%. G7 meet in Germany and predict further interest rate cuts and inflation free growth.

February-March 1995 - Lira falls to an all time low against the DM due to fears of an early General election. Italy introduces taxes and spending cuts to make up for the £8 billion shortfall in 1995 budget.

Mid April-May 1995 - G7 finance ministers meet in Washington with central bank governors and agree to cooperate in exchange markets over the weak Dollar.
July-August 1995 - On 15th August, the Dollar has the largest one day rise for 3 years following intervention from Japanese, German and US central banks.

September-November 1995 - EU Summit beginning on 22nd September highlights the row between EU and Germany over economic criteria for EMU. This brings the Deutschmark to a 9 year low against the Swiss Franc. On 8th October, G10 agree to IMF borrowing an extra £17 billion to cope with world financial crises.

Following Lira’s departure from the ERM in September 1992, it is well documented (Eichengreen and Wyplosz, (1993)) that the currency continued to depreciate against the Deutschmark. This is apparent from figure 7.9. The smoothed probability of being in regime 1 remains high until the end of the data set.

Figure 7.10 shows the smoothed probabilities for the Lira/Dollar rate. Here, the regime switching model is not as successful in finding major changes in policy. It omits 7 major events which are recorded by the policy change dummy and for which, there is a substantial movement in the exchange rate. Again, there is a clustering around particular events which I shall describe here.

Late January-February 1981 - Most large US banks cut prime rates from 20% to 19.5% while smaller banks reduced theirs to 19% on 3rd February.

Late July 1985 - Italian lira devalued by 8% within the EMS.

September 1985 - EC finance ministers approve an Ecu 32 billion draft budget which exceeds the self imposed ceiling.

September-October 1992 - This corresponds with the ERM crisis of 1992. Following the Italian departure on 16th September, the Lira depreciated further. This was accompanied by public sector pay freezes and the introduction of new taxes in Italy. IMF and World Bank meet in Washington. USA, Germany and Italy urged to cut deficits.

Late December 1992-February 1993 - Bill Clinton inaugurated as 42nd President of USA on 20th January.

It would appear that the regime switching model is only partially successful in detecting changes in regime for the Lira/Dollar exchange rate. It does not detect some significant realignments within the EMS which impacted on the Lira/Dollar. It also does not detect such large events as a change in Italian prime minister or changes in US economic policy.

In order to provide a more substantial test of the comparison, I construct a correlation coefficient for the policy switch dummy and smoothed probability of the regime switching model. This is calculated as follows:

$$\rho_{ij} = \frac{\sum_{t=1}^{n} (x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j)}{(n-1)S_i S_j}$$

where $\bar{x}_i$ and $\bar{x}_j$ are means and $S_i$ and $S_j$ are standard deviations of the data sets in question. The size of the data set is given by $n$. The result is seen in table 7.15.

Clearly, there is no substantial evidence to suggest that the regime switching model can be replaced by a policy change dummy in a GARCH setting. Indeed, this supports the argument made which suggests that each approach provides different information and that the optimal stance involves a combination of these two approaches.
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<th>St. Errors</th>
<th>t statistics</th>
</tr>
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Pound/DM

Mean Log Likelihood = -1.48843

Jarque-Bera test $\chi^2(2) = 627.13354$

Skewness = -0.63992
Kurtosis = 3.3621

Ljung-Box (1) = 0.014244
Ljung-Box (15) = 6.6995

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Pound/Dollar

Mean Log Likelihood = -1.73345

Jarque-Bera test $\chi^2(2) = 239.26343$

Skewness = -0.30793
Kurtosis = 2.1350

Ljung-Box (1) = 0.0010426
Ljung-Box (15) = 8.5295

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Lira/DM

Mean Log Likelihood = -0.988394

Jarque-Bera test $\chi^2(2) = 9630.3364$

Skewness = -1.8803
Kurtosis = 14.8655

Ljung-Box (1) = 0.14582
Ljung-Box (15) = 6.6473

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<td>0.1007</td>
<td>3.9424</td>
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Lira/Dollar

Mean Log Likelihood = -1.74333

Jarque-Bera test $\chi^2(2) = 222.34259$

Skewness = -0.22739
Kurtosis = 2.2851

Ljung-Box (1) = 0.65772
Ljung-Box (15) = 10.752

Table 7.6: The GARCH Model Including Position in the Band Data
<table>
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<tr>
<th>Date</th>
<th>Reason</th>
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<tbody>
<tr>
<td>7/10/81</td>
<td>Devaluation of Italian Lira by 3%. DM revalued by 5%.</td>
</tr>
<tr>
<td>3/2/82</td>
<td>US banks raise prime rate from 15.75% to 16.5%.</td>
</tr>
<tr>
<td>24/7/85</td>
<td>Lira effectively devalued by 8% within EMS.</td>
</tr>
<tr>
<td>16/4/86</td>
<td>Realignment of EMS currencies.</td>
</tr>
<tr>
<td>14/1/87</td>
<td>Realignment of EMS currencies.</td>
</tr>
<tr>
<td>16/9/92</td>
<td>Black Wednesday. Lira floats outside ERM.</td>
</tr>
<tr>
<td>7/10/92</td>
<td>German Bundestag ratifies Maastricht on condition that they have a say on the introduction of the single currency.</td>
</tr>
<tr>
<td>14/10/92</td>
<td>Birmingham summit of EC heads of state. Held in response to currency crisis and loss of confidence over Maastricht ratification.</td>
</tr>
<tr>
<td>6/1/93</td>
<td>Single European Market established.</td>
</tr>
<tr>
<td>28/4/93</td>
<td>Giulano Amato resigns as Italian prime minister on 22/4/93. Carlo Azeglio Ciampi, Governor of Bank of Italy is asked to form government.</td>
</tr>
<tr>
<td>8/3/95</td>
<td>Lira drops by further 4% against DM. The largest decline since crisis of Sept 1992. This follows tax increases and spending cuts to make up for £8 billion shortfall in 1995 budget.</td>
</tr>
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</table>

Table 7.7: Dates of Large Movements in Lira/DM Exchange Rate With Corresponding Events
### Lira/Dollar

<table>
<thead>
<tr>
<th>Date</th>
<th>Reason</th>
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<tbody>
<tr>
<td>19/3/80</td>
<td>US banks raise prime rate from 18.5% to 19%. This follows increases on 4th and 7th March 1980.</td>
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<td>28/1/81</td>
<td>UK to receive £262 million refund as the 2nd stage in reducing EC budget payments.</td>
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<tr>
<td>16/9/82</td>
<td>US Federal Reserve Board cuts discount rate from 11% to 10.5% and several major banks cut prime rates from 15% to 14.5%.</td>
</tr>
<tr>
<td>16/12/82</td>
<td>US Federal Reserve Board cuts discount rate from 9% to 8.5%.</td>
</tr>
<tr>
<td>27/3/85</td>
<td>Major reforms of the institutions and working of EC likely as Greece lifts its veto on terms of membership for Spain and Portugal.</td>
</tr>
<tr>
<td>24/7/85</td>
<td>Lira effectively devalued by 8% within EMS.</td>
</tr>
<tr>
<td>25/9/85</td>
<td>EC finance ministers approve an Ecu 32 billion (£18.4 billion) draft budget which exceeds the self imposed ceiling.</td>
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<tr>
<td>16/4/86</td>
<td>Realignment of EMS currencies.</td>
</tr>
<tr>
<td>14/1/87</td>
<td>Realignment of EMS currencies.</td>
</tr>
<tr>
<td>5/7/89</td>
<td>British PM addresses EC summit in Madrid setting terms for Britain’s entry to ERM. Interpreted as acceptance of Delors stage 1.</td>
</tr>
<tr>
<td>23/1/91</td>
<td>Gulf war started on 16/1/91. G7 meeting takes place on 20/1/91 to discuss this. The aim is to ‘maintain stability in international financial markets’.</td>
</tr>
<tr>
<td>24/4/91</td>
<td>EC and Mexico sign a deal on cooperation. Finance ministers and central bank governors meet at G7 meeting in Washington. They agree to cooperate in exchange markets despite disagreement between US and Germany on interest rates.</td>
</tr>
<tr>
<td>26/12/91</td>
<td>US cuts its discount rate by 1 point. France, Italy and Spain raise interest rates putting increasing pressure on UK.</td>
</tr>
<tr>
<td>16/9/92</td>
<td>Black Wednesday. Lira floats outside ERM.</td>
</tr>
<tr>
<td>7/10/92</td>
<td>German Bundestag ratifies Maastricht on condition that they have a say on the introduction of a single currency.</td>
</tr>
<tr>
<td>6/1/93</td>
<td>Single European Market established.</td>
</tr>
<tr>
<td>3/2/93</td>
<td>Bill Clinton inaugurated as 42nd President of US.</td>
</tr>
<tr>
<td>28/4/93</td>
<td>Giulano Amato resigns as Italian prime minister on 22/4/93. Carlo Azeglio Ciampi, Governor of Bank of Italy is asked to form government.</td>
</tr>
</tbody>
</table>

Table 7.8: Dates of Large Movements in Lira/Dollar Exchange Rate With Corresponding Events
<table>
<thead>
<tr>
<th>Date</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/3/76</td>
<td>Sterling Crisis.</td>
</tr>
<tr>
<td>27/10/76</td>
<td>Sterling Crisis - IMF to visit on 3/11/76 for discussion of $3.9 billion loan to UK.</td>
</tr>
<tr>
<td>12/11/80</td>
<td>Michael Foot wins leadership of labour party.</td>
</tr>
<tr>
<td>28/1/81</td>
<td>UK to receive £262 million refund as the second stage in reducing its EC budget payments.</td>
</tr>
<tr>
<td>25/2/81</td>
<td>EC provides UK with an additional £20 million for cheap loans to be used in job creation projects.</td>
</tr>
<tr>
<td>9/9/81</td>
<td>French banks to cut prime lending rate from 15.3% to 14.5%.</td>
</tr>
<tr>
<td>17/11/82</td>
<td>Sterling falls sharply against DM. Geoffrey Howe announces economic package on 8/11/82.</td>
</tr>
<tr>
<td>7/8/85</td>
<td>Britain’s banks cut base lending rates by 0.5% to 11.5%.</td>
</tr>
<tr>
<td>22/1/86</td>
<td>Bank base rates increase by 1% to 12%.</td>
</tr>
<tr>
<td>11/3/87</td>
<td>UK leading banks cut base rates by 0.5% to 10.5%.</td>
</tr>
<tr>
<td>23/9/92</td>
<td>Black Wednesday on 16/9/92. UK interest rate rose by 2% and then promised a further 3% the next day. This 2nd rise was cancelled as UK leaves ERM. On 18/9/92, Lamont blames Germany for problems with ERM. 22/9/92 saw a further 1% base rate cut in UK.</td>
</tr>
</tbody>
</table>

Table 7.9: Dates of Large Movements in Pound/DM Exchange Rate With Corresponding Events
<table>
<thead>
<tr>
<th>Date</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/3/76</td>
<td>Sterling Crisis.</td>
</tr>
<tr>
<td>20/3/85</td>
<td>Nigel Lawson announces Budget on 19/3/85. Leading banks reduce base rates by 0.5 percentage point to 13.5%.</td>
</tr>
<tr>
<td>27/3/85</td>
<td>2 of Britain’s leading clearing banks cut base lending rates by 0.5 percentage point to 13%. Major reform of institutions and working of EC likely as Greece lifts its veto on terms of membership for Spain and Portugal.</td>
</tr>
<tr>
<td>10/7/85</td>
<td>Chile announces 7.8% devaluation of the Peso.</td>
</tr>
<tr>
<td>7/8/85</td>
<td>Britain’s banks cut base lending rates by 0.5% to 11.5%.</td>
</tr>
<tr>
<td>29/9/85</td>
<td>EC finance ministers approve an Ecu 32 billion (£18.4 billion) draft budget which exceeds the self imposed ceiling.</td>
</tr>
<tr>
<td>24/4/91</td>
<td>UK interest rates drop by a further 0.5% to 12%.</td>
</tr>
<tr>
<td></td>
<td>EC and Mexico sign a deal on cooperation. Finance ministers and central bank governors meet at G7 meeting in Washington.</td>
</tr>
<tr>
<td></td>
<td>They agree to cooperate in exchange markets despite disagreement between US and Germany on interest rates.</td>
</tr>
<tr>
<td>16/9/92</td>
<td>Black Wednesday. UK interest rate rose by 2% and then promised a further 3% the next day. This 2nd rise was cancelled as UK leaves ERM.</td>
</tr>
<tr>
<td>23/9/92</td>
<td>On 18/9/92, Lamont blames Germany for problems with ERM. 22/9/92 saw a further 1% base rate cut in UK.</td>
</tr>
<tr>
<td>21/10/92</td>
<td>John Major claims he is ‘going for growth’ in shift in government economic policy. UK base rate is cut to 8%.</td>
</tr>
<tr>
<td>3/2/93</td>
<td>UK base rates cut to 6% - lowest level for 15 years.</td>
</tr>
<tr>
<td></td>
<td>Bill Clinton inaugurated as 42nd President of US.</td>
</tr>
</tbody>
</table>

Table 7.10: Dates of Large Movements in Pound/Dollar Exchange Rate With Corresponding Events
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pound/DM</td>
<td>(a_0) (-0.0136)</td>
<td>0.0292</td>
<td>(-0.4658)</td>
</tr>
<tr>
<td></td>
<td>(b_0) 0.4616</td>
<td>0.0607</td>
<td>7.6046</td>
</tr>
<tr>
<td></td>
<td>(b_1) 0.1367</td>
<td>0.0340</td>
<td>4.0206</td>
</tr>
<tr>
<td></td>
<td>(b_2) 0.3799</td>
<td>0.0676</td>
<td>5.6198</td>
</tr>
<tr>
<td></td>
<td>(d_0) 9.4915</td>
<td>4.1354</td>
<td>2.2952</td>
</tr>
<tr>
<td>Mean Log Likelihood</td>
<td>(-1.42686)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera test</td>
<td>(\chi^2(2) = 42.232683)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>(-0.022084)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.93251</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box (1)</td>
<td>0.19244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box (15)</td>
<td>22.6177</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Pound/Dollar | \(a_0\) \(-0.0034\) | 0.0374 | \(-0.0909\) |
|             | \(b_0\) 0.1303 | 0.0276 | 4.7210 |
|             | \(b_1\) 0.1052 | 0.0221 | 4.7602 |
|             | \(b_2\) 0.8107 | 0.0289 | 28.0519 |
|             | \(d_0\) 7.3191 | 1.6964 | 4.3145 |
| Mean Log Likelihood | \(-1.70275\) |
| Jarque-Bera test | \(\chi^2(2) = 27.587995\) |
| Skewness | \(-0.16154\) |
| Kurtosis | 0.68186 |
| Ljung-Box (1) | 3.2362 |
| Ljung-Box (15) | 20.5551 |

| Lira/DM | \(a_0\) \(-0.0405\) | 0.0175 | \(-2.3143\) |
|         | \(b_0\) 0.0270 | 0.0025 | 10.8 |
|         | \(b_1\) 0.2299 | 0.0229 | 10.0393 |
|         | \(b_2\) 0.7057 | 0.0178 | 39.6461 |
|         | \(d_0\) 6.6755 | 1.5497 | 4.3076 |
| Mean Log Likelihood | \(-0.862724\) |
| Jarque-Bera test | \(\chi^2(2) = 1295.7179\) |
| Skewness | \(-1.1618\) |
| Kurtosis | 5.1221 |
| Ljung-Box (1) | 0.33373 |
| Ljung-Box (15) | 10.5397 |

| Lira/Dollar | \(a_0\) \(-0.0449\) | 0.0402 | \(-1.1169\) |
|            | \(b_0\) 1.0556 | 0.1343 | 7.8600 |
|            | \(b_1\) 0.0822 | 0.0428 | 1.9206 |
|            | \(b_2\) 0.2211 | 0.0749 | 2.9519 |
|            | \(d_0\) 21.2881 | 5.1221 | 4.1561 |
| Mean Log Likelihood | \(-1.67899\) |
| Jarque-Bera test | \(\chi^2(2) = 1.0408528\) |
| Skewness | 0.0013517 |
| Kurtosis | 0.15939 |
| Ljung-Box (1) | 0.04756 |
| Ljung-Box (15) | 26.7350 |

Table 7.11: The GARCH Model Incorporating a Dummy Variable For Policy Shocks
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.0058</td>
<td>0.0125</td>
<td>0.464</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.0632</td>
<td>0.0041</td>
<td>15.4146</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.7001</td>
<td>0.0430</td>
<td>16.2814</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.3777</td>
<td>0.0158</td>
<td>23.9051</td>
</tr>
<tr>
<td>$d_0$</td>
<td>2.9677</td>
<td>0.3272</td>
<td>9.0700</td>
</tr>
<tr>
<td><strong>Lira/DM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Log Likelihood = -0.917768</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera test $\chi^2(2) = 4157.8752$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness = -0.25635</td>
<td>Kurtosis = 10.0624</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box (1) = 0.16604</td>
<td>Ljung-Box (15) = 3.2287</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>-0.0196</td>
<td>0.0422</td>
<td>0.4645</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.4050</td>
<td>0.0878</td>
<td>4.6128</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.2143</td>
<td>0.0453</td>
<td>4.7307</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.5781</td>
<td>0.0647</td>
<td>8.9351</td>
</tr>
<tr>
<td>$d_0$</td>
<td>5.7758</td>
<td>0.5947</td>
<td>9.7121</td>
</tr>
<tr>
<td><strong>Lira/Dollar</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Log Likelihood = -1.74152</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera test $\chi^2(2) = 65.633423$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness = -0.083922</td>
<td>Kurtosis = 1.2547</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box (1) = 2.7576</td>
<td>Ljung-Box (15) = 22.7482</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.12: The GARCH Model Incorporating a Dummy Variable For Realignment Dates For the Lira/DM and Lira/Dollar Exchange Rates
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.0027</td>
<td>0.0294</td>
<td>0.0918</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.5388</td>
<td>0.3683</td>
<td>1.4629</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.4075</td>
<td>0.0563</td>
<td>7.2380</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.1394</td>
<td>0.0333</td>
<td>4.1862</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.4298</td>
<td>0.0659</td>
<td>6.5220</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.0001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_0$</td>
<td>8.1025</td>
<td>2.8968</td>
<td>2.7971</td>
</tr>
</tbody>
</table>

Mean Log Likelihood = -1.42451
Jarque-Bera test $\chi^2(2) = 38.783017$
Skewness = -0.048379 Kurtosis = 0.96826
Ljung-Box (1) = 0.26904 Ljung-Box (15) = 21.5643

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.0054</td>
<td>0.0373</td>
<td>-0.1448</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.1267</td>
<td>0.0269</td>
<td>4.7100</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.1035</td>
<td>0.0211</td>
<td>4.6833</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.8125</td>
<td>0.0289</td>
<td>28.1142</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.2365</td>
<td>0.2348</td>
<td>1.0072</td>
</tr>
<tr>
<td>$d_0$</td>
<td>6.8277</td>
<td>1.7373</td>
<td>3.9301</td>
</tr>
</tbody>
</table>

Mean Log Likelihood = -1.70211
Jarque-Bera test $\chi^2(2) = 23.91284$
Skewness = -0.16871 Kurtosis = 0.68588
Ljung-Box (1) = 3.3267 Ljung-Box (15) = 20.6606

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.0228</td>
<td>0.0167</td>
<td>-1.3653</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.1075</td>
<td>0.0462</td>
<td>2.3268</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.0320</td>
<td>0.0032</td>
<td>10</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.2994</td>
<td>0.0282</td>
<td>10.6170</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.6442</td>
<td>0.0198</td>
<td>32.5353</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.0001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_0$</td>
<td>7.4279</td>
<td>2.2096</td>
<td>3.3616</td>
</tr>
</tbody>
</table>

Mean Log Likelihood = -0.85840
Jarque-Bera test $\chi^2(2) = 1252.1929$
Skewness = -1.1093 Kurtosis = 5.0646
Ljung-Box (1) = 0.11917 Ljung-Box (15) = 10.7807

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.0458</td>
<td>0.0400</td>
<td>-1.145</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.8994</td>
<td>0.1298</td>
<td>6.9291</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0870</td>
<td>0.0433</td>
<td>2.0092</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.2517</td>
<td>0.0756</td>
<td>3.3294</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.4385</td>
<td>0.2658</td>
<td>1.6497</td>
</tr>
<tr>
<td>$d_0$</td>
<td>20.3259</td>
<td>4.9401</td>
<td>4.1145</td>
</tr>
</tbody>
</table>

Mean Log Likelihood = -1.67704
Jarque-Bera test $\chi^2(2) = 0.9117429$
Skewness = -0.0011815 Kurtosis = -0.14918
Ljung-Box (1) = 0.12634 Ljung-Box (15) = 22.9577

Table 7.13: The GARCH Model Incorporating a Position in the Band Variable and a Dummy Variable For Policy Shocks
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>St. Errors</th>
<th>t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pound/DM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>-0.2148</td>
<td>0.0724</td>
<td>-2.9669</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.0521</td>
<td>0.0314</td>
<td>1.6592</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>1.5200</td>
<td>0.0515</td>
<td>29.5146</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.6331</td>
<td>0.0305</td>
<td>20.7574</td>
</tr>
<tr>
<td>( P )</td>
<td>0.8505</td>
<td>0.0330</td>
<td>25.7727</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.8902</td>
<td>0.0244</td>
<td>36.4836</td>
</tr>
<tr>
<td>Mean Log Likelihood = -1.43914</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Pound/Dollar |           |            |              |
| \( \mu_1 \) | -0.0350   | 0.0458     | -0.7642      |
| \( \mu_2 \) | 0.0079    | 0.0313     | 0.2524       |
| \( \sigma_1 \) | 1.5084    | 0.0203     | 74.3054      |
| \( \sigma_2 \) | 0.1709    | 0.0177     | 9.6554       |
| \( P \)    | 0.9922    | 0.0034     | 291.8235     |
| \( Q \)    | 0.8570    | 0.0578     | 14.8270      |
| Mean Log Likelihood = -1.75767 |

| Lira/DM     |           |            |              |
| \( \mu_1 \) | -0.2297   | 0.0886     | -2.5926      |
| \( \mu_2 \) | -0.0298   | 0.0121     | -2.4628      |
| \( \sigma_1 \) | 1.4283    | 0.0371     | 38.4987      |
| \( \sigma_2 \) | 0.2970    | 0.0089     | 33.3708      |
| \( P \)    | 0.8792    | 0.0225     | 39.0756      |
| \( Q \)    | 0.9476    | 0.0103     | 92           |
| Mean Log Likelihood = -0.819775 |

| Lira/Dollar |           |            |              |
| \( \mu_1 \) | -0.2750   | 0.2119     | -1.2978      |
| \( \mu_2 \) | -0.0223   | 0.0442     | -0.5045      |
| \( \sigma_1 \) | 2.4177    | 0.0771     | 31.3580      |
| \( \sigma_2 \) | 1.507     | 0.0367     | 31.3542      |
| \( P \)    | 0.9031    | 0.0354     | 25.5113      |
| \( Q \)    | 0.9776    | 0.0099     | 98.7475      |
| Mean Log Likelihood = -1.74292 |

Table 7.14: The Regime Switching Model Using Pound And Lira Exchange Rate Data

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lira/DM</td>
<td>0.16635</td>
</tr>
<tr>
<td>Lira/Dollar</td>
<td>0.351597</td>
</tr>
<tr>
<td>Pound/DM</td>
<td>0.194501</td>
</tr>
<tr>
<td>Pound/Dollar</td>
<td>0.150346</td>
</tr>
</tbody>
</table>

Table 7.15: Correlation Coefficients for the Policy Change Dummy and the Smoothed Probability of the Regime Switch Model
7.7 Conclusion

The purpose of this chapter was to compare the results from two different modelling processes with respect to exchange rate data covering ERM and non-ERM periods. The standard GARCH model provides an adequate description of the Lira and Pound data. For each of these rates, there was considerable evidence of persistence as indicated by $b_1 + b_2$. However, it has been argued that a currency’s position in its ERM band played an important part in determining the mean and variance of the exchange rates. Therefore, I incorporated this into the standard GARCH framework to test this hypothesis. For each exchange rate, I found evidence to suggest that the lagged position in the band played a part in the conditional variance function. For the Lira exchange rates, the current position in the band was also an important determining factor in the mean function. This was not so marked for the Pound/DM or Pound/Dollar rates.

It has also been argued that ‘news’ or major policy announcements have an impact on the dynamics of exchange rate changes. I introduce a dummy variable for policy changes into the conditional variance function. The parameter on the dummy variable is statistically significant in all cases. Furthermore, I still find evidence of GARCH in the Pound/DM and Lira/DM rates. However, persistence is greatly reduced for the Lira/Dollar rate and the specification tests suggest evidence of serial correlation for this rate.

In the case of the Lira, I include a realignment dummy in place of the policy change dummy to test for an effect in the conditional variance. The parameter on the dummy variable was significant in each case and there was still considerable evidence of persistence especially in the Lira/DM rate. The specification tests also proved favourable.

I then incorporate policy changes and position in the band information into the standard GARCH framework. For each of the Pound/DM and Lira/DM rates, the lagged position in the band does not impact upon the conditional variance function whereas the current position in the band is a determining factor of the mean function. By contrast, for the Pound/Dollar and Lira/Dollar rates, the position in the band did not enter into the mean
function, but the lagged variable did enter into the conditional variance. The Ljung Box portmanteau tests and tests for skewness and kurtosis were favourable for each exchange rate considered.

The regime switching model provides a good fit for the Pound/DM and Lira/DM rates but not such a good fit for the dollar rates as indicated by the t statistics. It identifies two regimes, one with a negative mean indicating a depreciating currency and the other with a higher mean. In the case, of the Pound, this is positive indicating an appreciating currency. For the Lira, this mean is still negative but not as negative as that for regime 1. I argue that this model provides a different type of information from that of the modified GARCH model. The correlation coefficient revealed that the policy change dummy and the smoothed probability of being in regime 1 were not closely related. This suggests that the regime switching model is not merely another way of producing a policy change dummy. Close inspection of the smoothed probabilities given in figures 7.7-7.10 shows that where the probability of being in regime 1 is the highest there have been significant economic events. However, it also highlights a number of dates for which the probability of being in regime 1 is high yet these do not appear to correspond with a significant economic event. This may suggest that the model is detecting expectations of a change in regime. In each of figures 7.7-7.10, there are a number of ‘spikes’ before an actual shift in regime occurs. Clearly, the GARCH policy change dummy identifies the major policy changes but not the possible expectations of a change in regime.

A main criticism of regime switching models is that they have proved to be poor forecasters of a change in regime. However, I argue that this is due to the fact that changes in probability only take place when there is an actual policy change. It follows that a regime switching model provides a good in-sample fit but not generally a good forecasting model.

The GARCH model, by contrast is useful in examining the impact of individual variables such as the currency’s position in the band or, as in Bekaert and Gray, a country’s level of foreign exchange reserves. However, it does not detect these pre-event increases in the probability of a regime shift. I argue that a more suitable approach would be a combination of
a modified GARCH framework with a regime switching element. In part, this has already been achieved in the work of Dueker (1994) among others. However, as yet, there is no work considering the Pound and Lira exchange rates incorporating both ERM and non-ERM periods. I argue that this would provide a valuable future line of research.
Chapter 8

Conclusion

This thesis has focused on three diverse areas of research with the common theme of currency crises and speculative behaviour. The aim of the thesis was to analyse the events of the 1992 currency crisis and discuss the implications of these findings. Therefore, it seemed logical to examine the theory and empirical evidence surrounding these events.

Initially, I examined the theory literature. There has been a vast literature written in the area of currency crises and I described some of the most influential papers in chapter 2. This starts with the pioneering work of Krugman (1979) into speculative attack models and then the contribution of Obstfeld (1986) with self fulfilling speculative attacks. However, I observed that there were a number of features of currency crises which had not been explained by the literature to date. Firstly, the previous work had not considered the issue of the timing of a currency crisis. Therefore, I provided a model of information externalities and search which offered an explanation for the timing of such an attack. I found that by imposing a Tobin tax, a government could delay the onset of a speculative attack on a currency.

Secondly, I observed that the duration of these crises were not explained by the literature. Therefore, I set up a ‘war of attrition’ model which showed the optimal time at which a government concedes and incurs a major policy change as a result of a currency crisis. In a third model, I showed how informational events and the lack of common knowledge of the value placed
by the government in remaining in the ERM can make it optimal for investors to abandon a currency.

Clearly, these models address particular features of currency crises. A potential future line of research would be to create a model which captures each of the features described.

The literature in foreign exchange markets is very diverse in that it covers a wide range of areas in economics. Recent approaches have concerned the application of models of informational cascades and herd behaviour to explain the behaviour in financial markets. Diamond and Dybvig (1983) provide an early example of such behaviour when they consider bank runs. While this area is growing, there has been very little work in terms of experimental economics to test the validity of these models. The exception to this is the model of Bikhchandani, Hirshleifer and Welch (1992) which has been tested in the laboratory by Anderson and Holt (1997). In chapter 5, I set up this model and discuss the experiment and its results. This acts as a preview to the experiment presented in chapter 6.

Chapter 6 is the result of joint work with Professor John Hey of the University of York. Initially, we investigate a theoretical point in the Banerjee model and demonstrate that the results of his model depend crucially on an assumption he makes concerning the behaviour of individuals. We then test the validity of the Banerjee framework by performing two experiments. The first contains the crucial assumption in question and the second omits this assumption. These results suggest firstly, that the assumption in question plays an important role in the Banerjee model. Secondly, we discover that in the experiment including assumption A, herding does not occur as frequently as predicted by his model.

In the final chapter, I examine the empirical evidence. Again, I discovered a tremendous literature in this area. However, this had largely focused on those currencies which were long standing members of the ERM. Since many currencies spent long periods outside the ERM, this represented an important omission. Therefore, I model the exchange rate data for two currencies, the British Pound and Italian Lira, for the periods before, during and after their membership in the ERM. Furthermore, I provide a comparison of two
different modelling techniques. Firstly, I model each data set using a GARCH model modified to include changes in economic policy and position in the ERM band. I then compare the outcome with a regime switching model. I find that these approaches are not interchangeable and that each provides a different form of information. Therefore, I argue that the optimal modelling strategy would involve a combination of these two approaches. Clearly, this offers a great deal of potential future research.
Bibliography


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